ADDIS ABABA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

APPLICATION OF STATISTICAL SAMPLING TO AUDITING AND ACCOUNTING

BEFEKADU AYELE

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FACULTY OF SCIENCE

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ABSTRACT

An inquiry arises in applying sampling techniques to auditing. Why do auditors sample? Possible reasons may be:

- When the nature and materiality of the balance or class of transactions does not demand a 100 percent checking.
- When a decision must be made about the balance or class of transactions.
- When the time and cost to audit 100 percent of the population would be so great.
- A 100 percent checking does not in general guarantee a definite assurance.

Therefore, auditors use sampling because they need to perform efficient audits on a timely basis and cost effectively where they can not do so by auditing 100 percent.

This paper was designed to show the advantages of applying statistical sampling methods in the field of auditing. Using a population of known characteristics the study was conducted to see how good the total values are estimated.

Repeated samples of same size were taken from unstratified and stratified designs, and sampling with probability proportional to monetary size (PPMS), usually called Monetary or Dollar Unit Sampling. The size of the sample was computed using sample size computational formula for each design with the objective of attaining the same planned error margin and same reliability level for the entire study. From the first two designs the mean per unit, difference and ratio estimators were considered. The audit objective assumed was to check whether the recorded book value was fairly stated. The audit

hypothesis approach was thus followed. Sample size was determined using corresponding formula for each design.

Large sample size was obtained for unstratified sampling but a comparatively small size for stratified and PPMS. The total audited amount was estimated very well using PPMS, the unstratified difference and ratio, and the stratified mean per unit cases. The auxiliary information estimators resulted in higher precision in both unstratified and stratified, and the mean per unit was the least. This may be due to the high correlation between the recorded and audited amounts. The total error was fairly estimated in the case of the PPMS, unstratified difference and ratio, and the stratified mean per unit cases. The unsratified mean per unit was the least. The result also reveals that if no errors are observed in the sample estimation using the difference and ratio estimators fail. In such cases the PPMS and the mean per unit are recommended.

From the point of view of sample evaluation in the study, the PPMS approach is recommended for audit application. Stratified sampling is also helpful provided stratification and computational complexities are handled by using computers.

CHAPTER ONE

Introduction

Accountants are concerned with collecting, classifying, summarizing, and reporting data relating to the operations of organizations. And auditors are responsible for reviewing the work of accountants in order to express an opinion on the accountants' principal product, financial statements and summary reports. Thus both accountants and auditors must analyze data. The accountants in the collection, classification and summarization process, and the auditors in the review process.

Since statistical sampling can be used as means of data analysis, it is not surprising that both accountants and auditors rely on statistical sampling as a means of accounting data analysis.

Applying statistical audit sampling to accounting data is also supported and recommended in the statement of auditing standards considering it as a scientific approach to supplement audit methodologies and reach a reasonable result to attain the proposed audit objectives.

Until shortly before the beginning of 1960's statistical sampling techniques had not been generally applied to the more complex field of auditing. These techniques have been developed to meet common audit objectives. Statistical sampling do not lessen in any way the need for the auditor to use his judgments, but rather they increase the reliability of the information on which he or she bases his or her opinion. Statistical techniques often require the auditor to specify more clearly his audit objectives.

The primary objective of an audit is to formulate and express an overall opinion showing the true and fair view of financial or other information based on an examination of records of transactions and other relevant information. However, detailed review of even more than a fraction of transaction documents of a large organization would become extremely costly and the time involved is so great that many of the benefits of the audit would have been lost by the time the audit work is completed.

Thus, with this greatly expanded volume of transaction it had become necessary to rely on, to a far greater extent, the advantage of using sampling techniques. In fact, in most cases, detailed checking serves no purpose at all or it may be practically impossible for the auditor. Montgomery (1985) came to an obvious conclusion that "no audit can or should embrace a complete verification of all the transactions of the period under review, neither the client nor the public expects the auditor to examine every transaction."

Eventhough, the intuitive judgmental sampling, also called non-statistical sampling, has been used by auditors for many years, there have always been question about:

- whether the sample used was adequate in size ,
- whether it was representative,

- what could be the chances that the major error might remain undetected in the unsampled portion.

These could be easily attained through statistical sampling guided by the auditor's judgement; the degree of risk involved, the reliability of the sample results, and other factors can be readily determined by means of statistical sampling.

The application of statistical sampling is very limited in Ethiopian audit profession. A selfparticipatory witness reveals that a study of this kind had not been conducted to support the auditing profession in Ethiopia. And as a result proper use of statistical sampling procedures in auditing is, in general, retarded. Eventhough, it is apparent that there are major problems in auditing from the stage of planning to performing and evaluating the audit work up to reporting. But this could have been minimized if the auditing methodologies were supported by researches that could supplement auditor's proficiency.

Infact some of the factors that hinder auditors from using statistical sampling in auditing are:

- lack of technical assistance to the auditor,
- lack of knowledge and use of the basic statistical concepts. This includes the underlying ideas, the selection procedures, the evaluation methods applied, and the computation formulas involved in sampling not being manageable for computation,

- lack of successive capacity building training, and therefore, scarcity of skilled personnel capable of carrying out their duties adequately ,
- lack of well developed and monitored accounting and auditing practice standards that support the auditor's conclusion based on the sample results.

Thus, this study attempts to:

a/ Suggest a convenient method of statistical sampling approach suitable for substantive procedures.

b/ Indicate optimum sample size determination methods and selection procedures .

c/ Indicate projection ways of sample results to total population results with a specified degree of reliability.

d/ Recommend possible solutions to the use of statistical sampling to auditing .

1.1 Objectives

1.1.1 General Objectives

To formulate appropriate sample procedure suitable for audit applications.

1.1.2 Specific Objectives

a/ show how statistical sampling is applied to accounting data.

b/ examine some of the single stage sample designs relevant to substantive tests .

c/ introduce Monetary Unit Sampling and its relative efficiency in audit sampling .

d/ examine estimation methods of sample results .

e/ provide valuable recommendations to interested users and beneficiaries .

The data for the study was obtained from an auditee organization. It was actually an audited data on a hundred percent basis. The aim here is to reach to the known results using some of the single stage sampling designs and to show how each design works for audit situations. The mean per unit, difference, and ratio estimation for unstratified and stratified designs and probability proportional to monetary size, PPMS, sample designs are applied to see which estimator gives a better result. The population of the study contains a set of payment vouchers and so the sampling unit is the payment voucher with its recorded monetary amount. Statistical procedures to be employed for data treatment are, theory of hypothesis testing and the sample design precision measurement to see how well the estimated audited values behave.

1.2 Limitation of the study

- The study attempts to compare the results obtained and not the performance of each estimator.
- Because of the similar structure on the nature of the concept between the difference and the regression estimators and the latter is biased the difference estimator was preferred for the study.
- Systematic Sampling was not also done since it doesn't have an unbiased variance estimator for the mean and the total.
- Since its application in audit sampling is rather limited cluster sampling design was not included.

1.3 Application of results and beneficiaries

Based on the findings, it is hoped that valuable suggestions are made so that useful achievements can be obtained that are relevant to the application of statistical sampling methods in auditing. These suggestions and recommendations can help auditors and chief audit personnel to plan their audit work scientifically and to get a better audit output and fair reporting.

The outcome of the study may also be useful to private auditors, internal auditors, Regional State auditors, and the Federal auditors. Other research groups on this and related topic may also benefit from the outcome of this research thesis for further related works in the field.

CHAPTER TWO

Literature Review

Applying statistical sampling to auditing has a great advantage for auditors to deal with a large volume of transactions and to estimate the total audited amount and total error from the audited sample of transactions. But, using a sample result, estimating error occurrences in an accounting population has been of great concern for researchers. Explaining the nature of the errors explicitly was not an easy task to accomplish. This is because occurrence of error in such populations is purely subjective and depends on accountants who are in charge of preparing, recording, finalising and summarising the transactions. The degree of making error in turn depends on the knowledge, experience, careful handling of their job. The occurrences of error may go from the limit of being immaterial at all to the extent of affecting the audit output materially. It is agreed that these errors are categorised as rare errors.

The other major problems arising during the audit of such a population are:

Can we detect all the errors?

Do we have a tool (working procedure) to face the problem?

How are we going to analyse this accounting population in such a way that we can develop a procedure to detect as many of the errors as possible in a cost-effective manner, and hence reach the true audited value?

Or what sampling design would serve as to detect as much of the errors in the population as possible?

Or on the application of detailed checking on a sample of an accounting population which estimation strategy would benefit us to get a better precision?

Precision is a measure of closeness between a sample estimate and the corresponding unknown population characteristic. By designing a sample and an estimation strategy we can make the deviation between the actual unknown population value and the estimated total error as small as possible and so increase our precision. This is done after sampling has been applied in auditing and detailed checking of transactions using suitable audit procedures is completed. But practical experience showed that it might not be easy to give a good estimate of the total occurrence of these rare errors in the accounting population from which the sample was drawn. This calls for the need to deal with the nature of the error distribution in an accounting population in order to design and apply a good statistical sampling method and a reliable estimation strategy to increase precision. Researchers who work on this issue have suggested possible approaches to give an upper bound for the total error estimate which in turn gives an accepted estimate of the total audited amount.

The basic assumption underlying audit sampling is that items in the population are, for sampling purposes, homogeneous in the sense that observations of some subset of items is useful for drawing conclusions about the remainder of the population. This assumption is essential for inferences based on samples.

Smith (1979) first identified the audit population as a well defined population of N items which comprise the transactions that enter into a set of accounts, which were labelled by him as vouchers, entries in a ledger, or entries on a computer file. In order to solve the auditor's problem in making a judgement about the accuracy of the total recorded amount, he defined the error as $E_i = Y_i - X_i$. And the total error denoted by T_E was considered as the sum of all E_i's, where Yi and Xi are the i^{th} recorded and audited values, respectively. He noted that when N is large it is too expensive to check the accuracy of all items. Therefore, the statistical problem is to design and draw a sample, to evaluate the results and to suggest ways of aiding the auditor's judgement. He also compared the upper error bound setting procedures formerly done by different researchers about the population total error amount T_E. Using the Poisson approximation for the distribution of these rare errors, in an accounting population he formulated the bound for the percentage of error in the population using the number of errors found in the sample. The approximation provided good estimate results. According to his conclusion, for a particular population all the procedures conservatively determine the upper error limit. And he put his views on the problem of estimating the monetary value of an error and realised that the Bayesian approach is the only satisfactory one from a statistical viewpoint.

