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SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

Cooperative Diversity for Broadband Vehicular Communication under Doubly-Selective Channel

By

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Declaration

I, the undersigned, declare that this thesis is my original work, has not been presented for a degree in this or any other university, and all sources of materials used for the thesis have been fully acknowledged.

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Place: Addis Ababa

Date of Submission: ________________

This thesis has been submitted for examination with my approval as a university advisor.

Dr.-Ing. Dereje Hailemariam ______________

Advisor’s Name Signature

Abstract

Previous works on vehicular communication assume the channel to be frequency-flat and quasi-static. But this assumption is unrealistic in now days where communication takes place at a high data rate and also when large number of reflectors are present between transmitter and receiver. These factors introduce frequency-selective behavior to the channel and also the movement of reflectors and/or communicating terminals makes the channel to be time-selective. Therefore it is better to assume our channel to be doubly-selective.

Mitigation techniques are required to have a reliable communication in this type of channel. Precoding, equalizer and cooperative communication are considered as mitigation techniques in this thesis work. Selection Combining (SC) and Equal Gain Combining (EGC) are used as diversity combining scheme. It is assumed that the relay just amplifies and forwards (AF) the information it has received from the source.

Simulation results show that cooperative diversity along with precoding and equalizer improves the performance of fading channel and hence provide a better service even in the worst type of fading channel i.e., doubly-selective channel. Taking a fixed bit error rate (BER) of 10-4, performance was improved by 2.5dB for EGC scheme and by 0.5dB for SC.

Key Words: Doubly-selective channels, Equalizer, Precoding, Cooperative communication, Diversity combining.

Acknowledgment

I would like to take this chance to express my gratitude to all those around me who made this work happen. Family and friends, thank you for being my courage throughout the course of this thesis. I also would like to extend my thanks to my advisor Dr.-Ing. Dereje Hailemariam for inspiring me to work on this area from the very start. His ideas and guidance had an immense contribution.

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ACI	Adjacent Channel Interference
AF	Amplify and Forward
AWGN	Additive White Gaussian Noise
BEM	Basis Expansion Model
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CE-BEM	Complex Exponential Basis Expansion Model
COFDM	Coded Orthogonal Frequency Division Multiplexing
CSI	Channel State Information
DF	Decision Feedback
DFE	Decision Feedback Equalizer
DFT	Discrete Fourier Transform
DPS	Discrete Spheroidal Sequence
DS/SS	Direct Sequence Spread Spectrum
EGC	Equal Gain Combining
FH/SS	Frequency Hopping Spread Spectrum
FIR	Finite Impulse Response
HDTV	High Definition Television
IID	Independent and Identically Distributed
ISI	Inter Symbol Interference
KL-BEM	Karhunen-Loeve Basis Expansion Model

LMS	Least Mean Square
LS	Least Square
LTI	Linear Time Invariant
MAP	Maximum A posterior Principle
MIMO	Multiple Input Multiple Output
ML	Maximum Likelihood
MLS	Maximum Likelihood Sequence
MLSD	Maximum Likelihood Sequence Detection
MMSE	Minimum Mean Square Error
MMSE-ZF	Minimum Mean Square Error Zero Forcing
MRC	Maximum Ratio Combining
MSE	Mean Square Error
OFDM	Orthogonal Frequency Division Multiplexing
PDF	Probability Density Function
PE	Polynomial Expansion
PEP	Pair wise Error Probability
PLL	Phase Locked Loop
PSD	Power Spectral Density
PSK	Phase Shift Keying
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase Shift Keying
RD	Relay to Destination

SD	Source to Destination
SR	Source to Relay
RLS	Recursive Least Square
RMS	Root Mean Square
SBEM	Slepian Basis Expansion Model
SNR	Signal to Noise Ratio
TV	Time Varying
V2R	Vehicle to Road
V2V	Vehicle to Vehicle
WSS	Wide Sense Stationary

1. Introduction

1.1 Background

A vehicle communicates with another vehicle (V2V) or sensor/access-point (V2R) using vehicular ad-hoc networks. A typical application could be to supply information on road and traffic conditions. Nowadays, with the advancement of technologies, the applications installed in a vehicle allows users to access high rate data like Internet, audio/video streaming, multi-player gaming and so on [3].

The communication at a high data rate introduces a frequency-selective behavior to the channel. Moreover, due to mobility of source and/or destination vehicular terminals or reflectors in the transmission environment, the principal characteristics of the wireless channel changes in time which results in time-varying fading of the received signal. The wireless channel under such condition is said to be *doubly-selective channel* to depict the combined time- and frequency-selective behavior of the channel [4].

Diversity techniques in time, frequency and/or space offer an effective countermeasure against fading by providing the receiver with multiple replicas of the same information with independent channel gains. One way of realizing special diversity for fading channel is using cooperative diversity or cooperative communication. Cooperative diversity is a scheme in which users share their own antennas to form a virtual multiple input multiple output (MIMO) system. This techniques is very applicable in terminals which cannot afford multiple receive/ transmit antennas due to cost or size [5]. Thus, cooperative communication systems with each wireless device having a single antenna only emerged. Cooperation among nodes has advantage of increasing the communication rate and/or increases the communication reliability of the system.

1.2 Problem Statement

Previous studies on cooperative vehicular communications build upon the assumption of frequency-flat and quasi-static fading channels. But this assumption holds true for narrowband systems in very slow traffic flow. But for broadband systems with a fast traffic flow, it is reasonable to consider doubly-selective channels to model the channel for communication systems. Doubly selective channels suffer from Doppler shifts due to the relative motion between the source and destination stations and frequency-selectivity results in inter-symbol interference (ISI) in which both of them severely degrade the performance in conventional transmissions (without cooperating nodes).

1.3 Objectives of the Thesis

1.3.1 General Objective

The main objective of this thesis is to propose a system model for cooperative communication in time- and frequency-selective channel and see the performance using MATLAB simulation.

1.3.2 Specific Objectives

The specific objectives to be accomplished in this thesis are:

- Understanding doubly-selective channels and using a proper model capture its characteristics.
- Studying the different mitigation techniques for doubly-selective channels and applying them to formulate a model for communication over doubly-selective channel.

- Studying the different cooperative schemes and diversity combining techniques and choosing among the available options to come up with a cooperative communication model for the thesis work.
- Analyzing the performance of the cooperative technique over doubly-selective channel using BER.

1.4 Literature Review

I. Barhumi, G. Leus and M. Moonen [1] proposed minimum mean-square error (MMSE) time-varying (TV) finite impulse response (FIR) equalizer for doubly-selective channels. They used basis exponential model (BEM) to capture the characteristics of the channel and also to design the equalizer. They were able to prove that the suggested equalizer has a performance close to zero forcing (ZF) and MMSE block linear equalizer while the implementation and design complexity are much lower.

S. P. Shenoy, I. Ghauri, and D.T.M. Slock [2] suggested a technique to reduce computational burden introduced by matrix inversion while using minimum mean-square error zero-forcing (MMSE-ZF) Equalizer which was designed previously specifically for doubly-selective channels. It was already known that this equalizer with proper precoding at the transmitter can help to achieve full diversity over doubly-selective channel so they suggested polynomial expansion (PE) to reduce the computational burden that will be experienced at the receiver side. They are able to show that this new approach was able to reduce the computational burden but does not affect the diversity order.

M. F. Feteiha and M. Uysal [3] considered a broadband precoded cooperative V2V system over doubly-selective fading channel. The proposed transmission model for the cooperative communication builds upon the precoding scheme introduced for the conventional direct transmission. The idea for the direct transmission is extended to single relay cooperative scenario assuming amplify-and-forward (AF) relaying and orthogonal cooperation protocol. The performance analysis from analytic derivation shows that, through proper precoding, the proposed system is able to extract maximum available diversity in time (through Doppler diversity), frequency (through multipath diversity) and spatial (through cooperative diversity) dimensions.

The maximum-diversity transmission over doubly selective wireless channels is presented in X. Ma and G. Giannakis [4]. In this paper, wireless channel suffering from time- and frequency-selective fading is analyzed. The analysis based on a basis expansion model (BEM) shows that for linearly precoded transmissions and uncorrelated taps doubly selective channels offer higher order diversity gains. The analysis was for a point-to-point communication link.

1.5 Methodology

The following methodology is employed.

Literature Review: Articles, books, research papers and information available through the internet on doubly-selective channels and cooperative diversity schemes were reviewed.

System Modeling: Includes choosing a channel model that can properly capture the behavior of doubly-selective channel and mitigation techniques for communication over doubly-selective channel and the type of cooperative scheme to be used along with the diversity combining methods.

Experiment: This phase deals with conducting simulation experiment using MATLAB for the point-to-point link with mitigation techniques and the cooperative network.

Evaluation, Conclusion and Recommendation: The last phase deals with explaining the result obtained in earlier phase and draw a conclusion based on the result. Recommendations for future work will be stated too.

1.6 Assumptions

Perfect channel state information (CSI) both at the transmitter and receiver is assumed. During reception the receiver is assumed to know the channel parameters the transmitter has used to send the data. Therefore, the receiver will not have a problem while decoding the information it has received.

We also assume to have a transmitter, a receiver and one relay node. The source to destination and relay to destination channels are assumed to be doubly-selective. As for the source to relay, assuming the vehicles move in the same direction with the same speed and there are no reflectors between them, the channel is assumed to be frequency-flat and slow-fading. And also another scenario can be assuming both the source and relay to be static with no multipath propagation between them then the channel can still be considered as frequency-flat and slow-fading. Orthogonal cooperation (time division scheme) is also assumed. By orthogonal cooperation we mean we will have two phases separated in time. The first is the broadcasting phase where the source transmits to both relay and destination and the second is the relaying phase where the relay forwards what it has received in the broadcasting phase to destination.

2. Doubly-Selective Channel Models

2.1 Wireless Channels

In wireless communications, signals often reach the receiver by two or more paths and this phenomenon is called *multipath propagation*. Multipath is caused by reflection of radio signals from objects in the environment like buildings and mountains and also from reflection and refraction from the ionosphere [11].

The signals from the different propagation paths add up constructively or destructively and also a shift in phase of the signal may occur due to delay. In a multipath propagation environment, several delayed and scaled versions of the transmitted signal arrive at the receiver. The span of path delays is called *delay spread.* If the delay spread is greater than symbol period, ISI will occur. A channel that exhibits ISI is called frequency-selective channel. Furthermore, time-selective fading due to scatterer or/and transmitter/receiver motion results in a *Doppler spread*.

The channel under consideration for this thesis is both frequency-and time-selective hence named doubly-selective. There are statistical models that are used to represent the effect of fading channel on signals. Rayleigh fading is one of such statistical models. It assumes that the power of a signal that has passed through a communication channel will vary randomly, or fade, according to a Rayleigh distribution. In Rayleigh fading channels, the in-phase and quadrature components of the received signal can be assumed as independent zero-mean Gaussian processes.

Now let us see some ways of generating Rayleigh fading channel.

2.2 Simulating Rayleigh Fading Channel

In any model simulating the Rayleigh fading channel, the envelope A of the received signal has to have Rayleigh probability density function (pdf) given by

$$f_A(a) = \begin{cases} \frac{a}{\sigma^2} exp\left(\frac{-a^2}{2\sigma^2}\right), & a \geq 0 \\ 0, & a < 0 \end{cases} \tag{2.1}$$

σ^2 being the time-average power of the received signal before envelope detection. The phase θ of the received signal is uniformly distributed with pdf [7]

$$f_\vartheta\,(\theta) = \frac{1}{2\pi}\,, \qquad \theta \,\epsilon\, [0,2\pi) \tag{2.2}$$

Some of the most commonly used methods for generating Rayleigh fading channels are stated below.

2.2.1 Jakes' Model

A Rayleigh fading channel can be modeled by generating the real and imaginary parts of a complex number according to independent normal Gaussian variables.

The Jakes' model assumes the received signal $y(t)$ at time t is

$$y(t) = E_0 \sum_{l=1}^{L} C_l \cos(w_c t + w_m t \cos A_l + \Phi_l\,) \tag{2.3}$$

Where E_0 is the amplitude of the transmitted cosine wave, C_l is a random variable representing the attenuation of the l-th path, A_l is a random variable representing the angle of arrival of the l-th ray with respect to the direction of motion of the receiver, Φ_l is a random variable representing the phase shift undergone by the l-th ray, w_c is the carrier radian frequency and w_m is the maximum doppler radian frequency spread. The stochastic signal $y\ (t)$ representing the flat fading signal can be characterized by L sets of triples $(C_l,\ A_l, \Phi_l)$. The random variables C_l, A_l, and Φ_l are assumed statistically independent [7].

To reduce the complexity, the simplified Jakes' model selects:

$$C_l = \frac{1}{\sqrt{L}} \tag{2.4a}$$

$$A_l = \frac{2\pi l}{L} \tag{2.4b}$$

$$\Phi_l = 0 \tag{2.4c}$$

Where $l = 1, 2, \ldots, L$. Also, L is of the form $L = 4M + 2$ where M is a positive integer.

However, the simplifying relationships forced in (2.4a-2.4c) make this simulation model deterministic and wide-sense non-stationary. Various modifications of Jakes' model have been proposed, which we call the family of Jakes' simulators. The normalized low-pass fading process of the statistical sum-of-sinusoids simulation model is defined by

$$y(t) = y_c(t) + j y_s(t) \tag{2.5a}$$

$$y_c(t) = \frac{2}{\sqrt{M}} \sum_{l=1}^{M} \cos(\vartheta_l) \cos(w_m t \cos\alpha_l + \ \phi) \tag{2.5b}$$

$$y_s(t) = \frac{2}{\sqrt{M}} \sum_{l=1}^{M} \sin(\vartheta_l) \cos(w_m t \cos\alpha_l + \ \phi) \tag{2.5c}$$

with

$$\alpha_l = \frac{2\pi l - \pi + \theta}{4M}, l = 1, 2, \dots, M$$

where α_l, ϕ, and ϑ_l are statistically independent and uniformly distributed over [-π,π) for all l. As $M \to \infty$, the envelope $|Y|$ is Rayleigh distributed and the phase $\theta_y(t)$ is uniformly distributed over [-π, π) for which the pdf's are given by

$$f_{|y|}(y) = y\, exp\left(\frac{-y^2}{2}\right), y \geq 0 \tag{2.6}$$

$$f_{\theta_{y(t)}}(\theta) = \frac{1}{2\pi}, \theta \epsilon [-\pi, \pi) \tag{2.7}$$

A problem occurs in model (2.5) when $w_m = 0$ or the Doppler spread is small: A Rayleigh distribution cannot be guaranteed. This problem can be easily resolved by replacing a common phase ϕ by ϕ_l, which is also uniformly distributed over [-π,π) for all l [7].

2.2.2 Basis Expansion Model

In basis expansion model (BEM), channel time variant impulse response is expressed as superposition of time-varying basis functions.

The different types of basis expansion models are classified based on the basis functions used. Candidate basis functions include complex exponential (Fourier functions), polynomials, wavelets and discrete prolate spheroidal sequence.

Basis expansion model is preferable because it has finite parameter and the coefficients evolve more slowly in time than the channel tap gains and hence can be tracked easily. And also it has low complexity.

Among the different BEM, Complex exponential-basis expansion model (CE-BEM) is used in this thesis for modeling doubly-selective channel. CE-BEM is the most commonly used because of simplicity and it is explained below.

Complex Exponential Basis Expansion Model (CE-BEM)

Statistical modeling of the channel is well motivated when time-varying path delays arise due to a large number of reflectors. Deterministic basis expansion models have gained popularity for wireless applications, especially when the multipath is caused by a few strong reflectors and path delays exhibit variations due to the mobiles. The time-varying taps are expressed as a superposition of time-varying bases (complex exponentials when modeling Doppler effects) with time invariant coefficients. By assigning time variations to the bases, rapidly fading channels with coherence time as small as a few tens of symbols can be captured.

