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Application of Mixed Model in Agricultural Field Experiment on Wheat

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Abstracts

This study was designed to explore the efficiency of mixed model over fixed effect model. Furthermore, it was designed to determine how the REML procedure was used to find the estimate of variance-covariance matrix of the model. The methods of restricted maximum likelihood are applied to the data from uniformity and multi-location trials. In this study, two data sets are used; uniformity trial which was used to determine optimum plot size and shape, and multi-location trial which is used to identify the effect of location on different varieties of wheat.

In uniformity trial, two sowing methods: row and broadcast sowing, are considered both in 1996 and 1997 in which the trial were conducted. These factors are considered as fixed effect of the model. But, plot sizes ranging between 1 and 24 m² with different shape are taken as random factors of the model.

Regarding Multi-location trial, it was conducted in 2001, 2002 and 2003 using 20 wheat varieties in 14 locations that were expected to have the same agro-ecological effect for wheat adaptations. All the three years in which the trail was conducted are taken as fixed effect whereas varieties and locations are considered as random factors of the model.

The result of the study based on restricted maximum likelihood (REML) revealed that small plot sizes have larger coefficient of variation in both 1996 and 1997 for both row and broadcast sown methods. This indicates that the coefficient of variation and plot sizes are inversely proportional. Nevertheless, the coefficients of variation for 1996 sown by broadcast method have higher values than the other year of the same method. This shows that the coefficient of variation was influenced not only by soil variability and plot orientation but also by crop geometry resulting from the sowing method. The test done for identification and adaptation of different varieties on different location shows that there is no average effect due to variety and hence all varieties have the same effect on the response variable. This implies that all varieties are contributing the same average effects for the yield obtained.

However, the test statistics for location shows that the coefficients for Bako, Bekoji, Debre Zeit, Ginchi, Hollota, Kulumsa, and Sinana are significantly different from the remaining locations. This significance indicates that the average productivity potential of these locations is higher than that of the remaining locations. In general, the contribution of each location to yield is not the same unlike that of variety. Hence, the average effect of location on the response variable is not the same.

Acronyms/ List of Abbreviations

AI :-	Average Information Matrix
AIC :-	Akaike's Information Criteria
ANOVA:-	Analysis of Variance
ASREML:-	Average Score Restricted/Residual Maximum Likelihood
BIC:-	Schwarz's Bayesian Criteria
BLUE:-	Best Linear Unbiased Estimator
BLUP:-	Best Linear Unbiased Prediction
CIMMYT	Centro Internacional de Mejoramiento de Maize y Trigo (International Maize and Wheat Improvement Center)
CRD:-	Completely Randomized Design
CSA:-	Central Statistical Agency
CV:-	Coefficient of Variations
DF:-	Degree of Freedom
DFREML:-	Derivative Free Restricted/Residual Maximum Likelihood
EBLUE:-	Empirically Best Linear Unbiased Estimator
EBLUP:-	Empirically Best Linear Unbiased Prediction
EIAR:-	Ethiopian Institutes of Agricultural Research
EM:-	Expectation Maximization
FI:-	Fisher Information Matrix
GDP:-	Gross Domestic Product
GLM:-	General Linear Model
H:-	Hessian Matrix
ML:-	Maximum Likelihood
MME:-	Mixed Model Equation
MSE:-	Mean Square Error
NASS:-	National Agricultural Statistics Services
NWRP:-	National Wheat Research Programme
OLS:-	Ordinary Least Square
PROC:-	Procedure
RCBD:-	Randomized Complete Block Design
REML:-	Restricted/Residual Maximum Likelihood
SAS:-	Statistical Analysis Software
SUSAN:-	Sub Saharan African Network
USAD:-	United States of Agricultural Department

Chapter One: Introduction

1.1. Background

As is true in most developing countries, agriculture is the dominant sector of the economy in Ethiopia. As a result of this, Ethiopian Agriculture contributes the lion share of the gross domestic product (GDP) and foreign currency earnings of the country from the sell of agricultural outputs abroad, as well as creates employment opportunity to the majority of the country's population. Hence, agriculture is the major sector expected to play a dominant role to bring about an overall sustainable economic growth to the country if strenuous efforts are made to modernize the farmer activity of the sector as a whole.

Among a number of efforts that should be made by the concerned stakeholders to meet the desired goal mentioned above, the availability of reliable comprehensive and timely research on the performance of the sector is considered essential for agricultural development. Hence, the researcher should know the most appropriate method that leads to the desired goals.

Wheat (*Triticum* spp.) is one of the most important cereal crops grown under rain fed conditions in the highlands of Ethiopia (Hailu, 1991). As far as wheat production is concerned, farmers have produced it since time immemorial. According to Vavilov (1951), Ethiopia is a secondary center of diversity for several wheat species. The highlands of Ethiopia possess unique environmental conditions that enhanced the adaptive diversity of many crop species, including coffee, barley, teff, and wheat. Therefore, it does not come as a surprise that wheat occupies a significant proportion of the land cultivated annually in Ethiopia, and according to the report of CSA 2005/06 the area cultivated under wheat was 1439187 hectares. The total yield produced from those cultivated area was 24082340 quintals. Traditionally, tetraploid¹ wheat has been

¹ Species of wheat are classified according to the number of chromosomes found in the vegetative cell. They are divided into three series: the diploid, which contains 14 chromosomes; the tetraploid, which contains 28 chromosomes; and the hexaploid, contains 42 chromosomes.

predominantly grown by the Ethiopian farmers, and currently occupies more land than hexaploid wheat. However, the area under bread wheat has significantly increased in recent years. The largest proportion of the wheat crop is grown at altitudes above 1600 m, predominantly in the highlands of Arsi, Bale and all Shewa zones.

Ethiopia is the largest wheat producer in sub-Saharan Africa with about 0.75 million hectare of durum and bread wheat. Wheat is one of the major cereal crops in the Ethiopian highlands, which range between 6 and 16°N, 35 and 42°E (Girma et al 2005). At present, wheat is produced solely under rain fed conditions. Of the current total wheat production area, 75.5% is located in Arsi, Bale and Shoa regions. Forty-six percent of the 13 million hectare classified as highly suitable for wheat production is located in Arsi and all Shoa zones.

Beside this, altitude plays an important role in the distribution of wheat production even though the influence of rainfall, temperature, and diseases also have a great impact on the production. Arsi, Bale and Shoa regions, where altitude is in the range of 1900-2300 m above sea level, are favorable environment for the production of early and intermediate maturing varieties of bread wheat. Because the nutrient contents of soil and the amount of moisture required for growing these varieties are favorable at that altitude range. This is estimated to comprise 25 % of the total wheat area, while the remaining 75 % falls in the 2300-2700 m altitude zone. Soil types used for wheat production vary from well-drained fertile soils to waterlogged heavy Vertisols.

1.2. Nutritional Values of Wheat

According to the report of Central Statistical Authority and Ethiopian Nutrition Research Institute, wheat has been one of the major cereals of choice, dominating the food habits and dietary practices of the highland population of Ethiopia. In nutritional terms, wheat is known to be a major source of energy and protein as documented by various surveys and studies conducted by the Ethiopian Nutritional research Institute. Chemical analysis of samples taken from different sites in Ethiopia has shown that wheat contains 339 kcal of energy, 10.3 g of protein, and 49 mg of calcium and 1.5 mg of iron/100 g of whole grain.

These levels are not significantly altered by milling. In terms of quality, wheat appears to provide an optimum amount of protein, energy, calcium and iron. The main limitation appears to be its deficiency in Vitamins A and C, both of which are known to be the main limiting nutrients in the Ethiopian diet. The contribution of wheat to infants' diets has also been documented from a comprehensive survey, involving representative sites in the country. Accordingly, in areas where wheat is used as a basic staple food, its contribution to the energy and protein intake of different age groups has ranged from 36 to 85% for energy and from 32 to 78% for protein. To ensure adequate intake of minerals and Vitamins A and C, supplementation with vegetable and fruit has been adopted.

The main source of calories for the majority of the Ethiopian population is undoubtedly derived from local staple foods. In most parts of Ethiopia, this is cereal: tef, maize, sorghum, wheat or barley. In fact, the high proportion of cereal production relative to food legume and the pattern of consumption in the general population make cereals the number one source of protein.

Wheat is an important food crop in the highlands of Ethiopia and an essential food choice during drought or famine. In addition to being ground for making flat bread locally called "injera" and the local bread, wheat can be roasted and eaten for breakfast and in between meals.

As far as the straw of wheat is concerned, it is one of the most important cereal straws produced in the mid-altitude and highland areas of Ethiopia, and is used for different purposes. The main use is as livestock feed, particularly during the dry season because in tradition cropping and livestock production have been closely integrated and complementary. Livestock greatly influence the ability of farmers to produce food and cash crops through draft power, cash availability and manure. On the other hand, crop residues plays a crucial role in livestock nutrition and this is likely to increase as more grazing land is put under cultivation due to rapidly increasing population pressure. Other uses of strew include thatching, bedding material, maintenance of soil fertility (being plowed back into the soil), and industrial uses (e.g. paper products, hardboard, egg trays, and for packaging in the glass industry) Capper et al (1988).

According to Coxworth et al (1981), when the nutritional value of wheat straw as animal feed is evaluated, the proportions and nutritive value of leaf, leaf sheath and stem vary considerably amongst wheat straws. In general, the stem comprises the largest fraction, is the most lignified and has the lowest potential digestibility. Leaf sheath is substantially less lignified, and has a much higher potential digestibility than the stem fraction. The grinding energy for the various straw fractions increases in the order of leaf blades (lowest), leaf sheaths and stems (highest). Consequently, leaf material is more acceptable to ruminants than stems. Varietal differences also influence wheat straw utilization as animal feed.

1.3. Research Activities on Wheat

Despite the long history of wheat production in Ethiopia, the mean national wheat yield is quite low. This low productivity can be ascribed to many factors, primarily those related to a low level of agricultural technology. The traditional farming community, accounting for over 95% of the cultivated land, relies on local varieties and crop production practices, and rarely utilizes improved crop and soil management techniques. By contrast, the state sector has benefited more from research on improved technology with respect to varieties as well as crop production and protection techniques. But even here, yield per hectare can be improved.

The research services, including the Ethiopian Institute of Agricultural Research and the universities and colleges, have conducted a large number of research trials aimed at developing agricultural technologies to significantly raise wheat productivity. The average rate of use of improved seed of all crops has been reported to be less than 5 kg/ha, while the average rate of fertilizer application is somewhat more than this.

It can be fairly concluded that the adoption of improved technology for wheat production is still at an early stage. The relevant issues in this regard to explored are:

- The availability of relevant improved technologies;
- The level of popularization of these technologies amongst farmers;
- The timely availability of the necessary inputs to farmers at affordable prices;

- Price incentives to encourage farmers to adopt improved technology and others

The priorities given by the EIAR for wheat research emphasized increased wheat production by concentrating on improved varieties with a package of cultural practices. EIAR's wheat research activities, in close collaboration with other organizations, have included:

- The use of international and national nurseries to identify desirable genotypes;
- The exploitation of the Ethiopian tetraploid wheat germplasm;
- The execution of an extensive national and regional variety testing program;
- The development of varieties through breeding;
- The coordination and execution of agronomic and crop management studies; and
- The multiplication and distribution of breeder and basic seed.

During the past 35 years, a comprehensive national wheat research program has been developed and more than 30 improved bread and durum wheat varieties have been released.

The National Wheat Research Program (NWRP) has a general objective of generating basic scientific information and applied technology to increase and sustain wheat production in Ethiopia.

1.4. Materials And Methods Used by Previous Workers

One of the major problems in the analysis of agricultural field trial is inappropriate accounting for field variability. Agricultural experimentation involves selection of experimental materials, selection of experimental units, planning of experiments, collection of relevant information, analysis and interpretation of the result. There is no perfect experimental design nor are they any perfect analysis procedures known to account for all variations encountered in practice. Hence the quality of the result depends on how well the selected experimental design or method of analysis helps in estimation of variation (Girma 2005). Design based approach failed to account for such variation, especially, when block cannot be laid out successfully. As a result, the experimental error

mean square of classical analysis may be severely inflated (As cited by Girma 2005). High error mean square affects sensitivity of the test thus leading to inappropriate conclusions. The severely inflated experimental error mean square is the totality of both experimental and random effects. Therefore, random effect has its own contribution to the total variations. Hence, to minimize the severely inflated experimental error mean square we have to use mixed model rather than fixed effect model.

The selection of experimental material and analysis techniques should be carefully considered in order to achieve optimal results. To optimize the use of resources the best method of describing and accounting for field variability should be used so that variety effects and difference between them are measured as precisely as possible. This requires improvement in both design and modeling.

Randomized complete block design (RCBD) has been widely used in agricultural field experiments compared to other known design such as Latin square and family of incomplete block designs. The major reasons for the popularity of this design are its computational convenience, flexibility and efficiency given that the required assumptions are fulfilled.

As quoted by Girma 2005, SUSAN proceeding (1996) reported that the majority of research trials in Sub-Saharan Africa were planned as RCBD regardless of the size of experiments. Based on survey conducted on the type of the design used in field trails in Ethiopia (Biometrics Report, 1996), it was found that about 78% of the design used in breeding and agronomy research programmes is RCBD with and without split plot. Such designs are often applied at all levels of research including nursery, regional variety trails and national variety trails. This resulted in squeezing a large number of treatment combinations into a complete block, which lead to loss of homogeneity within block (Girma et al 2005).

The statistical analyses used so far in wheat research as related to various factors were ordinary maximum likelihood estimator of variance component. But this takes no account of the degree of freedom used in estimating fixed effect and hence is biased. This

introduce down ward bias, which increases as the number of fixed effects in the model increases. Therefore it is more sensitive to use variance components, which take account of the degree of freedom for estimating treatment effects (Girma 2005).

1.5. Failure to properly account for field variability

Many research activities in Ethiopian Institutes of Agricultural Researchers (EIAR) are designed in RCBD. By using RCBD, it is possible to control only one factor in perpendicular direction to that factor. Hence, such experimental design did not allow researchers to control all factors. As a result, the sums of square due to the remaining factors are included in the error sum of squares. Therefore, the hypothesis test based on the error sum of squares is not properly evaluated and the conclusion drawn from such test statistics misleads the researchers.

On top of this there is inappropriate design method by some researchers. For example some researchers use completely randomized design (CRD) during data analysis and reporting instead of RCBD for trails designed as RCBD. However, if there are fertility trends then the trends cast problem on CRD application. Thus, in the presence of these trends, the use of CRD is misleading because there is a possibility that one of the varieties may receive all the good or the bad plots which ultimately leads to biased error estimate. In addition to this biased error estimate, we lose the efficiency gained due to using local control (blocking).

There is also a shift of trails first designed in split plot experiment during design layout then to RCBD during data analysis and reporting the results. As a result, if someone initially designs his/her experiment as split-plot, but later on analyze it as RCBD the major problem committed is that:

- It ignores the difference in randomization between the two,
- It loses the precision obtained for the subplot at the expense of larger plot, and
- The split-plot treatment will be affected, while the main-plot treatment is boosted incorrectly.

1.6. Motivation/Problem Statement

In field trial experiment, most researchers do not properly account field variability due to improper use of the model. Moreover, the model used for conducting research was fixed effect model. In fixed effect model, the levels of the independent variables that occur in the study are considered to be the only level of interest and have a fixed effect on the response variable. More likely, the variables in fixed effect model are determined or set by the experimenter. Hence, the only random variable is the error term and therefore the variance of the response variable and the variance of the error term are the same. Since the only source of randomness in a fixed effect model is the error terms, parameter estimation could be done using ordinary least squares (OLS) technique.

However, if the levels are considered to be random sample from the population of the possible levels, each level has a random effect on the response variable. As a result, the variance of the response variable in a model which contains both fixed and random effect factors with error terms is the sum of the variances of random effect and the error terms.

Thus, the estimation of parameters in the mixed model effect are based on the generalized least squares method, and the variance components corresponding to the different sources of randomness are estimated by restricted maximum likelihood criteria to get estimates for both fixed and random factors.

RCBD designs that are implemented by many researchers do not account for correlations of neighboring plot experimental area which plays a significant role in the variability of the experiment. This correlation coefficient was obtained indirectly from soil heterogeneity index. Smith's index is used for soil heterogeneity measurement. The value of the index indicates the degree of correlation between adjacent experimental plots. The larger the value of the index, the lower the correlation coefficient between adjacent plots indicating heterogeneity of the experimental area. Hence, if the correlation between adjacent plots is small, then large plot sizes again inflate the experimental error sum of square since there is no homogeneity within blocks.

Variability increases with the size of experiments and consequently the size of the block increases. With the increase in size of blocks, the possibility for controlling variability within a block diminishes leading to loss of precision. Therefore, the possible effects of blocks heterogeneity in field experiment, if not removed or minimized, could result in the form of overestimation of error mean square. Similarly standard deviations for comparing the differences among varieties are inflated and lead to inappropriate conclusions. Hence to increase the precision and minimize overestimation of error mean squares using mixed model instead of fixed i.e. RCBD is advantageous.

In general, unless, a proper model is used to analyze data, the conclusion drawn from such analysis may be misleading.

1.7. Objective of the Study

General Objective:

To study the effect of random variates and the estimation of different variance components.

Specific objectives:

- To access the benefits of mixed model in crop experiments (especially wheat)
- To adopt a suitable statistical model for agricultural wheat trials
- To compare performance of mixed effect model using different scoring methods.

1.8. Significance of The Result

The outcome of the research will help agricultural researchers especially breeders and agronomists to conduct cost-effective agricultural research with efficient use of limited research resources and to determine optimum plot size and shape for specific field crops, based on uniformity trials. Moreover, the result helps the researcher as a guideline for indicating possible sources of variation that might occur in research activities. On top of this, the result of the study will enable researchers to enhance the awareness of factors which

influence the efficiency of the model. The result of this study can also be used as a source of information to other researchers in the future.

We can also list the applications of the research as follows:

- The major findings from this research are expected to benefit different researchers working in the areas of agriculture.
- It is also believed that results of the research will be used as a basis for further study on wheat using mixed model.

1.9. Organization of the Paper

The paper is presented in six chapters. The first chapter, introduction, deals with general background and nutritional value of wheat. Previous research activities, failure to proper account variability, problem statement, objectives and significance of the research are also addressed in this chapter.

The second chapter reviews related works in various countries by different organizations and individuals to the major problem being investigated.

The study area and sources of data, concepts of random and fixed factors, analysis of data, model fitting, principle of maximum likelihood, the average information algorithm for REML estimation, estimation of the parameters are described in detail in the third chapter. Moreover, the theoretical and conceptual frameworks that help to analyze the problem under study are included in this chapter.

The fourth chapter presents issues of fitting models, assumption checking and appropriate transformation of the model to analyze the data.

Chapter five is devoted on the result and discussion of the estimated parameters of the model. Here, the contribution of each random variable, test of parameters, model selection and comparative advantage of mixed model over fixed effect model was described.

The last chapter gives conclusions drawn from the study. It culminates by giving recommendations to the major problems investigated.

Chapter Two: Literature Review

2.1. General Literature Review

Mixed model methodology takes its name from the fact that the elements of the model underlying the analysis can be a mixture of what are called *fixed* and *random* effects. A fixed effect would be an effect which can be specifically designed into the study that does not go beyond the study under consideration. In contrast, an experiment, which usually involves a random component from a large population taken randomly, is called *random effect*. Hence, mixed-effects model is a generalization of the standard linear model (a multiple regression model) that enables the analysis of data generated from several sources of variation instead of just one (As Cited in SAS, 1996). It associates one continuous dependent variable (a response, an outcome) with several explanatory variables (categorical or continuous). In relation to standard linear model, the unique aspect of the mixed-effects model is the inclusion of random factors in addition to fixed effect. Hence, in mixed model, fixed effects provide estimates of the average responses in the group, like in a common regression model, while random effects account for the natural heterogeneity in the responses of different variables and allow estimation of responses for each individual in the study. This leads to specific covariance structure, if there exists a dependence response in the model. Hence, the model allows the assumption of several covariance structures and enables estimation of the effects as well as variance parameters. In such modeling, the number of observations per subject can be either the same in case of balanced design or different in case of an unbalanced design. The time points can be either identical across subjects or not.

The Linear Mixed Models procedure expands the general linear model so that the data are permitted to exhibit correlated and non-constant variability. The mixed linear model, therefore, provides the flexibility of modeling not only by the means of the data but also by their variances and covariances as well.

Mixed models were applied with and without the fixed effects (without- is equivalent to the random-effects model), to determine the impact of the fixed effects upon the variance

component associated with the random effects. The estimated variance components of both models were compared.

While a fixed model allows one only to estimate unconditioned variance components, a mixed model enables estimation of between- and within-random effect variance components of the model while accounting for the fixed effects of the model characteristics. Consequently, we can identify the model characteristics affecting each variance component. Hence, mixed models were applied to the data sets with repeated measurements and auxiliary information of the model characteristics.

So far, in studies with uniformity trials, most researchers have used either; a one-way random-effects model to estimate variance components, ignoring other model characteristics and/or, multiple linear regression to model the effect of model characteristics on observed yield levels, ignoring the correlation between repeated observations from the same characteristics of the model (Kromhout *et al.*, 1987). Therefore, to overcome such problems and handling those effects simultaneously using mixed model is preferable.

In addition to handling unbalanced data, mixed-effect models are also used to estimate both the effects and the variance components in a more efficient way (Searle, 1988). By defining an effect as random, we are visualizing the set of units under investigation as a representative sample from a wider population.

A. Reverter, *et. al.* (1996) used mixed model in assessing the efficiency of multiplicative mixed model to account for heterogeneous variance across herds in carcass scan traits from beef cattle at The University of New England. The result shows that the variance of the error terms of mixed model is much smaller than the variance of the error terms of fixed effect model.

As cited by Methu *et al.*, 2001 mixed model was used to investigate different planting densities and thinning methods, and to identify particles that would increase forage production without compromising the harvest grain yield of maize for human consumption in the Central Kenya Highlands. In this analysis, planting densities and

thinning methods are used as fixed effect where as particles that would increase forage production is used as random factors.

The conditional (subject-specific) and marginal (population-averaged) formulations in the linear mixed model enable you to consider conditional residuals that use the estimated BLUPs of the random effects, and marginal residuals which are deviations from the overall mean. Residuals using the BLUPs are useful to diagnose whether the random effects components in the model are specified correctly, marginal residuals are useful to diagnose the fixed-effects components. According to the output obtained from this analysis, the estimates of the coefficients for planting densities and thinning methods are significantly different from zero at $\alpha=5\%$ level of significant. The test statistic done for the variance of the random component of the model have p-value <0.0001 . This value indicates that the model explains the assumed variation efficiently.

In line with this, Baker 1998 reported that an experiment was conducted to compare the genetic resistance to helminthiasis of two indigenous breeds of sheep - Dorper and Red Maasai - and to use this information alongside survival rate to compare the overall productive performance of each breed using mixed model. Helminthes constitute one of the most important constraints to livestock production in the tropics. The study was conducted in Kenya coast between 1991 and 1997.

According to Romney *et. al.* (2000) report the analysis that described how to determine a) the influence of household (farm) and cow factors on milk yield, and (b) the relationships between milk yield and concentrate fed at different phases of lactation were analyzed using mixed model effect in dairy production which is an important source of income for many smallholder households in the highlands of East Africa.

2.2. Application of Mixed Model to cereal Crops

United State Department of Agriculture (USAD) and National Agricultural Statistical Services (NASS) in collaboration with state governments have been publishing annual crop estimates since early 1930's. In this crop estimation, NASS implemented Bayesian inference using mixed linear model for small area estimation for county level along with

many other non model based techniques using model based techniques (As cited by Minilik T. 2004). The content of data sets that they use to make the Bayesian inference using mixed linear model methodology is as follows:

- a. Number of segments in each county
- b. The number of hectares of corn and soybeans for each sampled segment
- c. The number of pixels classified as corn soybeans for each sampled segment
- d. The county of pixels per segment classified as corn and soybeans

Minilik T. 2004 has fitted the same model as above using Central Statistical Authority data on area cultivated for barely from 18 woredas of North and South Gonder zones.

The study will try to generate estimates of the mean harvested area per holder for barley first using BLUP approach for mixed linear models and then using the weight for each woreda provided by CSA, an estimate of the total cultivated area for barley for each woreda will be calculated.

Finally comparing it with the 2001/2002 Ethiopian agricultural sample census estimates will assess the reliability of these estimates.

Based on the data from CSA on study area the small area estimates were obtained using mixed linear model and the results of the study have been compared to the result obtained from the 2001/2002 Ethiopian agricultural sample census which was conducted mainly to produce woreda level direct survey estimates.

Using the commodity based weight produced by the agricultural census total acreage for barley were obtained from small area model estimates of mean of acreage and then compared with the census estimate of acreage for barley. For all the woredas the difference is found to be insignificant. This strongly supports the usefulness of small area estimation techniques and their application in various aspects to get estimates at woreda level (small area level).

2.3. Specific Literature to Wheat

Linear models with multiple sources of error are widely used in designed experiments across many science fields. An example of such an experiment is described by Patterson and Nabugoomu (1992), from Patterson and Silvey (1980). Six varieties of wheat were grown at ten centers that formed a sample of the main types of growing area for wheat in Scotland, and the yields in tones per hectare were recorded. The experiment is unbalanced because, out of 60 possible variety-center combinations, only 46 were used. At seven centers, four varieties were grown and at the remaining three centers, all six varieties were grown. The result obtained from mixed model shows small variation than fixed effect model i.e. RCBD with two factors.

The illustration of the convergence performance of the average information algorithm relative to the Fishers scoring algorithm as implemented in GENSTAT (as cited by Payne 1993) for the analysis of a spring wheat variety trail was conducted at State Hall Farm, UK in 1976. The experiment was designed to compare the yields of 25 varieties. The experiment was conducted in six locations. Opposite to that of Patterson and Nabugoomu, the locations are considered as fixed effect where as varieties are considered as random effect of the mixed. The yield data obtained from this field experiment, measured in gram per meter square, was used as vector of observation. The result obtained from this analysis shows that Average Information algorithm fulfills convergence criteria with less number of iterations than Fisher algorithm to meet the same situations.

2.4. Research Activities on Wheat at EIAR

Wheat researchers in Ethiopia strive to improve the experimental precision of field trials in order to obtain unbiased and accurate estimates of treatment effects. However, information on the degree of soil heterogeneity and on optimum plot dimensions for wheat experimentation is lacking (Girma et al 2000). By convention, wheat breeders in Ethiopia manually sow wheat in rows within long and narrow plots; gross plot areas are 3.0 m² and net plot areas (i.e., harvested areas) are 2.0 m² for variety trials, and four replications are normally used (Desalegn et al., 1996). By contrast, wheat agronomists

conduct their research trials under small-holder conditions; the agronomists customarily establish trials by manually broadcasting wheat seed and basal fertilizer and subsequently incorporating the broadcast inputs with one pass of the local ox-plough (i.e., similar to the conventional practice of peasant farmers in Ethiopia). Agronomic trials, both on-farm and on-station, generally consist of individual plots characterized by gross and net plot areas of 25 and 9 m², respectively, with two or three replications being common by using RCBD design (Tanner et al., 1992).

2.5. Estimation of Optimum Plot Dimensions

The Ethiopian Institutes of Agricultural Research (EIAR) and collaborating institutions conduct a multi-disciplinary national wheat research programme directed towards improving the productivity, yield potential and quality of wheat in Ethiopia (Desalegn *et al.*, 1996).