Moors (1983) considered an accounting population of which a small proportion P of the elements is in error. For such a population out of the total unaudited recorded value X, the amount Y was considered incorrect, where X and Y are assumed to be positive. Hence, the correct value of the population is X-Y rather than X. The problem according to him was then to derive, from a sample, an upper confidence limit for the total error, that is the total of Y over the population. Equivalently, an upper bound for the total fractional error, which is the total of Y expressed as a fraction of the total of X. The crucial point of this problem was that P might be so small that the probability of an error free sample is not negligible. In sampling for auditing this problem is rather fundamental.

Taylor (1974) considered audit precision from the point of view of error analysis in audit tests. Identifying the type and nature of errors may help to reach a justifiable audit result, and precision. He tried to identify the nature of the errors as compliance deviations qualitatively and as monetary errors quantitatively. The auditor's final subjective conclusion, after the qualitative audit is accomplished, must simply be whether the internal control is strong enough to support his or her original decisions to rely on it. If not, the reduced reliability used for the substantive tests must be increased which in turn increases precision of the audit result.

A meaningful numerate error analysis would also help to reach a sound precision. Having selected and examined certain transactions or balances, the auditor must decide what he or she will do about any errors he or she has detected. In most cases, according to Taylor, it doesn't make sense for the error or errors to be statistically evaluated, because the results are frequently beyond acceptable limits.

Since the auditor usually knows the amounts recorded by the auditee organisation (his client) for each line item in the population called the book or recorded amount, this information can be utilised in estimating the total amount of error. One such estimator that incorporates this supplementary (auxiliary) information is the difference estimator, which for simple random sampling of line items, is the difference between the book and the audited amounts for the i^{th} sample line item. Other estimates that incorporate the information on book amounts are the ratio and regression estimators. Each of these can be used with simple random (unstratified) or stratified random sampling of line items. Still another means of incorporating the information about book amounts is through sampling with probabilities proportional to book amount and then utilising an unbiased estimator.

The search for alternative inference procedures that do not depend on large-sample theory has frequently involved Monetary Unit Sampling (MUS) also called Dollar-Unit Sampling (DUS). The procedure is based on the selection of a given sampling unit using sampling with probabilities proportional to aggregate monetary amount from the population. Since the auditor cannot audit a single monetary unit but only a line item to which the unit pertains, any error found is then prorated to the monetary units belonging to the line item. The prorated errors are called <u>taints</u> in auditing, and are denoted by $t_k = d_k / y_k$, where d_k is the difference between the audited and recorded amount and is not equal to zero, y_k is the sample recorded amount. An important case in auditing is when the errors in the population are all overstatement errors and the taints are restricted to positive values not exceeding 1.0. For this case, a conservative confidence bound for the total error amount can be constructed by assuming that all overstatement taints are at the maximum value of 1.0. The problem then becomes one of obtaining an upper confidence bound for the population proportion of monetary units that contain an error. Multiplying this bound by X, the total number of monetary book units in the population, yields a conservative upper confidence bound for the total error amount.

Neter and Loebbecke (1975) studied four actual accounting populations and from these they constructed a number of study populations with varying error rates. They utilised sample sizes of 100 and 200 with unstratified, stratified, and probability proportional to monetary size sampling. They found, for the supplementary information procedures, that the coverage for large sample upper confidence bounds for the total error amount (i.e. the proportion of times the bound is correct in repeated sampling) were frequently substantially below the nominal confidence level.

The above discussion indicates that analysis of the nature of errors possibly occurring in audit population was a relevant subject of research to equip the auditor with better audit sampling approach. The discussion that follows also strengthens this.

The paper by Hylas and Ashton (1982) is an empirical study of 281 errors requiring financial statement adjustment on 152 audits. It reports, among other things, on those audit areas in which the errors occurred, the audit procedure, circumstances or other events that initially signalled that an error had occurred, and the apparent causes of the errors, including whether they appeared to have been caused intentionally or unintentionally. The result suggests that client personnel problems, such as inexperience and insufficient knowledge of accounting, and cut-off or accrual problems, are relevant causes of errors. It was also noted that analytical review procedures and discussions with the client depict occurrence of a large proportion of errors. Thus, their results were consistent with an increased emphasis on such procedures in practice. But they could not conclude that the use of detailed tests should be reduced, since they also detect a large proportion of known errors. It appears that increased utilisation of less rigorous audit procedures might improve the auditor's effectiveness and/or efficiency in detecting errors, and also allow a 'fine' tuning of substantive tests of details.

The paper by Burgstahler and Jiambalvo (1986) examines the practice of *'isolating'* certain errors observed in an audit sample and subsequent failure to project these errors in estimating total error in an audit population. An abstract (normative) model of inference is developed to analyse this practice and related descriptive theories of human information processing are reviewed. Previous descriptive research suggests that auditors may be inclined to isolate errors that are considered to be 'unique'. However, the normative analysis suggests that isolation is seldom appropriate. The responses of auditors to a number of hypothetical cases are analysed to provide descriptive empirical evidence on the practice of error isolation. The responses indicate a strong tendency to isolate errors whose cause is perceived as unique. An analysis of isolation/projection decisions as a function of firm membership, experience, and training in statistics indicates that the tendency to isolate errors is, for individual cases, related to firm membership and experience. However, the observed firm membership and experience effects are not consistent across all cases. Thus, the results are not well explained by a simple model, which posits that inappropriate error isolation is limited to a few firms or to inexperienced auditors. While incorrect error isolation appears to be prevalent in practice, Burgstahler and Jiambalvo suggest several factors, which may mitigate its adverse effects. Their analysis suggests that isolation of errors by auditors may lead to inappropriate inferences.

One of the approaches to deal with the error analysis of the accounting population and a sample drawn from it was to use the Bayesian analysis to obtain an upper bound for the total population error amount as estimated from the sample output. The main logic behind using Bayesian approach was using the fact that auditors generally have good insight in the characteristics of the accounting population at hand. This is used as a base to choose for a manageable prior distribution to deal with. In fact it is hoped that using these results and similar resonates of this kind, the auditing profession may arrive at the position that official guidelines will be available prescribing the choice of the a priori distribution in any possible situation.

The Bayesian methodology has been proposed when monetary unit sampling is employed. Cox and Snell (1979) considered Bayes estimators for rare errors starting from an infinite population model and using cleverly chosen prior distributions. In this way an upper confidence limit for the total error in the population was obtained. They proposed a Bayesian bound for which they assumed that the population mean taint for monetary units in error are independent parameters. These and other assumptions lead to a posterior distribution of the total error amount that is a simple multiple of the F distribution. Hence, Bayesian bounds are very easy to obtain by this method. However, Moors (1983) showed that their result contains an error, even if they gave a correct formula with a full derivation. Moors gave the correct values and clearly showed that the Bayes upper limit given is far less conservative than the limits met in practice. And finally, the probability distribution of the new confidence limit is calculated for a theoretical population.

Neter and Godfrey (1985) also studied the behaviour of the Cox and Snell bound in repeated sampling from a given population for a sample of size 100. They found out that conservative prior parameter values exist so that the Cox and Snell bound has convergence near or above the Bayesian probability level for a wide range of populations. However, research in progress suggests that robustness may be a function of sample size, and not include all possible populations.

McCray(1984) presents a theoretical Quasi- Bayesian (QB) model for dollar-unit sampling, DUS. This model generates a discrete posterior probability distribution on the expected total error in an accounting population for any dollar unit sample and any given discrete or continuous prior probability distribution on the expected total error in the population. The model can be used with any sample size and any number of overstatements and understatements. In addition, it is the only dollar unit sampling evaluation procedure that can use data on the proportion of total dollars of each tainting found in the sample, or known or assumed to exist in the population. This is possible for this model even if this information is not a sample information. Based on all comparisons of the upper bounds of the proposed QB model and multinomial models, it appears that the proposed model is an acceptable model for evaluating dollar unit samples even if an informative prior probability distribution on the total expected error is not available.

Stringer (1963) developed a heuristic procedure to reduce the size of an upper confidence bound for the total error amount when all observed taints are not at the maximum value of 1.0. Simulation studies by Reneau (1978) have shown that the Stringer bound has a coverage exceeding the nominal level and often close to 100 percent, and that the bound is not very tight and it may involve substantial risks of making incorrect decisions when the bound is used for testing purposes.

Fienberg et al. (1977) developed a bound based on the multinomial distribution by viewing monetary unit sampling in discretized form. The multinomial classes represent the different possible taints rounded to a specified degree. The procedure involves obtaining a joint confidence region for the multinomial parameters and then maximising a linear function of the parameters, representing the total error amount in the population, over the confidence region. Because of the complexities of computation, Fienberg et al. utilised only a partial ordering of the sample outcomes for developing the joint confidence region. However, simulation studies like Plante et al. (1985) in their publication on auditing, have shown that coverage for the multinomial bound are near to or above the nominal level for a variety of populations, particularly if cell sampling is employed.

Neter et al. (1978) also presented a statistical sampling approach based on the multinomial distribution for obtaining a bound for either total population understatement or overstatement errors or both. The approach is non-parametric (distribution free) in nature and, unlike most of the techniques, it has known characteristics so that the auditor is assured of the specified confidence level regardless of the nature of the population and the nature of the error pattern. Results were presented which show the multinomial bound to give tighter bounds than the Stringer bound in all instances studied. The behaviour of the multinomial bound is studied with respect to the effects of sample size and error patterns.

The non-parametric approach and known confidence levels are assured for all situations, no matter what the nature of the population, the nature of the error pattern, or the frequency of the errors. This is due to the fact that very little was known at that time about the characteristics of audit populations and error patterns, and a non-parametric approach which provides known confidence levels regardless of the nature of the population gives auditors an important assurance.

Leitch et al. (1982) examined the characteristics of modified multinomial bounds for errors in an audit population and compared with those of the widely used Stringer bound. It was found that the modified multinomial bound was usually considerably tighter than the Stringer bound and should be useful in many audit applications. The confidence level of the modified multinomial bound was also investigated by sampling simulation, and it was found that the level exceeds or is close to the nominal level for all populations studied.

In their modification they grouped the observed tainting (percentage errors) into clusters and treat all tainting in a cluster as being equal to the largest one in the cluster. They presented, first, an overview of the modified multinomial bound for overstatement errors and characteristics of the modified multinomial bound for different overstatement tainting distributions commonly found in audit situations, and study the comparative tightness of the modified multinomial bound relative to the Stringer bound. Also they considered the effects on the modified multinomial bound of the error rate, sample size and number of clusters used. This bound was based on clustering of tainting when there are between 8 and 25 or 6 and 30 errors in the sample. Because it has confidence levels that exceed or are near the specified nominal level and is significantly tighter than the commonly used stringer bound over a wide range of error distributions encountered in auditing populations. Thus, the procedure presented in their paper helps to close the gap between use of the unclustered multinomial bound for eight or fewer errors and use of classical large-sample procedures when there are many errors in the audit sample.

