



**THE STRUCTURE AND EVOLUTION OF  
INTERMEDIATE-MASS STARS**

**By**

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**This Work is Dedicated**  
**to**  
**My father**  
**GEZIE WALELIGN**

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## Abbreviations

<b>ISM</b>	= <b>InterStellar Medium</b>
<b>GMC</b>	= <b>Giant Molecular Cloud</b>
<b>MRR</b>	= <b>Mass-Radius Relation</b>
<b>MTR</b>	= <b>Mass-Temperature Relation</b>
<b>LTR</b>	= <b>Luminosity-Temperature Relation</b>
<b>HRD</b>	= <b>Hertzprung Russell Diagram</b>
<b>AGB</b>	= <b>Asymptotic Giant Branch</b>
<b>ISM</b>	= <b>InterStellar Medium</b>
<b>ZAMS</b>	= <b>Zero Age Main Sequence</b>
<b>IMF</b>	= <b>Initial Mass Function</b>
<b>EOS</b>	= <b>Equation Of State</b>
<b>MS</b>	= <b>Main Sequence</b>

## Physical Constants

Solar radius	$R_{\odot} = 6.96 \times 10^{10} \text{cm}$
Gas constant	$\Re = 8.315 \times 10^7 \text{ergK}^{-1} \text{g}^{-1}$
Speed of Light	$c = 3 \times 10^8 \text{ms}^{-1}$
Mass of the Sun	$M_{\odot} = 1.99 \times 10^{33} \text{g}$
Solar luminosity	$L_{\odot} = 3.847 \times 10^{33} \text{ergs}^{-1}$
Mass of He atom	$m_{He} = 6.6463173 \times 10^{-24} \text{g}$
Mass of Be atom	$m_{Be} = 1.662747579 \times 10^{-24} \text{g}$
Atomic mass unit	$m_H = 1.6605 \times 10^{-24} \text{g}$
Solar temperature	$T_{\odot} = 5.8 \times 10^3 \text{K}$
Radiation constant	$\alpha = 7.566 \times 10^{-16} \text{Jm}^{-3} \text{K}^{-4}$
Boltzmann constant	$k = 1.3806505 \times 10^{-23} \text{JK}^{-1}$
Gravitational constant	$G = 6.6726 \times 10^{-8} \text{g}^{-1} \text{cm}^3 \text{s}^{-2}$

## Symbols

$\varepsilon$	Energy Generation factor
$\Omega$	Gravitationa Potential energy of the particle
U	Total energy density
R	Total star's radius
$d\omega$	Small solid angle
$S_\nu$	Source function interms of frequency
$\tau_\nu$	Optical depth interms of frequency
$M_r$	The mass in a sphere of radial distance
$L_r$	The rate of energy flow
$I(\theta)$	Specific intensity with incident angle

## Unit Conversion

Dyne	$dyn = g.cm.s^{-2}$
Jansky	$Jy = 10^{-26} J.s^{-1}m^{-2}$
Second	$s = 3.1556952 \times 10^{-7} yr$
Solar radius	$R_{\odot} = 6.96 \times 10^8 m$
Mass of Proton	$M_p = 1.67262178 \times 10^{-24} g$
Mass of the Sun	$M_{\odot} = 1.99 \times 10^{30} kg$
Solar luminosity	$L_{\odot} = 3.847 \times 10^{26} J.s^{-2}$
Electroretinogram	$erg = 10^{-7} J.s^{-1}$

## Abstract

We explore the structure and evolution of intermediate-mass stars. Under this, we focus on the mass-luminosity relation of the single main sequence intermediate-mass stars. We derive the stellar structure equations and put the fundamental stellar quantities in terms of mass. After this, we derive the lifetime of intermediate-mass stars in three nuclear burning phases and relate the stellar quantities with their lifetime. Thus we predict the relation MLR, MRR, and MTR in hydrogen core burning, hydrogen shell burning, and helium core burning. The mass-luminosity relation is in good agreement for O.Yu. Malkov's observational work of the absolute MLR relation. In the effects of temperature, gravity, and nuclear reaction, the star's structure was varied. To determine the final differential results of star's structure equations, we must use the derivative of interior star's structure equations with central radius  $r$ . This allows us to compute the  $M$ ,  $T$ ,  $P$ ,  $L$ , and  $R$  relations. Those are quantities derived from the stellar structure equations. From those equations, we have obtained the mass-luminosity relation equation and using nuclear reactions we studied the star's mass and age relation using the simplified form of the equations. We showed the graphical relationship of Mass to Luminosity, Mass to Temperature, and Mass to Radius of stars in terms of a parameter of a nuclear burning lifetime.

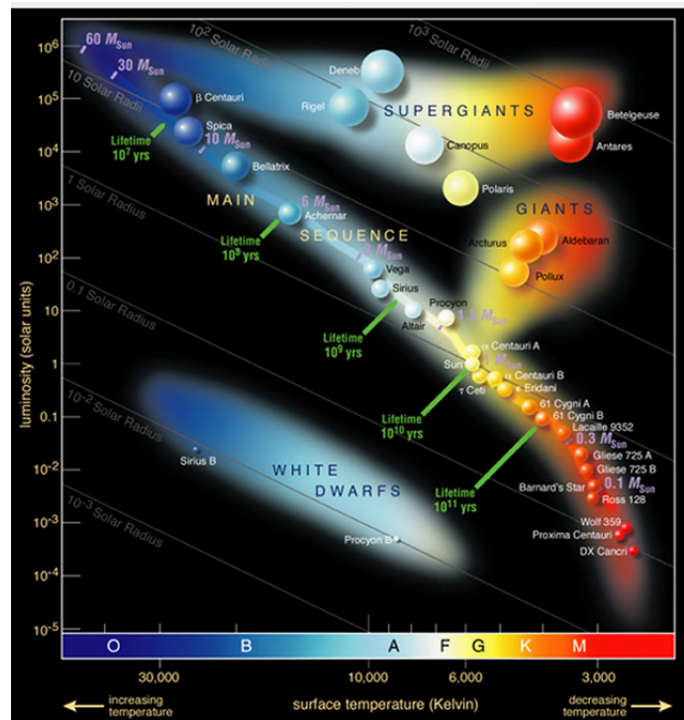
The lifetime equation of the star in hydrogen core burning and the mass-luminosity equation are good ways to measure the mass-luminosity relation of B-type stars.

## Introduction

All objects in the universe are stellar objects. Some of them are planets, galaxies, stellar populations, and the Stellar medium (interstellar space). The Stellar medium sometimes referred to as "stellar nurseries" or "star-forming regions". This region has clouds and different matters. In the case of a chemical reaction, the cloud's gas and dust are collapse. Then the nebula or molecular cloud is formed. As the cloud collapse, a molecular cloud breaks into smaller fragments. In each fragment, the collapsed gas release, as gravitational potential energy changes to heat energy. Then, as the temperature and pressure of the fragment increase, the collapse will be stopped in the central region [1] and the clouds condensed into a rotating sphere of superhot gas. This condensed gas is called **protostar**. Protostar is the pre-birth state of stars or, it is a young star. The condensed cloud has self-gravity and radiated energy supplied by an internal source. The body which has this condition is called a **star**. The structure of the star is measured by the sun's structure. That means, the object of reference in stellar physics naturally is the Sun see [2]. Stars are the building blocks of planetary systems, cluster associations, and galaxies. Galaxies are large systems of stars, which also contain interstellar clouds of gas and dust. Many of the stars in a Galaxy aggregated in clusters. The biggest among them containing quite 10<sup>5</sup> Stars. One type of Galaxy, which we live in, is called the Milky way galaxy. It is our Galaxy that has many stars. Those stars have variable energy. In the case of this, they evolve and release energy. Then, the necessary occurrences changed in their structure or composition, or both. These shows precisely the meaning of stellar evolution. The evolution of a star is the physical process in their interiors, which starts from the early stages of the Stellar's evolution. Principally, Stars evolution determined by their initial mass to a lower extent, by their composition, and by the presence of the time of birth to death, the stars have an evolutionary turning region, which is known as the evolutionary phase. Those phases are the main sequence, giant, and dwarf phase.

Before the main sequence , there is a Pre -main sequence. This provides the constraints for the input physics of early stellar evolution [3]. At every phase, the structure of the stars change. That means some stars property is changed. The main important property of the Star is Stellar Mass. Depending on Stellar-mass, the stars in our Galaxy in terms of

solar mass can be classified as; low mass, intermediate-mass, and massive stars. Those are the common inhabitants of galaxies throughout the Universe. Stars in this mass range are numerous for reasons of star formation processes. That favor the formation of intermediate-mass stars in addition to low mass stars over massive stars see [4]. The stellar structure depends on the stellar structure equations (we will see in section 2.3 ). In modern times the star's Sellar structure and evolution clarified in HRD. HRD is a diagram, that is done by the American astronomer, Henry Norris Russell and the Danish chemical engineer turned astronomer, Ejnar Hertzsprung, from 1911 to 1913 [5].



**Figure 1.1:** The stellar structure and evolution using Hertzsprung-Russell Diagram (HRD) with Luminosity vs. Temperature of Stars (in Solar neighborhood)

The two astronomers gain the same result with different techniques in 1933. HRD is the short written form of the Hertzsprung-Russell Diagram, [6]. As shown in the figure 1.1, the stars' structure changed from the pre-main sequence, red giant, super-giant to white dwarf phases. As the star's phase change, they form in binary. Most main sequence stars are in binary. Depended on the appearing of the star to observer we classify as follows

1. Visual binary:- When the stars periodic component seen to observer, that stars are called visual binary stars.
2. Astrometric-binary:- In the orbiting of the binary stars, when the motion of only single star is seen to the observer, the star is the Astrometric binary.
3. Spectroscopic binary:- When spectral lines detected in the star, the stars are called

the Spectroscopic binary stars.

4. Photometric-binary:- If the stars show the periodic variation of flux for observers, the stars are photometric binary stars.
5. Eclipsing binary:-During the stars orbit, one-star eclipsed by the other star. The system is eclipsing binary stars.

Every binary star has mass divisions. Primary stars are more massive and secondary stars are less massive. Each division is single stars, and they appear at different phase or the same phase, as shown in fig.1.1. Those single stars have different mass in different phases. One of them is an intermediate-mass star. Univers have element factory that produces the intermediate-mass stars The structure and evolution of main sequence intermediate-mass star calculated numerically [7]. The phase nucleosynthesis of the main sequence is less than the asymptotic giant branch and others. Their stellar structure depends on the stellar structure equations like mass conservation, hydrostatic equilibrium, energy generation, and energy transport equations. The stellar structure equations have different quantities. The fundamental quantities are mass, lumi- nosity, and radius see [8]. Some papers described sets of intermediate-mass stars with various chemical compositions. However, in the continuous improvement of the input physics, the computation of the same stellar models becomes necessary. Major contributions are the stellar quantities [9].

Several authors have studied the intermediate-mass star quantities and their relation ( $MLR$ ,  $MT_{eff}R$ , and  $MRR$ ) using diagram separately, and by collecting eclipsing binaries [60], to show the structure and evolution of intermediate mass stars.

The  $MLR$  relation is well constrained for solar-type and intermediate-mass stars see [57]. From the eclipsing binary star, the rapid rotator primary single stars appear on the main sequence. In binary star, a rapid rotator primary single intermediate-mass star property on the main sequence observed by O.Yu.Malkov, in 2007. It concluded that the  $MLR$  relation based on eclipsing binary data could not be used to derive the IMF of single stars [12]. But the data observed by. Initial mass function(IMF) is the distribution of stellar masses that form in a one-star formation event in a given volume of space [13]. Intermediate mass stars' structure and evolution consists of fundamental quantity such as mass, Luminosity, Radius, and Temperature. These facts provide the motivations for our study, which is generally aimed to show the relationship between- Mass to Luminosity of single intermediate-mass stars in terms of their evolution(age) on the main sequence theoretically. specifically

1. Determine the fundamental stellar quantities  $M, L, R,$  and  $T$  in terms of nuclear burning time, by derivating the stellar structure equations and nuclear burning lifetime.
2. Study the star's fundamental quantities relation with nuclear burning lifetime.
3. Produce the MTR and MRR relationship graphically using the stellar structure equations.

In this thesis, we will focus on the intermediate-mass star quantities relation in terms of their lifetime. For this purpose, we took detached main-sequence eclipsing binary components' data. Using this data, we showed the relationship of  $MRR, MT_{eff}R,$  and  $MLR$  graphically, then compare with the theoretical values that done theoretically.

First, we review this scenario briefly, by derivating all stellar structure equations (see section 2.3). We pay particular attention to the derivation of hydrostatic equilibrium equations and nuclear burning time in hydrogen core burning, hydrogen shell burning, and helium core burning that provide us to study mass-luminosity relation, mass-radius relation, and mass-temperature relation.

The outline of this thesis is as follows: In chapter 1 introduction. In Chapter 2 the stellar structure equations and evolution of intermediate-mass stars, energy transport of intermediate-mass stars, the stellar structure quantities, and their relation and the age of stars are explained. In Chapter 3, the stellar structure equation, determine the nuclear burning lifetime equations for intermediate-mass stars were derived. Chapter 4 using the stellar structure and evolution equations that derivate in chapter 3, result and discussion examined. Finally, we conclude in Chapter 5.

## Structure of stars

### 2.1 Introduction

In 19<sup>th</sup> century the structure of stars is described in different phases. One of them is the main sequence. The main sequence stars structure and evolution is well understood [14]. There are several observational results of mass-luminosity, mass-radius, and mass-temperature relations obtained recently related to the structure and evolution of intermediate-mass stars. In this chapter, we will explain the structure of stars, the equation of the stellar structure, Energy generation, energy transport, fundamental quantities of main-sequence intermediate-mass stars, the stellar evolution of intermediate-mass stars, and intermediate-mass star's age.

### 2.2 Structure of a star

Stellar structure equations describe the structure of the stars and star formations. The stellar structure of the stars depends on the star's mass in case of these stars are divided into three, those are low mass stars, intermediate-mass stars, and massive stars.

#### 2.2.1 Low- Mass Stars

Low Mass stars are the mass between  $0.8 - 2.3M_{\odot}$ . Those stars burn H in their core, and they have He ignition experience under the degenerate condition known as the core He flash. The characteristics of low mass stars are frequently derived from photometry. The validity of the theoretical relation for these band is thus of significant interest, but too few low mass stars with accurate masses have known luminosities [57].

#### 2.2.2 Intermediate-Mass Stars

Those stars are the mass range between  $2.3 - 8M_{\odot}$ , Intermediate Mass stars are not sufficiently massive to ignite C into their core. It has composed of C and O to follow He burning, and C ignited under degenerate conditions. Those mass go on to experience thermal pulses on the "super-AGB" to following He core exhaustion, and the more massive of them will experience the second dredge up as they begin their ascent of the AGB see [55].

### 2.2.3 Massive Stars

The star with mass greater than about  $8M_{\odot}$ , call as massive stars. Stars upper mass limit (stability limit) around  $100M_{\odot}$  are large convective cores on the main sequence (up to 80% or more of mass with  $M_{\odot}$ ). In this mass, the effect of convective overshooting is larger convective cores. Their lifetime on the main-sequence is extended, and it leads to higher luminosity and broader MS. Radiation pressure and electron scattering also dominant on the main sequence. In massive stars, there is a mass loss on the main sequence due to radiation driven winds from hot luminous surface. Both convection and mass loss are decisive for evolution [55]

## 2.3 Structural equation of stars

The stellar structure equation describes the structure of the stars. Those equations are hydrostatic equilibrium, mass continuity, energy generation, and energy transport equations. To calculate the star's structure, we solve those equations. It makes sense to write mass in terms of fractional radius  $r$ . By the assumption of spherical symmetry, all parameters describing the star depend only on the distance  $r$  from the center. It also emphasized the matter density, denoted by  $\rho$  at a generic point  $r$  within the star. The mass contained within a sphere of radius  $r$  centered gives the stellar structure equation that determined based on the fact that at every point of the stars in a steady-state, see [16].

### 2.3.1 Mass continuity equations

Using the hydrostatic equilibrium with acceleration due to gravity, the relation between  $M(r)$  and the mass density per unit volume  $\rho$  is the mass continuity equation. [22]

$$dM_r = 4\pi r^2 \rho dr \quad (2.1)$$

The mass continuity equation can be used to put the mass contained in the star. When nearly all the hydrogen in the core has fused, the main sequence star's mass change over their life. As the star slowly convert hydrogen in the core into helium main-sequence life of the star decrease. see more [18].

### 2.3.2 Hydrostatic equilibrium equation

Most stars are in long-lasting phases. In their evolution, structural change can not be observed at all. This shows the matter in a star cannot be accelerated noticeably.

Which means the forces acting on the star mass of element compensate each other. Then there is mechanical change in the stars that is known as "hydrostatic equilibrium" [19]. In hydrostatic equilibrium, the outward force due to the gas pressure balances the force from

its gravity infall [20].

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} \quad (2.2)$$

From this equation, four quantities should be available on demand by the model builder. The righthand variants of these equations are setdown[45]. It is the second stellar structure equation. They remain two stellar structure equations obtained from the conservation of energy.

## 2.4 Energy transport in intermediate mass stars

In stars, mostly the radiation and convection former mechanisms for energy transport are at play. In solar mass stars, energy is transported from the core by radiation until a distance of about  $r = 0.7R$ . Energy transport makes a very crude estimate of how long a star remains on the main sequence. The outline of the work is finding a brief-expression for the luminosity of the star. We know that luminosity is energy radiated away per unit of time. Assume how much energy the star has available to radiate away during its lifetime, we can divide this energy by the luminosity, and it used to find a lifetime. These show there were thermodynamic states. In the case of this, the energy density varies. That means the energy generation rate within the star diffuse outwards, and so the contribution to the outflow of energy from the shell of the radius( $r$ ) and thickness( $dr$ ) is;

$$dl = 4\pi r^2 \rho \varepsilon dr \quad (2.3)$$

where  $\rho$  is the surface density,  $\varepsilon$  is the energy generation coefficient. This equation is the total heat flux flowing through a spherical shell with the radius  $r$ . The most stellar energy-transportation ways in medium mass stars are the radiation and convection process.

### 2.4.1 Energy transport by radiation

In stars photons from the photon gas traveling outwards. The photons cannot travel directly from the core but are continuously scattered in various directions by collisions with other particles. After a large number of scatterings and direction changes, it will eventually escape at the surface. For a crude estimate of the luminosity, we always use the energy transport equation that was done in the nineteenth century. In terms of the differential equation, it is the temperature gradient which has two forms depending upon whether radiative or convective equilibrium exists at the point[17].

