ADDIS ABABA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

COLLEGE OF NATURAL AND COMPUTATIONAL SCIENCES

DEPARTMENT OF STATISTICS

DETERMINANTS OF TIME-TO-UNDER-FIVE MORTALITY IN ETHIOPIA: A COMPARISON OF VARIOUS PARAMETRIC SHARED FRAILTY MODELS

BY: ABEBE ARGAW

A THESIS SUBMITTED TO THE DEPARTMENT OF STATISTICS IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTERS OF SCIENCE (BIOSTATISTICS)

Addis Ababa University
Addis Ababa, Ethiopia
June, 2016

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Addis Ababa University
Addis Ababa, Ethiopia
June, 2016

Addis Ababa University

School of Graduate Studies

This is to certify that the thesis prepared by Abebe Argaw, entitled: *Determinants of Time-to-Under-Five-Mortality in Ethiopia: A Comparison of Various Parametric Shared Frailty Models* and summited in partial fulfillment of the requirements for the Degree of Master of Science (Biostatistics) complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

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Advisor_____Signature_____Date _____

Chair of Department or Graduate Program Coordinator

DECLARATION

I hereby declare that the thesis is my original work, to the best of my knowledge, this thesis has not been presented for degrees in any other University and all sources of materials used for the thesis have been duly acknowledged with proper citation.

Name: Abebe Argaw

Signature: ----------

Place of submission: Department of Statistics, College of Natural and computational Sciences, Addis Ababa University

Date of submission: June, 2016

This thesis has been submitted for examination with my approval as a University advisor.

Dr. Shibru Temesgen ----------

Advisor's Name signature

TABLE OF CONTENTS

Contents **Pages**

ABSTRACT

Analysis of clustering effect in modeling the determinants of time-to-under-five mortality in Ethiopia.

Abebe Argaw
Addis Ababa University, 2016

Under-five mortality is one of the critical indicator of development of a country. It tells of children's access to basic health interventions such as vaccinations, medical treatment and inadequate nutrition (WHO, 2013). According to preliminary estimates, the global U5MR has declined by more than half, dropping from 90 to 43 deaths per 1,000 live births between 1990 and 2015. But, at today's rate of progress, it will take about 10 more years to reach the global target (UNICEF, 2015). The main objective of this study is to identify the determinants of time to under-five mortality in Ethiopia. The data for the study were taken from the 2014 Ethiopian Mini Demographic and Health Survey of women in the age group15-49 years. Mothers' educational level, mothers' age at first birth, place of residence, household size, sex of child born, preceding birth interval, economic status of family, place of delivery, marital status of family, and source of drinking water were identified as determinant factors that affect the time to under-five mortality from the socio-economic and demographic variables, and environmental factors. Regions of study were used as clusters which was taken care of the frailty term at regional level and shared frailty models were explored. Comparison of the model was done by using AIC, and Weibull-Gamma shared frailty model was selected for time-to-under-five mortality in Ethiopia. Based on the result of selected model, except marital status of family and age of mothers' at first birth, all the identified predictor variables had significant effect on time to under-five mortality. Great attention should be given to these predictor variables while planning to increase child survival time.

Key words: *Heterogeneity, Frailty, Laplace transformations, penalized partial likelihood.*

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LIST OF ABBREVIATIONS (ACRONYMS)

AIC	Akaike's Information Criterion
CSA	Central Statistics Agency
DHS	Demographic and Health Survey
EAs	Enumeration Areas
EDHS	Ethiopian Demographic and Health Survey
MGDs	Millennium Development Goals
UNICEF	United Nations Children's Fund
UNIGME	United Nations Inter-Agency Group for Child Mortality Estimate
U5M	Under-Five Mortality
U5MR	Under-five Mortality Rate
WHO	World Health Organization

LIST OF TABLES

Pages

LIST OF FIGURES

CHAPTER ONE

INTRODUCTION

1.1. Background of the Study

Under-five mortality is one key indicator of development and a critical component of the millennium development goals (MDGs) for the reduction of child mortality. It tells of children's access to basic health interventions such as vaccinations, medical treatment and adequate nutrition (WHO, 2013). Decreasing childhood mortality is a focus of communities and governments all over the world. Between 1990 and 2015, 62 of the 195 countries with available estimates met the Millennium Development Goal (MDG) 4 target of a two-thirds reduction in the under-five mortality rate between 1990 and 2015. Among them, 24 are low and lower-middle income countries. According to preliminary estimates, the global U5MR has declined by more than half, dropping from 90 to 43 deaths per 1,000 live births between 1990 and 2015. The dramatic improvement in child survival is welcome news. Despite this impressive improvements, in most regions current trends were not sufficient to meet the MDG 4 target. At today's rate of progress, it will take about 10 more years to reach the global target (UNICEF, 2015).

Studies showed that mother's education is a powerful determinant of inequity in U5MR. Analysis of a subset of countries reveals that when overall child mortality declines, there is a gap between children born to mothers with secondary or higher education and those born to mothers with no education. Children of mothers with no education are on average about two and a half times more likely to die before their fifth birthday than children of mothers with secondary or higher education (Mahfouz, 2009).

Household survey data also suggest that children from poorer households remain disproportionately vulnerable compared with the wealthiest households. On average, the risks of under-five mortality are almost twice as high for children in the poorest households as for children in the richest. The survival time to death is lower for children in rural areas. These children are about 1.7 times more likely to die before their fifth birthday as those in urban areas (UNICEF, DHS data 2005-2013).

U5MR varies across different physical, ecological, and political structures within countries. One such contextual determinant is the regional environment (Antai, 2011, Montgomery and Hewett, 2005, Wang, 2002).

Since the study population is divided into clusters, subjects from the same cluster were behave more cohesively than subjects from different clusters. Due to this fact, in this research, shared frailty models were explored assuming that children with in the same cluster (region) shares similar risk factors, which will be taken care of the frailty term at regional level. This model is a conditional independence model where the frailty is common to all individuals in a cluster and therefore responsible for creating dependence between event times. This is because ignoring the full dependence among observations may lead to standard errors that are understated and parameter estimates that are both biased and inconsistent (Sastry, 1997).

Frailty is the variance in mortality caused by unobserved factors. Frailty models address the situation where the same individual may experience the hazard more than once, raising the possibility that due to some unmeasured and perhaps unknown cause (that is, a cause of "unobserved heterogeneity"), and some subjects may be more likely than others to experience repeated hazards.

The problem of the effect of unobserved heterogeneity on survival data was first addressed by Beard (1959). He used the term longevity factor to improve the effect of mortality models in populations. But the term frailty was introduced by Vaupel et al. (1979) that showed how unobserved heterogeneity of individual frailty has an impact on the dynamic of mortality at the population level. Lancaster (1979) introduced the model to the literature of economics and the model is called the mixed proportional hazards model. The concept, however, goes back to work of Greenwood and Yule on "accident proneness" in 1920. Clayton (1978) discussed the applications of the model to multivariate survival data in his seminar paper on chronic disease incidence in families. Since then, a growing body of research has developed and widened the frailty models that are used to take the individual hidden differences into account in the analysis of population survival (Vaupel et al. 1979), to explain deviant behavior of mortality rates at old ages like cross-over and deceleration (Vaupel and Yashin, 1983), to correct biased estimates of regression coefficients of proportional hazard models in duration dependence analysis (Chamberlain 1985) and to separate biological effects from compositional ones in studies about aging (Manton et al. 1986).

The major aim of this study is to identify the determinants of time-to- under-five mortality in Ethiopia and to compare various parametric shared frailty models using the 2014 Ethiopian Mini Demographic and Health Survey (EMDHS) data that was conducted under the aegis of the Ministry of Health and implemented by the Central Statistical Agency (CSA) to close the knowledge gap in the study subject.

1.2. Statement of the Problem

Under five mortality remains a great concern not only to Ethiopia but also to many countries in the sub Saharan region. Ethiopia made a commitment to reduce U5MR by two thirds by 2015. As a signatory to this Millennium Development Goal, Ethiopia is the one among 12 low-income countries who have reduced under-five mortality by two thirds and achieved the MDG 4 target set in 2000 (UNICEF, 2015). In addition to this, surveys conducted in 2000, 2005 and 2011 showed under-five mortality rates in Ethiopia have continuous declining trend.

Studies have been conducted to identify covariates of under-five mortality in Ethiopia by many scholars using logistic regression and Semi-parametric proportional hazard models. But Logistic regression does not account the censoring observations, that is, it does not hold for time-to-event data, and in demographic applications, nonparametric and semi-parametric models are often used to model transition data. In such applications, it is assumed that all heterogeneity is captured by theoretically relevant covariates (Trussell and Richards, 1985; Trussell and Rodriguez, 1990; Van den Berg, 2001). In many situations, however, there are ample reasons to suspect omitted or unmeasured factors. That is, while some individuals will be more at risk of experiencing the event, it is unlikely the underlying reasons for this variability will be fully captured by the observed covariates. If there is unmeasured frailty, the hazard will not only be a function of the covariates but also of the frailty.

To assess the true effects of the observed covariates under this circumstance, some have stressed the need to explicitly account for unobserved heterogeneity (Aalen 1994; Baker and Melino 2000; Baschieri and Hinde 2007; Blossfeld and Hamerle 1992; Heckman et al., 1985; Heckman and Singer, 1984; Jenkins, 2005; Lancaster, 1990; Manda and Meyer 2005; Vaupel et al., 1979).

Indeed, results from several empirical and simulation studies have shown that accounting for unobserved heterogeneity significantly improves overall model fitness (Heckman et al., 1985; Sastry, 1997; Trussell and Rodriguez, 1990; Blossfeld and Hamerle, 1992).

Therefore, in this study we argued that clustering (frailty) has an effect on modeling the determinants of time to under-five mortality, which might be due to the heterogeneity in regions of study. As a result, shared frailty model approach is relatively better to determine covariates related to under- five mortality in helping the concerned body of Ethiopian Government.

1.3. **Objective of the Study**

1.3.1. **General Objective**

The general objective of this study is to model determinants of time- to- under-five mortality.

1.3.2. **Specific Objectives**

- ✓ To compare various parametric shared frailty models.
- ✓ To identify variables that are significantly associated with time-to-under-five mortality.

1.4. **Significance of the Study**

The result of this study will provide information on determinants of time to under-five mortality by analyzing the impact of regional difference on survival time of under-five mortality.

Specifically;

- ✓ The results are expected to give some knowledge about the determinants or risk factors of under-five mortality in Ethiopia.
- ✓ This study could provide an information to government and other concerned bodies to make enabling environment for the intervention to reduce under-five mortality in Ethiopia
- ✓ The study will also add to the existing literature on determinants of time-to-under-five mortality, that is, it provides an input for further study in the area.

CHAPTER TWO

LITERATURE RIVIEW

2.1. Overview of Under-Five Mortality

U5MR is defined as the probability of dying between birth and age five, which is usually expressed per 1000 live births. Mortality of children under the age of five has been the main target of public health policies and is a common indicator of mortality levels, especially in developing countries (Gakusi and Garenne, 2006). It is also viewed as an indicator of the level of development, health care, nutrition, sanitation and safe drinking water, and socioeconomic status of the population.