Tsui et al. (1985) take a Bayesian approach to incorporate auditors prior information in constructing error bounds for the proportion of dollar amount overstated (equivalently, the total amount overstated) in an accounting population. Their analysis employs the multimomial distribution model for the distribution of dollar units in the sample. The auditor's prior knowledge of the error rate and the tainting distribution is incorporated via the Drichlet distribution. Theoretically, this method of incorporating prior information should greatly improve the accuracy of the upper error bound for the proportion of the amount in error. That is, the Bayesian bounds may be very tight, depending on the appropriateness of the prior distribution. Their numerical studies substantiate the benefits of this approach. Simulation results suggest that some of the Bayesian bounds have good repeated sampling properties. In particular, in repeated sampling, a Bayesian bound gives a significance level close to the nominal level for many typical accounting populations. Another advantage of the Multinomial-Dirichlet bound approach is computational efficiency, which is independent of the sample size and the number of errors found in the sample. This analysis not only provides upper error bound, for the proportion of the amount in error, but also provides its entire posterior distribution. The posterior distribution can be used to obtain other quantities of interest, such as the expected value, the variance, the median or the mode of the proportion of the amount in error. Moreover, cumulative posterior probabilities of the error amount can be tabulated quickly so that the auditor can appraise the magnitude of the true proportion of monetary amount in error.

Accounts, such as accounts receivable and inventory, often consist of thousands of individual line items that have a total value of millions of dollars. For such large accounts, it is not economical for the auditor to audit every line item. Consequently, auditors frequently select a sample of line items and audit these, on the basis of which inferences are made about the total error amount in the population. Thus, the characteristic of interest here is quantitative.

Auditors require empirical information about the characteristics of errors in audit populations. Johnson et al. (1981) examined the error characteristics in 55 accounts receivable and 26 inventory audits. First, the error rates present in these audits are analysed, and balances between overstatement and understatement errors were studied. The distribution of the error amounts and error tainting were then studied, as well as the relation between error amounts and book amounts. Among others they found out that:

- Most errors in receivable audits are overstatement errors while in inventory audits overstatement and understatement errors are more balanced in numbers.
- Analysis of the 20 audits with the largest numbers of errors indicated that receivable errors tend to be larger and less variable than inventory errors.
- The distribution of error amounts for both types of audits are far from normal, exhibiting both greater peakedness near that mean and "fatter" tails in the upper directions.
- Most of the distributions are positively skewed, and the standardised distributions of the error amounts for each types of audit tend to be very similar.

- A study of intra-audit data for 20 audits failed to disclose any strong linear relationship between error amount and book amount for most audits, but showed a definite tendency for errors for larger book amounts to be more variable, particularly for inventory audits.

Neter et al. (1985) extended the analysis of line-item error taints and error rates they presented in their former work by first considering the distribution of dollar unit taints, the relevant distributions was when simple random sampling is applied to monetary units. They found, among others, that:

- Dollar-Unit error rates for both types of audits tend to be higher than the line item error rates.
- There is great variability in the dollar-unit error rates among audits. Also, the dollar unit error rates tend to be substantially higher for inventory audits than for receivable audits. These findings go parallel with the earlier ones for line item error rates.

The distributions of the proportions of overstatement errors in audits when measured on a dollar unit basis are very similar to the corresponding distributions measured on a line-item basis.

On the projection of sample result to the population, Dworin and Grimlund (1986) gave results on the relative performance of two audit procedures for dollar unit sampling - the Quasi-Bayesian (QB) method and a slightly modified version of the Moment Method (MM) developed by

Dworin and Grimlund (1984). An extensive simulation study of 96 representative accounts receivable and inventory populations were conducted. Both methods were found to be reliable over the tested range, and both provided tighter confidence bounds than those provided by the Stringer bound. This was the first study that has been made of the reliability of the Quasi-Bayesian bound and of the comparative performance of the Quasi-Bayesian and the Moment Methods. The Moment Method provided a tighter confidence bound at low error rates. For populations with high error rates, for which the true mean error was either very nearly zero or very large, the Quasi-Bayesian method provided a tighter bound. Both bounds provided comparable result for the remaining intermediate cases. They concluded that the performance of neither method uniformly dominates the other.

Several methods of evaluating upper error bounds for dollar unit samples have been proposed for use in auditing. Grimlund and Felix (1987) present results of a simulation study that compared four of these methods, and proposed additional simulation on alternative forms of two of them . Evidence on the reliability and relative size of the upper error bounds for each method is provided for 2,160 different combinations of sample sizes, confidence levels, and types of accounting populations. The results provide a comprehensive analysis of the capabilities of these methods. All four methods have strengths and weaknesses. The Multinomial-Dirichlet bound is quite reliable but very conservative and (as formulated) not able to utilise understatement errors. The two versions of the Cox and Snell bound that were tested with different priors varied considerably in their reliability over a range of confidence levels. While not without an occasional reliability problem, test results for the Moment Method bound and one of the two Bayesian-Normal bounds were largely favourable.

CHAPTER THREE

Objective, Data and Methodology

3.1 Theory

In an audit sampling application whenever a statistical approach is used to project quantitative characteristics such as a monetary amount we say we are applying variable sampling methods. In designing such an application, the accountant or auditor must consider whether his or her audit objective is: (1) to make an independent estimate of some amount, or (2) to test the reasonableness of a financial statement representation, i.e. accuracy of recorded balances. When an accounting balance is to be determined using statistical sampling, the accountant should use an accounting estimation approach.

When the auditor desires to accept a client's record without adjustment if it is reasonably correct, or to propose an adjustment only if it is probable that there might be a material misstatement in the amount as stated by the client, an audit hypothesis approach is used. This statistical method discriminates between the null hypothesis that the amount as recorded is correctly stated with the alternative hypothesis that the amount is materially misstated. The actual testing of the hypothesis involves the design and application of statistical sampling plan, which meets certain specified objectives. These objectives include an expression of the reliability desired for the sampling plan in terms of the risk factors of incorrect rejection (the alpha risk)and/or of incorrect acceptance (the beta risk).

Beta risk, commonly known as detection risk in auditing is the risk of incorrectly accepting a book value that is materially in error. Statistically, the other counterpart of beta risk is alpha risk which is the risk of incorrectly rejecting a book value that is stated fairly. The two risks collectively form the sampling risk, the risk that the auditor's conclusions regarding the accounting population of interest are incorrect.

The theory of hypothesis testing as applied to assessing audit risk components may be represented as follows:

	T_o	F_o
Audit conclusion	Book value is correct	Book value is materially misstated
H_o: Book value is fairly stated	Correct Judgment	Beta risk
H_1: Book value is in error by a material amount	Alpha risk	Correct Judgment

Under the null hypothesis the book value is fairly stated, whereas under the alternative hypothesis the book value is materially misstated. If the null hypothesis is chosen and the book value is fairly stated (valid state, T_o), a correct judgment has been made. If, on the other hand, the book value is materially misstated (Invalid state, F_o) an incorrect decision has been made. The risk of accepting the null hypothesis when the alternate state exists is the beta risk. The risk of accepting the alternate hypothesis when the true or valid state exists is the alpha risk.

3.1.1 Quantifying The Risk Factors

Auditors do not necessarily need to quantify audit risk for every engagement. Pre-assigned quantities can also be used. Quantifying audit risk requires assessment of control risk; the risk that material errors or irregularities are not prevented or detected by the structure of underlying internal controls. Control risk is evaluated by performing a pre-test of the auditee organization.

Rule of thumb (Konrath,1996).

Complement of overall audit risk is reliability.

Alpha risk is assigned equal to overall audit risk.

Beta risk is evaluated from the relation that overall audit risk is viewed as the joint probability of beta risk and control risk. That is AR=CRxBR which implies BR = AR/CR,

where AR is an overall audit risk, CR the control risk and BR is the desired beta risk.

The risk related to the susceptibility of an account balance or class of transactions to errors that could be material, inherent risk, is considered to exist in every audit work.

The other concept in audit sampling application is the idea of materiality. In an accounting sense, a matter is material if its omission or misstatement would reasonably influence the decisions of the users of the auditor's report. From the audit point of view anything that would distort the view given on accounts must lead to report qualification, but only if it is material. A tolerable amount of misstatement, materiality M is set by the Audit body which can be used for any audit condition. It is set as a working policy for the audit organization. The concept of materiality is used in audit sampling application to help in determining the sample size required for detailed audit of an accounting population. The application of the above concepts to audit sampling will be seen later. The following statistical accounting estimation designs (models) will be discussed in this chapter. The approaches for analysis are unstratified, stratified and unequal probability treatment. We shall compare:

Unstratified mean-per unit, difference and ratio estimation.

Stratified mean per unit, difference and ratio estimation.

And from unequal probability, sampling with probabilities proportional monetary size will be dealt with.

3.2 The Unstratified Methods

The sampling design used here is simple random sampling. Then after taking a random sample from the study population we shall compare the precision of each estimator to see how good the true audited amount is estimated.

3.2.1 Unstratified Mean Per Unit Estimation

The unstratified mean per unit (MPU) model is used to project an estimated monetary value for a simple random sample. After a sample is selected and a value determined from each sample item, the sample mean of audited values is multiplied by the number of items in the population to get the estimate of the total audited monetary amount for the sampled population.

In an unstratified MPU application the objective is to calculate a sample mean to project as the population total which is obtained by multiplying the sample average audited amount by the total number of units in the population. Of course the $N\bar{x}$ projection will not correspond exactly to the true but unknown population total. But the projection plus and minus a precision limit should contain the true population total with a defined reliability level. Consequently, the point estimate so obtained must be paired with a reliability percentage and a precision limit. Symbolically the estimate is expressed as:

$$\hat{X}_{MPU} = N\bar{x} \qquad \text{with} \qquad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

where $\bar{x}$ is the sample mean of audited amount, $\hat{X}_{MPU}$ represents the mean per unit estimate of the total audited amount, and N the total number of transaction items in the population.

The sample size required for unstratified design estimators is given by, (Roberts,1978).

$$n = \left(\frac{N^2 Z^2_{\alpha/2} S^2_x}{A^2 + N Z^2_{\alpha/2} S^2_x} \right) \qquad \text{with} \qquad S_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

where $Z^2_{\alpha/2}$ is the square of a 100(1-α)% confidence level coefficient

S_x is the estimated standard deviation of the population audited amounts.

A is the planned or acceptable precision

The acceptable precision A in the formula for sample size is obtained from the amount assigned for materiality M by the audit institution, and multiplying this by a factor obtained from the alpha and beta risk expressed as, (Konrath, 1996).

$$A = \left(\frac{Z_{\alpha/2}}{Z_{\alpha/2} + Z_{\beta}} \right) M$$

where α and β denote the alpha and beta risks respectively and are as described above.

