Addis Ababa University

Addis Ababa Institute of Technology (AAiT)

School of Electrical and Computer Engineering

Harmonic Analysis of Addis Ababa Light Rail Transit Traction Converters

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A Thesis Submitted to Addis Ababa University, Institute of Technology, in Partial Fulfilment of the Requirements for the Degree of Masters of Science in Electrical Engineering (Railway)

August 2014

Addis Ababa, Ethiopia

Addis Ababa University

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DECLARATION

I, the undersigned declare that this thesis is my original work, and has not been presented for a degree in this or any other university, and all sources of materials used for the thesis have been fully acknowledged.

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Addis Ababa, Ethiopia August, 2014

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ABSTRACT

Traction power system is the main area of industrial sector where the application of power electronic technology is widely used. The uses of these devices create harmonics that cause a serious problem on the proper operation of the system. There have been different corrective methods proposed by different researchers. Among them, the AC side passive filters that provide low resistive path to eliminate the harmonics; and the active filter that produce harmonic currents in equal magnitude and 180^0 out of phase to suppress it, are the widely discussed ones.

This thesis provides the harmonic analysis of Addis Ababa Light Rail Transit (LRT) traction system. The analysis is performed on the Lideta rectifier substation. The traction system is modelled using Matlab/Simulink environment. Based on the data obtained from Ethiopian Railway Corporation (ERC), simulation is carried out and compared with the IEEE standard limits.

Taking into consideration the cost and simplicity of the operation, single tuned filters tuned at 11^{th} and 13^{th} harmonics and high pass filter adjusted for higher harmonics currents are designed for the AC side, and LC filter is proposed for the DC side of the system.

The simulation result comes with significant reduction in harmonic distortion. The Total Harmonic Distortion (THD) is reduced to 0.80% for the current and to 0.33% for voltage, which is far below the 5% limits of IEEE standard. In addition to that the system power factor is improved from 0.77 to 0.99, which avoids the installation of power factor corrector.

Keywords: – Traction power system, converters, harmonic analysis, harmonic filters

ACKNOWLEDGMENT

First, I would like to express my deepest appreciation and sincere gratitude to my advisor Dr. Mengesha Mamo for his guidance, support and invaluable contributions throughout the preparations for this thesis.

I would like to thank all the staffs of the Department of Electrical and Computer Engineering at AAiT and respective officials at Addis Ababa Light Rail Transit (LRT) project, for providing me all the invaluable materials and helpful pieces of advice.

Finally, I wish to thank all of my friends and family for their persistent support during my thesis work.

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ACRONYMS

AC	Alternate Current
CSI	Current Source Inverter
DC	Direct Current
EEPCo	Ethiopian Electric Power Corporation
emf	Electromagnetic force
ERC	Ethiopian Railway Corporation
EW	East-West
FFT	Fast Fourier Transform
HVDC	High Voltage Direct Current
IEEE	Institute of Electrical and Electronic Engineers
IGBT	Insulated Gate Bipolar Transistor
kVA	Kilo Volt-Ampere
LRT	Light Rail Transit
MI	Modulation Index
MMF	Magnetic Motive Force
MVA	Mega Volt-Ampere
OCS	Overhead Contact System
PCC	Point of Common Coupling
PF	Power Factor
PWM	Pulse Width Modulation
RMS /rms	Root Mean Square
THD	Total Harmonic Distortion
VSI	Voltage Source Inverter
VVVF	Variable Voltage Variable Frequency

CHAPTER ONE

1. INTRODUCTION

1.1. Motivation and Background

The emerging of electricity as the main power source of railway transportation makes the transportation convenient, fast and environmentally friendly. But, at the same time this brings new challenges to the power grid. Because of the rapid development in the power electronics technology and an increase in the application of these devices in railway sector, the railway supply system gets a distorted waveform that affect the overall performance of the system. This distorted waveform is created from the harmonic currents injected into the system by non-linear loads. The harmonic currents flow into the surrounding grid through the transmission line and bring the power quality problems that cannot be ignored. It is noteworthy that the harmonic currents will be amplified when resonance exist, and thus interfere with the neighbouring lines of communication and the signal system of the railway, causing overheating and instability of the power capacitors, brings a malfunction of protection device, and a series of questions exist. Therefore, the harmonic current flow must be assessed exactly in the designing and planning stage of the electric traction system [1]. It needs to be accurately modelled to analyze and assess the harmonic effect on the power-feeding system [2].

There are different correction actions taken worldwide in order to overcome these problems. Some of the classic solutions are the use of harmonic filters to eliminate specific frequency waveforms. Many of the harmonic filters used consist of parallel connected capacitor- inductor circuits to create a low impedance device. These types of filters are considered as passive devices. The low impedance devices attract harmonic current from all sources connected to the system (considered export harmonics) and those exterior to the system (considered imported harmonics). They then dissipate the harmonic current as heat. Note heat dissipation should not be considered as a loss caused by the harmonic filters as this waste heat already existed in the system in unusable frequencies. This heat is dissipated by the harmonic filter instead of being exported to the utility grid to be dissipated by other low impedance devices.

An alternative to the passive devices are active devices. They cancel harmonics by producing them at 180 degrees phase angle to the harmonics being created. Active filters give good system performance and current harmonics reduction. However, they are based on sophisticated power electronics components and thus they are much more expensive than passive filters.

In this thesis work harmonics created by power converters in railway traction of Addis Ababa Light Rail Transit (LRT) is studied. The performance of the system under different condition is analyzed. It is supported with analytical analysis and computer based simulation of different harmonics orders. Finally, the explanation and design of best harmonic mitigation technique available to solve these harmonic problems is provided.

1.2. Statement of Problem

A variety of different designs of traction power systems exist and many have been in place for years. However, they can all be grouped into one of two categories, AC or DC. In AC traction system the power is delivered to the load in the same form as its generation and for the control purpose the conversion to DC is processed at the load side.

But, in DC traction system the power is first converted into DC form and transported to load side through catenary system or third rail system. On the load side it is converted back to AC since loads are AC type. So, in DC traction system there will be more power conversion than AC traction. This conversion is facilitated by power electronics devices that operate with the principle of on/off switching mechanism [3]. Since the operation behaviour of these devices is non-linear, as a result the harmonic current is introduced to the system. These harmonics have effects on the linearity of the supply power from the utility grid which leads to the degradation of system performance. On the other word, because the harmonic current doesn't deliver any power, its presence simply uses up system capacity and reduces the number of loads that can be powered. Harmonic current occurring in a system can cause equipment malfunction, power loss, decrease in power factor, data distortion, transformer and motor insulation failure, overheating of neutral buses, tripping of circuit breakers, and solid-state component breakdown. In general the cost of these problems can be enormous.

Since the Addis Ababa LRT traction system is a DC traction type and installed with highly rated converters, the harmonic issue becomes the most interesting area that needs to be properly analyzed. So, in the proposed work, the harmonics characteristics of Addis Ababa LRT traction system is first studied and analyzed, and then a proper solution is provided in order to minimize the effect of the harmonics on the proper operation of system.

1.3. Objective

The main aim of this work is to study the harmonic analysis in the Addis Ababa LRT traction system and propose the best minimization technique. The proposed solution needs to comply with the IEEE Standard 519-1992 recommended harmonic limits.

To achieve this main objective the work is specifically aimed at;

- Studying and selecting of sections of Addis Ababa City LRT which is most prone to harmonic distortion
- Applying analytical harmonic analysis methods
- Developing best mitigation techniques
- Improve the power factor of the system
- Decrease the power loss
- Comparing the performance of system with and without proposed solution

1.4. Methodology

In order to achieve the main aim of the study there are various procedural tasks followed by the author. The first method towards processing the work is started with reviewing different literatures where all the theoretical information regarding the traction power system harmonic analysis is gathered and a comparison of previous similar research is studied. Alongside with literature reviewing, data collection from ERC is performed. This is followed by studying the characteristic and modelling of the traction power system components. Once the model is developed using MATLAB/Simulink, the analysis of the system is performed. Then based on the analysis result proper filter is designed. Finally, the performance of the filter is analyzed and a comparison is made. The general block diagram of the methodology is given below.

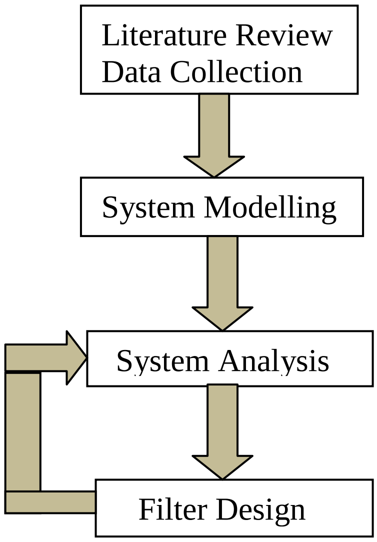


Figure 1.1 Summary of methodology

1.5. Related Works

In recent years, with the popularization of power electronics equipment in railway traction applications such as converter driven motor drives, battery chargers and auxiliary power supplies, harmonic pollution has become more serious. Thus, railway engineers pay more attention to power quality of traction power-supply systems. Harmonic contents in traction currents are one of the most important aspects of power quality. Various methods to deal with such problems were carried out.

- KH. Yuen, MH. Pong, WC. Lo and ZM. Ye [41] given a behavioural model in frequency domain to represent electric railway vehicle based on testing and measurement for harmonic analysis. The Authors treats a train as a black box. The behaviour of traction converter of railway vehicles is assumed as piecewise linear for each operation notch. By applying cubic curve-smoothing technique on interpolation of measurement, a simple model for railway vehicle is obtained. Because the model is obtained from real-world measured data, it is a good approximation to a real train. But, this method does not work to perform harmonic analysis at planning stage since measurement data do not available.
- The harmonic mitigation of DC underground traction systems is provided [14]. The Author stated that Active filter is the best in harmonic mitigation of traction system. This is true for the case when the cost of installation is not the main concern, but nowadays the cost and simplicity of the system installation needs to be within the range of interest.

- Ezgi UNVERD and Ali Bekir YILDIZ [33] have developed the equivalent circuit model for the rectifier. The equivalent circuit contains harmonic current on AC side and voltage harmonics on DC side of three-phase diode rectifiers. In this thesis, this idea is taken to determining the harmonic loss calculation.
- Kuldeep Kumar Srivastava, Saquib Shakil and Anand Vardhan Pandey [42] proposed “Harmonics & Its Mitigation Technique by Passive Shunt Filter”. This paper has presented a brief idea about harmonic & their consequences affect on the distribution & transmission system. It presented a brief idea about the designing process of passive shunt filter for two types of passive filter names single tuned filter & high pass filter for the mitigation of harmonic in the system. The basic idea of filter designing discussed in this paper is taken for this thesis.
- Seema P. Diwan, Dr. H. P. Inamdar, and Dr. A. P. Vaidya [44]” Simulation Studies of Shunt Passive Harmonic Filters: Six-Pulse Rectifier Load: Power Factor Improvement and Harmonic Control.” This paper presents a study of harmonic filters design procedure to minimize harmonic distortion caused by a harmonic source such as drives. Design and simulation procedure with respect to six-pulse rectifier load is explained. The design and performance of single-tuned and high-pass filters and the methodology used for the analysis is discussed. The shunt passive filter was designed to improve the power factor, reduce current and voltage distortion to standard limits, and reduce resonance problems.
- Young-Sik Cho and Hanju Cha [45]” Single-tuned Passive Harmonic Filter Design Considering Variances of Tuning and Quality Factor”, This paper presents how to select tuning factor and quality factor in designing of a single-tuned passive harmonic filter. If a single-tuned passive harmonic filter were detuned, its performance would be deteriorated substantially and resulted in a parallel resonance between grid inductance and filter capacitance. In order to avoid this side effect from detuning, the filter must be tuned on some preceded order not on the exact order. In other words, total filter impedance must have reactive impedance on a tuned frequency. In this paper, tuning factor is derived by using a bode-plot based method and then performance of filter is confirmed as a harmonic current absorption rate which harmonic source flows through filter; and quality factor is also derived by using the same method and then the performance is confirmed by the same filter current absorption rate. Finally, the performance of

proposed passive harmonic filter design using the tuning factor and quality factor is verified by experiment. Experimental results show that the 5th, 7th, 11th, and 13th current harmonic distortions meet IEEE-519 requirement.

In this thesis, the harmonic analysis is performed using Matlab/Simulink environment. Having cost and simplicity of operation in consideration, shunt passive filters are designed for harmonic mitigations. On the DC side of converters LC filter is proposed to avoid AC ripple.

1.6. Thesis Organization

This thesis is organized into five chapters with each chapter explains in detail about the research. The first chapter provides an introduction of the project and defines the subject of the thesis.

The second chapter generally covers about theoretical background of power system harmonics. This chapter presents a detail about harmonic generating devices, the consequence of this generated harmonics and finally explains the possible mitigation techniques.

In chapter three, the traction power system modelling to analyze the effect of harmonics and the filter designing procedure is included.

In the fourth chapter, based on the selected data the system is simulated and result analysis is performed. This includes the interpretation of the result obtained with respect to the standards.

In the final part, chapter five, the concluding remarks that summarize the research results and gives future work recommendations on subjects related to the thesis is presented.

CHAPTER TWO

2. POWER SYSTEM HARMONICS

2.1. Introduction

The term harmonic refers to a component of a waveform that occurs at an integer multiple of the fundamental frequency. Ideally, the electrical supply should be a perfect sinusoidal waveform without any kind of distortion. If the current or voltage waveforms are distorted from its ideal form it will be termed as harmonic distortion. This harmonic distortion could result because of many reasons.

Traditional electrical power distribution system design has very little need to deal with harmonics because the loads typically designed for were linear in nature. With the proliferation of variable speed drives, electronic device need to be adjusted because some order harmonic currents are being injected in to power distribution system. Over the years, essential approaches evolved and became widely used to minimize these harmonic currents.

2.2. Sources of Harmonics

Many types of electrical and electronic equipment which are adversely affected by harmonic voltages and currents are also producing them in varying degrees. The main equipments are non-linear loads that produce the voltage and current harmonics. The harmonics are created because of the variation of device's resistance with respect to time.

There are four main categories of harmonic sources;

- Devices that generate harmonics during their switching processes:-

The most commonly seen are power electronic devices, such as converters that are often used in DC traction system, HVDC links and motor drives [4] [5]. These devices are sensitive to supply voltage distortion and unbalance.

- Devices that generate harmonics due to their nonlinear voltage-current characteristics:-

Typical harmonic sources of this category are transformers, reactors, AC arc furnaces, and fluorescent lamps with electromagnetic ballasts. Harmonics generated from transformers and reactors occur when the core flux enters the saturated region of the magnetization curve driven by over-excitation [6] [7].

- Hybrid devices that include both types of aforementioned devices:-

For the third category of harmonic sources, it consists of DC arc furnaces and fluorescent lamps with electronic ballasts. The harmonics produced by a DC arc furnace concentrates at orders of (6h+1). h = 0, 1, 2 ….Due to the large amount of different types of electronic ballasts, the harmonics produced by a fluorescent lamp with electronic ballast vary dramatically. In general, these harmonics are of all orders with even harmonics being of small magnitudes [8].

- Rotating machines with non-sinusoidal flux distribution:-

Harmonics in these devices generate because of non sinusoidal flux distribution in the stator and the harmonic interaction between the stator and field windings. The harmonic sources of this category mainly include synchronous machines and induction motors [9].

2.3. Effects of Harmonic Distortion

The effect of current distortion on power distribution systems can be serious, primarily because of the increased current flowing in the system. In other words, because the harmonic current doesn't deliver any power, its presence simply uses up system capacity and reduces the number of loads that can be powered. Harmonic current occur in a facility's electrical system can cause equipment malfunction, data distortion, transformer and motor insulation failure, overheating of neutral buses, tripping of circuit breakers, and solid-state component breakdown. These effects can be described in more details as follows:

2.3.1. Effects of Harmonics on Rotating Machine

In attempting to understand the performance of a rotating machine, we consider that the air-gap flux wave is purely sinusoidal. It is from that assumption that the analysis of induced emf, sinusoidal currents, the expressions for generated torque etc. proceed. In practice, there are deviations from this idealistic picture.

The non-ideality is the presence of harmonics in the input supply given to the three phase machine. Some harmonics has negative sequence system where as other has positive sequence effect. The MMF caused by a negative sequence excitation causes backward revolving flux pattern (compared to the direction of the fundamental). The torque which it generates will act as an opposing torque to that generated by the fundamental.

The torque produced by positive sequence currents will be additive with respect to the fundamental component's torque.

The actual effect of these harmonics on the rotating machine would depend on the reactance of the machine since at high frequencies; it is the reactance component that dominates the inductance. Excitation voltage waveforms with considerable harmonic content may result when induction machines are controlled through inverters. Apart from the effects on torque, these harmonics cause considerable heating in the machine and iron and copper losses, hence cause for concern. These harmonics are called time harmonics since they are generated by a source that varies non-sinusoidal in time [10].

2.3.2. Effects of Harmonics on Transformer

Transformers are designed to deliver the required power to the connected loads with minimum losses at fundamental frequency. Harmonic distortion of the current, in particular, as well as the voltage will contribute significantly to additional heating. There are three effects that result in increased transformer heating when the load current includes harmonic components;

1. *Rms current*. If the transformer is sized only for the kVA requirements of the load, harmonic currents may result in the transformer rms current being higher than its capacity. The increased total rms current results increase conductor losses.

2. *Eddy-current losses*. These are induced currents in the transformer caused by the magnetic fluxes. These induced currents flow in the windings, in the core, and in the other connecting bodies subjected to the magnetic field of the transformer and cause additional heating. This component of the transformer losses increases with the square of the frequency of the current causing the eddy current. Therefore, this becomes a very important component of transformer losses for harmonic heating.

3. *Core losses.* The increase in core losses in the presence of the harmonics will be dependent on the effect of the harmonics on the applied voltage and the design of the transformer core. Increasing the voltage distortion may increase the eddy currents in

the core laminations. The net impact that this will have depends on the thickness of the core laminations and the quality of the core steel.

2.3.3. Effects of Harmonics on Converters

These equipments can be expressed as switches or on/off equipment because of the switching of current and voltage by some devices such as diodes and thyristors. These converters can switch the current, so creating notches in voltage waveforms, which may affect the synchronizing of the other converter equipment. These voltage notches cause misfiring of the thyristors and creating unarranged other firing instances of the other thyristors in the equipment.

2.3.4. Effects of Harmonics on Cables and Lines

The main problems on cables and lines associated with harmonics are: increased losses and heating, serious damages in the insulation of cables, appearance of the corona (the amount of the ionization of the air around the conductor or the transmission line) due to higher peak voltages and corrosion in aluminium cables due to DC current.

2.3.5. Effects of Harmonics on Protective Devices

The protective devices such as circuit breakers and fuses are designed to trip out in specific current and voltage and through very specific short time. The presence of the harmonics causes the difference on the voltage and current. So, this can cause false tripping of these protective equipments. Also, over current and over voltage can cause improper operation for relays. However, this cause the unsuitable tripping time so, causing some serious damages.

2.4. Harmonics of Traction Converters

In traction power system the power conversion is done at the rectifier substation. One of the main equipment of the rectifier substation is a rectifier or converter unit. The commonly used rectifiers in traction system are a six-pulse and twelve-pulse rectifiers with three phase input supply system. Six pulse converters are fed by two winding three phase transformer while the twelve pulse converters are supplied power with three winding transformers with two secondaries displaced by 30^0. In this section the operational principles of six-pulse and twelve pulse rectifiers with the analysis of its harmonic current and voltage waveforms is to be covered. The explanation of how to quantify this harmonics is also provided.

2.4.1. Six Pulse Diode Rectifier

Six-pulse diode rectifier is a type of AC/DC converter with uncontrolled three phase full wave rectifier of which the control of the output waveform is not possible. Rectifiers of such type are often found in DC power supply and HVDC power transmission system. A typical circuit diagram of a six- pulse three-phase rectifier is shown in Figure 2.1, assuming lossless diodes. The rectifier consists of six power diodes (D_1, D_2, D_3, D_4, D_5 and D_6) supplied from a three-phase supply system.

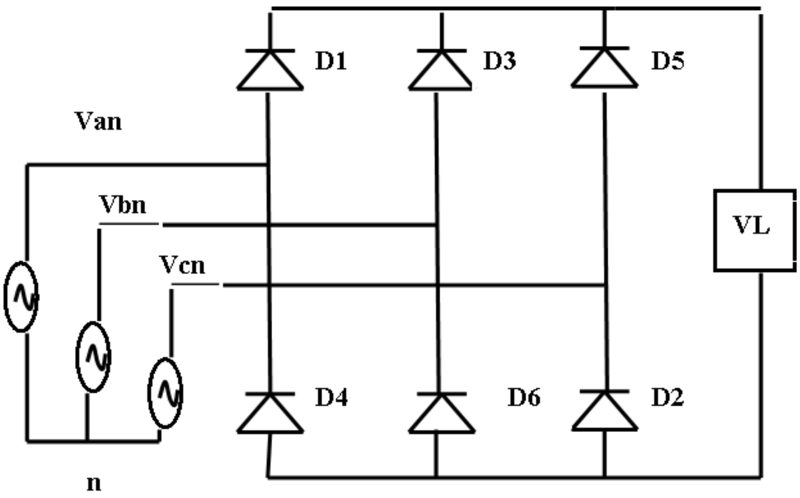


Figure 2.1 Circuit diagram of six-pulse rectifier [17]

For the six-pulse rectifier, the diodes conduct depending on the relative magnitude of the supply voltage. For any current to flow in the load at least one diode from the top group (D_1, D_3, D_5) and one from the bottom group (D_2, D_4, D_6) must conduct.

From symmetry consideration, it can be stated that each diode conducts for 120° of the input cycle (one third of a cycle). The diodes start conduction in the sequence of $D_1 \rightarrow D_2 \rightarrow D_3 \rightarrow D_4 \rightarrow D_5 \rightarrow D_6 \rightarrow D_1$ with an interval of 60° between each conducting diodes. Therefore diodes on the same phase leg are fired at an interval of 180° and hence cannot conduct simultaneously. This leaves only six possible conduction mode for the converter in the continuous conduction mode of operation. Each conduction mode is of 60° duration and appears in the sequence. The conduction table 2.1 shows the conducting diodes with their respective conduction interval.

