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SCHOOL OF MECHANICAL AND INDUSTRIAL ENGINEERING

Investigation on the Contact Fatigue of Railway Wheel Using a Critical Plane Model

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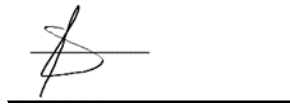
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APPROVAL

The undersigned have examined the thesis entitled ‘**Investigation on the Contact Fatigue of Railway Wheel Using a Critical Plane Model**’ presented by **AKLILU GETNET**, a candidate for the degree of **Master of Science** and hereby certify that it is worthy of acceptance.

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ABSTRACT

The contact areas between rail and wheel are highly critical to determine the performance of rail vehicles. So, the investigation of this highly stressed interface is important for the study of different damages mechanisms such as surface and sub-surface cracks, plastic deformation and wear. An easy and novel approach that predicts fatigue life and crack orientation is vital for the investigation of wheels against fatigue. This paper aims to investigate rolling contact fatigue (RCF) cracks and fatigue life in railroad wheels using Finite Element Model. The effect of different parameters such as wheel diameter, friction and vertical loads on the fatigue life of the wheel is also thoroughly studied.

The proper material model depicting the plastic shakedown, ratchetting and cyclic hardening is used for both the wheel and rail material. The problem is numerically modelled in Abaqus 2020 after the geometry is developed using the CAD section. Implementation of submodelling technique has highly reduced the computation demand. The stress and strain history outputs extracted from the submodel were used to predict fatigue life initiation period using critical plane models. The orientation of critical (crack) plane was found using a novel MATLAB algorithm by searching for a plane with maximum fatigue parameter ($FP_{\max}$).

Keywords: RCF, Critical Plane, Fatigue Parameter, Stress and Strain History Outputs, MATLAB, Abaqus

NUMENCLATURE

Symbol	Description and Units
a	Contact half width for line and point contact (mm)
a, b	Half-length of major and minor axis of the ellipse (mm)
E^*	Reduced modulus (GPa)
E	Elastic modulus (GPa)
$p(x), p(r)$	Contact pressure distribution (Pa)
$p(x, y)$	Contact pressure distribution along the elliptical area
P	Applied load (kN)
p_m	Mean contact pressure (Pa)
p_o	Maximum contact pressure (Pa)
ν	Poisson's ratio
R	Relative radius of curvature for wheel (mm)
k	Ellipticity parameter
R_x, R_y	Reduced radii along x and y axes (mm)
m, n	Elliptic Hertz coefficients
A, B	Geometrical constants in elliptic area calculation
τ_a	Shear stress amplitude
τ_{max}	Maximum shear stress
σ_h	Hydrostatic stress
τ_e	Positive fatigue limit
a_{DV}	Dang-Van parameter
$\Delta\gamma$	Shear strain range
$\Delta\epsilon$	Normal strain range
$\Delta\tau$	Shear stress range
$\sigma_{n, max}$	Maximum normal stress
σ_y	Yield stress
J	Material dependent constant in fatigue parameter calculation
t_{-1}, f_{-1}	Fatigue limits for pure uniaxial and torsional loading, respectively

t_{Nf}, f_{Nf}	Fatigue strength coefficients at finite life, N_f for pure uniaxial and torsional loading, respectively
N_f	Number of cycles to failure
α	Angle between crack and fracture plane
η_{nf}	Mean stress correction factor
σ_f', τ_f'	Fatigue strength coefficient in mode I and mode II crack types, respectively
ϵ_f', γ_f'	Fatigue ductility coefficient in mode I and mode II crack types, respectively
b, c	Fatigue strength exponents
FP	Fatigue parameter
FP_{max}	Maximum fatigue parameter
θ, φ	Critical plane orientation angles
Θ	Intermediate parameter
σ_{vm}	Von-mises stress
σ	Stress tensor
C_k, γ_k, R, Q	Wheel material parameters calibrated from uniaxial test data
α	Back stress tensor
$\bar{\epsilon}^p$	Plastic strain
$\dot{\bar{\epsilon}}^p$	Plastic strain rate
V	Translational velocity of the wheel
k_r, k_w	Stiffness of rail and wheel
c_r, c_w	Damping constant for rail and wheel
μ	Friction coefficient
l, m, n	Direction cosines

GREEK SYMBOLS

σ	Normal stress
γ	Shear strain
μ	Friction coefficient
φ, θ	Crack plane orientation
τ	Shear stress
ϵ	Normal strain

ABBREVIATIONS

Symbol	Description
RCF	Rolling contact fatigue
HCF	High cycle fatigue
LCF	Low cycle fatigue
EHL	Elasto-hydrodynamic lubrication
FE	Finite element
FEM	Finite element method
FEA	Finite element analysis
CPS	Critical plane search
FP	Fatigue parameter
GM	Global model
SM	Submodel

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CHAPTER 1 INTRODUCTION

1.1 Background

The performance of rail vehicles is highly affected by the contact areas where the wheels and the rails meet. This highly stressed region is the motive to most researches in vehicle-track interaction which range from the dynamic performance of the track vehicle to the maintenance of the infrastructure [1]. For example, to understand damage mechanisms such as cracks, wear and plastic deformation of the contact area between steel wheel and steel rail interactions are investigated [2]. Moreover, investigation of this contact between the wheel and the rail allows us to take the necessary measures such as corrective and preventive maintenance of the system. Therefore, a better understanding of the contact phenomenon between wheel and rail is crucial to design an optimized railway system of transportation.

Nowadays, with the growing demand of the market for higher speed and heavier axle loads, the high maintenance cost of both the rolling stock and the infrastructure became among the top concerns for track owners and train operators. The major portion of these maintenance costs results from the deterioration phenomena occurring in the notorious wheel-rail interface. Wear and rolling contact fatigue (RCF) mostly contribute for the occurrence of this phenomenon.

Rolling contact fatigue (RCF) has become one of the main failure mechanisms of rolling components, such as railway wheels. RCF occurs in a rail material due to a repetitive passage of wheel on rails under an impact loading condition [3]. Usually, the twin disc test, which is a rolling disc-on-disc test, is carried out in a laboratory environment to investigate rolling contact fatigue (RCF) for different contact conditions [4][5].

RCF manifests itself in a variety of different mechanisms that ultimately lead to final failure [6][7]. There are two most dominant RCF damage mechanisms; subsurface originated spalling and surface originated pitting. These are often competing failure modes, and the ultimate mechanism that prevails depends on a number of factors, such as surface quality, lubricant cleanliness, material quality, etc. [6].

Subsurface cracks occur when microcracks initiate below the surface at inhomogeneities of the material. Factors favoring subsurface cracks are smooth surfaces, the presence of nonmetallic inclusions in the material, and absence of surface shear. This mechanism is the dominant mode of failure in rolling element bearings that have smooth surfaces and operate under elastohydrodynamic lubrications (EHL) conditions. Surface originated pitting, on the other hand, occurs in cases where surface irregularities in the form of dents or scratches are present. Here, cracks initiation occurs at the surface stress concentrators and then propagate at a shallow angle to the surface. When a critical length or depth is reached, the cracks grow towards the free surface, removing a piece of material from the surface and resulting in pit formation. This mechanism of failure is common in wheel-rail contact and gears where substantial sliding occurs between the contacting surfaces[6][7].

In the past few years, there has been a substantial deal of research dedicated to comprehend and predict surface initiated RCF phenomena. Nevertheless, the primary step that enables investigating such mechanisms is to clearly know the shape and size of the contact patch, and the distribution of contact stresses within it [8][9]. These mainly depend on four primary factors: axle loads, wheel and rail profiles, track curvature and the friction coefficient [8][10].

When the fatigue crack is initiated, prediction of the crack propagation is important for estimating the crack growth and the wheel and rail lifetime. This can prevent the possible fracture of wheel and rail by applying in-time maintenance strategies [3]. Therefore, investigation of rolling contact fatigue is critical to better understand unexpected failures in wheel and rail.

1.2 Statement of the problem

The wheel-rail contact area is the most critical region in a railroad system as it is the pathway for the transfer of load from the bogie (train) to rail and the finally to the ground. Therefore, the wheel/rail contact region requires a thorough investigation while designing a railroad system. Especially, the wheel is exposed to low cycle fatigue (LCF) failure due cyclic loading caused by the rotation. As a result, fatigue life of a railroad wheel should be investigated, thoroughly.