The conduct of cost-effective agricultural research necessitates the efficient use of limited research resources; research trials should attain an acceptable level of experimental precision with minimal cost. The major factors that determine the cost of a field experiment are the size and shape of plots, blocks, and the number of replications used since these factors are directly proportional to the area occupied by the field trial. The choice of plot size and shape is influenced by field management techniques, which are related to prevailing agricultural technologies and the extent of within site variability, which is related to the degree of soil heterogeneity. Therefore, it is important to develop and utilize appropriate methodologies for field experimentation in order to optimize both experimental precision and the efficiency of research resource usage.

Research to determine optimum plot size and shape for specific field crops, based on uniformity trials, has been conducted for many years; considerable progress has been achieved after Smith (1938) has developed a method to estimate soil heterogeneity. Girma et al 2000 said that many researchers have modified Smith's methods of soil heterogeneity index to obtain better soil heterogeneity index.

The two most widely used methods for selecting optimum plot sizes are those suggested by Smith (1938) and Hatheway (1961). Utilizing these techniques, crop scientists have recommended optimal plot sizes and shapes for specific crop soil combinations.

The relationship between the coefficient of variability (CV) and plot size can be used for the optimization of plot size. Subsequent to representing this relationship graphically, the optimum plot size has been obtained in one or two ways by different authors. Narayana Reddy and Ramanatha Chelty (1985) considered optimum plot size to be represented by the point on the curve at which the maximum rate of change in the estimate of CV per incremental unit of plot size occurs. However, this method may be affected by the scale of the co-ordinates used in plotting. The second method (Lessman and Atkins, 1963) is based on a derivation of the variance per basic unit area, or similar measures of variability, in relation to plot size. This method was further simplified by introducing the concept of critical plot size: the critical plot size corresponds to the point of maximum curvature on the graphical relationship between variance and plot size (Lessman and Atkins, 1963).

Another method previously considered in the determination of optimum plot size was based on cost factors associated with the number and size of plots within a trial (Smith, 1938). However, this method often identified an impracticably narrow plot dimension as optimal (Narayana Reddy and Ramanatha Chelty, 1985). All these estimations of optimum plot size were based on fixed effect model i.e. RCBD.

2.6. Estimation of Optimum Plot Dimensions for Wheat in Ethiopia

The estimates of optimum plot dimension for wheat and the degree of soil heterogeneity index on a typical research station was done in the central highlands of Ethiopia, at Holeta using yield produced in 1996 and 1997 main crop season. The objective of that trial was to estimate the degree of soil heterogeneity and to determine the associated optimum plot sizes and shapes for broadcast and row sown wheat experiments.

The corresponding coefficients of soil heterogeneity indices denoted by b for broadcast

sown trials were 0.240 and 0.270 with coefficient of determination (R^2) values of 95% and 96% for 1996 and 1997, respectively. These coefficients indicate that the degree of heterogeneity within a trial is influenced not only by soil variability and plot orientation, but also by crop geometry resulting from the sowing method used. Similarly, the corresponding coefficients of soil heterogeneity indices for row sown trials were 0.476 and 0.455 with R^2 values of 74% and 98% for 1996 and 1997 respectively. The smaller the value of soil heterogeneity index the larger the value of the correlation coefficient. This implies that there is no difference among neighboring plot areas and hence we can use larger plot size for the trial. Adjusted values of soil heterogeneity indices were 0.504 and 0.324 for 1996 and 0.481 and 0.352 for 1997 for row and broadcast sown trials, respectively; mean values of soil heterogeneity indices were 0.493 and 0.338 for row and broadcast sown trials.

Heterogeneity coefficients due to rectangularity were non-significant ($P > 0.05$) in both years for both sowing methods. The heterogeneity of comparable variance for plots of equal size was found to be non-significant ($P > 0.05$) for both row and broadcast sown trials, suggesting that differing plot shapes of equal size are equally efficient for both sowing methods on the Holeta Nitisol. Hence, increasing the plot size of both rectangular and square plots reduced the magnitude of error variance at approximately the same rate.

This indicates that the researchers can use as large as the size used by wheat breeder or larger irrespective of the shape of plot.

Chapter Three: Method of Analyses

3.1. The study Area and sources of Data

The data in this study were collected from two sets of soil uniformity trials, broadcast and row sown, established on a soil classified as a Haplic Nitisol at Holeta Research Center (altitude 2400 m a.s.l., latitude 9°03' N, and longitude 38°30' E) which is located in a major wheat producing region of Ethiopia (Hailu, 1991). The trials were conducted during the 1996 and 1997 G.C. cropping seasons². Both sets of trials were hand sown using the popular local bread wheat (*T. aestivum*) cultivar "Qubsa" (=Attila, a CIMMYT derived, semi-dwarf with high yield potential in the central highlands of Ethiopia). The trials were sown on June 20 in 1996 and June 18 in 1997 with uniform application of the inputs and cultural practices recommended for the area (Tanner et al., 1991). For row sown trials, fertilizer, in the form of urea and triple super phosphate at the rate of 60 kg N and 60 kg phosphoric oxide (P_2O_5) per hectare, was placed in rows spaced 20 cm apart and then seeds were sown in the rows at the rate of 150 kg/ ha. Rows were subsequently covered manually. For broadcast trials, 60 kg Nitrogen, 60 kg phosphate and 175 kg seed per hectare were broadcast by hand and then were covered by one passes with the local ox-plough. Weeds were controlled by post-emergence spray application of fenoxaprop-p-ethyl at 69 g a.i. ha⁻¹ and 2,4-D at a rate of 1.08 kg a.i. ha⁻¹.

At emergence, each trial was subdivided into basic unit plots measuring 1 m x 1 m by fixing sisal strings on permanent pegs spaced at 1 m intervals along all four sides of the trial. At maturity, border strips measuring 3 m for broadcast sown and 2 m for row sown trials were manually removed along each trial perimeter leaving a net area of 12 x 60 m for broadcast sown and 12 x 80 m for row sown trials in 1996, and 10 x 72 m for both trials in 1997. Thus, a total of 720 and 960 basic unit plot areas were harvested from broadcast and row sown trials, respectively, in 1996. In 1997, a total of 720 basic units were harvested from each trial. Each basic unit plot was hand harvested separately by sickle, placed in cloth sacks, tagged, sun-dried in the open air, and then threshed by

² This trial was undertaken to compute optimum plot size and shape for wheat research activities by biometricians in collaboration with other researchers. The result obtained has been discussed in chapter two.

manually beating the sacks. Grain yield was determined for each basic unit plot area and recorded on a gram per meter square basis. The grain yields from adjacent basic unit plots were subsequently combined to derive values for different plot shapes and sizes. Plot sizes ranging from 1 to 240 m² with various shapes were considered in the study. In this paper we considered plot areas ranging from 1 to 24 m² only to ensure the homogeneity within experimental plots. Because as area of plot size increases heterogeneity is introduced and hence there is a yield difference obtained as factors beyond the control of experiment.

For this paper 11409 observations (3420 observed yield records were taken from row sown and 2651 observations were from broadcast sown in 1996, whereas 2715 observations were from broadcast sown and 2623 observations were from row sown in 1997) were used for analysis. This data includes observations from one of the two plot areas if the plots have the same area but the dimensions are exchanged i.e. yield obtained from plots size having area of 1X2 m² and 2X1m² are taken only once if there is no significant difference in yield obtained from those plots. Gomez and Gomez (1984) indicated that if there is no significant difference in yield obtained from the plots having the same area but different shapes, taking yield obtained from one of the two plots is sufficient. The SAS Output which helps us to check whether this condition is fulfilled or not is given in Appendix A.

As it is shown in Appendix A table 1, there is no evidence against the difference in yield obtained from plots having the same area but different shape. Hence, we can take only one of the two plots shapes. The test statistic gives the same result for all plots that have the same area but dimensions are exchanged. In general, for this trial we have three explanatory factors (year, Sown type and plot size and shape) and one response variable i.e. yield.

In addition to uniformity trial data, there is also multi-location trial data sets that come from different location having the same varieties was analyzed. This trial was undertaken to compare varieties germplasm at different location for adaptation purposes. The data were collected but not analyzed for the objectives stated. Multi-location trial data were

collected from Kumusa, Asasa, Arsi Nagelle, Holeta, Ginchi, Adet, Sinana, Kokate, Hosana, Alemaya and Bako in 2001. The trials at Asasa, Kulumsa, Arsi Nagelle and Sinana locations were sown in June in 2001. The trials for the remaining location the trial was sown in July in 2001. In case of year 2002, the trial data were collected from Kulumsa, Arsi Robe, Bekoji, Holeta, Ginchi, Adet, Sinana, Kokate, Debre Ziet and Bako. In this year, the trials have sown in June at Kulumsa, Bekoji, Arsi Robe, Holeta and Sinana locations. For the remaining locations, the trials were sown in July. The third year i.e. year 2003, trial data were collected from Kulumsa, Asasa, Arsi Nagelle, Arsi Robe, Bekoji, Holeta, Ginchi, Sinana, Hosana, Debre Zeit, Beko and Alemaya. The sowing activities have undergone in June 2003 at Kulumsa, Asasa, Arsi Nagelle, Bekoji and Sinana. For the remaining locations, the sowing activities were undergone in July 2003. The sowing activities were undergone with uniform application of the inputs and cultural practices recommended for the area. In all the three year the trials were sown by broadcast method by hand and then were covered by hand operating material. Weeds were controlled by post-emergence spray application according to recommendation.

All varieties in all locations were sown on plot area of 3x3 m². At maturity, central plot area measuring 2x2 m² were hand harvested separately by sickle, placed in cloth sacks, tagged, sun-dried in the open air, and then threshed by manually beating the sacks. Grain yield was determined for each 4 m² plot area and recorded on a gram per meter square.

In this multi-location trial we have also three explanatory variables (Year, Variety and Location) and one response variable. The locations considered are those that have the same potential for growing different variety of wheat as national variety verification. The locations are Arsi Negele, Arsi Robe, Adet, Alemaya, Asasa, Bako, Bekoji, Debre Zeit, Ginchi, Hollota, Hosana, Kokate, Kulumsa and Sinana. Among a number of wheat varieties, the varieties used for this analysis are: ENKOY/ISRAEL, ET12D4/HAR 604, ET12D4/HAR 604(2), HAR 1685/K6295-4A, HAR 416/DERESELIG, HAR 710/DERESELIG, HAR 710/HAR 719, HAR3442, HAR3603, HAR3643, HAR3646, HAR3658, K6295-4A/HAR 424(1), LOCAL CHECK, MAMBA/HAR 1384, MILAN/SHA7, RBC/HAR 800, RBC/HAR 921, SIMBA.

Regarding the difference between uniformity and multi-location trials, in multi-location the trial was sown in different location on different dates on 3x3 m² of land in all locations. In case of uniformity trial, plots were sown by the same variety on the same date. At emergence, each trial was subdivided into basic unit plots measuring 1 m x 1 m by fixing sisal strings on perimeter edges spaced at 1 m intervals along all four sides of the trial. At maturity, basic unit plot areas were harvested from both broadcast and row sown trials separately by sickle. Grain yield was determined for each basic unit plot area and recorded on a gram per meter square basis. The grain yields from adjacent basic unit plots were subsequently combined to derive values for different plot shapes and sizes. However, in case of multi-location there is no plot shape and size derived. Fixed size was used in all conditions.

In this paper, we use two different data sets: one from uniformity and the other from multi-location. Because the uniformity trial is used only to determine the optimum plot size that is used for research activity, it is not applied for variety adaptation that uses a number of locations. To overcome this problem we additionally use multi-location trial that was used for variety adaptation.

3.2. Concepts of Random and fixed factors

In modeling, if the level of the independent variable that occurs in the study is considered to be the only level of interest, each level has a fixed effect on the response variable and the independent variable is known as a fixed effect factor. Independent variables whose levels are determined or set by the experimenter are said to have fixed effects. However, if the levels are considered to be random sample from the population of the possible levels, each level has a random effect on the response variable and the variable is known as random effect factor. Random effects are classification effects where the levels of the effects are assumed to be randomly selected from infinite population of possible levels. A model which contains both fixed and random effect factors with error terms is called mixed model or mixed effect model. And if the relationship among these factors the error term and the response variable is linear we call such model a mixed linear model.

Since the only source of randomness in a fixed effect models are the error terms, parameter estimation could be done using ordinary least squares (OLS) technique. But simple ordinary least square criteria could not be employed in models containing two independent random variables. Thus, the estimation of the mixed effect model is therefore based on the generalized least squares method and the variance components corresponding to the different sources of randomness are estimated by restricted maximum likelihood criteria (Patterson and Thompson, 1971) to get estimates for both fixed and random factors.

As shown above, plot size ranges between 1 and 24 meter square with difference shapes. This shows that it is possible to generate a number plot sizes by changing the dimensions of the plots. As result, considering all plot size and shape in uniformity trial is difficult. Therefore, we randomly select plot size and shapes considered in uniformity trial and hence are random effect. But year and sown type are fixed effects since they are determined by the trial and used them exhaustively.

In multi-location trial, variety and locations are considered as random effects. Ethiopia is secondary a secondary center of diversity for several wheat species. As a result, there are a number of wheat varieties available. Therefore, considering all varieties are tedious. This leads us to use varieties as a random effect. Similarly, locations are considered as random effect since there are similar location found in the country which have similar production potential for variety verifications. Thus, variety and location are random effects. But year in which the trial was undertaken is considered as fixed effect since it was determined by the experiment.

3.3. The model

In this section we will try to describe the model used to analyze wheat trail data considered in this study, parameters of the model and the assumptions. Consider a mixed linear model represented by

$$Y=X\beta+Zu+e.....(3.0)$$

Where

- $Y=11409 \times 1$ and 660×1 vectors of yield of wheat data obtained from 73 different plot sizes in case of uniformity trial and from 20 different varieties as well as 14 location in 3 years in case of multi-location trials respectively.
- $\beta=4 \times 1$ vector of fixed effects (year and sown type in case of uniformity trial) and (year and the intercept in case of multi-location trial) respectively.
- $u=73 \times 1$ and 34×1 vectors of unobservable random effects for uniformity trial and multi-location trial respectively. The distribution of u is usually considered to be normal with mean vector zero and variance-covariance matrix $\sigma_u^2 I$ (where I represents 73×73 as well as 34×34 identity matrix for uniformity and multi-location trial respectively; and σ_u^2 an unknown scalar);
- $X=11409 \times 4$ and 660×4 design matrices relating the fixed effects that contains 0's and 1's. As usual there is no first column of the matrix with values 1's representing intercept of the model in case of uniformity trials. The model discussed for this data has no intercept. Because if there is an intercept it tells us the amount of yield obtained with out having any area for sown. But, multi-location trial is an interceptal model. In general the design matrix X is described as follows for both trials:

$$X_{i1} = \begin{cases} 1 & \text{if data is from 1996} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{i2} = \begin{cases} 1 & \text{if data is from 1997} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{i3} = \begin{cases} 1 & \text{if data is from Broadcast Sown} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{i4} = \begin{cases} 1 & \text{if data is from Row Sown} \\ 0 & \text{otherwise} \end{cases}$$

where $i=1,2,3,\dots,11409$ for uniformity trial.

In multi-location trial, the first column of design matrix relating the fixed effect have a value of 1's representing intercept of the model while the remaining columns will have the following form:

$$X_{i2} = \begin{cases} 1 & \text{if data is from 2001} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{i3} = \begin{cases} 1 & \text{if data is from 2002} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{i4} = \begin{cases} 1 & \text{if data is from 2003} \\ 0 & \text{otherwise} \end{cases}$$

where $i=1,2,3,\dots,660$

- $Z=11409 \times 73$ and 660×34 design matrix relating random effects (plot sizes and shape in case uniformity and Variety as well as location in case of multi-location trials) respectively. The elements of Z matrix will have the form:

$$Z_{i1} = \begin{cases} 1 & \text{if data is from } 1 \times 1 \text{m}^2 \text{ plot size sown in 1996 by row method} \\ 0 & \text{otherwise} \end{cases}$$

$$Z_{i2} = \begin{cases} 1 & \text{if data is from } 1 \times 2 \text{m}^2 \text{ plot size sown in 1996 by row method} \\ 0 & \text{otherwise} \end{cases}$$

$$Z_{i3} = \begin{cases} 1 & \text{if data is from } 1 \times 4 \text{m}^2 \text{ plot size sown in 1996 by row method} \\ 0 & \text{otherwise} \end{cases}$$

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$$Z_{i73} = \begin{cases} 1 & \text{if data is from } 10 \times 2 \text{m}^2 \text{ plot size sown in 1997 by broadcast method} \\ 0 & \text{otherwise} \end{cases}$$

The definition for the remaining Z is given in Appendix C Table 4.

Where $i=1,2,\dots,11409$ in case uniformity trial

$$\begin{cases} 1 & \text{if data is from ENKOY/ISRAEL Variety sown in } j^{\text{th}} \text{ year.} \\ 0 & \text{otherwise} \end{cases}$$

$$Z_{i1} =$$

$$Z_{i2} = \begin{cases} 1 & \text{if data is from ENKOY/ISRAEL Variety sown in } j^{\text{th}} \text{ year.} \\ 0 & \text{otherwise} \end{cases}$$

$$Z_{i3} = \begin{cases} 1 & \text{if data is from ENKOY/ISRAEL Variety sown in } j^{\text{th}} \text{ year} \\ 0 & \text{otherwise} \end{cases}$$

where $i=1,2,\dots,660$ in case multi-location trial

$j=1,2,3$ where $1=2001$ $2=2002$ and $3=2003$

The rest definition for the values of Z is given in Appendix C Table 4

- $\boldsymbol{\varepsilon}=11409 \times 1$ and 660×1 unobservable vector of random residuals of the model for uniformity and multi-location trials respectively. The distribution of $\boldsymbol{\varepsilon}$ is a 11409 and 660 dimensional normal with mean vector 0 and variance covariance matrix $\sigma_e^2 \mathbf{I}$ (where \mathbf{I} represents 11409×11409 identity matrix in case of uniformity trial, and 660×660 identity matrix in case of multi-location trial, σ_e^2 is an unknown scalar)
- The vectors $\boldsymbol{\varepsilon}$ and \mathbf{u} are statistically independent
- σ_u^2 and σ_e^2 are unknown scalar-value parameters called variance components
- The fixed components of the model and the error terms are uncorrelated, i.e. $\text{cov}(\mathbf{X}, \boldsymbol{\varepsilon})=0$

3.4. Analysis of the Data

In analyzing data, we are promptly faced with variances and covariance structure of the data. With each set of data we assume a mixed model that explains the observations. Thus, we make a distinction between fixed effects, that determine the level (expected

means) of observations, and random effects that determine variance. For every model at least there exists one fixed effect (mean) and one random effect (residual error variance). If observations are also influenced by factors that occur randomly but play a significant role for the output of the observation, then those factors are included randomly. Since plot size and shape, and variety and locations are random factors in case of uniformity and multi-location trial dataset respectively, there exists variance component for the respective random factors in addition to the error (residual) variance. In that situation, we have two components contributing to the total variance of the observations: different plots sizes and shape as well as a residual variance component in case of uniformity trial, and variety, location as well as residual variance component in case of multi-location. If we want to calculate variance of yields obtained either from such plot sizes and shape in case of uniformity trial or variety and location in case of multi-location trial, we make use of those parameters. In predicting both plot sizes and shape; and variety and location that gives optimum yield in case of uniformity and best variety adaptation in case of multi-location, appropriate model can be applied which uses both fixed and random effects and Best Linear Unbiased Prediction (BLUP).

3.4.1. When to estimate variance components?

In general, the estimation of variances and covariance has to be based on a sufficient amount of data. Depending on the data structure and the circumstances during measuring, estimation can be based on sufficiently large number of experimental observations (field recorded data). In general, we have to estimate variance if:

- We are interested in a new trial, from which no parameters are available;
- Variances and covariances might have changed over time
- Considerable changes have occurred in a population e.g. due to recent factors.

Therefore, we can undertake variance component estimation if there are sufficient reasons for regular estimation of covariance-variance components.

3.4.2. Methods used for estimating variance component of the parameters

There are different methods used to estimate variance component. Some of them are listed below.

3.4.2.1. Henderson methods

The method of Henderson uses Least Squares equations and variance components are estimated from the random terms in addition to the overall error terms. The variance of the estimates is not minimized (i.e. the estimation is not the most accurate) because sums of squares and expectations are not dependent on the variance-covariance structure of the data but rather on LS equations. Even though the estimates of variances from such methods are unbiased they may fall outside the parameter space (e.g. they can be negative). In addition to this, estimates also may not be unique because sums of squares due to random effects can be computed in several ways when there are several random effects, i.e. corrected for several combinations of other effects.

3.4.2.2. Maximum Likelihood Method

ML (Maximum Likelihood)-estimators maximizes the likelihood of the parameters given the density functions and the data. Estimates are not unbiased but they have smaller variance than the unbiased estimators. **REML** (Restricted ML) estimators maximize the likelihood of the parameters after correcting for the fixed effects (formally: in the space orthogonal to the fixed effects).

In ML methods, the loss in degrees of freedom due to correction for fixed effects is not taken into account. In REML, this loss in degrees of freedom is accounted for. Different quadratic forms are calculated based on the mixed model equations. In most algorithms, to obtain REML estimates, iteration is used. This process starts with a certain set of variance components and stops when the set of variance components that results in the highest likelihood is found. Although REML estimators are within the parameters space by definition, they are biased. Even though there are several algorithms to compute

REML, in practice some algorithms give even negative estimates. Such estimates are therefore formally not REML. For this paper REML estimators that meet convergence criteria will be implemented.

3.5. Model fitting

As discussed above, maximum likelihood based methods appeared to be most flexible to accommodate mixed models. In terms of REML or ML, estimation of variance components has changed from the expectation of mean squares and the interpretation of observational components of variance in trial data to a more direct approach of calculating a likelihood of a data vector for a given model with a given set of parameters, and maximizing this likelihood estimator.

Some REML packages may even allow estimation based on data that have single records per plot and not the whole data structure. Different derivations and algorithms have been implemented to analyze the data. Particularly (restricted) maximum likelihood has been used for the above-mentioned model.

Consider a mixed linear model for wheat trial data given above with all given assumptions. Assume all levels of \mathbf{u} pertain to the same source of variation, for example plot size and shape, and that $V(\mathbf{u}) = \sigma_u^2 \mathbf{I}$, $V(\mathbf{e}) = \sigma_e^2 \mathbf{I}$ and $\text{cov}(\mathbf{u}, \mathbf{e}') = 0$.

The Least Squares equations are:

$$\begin{bmatrix} X'X & X'Z \\ Z'X & Z'Z \end{bmatrix} \begin{bmatrix} \hat{b} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X'y \\ Z'y \end{bmatrix} \dots\dots\dots 3.1a$$

Absorbing the fixed effects reduces the equations to

$$\left. \begin{array}{l} \mathbf{Z}'\mathbf{M}\mathbf{Z}\hat{u} = \mathbf{Z}'\mathbf{M}\mathbf{y} \\ \text{With } \mathbf{M} = \mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}' \end{array} \right\} \dots\dots\dots 3.1b$$

When the inverse of $(X'X)$ does not exist, a generalized inverse can be used in its place. But, in our case X has full column rank and hence the inverse of the matrix exists for uniformity trial. However, we use generalized inverse for multi-location trial since the design matrix is not full column rank. Therefore, we can make estimation of variance components by using:

$$\left. \begin{aligned} \hat{\sigma}_e^2 &= \frac{y'y - \hat{u}'Z'y - \hat{b}'X'y}{N - r(X) - r(Z) + 1} \\ \hat{\sigma}_u^2 &= \frac{\hat{u}'Z'My - (r(Z) - 1)\hat{\sigma}_e^2}{tr(Z'MZ)} \end{aligned} \right\} \dots\dots\dots 3.2$$

With $r(\mathbf{X})=4$ and $r(\mathbf{Z})=73$ denoting the column rank of \mathbf{X} and \mathbf{Z} , respectively, $N=11409$, the number of observations in case of uniformity trial where as $r(\mathbf{X})=3$ and $r(\mathbf{Z})=34$ as well as $N=660$ observations in case of multi-location trial, and tr is the trace operator. In this method, any covariances between levels of \mathbf{u} (i.e. correlations between plot sizes and shape varieties as well as locations in case uniformity and multi-location trial respectively) are ignored.

An analogy with the ANOVA methods can be shown as follows:

The expression $y'y$ is a vector notation for 'total sum of squares'. Expressions like $\hat{b}'X'y$ (Solution multiplied by right hand side) can also be written as $y'X(X'X)^{-1}X'y$ since $\hat{b} = (X'X)^{-1}X'y$ (we ignore now for random effects). Since $X'y$ contains the class totals, and $X'X$ contains the number of observations per class, the expressions $y'X(X'X)^{-1}X'y$ gives the sum of the class totals squared, divided by the number of observations per plot. This is exactly the Sum of squares due to the b -effect. The expression $y'y - \hat{u}'Z'y - \hat{b}'X'y$ is therefore equal to the residual sum of squares.

3.5.1. Restricted Maximum Likelihood

General interest in Maximum Likelihood (ML) estimators of variance components has been propelled by their desirable statistical properties: they are consistent, asymptotically normal and efficient. Harville (1977) has given an extensive review of ML estimation.

Furthermore, the ML framework provides a great deal of flexibility, allowing for designs and models for analysis, which cannot be accommodated by ANOVA type of estimators. Initial interest in ML, to estimate both random effect parameters and fixed effects, was stimulated by concern about bias due to selection. A number of simulation studies have illustrated that selection can be accounted for by REML (Sorensen and Kennedy, 1984; Van der Werf and De Boer, 1990) when the complete mixed model is used with all data used for selection included.

Restricted Maximum Likelihood (REML) is a ML method that accounts for the loss of degrees of freedom due to fitting fixed effects. Patterson and Thompson (1971) formally described REML. The procedure requires that \mathbf{y} have a multivariate normal distribution although various authors have indicated that ML or REML estimators may be an appropriate choice even if normality does not hold (Meyer, 1990).

Over the last decade, extensive research effort has been directed towards the development of specialized and efficient algorithms for particular classes of models. In ML and REML the aim is to find the set of parameters that maximizes the likelihood of the data. The likelihood of the data for a given model can be written as a function of multivariate normal. From calculus we know that we can find the maximum of a function by taking the first derivative and set that equal to zero. Solving that would result in the desired parameters (assuming that we did not find the minimum, this can be checked using second derivatives). The first and second derivatives of the likelihood function are complicated formulas. Different algorithms have been developed which try to circumvent this problem. An overview of different methods is given by Meyer (1990).

3.5.2. Principle of Maximum Likelihood

Suppose we have a variable y with mean μ and standard deviation σ . The normal distribution of this variable can be represented as $y \sim N(\mu, \sigma^2)$. A mathematical representation of a density function for a normally distributed variable is

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right)$$

This is called the Probability Density Function (PDF) of y .