Table 2-1 Rectifier diodes conduction modes [17]

Period of conduction	Conducting diodes
0-π/6	D_1D_2
π/6 -π/2	D_2D_3
π/2- 5π/6	D_3D_4
5π/6-7π/6	D_4D_5
7π/6-3π/2	D_5D_6
3π/2-11π/6	D_6D_1
11π/6-2π	D_1D_2

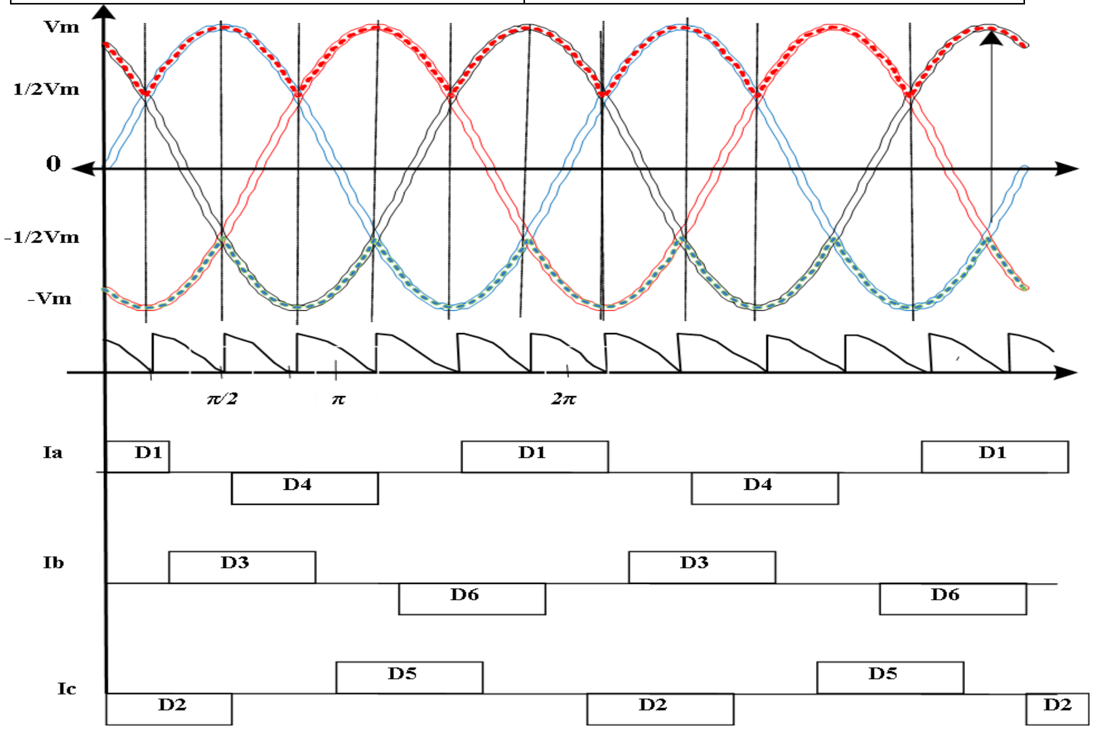


Figure 2.2 The current and voltage waveforms at the input side of rectifier [17]

The line-to-neutral three phase voltages are given below [17]:

$$v_{an} = V_m \sin \omega t$$
$$v_{bn} = V_m \sin (\omega t - \frac{2\pi}{3}) \qquad (2.1)$$
$$v_{cn} = V_m \sin (\omega t + \frac{2\pi}{3})$$

The corresponding line-to-line voltages will be:

$$v_{ab} = v_{an} - v_{bn} = \sqrt{3}\, V_m \sin (\omega t + \frac{\pi}{6})$$
$$v_{bc} = v_{bn} - v_{cn} = \sqrt{3}\, V_m \sin (\omega t - \frac{\pi}{2}) \qquad (2.2)$$
$$v_{ca} = v_{cn} - v_{an} = \sqrt{3}\, V_m \sin (\omega t + \frac{5\pi}{6})$$

Using the Fourier series expansion the output voltage waveform can be expressed in trigonometric form as [17]:

$$V_o = V_{dc} + \sum_{n=6,12,18,..}^{\infty} a_n Cos(n\omega t) + \sum_{n=6,12,18,..}^{\infty} b_n \sin(n\omega t) \qquad (2.3)$$

Where the average output voltage is found from:

$$V_{dc} = \frac{1}{T}\int_0^T V_{ab} d\omega t = \frac{3}{\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sqrt{3}\sin(\omega t + \frac{\pi}{6}))d\omega t$$

$$= \frac{3\sqrt{3}}{\pi} V_m \qquad (2.4)$$

The rms value of the output voltage is:

$$V_{rms} = \left[\frac{3}{\pi}\int_{\pi/6}^{\pi/2} v_{ab}^2 \, d(\omega t)\right]^{1/2} = \sqrt{3} V_m \, (\frac{1}{2} + \frac{3\sqrt{3}}{4\pi})^{1/2} = 1.6554 V_m \qquad (2.5)$$

The Fourier series coefficient is calculated as;

$$a_n = \frac{2}{T}\int_0^T V_{ab} \cos n\omega t d\omega t = \frac{6}{\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sqrt{3} V_m \sin(\omega t + \frac{\pi}{6})) \cos n\omega t d\omega t$$

$$= \frac{6\sqrt{3}}{\pi} V_m (\frac{1}{n^2 - 1}) \qquad (2.6)$$

$$b_n = \frac{2}{T}\int_0^T V_{ab} \sin(n\omega t) d\omega t = \frac{6}{\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sqrt{3} V_m \sin(\omega t + \frac{\pi}{6})) \sin n\omega t d\omega t = 0 \qquad (2.7)$$

Using equation (2.4), (2.6) and (2.7), the output voltage waveform is expressed as follows.

$$V_o = \frac{3\sqrt{3}}{\pi} V_m + \frac{6\sqrt{3}}{\pi} V_m \sum_{n=6,12,18,..}^{\infty} (\frac{1}{n^2 - 1}) \cos(n\omega t) \qquad (2.8)$$

$$V_o = \frac{3\sqrt{3}}{\pi} V_m + \frac{6\sqrt{3}}{35\pi} V_m \cos(6\omega t) + \frac{6\sqrt{3}}{143\pi} V_m \cos(12\omega t) + \frac{6\sqrt{3}}{323\pi} V_m \cos(18\omega t) + ...$$

So, output voltage of six pulse rectifier has harmonic components of integral multiples of 6th harmonics. Similarly, the input phase current I_a can be expressed in terms of a Fourier series by replacing the I_o with its average value for simplification purpose. This approximation will be valid provided that the ripple on I_o is small, i.e., the load is highly inductive. The modified input current waveform will then be I_a which can be expressed in terms of a Fourier series as

$$I_a \approx \frac{I_{dc}}{2} + \sum_{n=1}^{\infty} a_n Cos(n\omega t) + \sum_{n=}^{\infty} b_n \sin(n\omega t) \qquad (2.9)$$

Where the coefficients are determined as follows;

$$I_{dc} = \frac{1}{T}\int_0^T I_a d\omega t = \frac{1}{2\pi}\int_0^{2\pi} I_a d\omega t = 0 \qquad (2.10)$$

$$a_n = \frac{2}{T}\int_0^T I_a Cosn\omega t d\omega t = \frac{1}{\pi}\int_0^{2\pi} I_a \cos n\omega t d\omega t \qquad (2.11)$$

$$= \frac{4I_o}{n\pi}\cos\frac{n\pi}{6}\sin\frac{n\pi}{2}$$

$$= (-1^k)\frac{2\sqrt{3}I_o}{(6k \pm 1)\pi}\sin(k\pi \pm \frac{\pi}{2}), \text{ for n=6k}\underline{+}\text{1, k=1, 2, 3…}$$

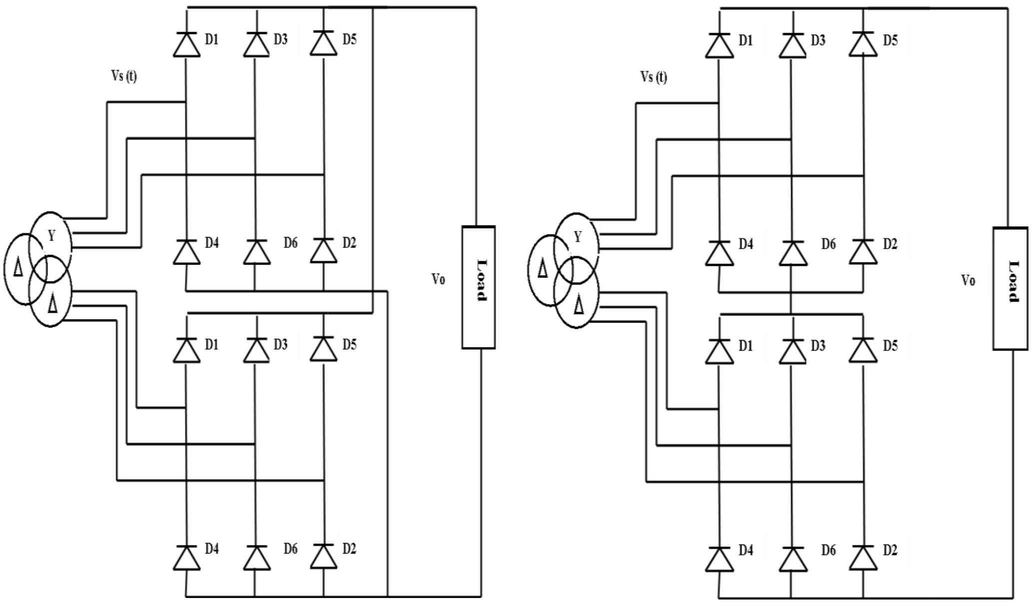
$$b_n = \frac{2}{T}\int_0^T I_a \sin n\omega t d\omega t = \frac{1}{\pi}\int_0^{2\pi} I_a \sin n\omega t d\omega t = 0 \qquad (2.12)$$

$$\therefore I_a = \sum_{k=1}^{\infty}(-1^k)\frac{2\sqrt{3}I_o}{(6k \pm 1)\pi}\sin(k\pi \pm \frac{\pi}{2})Cos((6k \pm 1)\omega t) \qquad (2.13)$$

$$I_a = \frac{2\sqrt{3}}{\pi}I_o(\cos\omega t - \frac{1}{5}\cos 5\omega t + \frac{1}{7}\cos 7\omega t - \frac{1}{11}\cos 11\omega t + \frac{1}{13}\cos 13\omega t - \frac{1}{17}\cos 17\omega t + ...)$$

2.4.2. Twelve Pulse Diode Rectifier

There are two combinations of six-pulse diode rectifiers, series and parallel, to form a twelve pulse rectifier. A typical example of 12-pulse rectifiers is given in the following Figure 2.3.



(a) Parallel (b) Series

Figure 2.3 Basic circuit diagram of 12-pulse rectifier for different combination [17]

The parallel connections of two six pulse diode rectifiers give in a 12 pulse rectifier with smoothed output waveform. The advantages of the 12 pulse rectifier are reduction in the AC input line current harmonics and dc output voltage ripple.

The output voltage and input current waveform can be calculated using the six-pulse rectifier waveforms. The output voltage waveform of 12 pulse rectifier, Figure 2.3a, is given as follows,

$$V_{o\Delta} = \frac{3\sqrt{3}}{\pi}V_m + \frac{6\sqrt{3}}{35\pi}V_m\cos(6\omega t) + \frac{6\sqrt{3}}{143\pi}V_m\cos(12\omega t) + \frac{6\sqrt{3}}{323\pi}V_m\cos(18\omega t) + ...$$

$$V_{oy} = \frac{3\sqrt{3}}{\pi}V_m + \frac{6\sqrt{3}}{35\pi}V_m\cos(6\omega t - 30^o) + \frac{6\sqrt{3}}{143\pi}V_m\cos(12\omega t - 30^o) + \frac{6\sqrt{3}}{323\pi}V_m\cos(18\omega t - 30^o) + ...$$

The output voltage of the 12-rectifier is approximated by the average of the two six-pulse rectifiers output voltages [17].

$$V_o = \frac{V_{oy} + V_{o\Delta}}{2} \qquad (2.14)$$

$$V_o = \frac{3\sqrt{3}}{\pi}V_m + \frac{6\sqrt{3}}{35\pi}V_m\cos(12\omega t) + \frac{6\sqrt{3}}{143\pi}V_m\cos(24\omega t) + \frac{6\sqrt{3}}{323\pi}V_m\cos(36\omega t) + ...$$

Applying similar procedure for the current, the AC line current is given as follows.

$$I_{ay} = \frac{2\sqrt{3}}{\pi} I_o (\cos\omega t - \frac{1}{5}\cos 5\omega t + \frac{1}{7}\cos 7\omega t - \frac{1}{11}\cos 11\omega t + \frac{1}{13}\cos 13\omega t - \frac{1}{17}\cos 17\omega t + ...)$$

$$I_{a\Delta} = \frac{2\sqrt{3}}{\pi} I_o (\cos\omega t + \frac{1}{5}\cos 5\omega t - \frac{1}{7}\cos 7\omega t - \frac{1}{11}\cos 11\omega t + \frac{1}{13}\cos 13\omega t + \frac{1}{17}\cos 17\omega t + ...)$$

$$I_A = I_{ay} + I_{a\Delta} = \frac{4\sqrt{3}}{\pi} I_o (\cos\omega t - \frac{1}{11}\cos 11\omega t + \frac{1}{13}\cos 13\omega t - \frac{1}{23}\cos 23\omega t + ..) \quad (2.15)$$

2.5. Harmonic Mitigation Techniques

Various harmonic reduction techniques have been developed to meet the requirements imposed by the current harmonic standards. The intent of these techniques is to make the input current a pure sinusoidal waveform, so as to reduce the overall current THD. In general these techniques can be classified into three broad categories:

1. Passive filters (line reactors / DC link chokes, series, shunt, and low-pass filters)

2. Active harmonic compensation systems (series and parallel)

4. Hybrid systems

2.5.1. Passive Filters

Passive filter uses passive elements. It is constructed using resistors, capacitors and inductors.

In passive filters, the flow of the undesired harmonic currents into the power system can be prevented by the usage of a high series impedance to block them or by diverting them to a low impedance shunt path. These two ideas represent the concept of the series and the shunt passive filters, respectively.

Series passive filters can be purely inductive type or LC tuned type. AC line reactor filter and DC link inductor filter are the two purely inductive type filters. AC line reactors offer a considerable magnitude of inductance that alters the way the current is drawn by the rectifier bridge. They make the current waveform less discontinuous, resulting in lower current harmonics. To maximize the input reactance while minimizing AC voltage drop both AC line reactors and DC link inductance (choke), shown in Figure 2.4, can be combined. The DC link inductance is electrically present after the diode rectifier and before the DC bus capacitor and it performs very similar to the three phase AC line reactors. Both AC line and DC link inductance insertion methods provide a limited amount of THD reduction that is not sufficient to comply with the IEEE 519 standards.

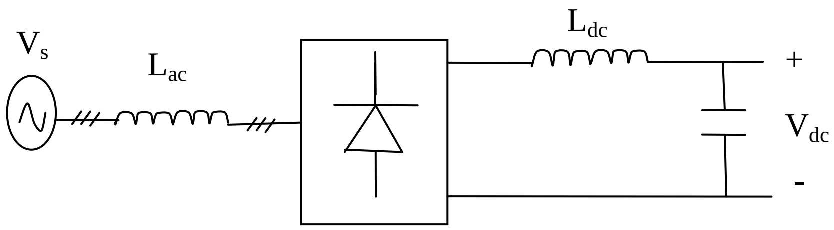


Figure 2.4 AC line reactor and DC line inductance based passive filtering [36]

The tuned series passive filter, shown in Figure 2.5, is connected in series with the load. The filter consists of parallel inductance and capacitance that are tuned to provide high impedance at a selected harmonic frequency. The high impedance then blocks the flow of harmonic current at the tuned frequency only. At fundamental frequency, the filter is designed to yield low impedance, thereby allowing the fundamental current to flow. For blocking multiple harmonics, multiple series filters are needed.

They must be designed to carry a full rated load current as they are connected in series to full line voltage. Therefore, they can create significant losses at the fundamental frequency.

In contrast, shunt passive filters carry only a fraction of the current that a series filter must carry. Given the higher cost of a series filter, and the fact that shunt filters may supply reactive power at the fundamental frequency, the most practical approach usually is the use of shunt filters.

A shunt filter offers very low impedance path at the frequency to which it is tuned and it shunts most of the harmonic current at that frequency. Most Common shunt filter types are the single tuned and high-pass filters. These two filters are the relatively simple to design and implement among the other shunt types. The layout of common shunt filter types is shown in Figure 2.6 [11].

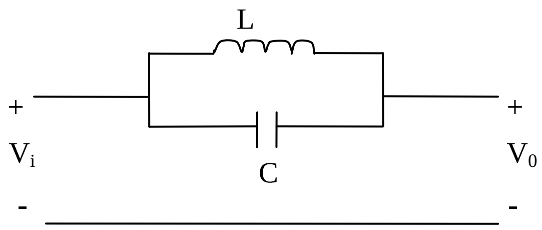


Figure 2.5 Series passive filter configuration [36]

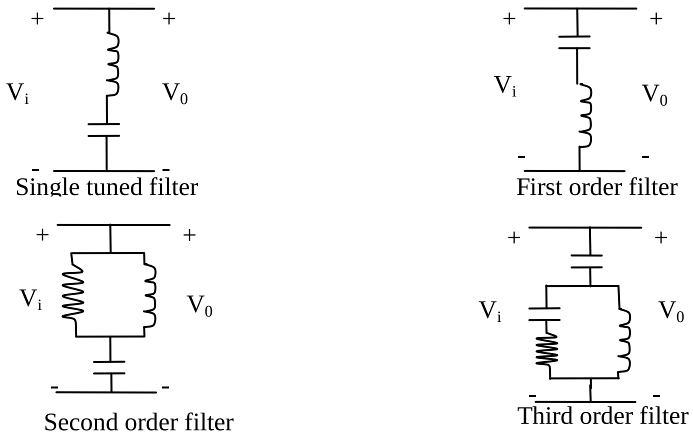


Figure 2.6 Common shunt passive filter configuration [36]

Unlike the shunt and series filters that have a narrow band of harmonic suppression, broadband filters have a wider range of harmonics suppression property. Broadband filters employ a combination of the two passive techniques, with a high series impedance to block the undesired current harmonics (from flowing through the grid) and a low shunt impedance path to divert their flow through the shunt filter. They can be in different structures, shown in Figure 2.7, LC and LLCL type [12] [13].

They are tuned to a low cut off frequency such that only fundamental component will pass from the input to the output. Therefore, they are called low pass broadband filters. On the contrary, classical shunt filters are tuned to a single harmonic frequency to be suppressed and multiple stages are used to suppress all injected current harmonics.

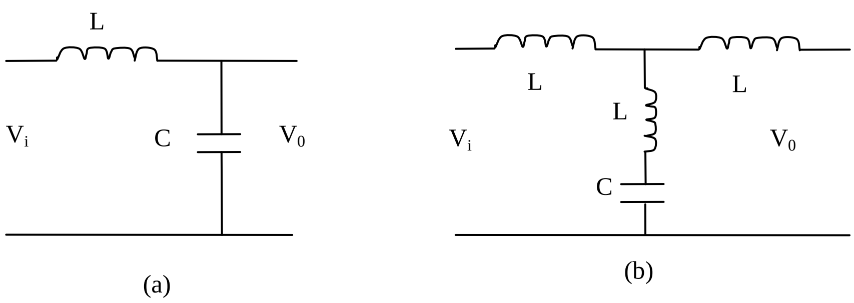


Figure 2.7 Low pass filter configuration (a) LC type, (b) LLCL type [36]

2.5.2. Active Filters

Active harmonic compensation (filtering) method is relatively a new method for eliminating current harmonics from the line. Active filters give good system performance and current harmonics reduction. However, they are based on sophisticated power electronics components and thus they are much more expensive than passive filters. In active filters the basic idea is to inject to the line equal magnitudes of the current/voltage harmonics generated by the nonlinear load and with 180 degrees phase angle difference so they cancel each other.

Active filters can be classified based on converter type, topology, and number of phases. The converter type can be either Current Source Inverters (CSI) or Voltage Source Inverters (VSI). CSI based active filters employ an inductor as the energy storage device whereas VSI-based active filters used a capacitor as the energy storage device. The topology can be shunt, series, or a combination of both. The third classification is based on the number of phases, such as two-wire (single-phase) and three- or four-wire (three phase) systems [14]. Three phase active filters are used for high-power nonlinear loads. Active filters of many configurations have been introduced and improved. Shown in Figure 2.8, are the fundamental configurations [15]. Of all various configurations, the parallel active filter using the voltage source inverter topology accompanied by high performance current regulation methods is the most frequently employed type.

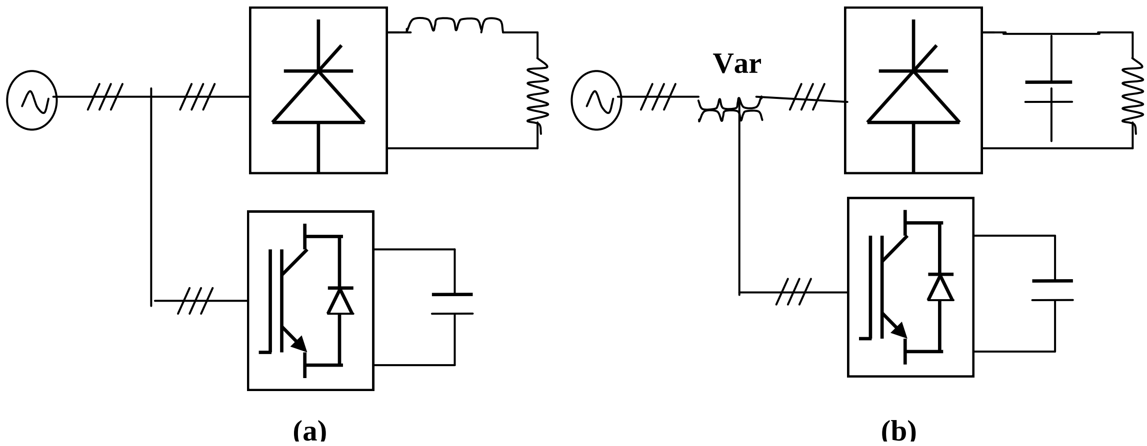


Figure 2.8 Active filter configurations (a) shunt, (b) Series [37]

2.5.3. Hybrid Filters

Hybrid active filters, as shown in Figure 2.9, combine active and passive filters in various configurations [15]. The main purpose of hybrid active filters is to reduce initial costs and to improve efficiency. They are also used to improve the compensation characteristics of passive filters and improve any series or parallel resonance due to supply or load respectively. Practically, more viable and cost effective hybrid filter topologies have been developed than stand-alone active filters.

They enable the use of significantly small rating active filters that is less than 5% of the load kVA compared to stand-alone parallel (25-30%) or series active filter solutions [16]. Usually, with shunt passive filter combinations, the passive filter is tuned up to a specific frequency to suppress the corresponding harmonic and decrease the power rating of the active filter. Another typical combination is of a series active filter and a series passive filter.