$Z_{\frac{\alpha}{2}}$ and Z_{β} are the tabulated values of the standard normal table.

Here both alpha and beta risk must be set by the auditor as a condition for determining sample size. Beta risk is a function of the auditors' study of the business, application of analytical procedures, and study and evaluation of internal control policies and procedures. Auditors usually consider a beta risk of less than 50 percent. On the other hand, most auditors simply equate alpha risk with overall audit risk, setting it at less than or equal to 10 percent.

The data on which this study is done were obtained from Amanuel Hospital Special Pharmacy for the 1989 budget year expenditure. For unstratified sample a 5 percent materiality level was taken. An alpha risk of 5 percent and a beta risk of 33 percent are considered. Using this in the above formula for n, a sample size of 312 was determined and taken from a population of 672 items of which 68 items are in error. Ten different random samples were considered to compare samples with number of errors equal to zero, one, two etc. and to test the efficiency of the estimators while estimating the total audited amount of the target population. The finite population correction (fpc) is ignored from the computation of precision A' if ((N-n)/N)≥ 0.95, otherwise it has to be included.

The evaluation of the MPU estimator of the total audited amount was done by using the achieved precision given by, (see Roberts,1978).

$$A'_{MPU} = \frac{\left(N Z_{\frac{\alpha}{2}} S_x \sqrt{\frac{N-n}{N}}\right)}{\sqrt{n}}$$

In order to accept or reject the sample result we compare this with the planned acceptable precision. If the value of A'_{MPU} is less we accept our sample result.

3.2.2 Unstratified Difference Estimation

The unstratified difference estimation can be employed by finding the estimate for the total difference value. The existence of a recorded amount for each sampling unit makes it possible to define a difference as the audited amount minus the recorded amount. This way of defining a difference is preferred since it shows the direction of adjusting the recorded amount. Thus the estimate is given by:

$$\hat{D} = N\bar{d} \qquad \text{with} \quad \bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$$

where d_i is the difference of the book value from the audited value, n is the sample size.

The estimate for the total audited value is:

$\hat{X}_D = Y + \hat{D}$ depending on the sign of the difference.

Y is taken to be the total recorded monetary value

$\hat{X}_D$ is the estimate for the total audited value using the estimated difference amount.

The sample evaluation is done by computing the achieved precision using the formula:

$$A_D' = \frac{(N Z_{\alpha/2} S_D \sqrt{\frac{N-n}{N}})}{\sqrt{n}} \qquad \text{where} \quad S_D^2 = \frac{\sum (d_i - \bar{d})^2}{n-1}$$

3.2.3 Unstratified Ratio Estimation

The population ratio is defined as the total audited amount divided by the total recorded amount. When the total audited amount is less than the total recorded amount this ratio is less than one which means the population recorded amount is overstated. Otherwise it is understated. From a random sample of n units, the auditor establishes the audited amounts together with the corresponding recorded amounts. The auditor can then form the estimated population ratio by dividing the total sample audited amount by the total sample recorded amount. In symbols it can be expressed as:

$$\hat{R} = \frac{\sum_{1}^{n} x_j}{\sum_{1}^{n} y_j}$$

where $\hat{R}$ is the estimate for the population ratio.

x_j is a sample audited value

y_j is a sample-recorded value

The estimate of the total audited amount is then computed by multiplying the estimated population ration times the known recorded amount. Therefore,

$$\hat{X}_R = \hat{R} Y$$

where $\hat{X}_R$ Is the estimated total audited amount using the ration estimated.

$\hat{R}$ the estimate of the population ratio

Y the total recorded amount.

The achieved precision of this estimate is calculated by the following formula, (see Roberts, 1978).

$$A'_R = \frac{(N Z_{\alpha/2} S_R \sqrt{\frac{N-n}{N}})}{\sqrt{n}} \quad \text{with} \quad S_R^2 = \frac{\sum_{i=1}^{n} x_i^2 + \hat{R}^2 \sum_{i=1}^{n} y_i^2 - 2\hat{R} \sum_{i=1}^{n} x_i y_i}{n-1}$$

where S_R is the estimated standard deviation of the population ratios,

A sample result is accepted if the achieved precision is less than the planned precision used to compute sample size.

3.3 The Stratified methods

Stratifying the variable of study is useful to improve efficiency by reducing sample sizes for desired levels of precision and reliability. Stratified random samples achieve efficiency by grouping sampling units with similar characteristic into one stratum. In this way the variability within a stratum is small and among strata is higher or units within a stratum are homogenous and those among different strata are heterogeneous.

For the purpose of this paper a stratum was formed in such away that approximately equal monetary amount is available within each stratum after each population unit is arranged with its monetary value from the smaller to the larger, (Roberts, 1978). Five strata were formed using this technique and the analysis was done using the result so obtained.

The allocation of the sample to the strata was done using the proportional allocation technique by multiplying the total sample size with the ratio of the stratum total monetary amount to the total recorded monetary amount.

For stratified application, the sample size from the study population of N = 672, was computed using, (see Roberts, 1978).

$$n = \frac{Z^2_{\frac{\alpha}{2}} Y \sum_{h=1}^{L} N_h^2 \frac{\sigma^2_{Yh}}{Y_h}}{A^2 + Z^2_{\frac{\alpha}{2}} \sum_{h=1}^{L} N_h \sigma^2_{Yh}}$$

where Y is the total recorded monetary amount.

Y_h is the total recorded monetary amount of the h^{th} stratum.

N_h is the h^{th} stratum total number of transactions.

The planned precision A is the same as used for unstratified case.

σ_{Yh} is the population standard deviation of units of the population or its estimate.

For the total population recorded amount of Birr 2,479,079.59 of size N = 672, a materiality of 5 percent was Birr 123,953.98 and a planned precision for an alpha risk of 5 percent and a beta risk of 33 percent was Birr 101,229.08. Using these and the sample size required for a 95 percent reliability level was computed to be n=50. The allocation will be n_h=10 for h=1,2,...5 at each stratum. That is we select 10 payment vouchers randomly from each stratum.

The estimation was done using the values of the above result for total and stratum sample sizes and the monetary amounts stated. The first estimation is:

3.3.1 Stratified Mean Per Unit Estimation.

The stratified mean per unit estimate of the total audited amount is computed from the sum of the mean per unit estimates over all strata. When there are L strata this is represented as:

$$\hat{X}_{MPUS} = N_1\bar{x}_1 + N_2\bar{x}_2 + \dots + N_L\bar{x}_L = \sum_{h=1}^{L} N_h\bar{x}_h$$

where $\hat{X}_{MPUS}$ is the stratified mean per unit estimate of the total audited amount.

X_h is the hth stratum sample mean of audited amounts.

The result obtained for the above total estimate is evaluated by computing the achieved precision of the estimated total audited amount as:

$$\hat{A}_{MS} = Z_{\frac{\alpha}{2}} \sqrt{\sum_{h=1}^{L} N_h (N_h - n_h) \frac{S_{xh}^2}{n_h}} \qquad \text{with } S_{xh}^2 = \frac{\sum_{i=1}^{n_h} (x_{hi} - \overline{x}_h)^2}{n_h - 1}$$

where n_h is the hth stratum sample size.

S_{xh} is the hth stratum standard deviation of the audited amount.

3.3.2 Stratified Difference Estimation

The stratified difference estimate of the total difference amount is composed of the sum of the difference estimates over all strata. If we deal with L strata, the estimated total difference is given by:

$$\hat{D}_S = \sum_{h=1}^{L} N_h \overline{d}_h = N_1 \overline{d}_1 + N_2 \overline{d}_2 + \dots + N_L \overline{d}_L$$

where $\hat{D}_S$ is the stratified difference estimator

d_h is the hth stratum mean difference.

The estimated total audited amount based on the stratified population difference estimator is given by:

$\hat{X}_{DS} = Y + \hat{D}_s$ Depending on the sign of the difference.

$\hat{X}_{DS}$ being the estimate for the total audited monetary amount using the stratified difference estimator, and Y total recorded monetary amount of the population.

The result of the difference estimation is evaluated by computing the achieved precision of the stratified difference estimator at a pre-specified reliability α as:

$$A_{DS} = Z_{\alpha/2}\sqrt{\sum_{h=1}^{L} N_h(N_h - n_h)\frac{S_{Dh}^2}{n_h}} \quad \text{with } S_{Dh}^2 = \sum_{j=1}^{n_h} \frac{d_{hj}^2 - n_h \bar{d}_h^2}{n_h - 1}$$

where S_{Dh} is the standard deviation of the differences in h^{th} stratum.

3.3.3 Stratified Ratio Estimation

The stratified ratio estimator most commonly used in auditing is the combined ratio estimator. Taking the ratio of the stratified mean estimator of the audited amounts to the stratified mean estimator of the recorded amounts forms the combined ratio estimator of the total audited amount. The estimate for the total audited amount is computed using this ratio multiplied by the total recorded amount. The expression is written as:

$$\hat{X}_{RC} = \left(\frac{N_1\bar{x}_1 + N_2\bar{x}_2 + \dots + N_L\bar{x}_L}{N_1\bar{y}_1 + N_2\bar{y}_2 + \dots + N_L\bar{y}_L}\right)Y = \left(\frac{\sum_{h=1}^{L} N_h\bar{x}_h}{\sum_{h=1}^{L} N_h\bar{y}_h}\right)Y = \hat{R}_C Y$$

where $\hat{R}_C$ is the combined ratio estimator.

Evaluation of the ratio estimation result is again done by computing the achieved precision of the combined ratio estimator at a specified reliability. It is expressed by means of the following formula, (see Roberts, 1978).

$$A'_{RC} = Z_{\alpha/2}\sqrt{\sum_{h=1}^{L} N_h(N_h - n_h\frac{S_{RCh}^2}{n_h}} \quad \text{with}$$

$$S_{RCh}^2 = \left(\frac{\sum_{i=1}^{n_h} x_{hi}^2 + \hat{R}_C^2 \sum_{i=1}^{n_h} y_{hi}^2 - 2\hat{R}_C \sum_{i=1}^{n_h} x_{hi} y_{hi}}{n_h - 1} \right)$$

where S_{RCH} is the estimated stratum variance of the population ratio.

3.4 Sampling with probability Proportional to Monetary Size (PPMS)

Selecting sample items with probabilities proportional to the recorded monetary amounts (a PPS sampling application) is an alternative to stratifying the population by recorded amounts. Both techniques give greater weight to items with large recorded amounts than to items with small-recorded amounts. Selecting items with probability proportional to recorded amount is somewhat simpler than stratified sampling. This, together with its use in populations where differences between audited and recorded amounts are rare, has made this selection technique increasingly popular in auditing practices.