A reliable and novel approach that forecasts fatigue life and orientation of crack is crucial to investigate fatigue in rotating wheel. This approach applies the stress and strain history results from FE analysis and a novel MATLAB algorithm to predict the orientation of crack in a wheel and its fatigue life to crack initiation.

1.3 Objectives

1.3.1 General objective

The major objective of this paper is to investigate rolling contact fatigue (RCF) crack of a railway wheel using a critical plane model.

1.3.2 Specific objective

- Analyzing the contact stresses and strains in the wheel-rail contact using three-dimensional finite element model.
- Investigation of RCF crack and fatigue life of the wheel.
- Predicting the orientation of crack (crack depth and crack angle) in the wheel.
- Investigating the effect of axle loads, friction and wheel diameter on the contact stress and the rolling contact fatigue.

1.4 Significance of the study

This research can be a useful tool to undertake preventive maintenance techniques and schedules before crack propagation happens.

The method proposed in this paper can also be applied for fatigue resistance design and maintenance planning of railroad wheels.

1.5 Limitations of the study

This study comes with some limitations due to different constraints which are listed below.

- The main limitation of this research is inaccessibility of high-performance computing devices to conduct the finite element analysis. As a result, the process demanded higher computational time.

- This study assumes wheel rolling along a straight track with no gradient (inclination). So, lateral loads and contact at flanges are not considered in this study.

CHAPTER 2 LITERATURE REVIEW

In this section previous works on general approach of contact problems has been reviewed. Different journal articles, books, book sections, conference proceedings etc. were referred. The topics are organized on theme-based approach. In the first section general behavior of contact mechanics and types of contact are investigated. In the second section the nature and mechanisms of the wheel rail contact are thoroughly investigated by exploring both analytical and numerical methods. In the final section the damages resulting from this highly critical contact stresses are dealt giving emphasis on the surface and subsurface initiated RCF.

2.1 Numerical analysis of Contact stresses

Srivastava et al[11] have formulated a finite element method (FEM) approach to determine contact areas (zones), contact pressures and contact stresses for different configuration of the wheel and rail profile. In geometric modelling UIC60 rail are used and imported to Ansys. Around the contact area, the mesh density of 1 mm element size has been implemented as it showed better accuracy in results than 0.5- and 1.5-mm element sizes. The SOLID 186 element was applied to discretize wheel and the rail geometry while the contact zones were meshed with contact (master) and target (slave) element CONTA174 and TARGE170, respectively. Since the wheels are assumed to roll on a straight and flat path, lateral loads to the system are neglected. Effects of wheel rotation are also ignored. The rail and wheel are assumed to have similar material properties. The results obtained in the FEM and the analytical method showed good convergence. The limitation of this paper is it only considered straight rails and curved rails which include traction forces besides normal loads were not taken into account.

Sladkowski et al[12], have also analyzed influence of wheel and rail profiles on the contact area and contact stresses. Besides, they investigated the deformation resulting from the contact forces and the influence of this deformation on redistribution of the contact stresses. A wheel with a standards wheel profile of GOST 9036-88 and rail P65 were used in the analysis. Discretization with semi-automatic mode was used as the FE grid created by the mesh generator gave less accurate results. The authors proposed to create regular FE grids for all zones near rail wheel contact to eliminate possible errors. As a result, to

minimize computation time, at first, sparse meshes of wheel and rail were considered to define displacements on the boundaries of considered areas. Then, the wheel rail contact problem was analyzed using dense mesh with the obtained boundary conditions. Next, near contact areas of the wheel and rail were positioned properly. Vertical and lateral force were applied on the borders of these areas as a boundary condition and the location and size of the contact zones was derived along with the stress distributions, iteratively.

Mehmet et al[13] paper proposes required procedures for more realistic 3D finite element analysis (FEA) solutions for rail wheel contact problems. Wheel, rail and axle geometries were created in CAD environment and the half symmetry is transferred to ANSYS for FE analysis. The 3D model is meshed with SOLID45 8-noded hex elements and the mesh has been refined very well at contact location for more accurate results. Contact173 elements and target170 elements are used for the wheel and the rail respectively. The results showed that maximum stresses occurred at some depth from the surfaces for both the wheel and the rail while the equivalent plastic strain results reveal most of the plastic deformation occurs at the rail and a very small plastic deformation occurs at the rail.

2.2 Rolling contact fatigue (RCF)

Rolling contact fatigue is dominant type of damage in wheel-rail contact. There are two types of RCF damage; surface initiated and sub-surface initiated RCF cracks. Usually surface initiated cracks starts at the surface while sub-surface initiated RCF cracks nucleate few mms below surface. Many literatures show surface-initiated cracks appear in surfaces with higher friction coefficient (lower/no lubrication) whereas subsurface-initiated cracks appear when the surface more lubricated (less friction coefficient) [10].

Many papers define fatigue life as a number of cycles a part undergoes before a crack initiation [4], [10], [14]–[16], [17]. This paper also assumes the same approach.

Various criterion can be applied to determine fatigue life of a machine element undergoing a cyclic loading like wheel and rail. There are mainly two categories of models in fatigue life prediction: bulk fatigue models and critical plane models.

2.2.1 Bulk fatigue models

These models utilize shear stress range and normal stress range averaged over all planes in the material. These models generally use stress invariants to determine the fatigue life of an element. Therefore, they are unable to determine the orientation of damage initiation and occurrence since stress invariants do not depend on transformation planes.

2.2.2 Critical plane models

The damage model under critical plane model is used to define the plane for crack initiation besides fatigue life.

2.2.2.1 Dang Van fatigue damage criterion

Dang-Vang shear-stress-based multiaxial fatigue limit criterion for high cycle fatigue (HCF) (greater than 10^5 cycles [18]) conditions, which is applied to steady state elastic shakedown in which no further damage happens [19]. So, Dang Van proposed that damage (crack initiation) will occur if one of the inequalities below are fulfilled.

$$\tau_a(t) + a_{DV}\sigma_h(t) > \tau_e \quad 2-1$$

$$\tau_a(t) - a_{DV}\sigma_h(t) < -\tau_e \quad 2-2$$

Where, $\tau_a(t)$ is the shear stress amplitude and $\sigma_h(t)$ is the hydrostatic stress, both at critical plane. τ_e is the positive fatigue limit of the material in pure shear and a_{DV} a positive dimensionless parameter accounting for the influence of the hydrostatic stress. This criterion is applied if the material is exhibiting perfectly elastic response or elastic shakedown response where no further damage occurs [10].

2.2.2.2 Findley criterion

This model is a stress-based approach. Findley proposed a critical plane model, which predict the crack plane is the plane with orientation θ with maximum Findley damage parameter [20],

$$\max_{\theta}(\tau_a + k\sigma_{n,max}) \quad 2-3$$

Where, τ_a is the shear stress amplitude and $\sigma_{n,max}$ is the maximum normal stress, both on plane with orientation, θ .

2.2.2.3 Brown-Miller criterion

This is a strain-based approach which considers both shear and normal strain on the plane of maximum shear.

$$\max_{\theta} \left(\frac{\Delta\gamma_{max}}{2} + S\Delta\epsilon_n \right) \quad 2-4$$

According to this model the critical plane is the plane of maximum shear stress range $\Delta\gamma_{max}$ and major value of normal strain range - $\Delta\epsilon_n$. Here, S is the normal strain effect coefficient.

2.2.2.4 *Fatemi–Socie criterion*

This model takes account both the strain and stress. The critical plane is the plane orientation θ with maximum F-S damage parameter given by [21]:

$$\max_{\theta} \left[\frac{\Delta\gamma_{max}}{2} \left(1 + k \frac{\sigma_{n,max}}{\sigma_y} \right) \right] \quad 2-5$$

Where, $\Delta\gamma_{max}$ is the maximum shear strain amplitude and $\sigma_{n,max}$ is the maximum normal stress both on plane θ , σ_y is material yield strength, and k is material constant.