A function for a multidimensional normal distribution $\mathbf{y} \sim N(\mathbf{Xb}, \mathbf{V})$ is

$$f(\mathbf{y}) = \frac{1}{(2\pi)^{N/2} |\mathbf{V}|^{1/2}} e^{-\frac{1}{2}(\mathbf{y} - \mathbf{Xb})' \mathbf{V}^{-1} (\mathbf{y} - \mathbf{Xb})} \dots\dots\dots(3.3)$$

Where N is the length of \mathbf{y} and $|\mathbf{V}|$ is the determinant of \mathbf{V} . The function $f(\mathbf{y})$ is called a density function of \mathbf{y} . The function gives the probability of finding a certain \mathbf{y} given the

parameters. The parameters are the means in \mathbf{Xb} ("*location parameters*") and the variances in \mathbf{V} ("*dispersion parameters*"). However, this function can also be used in the other way round i.e. if we have observed data, it gives us the probability of having such data for certain parameter values. Therefore, the probability density function can be used as a likelihood function as well. When the data \mathbf{y} is known, $f(\mathbf{y})$ is a likelihood function and this function can be maximized in the parameters, i.e. we want to find the parameters for which $f(\mathbf{y})$ has the highest value. Instead of maximizing $f(\mathbf{y})$ we can also maximize the natural log of $f(\mathbf{y})$; $L(\mathbf{b}, \mathbf{V} \mid \mathbf{X}, \mathbf{y})$, which is the log likelihood function:

$$L(b, V \mid X, y) = -\frac{1}{2} N \log(2\pi) - \frac{1}{2} \log|V| - \frac{1}{2} (y - Xb)' V^{-1} (y - Xb) \dots (3.4)$$

This function gives the likelihood of the unknown parameters \mathbf{b} and \mathbf{V} given the observed data \mathbf{y} and the design matrix \mathbf{X} . The matrix \mathbf{V} depends on the variance components we are interested in. \mathbf{V} has usually a known design but is proportional to unknown parameter values, e.g. $\mathbf{V} = \mathbf{ZAZ}'\sigma_a^2 + \mathbf{I}\sigma_e^2$. This can also be written as $\mathbf{V} = \mathbf{ZGZ}' + \mathbf{R}$ where $\mathbf{G} = \mathbf{A}\sigma_a^2$ and $\mathbf{R} = \mathbf{I}\sigma_e^2$. The maximum likelihood estimates of the parameters are obtained by maximizing the likelihood function.

In Restricted Maximum Likelihood as suggested by Patterson and Thompson (1971) the likelihood function of the data is maximized '*in the space of error contrasts*'. In other words, the density function is maximized after correcting all observations first for the fixed effects

Methods available to get REML estimates can be divided in the following groups:

- 1) Methods using first derivatives of the likelihood function.
- 2) Methods using first and second derivatives of the likelihood function.
- 3) Derivative free methods.

For models with a number of random factors it is more difficult to find the maximum and it is also more difficult to construct derivatives. In categories 1 and 2, the derivatives can be calculated exactly but in most methods approximations are used for sufficiently large data sets. Since we have a number of random effects we use approximation methods.

3.5.3. REML using derivatives

Methods which use both first and second derivatives, i.e. geometrically speaking information on slope and curvature of the function, have been found to converge quickly (Meyer, 1989). However, calculation of actual or expected second derivatives was initially computationally highly demanding if not prohibitive even for simple models. Therefore, initially many REML applications were based on the so-called Expectation-Maximization (EM) algorithm. This requires, implicitly, first derivatives of the likelihood to be evaluated. The resulting estimators then have the form of quadratics in the vector of random effects solutions, obtained by Best Linear Unbiased predictor (BLUP) for the assumed values of variances to be estimated, which are equated to their expectations. For the mixed model equations we have the following normal equations:

$$\begin{bmatrix} X'X & X'Z \\ Z'X & Z'Z + \hat{\alpha}A^{-1} \end{bmatrix} \begin{bmatrix} \hat{b} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X'y \\ Z'y \end{bmatrix} \dots\dots\dots(3.5)$$

Note that $\hat{\alpha}$ is a function of the variance parameters that need to be estimated.

Therefore, initially a prior (starting) value of α is used. The REML estimates of variance components using the EM algorithm can be obtained as:

$$\left. \begin{aligned} \hat{\sigma}_u^2 &= \frac{\hat{a}'A^{-1}a + tr(A^{-1}C)\hat{\sigma}_e^2}{q} \\ \hat{\sigma}_e^2 &= \frac{y'y - \hat{a}'Z'y - \hat{b}'X'y}{N - r(X)} \end{aligned} \right\} \dots\dots\dots 3.6$$

Where N is the number of observations i.e. 11409 and 660 in uniformity trial and multi-location trial respectively, q is the number of random effect levels i.e. 73 in case of uniformity trial and 34 in case of multi-location trial and C the part of the inverse of the mixed model equations that corresponds with the random effect.

The EM algorithms have the property of always yielding positive estimates as long as prior values (values that are used to start the calculations) are positive (Harville, 1977). The EM algorithm is not very difficult to program, because all elements, which are needed can be derived from the mixed model equations. What are needed for each round

of iteration are the solutions to the mixed model equations and the trace of the inverse of the random part of the coefficient matrix. This last element is computationally the most difficult part. Iterative methods can be used to obtain estimates for the fixed and random effect but the EM algorithm requires the direct inverse of a matrix of size equal to the number of levels of the random effects, in each round of iteration. This imposes restrictions on the kind of analyses feasible, especially for multivariate analyses.

The EM algorithm is an iterative procedure to get estimates. One starts the process with solving the equations for a given (prior) value of the variance components. These values are used in estimating the effects of the model (α depends on the assumed levels of the variance components). This results in a new value for the variance components and the corresponding value of α . In an iterative process, the old values and the new value of the next iteration round are becoming more and more similar, and ultimately converge (when the difference is very small) to a solution

3.5.4. Derivative Free REML (DFREML)

In the development of algorithms to compute REML an approach that did not make use of derivatives proved to be particularly successful to compute variance components from the given model. This approach is called a derivative free approach, and was first introduced by Smith and Graser (1986) and Graser et al. (1987). The maximum is found by comparing likelihood values of different parameter values.

The likelihood function for the model given by (3.0) can be re-written. First it is written after eliminating ("correcting for") the fixed effects. This is called the Restricted Maximum Likelihood.

Secondly, it is re-written in terms of elements that relate to the mixed model equations:

$$\text{Log } L = -\frac{1}{2} \left(\text{const} + q \log \sigma_u^2 + N \log \sigma_e^2 + y'Py + \log|W| + \log|A| \right)$$

Where \mathbf{W} is the coefficient matrix of the mixed model equations. The $\log |\mathbf{A}|$ is a constant which does not depend on the parameters of interest and does not has to be evaluated. The matrix \mathbf{P} is quite complicated, but Smith and Graser (1986) and Graser *et al.* (1987) showed that $\mathbf{y}'\mathbf{P}\mathbf{y}$ represents the sum of squares of residuals. The log determinant of the coefficient matrix ($\log |\mathbf{W}|$) can be evaluated simultaneously by augmenting \mathbf{W} by the

vector of right hand sides and the total SS ($\mathbf{y}'\mathbf{y}$) and absorbing all rows and columns into the latter. The augmented mixed model array is:

$$\begin{bmatrix} \mathbf{y}'\mathbf{y} & \mathbf{y}'\mathbf{X} & \mathbf{y}'\mathbf{Z} \\ \mathbf{X}'\mathbf{y} & \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{Z} \\ \mathbf{Z}'\mathbf{y} & \mathbf{Z}'\mathbf{X} & \mathbf{Z}'\mathbf{Z} + \alpha\mathbf{A}^{-1} \end{bmatrix}$$

Absorption, which is also referred to as Gaussian Elimination, is used to calculate the quantities $\mathbf{y}'\mathbf{P}\mathbf{y}$ and $\log |\mathbf{W}|$. The residual variance can be estimated as $\mathbf{y}'\mathbf{P}\mathbf{y}/(\mathbf{N}-r(\mathbf{X}))$ so that $\log L$ can be maximized with respect to one parameter only, the variance ratio α , estimating subsequently σ_a^2/α .

This principle has been extended to models including additional random effects, such as environmental effect due to other extraneous factors or application of fertilizers effect to multivariate analyses (Meyer, 1989).

The derivative free algorithm has been applied in the DFREML programs that are written and distributed by Karin Meyer. These programs can be used for uni- and multivariate analysis and for models with several random effects.

Groeneveld (1991) presented a second package for estimating variance components using a derivative free approach.

A more robust and efficient algorithm analysis is Average Information REML, now applied by the DFREML package. A very powerful program for parameter estimation is the ASREML package (Gilmour et al., 1996).

3.5.5. REML using the Average Information algorithm.

First we discuss more formally first and second derivatives of the likelihood function.

Then, the mechanism of an AI algorithm will be presented.

The partial derivative with respect to the vector of fixed effects, \mathbf{b} , in equation 3.4 can be calculated by using a general result for matrix derivatives (Lynch and Walsh, 1998),

$$\frac{\partial[(\mathbf{y} - \mathbf{X}\mathbf{b})'\mathbf{V}^{-1}(\mathbf{y} - \mathbf{X}\mathbf{b})]}{\partial \mathbf{b}} = -2\mathbf{X}'\mathbf{V}^{-1}(\mathbf{y} - \mathbf{X}\mathbf{b}) \dots \dots \dots (3.7)$$

It is noted that from equation 3.4 and 3.7, $\frac{\partial(Xb, \sigma_i^2)}{\partial b} = X'V^{-1}(y - Xb)$. The partial derivatives of $\ln |V|$ in equation 3.4 with respect to the variance of random effects, σ_i^2 [e.g. $i=u$ (random) and e (error)] can be obtained from matrix theory (Searle, 1982)

$$\frac{\partial}{\partial \sigma_i^2} \ln |V| = \text{tr} \left[V^{-1} \frac{\partial V}{\partial \sigma_i^2} \right] \text{ Differentiating } V^{-1} \text{ with respect to } \sigma_i^2 \text{ results in}$$

$$\frac{\partial V^{-1}}{\partial \sigma_i^2} = -V^{-1} \frac{\partial V}{\partial \sigma_i^2} V^{-1} \dots\dots\dots(3.8)$$

Where, tr (trace) is the sum of the diagonal elements of a square matrix.

To derive equation 3.8, from equation 3.4 we use the first derivatives of L from equation 3.4 with respect to the variance components can be obtained as,

$$\frac{\partial L(Xb, \sigma_i^2)}{\partial \sigma_i^2} = -\frac{1}{2} \text{tr}(V^{-1}V_i) + \frac{1}{2} (y - X\hat{b})' V^{-1}V_iV^{-1} (y - X\hat{b}) \dots\dots\dots(3.9a)$$

and from the second equation of 3.5, the second derivative is,

$$\frac{\partial^2 L(Xb, \sigma_i^2)}{\partial \sigma_i^2 \partial \sigma_j^2} = -\frac{1}{2} \text{tr}(V^{-1}V_iV^{-1}V_j) + \frac{1}{2} (y - X\hat{b})' V^{-1}V_iV^{-1}V^{-1}V_jV^{-1} (y - X\hat{b})$$

.....(3.9b)

where, $\frac{\partial V}{\partial \sigma_i^2}$ is denoted by V_i .

Suppose V is defined as $ZAZ' + I\sigma_e^2$, then the following equation evaluates the expression in equation 3.9a and 3.9 b.

$$\frac{\partial V}{\partial \sigma_i^2} = V_i = \begin{cases} ZAZ' & \text{if } i = u \\ I & i = e \end{cases} \dots\dots\dots(3.10)$$

As Lynch and Walsh (1998) stated, the ML estimators are obtained by making the first derivative of L (equation 3.9a) equal to zero and thus solving equation 3.9a gives

$$\text{tr}(V^{-1}V_i) = (y - X\hat{b})' V^{-1}V_iV^{-1} (y - X\hat{b}) = y' \hat{P}V_i\hat{P}y \dots\dots\dots(3.11)$$

Where $p = V^{-1} - V^{-1}X(X'V^{-1}X)^{-1}X'V^{-1}$

From equation 3.10 and 3.11, ML equations are

$$tr(V^{-1}(ZAZ')) = y' \hat{P}(ZAZ') \hat{P}y \text{ for } v_i = \sigma_u^2 \dots\dots\dots(3.12a)$$

$$tr(V^{-1}) = y' \hat{P} \hat{P}y \text{ for } v_i = \sigma_e^2 \dots\dots\dots(3.12b)$$

Similarly, but removing the fixed effects from the model, REML equations can be derived from ML equations. In REML we use a transformation matrix K such that,

$$KX = 0$$

Multiplying the K with the mixed linear model ($y = Xb + Za + e$) replaces

$$\left. \begin{array}{l} y \text{ by } Ky \sim N(0, KVK.) \\ X \text{ by } KX = 0 \\ Z \text{ by } KZ \\ V \text{ by } KVK' \end{array} \right\} \dots\dots\dots 3.13$$

Applying equation 3.13 to equation 3.12a or 3.12b yields,

$$tr((KVK')^{-1}KV_iK') = y' K'(KVK')^{-1}KV_iK'(KVK')^{-1}y \text{ Searle et al. (1992)}$$

proved that $P = K'(KVK')^{-1}K$ Therefore,

$$tr((KVK')^{-1}KV_iK') = y' K'(KVK')^{-1}KV_iK'(KVK')^{-1}y \text{ become}$$

$$tr(PV_i) = y' \hat{P}V_i \hat{P}y.$$

In similar way Equation 3.9a and 3.9b can also be transformed as,

$$\frac{\partial L}{\partial \sigma_i^2} = -\frac{1}{2} tr(PV_i) + \frac{1}{2} y' PV_i Py \dots\dots\dots(3.14a)$$

$$\frac{\partial^2 L}{\partial \sigma_i^2 \partial \sigma_j^2} = \frac{1}{2} tr(PV_i PV_j) - y' PV_i PV_j Py \dots\dots\dots(3.14b)$$

From equation 3.12, REML equations are

$$tr(P(ZAZ')) = y' \hat{P}(ZAZ') \hat{P}y \text{ for } \sigma_a^2 \dots\dots\dots(3.15a)$$

$$tr(P) = y' \hat{P} \hat{P}y \text{ for } \sigma_e^2 \dots\dots\dots(3.15b)$$

3.5.6. The Average Information Algorithm for REML estimation

Various techniques for solving ML / REML equations have been introduced (e.g. the Newton-Raphson algorithm, Fisher's scoring method and DF algorithm).

In this section, the Newton-Raphson algorithm and Fisher's scoring method are firstly described and then the Hessian matrix and the Fisher information matrix are derived. This may help to understand the property of the Average Information (AI) algorithm and the AI matrix. And then, the method for estimating the elements of AI matrix is described which is key process for the AI algorithm.

3.5.6.1. Average Information from the Hessian and the Fischer information matrix

The Newton-Raphson algorithm gives the REML estimate using the following equation (Lynch and Walsh, 1998).

$$\Theta^{(k+1)} = \Theta^k - (H^k)^{-1} \frac{\partial L}{\partial \Theta} \Big|_{\Theta^k} \dots\dots\dots(3.16)$$

where Θ is a vector of parameters, k is k^{th} iteration, $\frac{\partial L}{\partial \Theta}$ is a column vector of the first

derivatives of the log likelihood function with respect to each parameter, and H is the Hessian matrix elements of which are the second derivatives of the log likelihood function with respect to the variance components. From equation 3.16 and 3.14b, the Hessian matrix for variance components (residual variance and random effect variance) is,

$$H = \frac{\partial L}{\partial \sigma^2 \partial \sigma_j^2} = \frac{1}{2} \begin{bmatrix} tr(PP) - 2y' P P P y & tr(PA^* P) - 2y' P A^* P P y \\ tr(PA^* P) - 2y' P A^* P P y & 2tr(PA^* P A^*) - y' P A^* P A^* P y \end{bmatrix} \dots\dots\dots(3.17)$$

Where $A^* = ZAZ'$

In Fisher's scoring method, the inverse of the Hessian matrix in equation 3.17 is replaced by its expected value (Lynch and Walsh, 1998).

$$\Theta^{(k+1)} = \Theta^k - (F^k)^{-1} \frac{\partial L}{\partial \Theta} \Big|_{\Theta^k} \dots\dots\dots(3.18)$$

Where F is the Fisher information matrix and is given by

$$F = -E \left(\frac{\partial L}{\partial \sigma_i^2 \partial \sigma_j^2} \right) = \frac{1}{2} \begin{bmatrix} tr(PP) & tr(PA * P) \\ tr(PA * P) & tr(PA * PA*) \end{bmatrix} \dots\dots\dots(3.19)$$

Therefore, the average information from the Hessian and Fisher's information matrix is given as:

$$AI = \frac{(-H + F)}{2} = \frac{1}{2} \begin{bmatrix} y' PPPy & y' PA * PPy \\ y' PA * PPy & ty' PA * PA * Py \end{bmatrix} \dots\dots\dots(3.20)$$

From equation 3. 18 and 3.20,

$$\Theta^{(k+1)} = \Theta^k + (AI^{(k)})^{-1} \frac{\partial L}{\partial \Theta} \Big|_{\Theta^k} \dots\dots\dots(3.21)$$

The method for calculating the elements of the AI matrix requires term like $V_i Py$, which are referred to as working variates, $y(\sigma_i^2) = V_i Py$ (Gilmour *et al.*, 1995)

According to Johnson and Thompson (1995), the working variate for random effect variance and residuals are expressed as,

$$y(\sigma_u^2) = A * Py = \frac{1}{\sigma_u^2} Z\hat{u} \dots\dots\dots(3.22b)$$

$$y(\sigma_e^2) = Py = \frac{1}{\sigma_e^2} \hat{e} \dots\dots\dots(3.22a)$$

Where \hat{u} is vector of solutions from the mixed model equation (MME) and $\hat{e} = y - X\hat{b} - Z\hat{u}$

According to Johnson and Thompson (1995), the elements of the AI matrix can be calculated as a vector product of the working variates from the MME in which y is replaced by the working variates. For example, consider the element, y'PPy in the AI matrix where $y'PPPy = y(\sigma_e^2)' Py(\sigma_e^2)$, then the term, y'PPPy, can be obtained by multiplying the transpose of the column vector of $y(\sigma_e^2)$ (3.22 b) by the column vector

of residuals, $Py(\sigma_e^2)$. Therefore, all the elements in the AI matrix can be calculated in the same manner.

Alternatively, Gilmour *et al.* (1995) calculated the AI matrix using the Gaussian elimination of the M matrix.

$$M = \begin{bmatrix} y'R^{-1}y & y'R^{-1}X & y'R^{-1}Z \\ X'R^{-1}y & X'R^{-1}X & X'R^{-1}Z \\ Z'R^{-1}y & Z'R^{-1}X & Z'R^{-1}Z + A^{-1}\sigma_u^2 \end{bmatrix} \dots\dots\dots(3.23)$$

After performing Gaussian elimination, it is well known that the element of the first row and first column in the M matrix, $M^*(1,1)$, equals $y'Py$ (Graser *et al.*, 1987). If y is replaced by the working variate for σ_e^2 ($y(\sigma_e^2)$) from equation 3.22 b), then $M^*(1,1)$ after Gaussian elimination equals $y'PPPy$ in the AI matrix. If y is replaced by the working σ_u^2 ($y(\sigma_u^2)$), then $M^*(1,1)$ after Gaussian elimination equals $y'PA*PA*Py$ in the AI matrix. For a cross product (e.g. $y'PA*PPy$ in the AI matrix), the M matrix will be formed as,

$$M = \begin{bmatrix} y(\sigma_e^2)'(R^{-1}y(\sigma_e^2)) & y(\sigma_e^2)'R^{-1}X & y(\sigma_e^2)'R^{-1}Z \\ X'R^{-1}y(\sigma_e^2) & X'R^{-1}X & X'R^{-1}Z \\ Z'R^{-1}y(\sigma_e^2) & Z'R^{-1}X & Z'R^{-1}Z + A^{-1}\sigma_u^2 \end{bmatrix}$$

After performing Gaussian elimination of M, $M^*(1,1)$ equals $y'PA*PPy$ in the AI matrix. All elements in the AI matrix can be calculated in the same manner. After establishing

the AI matrix, $\frac{\partial L}{\partial \Theta}$ is required for equation 3.21.

$$\frac{\partial L}{\partial \sigma_i^2} = -\frac{1}{2} tr(PV_i) + \frac{1}{2} y'PV_iPy \dots\dots\dots from \quad 3.14 \text{ a. But, } y'PV_iPy \text{ can be}$$

calculated from equation 3.22 a and b. Thus,

$$\begin{aligned} \frac{\partial L}{\partial \sigma_u^2} &= -\frac{1}{2}(tr(PA^*) - y' PA^* Py) \\ &= -\frac{1}{2} \left[\frac{N_a}{\sigma_u^2} - \frac{tr(A^{-1}C^{aa})}{\sigma_u^4} - \left(\frac{\hat{e}}{\sigma_u^2} \right)' \left(\frac{Z\hat{a}}{\sigma_u^2} \right) \right] \dots\dots\dots (3.24a) \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial \sigma_e^2} &= -\frac{1}{2}(tr(P) - y' PPy) \\ &= -\frac{1}{2} \left[\frac{N - r(X)}{\sigma_e^2} - \left(N_u - \frac{tr(A^{-1}C^{aa})}{\sigma_u^4} \right) \frac{1}{\sigma_e^2} - \left(\frac{\hat{e}'\hat{e}}{\sigma_e^4} \right) \right] \dots\dots\dots (3.24b) \end{aligned}$$

3.5.7. The procedure of the AI algorithm

Efficient computational procedures for the AI algorithm for univariate case were described in several studies (Johnson and Thompson, 1995; Gilmour *et al.*, 1995). But the major ones are given as follows:

1. Construction of mixed model equation (MME) or matrix M [3.23]
2. Calculating log likelihood in current stage of iteration
3. The log likelihood from equation 3.5 can be calculated by the following equation (Meyer, 1989).

$$L = -\frac{1}{2} \left[\left(N - rank(X) - n \right) \log \hat{\sigma}_e^2 + \log |C| + \log |A| + n \log \hat{\sigma}_u^2 + y' Py \right]$$

Where n is the number of observations used, || is the determinant of the matrices.

4. Estimating \hat{b} , and \hat{u} . For the calculation of the working variates in equation 3.22a and b, the fixed effects (b) and random effects (u) are obtained from the MME. Alternatively, using intermediate terms formed during Gaussian elimination, \hat{b} and \hat{u} are efficiently obtained (see Gilmour *et al.*, 1995). Given the effects (\hat{b} , and \hat{u}) residuals of the model can be obtained by using $\hat{e} = y - X\hat{b} - Z\hat{u}$.
5. AI matrix using [3.20], [3.22 a and b] and [3.23]
6. First derivatives from [3.24]
7. Update from [3.21]
8. Convergence

3.6. Estimation of the parameters

After a convergence criterion is fulfilled, then the next point is the estimation of the parameters. For the mixed model given by equation (3.0), a key assumption in the foregoing analysis is that u and ε are normally distributed with

$$\begin{aligned} E \begin{bmatrix} u \\ \varepsilon \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ Var \begin{bmatrix} u \\ \varepsilon \end{bmatrix} &= \begin{bmatrix} G & 0 \\ 0 & R \end{bmatrix} \end{aligned} \dots\dots\dots 3.25$$

Where $G = A\sigma_u^2$ and $R = \sigma_\varepsilon^2 I_n$. Hence, the variance of y is, therefore, $V = ZGZ' + R$. As a result we can model V by setting up the random-effects design matrix Z and by specifying covariance structures for G and R , where Z containing dummy variables, G containing variance components in a diagonal structure, and $R = \sigma_\varepsilon^2 I_n$, where I_n denotes the $n \times n$ identity matrix. As it is shown above, for the foregoing analysis we need to know G and R . Since G and R are unknown parameter, we first find estimates of those parameters.

3.6.1. Estimating G and R in the Mixed Model

Estimation is more difficult in the mixed model than in the general linear model. Because we need not only β as in the general linear model, but also we have unknown parameters u , G , and R as well. Generalized least square (GLS) is more appropriate than Least Square (LS) and applied by minimizing $(y - X\beta)'V^{-1}(y - X\beta)$ or maximize function given by 3.3 after natural logarithm is applied for multivariate normal distribution.

To minimize $(y - X\beta)'V^{-1}(y - X\beta)$ or maximize function given by 3.3 it requires the knowledge of V and, therefore, knowledge of G and R . Lacking such information, one approach is to use estimated generalized least square (GLS), in which we insert some reasonable estimate for V into the minimization problem. The goal thus becomes finding a reasonable estimate of G and R and in turn V .

In many situations, the best approach is to use likelihood-based methods, exploiting the assumption that \mathbf{u} and $\boldsymbol{\varepsilon}$ are normally distributed (Hartley and Rao 1978; Patterson and Thompson 1971; Harville 1977; Laird and Ware 1982; Jennrich and Schluchter 1986). The SAS implements two likelihood-based methods: maximum likelihood (ML) and restricted/residual maximum likelihood (REML). Using calculus, it is possible to reduce the maximization problem to one over only the parameters in \mathbf{G} and \mathbf{R} (SAS 1996). The corresponding log-likelihood functions are as follows:

$$\begin{aligned} \text{ML: } l(\mathbf{G}, \mathbf{R}) &= -\frac{1}{2} \log |\mathbf{V}| - \frac{1}{2} \mathbf{r}' \mathbf{V}^{-1} \mathbf{r} - \frac{n}{2} \log(2\pi) \\ \text{REML: } l_R(\mathbf{G}, \mathbf{R}) &= -\frac{1}{2} \log |\mathbf{V}| - \frac{1}{2} \log |\mathbf{X}' \mathbf{V}^{-1} \mathbf{X}| \\ &\quad - \frac{1}{2} \mathbf{r}' \mathbf{V}^{-1} \mathbf{r} - \frac{n-p}{2} \log(2\pi) \end{aligned}$$

where $\mathbf{r} = \mathbf{y} - \mathbf{X}(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}^{-1}\mathbf{y}$ and p is the rank of \mathbf{X} . In mixed model analysis, the maximum likelihood (ML), Restricted/residual maximum likelihood (REML), or Type I to Type III SS provide estimates of \mathbf{G} and \mathbf{R} , which are denoted by $\hat{\mathbf{G}}$ and $\hat{\mathbf{R}}$, respectively.