The main objective of PPMS sampling is to get an estimate of the total error amount from the population of interest. And the auditor compares this maximum error limit to the population tolerable misstatement, materially. PPMS sampling is commonly known to be Monetary Unit Sampling.

3.4.1 Selection Method

For the purpose of this paper, sample selection is done using a form of systematic selection with unequal probability. The method is started by cumulating the recorded monetary amounts on each unit of the population i.e. Y_1, Y_2, ... Y_N and assign them the ranges 1 to Y_1, Y_1+ 1 to $Y_1 + Y_2$, $Y_1 + Y_2$ + 1 to $Y_1 + Y_2 + Y_3$, and so on. Then a random sample of size n is selected. To do this we first draw a random number between 1 and k =Y/n, Y being the total monetary amount. The units in the sample are those in whose range lie the random number i and all other numbers i+k, i+2k, i+3k, and so on are obtained by adding k successively to i till the total sum is reached. Any unit whose measure of size greater than or equal to Y/n, it is definitely included in the sample.

The probability that any unit Y_i is included in the sample is obviously $(Y_i/(Y/n)) = n(Y_i/Y) = np_i$, (see Raj, 1979).

3.4.2 Determining the Sample Size

The formula for determining sample size required to achieve a desired precision at a specified reliability is given by, (see Roberts, 1978).

$$n = \frac{Y^2 Z^2_{\alpha/2} S^2_p}{A^2} \quad \text{with} \quad S^2_p = \frac{\sum_{i=1}^{n}\left(\frac{x_i}{y_i}\right)^2 - \frac{1}{n}\left(\sum_{i=1}^{n}\frac{x_i}{y_i}\right)^2}{n-1}$$

A is the planned Accepted Precision (used earlier).

When the population error rate is expected to be as high as 10 percent or more, the appropriate PPMS estimate of the total audited amount is given by:

$$\hat{X}_{PPMS} = Y\left(\frac{1}{n}\sum_{i=1}^{n}\frac{x_i}{y_i}\right)$$

x_i the i[th] unit of sample audited amount.

y_i the i[th] unit of sample recorded amount.

It can be shown that the total difference can be estimated as follows:

$$D_{PPMS} = \hat{X}_{PPMS} - Y$$

The achieved precision for the estimate of the total audited amount is given by:

$$\acute{A}_{PPMS} = \frac{Z_{\frac{\alpha}{2}} Y S_p}{\sqrt{n}}$$

where S_p is as given above, Y is the total recorded value and n the sample size determined above.

3.5 Summary of the Foregoing Discussion

The application of any statistical sampling techniques in auditing commonly involve three phases, namely planning, execution, and evaluation. In this chapter three unstratified variable sampling methods and three stratified variable sampling methods from equal probability sampling, and one from unequal probability sampling methods are discussed theoretically. In addition to these, concepts relevant to auditing in relation to monetary amounts of transaction items like the risk factors, materiality or tolerable misstatement was also incorporated. The study commenced with these concepts in mind.

The planning aspect is common in auditing. It includes specifying the amount of monetary misstatement that can be tolerated, the risks involved, the planned precision, the determination of sample size, the number of strata, location of stratum boundaries, and the allocation of the sample to the strata.

Some theoretical results particular to audit sampling developed based on the nature of an accounting population were also taken from publications. These usually have to do with the error distribution in an accounting population. According to such theoretical results the study population for this paper can be considered as a high error rate population, (see Guy, 1994).

CHAPTER FOUR

Data Analysis and Result

In a situation requiring the projection of a total audited amount accounting estimation is the appropriate tool to use. By using the point estimate of the sample a projection can be made of the estimated population value. The estimated population value plus and minus calculated precision is expected to contain the true but unknown value at a defined pre specified reliability level.

To compare the efficiency of the point estimators used in statistical sampling, the expenditure data of 672 transaction items were considered.

The following summary result was obtained for the population:

TABLE 1

Description	Recorded Amount in Birr	Audted Amount in Birr	Error Amount in Birr
Total Sum	2479079.59	2457708.17	21371.42
Average	3689.24	3657.30	31.80
St.Deviation	1851.24	1858.53	234.38
C.V.	0.50	0.51	7.37
Skewness	0.54	0.54	15.64
Min.Value	248.95	248.95	0.00
Max. Value	10133.25	10133.25	5000.00

4.1 Unstratified Random Sample

The unstratified random sampling is appropriate when a book value for each population item is not available or when the recorded total of the book value is not accurate. The sample design considered here were simple random sampling. Then the unstratified mean per unit estimation, difference estimation and ratio estimation were considered with their corresponding point estimate and achieved precision. To compute the sample size a 5 percent materiality level of the total recorded monetary amount was taken. The planned precision considered was obtained by using a beta risk of 33 percent and an associated alpha risk of one minus reliability where reliability was considered to be 95 percent. The computation is summarized as follows:

Total monetary Amount	2,479,079.59
Materiality	123,953.98
Planned precision, A	101,229.08
Correlation between recorded and audited	0.992012
N	672

Sample size, $n = \frac{N^2 Z^2_{\frac{\alpha}{2}} S^2_x}{A^2 + N Z^2_{\frac{\alpha}{2}} S^2_x} = 311.36 \approx \underline{312}$

A random sample of 312 transactions was selected and after finding the point estimate for the total audited amount with its corresponding 95 percent reliability level for the precision we have the following results given in tables 2-4.

TABLE 2

Unstratified Mean Per Unit result/figures in Birr/				
Description	Total Recorded Amount	Estimated Audited Amount	Difference	Precision
Sample1	2479079.59	2480083.20	1003.61	100924.62
Sample2	2479079.59	2457336.00	-21743.59	107923.50
Sample3	2479079.59	2455467.80	-23611.79	100625.77
Sample4	2479079.59	2511169.90	32090.31	102148.95
Sample5	2479079.59	2485479.40	6399.81	101078.86
Sample6	2479079.59	2497373.80	18294.21	101936.86
Sample7	2479079.59	2400444.50	-78635.09	102110.38
Sample8	2479079.59	2457430.10	-21649.49	97376.98
Sample9	2479079.59	2512473.60	33394.01	104077.01
Sample10	2479079.59	2462665.00	-16414.59	99497.86

TABLE 3

Unstratified Difference Result/figures in Birr/				
Description	Total Recorded Amount	Estimated Audited Amount	Difference	Precision
Sample1	2479079.59	2453739.22	-25340.37	16465.99
Sample2	2479079.59	2454830.99	-24248.60	16670.95
Sample3	2479079.59	2449099.98	-29979.61	17337.82
Sample4	2479079.59	2446932.64	-32146.95	17967.46
Sample5	2479079.59	2467449.92	-11629.67	5434.26
Sample6	2479079.59	2459884.68	-19194.91	7216.05
Sample7	2479079.59	2452418.16	-26661.43	16813.67
Sample8	2479079.59	2460367.68	-18711.91	7856.87
Sample9	2479079.59	2451489.10	-27590.49	17374.11
Sample10	2479079.59	2448702.82	-30376.77	17862.12

TABLE 4

Unstratified Ratio Result /figures in Birr/				
Description	Total Recorded Amount	Estimated Audited Amount	Difference	Precision
Sample1	2479079.59	2450005.70	-29073.89	16925.61
Sample2	2479079.59	2454855.30	-24224.29	16669.10
Sample3	2479079.59	2449181.40	-29898.19	17296.19
Sample4	2479079.59	2447744.50	-31335.09	17936.39
Sample5	2479079.59	2467534.00	-11545.59	5421.74
Sample6	2479079.59	2460170.40	-18909.19	7257.73
Sample7	2479079.59	2451847.20	-27232.39	16797.36
Sample8	2479079.59	2460328.10	-18751.49	7893.01
Sample9	2479079.59	2452151.30	-26928.29	17384.07
Sample10	2479079.59	2448872.80	-30206.79	17832.69

In unstratified mean per unit a large sample size relative to other designs resulted. The true audited amount Birr 2,457,708.17 is estimated with a corresponding achieved precision. But the figures obtained for precision are very large, and some of them exceed the planned precision but the point estimates for the total audited amount are fair. Also see appendices 1-1 and 1-2. In table 2 five of the errors are overstatements i.e. all recorded values exceed the true values, where in table 3 and table 4 all differences depict overstatements, while the rest show understatement.

The precisions obtained due to unstratified difference and ratio estimators are almost equivalent. The point estimates for both are closer to the true audited value. The errors are all overstatement errors and comply with the actual result. This is due to a high correlation between the population recorded and audited amounts.

4.2 Stratified Random Sample

Stratified sampling is applied when the population elements vary in amount. It is efficient if variation among strata is high and variation within a stratum is small. Using the sample data for reliability and precision as in the unstratified case the stratified sample size determined was:

$$n=\frac{Z_{\alpha/2}^{2}Y\sum_{h=1}^{L}N_h^2\frac{\sigma_{Yh}^{2}}{Y_h}}{A^2+Z_{\alpha/2}^{2}\sum_{h=1}^{L}N_h\sigma_{Yh}^{2}}=49.3\approx 50$$

The results of the analysis are presented in tables 5-7 below.

TABLE 5

Stratified Mean Per Unit Result /figures in Birr/				
Description	Total Recorded Amount	Estimated Audited Amount	Difference	Precision
Sample1	2479079.59	2498945.30	19865.71	88022.75
Sample2	2479079.59	2466570.00	-12509.59	94960.39
Sample3	2479079.59	2403257.60	-75821.99	121084.96
Sample4	2479079.59	2331291.30	-147788.29	76572.02
Sample5	2479079.59	2450717.70	-28361.89	85727.36
Sample6	2479079.59	2452359.10	-26720.49	98389.89
Sample7	2479079.59	2464691.60	-14387.99	109624.84
Sample8	2479079.59	2449161.10	-29918.49	134221.08
Sample9	2479079.59	2534837.50	55757.91	94174.74
Sample10	2479079.59	2507451.80	28372.21	89039.18

TABLE 6

Stratified Difference Result /figures in Birr/				
Description	Total Recorded Amount	Estimated Audited Amount	Difference	Precision
Sample1	2479079.59	2477346.69	-1732.90	2689.86
Sample2	2479079.59	2469274.94	-9804.65	13776.55
Sample3	2479079.59	2475697.59	-3382.00	6245.41
Sample4	2479079.59	2464975.75	-14103.84	17629.22
Sample5	2479079.59	2460504.99	-18574.60	18986.90
Sample6	2479079.59	2462574.16	-16505.43	24013.35
Sample7	2479079.59	2449111.75	-29967.84	32626.74
Sample8	2479079.59	2452769.95	-26309.64	26158.21
Sample9	2479079.59	2464863.13	-14216.46	21972.95
Sample10	2479079.59	2473002.89	-6076.70	7790.05

TABLE 7

Stratified Ratio Result /figures in Birr/				
Description	Total Recorded Amount	Estimated Audited Amount	Difference	Precision
Sample1	2479079.59	2477362.10	-1717.49	2795.07
Sample2	2479079.59	2469257.00	-9822.59	14377.12
Sample3	2479079.59	2475595.80	-3483.79	6520.21
Sample4	2479079.59	2464417.12	-14662.47	17393.43
Sample5	2479079.59	2460376.20	-18703.39	5230.78
Sample6	2479079.59	2462505.90	-16573.69	25952.87
Sample7	2479079.59	2449299.30	-29780.29	34000.53
Sample8	2479079.59	2455868.10	-23211.49	28250.07
Sample9	2479079.59	2465253.40	-13826.19	23123.04
Sample10	2479079.59	2473086.20	-5993.39	8024.35

The point estimate for the total audited amount is far from the true result in about four of the cases. The total difference is not also a good estimate of the true error amount. The achieved precision exceed the planned precision in three of the samples for stratified mean per unit sampling. This calls for the amendment of the sampling plan. In the case of the difference and ratio estimators, the point estimate for the total audited amount is not closer to the true amount. The precision obtained is not as such satisfactory since in some of the cases it is very small. The error part is not also good in about four of the cases. See appendices 2-1 and 2-2.