2.2.2.5 *S-W-T criterion*

Smith, Watson and Topper formulated a model that predicts crack plane is a plane with orientation θ with maximum normal stress (maximum principal stress) [22].

$$\max_{\theta} \left(\sigma_n \frac{\Delta\epsilon_n}{2} \right) \quad 2-6$$

Where, σ_n is the normal stress and $\Delta\epsilon_n$ is the principal strain range both on plane θ .

The relation is applicable for materials that fail with (or initiate) cracks perpendicular to the principal strain amplitude (tensile mode). So, it is only valid only when $\sigma_n > 0$.

2.2.2.6 *Coffin-Manson's criterion*

This model is proposed for shear strain dominated damage. The crack plane is the plane with orientation θ with maximum shear strain amplitude.

$$\max_{\theta} \left(\frac{\Delta\gamma}{2} \right) \quad 2-7$$

Here, $\Delta\gamma$ is shear strain range.

2.2.2.7 *Jiang-Sehitoglu criterion*

Jiang and Sehitoglu proposed a model that predicts a fatigue crack plane orientation θ with maximum J-S fatigue parameter FP given by [23], [24].

$$\max_{\theta} \left[\frac{\Delta \varepsilon_n}{2} \sigma_{n,max} + J \Delta \gamma \Delta \tau \right] \quad 2-8$$

Where, $\Delta \varepsilon_n$ is the normal strain range, $\sigma_{n,max}$ is the maximum normal stress, $\Delta \gamma$ is shear strain range, $\Delta \tau$ is the shear stress range, and J is the material dependent constant all on plane θ . It is important to look that this model reduces to S-W-T model when $J = 0$, for normal cracking materials. When J is large the influence of the first term in equation 2-8 reduces and the model represents shear cracking fatigue.

2.2.2.8 Liu's criterion

This criterion is used for wide variety of material properties, ductile to brittle. Two planes are defined in this model; fracture plane and critical crack plane. The fracture plane is assumed to be the plane which experiences the maximum principal stress amplitude and form an angle α with the crack plane [14], [25]. The first procedure is to determine the fracture plane which is a plane with the maximum principal stress. This is achieved by enumeration, by changing the angle by 1° increments. Then, the angle α and material parameters are determined for different materials according to equation 2-9.

$$\cos \alpha = \frac{-2 + \sqrt{4 - 4\left(\frac{1}{s^2} - 3\right)\left(5 - \frac{1}{s^2} - 4s^2\right)}}{2\left(5 - \frac{1}{s^2} - 4s^2\right)} \quad s = \frac{t_{-1}}{f_{-1}} \leq 1 \quad 2-9$$

$$\alpha = 0 \quad s = \frac{t_{-1}}{f_{-1}} > 1 \quad 2-10$$

Finally, the stress components on the critical plane are calculated and the fatigue damage is evaluated using equation 2-23.

$$\sqrt{\left(\frac{\sigma_{a,c}}{f_{-1}}\right)^2 + \left(\frac{\tau_{a,c}}{t_{-1}}\right)^2 + k \left(\frac{\sigma_{a,c}^H}{f_{-1}}\right)^2} = \beta \quad 2-11$$

Where, $\sigma_{a,c}$, $\tau_{a,c}$ and $\sigma_{a,c}^H$ are the normal stress amplitude, shear stress amplitude and hydrostatic stress amplitude acting on the critical plane respectively. f_{-1} and t_{-1} are fatigue limits for pure torsional and uniaxial loading, respectively.

Here, k and β are constants defined by,

$$k = 0, \text{ and } \beta = [\cos^2(2\alpha)s^2 + \sin^2(2\alpha)]^{\frac{1}{2}}, \quad \text{for } s \leq 1 \quad 2-12$$

$$k = 9(s^2 - 1), \text{ and } \beta = s, \text{ for } s > 1 \quad 2-13$$

Liu modified the above equation for non-proportional loading to account for the mean stress effect, $\sigma_{m,c}$.

$$\sqrt{\left(\frac{\sigma_{a,c}(1+\eta\frac{\sigma_{m,c}}{f_{-1}})}{f_{-1}}\right)^2 + \left(\frac{\tau_{a,c}}{t_{-1}}\right)^2 + k\left(\frac{\sigma_{H,a,c}}{f_{-1}}\right)^2} = \beta \quad 2-14$$

Here, η (mean stress factor) is a material parameter defined by,

$$\eta = \frac{3}{4} + \frac{1}{4}\left(\frac{\sqrt{3}-\frac{f_{-1}}{t_{-1}}}{\sqrt{3}-1}\right); \quad \frac{f_{-1}}{t_{-1}} > 1, \quad \eta = 1; \quad \frac{f_{-1}}{t_{-1}} \leq 1 \quad 2-15$$

2.3 Litratues on fatigue life analysis

Srivastava et al[11] theoretically investigated the fatigue life of the wheel rail interaction using the results obtained from the finite element analysis. The life of a fatigue crack is categorized into three phases including crack initiation and growth. The first phase is shear stress-driven initiation at the surface, the second phase is transient crack growth behavior, and the last phase is subsequent tensile and shear driven crack growth.

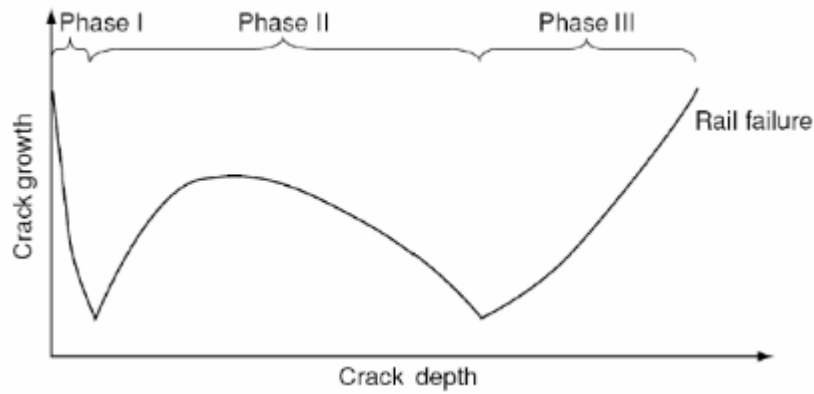


Figure 2-1 Three phases of RCF crack growth

Crack occurs at rolling contacts due to the accumulation of shear stresses from repeated rolling sliding contact loading. They calculated the fatigue life to initiation using the F-S damage criterion:

$$\text{Fatigue Life} = \left(\langle \sigma_{max} \rangle \frac{\Delta \epsilon}{2} + J \Delta \tau \Delta \gamma \right)_{max} = \frac{(\sigma'_f)^2}{E} (2N_f)^{2b} + \sigma'_f \epsilon'_f (2N_f)^{b+c} \quad 2-16$$

$$\langle \sigma_{max} \rangle = 0.5(|\sigma_{max}| + \sigma_{max}) \quad 2-17$$

Where, $\Delta \tau$ is the shear stress range perpendicular to the crack plane and $\Delta \gamma$ is the shear strain range found on the crack plane. J is the material and load-dependent constant that should be found from tension/torsion tests. The σ_{max} is the maximum normal stress in the plane, σ'_f and ϵ'_f denote the coefficient of fatigue strength and fatigue ductility respectively, E is the Young's modulus. N_f is the number of cycles to failure, b and c are the fatigue strength and the fatigue ductility exponents, respectively.

In *Ringsberg et al* [10] paper, a twin disc test was simulated in a 2D elastic-plastic FE simulations of a wheel/rail contact by ABAQUS. Fatigue crack initiation criteria for different material responses such as elastic shakedown, plastic shakedown and ratchetting material were estimated for a pearlitic rail steel BS11 normal grade using F-S damage criterion. The paper mainly demonstrated the effect of friction on fatigue life as illustrated using Figure 2-2.