The standard equation of radiative transport gives the luminosity ( $L_{rad}$ ) that transported by radiation through the atmosphere as a function of the temperature gradient, and the

radiative temperature gradient [23], the equation is;

$$\frac{dT}{dr} = -\frac{3\kappa\rho}{16\sigma T^3} \frac{L_{rad}}{4\pi r^2} \quad (2.4)$$

Where  $\kappa$  is the opacity, which in general depends on both  $\rho$  and  $T$ , and  $\sigma$  is the Stefan-Boltzmann constant.

### 2.4.2 Energy transport by Convection

The masses larger than the low mass of the hot gas may stream outward, while the cooler gas falls inwards. In this way, the heat and thereby the energy is transferred outwards. This method is known as energy transported by convection [24]. Convection is a much more efficient way of energy transport than radiation. In some stars, energy is transported by convection rather than radiation. In such stars, local density fluctuation produces bubbles of gas. Those are hotter and less dense than their surroundings. Those rise until they eventually reach equilibrium and mix with surrounding gas towards the outside of the star. The transport of energy by convective motion is relatively inefficient. But, radiation dominates and transports more than 95% of the total flux. This is because the density is very low and the mean free path of photons correspondingly long.

In this situation, convection is significantly superadiabatic like this;

$$d\ln T = \frac{\gamma - 1}{\gamma} (d\ln P) \quad (2.5)$$

where  $\gamma$  constant polytropic. The ratio  $d\ln T/d\ln P \equiv \nabla$  (gradient) is explicitly calculated from the mixing length equation [25]. The adiabatic stellar structure equation is.

$$\frac{dT}{dr} = -\frac{GM_r\mu}{Rr^2} \nabla_{ad} \quad (2.6)$$

where  $R$ ,  $r$ ,  $\mu$ , and  $G$  are the outer radius, internal radius, magnetic moment, and gravitational constant, respectively. This equation is done in the nineteenth century using the ideal gas law. Which is called an adiabatic temperature gradient[17]. When the gas steeper against convection, the adiabatic gradient is steeper than the radiative gradient, and convection does not occur, the gas will be rely on radiative transfer to transport energy. When the adiabatic gradient is shallower than the radiative gradient, convection will occur [26]. But, the star will use the energy transport process that has a shallower gradient.

## 2.5 Energy generation in intermediate-mass stars

The matter in stellar objects is not stable. They form different energy by the rate of energy[27]

As shown on equation 2.3, the energy generation rates( $\epsilon$ ) is commonly formalize energy conservation with three kinds of energy generation rates ( $\epsilon = \epsilon_{grv} + \epsilon_{nuc} - \epsilon_{\nu}$ ). In the

equation of energy conservation, the terms  $\epsilon$  controls the time evolution of a star throughout its life.

$$\frac{dL}{dM_r} = \epsilon_{grv} + \epsilon_{nuc} - \epsilon_{\nu} \quad (2.7)$$

Where  $L$ ,  $dM$ ,  $\epsilon_{grv}$ ,  $\epsilon_{nuc}$  and  $\epsilon_{\nu}$  are the luminosity, the enclosed mass at the radius  $r$ , the gravothermal energy rate, the nuclear energy rate and the neutrino cooling rate, respectively.

On the other name,  $\epsilon$  is the energy generation coefficient. That contains energy per unit time and unit mass generated by nuclear reactions ( $\epsilon_n$ ), energy per unit time, and unit mass generated by the thermodynamical transformations experienced by stellar matter during the star's evolution ( $\epsilon_g$ ). This contribution is usually named the gravitational energy term and energy per unit time and unit mass associated with neutrino production processes ( $\epsilon_{\nu}$ ) that effectively subtracted from the stellar energy budget.

Since neutrinos barely interact with the surrounding stellar matter. These show stellar structure depends on energy generation, which has opacity ( $\kappa$ ) and energy generation coefficient ( $\epsilon$ ). That needed to solve the equations of stellar structure. Finally, it is the radiative temperature gradient that generates energy per gram. The most stellar energy production mechanisms are Gravitational contraction and Nuclear reaction energy generations.

### 2.5.1 Energy generation by Gravitational Contraction

In the absence of nuclear reactions the thermodynamical transformations can generate energy due to the first principle of thermodynamics. It relates the heat energy ( $dQ$ ) added to the star (per unit mass) to the internal energy per unit mass  $U$  and the specific volume ( $\nu$ ), which related with  $1/\rho$ . A star under going gravitational contraction gravitational potential energy ( $\Omega$ ) with internal radius  $r$  and internal mass  $M_r$  will release in the process;

$$d\Omega = -\frac{GM_r}{r}dM_r \quad (2.8)$$

But, thermal energy( $T$ ) of a star using ideal gas law and hydrostatic equilibrium equation gives as,

$$\frac{dT}{dr} = \frac{3}{2}(4\pi r^2)P \quad (2.9)$$

Then total energy in gravitational contraction at  $P(R) = 0$ , and  $T = -\Omega/2$ .

This shows the thermal energy of the star is one half of the magnitude of the star's gravitational potential energy. Furthermore, reducing the potential energy through the gravitational contraction will increase the thermal energy. But one half of the amount of potential energy released. Using the statement of the Virial-theorem, one half of the gravitational potential energy of the star is available to be converted into thermal energy. The remain is radiated away and lost to the star [28].

$$\Delta T = -\frac{1}{2} \Delta \Omega \quad (2.10)$$

### 2.5.2 Energy generation by Nuclear Reaction

The nuclear reaction does not only release energy but also change the chemical composition of the stellar interior. As chemical composition change, the stellar evolution for a generic reaction between a particle 'a' and a nucleus 'x' produce a nucleus 'y' and a particle 'b'; usually, both 'a' and 'b' can be either another nucleus, a proton, a neutron, or an  $\alpha$  particle. Symbolically,



In all nuclear reaction charges, nucleon number, momentum, and energy must be conserved. That means the sum of the reactant mass differs from the sum of the product mass. It calls the mass excess and linked with the binding energy ( $E$ ) of the nuclei involved in the nuclear reaction. The binding energy is the energy required to separate the nucleons against their mutual attraction by the strong nuclear forces. It is also called Einstein's famous mass-energy equation.

$$\begin{aligned} E &= [(a + x)M - (b + y)M]c^2 \\ E &= [Ma + Mx - Mb - My]c^2 = \Delta Mc^2 \end{aligned}$$

Where;  $c$ ,  $M$ , and  $E$  are speed of light, the mass of the particle, and the binding energy difference of the product and reactant in the reaction, respectively. This is the stellar structure mass excess equation. One of the three major time-scales associated with nuclear burning and hydrodynamic flow is called nuclear time-scale [29]. Symbolically it gives as;

$$t_{nuc} = \frac{\Delta Mc^2}{L} \quad (2.12)$$

Where  $\Delta M$  and  $L$  are the mass difference between reactant and product, and luminosity.

If its relevant timescales are shorter than the timescales imposed by the drivers, the star can be considered in a quasi-steady state. The relevant timescale is the Eddington-sweet timescale, which corresponds to the time required for the redistribution of mass,

that is the nuclear mass and mass loss in the rate of change of the hydrogen mass in the convective core see [30]. The evolutionary properties of intermediate-mass stars strongly influenced by two main open problems are, hydrogen-burning and helium burning [31] Stars typically less than  $0.5M_{\odot}$  are unable to develop temperatures high enough to ignite Helium and terminate their evolution as degenerate He white dwarf. Stars above that critical mass and between  $2 - 2.5M_{\odot}$  develop degenerate cores and ignite helium off-center explosively during the so-called He-flash. Stars belong to the group of low-mass stars which differ from intermediate-mass stars ( $2 - 2.5M_{\odot} < M < 6 - 8M_{\odot}$ ) ignites He quietly at the center [32]. Bellow this we will see hydrogen and helium burnings;

## I. Hydrogen Burning

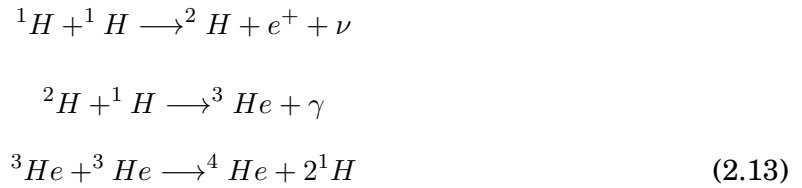
The energy source of the main sequence star is Hydrogen burning, and Helium is formed. The energy formed in this reaction is up to  $26.73 \text{ Mev}$ . Depend on whether a star occupies the lower or upper main sequence, Hydrogen burning occurs through the PP chain (proton-proton chain) or CNO cycle.

### (PP) chain:

It is the chemical reaction process of Proton-Proton, and it operates inside the star. There are three PP chains.

#### (PPI) chain

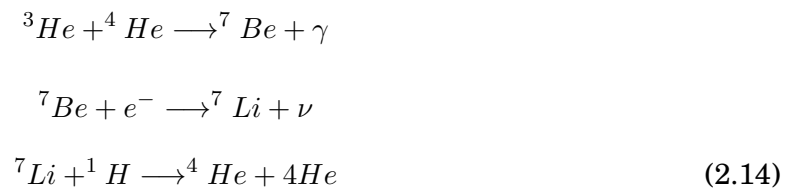
It involves a weak interaction to form deuterium and  $e^-$ ,  $e^+$  annihilations. This is smash two protons together to make deuterium.



Where  $e^+$ ,  $\nu$  are positron (electron with positive charge) and neutrino (very light and hard to detect particles), respectively. In second step an isotop of He is formed. Once enough  ${}^3He$  is accumulated it follows the so called PPI chain.

#### (PPII) chain

For the temperatures typical of main sequence stars  ${}^3He$  is also burned through the PPII chain the proton crashes in to a deuterium nucleus, making helium-3 and the reaction to be relevant for the temperature above  $\sim 1.4 \times 10^7 k$ .



Where,  $\gamma$  is gamma ray that have very high photon energy.

(PPIII) chain

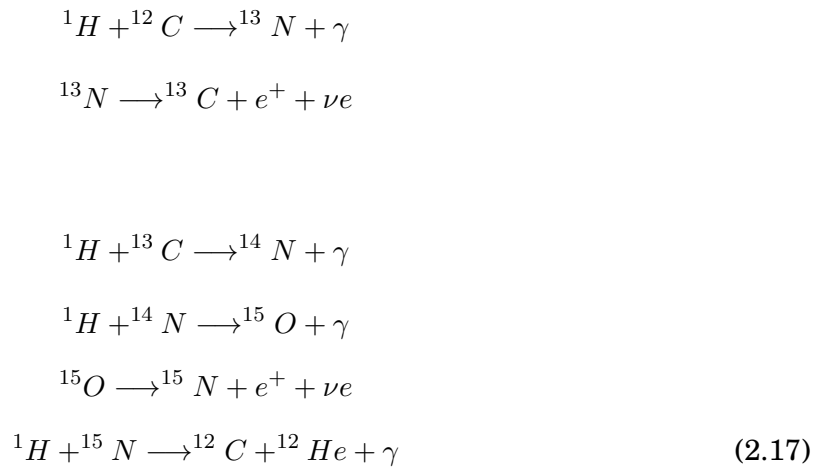
In this chain in which an excited state of beryllium decays to form two Helium nuclei and the relevant temperature is larger than  $\sim 2.3 * 10^7 K$



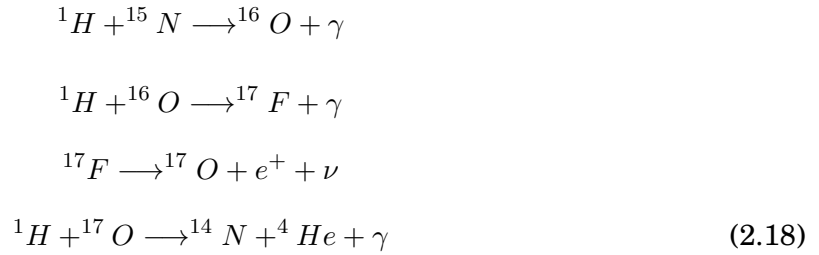
The net effect of the proton-proton chain is the conversion of four H one atom into one  ${}^4\text{He}$  atom with an energy release of about  $26\text{MeV}$  per  ${}^4\text{He}$  atom. The temperature on both PPII and PPIII is important to realize all the nuclei involved in that chain [59].



But, Helium and Hydrogen are unstable. So the reaction is rather efficient, Since CNO cycle is an efficient way to produce He. CNO cycle is a bicycle of CN and NO. CN Cycle dominant the reactions at low temperatures, and NO Cycle, mostly operate at high temperature. Both Cycles need a seed of carbon, nitrogen, and oxygen to be operative, and these nuclei catalyze the conversion of Hydrogen to He. That conversion of the carbon into nitrogen in the envelope requires extended mixing (so-called "extra mixing") in the radiative zone below the surface convection [34]. The CNO cycle becomes dominant energy source for stars more massive than the Sun[60].



The reaction with the smallest cross section is the fourth one. In equilibrium the over all reaction is governed by the reaction rate of  ${}^{14}\text{N}$ . Also the last reaction of the CN cycle can be substituted by the suite nuclear reaction  ${}^{16}\text{O}$  :



The reaction cycle is ON cycle. Using the previous discussion, the abundances equilibrium CNO determined by the nuclear cross-section of  ${}^{14}N$ , and thus the CNO cycle also converts Carbon and Oxygen into Nitrogen. Those were the relation of core mass to luminosity. It shows there is a maximum luminosity, and determined by the maximum possible core mass of  $\approx 1.4M_{\odot}$ . Subsequent calculations of main-sequence intermediate-mass stars revealed that hot bottom burning CNO cycle violate the conditions of the core mass-luminosity relationship, see [55].

## II. Helium Burning reaction

The main sequence class are the stars at the main sequence, sustaining themselves through the conversion of hydrogen to helium by nuclear fusion in the core [32] Hydrogen is exhausted in the core of the main-sequence star. The pressure gradient can't be balanced by the energy release of nuclear reactions, and the Hydrogen exhaust core that contracts and becomes hotter. At the same time, Hydrogen shell burning were produced. The envelope expands, the stellar becomes convectively unstable, and the products of nuclear reactions produced deep in the star are dredge-up to the surface. In this way, they're the temperature change.

Where helium burns in a thin shell ( $r \sim 5 \times 10^8 cm$ ), and at a period of  $\sim 200s$ , the bubbles cover the complete height of the convection zone ( $R \sim 4.7 \times 10^8 cm$ ). When the value of the total kinetic energy in the reaction rises, hot gas near the position of temperature maximum begins to move and carries the liberated nuclear energy off the burning region, thereby inhibiting a thermonuclear runaway [29].

Then the temperature reaches to  $\sim 10^8 K$  Helium is ignited through the so-called triple-alpha process, and contraction stopped. This nuclear reaction actualy be regarde as a three body interaction, there by its name and not a normal nuclear reactions. The suite of reaction is;





The nuclear isotope  ${}^8\text{Be}^*$  is very short lived at this very high temperature. This is the reason why this reaction is usually considered a three body interaction, and also why as soon as some carbon is  $\alpha$  captures on it rapidly set in this equation;



The reaction product of He burning are carbon and oxygen, and the ratio between their abundances being extremely dependent on the temperature at which burning proceeds, and 3 to 4 Mev energy per  ${}^4\text{He}$  nucleus is produced, see [59].

## 2.6 Fundamental Quantities of main-sequence Intermediate mass stars and their relation

As shown in figure 1.1 among the HRD phases is that the most sequence phase. It is the foremost prominent feature of the H-R diagram. In intermediate-mass stars, the central temperature reaches  $\sim 10^7 K$ . At this temperature, hydrogen-burning is ignited within the stellar core and also the so-called Main Sequence (MS) phase. Stars that have a strong correlation between luminosity and temperature are the most sequence, and together with the MS, Hotter stars are brighter than cooler stars. About 85% of nearby stars including, the sun are on the MS. That is categorized by spectral types. Astronomers concocted a spectral typing that supported the alphabet. However, the initial 20-odd classes from the foremost popular stars ( $T \simeq 50,000K$ ) to the simplest ones ( $T \simeq 2200K$ ) are combined as O, B, A, F, G, K, and M.

The B-type stars are main-sequence(hydrogen-burning) stars of spectral group B and luminosity(Mv). Some B-type stars that we decide on their apparent magnitudes within the optical region are HD 4881, HD 5839, HD 32509, HD 179218, HD 184761, and HD 224648 see[35] and [36]. Their properties observed with gratings onboard XMM-Newton and Chandra [36]. These stars have 2 to 16 times the mass of the sun, and surface temperature between 10,000 and 30,000K. So, those are medium mass stars. The structure of B-type stars depends on the stellar structure equations that we've seen above. Those equations have fundamental quantities like mass (M), radius (R), luminosity (L), temperature (T), and time (t). But, first-order fundamental quantities of any stars are the mass M, the luminosity L, and therefore the radius R. The remains the temperature(T), surface gravity (g) and the mean density ( $\rho$ ) are dependent quantity of the set  $M, L, R$  or sometimes

$M, L, \text{ or } R$ , (see for more [37]). Bellow this we explain the first order fundamental quantities of intermediate mass stars;

### 2.6.1 Mass (M)

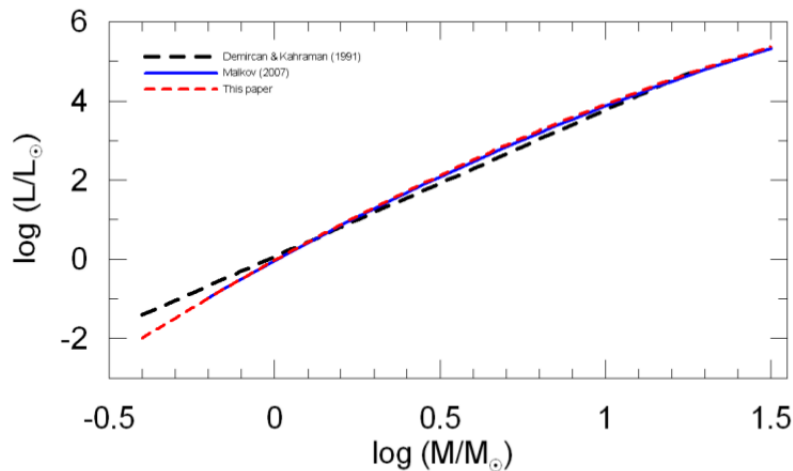
The fundamental stellar quantity with an important impact on the evolution and also the internal structure of stars is named mass. Stellar Mass is both a property of a frame and a measure of its resistance to acceleration (a change in its state of motion). When a net force is applied, an object's mass also determines the strength of its gravity to other bodies. Within the astrophysical situations (e.g. the IMF, the MLR, etc) are a necessary a part of the mass see more [35].