Improving child survival has been a priority for both policy makers and health advocates worldwide. One of these priorities is the development of the United Nations Millennium Development Goals (UN MDGs) of which one of them calls for a reduction in under-five deaths by two thirds by 2015. There has been a significant decline in under-five mortality in the twentieth century in almost all countries regardless of initial levels and socio-economic factors although the rate of decline differs across regions (UNIGME, 2012). A health transition observed globally indicates that under-five mortality had declined by 1.8 per cent per year between 1990 and 2000 (UNICEF 2012). This improvement increased to 3.2 per cent between 2000 and 2011 signifying a significant progress in child survival. However, under-five deaths remain high in the sub-Saharan Africa region where 1 in 9 children dies before the age of five (UNIGME, 2012). The U5MR has fallen from 179 deaths per 1,000 live births in 1990 to 86 in 2015 in this region (UNICEF, 2015).

Although the decline in U5MR in Ethiopia is encouraging, still nearly one in every 11 children dies before the fifth birthday. Between the 2000 and the 2011 EDHS, under-five mortality has declined by 47 percent; from 166 deaths per 1,000 live births to 88 deaths per 1,000 live births. Programs to upgrade the status of women through education and enhanced participation in the labor force can help to improve the probability of survival of young children (EDHS, 2011).

2.2. Empirical Literature

Empirically, many studies have shown that time-to-under-five mortality is influenced by a number of socio economic and demographic factors such as sex of the child, mother's educational level, mother's age at birth, birth order, preceding birth interval among others. Among the socio-economic factors, the importance of mothers' education on child survival and health has been well documented. Educated mothers are less likely to experience a child death compared to their non-educated counterparts. This could be explained by the benefits of education, which include economic advantage and the ability of educated women to access health services and live a modern life, both of which are associated with increased child survival (Caldwell 1990; UNICEF, 2012). Education also gives women power and confidence to make their own decisions (Ware, 1984).

For instance, Kumar and File (2005) used data from the Ethiopia Demographic and Health Survey [EDHS] conducted in 2005 to investigate the predictors of child U5MR mortality in Ethiopia. The cross tabulation technique was used to estimate the predictors of child mortality. Results revealed that birth interval with previous child and mother standard of living index were the vital factors associated with time to child mortality. Furthermore, the study concluded that Mother's education and birth order were found to have substantial impact on child mortality in Ethiopia.

Fathers' education, like mothers' education is also associated with U5MR. Educated fathers are more likely to support their wives in taking care of their children. Kembo and van Ginneken (2009) reported that father's education has a substantial impact on under-five mortality. Completing secondary education reduces the relative risk of child mortality by 33 per cent relative to children whose fathers had no formal education.

The mothers' place of residence is also the factor that indirectly affects infant and child mortality by determining a woman's access to a health care facility, ability to find transportation and even ability to have money for paid health care services (Mahy, 2003).

Mondal *et al.,* (2009) using the logistic regression model, investigated factors influencing infant and child mortality in Rajshahi District of Bangladesh. Findings revealed that the most significant predictors of neonatal, post-neonatal and child mortality levels are immunization, ever breastfeeding, mother's age at birth and birth interval. In a similar vein, Chowdhury *et al.* (2010) examined the effects of demographic characteristics on neonatal, post neonatal, infant and child mortality also using the logistic regression model. They identified the important predictors of infant and child mortality as mother's age at marriage, duration of marriage, birth interval, birth order and breast feeding practice.

Uddin *et al*. (2009) in their study, investigated child mortality in Bangladesh also using the logistic regression. Results of analysis showed that father's education, occupation of father, occupation of mother, standard of living index, breastfeeding status and birth order were significant determinants of child mortality in Bangladesh.

2.3. Over view of Unobserved Heterogeneity of Frailty

Ordinary methods in survival analysis are implicitly based on the assumption that the study population is homogeneous up to some observed covariates (Wienke, 2011). The concept of frailty provides a convenient way of introducing unobserved heterogeneity into models for survival data through the introduction of a random effect (Wienke, 2011).

The problem of the effect of unobserved heterogeneity on survival data was first addressed by Beard (1959); he used the term longevity factor to improve that of mortality models in populations and the term frailty was introduced by Vaupel *et a*l. (1979), which showed how unobserved heterogeneity of individual frailty has an impact on the dynamic of mortality at the population level.

Since then, a growing body of research has developed and widened the frailty models that are used to take the individual hidden differences into account in the analysis of population survival (Vaupel et al. 1979), to explain deviant behavior of mortality rates at old ages like cross-over and deceleration (Vaupel and Yashin, 1983), to correct biased estimates of regression coefficients of proportional hazard models in duration dependence analysis (Chamberlain, 1985) and to separate biological effects from compositional ones in studies about aging (Manton et al. 1986) and compared the inverse normal and gamma models, together with Gompertz and Weibull baseline hazards, in a study of survival at advanced ages, based on the data from US medicare insurance. Lancaster (1979) independently developed the same concept, but with a different name, in econometrics. He developed the same model and applied it to the analysis of unemployment spells duration data.

Frailty models (Clayton and Cuzick, 1985) are increasingly popular for analyzing clustered survival data, where frailties or random effects often enter into the baseline hazard multiplicatively to model the correlation among observations within the same cluster (YI, 2000). It has been applied to the analysis of event- history data in a number of research areas, including the study of age at time of death for individuals in terms of population (Zelterman, 1992), unemployment durations (McCall, 1994), consumer purchase behavior (Goniil and Srinivasan, 1993), spells on welfare (Blank, 1989), migration (Lindstrom, 1996), fertility (Larsen and Vaupel, 1993), and marriage and divorce (Lillard *et al*., 1995).

Ulviya (2013), employed frailty models for modeling heterogeneity and Suggested as a Semi-parametric regression model is an important way to handle heterogeneity. Regression models take lifetime as the dependent variable and explanatory variables as regressors. Sometimes these models may not provide adequate fit to the data. One of the reasons is due to omission of important covariates.

Zareand (2012) applied parametric frailty and shared frailty models to waiting time to first pregnancy and found that height, age at marriage and menstruation regularity to be important predictors of waiting time to pregnancy (WTTP).

Wienke (2010) considered Halle Lung Cancer (Halluca) study data and applied two different parametric shared gamma frailty models with exponential and Weibull baseline hazards. The results showed that the exponential hazard function was not flexible enough and Weibull model shows a significantly better fit to the data with respect to the likelihood ratio test. Duchateau and Janssen (2008) fit the inverse Gaussian frailty model with Weibull hazard to the udder quarter infection data.

Estimation of the frailty model can be parametric or semi-parametric. In the parametric case, a parametric density is assumed for the event times, resulting in a parametric baseline hazard function. Estimation is then conducted by maximizing the marginal log-likelihood (Gutierrez, 2002). In the semi-parametric case, the baseline hazard is left unspecified and more complex techniques are available to approach that situation (Abrahantes *et al*., 2007). The parametric estimation will be more powerful if the form of the baseline hazard is somehow known in advance (Munda, 2012). Although the baseline hazard function may be modeled parametrically, some have argued that the parameters of the frailty distribution may be sensitive to the choice of distribution for the hazard, and that the choice of distribution for the hazard may, in fact, be more important than the choice of frailty distribution (Santos, 1995).

2.4. Consequence of Ignoring Frailties

Ignoring the existence of heterogeneity will produce incorrect estimation of parameters and their standard errors in survival analysis. According to Keyfitz and Littman (1979), ignoring heterogeneity overestimates life expectancy based on their study on estimating life expectancy in a heterogeneous population. Lancaster (1990) showed that when heterogeneity is ignored, it caused underestimation of covariate effects in his study of unemployment rates. Frailty models are used to make adjustments for over dispersion/under dispersion. When unobserved or unmeasured effects are ignored, the estimates of survival may be misleading. Therefore, corrections for this over dispersion/under dispersion is needed in order to allow for adjustments for those important frailties. Henderson and Oman (1999) showed that ignoring frailty leads to regression coefficient estimates biased towards zero by an amount depending on the distribution and the variability of the frailty terms.

CHAPTER THREE

DATA AND METHODOLOGY

3.1. Data Source

As it is known the first Ethiopia DHS (EMDHS) was conducted in 2000, and then subsequently 2005, 2011 and finally Mini data in 2014. The data set in this study was obtained from Ethiopian Mini Demographic and Health Survey conducted in 2014. The data provide information on sections that covered infant, child, adult, and maternal mortality, child health, nutrition, family planning practices, women's empowerment, and HIV/AIDS related knowledge.

The EMDHS interviewed 8,070 women age 15-49 from a nationally representative sample of 8,475 households. In the collected data key health indicators are provided for the country as a whole, for urban and rural areas, and for each of the nine regional states and two city administrations.

3.2. Variables in the Study

3.2.1. Dependent variable

The response (outcome) variable in this study is time to under-five mortality in days. During the survey all women were asked a series of questions regarding to their children's time of births, time of death and age at death.

3.2.2. Independent (predictor) variables

Several variables are expected to be associated with time to under-five mortality. The following predictor variables were analyzed in this study as possible determinants of time to under-five mortality.

Table 3.1: Description and categories of explanatory variables

Variables	Explanations	Categories
Region	This variable is classified as: Tigray, Affar, Amhara, Oromiya, Somali, SNNP, Benishangul-Gumuz, Gambela, Harari, Addis Ababa and Dire Dawa.	0=Tigray 1=Affar 2=Amhara 3=Oromiya 4=Somali 5=Benishangul-Gumuz 6=SNNP 7=Gambela 8=Harari 9=Addis Ababa 10=Dire Dawa
Residence	Place of residence with categories: Urban and Rural	0=Rural 1=Urban
Family size/house hold size	Total number of family members which is measured in a continuous scale	Continuous
Mother's educational level	The highest level of education women attained with categories: No education, Primary, Secondary and higher	0=No education 1=Primary 2=Secondary and higher
Mother's age at first birth	It is the age of the mother at the time of her first birth. Coding is done as: <20 years, 20-29 years and above 29 years	0=Less than 20 1=20-29 2=Above 29

Sex	Sex of child born with categories: Male and Female	0=Male 1=Female
Wealth index	This is the measure that indicates inequalities in household characteristics, in the use of health and other services. It was categorized as Poor, Middle and Rich.	0=Poor 1=Middle 2=Rich
Preceding birth interval	The interval between the previous birth and the child that is considered. It was coded in months as: 9-23, 24-35, 36-47, 48-59, 60-259, and first born.	0=9-23 months 1=24-35 months 2=36-47 months 3=48-59 months 4=60-259 months 5=first born
Place of delivery	It is the place where child born with categories: Home, Governmental health institutions and private health institutions.	0=Home 1=Governmental health institutions 2=Private health institutions
Source drinking water	It is source of drinking water they used with categories: Piped and other protected sources, and unprotected sources.	0=Piped and other protected sources 1=Unprotected sources
Marital status	This is the variable that shows current marital status of family. It was coded as currently not in union and currently in union.	0=currently not in union 1=currently in union

3.3. Methods of Data Analysis

3.3.1. Survival Analysis

Survival analysis is a collection of statistical procedures for data analysis for which the outcome variable of interest is time until an event occurs. By time**,** we mean years, months, weeks, or days from the beginning of follow-up of an individual until an event occurs; alternatively, time can refer to the age of an individual when an event occurs. By event**,** we mean death, disease incidence, relapse from remission, recovery (e.g., return to work) or any designated experience of interest that may happen to an individual. In a survival analysis, we usually refer to the time variable as survival time**,** because it gives the time that an individual has "survived" over some follow up period.