Consider a time-varying channel with impulse response $h(t;\tau)$ (response at time t to a unit impulse at time $t-\tau$ which includes transmit-receive filters as well as doubly-selective propagation effects). Let $x(t)$ denote the complex baseband, continuous-time input signal (with symbol duration Ts), and $y(t)$ denote the complex baseband, continuous-time received signal. The noise-free received signal $y(t)$ is the convolution of $x(t)$ and $h(t;\tau)$:

$$y(t) = \int_0^\infty h(t;\tau)x(t-\tau)d\tau \qquad (2.8)$$

Let $H(f;\tau) = \int_{-\infty}^{\infty} h(t;\tau)\, e^{-j2\pi f t}dt$ be the Fourier transform of $h(t;\tau)$; $H(f;\tau)$ is the delay-Doppler spreading function of the channel. If $|H(f;\tau)| \approx 0$ for $|\tau| > T_d$, then T_d is called the (Multipath) delay-spread of the channel; if $|H(f;\tau)| \approx 0$ for $|f| > f_d$, then f_d is called the Doppler spread of the channel. If $x(t)$, $y(t)$ and $h(t;\tau)$ in (2.7) are sampled at symbol rate, then for $t = nT_s\, \epsilon [t_0, t_0 + NT_s]$, the sampled signal $y(n) = y(t)|t = nT_s$ has the representation

$$y(n) = \sum_{l=0}^{L} h(n;l)\, x(n-l) \tag{2.9}$$

where Ts is the symbol duration. Over the block interval of $[t_0, t_0 + NT_s)$, the channel impulse response $\mathrm{h(n;l)}$ can be represented using $Q_F + 1$ coefficients $\{w_q(l)\}_{q=0}^{Q_F}$, which remain invariant during this block but are allowed to change for the next block, and $Q_F + 1$ Fourier basis functions that are used to describe the temporal variation of the channel and are common for each block [7].

Then for the block of $[t_0, t_0 + NT_s)$, the discrete-time baseband equivalent channel model based on complex exponential basis expansion can be described as:

$$h(n;l) = \sum_{q=0}^{Q_F} w_q(l) e^{jw_q n} \tag{2.10}$$

where

$$w_q = \frac{2\pi}{N}\left(q - \frac{Q_F}{2}\right), q = 0, 1, \dots, Q_F \tag{2.11a}$$

$$L = \left\lceil {}^{T_d}\!/_{T_s} \right\rceil \tag{2.11b}$$

$$Q_F \geq 2\lceil f_d N T_s \rceil \tag{2.11c}$$

The channel model used for this thesis is CE-BEM.

3 Cooperative Transmission Protocols and Diversity Combining Techniques

3.1 Cooperative Diversity

Cooperative diversity is a cooperative multiple antenna technique for improving reliability of a communication by using the combined signal of the relayed signal and the direct signal in wireless networks. A single hop system uses direct transmission, where a receiver decodes the information only based on the signal it has received from a source while regarding signals from other nodes as interference. But in cooperative diversity, the signals from relays will be sought as contributions and will be combined with information from source to be decoded by the receiver.

Cooperative diversity is a scheme in which users share their own antennas to form a virtual MIMO system. It is most advantageous when devices cannot afford many antennas due to their size or cost.

Let us consider a three node network scenario with one source, one relay and one destination. In the first, the source broadcasts to the relay and destination nodes. In the second phase, the relay re-transmits to the destination. For cooperative diversity transmission, we model the channel during the first half of the block as

$$y_r[n] = \partial_{s,r}\, x_s[n] + z_r[n] \tag{3.1}$$

$$y_d[n] = \partial_{s,d}\, x_s[n] + z_d[n] \tag{3.2}$$

where $x_s[n]$ is the source transmitted signal and $y_r[n]$ and $y_d[n]$ are the relay and destination received signals, respectively. For the second half of the block, we model the received signal as

$$y_d[n] = \partial_{r,d}\, x_r[n] + z_d[n] \tag{3.3}$$

where $x_r[n]$ is the relay transmitted signal and $y_d[n]$ is the destination received signal.

In (3.1)-(3.3), $\partial_{i,j}$ captures the effects of path-loss, shadowing, and frequency non-selective fading, and $z_j[n]$ captures the effects of receiver noise and other forms of interference in the system, where $i \,\epsilon\, \{s, r\}$ and $j \,\epsilon\, \{r, d\}$. Statistically, we model $\partial_{i,j}$ as zero-mean, independent, circularly-symmetric complex Gaussian random variables with variances ${\sigma_{i,j}}^2$, so that the magnitudes $|\partial_{i,j}|$ are Rayleigh distributed ($|\partial_{i,j}|^2$ are exponentially distributed with zero mean and variance ${\sigma_{i,j}}^2$) and the phases $\measuredangle\partial_{i,j}$ are uniformly distributed on $[0, 2\pi)$. Furthermore, we model $z_j[n]$ as zero-mean mutually independent, circularly-symmetric, complex Gaussian random sequences with variance N_0 [8].

3.2 Cooperative Communication Protocols

The different types of cooperative transmission strategies are explained in this section. These protocols employ different types of processing by the relay terminals, as well as different types of combining at the destination terminals. The protocols are divided in to three namely fixed protocols, adaptive protocols (selection relaying) and protocols with limited feedback (incremental relaying).

For fixed cooperative protocols, we allow the relays to either amplify their received signals subject to their power constraint, or to decode, re-encode, and re-transmit the messages. These two options are referred as *amplify-and-forward* and *decode-and-forward*, respectively. Other than fixed strategies, there are also adaptive strategies in which the cooperating terminals accurately estimate the realized signal-to-noise ratio (SNR) between them and use this estimate to select a suitable cooperative action; the terminals decide between continuing their own transmission or relaying the transmissions of the other terminal using amplify-and-forward or decode-and-forward [8].

Destination radios can appropriately combine their received signals by exploiting control information in the protocol headers. We stress that adaptation is performed in the absence of feedback from the destination terminal; we also describe one simple protocol that exploits limited feedback from the destination.

The relaying scenario used is as shown below.

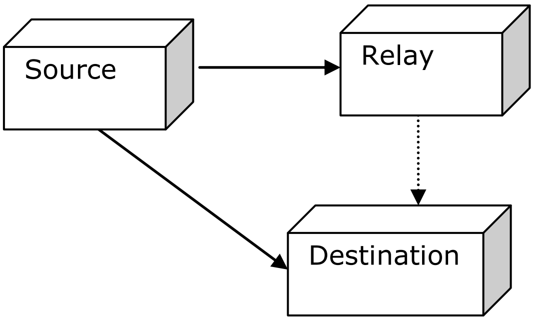

Figure 3-1: Signal Transmit paths in a Wireless Network with a Single Source and Destination.

3.2.1 Fixed Protocols

3.2.1.1 Amplify-and-Forward Transmission

For amplify-and-forward transmission, the appropriate channel model is (3.1)-(3.3). The source terminal transmits its information as $x_s[n]$. The relay processes $y_r(n)$, and relays the information by transmitting

$$x_r(n) = \beta y_r[n] \tag{3.4}$$

To remain within its power constraint (with high probability), an amplifying relay must use gain

$$\beta \leq \sqrt{\frac{P}{\left|\partial_{s,r}\right|^2 P + N_0}}$$

where we allow the amplifier gain to depend upon the fading coefficient $\partial_{s,r}$ between the source and relay, which the relay estimates to high accuracy. This transmission scheme can be viewed as repetition coding from two separate transmitters, except that the relay transmitter amplifies its own receiver noise. The destination can decode its received signal $y_d(n)$ for $n = 1, \ldots, N/2$ by first appropriately combining the signals from the two sub blocks using a suitably designed matched filter (maximum-ratio combiner) [8].

3.2.1.2 Decode-and-Forward Transmission

The source terminal transmits its information as $x_s(n)$. During this interval, the relay processes $y_r(n)$ by decoding an estimate $\hat{x}_s[n]$ of the source transmitted signal.

Under a repetition coded scheme, the relay transmits the signal

$$x_r[n] = \hat{x}_s[n] \tag{3.5}$$

Decoding at the relay can take on a variety of forms. For example, the relay might fully decode the source message by estimating the source codeword, or it might employ symbol-by- symbol decoding and allow the destination to perform full decoding. These options allow for trading of performance and complexity at the relay terminal [8].

3.2.2 Adaptive Protocols

Fixed decode-and-forward is limited by direct transmission between the source and relay. However, since the fading coefficients are known to the appropriate receivers, $\partial_{s,r}$ can be measured to high accuracy by the cooperating terminals; thus, they can adapt their transmission format according to the realized value of $\partial_{s,r}$.

This observation suggests the following class of adaptive algorithms. If the measured $\left|\partial_{s,r}\right|^2$ falls below a certain threshold, the source simply continues its transmission to the destination, in the form of repetition or more powerful codes. If the measured $\left|\partial_{s,r}\right|^2$ lies above the threshold, the relay forwards what it received from the source, using either amplify-and-forward or decode-and-forward, in an attempt to achieve diversity gain.

Adaptive protocols of this form should offer diversity because in either case, two of the fading coefficients must be small in order for the transmission to be lost. Specifically, if $\left|\partial_{s,r}\right|^2$is small, then $\left|\partial_{s,d}\right|^2$must also be small for the transmission to be lost when the source continues its transmission. Similarly, if $\left|\partial_{s,r}\right|^2$ is large, then both $\left|\partial_{s,d}\right|^2$and $\left|\partial_{r,d}\right|^2$ must be small for the transmission to be lost when the relay employs amplify-and-forward or decode-and-forward [8].

3.2.3 Limited Feedback Protocols

The fixed and adaptive protocols described above can make inefficient use of the degrees of freedom of the channel, especially for high transmission rates, because the relays essentially repeat the transmissions all the time. In this section, we describe a very simple protocol that exploits limited feedback from the destination terminal, e.g., a single bit indicating the success or failure of the direct transmission, that we will see can dramatically improve spectral efficiency over the fixed and adaptive protocols.

As one example, consider the following protocol utilizing feedback and amplify-and forward transmission. Protocols based upon feedback and decode-and-forward transmission are also possible, but the analysis is more involved and their performance is slightly worse than the following protocol.

First, the source transmits its information to the destination at spectral efficiency R. The destination indicates success or failure by broadcasting a single bit of feedback to the source and relay, which we assume is detected reliably by at least the relay. If the SNR between the source and destination is sufficiently high, the feedback indicates success of the direct transmission, and the relay does nothing. If the SNR between the source and destination is not sufficiently high for successful direct transmission, the feedback requests that the relay amplify-and-forward what it received from the source. In the latter case, the destination tries to combine the two transmissions. Protocols of this form make more efficient use of the degrees of freedom of the channel, because they repeat only rarely.

Such an assumption is reasonable if the destination encodes the feedback bit with a very low-rate code.

Even if the relay cannot reliably decode, useful protocols can be developed and analyzed. For example, a conservative protocol might have the relay amplify-and-forward what it receives from the source in all cases except when the destination reliably receives the direct transmission and the relay reliably decodes the feedback bit [8].

3.3 Outage Behaviour

For fixed fading values, the effective channel models induced by the transmission protocols described above are variants of well-known channels with additive white Gaussian noise. In this section, the performance of the various transmission protocols in terms of outage events and outage probabilities are compared, and focus is given on performance in the high SNR regime. Outage events specified in terms of the fading random variables $\left|\partial_{i,j}\right|^2$ have useful interpretations in both coded and uncoded settings, but results will be developed from a coded perspective and determine events in which the realized mutual information of the channel falls below a target transmission rate. This event will be converted into an equivalent event defined in terms of the fading coefficients of the channel.

Since the channel average mutual information I is a function of the fading coefficients of the channel, it too is a random variable. The event $I < R$ that this mutual information random variable falls below some fixed spectral efficiency R, is referred to as an *outage event*, because reliable communication is not possible for realizations in this event. The probability of an outage event, $Pr\,[I < R]$, is referred to as the *outage probability* of the channel.

Outage events are independent of the distribution of the underlying random variables, while outage probabilities are intimately tied to them. For example, if the outage event of a scheme at a particular rate is a strict subset of the outage event of another scheme at that rate, then the first scheme has smaller outage probability regardless of the probability distribution on the channel parameters. Furthermore, as it will be seen, several of the cooperation strategies have similar outage probabilities, but the structure of their outage events is sufficiently different that we might prefer one over the other in various regimes.

As a result, both outage events and outage probabilities are useful for characterizing our transmission protocols. Below outage events and outage probabilities will be developed for the transmission protocols. When specified in terms of the fading random variables of the channel, the outage event is sometimes called an outage region [8].

3.3.1 Direct Transmission

The maximum average mutual information between input and output in this case, achieved by independent and identically-distributed (*IID*) zero-mean, circularly-symmetric complex Gaussian inputs, is given by

$$I_D = \log\left(1 + SNR\left|\partial_{s,d(s)}\right|^2\right) \tag{3.6}$$

as a function of the fading coefficient $\partial_{s,d(s)}$. The outage event for spectral efficiency R is given by $I_D < R$ and is equivalent to the event

$$\left|\partial_{s,d(s)}\right|^2 < \frac{2^R - 1}{SNR} \tag{3.7}$$

For Rayleigh fading, i.e. $\left|\partial_{s,d(s)}\right|^2$ exponentially distributed with parameter $\left|\sigma_{s,d(s)}\right|^{-2}$, the outage probability satisfies [8]

$$P_D{}^{out} \triangleq P_r[I_D < R] = P_r\left[\left|\partial_{s,d(s)}\right|^2 < \frac{2^R - 1}{SNR}\right]$$

$$= 1 - exp\left(-\frac{2^R - 1}{SNR\,\sigma^2{}_{s,d(s)}}\right)$$

$$\sim \frac{1}{\sigma^2{}_{s,d(s)}} \cdot \frac{2^R - 1}{SNR}\text{ , SNR large,} \tag{3.8}$$

3.3.2 Fixed Protocols

3.3.2.1 Amplify-and-Forward Transmission

The amplify-and-forward protocol produces an equivalent one-input, two-output complex Gaussian noise channel with different noise levels in the outputs. The maximum average mutual information between the input and the two outputs, achieved by IID complex Gaussian inputs, is given by

$$I_{AF} = \frac{1}{2}\log\left(1 + SNR\left|\partial_{s,d(s)}\right|^{2} + f\left(SNR\left|\partial_{s,r}\right|^{2}, SNR\left|\partial_{r,d(s)}\right|^{2}\right)\right) \tag{3.9}$$

as a function of the fading coefficients, where

$$f(x,y) \triangleq \frac{xy}{x+y+1} \tag{3.10}$$

The outage event for spectral efficiency R is given by $I_{AF} < R$ and is equivalent to the event

$$\left|\partial_{s,d(s)}\right|^{2} + \frac{1}{SNR} f\left(SNR\left|\partial_{s,r}\right|^{2}, SNR\left|\partial_{r,d(s)}\right|^{2}\right) < \frac{2^{R}-1}{SNR} \tag{3.11}$$

For Rayleigh fading, i.e., $\left|\partial_{i,j}\right|^{2}$ independent and exponentially distributed with parameters ${\sigma_{i,j}}^{-2}$, analytic calculation of the outage probability becomes involved, but the high SNR behavior can be approximated as

$${P^{out}}_{AF}(SNR,R) \triangleq P_r[I_{AF} < R] \sim \left(\frac{1}{2{\partial^{2}}_{s,d(s)}} . \frac{{\partial^{2}}_{s,r} + {\partial^{2}}_{r,d(s)}}{{\partial^{2}}_{s,r}{\partial^{2}}_{r,d(s)}}\right) . \left(\frac{2^{2R}-1}{SNR}\right)^{2}, SNR\ large \tag{3.12}$$

The $1/SNR^2$ behavior in (3.12) indicates that amplify-and-forward achieves full second order diversity. Thus, increasing SNR by 10 dB reduces the outage probability by a factor of 100 [8].

3.3.2.2 Decode-and-Forward Transmissions

To analyze decode-and-forward transmission, let us examine a particular decoding structure at the relay. Specifically, we require the relay to fully decode the source message; examination of symbol-by-symbol decoding at the relay becomes involved because it depends upon the particular coding and modulation choices. Requiring both the relay and destination to decode perfectly, the maximum average mutual information for repetition-coded decode-and- forward can be readily shown to be

$$I_{DF} = \frac{1}{2} min\left\{\log\left(1 + SNR\,\left|\partial_{s,r}\right|^2\right), \log\left(1 + SNR\,\left|\partial_{s,d(s)}\right|^2\right) + \log\left(1 + SNR\,\left|\partial_{r,d(s)}\right|^2\right)\right\} \tag{3.13}$$

as a function of the fading random variables. The first term in (3.13) represents the maximum rate at which the relay can reliably decode the source message, while the second term in (3.13) represents the maximum rate at which the destination can reliably decode the source message given repeated transmissions from the source and destination. We note that such mutual information forms are typical of relay channels with full decoding at the relay.