### 2.6.2 Radius (R)

A star's radius is half the star's diameter. Stars are large balls of gas held together by gravity, and they are approximately spherical. Radii of the star measured in meters, but because stars are so very large that it's much more convenient to measure stellar radii in units of the Sun's radius, were  $1 R_{\odot} = 6.96 \times 10^8 m$ .

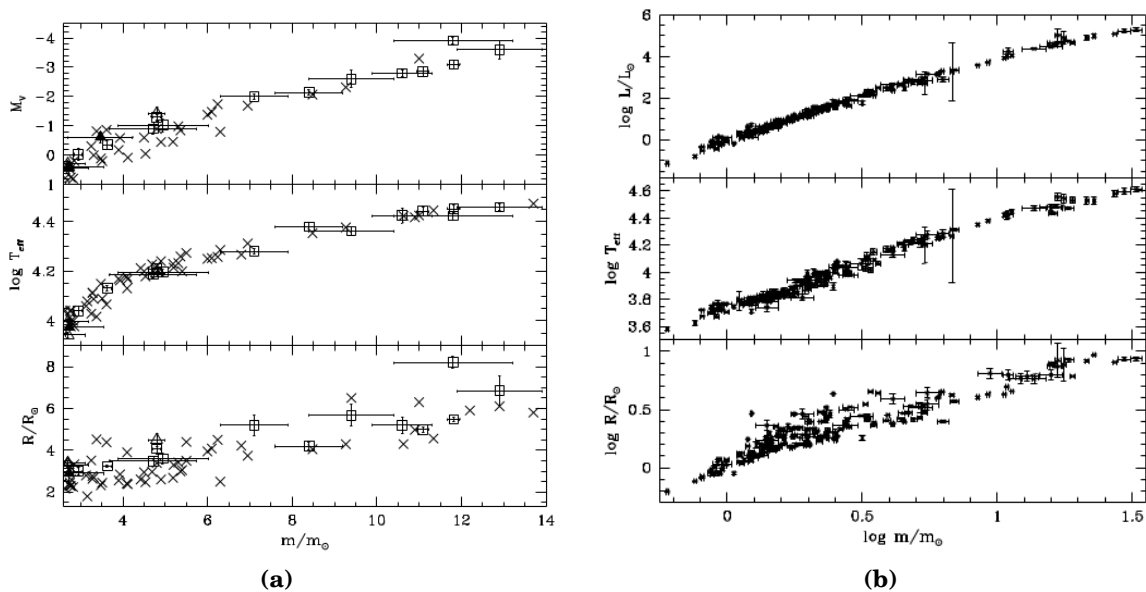
### 2.6.3 Luminosity (L)

The Luminosity(L) of a star (the total energy emitted per second) depends on two quantities. Such as the effective temperature and Radius. The "effective temperature" of the stellar photosphere  $T_{eff}$  and the total surface area of the star (which depends on its radius R) are stellar structure changers. Because stars are hot, the gas makes a dense ball. They produce approximately continuous radiation, and they obey the Stefan-Boltzmann Law. The equations, what we done on 2.3, have different quantity like; mass, luminosity, radius-temperature, density, pressure, metallicity and opacity etc. [38]. One of the most fundamental



**Figure 2.1:** Comparison of mass-luminosity relation that done by T.Senyuzand Demircan, Malkov(2007) and Demircan Kahraman(1991) [60].

confirmation on the main sequence and universally recognized astronomical relation is Mass-luminosity. The relation can be expressed in different ways [49]. For stars with masses greater than  $M_{\odot}$ , the relation written as  $L \propto M^{\alpha}$ . This equation and the usual value of  $\alpha = 3.5$  only applies to main-sequence stars with masses  $2M_{\odot} < M < 55M_{\odot}$ . The exponent  $\alpha$  is smaller for stars with masses greater than  $10M_{\odot}$  and also for stars less massive than the Sun [64]. Some mass-luminosity relation done by Z. Eker et al. 2015 [40] as shown on figure 2.1, and figure 2.2 that done by comparing the Malkove 2007 and Demircan and Kahraman1991 and the O.Yu Malkove mass-luminosity relation of detached main sequence intermediate mass stars. The relation between the mass of the star and luminosity on the main sequence is used in various fields of astrophysics [38]. Most of the time mass and luminosity relation of single intermediate-mass stars done observationally. Besides, in actual observations, the measurement of  $R$  depends on accurate distance measurement, and  $T_{eff}$  does not depend on distance. It shows the effective temperature modifier leads mass estimating equation [41]. Mass-radius relation applies on star formation [42].



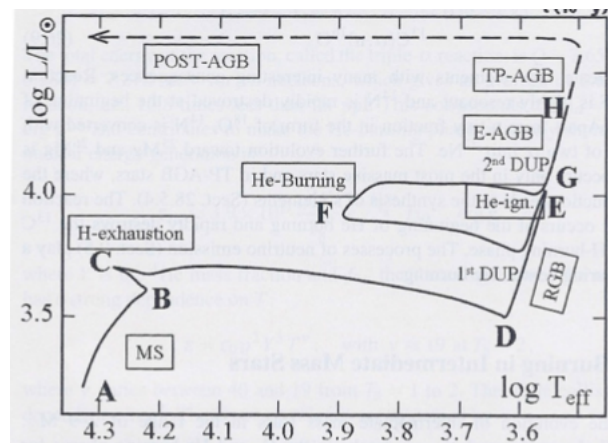
**Figure 2.2:** (a).mass-radius relation, mass-temperature and mass-bolometric luminosity relations.The square indicates non-synchronized double-lined eclipsing binary star, and the crosses synchronized double-lined eclipsing binary stars. (b). the logarithm of mass mass-logarithm of luminosity, logarithm of mass- logarithm of temperature and logarithm of mass-logarithm of radius relations for detached main sequence stars [38].

An effective temperature for a star may be, calculated using the well known Stephan-Boltzmann relation ( $L = 4\pi R^2 T_{eff}^4$ ) [60]. The luminosity of a star is proportional to its effective temperature to the 4<sup>th</sup> power, and its radius squared.

## 2.7 Evolution of intermediate-mass stars

Stellar evolution is the process by which a star changes over time, counting on the mass of the star. Their lifetime can change from some million years for the foremost massive, and trillions of years for the smallest amount massive, which is considerably longer than the age of the universe. Like stellar structure, the mass of stars determines its evolution and eventual fate. When the temperature has reached about  $10K$ , helium ignited. The core has thus tapped an oversized new energy source. That stops its rapid contraction, and also the star again reaches a stage of complete (thermal and hydrostatic equilibrium).

As shown in bellow figure 2.3 the entire core contraction from C to D has proceeded roughly on the KelvinHelmholtz timescale of the core. At the identical time, the outer layers have rapidly expanded, and therefore the stellar radius is increased. Star's luminosity increases quickly as effective temperature increases and time decrease. It climbs the red giant star Branch (RGB) within the HR diagram (HRD). Elements can bring up from the core to the surface of the star. Stars have different evolutionary phases some of them are: Main



**Figure 2.3:** The evolution of intermediate-mass stars [60].

sequence phase, giant phase, and dwarf phases.

### I. Main sequence phase

The MS phase of stars is a very stable phase. In this phase, nuclear fusion steadily turns hydrogen into helium.

Its change leads to a very slow change in the structure of MS stars. All-stars undergo the main sequence phase, characterized by core hydrogen burning. The major product is helium(He). Other important, nitrogen and oxygen isotopes are synthesized during the main sequence phases. But, resulting from proton capture on carbon nuclei in the stellar core and isotopes,  $^3\text{He}$  and  $^{13}\text{C}$  produced in cooler regions outside the core.

## II. Red giant phase:

As the MS phase advances a hydrogen depleted core grows gradually in mass, the hydrogen-burning shell surrounding the core separates from the envelope. Thus, as the burning shell moves outward, the core becomes isothermal. While its mass and temperature increases. Then reaches to new phase RGB phase. A red giant phase is a star that exhausted the supply of hydrogen in its core and has to begin thermonuclear fusion of hydrogen in a shell surrounding the core. Almost all stars in the mass range from  $1M_{\odot}$  to  $8M_{\odot}$  evolve through the Asymptotic Giant Branch, preplanetary nebula (PPN), and planetary nebula (PN) evolutionary phases [43].

## III. White dwarf phases:

After the red giant phase, the star's temperature increases, at some constant luminosity, and the star's age decreases. Then at high and some constant temperature, the luminosity rapidly decreased. That phase is known as the white dwarf phase. When the mass of the contracting star along the Hayashi track is below  $\sim 0.08M_{\odot}$ , the contraction and temperature increase. Due to the virial theorem is halted before reaching the hydrogen-burning temperatures due to the onset of electron degeneracy. The objects that will never produce energy from nuclear reactions are called brown dwarfs. Their evolution is similar to the much more advanced white dwarf evolutionary phase-type of stars, like the Sun.

## 2.8 Intermediate mass star's age

The ISM is a form of about 89% of hydrogen, 10% of helium gases, and 1% is traces of heavy elements (dust) that in the form of grains. It also has large molecular clouds of masses in the range of  $10^5 - 10^6$  solar masses, temperature between  $10K$  and  $100K$ , and densities between 10 and  $10^2$  particles  $cm^3$ . In the Milky Way Galaxy ISM contains several gas phases that exist in rough pressure equilibrium, at temperatures ranging from  $\approx 100$  to  $10^6K$ , and densities between 10 and  $10^3$  particles per  $cm^3$ . These clouds are the star-forming regions. Before reaching the hydrostatic equilibrium stage, stars must have formed out of interstellar matter (ISM) in a diffuse form.

If a cloud is a hug enough that the gas pressure level is insufficient to support it, the cloud will undergo implosion. All stars formed from collapsing clouds of gas and mud often called nebulae or molecular clouds. Throughout countless years, these protostars relax into a state of equilibrium. That is known as a main-sequence star. When a star is born, its situation on the so-called (ZAMS) curve. It represents the star's position in the H-R diagram at the onset of Hydrogen fusion in their center. While on the main-sequence, as Hydrogen progressively fused into Helium within the star's central region, the star's structure readjusts and slowly move aloof from ZAMS [44]. ZAMS stars are computed

by solving the stellar structure equations numerically. The amount that's changed by nuclear processes, besides abundances, is a (small). The fraction of the rest-mass energy given by Einstein's famous relation  $E = mc^2$ . This fraction, which may be turning into other forms of energy, constitutes the nuclear potential energy. Hence we may take  $\Phi = \varepsilon Mc^2$ , where  $\varepsilon$  can be estimated by the typical binding energy of a nucleon divided by the nucleon's rest-mass energy, which amounts to a few  $10^{-3}$ . The rate of change of the nuclear potential energy is, obviously, the nuclear luminosity  $L_{nuc}$ , and since we are allowed to assume thermal equilibrium, we may take  $\Phi = L_{nuc} = L$ . Hence  $\Phi = \varepsilon Mc^2$  [45] Medium mass star's evolution on the MS and beyond occur either on a nuclear time scale ( $t_{nuc}$ ) or on kelvinhelmholtz time scale ( $t_{KH}$ ). The nuclear time scale reads  $t_{nuc} \approx (available\ fuel)/(Power) \approx (M_{fuel}c^2)/(L)$ . The nuclear time duration of hydrogen burning on the main sequence is:

$$t_{nuc} \propto \frac{M_{core}X_Hc^2}{L} \quad (2.21)$$

Where,  $M_{core}X_H$  is the total mass amount of hydrogen burned during the MS. The burning of H in the core of main-sequence stars will last a period according to the relation.

$$t_{MS} \simeq 10^{10} \frac{M}{L} yr \quad (2.22)$$

M and L are in solar units,  $10^{10}yr$  is solar lifetime. This is the duration of H-burning on the MS as well as the duration of He burning [46].

During the core H-burning phase, the luminosity and the radius of the star increase. Whereas Effective temperature decreases. The core mass is defined here as the mass within which the hydrogen mass fraction is  $X_H < 0.37$ . After hydrogen disappears at the center, the H-exhausted core of the star contract, and drove by the energy generated at the bottom of the hydrogen-burning shell [47]. Then the envelope expands. He-core burning sets before the star reach the giant branch and temporarily reverses that evolution. On the main sequence phase, in addition to the ZAMS (core-H exhaustion), there are two additional loci, 50% of core-H depletion and the TAMS [48]. The helium mass fraction of low and medium mass stars on the main sequence are  $Y_{ini} = 0.248, 0.4$  and  $0.8$  with  $12.9Gyr, 4.62Gyr$  and  $158Myr$ (90%, 86% and 60% of total lifetime) respectively for their total evolution [64].

## 2.9 Summary

Naturally, the stellar structure of stars is not yet understood. We call daily or by guessing. Several methods have been using so far, and a large number of indirect shreds of evidence suggesting that the mass of stars is the factor for stellar structure, and mostly the stars' stellar structure depends on HRD. HRD has different phases with different types of star mass. One of them is the main sequence phase, and one of the mass division of stars is the intermediate-mass star. The stellar structure of intermediate-mass star was calculated, with stellar structure equations. The fundamental quantities of intermediate-mass stars relation are the main factor for the stellar structure of intermediate-mass stars. The stars change their stellar structure over time is called stellar evolution. In three evolutionary phase of intermediate-mass stars, the stars' age changed from ZAMS to TAMS in the case of nuclear burning.

## Derivation of stellar structure equation for intermediate mass stars

### 3.1 Introduction

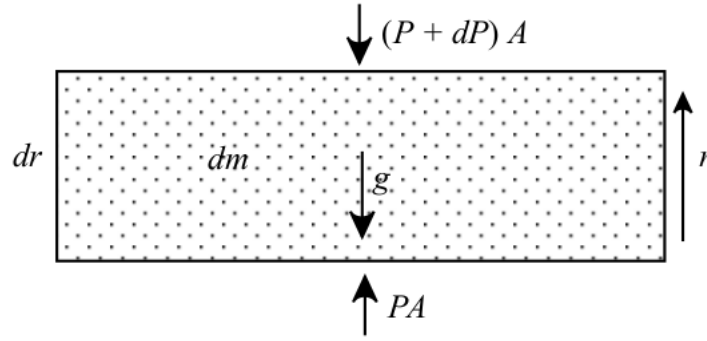
In this chapter, we will follow the methods used for our work. The stellar structure describes the run of pressure, temperature, luminosity, radius, and chemical element abundances as a function of  $M_r$  at a given time  $t$  of their evolution. [50]. To show the Mass to Luminosity relation; first, we must derive the stellar structure equation and the lifetime equation for the main sequence intermediate-mass stars. After deriving the nuclear burning lifetime equation, we divide their age into three phases as hydrogen core burning, hydrogen shell burning, and helium core burning by call as phase-A, phase-B, and phase-C, respectively.

### 3.2 Derivations of stellar structure equation for intermediate mass stars

We have seen four types of stellar structure equations in chapter 2. In this section, we will show how they determined. Using those equations, we find the interior stellar quantities, such as  $M$ ,  $P$ ,  $\rho$ ,  $T$ , and  $L$ . To construct the star's structure, we specify the total star's mass and the runing of composition as a function as interior mass or radius. At the end of the calculation the mass versus radius (or the other way around), and the corresponding local values of pressure, density, temperature, and luminosity are present. Stars are self-gravity objects of hot plasma. That emits energy in the form of photons from the surface, and spherical symmetry (absence of rotation and magnetic fields). The one-dimensional problem with radius  $r$  being the natural coordinate (Euler description) [18]. Using this concept, we will drive the fundamental stellar structural equations. One of them is Hydrostatic equilibrium equation.

As shown in Fig.3.1, the differential mass element  $dm$ , vertical thickness  $dr$ , and horizontal area  $A$  are given.

The vertical forces on this mass element  $dm$  are produced by gravity( $g$ ) and pressure forces



**Figure 3.1:** Force diagram for stellar mass element [26].

( $P$ ) acting on the bottom and at the top ( $P + dP$ ) of the element. If the atmosphere is in hydrostatic equilibrium, there will be no vertical acceleration, and we can formulate an equation by requiring that the upward forces and downward forces are balanced. That means  $(P + dP)A + gdm = PA$ , after rearranging it  $dPA = -gdm$ . But the mass distribution in terms of mass density,  $r$  and  $dr$  inside the star at constant time is,  $dm = 4\pi r^2 \rho dr = \rho A dr$ . Using chain rule  $dPA$  gives  $dPA/dr$ . Then substitute  $dm$  in to  $dPA$

$$\frac{dP}{dr} + g\rho = 0 \quad (3.1)$$

Which gives the condition of hydrostatic equilibrium and the balance of force from pressure and gravity. Both are per unit volume of the thin shell. That is the equation of hydrostatic equilibrium on the atmosphere of the star. One dependent idea to hydrostatic equilibrium is ideal gas law.

### Ideal gas pressure

The gas pressure is just the product of number density and temperature, which is called a statement of the ideal gas law or EOS. Or, symbolically;

$$P_g = n\kappa T.$$

Where,  $\kappa$  is boltzman constant, and  $n = \rho/(\mu m_H)$  is molecular density number

or number of moles. Using the molecular density number the value of the gas pressure gives as,

$$P_g = \frac{\rho k T}{\mu m_H} = \frac{\mathfrak{R} \rho T}{\mu} \quad (3.2)$$

Using their constant values gas constant ( $\mathfrak{R}$ ) =  $8.315 \times 10^7 \text{ erg K}^{-1} \text{ g}^{-1} \text{ mole}^{-1}$ , and average particle mass in units of  $m_H$  ( $\mu$ ) = 0.03, the gas pressure interms of density and temperature gives as;

$$P_g = 2.495 \times 10^6 \text{ erg K}^{-1} \text{ g}^{-1} \text{ mole}^{-1} \rho T \quad (3.3)$$

This shows gas pressure depends on the molecular density of gas and temperature. When the stars cold gas pressure decrease, and as temperature hot gas pressure rapidly increases.

### 1. Mass Conservation equation

In star formation, the star comes to stable in which the force of gravity is balanced by the pressure gradient of the hot gas, as shown in fig. 3.1, i.e. radius, mass, and density are dependent. According to the assumption of spherical symmetry, all quantities describing the star depend only on the distance  $r$  from the center, or mass( $M_r$ ) and gravity( $g_r$ ) are changed as follows.

$$g(r) = \frac{GM_r}{r^2} \quad (3.4)$$

In spherical symmetry with thickness( $dr$ ), small mass( $dm$ ), volume( $Adr$ ) and density ( $\rho$ ), mass is conserved as,

$$dM_r = 4\pi\rho r^2 dr \quad (3.5)$$

These called the mass conservation equation, and it is the same equation that we review in section 2.3.1. Then at constant density, the total mass of medium mass stars in terms of total radius and density give as;

$$M = \frac{4\pi\rho R^3}{3} \quad (3.6)$$

This shows, in the middle of the main sequence phase star's volume is large as radius  $R$  is large.