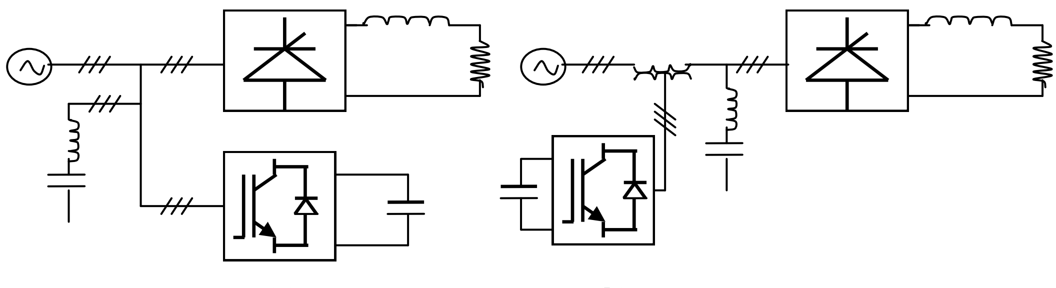


Figure 2.9 Hybrid active filters common configurations: (a) Shunt active filter and shunt passive filter, (b) Series active filter and shunt passive filter [37]

High fundamental component current through the series active filter and the fundamental component voltage across the shunt active filter are problematic. High initial and running cost and complexity are major drawbacks of the active harmonic filtering technique.

To conclude, most of the mentioned filtering techniques have common drawback of higher cost compared to passive filtering techniques. The passive harmonic filtering techniques to a large extent are still the most commonly used techniques for current harmonic mitigation of power system applications. The passive filter is also simple in design and easy for operation purpose. So, in this work shunt passive harmonic filtering techniques is selected as a filter to eliminate harmonic currents injected by the converters to the Addis Ababa LRT railway network. This filter is designed to its optimum value to effectively minimize the harmonics in the system.

2.6. Performance Specification

2.6.1. Total Harmonic Distortion

Total harmonic distortion of a signal is a measurement of the harmonic distortion and is defined as the ratio of the sum of the powers of all harmonic components to the power of the fundamental frequency. It is an important figure of merit used to quantify the level of harmonics in voltage or current waveforms. In a power system, total harmonic distortion current can be used to indicate the harmonic levels [18].

$$THD_I = \sqrt{\sum_{n=2}^{\infty} \frac{I_n^2}{I_1^2}} = \frac{\sqrt{\sum_{n=2}^{\infty} I_n^2}}{I_1} \tag{2.16}$$

Similarly the total harmonic distortion voltage is given as,

$$THD_v = \frac{\sqrt{\sum_{n=2}^{\infty} V_n^2}}{V_1} \tag{2.17}$$

2.6.2. Power Factor

In power systems which contain nonlinear loads, there are essentially two power factors; the displacement power factor and the distortion power factor. The displacement power factor is the displacement component of the power factor and is given by the ratio of the active power of the fundamental wave, in watts, to the fundamental wave, volt-amperes. The distortion power factor is the distortion component associated with the harmonic voltages and currents present. It is defined as the ratio of the fundamental component of the AC line current to the total line current.

The true power factor of a system is the product of these two power factors and it is the ratio of average power to apparent power [18].

$$S(kVA) = \sqrt{(P^2 + Q^2 + D^2} = \sqrt{kW^2 + kVAr^2 + kVArH^2} \tag{2.18}$$

Where,

S is apparent power

P is the active power

Q is the reactive power

D is the distortion power

The active power, *P* can be calculated using:

$$P = \sum_{h=0}^{\infty} V_h I_h \cos\varphi_h = P_0 + P_1 + \sum_{h=2}^{\infty} P_h \tag{2.19}$$

Where,

Φ_h is the phase of the h[th] harmonic

P_0 is the dc component of active power

P_1 is the fundamental component of active power

P_h is the active component of individual harmonics

Similarly, the reactive power, *Q*, is defined as:

$$Q = \sum_{h=0}^{\infty} V_{Hrms} I_{Hrms} \sin\varphi_h = Q_1 + \sum_{h=2}^{\infty} Q_h \tag{2.20}$$

Where,

Q_1 - fundamental component of reactive power

Q_h - reactive component of individual harmonics

The apparent power, *S*, can also be expressed using the current and voltage total harmonic distortion.

$$S = V_{rms} * I_{rms} = \sqrt{\sum_{h=1}^{\infty} V_{hrms}^2 I_{hrms}^2} = V_{1rms} * I_{1rms} \sqrt{1+THD_V^2}\sqrt{1+THD_I^2}$$

$$= S_1 \sqrt{1+THD_V^2}\sqrt{1+THD_I^2} \qquad (2.21)$$

Where,

S_1 - is the apparent power at the fundamental frequency

THD_I/THD_V is Current and Voltage total harmonic distortion

The distortion power, D, can be calculated as:

$$D^2 = S^2 - (P^2 + Q^2) \qquad (2.22)$$

As detailed above, the power factor is the ratio of the active power in kW to the apparent power in kVA.

Power factor: $$Cos\phi = \frac{P}{S} = \frac{P}{S_1}\frac{1}{\sqrt{1+THD_V^2}\sqrt{1+THD_I^2}} = Cos\phi_{disp} * Cos\phi_{disp} \qquad (2.23)$$

Displacement power factor: $$cos\emptyset_{disp} = \frac{P}{S_1} \qquad (2.24)$$

Distortion power factor: $$Cos\phi_{dist} = \frac{1}{\sqrt{1+THD_V^2}\sqrt{1+THD_I^2}} = \frac{V_{1rms} I_{1rms}}{V_{rms} I_{rms}} = \frac{S_1}{S} \qquad (2.25)$$

Where,

$\cos\varphi_{disp}$ = displacement power factor (fundamental components)

$\cos\varphi_{dist}$ = distortion power factor (harmonic components)

2.6.3. Power Loss

Voltage and current harmonics produced by nonlinear loads increase power losses and, therefore, have a negative impact on electric utility distribution systems and components. Most of the losses associated with harmonics are transformer losses and transmission line losses. The presence of harmonic currents in power system causes extra loss in transformer winding and thus, leads to increase in temperature, reduction in insulation life, increase to higher losses and finally reduction of the useful life of transformer [19]. Harmonic voltage increase losses in its magnetic core while harmonic currents increased losses in its winding and structure. In general, harmonics losses occur from increased heat dissipation in the windings and skin effect both are a function of the square of the rms current, as well as from eddy currents and core losses. This extra heat can have a significant impact in reducing the operating life of the transformer insulation the increased of eddy current losses that produced by a non-sinusoidal load current can cause abnormal temperature rise and hence excessive winding losses. The transformer harmonic loss is the sum of eddy current loss, stray current loss and dc component loss.

- Eddy Current Loss in Windings

This loss is caused by time variable electromagnetic flux that covers windings. Skin effect and proximity effect are the most important phenomenon in comparison to external windings, internal loss. The reason is the high electromagnetic flux intensity near the core that covers these windings.

As mentioned above, eddy current loss of windings is square of harmonic frequency in harmonic condition. In following equation, this loss is calculated:

$$P_{eddyloss} = P_{\mathrm{Re}ddy} \sum_{h=1}^{\infty} h^2 [\frac{I_h}{I_R}]^2 \tag{2.26}$$

- Stray current loss

Due to the linkage between electromagnetic flux and conductor, a voltage induces in the conductor and this will lead to producing eddy current Eddy current produces loss and increases temperature. A part of eddy current loss which is produced in structural parts of transformers (except in the windings) is called other stray loss [20] [21]. Many factors such as size of core, class of voltage of transformer and construction of materials used to build tank and clamps [22]. To determine the effect of frequency on the value of other stray loss, different tests have been fulfilled. The other stray losses are assumed to vary with the square of the rms current and the harmonic frequency to the power of 0.8:

$$P_{strayloss} = P_{Rstray} \sum_{h=1}^{\infty} h^{0.8} [\frac{I_h}{I_R}]^2 \tag{2.27}$$

- Effect of Harmonics on DC Losses

This loss can be calculated by measuring winding dc resistance and load current. If RMS value of load current increases due to harmonic component, this loss will be increased by square of load current [23]. If the rms value of the load current is increased due to harmonic components, then will increase by square of RMS of load current condition is shown by:

$$P_{dc} = R_{dc} * I^2 = R_{dc} * \sum_{h=1}^{\infty} I_h^2 \qquad (2.28)$$

2.6.4. Resonance Effect

All circuits containing both capacitances and inductances have one or more natural frequencies. When one of those frequencies lines up with a frequency that is being produced on the power system, a resonance may develop in which the voltage and current at that frequency continue to persist at very high values. This is the root of most problems with harmonic distortion on power systems. There are two forms of resonance which need to be considered: series resonance and parallel resonance, as shown below in, Figure 2.10.

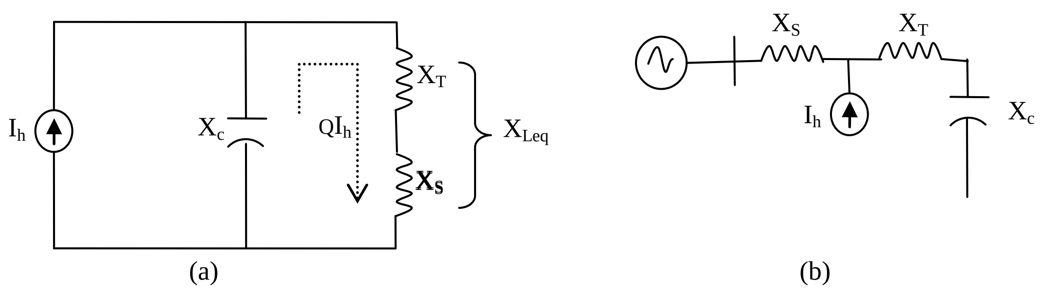


Figure 2.10 Parallel and series resonance circuit [18]

I) Parallel resonance

From the perspective of harmonic sources the shunt capacitor appears in parallel with the equivalent system inductance (source and transformer inductances) at harmonic frequencies.

Parallel resonance occurs when the reactance of X_C and the power system cancel each other out. The frequency at which this phenomenon occurs is called the parallel resonant frequency. It can be expressed as follows:

$$f_p = \frac{1}{2\pi}\sqrt{\frac{1}{L_{eq}C} - \frac{R^2}{4L_{eq}^2}} \approx \frac{1}{2\pi}\sqrt{\frac{1}{L_{eq}C}} \quad (2.29)$$

Where,

R-resistance of combined equivalent source and transformer

L_{eq}-inductance of combined equivalent source and transformer

C-capacitance of capacitor bank

At the resonant frequency, the apparent impedance of the parallel combination of the equivalent inductance and capacitance as seen from the harmonic current source becomes very large, i.e.

$$Z_p = \frac{X_c(X_{Leq} + R)}{X_C + X_{Leq} + R} = \frac{X_C(X_{Leq} + R)}{R} \quad (2.30)$$

II) Series resonance

There are certain instances when a shunt capacitor and the inductance of a transformer or distribution line may appear as a series LC circuit to a source of harmonic currents. If the resonant frequency corresponds to a characteristic harmonic frequency of the nonlinear load, the LC circuit will attract a large portion of the harmonic current that is generated in the distribution system. A customer having no nonlinear load, but utilizing power factor correction capacitors, may in this way experience high harmonic voltage distortion due to neighbouring harmonic sources.

During resonance, the power factor correction capacitor forms a series circuit with the transformer and harmonic sources. The harmonic source shown in figure 2.10 represents the total harmonics produced by other loads. The inductance in series with the capacitor is that of the service entrance transformer. The series combination of the transformer inductance and the capacitor bank is very small (theoretically zero) and only limited by its resistance. Thus the harmonic current corresponding to the resonant frequency will flow freely in this circuit. The voltage at the power factor correction capacitor is magnified and highly distorted. This is apparent from the following equation:

$$V_{S(atCapacitorBank)} = \frac{X_C}{X_T + X_C + R} V_h \approx \frac{X_C}{R} V_h \tag{2.31}$$

2.6.5. Standards of Harmonics

Harmonic voltage or current distortions can affect the operation of other devices connected to the same power grid; therefore various standards have been devised to judge the severity of harmonic distortion. One of these is IEEE 519-1992 by the American Institute of Electrical and Electronic Engineers. This standard puts a limit on individual and total distortion for current and voltage harmonics.

In order to apply the standard limit the concept of Point of Common Coupling (PCC) must be analyzed first. PCC is generally defined as the utility/customer connection point. It is the point at which the current distortion limits apply. The idea behind the standard is that harmonic limits are placed on a customer side, on the basis of current distortion relative to the total plant load. The limits do not apply to a specific non-linear load in the plant. The harmonic current limits change depending on the ratio of short circuit current to maximum demand load current at the PCC.

$$Ratio = \frac{I_{sc}}{I_L} \tag{2.32}$$

Where:-

I_{sc} - maximum short circuit current at the PCC

I_L - maximum demand load current at PCC

The overall aim of the standard is to keep voltage distortion at the PCC below the limit.

According with IEEE-519 (Institute of Electrical and Electronic Engineers), the table 2.2 lists the limits on the harmonic currents that a user of power electronics equipment and other nonlinear loads is allowed to inject into the utility system. Table 2.3 lists the quality of voltage that the utility can provide the user.

Table 2-2 Current Distortion Limits for (120V-69kV) Distribution Systems [24]

Maximum Harmonic Current Distortion in Percent of I_L						
Individual Harmonic Order(Odd Harmonics)						
I_{sc}/I_L	<11	$11 \le h < 17$	$17 \le h < 23$	$23 \le h < 35$	$35 \le h$	TDD
<20	4.0	2.0	1.5	0.6	0.3	5.0
20-50	7.0	3.5	2.5	1.0	0.5	8.0
50-100	10.0	4.5	4.0	1.5	0.7	12.0
100-1000	12.0	5.5	5.0	2.0	1.0	15.0
>1000	15.0	7.0	6.0	2.5	1.4	20.0

Even harmonics are limited to 25% of the odd harmonic limits above.

Table 2-3 Harmonic Voltage limits in percent of fundamental [24]

Voltage level	2.3-69kV	69-138kV	>138kV
Maximum for individual harmonics	3.0	1.5	1.0
Total Harmonic Distortion(THD)	5.0	2.5	1.5

Table 2.3 specifies the individual harmonics and the THD limits on the voltage that the utility supplies to the user at the connection point.

CHAPTER THREE

3. TRACTION POWER SYSTEM MODELLING

3.1. Introduction

All analysis in the engineering starts with the formulation of appropriate models. A model in power system analysis is a mathematical model, a set of equations or relations, which appropriately describes the interactions between different quantities in the time frame studied and with the desired accuracy of a physical component.

Depending on the purpose of the analysis different models of the same physical system or components might be valid. In this section the traction power system components are modelled in order to perform the harmonics distortion analysis throughout the system. The traction power system components will be independently modelled and finally cascaded to represent the entire system.

3.2. Traction Supply Network

The Addis Ababa Light Rail Transit has two routes, East-West and North-South. Each route receives power from four different EEPCo substations that are located at the four edges of rail routes. These substations are supplied from five grid substations; Gefersa, Sebeba-I, Kality-I, Legetafo and Sululta which are connected in a ring form. The four EEPCo side railway substations; Torhailoch, Kality, Ayat and Menelik II, are installed with double circuits of transformer capacity 25MVA, 132kV/15kV. The general topology of AC source supply system is given in figure 3.1.

Each EEPCo substation provides power supply for half of the LRT rectifier substations found on the route. In this work the power system analysis is performed on Lideta rectifier substation. The Lideta substation is found on the common rail section where more traffic flow exists and has a higher loading capacity. This rectifier substation is supplied from Torhailoch substation. In addition to this the Torhailoch substation supplies four different substations with three outgoing circuits.

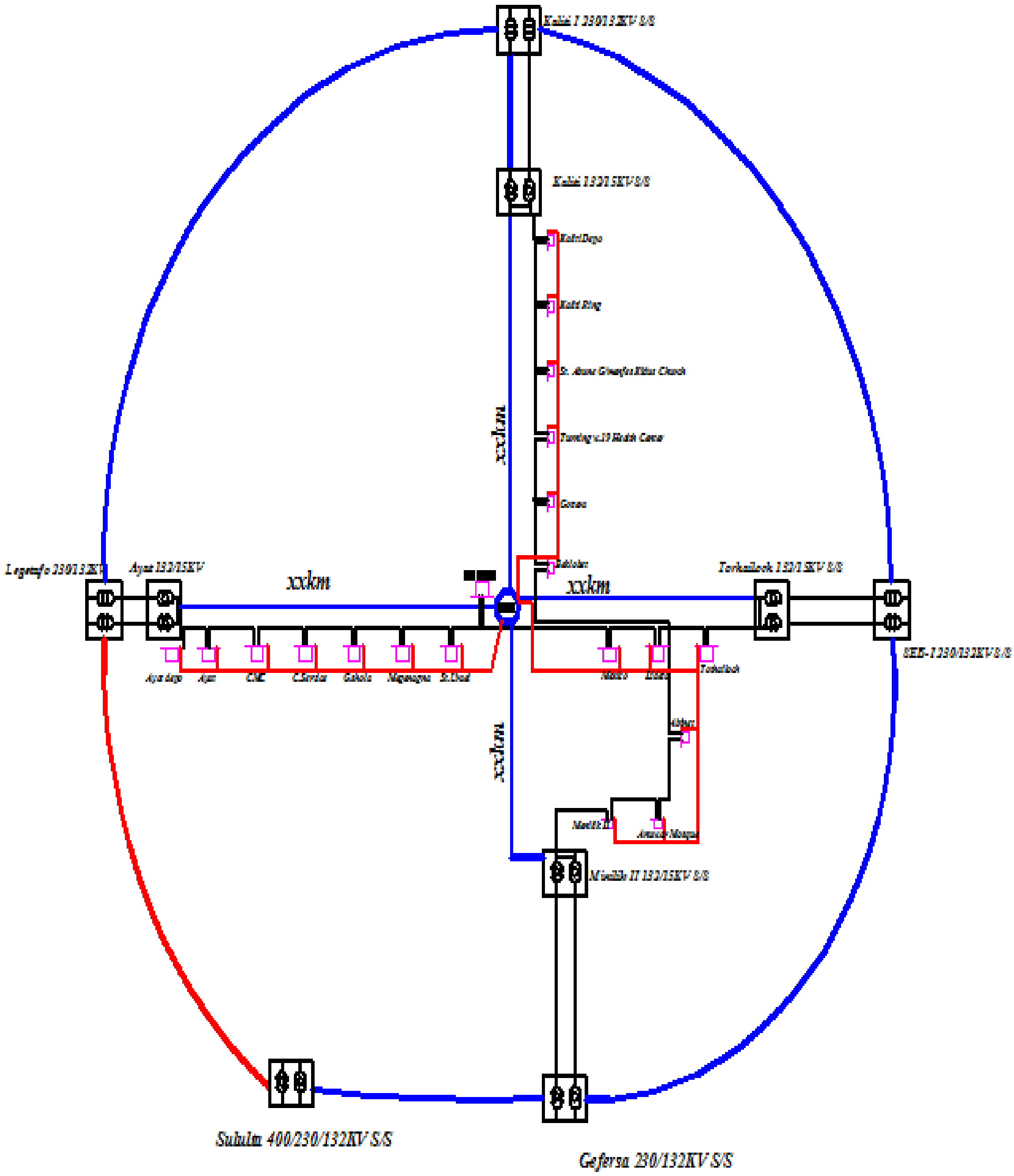


Figure 3.1 The LRT AC source supply system [34]

Each circuit feeds the rectifier substations in such a way that the consecutive substations are not fed from the same circuit. The first circuits provide supply for the Torhailoch rectifier substation (EW22).The second circuit supplies the Lideta (EW20) and Urael (EW13) substations, where as the third circuit is for the near DebreWork Building(EW18) substation, Saint Joseph(EW16) substation and near Legese Feleke(EW10) substation. The single line diagram of the feeding system of Torhailoch substation is given in figure 3.2.

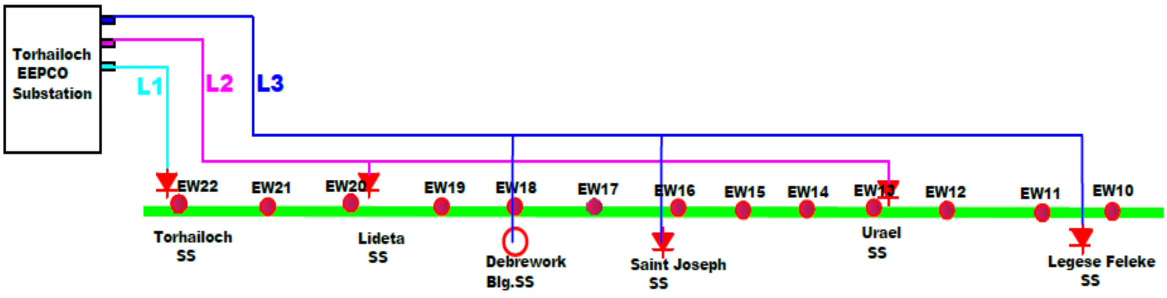


Figure 3.2 Torhailoch Substation feeding system [34]

There are a total of 20 railway substations on both the East-West and North-South routes of the line. Nineteen substations are a combined step down and rectifier substations, where as one is step down substation that provides supply for station service. Each rectifier substation converts the incoming 15kV AC voltage to the 750V DC.

The configuration of the rectifier substation consists of power transformation equipments for traction units which convert AC electricity to DC. Rectification is made through two three phase diode rectifiers connected in parallel to form a 12-pulse rectifier. DC terminals of the rectifiers are connected in parallel to a positive and a negative DC bus, respectively. The positive bus is connected to the line and the negative to the rail. The traction supply system is bidirectional supply system; with the equal voltage DC is supplied by rectifier substations from both ends of the section. Figure 3.3, shows the AUTOCAD design of a feeding section between Lideta and Saint Joseph rectifier substation.

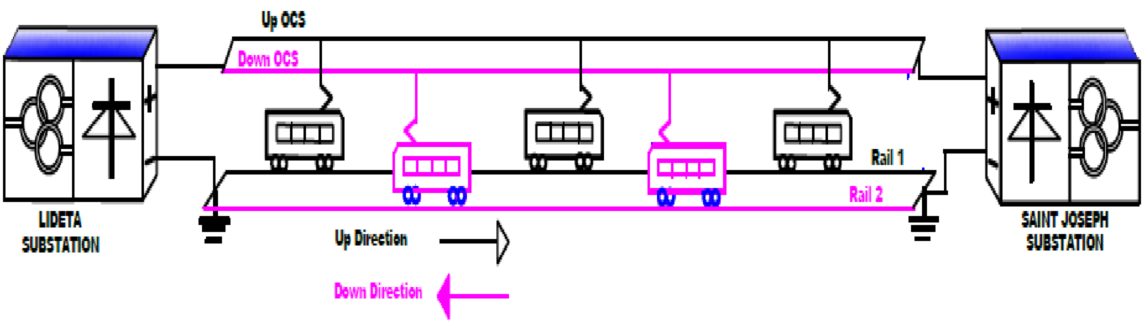


Figure 3.3 Lideta to Saint Joseph section feeding system [Author]

3.3. Distribution Line Modelling

There are two categories of distribution lines in traction power system, AC and DC. The AC distribution line transmits electrical power from national grid network to the railway substation, where as the DC distribution lines delivers power to load side. In both cases, the power transmission depends on the line physical properties to reach the receiving end in the preferred condition. The line performance is determined by its voltage drop, line losses and efficiency. A common method of analyzing this behaviour is through parameterization and modelling of the transmission lines.