We also typically refer to the event as a failure**,** because the event of interest usually is death, disease incidence, or some other negative individual experience. However, survival time may be "time to return to work after an elective surgical procedure," in which case failure is a positive event (Kleinbaum and Klein, 2005).

3.3.2. Non- parametric survival analysis

Non-parametric survival analyses are more widely used in situations where there is doubt about the exact form of distribution. In survival analysis, the data are conveniently summarized through estimates of the survival function and hazard function. The estimation of the survival distribution provides estimates of descriptive statistics such as the median survival time. These methods are said to be non-parametric methods since they require no assumptions about the distribution of

survival time. The Kaplan-Meier, Nelson-Aalen and Life Tables are the most widely used to estimate the survival and hazard functions (Collet, 1994).

3.3.3. **Median Survival Time**

Median survival time is the time beyond which 50% of the individuals in the population under study are expected to survive and is given by that value t(50) which is such that $S\{t(50)\} = 0.5$. Due to the fact that the non - parametric estimates $S(t)$ are step functions, it will not usually be possible to realize an estimated survival time that makes the survival function exactly equal to 0.5. Instead, the estimated median survival time, is defined to be the smallest observed survival time for which the value of the estimated survival function is less than 0.5.

In mathematical terms,

$$\hat{t}(50) = \min\{t_{(i)} \setminus \hat{S}(t_{(j)}) < 0.5\}, \text{----------3.1}$$

where $t_{(i)}$ is the observed survival time for the i^{th} individual, $i = 1,2, \dots, n$ and $t_{(j)}$ is the j^{th} ordered death time, $j = 1,2, \dots, r$

3.3.4. The Kaplan-Meier estimate of the survival function

The Kaplan-Meier (KM) estimator is the standard non-parametric estimator of the survival function used for estimating the survival probabilities from observed survival times both censored and uncensored (Kaplan and Meier, 1958).

Suppose that *r* individuals have failures in a group of individuals, let $0 \leq t_{(1)} \dots < t_{(r)} < \infty$ be the observed ordered death times. Let $r_{(j)}$ be the size of the risk set at $t_{(j)}$, where risk set denotes

the collection of individuals alive and uncensored just before $t_{(j)}$. Let $d_{(j)}$ be the number of observed events at $t_{(j)}$, $j = 1, \dots, r$. Then the K-M estimator of $S(t)$ is defined by

$$\hat{S}(t) = \prod_{j:t_{(j)}<t} \left[1 - \frac{d_{(j)}}{r_{(j)}}\right] \text{----------3.2}$$

This estimator is a step function that changes values only at the time of each death of children before their fifth birth date. The cumulative hazard function of the KM estimator can be estimated as:

$\widehat{H}(t) = -ln[\hat{S}(t)]$, where $\hat{S}(t)$ is KM estimator

3.4. Modelling Frailty

Frailty models extend Cox proportional hazards model (Cox, 1972) by introducing unobserved "frailties" to the model. In this case, the hazard rate will not be just a function of covariates, but also a function of frailties. A frailty model is a random effects model which has a multiplicative effect on the hazard rates of all the members of the subgroups. In univariate survival models, it can be used to model the heterogeneity among individuals, which is the influence of unobserved risk factors in a proportional hazards model. In multivariate survival models, shared frailty model is used to model the dependence between the individuals in the group. In the multivariate case, unobserved frailty is common to a group of individuals.

3.4.1. Shared Frailty Models

Multivariate or shared frailty model is a conditional independence model in which frailty is common to all subjects in a cluster. This model is responsible for creating dependence between event times. It is also known as a mixture model because the frailties in each cluster are assumed to be random. It assumes that, given the frailty, all event times in a cluster are independent.

Shared frailty model was introduced by Clayton (1978) without using the notion frailty and extensively studied in Hougaard (2000), Therneau and Grambsch (2000), Duchateau et al. (2002, 2003), and Duchateau and Janssen (2004).

Multivariate frailty model is an extension of the univariate frailty model which allows the individuals in the same cluster to share the same frailty value. When frailty is shared, dependence between individuals who share frailties is generated.

Suppose we have j observations and i subgroups. Each subgroup consists of n_i observations and $\sum_{i=1}^{G} n_i = n$ where n is the total sample size. The hazard rate for the j^{th} individual in the i^{th} subgroup is given by

$$h_{ij}(t) = h_o(\mathrm{t}) u_i \exp\left(Z_{ij}{}^t \beta\right),\ i = 1, \dots, G,\ \ j = 1, \dots, n_i \text{----------3.3}$$

where u_i are frailty terms for subgroups and their distribution is again assumed to be independent with a mean of 0 and a variance of 1. If the number of subjects n_i is 1 for all groups, the univariate frailty model is obtained (Wienke, 2011); otherwise the model is called the shared frailty model (Hougaard, 2000; Duchateau and Janssen, 2008) because all subjects in the same cluster share the same frailty value.

3.4.2. Frailty Distribution

3.4.2.1. Gamma Distribution

Gamma frailty model belongs to the power variance function family (Hougaard, 1986b) and can be expressed in terms of its Laplace transform from which properties such as mean and variance are easily derived (Duchateau and Janssen, 2008). From a computational and analytical point of view, it fits very well to failure data. It is widely used due to mathematical tractability (Wienke, 2011). Assuming a two-parameter gamma density with $\delta > 0$ and $\gamma > 0$ as shape and scale parameters respectively, the density function is given by

$$f_Z(zi) = \frac{\gamma^{\delta} zi^{\delta-1} \exp(-\gamma zi)}{\Gamma(\delta)} \text{----------3.4}$$

with $\delta > 0$ and $\gamma > 0$ and where $\Gamma(.)$ is the Gamma function. The corresponding Laplace transformation is

$$L(s) = \gamma^{\delta}(s+\gamma)^{-\delta} \text{----------3.5}$$

In gamma frailty models, restriction $\delta = \gamma$ is used, which results in expectation of 1. The variance of the frailty variable is then 1. Assuming that the frailty term z_i is a gamma with $E(Z) = 1$ and $Var(Z) = \theta$, then $\delta = \gamma = \frac{1}{\theta}$ (Ulviya, 2013). The distribution function of the frailty term z_i is therefore a one-parameter gamma distribution given by

$$f_Z(zi) = \frac{z_i^{\left(\frac{1}{\theta}\right)-1} \exp(^{-z_i}/_{\theta})}{\Gamma(\frac{1}{\theta})\theta^{\frac{1}{\theta}}} \text{----------3.6}$$

Where $\theta > 0$ and $z_i > 0$ indicates that individuals in group i are frail, whereas $z_i < 0$ indicates that individuals are strong and have lower risk. The corresponding Laplace transform is given by;

$$L(s) = (1 + \frac{s}{\theta})^{-\theta} \text{----------3.7}$$

Note that if θ > 0, there is heterogeneity. So the large values of θ reflect a greater degree of heterogeneity among groups and a stronger association within groups.

The conditional survival function of the gamma frailty distribution is given by: (Gutierrez, 2002).

$$S_\theta(t) = [(1 - \theta ln\{S(t)\})]^{-1/\theta} \qquad \theta > 0 \text{ ----------3.8}$$

The conditional hazard function of the gamma frailty distribution is given by: (Gutierrez, 2002)

$$h_\theta(t) = h(t)[1 - \theta \ln\{S(t)\}]^{-1} \text{----------3.9}$$

where S (t) and h(t) are the survival and the hazard functions of the baseline distributions.

The variance θ of the frailty term represents the heterogeneity among clusters while the mean is constrained to 1 in order to make the average hazard identifiable (Duchateau et al., 2002; Nguti, 2003; Glidden and Vittinghoff, 2004; Duchateau and Janssen, 2008). Larger variance indicates a stronger association within groups.

For the Gamma distribution, the Kendall's Tau (Hougaard, 2000), measures the association between any two event times from the same cluster in the multivariate case and given by:-

$$\tau = \frac{\theta}{(\theta+2)} \text{ , where } \tau\epsilon(0,1) \text{ ----------3.10}$$

3.4.2.2. Inverse Gaussian frailty distribution

The inverse Gaussian (inverse normal) distribution was introduced as a frailty distribution alternative to the gamma distribution by (Hougaard, 1984) and has been used by Klein *et al.*, (1992), and Duchateau and Janssen (2008). Similar to the gamma frailty model, simple closed-form expressions exist for the unconditional survival and hazard functions, this makes the model attractive. The probability density function of an inverse Gaussian shared distributed random variable with parameter θ > 0 is given by

$$f_Z(Z_i) = \frac{1}{\sqrt{2\pi}} {Z_i}^{-3/2} exp\left(\frac{-(z_i-1)^2}{2\theta z_i}\right), \theta > 0, z > 0 \text{ ----------3.11}$$

It has a mean 1 and variance θ, and the Laplace transformation is given by

$$L(s) = exp\left(\frac{1-\sqrt{1+2\theta s}}{\theta}\right) \text{----------3.12}$$

Its conditional survival function is given by:

$$S_\theta(t) = exp\left\{\frac{1}{\theta}\left(1 - [1 - 2\theta ln\{S(t)\}]^{1/2}\right)\right\} \qquad \theta > 0 \text{ ----------3.13}$$

And the conditional hazard function is given by:

$$h_\theta(t) = h(t)[1 - 2\theta ln\{S(t)\}]^{-1/2} \qquad \theta > 0 \text{ ----------3.14}$$

where S (t) and h(t) are the survival and the hazard functions of the baseline distributions.

3.4.2.3. Positive stable distribution

The positive stable (PS) model (Hougaard, 2000) is a useful alternative to gamma distributions, because it has the attractive feature that predictive hazard ratio decrease to 1 over time (Oakes, 1989). The property is observed in familial associations of the ages of onset of diseases with etiologic heterogeneity, where genetic cases occur early and long-term survivors are weakly correlated. The gamma model has predictive hazard ratios which are time invariant and may not be suitable for these patterns of failures (Fine et al., 2003). The probability density function (pdf) of positive stable distribution with two parameters α and δ, restricting the parameters ($\alpha = \delta$) in order to solve the non-identifiably problem is given by: (Hougaard, 2000, p. 503).

$$f(z) = \frac{1}{\pi}\sum_{k=1}^{\infty}\frac{\Gamma(\kappa\alpha+1)}{\kappa!}\left(-\frac{1}{z}\right)^{\alpha\kappa+1}\sin(\alpha\kappa\pi),\ Z > 0, 0 < \alpha < 1, \delta > 0 \text{----------3.15}$$

And its Laplace transformation is given by:

$$L(s) = E(e^{-sz_i}) = e^{-s^{\alpha}} \text{----------3.16}$$

The unconditional survival function of the lifetime T with PS frailty is given by

$$S(t) = \exp(-H(t))^{\alpha} \text{----------3.17}$$

The positive stable model has the advantage that it fits proportional hazards which means that if the conditional model has proportional hazards, so does the marginal distribution. This is an advantage, when considering the model as a random effects model.