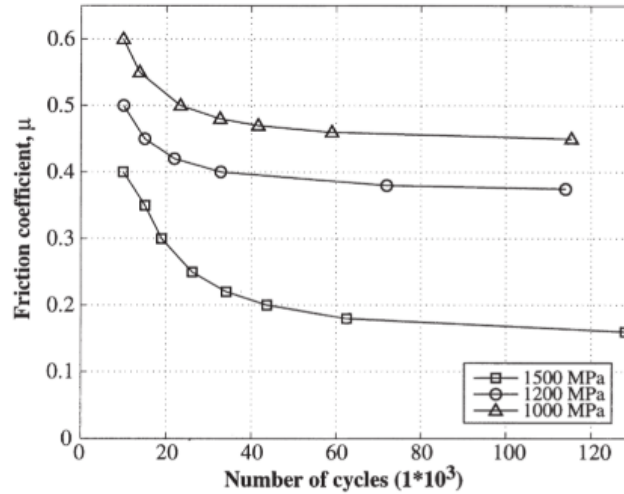


Figure 2-2 The effect of friction on fatigue life at different contact pressure

Liu et al [14] multiaxial high-cycle fatigue damage is used to predict both the initiation crack plane orientation and fatigue initiation life of a railway wheel using *Liu's damage criterion* explained in the previous section. The stress results from the wheel FE model are used to estimate fatigue life. He also studied the effect of different parameters on wheel damage, one of which was the wheel diameter. Figure 2-3 represents the relationship between wheel diameters and damage cycles [14].

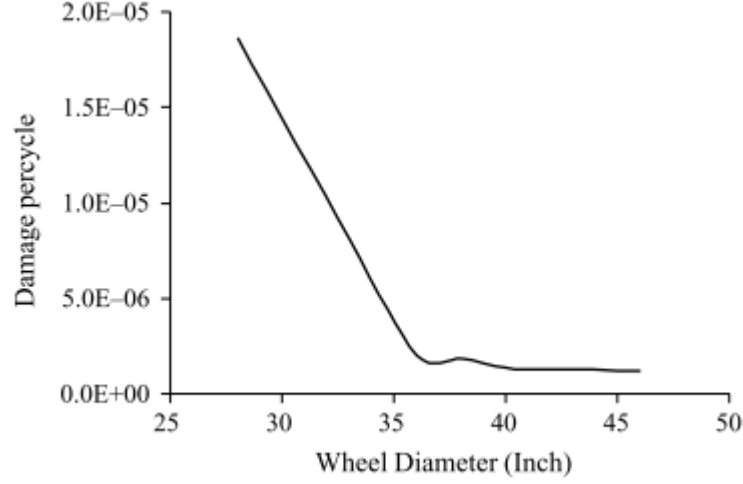


Figure 2-3 The effect of wheel diameter on damage

El-sayed et al [15] developed two models: the global model and the submodel. The global model was applied to extract boundary conditions for the submodel at the cut surfaces, whereas the submodel was implemented to calculate the stress field more accurately and efficiently by applying finer meshes. Besides, results from the submodel like stress and strain history were used to calculate the orientation of critical crack plane and the fatigue life to crack initiation. Therefore, the critical plane approach was proposed by this paper with multiaxial low cyclic fatigue (LCF) model. The J-S fatigue parameter (FP) was used to know the amount of damage.

$$FP = \left(\langle \sigma_{max} \rangle \frac{\Delta \epsilon}{2} + J \Delta \tau \Delta \gamma \right)_{max} \quad 2-18$$

The material plane with the largest fatigue parameter (FP_{max}) is called critical plane (crack plane). This equation consists of two forms: formula that includes shear form is adopted to evaluate Nf if shear (second mode) cracking governs the fatigue life. However, for material exhibiting tensile (first mode) cracking, tensile form should be used.

$$FP_{max} = \begin{cases} \frac{(\sigma'_f)^2}{E} (2N_f)^{2b} + \sigma'_f \epsilon'_f (2N_f)^{b+c} \\ \frac{(\tau'_f)^2}{G} (2N_f)^{2b} + \tau'_f \gamma'_f (2N_f)^{b+c} \end{cases} \quad 2-19$$

El-sayed et al tend to use the shear model because all the normal stress components they find were in compression. In their paper, FP_{max} and the corresponding critical crack plane was investigated for by computing the transformation matrices for every set of angles where the von-mises stress is maximum, because this is where damage may initiate. Two spherical angles (θ, φ) were utilized to determine the orientation of the critical plane.

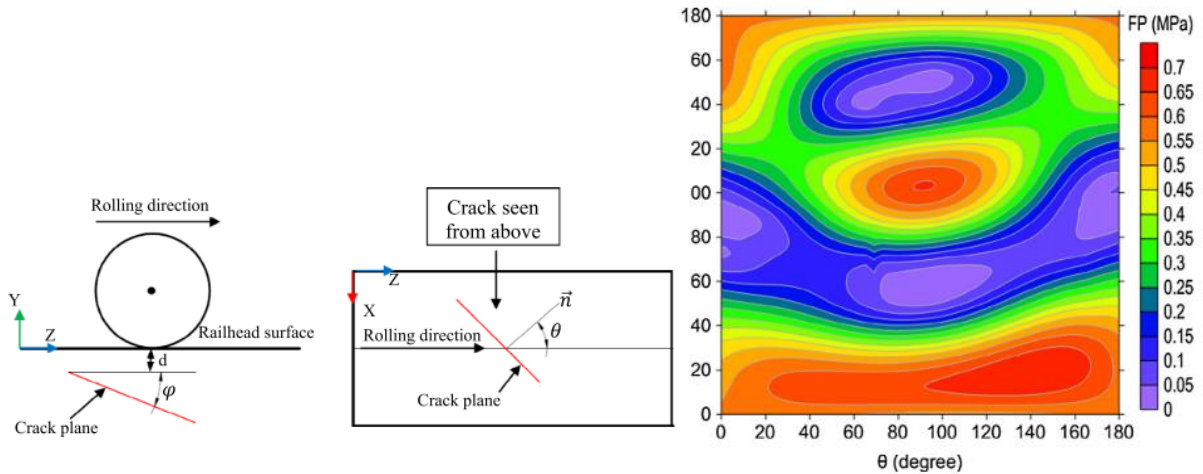


Figure 2-4 Critical plane orientation

Taraf et al [16] paper models rolling contact fatigue in railroad wheels using a moving Hertzian contact load using a two dimensional FE model. This paper conducted parametric study to see the effect of parameters like shape and size of defects, friction, and vertical load on the fatigue life of a wheel. For analyzing the effect of friction on fatigue damage defect free materials are implemented. They also showed that for the defect free material only the shear stress and shear strain ranges operate in fatigue parameter calculations because the normal stress is compressive and its effect is negligible.

Another work on rolling contact fatigue investigation is an experimental work done by Jiang et al [26]. They used thin-wall tubular specimens made from structural steel S460N. the specimens were exposed to different loading paths. Among the specimens those under loading path *h* and *i* reproduce the loading conditions in wheel-rail interactions.

2.4 Research gap

Most of the works done by scholars focus on the fatigue analysis of the rail and those which are conducted on the wheel are mostly two dimensional. This can compromise the reliability and accuracy of numerical results and fatigue life estimation.

There are few papers that have conducted the three-dimensional numerical analysis on railroad wheels to investigate fatigue life. But this paper suggests a more reliable and integrated approach to have more complete information about fatigue in railroad wheels. The new approach to these problems involves an integration of a numerical software Abaqus 2020 with MATLAB. Besides, emphasis is given in investigating the orientation of damage (crack initiation) because it is as important as determining its life time.

CHAPTER 3 MATERIALS AND METHODS

3.1 Numerical model of wheel rail contact

In this section, a finite element models using Abaqus CAE 2020 software, is implemented for numerical analysis of the contact between the rail and wheel. First, the geometrical model of both the wheel and the rail are developed according to the parameters from the ERC data. Then, this model was used for finite element analysis.