The outage event for spectral efficiency R is given by $I_{DF} < R$ and is equivalent to the event

$$min\left\{\left|\partial_{s,r}\right|^2, \left|\partial_{s,d(s)}\right|^2 + \left|\partial_{r,d(s)}\right|^2\right\} < \frac{2^{2R}-1}{SNR} \tag{3.14}$$

For Rayleigh fading, i.e., $\left|\partial_{i,j}\right|^2$ independent and exponentially distributed with parameters $\sigma_{i,j}{}^{-2}$, the outage probability for repetition-coded decode-and-forward can be computed as

$$P^{out}{}_{DF}(SNR,R) \triangleq P_r[I_{DF} < R]$$

$$= P_r\left[\left|\partial_{s,r}\right|^2 < g(SNR)\right] + P_r\left[\left|\partial_{s,r}\right|^2 \geq g(SNR)\right] P_r\left[\left|\partial_{s,d(s)}\right|^2 + \left|\partial_{r,d(s)}\right|^2 < g(SNR)\right] \tag{3.15}$$

Where $g(SNR) = [2^{2R} - 1]/SNR$. For large SNR the expression in (3.15) becomes

$$P^{out}{}_{DF}(SNR,R) \sim \frac{1}{\partial^2{}_{s,r}} . \frac{2^{2R}-1}{SNR}, SNR\ large \tag{3.16}$$

The $1/SNR$ behavior in (3.16) indicates that fixed decode-and-forward does not offer diversity gains for large SNR, because requiring the relay to fully decode the source transmissions limits the performance of decode-and-forward transmission to that of direct transmission between the source and relay [8].

3.3.3 Adaptive Protocols

To overcome the shortcomings of decode-and-forward transmission, let us describe adaptive versions of the amplify-and-forward and decode-and-forward protocols, both of which fall back to direct transmission if $\left|\partial_{s,r}\right|^2$ falls below some threshold.

As an example the performance of adaptive decode-and-forward transmission will be analyzed. The mutual information of this adaptive hybrid is somewhat involved to write

$$I_{ADF} = \begin{cases} \frac{1}{2}\log\left(1 + 2SNR\left|\partial_{s,d(s)}\right|^2\right) & \left|\partial_{s,r}\right|^2 < g(SNR) \\ \frac{1}{2}\log\left(1 + SNR\left|\partial_{s,d(s)}\right|^2 + SNR\left|\partial_{r,d(s)}\right|^2\right) & \left|\partial_{s,r}\right|^2 \geq g(SNR) \end{cases} \tag{3.17}$$

where $g(SNR) = [2^{2R} - 1]/SNR$. The first case in (3.17) corresponds to the maximum average mutual information of repetition coding from the source to the destination, hence the extra factor of 2 in the SNR. The second case in (3.17) corresponds to the maximum average mutual information of repetition coding from the source and relay to the destination, assuming the relay can fully decode the source transmission.

The outage event for spectral efficiency R is given by $I_{ADF} < R$ and is equivalent to the event

$$\left(\left\{\left|\partial_{s,r}\right|^2 < g(SNR)\right\} \cap \left\{2\left|\partial_{s,d(s)}\right|^2 < g(SNR)\right\}\right)$$

$$\cup \left(\left\{\left|\partial_{s,r}\right|^2 \geq g(SNR)\right\} \cap \left\{\left|\partial_{s,r}\right|^2 + \left|\partial_{r,d(s)}\right|^2 < g(SNR)\right\}\right) \tag{3.18}$$

The first (respectively also the second) event of the union in (3.18) corresponds to the first (respectively also the second) case in (3.17) We can observe that adapting to the realized fading coefficient ensures that the protocol performs no worse than direct transmission, except for the fact that it potentially suffers the bandwidth inefficiency of repetition coding.

Because the events in the union of (3.18) are mutually exclusive, the outage probability becomes the sum

$$P^{out}{}_{ADF}(SNR, R) \triangleq P_r[I_{ADF} < R]$$

$$= P_r\left[\left|\partial_{s,r}\right|^2 < g(SNR)\right] P_r\left[2\left|\partial_{s,d(s)}\right|^2 < g(SNR)\right]$$
$$+ P_r\left[\left|\partial_{s,r}\right|^2 \geq g(SNR)\right] P_r\left[\left|\partial_{s,d(s)}\right|^2 + \left|\partial_{r,d(s)}\right|^2 < g(SNR)\right] \tag{3.19}$$

At large SNR (3.19) becomes

$$\rightarrow \frac{1}{2\left|\partial_{s,d(s)}\right|^2} \frac{\left|\partial_{s,r}\right|^2 + \left|\partial_{r,d(s)}\right|^2}{\left|\partial_{s,r}\right|^2 \left|\partial_{r,d(s)}\right|^2}$$

Thus, it can be concluded that the large SNR performance of adaptive decode-and-forward transmission is identical to that of fixed amplify-and-forward transmission [8].

3.4 Diversity Combining Schemes

As explained above, one way of overcoming the shortcomings of fading channel is using Diversity schemes. It is an efficient technique to exploit the random nature of radio propagation by finding methods to generate and extract independent signal paths for communication [13].

The advantage of using cooperative diversity is that if one signal path undergoes a deep fade at a particular point of time, another independent path may have a strong signal. Having multiple paths to choose from will improve the instantaneous and average SNR at the receiver. The various types of diversity used in communication systems operating over fading channels are:

- Space Diversity.

- Frequency Diversity.

- Time Diversity.

- Polarization Diversity.

- Multipath Diversity.

Whatever diversity technique we use, the receiver has to have a mechanism to process the multiple signals it has received in a way that maximizes the efficiency of the system. The diversity reception methods employed in communication receivers are:

- Selection Combining (SC).
- Equal Gain Combining (EGC).
- Maximal Ratio Combining (MRC).

These methods are discussed in the following subsections:

3.4.1 Selection Combining (SC)

Selection combining is one of the simplest diversity reception techniques. The receiver chooses the signal with the largest SNR as shown in Fig (3-2). The combining strategy for selective combining is given by:

$$Y_K = \begin{cases} Y_{1K} & if\ |Y_{1K}| > |Y_{2K}| \\ Y_{2K} & if\ |Y_{2K}| > |Y_{1K}| \end{cases} \tag{3.20}$$

Where, Y_{1K} and Y_{2K} are decision variables at the first and second diversity paths and Y_K is decision variable at the output of the diversity combiner. Let ρ be the threshold SNR that must be achieved for proper demodulation and detection of the received signal. Let there be L diversity branches and let k be the instantaneous SNR for the K_{th} branch. Using the expression for the pdf for the instantaneous SNR we can get an expression for the outage probability (probability that the SNR falls below the threshold) for all the branches as:

$$P(\gamma_1, \dots, \gamma_L \le \rho) = \left(1 - e^{-\rho/\bar{\gamma}_c}\right)^L \tag{3.21}$$

Where, $\hat{\gamma}_c$ is the average SNR at every diversity branch, assuming all the branches have the same averageSNR. It is given by:

$$\bar{\gamma}_c = \frac{E_b}{N_0} E\{R^2\} \tag{3.22}$$

The improvement in the SNR due to selective combining is evident from equation (3.21) [13].

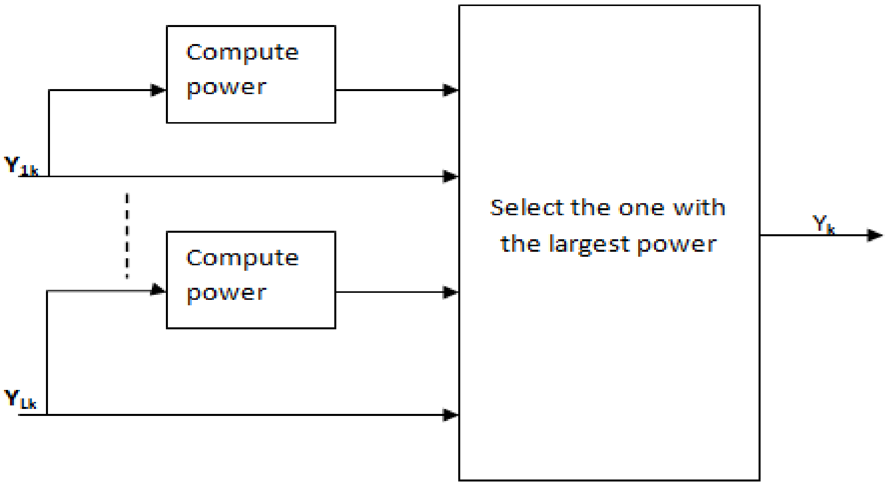

Figure 3-2: Selection Combining.

3.4.2 Equal Gain Combining (EGC)

In Equal Gain Combining (EGC), all the received signals are co-phased at the receiver and added together without any weighting.

The performance of EGC is only inferior to the optimal maximal ratio combiner. In case of a two-fold diversity scheme, the combining equation is given by [13]:

$$Y_K = Y_{1K} + Y_{2K} \tag{3.23}$$

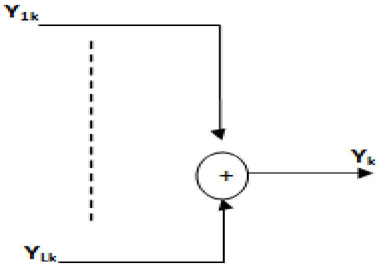

Figure 3-3: Equal Gain Combining.

3.4.3 Maximal Ratio Combining (MRC)

In Maximal Ratio Combining (MRC), the signal from all branches are co-phased and individually weighed to provide the optimal SNR at the output.

It can be shown that the output SNR is maximized when the signals in each of the diversity branches are weighed by their own envelopes. In case of a two-fold diversity scheme, the combining equation is given by:

$$Y_K = \gamma_{1K} Y_{1K} + \gamma_{2K} Y_{2K} \tag{3.24}$$

Where, γ_{1K} and γ_{2K} represent the instantaneous envelopes of the signals received at each of the diversity branches. The SNR per bit at the output of the maximal ratio combiner (γ_b) can be written as:

$$\gamma_b = \sum_{k=1}^{L} \gamma_k = \frac{E_b}{N_0} \sum_{k=0}^{L} R^2{}_k \qquad (3.25)$$

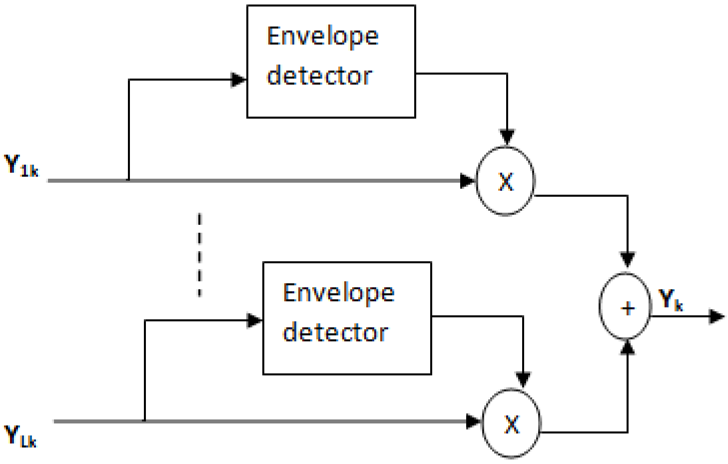

Figure 3-4: Maximal Ratio Combining.

Where $\gamma_k = R^2\,(E_b/N_0)$ is the instantaneous SNR in the k_{th} diversity branch. The pdf of the output SNR can be written as:

$$f_{\gamma_b}(\gamma_b) = \frac{1}{(L-1)!\gamma_c{}^L}\, \gamma_b{}^{L-1}\, e^{-\gamma_b/\bar{\gamma}_c} \qquad (3.26)$$

Where $\bar{\gamma}_c$ is the average SNR per channel given by Equation (3.22) [13].

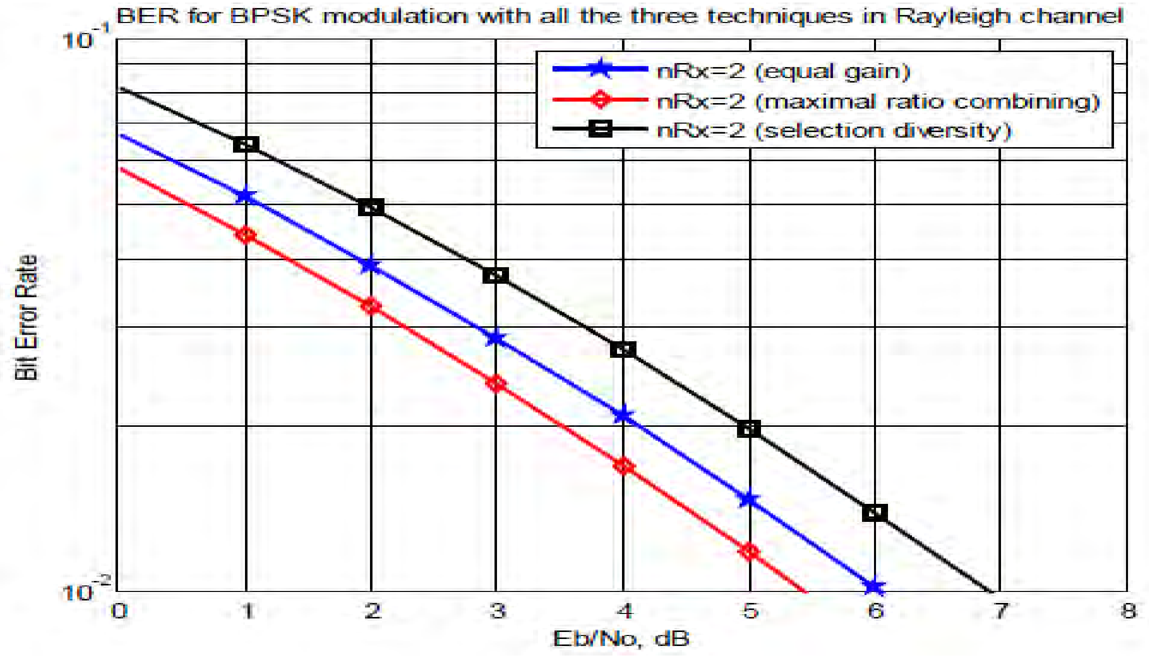

Figure 3-5: Performance comparisons between the diversity combining schemes in Rayleigh channel [17].

4 Mitigation Techniques for Doubly-Selective Channels

4.1 Frequency Selective Channels

Fading is the term used to describe the effect of channel on a signal in short period of time. It is characterized by change in amplitude and phase of a signal. It also describes the delay experienced by the signal due to reflectors in the environment. Fading is also caused by movement of reflectors, transmitter and receiver. The wireless channel in these situations can be modeled as a linear filter having a time varying impulse response $h(t,\tau)$. The filtering characteristic of the channel is due to the summation of amplitudes and delays of the multipath components of a signal arriving at the same instances of time [15].

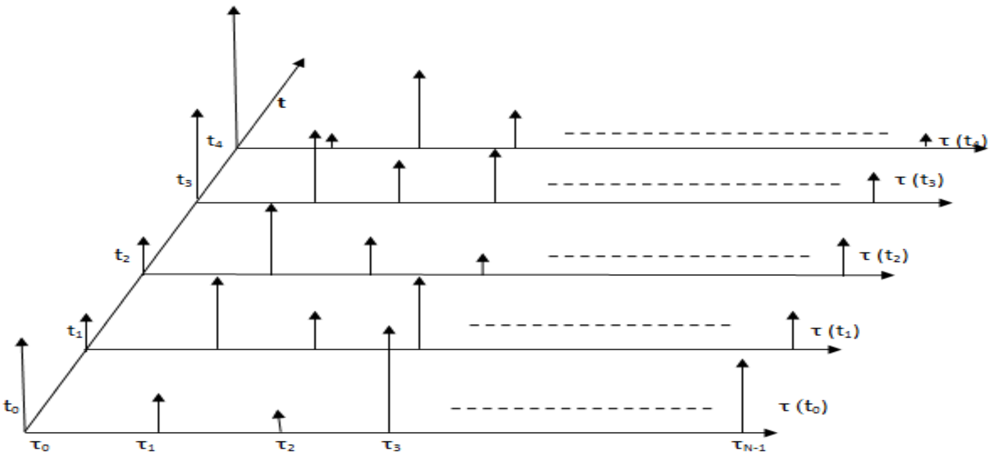

Figure 4-1: Time Varying Impulse Response, $h(t,\tau)$, of a Multipath Wireless Channel.