3.4.3. Baseline hazard distributions for parametric frailty models

As in the proportional hazards model, parametric or non-parametric forms of baseline hazard can be assumed in frailty models. If non-parametric form is assumed for $h_o(t)$, then semi parametric proportional hazards model is considered and the estimates are usually obtained by using Expectation-Maximization (EM) algorithm.

If parametric form for $h_o(t)$ is assumed, the baseline hazard function is defined as a parametric function and the vector of its parameters that estimated together with the regression coefficients and the frailty parameter(s), and then maximum likelihood estimates can be obtained by maximizing the likelihood function. This study was only consider the parametric forms of baseline hazard for simplicity. Using parametric baseline hazards not only makes the estimation easier, but it can also describe explicitly the effect of the frailty on hazard ratios over time

Let T be a random variable associated with the survival times, *t* be the realization of the random variable T and *f* (*t*) be the underlying probability density function of the survival time t. The cumulative distribution function *F* (*t*), which represents the probability that a subject selected at random will have a survival time less than some stated value *t,* is given by:

$$F(t) = P(T \leq t) = \int_0^t f(u)du, t \geq 0 \text{----------3.21}$$

The survivor function, denoted by $S(t)$, is defined to be the probability of an individual surviving or being event-free beyond time t (experiencing the event after time t). It is defined as

$$S(t) = P(T > t) = \int_t^\infty f(u)du \text{----------3.22}$$

The survival function is merely the complement of the cumulative distribution function, that is $S(t) = 1 - F(t)$ and density function is:-

$$f(t) = \frac{-dS(t)}{dt} \; t > 0 \text{----------3.23}$$

The hazard function is a measure of the probability of failure during a very small interval, assuming that the individual has survived at the beginning of the interval. It is defined as:-

$$h(t) = \frac{f(t)}{s(t)} = \frac{-dlns(t)}{dt} \text{----------3.24}$$

Survival model is usually expressed in terms of hazard function. The cumulative hazard function is defined as:- $H(t) = \int_0^t h(u)du$----------3.25

In this research the following commonly used baseline hazard distributions are considered.

Table 3.2: Parametric distributions for the baseline hazards

Distributions	$\boldsymbol{f(t)}$	$\boldsymbol{S(t)}$	$\boldsymbol{h(t)}$	$\boldsymbol{H(t)}$	**Parameter Space**
Exponentials	$\lambda e^{-\lambda t}$	$e^{-\lambda t}$	λ	$-\ln[S(t)]$	$\lambda > 0$
Weibull	$\lambda\rho t^{\rho-1}\exp(-\lambda t^{\rho})$	$\exp(-\lambda x^{\rho})$	$\lambda\rho t^{\rho-1}$	λt^{ρ}	$\lambda > 0, \rho > 0$

3.5. Parameter Estimation

Frailty models account for the clustering present in grouped event time data. For a right-censored clustered survival data, the observation for subject j $\epsilon J_i = \{1, \dots, n_i\}$ from cluster $i \epsilon I = \{1, \dots, s\}$ is the couple (y_{ij}, δ_{ij}), where $y_{ij} = \min(t_{ij}, c_{ij})$ is the minimum between the survival

time t_{ij} and the censoring time c_{ij}, and the indicator $\delta_{ij} = I(t_{ij} \leq c_{ij})$ is one for a subject where the event has taken place, while $\delta_{ij} = 0$ for a censored observation.

When covariate information's been collected the observation will be $(y_{ij}, \delta_{ij}, X_{ij})$, where X_{ij} denote the vector of covariates for the ij^{th} observation. In the parametric setting, estimation is based on the marginal likelihood in which the frailties have been integrated out by averaging the conditional likelihood with respect to the frailty distribution.

Under the assumption of right-censoring and of independence between the censoring time and the survival time of random variables, given the covariate information, the marginal log-likelihood of the observed data can be given as:

$$l_{marg}(\varphi, \beta, \theta; Z, X) = \sum_{i=1}^{s} \left\{ \left[\sum_{j=1}^{ni} \delta_{ij} \left(log\left(h_o(y_{ij})\right) + {X_{ij}}^T \beta \right) \right] + log \left[(-1)^{(di)} L^{(di)} \left(\left[\sum_{j=1}^{ni} H_o(y_{ij}) exp\left({X_{ij}}^T \beta\right) \right] \right) \right] \right\}$$

where $di = \sum_{j=1}^{ni} \delta ij$ is the number of events in the i^{th} clusters and $L^{(q)}(.)$ is the q^{th}derivative of the Laplace transform of the frailty distribution Z is defined as:-

$$L_{(s)} = E[exp(-Zs)] = \int_0^\infty exp(Z_i s) f(Z_i) dz_i \,, s > 0 \quad \text{and}$$

$$L^{(q)}(s) = (-1)^q \int_0^\infty Z^q exp(-Zs) f(z) dz \,, \; q \geq 0$$

where φ represents a vector of parameters of the baseline hazard function, β the vector of regression coefficients and θ the variance of the random effect. The estimates of φ , β, θ are obtained by maximizing the marginal log-likelihood of the above. This can be done if one is able to compute higher order derivatives $L^{(q)}(.)$ of the Laplace transform up to $q = max\{d_1, .., d_s\}$.

3.6. Comparison of the Models

There are several methods of model selection. One of the most commonly used model selection criteria is Akaike Information Criterion (AIC). A data-driven model selection method such as an adapted version of Akaike's information criterion AIC (Akaike, 1974) is used to find the truncation point of the series. This study used the AIC criteria to compare various candidates of parametric frailty models. The model with the smallest AIC value is considered a better fit.

3.7. Model Diagnostics

For model diagnostics the study used the graphical method of whether or not the distribution fits the observed data. For instance, in case of weibull baseline hazard function, the plot of $log\ (-log\ (\hat{s}(t)))$ versus $log\ (t)$ is used, where $\hat{S}$(t) is Kaplan-Meier survival estimate (Dätwyler and Timon Stucki, 2011).

For the model with the exponential baseline, the plot of $-log\ (\hat{s}(t))$ versus t is used, where $\hat{s}(t)$ is Kaplan-Meier survival estimate. This plot should be linear and goes through the origin (Klein, 1992). Because for exponential distribution, $\hat{S}$(t) = $exp\ (-\lambda t)$, and hence, -log ($\hat{S}$(t)) = λt is linear with time.

3.7.1. The Cox- Snell Residuals

The Cox-Snell residuals method can be applied to any parametric model and the residual plots can be used to check the goodness of fit of the model. For the parametric regression problem, analogs of the semi-parametric residual plots can be made with a redefinition of the various residuals to incorporate the parametric form of the baseline hazard rates (Klein and Moeschberger, 2003).

The Cox-Snell residual for the j^{th} individual with observed survival time t_j is given by $r_{j=\hat{H}(T_j/X_j)=-\log \hat{S}(T_j/X_j)}$, where $\hat{H}$ and $\hat{S}$ are the estimated values of the cumulative hazard and survivor function of the j^{th} subject at time t_j respectively. If the model fits the data, then the $r_j's$ should have a standard (λ=1) exponential distribution, so that a hazard plot of r_j versus the Nelson–Aalen estimator of the cumulative hazard of the $r_j's$ should be a straight line with slope unity and zero intercept. If yes, the fitted model is adequate. In general, Cox-Snell residual that provides a check of the overall fits of the model (Cox and Snell, 1968).

Table 3.3: The Cox–Snell residuals for the common baseline hazard functions that are considered in this study.

Baseline hazard functions	r_j
Exponential	$\hat{\lambda}t_j\exp(\hat{\beta}'X_j)$
Weibull	$\hat{\lambda}t_j^{\hat{\gamma}}\exp(\hat{\beta}'X_j)$

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.1. Descriptive Summary

The descriptive summaries of baseline categorical covariates is given in table 4.1. A total number of 5579 children under five age were included in the study during the data collection. From this 4580 were included in this study due to the fact that time to under-five mortalities of the rest were recorded as zero and some are censored, they were excluded from the study. Among those included in the study, only 354 (7.73%) experienced the event or died before age five. Relatively among the region of Ethiopia, Affar region experienced highest death of under-five 52 (9.98%) followed by Benishangul-Gumuz 38 (9.98%) and Somali region 54 (9.66%). Addis Ababa city administration was the one that experienced relatively the lowest under-five age death 4 (2.36%) followed by Dire Dawa city administration 20 (5.60%). Addis Ababa city administration has also the highest median time to deaths (240 days) among the other region. Regarding mother's educational attainment, relatively about 8.92% have no education, 6.04% attend primary school and 1.05% attended secondary and above. Children from mothers who attended secondary and above has highest median time to death (120 days) than children from other mothers. Children who were residing in the rural (8.67%) Ethiopia experienced death before their fifth birth days than those residing in urban (6.5%) Ethiopia. From the results we can also observe that relatively children who were delivered at their home 306 (8.28%) have highest experience of dying before their fifth age than those delivered at private health institutions 6 (4.51%). Finally, the result also showed that source of drinking water seems to have an impact on survival time of under-five

mortality, thus children who were using unprotected source (7.97%) of water have more chance to die before their fifth age than those who were using Piped and other protected source (7.43%).

Table 4.1: Descriptive summary of covariate variables of under-five mortality in Ethiopia, (EMDHS, 2014)

Covariate Variable	Category	Number of under-five age	Event occurred (r.f %)	Mean time (in days)	Median time (in days)
Region	Tigray	379	25(6.59)	76	20
	Affar	521	52(9.98)	186	35.3
	Amhara	434	28(6.45)	130	19
	Oromiya	552	33(5.97)	128	30
	Somali	559	54(9.66)	162	30
	Benishangul-Gumuz	384	38(9.89)	143	30
	SNNP	604	52(8.61)	188	135
	Gambela	357	26(7.28)	168	90
	Harari	294	22(7.48)	189	28.5
	Addis Ababa	169	4(2.36)	398	240
	Dire Dawa	327	20(6.11)	105	22.5
Residence	Urban	763	23(6.5)	122	30
	Rural	3817	331(8.67)	159	30
Mother's Highest Level of Education	No Education	3183	284(8.92)	163	60
	Primary	1110	67(6.04)	133	21
	Secondary and above	287	3(1.05)	141	120

Age at first birth	Less than 20	2840	230(8.10)	139	30
	20-29	1672	116(6.93)	188	60
	Above 29	68	8(11.76)	118	105
Sex of child	Male	2385	217(9.10)	143	30
	Female	2195	137(6.24)	178	30
Preceding birth intervals	9-23 months	1016	118(11.61)	146	30
	24-35 months	1158	89(7.68)	152	60
	36-47 months	722	50(6.93)	156	75
	48-59 months	373	15(4.02)	157	15
	60-259 months	452	14(3.09)	214	105
	First born	859	67(7.79)	171	285
Wealth index	Poor	2462	209(8.49)	166	60
	Middle	716	58(8.10)	151	21
	Rich	1402	87(6.21)	140	27
Marital Status	Currently not in union	282	21(8.51)	125	60
	Currently in union	4298	330(7.67)	159	30
Place of Delivery	Home	3696	306(8.28)	163	30
	Governmental Hospitals	761	42(5.51)	123	20
	Private Hospitals	133	6(4.51)	82	5
Source of Drinking water	Piped and other protected source	2097	156(7.43)	160	30
	Unprotected Source	2483	198(7.97)	154	60

Source: Ethiopian Mini Demographic and Health Survey, 2014

4.2. Tests of unobserved heterogeneity

In frailty models, θ is estimated to get an idea on heterogeneity in the outcome among clusters. When θ is large and differs significantly from zero; it reflects heterogeneity among clusters and a strong association among individuals in the same cluster. On the other hand, when θ is equal to zero, the frailties are identically equal to one which implies that the cluster effects are not present and events are independent within and across centers (Glidden and Vittinghoff, 2004). The likelihood ratio test is used for comparing the models with and without frailties. In other words, it is used for testing the null hypothesis H_0: θ = 0 versus the alternative hypothesis H_0: θ > 0. This heterogeneity parameter θ from the frailty models was estimated using the Penalized Partial Likelihood (PPL) technique. Since the null hypothesis is at the boundary of the parameter space, the LR test statistic is not the usual χ_1^2 but rather is a 50:50 mixture of chi-square distribution with 0 and 1 degree of freedom, denoted as $\bar{\chi}_{01}^2$ was used, and thus requires careful consideration concerning the calculation of its p-value (Gutierrez, Carter, and Drukker, 2001; Duchateau and Janssen, 2008).