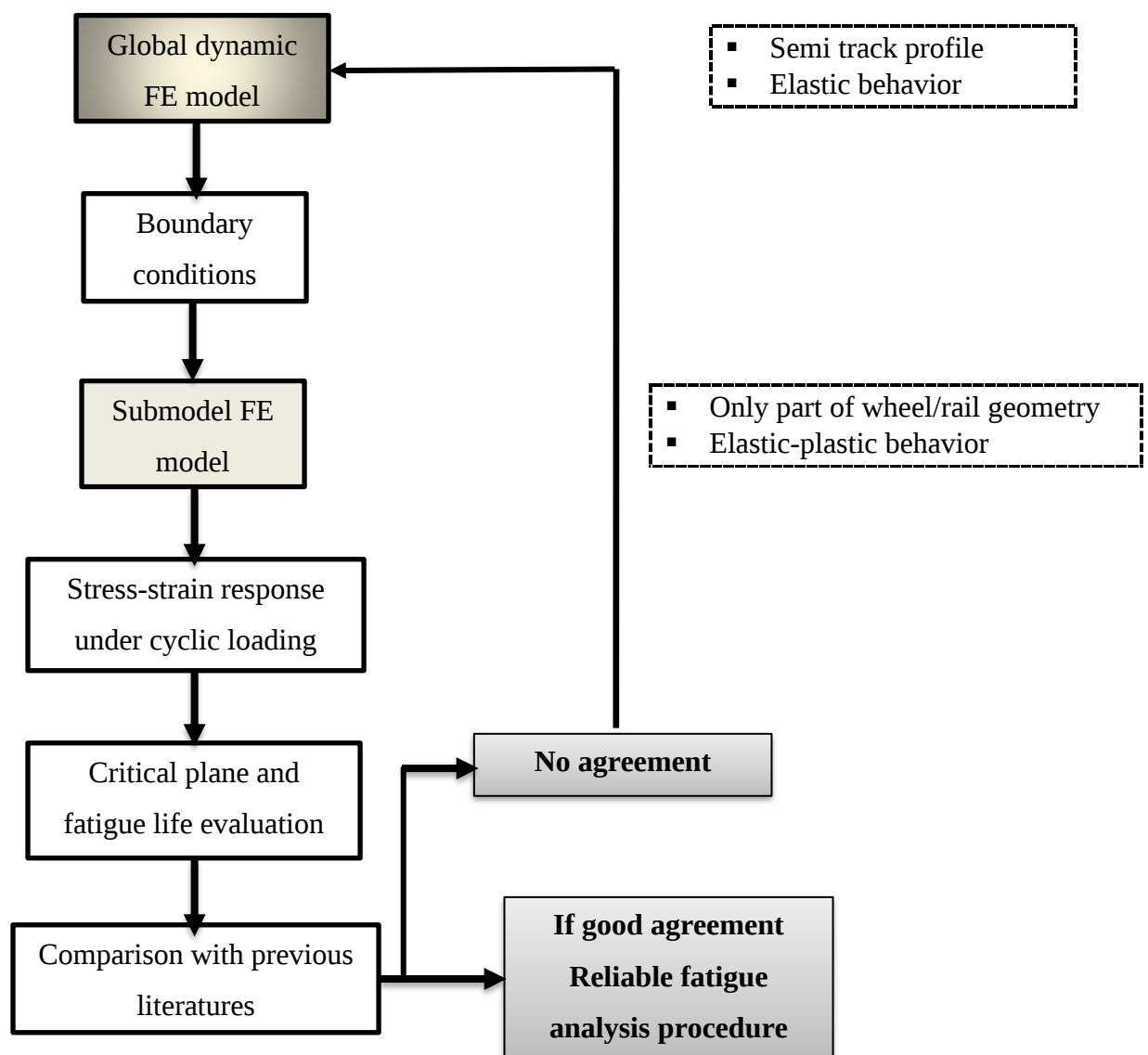


Figure 3-1 Flow chart of wheel fatigue analysis

3.1.1 Geometrical modelling

The two-dimensional curve the wheel profile is revolved within a radius of 330 mm to create the three-dimensional geometric model of the wheel. Whereas, the 3D rail model is generated by extruding railhead profile curve by a distance of 600 mm (the distance between slippers).

3.1.1.1 Wheel and rail geometries

The point of contact is established so that the wheel tread center and the rail head center are aligned, as shown in Figure 3-2.

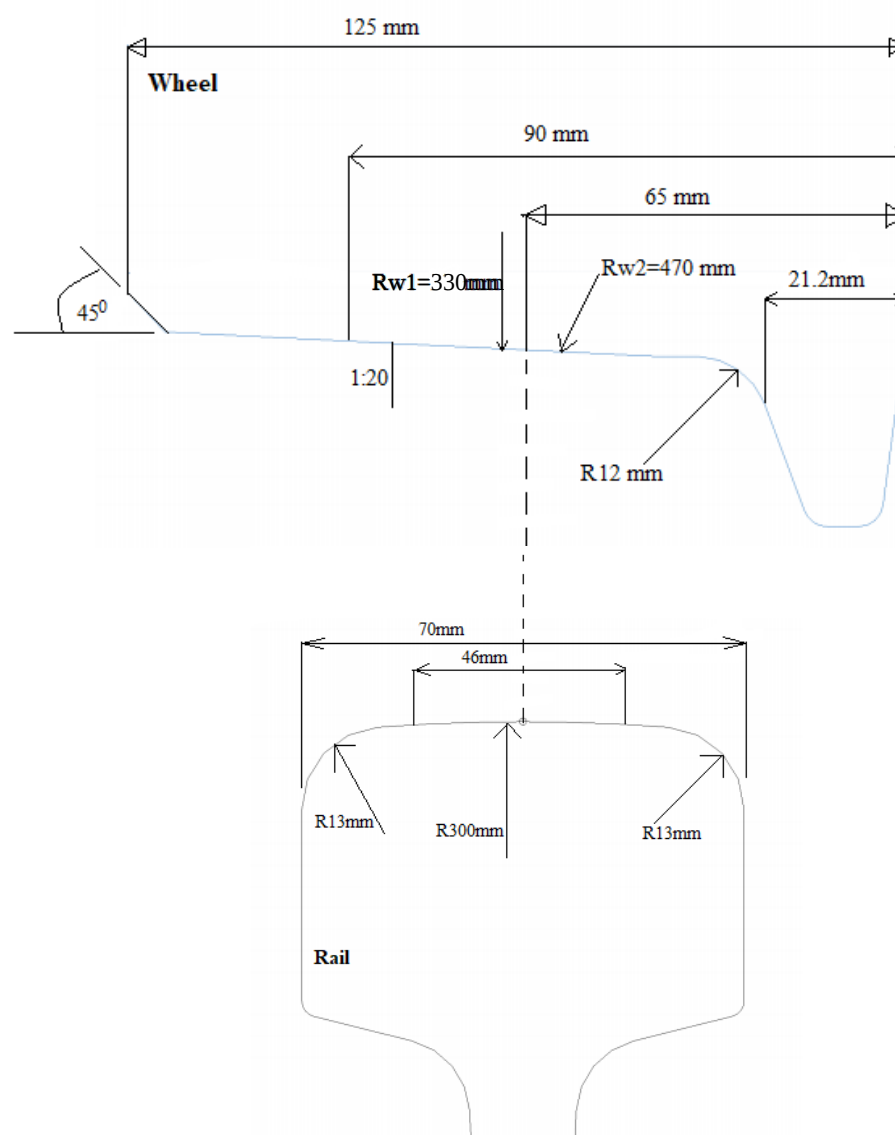
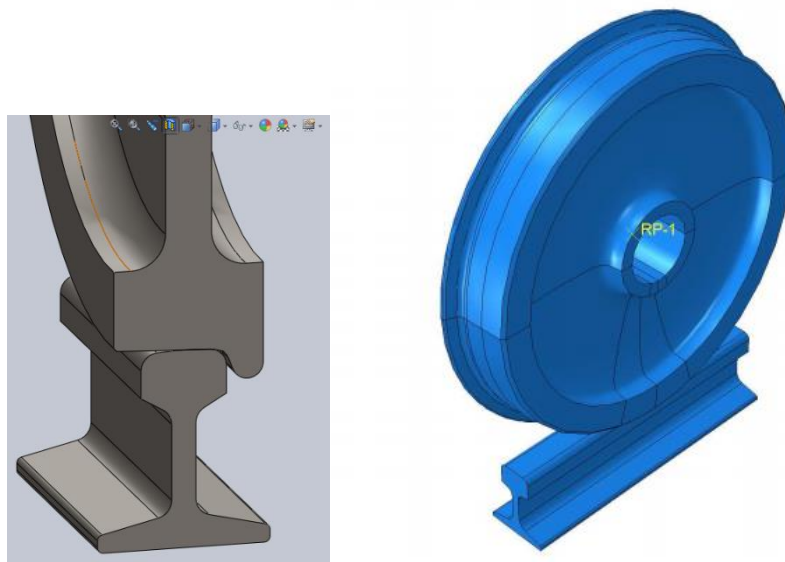


Figure 3-2 Wheel and Rail geometry model

Table 3-1 Geometry parameters of wheel and rail [27]

Wheel					Rail	
Radius(Rw1)	Contact curvature(Rw2)	Flange thickness(e)	Flange height(h)	Rim nominal width(L)	Radius (Rr1)	Contact curvature (Rr2)
330 mm	470mm	21.21mm	28mm	130mm	∞	300mm

First the rail and the wheel are created separately using the Abaqus CAD section. The 3D part files were generated by extruding (for the rail) and revolving (for the wheel) using the 2D drawing primarily created as in Figure 3-3. The rail was extruded for length of 600mm (the length between the sleepers). The assembly tool is used to bring the files together as shown in Figure 3-3 to create the whole model.