AC lines are modelled using its series resistance, series inductance, shunt capacitance, and shunt conductance. There are three ways in common practice to model power lines. The three models are the short line model, medium line model and the long line models. A line is defined as a short-length if its length is less than 80 km (50 miles), or medium length for the length between 80 km (50 miles) and 240 km (150 miles), and long line for length above 240km [25].

Both short and medium-length lines are approximated by lumped-parameter models [25]. However, if the line is larger than 240 km, the model must consider parameters uniformly distributed along the line [25][26]. Since the AC distribution line of the Addis Ababa LRT supply system has a length of less than 80 km it is modelled using the short line model method. Assuming there is a balanced three phase transmission line, the section is modelled using three phase pi circuit with lumped parameters. The equivalent three phase circuit diagram of a three phase pi circuit is given as shown in the figure 3.4.

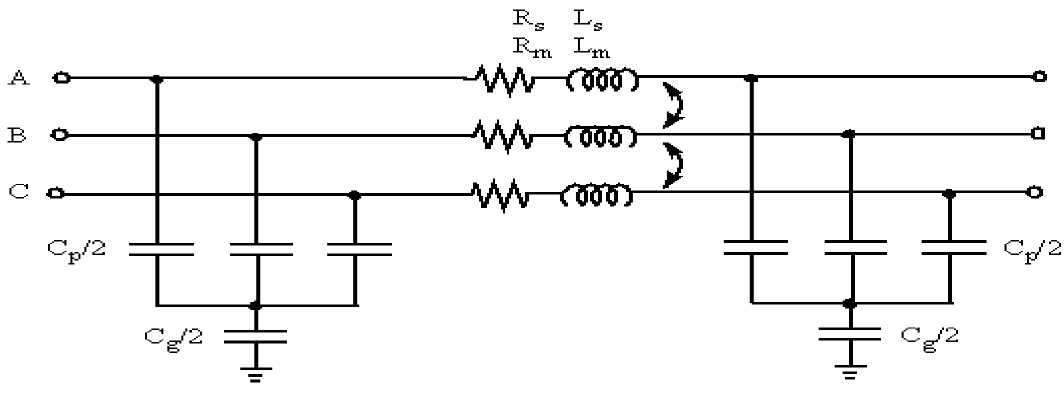


Figure 3.4The equivalent circuit of three phase pi-circuit model [27]

The line parameters R, L, and C are specified as positive- and zero-sequence parameters that take into account the inductive and capacitive couplings between the three phase conductors, as well as the ground parameters. This method of specifying line parameters assumes that the three phases are balanced.

The self and mutual resistances (*Rs*, *Rm*), self and mutual inductances (*Ls*, *Lm*) of the three coupled inductors, as well as phase capacitances *Cp* and ground capacitances *Cg*, are deduced from the positive- and zero-sequence RLC parameters as follows.

If the line parameters are defined as follows:

r_1, r_0 - positive- and zero-sequence resistances per unit length (Ω/km)

l_1, l_0 - positive- and zero-sequence inductances per unit length (H/km)

c_1, c_0 - positive- and zero-sequence capacitances per unit length (F/km)

*l*sec - line section length (km)

The total positive and zero-sequence RLC parameters for the short line modelling are evaluated as;

$$R_1 = r_1 * l_{sec} \quad , \quad R_0 = r_0 * l_{sec}$$
$$L_1 = l_1 * lsec \quad , \quad L_0 = l_0 * l_{sec} \qquad (3.1)$$
$$C_1 = c_1 * l_{sec} \quad , \quad C_0 = c_0 * l_{sec}$$

Then RLC line section parameters are then computed as follows:

$$R_m = (Ro - R_1)/3 \qquad R_s = (2R_1 + R_0)/3$$
$$L_m = (Lo - L_1)/3 \qquad L_s = (2L_1 + L_0)/3 \qquad (3.2)$$
$$C_p = C_1 \qquad C_g = 3C_1C_0/\,(C_1 - C_0)$$

The characteristic behavior of DC line is purely resistive. The DC traction line is modelled using resistive elements. [39]

3.4. Rectifier Substation Modelling

The main components of the rectifier substation are rectifier units and rectifier-transformers.

A. Rectifier Unit Model

The LRT traction rectifier is a 12-pulse rectifier that has a combination of two six pulse diode rectifiers connected in parallel, as shown in figure 3.5.

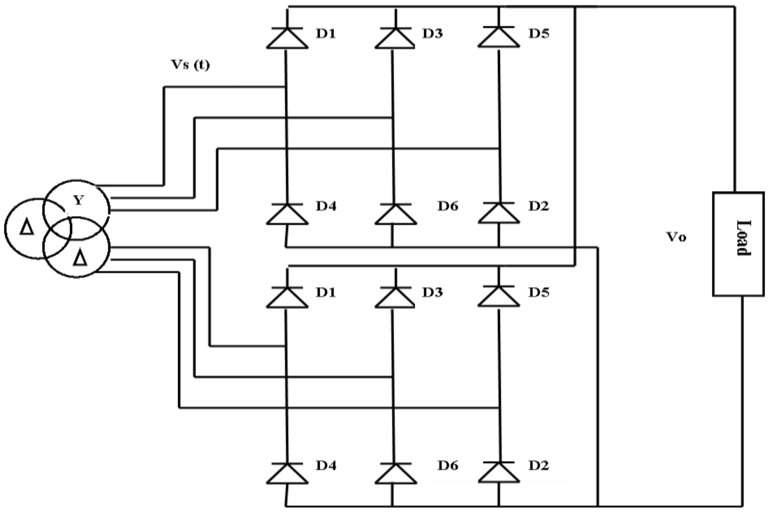


Figure 3.5 Basic circuit of 12- pulse diode rectifier [17]

Many harmonic models have been proposed for representing rectifier unit [28] [30]. The most common model is in the form of harmonic current source, which is specified by its magnitude and phase spectrum. The amplitude of harmonic currents generated by a diode rectifier on the DC side is almost constant even if source impedance varies, because the impedance on the DC side is much larger than the source impedance on the AC side. Therefore, in conventional equivalent circuits, the rectifier has been widely considered as an ideal current source for harmonics [31][32]. But, in rectifier circuits, there are both current harmonics on AC side and voltage harmonics on DC side. For an accurate analysis, it is necessary to deal with both harmonics. For this purpose, an equivalent circuit model containing both current harmonics and voltage harmonics are used in representing a 12-pulse rectifier [33].

In a rectifier circuit AC source is processed through a set of switches to create a well defined waveform. In this equivalent circuit the combined action of an actual source (AC source) and a set of switches are represented by an equivalent source. Based on superposition, a term-by-term for the Fourier series of the current and voltage in the rectifier circuits can be solved. The equivalent voltage source, Vo(t), contains both the fundamental component and harmonic components of the voltage. The Fourier series for the voltage $V_O(t)$ can be expressed in trigonometric form as, (detail is given in section 2.4) [17]

$$V_o = V_{dc} + \sum_{n=6,12,18,..}^{\infty} a_n Cos(n\omega t) + \sum_{n=6,12,18,..}^{\infty} b_n \sin(n\omega t)$$

$$V_o = \frac{3\sqrt{3}}{\pi} V_m + \frac{6\sqrt{3}}{35\pi} V_m \cos(12\omega t) + \frac{6\sqrt{3}}{143\pi} V_m \cos(24\omega t) + \frac{6\sqrt{3}}{323\pi} V_m \cos(36\omega t) + ...$$

Every component of Fourier series corresponds to a voltage source can be symbolized as V_{dc}, $V_{12}(\omega_{12}t)$,$V_{24}(\omega_{24}t)$. Second, when considering the AC side of the system, the rectifier is modelled by its harmonic current .The source currents are dependent on the load current. Therefore, these currents are modelled by a current controlled current source, which is controlled with load current. To derive the whole equivalent circuit containing harmonic currents on the AC side and harmonic voltages on the DC side of rectifiers, per phase equivalent circuit is represented as follows.

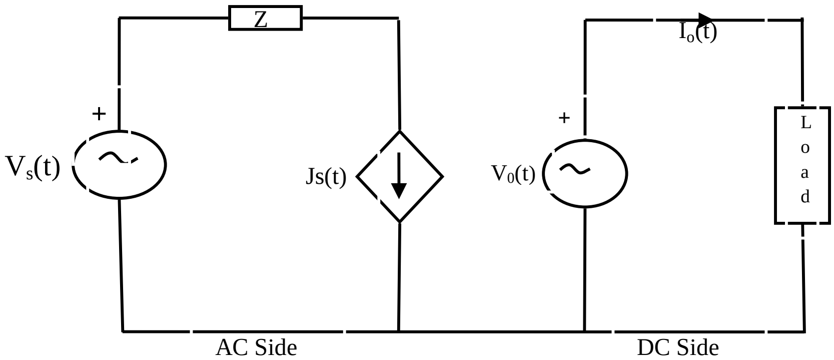


Figure 3.6 Simplified model of rectifier circuit [33]

From the point of harmonic studies, the equivalent circuit can be divided into two circuits: one is an equivalent circuit to the fundamental component and the other is an equivalent circuit to harmonics. The equivalent circuit to the fundamental components on both the AC and DC sides of the rectifier are represented by omitting the harmonic components from the current and voltage sources.

But, the equivalent circuit to harmonic components on both the AC side and the DC side of the rectifier is obtained under the condition that the source voltage $Vs(t)$= 0, the first component of harmonic current source J1(ω1t)= 0 and the first component of harmonic voltage source v1(ω1t)= 0. The equivalent circuit to harmonics is shown as,

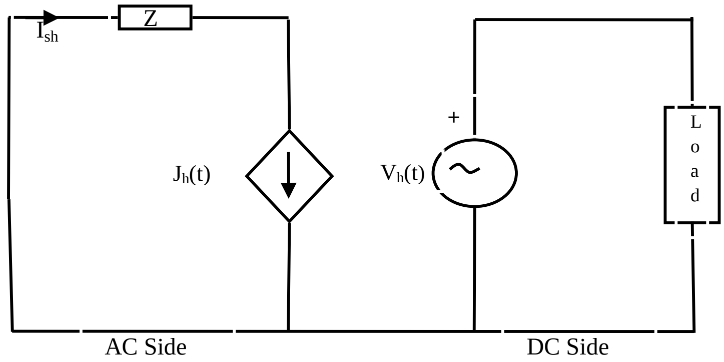


Figure 3.7 Equivalent circuits to harmonic components on the DC and AC side [33]

B. Rectifier Transformer Model

The rectifier transformers are special transformers with delta primary winding and both delta and star connected secondaries. The two secondaries are connected in such a way that a phase shift of 30^0 for current and voltage waveform appears at the output side. Such transformers are used in order to enhance some harmonic order cancellation.

To model a transformer accurately, we need to account for the following losses:

1. Copper losses – resistive heating in the windings: I^2R.

2. Eddy current losses – resistive heating in the core: proportional to the square of voltage applied to the transformer.

3. Hysteresis losses – energy needed to rearrange magnetic domains in the core: nonlinear function of the voltage applied to the transformer.

4. Leakage flux – flux that escapes from the core and flux that passes through one winding only.

For the balanced three phases system per phase equivalent circuit of the rectifier transformer is given as shown in figure 3.8.

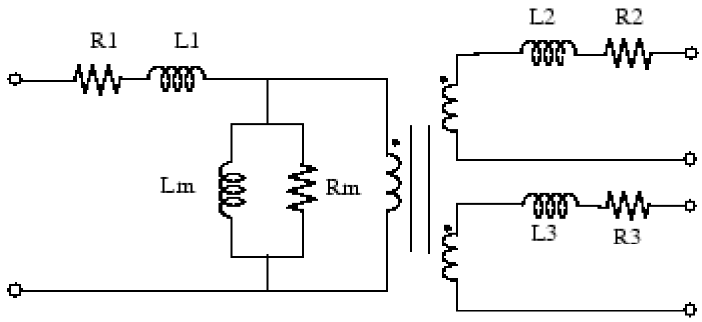


Figure 3.8 Equivalent circuit of three winding transformer [27]

3.5. Traction Load Modelling

The LRT tramcar unit consists of three wagons, two with cabin and one intermediate. Each wagon with cabin has two motors that drive the system. Squirrel-cage three-phase asynchronous traction motor type is used to drive the system. The DC current is collected from the overhead contact system (OCS) through the pantograph mounted on the intermediate wagon. This power is converted to AC with the use of inverter. The general block diagram of the traction load is given in figure 3.9.

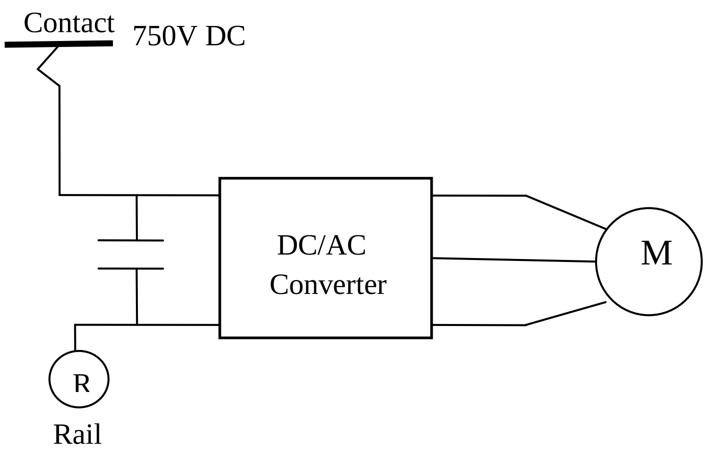


Figure 3.9 Traction Load diagram [42]

The Variable Voltage Variable Frequency (VVVF) inverter with Pulse Width Modulation (PWM) technology is applied to control the train speed. There is one inverter on each motorized cabin that feed two motors. So, the traction unit contains a total of two VVVF inverters that feed four motors and an auxiliary static inverter that provide power supply for on board train auxiliary devices. The drive system uses a six pulse inverter with a high switching frequency device, Insulated Gate Bipolar

Transistor (IGBT). With the high switching speeds and advanced PWM schemes, the performance of an inverter is significantly improved.

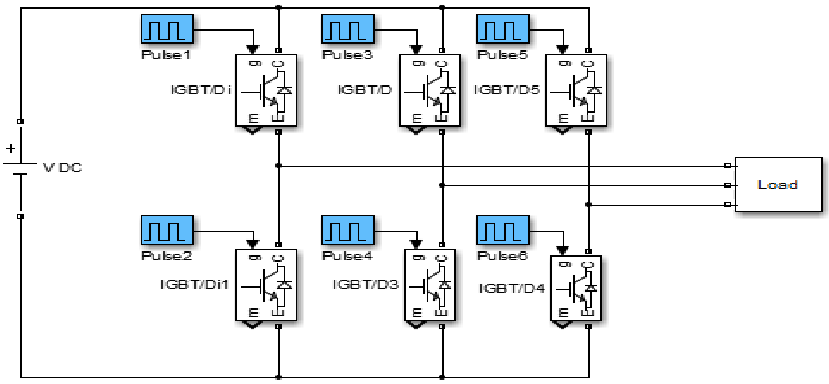


Figure 3.10 Circuit diagram of six pulse IGBT/PWM Inverter [Author]

The sinusoidal PWM uses a carrier comparison scheme to produce the gate signals of the inverter. To obtain a balanced three phase output voltages, the saw tooth voltage waveform (carrier wave V_c, frequency of 4kHz) is compared with three sinusoidal control voltages (reference wave $V_{r,}$ frequency f_r) that are 120 degrees out of phase. The inter section of V_c and Vr waves determine the switching instants and commutation of the modulated pulse.

The carrier and reference waves are mixed in a comparator. When sinusoidal wave has magnitude higher than the saw tooth wave, the comparator output is high, otherwise it is low. The figure 3.11 shows the simulation circuit of PWM in MATLAB.

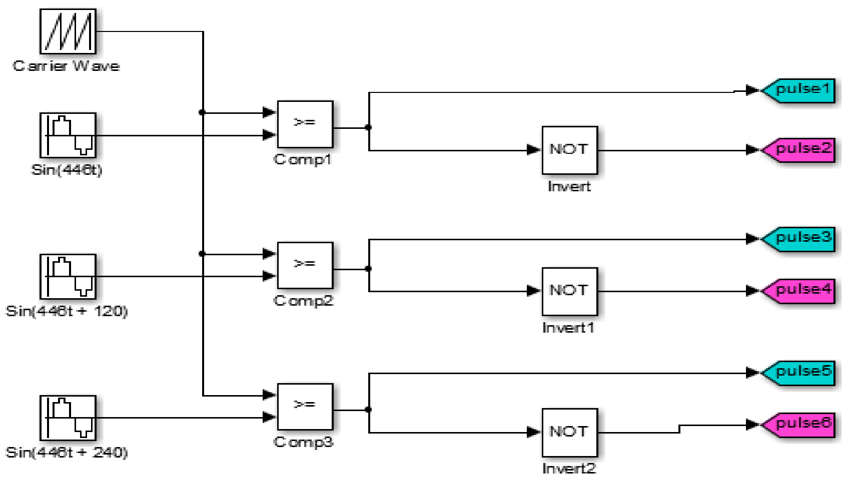


Figure 3.11 Sinusoidal PWM modelling[Author]

The ratio of Vr / Vc is called the modulation index (MI) and it controls the harmonic content of the output voltage waveform. The magnitude of fundamental component of output voltage is proportional to MI. Thus the output voltage is controlled by varying MI.

Frequency of the output wave is the same as frequency of the reference sinusoidal wave. The LRT tramcar induction motor has a rating of 130kW, 500V and 71Hz. The magnitude of the MI for six pulse inverter is calculated using the formula [17]:

$$V_{ab}\,peak = \frac{MI}{2} * V_{dc} * \sqrt{3} \qquad (3.3)$$

Where,

V_{ab} - phase to phase rms voltage of inverter

V_{dc} - dc voltage

So, the value of MI that yield 500V output voltage is calculated as;

$$MI = 2*(V_{ab}rms*\sqrt{2})/(V_{dc}*\sqrt{3})$$

$$= 2*(500*\sqrt{2})/(750*\sqrt{3})$$

$$= 1.09$$

For the Sinusoidal PWM, the moduletion index is the ratio of amplitude of the reference voltage to the amplitude of the saw tooth wave. By assuming the amplitude of all the reference waves equal to unity, the amplitude of the saw tooth wave can be calculated as;

$$V_c = \frac{V_r}{MI} = \frac{1}{1.09} = 0.9 \qquad (3.4)$$

The comparator compares the magnitude of the sinusoidal and the saw tooth waves and outputs one if sinusoidal wave is higher than saw tooth wave otherwise outputs zero.

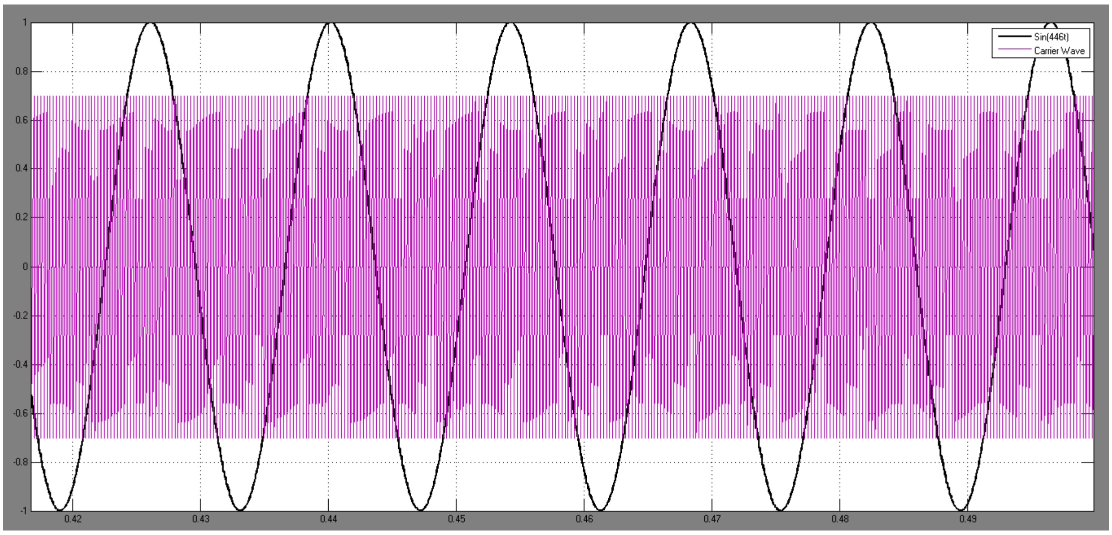


Figure 3.12 Comparator input, sinusoidal and sawtooth waveforms

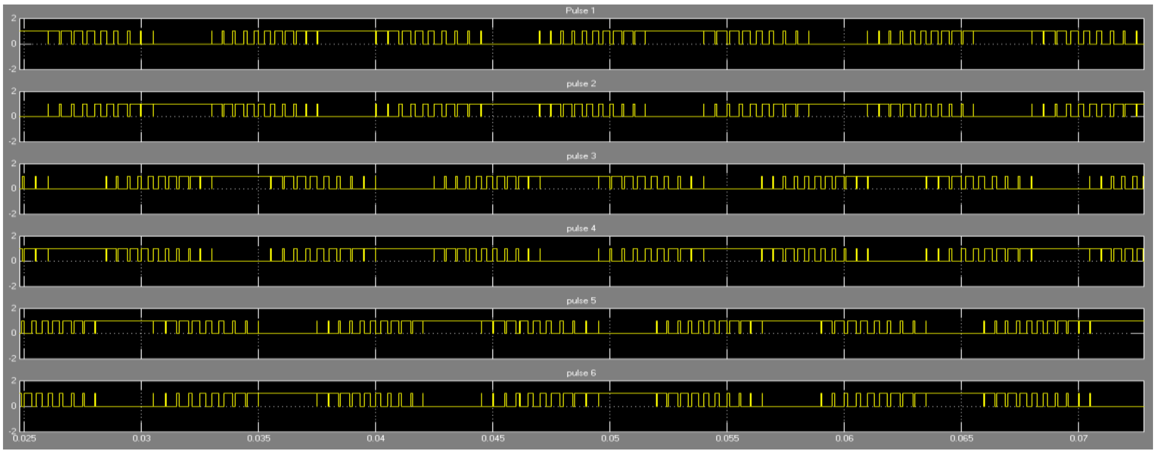


Figure 3.13 VVVF inverter gate trigger pulses

The auxilary static inverter converts the DC 750V to 380V three phase or 220V single phase. The value of MI for 380V voltage is calculated as;

$$MI = 2 * (V_{ab}rms * \sqrt{2})/(V_{dc} * \sqrt{3})$$

$$= 2 * (380 * \sqrt{2})/(750 * \sqrt{3})$$

$$= 0.83$$

By assuming the amplitude of all the reference waves equal to unity, the amplitude of the saw tooth wave can be determined using the calculated MI;

$$V_c = \frac{V_r}{MI} = \frac{1}{0.83} = 1.2 \quad (3.5)$$

The pulses generated by comparing three single phases (with 120^0 out of phase and 50Hz) with saw tooth wave of amplitude 1.2 is shown in the following figure.