3.6.2. Estimating $\boldsymbol{\beta}$ and \mathbf{v} in the Mixed Model

To obtain estimates of $\boldsymbol{\beta}$ and \mathbf{u} , the standard method is to solve the normal equation of mixed model equations given by equation 3.5. From this equation $\boldsymbol{\beta}$ is the vector of fixed effects and \mathbf{v} is vector of random effects. In this regard the random effect and the error terms are assumed to have zero means, assumed to be uncorrelated and, have variance-covariance matrices given by $\text{Var}(\mathbf{u}) = \text{E}(\mathbf{u}\mathbf{u}') = \mathbf{G}$ and $\text{Var}(\boldsymbol{\varepsilon}) = \text{E}(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \mathbf{R}$. These variance-covariance are assumed known. Therefore, the variance of the response variable, yield, is given by $\mathbf{V} = \text{var}(\mathbf{y}) = \mathbf{Z}\mathbf{G}\mathbf{Z}' + \mathbf{R}$. On top of this, we also assume that \mathbf{V} is non-singular. The normal equations stemming from generalized least squares are then $\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{X}'\mathbf{V}^{-1}\mathbf{y}$ with solution $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\hat{\mathbf{V}}^{-1}\mathbf{X})^{-1}\mathbf{X}'\hat{\mathbf{V}}^{-1}\mathbf{y}$. If \mathbf{V} is singular \mathbf{V}^{-1} is replaced by \mathbf{V}^- . In the fixed effects case \mathbf{V} usually has the form $\sigma_e^2 \mathbf{I}$ where \mathbf{I} is the identity matrix having N

dimension, may be diagonal. In either case, inversion of V is simple. But, $V=ZG Z' +R$ is difficult to invert even if G and R are diagonal. However, as indicated by Henderson et al (1959), a set of equations not involving V^{-1} can be established, alternative to

$X'V^{-1}X\hat{\beta} = X'V^{-1}y$ for deriving $\hat{\beta}$. Suppose that in (3.5) the effects represented by u were in fact fixed and not random. Then, the normal equations for completely fixed effects model would be $\begin{bmatrix} X'R^{-1}X & X'R^{-1}Z \\ Z'R^{-1}X & Z'R^{-1}Z \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X'R^{-1}y \\ Z'R^{-1}Y \end{bmatrix}$ (Searle et al 1992). The solutions for

these normal equations can also be written as:

$$\begin{aligned} \hat{\beta} &= (X'\hat{V}^{-1}X)^{-1} X'\hat{V}^{-1}y = (X'(Z'GZ + R)^{-1}X)^{-1} X'(Z'GZ + R)^{-1}y \\ \hat{u} &= \hat{G}Z'\hat{V}^{-1}(y - X\hat{\beta}) = (Z'R^{-1}Z + G^{-1})^{-1}(Z'R^{-1}(y - X\hat{\beta})) \end{aligned}$$

However, G and R are usually unknown and are estimated using one of the aforementioned methods. These estimates, \hat{G} and \hat{R} , are therefore simply substituted into the preceding expression to obtain the approximate variance-covariance matrix of $(\hat{\beta} - \beta, \hat{u} - u)$.

For inferences concerning the fixed- and random-effect parameters in the mixed model, consider estimable linear combinations of the following form:

$$L \begin{bmatrix} \beta \\ u \end{bmatrix} \dots\dots\dots 3.26 \quad (\text{SAS 1999})$$

The estimability requirement (Searle 1971) applies only to the β -portion of L , as any linear combination of u is estimable. Such a formulation in terms of a general L matrix encompasses a wide variety of common inferential procedures such as those employed with Type I -Type III tests and LS-means. Typically, inference on fixed-effects is the focus, and, in this case, the u -portion of L is assumed to contain all 0s.

Statistical inferences are obtained by testing the hypothesis

$$H: L \begin{bmatrix} \beta \\ u \end{bmatrix} = 0 \quad \dots\dots\dots 3.27$$

or by constructing point and interval estimates.

$$t = \frac{L \begin{bmatrix} \hat{\beta} \\ \hat{u} \end{bmatrix}}{\sqrt{S}} \quad \dots\dots\dots 3.28$$

where S is the covariance matrix of $(\hat{\beta} - \beta, \hat{u} - u)$. Under the assumed normality of u and ϵ , t has an exact t-distribution only for data exhibiting certain types of balance and for some special unbalanced cases. In general, t is only approximately t-distributed, and its degrees of freedom must be estimated. With $\hat{\nu}$ being the approximate degrees of freedom, the associated confidence interval is

$$L \begin{bmatrix} \beta \\ u \end{bmatrix} \pm \sqrt{S} t_{\hat{\nu}, \alpha/2} \quad \dots\dots\dots 3.30$$

Where $t_{\hat{\nu}, \alpha/2}$ is the $(1-\alpha/2)$ 100th percentile of the $t_{\hat{\nu}}$ -distribution.

The t-statistic enables us to make inferences about our fixed effects, which account for the variance-covariance model we have selected so far. An alternative is the χ^2 statistic associated with the likelihood ratio test. This statistic compares two fixed-effects models by considering one as a special case of the other.

Chapter Four: Model fitting and Diagnostics

4.1. Model Diagnostics for Original Model

As discussed in chapter three the error term ε for mixed model given by $Y=X\beta+Z\upsilon+\varepsilon$, is assumed to be independent and normally distributed with zero mean vector and constant variance. If the model is appropriate for data under consideration, the observed residuals of the fitted model, say, $\hat{\varepsilon}_i = y_i - \hat{y}_i$ would have zero mean and constant variance. To check whether these assumptions have fulfilled or not SAS software was used. The output of the software is given in Appendix B, summary 1 and 2. From these result we conclude that the observed residual did not fulfill the normality assumptions. Because the homogeneity tests for variance in all tests statistic indicates non constant variance for both data sets. Hence, it requires transformation of the response variable.

Although small departures from normality do not affect the model greatly, gross non normality is potentially more serious as the t- or F- statistics, and confidence and prediction intervals depend on normality assumptions. Further if the errors come from a distribution with thicker or heavier tails than the normal, then the model fit may be sensitive to small subset of the data. Heavy tail error distribution often generates outliers that “pull” the model fit too much in their direction.

To check whether there is a departure from the stated assumption or not there are a number of model assumption checking mechanism. Among them, a very simple method of checking the normality assumption is the normal probability plot. If the sketch of this probability plot is scattered around the straight line, then we can say the observed error satisfies the stated normality assumptions. The straight line is usually determined visually, with emphasis on the central values (e.g. the 0.33 and 0.67 cumulative probability points) rather than the extremes. Substantial departure from a straight line indicates that the distribution is not normal. But if plot shows a sharp upward and/or downward curve at both extremes, then there is indication that the tails of these distribution are too heavy for it to be considered as normal. Conversely, if the plot shows flattening at the extremes, then it shows a typical pattern of samples from a distribution with thinner tails than

normal. On the other hand, if the plot shows certain pattern instead of following the straight line randomly then we can conclude that the samples are taken from either positive or negative skewed distribution based on the pattern of plot (Douglas *et al.*, 1991).

Normal Probability Plot (PP-plot) is the plot sketched by using each residual against the expected value under normality. A plot that is nearly linear suggests that the normality assumption is valid, whereas a plot that departs substantially from linearity suggests that the distribution of the errors are not normal.

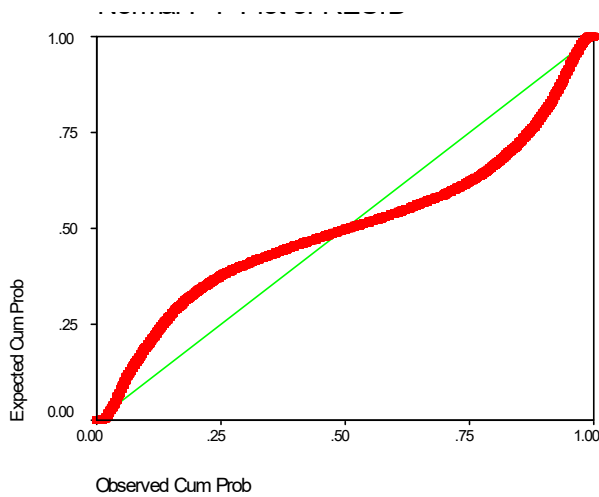


Figure 1a The Normal PP-plot of the residual uniformity trial

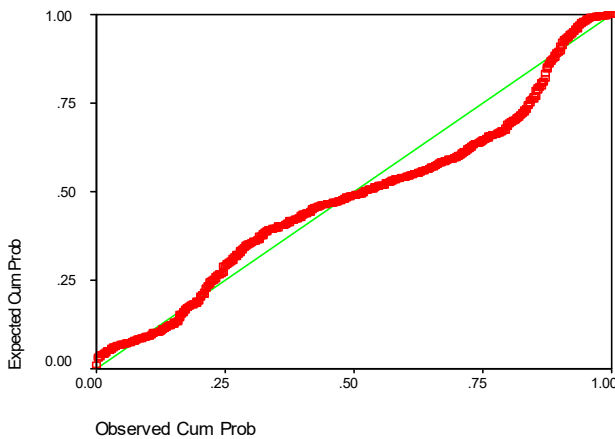


Figure 1b The Normal PP-plot of residual for Multi-location trial

The normal probability plot (Figure 1a and 1b) given above show that both uniformity and multi-location trial data are substantially deviated from the given hypothetical linear line and this indicates that the distribution of the error term and the response variable is not normal.

The normality assumption may also be checked by constructing a histogram of the residuals. However, if the number of residuals is too small it is too difficult to allow easy visual identification of the shape of the normal distribution for small sized sample.

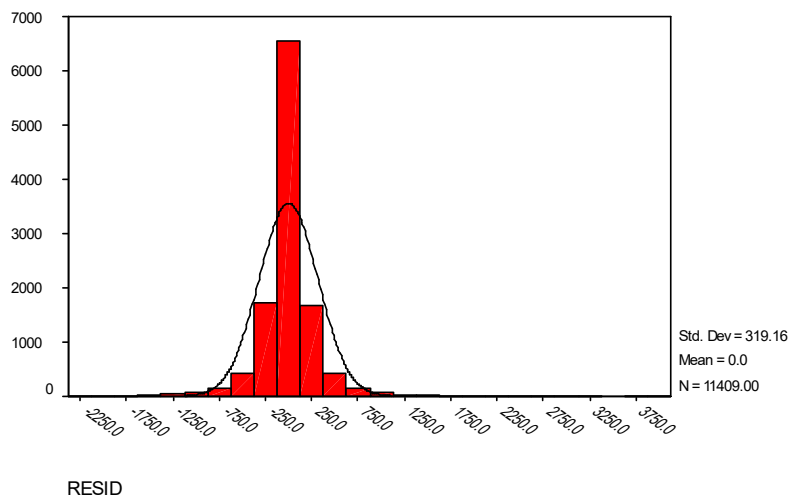


Figure 2a Histogram plot for residual of Uniformity trial

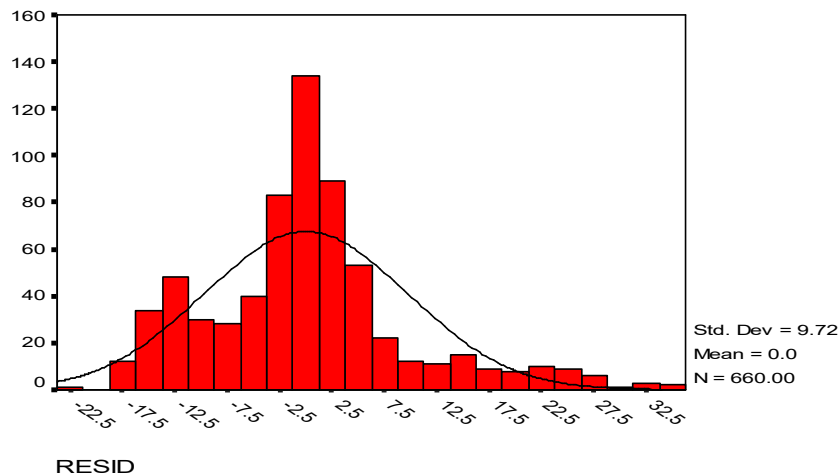


Figure 2b Histogram plot for residual of Multi-location trial data.

If the line of normal curve is almost symmetric around the mean of the residual, then there is an indication of the satisfaction of the normality assumption. However, if the normal curve is tailed to either left or right, the assumption of normality is failed. The plotted histogram of wheat data for Uniformity trial (Figure 2a) shows that a considerable number of observations are on the center of the plot where as multi-location trial data (Figure 2b) shows that more observations are on the right of the center than on center of the plot and shows the existence of skewness. On the other hand, both figures show very peaked curve. Such very peaked curve, which is known as leptokurtosis, is not the same as that of mesokurtic curve, which is the bell-shaped (normal) distribution. Hence, to stabilize these we have to transform the response variable.

The standardized (or studentized) residuals are also useful tool in detecting departures of the error term from the normality. A plot of residuals e_i (or the scaled residuals d_i or r_i

where $d_i = \frac{e_i}{\sqrt{MSE}}$ and $r_i = \frac{e_i}{\sqrt{MSE \left[1 - \left(\frac{1}{n} + \frac{(X_i - \bar{X})^2}{S_{xx}} \right)^2 \right]}}$) against the corresponding fitted

values \hat{y} (predicted value \hat{y}) is useful for detecting several common types of model inadequacy. Generally if the plot of residual (standardized residuals) versus the predict values lie within ± 2 horizontal band, and then there are no model deficit or defects. In general, if the errors are normally distributed, then approximately 68 percent of the standardized residuals should be fall between -1 and +1, approximately 95 percent of them should fall between -2 and +2, and approximately 99 percent of them should fall between -3 and 3. Substantial deviation from these limits indicates potential violation of the normality assumption. Opposite to this, if scatter plot of standardized residual versus predicted value lie outside the specified horizontal band with the large number of observation, then we can say that there is a model deficiency (Douglas *et al*, 1991).

The scatter plot of the data are shown below

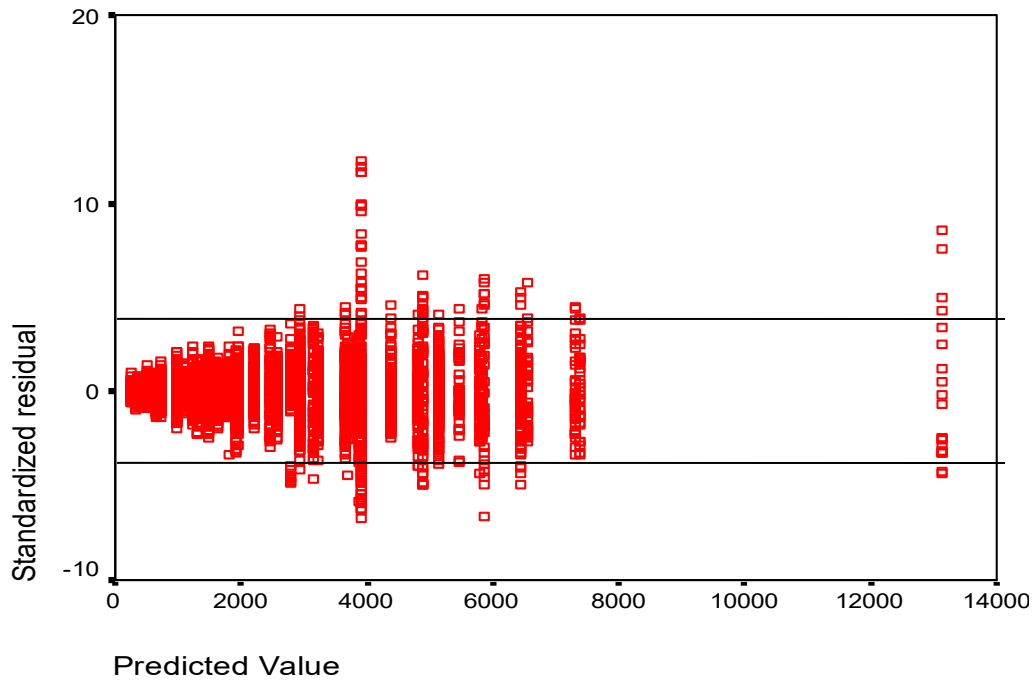


Figure 3a Scatter plot Standardized Residual versus predicted value for uniformity trials

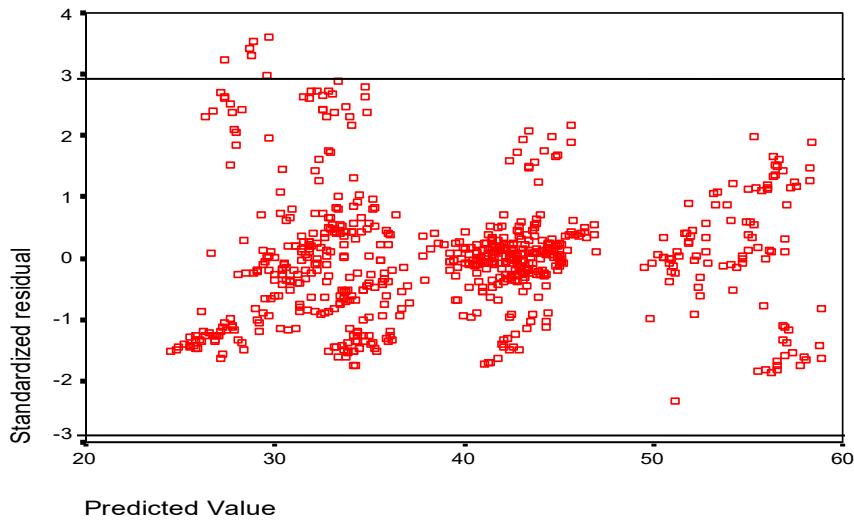


Figure 3b Scatter plot of Standardized residual Versus Predicted value for Multi-location trial.

As shown above in figure 3a, a plot of standardized residuals against the predicted value for uniformity trial data shows that much of the observations are above the horizontal line. And hence they are not contained in the suggested horizontal bands. In case of multi-location, almost all observations are contained within the indicated horizontal band.

A line plot of standardized residuals versus cases of observations, when the number of observation is considerably large, is also helpful for obtaining summery observation about the symmetry of residuals and possible outliers. Symmetry of the plots about a horizontal line at zero conforms that the mean of the error term is zero. But this type of plot is only possible for a maximum of 3000 observations. Since the number of observations for uniformity trial is 11409 it is impossible to use this testing mechanism. Hence, only plot for multi-location trail is given below.

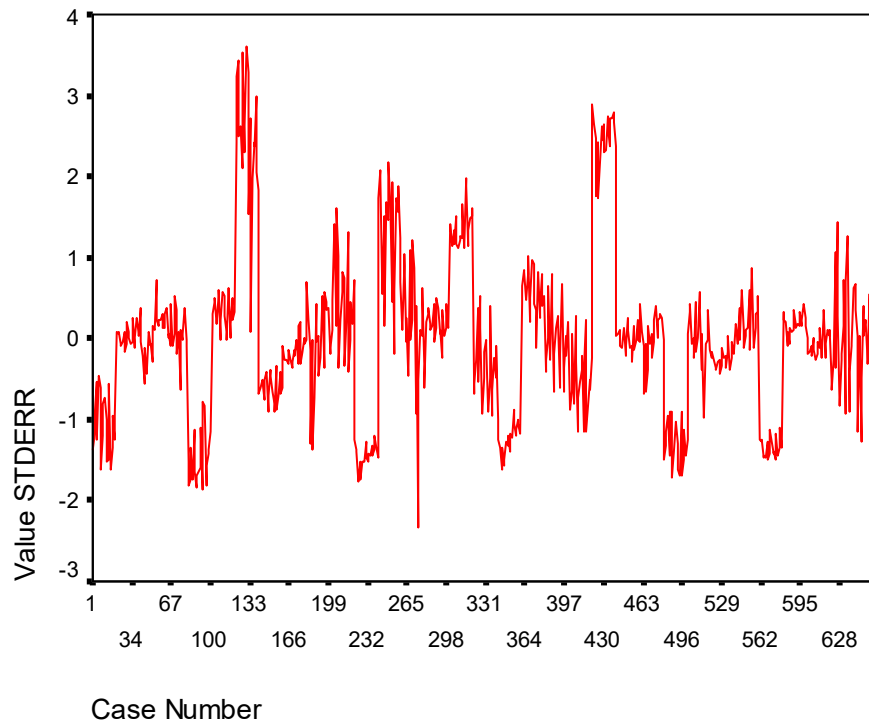


Figure 4 Plot of Standardized residual versus case number for multi-location trail.

As shows in figure 4, more observations lie above a horizontal line passing through zero. Hence, the multi-location data do not conform to the symmetry property which is an

indication of zero mean of the error term and constant variance. A more general test using SAS for normality of the two data are given in Appendix B Summary 1 and 2.

From Appendix B Summary 1 and 2 model diagnostic results, we can conclude that the fitted model does not fulfill the normality and constant variance assumption for the error term. Hence, it requires appropriate transformation on the response variable. Unequal error variance and non normality of error terms frequently appear together. The major solution to these departures from the normality is transforming the response variable, since the shape and the distribution of the response variable needs to be changed. Then the model can be refitted using the transformed response variable for further analysis.

4.2. Variance Stabilizing Transformations

The assumption of constant variance is a basic requirement for model analysis. A common reason for the violation of this assumption is the response variable to follow a probability distribution in which the variance is functionally related to mean. For example, if the response variable is Poisson random variable, then the variance of response variable is equal to the mean. Since the mean of the response variable is related to the regressor variable, the variance of dependent variable will be proportional to regressor variable; variance-stabilizing transformations are often useful in these cases. Thus if the distribution of response variable is Poisson random variable, we regress the square root of the response variable against regressor variable since the variance of the square root of a Poisson random variable is independent of the mean. Another example, if the response variable is a proportion ($0 \leq \text{response variable} \leq 1$) and the plot of the residuals versus fitted response variable has double-bow pattern then the arcsine transformation is appropriate (Douglas *et al*, 1991.)

The above model diagnostic results such as the PP-plot, the residual versus predicted scatter plot, normal curve and univariate normality test procedures suggest Square root transformation for uniformity trail. Because the scatter plot of standardized residual versus predicted in Figure 3a shows that the residual and predicted values are proportional. This means as predicted value increases the value of standardized residual also increases. Regarding multi-location trial the scatter plot of standardized residual

versus predicted in Figure 3b shows a mega phonic shape. This means on the left side of the graph the scattered point covers large area but as one moves to right the area decreased compared to the left side. Hence to stabilize variance component and to make the scattered point randomly, logarithmic transformation for multi-location trial of the response variable is appropriate.

4.3. Model Diagnostic for Transformed Model

Model diagnostics for transformed response variable is given below.

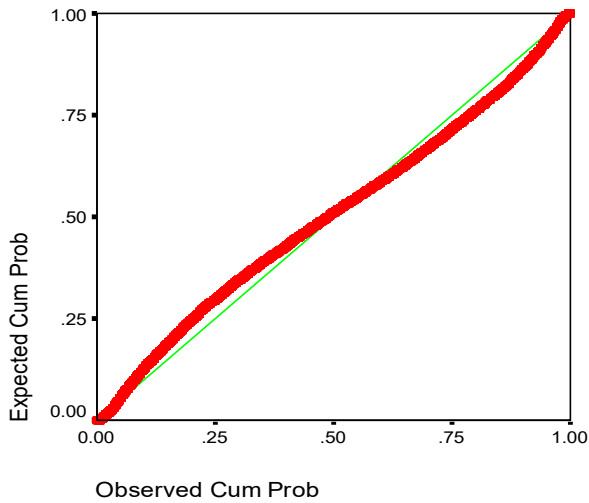


Figure 5a PP-plot of the residual of uniformity trail after Square root transformation

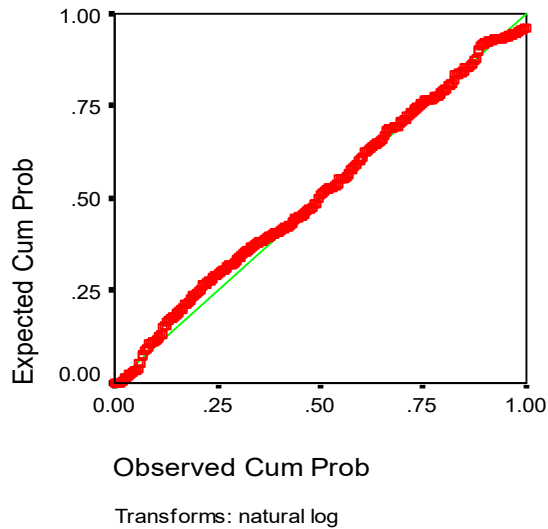


Figure 5b PP-plot of the residual of multi-location trail after natural logarithmic transformation.

The PP-plot (figure 5a and 5b) of the transformed model shows nearly linear line and this tells us that the distribution of the error term and hence the response variable is normal. To strengthen this conclusion we also plot the normal histogram curve of the transformed model as shown below for both data:

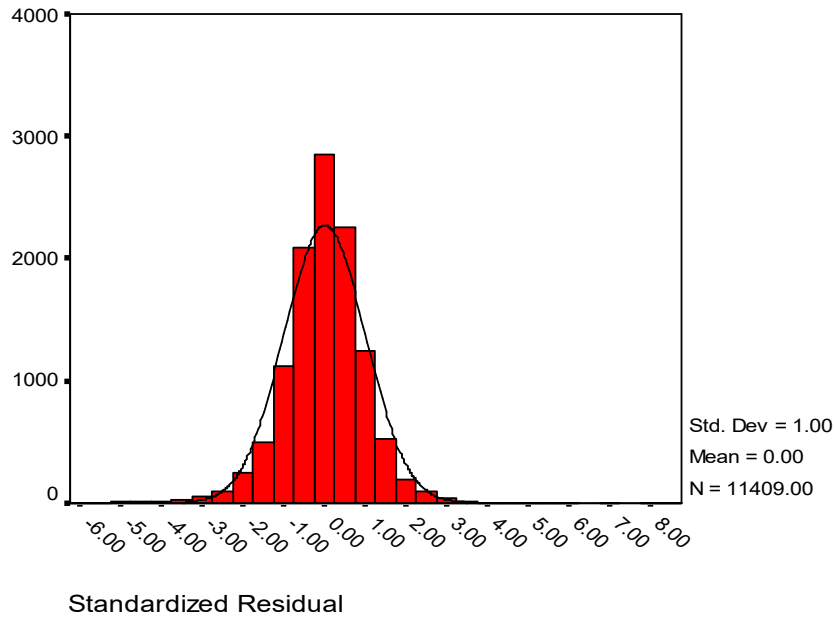


Figure 6a Histogram plot for residual of Uniformity trial after Square root transformation

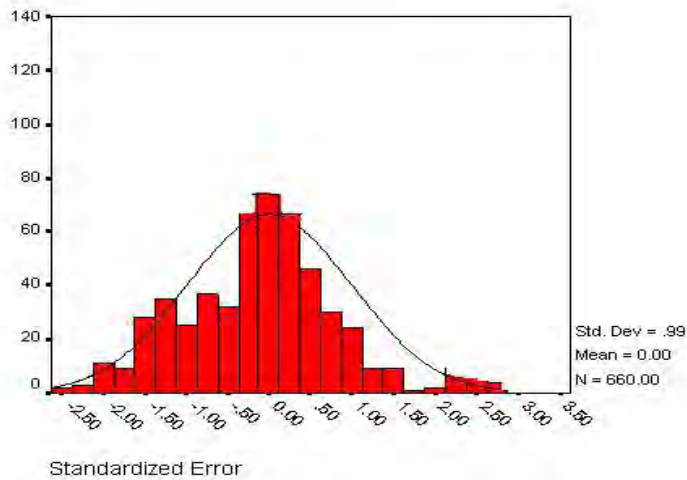


Figure 6b Histogram plot for residual of Multi-location trial after natural logarithmic transformation

In addition to PP-plot, the histogram (figure 6a and 6b) of the transformed model shown above also show that almost all of the residual points are contained within the normal curve line. This also confirms the validity of the normality assumptions.