Table 4.2: Tests of unobserved heterogeneity by using likelihood ratio test

Shared frailty models	**LRT**	θ	τ	**p-value**
Weibull-Gamma	7.77	0.863	0.301	0.003
Weibull-Inverse-Gaussian	7.79	0.670	0.251	0.003
Weibull-Positive Stable	11.23	0.052	0.025	0.000
Exponential- Gamma	10.41	0.067	0.032	0.001
Exponential- Inverse-Gaussian	10.36	0.080	0.038	0.001
Exponential- Positive Stable	11.65	0.046	0.022	0.000

Source: Ethiopian Mini Demographic and Health Survey, 2014

LRT=Likelihood-ratio test of theta=0 at chi-square with 0 and 1 degrees of freedom,
θ =theta (variance of random terms), τ=Kendaell's tau

Multivariable analysis was done by assuming the exponential and Weibull baseline hazard functions for gamma, Inverse-Gaussian, Positive Stable shared frailty distributions. The results of this study given in Table 4.2 shows that the likelihood ratio tests of variance of random term (theta) for Weibull-gamma, Weibull-Inverse-Gaussian, Weibull-Positive Stable, Exponential-gamma, Exponential-Inverse-Gaussian and Exponential-Positive Stable shared frailty models were 7.77, 7.79, 11.23, 10.41, 10.36, and 11.65 with p-values 0.003, 0.003, 0.000, 0.001, 0.001, 0.000 respectively. Thus from this results we can conclude that unobservable heterogeneity is significant in all models at 5% level of significance.

This heterogeneity parameter (variance of random term) is highest for Weibull-gamma shared frailty model (θ =0.863) followed by Weibull-Inverse-Gaussian shared frailty model (θ = 0.670) and lowest (θ =0.046) for Positive Stable shared frailty model with exponential baseline hazard. The Kendall's tau (τ) is used to measure the dependence within the clusters (regions) and it is higher for the higher variance of random effect (θ) values. From the results of this study the values of Kendall's tau (τ) for the Weibull-gamma, Weibull-inverse Gaussian, Weibull-Positive Stable, Exponential-gamma, Exponential-inverse Gaussian and Exponential-Positive Stable shared frailty models were 0.301, 0.251, 0.025, 0.032, 0.038, 0.022, respectively (see Annex I). From this evidence we can conclude that, on average, there is a positive correlation between times to deaths within the clusters (regions).

4.3. Multivariable analysis and model comparisons

In order to select variables to be included in multivariable analysis, stepwise variable selection is used. Accordingly, except marital status of family and age of mothers at first birth all variables were statistically significant at 5% level of significance in all shared frailty models.

Several model selection methods have been proposed in the literature. The most commonly used methods include information and likelihood based criteria. For shared frailty models information based criteria is used while for the nested frailty model likelihood ratio test is used.

Therefore, to compare the Gamma, Inverse-Gaussian and Positive Stable shared frailty models with Exponential and Weibull hazard functions, this study used information based criteria. The most commonly used model selection criteria are the Akaike information criterion (AIC) and Bayesian information criterion (BIC). The model with the smallest AIC value is considered a better fit.

Table 4.3. The value of AIC, BIC and LRT for Multivariable Parametric Shared Frailty Models, EMDHS, 2014

Baseline hazard		**Frailty distributions**		
		Gamma	**Inverse Gaussian**	**Positive stable**
Exponential	AIC	7071.493	7071.599	7075.019
	LRT	319	320	322
Weibull	AIC	6270.935	6270.984	6273.850
	LRT	297	298	300

AIC= Akaike's Information Criteria LRT=likelihood ratio test

Table 4.3 summarizes all the results of the three shared frailty models with two baseline hazard functions. Among those models, Gamma shared frailty model with Weibull baseline hazard function has the smallest AIC (6270.935). This indicates that under the given scenario, it is relatively the most appropriate model to describe time-to-under-five mortality in Ethiopia.

Table 4.4. Results of Weibull-gamma multivariable shared frailty model.

Covariates	Coeff	St. err	ϕ	P-value	[95% Conf. Interval]	
Residence						
Urban(ref)	1					
Rural	-2.48126	.7922175	.0927	0.002	-4.033986	-.9285506
Education						
No education(ref)	1					
Primary	1.051828	.4668377	2.9265	0.024	.1368424	1.966813
Sec.and higher	5.825728	1.931444	6.6692	0.003	2.040167	9.61129
Hhsize						
	.5225479	.089177	1.6746	0.000	.3477641	.6973317
Sex						
Male (ref)	1					
Female	1.209173	.3492477	2.9900	0.001	.5246604	1.893686
Preceding						
9-23 months(ref)	1					
24-35 months	1.560625	.4527167	3.0001	0.001	.6733161	2.447933
36-47 months	1.724853	.5485246	3.0424	0.002	.6497648	2.799942
48-59 months	3.63329	.8830749	4.6731	0.000	1.902495	5.364085
60-259 months	3.853066	.9167747	4.6601	0.000	2.05622	5.649911
First born	1.564544	.5301022	3.0153	0.003	.5255627	.603525
Wealth index						
Poor (ref)	1					
Middle	-1.39030	.4649065	.24752	0.003	-2.301501	-.4791008
Rich	-2.46371	.4860436	.08563	0.000	-3.416346	-1.51109
Delivery						

Home (ref)	1					
Gov. health institutions	-.769434	.5811873	.46501	0.180	-1.90854	.369672
Private health institutions	-3.57472	1.41216	.02997	0.011	-6.342504	-.8069394
Water						
Piped or protected source(ref)	1					
Unprotected source	-5.11692	.5098087	.0064	0.000	-6.116133	-4.11772
$\tau = 0.301$ $\theta = 0.863$ $\lambda = 0.006$ $\gamma = 3.14$						

Likelihood-ratio test of theta=0: chibar2 (01) = 10.39 Prob>=chibar2 = 0.000

Source: Ethiopian Mini Demographic and Health Survey, 2014

Coeff= coefficient, St. err= standard error, ϕ = acceleration factor, τ=Kendaell's tau, θ=variance of random effect, λ= scale, γ =shape, chibar2 =Chis-square, chibar2 (01) = Chis-square distribution with 0 and 1 degrees of freedom

From Table 4.4, all categorical variables are statistically significant in determining time-to-under-five mortality except for one categories of place of delivery at 5% level of significant.

An acceleration factor of greater than 1 indicates prolonging the time of death. Accordingly, children from mothers who attended primary school (ϕ =2.9265), from mothers who attended secondary and higher education (ϕ =6.6692), female children (ϕ =2.9900), children whose preceding birth intervals were 24-35 months (ϕ =3.0001), 36-47 months (ϕ =3.0424), 48-59 months (ϕ =4.6731), 60-259 months (4.6601), and those born first (ϕ =3.0153) have prolonging time to death when compared to their corresponding reference categories. In other words, they have higher expected survival time than their corresponding reference categories.

The variability (unobserved heterogeneity) in the population of clusters (region) estimated by the selected model (Weibull-gamma shared frailty model) is θ = 0.863, and the dependence within clusters is about τ =30.1% which is the highest unobserved heterogeneity than the other models.

The estimate value of shape parameter in this selected model is (γ=3.14). This value is greater than unity that indicates the shape of hazard function is unimodal, that is, it increases up to some time and then decreases.

4.4. Survival function of different groups

4.4.1. Survival Function by Mothers' Educational attainment

Descriptive graphs of survivor function would be used for the purpose of comparing the event experiencing time of two or more groups and the survival quantities of covariates to describe the survival experience of an individual at specific times.

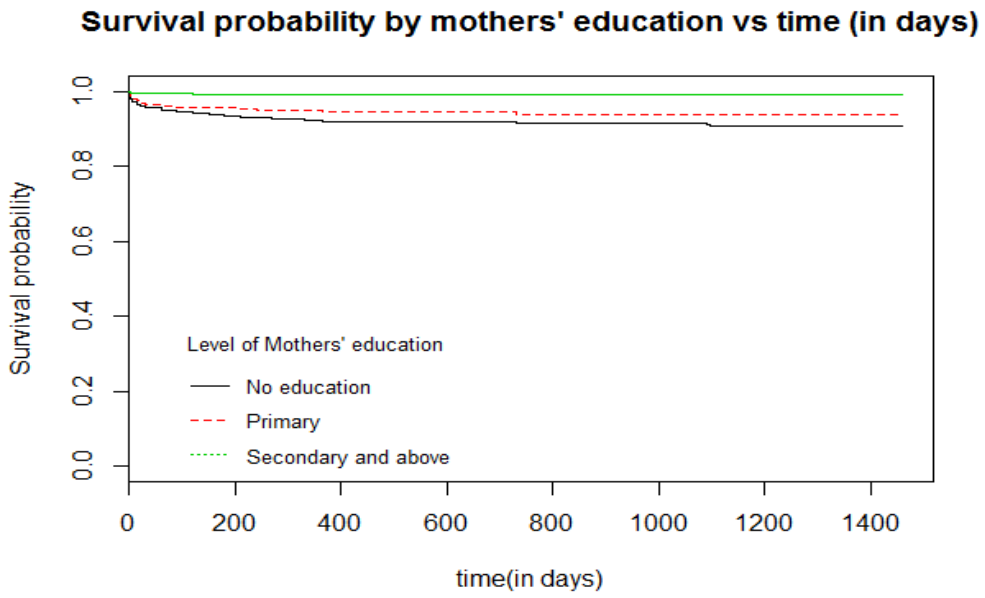

Figure 1: Survival function of time-to-under-five mortality by mother's educational attainment

Figure 1 displays the survival function against survival time for time-to-death by mothers' educational attainments. In all frailty models the categories of mother's educational level were

significant at 5% level of significance when compared with the reference category (no education). The gap between the three curves distinguishes that the survival distribution of time-to-under-five mortality in Ethiopia by mother's educational level. The differences that are displayed in survival curve shows that children from mothers who attend at least primary school have higher survival time when compared to those from mothers who have no education.