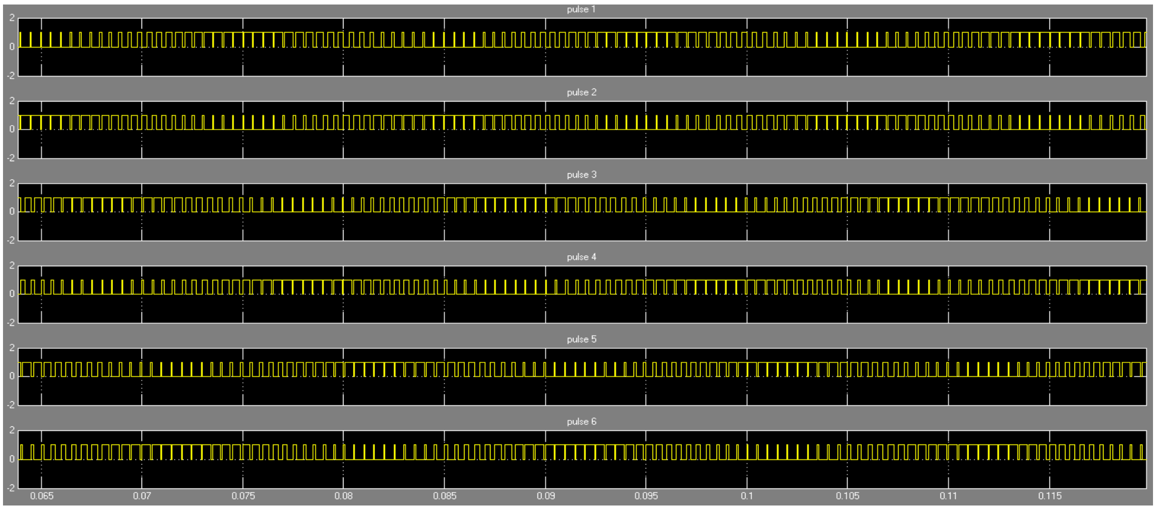


Figure 3.14 Auxiliary static inverter gate trigger pulses

The induction motor is modeled using the simplified equivalent circuit. The equivalent circuit of an induction motor contains a seriesly connected resistor and inductor.

The value of the resistor is determined in such a way that it can disippate equivalent power to that of total active power of a motor and the inductor is similary calculated for the reactive power.

According to the LRT data and calculated parameters (see Appendix A), the active power of a motor is 130kW and reactiv power is 63kVAr.

The auxilary loads are modelled as a resistive load with power of 34kW. The general Simulink model of traction load is given in figure 3.15. There are two identical IGBT/PWM inverters that feed four induction motors and one inverter for on train auxiliary power supply.

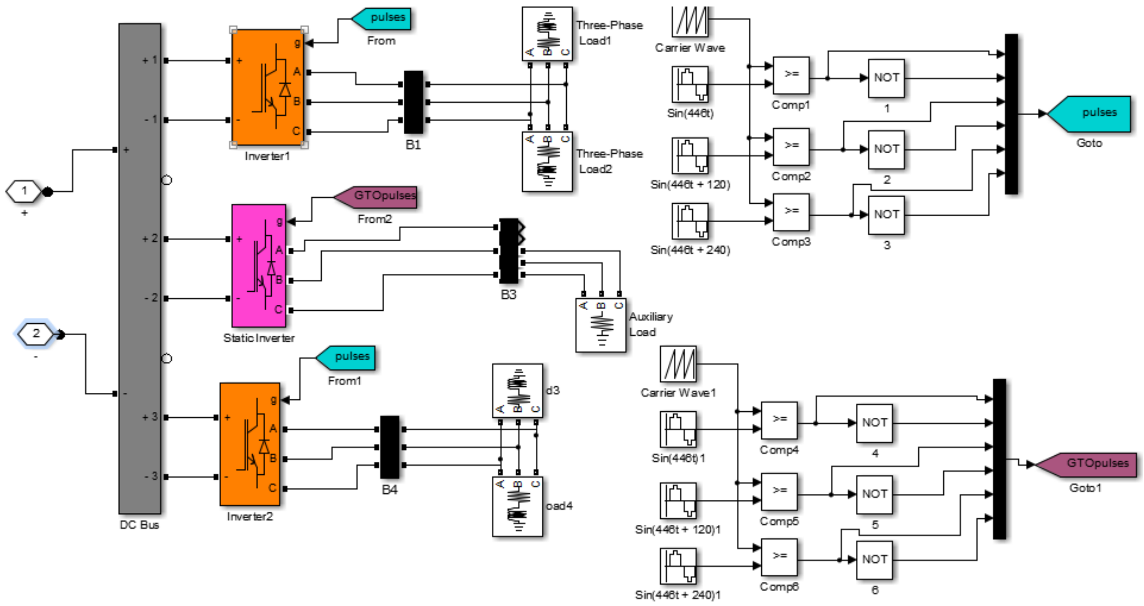


Figure 3.15 Traction load model [Author]

3.6. Harmonic Filter Design

The railway converters inject harmonics to the AC and DC sides of the system. The AC side harmonics enters the grid network where it can easily flow to other loads connected to the network. The DC side harmonics appears on the load side where it affects the proper operation of the motors. In the following sub-sections the AC side and DC side harmonic filters are designed for Addis Ababa LRT traction system to overcome such problems.

3.6.1. AC Harmonic Filter

In this thesis single tuned and high pass shunt passive filters are proposed in order to eliminate the harmonic currents injected by the converters. Passive filter offer a very low impedance in the network at the tuned frequency to divert all the related current at given tuned frequency. Because of passive filter always have tendency of offering some reactive power in the circuit, the design of it takes place for the two purposes one is the filtering purpose & another one is to provide reactive power compensation for correcting power factor in the circuit at a desired level. The advantage with the passive shunt type filter is that it only carry fraction of current so the whole system AC power losses are reduced compare to series type filter [12].

Two single tuned filters for 11th and 13th harmonics and high pass filter for higher order harmonics are designed for the section. The designing of these filters is in such a way that they will precisely employ for each specified harmonic frequency for which it has been tuned. These filters would normally be tuned below the respective characteristic frequencies. This is done for several practical reasons.

- One is that a perfect tuning would attract the dominant harmonics of the neighbouring nonlinear loads and result in over-current condition in the filter and fail.
- Another reason is that the filter components, in particular the capacitor C parameter decreases due to aging and the tuning frequency moves upwards and design at or above the tuning frequency would result in degraded filter performance as the capacitors age. With lower frequency detuning, the series resonance frequency increases and shift the minimum impedance point closer to the harmonic frequency. This increases the effectiveness of the filters by suppressing more current harmonics.
- Third, lower frequency detuning may be necessary to move the parallel resonance frequency away from the dominant harmonic frequency to be compensated for. Depending on the line impedance parameters, this may be necessary to avoid large overvoltage stresses on the rectifier terminals due to parallel resonance at the discussed harmonic frequency.

The shifted resonance frequency is usually chosen 3-8 % below the dominant harmonic frequency considered to be compensated for [35]. In some guidance notes a typical values for each filter is proposed to be detuned to 4.7^{th} (for 5^{th} harmonic), 6.6^{th} (for 7^{th} harmonic), 10.5^{th} (for 11^{th} harmonic) and 12.4^{th} (for 13^{th} harmonic) [18].

In this thesis the 11^{th} and 13^{th} harmonics is tuned to 4.5% below the dominant harmonic frequency of each harmonic. In designing a single tuned filter, generally the filter capacitor is sized for a known reactive power compensation required to improve the line power factor. Consequently, the filter reactor is defined to provide series resonance impedance (low impedance) at the harmonic frequency to be suppressed. At this resonance frequency, capacitor and inductor impedances are approximately equal in magnitude with opposite signs. Therefore they cancel each other. This impedance is given by;

$$Z_0 = j(\omega L - \frac{1}{\omega C}) \qquad (3.6)$$

Where,

Z_0 -is the resonance impedance,

L -is the filter reactance and

C-the filter capacitance

And the corresponding series resonance frequency of the filter is determined by;

$$f_s = \frac{1}{2\pi\sqrt{LC}} \qquad (3.7)$$

Where,

f_s -is the series resonance frequency

The parallel resonance frequency (f_p) that occurs between the single filter components and the total line reactor (supply and L) is calculated using the following formula.

$$f_p = \frac{1}{2\pi\sqrt{(L_s + L)C}}) \qquad (3.8)$$

Where,

f_p -is the parallel resonance frequency,

L_s -is the total line reactance,

L -is the filter reactance and

C -the filter capacitance

The harmonic filters can also provide a large percentage of reactive power for the power factor correction. When the capacitor, Q_{com} kVA, is installed in a system with a real power load P kW, the power factor can be improved from pf_0 to pf_1,

Where,

$$Q_{com} = p * (\tan(\cos^{-1} pf_0) - \tan(\cos^{-1} pf_1)) \qquad (3.9)$$

The capacity of a single-tuned filter can be set to,

$$Q_f = Q_{com} \qquad (3.10)$$

But, for multiple parallel single-tuned filters, the capacitor corresponding to the h^{th} harmonic filter can be distributed approximately by,

$$Q_{hf} = Q_{com} * \frac{I_h}{\sum I_h}, \text{ h=2, 3 } \ldots \qquad (3.11)$$

Where,

I_h -denotes the h^{th} harmonic current and

Q_{hf}- represents the capacity of the h^{th} harmonic filter.

Also, the filter capacity Q_{hf} contains the capacity of capacitor Q_C, and inductor Q_L. They have the following relationships.

$$Q_c = \frac{h^2}{h^2 - 1} Q_{hf} \qquad (3.12)$$

$$Q_L = Q_c - Q_{hf} \qquad (3.13)$$

$$Q_L = \frac{1}{h^2} Q_c \qquad (3.14)$$

Other important term, which is tentatively necessary to keep in mind during the designing of a filter, is quality factor. It determines the “sharpness” of the “tuning” of the passive filter and is given by the ratio of reactance at the resonant condition & resistance of the circuit as follows in the equation,

$$q_f = \frac{X_C or X_L}{R} \qquad (3.15)$$

Where,

q_f = quality factor,

R = resistance of filter in ohms, typical value of quality factor fluctuates in between 15 to 80 for single tuned filter and between 0.5-5 for high pass [37].

For the Lideta substation, the filter parameters are determined as follows. This rectifier substation has a rectifier transformers capacity of 2000kVA. Taking the power factor of 0.9, the filter parameters are determined using the above equations. For the case when the transformer is fully supplying power, the reactive power will be,

$$Q_{tr} = S * \sin(\cos^{-1} pf)$$

$$= 2000 * \sin(\cos^{-1} 0.9) = 871.78kVAr$$

To correct the power factor to unity, the reactive power to be compensated is equal to the calculated reactive power of the system.

$$Q_{com} = 871.78kVAr$$

The capacity of each filter is determined using equation (3.11).

Using the current Fourier transform, the expression for individual harmonics can be express as follows using equation 2.9.

$$I_A = \frac{4\sqrt{3}}{\pi} I_o (\cos\omega t - \frac{1}{11}\cos 11\omega t + \frac{1}{13}\cos 13\omega t - \frac{1}{23}\cos 23\omega t + ..)$$

$$= 2.2 I_o \cos\omega t - 0.2 I_0 \cos 11\omega t + 0.17\cos 13\omega t - 0.1\cos 23\omega t + ..$$

From the above equation, the rms value for each harmonic current can be calculated as follows.

$$I_1 = \frac{2.2 I_o}{\sqrt{2}} = 1.56 I_0 Arms$$

$$I_{35} = \frac{0.063 I_o}{\sqrt{2}} = 0.0446 I_0 Arms$$

$$I_{11} = \frac{0.2 I_o}{\sqrt{2}} = 0.142 I_0 Arms$$

$$I_{37} = \frac{0.0596 I_o}{\sqrt{2}} = 0.042 I_0 Arms$$

$$I_{13} = \frac{0.17 I_o}{\sqrt{2}} = 0.12 I_0 Arms$$

$$I_{47} = \frac{0.0469 I_o}{\sqrt{2}} = 0.033 I_0 Arms$$

$$I_{23} = \frac{0.1 I_o}{\sqrt{2}} = 0.066 I_0 Arms$$

$$I_{49} = \frac{0.0438 I_o}{\sqrt{2}} = 0.031 I_0 Arms$$

$$I_{25} = \frac{0.0882 I_o}{\sqrt{2}} = 0.062 I_0 Arms$$

The sum of all harmonic currents up to 50th order becomes;

$$I_{tot} = \sum_{n=11,13,23,25,..}^{49} I_n = 0.549 I_0 Arms$$

For single tuned 11th harmonic,

$$Q_{11f} = Q_{com} * \frac{I_{11}}{\sum I_h}, h = 2,3, \dots \dots$$

$$= 871.78 * \frac{0.142 I_0}{0.549 I_0} = 225.49 kVAr$$

The capacity of capacitor and inductor of 11th harmonic filter is calculated as,

$$Q_{11c} = \frac{h^2}{h^2 - 1} Q_{11f} = \frac{11^2}{11^2 - 1} 225.49 = 227.37 KVAr$$

The capacity of inductor is $Q_{11L} = Q_{11c} - Q_{11f} = 227.37 - 225.49 = 1.88 KVAr$

The capacitive and inductive reactance are calculated using the formula,

$$X_{11c} = \frac{KV^2}{Q_{11c}} = \frac{15000^2}{227370} = 989.58\Omega$$

$$X_{11L} = \frac{X_{11c}}{n^2} = \frac{989.58}{11^2} = 8.18\Omega$$

The value of capacitor and inductor is determined from their respective reactance equation.

$$C_{11} = \frac{1}{2 * \pi * f * X_{11c}} = \frac{1}{2 * \pi * 50 * 989.58} = 3.22 \mu F$$

$$L_{11} = \frac{X_{11L}}{2 * \pi * f} = \frac{8.18}{2 * \pi * 50} = 26.04 mH$$

The values of the capacitor and inductor calculated here are tuned to the exact 11th resonant frequency. To have a detuned filter the value of the capacitor needs to be recalculated for the detuned frequency of 4.5% below resonant one.

$$f_{11detuned} = 0.955 * f_{11} = 0.955 * 550 = 525.25 Hz$$

$$f_{11detuned} = \frac{1}{2\pi\sqrt{L_{11} C_{11new}}}$$

$$C_{11new} = \frac{1}{4 * \pi^2 * f^2{}_{11detuned} * L_{11}} = \frac{1}{4 * \pi^2 * 525.25^2 * 0.02604}$$

$$= 3.53 \mu F$$

For single tuned 13th harmonic filter the capacity of filter is determined as,

$$Q_{13f} = Q_{com} * \frac{I_{13}}{\sum I_h}, h = 2,3, \dots \dots$$

$$= 871.78 * \frac{0.12 I_0}{0.549 I_0} = 190.55 kVAr$$

The capacity of capacitor and inductor of 13th harmonic filter is calculated as,

$$Q_{13c} = \frac{h^2}{h^2-1} Q_{13f} = \frac{13^2}{13^2-1} 190.55 = 191.69 KVAr$$

The capacity of inductor is $Q_{13L} = Q_{13c} - Q_{13f} = 191.69 - 190.55 = 1.14 KVAr$

The capacitive and inductive reactance are calculated using the formula,

$$X_{13c} = \frac{kV^2}{Q_{13c}} = \frac{15000^2}{191690} = 1173.77 \Omega$$

$$X_{13L} = \frac{X_{13c}}{n^2} = \frac{1173.77}{13^2} = 6.95 \Omega$$

The value of capacitor and inductor is determined from their respective reactance equation.

$$C_{13} = \frac{1}{2 * \pi * f * X_{13c}} = \frac{1}{2 * \pi * 50 * 1173.77} = 2.7 \mu F$$

$$L_{13} = \frac{X_{13L}}{2 * \pi * f} = \frac{6.95}{2 * \pi * 50} = 22.12 mH$$

The detuned values of the capacitor and inductor are calculated as,

$$f_{13detuned} = 0.955 * f_{13} = 0.955 * 650 = 620.75 Hz$$

$$f_{13detuned} = \frac{1}{2\pi \sqrt{L_{13} C_{13new}}}$$

$$C_{13new} = \frac{1}{4 * \pi^2 * f^2{}_{13detuned} * L_{13}} = \frac{1}{4 * \pi^2 * 620.75^2 * 0.02212}$$

$$= 2.97 \mu F$$

For higher order harmonics above 23, high pass filter is designed. Because high pass filter provides low impedance for a wide spectrum of harmonics without the need for subdivision of parallel branches with increased switching and maintenance problems.

For a high pass filter, the capacity will becomes the total reactive power to be compensated minus the capacities of 11th and 13th harmonic filters.

$$Q_{hp} = Q_{com} - Q_{11f} - Q_{13f} = 871.78 - 225.49 - 190.55 = 455.74KVAr$$

The capacitive reactance is calculated from the line voltage and compensative reactive power.

$$X_{cHP} = \frac{KV^2}{Q_c} = \frac{15000^2}{455740} = 493.7\Omega$$

The inductive reactance will be,

$$X_{lHP} = \frac{X_{cHP}}{n^2} = \frac{493.7}{23^2} = 0.93\Omega$$

The characteristic impedance, X_n, is determined using the capacitive and inductive reactance.

$$X_n = \sqrt{X_{cHP}X_{lHP}} = \sqrt{493.7 * 0.93} = 21.43\Omega$$

Choosing the quality factor of five, the maximum allowable value to have best tuning effect, the resistor value is calculated.

$$R = X_n * Q = 21.43 * 5 = 107.14\Omega$$

$$C_{HP} = \frac{1}{2 * \pi * f * X_{HPc}} = \frac{1}{2 * \pi * 50 * 493.7} = 6.45\mu F$$

$$L_{hp} = \frac{X_{lhp}}{2 * \pi * f} = \frac{0.93}{2 * \pi * 50} = 2.96mH$$

Detuning the filter to 4.5% below resonant, the capacitor and inductor value will calculated as,

$$f_{23detuned} = 0.955 * f_{23} = 0.955 * 1150 = 1098.25Hz$$

$$C_{HPnew} = \frac{1}{4 * \pi^2 * {f_{detun}}^2 * L_{hp}} = \frac{1}{4 * \pi^2 * 1098.25^2 * 2.96m} = 7.1\mu F$$

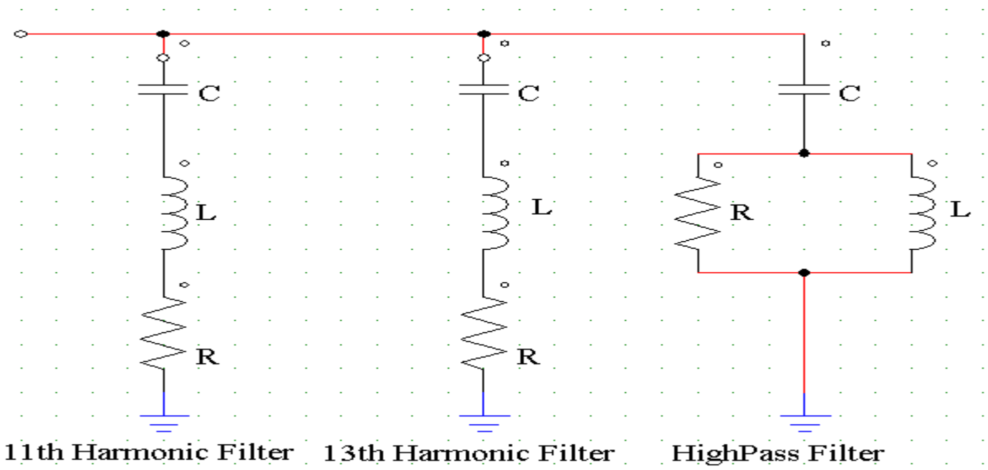


Figure 3.16 AC side filter circuit

Table 3-1 Summary of harmonic filters components

Filters	Capacity (kVAr)	Frequency (Hz)	Capacitor (μF)	Inductor (mH)	Quality factor	Characteristic Impedance(Ω)
11^{th} harmonic	225.49	525.25	3.22	26.04	80	89.93
13^{th} harmonic	190.55	620.75	2.97	22.12	80	86.3
High pass Filter	455.74	1098.25	7.1	2.96	5	21.3

The load side high pass filter designing procedure is given below. Using the line Voltage of 750V and capacity of 207.9kVAr

The capacitive reactance is calculated from the line voltage and compensative reactive power.

$$X_{cHP} = \frac{KV^2}{Q_c} = \frac{750^2}{207900} = 270.56\Omega$$

The inductive reactance will be,

$$X_{lHP} = \frac{X_{cHP}}{h^2} = \frac{270.56}{78^2} = 0.045\Omega$$

The characteristic impedance, X_n, is determined using the capacitive and inductive reactance.

$$X_n = \sqrt{X_{cHP}X_{lHP}} = \sqrt{270.56 * 0.045} = 3.49\Omega$$

Choosing the quality factor of 5, the maximum allowable value to have best tuning effect, the resistor value is calculated.

$$R = X_n * Q = 3.49 * 5 = 17.45\Omega$$

$$C_{HP} = \frac{1}{2 * \pi * f * X_{HPc}} = \frac{1}{2 * \pi * 50 * 270.56} = 11.76\mu F$$

$$L_{hp} = \frac{X_{lhp}}{2 * \pi * f} = \frac{0.045}{2 * \pi * 50} = 143\mu H$$

The load side filter has a capacitor 11.75μF, inductor 143μH and 17.45Ω.

3.6.2. DC Harmonic Filter

Harmonic voltages that occur on the DC side of a converter station cause AC currents, which are superimposed on the direct current distribution line. This undesirable AC component is called ripple. In a 12-pulse scheme the ripples has a lowest frequency of 600Hz, but because of the imperfections in the AC system and the converter circuit, harmonic voltages of other orders will also be present[40]. The flow of this harmonic current in the distribution line can cause interference due to electro-magnetic induction on the open-wire telecommunication n lines in the surrounding of the distribution lines.

In general, there are three methods of controlling ripples in DC circuits.