### 2. Hydrostatically equilibrium

From hydrostatic equilibrium in stellar interior  $g$  varies with  $r$ . As shown on equation 3.4 acceleration due to gravity varies as  $g \approx 1/r^2$ . Then hydrostatic equilibrium equation is obtained by substituting equation 3.4 into equation 3.1,

$$\frac{dP}{dr} = -\frac{GM_r\rho(r)}{r^2} \quad (3.7)$$

This is hydrostatic equilibrium equations. From this equation we can obtain an estimate central pressure ( $P_c$ ) of a star with mass  $M$  and radius  $R$ . Using the approximations  $dP/dr \approx (P_{surface} - P_c)/R$ , replace  $dP/dr$  by  $P(M) - P_c$ , or by substituting equation 3.6 into equation 3.7 we obtain;

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4} \quad (3.8)$$

Then equation 3.7 concerning  $m$  gives, the pressure between the surface and core gives;

$$P(M) - P_c = -\int_0^M \frac{Gm}{4\pi r^4} dm \quad (3.9)$$

On the left-hand side,  $P(M)$  is the pressure at the star's surface, and  $P(c)$  is the pressure at center of the star. The surface pressure is tiny ( $P_{surface} = P(M) = 0$ ). So we can drop it. For the right-hand side, we know that  $r$  is always smaller than  $R$ , so  $Gm/4\pi r^4$  is always larger than  $Gm/4\pi R^4$  or  $dP_c/dm > Gm/4\pi r^4$ . Thus we can write the integral form of total pressure from the internal mass(0) to external mass.

$$P_c = \int_0^M \frac{Gm}{4\pi r^4} dm > \int_0^M \frac{Gm}{4\pi R^4} dm \quad (3.10)$$

Then pressure at the surface ( $R \sim \infty$ ) is zero, and  $r \sim R$ , central pressure with respect to outer radius( $R$ ) gives as.

$$P_c = \frac{GM^2}{8\pi R^4} \quad (3.11)$$

$$P_c = \frac{6.674 \times 10^{-8} g^{-1} cm^3 s^{-2} M^2}{8\pi R^4}$$

$$P_c = 4.4 \times 10^{13} \left(\frac{M}{M_\odot}\right)^2 \left(\frac{R_\odot}{R}\right)^4 Nm^{-2} \quad (3.12)$$

Where, gravitational force constant ( $G$ ) =  $6.674 \times 10^{-8} g^{-1} cm^3 s^{-2}$ ,  $M_\odot$  represented for  $2 \times 10^{33} g$ , and  $R_\odot$  represented for  $7 \times 10^{10} cm$ . Central pressure of the star is inversly proportion to their radius. Then, using the mechanical equilibrium of a star, we determine the average pressure within it. After substituting equation 3.4 in to the integral form of equation 3.1, average pressure give as;

$$P = \frac{3G M^2}{4\pi R^4} = \frac{3 \times 6.674 \times 10^{-8} g^{-1} cm^3 s^{-2} M^2}{4 \times 3.14 R^4}$$

$$P \approx 3 \times 10^{-8} \left(\frac{M^2}{R^4}\right) Nm^{-2} \quad (3.13)$$

### 3. Derivation of Radiative diffusion equation

The intermediate-mass stars have photons with frequency( $\nu$ ) in the radiation field. That is symbolized by a physical quantity  $I_\nu$  called specific intensity.

Specific intensity is the directional value of the radiation field. It is defined by per unit of solid angle and radiative transfer. Specific intensity with incident angle  $I(\theta)$  corresponds to the energy density  $v(\theta)d\omega$  and light passing through a solid angle of  $d\omega$ . A solid angle is the two-dimensional equivalent angle. Its unit is steradians( $sr$ ), and which defines the direction of intensity. Spherical the angle subtended between  $\theta$  and  $\theta + d\theta$ , and  $\varphi$  and  $\varphi + d\varphi$ .

Then the radiative energy flux is  $I(\theta)d\omega$  in ( $erg/cm^2s$ ).  $\theta$  is colatitude angle(in spherical coordinates) at some position  $r$  or, in plane-parallel geometry to  $z$ . This is related to  $I(\theta)$  by considering how much radiant energy contained in a tube of unit cross-section and length  $1sec \times c$  in the direction  $\theta$ . Thus,

$$v(\theta)d\omega = \frac{I(\theta)}{c}d\omega \quad (3.14)$$

By integrating over  $4\pi sr$  we get total energy density  $U$ ,

$$U = \int_{4\pi} v(\theta)d\omega = \frac{2\pi}{c} \int_{-1}^1 I(\mu)d\mu \quad (3.15)$$

Where azimuthal symmetry (for around star or flat plane) in  $\varphi$  has been assumed, and  $\mu = \cos\theta$  with  $-1 \leq \mu \leq 1$ . It represents a unit area ( $1cm^2$ ) perpendicular to the  $z$ -direction. The projection of  $I(\theta)$  onto this area is  $I(\theta) \cos\theta \times 1cm^2$ .

If we integrat this over  $d\omega$  and divide the result by  $1cm^2$ , total flux in the  $z$ -direction gives,

$$F = \int_{4\pi} I(\theta)\cos\theta d\omega = 2\pi \int_{-1}^1 I(\mu)\mu d\mu \quad (3.16)$$

At the same amount of radiation comes as goes out,  $I$  is constant. Then total flux is zero. Hence  $I$  must vary with  $\mu$  or  $\theta$ , radiant energy to be transported. Then,  $I(\theta)$  must be anisotropic. At any location  $z$  or  $r$ ,  $I(\theta)$  defined by radiation being scattered from other directions into  $\theta$  or by direct emission from local atoms. Then the atom emits light spontaneously, without any particular provocation. We can account this process by defining the spontaneous emission coefficient  $j(\theta)$ . As light emit, the energy emitted per unit volume per unit time per frequency interval per solid angle is constructed. If the direction along  $I(\theta)$  over which  $I(\theta)$  is augmented by the amount  $dI(\theta)$  due to scattering or emission,  $dI(\theta)$  interms of emission( $j(\theta)$ ) defined by;

$$dI(\theta) = j(\theta)\rho ds \quad (3.17)$$

Those processes are accounted by the opacity (or mass absorption coefficient( $\kappa$ )). So that the amount of intensity removed from  $I(\theta)$  takes out;

$$dI(\theta) = -\kappa\rho I(\theta)ds \quad (3.18)$$

For absence of emission, on equation 3.17  $j(\theta)$  is zero,  $\kappa$  and  $\rho$  are both constant, and equation 3.18 gives  $\frac{dI(\theta)}{I(\theta)} = -\kappa\rho ds$ . Exponentialy it forms as  $I \propto e^{-\kappa\rho s}$ . Here the negative sign signifies that the intensity decreases as  $s$  increases, and as the ray travels deeper into the matter. This relation defines an absorption coefficient with frequency  $\kappa_\nu$ . The product

$\rho\kappa$  is typically mean free path and net change in  $I(\theta)$  per unit path length is the sum of equation 3.17 and equation 3.18.

$$\frac{1}{\rho} \frac{dI(\theta)}{ds} = j - \kappa I(\theta) \quad (3.19)$$

It is a radiative transfer form holds in planar geometry. If local thermodynamic equilibrium(LTE) complete isotropy and spatial uniformity of the radiation field form. Then  $dI/ds = 0$  and  $I = j/\kappa$  would be constant with no radiant energy transported. In that case the energy density of the radiation field  $U$  on equation 3.15 is  $4\pi/cI$  which call as radiative energy  $E_{rad} = \alpha T^4$ . Then, the intensity in terms of opacity and mean intensity gives as;

$$I = \frac{j}{\kappa} = \frac{c}{4\pi} \alpha T^4 = \frac{\sigma}{\pi} T^4 = B(T) \quad (3.20)$$

Where,  $B(T)$  is Plunck function (in  $erg\,cm^{-2}$ ), that depend on frequency;

$$B(T) = \frac{2h\nu^3}{c^2} \left[ \frac{1}{e^{(h\nu)/(\kappa T)} - 1} \right] \quad (3.21)$$

Where,  $h$  and  $T$  are Planck constant and temperature. The radiation field is nearly isotropic, then the intensity should closely resemble  $B(T)$ . At this point we introduce frequency  $\nu$  into all expressions and realize that quantities such as  $I$ ,  $j$  and  $\kappa$  must all depend on  $\nu$ . So, that we can talk about photons of a given frequency being added to or substracted from a beam in direction  $\theta$ . In addition, we introduce the source function( $S_\nu$ ) (which, is a computational device),

$$S_\nu = \frac{j_\nu}{\kappa_\nu} \quad (3.22)$$

The source function  $S_\nu$  is the property of the matter in its particular thermodynamic state. There is also optical depth  $\tau_\nu$  and small optical depth with  $r$   $d\tau_\nu(r) = -\kappa_\nu \rho dr$  integrate it from partial refference level to its surface ( $r_0$  to  $r$ )

$$\tau_\nu(r) = \tau_{\nu,0} - \int_{r_0}^r \kappa_\nu \rho dr \quad (3.23)$$

Where  $r_0$  is some spatial reference level,  $\tau_{\nu,0}$  is the optical depth evaluated at that level, and  $\tau_\nu(r)$  is depth from the surface in curious (dimensionless).

If  $r_0$  corresponds to the true surface of the star, where, density and pressure presumably go to zero. Then  $\tau_{\nu,0}$  is taken to be zero, and equation 3.18 gives as,

$$\begin{aligned} \frac{dI(\nu)}{I(\nu)} &= -\kappa \rho ds \\ \ln I(\nu) - \ln I(0) &= - \int_0^r \kappa_\nu \rho dr \\ I_\nu &= e^{\tau_\nu} \end{aligned} \quad (3.24)$$

If we recall  $ds$  is measured on the direction of  $I(\theta)$ , then  $dr = \cos\theta ds = \mu ds$ . By combining the equations 3.19, 3.22 and 3.23 the radiative transfer equation become;

$$\mu \left( \frac{dI_\nu(\tau, \mu)}{d\tau_\nu} \right) = I_\nu(\tau, \mu) - S_\nu(\tau, \mu) \quad (3.25)$$

The special case to consider is that with no emission, which we can obtain by setting  $j_\nu$  equal to zero then  $S_\nu$  also zero. The transfer equation simplifies as  $\exp(-\tau/\mu)$ . Thus multiply the equation 3.25 through by the factor;

$$e^{-\tau/\mu} \left( \frac{dI_\nu(\tau, \mu)}{d\tau_\nu} \right) = e^{-\tau/\mu} \frac{I_\nu(\tau, \mu)}{\mu} - e^{-\tau/\mu} \frac{S_\nu(\tau, \mu)}{\mu} \quad (3.26)$$

Here the subscript  $\nu$  and the arguments  $\theta$  or  $\mu$  will be deleted for visual clarity. The recognized perfect differential form is;

$$\frac{d}{d\tau} \left( e^{-\tau/\mu} I \right) = (e^{-\tau/\mu}) \frac{S}{\mu} \quad (3.27)$$

Integrate from some reference level, initial optical depth  $\tau_0$  to a general level  $\tau$ ,

$$I(\tau, \mu) = \left( e^{-\frac{(\tau_0 - \tau)}{\mu}} \right) I(\tau_0, \mu) + \int_\tau^{\tau_0} \left( e^{-\frac{(t - \tau)}{\mu}} \right) \frac{S(t)}{\mu} dt \quad (3.28)$$

Where  $t$  is a dummy integration variable. Depending on the range of  $\mu$  (or  $\theta$ ), different values for  $\tau_0$  are chosen for  $I(\tau, \mu)$ . For forward-directed radiation (heading out toward the surface) with  $\mu \geq 0$  and ( $0 \leq \theta \leq \pi/2$ ), choose  $\tau_0$  to be very large. Then the equation of radiative transfer becomes

$$I(\tau, \mu \geq 0) = \int_\tau^\infty e^{-\frac{(t - \tau)}{\mu}} \frac{S(t)}{\mu} dt \quad (3.29)$$

For inward direction radiation  $\mu < 0$  and  $\tau_0 = 0$  for the true surface of the star,

$$I(\tau, \mu < 0) = \int_\tau^0 e^{-\frac{(t - \tau)}{\mu}} \frac{S(t)}{\mu} dt \quad (3.30)$$

In the last expression, advantage has been taken of the fact that the level  $\tau_0 = 0$

for  $\tau = 0$  and  $I(0, \mu < 0) = 0$  this shows, the surface can't gain incoming radiation. If the deep interior is to be nearly in LTE, we expect the source function  $S = j/\kappa$  to be almost independent of directional angle, and equation 3.15 to be near its Planckian value  $B(T)$  at depth  $\tau$ . Then the first order Taylor series of  $S(t)$  in  $\tau$  is;

$$S(t) = B(\tau) + (t - \tau) \left( \frac{\partial B}{\partial \tau} \right)_\tau \quad (3.31)$$

Where  $B(\tau)$  stands for  $B(T_\tau)$ . Substitute equation 3.21 in to the difference of equation 3.30 and 3.29 then

$$I(\tau, \mu \geq 0) = B(\tau) + \mu \left( \frac{\partial B}{\partial \tau} \right)_\tau$$

$$I(\tau, \mu < 0) = B(\tau)[1 - e^{\tau/\mu}] + \mu \left( \frac{\partial B}{\partial \tau} \right)_\tau [e^{\tau/\mu}(\tau/\mu - 1) + 1] \quad (3.32)$$

Since  $\mu < 0$  in equation 3.30  $e^{\tau/\mu}$  is neglect for  $\tau$  large and eqn.3.29 is valid for all  $\mu$ . In case of the presence of the factor  $\mu$  in above equations, the outwardly directed intensity (with  $\mu \geq 0$ ) is enhanced over  $B(\tau)$  and the intensity directed inward (with  $\mu \leq 0$ ) is reduced. The net result is a flow of radiation outward, and to compute the flux of this radiation we use equation 3.16

$$F(\tau) = 2\pi \int_{-1}^1 \left( B(\tau) + \mu \frac{\partial B(\tau)}{\partial \tau} \right) \mu d\mu = \frac{4\pi}{3} \frac{\partial B(\tau)}{\partial \tau} \quad (3.33)$$

Its unit is  $ergcm^{-2}s^{-1}$ , this shows Planck function directly related with  $T^4$  (as shown on equation 3.20), and the amount of energy flux carried by radiation depends only on how rapidly temperature varies with optical depth. Then the luminosity for a spherical star at radius  $r(\tau)$  is

$$L_r = 4\pi r^2 F(r) \quad (3.34)$$

The flux with frequency dependence ( $F_\nu$ ) as shown on equation 3.33 is

$$F_\nu = \frac{4\pi}{3} \frac{\partial B_\nu(\tau)}{\partial \tau_\nu}$$

which may be rewritten using the definition of  $d\tau_\nu$  as  $-\kappa_\nu \rho dr$ , then frequency dependent flux interms of density and opacity is

$$F_\nu = -\frac{4\pi}{3} \frac{\partial B_\nu(\tau)}{\kappa_\nu \rho dr} \quad (3.35)$$

The derivative of  $B_\nu(\tau)$  is cast into more a convenient form by using the chain rule, so that;

$$\frac{dB_\nu}{dr} = \frac{\partial B_\nu}{\partial T} \frac{dT}{dr} \quad (3.36)$$

$F_\nu$  is also integral of  $F_\nu d\nu$ . So, the integral form of equation 3.35 over frequency is;

$$\int_0^\infty F_\nu d\nu = -\frac{4\pi}{3\rho} \frac{dT}{dr} \int_0^\infty \frac{1}{\kappa_\nu} \frac{dB_\nu(T)}{dT} d\nu = -\frac{4\pi dT}{3\rho dr} \int_0^\infty \frac{dB_\nu(T)}{\kappa_\nu dT} d\nu \quad (3.37)$$

but

$$\int_0^\infty \frac{1}{\kappa_\nu} \frac{dB_\nu(T)}{dT} d\nu = \frac{1}{\bar{\kappa}} \int_0^\infty \frac{dB_\nu(T)}{dT} d\nu. \quad (3.38)$$

Where  $\bar{\kappa} = \kappa_{ff} + \kappa_{bf} + \kappa_e$  is the average opacity with the sum of free-free opacity, bound-free opacity and electron opacity, respectively. Then by substituting equation 3.20 in to the derivative of equation 3.38, we get;

$$\int_0^\infty \frac{dB\nu(T)}{dT} d\nu = \frac{d}{dT} \int_0^\infty B\nu(T) d\nu = \frac{dB(T)}{dT} = \frac{d}{dT} \frac{\sigma T^4}{\pi} = \frac{4\sigma T^3}{\pi} \quad (3.39)$$

By inserting this equation into equation 3.35 then  $F_\nu \sim F_r$

$$F_r = -\frac{16\sigma T^3}{3\rho\bar{\kappa}} \frac{dT}{dr} \quad (3.40)$$

By substituting this equation in to equation 3.34 of the value  $F_r$  it gives,

$$\frac{dT}{dr} = -\frac{3\bar{\kappa}\rho L_r}{16\alpha c\pi T^3 r^2} \quad (3.41)$$

This equation is the third stellar structure equation, commonly called the radiative energy transport equation or radiative temperature gradient. It shows how temperature change with radius in the region. The integral form of this at  $L_r \approx L$  with the boundary 0 to  $r$ .

$$\int_0^r dT = -\int_0^r \frac{3\bar{\kappa}\rho}{4\alpha c\pi T^3} \frac{L_r}{4\pi r^2} dr \quad (3.42)$$

$$T = \left( \frac{9\bar{\kappa}\rho L}{16\alpha c\pi r^3} \right)^{1/4} \quad (3.43)$$

Where,  $\alpha$ , and  $\bar{\kappa}$  are the Planck constant, and average opacity.

#### 4. Derivation of convectonal Energy transport equation

As energy and chemical elements of stars transport, the large-scale motions of matter in the stellar interior organized. Using the first law of thermodynamics as shown bellow,

$$PV = n\Re T$$

The ideal gase constant,  $\Re = C_p - C_v$ . Where,  $C_p$  is specific heat at constant pressure and  $C_v$  specific heat at constant volume. The ratio of  $C_p$  and  $C_v$  is the polytropic constant ( $C_p/C_v = \gamma$ ).