4.4.2. Survival function by place of delivery

Place of delivery is one factor of interest in determining time-to-under-five-mortality. That is, many literature suggests that children who have antenatal care follow up have less chance to die before their fifth birth dates. From the results displayed in Figure 2, it seems that there is an effect on time-to-death due to place of delivery. That is, children delivered in governmental and private hospital seems to have higher survival time to death than those delivered at their home.

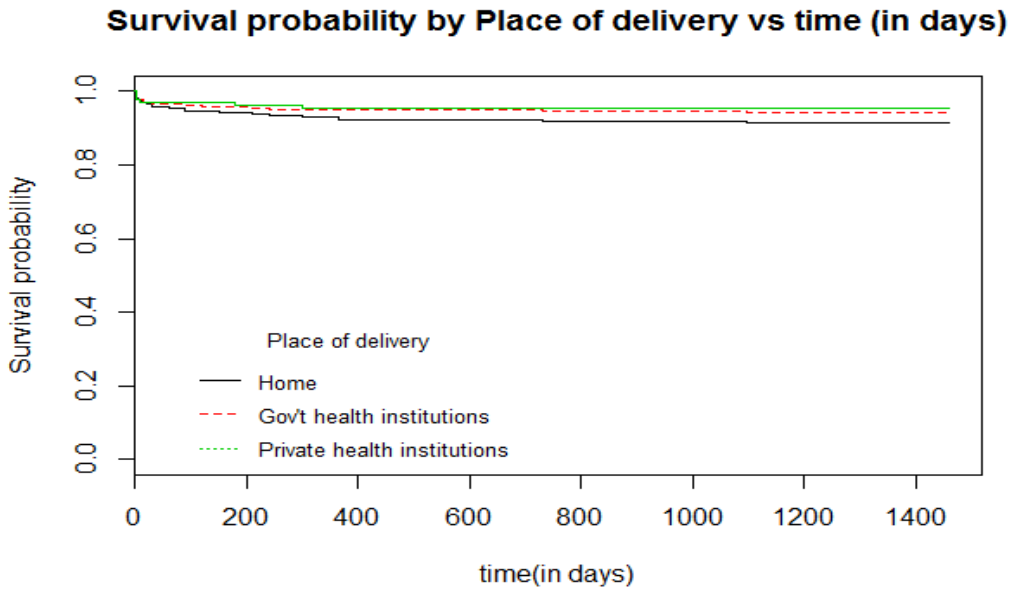

Figure 2: Survival function of time-to-under-five mortality by place of delivery.

4.5. Checking for overall goodness of fit

4.5.1. Diagnostic Plots of the Parametric Baselines

The final step in the model assessment is to see the overall goodness of fit. Therefore, it is desirable to determine whether a fitted parametric model adequately describes the data or not. In other words, the preliminary final model shall be diagnosed for describing our data optimally or not. To check the adequacy of the baseline hazards, the exponential baseline hazard is plotted by using the estimated cumulative hazard function versus time of study. To be more appropriate baseline hazard, the plot must be straight line in time with the slope (λ) and intercept zero. For Weibull we used the plot of logarithms of estimated cumulative hazard function versus logarithms of time to see whether it is a straight line in log(t) with slope (γ) and intercept log(λ) or not.. From the two plots it seems that Weibull is more linear than exponential. This indicates that Weibull is more appropriate baseline hazard in the models.

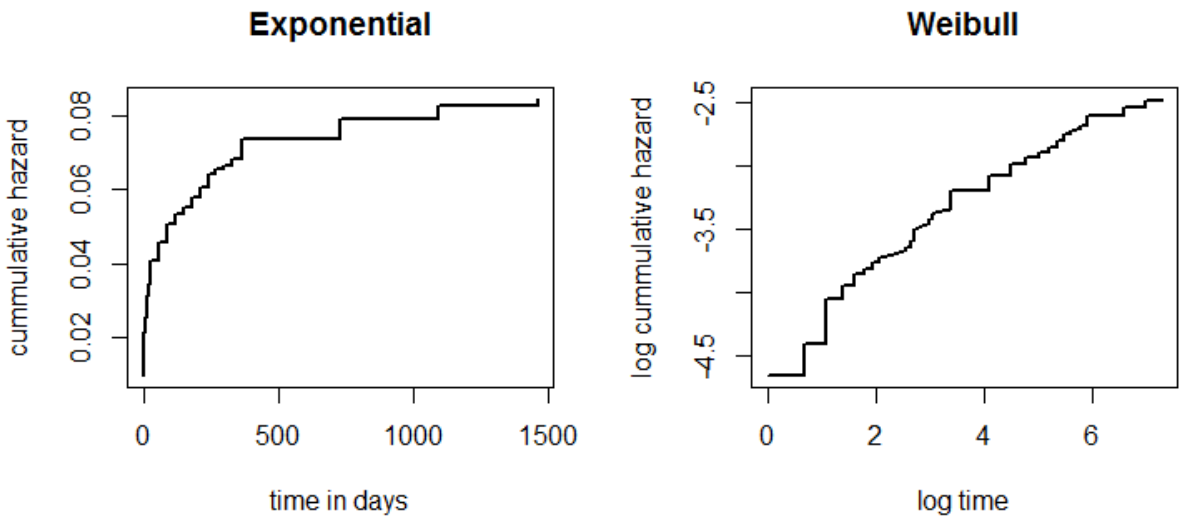

Figure 3: Evaluation of exponential and Weibull assumption**.**

4.5.2. Cox Snell Residual plots

The Cox-Snell residual is one way to investigate how well the model fits the data. If model fits, residuals should look like a censored sample from a unit exponential distribution. That is, deviations from expected should be small. The plot for fitted model of residuals for exponential and Weibull to the data via maximum likelihood estimation with cumulative hazard functions are given in Figure 4. It can be seen that the plot of the residuals for Weibull hazard function is fairly close to the 45^0 straight line through the origin than that of exponential hazard. Thus, the plot gives us an evidence that the Weibull model fitted to the data is relatively satisfactory. This result is similar with the result obtained from the Weibull plot in Figure 3.

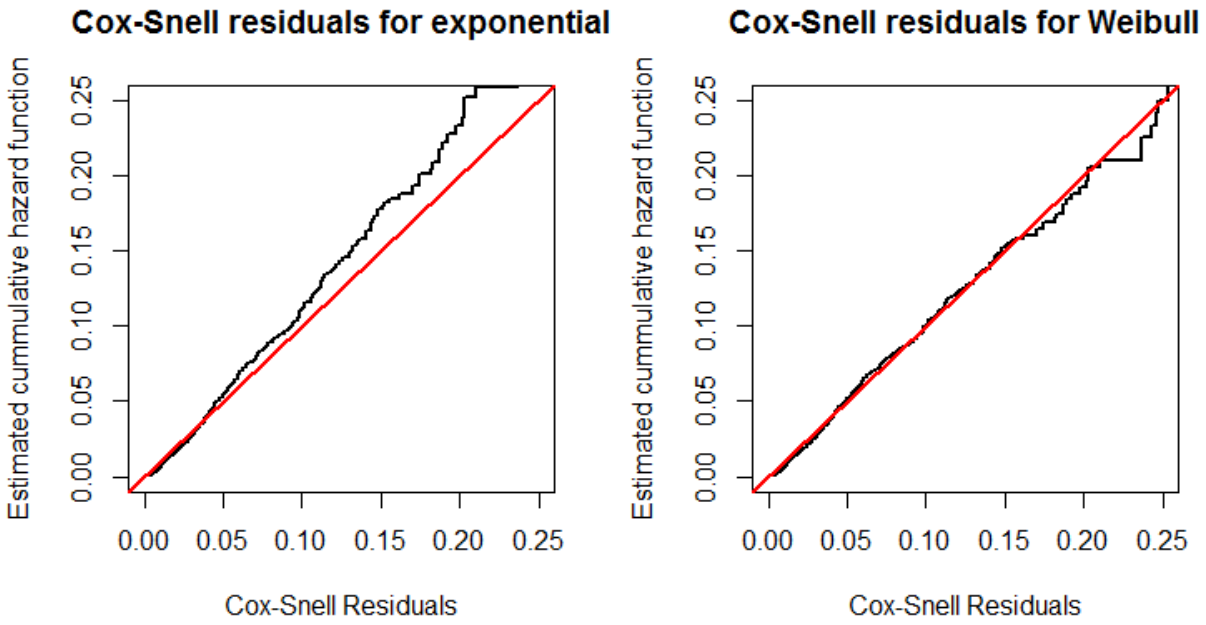

Figure 4: Estimated cumulative hazard plot of the Cox-Snell residuals for the exponential and Weibull models

4.6. Discussion of the result

The main aim of this study was to identify the determinants of time-to-under-five mortality in Ethiopia by using Gamma, Inverse-Gaussian and Positive Stable shared frailty models among the various parametric frailty models with Exponential and Weibull hazard as a baseline.

In this study, since population in the same region relatively have some shared factors such as environment, health facilities and others in determining time-to-under-five mortality in Ethiopia, regions were used as clusters. The effect of clustering (unobserved heterogeneity) between the clusters was tested by likelihood ratio test and the result reveled that in all shared frailty models the variance of the random term is statistically significant at 5% level of significance (see Annex I). This showed that we have to include the effect of random terms in our models since the correlation within regions cannot be ignored. Clusters with minimum median time have smaller frailties, so that these clusters are predicted to have a high hazard; more probably they have shorter time to death (Duchateau and Janssen, 2008). Clusters which are frail more are more likely to die than less frail since the event is positive. These nuisance terms modify the hazard function, so that the hazard function should be evaluated conditionally on this effect.

There are three types of frailty models: shared, nested and joint frailty models. For the comparison of distributions of the models, AIC is used for shared frailty model while LRT is used for nested and joint frailty models (Virginie, *et al*., 2012). Therefore, for this study, the comparison was done by using the AIC criteria, where a model with minimum AIC is accepted to be the best (Munda, 2012).

Accordingly, Weibull-Gamma shared frailty model which have AIC value of 6270.935 was the most appropriate model to describe time-to-under-five mortality data (Table 4.3). The result confirmed that Weibull is the most acknowledged parametric model as it allows the proportional hazards and accelerated life time model when compared to other parametric models (Hougaard, 2000).

The prognostic factors considered in this study were the place of residence, age of mothers at first birth, sex of child, mothers' educational attainment, household size, preceding birth interval, wealth index, place of delivery, source of drinking water and marital status of family.

All these variables were included in the model and finally by using variable selection strategy (stepwise method) significant predictors were selected and multivariable analysis was done. Accordingly, except marital status of families and age of mothers' at first birth, all the included variables are statistically significant at 5% level of significance (see Annex I).