1. Capacitor filter
2. Inductor filter
3. Capacitor-Inductor filter

I. Capacitor Filter

A ripple in the DC circuit can be reduced if some of the energy can be stored in a capacitor. The capacitor will react to any change in circuit voltage. Because only the impedance in the source side, which is low, limits the rate of capacitor charging the voltage across the capacitor can rise nearly fast. In other words, the RC charge time is relatively short. The charge on the capacitor represents storage of energy. When the rectifier output drops to zero, the voltage across the capacitor does not fall immediately. Instead, the energy stored in the capacitor is discharged through the load during the time that the rectifier is not supplying energy.

The voltage across the capacitor and the load falls off very slowly if it is assumed that a large capacitor and relatively large value of load resistance are used. However, if the resistance of the load is small, the load will draw a heavy current and the average output voltage will fall. In addition, the filter capacitor acts like a short circuit across the rectifier while the capacitor is being charged. Due to these reasons, a simple capacitor filter is not suitable for rectifiers in higher power applications. In practice, the ripple factor can be found from [40].

$$RF = \frac{1}{[\sqrt{2}(2 f_r RC - 1)]} \tag{3.16}$$

Where f_r is the output ripple frequency

II. Inductor Filter

The other method of controlling DC harmonic distortion is limiting the current pulses. Coils that may also be called reactors, inductors, or chokes generally accomplish this. The inductance of a coil creates a back electromotive force (emf, or voltage) as the current pulse passes through it. This reduces the current pulsation.

We have seen that a capacitor is a device that reacts to variation in voltage and are connected across the load while an inductor is a device that reacts to changes in current. The inductor causes delay in current. Since the current is same in all parts of the series circuit, an inductor is connected in series with the load.

The smoothing reactors are used to;

- decrease harmonic voltages and currents in the DC line
- serves as a buffer between the converters and the DC line
- smooth the ripple in the direct current in order to prevent the current becoming discontinuous at light loads
- limit the crest current in the rectifier due to a short circuit on the DC line
- limit the current in the valves during the converter bypass pair operation, due to the discharge of the shunt capacitances of the DC line

The use of an inductor prevents the current from building up or dying down too quickly. If the inductor is made large enough, the current becomes continuous and nearly constant. The inductor prevents the current from ever reaching the peak value, which would otherwise be reached without a filter inductor. Consequently, the output voltage never reaches the peak value of the applied sine wave. Thus, a rectifier whose output is filtered by an inductor cannot produce as high a voltage as that could be produced by a rectifier filtered by a capacitor. However, this disadvantage is partly compensated because the inductor filter permits a larger current without a serious change in output voltage. This is the reason that an inductor filter is suitable for high power applications.

III. Capacitor-Inductor Filter

A capacitor-inductor filter is used to improve the filtering action of rectified voltage and current. As it was discussed in the above sections, the capacitor alone or the inductor alone could not perform the filter action satisfactorily as former is suitable for low-power applications and the latter is suitable for high-power applications. However, if both the capacitor and inductor are combined, they produce high quality dc voltage and current. The function of the capacitor is to smooth out the variations in voltage while the inductor is used to smooth out the variations in current. Because of the uniform flow of current, the capacitor-inductor filter is used widely in high-power applications.

In this proposed work, the smoothing reactor alongside with the DC bus capacitor is used to minimize the AC ripple in the DC line. The smoothing reactors are connected before the DC bus capacitor in series with the converter and the capacitor bus is connected in parallel to the station poles to have an effective reduction of the harmonics.

Since there is no mathematical formula available to calculate the values of capacitor and an inductor in Capacitor-inductor filter [40], their values are estimated through several iteration until the minimum ripple is obtained. The iteration is started from the case for capacitor filter only and continued until a ripple below 1% is obtained for both current and voltage. After several iterations for the capacitor value of 50μF and an inductor value of 0.5mH, the ripple becomes below 1%. [40]

CHAPTER FOUR

4. SIMULATION AND ANALYSIS

4.1. Introduction

In this chapter, the simulation of the traction power system is done using the Matlab Simulink environment. SIMULINK® is a toolbox extension of the MATLAB program. It is a program for simulating dynamic systems. Simulink has the advantages of being capable of complex dynamic system simulations, graphical environment with visual real time programming and broad selection of toolboxes. The program is capable of solving both linear and nonlinear processes so it is perfectly suited to simulate traction power system.

The first step in simulating a traction supply system is creating a model that represents components of a system by using an existing blocks in the Simulink library or from those created by the user. The models presented in previous chapter are represented by their respective equivalent Simulink blocks. Then the analysis is done for three different cases.

First, the simulation is done for the case when there is no harmonic filter for the system. Followed by, the simulation with the insertion of the load side's filter. This analysis is done just to limit the harmonics generated by the load converters. Finally, the simulation is done with the use of AC side and DC side harmonic filters for the proposed model. In all cases, the three-phase supply source is assumed to be balanced and the system is at its rated load.

Finally analysis for the simulation results with respect to total harmonic distortion, ripple, power factor and power loss have been discussed.

4.2. Simulink Model

A Simulink Model of the traction supply system is developed for the maximum possible number of traction loads. Depending on the substation capacity, the model was developed for three traction loads. Each traction load is at its rated capacity and has equal spacing of 0.8km between them.

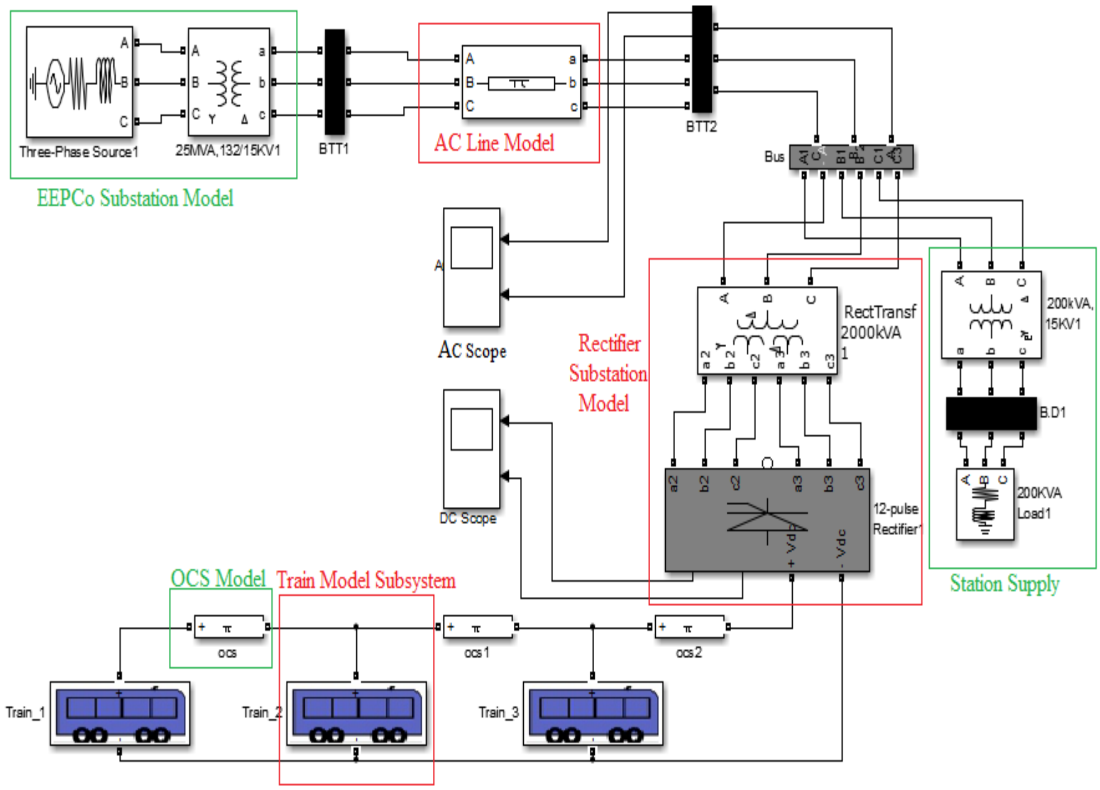


Figure 4.1 Traction power system model, without filter

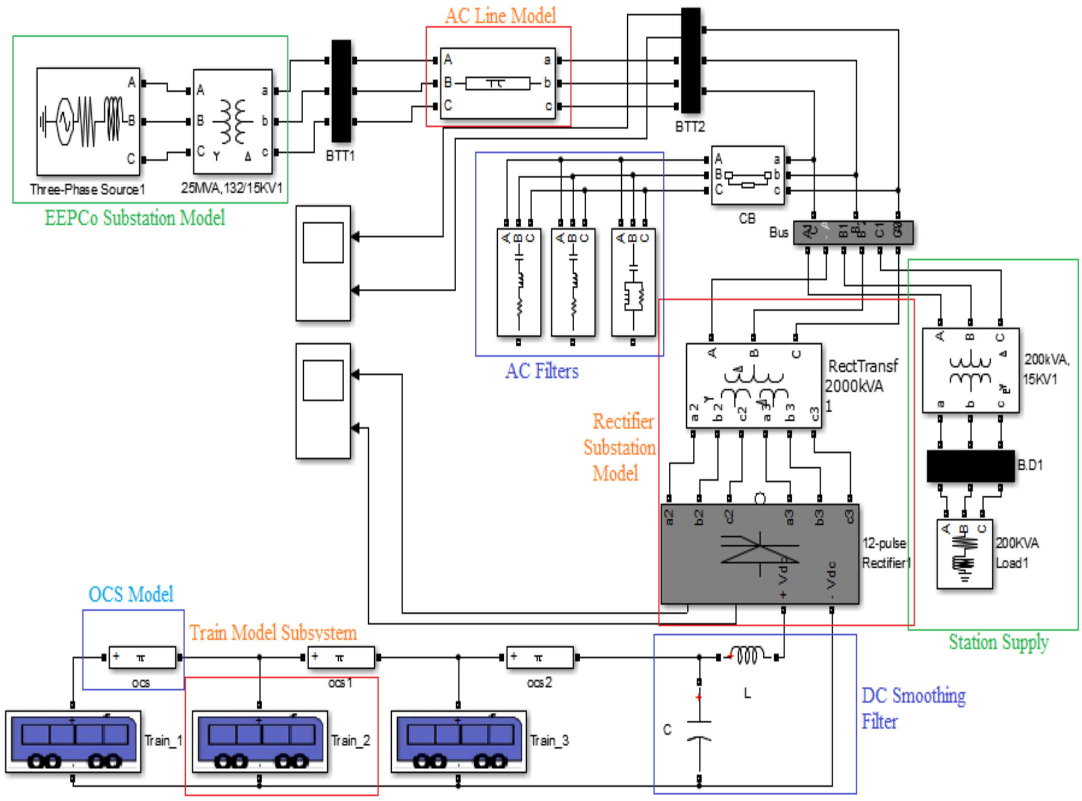


Figure 4.2 Traction power system model, with filter

4.3. Simulation Parameters

The simulation parameters used in this thesis are based on the actual data obtained from ERC and some approximate calculation. Some methods for determining the parameters through calculation are given in Appendix A. Summary of the simulation parameters is given in the following table.

Table 4-1 Simulation parameters

Voltage source	Vph-ph	132kV
	Frequency	50Hz
	3 phase short circuit level	250MVA
	X/R ratio	24
Rectifier transformer	Nominal power	2MVA
	Winding 1	1500Vrms,R1=0.0233,L1=0.0077
	Winding 2	590Vrms, R2=0.01165, L2=0.004
	Winding 3	590Vrms, R3=0.01165,L3=0.004
Rectifier	diodes forward voltage	0.8V
	Snubber resistance	100Ω
	snubber capacitance	5μF
AC distribution line	Positive and zero sequence resistance Ω/km)	r_1=0.6969 r_0=7.738
	Positive and zero sequence inductance (H/km)	l_1=0.001566 l_0=0.0246
	Positive and zero sequence capacitance (F/km)	$c_1=6.79*10^{-9}$ $c_0=6.79*10^{-12}$
DC OCS	Resistance(Ω/km)	0.123
Traction motor	Rated power, frequency	145kVA,71Hz
	Rated voltage, power factor	500Vrms,0.9

4.4. Simulation Result

The simulation of the models developed in section 4.2 is done by using the above stated simulation parameters. In this section, the graphic results for the following cases are presented:

Case 1: When there is no harmonic filter for the system

Case 2: With only load side high pass harmonic filter for each load

Case 3: With the addition of AC side and DC side harmonic filters on Case2.

I. Case 1

The simulation result of current and voltage waveforms at the point of common coupling is highly distorted. The point of common coupling on this system is the point at which the Lideta rectifier substation is connected to other nearby substations.

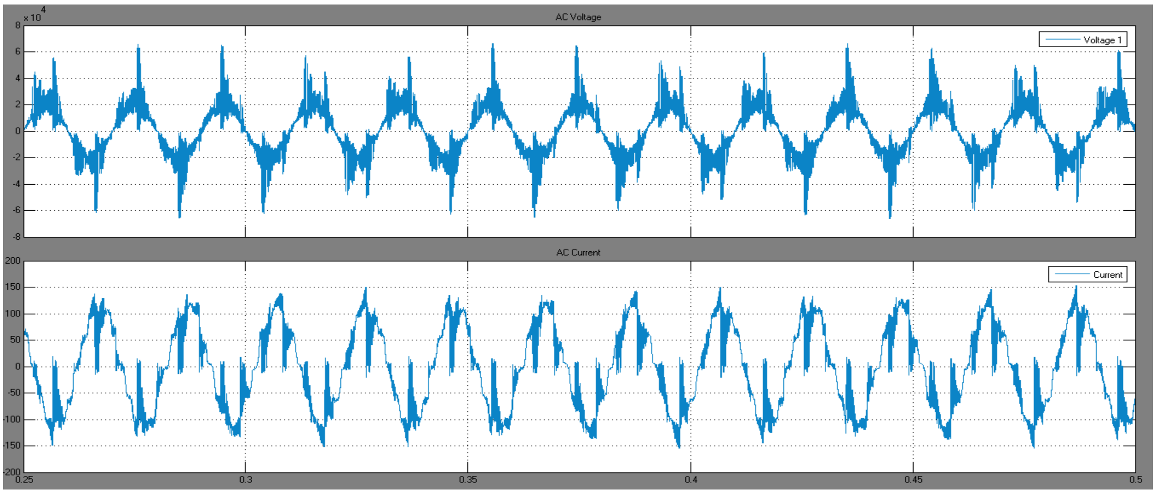


Figure 4.3 Voltage and Current waveform at PCC, without filter

The results obtained in the harmonic distortion analysis of voltage and current at the PCC, without the harmonic filter are shown in Figure 4.4. The THD value corresponding to the voltage is 57.08% while the current has 34.83%. The dominant individual harmonics are those with the order of 80 and 160. The frequency of these harmonics are an integer multiples of 4000Hz which is the same to PWM carrier frequency. So, it can be deduced from this that these harmonics are generated by load side inverters.

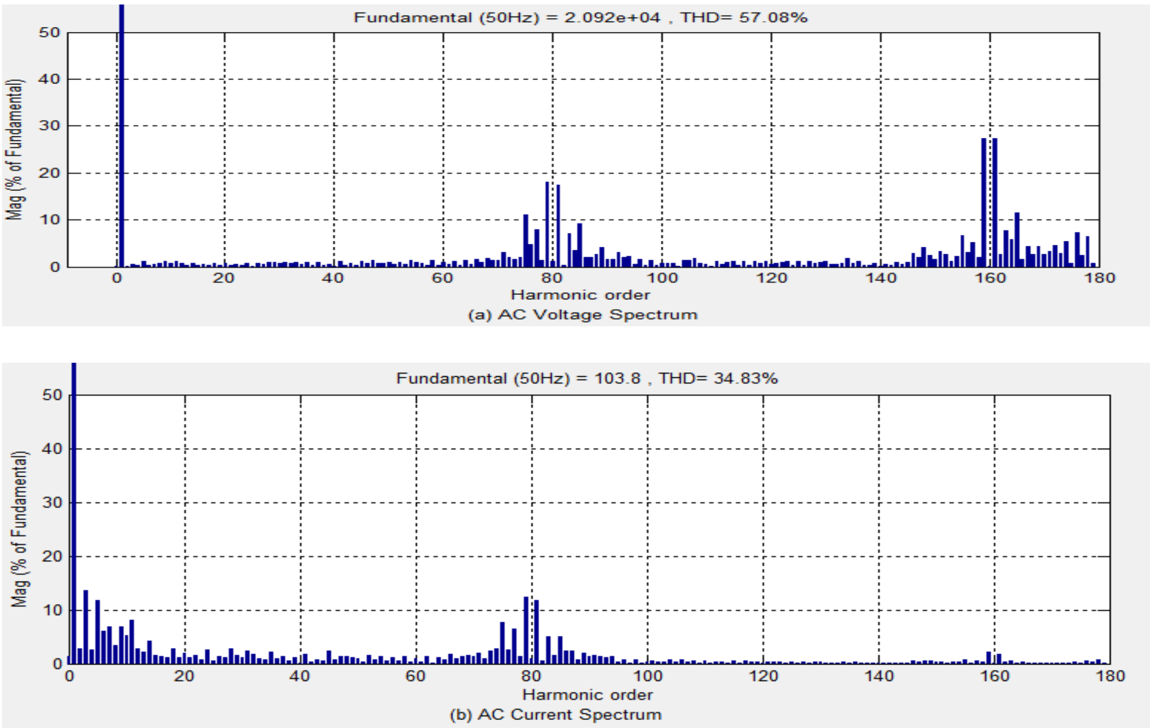


Figure 4.4 Voltage and Current harmonic spectrums at PCC, without filter

In Figure 4.5, the DC output voltage and current waveforms are shown. The voltage and current waveforms have no DC nature; it pulsates between near zero magnitude and expected maximum DC output. The DC behaves like this because of the harmonics generated by load and that of rectifier substations are not filtered. Since the ripple has a high magnitude it needs a DC smoothing filter.

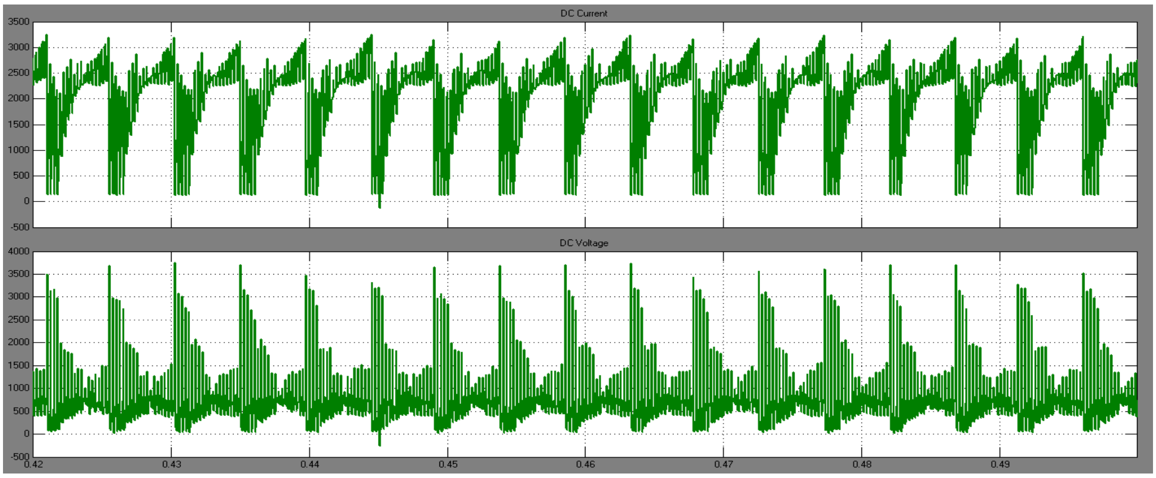


Figure 4.5 DC voltage and current waveform at PCC, without filter

A high pass filter tuned to a corner frequency of 78^{th} harmonic order is designed for each load. This corner frequency is selected to eliminate harmonic frequency above 80^{th} order. From the FFT analysis, the fundamental component of voltage on DC side has rms value of 40.92V and current has 13.27Arms. The magnitudes of the harmonic current with an order of 80 and 160, with respect to fundamental are about 999% and 531.67% respectively.

The magnitude of the current is determined from percentage harmonics and fundamental component.

$$I_{80} = I_{Fundamental} * \%I_{80} = 13.27 * 9.99 = 132.57A$$
$$I_{160} = 13.27 * 5.3167 = 70.55A$$

Similarly calculating for the voltage, V_{80}=2715% and V_{160}=2100%.

$$V_{80} = V_{Fundamental} * \%V_{80} = 40.92 * 27.15 = 1110.99V$$
$$V_{160} = 40.92 * 21 = 859.32V$$

The power will be

$$P = V_{80} * I_{80} + V_{160} * I_{160} = 132.57 * 1110.99 + 70.55 * 859.32 = 207.9KVA$$

Since this current and voltages pass through filter, the filter is designed to this capacity. Using this capacity, the filter is designed (see section 3.6.1). The load side filter has a capacitance 11.75μF, inductance 143μH and 17.45Ω.

II. Case 2

The use of high pass harmonic filter for each load limits the harmonics generated by load from entering grid network. In this case, the voltage and the current have sinusoidal nature. The voltage is purely sinusoidal while the current is still distorted.

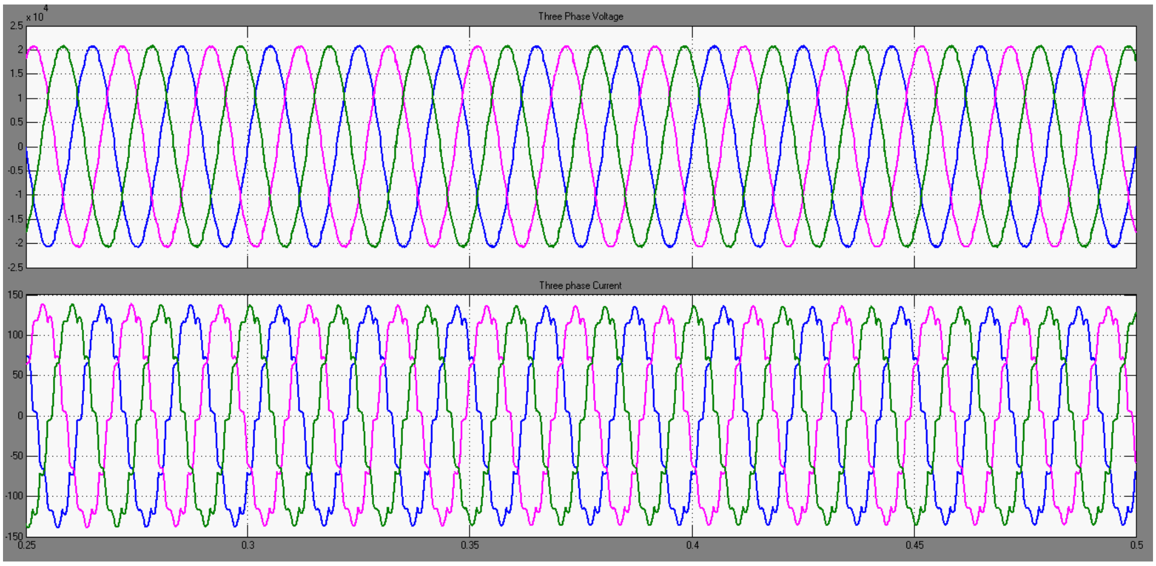


Figure 4.6 Voltage and Current waveform at PCC, with load filter

As we can see from Figure 4.7, the total harmonic distortion is reduced to 2.20% for voltage and 8.30% for current. The dominant individual harmonics of the graphs are 11th, 13th, 23rd, 25th, 35th, 37th and so on. These harmonics are those that are injected by the 12-pulse rectifier. Therefore, the use of high pass harmonic filter tuned at frequency of PWM carrier frequency limits the harmonics of load side.