Then differentiate the ideal gas law with respect to temperature(T) and using chaine rule,

$$\Re = \frac{PdV}{dT} + \frac{VdP}{dT}$$

By substituting the specific heat into this equation, the gas pressure in terms of temperature gives;

$$\begin{aligned}
\frac{dT}{T} &= \left( \frac{C_p - C_v}{C_p} \right) \frac{dP}{P} \equiv \left( \frac{\gamma - 1}{\gamma} \right) \frac{dP}{P} \\
d \ln T &= \left( \frac{\gamma - 1}{\gamma} \right) d \ln P \\
d \ln P &= \frac{\gamma}{\gamma - 1} (d \ln T) \\
P &= T^{(\gamma)/(\gamma-1)}
\end{aligned} \tag{3.44}$$

This shows pressure is depend on temperature. Then by substituiting equation 3.2 into 3.44

$$\begin{aligned}
P &= \left( \frac{P\mu}{\rho\mathfrak{R}} \right)^{(\gamma)/(\gamma-1)} \equiv \left( \frac{\mathfrak{R}}{\mu} \right) \rho^{(\gamma-1)/(\gamma)} \\
P &= K\rho^{(1+1/n)}
\end{aligned} \tag{3.45}$$

where,  $K$  is constant and  $n$  is the polytropic index, and  $\gamma$  interms of polytropic index,  $\gamma = (n + 1)/n$  and  $(\gamma - 1) = 1/n$  the ratio of  $\gamma - 1$  and  $\gamma$  is  $1/(n + 1)$ . It is adiabatic gradiant, which call as EOS. As pressure depends on density, the EOS is related to the hydrostatic equation i.e. it is related to the integral form of equation 3.7. So,

$$P = \frac{2GM\rho}{R^3} = K\rho^{(1+1/n)} \tag{3.46}$$

Then, mass interms of density and polytropic index.

$$M = \frac{KR^3}{2G}\rho^{(1/n)} \tag{3.47}$$

At high temperature polytropic index is large. But, in the case of completely degenerate electron gas or a nonrelativistic ideal gas the adiabatic exponent  $\gamma$  is less than one[45].

$$M = K \left( \frac{R^3}{2G} \right) = \left( \frac{\mathfrak{R}}{2\mu G} \right) R^3 \tag{3.48}$$

The constant  $K \sim \mathfrak{R}/\mu$ , this equation is the mass to radius relation equation. The equation of state (EOS) as shown on equation 3.2. Using the mass conservation equation  $P_g$  gives as,

$$P_g = \frac{\mathfrak{R}\rho T}{\mu} = \frac{\mathfrak{R}}{\mu} \left( \frac{3MT}{4\pi R^3} \right) \tag{3.49}$$

This equation shows, gas pressure is increase as mass and temperature increase. Using the star's fluid property, the pressure on mechanical equilibrium is similar to the ideal gas law ( $P_g = P$ ). Then we substitute equation 3.13 in to equation 3.49;

$$\frac{3G}{4\pi} \frac{M^2}{R^4} = \frac{\mathfrak{R}}{\mu} \left( \frac{3MT}{4\pi R^3} \right) \tag{3.50}$$

divide both side by  $(4\pi\mu R^3/3\Re M)$ . Then, the temperature interms of mass and radius gives as,

$$T = \frac{\mu G}{\Re} \left( \frac{M}{R} \right) \quad (3.51)$$

This is the temperature required to ballance the mechanical pressure and gas pressure. By substituiting equation 3.43 in to equation 3.51

$$\begin{aligned} \left[ \frac{\mu G}{\Re} \left( \frac{M}{R} \right) \right]^4 &= \left( \frac{6\bar{\kappa}\rho L}{16ac\pi R^3} \right) \\ \left[ \frac{\mu G}{\Re} \left( \frac{M}{R} \right) \right]^4 &= \left( \frac{6\bar{\kappa}L}{16ac\pi R^3} \right) \left( \frac{3M}{4\pi R^3} \right) \\ \left[ \frac{\mu G}{\Re} \right]^4 (M^3 R^2) &= \left( \frac{18}{16(4\pi)^2} \right) \left( \frac{\bar{\kappa}L}{\sigma} \right) \end{aligned} \quad (3.52)$$

By substituiting the equation 3.48 (R interms of M) into equation 3.52 luminosity interms of mass gives,

$$\begin{aligned} \left[ \frac{\mu G}{\Re} \right]^4 (M^3) \left( \frac{2\mu GM}{\Re} \right)^{2/3} &= \left( \frac{18}{16(4\pi)^2} \right) \left( \frac{\bar{\kappa}L}{\sigma} \right) \\ L &= (2)^{2/3} \left( \frac{(16)^2}{18} \right) \left( \frac{(\pi)^2 \sigma}{\bar{\kappa}} \right) \left[ \frac{\mu G}{\Re} \right]^{14/3} M^{11/3} \end{aligned} \quad (3.53)$$

Using the constant values, we may carry this further by assuming the opacity due to primarily, Thomson scattering, which is  $\kappa_e = 0.2(1+x) = \bar{\kappa} = 0.34$  (at  $x = 0.7$ ),  $G = 6.674 \times 10^{-8} g^{-1} cm^3 s^{-2}$  gravitational constant,  $\Re = 8.3 \times 10^3 Jg^{-1} mole^{-1} K^{-1}$  ideal gas constant,  $\mu = 0.03/mole$  mean molecular weight, and  $\alpha = 4\sigma/c$ ,  $\sigma = 5.67 \times 10^{-5} ergm^{-2} K^{-4} s^{-1}$ .

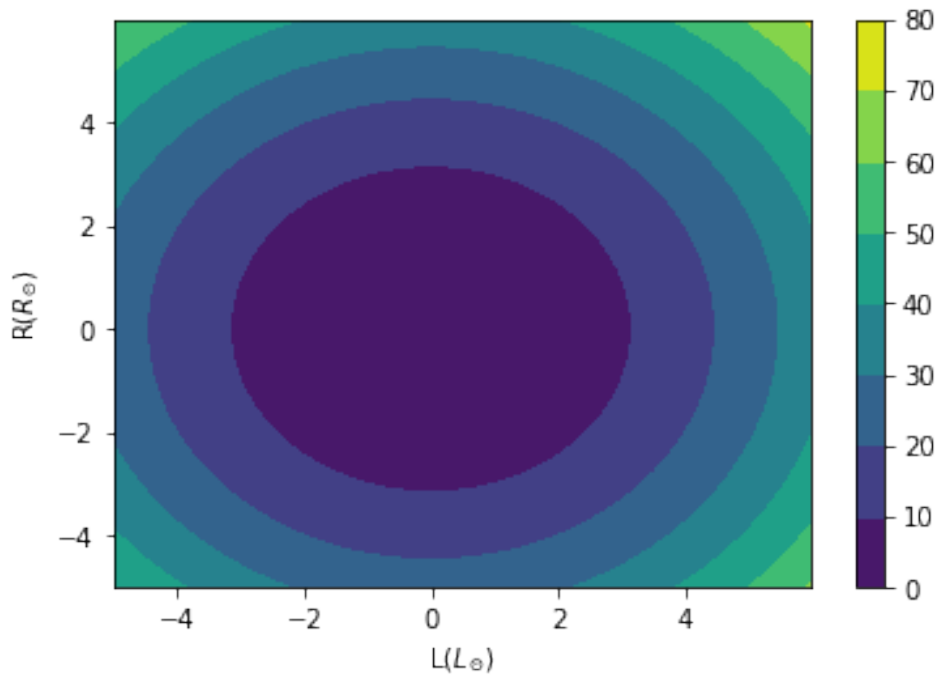
$$\begin{aligned} L &= \left( \frac{(1.587)(256)(9.8569)\sigma}{18 \times 0.34} \right) \left[ \frac{\mu G}{\Re} \right]^{14/3} M^{11/3} \\ L &= \left( \frac{(5105.27768 \times 10^{-8})(5.67 \times 10^{-5} ergm^{-2} K^{-4} s^{-1})}{6.120} \right) \left[ \frac{6.674 \times 10^{-8} g^{-1} cm^3 s^{-2}}{8.3 \times 10^7 ergg^{-1} K^{-1}} \right]^{14/3} M^{11/3} \\ &= 2.894 \times 10^{-9} ergm^{-2} K^{-4} s^{-1} \left[ 0.804 \times 10^{-15} \left( \frac{cm^3 K}{ergs^2} \right) \right]^{14/3} M^{11/3} \\ &= (2.894 \times 10^{-9})(0.804 \times 10^{-15})^{14/3} \left[ ergcm^{-2} K^{-4} s^{-1} \left( \frac{cm^3 K}{ergs^2} \right) \right]^{14/3} M^{11/3} \\ &= 2.3272 \times 10^{-79} \left[ cm^{-2} \left( \frac{s^3}{gcm^2} \right)^{11/3} s^{-14} \right] M^{11/3} \end{aligned}$$

$$\begin{aligned}
&= 2.3272 \times 10^{-79} \text{cm}^2 \text{gs}^{-3} \left(\frac{M}{g}\right)^{11/3} \\
&= 2.3272 \times 10^{-79} \left(\frac{L_{\odot}}{3.8 \times 10^{33}}\right) \left(\frac{2 \times 10^{33} M}{M_{\odot}}\right)^{11/3} \\
L &\approx (12.248 \times 10^8) L_{\odot} \left[\frac{M}{M_{\odot}}\right]^{3.66} \tag{3.54}
\end{aligned}$$

This equation is the mass-luminosity relation of intermediate-mass stars in the main sequence phase.

### 3.3 Intermediate mass stars lifetime in H and He burning phases

As shown in section 2.5.2, the age of medium mass stars on MS depends on the chemical conversion from hydrogen to helium. This conversion also came from the nuclear burning, as shown in below figure 3.2.



**Figure 3.2:** Nuclear burning. The blue-black color is the core of helium after hydrogen shell burn, the blue color the burn hydrogen shell, and the outer circles sims like water wave the burning of gas and changed to the next heavier element. The vertical bar is the color bar

The color bar shows as the rate of energy increase density are decrease and color of gas change to bright. Because, as the core burn nuclear fuel at a particular equilibrium rate,

the burning rate perturbed by some amount of energy [28]. Then the energy produced in the core increase, and it causes the expansion of the core as the core expanded potential energy reduces, and it will cool since the rate of burning depends on temperature, the burning factor, and the radius of stars.

Depend on the core and shell burning of hydrogen and helium, we divide the star's structure into three nuclear burning phases like  $H_{core}$  burning (phase A),  $H_{shell}$  burning (phase B), and  $He_{core}$  burning (phase C). Nuclear energy determines using the Einstein energy equation as follows;

$$E = fMc^2 \quad (3.55)$$

where  $E$ ,  $f$ ,  $M$ , and  $C$  are the total energy released by star, fraction of mass convert to energy, the mass of the star, and speed of light, respectively. But, the rate of energy is luminosity. So, lifetime interms of luminosity gives as;

$$t = \frac{fMc^2}{L} \quad (3.56)$$

But, using  $f = \phi f_{nuc}$ , where  $\phi$  is the small fraction of rest mass of the reacting nuclei into energy with the value 0.001, and  $f_{nuc}$  is the fraction of the mass of the star that serves as nuclear fuel, and for thermal equilibrium the nuclear timescale calculated as;

$$\begin{aligned} t &= \frac{\phi f_{nuc} Mc^2}{L} = \frac{(0.001 \times 0.007)M(3 \times 10^{10} \text{ cm/s})^2}{L} \\ t &= (63 \times 10^{14} \text{ cm}^2/\text{s}^2) \frac{M}{L} \\ t &= \left[ 63 \times 10^{14} \left( \frac{L_{\odot}}{3.826 \times 10^{33}} \right) \left( \frac{2 \times 10^{33}}{M_{\odot}} \right) \frac{M}{L} \right] \\ t &\approx 10^{10} \text{ yr} \frac{M}{M_{\odot}} \frac{L_{\odot}}{L} \end{aligned} \quad (3.57)$$

We have used  $\text{cm}^2/\text{s}^2 = L_{\odot} s / 3.826 \times 10^{33} g$ ,  $g = M_{\odot} / (2 \times 10^{33})$ , and  $s = 3.1556952 \times 10^{-7} \text{ yr}$ . In main sequence stars for fusion of hydrogen, helium and heavier elements  $\phi$  is very small and  $f \approx f_{nuc}$  [51]. Then by substituiting equation 3.54 into equation 3.56, the lifetime of main sequence star gives as;

$$\begin{aligned} t &= \left[ \frac{\phi f_{nuc} Mc^2}{12.248 \times 10^8 L_{\odot} (M/M_{\odot})^{3.66}} \right] = \left[ \frac{(9 \times 10^{20} \text{ cm}^2 \text{ s}^{-2}) 7 \times 10^{-3}}{12.248 \times 10^8 \times 3.826 \times 10^{33} \text{ g cm}^2 \text{ s}^{-3}} \right] \left( \frac{M}{(M/M_{\odot})^{3.66}} \right) \\ &= 2.518 \times 10^{-20} \text{ s} \left[ \frac{2 \times 10^{33}}{M_{\odot}} \right] \left( \frac{M}{(M/M_{\odot})^{3.66}} \right) = (5.14369 \times 10^{16}) (3.155695 \times 10^{-7} \text{ yr}) \left( \frac{M/M_{\odot}}{(M/M_{\odot})^{3.66}} \right) \\ t &= 16.231938684 \times 10^9 \text{ yr} \left( \frac{(M/M_{\odot})}{(M/M_{\odot})^{3.66}} \right) \approx 10^{10} \text{ yr} \left( \frac{M}{M_{\odot}} \right)^{-2.66} \end{aligned} \quad (3.58)$$

We have used,  $gcm^2s^{-2} = L_{\odot}/3.847 \times 10^{33}$  and the fractional factor nuclear burning ( $f_{nuc} = 0.007$ ). Where  $10^{10}yr$  is the solar lifetime,  $t$  is the lifetime of the stars on the main sequence ( $t_{ms}$ ), and  $M$  is the total mass of the stars. By substituting equation 3.48, 3.54 and 3.56 into equation 3.58, we determine the temperature to time, radius to time and luminosity to time relation equations as follows.

$$\begin{aligned}
t &\approx 10^{10}yr \left( \frac{\mathfrak{R}}{2\mu GM_{\odot}} \right)^{-2.66} R^{-8} \\
&\approx 10^{10}yr \left[ \frac{8.3 \times 10^7 ergK^{-1}g^{-1}}{4(0.34 \times 6.674 \times 10^{-8}g^{-1}cm^3s^{-2} \times 10^{33}g)} \right]^{-2.66} R^{-8} \\
&\approx 10^{10}yr \left[ \frac{8.3 \times 10^3 gcm^2s^{-2}g^{-1}}{90.766 \times 10^{25}g^{-1}cm^3s^{-2}g} \right]^{-2.66} R^{-8} \\
&\approx 10^{10}yr \left[ \frac{0.64 \times 10^{-12}}{R_{\odot}} \right]^{-2.66} R^{-8} \\
t &\approx (10.444)^{-40}yr \left( \frac{R}{R_{\odot}} \right)^{-8} \tag{3.59}
\end{aligned}$$

This is the main sequence stars time in terms of radius that calculated by substituting equation 3.48 into equation 3.59. By substituting equation 3.51 into 3.59 the lifetime of main-sequence intermediate-mass stars with temperature is obtained;

$$\begin{aligned}
t &\approx 10^{10}yr \left( \frac{\mathfrak{R}}{\mu GM_{\odot}} \left( \frac{T^3}{2} \right)^{1/2} \right)^{-2.66} \\
&\approx 10^{10}yr \left[ 2.419 \times 10^{-18} \left( \frac{T}{T_{\odot}} \right)^{3/2} \right]^{-2.66} \\
t &\approx (2.05)^{38}yr \left( \frac{T}{T_{\odot}} \right)^{-3.99} \tag{3.60}
\end{aligned}$$

This is the duration for nuclear burning main sequence intermediate-mass stars with a given temperature.

$$\begin{aligned}
t &\approx 10^{10}yr \left[ \left( \frac{1}{5.749 \times 10^{-8}M_{\odot}} \right)^{3/11} \left( \frac{L}{L_{\odot}} \right)^{3/11} \right]^{-2.66} \\
t &\approx 10^{10}yr \left[ \left( \frac{1}{5.749 \times 10^{-8}M_{\odot}} \right)^{3/11} \left( \frac{L}{L_{\odot}} \right)^{3/11} \right]^{-2.66} \\
t &\approx (0.1739)^{-8.545}yr \left( \frac{L}{L_{\odot}} \right)^{-0.727} \tag{3.61}
\end{aligned}$$

This shows the time required for the rate of energy generated in the nuclear burning of main-sequence stars.

To determine the age of the main sequence medium mass stars, we divide into three phases depending on hydrogen and helium fusion. For each phase, we calculate the age of the stars. Hydrogen burning is the fusion of four protons into single protons(2 protons + 2 neutrons) which, is a helium nucleus. This fusion process is a series of reactions.

The reactions depend on mass, core temperature, and density. In intermediate-mass stars, there are different sequences of nuclear reactions, which are called the fuse of hydrogen into helium. As shown in equation 3.56, the main sequence star's nuclear lifetime is the ratio of the amount of nuclear energy available to the rate of energy use.