Figure 4-1 shows different snapshots of $h(t,\tau)$where t varies into the page and the multipath delay axis is quantized into excess delay bins of width $\Delta\tau$. The relative delay of the i^{th} multipath component as compared to the first arriving component is called *excess delay* and is denoted by τ_i, where $\tau_i = i\Delta\tau$. The first arriving multipath component has an excess time delay $\tau_0 = 0$. Any number of multipath signals received within the i^{th} bin is represented by a single resolvable multipath component. If we have N number of multipath components, then the maximum excess delay will be $N\Delta\tau$. The baseband impulse response $h(t,\tau)$ of a multipath channel can be expressed as the vector sum of a series of delayed, phase shifted replicas of the transmitted signal. i.e.

$$h(t,\tau) = \sum_{i=0}^{N-1} a_i(t,\tau) \exp\,[j\theta_i(t,\tau)\delta(t-\tau_i(t))] \tag{4.1}$$

where $a_i(t,\tau)$, $\tau_i(t)$, $\theta_i(t,\tau)$ are the real amplitudes, excess delays and the phase shifts of a single multipath component within the i^{th} excess delay bin.

Power Delay Profile

Power delay profile gives the average power at the channel output as a function of the time delay. It is obtained by taking the spatial average of $|h(t,\tau)|^2$ over a local area. By making several local area measurements of $|h(t,\tau)|^2$ in different locations, it is possible to build an ensemble of power delay profile, each one representing a possible small-scale multipath channel state.

The power delay profile at time t_0 for a probing pulse $p(t)$ at the channel input is given by

$$P(\tau_0) = |p(t_0)|^2 = \sum_{k=0}^{N-1} a_k{}^2(t_0) \tag{4.2}$$

Mean excess delay, *root mean square (rms) delay* and *maximum excess delay* are small scale multipath channel parameters which characterizes the time dispersive properties of the channel. They are calculated from the power delay profile.

Mean Excess Delay: It is the first moment of the power delay profile and is defined as

$$\bar{\tau} = \frac{\sum_k a_k{}^2 \tau_k}{\sum_k a_k{}^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)} \tag{4.3}$$

Root Mean Square (rms) Delay: It is the square root of the second central moment of the power delay profile and is defined as

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2} \tag{4.4}$$

Where

$$\bar{\tau} = \frac{\sum_k a_k{}^2 \tau_k{}^2}{\sum_k a_k{}^2} = \frac{\sum_k P(\tau_k)\tau_k{}^2}{\sum_k P(\tau_k)} \tag{4.5}$$

These delays are measured relative to the first detectable signal arriving at the receiver at $\tau_0 = 0$.

Maximum Excess Delay: The maximum excess delay of the power delay profile is defined as the time delay during which the multipath energy falls to $X\ dB$ below the maximum. It is defined as $\tau_x - \tau_0$, where τ_0 is the first arriving signal and τ_x is the maximum delay at which a multipath component is within $X\ dB$ of the strongest multipath signal [15].

The above parameters characterize the channel in time domain. The channel in frequency domain is defined by *coherence bandwidth* (B_c).

Coherence Bandwidth: Coherence bandwidth is the range of frequencies over which the signal strength remains more or less unchanged. If it is defined as the bandwidth over which the frequency correlation function is above 0.9, then it can be mathematically obtained as

$$B_c \approx \frac{1}{50\ \sigma_\tau} \tag{4.6}$$

The coherence bandwidth for frequency correlation above 0.5 is given by

$$B_c \approx \frac{1}{5\ \sigma_\tau} \tag{4.7}$$

where σ_τ is the rms delay.

If the bandwidth of the transmitted signal is greater than coherence bandwidth of the channel, then the signal undergoes frequency selective fading.

In such cases, the multipath delay spread is greater than the symbol interval. Within single symbol duration we will have attenuated and delayed version of a previous symbols. This phenomenon is called Inter-symbol Interference (ISI).

Frequency Selective Fading channels are also called wideband channels since the symbol bandwidth (B_s) is greater than the coherence bandwidth [15].

Thus, a channel undergoes frequency selective fading if

$$B_s > B_c \tag{4.8}$$

and

$$T_s < \sigma_T \tag{4.9}$$

where T_s is the symbol duration.

To understand what ISI is, let us consider the transmission of a sequence of symbols with the basic waveform $u(t)$. To send the n^{th} symbol b_n, we send $b_n u(t - nT)$, where T is the symbol interval.

Therefore, the transmitted signal is

$$\sum_n b_n u(t - nT) \tag{4.10}$$

If the channel is dispersive, the received signal will be

$$y(t) = \sum_n b_n v(t - nT) + n(t) \tag{4.11}$$

where $v(t) = u * h_c(t)$ is the received waveform for a symbol. $h_c(t)$ is impulse response of the channel and $n(t)$ is AWGN with power spectral density $N_0/2$.

If a single symbol, say the symbol b_0, is transmitted, the optimal demodulator is the one that employs the matched filter, i.e., we can pass the received signal through the matched filter $\tilde{v}(t) = v(-t)$ and then sample the matched filter output at time $t = 0$ to obtain the decision statistic.

When a sequence of symbols is transmitted, we can still employ this matched filter to perform demodulation. A reasonable strategy is to sample the matched filter output at time $t = mT$ to obtain the decision statistic for the symbol b_m. At $t = mT$, the output of the matched filter is

$$z_m = \sum_n b_n \, v * \tilde{v}(mT - nT) + n_m$$

$$= b_m \|v\|^2 + \textstyle\sum_{n \neq m} b_n \, v * \tilde{v}(mT - nT) + n_m \qquad (4.12)$$

Where n_m is a zero-mean Gaussian random variable with variance $N_0 \|v\|^2/2$. The first term in (4.12) is the desired signal contribution due to the symbol b_m and the second term contains contributions from the other symbols. These unwanted contributions from other symbols are called $Intersymbol\ interference\ (ISI)$ [14].

Suppose $v(t)$ is time limited, i.e., $v(t) = 0$ except for $0 \leq t \leq T$. Then it is easy to see that $v * \bar{v}(t) = 0$ except for $-T < t < T$. As a result, the demodulation strategy above can be interpreted as matched filtering for each symbol.

Unfortunately, a time limited waveform is never band limited. Therefore, for a band limited channel, $v(t)$ and, hence, $v * \bar{v}(t)$ are not time limited and hence ISI is, in general, present.

ISI is undesired effect of a channel and hence needs to be mitigated. The following section revises some mitigation techniques for frequency-selective fading channel.

4.2 Mitigation Techniques for Frequency-Selective Distortion

Equalization can mitigate the effects of channel-induced ISI brought on by frequency-selective fading. The process of equalizing for mitigating ISI effects involves using methods to gather the dispersed symbol energy back into its original time interval. In effect, an equalizer is an inverse filter of the channel. If the channel is frequency selective, the equalizer enhances the frequency components with small amplitudes and attenuates those with large amplitudes.

The goal is for the combination of channel and equalizer filter to provide a flat composite-received frequency response and linear phase. Because in a mobile system the channel response varies with time, the equalizer filter must also change or adapt to the time-varying channel characteristics. Such equalizer filters are therefore adaptive devices that accomplish more than distortion mitigation; they also provide diversity. Since distortion mitigation is achieved by gathering the dispersed symbol's energy back into the symbol's original time interval so that it doesn't hamper the detection of other symbols, the equalizer is simultaneously providing the receiver with symbol energy that would otherwise be lost.

Direct-sequence spread-spectrum (DS/SS) techniques can be used to mitigate frequency-selective ISI distortion because the hallmark of spread-spectrum systems is their capability of rejecting interference, and ISI is a type of interference. Consider a DS/SS binary phase-shift keying (PSK) communication channel comprising one direct path and one reflected path. Assume that the propagation from transmitter to receiver results in a multipath wave that is delayed by τ compared to the direct wave. The received signal, $y(t)$, neglecting noise, can be expressed as follows:

$$y(t) = Ax(t)g(t)cos(2\pi f_c t) + \alpha Ax(t-\tau)g(t-\tau)cos(2\pi f_c t + \theta) \qquad (4.13)$$

where $x(t)$ is the data signal, $g(t)$ is the pseudo noise (PN) spreading code, and τ is the differential time delay between the two paths. The angle θ is a random phase, assumed to be uniformly distributed in the range $(0, 2\pi)$, and α is the attenuation of the multipath signal relative to the direct path signal. The receiver multiplies the incoming $y(t)$ by the code $g(t)$. If the receiver is synchronized to the direct path signal, multiplication by the code signal yields the following:

$$y(t)g(t) = Ax(t)g^2(t)cos(2\pi f_c t) + \alpha Ax(t-\tau)g(t)g(t-\tau)cos(2\pi f_c t + \theta) \qquad (4.14)$$

where $g^2(t) = 1$. If τ is greater than the chip duration, then

$$\left|\int g(t)g(t-\tau)dt\right| \ll \left|\int g^2(t)dt\right| \qquad (4.15)$$

over some appropriate interval of integration (correlation). Thus, the spread spectrum system effectively eliminates the multipath interference by virtue of its code-correlation receiver. Even though channel-induced ISI is typically transparent to DS/SS systems, such systems suffer from the loss in energy contained in the multipath components rejected by the receiver. The need to gather this lost energy belonging to a received chip was the motivation for developing the Rake receiver. The Rake receiver dedicates a separate correlator to each multipath component (finger), and coherently adds the energy from each finger by selectively delaying each (the earliest component gets the longest delay) so that they can all be coherently combined.

A channel that is classified as flat fading can occasionally exhibit frequency selective distortion when the null of the channel's frequency-transfer function occurs at the centre of the signal band. The use of DS/SS is a practical way of mitigating such distortion because the wideband SS signal can span many lobes of the selectively faded channel frequency response. Hence, a great deal of pulse energy is passed by the scatterer medium, in contrast to the channel-nulling effect on a relatively narrowband signal.

The ability of the signal spectrum to span over many lobes of the frequency-selective channel transfer function is the key to how DS/SS signaling can overcome the degrading effects of a multipath environment. This requires the spread-spectrum bandwidth, W_{ss} (or the chip rate, R_{ch}), to be greater than the coherence bandwidth, f_0. The larger the ratio of W_{ss} to f_0, the more effective the mitigation.

A time-domain view of such mitigation can be similarly described. That is, to resolve multipath components requires that the spread-spectrum signal dispersion be greater than a chip time.

Frequency-hopping spread-spectrum (FH/SS) can be used as a technique to mitigate the distortion caused by frequency-selective fading, provided that the hopping rate is at least equal to the symbol rate. Compared to DS/SS, mitigation takes place through a different mechanism. FH receivers avoid the degradation effects due to multipath by rapidly changing in the transmitter carrier-frequency band, thus avoiding the interference by changing the receiver band position before the arrival of the multipath signal.

Orthogonal frequency-division multiplexing (OFDM) can be used for signal transmission in frequency-selective fading channels to avoid the use of an equalizer by lengthening the symbol duration. The approach is to partition (demultiplex) a high symbol-rate sequence into N symbol groups, so that each group contains a sequence of a lower symbol rate (by the factor $1/N$) than the original sequence.

The signal band is made up of N orthogonal carrier waves, and each one is modulated by a different symbol group. The goal is to reduce the symbol rate (signaling rate), $W \approx 1/Ts$, on each carrier to be less than the channel's coherence bandwidth f_0. OFDM, originally referred to as *kineplex*, is a technique that has been implemented in the United States in mobile radio systems, and has been chosen by the European community, under the name coded OFDM(COFDM), for high-definition television (HDTV) broadcasting.

Pilot signal is the name given to a signal intended to facilitate the coherent detection of waveforms. Pilot signals can be implemented in the frequency domain as in-band tones or in the time domain as digital sequences that can also provide information about the channel state and thus improve performance in fading conditions.

Among the above mentioned techniques equalizer is used for this thesis and hence will be discussed below.

4.2.1 Equalization

For many physical channels, such as telephone lines, not only are they bandlimited, but they also introduce distortions in their passbands. Such a channel can be modeled by an

LTI filter followed by an AWGN source. When a channel is dispersive in time, ISI is often introduced. For a communication system employing a linear modulation, such as BPSK, through a dispersive channel, the whole system can be described by the conceptual model shown below

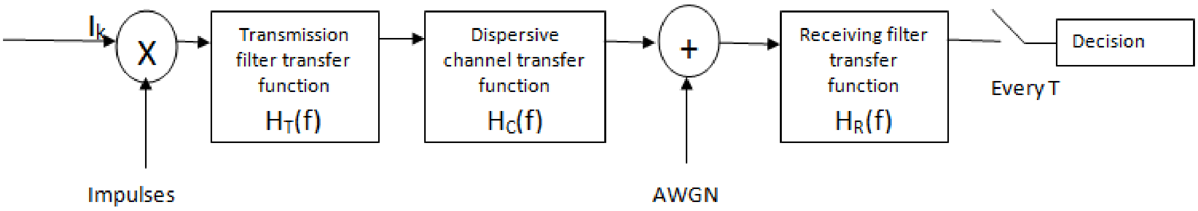

Figure 4-2: Model for Communication over a Dispersive Channel.

In Figure 4-2, in which the sequence of information symbols is denoted by $\{I_k\}$ and $H_T\,(f)$, $H_c\,(f)$, and $H_R\,(f)$ are the transfer functions of the transmission (pulse-shaping) filter, the dispersive channel and the receiving filter, respectively [14].

Letting $X(f) = H_T\,(f)H_c(f)H_R(f)$, the condition for no ISI is that the folded spectrum $X(f)$, is constant for all frequencies, i.e.,

$$\sum_{n=-\infty}^{\infty} x\left(f - \frac{n}{T}\right) = T \qquad (4.16)$$

One method to achieve the Nyquist condition is to fix the receiving filter to be the matched filter, i.e., set $H_R\,(f) = H^*{}_T\,(f)H^*{}_c\,(f)$, and choose the transmission filter so that (4.16) is satisfied. The major disadvantage of this pulse-shaping method is that it is in general difficult to construct the appropriate analog filters for $H_T\,(f)$ and $H_R\,(f)$ in practice. Moreover, we have to know the channel response $H_C\,(f)$ in advance to construct the transmission and receiving filters.

An alternative method is to fix the transmission filter and choose the receiving filter $H_R\,(f)$ to satisfy the condition in (4.16). As for the previous method, it is also difficult to build the appropriate analog filter $H_R\,(f)$ to eliminate ISI. However, notice that what we want eventually are the samples at intervals T at the receiver. Therefore, we may choose to build a simpler (practical) filter $H_R\,(f)$, take samples at intervals T, and put a digital filter, called *equalizer*, at the output to eliminate ISI as shown below in Figure 4-3. This approach to remove ISI is usually known as *equalization*. The main advantage of this approach is that a digital filter is easy to build and is easy to alter for different equalization schemes, as well as to fit different channel conditions [14].

The goal is to design the equalizer which can remove (or suppress) ISI. To do so, the continuous-time communication system model in Figure 4-3 has to be translated in to an equivalent discrete-time model that is easier to work with. The translation process is described by the following steps.

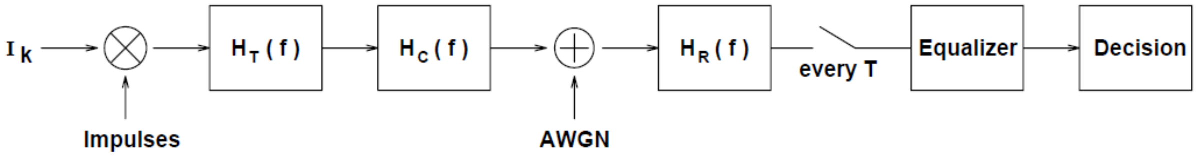

Figure 4-3: Communication System with Equalizer.

Instead of considering AWGN being added before the receiving filter $H_R\,(f)$, an equivalent colored Gaussian noise being added after $H_R\,(f)$ can be considered. The equivalent colored noise is the output of $H_R\,(f)$ due to AWGN. The resulting model is shown in Figure 4-4.