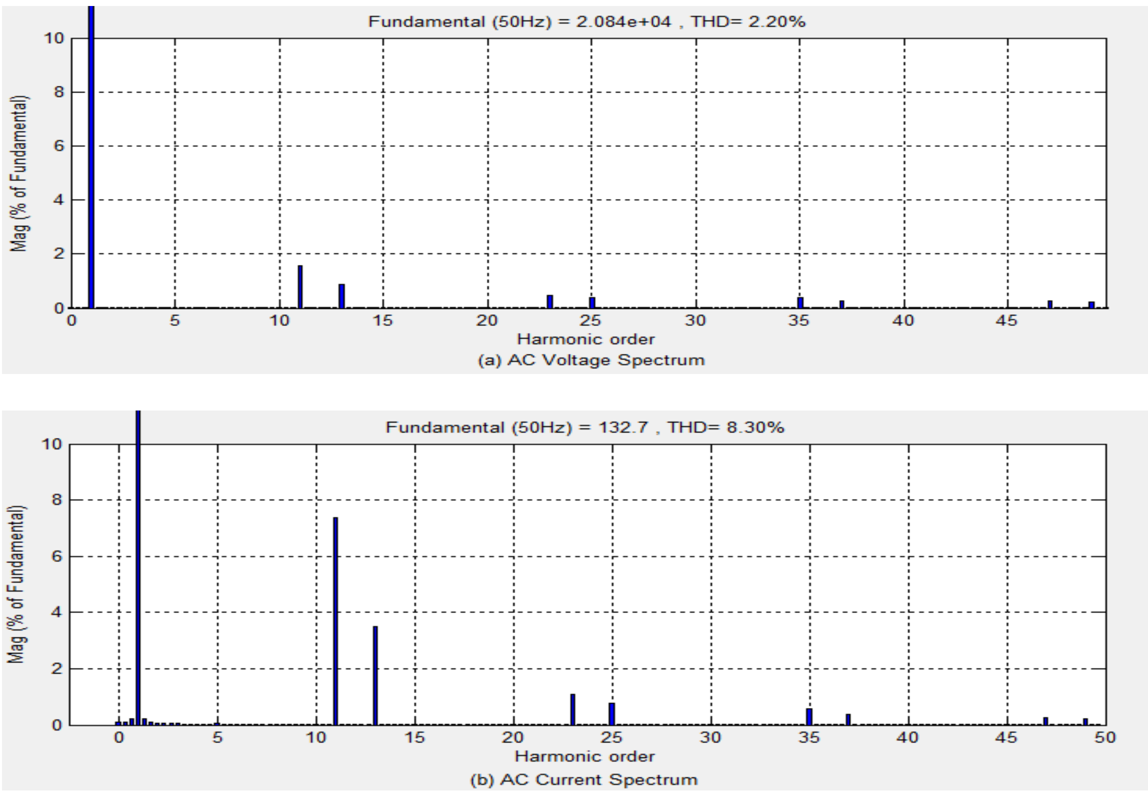


Figure 4.7 Voltage and Current harmonic spectrums at PCC, with load filter

The DC output voltage and current, in figure 4.8, becomes smooth with some ripples. The ripple has a frequency of 12, 24, 36 etc. The ripple factor of the output waveform is determined by assuming the ripple has a sinusoidal waveform [40]. From the above graph the voltage ripple is calculated using Vmax=772V,Vmin=752.5V and Vo= 764.6V obtained from the FFT analysis result.

Percentage of ripple = (RMS value of ripple/Average DC output) x 100

$$\%r = (\frac{V_{\max} - V_{\min}}{\sqrt{2}})/Vo)*100$$

$$= (\frac{772 - 752.5}{\sqrt{2}})/764.6)*100 = 1.8\%$$

The current ripple for Imax=2765A, Imin=2561A and Io=2686A becomes;

$$\%r = (\frac{I_{\max} - I_{\min}}{\sqrt{2}})/Io)*100$$

$$= (\frac{2765 - 2561}{\sqrt{2}})/2686)*100 = 5.37\%$$

Since the AC side current has a THD of 8.30% which is above the IEEE standard limit and the DC current has a ripple of 5.37% which is high, shunt passive filters are designed for AC side and LC filter is designed for DC side. The detail design procedure and values of each filter prameters is given in section 3.6.

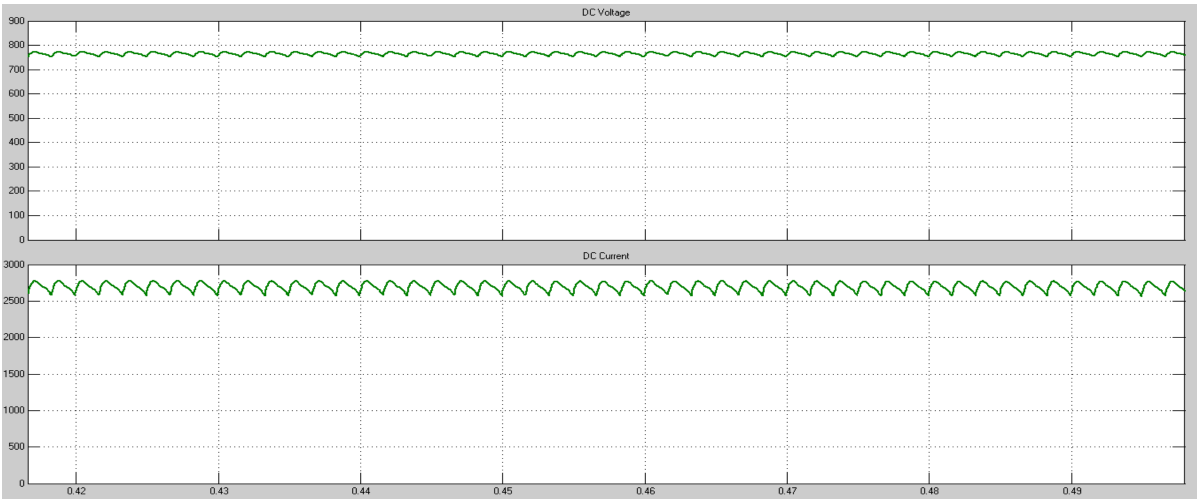


Figure 4.8 DC voltage and current waveform at PCC, with load filter

III. Case 3

With the insertion of designed AC side and DC side filters (see section 3.6.1); the current waveform becomes pure sinusoidal. The THD of the current is reduced to 0.80% and THD of voltage becomes 0.33%.

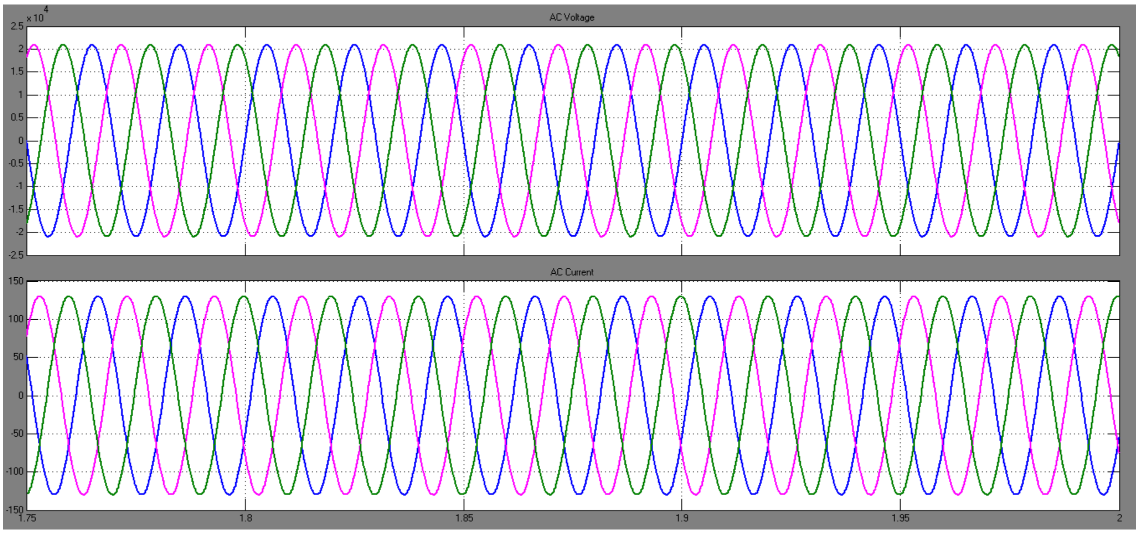


Figure 4.9 Voltage and current waveform at PCC, with AC and DC side filters

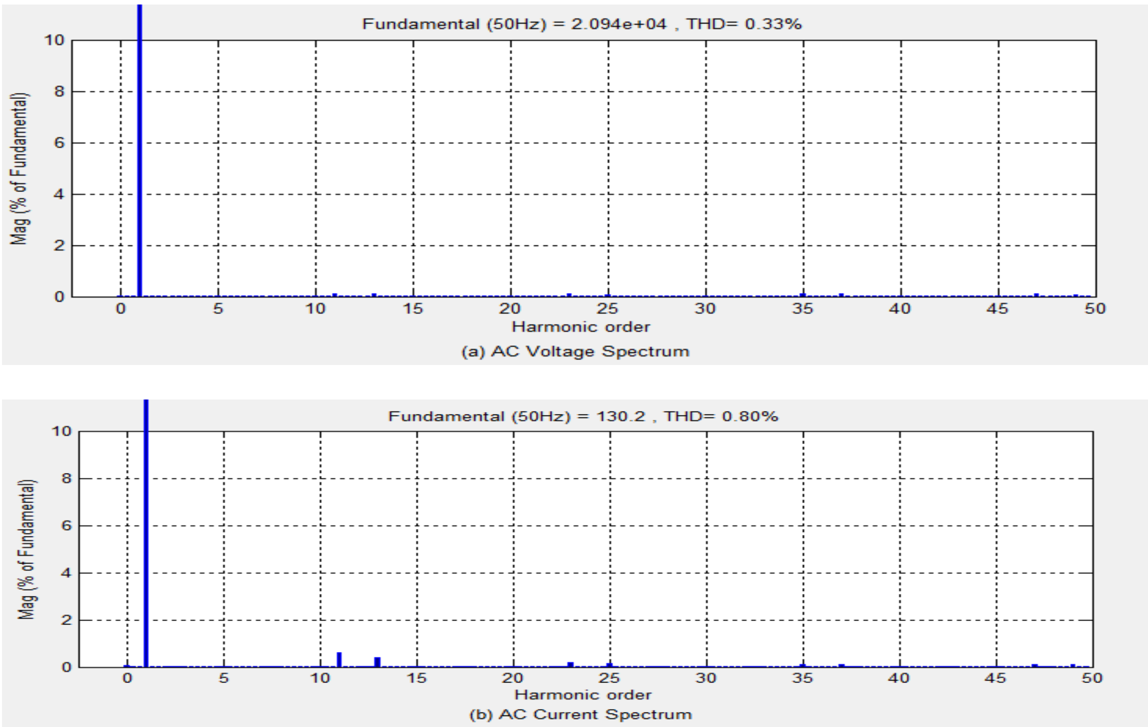


Figure 4.10 Voltage and Current spectrums at PCC, with AC and DC side filters

Figure 4.11 shows the simulation result of the DC output voltage and current by including the AC side and DC side filters. The ripple voltage and current is significantly reduced. The ripple factor of the Voltage and current is determined as follows.

From the graph, Vmax=769.55V, Vmin=768.45V and Vo= 769.2V.

Percentage of ripple = (RMS value of ripple/Average DC output) x 100

$$\%r = (\frac{769.5 - 768.45}{\sqrt{2}}) / 769.2) * 100 = 0.1\%$$

The current ripple is determined using Imax=2709A, Imin=2693A and Io=2702A.

The current ripple becomes;

$$\%r = (\frac{2709 - 2693}{\sqrt{2}}) / 2702) * 100 = 0.04\%$$

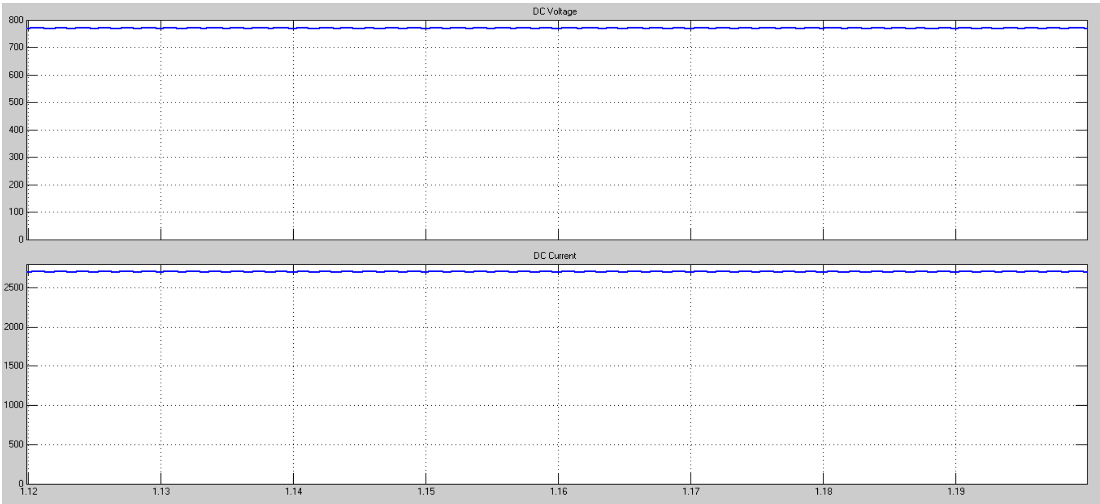


Figure 4.11 DC voltage and current waveform at PCC, with AC and DC side filters

4.5. Result Analysis

A. Total Harmonic Distortion

Different simulation results were obtained in the previous section (section 4.4). These results need to be compared with the available standard in order to evaluate the performance of the filters. According to the IEEE-519 standard, the total harmonic distortion of voltage at point of common coupling is limited to 5% [24]. To determine total harmonic distortion limit for the current, the ratio of short circuit current to full load current is needed. The currents are determined for the transformer full load condition.

For this analysis, the short circuit and full load currents are determined as follows [18]:

Full load current, $I_{FL} = \dfrac{Transformer(kVA)}{SecondaryVoltage * \sqrt{3}}$

For 2000kVA transformer capacity with impedance percentage of 8% and 590V secondary voltage, the currents will be determined as:

$$I_{FL} = \frac{2000kVA}{590V * \sqrt{3}} = 1.96kA$$

Short circuit current, $I_{SC} = \dfrac{FullLoadCurrent}{\%impedance} = \dfrac{1.96kA}{0.08} = 24.5kA$

Then, the ratio of short circuit current to full load current becomes,

$$\frac{I_{SC}}{I_{FL}} = \frac{24.5kA}{1.96kA} = 12.5$$

According to the IEEE-519 standard, the total harmonic distortion for current ratio of 12.5 is 5%. (See Table 2.2 on section 2.6)

Table 4-2 Summary of Voltage and Current THD

Analysis	THD_I	THD_V	THD Standard limit	Remark
Case 1	34.83%	57.08%	5%	Both above limit
Case 2	8.30%	2.20%	5%	Voltage below limit Current above limit
Case 3	0.80%	0.33%	5%	Both below limit

B. Power Factor

The true power factor depends on the displacement and distortion power factors of the system. From the calculated total harmonic distortion, the distortion power factor is determined as follows:

$$pf_{dist} = \frac{1}{\sqrt{1+THD_V^2}} * \frac{1}{\sqrt{1+THD_I^2}}$$

Case 1: $pf_{dist1} = \frac{1}{\sqrt{1+0.5708^2}} * \frac{1}{\sqrt{1+0.3483^2}} = 0.82$

Case 2: $pf_{dist2} = \frac{1}{\sqrt{1+0.022^2}} * \frac{1}{\sqrt{1+0.083^2}} = 0.996$

Case 3: $pf_{dist3} = \frac{1}{\sqrt{1+0.0033^2}} * \frac{1}{\sqrt{1+0.008^2}} = 0.999$

The displacement power factor is the ratio of active power to the fundamental apparent power.

$$pf_{disp} = \frac{ActivePower}{FundamentalApparentPower} = \frac{P}{S_1}$$

Active power is calculated by using the FFT analysis of current and voltage. The individual harmonics for dominant orders are considered to avoid the complexity of calculation. Detail calculation is provided in Appendix B.

Case 1: $pf_{disp1} = 0.947$

Case 2: $pf_{disp2} = 0.96$

Case 3: $pf_{disp3} = 0.994$

The true power factor is the product of the displacement and distortion power factors.

Table 4-3 Distortion, displacement and true power factor of the system

Analysis	Distortion pf	Displacement pf	True pf
Case 1	0.82	0.947	0.776
Case 2	0.996	0.96	0.95
Case 3	0.999	0.994	0.99

C. Resonant Analysis

The harmonic filter impedance vs frequency plot shows the filter has high impedance at 50Hz and lower impedance at resonant frequencies. Therefore, it blocks fundamental frequency waveforms using its high impedance and permits resonant frequency harmonic waveforms to pass.

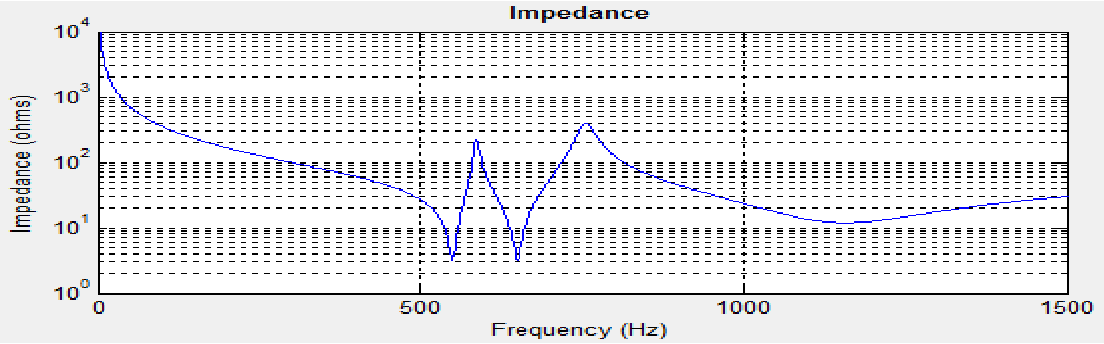


Figure 4.12 Impedance vs frequency plot of harmonic filter

As it is shown in figure 4.13, peak impedance does not occur at resonant frequencies. It has lower impedances at fundamental frequencies. Parallel resonances (i.e. peaks) do not occur at frequencies where harmonic currents exist.

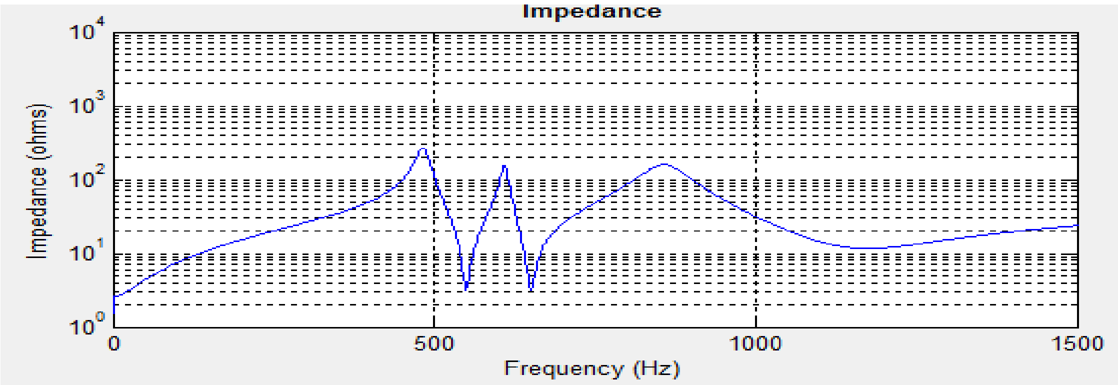


Figure 4.13 Impedance vs frequency plot of system

D. Power Loss

Harmonic power loss is calculated for AC and DC side of the rectifier. The AC side is represented with current harmonic sources while the DC side is modelled with harmonic voltage sources [33]. The dominant harmonic currents (11^{th}, 13^{th}, 23^{rd} and 25^{th}) and harmonic voltages (12^{th}, 24^{th} and 36^{th}) which have large magnitude compared to other harmonics are considered for loss calculation.