An alternative test for normality, as discussed above, is the scatter plot of standardized residual versus predicted value which helps to identify whether there is a gross departure from normality assumption or not. To confirm this, the scatter plots of the transformed model of the two data are given below:

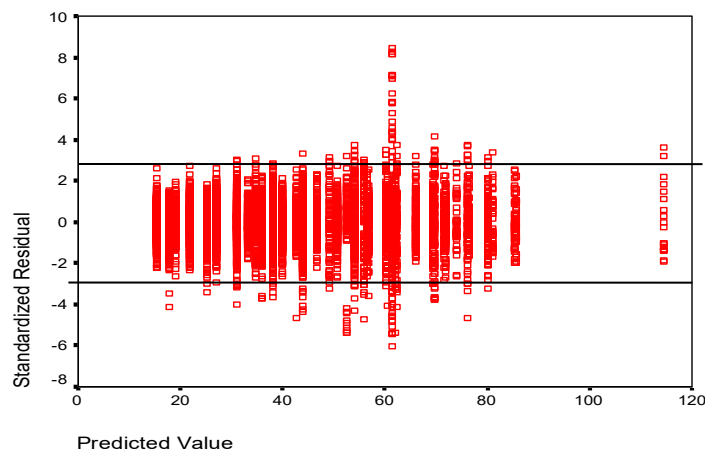


Figure 7a Scatter plot of Standardized Residuals versus predicted values for uniformity trials after Square root transformation.

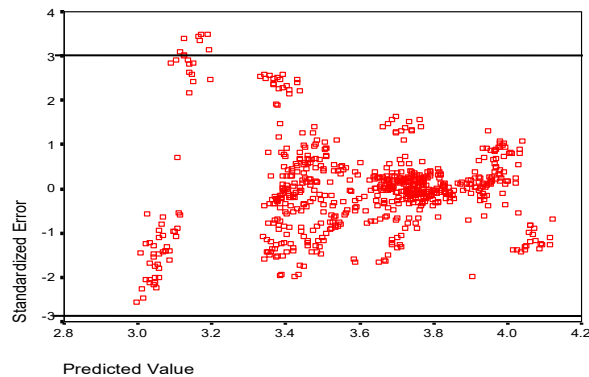


Figure 7b Scatter plot of Standardized Residuals versus predicted values for Multi-location trials after Natural logarithmic transformation.

The plot of standardized residuals against the predicted value for the transformed model for both data shows that the observations are randomly distributed around a horizontal line passing through zero and contained in the suggested horizontal bands. From this we realize that the variance of the error term is constant.

The case plot, as shown in figure 8 below, of the transformed model for Multi-location trail data approximately exhibits symmetry property indicating that the mean of the error term is zero and the constant variance. As given above the plot for uniformity trial is not given here since the number of observation is beyond the maximum number that the software can plot. As a result, only plot of multi-location trial data is given below.

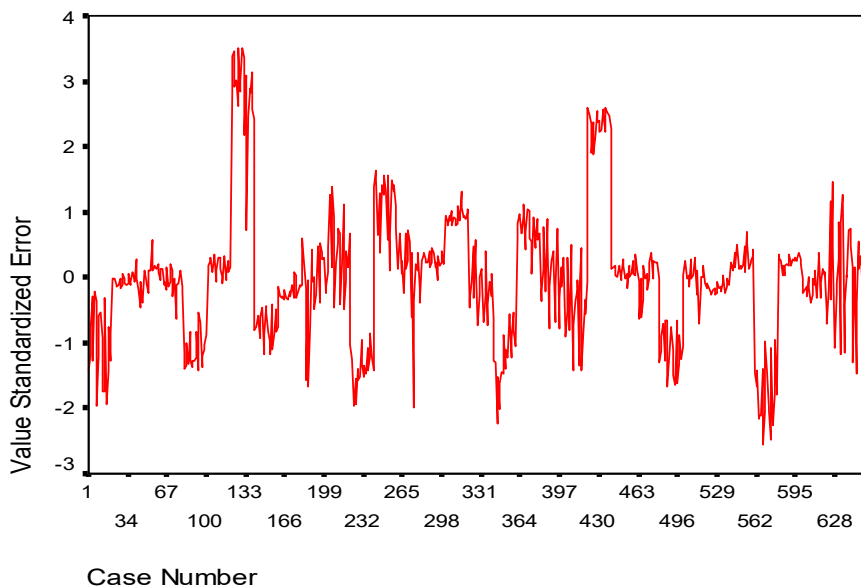


Figure 8: Plot of Standardized residual versus case number for multi-location trail after Natural logarithmic transformation.

To more realize this, the SAS procedure which checks the normality assumption is given in Appendix B, Summary 3 and 4 for transformed model. From these test we conclude that both the central location and test of normality i.e. Constant variance, independence and linearity are fulfilled.

From the result of model diagnostics, we can say that the transformed model is more appropriate than original data to apply t- and F statistics, and confidence and prediction intervals of the parameter. Based on this fact, the transformed model is employed for the analyses of both uniformity and multi-location trials data. After estimates of parameters are obtained for transformed model, we apply square and exponent transformation to uniformity and multi-location trials, respectively, to have original data for analysis and discussion.

Chapter Five: Result and Discussion

5.1. Introduction

This chapter provides outputs (results) and discussion of a restricted/residual likelihood (REML)-based approach to general linear mixed models. This approach simplifies and unifies many common statistical analyses, including those involving repeated measures, random effects, and random coefficients. This data analysis is concerned with mixed model effect analysis. The basic assumption in this data analysis is that the response variable is linearly related to unobserved multivariate normal random variables.

For this data analysis, SAS software was implemented as major software among others. As an additional one I have also used SPSS in constructing PP-plot, Histogram and Scatter plots. GENSTAT was used only in calculating AI matrix.

Although a generalized linear model given by $y = X\beta + \varepsilon$ where y denotes the vector of observed y_i 's, X is the known matrix of x_{ij} 's, β is the unknown fixed-effects parameter vector, and ε is the unobserved vector of independent and identically distributed Gaussian random errors it is certainly a useful one (Searle 1971), and it is the one fitted by the GLM procedure. However, many times the distributional assumption about ε is too restrictive. The mixed model extends the general linear model by allowing a more flexible specification of the covariance matrix of ε . In other words, it allows for both correlation and heterogeneous variances, although we still assume normality.

5.2. Estimation of Variance-Covariance parameter

For the mixed model given by equation (3.0) a key assumption in the foregoing analysis is that u and ε are normally distributed with zero mean vector and covariance matrix of G and R given by 3.25 in chapter three. The estimates of G and R are calculated using REML method as discussed in the previous chapter. Since $Cov(u_i, u_j)=0$ for $i \neq j$ and $Cov(u, \varepsilon)=0$, the estimates of G and R are a diagonal matrix. As a result, the estimate of G and R are $389.54I$ and $10.3168I$ where I 's are an identity matrix of 73×73 and

11409x11409 respectively. Therefore, the estimates of the variance component in the model are:

$$Var \begin{bmatrix} u \\ \varepsilon \end{bmatrix} = \begin{bmatrix} G & 0 \\ 0 & R \end{bmatrix} = \begin{bmatrix} 389.54I & 0 \\ 0 & 10.3168I \end{bmatrix} \text{ for uniformity trial.}$$

From this matrix we observed that plot size and shape is more variable than the random error term. If we include plot size and shape as fixed effect in the model we will have the variance of the random error term only. Since the variance of the response variable, yield, is based on the variance of the random effects in the model i.e. the error term, the estimates based on this variance will result in over estimation. Moreover, the confidence interval, F- and t- test will be unreal. But, the inclusion of the random effect increases the variance and hence will balance this unreal test effect.

For multi-location trial the estimates of G is not as that of uniformity trial since there are two random effects in the model one for variety and the other for locations in which the trial is conducted. Therefore, the estimate of G is a diagonal matrix having 0.002004 on main diagonal for the first 19 elements and 0.06170 on the main diagonal for the last 14 elements. The estimate of the variance of the residual term, R, is 0.002004I. Hence, the estimates of the variance of the random effects including residual terms are given as follows:

$$Var \begin{bmatrix} u_1 \\ u_2 \\ \varepsilon \end{bmatrix} = \begin{bmatrix} 0.002004I & 0 & 0 \\ 0 & 0.06170I & 0 \\ 0 & 0 & 0.07977I \end{bmatrix}$$

Where the I's on the main diagonal along with numbers are the identity matrix of 19x19, 14x14 and 660x660 dimensions.

Even though the variance of the random effects i.e. variety and location, seem very small they play a significant role in controlling the over estimation and down ward bias of the variance of the response variable since the variance of the response variable is the sum of

the variances of all random effects in the model instead of the variance of the random error terms.

Since mixed model equations are extended normal equations, the solutions to these normal equation expressions assume that \hat{G} is nonsingular. Since \hat{G} is non singular, the inverse of \hat{G} exists and hence we obtain unique eigen values of \hat{G} . These eigen values indicate the contribution of each random variable of the model.

For the extreme cases, when the eigen values of \hat{G} are very large, \hat{G}^{-1} contributes very little to the equations and \hat{u} is close to what it would be if u actually contained with fixed-effects parameters in the model. On the other hand, when the eigen values of \hat{G} are very small, \hat{G}^{-1} dominates the equations and \hat{u} is close to 0. For intermediate cases, \hat{G}^{-1} can be viewed as shrinking the random-effect estimates of u towards 0 (Robinson 1991).

From the two matrices given above, \hat{G} is a diagonal matrix in both data sets. Hence, the inverse of \hat{G} is simply the inverse of those elements on the main diagonal. As a result, the inverse of \hat{G} for uniformity trial data is $0.00257\mathbf{I}$ where \mathbf{I} is an identity matrix of 73x73; and that of multi-location trial is \mathbf{BI} where \mathbf{B} is a column vector containing 499 for the first 19 elements and 14.93 for the last 14 elements, and \mathbf{I} is an identity matrix of 34x34. Beside this, the eigen values of \hat{G} are the same as that of the elements of the matrix of \hat{G} on the main diagonal since the solution of the characteristic equation given by $|\hat{G} - \lambda\mathbf{I}| = 0$ gives the same results as that of \hat{G} elements. Hence the values of all λ 's are 389.54 for uniformity trial, 0.002004 for the first 19 elements and 0.06170 for the last 14 elements of the multi-location trial.

From these values we conclude that the inclusion of the random effect, \hat{u} , plays a significant role in explaining the variance covariance effects in case of uniformity trial. But for multi-location trial data \hat{G}^{-1} dominates the equations because the value of \hat{G}^{-1} is approximately 499 and 15 and therefore \hat{u} is close to 0. Especially the variety components of the mixed model have very small eigen values that dominates the equation where as the location part is viewed as a shrinking effect of the random component of

the model due to the fixed effect since the value of eigen values for those factors are the intermediate one. According to Robinson 1991, a random variable whose eigen value is in the range of 10 to 100 is considered as intermediate.

5.2.1. Advantage of using Newton-Raphson Algorithm In Variance Estimation

One advantage of using the Newton-Raphson algorithm is that the second derivative matrix of the objective function evaluated at the optima is available upon completion. Denoting this matrix \mathbf{H} , which is the Hessian matrix, the asymptotic theory of maximum likelihood (Serfling 1980) shows that $2\mathbf{H}^{-1}$ is an asymptotic variance-covariance matrix of the estimated parameters of \mathbf{G} and \mathbf{R} . Thus, tests and confidence intervals based on asymptotic normality can be obtained. However, these can be unreliable in small samples, especially for parameters such as variance components which have sampling distributions that tend to be skewed to the right (SAS manual, 1999). Hence we can not obtain a reliable variance component for multi-location trial unless it is stabilized by appropriate transformation since the graph for multi-location trial is skewed to the right (figure 2b). But the transformed model satisfies the required assumptions. As a result we can estimate different variance components based on this transformed model.

Next to this, we are going to estimate Fisher information, Hessian and Average information matrices. The $2\mathbf{H}^{-1}$ matrix, which is the observed Fisher information matrix evaluated at the final covariance parameter estimate i.e. evaluated after six iteration in uniformity trial and four iteration in case of multi-location trial, is the asymptotic variance-covariance matrix of the estimated parameters for the two trials are given as follows:

$$F = \begin{bmatrix} 4334.52 & -6.93 \times 10^{-6} \\ -6.93 \times 10^{-6} & 0.01878 \end{bmatrix} \text{ for uniformity trial data set}$$

$$F = \begin{bmatrix} 2.169 \times 10^{-6} & 9.767 \times 10^{-9} & -6.27 \times 10^{-7} \\ 9.767 \times 10^{-9} & 0.000615 & -3.17 \times 10^{-7} \\ -6.27 \times 10^{-7} & -3.17 \times 10^{-7} & 0.000020 \end{bmatrix} \text{ for multi-location trial data}$$

set. Since these matrices are Fishers information matrices, we have to find the Hessian matrices by finding the inverse of half of Fisher information matrix. Because obtaining Hessian matrices allow us to calculate Average Information (AI) matrix (Gilmour, 1995). Therefore, Hessian matrices for these data sets are given as follows:

$$H = \begin{bmatrix} 4.61 \times 10^{-4} & 1.70 \times 10^{-7} \\ 1.70 \times 10^{-7} & 106.496 \end{bmatrix} \text{ for uniformity trial}$$

$$H = \begin{bmatrix} 930517 & 0.2587 & 29172 \\ 0.2587 & 3252.1 & 51.553 \\ 29172 & 51.553 & 100915 \end{bmatrix} \text{ for multi-location trial}$$

As given by 3.20 of chapter three, since Average Information (AI) matrix is half of the sum of Fisher information matrix and Hessian matrix it is given as follows:

$$AI = \begin{bmatrix} 2167.26 & -3.38 \times 10^{-6} \\ -3.38 \times 10^{-6} & 53.2574 \end{bmatrix} \text{ for uniformity trial data set}$$

$$AI = \begin{bmatrix} 465258.5 & 0.12935 & 14586 \\ 0.12935 & 1626.05 & 25.7765 \\ 14586 & 25.7765 & 50457.5 \end{bmatrix} \text{ for multi-location trial.}$$

Since a residual variance σ_e^2 is a part of our mixed model, it can usually be profiled out of the likelihood. This means solving analytically for the optimal σ_e^2 and plugging this expression back into the likelihood formula (Wolfinger, Tobias, and Sall 1994). This reduces the number of optimization parameters by one and can improve convergence properties. The mixed model procedure, when we use SAS, profiles the residual variance out of the log likelihood whenever it appears reasonable to do so. Since we have got an

asymptotic estimator for variance components, the next step is testing the significance the estimated variances of the random effects.

5.2.2. Inference and Test Statistics of Variance estimates

For inferences concerning the variance-covariance parameters in the mixed model, it is possible to use the likelihood-based statistics. One common likelihood-based statistic is the *Wald Z*, which is computed as the parameter estimate divided by its asymptotic standard error. The asymptotic standard errors are computed from the inverse of the second derivative matrix of the likelihood with respect to each of the covariance parameters. The *Wald Z* is valid for large samples, but it can be unreliable for small data sets and for parameters such as variance components, which are known to have a skewed or bounded sampling distribution. The *Wald Z* test statistics of the two data sets are given as follows:

Table 1 Estimation and Test of Variance Parameter for uniformity trial

Covariance Parameter	Estimate	Standard Error of estimate	Z – value	Pr Z
Plot size and Shape	389.54	65.8371	5.92	<.0001
Residual	10.3168	0.1370	75.29	<.0001

Table 2: Estimation and Test of Variance Parameter for multi-location trial

Covariance Parameter	Estimate	Standard Error of estimate	Z – value	Pr Z
Variety	0.002004	0.001473	1.36	0.0868
Location	0.06170	0.02479	2.49	0.0064
Residual	0.07977	0.004510	17.69	<.0001

As shown in Table 1 the Wald Z^3 test statistics for the random effect, plot size and shape, and the residual are significant at α equals 0.001 level of significance. Whenever we observe the Wald's Z test for multi-location trial, given in Table 2, the estimates of covariance parameter for variety is not significant at α equals 0.05. But other covariance estimates such as covariance estimates of location and residual are significant at 5% α level of significance.

³ Wald tests is an asymptotic approximation i.e. it is reliable only for large samples and underestimates the probabilities in other cases.

In relation to Wald Z test statistics, a better alternative is the Chi-square likelihood ratio test, χ^2 . This statistic compares two covariance models in which one is a special case of the other. If the reduced model does not occur on the boundary of the covariance parameter space, the χ^2 statistic computed in this fashion has a large-sample sampling distribution that is χ^2 with degrees of freedom equal to the difference in the number of covariance parameters between the two models. If the reduced model does occur on the boundary of the covariance parameter space, the asymptotic distribution becomes a mixture of χ^2 distributions. This commonly exists when we are testing a variance component equals its lower boundary constraint of 0 (Self and Liang 1987). Since we are assuming Wald Z statistic is approximately normal, then Z^2 of Wald statistic is approximately Chi-square distribution with DF is 1. Therefore, the χ^2 for plot size and shape, and residual are 35.0464 and 5668.5841 respectively for uniformity trial where as for multi-location trial the χ^2 for variety, location and residual are 1.8496, 6.2001 and 312.9361 respectively. The 95th and the 99th percentile of the chi-square distribution with 1 degree of freedom are 3.84 and 5.02. Therefore, the test statistic with $\alpha=0.05$ and 0.01, for the variance of all random variable are significant except for variety effect. Both test statistics confirms that the estimate of the variance is above the lower boundary constraint of 0 except for variety effect. This test statistic ensures that whether using random effect really increases the variance of the response variable or not. If the test statistic shows that the variance is not significantly different from zero then the inclusion of the random variable is meaning less. But if the test statistic indicates that the variance is above the lower boundary constraints then the variance of the response variable also increases and this in turn reduces the over estimation, confidence interval, prediction interval and test statistics based on this variance. On top of this, unless the variance of the response variable, which is the sum of the variances of the random factor and the residual terms, that is used in determining the optimum plot sizes and shape are significant at a given level of significance will result in unreliable plot size and shape.

5.3. Application of Mixed model in the process of Optimum plot size and shape Determination

In fixed effect model i.e. RCBD, variance among plots for grain yield (GY) that was used in optimum plot size and shape determination was computed as the variance among plots for each specific plot area directly from grain yield obtained. From this value, the variance per basic unit area was determined by the formula $V_x = V(x)/X^2$. where V_x is variance per basic unit area, $V(x)$ is variance among plot area, and X is specific plot area. Then the comparable variance, V , was calculated as: $V = V(x)/X$ (Girma et. al, 2000).

However in the case of mixed model, the variance among plots for grain yield was the sum of the variance of the random factor i.e. plot size and shape, and the random error terms. This implies that the variance among plot areas were determined from random factors that represent the population and the unavoidable disturbance term of the model.

Hence, the basic difference between the two methods of calculating the variance among plots is that the accountability of all variability that was assumed to be included in the model in case of mixed model analysis. This is to mean that the variance of fixed effect is less than the variance of mixed model for the response variable. As a result, the comparable variance calculated from the variance per basic unit area was minimum. Hence, to balance this mixed model is appropriate. In addition to this, the result obtained was also limited to the sample under consideration in case of fixed effect where as the conclusion obtained from mixed model was extended to whole population from where the sample was taken.

5.4. Model Selection

The previous section on estimation assumes the specification of a mixed model in terms of \mathbf{X} , \mathbf{Z} , \mathbf{G} , and \mathbf{R} . Even though \mathbf{X} and \mathbf{Z} have known elements, their specific form and construction is flexible, and several possibilities may present themselves for a particular data set. Likewise, several different covariance structures for \mathbf{G} and \mathbf{R} might be reasonable. First, subject matter consideration and objectives are of great importance when we are going to select the appropriate model (Diggle 1988, Lindsey 1993).

Second, when the graphical methods and the diagnostics results of the general linear model is examined, it is the data itself that extend to the mixed model setting as well (Christensen, Pearson, and Johnson 1992).

Finally, a likelihood-based approach to the mixed model provides several statistical measures for model adequacy as well. The most common of these are the likelihood ratio test and Akaike's and Schwarz's criteria (Bozdogan 1987; Wolfinger 1993).

When we analyze uniformity trial data, it is difficult to consider all possible plot size and shape constructed by taking different dimensions. Because there are a number of plots areas constructed by using different dimensions. Hence considering plot size and shape as a random sample taken from population simplifies the problem. Moreover, it is a sample that is taken randomly represents the population under investigation. Similarly, in multi-location trial there are a number of variety and different location for the trial. As a result considering all location and variety as fixed effect is very difficult. Therefore, taking random sample that represent the domain simplifies the problem. Hence, we conclude that, it is the data themselves that guide us to use mixed model instead of fixed effect of generalized linear model.

There are so many statistical procedures which allow us to compare one model with the other model. Among these the most and well known model comparing procedures are given below.

Akaike's Information Criterion (AIC): is used to compare models with the same fixed effects but different variance structures; the model having the smallest AIC is deemed best (Akaike 1974).

Schwarz's Bayesian Criterion (BIC): compares two models and select models with smaller Schwarz's Bayesian Criterion (BIC). Even though model with smaller BIC is preferred, BIC penalizes models with a greater number of covariance parameters more than AIC does, and the two criteria may not agree as to which covariance model is best (Schwarz 1978).

Restricted Log Likelihood: is used to identify whether the random effect of the model has a significant effect or not. In order to apply this, we have to re-run the model but dropping out the random effect and then examine the difference between the -2ResLog likelihood. This difference is a chi-square distribution with degree of freedom equal to 1. If this calculated value is greater than the tabulated value at the specified α level of significance, then we conclude that random effect plays a significant role in explaining the model.

Therefore, to test the significance of random effects, the fit statistics for both data sets are given in Appendix D. Hence, the value of the Fit Statistics for uniformity trial by making land shape and area as random effect for -2 Res log Likelihood, AIC, AICC and BIC are 59591.1, 59595.1, 59595.1 and 59599.6 respectively. But the value of Fit Statistics for uniformity trial after ignoring the random effect for -2 Res log Likelihood, AIC, AICC and BIC are 97649.3, 97651.3, 97651.3 and 97658.6 respectively.

Since the smaller the value of AIC, AICC and BIC, the better the model is, model that include plot size and shape as random factor shows the small the values of AIC, AICC and BIC than model fitted by ignoring plot size and shape as random effect. Therefore, random effect has a significant effect in explaining the model. Moreover, the difference between -2Res Log Likelihood of the models that ignore plot size and shape and that include this factor as a random, is 38058.2. Thus, we have a χ^2 of 38058.2 for 1 degree of freedom. The critical tabulated value for a χ^2 with 1 d.f. and a probability of 5% is 3.84

and that 1% is 6.63. Hence, we conclude that the effect of plot size and shape is quite significant and should be maintained in the model; therefore, model that includes plot size and shape as random factor would be an appropriate model.

Regarding multi-location trial the Fit Statistics by making Variety and location as random effect for -2 Res log Likelihood, AIC, AICC and BIC are 277.1, 283.1, 283.2 and 286.0 respectively. Consider this as the first fit. The value of Fit Statistics for multi-location trial after ignoring variety as random effect for -2 Res log Likelihood, AIC, AICC and BIC are 320.8, 324.8, 324.8 and 326.0 respectively. This is considered as second fit. The fit statistics for multi-location trial after ignoring location as random effect -2 Res log Likelihood, AIC, AICC and BIC are 621.1, 623.1, 623.1 and 624.1 respectively. Consider this as third fit. Finally, the Fit Statistics by ignoring both variety and location effects for -2 Res log Likelihood, AIC, AICC and BIC are 664.9, 666.9, 666.9 and 671.4 respectively. This is the fourth fit.

When we compare the values of AIC, AICC and BIC for the three fits, the first model has the smallest value among the three fit statistics. Therefore, both random effects; variety and locations, have a significant effect in explaining the model even though the coefficients of varieties are not significantly different from zero. Moreover, when we compare the -2 Res log likelihood the differences between the first and the second model, the third and the second models, and the third and the first models, the results are 43.7, 344.1 and 387.8 respectively. Thus, the calculated values of the χ^2 for these differences are 43.7, 344.1 and 387.8 respectively. The critical tabulated value as given above for a χ^2 with 1 d.f. and a probability of 5% is 3.84 and that of 1% is 6.63. Thus, we conclude that the effect of variety and location is quite significant and should be maintained in the model; therefore the first analysis is the one that we should use.

5.5. Comparative advantage of Mixed model over Fixed Effect model

As it is generally accepted, general linear model (GLM), be it is a regression or a classification, does not include the random effect except the random error terms of the model. Hence, there is no variance due to the random effect other than the variance of

random error terms when calculating the variances (standard errors) of the response variable, estimates of the parameters, test statistics and constructing confidence interval. As a result all variances expected from the model, if any, are included in the variance of the disturbance term inappropriately since all factors in GLM is a fixed effects which do not endure the inclusion random effects. In such kinds of models the variance of the error terms boost very much. This illustrated that the danger of miss using the statistical models and coming up with irrelevant conclusion.

However, there are random factors that included in the mixed model that contributed its lion share in estimating the variance of the response variable. The inclusion of the random effects in the model removes the down ward bias of variance of the response variable and the boosted variance of the error terms of GLM.

To lay out the advantage of mixed model over GLM, we use the SAS outputs given in Appendix D. From summary 3 and 8 of this Appendix, we observe that the standard error of uniformity trial for the GLM is 5.211969 and that of multi-location is 0.5282395. But, the standard error for uniformity trial from mixed model is 3.21198 and that of multi-location trial is 0.2825. The ratio of the standard errors of the GLM to that of mixed model are 1.62 and 1.87 for uniformity and multi-location trials respectively. From this we conclude that the standard error of GLM is approximately 1.6 and 1.9 times that of mixed model for uniformity and multi-location trials respectively. These results tell us that mixed model is more advantageous than GLM.

Coefficient of variation is also used as criteria for comparing the advantage or the efficiency of the model. From the SAS outputs given at the end of Appendix D, the coefficient of variation for uniformity trial is 8.67% and 6.61% respectively for general linear model and mixed model. Similarly, the coefficient of variation for multi-location trial is 7.818% and 1.70% when we use general linear model and mixed model respectively. As a principle, the small the coefficient of variation the better the model is. Therefore, from these results we can conclude that using mixed model in field trial allows the researchers to estimate variation better than the fixed effect model.

5.6. Estimation and Statistical Properties of β and u in Mixed Model

The maximum likelihood (ML), Restricted/residual maximum likelihood (REML), or Type I to Type III SS provide estimates of \mathbf{G} and \mathbf{R} , which are denoted $\hat{\mathbf{G}}$ and $\hat{\mathbf{R}}$, respectively. As described in chapter three, to obtain estimates of β and u , the standard method is to solve the normal equation of mixed model equations.