Among the socio-economic factors, the importance of mothers' education on child survival and health has been well documented. Educated mothers are less likely to experience a child death as compared to their non-educated counterparts. This could be explained by the benefits of education, which include economic advantage and the ability of educated mother to access health services and live a modern life, both of which are associated with increased child survival (Caldwell 1990; UNICEF, 2012). Education also gives women power and confidence to make their own decisions (Ware, 1984). The results of this study also revealed that relatively children from educated mothers have more survival time to death when compared to mothers with no education. That is, from the selected model we can observe that the prognostic time is increased by a factor (ϕ =6.6692) for children from mothers who attended secondary and higher education when compared to those

whose mothers have no education (Table 4.4). In other words, children from mothers who attended secondary and higher education have 66.69% more prolonged time to death than those from mothers who have no education. From this we can conclude that as educational level of mothers' increase, the survival time to death for children also increase. A study using data from 175 countries revealed that almost 4.2 million deaths averted between 1970 and 2009 can be attributed to the increase in women's education (Gakidou *et al.* 2010). This concurs with assertions by UNICEF that more than half of recent reductions in child deaths are linked to gains in women's educational attainment (UNICEF, 2012). The results by (Clara Lemani, 2013) using logistic regression of 2004 DHS data, under-five mortality of Malawi showed also that children whose mothers had primary education had lower risks of death compared to those whose mothers had no education. This is in agreement with what Basu and Stephenson (2005) suggested that even a little schooling may have an impact on child survival.

Mothers' place of residence is another variable of interest in studying determinants of child survival time. Provision of social services, which include schools, health services, transport services, economic wellbeing of the household and even the reproductive behavior of the people in urban areas is different from that of rural areas. The mothers' place of residence, therefore, indirectly affects time to under-five mortality by determining a woman's access to a health care facility, ability to find transportation and even ability to have money for paid health care services (Mahy, 2003). Studies have found an increase in the risk of death among children whose mothers reside in rural areas compared to those who stay in urban areas (Kayode, Adekanmbi and Uthman 2012; Kembo and van Ginneken, 2009). The results of this study concedes with this idea, that is relatively the risk of death were higher in rural areas 331 (8.67%) than those resides in urban areas

23 (6.5%). In Nigeria, living in rural areas increased the risk of under-five mortality by 53 per cent compared to children in urban settlements (Kayode, Adekanmbi and Uthman, 2012).

Concerning sex of child, biologically males have higher risk of death relative to females (Elizabeth, 2010), which also has been confirmed in the results of this study. That is the median survival time to death for female is 178 days while for male is 143 days. Besides to this, from Weibull-gamma shared frailty model the acceleration factor for females is greater than 1 (ϕ=2.9900) which indicates females have 2.9900 more prolonging times to death than males. In Kenya, Mutunga (2007) found that male children had a lower probability of surviving than females. This is consistent with other studies (Bolstad and Manda 2001; Kabir, Islam, Ahmed *et al.* 2001). Since differentials in mortality between males and females are endogenous (biological), it is less likely to be improved through an intervention (Barnet, *et.al*., 1989).

From the findings of this study, we can also observe that the length of the preceding birth interval is significantly affect time to under-five mortality of Ethiopia since p-values of all its categories were significant at 5% level of significance. The result showed that the survival time to death is increased as the length of preceding birth interval increased, that is, the survival time to death is increased by 3.0001, 3.0424, 4.6731 and 4.6601 for the length of birth intervals 24-35, 36-47, 48-59 and 60-259 months, respectively, when compared to those born within 9-23 months birth intervals. This is consistent with other studies (Das Gupta 2010; Kayode, *et.al*, 2012). Short preceding birth interval is clearly a risk factor for both the child and mother. Mothers who wait for at least 24 months have enough time to regain the nutrients and blood loss during the previous child birth and breast feeding. This reduces the risks of obstetric complications and also enhances the full development of fetus in the next pregnancy thereby reducing the risks of death early in

life. Majumder and May (1997) also argued that an additional child implies an added responsibility to parents and may lead to a reduction in resources and care per child. Closely spaced children are therefore, more likely to face greater competition with the younger ones being more likely to suffer.

Wealth index of family is also another factor that determine time-to-under-five mortality in Ethiopia. This is due to the fact that household economic status may lead to good health care and nutrition for children since the parents can afford private health care and nutritious food. Children from households with a good economic status are therefore more likely to receive quality care in terms of nutrition and health services, which lead to an increase in child survival. The results of this study also acknowledges children from rich households had an increased survival time when compared to those from poor family. The study by (Blakely, Atkinson, Kiro *et al.* 2003; Mahfouz, Surur, Ajak *et al.* 2009) also confirms children living in low occupational class households experienced a lower survival time to death when compared with children from a high occupational class.

Another important factor is the place of delivery. Children born in government and private hospitals (maternity clinics) have a lower risk of mortality due to proper health care and attention they receive during and after delivery. This has been supported by different studies (Abimbola, Adepoju, Akanni *et al.* 2012; Rutstein 2000). In relation to that, a study in Ethiopia found that children whose mothers had no antenatal care follow up were 2.3 times more likely to die than those whose mothers had at least one follow up (Dube, *et.al*, 2013). From descriptive summary of this study we can also observe that the risk of death is lower for those delivered in governmental

and private health institutions than those born at their home, but, from the selected model we cannot observe this result. This inconsistency might be due to data or any other factors.

Finally the findings of this revealed that source of drinking water have a significant effect on time-to-under-five mortality in Ethiopia, since p-value =0.000. The relative percentage of risk of death of children from families whose water source were piped and other protected source is 7.43% and 7.97% for those using unprotected source of water (Table 4.1). Furthermore, the results from the selected model showed that the survival time is decreased by 0.0064 for children from family using unprotected source of water. This is due to the fact that access to good sanitation facilities is believed to reduce morbidity and many waterborne diseases such as diarrhea, which is one of the major causes of under-five mortality in Sub- Saharan Africa countries. It is therefore thought that water source influences mortality both directly and indirectly through omitted variables that are correlated with water source and under-five mortality.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1. Conclusions

Increasing child survival time is the major goal almost in every country. To do that it is better to identify determinants that are related to time to under five mortality. A number of socioeconomic, bio-demographic and environmental factors were identified from the literature as control variables. In this study the major factors identified were mothers' educational level, mothers' age at first birth, place of residence, household size, sex of child born, preceding birth interval, economic status of family, place of delivery, marital status of family, and source of drinking water. The study has found relationships between these variables and time-to- under-five mortality which are generally similar to findings from previous studies. In general, the survival time to death was higher for children of educated mothers, for children of young mothers, for female children, for children of small family size, for children of born with high birth intervals, and for children of rich family using protected and piped source of water.

The findings of this study revealed that clustering effect (effect of unobserved heterogeneity) between the regions of Ethiopia was significant in describing time-to-under-five mortality. Therefore, the effects of this unobserved heterogeneity (random term) was included in the model. By using AIC criteria, relatively, Weibull-gamma shared frailty model is the most appropriate model among the various shared frailty models for time-to-under-five mortality in Ethiopia under the given scenario (based on the 2014 EMDHS data). Finally, the model adequacy checking was done by using Cox-Snell residual and the result showed that the Weibull distribution seems better to describe the data.

5.2. Recommendations

The findings of this study have important policy implications. Hence, based on the results of this study we make the following recommendations:

- ✓ Mothers' educational level, place of residence, household size, sex of child born, preceding birth interval, economic status of family, place of delivery, and source of drinking water were significant factors and need to be considered when planning and developing policies against under-five mortality in order to successfully work towards achieving the goal of increasing child survival time.
- ✓ According to the findings of this study, child spacing will help to increase time-to-under five-mortality. There is therefore, a need to sensitize people on the importance of child spacing and implement health programs that will encourage use of contraception for child spacing.
- ✓ Although Ethiopia has managed to achieve a significant increase in survival time of children, the risks of death were not equal in all regions. It remained high for regions such as Affar, Benishangul-Gumuz and Somali regions when compared to others. Therefore, more effort is still required to reduce these mortality rates in those regions.
- ✓ For those who have an interest in comparing parametric shared frailty models, we recommend that to carry out simulation studies to get more appropriate results.

5.3. Limitation of the study

The study faced a number of limitations ranging from those in the data and even the methodology adopted which may have affected the results to some extent. First, in the collected data (EMDHS, 2014), for some observations, time to death were recorded as zero. As it is known time must be positive in survival analysis. Due to this fact observations with zero time to death were ignored from this study. This might mislead the findings of the study.

Second, this study also did not include some of the child care variables like breastfeeding, HIV status of mothers, and occupational status of family. These factors have been found to be significantly affect childhood mortality in different settings (Abimbola, Adepoju, Akanni *et al.* 2012; Mustafa and Odimegwu 2008). They were, however, not included in Mini Demographic and Health Surveys conducted on 2014. Future research may include these and other factors to determine their effect on time-to-under-five mortality in Ethiopian setting.

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ANNEX I

A. Exponential -Gamma multivariable Frailty Model

Covariate	Coeff	St. err	ϕ	P-value	[95% Conf. Interval]	
Residence						
Rural (ref)	1					
Urban	-.830303	.249054	.4412	0.001	-1.318442	-.3421652
Education						
No education (ref)	1					
Primary	.332125	.1467339	1.4025	0.024	.0445318	.6197182
Sec.and higher	1.862408	.6047077	4.2561	0.002	.6772024	3.047613
Hh size						
	.1922675	.0290434	1.2248	0.000	.1353434	.2491915
Sex						
Male (ref)	1					
Female	.3808595	.1097034	1.4567	0.001	.1658448	.5958741
Preceding						
9-23 months(ref)	1					
24-35 months	.5036788	.1424922	1.6231	0.000	.2243993	.782958
36-47 months	.5561272	.1727122	1.7781	0.001	.2176179	.894636
48-59 months	1.175944	.2773974	3.2525	0.000	.6322551	1.71963
60-259 months	1.237646	.2880146	3.4529	0.000	.6731474	1.80214
First born	.5614139	.1671877	1.7556	0.001	.2337321	.889095
Wealth						
Poor (ref)	1					
Middle	-.414138	.1457209	.6524	0.004	-.6997462	-.128530
Rich	-.801552	.1528185	.4215	0.000	-1.101071	-.502033

Delivery						
Home (ref)	1					
Gov. hospitals	-.339229	.1821015	0.7265	0.062	-.696142	.017682
Private hospitals	-1.29827	.4448618	0.2548	0.004	-2.170184	-.426357
Water						
Piped or protected source(ref)	1					
Unprotected source	-1.70238	.1608258	0.6216	0.000	-2.017599	-1.38717
$\tau = 0.032$ $\theta = 0.067$ $\lambda = 0.042$ $\gamma = 1$						

Likelihood-ratio test of theta=0: chibar2(01) = 10.41 Prob>=chibar2 = 0.001

Source: Ethiopian Mini Demographic and Health Survey, 2014

Coeff= coefficient, St. err= standard error, ϕ = acceleration factor, τ=Kendaell's tau, θ=variance of random effect, λ= scale, γ =shape, chibar2 =Chis-square, chibar2 (01) = Chis-square distribution with 0 and 1 degrees of freedom