**Figure 3-3 Assembly geometry of wheel and rail**

3.1.1 Finite element model (FEM)

In this part, the geometry developed in the CAD environment of Abaqus is used for numerical analysis. In order to combine the global response with the local response two models were used. The first model is called the global model in which one of the wheels is permitted to roll over 600 mm (the length between the sleepers) of the rail. The second one is called the submodel which is extracted from the global model using the submodelling technique in Abaqus. Time dependent boundary condition was implemented to connect both models.

3.1.1.1 Global model (GM)

The global model consists of one wheel and the rail. This model uses a relatively coarse mesh to obtain results. The submodel is driven by the data from the output database that is generated by the global model. The main target of the global model in this analysis is to drive the boundary conditions for the submodel analysis.

The solution process in this model consists of three steps.

Initial step: In this step the interaction is initiated between the wheel and rail surface

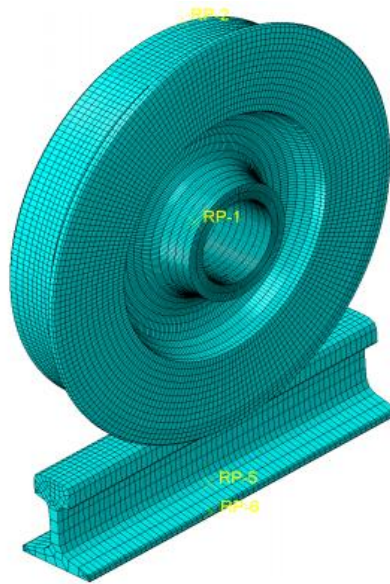


Figure 3-4 Global Model

Step 1: This step is a stabilization step which introduces gradual loading application at the axle point to avoid diverging, bouncing back and numerical instability. In this step the contact between the wheel and the rail is established, concentrated load at a reference point in the wheel center was applied, field outputs are requested and most boundary conditions are created. The step time was given 0.001 s.

Step 2: The wheel was allowed to roll along the track for 600mm, with a translational velocity of 70km/hr (the maximum track speed) and rotational velocity of 64.38rad/s.

The rotational velocity is calculated from:

$$\omega = V/W_r \quad 3-1$$

Where ω is the rotational speed, V is the translational velocity and W_r is the wheel radius at the point of contact.

The step time for the second step (t_2) is calculated using the rolling distance (L) and the translational velocity (V) which is, $t_2 = L/V = 0.6m/19.44m/s = 0.03 s$

Step 3: The load was removed from the wheel

3.1.1.2 Submodel (SM)

The submodel was required to perform more accurate analysis of the contact stresses near contact patches with quite fine meshes because using finer meshes in the global model will result in great deal of computational cost and time. Both models were connected together using time dependent boundary conditions. In submodel one the stress field, contact pressure von-mises stress and shear stress will be determined with acceptable accuracy. Whereas, in submodel two the initiation crack plane and fatigue life are predicted.

Each cycle in this analysis consists of four steps.

Step 1: the load was applied, and wheel/rail contact was established.

Step 2: the wheel is allowed to rotate along the track with scaled distance by applying the same translational and rotational velocity as the global model.

Step 3: the gravity load was removed from the wheel

Step 4: the system is allowed to return to its initial position.

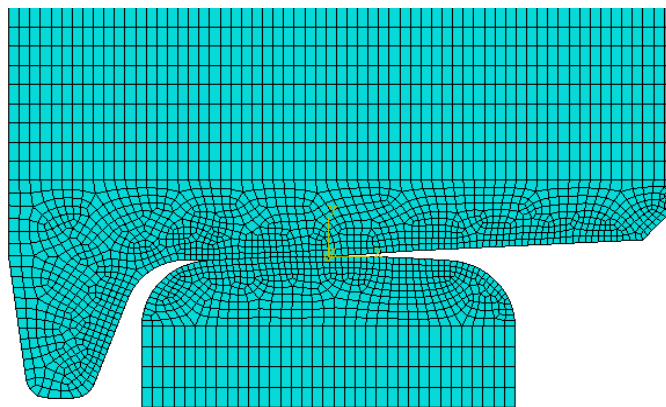


Figure 3-5 Submodel

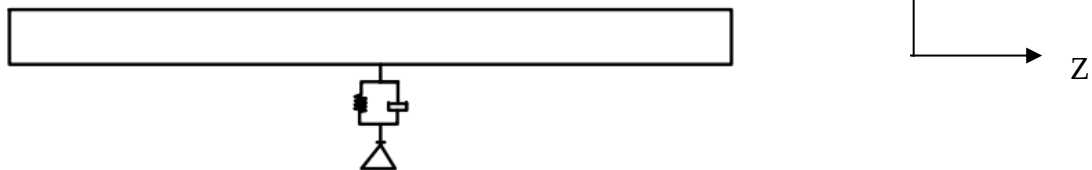
Modelling dynamic wheel-rail Interaction:

There are three main methods for rail bedding constraints

1. Simply fixed bedding



2. Rigid constraint with one spring-damper (S/D) element



3. Multiple S/D elements for every sleeper

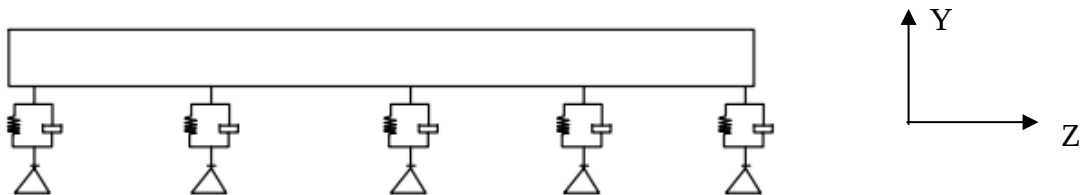


Figure 3-6 Interaction Model

The multiple S/D element method gives the most accurate result at an expense of high computational time. Using rigid constraint with one S/D element provides an acceptable accuracy with lower computational time. Therefore, due to the limitations in computational capability the second method was implemented.

The whole interaction model from bogie to sleeper can be modelled in simplified way as shown in Figure 3-7:

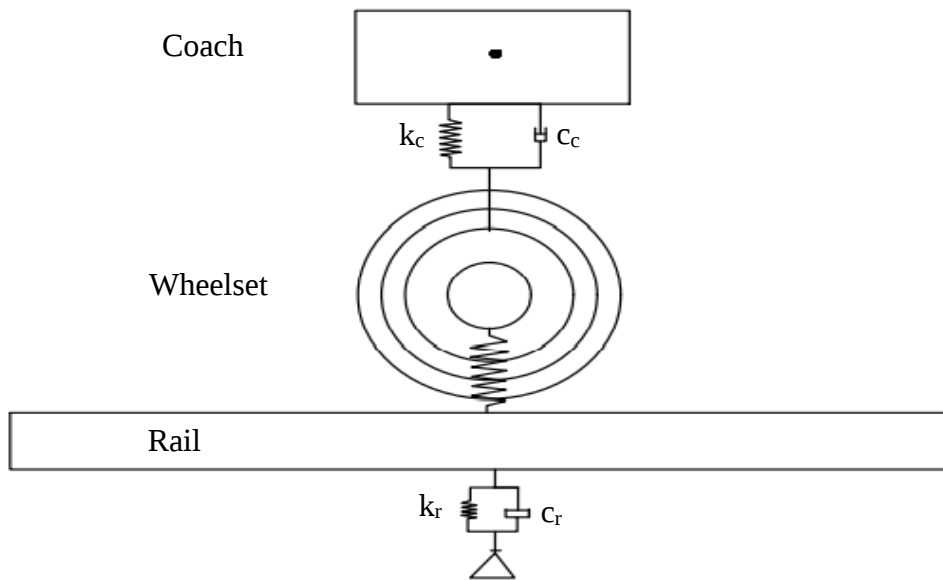


Figure 3-7 Dynamic interaction model

A reference point (pilot node) was created at the center of the axle joint hole and was used to create multi-point constraint (MPC) beam constraint between the internal surface of the hole. The second MPC beam constraint was created between rail bottom surface and a reference point created at the center of this surface.