When there are no errors or when the recorded and audited amounts agree in both the stratified and unstratified case, the difference and the ratio estimators do not give a result different from those obtained by the mean per unit estimators. Their computational formula also reveals this fact. From the stratified sample analysis, these estimators are not also efficient when only a few errors are realised in the sample, or when the population under audit is only of a low error rate.

4.3 Sampling With Probability Proportional To Size

The unequal probability sampling in auditing is applied for both small error and large error occurrences in the population. For this study the large error rate population application is considered. The result of the data analysis is tabulated as follows. And the sample size computed was 46 using the formula:

$$n=\frac{Z^2_{\alpha/2}Y^2S^2_p}{A^2}=45.6\approx46$$

Table 8 gives the results of the analysis obtained from PPMS.

TABLE 8

Sampling Proportional to Monetary Size Result/figures in Birr/				
Description	Total Recorded Amount	Estimated Audited Amount	Difference	Precision
Sample1	2479079.59	2421018.73	-58060.86	95425.73
Sample2	2479079.59	2478335.59	-744.00	1458.23
Sample3	2479079.59	2383828.46	-95251.13	87999.61
Sample4	2479079.59	2466555.35	-12524.24	17452.27
Sample5	2479079.59	2477740.68	-1338.91	2016.69
Sample6	2479079.59	2444582.35	-34497.24	40065.34
Sample7	2479079.59	2477015.74	-2063.85	2232.22
Sample8	2479079.59	2477015.74	-2063.85	2232.22
Sample9	2479079.59	2461521.52	-17558.07	22020.85
Sample10	2479079.59	2465474.22	-13605.37	154790.79

On the average in this case the population audited amount estimate is better than the other designs. But the point estimate in each sample has some exaggerated differences. Even if the number of errors appeared in the sample are less, good estimate of the error or difference is obtained. See the appendices 1-1, 1-2, 2-1 and 2-2.

CHAPTER FIVE

Discussion and Recommendation

The main objective of this paper is to describe a possible way to integrate statistical sampling into the audit planning process. One of the purposes of planning is to reduce the audit risk to a tolerable level.

Using statistical sampling contributes positively in the planning process by allowing the auditor to control one type of risk - the sampling risk that occurs when a sample of items from an audit population are observed instead of all. Integrating statistical sampling into the audit planning process helps the auditor in his or her effort to control the sampling risk associated with developing a fair opinion on the financial statements.

One way to achieve an integrated sampling plan is to establish a material amount of monetary error for each account balance or class of transactions and consider the risk that all substantive tests taken together would fail to detect a material monetary error.

The next stage of the planning process is the auditors' plan of detailed tests. Incorporating statistical sampling to the audit tests will help to select an appropriate audit objective using statistical tools, like determining an optimum sample size, deciding an appropriate sampling unit and frame, and deciding the sample design and the corresponding selection method. The sample planning process is a tentative audit program in which first, it is based on the auditors' review and preliminary evaluation of the system of internal accounting control procedures and past audit results. Secondly, the tentative nature is that information from some of the planned substantive tests may cause the auditor to revise the audit program. Consequently, the audit program actually followed may be different from the planned program.

The variable sampling methods discussed in this paper are of two natures, equal probability and unequal probability approaches. Each design was evaluated from the point of view of stratification and without stratification. The result for unstratified sampling design, stratified sampling design and sampling with probabilities proportional to monetary size commonly called Monetary Unit Sampling in auditing were discussed. The auxiliary information estimators namely the difference and ratio estimators have also been considered. In each sampling design sample size was computed using the corresponding sample size formula provided that same planned error margin (planned precision) and same reliability level is attained.

When an amount recorded is greater than the amount that should actually be it is said to be overstated. The auxiliary information estimators and the PPMS correctly show that the study population has an overstated recorded amount but for the mean per unit case sometimes this fails. The result also shows that the point estimate of the audited value with its computed precision limit does not contain the true audited amount. This is mostly the case for the PPMS where some exaggerated amounts are seen. In the case of the mean per unit the achieved precision exceed the planned precision for some of the samples. This may call for adjusting the sampling plan for these designs.

Theoretically, in estimating the unknown population parameter the performance of all estimators is equivalent. The difference arises when we deal with the efficiency due to sample size. Checking only a comparatively smaller sample of units in the stratified and PPMS case will easily attain the result obtained by auditing a larger sample of transactions in the unstratified case. The result of this study shows that the auxiliary information estimators namely the difference and ratio estimators have almost similar results in estimating the total audited amount. See appendices 1-1 and 2-1.

The estimated total error or difference was satisfactory in the case of the unstratified difference and ratio estimators than the stratified one. This is in line with the theoretical result, that is stratification is effective when we have a heterogeneous population with highly varying values or where the coefficient of variation exceeds 70percent, (see GAO, 1992).

On the average the PPMS estimator was the best in estimating the total audited amount. It was also efficient because the sample size was smaller than even the stratified case. For the unstratified case the sample size is too large but on the average the point estimate for the total audited amount was as good as the PPMS. But the population error amount was effectively estimated using the difference and the ratio estimator under this approach. See appendices 1-1 and 2-2.

When there are no errors in the sample or the population is with a low error rate the difference and ratio estimators are not efficient. Therefore, further study and researches are needed to improve the efficiency of these estimators under this condition by studying populations of different error rates. Such a study may help whenever an error free sample or a low error rate population is to be audited. In the case of PPMS the estimation result obtained does not entirely depend on the number of errors found in the sample.

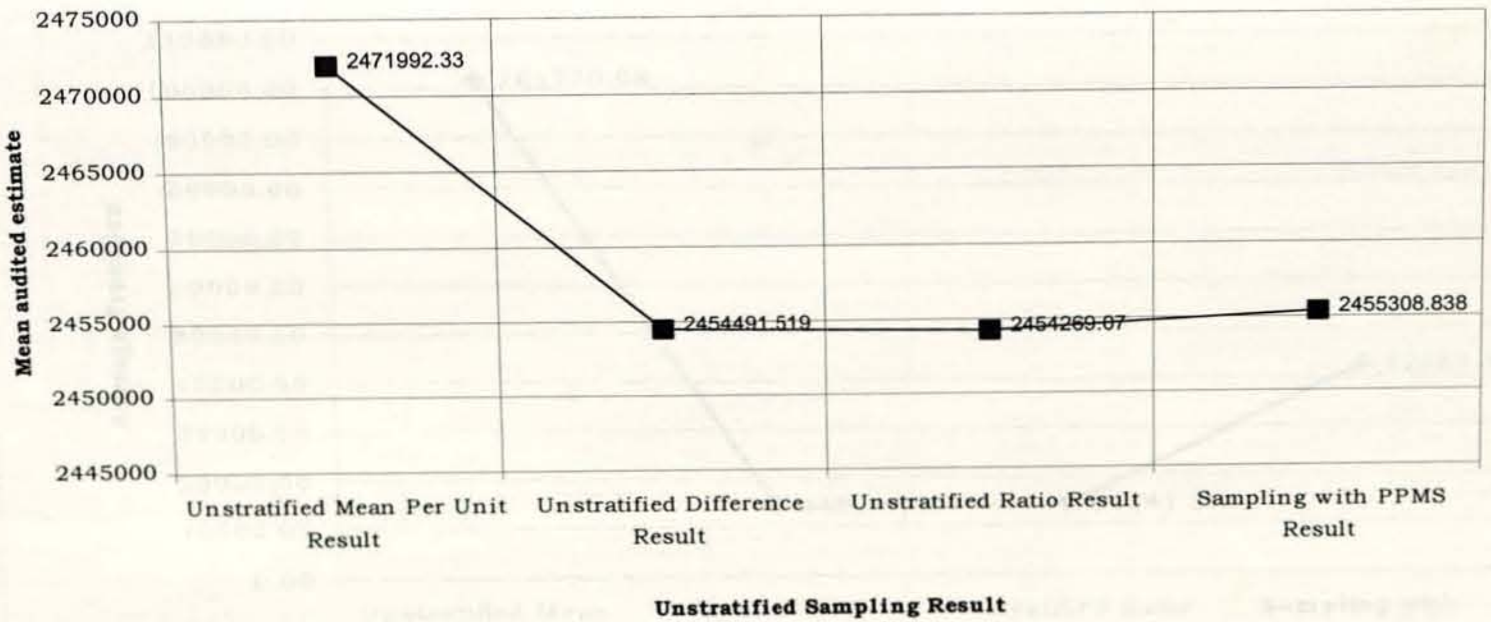
Statistical audit sampling is not properly used in our country. Since this study was based on the objective of ascertaining whether the recorded monetary amount on the book of records of the client auditee organisation, it is not impractical to apply it in our country's audit condition since the audit objective is usually the same. The result of this study is encouraging that the use of statistical sampling is much more advantageous whenever the audit of the whole accounting population becomes very tedious. Even if this is not the case proper use of statistical sampling has great advantage to save time and minimise cost. Therefore, it is recommended that audit organisations in the country, private auditors and/or governmental auditors, use statistical sampling in their audit work.

To overcome the computational inconvenience by using electronic media applications, it is advisable to develop sampling application software in the future studies. They help to overcome manual sample selection, evaluation and related computational difficulties for auditors. Their application in statistical sampling for auditing can save time and minimise cost.