To obtain the solution for fixed effect in case of multi-location trial we use the generalized inverse of $X'\hat{V}^{-1}X$ since the matrix is not full column rank. Because, the sum of the last three columns of the design matrix for fixed effect will give the first column, which represents the intercept. As a result, the true inverse does not exist. Hence, we are obligated to use generalized inverse of the design matrix

Based on the discussion of chapter three, if \mathbf{G} and \mathbf{R} are known, $\hat{\beta}$ is the *best linear unbiased estimator* (BLUE) of β , and \hat{u} is the *best linear unbiased predictor* (BLUP) of u (Searle 1971; Harville 1988, 1990; Robinson 1991; McLean, Sanders, and Stroup 1991). Here, "best" means minimum mean squared error. However, \mathbf{G} and \mathbf{R} are usually unknown and are estimated using one of the aforementioned methods. These estimates, $\hat{\mathbf{G}}$ and $\hat{\mathbf{R}}$, are therefore simply substituted into the approximate variance-covariance matrix of $(\hat{\beta} - \beta, \hat{u} - u)$. In this case, the BLUE and BLUP acronyms no longer apply, but the word *empirical* is often added to indicate such an approximation. The appropriate acronyms thus become EBLUE and EBLUP (McLean and Sanders, 1988). Therefore EBLUE of uniformity and multi-location trial are given as follows:

Table 3: Solution for Fixed Effects of uniformity trial

Effect	Estimate	Standard Error of estimate	Degree of Freedom	t- value	Pr > t
Year 1996	45.9326	4.0224	11E3	11.42	<.0001
Year 1997	52.8266	4.0224	11E3	13.13	<.0001
Row Sown	4.9040	4.6250	11E3	1.06	0.2890
Broadcast sown	0

Table 4: Solution for Fixed Effects of multi-location trial

Effect	Estimate	Standard Error of estimate	Degree of Freedom	t- value	Pr > t
Intercept	3.5908	0.06980	13	51.45	<.0001
Year 2001	0.09299	0.02787	626	3.34	0.0009
Year 2002	0.01052	0.02903	626	0.36	0.7172
Year 2003	0

The solution to the random effect of the model for uniformity trial is given in Appendix C Table 1 where as that of multi-location data is given in table 2 and 3

The preceding tables (Table 3 and 4) list the solution vector for the fixed effects. SAS system sets zero as the estimates of broadcast sown in uniformity trial and year 2003 in multi-location trial data. However, these are not the actual estimates of those factors. In mixed model, if the model under consideration is non-intercepted model and the variable under consideration has different levels, the SAS set the estimates of each level of the variable by subtracting the estimates of the last level from the estimates of the remaining levels of the variable. But, if the model is interceptal model, the estimates of each variable is the sum of the intercept and the estimate set for specific level of the variable under consideration (SAS 1999).

As a result, the actual estimates for broadcast sown is 4.9040 since both broadcast and row sown are the levels of sown type as well as non-intercepted model. Similarly the actual estimates of year 1997 is the sum the estimates of year 1996 and year 1997. Therefore, the estimated value for year 1997 is 98.7592. In case of multi-location trial since the model has an interceptal model, the actual estimates of year 2003 is the same as that of the value of the intercept. But the estimates of year 2001 and year 2002 are $3.5908 + 0.09299 = 3.68$ and $3.5908 + 0.01052 = 3.60132$ respectively. That is why the estimates of the last level of the effects are set to zero.

5.7. Test Statistics and inferences

As it is mentioned in the preceding section, the estimates of average effect of year 1996, year 1997, row sown and broadcast sown are 45.9326, 98.7592, 4.9040 and 4.9040 respectively. This shows that the contributions of the first two factors are very high in relation to the last two factors. The inferences and test statistics concerning the fixed part of uniformity trial is that the coefficients of year 1996 and year 1997 are significant at

$\alpha=0.01$ level of significance, but the coefficients for both row and broadcast sown are not significantly different from zero at α -level of 0.05. On the other hand, Type III SS, which is the marginal sum of squares, tests of Fixed Effects for uniformity trial shows no significant difference at α -level of 0.05. This indicates that the average effects of the fixed effect are the same.

Similarly, the estimates of the year 2003 and the intercept of the model is 3.5908 while that of year 2001 and year 2002 are $3.5908 + 0.09299=3.68379$ and $3.5908 + 0.09299 + 0.01052 = 3.69431$ respectively. Since model for multi-location trial is interceptal model, the estimates of each effect of the variable in the model is calculated as a difference from the last effect in mixed models. This indicates that the first effect of the model take large contribution in estimating the model. Due to this fact, the intercept of the model take large average effects in estimating the model in multi-location trial.

On top of this, the t-test statistics for multi-location trial given in table 4 shows that the intercept of the model and the coefficient of year 2001 are significantly different from zero for α -level greater than 0.01. Since the estimated values of year 2001 and 2003 are 3.68379 and 3.69431, which are greater than the estimated value of the intercept i.e. 3.5908, both these factors are as significant as that of the intercept.

The estimated coefficients of random effects, plot size and shape, the t-test statistic and the prediction of the coefficient of random-effects and coefficient of variation for uniformity trial and multi-location trial were given in Appendix C, Table 1, 2, and 3 respectively. From table 1 of Appendix C we observe that all the coefficients are significantly different from zero expect for: 2x4, 2x10, 3x2, 6x1 and 6x4 m² of year 1996 sown by row method; 2x3, 2x4, 6x1 m² of year 1996 sown by broadcast method; and 1x8, 2x4, 5x2 and 10x1 m² of 1997 sown in both methods. Especially the coefficients of plot sizes having small values sown in both methods in both year shows the most significant difference from zero. However, small plot sizes consistently shows higher degrees of variability than larger plot sizes as indicated by the coefficients of variations given in Appendix C Table 1. The coefficient of variation decreased markedly for both row and broadcast sown trial as plot sizes increases linearly. Especially, for plot size increasing from 1m² to 5m², the coefficient of variation decreases from 21.17% to 14.57% for the

year 1996 sown by row method. Besides this, if plot sizes increases from 1m^2 to 24m^2 , the coefficient of variation decreases from 21.17% to 7.42%.

Similarly, for broadcast sown in 1996 the coefficient of variation decreases from 30.82% to 23.87% as plot size increases from 1m^2 to 5m^2 . Moreover, if the plot size increases from 1m^2 to 20m^2 , the coefficient of variation decreases from 30.82% to 19.86%. This shows that the maximum the size of plot gives the minimum coefficient of variation.

Regarding the 1997 trials, the coefficient of variation decreases at the rate of 19.25% to 11.52 % as plot size increases from 1m^2 to 8m^2 for row sown. If we continue increasing plot size from 1m^2 to 20m^2 , the coefficient of variation markedly decreases from 19.25% to 8.76%. These results also confirm that the minimum coefficient of variation is obtained at the maximum plot size. For trial sown by broadcast method in 1997, the coefficient of variation ranges between 9.22% and 18.57% for plot sizes ranging between 1m^2 and 20m^2 . The highest CV was obtained at the smallest plot size and vice versa.

When we compare the variability of the two sowing methods, i.e. broadcast and row sown, which have the same plot size in the same year i.e. 1996, the variability of broadcast method have higher coefficient of variations than row sown method as shown in Appendix C Table 1. Even though the variability for broadcast sown method has higher coefficient variation, the difference between coefficient of variation for plot sizes ranging from 1×1 to 1×5 are 6.60% and 6.35% for row and broadcast sown respectively. Hence the rate of change of coefficient of variation as plot size increases is approximately equal.

Regarding the 1997 trials, the range of coefficient of variation for row and broadcast methods are 10.49 and 9.35 respectively. This shows that both methods have approximately the same maximum and minimum values of coefficient of variation. This implies that the differences are not significantly apart from each other. This shows that methods have relatively similar variations.

Generally the above mentioned results show that plot sizes and coefficient of variation are inversely related to each other. Hence, we have to construct line graph using CV

versus plot size in order to find optimum plot at a considerable coefficient of variations. According to Gomez and Gomez (1984) optimum plot size is obtained at the turning point of the graph. Graph of CV versus Plot size for the two methods, row and broadcast sown, are given in Appendix F. From these graphs, the optimum is found approximately at 10m² and 8m² for row sown and broadcast sown respectively. These plot sizes are from plot shape of 2x5 for row sown and 2x4 for broadcast sown methods.

The empirically best linear unbiased predictor (EBLUP) of multi-location trial data given in Appendix C Table 2 and 3 shows that all coefficients for varieties are not significant at 5% level of significance. This test statistics tests the overall mean plus average effect of each variety. Therefore, there is no average effect due to variety and hence all varieties have the same effect on the response variable. This implies that all varieties are contributing the same average effects for the yield obtained. More over the test statistics confirms that the local check is as important as other varieties.

But in the case of location, the coefficients for Bako, Bekoji, Debre Zeit, Ginchi, Holeta, Kulumsa, and Sinana are significantly different from zero at α 5% level of significance. The significance of the test indicates that the average productivity potential of those locations is higher than that of the remaining location. For the remaining location, the coefficients are not significant at the normal level of significance. In general the contribution of each location to yield is not the same unlike that of variety. Hence, the average effect of location on the response variable is not the same.

Regarding the interaction effect among variables in multi-location data, the SAS output is given in Appendix E Summary 1. The result shows that effect, year*variety and year*location are significant. But, the genotype and environment interaction effect is not significant. This tells us that the overall interaction of genotype and environment is not significant even though individual interaction of genotype and environments are rarely significant. As a result, combined analysis of variance which identifies the significance is not necessary to evaluate genotype environment interaction effects.

Chapter Six: Conclusions and recommendations

6.1. Conclusion

The estimation of parameters in mixed model requires the specification of \mathbf{X} , \mathbf{Z} , \mathbf{G} , and \mathbf{R} even though \mathbf{X} and \mathbf{Z} have their own specific form and known elements. This is because the knowledge of the structure of \mathbf{G} and \mathbf{R} will help the estimation of the variance of the random factors in the model. There are several covariance structures for \mathbf{G} and \mathbf{R} that might occur based on the specified model, subject matter under consideration and objectives stated to undertake analysis. The most widely used and the forward structure of \mathbf{G} and \mathbf{R} are the one whose matrix is diagonal and off diagonals elements are zero.

Since mixed model equations are extended normal equations, the solutions to these normal equation expressions are assumed to be nonsingular. As a result, it is more useful to use the Newton-Raphson algorithm, which is the second derivative matrix of the objective function evaluated at the optimal point up on completion. Because, it simplifies the test and confidence interval calculations that were based on asymptotic normality for normally distributed large sample sizes.

Moreover, the convergence criteria of the model are the primary point that was raised in mixed model. Since parameter estimation and test statistics of the model are based on these convergence criteria, we need the residual variance that fulfills these properties. In addition, a residual variance, σ^2 , is parts of the mixed model which is usually profiled out of the likelihood and solved analytically for the optimal σ^2 and then plug this expression back into the likelihood formula. Because this reduces the number of optimization parameters by one and can improve convergence properties and, also used to obtain an asymptotic estimator for variance components that helps one in estimating and testing the unknown coefficients of the model.

The contribution of each random factor in the model is determined by the corresponding eigen values of the variance-covariance of random variables. The larger the eigen values of the random variable the larger the contribution of that random variable. The eigen

values of plot size and shape in uniformity trial is 389.54. This shows that plot size and shape, which is used as random factor, have very large contribution in explaining the response variable i.e. the yield. Similarly, the eigen values of varieties and location in multi-location trial are 0.002004 and 0.067. This shows that the contribution of both variety and location are very low in relation to that of plot size and shape in uniformity trial. However, these factors have a lion share in explaining the response variable when used as random factors than fixed effect factors. Because the standard error of the response variable is 0.06496 when we use varieties and location as random factors but the standard error is 0.282395 when these factors are used as fixed effects. This shows that the standard error of the response variable i.e. yield, when we use these factors as fixed effect model is more than 4 times when we use them as random effect. The variability of the response variable is larger in fixed effect model than mixed effect. This is because: 1) fixed effect models do not properly account for field variability; 2) in the fixed effect model, the levels of the independent variables that occur in the study are considered to be the only level of interest and have a fixed effect on the response variable; and 3) in a fixed effect model, the estimation of the parameter is done using ordinary least squares (OLS) technique since estimation of parameters using OLS do not include variance components corresponding to the different sources of randomness. On the other hand, the estimation of parameters in mixed model effect is undertaken by restricted maximum likelihood criteria which accounts for different sources of randomness to get estimates of the parameters efficiently.

For the uniformity trial, small plot size for any shape consistently exhibited higher degrees of variability than larger plots. The coefficient of variation decreased markedly for both row and broadcast sown trials as plot size increased. However, the rate of change was more pronounced for row sown than for broadcast sown trials. Variability was higher for broadcast sown than for row sown trials for the larger plot sizes in both years for all plot sizes. In the broadcast sown trials, the coefficient of variation could have been increased due to: 1) lack of uniformity in plant density within plots which amplified crop geometry effects; and 2) uneven distribution of broadcast fertilizer which introduced heterogeneity and affected crop performance within plots.

Soil heterogeneity index (Smith's index) has a lion share in optimum plot size and shape determination. However, the coefficients soil heterogeneity indices which indicate the degree of heterogeneity within a trial is influenced not only by soil variability and plot orientation, but also by crop geometry resulting from the sowing method used. Since soil heterogeneity is randomly occurred, plot size and shape that was determined from such experiment should be added in the model as random effects. Hence using plot size and shape obtained from mixed model in field trial is reasonably acceptable than those obtained from fixed effects as it includes all variabilities. Therefore, plot size and shape obtained from mixed model is 10m² and 8m² for row sown and broadcast sown respectively. These plot sizes are from plot shape of 2x5 for row sown and 2x4 for broadcast sown methods respectively.

In multi-location trial, the average effect of each variety is the same and hence all varieties have the same effect on the response variable. This implies that all varieties are contributing the same average effects for the yield obtained. Unlike that of variety, there are differences among locations. Especially trials conducted at Bako, Bekoji, Debre Zeit, Ginchi, Holeta, Kulumsa, and Sinana are significantly different from trial conducted the remaining locations. This indicates that the average productivity potential of those locations is higher than that of the other locations. All the remaining locations such as Arsi Negele, Arsi Robe, Adet, Alemaya, Asasa, Hosana and Kokate, have productivity potential unless there are external factors that influence them. In general the contribution of each location to yield is not the same unlike that of variety. Hence, the average effect of location on the response variable is not the same.

6.2. Recommendations

Based upon the major findings of this study, the researcher would like to recommend the following major points to the problems addressed by the study. The recommendations include measures to be taken at researchers' level, at center level and/or Institutes of Agriculture Research level.

- Agricultural researchers, both breeders and agronomists should give due attention to appropriate statistical design and analysis that properly account field variability, i.e. mixed model in uniformity and multi-location trials.
- Both breeders and agronomists should use the optimum plot size determined using mixed model, to be a cost effective in agricultural research which use limited research resources to attain an acceptable level of experimental precision with minimum cost.
- All national and regional variety trial activities conducted in different location should use mixed model, which considers location as random factor for recommendation instead of fixed effect model. Because in fixed effect model, the recommendation for adaptation of the given variety is only for the specific location under consideration but in the case of mixed model the recommendation domain will apply to all locations under the same agro-ecological zones, if location is considered as random factor.
- All researchers should try their level best to serve the community with commitment and professional ethics regarding research methodology. They should teach both technical assistant and field assistant on various techniques of statistical designs, gross and net plot size determination, data collection, and data entry. The institute should also give the necessary technical and logistic support to researchers. Various up-dating trainings, field trial schemes for research outputs, information on farm technology innovations should be facilitated for the researchers to have quality data which gives best result.
- Sufficient biometricians that are equipped by sufficient training and advisory services should be extended to all centers in which the research is undertaken.

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Appendix A

Table 1: The T test statistics for row sown trial 1996 on plot dimensions of 1X2 and 2X1

variable	Plot	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err
Yield	1x2	480	633.59	643.89	654.19	108.01	114.84	122.61	5.2418
Yield	2x1	480	633.92	643.89	653.85	104.5	111.11	118.62	5.0714
difference	1x2-2x1		-14.31	0	14.313	108.15	112.99	118.29	7.2935

T-Tests

Variable	Method	Variances	DF	t Value	Pr > t
Yield	Pooled	Equal	958	0.00	1.0000
Yield	Satterthwaite	Unequal	957	0.00	1.0000

Equality of Variances

Variable	Method	Num DF	Den DF	F Value	Pr > F
Yield1	Folded F	479	479	1.07	0.4699

Table 2: The T test statistics for row sown trial 1996 on plot dimensions of 1X4 and 4X1

variable	Plot	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err
Yield	1x4	240	1264.8	1287.8	1310.7	165.55	180.37	198.13	11.643
Yield	4x1	240	1264.3	1287.8	1311.3	169.67	184.86	203.07	11.933
difference	1x4-4x1		-32.76	0	32.759	171.75	182.63	194.99	16.672

T-Tests

Variable	Method	Variances	DF	t Value	Pr > t
Yield	Pooled	Equal	478	0.00	1.0000
Yield	Satterthwaite	Unequal	478	0.00	1.0000

Equality of Variances

Variable	Method	Num DF	Den DF	F Value	Pr > F
Yield	Folded F	239	239	1.05	0.7041

Table 3: The T test statistics for row sown trial 1996 on plot dimensions of 2X4 and 4X2

variable	Plot	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err
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Yield	2x4	120	2527.4	2577.8	2628.2	247.62	179.02	319.59	25.47
Yield	4x2	120	2517.9	2575.5	2633.2	282.87	318.72	365.08	29.095
difference	2x4-4x2		-73.93	2.25	78.427	274.87	299.09	329.09	38.669

T-Tests

Variable	Method	Variiances	DF	t Value	Pr > t
Yield	Pooled	Equal	238	0.06	0.9536
Yield	Satterthwaite	Unequal	234	0.06	0.9537

variable	Plot	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err
Yield	X12	360	475.45	489.51	503.56	126.39	135.62	146.33	7.15
Yield	X21	360	476.06	489.51	502.95	120.88	129.72	139.96	6.84
difference	X12-X21		-19.42	0	19.42	126.18	132.7	139.94	9.89

Equality of Variances

Variable	Method	Num DF	Den DF	F Value	Pr > F
Yield	Folded F	119	119	1.30	0.1481

Table 4: The T test statistics for Broadcast sown trial in 1996 on plot dimensions of 1x2 and 2x1

T-Tests

Variable	Method	Variiances	DF	t Value	Pr > t
Yield	Pooled	Equal	718	0.00	1.0000
Yield	Satterthwaite	Unequal	717	0.00	1.0000

Equality of Variances

Variable	Method	Num DF	Den DF	F Value	Pr > F
Yield	Folded F	359	359	1.09	0.3993

Table 5: The T test statistics for Broadcast sown trial in 1996 on plot dimensions of 1x3 and 3x1

variable	plot	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err
Yield	1x3	240	710.17	734.26	758.34	173.83	189.4	208.05	12.226
Yield	3x1	240	711.4	734.26	757.11	164.97	169.75	197.44	11.602
difference	1x3-3x1		-33.12	0	33.118	173.64	184.63	197.13	16.855

T-Tests

Variable	Method	Variiances	DF	t Value	Pr > t
Yield	Pooled	Equal	478	0.00	1.0000
Yield	Satterthwaite	Unequal	477	0.00	1.0000

Equality of Variances

Variable	Method	Num DF	Den DF	F Value	Pr > F
Yield	Folded F	239	239	1.11	0.4193

Table 6: The T test statistics for Broadcast sown trial 1996 on plot dimensions of 1x4 and 4x1

variable	plot	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err
Yield	1x4	180	938.81	975.29	1011.8	224.79	248.04	276.7	18.488
Yield	4x1	180	944.88	979.01	1013.1	210.33	232.08	258.89	17.298
difference	1x4-4x1		-53.51	-3.717	46.075	223.83	240.19	259.19	25.319

T-Tests

Variable	Method	Variiances	DF	t Value	Pr > t
Yield	Pooled	Equal	358	-0.15	0.8834
Yield	Satterthwaite	Unequal	356	-0.15	0.8834

Equality of Variances

Variable	Method	Num DF	Den DF	F Value	Pr > F
Yield	Folded F	179	179	1.14	0.3744

Table 7: The T test statistics for Broadcast sown trial in 1996 on plot dimensions of 1x6 and 6x1

variable	plot	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err
Yield	1x6	120	1409.9	1468.5	1527.2	287.92	324.42	371.6	29.615
Yield	6x1	120	1406.8	1468.5	1530.2	302.9	341.3	390.94	31.157
difference	1x6-6x1		-84.68	0	84.68	305.55	332.97	365.83	42.986

T-Tests

Variable	Method	Variiances	DF	t Value	Pr > t
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Yield Pooled Equal 238 0.00 1.0000
 Yield Satterthwaite Unequal 237 0.00 1.0000

Equality of Variances

Variable Method Num DF Den DF F Value Pr > F
 Yield Folded F 119 119 1.11 0.5808

Table 8: The T test statistics for Broadcast sown trial in 1996 on plot dimensions of 2x3 and 3x2

variable	plot	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err
Yield	2x3	120	1407.3	1468.5	1529.8	300.8	338.93	388.22	30.94
Yield	3x2	120	1408.1	1468.5	1529	296.82	334.45	383.09	30.531
difference	2x3-3x2		-85.63	0	85.63	308.98	336.7	369.93	43.468

T-Tests

Variable Method Variances DF t Value Pr > |t|
 Yield Pooled Equal 238 0.00 1.0000
 Yield Satterthwaite Unequal 238 0.00 1.0000

Equality of Variances

Variable Method Num DF Den DF F Value Pr > F
 Yield Folded F 119 119 1.03 0.8848

Table 9: The T test statistics for Broadcast sown trial in 1996 on plot dimensions of 2x3 and 3x2

variable	plot	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err
Yield	2x4	90	1866.1	1958	2050	383	439.11	514.64	46.286
Yield	4x2	90	1867.3	1958	2048.8	377.91	433.27	507.8	45.671
difference	2x4-4x2		-128.3	0	128.3	395.21	436.2	486.75	65.025

T-Tests

Variable Method Variances DF t Value Pr > |t|
 Yield Pooled Equal 178 0.00 1.0000

variable	Plot	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err
Yield	1x2	360	632.33	643.14	653.96	97.232	104.34	112.57	5.499
Yield	2x1	360	632.992	643.14	653.36	91.885	98.599	106.38	5.1966
difference	1x2-2x1		-14.85	0	14.854	96.52	101.51	107.05	7.566

Yield Satterthwaite Unequal 178 0.00 1.0000

Equality of Variances

Variable Method Num DF Den DF F Value Pr > F
 Yield Folded F 89 89 1.03 0.8998

variable	plot	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err
Yield	1x2	360	718.08	729.61	741.14	103.67	111.24	120.02	5.8629
Yield	2x1	360	722.17	736.99	715.82	120.81	130.49	141.86	7.5336
difference	1x2-2x1		-25.86	-7.382	11.094	114.2	120.37	127.24	9.4096

Table 10: The T test statistics for row sown trial 1997 on plot dimensions of 1x2 and 2x1

T-Tests

Variable Method Variances DF t Value Pr > |t|
 Yield1 Pooled Equal 658 -0.78 0.4330
 Yield1 Satterthwaite Unequal 590 -0.77 0.4396

Equality of Variances

Variable Method Num DF Den DF F Value Pr > F
 Yield1 Folded F 299 359 1.38 0.0038

Table 11: The T test statistics for Broadcast sown trial in 1997 on plot dimensions of 1x2 and 2x1

T-Tests

Variable Method Variances DF t Value Pr > |t|
 yield2 Pooled Equal 718 0.00 1.0000

yield2 Satterthwaite Unequal 716 0.00 1.0000

Equality of Variances

Variable	Method	Num DF	Den DF	F Value	Pr > F
yield2	Folded F	359	359	1.12	0.2843

Appendix B

Summary 1: The UNIVARIATE Procedure for Uniformity trial Data

Variable: Residual

Moments

N	11409	Sum Weights	11409
Mean	0	Sum Observations	0
Std Deviation	319.157	Variance	101861.281
Skewness	1.0658	Kurtosis	18.2018132
Uncorrected SS	1162033492	Corrected SS	1162033492
Coeff Variation	∞	Std Error Mean	2.98800126

Basic Statistical Measures

Location

Mean	0.0000
Median	-1.7074
Mode	-13.9673

Variability

Std Deviation	319.15714
Variance	101861
Range	6065
Interquartile Range	199.55265

NOTE: The mode displayed is the smallest of 2 modes with a count of 13.