### i. Phase A (Hydrogen core burning)

The first main hydrostatic burning phase of the star is Hydrogen burning. The reaction is from hydrogen to helium which gives as;



from this reaction, the mass fraction energy change of hydrogen core is calculated, as using the Einstien equation:

$$\frac{\Delta E}{E} = \frac{4m_H c^2 - m_{He} c^2}{4m_H c^2} \approx 0.007 \quad (3.63)$$

Where,  $m_H = 1.6605 \times 10^{-24}g$  mass of proton, and  $m_{He} = 6.6463173 \times 10^{-24}g$  mass of helium atom. This is the nuclear fusing from H to He, which liberates roughly 0.7% of the available mass-energy. Nuclear burning lifetime interms of mass(M)

$$\begin{aligned} t_{nuc} &= \frac{M f_{H,core} c^2}{L} = \frac{0.007 \times (3 \times 10^8 m/s)^2 M}{(12.248 \times 10^8) L_{\odot} \left[ \frac{M}{M_{\odot}} \right]^{3.66}} \\ &= \frac{0.014 \times 10^{41} s}{3.8 \times 10^{29}} \left( \frac{M}{M_{\odot}} \right) \left[ \frac{M_{\odot}}{M} \right]^{3.66} \\ t_{nuc} &= 2.708 \times 10^9 s \left[ \frac{M}{M_{\odot}} \right]^{-2.66} \end{aligned} \quad (3.64)$$

After rearranging equation 3.54 (mass interms luminosity), nuclear lifetime interms of luminosity gives as;

$$\begin{aligned} t_{nuc} &= \frac{M f_{H,core} c^2}{L} = \left[ \frac{0.007 \times (3 \times 10^8 m/s)^2 M_{\odot}}{L} \right] \left( \frac{L}{12.248 \times 10^8 L_{\odot}} \right)^{3/11} \\ &= \left[ \frac{(0.014)(3 \times 10^8)^2 (10^{37}) cm^2 gs^{-2}}{(12.248 \times 10^8)^{3/11}} \right] \left( \frac{L^{-8}}{L_{\odot}^3} \right)^{1/11} \end{aligned}$$

$$\begin{aligned}
&= \left[ \frac{0.126 \times 10^{53}}{(12.248 \times 10^8)^{3/11}} \frac{L_{\odot}}{3.8 \times 10^{33}} \right] \left( \frac{L^{-8}}{L_{\odot}^3} \right)^{1/11} \\
t_{nuc} &= (0.027)^{13/11} s \left( \frac{L}{L_{\odot}} \right)^{-8/11} \tag{3.65}
\end{aligned}$$

Then, mass and luminosity relation in terms of each star's age at hydrogen core burning calculated as;

$$\begin{aligned}
t_{nuc} &= \frac{M f_{H,core} c^2}{L} = \frac{0.007 \times (3 \times 10^8 m/s)^2 M}{L} \\
&= 63 \times 10^{13} s \left( \frac{L_{\odot}}{3.847 \times 10^{23}} \right) \left( \frac{2 \times 10^{33}}{M_{\odot}} \right) \left( \frac{M}{L} \right) \\
t_{nuc} &\approx 10.336 \times 10^{13} yr \left( \frac{M}{M_{\odot}} \right) \left( \frac{L_{\odot}}{L} \right) \tag{3.66}
\end{aligned}$$

Where,  $m^2/s^2 = (L_{\odot}/3.847) \times 10^{-23} g^{-1} s^{-1}$ ,  $g = (2 \times 10^{33})/(M_{\odot})$ , and  $s = 3.1556952 \times 10^{-7} yr$

By substituting the Stephan Boltzman equation and 3.51 into equation 3.56, we obtain the mass to radius relation in terms of each star's age at hydrogen core burning. i.e.

$$\begin{aligned}
t_{nuc} &= \frac{M f_{H,core} c^2}{L} = 63 \times 10^{13} m^2/s^2 \left[ \frac{M}{4\pi\sigma R^2 (12.248 \times 10^9 K (M/R))^4} \right] \\
t_{nuc} &= \left[ \left( \frac{63 \times 10^{13} m^2/s^2}{7.1215 erg/cm^2 s (12.248 \times 10^8)^4} \right) \left( \frac{R^2}{M^3} \right) \right] \\
t_{nuc} &\approx 0.471 yr \left( \frac{R}{R_{\odot}} \right)^2 \left( \frac{M_{\odot}}{M} \right)^3 \tag{3.67}
\end{aligned}$$

Where,  $\sigma = 5.67 \times 10^{-5} erg/cm^2 s K^4$ , then replace  $6.96 \times 10^8 m$  with  $R_{\odot}$  and  $2 \times 10^{30} kg$  with  $M_{\odot}$ .

Then by substituting equation 3.48 in to equation 3.65 nuclear burning lifetime interms of radius gives as;

$$\begin{aligned}
t_{nuc} &\approx 0.471 yr \left( \frac{R}{R_{\odot}} \right)^2 \left( \frac{2\mu G M_{\odot}}{\mathfrak{R} R^3} \right)^3 \\
&\approx 0.471 yr \left( \frac{R}{R_{\odot}} \right)^2 \left[ \frac{M_{\odot}}{5.1875 \times 10^{13} M_{\odot}} \left( \frac{R_{\odot}}{R} \right)^3 \right]^3 \\
&\approx \frac{0.471 yr}{5.1875 \times 10^{39}} \left( \frac{R_{\odot}}{R} \right)^3 \\
t_{nuc} &\approx 0.09 \times 10^{-39} yr \left( \frac{R_{\odot}}{R} \right)^3 \tag{3.68}
\end{aligned}$$

To determine mass to the temperature relation in terms of each star's age at hydrogen core burning, we substitute Stephan Boltzmann equation and equation 3.48 into equation 3.56

$$\begin{aligned}
t_{nuc} &= \frac{M f_{H,core} c^2}{L} = 63 \times 10^{13} m^2/s^2 \left[ \frac{M}{4\pi\sigma(M^{1/3})^2 T^4} \right] \\
&= \left( \frac{63 \times 10^{13} m^2/s^2}{672.6833 \times 10^{48} erg/K^4 s} \right) \left[ \frac{M}{(M/M_\odot)^{2/3} T^4} \right] \\
&= (0.0936 \times 10^{-33} (cm/s)^2 / gcm^2 / K^4 s^3) \left[ \frac{M}{(M/M_\odot)^{2/3} T^4} \right] \\
&= 0.0936 \times 10^{-33} s \left( \frac{2 \times 10^{33}}{M_\odot} \right) \left( \frac{T_\odot}{5.8 \times 10^3} \right)^4 \left[ \frac{M}{(M/M_\odot)^{2/3} T^4} \right] \\
t_{nuc} &\approx 4.155 yr \left( \frac{M}{M_\odot} \right)^{1/3} \left( \frac{T_\odot}{T} \right)^4 \tag{3.69}
\end{aligned}$$

Where,  $K^4 = (T_\odot/5.8 \times 10^3)^4$ , and  $erg = gcm^2/s^3$ .

By substituiting equation 3.51 in to equation 3.69 we obtain nuclear lifetime interms of temperature as follows;

$$\begin{aligned}
t_{nuc} &\approx 4.155 yr \left( \frac{M}{M_\odot} \right)^{1/3} \left( \frac{T_\odot}{T} \right)^4 \\
t_{nuc} &\approx \frac{4.155 yr}{7.013 \times 10^6} \left( \frac{T}{T_\odot} \right)^{1/2} \left( \frac{T_\odot}{T} \right)^4 \\
t_{nuc} &\approx 0.5924 \times 10^{-6} yr \left( \frac{T_\odot}{T} \right)^{3.5} \tag{3.70}
\end{aligned}$$

## ii. Phase B (Hydrogen shell burning)

Hydrogen burning in shell around inert core (shell burning phase) is the develop of core until  $M_{core}/M \sim 0.1$  (Schonberg Chandrasekhar limit).

$$\begin{aligned}
t_{nuc} &= \frac{M f_{H,shell} c^2}{L} = \frac{0.100 \times (3 \times 10^8 m/s)^2 M}{(12.248 \times 10^8) L_\odot \left[ \frac{M}{M_\odot} \right]^{3.66}} \\
t_{nuc} &= \frac{0.300 \times 10^{41} s}{3.8 \times 10^{29}} \left( \frac{M}{M_\odot} \right) \left[ \frac{M_\odot}{M} \right]^{3.66} \\
t_{nuc} &= 7.894 \times 10^9 s \left[ \frac{M}{M_\odot} \right]^{-2.66} \tag{3.71}
\end{aligned}$$

After rearranging equation 3.54 (mass interms luminosity), nuclear lifetime interms of luminosity. Nuclear lifetime interms of luminosity gives as;

$$\begin{aligned}
t_{nuc} &= \frac{M f_{H,shell} c^2}{L} = \left[ \frac{0.100 \times (3 \times 10^8 m/s)^2 M_{\odot}}{L} \right] \left( \frac{L}{12.248 \times 10^8 L_{\odot}} \right)^{3/11} \\
&= \left[ \frac{(0.200)(3 \times 10^8)^2 (10^{37}) cm^2 gs^{-2}}{(12.248 \times 10^8)^{3/11}} \right] \left( \frac{L^{-8}}{L_{\odot}^3} \right)^{1/11} \\
&= \left[ \frac{1.8 \times 10^{53}}{(12.248 \times 10^8)^{3/11} 3.8 \times 10^{33}} \right] \left( \frac{L^{-8}}{L_{\odot}^3} \right)^{1/11} \\
t_{nuc} &= (0.1469)^{13/11} s \left( \frac{L}{L_{\odot}} \right)^{-8/11} \tag{3.72}
\end{aligned}$$

Then, mass and luminosity relation in terms of each star's age at hydrogen shell burning gives as,

$$\begin{aligned}
t_{nuc} &= \frac{M f_{Hshell} c^2}{L} = 33.3 \times 10^{15} m^2/s^2 \left( \frac{M}{L} \right) \\
&= 33.3 \times 10^{15} s \left( \frac{L_{\odot}}{3.847 \times 10^{23}} \right) \left( \frac{2 \times 10^{33}}{M_{\odot}} \right) \left( \frac{M}{L} \right) \\
t_{nuc} &\approx 0.0546 \times 10^{13} yr \left( \frac{M}{M_{\odot}} \right) \left( \frac{L_{\odot}}{L} \right) \tag{3.73}
\end{aligned}$$

Where,  $f_{Hshell}$  is the fraction hydrogen shell burning = 0.37 and  $c^2 \approx 9 \times 10^{16} m^2/s^2$ . By substituting the Stepha Boltzman equation and 3.51 into equation 3.73, we obtain the mass to temperature relation in terms of each star's age at hydrogen shell burning, as shown below;

$$\begin{aligned}
t_{nuc} &= \frac{M f_{Hshell} c^2}{L} = 33.3 \times 10^{15} m^2/s^2 \left[ \frac{M}{4\pi\sigma R^2 (4.5319 \times 10^{-13} K(M/R))^4} \right] \\
t_{nuc} &= \left[ \left( \frac{3.33 \times 10^{15} m^2/s^2}{7.1215 erg/cm^2 s (12.248 \times 10^8)^4} \right) \left( \frac{R^2}{M^3} \right) \right] \\
t_{nuc} &\approx 0.11 yr \left( \frac{R}{R_{\odot}} \right)^2 \left( \frac{M_{\odot}}{M} \right)^3 \tag{3.74}
\end{aligned}$$

We substitute equation 3.48 in to equation 3.72 nuclear burning lifetime interms of radius gives as;

$$\begin{aligned}
t_{nuc} &\approx 0.11yr \left( \frac{R}{R_{\odot}} \right)^2 \left( \frac{2\mu GM_{\odot}}{\mathfrak{R}R^3} \right)^3 \\
&\approx 0.11yr \left( \frac{R}{R_{\odot}} \right)^2 \left[ \frac{M_{\odot}}{5.1875 \times 10^{13} M_{\odot}} \left( \frac{R_{\odot}}{R} \right)^3 \right]^3 \\
&\approx \frac{0.11yr}{5.1875 \times 10^{39}} \left( \frac{R_{\odot}}{R} \right)^3 \\
t_{nuc} &\approx 2.12 \times 10^{-39} yr \left( \frac{R_{\odot}}{R} \right)^3 \tag{3.75}
\end{aligned}$$

Then, to obtain the mass to temperature relation in terms of each star's age at hydrogen shell burning, we substitute the Stephan Boltzmann equation and equation 3.48 into equation 3.74.

$$\begin{aligned}
t_{nuc} &= \frac{M f_{Hshell} c^2}{L} = 33.3 \times 10^{13} m^2/s^2 \left[ \frac{M}{4\pi\sigma(M^{1/3})^2 T^4} \right] \\
&= \left( \frac{33.3 \times 10^{13} m^2/s^2}{672.6833 \times 10^{48} erg/K^4 s} \right) \left[ \frac{M}{(M/M_{\odot})^{2/3} T^4} \right] \\
&= (0.0495 \times 10^{-33} (cm/s)^2 / gcm^2 / K^4 s^3) \left[ \frac{M}{(M/M_{\odot})^{2/3} T^4} \right] \\
&= (0.0495 \times 10^{-33} s \left( \frac{2 \times 10^{33}}{M_{\odot}} \right) \left( \frac{T}{5.8 \times 10^3} \right)^4 \left[ \frac{M}{(M/M_{\odot})^{2/3} T^4} \right] \\
t_{nuc} &= 0.6175yr \left( \frac{M}{M_{\odot}} \right)^{1/3} \left( \frac{T_{\odot}}{T} \right)^4 \tag{3.76}
\end{aligned}$$

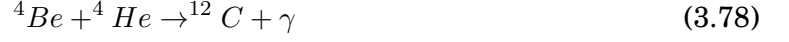
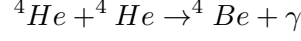
Where,  $K^4 = (T_{\odot}/5.8 \times 10^3)^4$ , and  $erg = gcm^2/s^3$ .

By substituting equation 3.51 in to equation 3.76 we obtain nuclear lifetime interms of temperature as follows;

$$\begin{aligned}
t_{nuc} &\approx 0.6175yr \left( \frac{M}{M_{\odot}} \right)^{1/3} \left( \frac{T_{\odot}}{T} \right)^4 \\
t_{nuc} &\approx \frac{0.6175yr}{7.013 \times 10^6} \left( \frac{T}{T_{\odot}} \right)^{1/2} \left( \frac{T_{\odot}}{T} \right)^4 \\
t_{nuc} &\approx 0.088 \times 10^{-6} yr \left( \frac{T_{\odot}}{T} \right)^{3.5} \tag{3.77}
\end{aligned}$$

### iii. Phase C (Helium core burning)

The main sequence stars accumulate helium in their cores from the result of hydrogen fusion. But, the core does not become hot enough to initiate helium fusion. *He* fuses to  $^{12}C$  as shown below;



The mass fraction change from this equation is calculated as;

$$\frac{\Delta E}{E} = \frac{4m_{\text{He}}c^2 - 12m_{\text{C}}c^2}{4m_{\text{He}}c^2} \approx 0.286 \quad (3.79)$$

Where,  $m_{\text{He}} = 6.6463173 \times 10^{-24} \text{g}$  and  $m_{\text{C}} = 1.998467052 \times 10^{-23} \text{g}$

$$\begin{aligned} t_{\text{nuc}} &= \frac{Mf_{\text{H,core}}c^2}{L} = \frac{0.286 \times (3 \times 10^8 \text{m/s})^2 M}{(12.248 \times 10^8) L_{\odot} \left[ \frac{M}{M_{\odot}} \right]^{3.66}} \\ t_{\text{nuc}} &= \frac{0.858 \times 10^{41} \text{s}}{3.8 \times 10^{29}} \left( \frac{M}{M_{\odot}} \right) \left[ \frac{M_{\odot}}{M} \right]^{3.66} \\ t_{\text{nuc}} &= 22.578 \times 10^9 \text{s} \left[ \frac{M}{M_{\odot}} \right]^{-2.66} \end{aligned} \quad (3.80)$$

After rearranging equation 3.54 (mass interms luminosity), nuclear lifetime interms of luminosity. Nuclear lifetime interms of luminosity gives as;

$$\begin{aligned} t_{\text{nuc}} &= \frac{Mf_{\text{H,core}}c^2}{L} = \left[ \frac{0.286 \times (3 \times 10^8 \text{m/s})^2 M_{\odot}}{L} \right] \left( \frac{L}{12.248 \times 10^8 L_{\odot}} \right)^{3/11} \\ &= \left[ \frac{(0.572)(3 \times 10^8)^2 (10^{37} \text{cm}^2 \text{gs}^{-2})}{(12.248 \times 10^8)^{3/11}} \right] \left( \frac{L^{-8}}{L_{\odot}^3} \right)^{1/11} \\ &= \left[ \frac{5.148 \times 10^{53}}{(12.248 \times 10^8)^{3/11} 3.8 \times 10^{33}} \frac{L_{\odot}}{L} \right] \left( \frac{L^{-8}}{L_{\odot}^3} \right)^{1/11} \\ t_{\text{nuc}} &= (0.4203)^{13/11} \text{s} \left( \frac{L}{L_{\odot}} \right)^{-8/11} \end{aligned} \quad (3.81)$$

Then, mass and luminosity relation in terms of each star's age at helium core burning is;

$$\begin{aligned} t_{\text{nuc}} &= \frac{Mf_{\text{He,core}}c^2}{L} = \frac{0.286 \times (3 \times 10^8 \text{m/s})^2 M}{L} \\ t_{\text{nuc}} &= 25.74 \times 10^{15} \text{s} \left( \frac{L_{\odot}}{3.847 \times 10^{23}} \right) \left( \frac{2 \times 10^{33}}{M_{\odot}} \right) \left( \frac{M}{L} \right) \\ t_{\text{nuc}} &\approx 0.0422 \times 10^{13} \text{yr} \left( \frac{M}{M_{\odot}} \right) \left( \frac{L_{\odot}}{L} \right) \end{aligned} \quad (3.82)$$

In the main sequence phase, the ZAHB corresponds to the point where the helium core mass fraction of the star is 0.28 [52]. By substituting Stephan boltzman equation and equation 3.51 into equation 3.82, we obtain the mass to radius relation in terms of each star's age at helium core burning

$$\begin{aligned}
 t_{nuc} &= \frac{M f_{He,core} c^2}{L} = 25.74 \times 10^{15} m^2/s^2 \left[ \frac{M}{4\pi\sigma R^2 (4.5319 \times 10^{-13} K (M/R))^4} \right] \\
 t_{nuc} &= \left[ \left( \frac{25.74 \times 10^{15} m^2/s^2}{7.1215 erg/cm^2 s (12.248 \times 10^8)^4} \right) \left( \frac{R^2}{M^3} \right) \right] \\
 t_{nuc} &= 0.22 yr \left( \frac{R}{R_\odot} \right)^2 \left( \frac{M_\odot}{M} \right)^3 \tag{3.83}
 \end{aligned}$$

Then to obtain nuclear burning lifetime interms of radius, we substituting equation 3.48 in to equation 3.80 as follows;

$$\begin{aligned}
 t_{nuc} &\approx 0.222 yr \left( \frac{R}{R_\odot} \right)^2 \left( \frac{2\mu GM_\odot}{\mathfrak{R} R^3} \right)^3 \\
 &\approx 0.222 yr \left( \frac{R}{R_\odot} \right)^2 \left[ \frac{M_\odot}{5.1875 \times 10^{13} M_\odot} \left( \frac{R_\odot}{R} \right)^3 \right]^3 \\
 &\approx \frac{0.222 yr}{5.1875 \times 10^{39}} \left( \frac{R_\odot}{R} \right)^3 \\
 t_{nuc} &\approx 0.043 \times 10^{-39} yr \left( \frac{R_\odot}{R} \right)^3 \tag{3.84}
 \end{aligned}$$

By substituting Stephan boltzman equation and 3.48 into equation 3.83, we obtain the mass to effective temperature relation interms of each star's age at helium core burning is,

$$\begin{aligned}
 t_{nuc} &= \frac{M f_{He,core} c^2}{L} = 25.74 \times 10^{15} m^2/s^2 \left[ \frac{M}{4\pi\sigma (M^{1/3})^2 T^4} \right] \\
 &= \left( \frac{25.74 \times 10^{15} m^2/s^2}{672.6833 \times 10^{48} erg/K^4 s} \right) \left[ \frac{M}{(M/M_\odot)^{2/3} T^4} \right] \\
 &= (0.0382 \times 10^{-33} (cm/s)^2 / gcm^2 / K^4 s^3) \left[ \frac{M}{(M/M_\odot)^{2/3} T^4} \right] \\
 &= (0.0382 \times 10^{-33} s \left( \frac{2 \times 10^{33}}{M_\odot} \right) \left( \frac{T}{5.8 \times 10^3} \right)^4 \left[ \frac{M}{(M/M_\odot)^{2/3} T^4} \right] \\
 t_{nuc} &\approx 0.46 yr \left( \frac{M}{M_\odot} \right)^{1/3} \left( \frac{T_\odot}{T} \right)^4 \tag{3.85}
 \end{aligned}$$

Where,  $K^4 = (T_{\odot}/5.8 \times 10^3)^4$ , and  $erg = gcm^2/s^3$ .