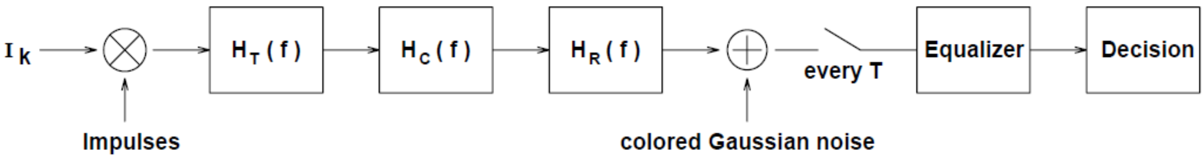

Figure 4-4: Equivalent Communication System with Colored Gaussian Noise.

Then we input a bit or a symbol to the communication system every T seconds, and get back a sample at the output of the sampler every T seconds. Therefore, the communication system in Figure 4-4 from the information source to the sampler can be represented as a digital filter. Since $H_T\ (f)$, $H_C\ (f)$ and $H_R\ (f)$ are LTI filters, they can be combined and represented by an equivalent digital LTI filter [14].

Let us denote the transfer function of this LTI filter by $H(z)$ and its impulse response $\{h_k\}_{k=-\infty}^{\infty}$. The resulting *discrete time-linear filter model* is shown in Figure 4-5.

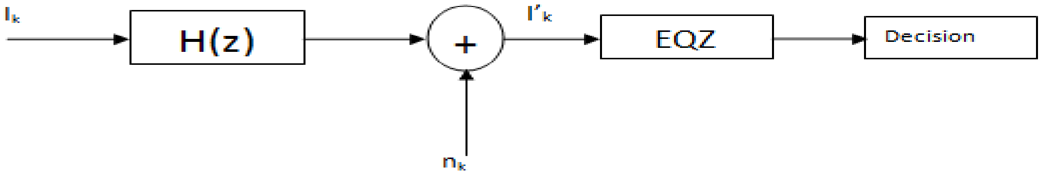

Figure 4-5: Equivalent Discrete-Time Communication System Model with Colored Gaussian Noise.

The output sequence I'_k is given by

$$I'_k = \sum_j I_j h_{k-j} + n_k$$

$$= I_k h_0 + \sum_{j \neq k} I_j h_{k-j} + n_k \tag{4.17}$$

Where $h_j \neq 0$ for some $j \neq 0$. Therefore, ISI is present. Notice that the noise sequence $\{n_k\}$ consists of samples of the colored Gaussian noise (AWGN filtered by $H_R\ (f)$ and is not white). Usually, equalizer consists of two parts, namely, a *noise-whitening* digital filter $H_W\ (Z)$ and an equalizing circuit that equalizes the noise-whitened output as shown in Figure 4-6. The effect of $H_W\ (Z)$ is to "whiten" the noise sequence so that the noise samples are uncorrelated.

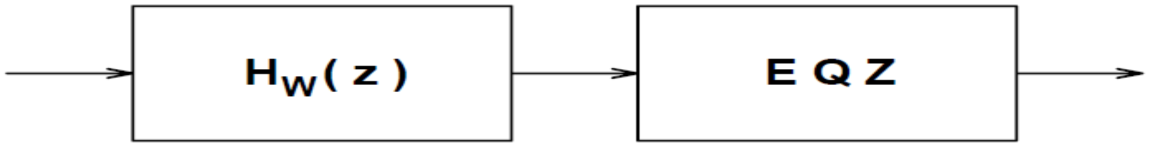

Figure 4-6: An Equalizer.

Notice that $H_W\ (Z)$ depends only on $H_R\ (f)$, and can be determined a prior according to our choice of $H_R\ (f)$. At the output of $H_W\ (Z)$, the noise sequence is white. Therefore, equivalently, we can consider the equivalent discrete-time model shown in Figure 4-7, in which $\{\widetilde{n_k}\}$ is an AWGN sequence [14].

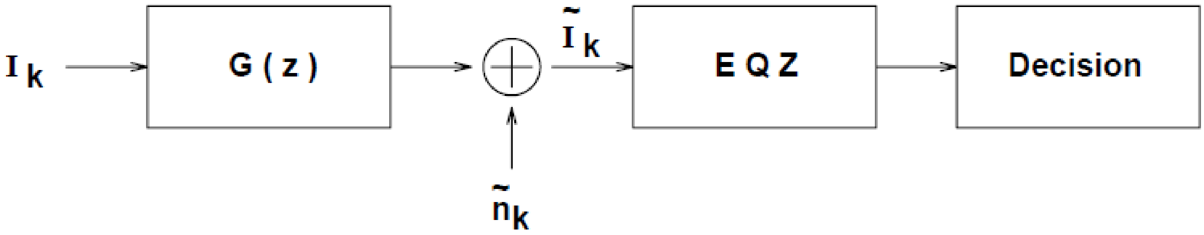

Figure 4-7: Equivalent Discrete-Time White-Noise Linear Filter Model.

Now letting $G(Z) = H(Z)H_W(Z)$ the communication system from the information source to the output of the noise whitening filter can now be represented by the *discrete-time white-noise linear filter model* in Figure 4-7. The output sequence $\tilde{I}_k$ is given by

$$\tilde{I}_k = \sum_j I_j g_{k-j} + \tilde{n}_k \tag{4.18}$$

where $\{g_k\}$ is the impulse response corresponding to the transfer function $G(z)$, and $\{\tilde{n}_k\}$is an AWGN sequence. This discrete-time model will be used for the discussions below.

At the end the equalizer attempts to remove ISI from the output of $G(z)$. The focus of the next section is the design of this equalizer.

Suppose that the equalizer is also an LTI filter with transfer function $H_E(z)$ and corresponding impulse response $\{h_{E,j}\}$, then the output of the equalizer is given by

$$\hat{I}_k = \sum_j \tilde{I}_{k-j} h_{E,j} \tag{4.19}$$

Ideally, $\hat{I}_k$ contains only contributions from the current symbol I_k and the AWGN sequence with small variance [14].

4.2.1.1 Zero-Forcing Equalizer

First, let us consider the use of a linear equalizer, i.e., we employ an LTI filter with transfer function $H_E\ (f)$ as the equalizing circuit. The simplest way to remove the ISI is to choose $H_E\ (f)$ so that the output of the equalizer gives back the information sequence, i.e., $\hat{I}_k = I_k$ for all k if noise is not present. This can be achieved by simply setting the transfer function $H_E(z) = 1/G(z)$. This method is called *zero-forcing* equalization since the ISI component at the equalizer output is forced to zero.

In general, the corresponding impulse response $\{h_{E,k}\}$ can be an infinite length sequence. Suitable truncation and delay is applied to get an approximation.

We can notice that the effect of the equalizing filter on the noise is neglected in the development of the zero-forcing equalizer above. In reality, noise is always present. Although the ISI component is forced to zero, there may be a chance that the equalizing filter will greatly enhance the noise power and hence the error performance of the resulting receiver will still be poor. To see this, let us evaluate the signal-to-noise ratio at the output of the zero-forcing equalizer when the transmission filter $H_T\ (f)$ is fixed and the matched filter is used as the receiving filter, i.e.,

$$H_R(f) = {H_T}^*(f){H_C}^*(f) \tag{4.20}$$

In this case, it is easy to see that the digital filter $\mathrm{H}\ (\mathrm{z})$ is given by

$$H\left(e^{j2\pi fT}\right) = \frac{1}{T}\sum_{n=-\infty}^{\infty}\left|H_T\left(f-\frac{n}{T}\right)H_c(f-\frac{n}{T})\right|^2 \tag{4.21}$$

and the PSD of the colored Gaussian noise samples $n_k$in Figure 4-5 is given by

$$\Phi_{nk}\left(e^{j2\pi fT}\right) = \frac{N_0}{2T}\sum_{n=-\infty}^{\infty}\left|H_T\left(f-\frac{n}{T}\right)H_c(f-\frac{n}{T})\right|^2 \tag{4.22}$$

Hence, the noise-whitening filter $H_W(z)$ can be chosen as

$$H_W(e^{j2\pi fT}) = \frac{1}{\sqrt{H(e^{j2\pi fT})}} \tag{4.23}$$

and then the PSD of the whitened-noise samples $\tilde{n}_k$ is simply $N_0/2$. As a result, the overall digital filter $G(z)$ is

$$G\left(e^{j2\pi fT}\right) = H\left(e^{j2\pi fT}\right)H_W(e^{j2\pi fT}) = H(e^{j2\pi fT}) \tag{4.24}$$

Now, we choose the zero-forcing filter $H_E(z)$ as

$$H_E(e^{j2\pi fT}) = \frac{1}{G(e^{j2\pi fT})} = \frac{1}{\sqrt{H(e^{j2\pi fT})}} \tag{4.25}$$

Since the zero-forcing filter simply inverts the effect of the channel on the original information symbols I_k, the signal component at its output should be exactly I_k.

If we model the $\mathrm{I_k}$ as *IID* random variables with zero mean and unit variance, then the PSD of the signal component is 1 and hence the signal energy at the output of the equalizer is just $\int_{-1/2T}^{1/2T} df = 1/T$. On the other hand, the PSD of the noise component at the output of the equalizer is $\frac{N_0}{2}\left|H_E(e^{j2\pi fT})\right|^2$. Hence the noise energy at the equalizer output is $\int_{-1/2T}^{1/2T} \frac{N_0}{2}\left|H_E(e^{j2\pi fT})\right|^2 df$. Defining the SNR as the ratio of the signal energy to the noise energy, we have

$$SNR = \left\{\frac{N_0T^2}{2}\int_{-1/2T}^{-1/2T}\left[\sum_{n=-\infty}^{\infty}\left|H_T\left(f-\frac{n}{T}\right)H_c(f-\frac{n}{T})\right|^2\right]^{-1} df\right\}^{-1} \tag{4.26}$$

Notice that the SNR depends on the folded spectrum of the signal component at the input of the receiver. If there is a certain region in the folded spectrum with very small magnitude, then the SNR can be very poor [14].

4.2.1.2 Minimum Mean Square Error Equalizer

The zero-forcing equalizer, although removes ISI, may not give the best error performance for the communication system because it does not take into account noises in the system. A different equalizer that takes noises into account is the minimum mean square error (MMSE) equalizer. It is based on the mean square error (MSE) criterion.

Without knowing the values of the information symbols I_k beforehand, we model each symbol I_k as a random variable. Assume that the information sequence $\{I_k\}$ is WSS. We choose a linear equalizer $H_E(z)$ to minimize the MSE between the original information symbols I_k and the output of the equalizer $\hat{I}_k$:

$$MSE = E[{e_k}^2] = E\left[(I_k - \hat{I}_k)^2\right] \tag{4.27}$$

Let us employ the FIR filter of order $2L + 1$ shown in Figure 4-7 as the equalizer. We note that a delay of L symbols is incurred at the output of the FIR filter. Then

$$MSE = E\left[\left(I_k - \sum_{j=-L}^{L} \tilde{I}_{k-j} h_{E,j}\right)^2\right]$$

$$= E\left[(I_k - {\tilde{I}_k}^T h_E)^2\right] \tag{4.28}$$

Where

$$\tilde{I}_k = [\tilde{I}_{k+L}, \dots\dots, \tilde{I}_{k-L}]^T \tag{4.29}$$

$$h_E = [h_{E,-L}, \dots, h_{E,L}]^T \tag{4.30}$$

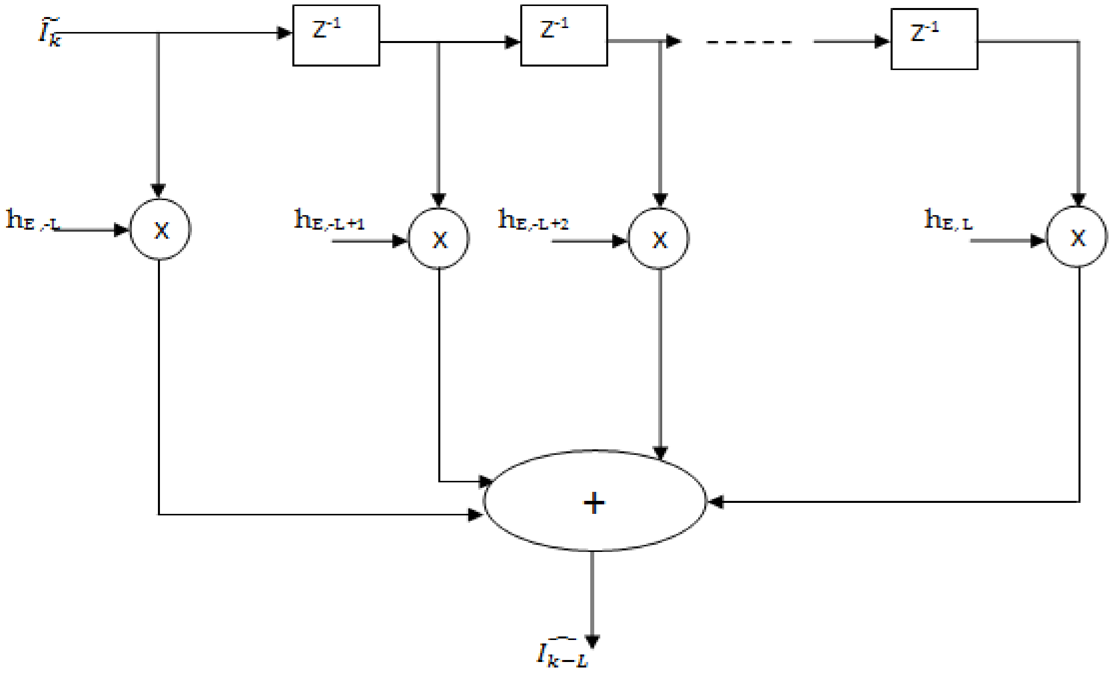

Figure 4-8: FIR Filter as an MMSE Equalizer.

We want to minimize MSE by suitable choices of $h_{E,-L}, \dots, h_{E,L}$. Differentiating with respect to each $h_{E,j}$ and setting the result to zero, we get

$$E\left[\tilde{I}_k(I_k - \widetilde{I_k}^T h_E)\right] = 0 \tag{4.31}$$

Rearranging, we get

$$Rh_E = d \tag{4.32}$$

where

$$R = E[\tilde{I}_k \widetilde{I_k}^T] \tag{4.33}$$

$$d = E[I_k \widetilde{I_k}] \tag{4.34}$$

If R and d are available, then the MMSE equalizer can be found by solving the linear matrix equation (4.32). It can be shown that the signal-to-noise ratio at the output of the MMSE equalizer is better than that of the zero-forcing equalizer.

The linear MMSE equalizer can also be found iteratively. First, notice that the MSE is a quadratic function of h_E. The gradient of the MSE with respect to h_E gives the direction to change h_E for the largest increase of the MSE. In our notation, the gradient is $-2(d - Rh_E)$. To decrease the MSE, we can update $\mathrm{h_E}$in the direction opposite to the gradient. This is the *steepest descent algorithm:*

At the k^{th} step, the vector $h_E(k)$ is updated as

$$h_E(k) = h_E(k-1) + \mu[d - Rh_E(k-1)] \tag{4.35}$$

where μ is a small positive constant that controls the rate of convergence to the optimal solution.

In many applications, we do not know R and d in advance. However, the transmitter can transmit a *training sequence* that is known *a priori* by the receiver. With a training sequence, the receiver can estimate R and d. Alternatively, with a training sequence, we can replace R and d at each step in the steepest descent algorithm by the rough estimates $\tilde{I}_k \widetilde{I_k}^T$ and $I_k \tilde{I}_k$, respectively. The algorithm becomes:

$$h_E(k) = h_E(k-1) + \mu[I_k - \widetilde{I_k}^T h_E(k-1)]\tilde{I}_k \tag{4.36}$$

This is a stochastic steepest descent algorithm called the *least mean square (LMS) algorithm* [14].