Appendix 1

	Unstratified Result /amounts in Birr/							
	Unstratified Mean Per Unit Result		Unstratified Difference Result		Unstratified Ratio Result		Sampling with PPMS Result	
	Estimated Audited Amount	Precision	Estimated Audited Amount	Precision	Estimated Audited Amount	Precision	Estimated Audited Amount	Precision
Sample 1	2480083.20	100924.62	2453739.22	16465.99	2450005.70	16925.61	2421018.73	95425.73
Sample 2	2457336.00	107923.50	2454830.99	16670.95	2454855.30	16669.10	2478335.59	1458.23
Sample 3	2455467.80	100625.77	2449099.98	17337.82	2449181.40	17296.19	2383828.46	87999.61
Sample 4	2511169.90	102148.95	2446932.64	17967.46	2447744.50	17936.39	2466555.35	17452.27
Sample 5	2485479.40	101078.86	2467449.92	5434.26	2467534.00	5421.74	2477740.68	2016.69
Sample 6	2497373.80	101936.86	2459884.68	7216.05	2460170.40	7257.73	2444582.35	40065.34
Sample 7	2400444.50	102110.38	2452418.16	16813.67	2451847.20	16797.36	2477015.74	2232.22
Sample 8	2457430.10	97376.98	2460367.68	7856.87	2460328.10	7893.01	2477015.74	2232.22
Sample 9	2512473.60	104077.01	2451489.10	17374.11	2452151.30	17384.07	2461521.52	22020.85
Sample 10	2462665.00	99497.86	2448702.82	17862.12	2448872.80	17832.69	2465474.22	154790.79
Average	2471992.33	101770.08	2454491.52	14099.93	2454269.07	14141.39	2455308.84	42569.40

Appendix 1-1


Mean audited estimate
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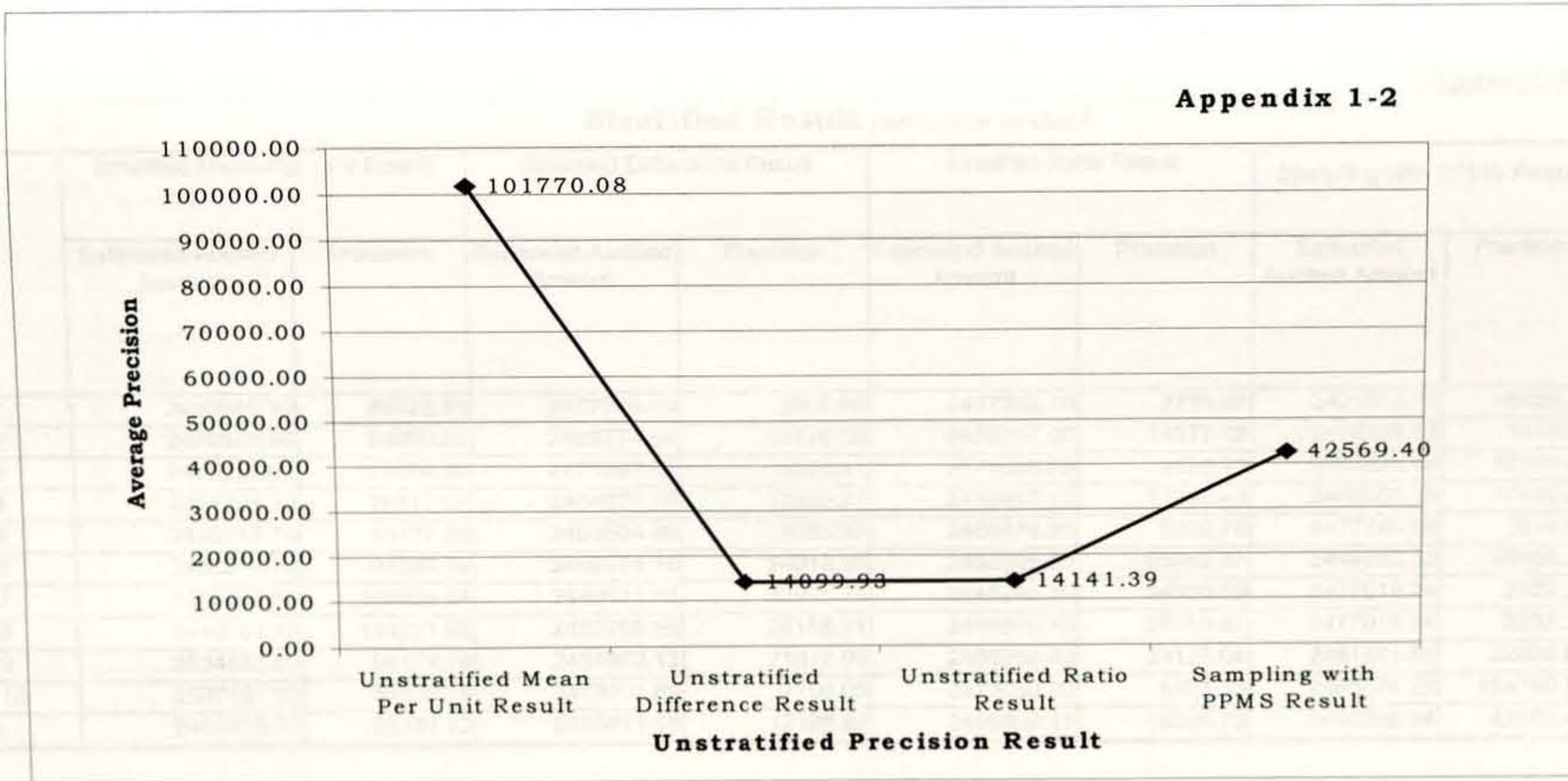
2470000
2465000
2460000
2455000
2450000
2445000
2471992.33
2454491.519
2454269.07
2455308.838
Unstratified Mean Per Unit Result
Unstratified Difference Result
Unstratified Ratio Result
Sampling with PPMS Result
Unstratified Sampling Result

Appendix 1-2

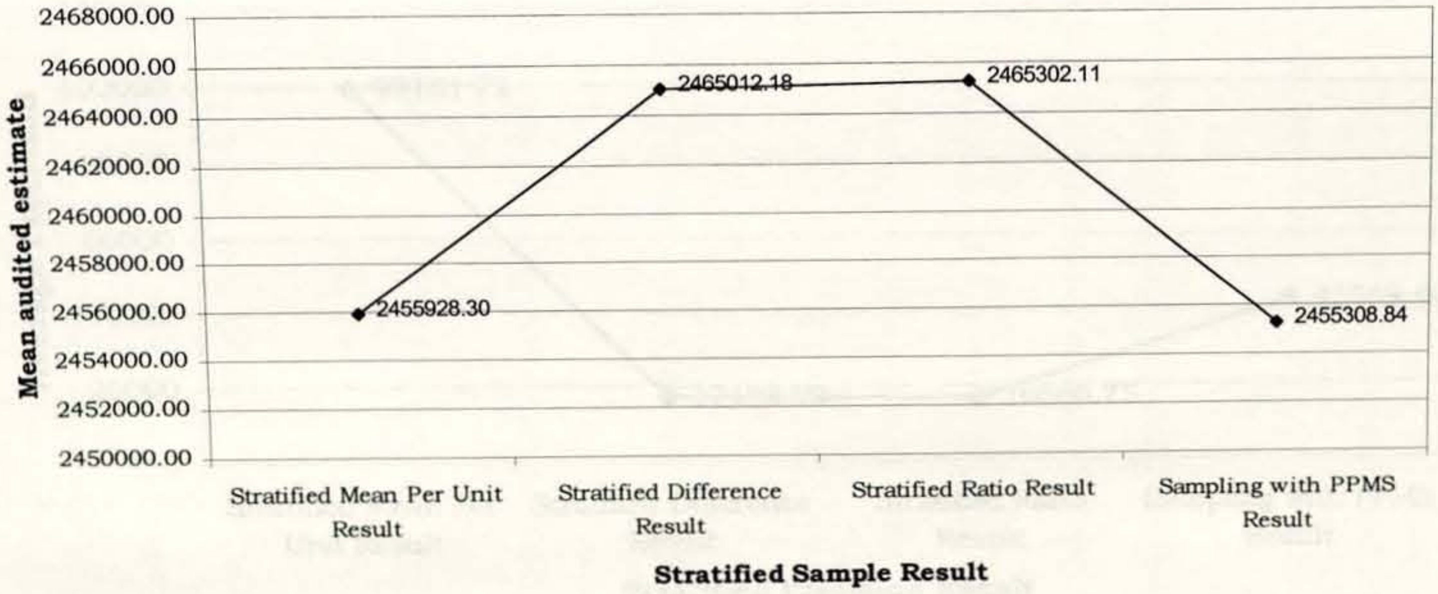


Average Precision
110000.00
100000.00
90000.00
80000.00
70000.00
60000.00
50000.00
40000.00
30000.00
20000.00
10000.00
0.00
101770.08
14099.98
14141.39
42569.40
Unstratified Mean Per Unit Result
Unstratified Difference Result
Unstratified Ratio Result
Sampling with PPMS Result
Unstratified Precision Result

Appendix 2

	Stratified Result /amounts in Birr/							
	Stratified Mean Per Unit Result		Stratified Difference Result		Stratified Ratio Result		Sampling with PPMS Result	
	Estimated Audited Amount	Precision	Estimated Audited Amount	Precision	Estimated Audited Amount	Precision	Estimated Audited Amount	Precision
Sample 1	2498945.30	88022.75	2477346.69	2689.86	2477362.10	2795.07	2421018.73	95425.73
Sample 2	2466570.00	94960.39	2469274.94	13776.55	2469257.00	14377.12	2478335.59	1458.23
Sample 3	2403257.60	121084.96	2475697.59	6245.41	2475595.80	6520.21	2383828.46	87999.61
Sample 4	2331291.30	76572.02	2464975.75	17629.22	2464417.12	17393.43	2466555.35	17452.27
Sample 5	2450717.70	85727.36	2460504.99	18986.90	2460376.20	5230.78	2477740.68	2016.69
Sample 6	2452359.10	98389.89	2462574.16	24013.35	2462505.90	25952.87	2444582.35	40065.34
Sample 7	2464691.60	109624.84	2449111.75	32626.74	2449299.30	34000.53	2477015.74	2232.22
Sample 8	2449161.10	134221.08	2452769.95	26158.21	2455868.10	28250.07	2477015.74	2232.22
Sample 9	2534837.50	94174.74	2464863.13	21972.95	2465253.40	23123.04	2461521.52	22020.85
Sample 10	2507451.80	89039.18	2473002.89	7790.05	2473086.20	8024.35	2465474.22	154790.79
Average	2455928.30	99181.72	2465012.18	17188.92	2465302.11	16566.75	2455308.84	42569.40