B. Weibull-gamma multivariable shared frailty model

Covariates	Coeff	St. err	ϕ	P-value	[95% Conf. Interval]	
Residence						
Urban (ref)	1					
Rural	-2.48126	.7922175	.0927	0.002	-4.033986	-.9285506
Education						
No education(ref)	1					
Primary	1.051828	.4668377	2.9265	0.024	.1368424	1.966813
Sec.and higher	5.825728	1.931444	6.6692	0.003	2.040167	9.61129
Hhsize	.5225479	.089177	1.6746	0.000	.3477641	.6973317
Sex						
Male (ref)	1					

Female	1.209173	.3492477	2.9900	0.001	.5246604	1.893686
Preceding						
9-23 months(ref)	1					
24-35 months	1.560625	.4527167	3.0001	0.001	.6733161	2.447933
36-47 months	1.724853	.5485246	3.0424	0.002	.6497648	2.799942
48-59 months	3.63329	.8830749	4.6731	0.000	1.902495	5.364085
60-259 months	3.853066	.9167747	4.6601	0.000	2.05622	5.649911
First born	1.564544	.5301022	3.0153	0.003	.5255627	.603525
Wealth index						
Poor (ref)	1					
Middle	-1.39030	.4649065	.24752	0.003	-2.301501	-.4791008
Rich	-2.46371	.4860436	.08563	0.000	-3.416346	-1.51109
Delivery						
Home (ref)	1					
Gov.hospitals	-.769434	.5811873	.46501	0.180	-1.90854	.369672
Private hospitals	-3.57472	1.41216	.02997	0.011	-6.342504	-.8069394
Water						
Piped or protected source(ref)	1					
Unprotected source	-5.11692	.5098087	.0064	0.000	-6.116133	-4.11772
$\tau = 0.301$ $\theta = 0.863$ $\lambda = 0.006$ $\gamma = 3.14$						

Likelihood-ratio test of theta=0: chibar2 (01) = 10.39 Prob>=chibar2 = 0.000

Source: Ethiopian Mini Demographic and Health Survey, 2014

Coeff= coefficient, St. err= standard error, ϕ = acceleration factor, τ=Kendaell's tau, θ=variance of random effect, λ= scale, γ =shape, chibar2 =Chis-square, chibar2 (01) = Chis-square distribution with 0 and 1 degrees of freedom

C. Exponential- Inverse Gaussian Multivariable frailty model

Covariate	Coeff	St. err	ϕ	P- value	[95% Conf. Interval]	
Residence						
Urban (ref)	1					
Rural	-.8250818	.2489591	0.4389	0.001	-1.313033	-.337131
Education						
No education (ref)	1					
Primary	.3355294	.1467132	1.3898	0.022	.0479769	.6230819
Sec. and higher	1.865155	.6046594	4.4214	0.002	.6800442	3.050266
Hh size						
	.1923092	.02905	1.2652	0.000	.1353722	.2492461
Sex						
Male (ref)	1					
Female	.3808293	.1097088	1.3451	0.001	.1658041	.5958545
Preceding						
9-23 months(ref)	1					
24-35 months	.5579582	.1726606	1.6975	0.000	.2254218	.7839301
36-47 months	.5579582	.1726606	1.7547	0.001	.2195496	.8963668
48-59 months	1.17799	.2774028	3.2475	0.000	.6342903	1.721689
60-259 months	1.238968	.2879712	3.4639	0.000	.6745545	1.803381
First born	.5622723	.1671601	1.7954	0.001	.2346444	.8899001
Wealth						
Poor (ref)	1					
Middle	-.4132705	.1457651	0.6421	0.005	-.6989649	-.1275761
Rich	-.8001402	.1528661	0.4985	0.000	-1.099752	-.5005281
Delivery						

Home (ref)	1					
Gov.hospitals	-.3397424	.1820947	0.7124	0.062	-.6966414	.0171566
Private hospitals	-1.298769	.4448477	0.2942	0.004	-2.170654	-.4268832
Water						
Piped or protected source(ref)	1					
Unprotected source	-1.703809	.1609494	0.1952	0.000	-2.019264	-1.388354
$\tau = 0.038$ $\theta = 0.080$ $\lambda = 0.046$ $\gamma = 1$						

Likelihood-ratio test of theta=0: chibar2 (01) = 10.36 Prob>=chibar2 = 0.001

Source: Ethiopian Mini Demographic and Health Survey, 2014

Coeff= coefficient, St. err= standard error, ϕ = acceleration factor, τ=Kendaell's tau, θ=variance of random effect, λ= scale, γ =shape, chibar2 =Chis-square, chibar2 (01) = Chis-square distribution with 0 and 1 degrees of freedom

D. Weibull- Inverse Gaussian Multivariable frailty model

Covariates	Coeff	St. err	ϕ	P-value	[95% Conf. Interval]	
Residence						
Urban(ref)	1					
Rural	-2.46873	.7920087	0.0912	0.002	-4.02103	-.916421
Education						
No education(ref)	1					
Primary	1.060629	.4667297	2.6452	0.023	.1458554	1.975402
Sec.and higher	5.831528	1.930865	6.4789	0.003	2.047102	9.615955
Hhsize	.5225298	.0891832	1.2854	0.000	.347734	.6973257
Sex						
Male (ref)	1					

Female	1.209426	.3492368	2.4120	0.001	.5249348	1.893918
Preceding						
9-23 months(ref)	1					
24-35 months	1.561941	.4526066	2.8512	0.001	.6748481	2.449034
36-47 months	1.729109	.5483282	2.9110	0.002	.6544052	2.803812
48-59 months	3.638275	.8829496	4.6502	0.000	1.907725	5.368824
60-259 months	3.85615	.9164913	4.6692	0.000	2.05986	5.652441
First born	1.565955	.5299861	2.8121	0.003	.5272017	2.604709
Wealth index						
Poor (ref)	1					
Middle	-1.38851	.4650184	0.2501	0.003	-2.299934	-.477095
Rich	-2.45952	.4860282	0.0910	0.000	-3.412118	-1.50692
Delivery						
Home (ref)	1					
Gov.hospitals	-.771655	.5810853	0.4245	0.184	-1.910561	.367251
Private hospitals	-3.57351	1.412072	.02178	0.011	-6.341126	-.805905
Water						
Piped or protected source(ref)	1					
Unprotected source	-5.12071	.5099294	.0069	0.000	-6.120154	-4.12126
$\tau = 0.251$ $\theta = 0.670$ $\lambda = 0.315$ $\gamma = 3.13$						

Likelihood ratio of theta=0: chibar2 (01) = 7.79 Prob>=chibar2 = 0.003

Source: Ethiopian Mini Demographic and Health Survey, 2014

Coeff= coefficient, St. err= standard error, ϕ = acceleration factor, τ=Kendaell's tau, θ =variance of random effect, λ= scale, γ =shape, chibar2 =Chis-square, chibar2 (01) = Chis-square distribution with 0 and 1 degrees of freedom

E. Exponential-Positive Stable Multivariable frailty model

Covariates	Coeff	St. err	ϕ	P-value	[95% Conf.Interval]	
Residence						
Urban(ref)	1					
Rural	-0.830	0.02064	0.4478	0.000	-0.87077	-0.78985
Education						
No education(ref)	1					
Primary	0.331	0.09270	1.3354	0.003	0.149433	0.512802
Sec.and higher	1.862	0.18986	3.9783	0.000	1.489499	2.233719
Hhsize						
	0.192	0.00208	1.2201	0.000	0.188068	0.196206
Sex						
Male (ref)	1					
Female	0.380	0.03061	1.4751	0.000	0.320282	0.440284
Preceding						
9-23 months(ref)	1					
24-35 months	0.504	0.03245	1.6620	0.000	0.440314	0.567510
36-47 months	0.557	0.09377	1.7563	0.001	0.373692	0.741280
48-59 months	1.179	0.11991	3.2643	0.000	0.944160	1.414193
60-259 months	1.240	0.02326	3.4752	0.000	1.194683	1.285868
First born	0.562	0.03494	1.7690	0.001	0.493327	0.630280
Wealth index						
Poor (ref)	1					
Middle	-0.415	0.02606	0.6756	0.000	-0.46584	-0.363688
Rich	-0.806	0.01686	0.4542	0.000	-0.83929	-0.773211
Delivery						
Home (ref)	1					

Gov.hospitals	-0.338	0.02676	0.7215	0.059	-0.39006 -0.285167
Private hospitals	-1.297	0.09009	0.2799	0.001	-1.47378 -1.120634
Water					
Piped or protected source(ref)	1				
Unprotected source	-1.706	0.01160	0.0020	0.000	-1.728536 -1.68305
$\tau = 0.022$ $\nu = 0.046$ $\lambda = 0.002$ $\gamma = 1$					

Likelihood-ratio test of theta=0: chibar2 (01) =11.65 Prob>=chibar2 = 0.000

Source: Ethiopian Mini Demographic and Health Survey, 2014

Coeff= coefficient, St.err= standard error, ϕ = acceleration factor, τ=Kendaell's tau, ν=variance of random effect, λ= scale, γ =shape, chibar2 =Chis-square, chibar2 (01) = Chis-square distribution with 0 and 1 degrees of freedom

F. Weibull-Positive Stable Multivariable frailty model

Covariates	Coeff	St. err	ϕ	P-value	[95% Conf.Interval]
Residence					
Urban(ref)	1				
Rural	-2.435	0.7894	0.0899	0.000	-3.9827 -0.88818
Education					
No education(ref)	1				
Primary	1.037	0.4631	2.8224	0.002	0.12979 1.94519
Sec.and higher	5.739	1.9196	6.4992	0.002	1.97674 9.50153
Hhsize					
	0.514	0.0907	1.6881	0.000	0.33649 0.69207
Sex					
Male (ref)	1				

Female	1.190	0.3486	3.2978	0.000	0.50714	1.87357
Preceding						
9-23 months(ref)	1					
24-35 months	1.537	0.4519	4.6588	0.001	0.65120	2.42260
36-47 months	1.700	0.5466	5.4110	0.001	0.62882	2.77135
48-59 months	3.595	0.8857	6.0012	0.000	1.85941	5.33126
60-259 months	3.806	0.9209	6.0124	0.000	2.00059	5.61052
First born	1.538	0.5264	4.6591	0.003	0.50642	2.56981
Wealth index						
Poor (ref)	1					
Middle	-1.387	0.4608	0.2102	0.004	-2.2897	-0.48352
Rich	-2.456	0.4899	0.0819	0.000	-3.4163	-1.49589
Delivery						
Home (ref)	1					
Gov. hospitals	-0.751	0.5725	0.4952	0.200	-1.8731	0.37106
Private hospitals	-3.509	1.3987	0.0326	0.018	-6.2501	-0.76727
Water						
Piped or protected source(ref)	1					
Unprotected source	-5.081	0.5474	0.0065	0.000	-6.1535	-4.00767
$\tau = 0.025$ $\nu = 0.052$ $\lambda = 0.007$ $\gamma = 0.315$						

Likelihood-ratio test of theta=0: chibar2 (01) = 11.23 Prob>=chibar2 = 0.000

Source: Ethiopian Mini Demographic and Health Survey, 2014

Coeff= coefficient, St.err= standard error, ϕ = acceleration factor, τ=Kendaell's tau, ν =variance of random effect, λ= scale, γ =shape, chibar2 =Chis-square, chibar2 (01) = Chis-square distribution with 0 and 1 degrees of freedom