Tests for Location: $\mu_e=0$

Test	-Statistic-	-----p Value-----
Student's t	t =0	Pr > t 1.0000
Sign	M=-51.5	Pr >= M 0.3396
Signed Rank	S =-404402	Pr >= S 0.2504

Tests for Normality

<u>Test</u>	<u>Statistic</u>	<u>p Value</u>
Kolmogorov-Smirnov	D 0.136253	Pr > D <0.0100
Cramer-von Mises	W-Sq 95.36105	Pr > W-Sq <0.0050
Anderson-Darling	A-Sq 505.8778	Pr > A-Sq <0.0050

Summary 2: The UNIVARIATE Procedure for multi-location trial
Variable: Residual

Moments

N	660	Sum Weights	660
Mean	0	Sum Observations	0
Std Deviation	9.72051155	Variance	94.488344
Skewness	0.79543371	Kurtosis	1.1885363
Uncorrected SS	62267.8193	Corrected SS	62267.8193
Coeff Variation	∞	Std Error Mean	0.3783704

Basic Statistical Measures

<u>Location</u>		<u>Variability</u>	
Mean	0.00000	Std Deviation	9.72051
Median	-0.25423	Variance	94.48834
Mode	∞	Range	57.68056
		Interquartile Range	8.97529

Tests for Location: $\mu_e = 0$

<u>Test</u>	<u>Statistic</u>	<u>p Value</u>
Student's t	t = 0	Pr > t 1.0000
Sign	M=-12	Pr >= M 0.3707
Signed Rank	S =-6684	Pr >= S 0.1727

Tests for Normality

<u>Test</u>	<u>Statistic</u>	<u>p Value</u>
Shapiro-Wilk	W=0.941523	Pr < W <0.0001
Kolmogorov-Smirnov	D=0.120138	Pr > D <0.0100
Cramer-von Mises	W-Sq=2.040222	Pr > W-Sq <0.0050
Anderson-Darling	A-Sq=11.85788	Pr > A-Sq <0.0050

Summary 3: The UNIVARIATE Procedure
Variable: standardized error Uniformity trial

Moments

N	11409	Sum Weights	11409
Mean	0	Sum Observations	0
Std Deviation	1.00000009	Variance	1.00000018
Skewness	0.00039512	Kurtosis	0.00000002
Uncorrected SS	11408.0021	Corrected SS	11408.0021
Coefficient of Variation	∞	Std Error Mean	0.00936216

Basic Statistical Measures

<u>Location</u>		<u>Variability</u>	
Mean	0.00000	Std Deviation	1.00000
Median	0.00693	Variance	1.00000
Mode	-0.00755	Range	14.52311
		Interquartile Range	1.10405

Tests for Location: $\mu_e=0$

<u>Test</u>	<u>-Statistic-</u>	<u>p Value</u>
Student's t t	0	Pr > t 1.0000
Sign M	131.5	Pr >= M 0.1142
Signed Rank S	577574.5	Pr >= S 0.1007

Tests for Normal

Test	Statistic	p Value
Kolmogorov-Smirnov D	0.050134	Pr > D 0.1000
Cramer-von Mises W-Sq	10.96606	Pr > W-Sq 0.5000
Anderson-Darling A-Sq	68.24961	Pr > A-Sq 0.5000

Summary 4: The UNIVARIATE Procedure

Variable: Standardized error of Multi-location

Moments

N	660	Sum Weights	660
Mean	0	Sum Observations	0
Std Deviation	1.00000045	Variance	1.0000009
Skewness	0.0005692	Kurtosis	0.00004783
Uncorrected SS	659.000596	Corrected SS	659.00059
Coefficient of Variation	∞	Std Error Mean	0.03892496

Basic Statistical Measures

<u>Location</u>		<u>Variability</u>	
Mean	0.00000	Std Deviation	1.00000
Median	0.00001	Variance	1.00000
Mode		Range	6.10184
		Interquartile Range	0.84095

Tests for Location: $\mu_e = 0$

Test		-Statistic	p Value
Student's t	t	0	Pr > t 1.0000
Sign	M	-3	Pr >= M 0.8457
Signed Rank	S	-4255	Pr >= S 0.3856

Tests for Normality

<u>Test</u>		<u>Statistic</u>	<u>p Value</u>
Shapiro-Wilk	W	0.938416	Pr < W 0.1035
Kolmogorov-Smirnov	D	0.120932	Pr > D 0.2103
Cramer-von Mises	W-Sq	2.324172	Pr > W-Sq 0.3520
Anderson-Darling	A-Sq	13.16368	Pr > A-Sq 0.7509

Appendix C

Appendix C: Table 1 Plot shape, Area, Estimate, Standard Error, t-Value and P-value of EBLUP for uniformity trial

Plot shape	Area in meter Square	Var	Estimate	Standard Error of Precision	Degree of Freedom	t-Value	Pr> t	CV in Percentage
1x1	1	1	-33.0064	4.0988	1.10E+03	-8.05	<0.0001**	21.17
1x2	2	2	-25.5707	4.1	1.10E+03	-6.24	<0.0001**	17.84
1x4	4	3	-15.0399	4.1024	1.10E+03	-3.67	0.0002**	14.01
1x5	5	4	-10.8148	4.1036	1.10E+03	-2.64	0.0084**	13.96
2x2	4	5	-15.0525	4.1024	1.10E+03	-3.67	0.0002**	14.57
2x4	8	6	-0.1395	4.1072	1.10E+03	-0.03	0.9729	10.82
2x5	10	7	20.891	4.1167	1.10E+03	5.07	<0.0001**	8.62
2x8	16	8	29.3081	4.1215	1.10E+03	7.11	<0.0001**	11.08
2x10	20	9	5.8031	4.1095	1.10E+03	1.41	0.1579	9.11
3x1	3	10	-19.8618	4.1012	1.10E+03	-4.84	<0.0001**	15.73
3x2	6	11	-6.9775	4.1048	1.10E+03	-1.7	0.0892	13.46
3x4	12	12	11.2385	4.1119	1.10E+03	2.73	0.0063**	9.92
3x5	15	13	18.5581	4.1155	1.10E+03	4.51	<0.0001**	10.06
4x4	16	14	20.8866	4.1167	1.10E+03	5.07	<0.0001**	8.77
6x1	6	15	-6.9842	4.1048	1.10E+03	-1.7	0.0889	12.88
6x2	12	16	11.2109	4.1119	1.10E+03	2.73	0.0064**	11.08
6x4	24	17	3.0657	4.1167	1.10E+03	0.74	0.4565	7.42
1x1	1	18	-30.4873	4.0241	1.10E+03	-7.58	<0.0001**	30.82
1x2	2	19	-24.0252	4.0257	1.10E+03	-5.97	<0.0001**	27.71
1x4	4	20	-14.8829	4.029	1.10E+03	-3.71	0.0002**	25.43
1x5	5	21	-11.1995	4.0306	1.10E+03	-2.78	0.0055**	23.87
2x2	4	22	-14.8829	4.029	1.10E+03	-3.69	0.0002**	24.47
2x3	6	23	-7.8734	4.0322	1.10E+03	-1.95	0.0509	23.08
2x4	8	24	-1.9634	4.0355	1.10E+03	-0.49	0.6266	22.43
2x5	10	25	3.2521	4.0387	1.10E+03	0.81	0.4207	21.53
2x10	20	26	23.6596	4.055	1.10E+03	5.83	<0.0001**	20.31
3x1	3	27	-19.041	4.0273	1.10E+03	-4.73	<0.0001**	24.48
3x4	12	28	7.9614	4.042	1.10E+03	1.97	0.0489*	21.01
3x5	15	29	14.348	4.0469	1.10E+03	3.55	0.0004**	20.21
4x4	16	30	16.2968	4.0485	1.10E+03	4.03	<0.0001**	20.82

4x5	20	31	23.6712	4.055	1.10E+03	5.84	<0.0001**	19.86
6x1	6	32	-7.846	4.0322	1.10E+03	-1.95	0.0517	22.09
6x2	12	33	9.8224	4.0371	1.10E+03	2.43	0.015*	20.86
6x4	24	34	30.323	4.0614	1.10E+03	7.47	<0.0001**	19.65
12x1	12	35	15.4215	4.0355	1.10E+03	3.82	0.0001**	20.66
1x1	1	36	-38.7195	3.8818	1.10E+03	-9.97	<0.0001**	19.25
1x2	2	37	-30.7941	3.8835	1.10E+03	-7.93	<0.0001**	15.25
1x3	3	38	-24.7219	3.8852	1.10E+03	-6.36	<0.0001**	13.69
1x4	4	39	-19.5994	3.8869	1.10E+03	-5.04	<0.0001**	12.42
1x6	6	40	-11.0187	3.8903	1.10E+03	-2.83	0.0046**	11.52
1x8	8	41	-5.3613	3.8937	1.10E+03	-1.38	0.1686	20.12
1x12	12	42	8.3413	3.9004	1.10E+03	2.14	0.0324*	10.36
1x18	18	43	23.1881	3.9106	1.10E+03	5.93	<0.0001**	10.21
2x2	4	44	-19.615	3.8869	1.10E+03	-5.05	<0.0001**	13.67
2x3	6	45	-11.0312	3.8903	1.10E+03	-2.84	0.0046**	12.43
2x4	8	46	-3.7976	3.8937	1.10E+03	-0.98	0.3294	11.13
2x6	12	47	8.3413	3.9004	1.10E+03	2.14	0.0325*	10.53
2x8	16	48	18.562	3.9207	1.10E+03	4.75	<0.0001**	9.48
2x12	24	49	56.6709	3.9408	1.10E+03	14.38	<0.0001**	9.48
5x1	5	50	-15.1325	3.8886	1.10E+03	-3.89	0.0001**	14.98
5x2	10	51	2.5714	3.897	1.10E+03	0.66	0.5094	12.05
5x3	15	52	16.1373	3.9055	1.10E+03	4.13	<0.0001**	10.85
5x4	20	53	27.5786	3.914	1.10E+03	7.05	<0.0001**	9.67
10x1	10	54	2.868	3.9004	1.10E+03	0.74	0.4622	11.59
10x2	20	55	28.0134	3.9207	1.10E+03	7.15	<0.0001**	8.76
1x1	1	56	-34.957	4.0241	1.10E+03	-8.69	<0.0001**	18.57
1x2	2	57	-27.5493	4.0257	1.10E+03	-6.84	<0.0001**	16.22
1x3	3	58	-21.8498	4.0273	1.10E+03	-5.43	<0.0001**	14.71
1x4	4	59	-17.042	4.029	1.10E+03	-4.23	<0.0001**	13.73
1x6	6	60	-8.9945	4.0322	1.10E+03	-2.23	0.0259*	13.07
1x12	12	61	9.1992	4.042	1.10E+03	2.28	0.0229*	10.81
1x18	18	62	23.1581	4.0517	1.10E+03	5.72	<0.0001**	9.22
2x2	4	63	-17.042	4.029	1.10E+03	-4.23	<0.0001**	13.49
2x3	6	64	-8.9823	4.0322	1.10E+03	-2.23	0.0259*	12.20
2x4	8	65	-2.1836	4.0355	1.10E+03	-0.54	0.5884	11.85
2x6	12	66	9.1883	4.042	1.10E+03	2.27	0.023*	11.32
2x12	24	67	18.8117	4.0485	1.10E+03	4.65	<0.0001**	10.42
5x1	5	68	-12.8082	4.0306	1.10E+03	-3.18	0.0015**	12.75
5x2	10	69	3.7847	4.0387	1.10E+03	0.94	0.3487	11.57

5x3	15	70	16.5178	4.0469	1.10E+03	4.08	<0.0001**	10.71
5x4	20	71	27.2412	4.055	1.10E+03	6.72	<0.0001**	10.53
10x1	10	72	3.7867	4.0387	1.10E+03	0.94	0.3485	11.44
10x2	20	73	27.2381	4.055	1.10E+03	6.72	<0.0001**	10.62

Appendix C: Table 2; Estimate, Standard Error, t-Value and P-value of EBLUP for variety effect multi-location trial

Effect	Variety name	Estimate	Standard Error of Prediction	Degree Of Freedom	t-Value	Pr> t
Variety	ENKOY/ISRAEL	0.02529	0.0338	626	0.75	0.4546
Variety	ET12D4/HAR 604	0.04632	0.0338	626	1.37	0.171
Variety	ET12D4/HAR 604(2)	0.0235	0.0338	626	0.7	0.4872
Variety	HAR 1685/K6295-4A	-0.0063	0.0338	626	-0.19	0.8521
Variety	HAR 416/DERESELIG	-0.00511	0.0338	626	-0.15	0.8799
Variety	HAR 710/DERESELIG	-0.03822	0.0338	626	-1.13	0.2587
Variety	HAR 710/HAR 719	0.001449	0.0338	626	0.04	0.9658
Variety	HAR3442	-0.05345	0.0338	626	-1.58	0.1143
Variety	HAR3603	-0.01806	0.0338	626	-0.53	0.5934
Variety	HAR3643	0.0528	0.0338	626	1.56	0.1188
Variety	HAR3646	-0.03021	0.0338	626	-0.89	0.3718
Variety	HAR3658	-0.0414	0.0338	626	-1.22	0.221
Variety	K6295-4A/HAR 424(1)	0.04773	0.0338	626	1.41	0.1584
Variety	L.CHECK	0.005588	0.0338	626	0.17	0.8688
Variety	MAMBA/HAR 1384	-0.00553	0.0338	626	-0.16	0.87
Variety	MILAN/SHA7	0.01803	0.0338	626	0.53	0.5939
Variety	RBC/HAR 800	-0.01018	0.0338	626	-0.3	0.7634
Variety	RBC/HAR 921	-0.01947	0.02903	626	-0.67	0.5026
Variety	SIMBA	0.007227	0.0338	626	0.21	0.8308

Appendix C: Table 3; Estimate, Standard Error, t-Value and P-value of EBLUP for location effect multi-location trial

Effect	Location	Estimate	Standard Error of Prediction	Degree Of Freedom	t-Value	Pr> t
Location	A. Negele	0.08894	0.07901	626	1.13	0.2607
Location	A.Robe	0.1156	0.07908	626	1.46	0.1444
Location	Adet	0.08259	0.07912	626	1.04	0.297
Location	Alemaya	-0.08986	0.07901	626	-1.14	0.2558
Location	Asasa	0.09435	0.07901	626	1.19	0.2329
Location	Bako	-0.1996	0.07514	626	-2.66	0.0081**
Location	Bekoji	0.3411	0.07908	626	4.31	<0.0001**
Location	D. Zeit	-0.1948	0.07908	626	-2.46	0.0140*
Location	Ginchi	0.384	0.07514	626	5.11	<0.0001**
Location	Hollota	0.1651	0.07514	626	2.2	0.0283*
Location	Hosana	0.1032	0.07901	626	1.31	0.1921
Location	Kokate	-0.144	0.07912	626	-1.82	0.0693
Location	Kulumsa	-0.2065	0.07514	626	-2.75	0.0062**
Location	Sinana	-0.5401	0.07514	626	-7.19	<0.0001**

* Significant at 5%

** Significant at 1%

Appendix C: Table 4 values of Z in design matrix for uniformity trial

Z	Values of Z in design matrix
1	1 if data is from 1x1m ² plot size sown in 1996 by row method 0 otherwise
2	1 if data is from 1x2m ² plot size sown in 1996 by row method 0 otherwise
3	1 if data is from 1x4m ² plot size sown in 1996 by row method 0 otherwise
4	1 if data is from 1x5m ² plot size sown in 1996 by row method 0 otherwise
5	1 if data is from 2x2m ² plot size sown in 1996 by row method 0 otherwise
6	1 if data is from 2x4m ² plot size sown in 1996 by row method 0 otherwise
7	1 if data is from 2x4m ² plot size sown in 1996 by row method 0 otherwise
8	1 if data is from 2x8m ² plot size sown in 1996 by row method 0 otherwise
9	1 if data is from 2x10m ² plot size sown in 1996 by row method 0 otherwise
10	1 if data is from 3x1m ² plot size sown in 1996 by row method 0 otherwise
11	1 if data is from 3x2m ² plot size sown in 1996 by row method 0 otherwise
12	1 if data is from 3x4m ² plot size sown in 1996 by row method 0 otherwise
13	1 if data is from 3x5m ² plot size sown in 1996 by row method 0 otherwise
14	1 if data is from 4x4m ² plot size sown in 1996 by row method 0 otherwise
15	1 if data is from 6x1m ² plot size sown in 1996 by row method 0 otherwise
16	1 if data is from 6x2m ² plot size sown in 1996 by row method 0 otherwise
17	1 if data is from 6x4m ² plot size sown in 1996 by row method 0 otherwise
18	1 if data is from 1x1m ² plot size sown in 1996 by broadcast method 0 otherwise
19	1 if data is from 1x2m ² plot size sown in 1996 by broadcast method 0 otherwise
20	1 if data is from 1x4m ² plot size sown in 1996 by broadcast method 0 otherwise
21	1 if data is from 1x5m ² plot size sown in 1996 by broadcast method 0 otherwise
22	1 if data is from 2x2m ² plot size sown in 1996 by broadcast method 0 otherwise
23	1 if data is from 2x3m ² plot size sown in 1996 by broadcast method 0 otherwise
24	1 if data is from 2x4m ² plot size sown in 1996 by broadcast method 0 otherwise
25	1 if data is from 2x5m ² plot size sown in 1996 by broadcast method 0 otherwise
26	1 if data is from 2x10 m ² plot size sown in 1996 by broadcast method 0 otherwise
27	1 if data is from 3x1m ² plot size sown in 1996 by broadcast method 0 otherwise
28	1 if data is from 3x4m ² plot size sown in 1996 by broadcast method 0 otherwise
29	1 if data is from 3x5m ² plot size sown in 1996 by broadcast method 0 otherwise
30	1 if data is from 4x4m ² plot size sown in 1996 by broadcast method 0 otherwise
31	1 if data is from 4x5m ² plot size sown in 1996 by broadcast method 0 otherwise
32	1 if data is from 6x1m ² plot size sown in 1996 by broadcast method 0 otherwise
33	1 if data is from 6x2m ² plot size sown in 1996 by broadcast method 0 otherwise
34	1 if data is from 6x4m ² plot size sown in 1996 by broadcast method 0 otherwise

35	1 if data is from 12x1m ² plot size sown in 1996 by broadcast method 0 otherwise
36	1 if data is from 1x1m ² plot size sown in 1997 by row method 0 otherwise
37	1 if data is from 1x2m ² plot size sown in 1997 by row method 0 otherwise
38	1 if data is from 1x3m ² plot size sown in 1997 by row method 0 otherwise
39	1 if data is from 1x4m ² plot size sown in 1997 by row method 0 otherwise
40	1 if data is from 1x6m ² plot size sown in 1997 by row method 0 otherwise
41	1 if data is from 1x8m ² plot size sown in 1997 by row method 0 otherwise
42	1 if data is from 1x12m ² plot size sown in 1997 by row method 0 otherwise
43	1 if data is from 1x18m ² plot size sown in 1997 by row method 0 otherwise
44	1 if data is from 2x2m ² plot size sown in 1997 by row method 0 otherwise
45	1 if data is from 2x3m ² plot size sown in 1997 by row method 0 otherwise
46	1 if data is from 2x4m ² plot size sown in 1997 by row method 0 otherwise
47	1 if data is from 2x6m ² plot size sown in 1997 by row method 0 otherwise
48	1 if data is from 2x8m ² plot size sown in 1997 by row method 0 otherwise
49	1 if data is from 1x12m ² plot size sown in 1997 by row method 0 otherwise
50	1 if data is from 5x1m ² plot size sown in 1997 by row method 0 otherwise
51	1 if data is from 5x2m ² plot size sown in 1997 by row method 0 otherwise
52	1 if data is from 5x3m ² plot size sown in 1997 by row method 0 otherwise
53	1 if data is from 5x4m ² plot size sown in 1997 by row method 0 otherwise
54	1 if data is from 10x1m ² plot size sown in 1997 by row method 0 otherwise
55	1 if data is from 10x2m ² plot size sown in 1997 by row method 0 otherwise
56	1 if data is from 1x1m ² plot size sown in 1997 by broadcast method 0 otherwise
57	1 if data is from 1x2m ² plot size sown in 1997 by broadcast method 0 otherwise
58	1 if data is from 1x3m ² plot size sown in 1997 by broadcast method 0 otherwise
59	1 if data is from 1x4m ² plot size sown in 1997 by broadcast method 0 otherwise
60	1 if data is from 1x6m ² plot size sown in 1997 by broadcast method 0 otherwise
61	1 if data is from 1x12m ² plot size sown in 1997 by broadcast method 0 otherwise
62	1 if data is from 1x18m ² plot size sown in 1997 by broadcast method 0 otherwise
63	1 if data is from 2x2m ² plot size sown in 1997 by broadcast method 0 otherwise
64	1 if data is from 2x3m ² plot size sown in 1997 by broadcast method 0 otherwise
65	1 if data is from 2x4m ² plot size sown in 1997 by broadcast method 0 otherwise
66	1 if data is from 2x6m ² plot size sown in 1997 by broadcast method 0 otherwise
67	1 if data is from 2x12m ² plot size sown in 1997 by broadcast method 0 otherwise
68	1 if data is from 5x1m ² plot size sown in 1997 by broadcast method 0 otherwise
69	1 if data is from 5x2m ² plot size sown in 1997 by broadcast method 0 otherwise
70	1 if data is from 5x3m ² plot size sown in 1997 by broadcast method 0 otherwise
71	1 if data is from 5x4m ² plot size sown in 1997 by broadcast method 0 otherwise
72	1 if data is from 10x1m ² plot size sown in 1997 by broadcast method 0 otherwise
73	1 if data is from 10x2m ² plot size sown in 1997 by broadcast method 0 otherwise

Appendix C: Table 5 values of Z in design matrix for multi-location trial

Z	Values of Z in design matrix
Z ₁	1 if data is from ENKOY/ISRAEL Variety 0 otherwise
Z ₂	1 if data is from ET12D4/HAR 604 Variety 0 otherwise
Z ₃	1 if data is from ET12D4/HAR 604 (2)Variety otherwise
Z ₄	1 if data is from HAR 1685/K6295-4A Variety 0 otherwise
Z ₅	1 if data is from HAR 416/DERESELIG Variety 0 otherwise
Z ₆	1 if data is from HAR 710/DERESELIG Variety 0 otherwise
Z ₇	1 if data is from HAR 710/HAR 719 Variety 0 otherwise
Z ₈	1 if data is from HAR3442Variety otherwise
Z ₉	1 if data is from HAR3603 Variety 0 otherwise
Z ₁₀	1 if data is from HAR3643 Variety 0 otherwise
Z ₁₁	1 if data is from HAR3646 Variety 0 otherwise
Z ₁₂	1 if data is from HAR3658 Variety 0 otherwise
Z ₁₃	1 if data is from K6295-4A/HAR 424(1) Variety 0 otherwise
Z ₁₄	1 if data is from L.CHECK Variety 0 otherwise
Z ₁₅	1 if data is from MAMBA/HAR 1384 Variety 0 otherwise
Z ₁₆	1 if data is from MILAN/SHA7 Variety 0 otherwise
Z ₁₇	1 if data is from RBC/HAR 800 Variety 0 otherwise
Z ₁₈	1 if data is from RBC/HAR 921 Variety 0 otherwise
Z ₁₉	1 if data is from SIMBA Variety 0 otherwise
Z ₂₀	1 if data is from Arsi Negelle location 0 otherwise
Z ₂₁	1 if data is from Arsi Robe location 0 otherwise
Z ₂₂	1 if data is from Adet location 0 otherwise
Z ₂₃	1 if data is from Alemaya location otherwise
Z ₂₄	1 if data is from Asasa location 0 otherwise
Z ₂₅	1 if data is from Bakoji location 0 otherwise
Z ₂₆	1 if data is from Bako location 0 otherwise
Z ₂₇	1 if data is from Debre Zeit location 0 otherwise
Z ₂₈	1 if data is from Ginchi location 0 otherwise
Z ₂₉	1 if data is from Holeta location 0 otherwise
Z ₃₀	1 if data is from Hosana location 0 otherwise
Z ₃₁	1 if data is from Kokate location 0 otherwise
Z ₃₂	1 if data is from Kulumsa location 0 otherwise
Z ₃₃	1 if data is from Sinana location 0 otherwise

Appendix D

Summary 1: Mixed model SAS out put for uniformity trial by making plot shape and area as random effect

```

The Mixed Procedure
Model Information
Data Set          WORK.HUNDE
Dependent Variable  Sqroot
Covariance Structure  Variance Components
Estimation Method    REML
Residual Variance Method  Profile
Fixed Effects SE Method  Model-Based
Degrees of Freedom Method  Containment

Class Level Information
Class      Levels  Values
Year       2      1 2
Sown_Type  2      1 2
Var        73     1 2 3 4 5 6 7 8 9 10 11 12 13
          14 15 16 17 18 19 20 21 22 23
          24 25 26 27 28 29 30 31 32 33
          34 35 36 37 38 39 40 41 42 43
          44 45 46 47 48 49 50 51 52 53
          54 55 56 57 58 59 60 61 62 63
          64 65 66 67 68 69 70 71 72 73

Dimensions
Covariance Parameters      2
Columns in X                4
Columns in Z                73
Subjects                    1
Max Obs Per Subject        11409
Observations Used          11409
Observations Not Used      0
Total Observations         11409

Iteration History
Iteration  Evaluations  -2 Res Log Like  Criterion
0          1      97649.27519554
1          2      59625.17903035  0.00088000
2          1      59603.27924612  0.00037944
3          1      59594.08312668  0.00011529
4          1      59591.42293355  0.00001690
5          1      59591.06322396  0.00000052
6          1      59591.05305620  0.00000000
Convergence criteria met.

```

Covariance Parameter Estimates

Cov Parm Estimate
Var 389.54
Residual 10.3168

Fit Statistics

-2 Res Log Likelihood 59591.1
AIC (smaller is better) 59595.1
AICC (smaller is better) 59595.1
BIC (smaller is better) 59599.6

Type 3 Tests of Fixed Effects

Num Den
Effect DF DF F Value Pr > F
Year 1 11E3 2.22 0.1364
Sown Type 1 11E3 1.12 0.2890

Summary 2: Mixed model SAS output for uniformity trial after ignoring random effect

The Mixed Procedure

Model Information

Data Set WORK.HUNDE
Dependent Variable Sqroot
Covariance Structure Diagonal
Estimation Method REML
Residual Variance Method Profile
Fixed Effects SE Method Model-Based
Degrees of Freedom Method Residual

Class Level Information

Class Levels Values
Year 2 1 2
Sown_Type 2 1 2
Var 73 1 2 3 4 5 6 7 8 9 10 11 12 13
14 15 16 17 18 19 20 21 22 23
24 25 26 27 28 29 30 31 32 33
34 35 36 37 38 39 40 41 42 43
44 45 46 47 48 49 50 51 52 53
54 55 56 57 58 59 60 61 62 63
64 65 66 67 68 69 70 71 72 73

Dimensions

Covariance Parameters 1
Columns in X 4
Columns in Z 0
Subjects 1
Max Obs Per Subject 11409
Observations Used 11409

Observations Not Used 0
 Total Observations 11409

Covariance Parameter Estimates

Cov Parm Estimate
 Residual 305.25

Fit Statistics

-2 Res Log Likelihood 97649.3
 AIC (smaller is better) 97651.3
 AICC (smaller is better) 97651.3
 BIC (smaller is better) 97658.6

The Mixed Procedure

Type 3 Tests of Fixed Effects

Effect	Num Den		F Value	Pr > F
	DF	DF		
Year	1	11E3	150.87	<.0001
Sown_Type	1	11E3	91.57	<.0001

Summary 3: GLM model SAS output for uniformity trial by making plot shape and area as fixed effect

The GLM Procedure
Class Level Information

Class	Levels	Values	
Year	2	1 2	
Sown_Type	2	1 2	
Var	73	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27	
	28 29		
		30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52	
		53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73	

Number of observations 11409

Dependent Variable: yield

		Sum of			
Source	DF	Squares	Mean Square	F Value	Pr > F
Model	72	9044606.117	125619.5294	4624.38	<.0001
Error	11336	116950.603	27.164		
Corrected Total	11408	3551965.232			

R-Square	Coeff Var	Root MSE	Sqroot Mean
0.967074	8.72	5.211969	45.42626

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Year	1	42335.005	42335.005	4103.52	<.0001
Sown Type	1	27952.736	27952.736	2709.45	<.0001
Var	70	3364726.888	48067.527	4659.18	<.0001

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Year	0	0.000	.	.	.
Sown Type	0	0.000	.	.	.
Var	70	3364726.888	48067.527	4659.18	<.0001

Summary 4: Mixed model SAS output for multi-location trial by making both variety and location as random effect

The Mixed Procedure

Model Information

Data Set WORK.HINDU
 Dependent Variable yield
 Covariance Structure Variance Components
 Estimation Method REML
 Residual Variance Method Profile
 Fixed Effects SE Method Model-Based
 Degrees of Freedom Method Containment

Class Level Information

Class	Levels	Values
year	3	2001 2002 2003
variety	19	ENKOY/ISRAEL ET12D4/HAR 604 ET12D4/HAR 604(2 HAR 1685/K6295-4 HAR 416/DERESLI HAR 710/DERESLI HAR 710/HAR 719 HAR3442 HAR3603 HAR3643 HAR3646 HAR3658 K6295-4A/HAR 424 L.CHECK MAMBA/HAR 1384 MILAN/SHA7 RBC/HAR 800 RBC/HAR 921 SIMBA
location	14	A.Negele A.Robe Adet Alemaya Asasa Bako Bekoji D.zeit Ginchi Hollota Hosana Kokate Kulumsa Sinana

Dimensions

Covariance Parameters	3
Columns in X	4
Columns in Z	33
Subjects	1
Max Obs Per Subject	660
Observations Used	660
Observations Not Used	0
Total Observations	660

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	621.10703571	
1	3	277.62714709	0.00079584
2	1	277.19126060	0.00010131
3	1	277.13980234	0.00000245
4	1	277.13864173	0.00000000

Convergence criteria met.

Covariance Parameter Estimates

Cov Parm	Estimate
variety_name	0.002004
location	0.06170
Residual	0.07977

Fit Statistics

-2 Res Log Likelihood	277.1
AIC (smaller is better)	283.1
AICC (smaller is better)	283.2
BIC (smaller is better)	286.0

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
year	2	626	6.32	0.0019

Summary 5: Mixed model SAS output for multi-location trial by making variety as fixed but location as random effect

The Mixed Procedure

Model Information

Data Set	WORK.HINDU
Dependent Variable	Natlog
Covariance Structure	Variance Components
Estimation Method	REML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Containment
Class Level Information	
Class	Levels Values
year	3 2001 2002 2003
variety_name	19 ENKOY/ISRAEL ET12D4/HAR 604 ET12D4/HAR 604(2 HAR 1685/K6295-4 HAR 416/DERESEL HAR 710/DERESEL HAR 710/HAR 719 HAR3442 HAR3603 HAR3643 HAR3646 HAR3658 K6295-4A/HAR 424 L.CHECK MAMBA/HAR 1384 MILAN/SHA7 RBC/HAR 800 RBC/HAR 921 SIMBA
location	14 A.Negele A.Robe Adet Alemaya Asasa Bako Bekoji D.zeit Ginchi Hollota Hosana Kokate

Kulumsa Sinana
Dimensions

Covariance Parameters	2
Columns in X	4
Columns in Z	14
Subjects	1
Max Obs Per Subject	660
Observations Used	660
Observations Not Used	0
Total Observations	660

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	621.10703571	
1	3	281.60108203	0.00079438
2	1	281.16406859	0.00010698
3	1	281.10985779	0.00000272
4	1	281.10857523	0.00000000

Convergence criteria met.