4.2.1.3 Least square Equalizer

In the training period for the MMSE equalizer, the "data" sequence, i.e., the training sequence is known to the equalizer. Instead of minimizing the MSE, which is a statistical average, we can actually minimize the sum of the square errors. This is called the *least squares (LS)* criterion. Suppose that the known sequence lasts for K symbols. Then the sum of the square errors is given by

$$e_k{}^2 = \sum_{k=1}^{K}(I_k - \hat{I}_k)^2$$

$$= \sum_{k=1}^{K}[I_k - \widetilde{I_k}^T h_E(k)]^2 \qquad (4.37)$$

Differentiating with respect to $h_E(k)$ and setting the result to zero, we get

$$R(k)h_E(k) = d(k) \qquad (4.38)$$

This time,

$$R(k) = \sum_{k=1}^{K} \tilde{I}_k \, \widetilde{I_k}^T \qquad (4.39)$$

$$d(k) = \sum_{k=1}^{K} I_k \tilde{I}_k \qquad (4.40)$$

Suppose that we are given one more training symbol. Apparently, we have to recalculate $R(K + 1)$ and $d(K+1)$, and solve the matrix equation all over again. However, actually, there is a more efficient approach. Assuming $R(K)$ is non-singular, $h_E(K) = R^{-1}(K)d(K)$. Notice that

$$R(k+1) = R(k) + \tilde{I}_{k+1}\widetilde{I^T}_{k+1} \tag{4.41}$$

$$d(k+1) = d(k) + I_{k+1}\tilde{I}_{k+1} \tag{4.42}$$

Using a matrix inversion, we get

$$R^{-1}(k+1) = R^{-1}(k) - \frac{R^{-1}(k)\,\tilde{I}_{k+1}\,\widetilde{I^T}_{k+1}\,R^{-1}(k)}{1+\widetilde{I^T}_{k+1}\;R^{-1}(k)\,\tilde{I}_{k+1}} \tag{4.43}$$

$$h_E(k+1) = h_E(k) + \frac{I_{k+1} - \;\widetilde{I^T}_{k+1}\,h_E(k)\,R^{-1}(k)\,\tilde{I}_{k+1}}{1+\widetilde{I^T}_{k+1}\;R^{-1}(k)\,\tilde{I}_{k+1}} \tag{4.44}$$

The procedure is called the recursive least squares (RLS) algorithm. In many cases, the RLS algorithm converges much faster than the steepest descent algorithm at the expense of more complex computation [14].

4.2.1.4 Decision Feedback Equalizer

Recall from the equivalent discrete-time model in Figure 4-7 that

$$\tilde{I}_k = I_k g_0 + \textstyle\sum_{j\neq k} I_j\, g_{k-j} + \tilde{n}_k \tag{4.45}$$

The current symbol we want to determine is I_k. If we had known the other symbols exactly, an obvious approach to eliminate ISI would be to subtract their effects off, i.e., the equalizer would give

$$\hat{I}_k = \tilde{I}_k - \textstyle\sum_{j\neq k} I_j g_{k-j} \tag{4.46}$$

In general, we do not know all the symbols that are affecting the reception of the current symbol.

However, it is possible to use previously decided symbols (output from the decision device) provided that we have made correct decisions on them. This approach is called *decision feedback equalization (DFE).*

With decision feedback, we can think of the equalizer to contain two parts a *feedforward* part and a *feedback* part. Suppose that the feedforward filter is of order $L_1 + 1$ and the feedback filter is of order L_2. Then

$$\hat{I}_k = \sum_{j=-L}^{0} \tilde{I}_{k-j} h_{E,j} + \sum_{j=1}^{L_2} I_{k-j}{}^{d}\, h_{E,j} \tag{4.47}$$

where $I_j{}^d$ are the decided symbols. Again, the filter coefficients $h_{E,j}$ can be found by minimizing the MSE. In general, significant improvement over linear equalizers can be obtained with the decision feedback equalizer.

Consider a DFE with a feedforward filter of order $L_1 + 1$ and a feedback filter of order L_2. Assume perfect decision feedback, i.e., $I_j{}^d = I_j$. Then

$$\hat{I}_k = I_F{}^T\, h_{E,F} + I_B{}^T\, h_{E,B} \tag{4.48}$$

Where

$$I_F = [\tilde{I}_{k+L1}\ \tilde{I}_{k+L1-1} \dots \tilde{I}_k]^T \tag{4.49}$$

$$I_B = [I_{k-1}\ I_{k-2} \dots I_{k-L2}]^T \tag{4.50}$$

$$h_{E,f} = [h_{E,-L1}\ h_{E,-L1+1} \dots h_{E,0}]^T \tag{4.51}$$

$$h_{E,B} = [h_{E,1}\ h_{E,-2} \dots\ h_{E,L2}]^T \tag{4.52}$$

Further assume that the data symbols I_k are zero-mean unit-variance iid random variables. We seek the filters $h_{E,F}$ and $h_{E,B}$ that minimize the MSE given by

$$E\left[(I_k - \hat{I}_K)^2\right] = E\left[(I_k - {I_F}^T h_{E,F} - {I_B}^T h_{E,B})^2\right] \tag{4.53}$$

Differentiating with respect to $h_{E,F}$ and $h_{E,B}$, we get

$$E\left[I_F\left(I_k - {I_F}^T h_{E,F} - {I_B}^T h_{E,B}\right)\right] = 0 \tag{4.54}$$

$$E\left[I_B\left(I_k - {I_F}^T h_{E,F} - {I_B}^T h_{E,B}\right)\right] = 0 \tag{4.55}$$

Notice that $E[I_k I_B] = 0$ and $E\left[I_k {I_B}^T\right] = I_{L2\times L2}$, i.e., the identity matrix. The equations for optimal $h_{E,F}$ and $h_{E,B}$ reduce to

$$E\left[I_F {I_F}^T\right]h_{E,F} + E\left[I_F {I_B}^T\right]h_{E,B} = E[I_k I_F] \tag{4.56}$$

$$E\left[I_B {I_F}^T\right]h_{E,F} + h_{E,B} = 0 \tag{4.57}$$

Solving these equations, we have

$$h_{E,F} = (E\left[I_F {I_F}^T\right] - E\left[I_F {I_B}^T\right] - E\left[I_B {I_F}^T\right])^{-1}\, E[I_K I_F] \tag{4.58}$$

$$h_{E,B} = -E[I_K I_F]h_{E,F} \tag{4.59}$$

Similar to the case of the MMSE equalizer, we can also solve for the feedforward and feedback filters using the steepest descent approach. If we do not know the expectations of the matrices above a *priori*, we can send a training sequence to facilitate their estimation [14].

4.2.1.5 Maximum Likelihood Sequence Receiver

Whenever it is not practical to construct ISI-free transmission pulses, we can use an equalizer to eliminate (or reduce) ISI, and then make a decision on the current symbol based on the equalizer output. Although this approach is simple and practical, we have no idea whether it is optimal in terms of minimizing the average symbol error probability. In fact, it turns out that all the equalization methods discussed in the previous section are not optimal. Because of the fact that the effect of a symbol is spread to other symbols, it is intuitive that the optimal receiver should observe not only the segment of received signal concerning the desired symbol, but the whole received signal instead. As a matter of fact, this strategy is also employed in the equalization techniques described previously.

Using the whole received signal, we can employ the maximum a posterior (MAP) principle to develop the optimal *symbol-by- symbol detector*, which decides one transmitted symbol at a time, to minimize the average symbol error probability. The development is rather involved and the optimal symbol-by-symbol detector is usually too complex to implement. Here, we opt for another possibility.

Instead of deciding a transmitted symbol at a time, we can consider to decide the whole transmitted symbol sequence simultaneously from the received signal. In this way, we aim at minimizing the probability of choosing the wrong sequence of symbols instead of the average symbol error probability.

With this sequence detection approach, we can employ the ML principle to achieve our goal. The resulting "optimal" receiver is referred to as the *maximum likelihood sequence (MLS)* receiver.

Although the DFE outperforms a linear equalizer, it is not the optimum equalizer from the viewpoint of minimizing the probability of error in the detection of the information sequence $\{I_k\}$ from the received signal samples $\{y_k\}$. In a digital communication system that transmits information over a channel that causes ISI, the optimum detector is a maximum-likelihood symbol sequence detector which produces at its output the most probable symbol sequence $\{\tilde{I}_k\}$ for the given received sampled sequence $\{y_k\}$. That is, the detector finds the sequence $\{\tilde{I}_k\}$ that maximizes the *likelihood function*

$$\Lambda(\{I_k\}) = lnP(\{y_k\}/\{I_k\}) \tag{4.60}$$

Where $P(\{y_k\}/\{I_k\})$is the joint probability of the received sequence $\{y_k\}$ conditioned on $\{I_k\}$). The sequence of symbols $\{\tilde{I}_k\}$ that maximizes this joint conditional probability is called the maximum likelihood sequence detector [9].

An algorithm that implements maximum-likelihood sequence detection (MLSD) is the *Viterbi algorithm, which was originally devised for decoding convolutional codes.*

The major drawback of MLSD for channels with ISI is the exponential behaviour in computational complexity as a function of the span of the ISI. Consequently, MLSD is practical only for channels where the ISI spans only a few symbols and the ISI is severe, in the sense that it causes a severe degradation in the performance of a linear equalizer or a decision-feedback equalizer.

4.3 Fast-Fading Channels

When viewed in the time domain, a channel is referred to as *fast fading* whenever $T_0 < T_s$, where T_0 is the channel coherence time and T_s is the symbol time.

Fast fading describes a condition in which the time duration that the channel behaves in a correlated manner is short compared to the time duration of a symbol. Therefore, it can be expected that the fading character of the channel will change several times during the time that a symbol is propagating. This leads to distortion of the baseband pulse shape, because the received signal's components are not all highly correlated throughout time. Hence, fast fading can cause the baseband pulse to be distorted, resulting in a loss of SNR that often yields an irreducible error rate.

Such distorted pulses typically cause synchronization problems, such as failure of phase locked- loop (PLL) receivers.

Viewed in the time domain, a channel is generally referred to as introducing *slow fading* if $T_0 > T_s$. Here, the time duration that the channel behaves in a correlated manner is long compared to the symbol time. Thus, one can expect the channel state to remain virtually unchanged during the time that a symbol is transmitted.

When viewed in the Doppler-shift domain, a channel is referred to as *fast fading* if the symbol rate, $1/Ts$, or the signal bandwidth, W, is less than the fading rate, $1/T_0$ or f_d. Conversely, a channel is referred to as *slow fading* if the signalling rate is greater than the fading rate. In order to avoid signal distortion caused by fast fading, the channel must be made to exhibit slow fading by ensuring that the signalling rate exceeds the channel fading rate.

4.4 Mitigation Techniques for Fast-fading Channels

Fast-fading distortion calls for the use of a ***robust modulation*** (noncoherent or differentially coherent) scheme that does not require phase tracking, and reduces the detector integration time [16]. Another technique is to increase the symbol rate, $W \approx 1/Ts$, to be greater than the fading rate, $f_d \approx 1/T_0$, by adding signal redundancy.

Error-correction coding can also provide mitigation; instead of providing more signal energy, a code reduces the required energy per bit to noise power spectral density ratio (E_b/N_0) for a desired error performance. For a given E_b/N_0 with coding present, the error floor out of the demodulator will not be lowered, but a lower error rate out of the decoder can be achieved.

Thus, with coding, one can get acceptable error performance and in effect withstand a large error floor from the demodulator that might have otherwise been unacceptable. To realize these coding benefits, errors out of the demodulator should be uncorrelated (which will generally be the case in a fast-fading environment) or an interleaver must be incorporated into the system design.

An interesting ***filtering technique*** can provide mitigation when fast-fading distortion and frequency-selective distortion occur simultaneously. The frequency-selective distortion can be mitigated by the use of an OFDM signal set. Fast fading, however, will typically degrade conventional OFDM because the Doppler spreading corrupts the orthogonality of the OFDM subcarriers. A poly phase filtering technique is used to provide time-domain shaping and partial response coding to reduce the spectral side lobes of the signal set, and thus help preserve its orthogonality. The process introduces known ISI and adjacent channel interference (ACI) which are then removed by a post-processing equalizer and cancelling filter.

5 Simulation Results and Discussions

5.1 Introduction

In this chapter, the performance of cooperative diversity for broadband vehicular communication when the channel is both frequency-selective and time-selective is investigated. First to validate the proposed system, simulation was carried out using same channel parameters as a previous work which was taken as a bench mark. Next the channel parameters were varied to study how the proposed system behaves for different channel conditions. The channels that were simulated are frequency-flat fast-fading, frequency-selective slow-fading and frequency-selective fast-fading (doubly-selective) channel. Finally the main focus of the thesis i.e. cooperative diversity performance over doubly-selective channel was investigated.

The main ideas that are needed to design the system model for the thesis work have been discussed in the previous chapters. After coming up with a possible system model, MATLAB simulation was carried out to study the performance of cooperative diversity for vehicular communication under doubly-selective channels. First the suggested system model will be explained and later the outputs from different scenarios will be shown.

5.2 Simulation Models for point-to-point and cooperative communication

The simulation model for Cooperative diversity for broadband vehicular communication under doubly selective channel is depicted in the diagrams below. Figure 5-1 shows a system model for direct point-to-point communication without any cooperative schemes then in Figure 5-2 the above idea was extended to include cooperative communication having a single relay node. AF relaying technique is chosen because of simplicity.

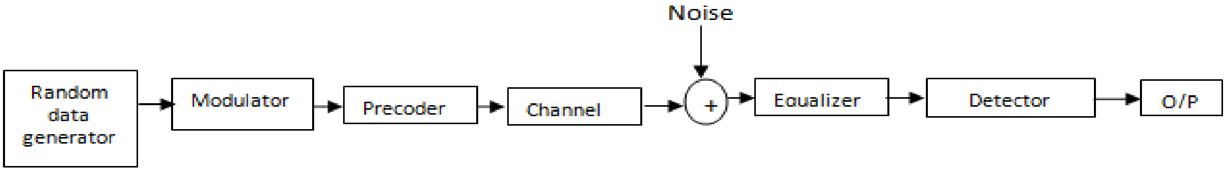

Figure 5-1: System Model for Point-to-point Communication for Vehicular Communication under Doubly-selective Channel.

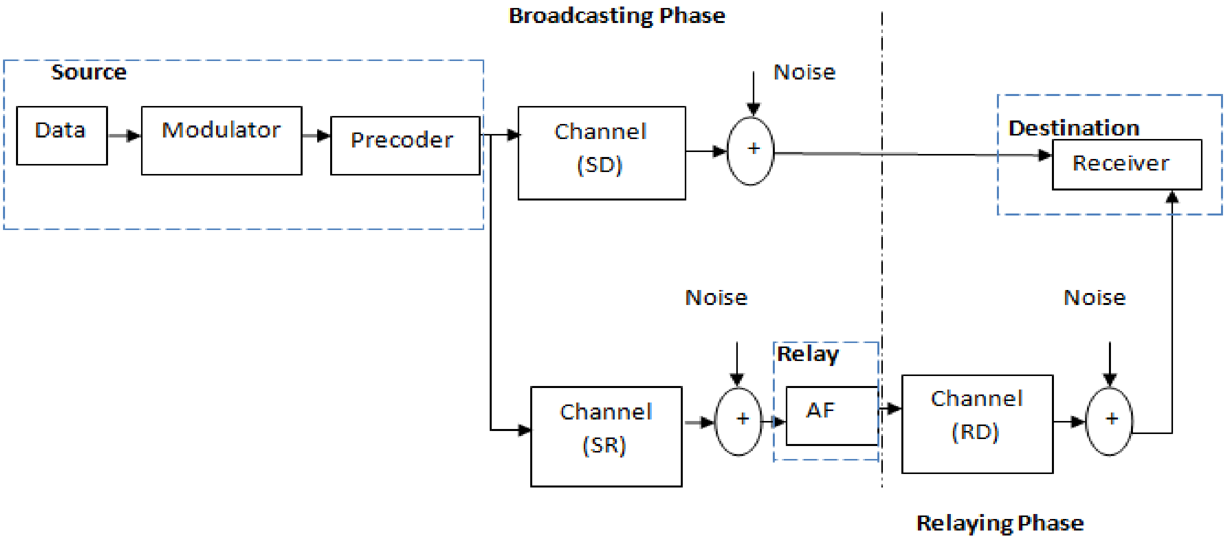

Figure 5-2: System Model for Cooperative Communication for Vehicular Communication under Doubly-Selective Channel.