By substituting equation 3.51 in to equation 3.82 we obtain nuclear lifetime interms of temperature as follows;

$$\begin{aligned}
 t_{nuc} &\approx 0.46yr \left( \frac{M}{M_{\odot}} \right)^{1/3} \left( \frac{T_{\odot}}{T} \right)^4 \\
 t_{nuc} &\approx \frac{0.46yr}{7.013 \times 10^6} \left( \frac{T}{T_{\odot}} \right)^{1/2} \left( \frac{T_{\odot}}{T} \right)^4 \\
 t_{nuc} &\approx 0.0655 \times 10^{-6} yr \left( \frac{T_{\odot}}{T} \right)^{3.5}
 \end{aligned} \tag{3.86}$$

### 3.4 Summary

This chapter brought us to the final stage, where we have all the quantities that are necessary to accomplish our objectives. We derivate the stellar structure equations and the lifetime equation in nuclear burning phases of the stars for our work. In the next chapter, the results and the discussions will present.

## Result and Discussion

### 4.1 Introduction

For studying the MLR relation of main-sequence intermediate-mass stars, the stellar structure equation of intermediate-mass stars on the main sequence, and their fundamental quantities classical relationship, and M, L, R, and T in terms of a nuclear lifetime are discussed in chapter 3 section 3.2 and 3.3, respectively. In this chapter, we construct the MLR relation of the main-sequence intermediate-mass stars. using the final derived equations obtained in chapter 3. We plot the graph of MLR, MRR, MTR, in terms of time parameter, and the relation of M, L, R, and T with lifetime. In the first section, the M, R, T, and L in terms of a lifetime, will do. In the second section, the MLR, MRR, MTR relations with the parameter lifetime are presented. Finally, we have dealt with the integrated comparison of MLR, MRR, and MTR relations with the observational works done by O.Yu. Malkov 2007 [38] and Z. Eker et al. 2015 [60] for mass-luminosity relation of medium mass stars.

### 4.2 M, L, R and T relation with lifetime

In this subsection, we will show the mass to time, luminosity to time, radius to time, and temperature to time relations.

#### 4.2.1 Mass to lifetime relation

In nuclear burning the heat released by fusing of hydrogen and helium into helium and carbon respectively. The time required to exhaust the hydrogen or helium in main sequence intermediate mass stars obtained from equation 3.61, 3.71, and 3.77.

$$t_{nuc} \approx b \times 10^9 s \left( \frac{M}{M_{\odot}} \right)^{-2.66} \quad (4.1)$$

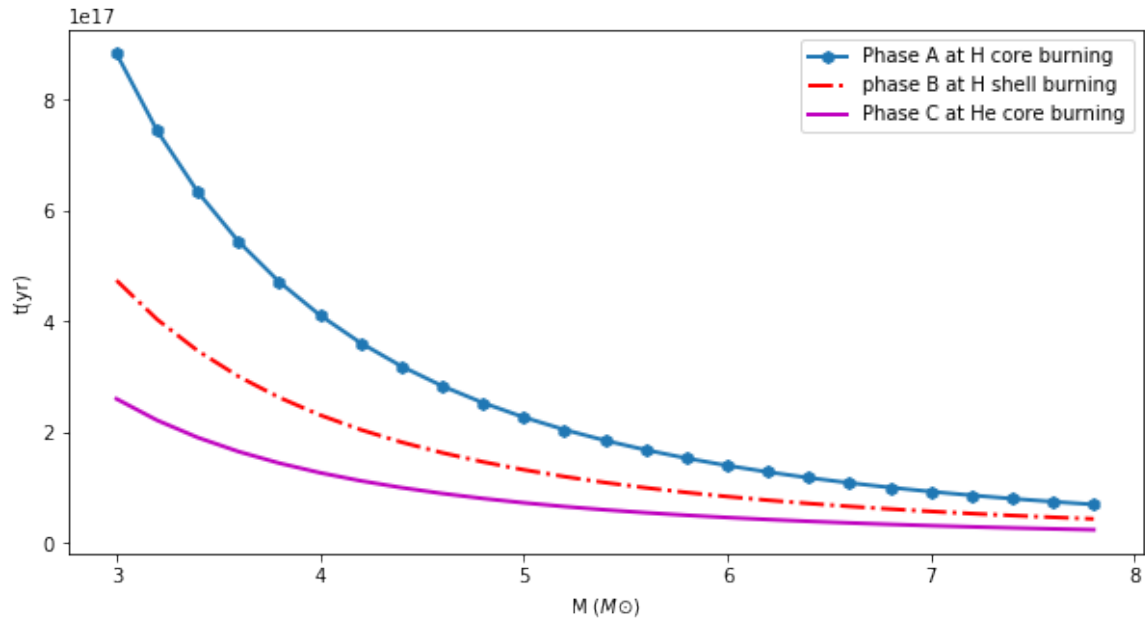
Where,  $b = (2.708, 7.894, 22.578)$ , in phase A, phase B and phase C, respectively.

This equation shows the lifetime of the main-sequence star is inversely proportional to its mass.

The actual lifetime of the stars will be about one-tenth solar lifetime times the inverse of the mass of stars. The time scale depends on the mass of the star.

For the star through the fuel consumption stage, the evolution will be fast.

This figure shows, Mass to lifetime relation of main-sequence stars within mass range



**Figure 4.1:** Mass to lifetime relation of main-sequence stars

The blue line is the hydrogen core burning, the red line shows the hydrogen shell burning, and the purple line shows helium core burning.

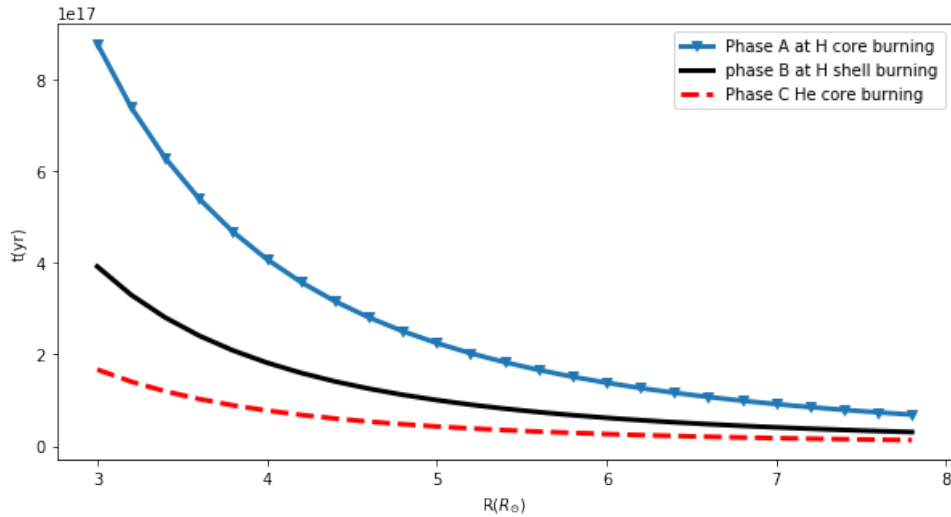
2.3 to 8 solar masses. H core burning stars have a large lifetime to change, comparably more than billion years ( $10^{12} years$ ). But helium core burning stars live for the shortest period (2 to 4 years) where they are the fastest to change the structure of the stars within 2 years of lifetime. Helium core burning is the fastest to change the structure of the stars than hydrogen core burning and hydrogen shell burning. After the hydrogen core burn, new stars begin in the main sequence with a high amount of helium which, calls as helium mass stars. Those stars are mature to undergo a remarkable transformation after they consume all the hydrogen in their core..

#### 4.2.2 Radius to lifetime relation

In nuclear burning, as core burn or shell expand the size also increase. The radius increase from core to shell ( $r \rightarrow R$ ) from equation 3.47, 3.72, and 3.81.

$$t_{nuc} \approx d \times 10^{-39} yr \left( \frac{R}{R_{\odot}} \right)^{-3} \quad (4.2)$$

Where,  $d = (0.09, 2.12, 0.043)$ , in phase A, phase B and phase C, respectively. For later applications, we note that the radius of main-sequence intermediate-mass stars in helium core burning is rapidly changing than the hydrogen core and shell burning. The duration increased very slowly until the core helium content has dropped. Mass to lifetime relation of main-sequence stars within mass range 3 to 8 solar masses. As shown in bellow graph 4.2, H core burning stars have a long life as they have a large size to change, comparably more than a billion years, but helium core burning stars live for the shortest time. Where they are the fastest to change the radius structure of the stars within 2 years lifetime.



**Figure 4.2:** Radius to lifetime relation of main sequence stars  
Blue line is the hydrogen core burning, the black line shows the hydrogen shell burning, and red broken line shows helium core burning.

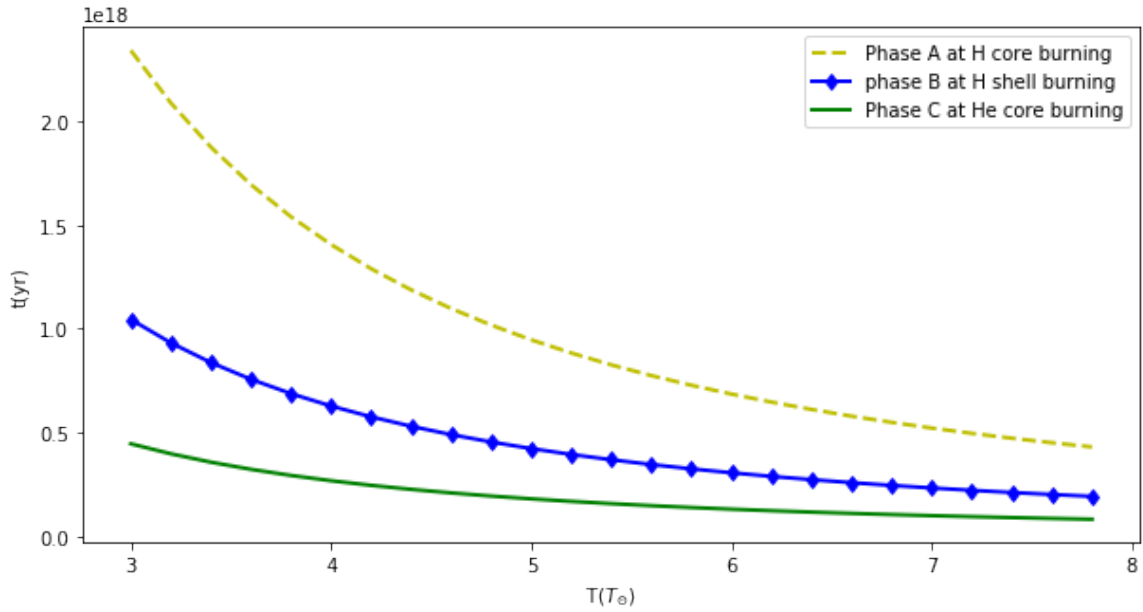
#### 4.2.3 Effective temperature to lifetime relation

The sequence hydrogen to helium infusion takes place in well-separated phases of burning [53] from equation 3.70, 3.74, and 3.83.

$$t_{nuc} \approx P \times 10^{-6} yr \left( \frac{T}{T_{\odot}} \right)^{-3.5} \quad (4.3)$$

Where,  $P = (0.592, 0.088, 0.0655)$ , in phase A, phase B and phase C, respectively). The temperature needed to burn lighter element is smaller than the heavier ones.

As shown in figure 4.3, the temperature required for nuclear burning depends on the mass of the stars.



**Figure 4.3:** Temperature to lifetime relation of main sequence stars  
The yellow line is the hydrogen core burning, the blue line hydrogen shell burning, and the green line helium core burning.

Because temperature is directly related to the mass of the stars, as shown in equation 3.51. The Effective temperature is inverse proportion to the time required to change from one phase to another, i.e., Most thermal (have a high temperature) stars burn for a long time than the stars which have low temperature. In this figure, the Effective temperature that burns the helium core is fastest than the hydrogen core or hydrogen shell burnings.

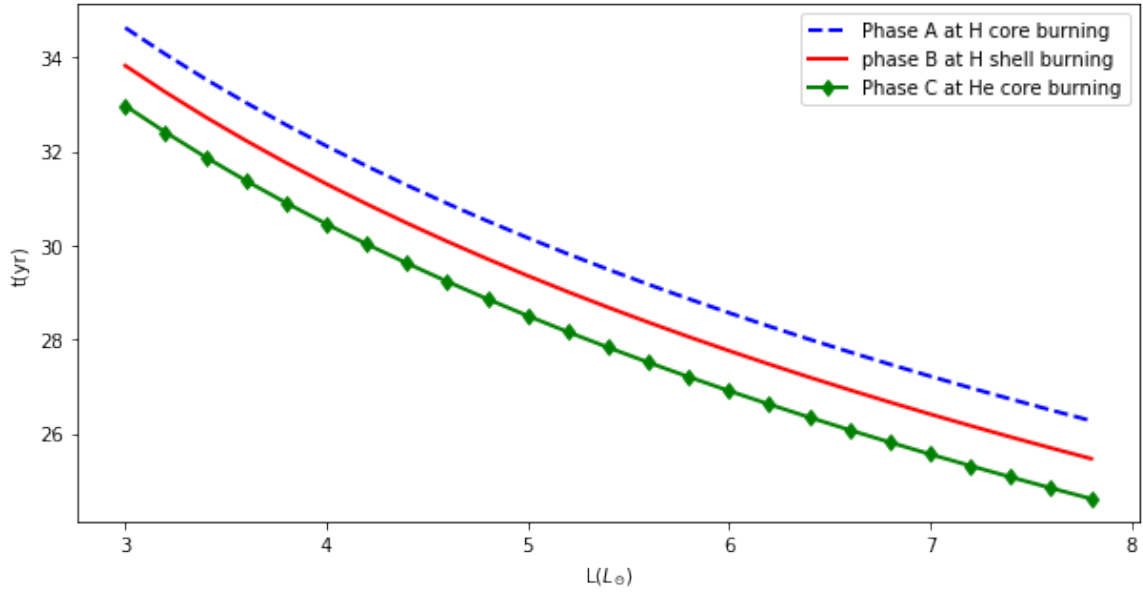
#### 4.2.4 Luminosity to lifetime relation

Since the luminosity of the star determined, by its mass and composition. Then the burning time in each phase is obtained by the fractional values, which depends on the mass of the star. Luminosity is the rate of energy it increases the fractional energy that change and leads a short time change lead shortly. from equation 3.47, 3.69, and 3.81.

$$t_{nuc} \approx Z \times 10^{1.18} s \left( \frac{L}{L_{\odot}} \right)^{-0.72} \quad (4.4)$$

Where,  $Z = (0.027, 0.1469, 0.420)$ , in phase A, phase B and phase C, respectively).

Figure 4.4 shows, hot and more luminous intermediate-mass stars lived for a short time, while the cool, and less luminous stars lived for a long time. So, luminous stars burn hydrogen core fastly than helium core and put out energy at a higher rate.



**Figure 4.4:** Luminosity to lifetime relation of main-sequence stars. The blue line is the hydrogen core burning, red line hydrogen shell burning, and the green line helium core burning

### 4.3 MLR, MRR, and MTR relationship in terms of lifetime

Luminosity is the basic property of a star. It is directly linked to the mass and evolution of a star. Thus, mass determines their luminosity, surface temperature, radius, and lifetime. Mass of a star most uniquely determines a star on the main sequence. We will study the details of MLR, MRR, and MTR relationship in terms of a lifetime in the following subheadings.

#### 4.3.1 MLR relation

By combining equation 3.66, 3.73, and 3.82, we produce below equation,

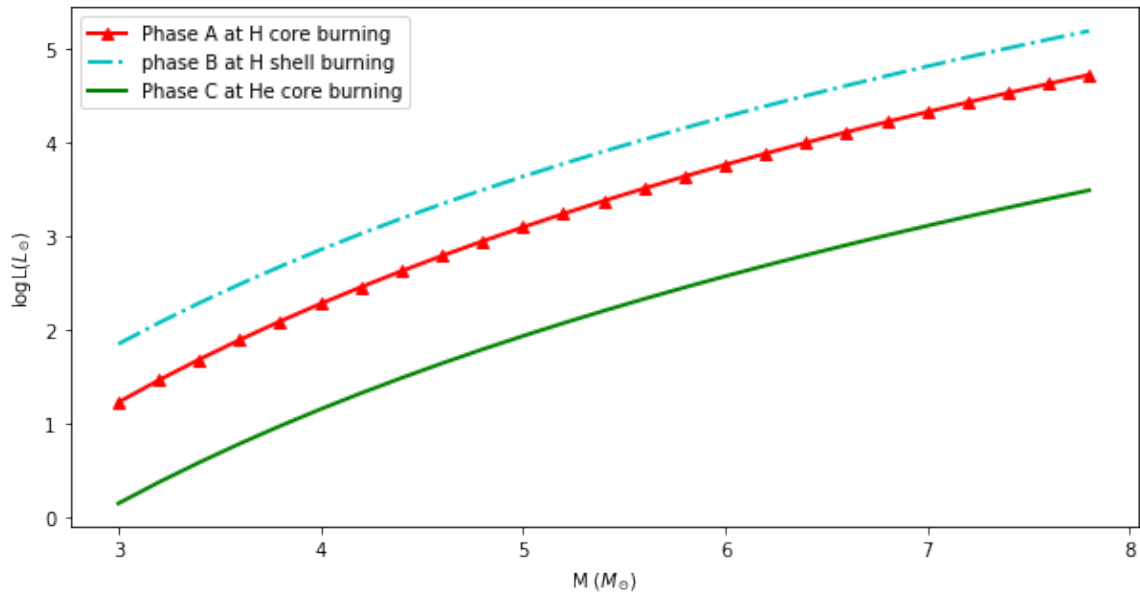
$$L \approx Y \times 10^{13} \text{yr} \left( \frac{L_{\odot}}{t_{nuc}} \right) \left( \frac{M}{M_{\odot}} \right) \quad (4.5)$$

Where,  $Y = (10.336, 0.0546, 0.0422, \text{ in phase A, phase B and phase C, respectively})$ , and  $t_{nuc} \approx t_{ms}$  (equation 3.58, main sequence lifetime). Using this equation, we plotted the mass- luminosity relation ( $L/L_{\odot} = (M/M_{\odot})^a$ ).