Covariance Parameter Estimates

Cov Parm	Estimate
location	0.06167
Residual	0.08167

Fit Statistics

-2 Res Log Likelihood	281.1
AIC (smaller is better)	285.1
AICC (smaller is better)	285.1
BIC (smaller is better)	286.4

Type 3 Tests of Fixed Effects

Effect	Num		F Value	Pr > F
	DF	Den		
year	2	644	6.17	0.0022

Summary 6: Mixed model SAS output for multi-location trial by making location as fixed but variety as random effect

The Mixed Procedure
Model Information

Data Set WORK.SIYAM
 Dependent Variable yield
 Covariance Structure Variance Components
 Estimation Method REML
 Residual Variance Method Profile
 Fixed Effects SE Method Model-Based
 Degrees of Freedom Method Containmentment

Class Level Information

Class	Levels	Values
year	3	2001 2002 2003
variety_name	19	ENKOY/ISRAEL ET12D4/HAR 604 ET12D4/HAR 604(2 HAR 1685/K6295-4 HAR 416/DERESELI HAR 710/DERESELI HAR 710/HAR 719 HAR3442 HAR3603 HAR3643 HAR3646 HAR3658 K6295-4A/HAR 424 L.CHECK MAMBA/HAR 1384 MILAN/SHA7 RBC/HAR 800 RBC/HAR 921 SIMBA
location	15	A.Nagelle A.Negele A.Robe Adet Alemaya Asasa Bako Bekoji D.zeit Ginchi Hollota Hosana Kokate Kulumsa Sinana

Dimensions

Covariance Parameters	2
Columns in X	4
Columns in Z	19
Subjects	1
Max Obs Per Subject	660
Observations Used	660
Observations Not Used	0
Total Observations	660

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	5265.19488694	

1 1 5265.19488694 0.00000000

Convergence criteria met.

Covariance Parameter Estimates

Cov Parm	Estimate
variety	0
Residual	172.69

Fit Statistics

-2 Res Log Likelihood	5265.2
AIC (smaller is better)	5267.2
AICC (smaller is better)	5267.2
BIC (smaller is better)	5268.1

Type 3 Tests of Fixed Effects

Effect	Num	Den	F Value	Pr > F
	DF	DF		
year	2	639	0.33	0.7225

Summary 7: Mixed model SAS output for multi-location trial after ignoring both variety as fixed but location as random effect

The Mixed Procedure

Model Information

Data Set WORK.HINDU
 Dependent Variable yield
 Covariance Structure Diagonal
 Estimation Method REML
 Residual Variance Method Profile
 Fixed Effects SE Method Model-Based
 Degrees of Freedom Method Residual

Class Level Information

Class	Levels	Values
year	3	2001 2002 2003
variety_name	19	ENKOY/ISRAEL ET12D4/HAR 604 ET12D4/HAR 604(2 HAR 1685/K6295-4 HAR 416/DERESEL HAR 710/DERESEL HAR 710/HAR 719 HAR3442 HAR3603 HAR3643 HAR3646 HAR3658 K6295-4A/HAR 424 L.CHECK MAMBA/HAR 1384 MILAN/SHA7 RBC/HAR 800 RBC/HAR 921 SIMBA
location	14	A.Negele A.Robe Adet Alemaya Asasa Bako Bekoji D.zeit Ginchi Hollota Hosana Kokate Kulumsa Sinana

Dimensions

Covariance Parameters	1
Columns in X	4
Columns in Z	0
Subjects	1
Max Obs Per Subject	660
Observations Used	660
Observations Not Used	0
Total Observations	660

Covariance Parameter Estimates

Cov Parm	Estimate
Residual	0.1470

Fit Statistics

-2 Res Log Likelihood	621.1
AIC (smaller is better)	623.1
AICC (smaller is better)	623.1
BIC (smaller is better)	627.6

Type 3 Tests of Fixed Effects

	Num	Den		
Effect	DF	DF	F Value	Pr > F
year	2	657	2.56	0.0782

Summary 8: GLM model output for multi-location trial by making both variety and location as fixed effect mistakenly

The GLM Procedure
Class Level Information

Class	Levels	Values
year	3	2001 2002 2003
variety	19	ENKOY/ISRAEL ET12D4/HAR 604 ET12D4/HAR 604(2 HAR 1685/K6295-4 HAR 416/DERESELI HAR 710/DERESELI HAR 710/HAR 719 HAR3442 HAR3603 HAR3643 HAR3646 HAR3658 K6295-4A/HAR 424 L.CHECK MAMBA/HAR 1384 MILAN/SHA7 RBC/HAR800 RBC/HAR 921 SIMBA
location	14	A.Negele A.Robe Adet Alemaya Asasa Bako Bekoji D.zeit Ginchi Hollota Hosana Kokate Kulumsa Sinana
		Number of observations 660

Dependent Variable: yield

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	33	166.3501884	5.43723189	18.02	<.0001
Error	626	49.92174353	0.27974719		
Corrected Total	659	97.35039600			

R-Square	Coeff Var	Root MSE	yield Mean
0.487195	14.6434956	0.528911325	3.611933

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Year	2	0.75225059	0.37612529	4.72	0.0093
Variety	18	2.67986269	0.14888126	1.87	0.0160
Location	13	43.99653920	3.38434917	42.44	<.0001

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Year	2	1.02045849	0.51022924	6.40	0.0018
Variety	18	2.67986269	0.14888126	1.87	0.0160
Location	13	43.99653920	3.38434917	42.44	<.0001

Appendix E

Summary 1: SAS output for multi-location trial to check the interaction effect among variables

```

Data Set           WORK.HINDU
Dependent Variable  yield
Covariance Structure Diagonal
Estimation Method   REML
Residual Variance Method Profile
Fixed Effects SE Method Model-Based
Degrees of Freedom Method Residual

Class Level Information
Class      Levels  Values
year              3  2001 2002 2003
variety_name      19  ENKOY/ISRAEL ET12D4/HAR 604
                  ET12D4/HAR 604(2 HAR
                  1685/K6295-4 HAR 416/DERESEL
                  HAR 710/DERESEL HAR 710/HAR
                  719 HAR3442 HAR3603 HAR3643
                  HAR3646 HAR3658 K6295-4A/HAR
                  424 L.CHECK MAMBA/HAR 1384
                  MILAN/SHA7 RBC/HAR 800 RBC/HAR
                  921 SIMBA
location          14  A.Negele A.Robe Adet Alemaya
                  Asasa Bako Bekoji D.zeit
                  Ginchi Hollota Hosana Kokate
                  Kulumsa Sinana

```

Number of observations 660

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Year	2	335.18387	167.59193	3.01	<.0001
Variety	18	2907.91586	161.55088	1.74	.0451
Location	14	47797.27064	3414.09076	1283.63	<.0001
Year*variety	36	305.85896	8.49608	3.19	0.0005
Year*location	16	51001.07249	3187.56703	1198.46	<.0001
Variety*location	252	5883.56470	23.34748	2.78	.0861
Year*variety*location	288	2186.14812	7.59079	2.85	0.893

Summary: About Type I II III and IV Analysis of SAS

From <http://www.asu.edu/sas/sasdoc/sashtml/chap12/sec9.htm>

Type I SS and Estimable Functions

The Type I SS and the associated hypotheses they test are by-products of the modified sweep operator used to compute a generalized inverse of $\mathbf{X}'\mathbf{X}$ and a solution to the normal equations. For the model $E(Y) = X1 \times B1 + X2 \times B2 + X3 \times B3$, the Type I SS for each effect correspond to

Effect	Type I SS
$B1$	$R(B1)$
$B2$	$R(B2 B1)$
$B3$	$R(B3 B1, B2)$

The Type I SS are model-order dependent; each effect is adjusted only for the preceding effects in the model.

There are numerous ways to obtain a Type I hypothesis matrix \mathbf{L} for each effect. One way is to form the $\mathbf{X}'\mathbf{X}$ matrix and then reduce $\mathbf{X}'\mathbf{X}$ to an upper triangular matrix by row operations, skipping over any rows with a zero diagonal. The nonzero rows of the resulting matrix associated with $X1$ provide an \mathbf{L} such that

$$\mathbf{SS}(H_0: \mathbf{L}\beta = 0) = R(B1)$$

The nonzero rows of the resulting matrix associated with $X2$ provide an \mathbf{L} such that

$$\mathbf{SS}(H_0: \mathbf{L}\beta = 0) = R(B1|B2)$$

The last set of nonzero rows (associated with $X3$) provide an \mathbf{L} such that

$$\mathbf{SS}(H_0: \mathbf{L}\beta = 0) = R(\beta_3|\beta_1, \beta_2)$$

Another more formalized representation of Type I generating sets for β_1 , β_2 and β_3 respectively, is :

$$\mathbf{G}_1 = (\mathbf{X}'_1\mathbf{X}_1 \mid \mathbf{X}'_1\mathbf{X}_2 \mid \mathbf{X}'_1\mathbf{X}_3)$$

$$\mathbf{G}_2 = (0 \mid \mathbf{X}'_2\mathbf{M}_2\mathbf{X}_2 \mid \mathbf{X}'_2\mathbf{M}_2\mathbf{X}_3)$$

$$\mathbf{G}_3 = (0 \mid 0 \mid \mathbf{X}'_3\mathbf{M}_3\mathbf{X}_3)$$

Where $\mathbf{M}_1 = \mathbf{I} - \mathbf{X}_1(\mathbf{X}'_1\mathbf{X}_1)^{-1}\mathbf{X}'_1$ and $\mathbf{M}_2 = \mathbf{M}_1 - \mathbf{M}_1\mathbf{X}_2(\mathbf{X}'_2\mathbf{M}_1\mathbf{X}_2)^{-1}\mathbf{X}'_2\mathbf{M}_1$

Using the Type I generating set \mathbf{G}_2 (for example), if an \mathbf{L} is formed from linear combinations of the rows of \mathbf{G}_2 such that \mathbf{L} is of full row rank and of the same row rank as \mathbf{G}_2 , then $SS. (\mathbf{H}_0 : \mathbf{L}\boldsymbol{\beta} = 0) = R(\mathbf{B}_2|\mathbf{B}_1)$

There are, however, a number of models for which the Type I hypotheses are considered appropriate. These are

- balanced ANOVA models specified in proper sequence (that is, interactions do not precede main effects in the MODEL statement and so forth)
- purely nested models (specified in the proper sequence)
- polynomial regression models (in the proper sequence).

Type II SS and Estimable Functions

For main effects models and regression models, the general form of estimable functions can be manipulated to provide tests of hypotheses involving only the parameters of the effect in question. The same result can also be obtained by entering each effect in turn as the last effect in the model and obtaining the Type I SS for that effect. These are the *Type II SS*. Using a modified reversible sweep operator, it is possible to obtain the Type II SS without actually rerunning the model.

Thus, the **Type II SS correspond to the R notation in which each effect is adjusted for all other effects possible**. For a regression model such as

$$E(Y) = X_1 \times B_1 + X_2 \times B_2 + X_3 \times B_3$$

the Type II SS correspond to

Effect	Type II SS
B_1	$R(B_1 B_2, B_3)$
B_2	$R(B_2 B_1, B_3)$
B_3	$R(B_3 B_1, B_2)$

For a main effects model (A , B , and C as classification variables), the Type II SS correspond to

Effect	Type II SS
A	$R(A B, C)$
B	$R(B A, C)$
C	$R(C A, B)$

As the discussion in the section ["A Three-Factor Main Effects Model"](#) indicates, for regression and main effects models the Type II SS provide an MRH for each effect that does not involve the parameters of the other effects.

For models involving interactions and nested effects, in the absence of a priori parametric restrictions, it is not possible to obtain a test of a hypothesis for a main effect free of parameters of higher-level effects with which the main effect is involved.

It is reasonable to assume, then, that any test of a hypothesis concerning an effect should involve the parameters of that effect and only those other parameters with which that effect is involved.

Contained Effect

Given two effects $F1$ and $F2$, $F1$ is said to be *contained in* $F2$ provided that

- both effects involve the same continuous variables (if any)
- $F2$ has more CLASS variables than does $F1$, and if $F1$ has CLASS variables, they all appear in $F2$

Note that the interaction effect μ_{jk} is contained in all pure CLASS effects, but it is not contained in any effect involving a continuous variable. No effect is contained by μ_{jk} .

Type II, Type III, and Type IV estimable functions rely on this definition, and they all have one thing in common: the estimable functions involving an effect $F1$ also involve the parameters of all effects that contain $F1$, and they do not involve the parameters of effects that do not contain $F1$ (other than $F1$).

Hypothesis Matrix for Type II Estimable Functions

The Type II estimable functions for an effect $F1$ have an \mathbf{L} (before reduction to full row rank) of the following form:

- All columns of \mathbf{L} associated with effects not containing $F1$ (except $F1$) are zero.
- The submatrix of \mathbf{L} associated with effect $F1$ is $(\mathbf{X}_1' \mathbf{M} \mathbf{X}_1)^-(\mathbf{X}_1' \mathbf{M} \mathbf{X}_1)$.
- Each of the remaining submatrices of \mathbf{L} associated with an effect $F2$ that contains $F1$ is $(\mathbf{X}_1' \mathbf{M} \mathbf{X}_1)^-(\mathbf{X}_1' \mathbf{M} \mathbf{X}_2)$.

Type III and IV SS and Estimable Functions

When an effect is contained in another effect, the Type II hypotheses for that effect are dependent on the cell frequencies. The philosophy behind both the Type III and Type IV hypotheses is that the hypotheses tested for any given effect should be the same for all designs with the same general form of estimable functions.

To demonstrate this concept, recall the hypotheses being tested by the Type II SS in the balanced 2×2 factorial shown in [Table 12.6](#). Those hypotheses are precisely the ones that the Type III and Type IV hypotheses employ for all 2×2 factorials that have at least one observation per cell. The Type III and Type IV hypotheses for a design without missing cells usually differ from the hypothesis employed for the same design with missing cells since the general form of estimable functions usually differs.

Type III Estimable Functions

Type III hypotheses are constructed by working directly with the general form of estimable functions. The following steps are used to construct a hypothesis for an effect $F1$:

1. For every effect in the model except $F1$ and those effects that contain $F1$, equate the coefficients in the general form of estimable functions to zero.

If $F1$ is not contained in any other effect, this step defines the Type III hypothesis (as well as the Type II and Type IV hypotheses). If $F1$ is contained in other effects, go on to step 2. (See the section "[Type II SS and Estimable Functions](#)" for a definition of when effect $F1$ is contained in another effect.)

2. If necessary, equate new symbols to compound expressions in the $F1$ block in order to obtain the simplest form for the $F1$ coefficients.
3. Equate all symbolic coefficients outside of the $F1$ block to a linear function of the symbols in the $F1$ block in order to make the $F1$ hypothesis orthogonal to hypotheses associated with effects that contain $F1$.

By once again observing the Type II hypotheses being tested in the balanced 2×2 factorial, it is possible to verify that the A and $A*B$ hypotheses are orthogonal and also that the B and $A*B$ hypotheses are orthogonal. This principle of orthogonality between an effect and any effect that contains it holds for all balanced designs. Thus, construction of Type III hypotheses for any design is a logical extension of a process that is used for balanced designs.

The Type III hypotheses are precisely the hypotheses being tested by programs that re-parameterize using the usual assumptions (for example, all parameters for an effect

summing to zero). When no missing cells exist in a factorial model, Type III SS coincide with Yates' weighted squares-of-means technique. When cells are missing in factorial models, the Type III SS coincide with those discussed in Harvey (1960) and Henderson (1953).

The following steps illustrate the construction of Type III estimable functions for a 2×2 factorial with no missing cells.

To obtain the Type III hypothesis for A , first start with the general form and equate the coefficients for effects μ and β to zero (let $L_1=L_4=0$). Next let $L_6=K*L_2$, and find the value of K that makes the A hypothesis orthogonal to the $A*B$ hypothesis.

The prime difference between the Type II and Type III hypotheses for A is the way K is determined. Type II chooses K as a function of the cell frequencies, whereas Type III chooses K such that the estimable functions for A are orthogonal to the estimable functions for $A*B$.

Type IV Estimable Functions

By once again looking at the Type II hypotheses being tested in the balanced 2×2 factorial you can see another characteristic of the hypotheses employed for balanced designs: the coefficients of lower-order effects are averaged across each higher-level effect involving the same subscripts. For example, in the A hypothesis, the coefficients of AB_{11} and AB_{12} are equal to one-half the coefficient of A_1 , and the coefficients of AB_{21} and AB_{22} are equal to one-half the coefficient of A_2 . With this in mind then, the basic concept used to construct Type IV hypotheses is that the coefficients of any effect, say F_1 , are distributed equitably across higher-level effects that contain F_1 . When missing cells occur, this same general philosophy is adhered to, but care must be taken in the way the distributive concept is applied.

Construction of Type IV hypotheses begins as does the construction of the Type III hypotheses. That is, for an effect F_1 , equate to zero all coefficients in the general form that do not belong to F_1 or to any other effect containing F_1 . If F_1 is not contained in any other effect, then the Type IV hypothesis (and Type II and III) has been found. If F_1 is contained in other effects, then simplify, if necessary, the coefficients associated with F_1 so that they are all free coefficients or functions of other free coefficients in the F_1 block.

To illustrate the method of resolving the free coefficients outside of the F_1 block, suppose that you are interested in the estimable functions for an effect A and that A is contained in AB , AC , and ABC . (In other words, the main effects in the model are A , B , and C .)

With missing cells, the coefficients of intermediate effects (here they are AB and AC) do not always have an equal distribution of the lower-order coefficients, so the coefficients of the highest-order effects are determined first (here it is ABC). Once the highest-order

coefficients are determined, the coefficients of intermediate effects are automatically determined.

The following process is performed for each free coefficient of A in turn. The resulting symbolic vectors are then added together to give the Type IV estimable functions for A .

1. Select a free coefficient of A , and set all other free coefficients of A to zero.
2. If any of the levels of A have zero as a coefficient, equate all of the coefficients of higher-level effects involving that level of A to zero. This step alone usually resolves most of the free coefficients remaining.
3. Check to see if any higher-level coefficients are now zero when the coefficient of the associated level of A is not zero. If this situation occurs, the Type IV estimable functions for A are not unique.
4. For each level of A in turn, if the A coefficient for that level is nonzero, count the number of times that level occurs in the higher-level effect. Then equate each of the higher-level coefficients to the coefficient of that level of A divided by the count.

A Comparison of Type III and Type IV Hypotheses

For the vast majority of designs, Type III and Type IV hypotheses for a given effect are the same. Specifically, they are the same for any effect $F1$ that is not contained in other effects for any design (with or without missing cells). For factorial designs with no missing cells, the Type III and Type IV hypotheses coincide for all effects. When there are missing cells, the hypotheses can differ. By using the GLM procedure, you can study the differences in the hypotheses and then decide on the appropriateness of the hypotheses for a particular model.

The Type III hypotheses for three-factor and higher completely nested designs with unequal N s in the lowest level differ from the Type II hypotheses; however, the Type IV hypotheses do correspond to the Type II hypotheses in this case.

When missing cells occur in a design, the Type IV hypotheses may not be unique. If this occurs in PROC GLM, you are notified, and you may need to consider defining your own specific comparisons.

Appendix F

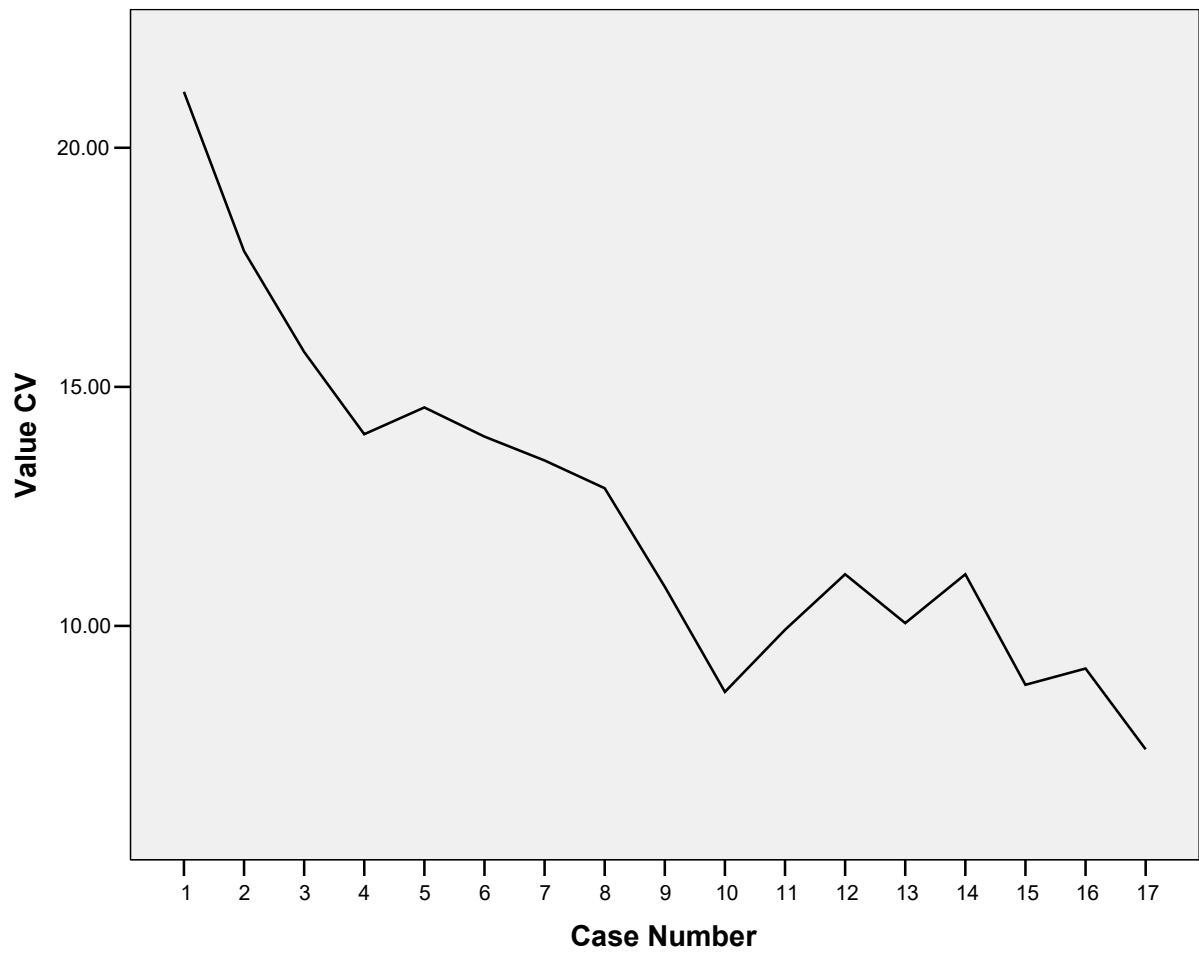


Figure1: Plot residual versus plot size for row Sown method

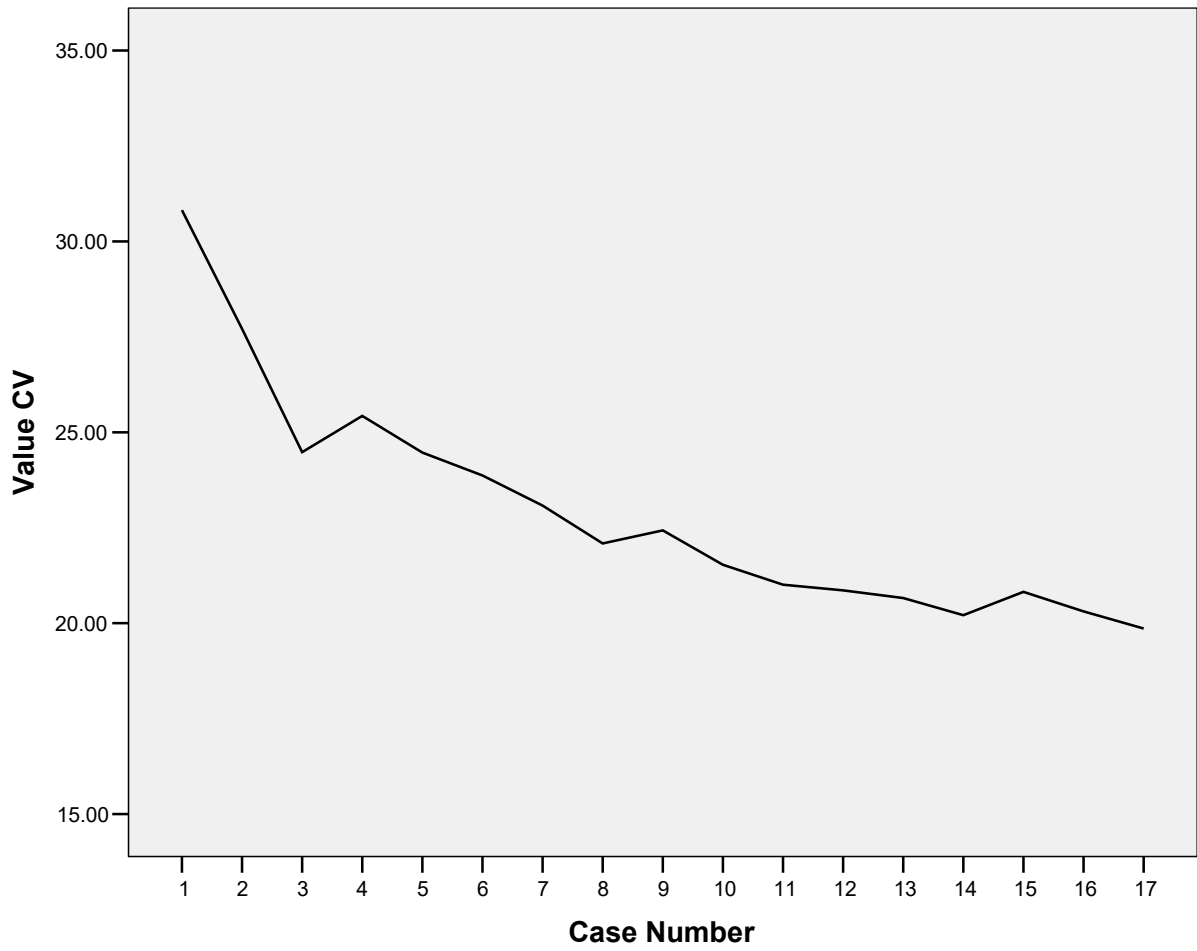


Figure 2 Plot residual versus plot size for broadcast sown method

DECLARATION

I, the undersigned, declare that the thesis is my original work, has not been presented for degrees in other university and sources of materials used for the thesis have been duly acknowledged

Name: Abbas Kedir

Signature.....

Place: Faculty of Science, Addis Ababa University

Date January 2008

This thesis has been submitted for examination with my approval as a University Advisor.

.....

Girma Taye (PhD)