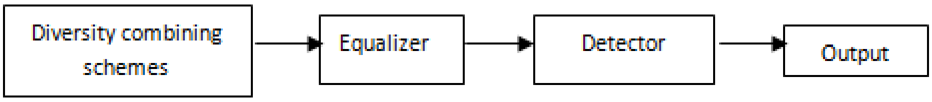

Figure 5-3: Destination Structure for Cooperative Communication.

In Figure 5-1, the random data generator generates the data that is to be transmitted. The data generated depends on the type of modulation we use. For example for Binary Phase Shift Keying (BPSK) we transmit two symbols i.e. 0 and 1 with integer equivalent same as the symbols transmitted, but for Quadrature Phase Shift keying (QPSK) the symbols to be transmitted are 00, 01, 10 and 11 with integer equivalent of 0, 1, 2 and 3 respectively. The generated data is fed to the modulator to be converted to digitally modulated symbols. Since the communication employed here is block transmission, the modulated symbols are converted to a block of certain length by the serial to parallel convertor.

After the data is divided in to blocks, each block will be multiplied by a precoder which adds redundancy to serve as channel coding to mitigate the effect of fast fading channels. Then the data will go through a channel which is modeled using Basis Expansion Model specifically Complex Exponential Basis Expansion Model (CE_BEM). The channel model is originally suggested to model the behavior of doubly-selective channel but by varying some of the parameters, other types of channels can be simulated like frequency-flat fast-fading and frequency-selective slow-fading channels.

We know that in real time wireless communication noise is among the many factors that corrupt the data we receive. So to include the effect of noise, additive white Gaussian noise is added to the output of the data from the channel. After this Equalizer specifically MMSE-ZF is applied at the receiver to mitigate the ISI introduced by the frequency-selective behavior of the channel. Note that all the above procedures are applied to each block of the transmitted data.

As mentioned before, Figure 5-1 shows the direct link communication without any cooperating nodes. The model with one relaying node is shown in Figure 5-2.

As shown in figure 5-2, we have two phases to transmit a single block of data, the broadcasting phase and the relaying phase. In the broadcasting phase the transmitter sends its data to the intended receiver and the relaying node. In the second phase i.e. the relaying phase the cooperating node sends the data it has received from the transmitter to the receiver. The cooperative scheme that is chosen is Amplify and forward (AF) because it is simple to implement. In AF, the relay sends out the data it has received without applying any kind manipulation to it except multiplying it with some scaling factor.

Now the receiver has double copies of the same data. The big question is what does the receiver do to solve this dilemma. At this point the receiver employs a combining technique to get one single data from the two. Among the three combining techniques SC and EGC are used. The third one which is MRC is not included in this work because it needs channel estimation to find the weight for the components to be added which is beyond the scope of this thesis.

In SC the receiver chooses the data with high SNR among the two where as in EGC the two copies will simply be added to form one single data. After combining, the resulting data will go through equalizer to remove the ISI induced by the channel and finally parallel to serial block converts the data to one single stream.

5.3 Simulation Parameters

The assumption is that we are operating at licensed band of 5.9GHZ for vehicular communication which is in line with IEEE 802.11p standard. We will have a corresponding wavelength of 0.051m.

As mentioned before the channel model used is Basis expansion model particularly CE-BEM. CE-BEM is the most commonly used one because of simplicity. In CE-BEM the time varying channel taps are expressed as superposition of time-varying basis functions. The time varying basis functions are the number of Doppler shifts that will be experienced by a single block i.e. it represents the number of frequency components available. The time varying channel taps, in our case the different multipath components due to reflectors will be linear combination of these basis functions. The coefficients that are used to combine the basis functions are assumed to be independent and identically distributed so they can be randomly generated. The basic parameters and mathematical relationships that are used in the thesis are listed in the table below.

Table 5-1: Simulation Parameters for Vehicular Communication.

No	Parameter	value
1	Carrier frequency (f_c)	5.9 GHz
2	Wavelength (λ)	0.05084 meter
3	Speed (V)	80 km/h
4	Maximum Doppler spread (f_{max})	431.7 Hz
5	Number of Doppler shifts experienced over data block (Q)	$Q = 2\lceil f_{max} M\, T_s \rceil$
6	Number of resolvable multipath	$L = \left\lceil {T_{max}}/{T_s} \right\rceil$

	components (L)	
7	Modulation scheme	4 QAM, 16-PSK
8	Number of symbols input to the precoder (N)	N=P*K, P,K>1
9	Number of symbols output from precoder (M)	M=(P+Q)*(K+L)

Where T_s is the symbol period, T_{max} is the maximum delay spread of the channel and M is block length after precoding.

5.4 Performance of Point-to-Point System Model under Doubly-selective Channel

This part of the simulation is done to validate the system model for the direct link with no cooperating nodes. To do this, parameters were chosen same as a system set as benchmark mentioned as reference 4. The precoder used for this thesis was first proposed in reference 4 and the performance with equalizers (but not with the one used in this thesis) was shown for point-to-point communication in the same work.

The parameters that were varied are Q and L which dictates the behavior of the doubly-selective channel and the performance of the channel is shown using Bit Error Rate (BER) verses Signal to Noise (SNR) curve.

The input to the precoder has a size of N which should fulfill a constraint $N = PK$ where P and K can be easily chosen as $P, K > 1$. And the output from the precoder has a length of M where $M = (P + Q)(K + L)$. Q and L are the parameters which will be changed and we will see how this affects the performance of the channel.

The choice of Q and L depends on the application-specific performance vs. complexity vs. decoding delay tradeoffs. This is because the design of the precoder and equalizer depends on the parameters so the larger we make them the longer it takes for the decoder and receiver to process.

P and K were set as 4 for the three simulations and initially Q is set as 2 and L as 1. Next L is increased to 2 while keeping Q at2. Finally Q is increased to 4 while keeping L at its initial value of 1.The simulation result is shown in the figure below.

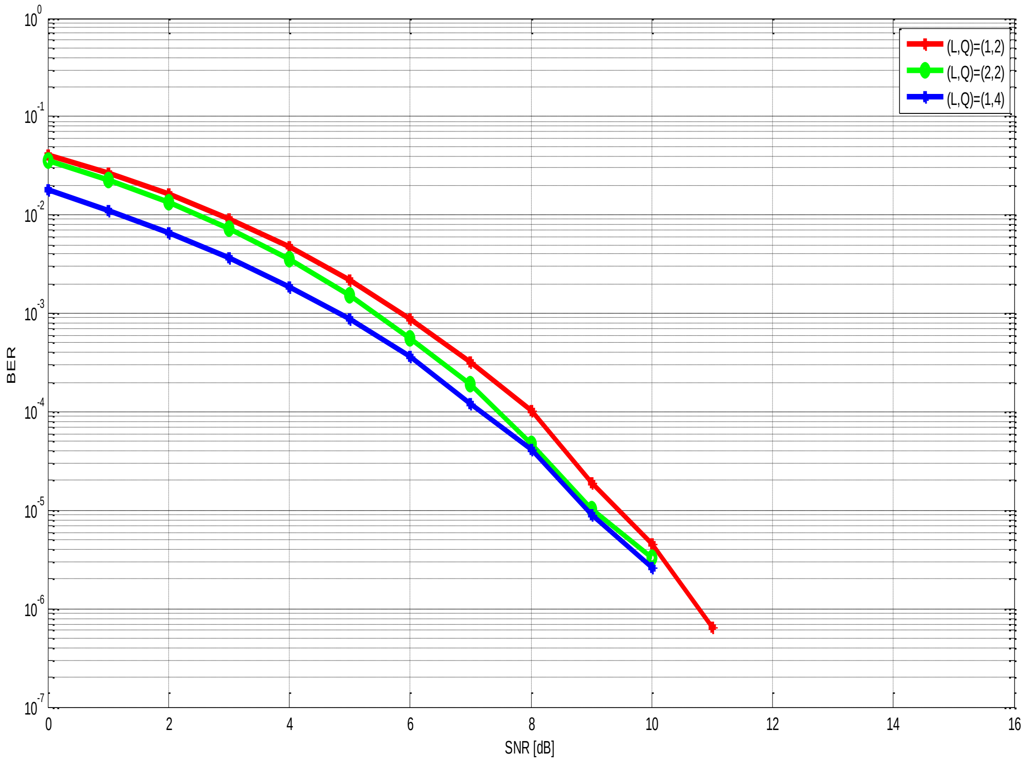

Figure 5-4: Performance of the Point-to-Point System Model for different types of Doubly-Selective Channels.

We can clearly see that the values of Q and L have an effect on the performance. This is because adjusting these values changes the output from the precoder.

The precoder is a matrix with complex values and it adds redundancy to the input symbol stream to improve performance in our case to mitigate the time-selective behavior of the channel. So as these values are increased, the redundancy added to the data increase thereby improving performance. But since the values of P and K are kept constant the input length remains the same.

One thing to mention here is that even if the number of Doppler spread and resolvable paths of the doubly-selective channels are increased we are able to get better performance at a lower coding rate. That is to say the symbols that should have been used to transmit useful data are instead used as redundancy to mitigate the fading induced by the channel. The input to the precoder for the three curves is the same which is P*K=4*4=16 complex modulated symbols but the output from the precoder varies according to the formula M= (P+Q)*(K+L). So as parameters L and/or Q are increased, the redundancy added to the data increases thereby decreasing coding rate.

5.5 Performance of Point-to-Point System Model for different Types of Fading Channels

Here the model suggested in figure 5-1 will be compared for different types of channels. The channels to be compared are doubly-selective, frequency-selective slow-fading and frequency-flat fast-fading channels. As discussed above we can change the channel to be doubly-selective, frequency-selective or time-selective by adjusting the parameters Q and L. For the first channel type both Q and L are set to be different from zero. For the second one only Q is set to zero and as for the final one only L is set to zero.

The simulation result is shown below.

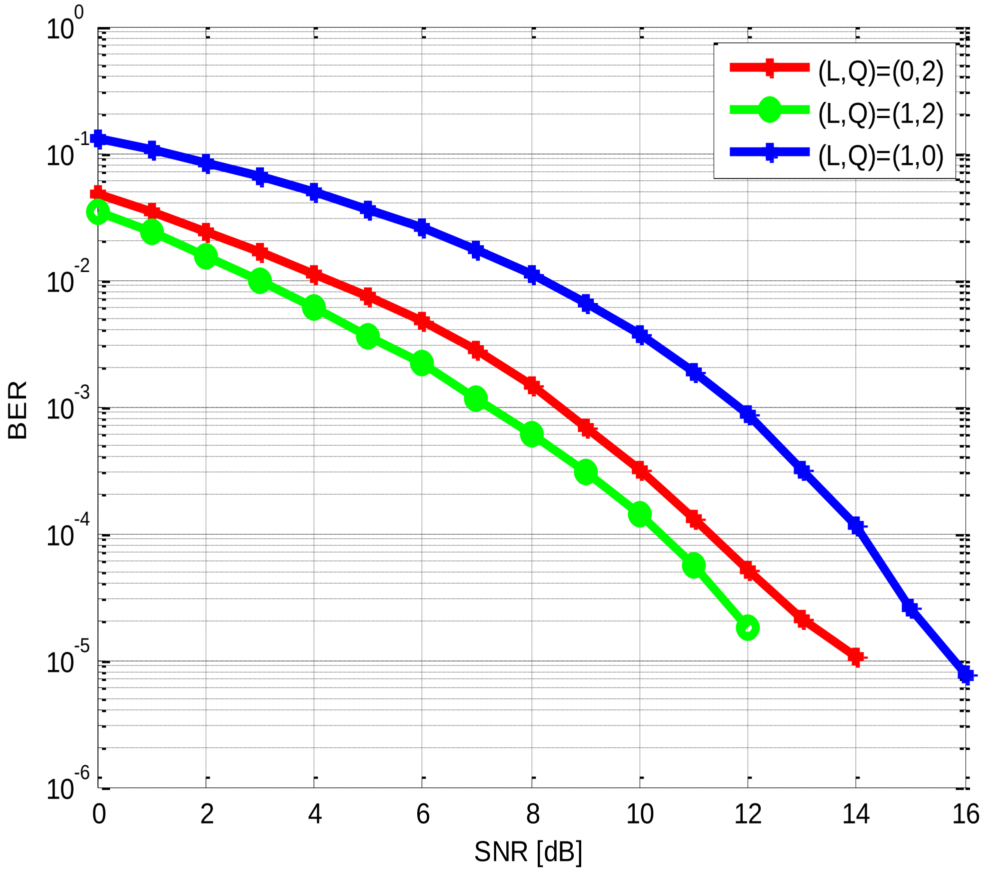

Figure 5-5: Performance of the Point-to-Point System Model for Doubly-Selective, Frequency-Selective and Fast-Fading Channels.

Note that the blue colored curve is for frequency-selective channel, the red one for fast fading and finally the green one for doubly-selective channel.

Let us analyze the result by taking a fixed BER value. For BER of 10^{-4} there is a about 2.5dB difference between frequency-selective and fast fading channel, and 3.5dB difference between frequency-selective and doubly-selective channel. This improvement as mentioned above is because the redundant symbols added to the data stream increases as the channel parameters are increased at the expense of throughput.

5.6 The effect of Cooperative Communication

The cooperative scheme suggested is AF. There will be two phases for transmission, broadcasting phase and relaying phase. The contribution for this type of system is from the relay which sends out what it has received from source to destination there by providing the receiver with double copy of the same data. The next step will be for the receiver to decide how to use the available copies.

Among the three combining schemes EGC and SC are employed in this work. For SC the receiver has to decide which symbol to take from the two. In order to do this the receiver computes the total power of each symbol and take the one with the highest power as the best one. The logical thing would have been to measure SNR of each symbol and take the one with the highest value but since it is difficult to measure SNR the total power is computed instead. The shortcoming with this method is the power of noise is included in the power computation so which ever has the highest power does not necessarily mean the one with high SNR. In fact it may mean the noise power is large in one symbol than other. That is the reason why this combining scheme has a small performance compared to other.

In equal gain combining the corresponding symbols from both phases are just added with equal weight. We can simply take the value of the weight to be one.

Another thing to consider is the behavior of the three channels that are involved in the cooperative scheme. The channels are source-to-destination, source-to-relay and relay-to-destination. The source-to-destination and relay-to-destination channels are assumed to be doubly-selective using the fact that there is a relative motion and multipath propagation between the communicating nodes.

The source-to-relay channel is assumed to be frequency-flat and slow-fading. In order for the channel to be like this we can assume either both source and relay are static or both are moving with the same speed in the same direction. In the first scenario since both stations are static there will not be any Doppler spread so the channel will be slow fading. In the second scenario if both stations are moving with same speed in the same direction still the relative motion between them will be zero so the channel will still be slow fading.

So using these ideas in mind, the cooperative scheme was simulated.

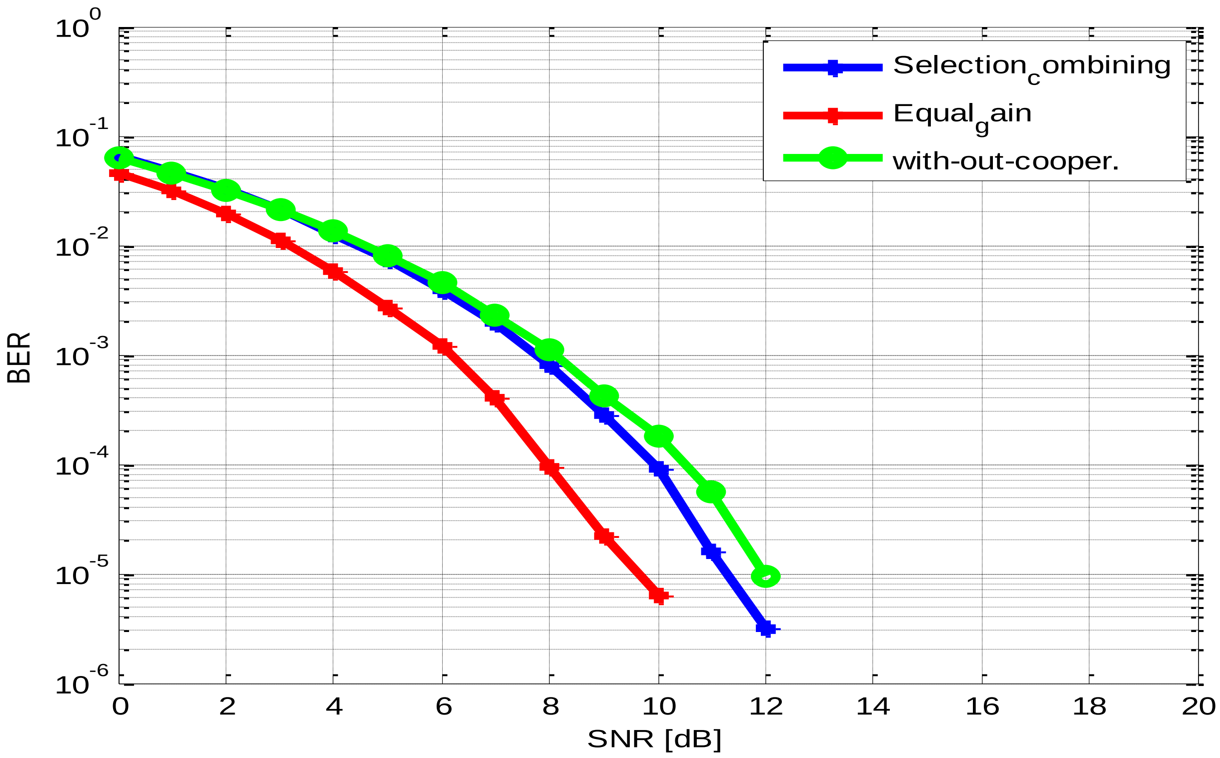

Figure 5-6: Performance of Cooperative Communication under Doubly-Selective Channel for Vehicular Communication.