Appendix 2-1



Mean audited estimate
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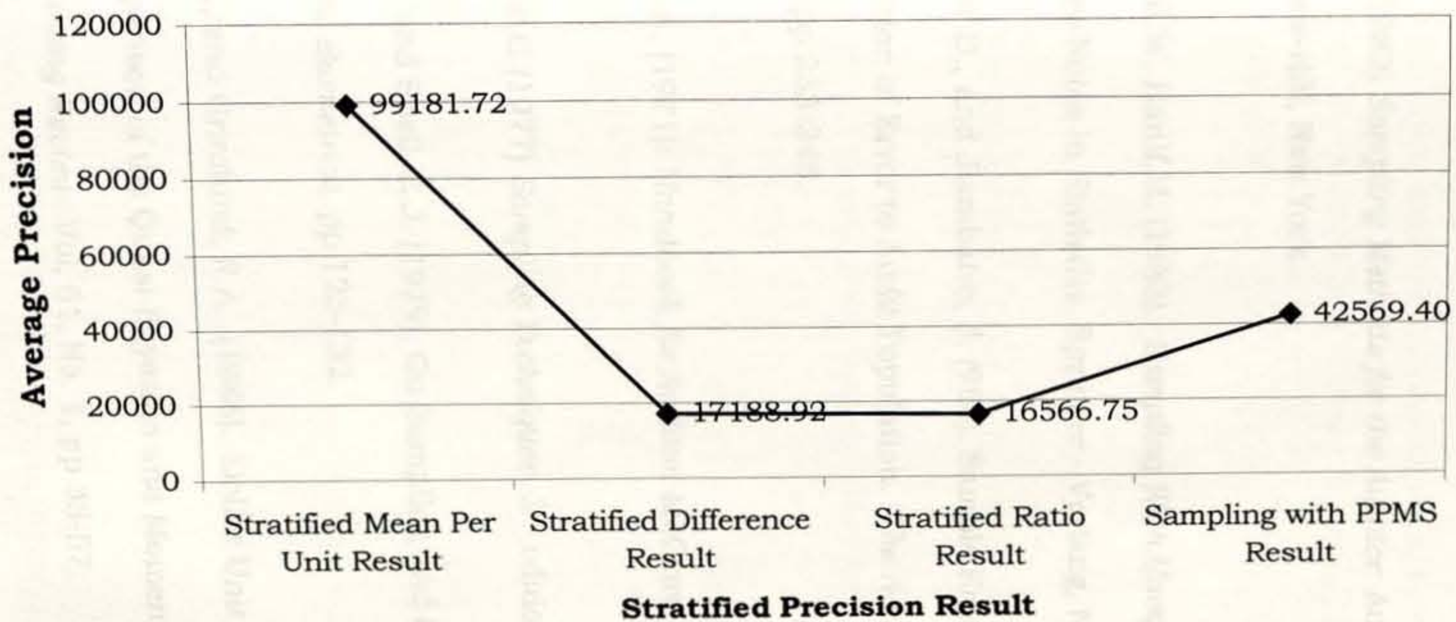
2458000.00
2456000.00
2454000.00
2452000.00
2450000.00
2455928.30
2465012.18
2465302.11
2455308.84
Stratified Mean Per Unit Result
Stratified Difference Result
Stratified Ratio Result
Sampling with PPMS Result
Stratified Sample Result

Appendix 2-2


120000
100000
80000
60000
40000
20000
0
Average Precision
99181.72
17188.92
16566.75
42569.40
Stratified Mean Per Unit Result
Stratified Difference Result
Stratified Ratio Result
Sampling with PPMS Result
Stratified Precision Result

BIBLIOGRAPHY

Arkin, H. (1984). *Handbook of Sampling for Auditing and Accounting.* 3rd edition. McGraw-Hill, New York.

Arkin, H. (1982) *Sampling Methods for the Auditor.* An Advanced Treatment. McGraw-Hill, New York.

Brewer, K.R.W, Hanif,M. (1983). *Sampling With Unequal Probabilities*: Lecture Notes in Statistics. Springer - Verlang, New York.

Burgstahler D., and Jiambalvo, J. (986). Sample Error Characteristics and Projection of Error to Audit Population. *The Accounting Review*. vol 61, No. 2 pp 233-248.

Cashin , J.A. (1971): *Handbook for Auditor.* McGraw-Hill. New York.

Cochran, W.G.(1977) *Sampling Techniques.* 3rd edition, Wiley, New York.

Cox, D. R. and Snell, E.J. (1979). On Sampling and Estimation of Rare Errors. *Biometrica.* pp 125-132.

Dworin, L., and Grimlund, R.A. (1986). Dollar Unit Sampling (DUS): A Comparison of the Quasi-Bayesian and Moment Bounds. *The Accounting Review*: Vol. 61, No. 1, pp 35-57.

Dworin, L. and Grimlund, R.A. (1984). DUS for Accounts receivable (A Moments Method approach). *The Accounting Review.* Vol. 59, No. 2 , pp 218-241.

Fienberg, S.E., Neter, J. and Leitch, R.A. (1977). Estimating the Total Overstatement Error in Accounting population. *Journal of the American Statistical Association.* Vol. 72, No. 358, pp 295-302.

General Accounting Office (1992): *Using Statistical Sampling.* From Program Evaluation and Methodology Division (PEMD). GAO publication, Washington D.C.

Grimlund, R.A. and Felix, W.L. (1987). Simulation Evidence and Analysis of Alternative Methods of Evaluating Dollar Unit Sampling. *The Accounting Review.* Vol. 62, No. 3, pp 455-479.

Gupta , K. (1991). *Contemporary Auditing*: For Students 4th edition, Tata McGraw-Hill. New Delhi.

Guy , D. M.(1981). *An Introduction to Statistical Sampling in Auditing.* John Wiley and Sons, New York.

Guy, D.M., Carmichael, D. and Whittinton, O.R. (1994): *Audit Sampling : An Introduction.* 3rd edition. John Wiley and Sons, New York.

Hill , H. P. (1962).*Sampling in Auditing. A Simplified Guide and Statistical Tables.* Krieger Publishing Company. Malabar, Florida.

Hylas, R.E. and Ashton, R.H. (1982). Audit Detection of Financial Statement Errors: *The Accounting Review.* Vol. 57, No. 4, pp 751-765.

Johnson, J.R., Leitch, R.A. and Neter, J. (1981). Characteristics of Error in Accounts Receivable and Inventory Audits. *The Accounting Review*. Vol. 56 No. 2, pp 270-293.

Johnson , S. R.(1995). *Quality Review Schemes For Auditors: Their Potential For Sub-Saharan Africa.* World Bank Technical Paper No 276. Washington, D.C.

Knight, P. (1979) Statistical Sampling in Auditing: An Auditor's Viewpoint. *The Statistician*, Vol. 28, No. 4, pp253-266.

Konrath, L.F.(1996). *Auditing Concepts and Applications: A Risk Analysis Approach.* 3rd edition, West Publishing Co., New York.

Leitch, R.A., Neter, J., Plante, R. and Sinha, P. (1982). Modified Multinomial bounds for larger numbers of Errors in Audits. *The Accounting Review*. vol. 57 No. 2, PP 385-400

Leslie, D.A. Teitlebaum, A.D. and Anderson, R.J. (1979). *Dollar-Unit Sampling. A Practical Guide for Auditors.* Cop Clark Pitman. TORONTO.

McCray, J.H. (1984). A Quasi-Bayesian Audit Risk Model for Dollar Unit Sampling (DUS). *The Accounting Review*. Vol. 59 No.1, pp 35-51.

Montgomery, R. (1985). *Montgomery's Auditing.* 10th edition, John Wiley and Sons, New York.

Moors, J.J.A. (1983). Bayes Estimation in Sampling for Auditing. *The Statistician.* vol.32 pp 281-288.

Neter, J. and Godfrey, J. (1985). Applied Statistics. *Journal of Royal Statistical Society : part C,* Vol. 34 , pp157-168.

Neter, J., Johnson, J.R. and Leitch, R.A. (1985). Characteristics of Dollar-Unit Taints and Error rates in Accounts Receivable and Inventory. *The Accounting Review.* Vol. LX, No.3, pp 488-499.

Neter, J., Leitch, R.A. and Fienberg, S.E. (1978). Dollar Unit Sampling: Multinomial Bounds for Total overstatement and Understatement Errors. *The Accounting Review.* Vol. 53 No l, PP 77-93.

Neter, J. and Loebbecke, J.K. (1975). Behaviour of Major Statistical Estimators in Sampling Accounting Population. *The American Institute of Certified Public Accountants(AICPA),* New York.

Plante, R.,Neter, J. and Leitch, R.A. (1985). Comparative Performance of Multinomial, Cell, and Stringer Bounds. *Auditing.* Vol.5 pp 40-56.

Raj, D. (1979). *Sampling Theory.* TATA McGraw-Hill Publishing Co. LTD, New Delhi.

Renau, J.H. (1978). Combined Attributes and Variables (CAV) Bounds in Dollar Unit Sampling (DUS): Some Simulation Results. *The Accounting Review.* Vol. 53, pp 669-680.

Roberts, D. M.(1978). Statistical Auditing. . *The American Institute of Certified Public Accountants (AICPA)* Publication. New York.

Robertson, J.C.(1993) *Auditing.* 7th edition. IRWIN,Boston.

Scheaffer, R.L., Mendenhall, W. and Ott, L. (1990) *Elementary Survey sampling.* 4th edition, Duxbury Press,California.

Scott, A. and Chien-Fu Wu (1981) On the Asymptotic Distribution of Ratio and Regression Estimators. *JASA.* Vol.76, pp 98-102

Smith, T.M.F. (1979). Statistical Sampling in Auditing: A Statistician View Point. *The Statistician.* vol. 28, No. 4, pp267-280.

Stringer, K.W. (1963). Practical Aspects of Statistical Sampling in Auditing. In Proceedings of the Business and Economic Section, *American Statistical Association.* pp405-411.

Taylor, R.G. (1974). Error Analysis is Audit Tests. *Journal of Accountancy.* Vol.137, pp78-82.

Tryfos, P. (1996): *Sampling Methods For Applied Research: Text and Cases.* John Wiley and Sons, New York.

Tsui, K.W., Matsumura, E.M. and Tsui, K.L. (1985). Multinomial- Drichlet Bounds for DUS in Auditing. *The Accounting Review.* Vol. 60, No. 1, pp 76-96.

Vanasse , R. W. (1976). *Statistical Sampling For Auditing And Accounting Decisions.* A Simulation. 2nd edition , McGraw-Hill, New York.

DECLARATION

I, the undersigned, certify that this thesis is my original work, has not been presented for a degree in any other University and that all sources of materials used for the thesis have been duly acknowledged.

NAME Befekadu Ayele

SIGNATURE