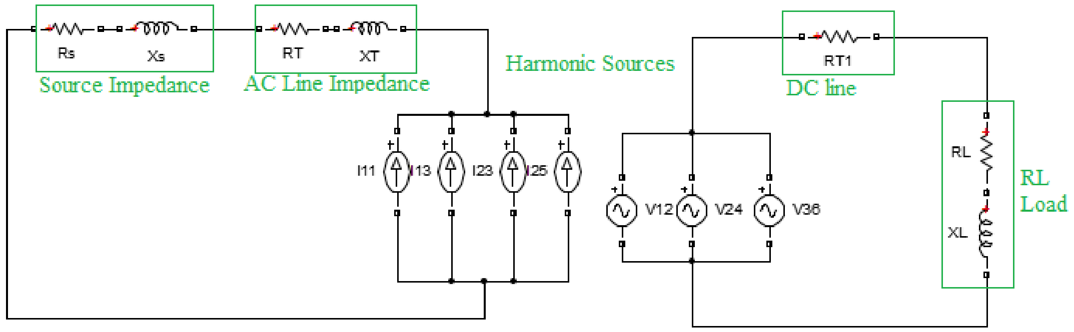


Figure 4.14 Per phase Equivalent circuit for harmonic power loss calculation

I. AC power loss

The AC side harmonic loss depends on the magnitude of harmonic currents and system impedance.

$$P_{AC} = I^2 Z_{AC}$$

The AC side equivalent impedance is the sum of the supply impedance and the transmission line impedance.

$$Z_{AC} = Z_s + Z_T$$

According to the data (Appendix A), supply impedance (Zs) =0.0375+j0.899 Ω and transmission line impedance (Z_T) =0.977+j0.096 Ω/km. The AC line has a length of 1.6km (from Lideta to Torhailoch). So, the AC impedance will be,

$$Z_{AC} = (0.0375 + 0.977 * 1.6) + j(0.899 + 0.096 * 1.6)\Omega$$

$$= 1.6 + j1.05\Omega$$

The harmonic current obtained from FFT analysis is expressed as (for more information Appendix B Table 2B);

$$I_h = \sqrt{2}(6.82\sin(2\pi 550 - 32.6) + 3.23\sin(2\pi 650 - 85.4) + \sin(2\pi 1150 + 26.6) + 0.74\sin(2\pi 1250 + 6.1)$$

Since the reactance of the system increase proportionally with harmonic order and the resistance increase with square root of harmonic order due to skin effect, the system impedance would be different for different harmonics.

$$Z_{11} = \sqrt{11}*1.6 + j11*1.05 = 5.31 + j11.55 = 12.71\angle 65.3^0$$

$$Z_{13} = \sqrt{13}*1.6 + j13*1.05 = 5.77 + j13.65 = 14.82\angle 67^0$$

$$Z_{23} = \sqrt{23}*1.6 + j23*1.05 = 7.67 + j24.15 = 25.34\angle 72.38^0$$

$$Z_{25} = \sqrt{25}*1.6 + j25*1.05 = 8 + j26.25 = 27.44\angle 73^0$$

The respective harmonic voltage is the product of the harmonic impedance and harmonic current.

$$V_h = I_h * Z_h$$

$$V_{11} = 6.82\angle -32.6^0 * 12.71\angle 65.3^0 = 87\angle 32.7^0$$

$$V_{13} = 3.23\angle -85.4^0 * 14.82\angle 67^0 = 47.87\angle -18.4^0$$

$$V_{23} = 1\angle 26.6^0 * 25.34\angle 72.38^0 = 25.34\angle 98.98^0$$

$$V_{25} = 0.74\angle 6.1^0 * 27.44\angle 73^0 = 20.30\angle 79.1^0$$

The per phase average power is given by,

$$P = \sum_{h=11,13,23,25}^{4} V_h I_h \cos\Phi_L = V_{11} I_{11} \cos\varphi_{11} + V_{13} I_{13} \cos\varphi_{13} + ...$$

$$= 2(6.82*87\cos 32.7 + 3.23*47.87*\cos 18.4 + 25.34\cos 98.98 + 0.74*20.30\cos 79.1)$$

$$= 1289W$$

The three phase power will be=3*1289=3867W.

II. DC power loss

The DC overhead contact system has an impedance of 0.123Ω/km. Taking the section between Lideta and Meskel square with a length of 2.65km, the line impedance will be,

$$Z_{T1} = 0.123*2.65 = 0.326\Omega$$

A single train consumes a power of 554kW. Assuming an inverter efficiency of 95%, the dc equivalent power will be 554/0.95=583.16kW. Taking the power factor of 91%, the reactive power will be determined as;

$$Q = \sqrt{(583.16/0.91)^2 - 583.16^2} = 265.7kVAr$$

The equivalent impedance that dissipates the same power to train power is determined by using DC voltage of 750V.

$$R = \frac{V^2}{P} = \frac{750^2 V^2}{583.16kW} = 0.965\Omega$$

$$X = \frac{V^2}{Q} = \frac{750^2 V^2}{265.7kVAr} = 2.116\Omega$$

The total DC side equivalent impedance is,

$$Z_{DC} = Z_{T1} + Z_L = 0.326 + 0.965 + j2.11\Omega = 1.291 + j2.11\Omega = 2.47\angle 58.54^0$$

The harmonic voltage obtained from simulation is expressed as;

$$V_h = \sqrt{2}(7.1\sin(2\pi * 600) + 2.9\sin(2\pi * 1200) + 1.15\sin(2\pi * 1800)$$

The respective harmonic current is the ratio of harmonic voltage by the harmonic impedance.

$$I_h = \frac{V_h}{Z_h}$$

$$I_{12} = \frac{V_{12}}{Z} = \frac{7.1}{2.47}\angle -58.54^0 = 2.87\angle -58.54^0$$

$$I_{24} = \frac{V_{24}}{Z} = \frac{2.9}{2.47}\angle -62.8^0 = 1.17\angle -62.8^0$$

$$I_{36} = \frac{V_{36}}{Z} = \frac{1.15}{2.57}\angle -64^0 = 0.45\angle -64^0$$

The per train average power is given by,

$$P = \sum_{h=1212436}^{3} V_h I_h \cos\Phi_L = V_{12} I_{12}\cos\varphi_{12} + V_{24} I_{24}\cos\varphi_{24} + V_{36} I_{36}\cos\varphi_{36}$$

$$= 2 * 7.1 * 2.87\cos 58.54 + 2 * 2.9 * 1.17 * \cos 32.8 + 2 * 1.15 * 0.45\cos 64$$

$$= 74.82W$$

For three train, power will be=3*74.82=224.5W

Total power loss, $P_{tot} = P_{AC} + P_{DC} = 3867 + 224.5 = 4091.5W$

After filters inserted, using the Table B3, following the same step, the power loss becomes 53.84W which is negligible. The saved power loss 4091.5-53.84=4037.86W.

III) Power factor improvement

With the use of AC side and DC side filters, the power factor of the system improved. As a result the apparent power of the system decreases. The following table gives the active, reactive and apparent power of the system.

Table 4-4 Active, Reactive and Apparent power before and after filter

Before filter			After filter			
KW	KVAR	KVA	KW	KVAR	KVA	KVA Saved
2130	640	2224	2160	220	2171	53

IV) The rectifier transformer harmonic loss (eddy current, stray current and dc loss)

The loss are measured across the rectifier transformer terminal and summarized in the following table.

Table 4-5 Rectifier Transformer loss

Before filter			After filter			
KW Primary	KW Sec.	KW loss	KW Primary	KW Sec.	KW loss	KW saved
1950	1912.2	37.2	2107.8	2081.9	25.9	11.3

V) Energy Saving

Energy is the power consumed for some duration of time. In this thesis the energy saving is calculated per day for train operation time. The LRT train operation hour starts at 5:00 morning and ends at 23:00 evening with a total of 18 hrs.

$$E_{day} = P_{section} * T_{hrs}$$

Where, E_{day}- Energy loss per day

$P_{section}$ - per section power loss

T_{hrs}- per day working hours

Table 4-6 Energy saving per section per month

S. No	Item description	Power saved (kW)	Energy saved /day/section (kWh)	Energy saved per month (kWh)	Energy saved in Birr
1	Harmonic loss	4.038	72.68	2180.52	1509.66
2	Power factor improvement	53	954	28620	19,866.59
3	Transformer loss	11.3	203.4	6102	4232.34
			Total	36902.52	25,608.4

Total energy saved by installing the filter per section is around 25,608.4birr, which is very high. The tariff calculation is done using Table 4-7 given below. The EEPCo tariff rate for general category is selected. It is calculated by 0.6088birr/kWh for the first 50kWh and 0.6943birr/kWh for the remained.

Table 4-7 Ethiopian Electric Power Corporation Tariff Structure

Tariff Category	Consumption (kWh /Month)	Tariff Rate (Birr/kWh)
I. Domestic		
Equivalent Flat Rate		0.4735
First Block	First 50 kWh	0.2730
Second Block	Next 50 kWh	0.3564
Third Block	Next 100 kWh	0.4993
Fourth Block	Next 100 kWh	0.5500
Fifth Block	Next 100 kWh	0.5666
Sixth Block	Next 100 kWh	0.5880
Seventh Block	Above 500 kWh	0.6943
II. General		
Equivalent Flat Rate		0.6723
First Block	First 50 kWh	0.6088
Second Block	Above 50 kWh	0.6943

CHAPTER FIVE

5. CONCLUSION, RECOMMENDATION AND FUTURE WORK

Based on the results of the harmonic analysis assessment carried out at selected substation of Addis Ababa LRT traction system, the following major conclusions are drawn. Moreover, useful recommendations are forwarded and the main areas of future work are suggested.

5.1 Conclusion

In this thesis, harmonic analysis of Addis Ababa LRT traction system has been studied. Traction power system was modeled and simulated using the Matlab /Simulink. Based on the simulation result obtained the analysis with respect to THD, power factor and power loss was discussed.

According to the analysis performed, the harmonic content of the system for without filter case is very high which exceed the limits recommended by IEEE Standard 519-1992. The significant harmonics are those with an integer multiples of an inverter PWM carrier frequency. This implies that those harmonics are generated by the load. So, based on the principle of limiting the harmonics near to their source, a high pass filter was designed for each load.

The simulation result with the use of high pass filter for each load shows a considerable harmonic reduction. At this time, the remained harmonic components have harmonic order of $12n\pm1$ which is the same to those harmonics created by 12-pulse rectifiers. This implies that the designed high pass filter performed best in eliminating those harmonics generated by the load. The THD for voltage is 2.2% which obeys the IEEE Standard limits, but THD for current is 8.30% which still above the recommended IEEE limits (5%). The DC part has a current ripple of 5.37% and voltage ripple of 1.8%. In both sides, AC and DC, the voltage waveforms are nearly smooth and within acceptable region. But the current needs AC side and DC side filters that minimize harmonics.

With the designing of single tuned filters at 11^{th} and 13^{th} harmonics and a high pass filter tuned at corner frequency of 23^{rd} harmonic for the AC side, the THD for current is reduced to 0.80% and THD for voltage becomes 0.33%. This result obeys the IEEE Standard limits. The LC filter designed for the DC part reduced the current ripples to

0.04% and the voltage ripples to 0.1%. In addition to reducing the harmonic distortions and ripples, the designed filters improved the power factor from 0.77 to 0.99 and saved 36.9MWh energy per section per month.

Finally, from the result obtained it can be concluded that the proposed filters performed best in fulfilling the objective of the study.

5.2 Recommendation

Based on the result of this thesis work, it is strongly recommended that ERC should install AC and DC side harmonic filters to its traction power system to maintain the system stability thereby improving overall system performance.

5.3 Future Work

Some suggestions for further works that can be used as input or idea to formulate new research are the traction system harmonic analysis with the consideration of unbalanced supply system, train speed profile and regenerative issue.

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APPENDIX A: PARAMETER DETERMINATION

I) EEPCo Substation Parameters

The EEPCo network that provide supply for railway substations have a transformer with capacity of 25MVA, impedance percentage 10% and132kV/15kV. Depending on these data the grid short circuit capacity and equivalent impedance are calculated as follows.

Transformer full load current, $I_{FL} = \frac{TransfotmerkVA}{\sqrt{3} * V_{secondary}} = \frac{25MVA}{\sqrt{3} * 15kV} = 962.25A$

The short circuit current referred to secondary side becomes the ratio of full load current to impedance percentage.

$$I_{SC} = \frac{FullLoadCurrent}{\%impedance} = \frac{962.25A}{0.1} = 9622.5A$$

The short circuit capacity MVA,

$$MVA_{SC} = \sqrt{3}V_{sec} * I_{SC} = \sqrt{3} * 15000 * 9622.5 = 250MVA$$

The equivalent supply reactance referred to the transformer secondary side is given by;

$$Z_S = \frac{V_{Sec}^2}{MVA_{SC}} = \frac{15000^2 V^2}{250MVA} = 0.9\Omega$$

From the ANSI Standard C37.010, the X/R ratio for the transformer capacity of 25MVA is equal to 24. Taking this value for X/R, the equivalent source reactance and resistance will be determined from the equation, $Z^2 = R^2 + X^2$ and through calculation R=0.0375Ω and X=0.899Ω.

II) Transmission Line Parameters

The transmission line parameters considered in this thesis are taken from the catalogues of the Nexans cable manufacturers for medium voltage underground cables. For the underground cable with 15kV and 240mm^2 specifications, the parameters are X=0.096Ω/km and R=0.977Ω/km. Since there is no full of data about the cables spacing and physical arrangements to determine positive and zero sequence impedances, these quantities are taken from the research performed on the "Harmonic

Distortion in Renewable Energy Systems". For the same voltage and cross sectional area, the line parameters are,

Positive sequence impedance: 0.6969 +j 0.492Ω/km

Zero sequence impedance: 5.945 + j 7.738Ω/km

Zero sequence susceptance: 2.13μS/km

Using inductive and capacitive reactance, the values of inductance and capacitance are determined as follows:

$$X_C = \frac{1}{2\pi f C}, \; C = \frac{1}{2\pi f X_C}$$

$$X_L = 2\pi f L \; , \; L = \frac{X_L}{2\pi f}$$

Using frequency of 50Hz and the reactance values of line parameters mentioned above, the following positive and zero parameters are calculated.

- Resistance Ω/km) - r_1=0.6969 r_0=7.738
- Inductance (H/km) - l_1=0.001566 l_0=0.0246
- Capacitance (F/km) - $c_1=6.79*10^{-9}$ $c_0=6.79*10^{-12}$

III) Rectifier Transformer parameters

Transformer parameters are determined from the short circuit and open circuit data. Using the following equivalent circuit diagram of the three phase three winding transformer, the components are determined from test data.

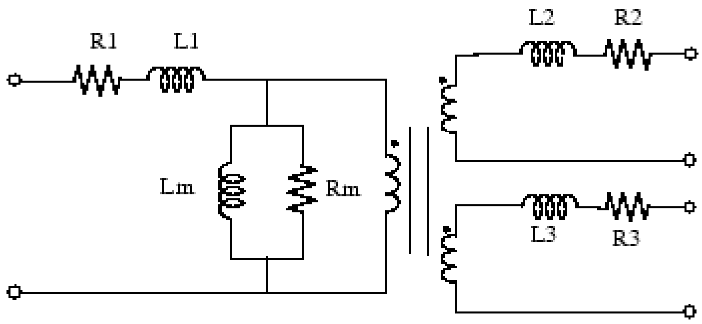

Figure A1. Equivalent circuit of three winding transformer [35]

The shunt admittance is determined from open circuit data. To evaluate Rm and X_M, we define the conductance of the core-loss resistance and the susceptance of the magnetizing inductor as follows:

$$G_C = \frac{1}{R_m}, \;\; B_m = \frac{1}{X_m}$$

Where, G_C is conductance of core-loss resistance

B_m is susceptance of the magnetizing effect

Since both elements are in parallel, their admittances add. Therefore, the total excitation admittance is:

$$Y_E = \quad G_c - jB_m = \frac{1}{R_c} - j\frac{1}{X_m}$$

The magnitude of the excitation admittance in the open-circuit test is given as:

$$Y_E = \frac{I_{oc}}{V_{oc}}$$

From the short-circuit data the magnitude of the series impedance referred to the primary side of the transformer is calculated. The magnitude of the series impedance referred to primary side is given by the equation:

$$Z_E = \frac{V_{sc}}{I_{sc}}$$

$$Z_E = R_{eq} + jX_{eq}$$

$$Z_E = (R_p + a^2 R_s) + j(X_p + a^2 X_s)$$

The short circuit and open circuit data of Addis Ababa LRT transformer is given as follows [34].

V_{oc} =15kV	Z_{sc}%=8,pf=0.9
I_{oc} =0.5%In=9.785A	Primary Voltage=15000V
Transformer rating=2000kVA	Secondary Voltage=590V

The short circuit parameters, I_{sc} and V_{sc}, can be determined using the rated transformer data and percentage of impedance.

Transformer short circuit current (I_{sc}),

$$I_{sc} = \frac{Transformer\ Full\ load\ Current}{Transformer\ Impedance}$$

Transformer full load current, I_{FL};

$$I_{FL} = \frac{Transformer\ KVA}{\sqrt{3} * Secondary\ Voltage}$$

$$I_{FL} = \frac{2000\ KVA}{\sqrt{3} * 590} = 1.957KA$$

Then short circuit current will be, $I_{sc} = \frac{1.957KA}{0.08} = 24.46KA$

The short circuit voltage is,

$$Percentage\ Impedance = \frac{Impedance\ Voltage}{Rated\ Voltage} * 100$$

$$Z_{sc}\% = \frac{V_{sc}}{V_{pri}} * 100$$

$$V_{sc} = \frac{V_{pri} * Z_{sc}\%}{100} = \frac{15000 * 8}{100} = 1200V$$

Using the test data and the above calculated parameters, the rectifier transformer parameters are estimated as given in the following table.

Table 1A. Calculated parameters of rectifier transformer

Y_E=6.523*10^{-4}	R_c=1613Ω	X_m=4909.65Ω
Z_E=0.04906	R_{eq}=0.0466Ω	X_{eq}=0.01532Ω

The per-unit Conversion

This allows us to specify the resistance and inductance of the windings in per unit (pu) value. The values are based on the transformer rated power Pn, in VA, nominal frequency fn, in Hz, and nominal voltage Vn, in Vrms, of the corresponding winding.

The base impedance, base resistance, base reactance, and base inductance are estimated by

$$Z_{base} = R_{base} = X_{base} = \frac{{V_n}^2}{p_n}$$

$$L_{base} = \frac{X_{base}}{2\pi fn}$$

Given the primary winding rated voltage 15000V, 2000kVA and 50Hz, the base parameters will be,

$$Z_{base} = R_{base} = X_{base} = \frac{15000^2}{2000000} = 112.5\Omega$$

$$L_{base} = \frac{112.5}{2\pi * 50} = 0.358H$$

For each winding, the per unit resistance and inductance are defined as

$$R_{(pu)} = \frac{R(\Omega)}{R_{base}}, \qquad L_{(pu)} = \frac{L(\Omega)}{L_{base}}$$

Assuming that the two secondary windings are identical and each parameters of the secondary is half of the primary winding, the per unit values are calculated.

Table 2A. Calculated parameters of rectifier transformer in per unit form

R_m (pu)=1613/112.5=14.34	X_m (pu)=4909.65/112.5=43.64
R1=Req/2=0.0466/2=0.0233	X1=Xeq/2=0.01532/2=0.0077
R2=R3=R1/2=0.0233/2=0.01165	X2=X3=X1/2=0.0077/2=0.004

IV) Traction load Parameters

According to the LRT data, an induction motor has the following parameters.

Table A.1 Traction Motor Performance Parameters [36]

Rated power	130KW
Rated voltage	3-ϕ,500V AC
Number of poles	4
Rated frequency	71Hz
Power factor	90%
Efficiency	91%
Rated rotation Speed	1800r/min
Maximum rotation speed	4377r/min

The total power of the tramcar is the sum of the cumulative power of traction motors and train auxiliary powers (34kW).

$$TotalPower = 4 * motorpower + AuxiliaryPower$$

$$= 4 * 130kW + 34kW = 554kW$$

The apparent power with power factor of 90%;

$$S = \frac{P}{pf} = \frac{554kW}{0.9} = 615.56kVA$$

The reactive power is, $Q = \sqrt{S^2 - P^2} = \sqrt{615.56^2 - 554^2} = 268.32kVAr$

The per motor parameters active and reactive powers; $P = 130kW$

$$Q = 130\sqrt{\frac{1}{0.9^2} - 1} = 63kVAr$$

APPENDIX B: DISPLACEMENT POWER FACTOR CALCULATION

The displacement power factor is the ratio of active power to the fundamental apparent power.

$$pf_{disp} = \frac{ActivePower}{FundamentalApparentPower} = \frac{P}{S_1}$$

$$P = V_{rms} * I_{rms} \sum_{h=1}^{\infty} \%V_h * \%I_h * \cos\varphi_h$$

Case 1: In the first case only the dominant individual harmonics are used, for the reason that the effect of these harmonics is higher on power factor. From Matlab FFT analysis, the fundamental rms voltage ph- neutral is 8537V and current rms is 73.4A.

Table 1B: Voltage and Current individual harmonics, without filter

Order	%V	φ_v	%I	φ_I	$\varphi(\varphi_{v-}\ \varphi_I)$	cos φ_h	Real power
1	100	-30.7	100	-39.2	8.5	0.98901	619733
79	17.24	153	12.32	244.8	-91.8	-0.03141	-909.861
81	16.84	-89.3	11.73	2.6	-91.9	-0.03315	-893.185
159	25.87	65	2.06	167.7	-102.7	-0.21984	-1597.89
161	26.97	180.8	1.73	-71.1	251.9	-0.31068	-1977.01
						Total	**614355**

$$pf_{disp1} = \frac{P}{V_{1rms} * I_{1rms}} = \frac{614355}{8537*76} = 0.947$$

Case 2: The dominant harmonics in case 2 are those created by 12 pulse rectifiers. The analysis considers harmonics up to 50 order (an exact analysis is used for up to 50^{th} harmonics) [18]. In this case the phase to neutral rms voltage is 8509V and rms current is 93.09.

Table 2B: Voltage and Current individual harmonics, with load filter

Order	%V	φ_v	%I	φ_I	$\varphi(\varphi_{v-}\ \varphi_I)$	cos φ_h	Real power
1	100	151.8	100	-40.4	192.2	-0.97742	-765897
11	1.53	233.3	7.41	-32.6	265.9	-0.0715	-110.551
13	0.86	171.8	3.51	265.4	-93.6	-0.06279	-25.8505
23	0.47	-65.6	1.09	26.6	-92.2	-0.03839	-2.68218
25	0.38	-86	0.8	6.1	-92.1	-0.03664	-1.5193
35	0.39	-19.9	0.57	71.8	-91.7	-0.02967	-0.89944
37	0.27	-54.6	0.37	37	-91.6	-0.02792	-0.38043
47	0.25	31.7	0.27	123.3	-91.6	-0.02792	-0.25705
49	0.21	10	0.21	101.5	-91.5	-0.02618	-0.15744
						Total	**766039**

$$pf_{disp2} = \frac{P}{V_{1rms} * I_{1rms}} = \frac{766039}{8509 * 92.09} = 0.96$$

Case 3: The same to case 2 the dominant harmonics are those created by 12 pulse rectifiers. The phase to neutral rms voltage is 8550V and rms current is 92.09.

Table 2B: Voltage and Current individual harmonics, with AC and DC side filters

Order	%V	φ_v	%I	φ_I	$\varphi(\varphi_{v-}\ \varphi_I)$	cos φ_h	Real power
1	100	-30.2	100	-24.2	-6	0.994596	783056.2
11	0.12	-22.9	0.6	71.2	-94.1	-0.071416	-0.70209
13	0.1	-85.6	0.4	7.9	-93.5	-0.061012	-0.33304
23	0.08	43.9	0.2	136.2	-92.3	-0.040165	-0.08757
25	0.08	-0.7	0.16	91.5	-92.2	-0.038382	-0.06701
35	0.08	123	0.12	214.7	-91.7	-0.029617	-0.03884
37	0.09	42	0.12	133.7	-91.7	-0.029617	-0.0437
47	0.09	176.2	0.1	267.6	-91.4	-0.024451	-0.02999
49	0.08	96.7	0.09	188.2	-91.5	-0.026121	-0.02571
						Total	**783054.9**

$$pf_{disp3} = \frac{P}{V_{1rms} * I_{1rms}} = \frac{783054.9}{8550 * 92.09} = 0.994$$