From the figure we can observe that equal gain combining has superior performance that selection combining. At lower SNR value the performance improvement by selection is insignificant but as the SNR value increases it gets improved.

Taking a fixed BER value of 10^{-4} we can observe that there is a 2.5dB difference between point-to-point communication and cooperative communication using Equal gain combining scheme where as in the case of selection combining there is a 0.5dB improvement. Since the receiver is provided with two copies of the same symbol and by using the mentioned combining technique, the decision made by the detector while demodulating will be enhanced.

And also as shown in Figure 3-5 EGC has superior performance as compared to SC which is also shown here.

5.7 The effect of increasing Throughput and Coding Rate on Performance of the Cooperative Scheme

To demonstrate these two simulations were carried out. In the first one throughput was increased by varying parameters L and Q. In the second one the throughput was improved by using higher modulation schemes. The result from the simulation is as shown below.

For the first case throughput was increased by using relatively higher modulation scheme namely 16-PSK.

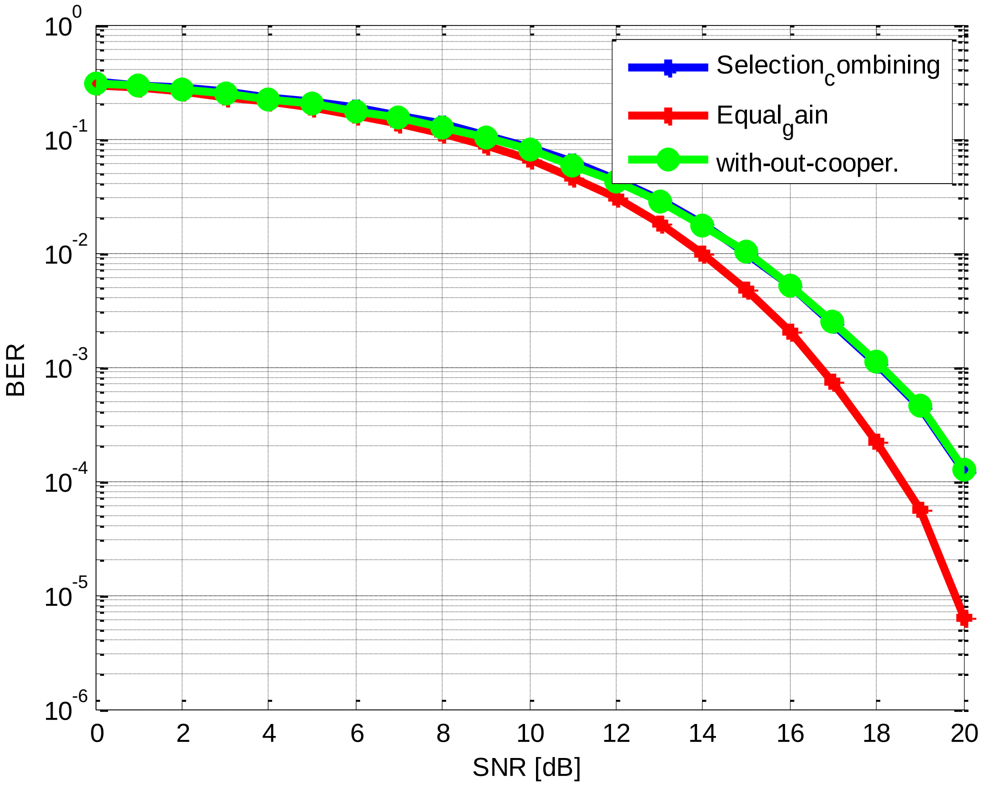

Figure 5-7: Performance of Cooperative Communication for 16-PSK using same Channel Parameters as Figure 5-6.

The channel parameters L and Q, and also P and K are all kept constant for results shown in Figure 5-6 and Figure 5-7. They values are 1, 2, 4 and 4 respectively. These values give a coding rate of N/M where N=P*K=16 and M= (P+Q)*(K+L) = (4+2)*(4+1) = 30.Therefore, coding rate=16/30. Although it is possible to increase throughput by using higher modulation schemes, the performance decreases. As shown in the figure a BER of 10-4 was achieved at SNR of 18.5dB for EG and 20dB for SC. But for the same BER 4-QAM was able to achieve these at 8dB and 10dB respectively (Figure 5-6). And also the performance curve of SC has almost overlapped with the curve for point-to-point communication.

For the second case the same modulation was used as Figure 5-6 but what was varied here was the number of symbols that were entered in to the Precoder. The coding rate for the precoder in Figure 5-6 was $16/30$ as calculated above now this was increased to $36/56$. The new coding rate was achieved for the same values of L and Q which are 1 and 2 respectively as mentioned above but what changed are the values of P and K which determine the number of input symbols to the precoder. Here, a value of 6 is used for both P and K.

A BER of 10^{-4} was achieved at SNR of 8dB and 10dB for the previous case with EG and SC respectively. But the same BER was reached at 9dB and11dB using a higher coding rate.

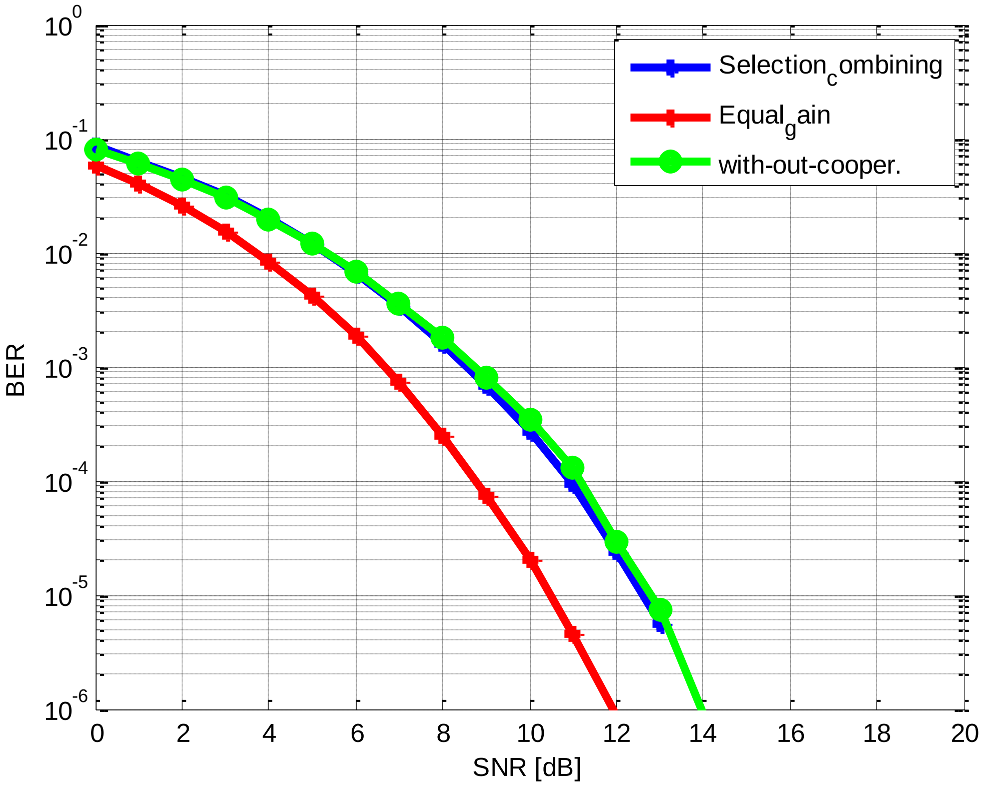

Figure 5-8: Performance of Cooperative Communication for 4-QAM with a Higher Coding Rate as Compared to Figure 5-6.

5.8 The effect of Doppler Spread and Multipath Components on Coding Rate

Here, what has been tried to show is how the channel parameters Q and L affect coding rate of the system. First, parameter L was fixed and other parameters Q, P and K were varied to show how the coding rate is affected at a certain value of L. The result is shown in Figure 5-9 and we can observe that as the value of L increases the coding rate decreases.

Next, the effect of varying Q and L at the same time was simulated. This is done by keeping Q and L at a constant value and by varying parameters P and K. Then finally the average value was taken as the coding rate at a fixed Q and L. the result cements the conclusion mentioned above i.e. increasing Q and L increases the system performance by decreasing the coding rate.

The choice of which values of the parameters to use depends on what qualities we want our system to have, reliability or high data rate.

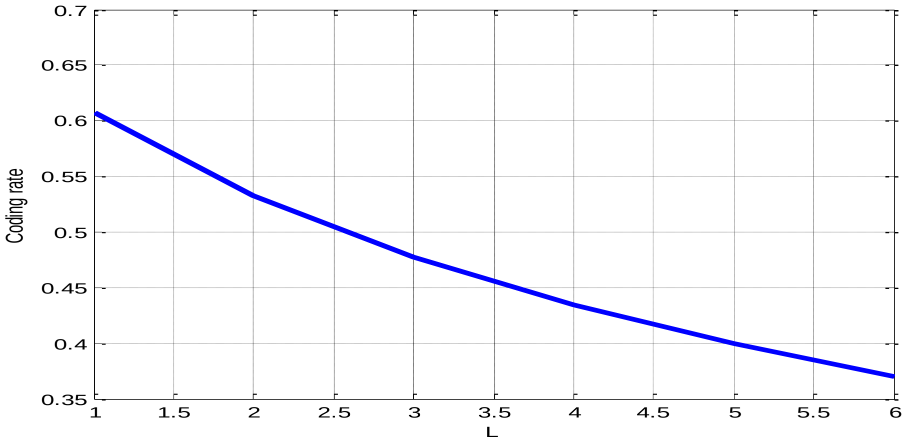

Figure 5-9: Effect of Channel Parameter L on Coding Rate.

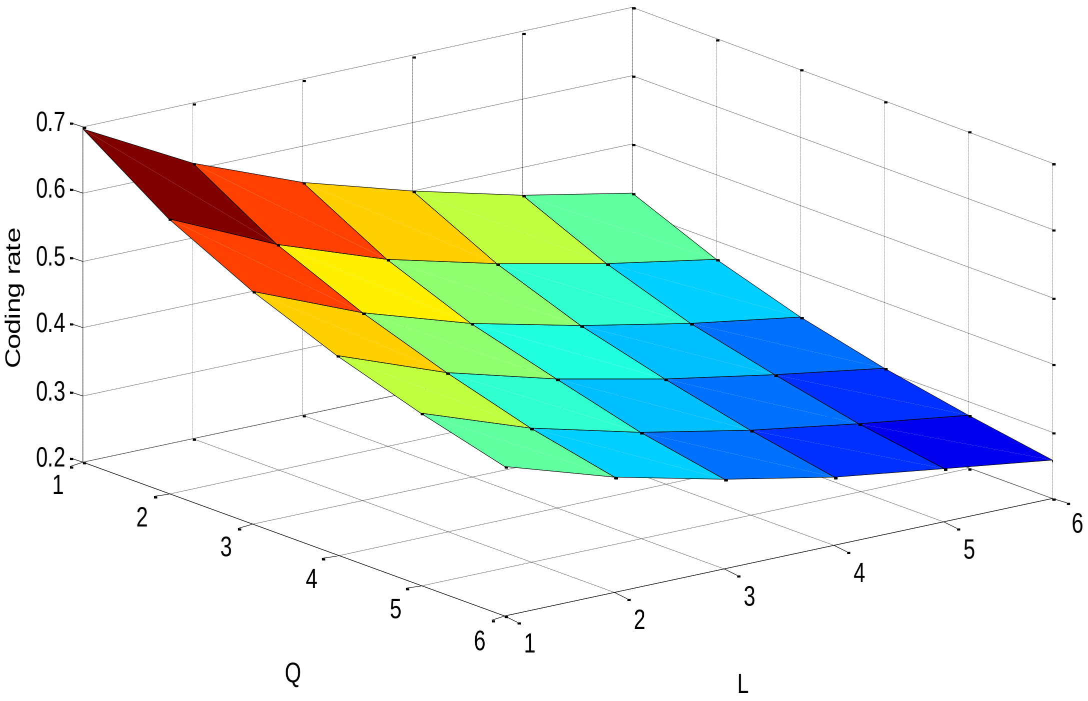

Figure 5-10: Effect of channel Parameters L and Q on coding rate.

6 Conclusions and Recommendation for Future Works

6.1 Conclusions

The performance of cooperative diversity for broadband vehicular communication was investigated in this thesis. Simulation results from the proposed cooperative communication system and point-to-point network topology were explained.

Identical parameters that were used in previous work were also considered for the point-to-point topology. The result for the point-to-point communication using precoder and equalizer shows performance improvement as the channel parameters are increased. This is because the multipath-doppler diversity provided by the doubly-selective channel increases with the increment of these parameters. Taking a BER of 10^{-4}, we can see a performance improvement of 0.5dB and 1dB by increasing the channel parameters from a reference value considered for the particular simulation.

The proposed mitigation techniques for doubly-selective channel allow the performance of the channel to be superior as compared to frequency-selective and time-selective channel but this comes at the expense of reduced throughput. This is because in doubly-selective channels the channel parameters will be higher as compared to frequency-selective and time-selective channels. These parameters and the coding rate of the precoder are inversely related. Increasing channel parameters will lead to adding more redundant bits to the information we are sending. Again by taking a BER of 10^{-4}, there is a performance improvement for doubly-selective channel by 3.5dB and 1dB as compared to frequency-selective channel and time-selective channel respectively for the channel parameters specifically taken for the simulation.

Cooperative diversity which is an example of spatial diversity is also one way of mitigating fading introduced by a channel. Extending the point-to-point topology to cooperative communication improves performance by 2.5dB for equal gain combining scheme and by 0.5dB for selection combining at BER of 10-4.

To conclude even if performance was improved by applying mitigation techniques on fading channel, there is always tradeoff between performance and throughput. The choice is always application specific and one has to decide which is more necessary before designing a system.

6.2 Recommendation for Future Works

This thesis can be developed further by incorporating other factors that were not taken in to account. Some of the suggestions are listed below.

- The thesis assumes there is a perfect feedback channel between transmitter and receiver there by both communicating parties have clear knowledge of CSI. One can assume the feedback channel to be erroneous and can investigate the impact.
- Only AF diversity scheme is employed. The performance of other schemes like decode and forward, incremental relaying and selection relaying can also be taken as future enhancements to this work.
- This work also assumes only one relaying node. Many numbers of nodes can be taken in to consideration.

- We have seen that the throughput is affected even if the performance is increased. To mitigate this one can apply higher modulation schemes in higher SNR regions. So one can employing adaptive modulation and study the effect.
- Among the many BEM channel modeling techniques only CE-BEM was chosen for this work. Using other modeling techniques can also be taken as an interest for future works.
- And also the performance of one equalizer namely MMSE-ZF was shown here. Comparing the many equalizers which have been proposed for doubly-selective channels on previous works can also be work of interest.

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