A concentrated load consisting of the sum of the weight of train body, the passengers and the axle was applied at the reference node created at the center of the axle joint hole.

3.1.1.3 Numerical approach of wheel-rail contact

A finite element code – capable of treating contact problems – should include the following steps: contact detection, construction and update of “contact elements”, their incorporation in associated residual vectors and tangential matrices and finally resolution of the resulting problem[28].

Contact detection:

Contact elements are a sort of “bridge elements” between locally separated but potentially interacting bodies. Each contact element contains components (nodes, edges, faces) of both surfaces; the composition of these components depends upon the contact discretization method. The aim of contact detection is to create contact elements containing

the proximal components of both surfaces that may come into contact during the current solution step. Contact detection depends on the type of contact discretization.

Contact resolution

Besides structural degree of freedom, contact element may have their proper degree of freedom. These unknowns and the structure of the residual vector and the tangential matrix are evaluated by the resolution method.

Contact discretization

The structure of contact elements that transfer loads from one contacting surface to another is predetermined by contact discretization. Three types of discretization are identified:

1. Node to Node (NTN)

This type of discretization introduces limitations on mesh generations and does not allow any finite sliding or large deformations and node to node misalignment. However, it passes the contact patch test – contact pressure is transferred correctly through the conforming contact interface. NTN discretization can be applied to linear and quadratic elements in 2D case and linear elements in 3D case.

2. Node to Surface (NTS)

This technique is valid for non-conforming meshes and large deformations or finite sliding. However, it doesn't pass contact patch test for non-conforming meshes - a uniform contact pressure cannot be transferred correctly through the contact interface. So, some modification might be needed.

3. Surface to Surface (STS)

STS discretization technique has been efficiently implemented to two- and three-dimensional problems coupled with the mortar method for non-conforming meshes.

In the current study, Wheel/rail contact was defined using surface to surface discretization method. This is because surface to surface approach has many benefits over the node-to-node approach such as[28]:

- reduces likelihood of large localized penetrations

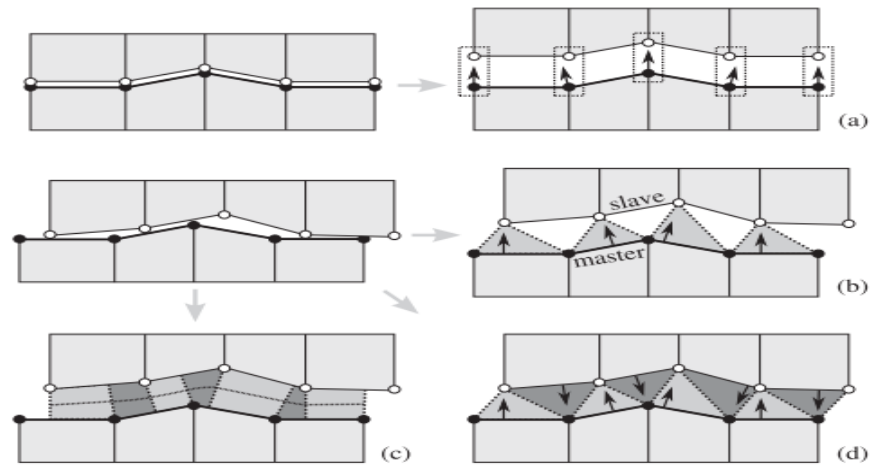


Figure 3-8 Graphical representation of different contact discretization: for small deformations/sliding (a) node-to-node, associated pairs of nodes and normals; for large deformations/sliding (b) node-to-segment, slave nodes and associated master segment

- decreases sensitivity of results to master and slave roles and node alignment
- provides more accurate contact stress results
- has better convergence

In the contact property section, penalty friction formulation was applied to the model to enforce the contact constraints.

Relative sliding formulation

Abaqus offers finite- and small-sliding versions of S-to-S and N-to-S contact formulations. *Small sliding formulation* assumes relative motion per slave node remains small compared to local curvature of the master surface and facet size of the master surface. This formulation has a potential for reduced cost per iteration and finding a converged solution in fewer iterations. However, results obtained can be non-physical if relative tangential motion does not remain small [15]. *Finite sliding formulation* is appropriate for this study to accurately represent the problem. Coefficient of friction of 0.3 was used, accounting the lubrication. Coulomb's friction law (Equation 3-6) was used to describe to describe frictional rolling contact. In addition to this, in this study, the initial contact point is assumed to occur at the rail head center and wheel tread center Figure 3-2.

$$\tau_{lim} = \mu P_o, \|\tau\| = \tau_{lim} \quad 3-2$$

Where P_o is the maximum contact pressure; μ is the friction coefficient; τ_{lim} is the limiting frictional stress; τ is the frictional stress.

Boundary conditions (constraints):

Two amplitudes were created. Amplitude-1 is used for every boundary condition with gradual increase of magnitude to avoid divergence error by eliminating sudden increase of movement and loading.

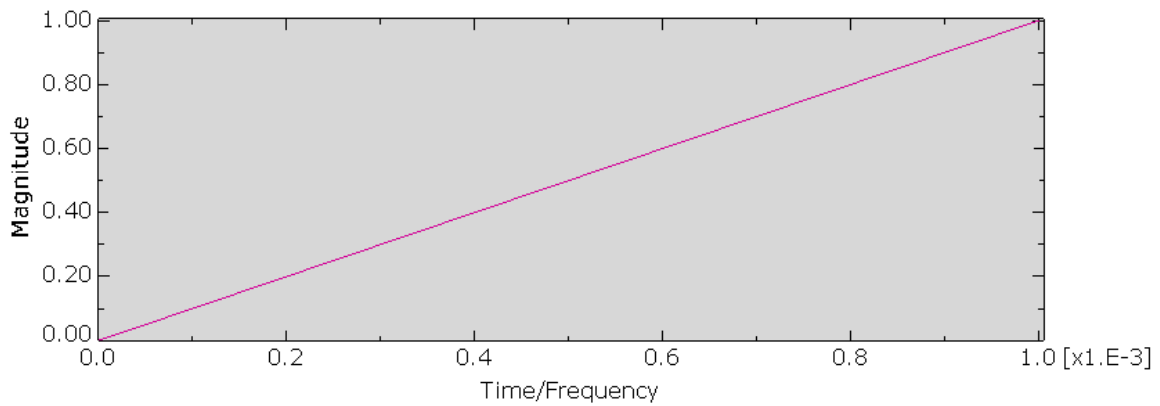


Figure 3-9 Amplitude 1

Amplitude-2 is for the loading and represents smooth increase of the concentrated load in the first step (stabilization step) and constant magnitude throughout the second step (rolling step).