Thus, for intermediate-mass stars with eight solar mass, we found that thousand's higher luminosity than the solar luminosity, that is  $L = 5.589 \times 10^{36} \text{erg/s}$  as shown in figure 4.5. Notice that the range of masses corresponds to a luminosity range from 1 to  $10^5$  solar luminosities.

The basic scenario of mass- luminosity relation is that a H shell burning stars have higher luminosity whereas a helium core burning stars have low luminosity. Hydrogen shell burning ( $10^{3.2} \text{ergs}$ ) generates more energy than the core burning ( $10^{2.8} \text{ergs}$ ), and hydrogen core burning generates low energy than helium core burning ( $10^{2.3} \text{ergs}$ ).

Then the star increases sharply in luminosity and expands in size to become a red giant, and have the shortest lifetimes. Hot stars are more luminous and give off high total energy.



**Figure 4.5:** Mass to luminosity relation of intermediate main sequence stars the red triangular line is the hydrogen core burning, blue broken line is the hydrogen shell burning and the green line is He core burning

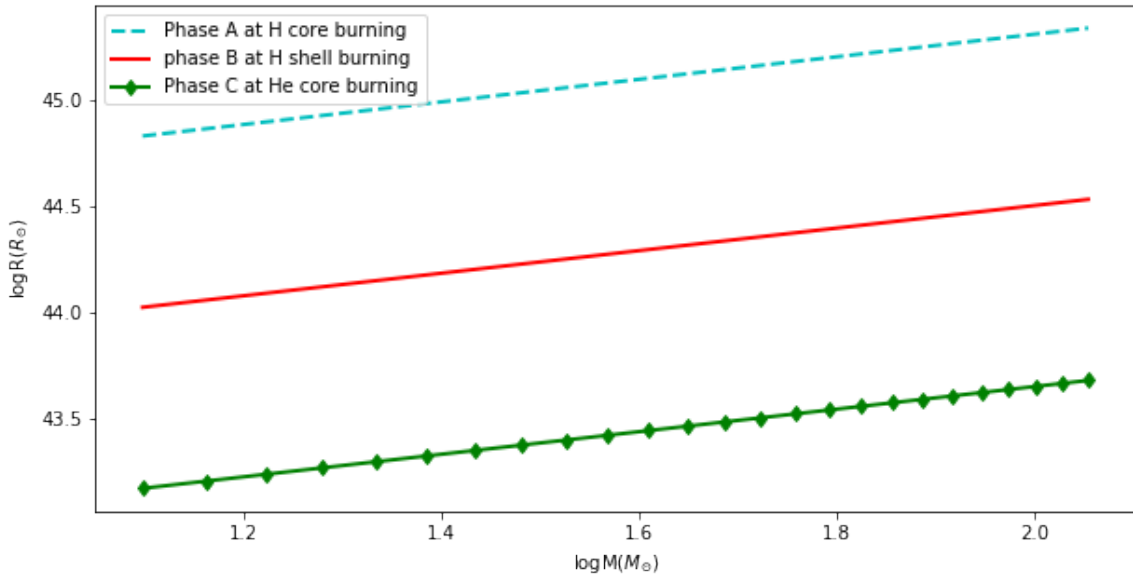
A star can be luminous either because it is hot or because it is massive. As the mass increases, the luminosity also rapidly increases. The plot also calls as the mass-luminosity ratio. Figure 4.5 shows, in the helium core burning of intermediate-mass stars are low luminous than the hydrogen core burning and shell burning. But fastly luminous, while at hydrogen core burning natural logarithm of luminosity mostly increases as mass increase. So, hydrogen shell burning of the main sequence intermediate-mass stars is more luminous than the hydrogen core burning and helium core burning phases.

### 4.3.2 MRR relation

Mass and radius are directly related with invers parameter of nuclear time. From equation 3.67, 3.74, and 3.80.

$$M \approx \left[ (W)yr \frac{M_{\odot}^3}{t_{nuc}} \left( \frac{R}{R_{\odot}} \right)^2 \right]^{1/3} \quad (4.6)$$

Where,  $W = (0.471, 0.11, 0.22, \text{ in phase A, phase B and phase C, respectively})$ .



**Figure 4.6:** Mass to radius relation of intermediate main sequence stars blue line is .

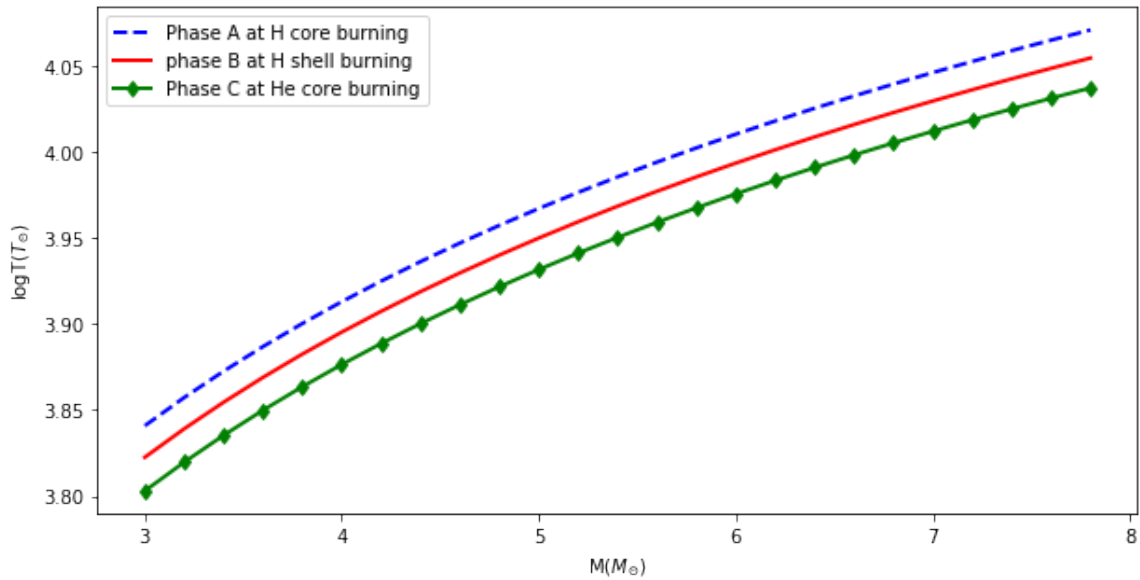
Figure 4.6 shows, the radius of main-sequence stars change slowly, i.e., the increased mass effect to increased density with little change of radius. The mass-radius relation of main-sequence stars obeys the structure of stars. As stars with convectively and nuclear burn, the mass-radius is directly related, but mass and nuclear time are inverses. As the central energy increase, the radius is bound from  $r$  to  $R$  and fastly compressed or decrease. Then, the nuclear burn changes to the next nuclear burn( as shown in fig.4.6). For non-convective stars, the contraction increases, and the central temperature also moves to the surface of the star. This shows, as mass increase the radius of the star more expand fastly than at hydrogen burning, and as a mass increase to  $8M_{\odot}$  size also increase, hotter are more luminous. The hydrogen-burning shell generates more energy than the core burning and the star increases sharply in luminosity and expands in size( $R$ ) to become a red giant.

### 4.3.3 MTR relation

Mass temperature relation of intermediate-mass stars in nuclear burning gives as; from equation 3.55, 3.73, and 3.82.

$$M \approx \left[ (E)yr \frac{(M_{\odot}^{-1/3})}{t_{nuc}} \left( \frac{T_{\odot}}{T} \right)^4 \right]^3 \quad (4.7)$$

Where,  $E = (4.155, 0.6175, 0.46, \text{ in phase A, phase B and phase C, respectively})$ . High mass stars are big, hotter, more luminous, and less dense than low mass stars, and these properties affect their energy generation, their structure, and their future evolution. The hydrogen core burning of medium mass stars requires high temperature than helium core burning, and it burns slower than hydrogen shell burning or helium core burning.



**Figure 4.7:** Mass to temperature relation of intermediate main sequence stars. Blue line the hydrogen core burning, red line hydrogen shell burning and green line the helium core burning

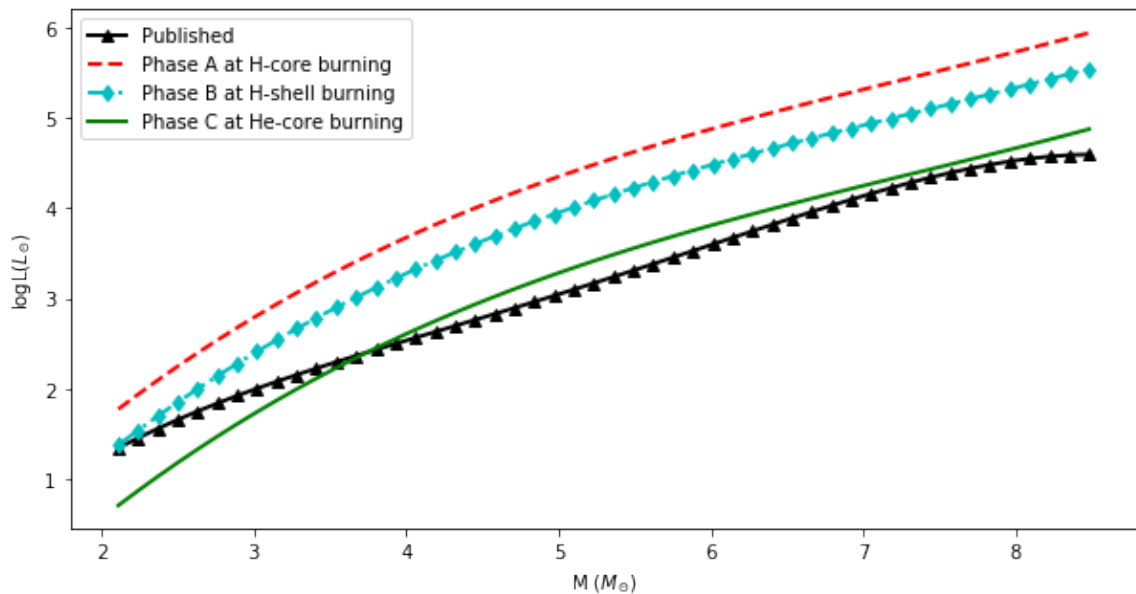
In helium core burning mass of stars and temperature near to direct proportion than hydrogen core burning or hydrogen shell burning(as figure 4.7 shown). This indicates that the lifetime of most intermediate-mass stars or the mass of stars that greater than the Sun depends on the core temperature and the production of heavy elements.

## 4.4 Comparison with observational works

The MLR relation is also explained for particular intermediate-mass stars by using observational data. For instance, O. Yu. Malkov 2007 analyzed the observational data of

detached main- sequence double-lined eclipsing binary, and demonstrate that the absolute MLR relation [54]. Here we used the data of B-type stars and explained in analytical curve fitting. Using this mechanism, we have compared mass to luminosity, mass to the radius, and mass to effective temperature relations of our work in hydrogen burning and helium burning phases with the fitted data. Thus, as shown the below mass grows linearly with luminosity and temperature. Moreover, data of the well-known HRD significantly explain the observational output that best fitted analytical formulations. It absolutely was found that rapid rotators exhibit slightly higher luminosities. Our analytical investigation (especially in hydrogen core burning) largely confirms as there is a good agreement between observations and theoretical calculations. It is shown in figures 4.8 and 4.9.

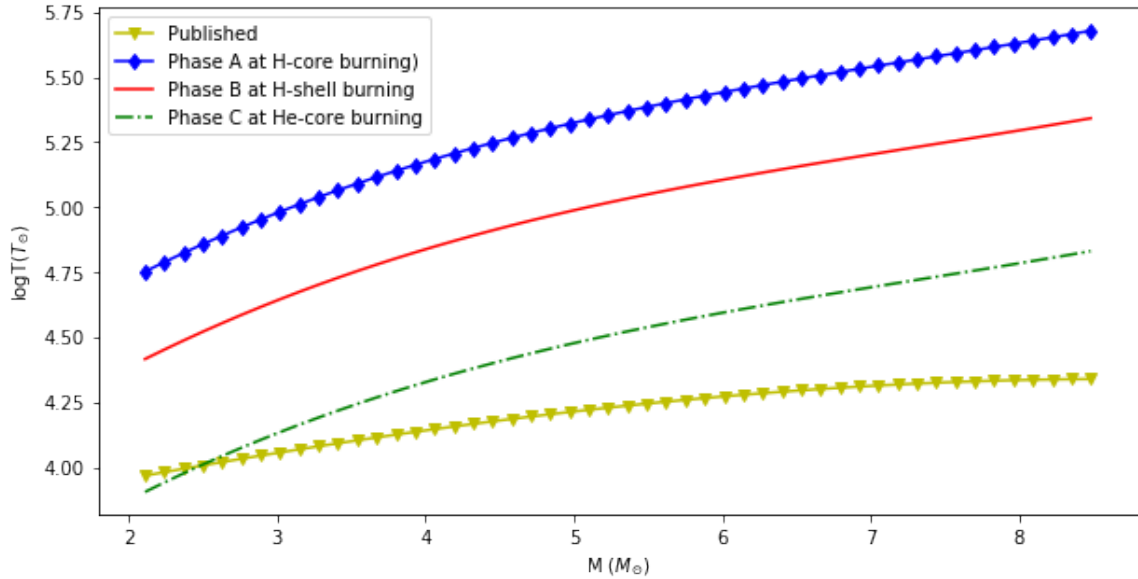
The detached main sequence type-B stars mass-luminosity relation is near to the hydrogen



**Figure 4.8:** Mass to luminosity relation of the main sequence intermediate mass stars. Black line is the observational work the other lines like red broken line, blue dimend line, and green line are the mass-luminosity relation of stars at three nuclear burning phases [12].

core burning star's mass-luminosity relation.

While phase C intermediate stars with eight solar mass attain one million degree temperature and phase A at H core burning has lower temperature of  $(10^{4.6})$  degree.



**Figure 4.9:** Mass to temperature relation of main sequence intermediate mass stars, the yellow line is the observational work, the other lines like red line, blue diamond line, and green broken line are the mass to temperature relation of stars at three nuclear burning phases[12].

## 4.5 Summary

In this chapter, we have presented the results we have obtained from the derivation equations in chapter 3. The results enable us to produce a good resolution nuclear burning phase that allows us to look for possible fundamental stellar quantities relation (MLR, MTR, MRR, and their relation with lifetime) which are required to obtain the expected stellar structure and evolution of intermediate-mass stars. The lifetime of the star's mass, radius, luminosity, and temperature, respectively, are plotted. They have shown the stellar structure and evolution of intermediate-mass stars. The mass to luminosity and mass to temperature relations in the nuclear burning phase is fitting with the observational work done by O.Yu Malkove of B-type main-sequence stars[38]. The comparison of our result and observational work mass-luminosity relation is one of the stellar structure and evolution. Like the figures, 4.8 and 4.9, the mass-luminosity relation in helium core burning is a good agreement to the observational work.

In general, the graphs that we show are fair, linear, and show a clear direction. As mass increases, the main sequence quantities, like, radius, luminosity, and temperature all increase while the mean density decreases. Our work for He core burning is in good

agreement with Published (the observational work of O.Yu Malkove) with a confidence level of 89% (as seen from 4.9 ). Thus, the H core burning star has an increasingly higher temperature. whereas He cores burning lowest temperature.

## Conclusion

The stellar structure of the stars depends on the stellar structure equations. Intermediate mass stars are the dominant stars on the main sequence. Those stars have a great role in the nuclear burning of hydrogen and helium. Nevertheless, their true structure is still elusive. We have investigated the stellar structure and evolution of intermediate-mass stars. We computed the mass-luminosity relation of intermediate-mass stars by derivating the stellar structure equation. Using Stephan Boltzmann equation we have produced a good resolution map of the luminosity to radius relation that allows us to look at the lifetime of stars in three nuclear burning phases for possible structures and substructures changes. Then, we developed the nuclear burning time to luminosity, mass, radius, and effective temperature relations and vice versa. As the mass of stars increases, their lifetime in helium core burning is fastly decreased than the hydrogen shell burning and hydrogen core burning.

Our result for the integrated mass-luminosity relation in helium core burning is in good agreement with the observed mass-luminosity relation of B-type intermed - iate-mass stars. While the mass-luminosity relation of intermediate-mass stars obtained in hydrogen core burning and hydrogen shell burning is almost linearly proportion with the observational works. But, helium core burning is a good identification way to mass-luminosity relation of B-type main sequence intermediate-mass stars that are done by O.Yu. Malkov (demonstrate that the absolute MLR relation). Here we used the data of B-type main sequence intermediate-mass stars by analytical curve fitting. Thus, as shown in figure 4.5 mass grows linearly with luminosity. Moreover, data of the well-known HRD (Henry Norris Russell and Ejnar Hertzsprung) significantly explain the observational output that best fitted analytical formulations. We found that rapid rotators exhibit slightly higher luminosities. Our analytical investigation largely confirms as there is a good agreement between observations and theoretical calculations. For the three phases, what we call nuclear-burning phases, the mass-luminosity relation has shown directly for an inverse proportion of their lifetime. The Mass-luminosity relation of the main sequence B-type intermediate-mass stars is dominantly determined by helium core burning. It is appropriate to consider the simplification form of the stellar structure equation and the lifetime equation of stars in helium core

burning. But ignoring the other nuclear burning phases is not important. Because the main sequence medium mass stars are mostly dominated in hydrogen core burning.

So we have planned to extend this work: 1. using the age of stars in the nuclear burning phase and fundamental quantities with the best fit to the observed mass-luminosity relations and 2. considering a new equation that all the suggested equations according to their significance.

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**DECLARATION**

ADDIS ABABA UNIVERSITY  
COLLEGE OF NATURAL AND COMPUTATIONAL SCIENCES  
DEPARTMENT OF PHYSICS

MSc Thesis

The Structure and Evolution of Intermediate-Mass Stars

Name of Candidate: Dessalegn Gezie Walelign

I the under signed declare that the thesis is my original work and no part of it can be claimed as an intellectual property of anybody else except me and my advisors.

Signature: \_\_\_\_\_