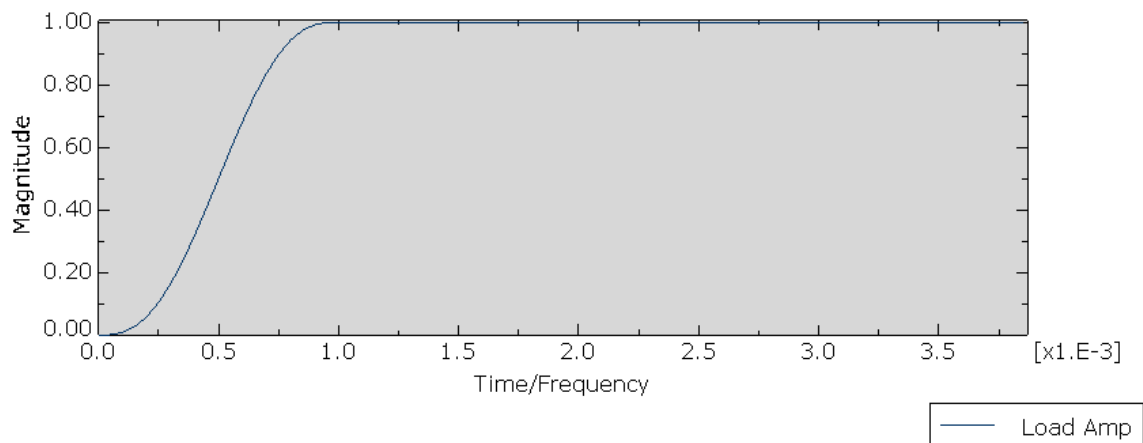


Figure 3-10 Amplitude 2

Discretization

Coarse and fine mesh: The global model uses coarse meshes compared to the submodel because we only need the global model to extract the boundary conditions at the cut section in the submodel. Moreover, the use of the submodel can save up to more than 100 times computational time [14]. The submodel (section of the global model) only takes account for the geometry near the area of contact. Therefore, finer meshes are applied for the submodel. The stress and strain results for the two models need not be the same. Wide difference in element size between the two models accounts for the divergence in the numerical results. Generally, relatively even finer meshes are applied in areas of contact between the wheel and the rail to get more accurate numerical results.

Element type and element size. The element type used for the global and the submodel is a fully integrated eight-node linear brick (C3D8) elements.

3.1.1.4 Convergence analysis.

In the submodel, higher level of accuracy is required at the lower wheel section and the upper rail part (rail head zone) where several strains and stresses are anticipated with reasonable computational time. As a result, mesh sensitivity analysis needs to be performed to determine the optimum mesh size. Mesh refinement is carried out only at lower wheel part and the upper rail part. Various element sizes were taken and the element size where numerical results start converging was implemented to conduct the rest of the analysis.

The element sizes of 1.2 mm, 1.4 mm, 1.6 mm, 1.8 mm, 2.0 mm and 3.0mm are used. The highest contact pressure was obtained at element sizes between 1.2 mm and 1.8 mm, whereas the lowest was obtained at 3.0mm element size. This implies, the finer the mesh the higher the numerical results will be. The variation in result with in the element size range between 1.2 mm and 1.8 mm is less than 1.8%. Therefore, we can get a result with an acceptable accuracy and much lower computational effort at mesh size 1.8 mm x 1.8 mm. So, 1.8mm element size was used for the rest of the analysis.

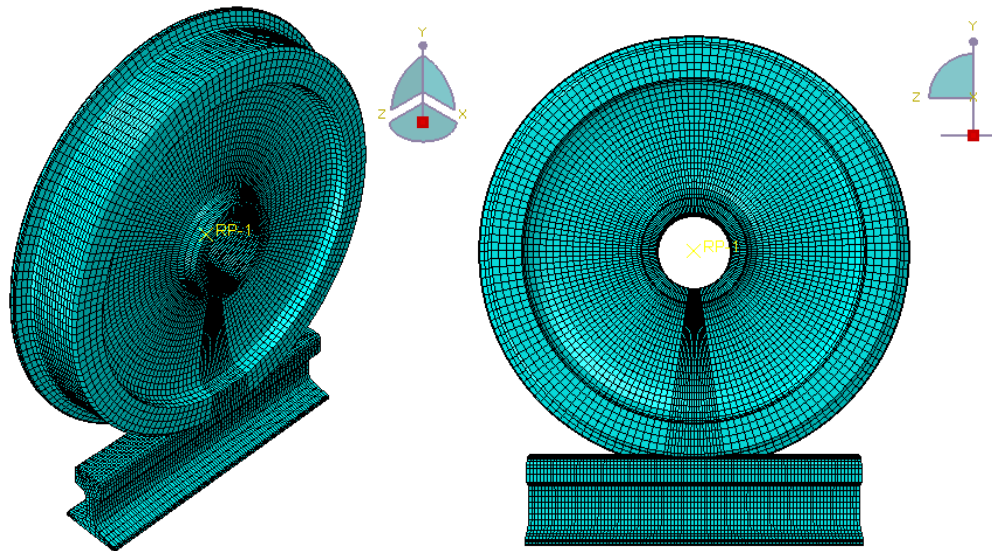


Figure 3-11 Mesh of Wheel-rail assembly in Abaqus CAE

Table 3-2 Mesh Convergence

Element size	Von-mises stress (MPa)
3.0mm	520
2.0mm	537
1.8mm	550
1.6mm	540
1.4mm	543
1.2mm	545

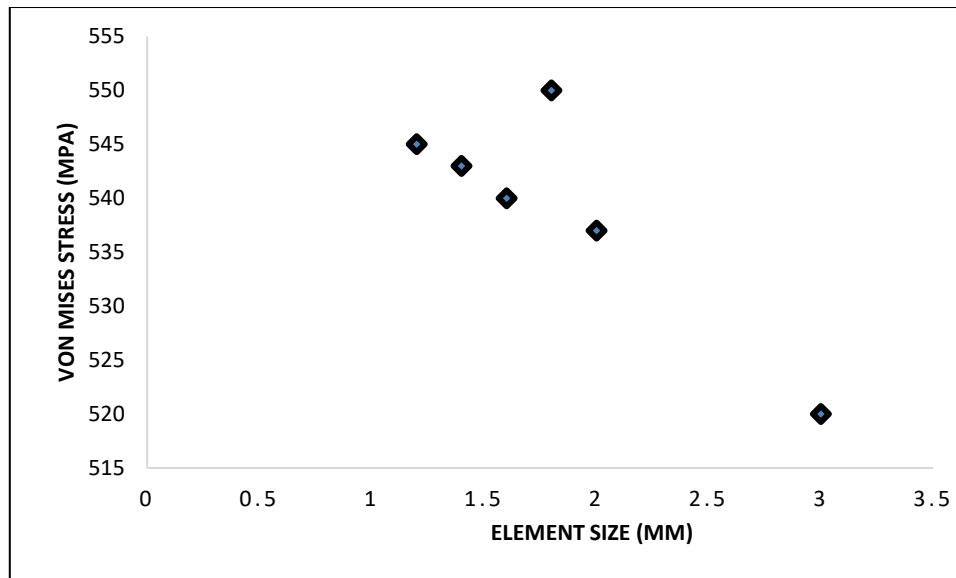


Figure 3-12 Convergence plot

3.2 Materials model

In this section of the paper, first the material properties of the wheel and the rail and the loading conditions of the rail stock are presented. The material response at the wheel-rail contact region over cyclic loading is also thoroughly studied. For example, there are four possible ways the material at wheel-rail contact region might respond; Perfectly elastic, elastic shakedown, plastic shakedown and ratcheting responses. The loading condition, material hardening, and the variation of contact conditions due to wear and/or plastic deformation are major factors in deciding in which way the material might respond.

3.2.1 Wheel and rail material model

In this paper, standard wheel and rail materials models are implemented to conduct the analysis.

Table 3-3 Chemical composition [29]

	C%	Si%	Mn%	Po%	S%	Cr%	Co%	Mb%	Ni%
R8T	0.54	0.36	0.80	0.02	0.006	0.30	0.30	0.08	0.14

Table 3-4 Mechanical properties of wheel [30]

Wheel type	Tensile strength (MPa)	Yield strength (MPa)	Elastic modulus (GPa)	Poisson's ratio
R8T	980	540	206	0.29

Table 3-5 Mechanical properties of rail

Rail type	Tensile strength (MPa)	Yield strength (MPa)	Elastic modulus (GPa)	Poisson's ratio
50kg/m	960-980	543	210	0.3

Table 3-6 Loading conditions of R8T wheel

Loading condition	Axle load (KN)
L1	110
L2	140
L3	160
L4	180
L5	200
L6	220

3.2.2 Material Response

Repeated rolling or sliding contacts stresses the material cyclically. Dependent on the character and intensity of applied contact loads, the material hardening and the variation of contact conditions due to plastic deformation and wear, the material in wheel-rail contact region responds in one of the following four ways [4], [15], [16].

- Perfectly elastic behavior if the contact load is sufficiently low and does not exceed the elastic limit and the material is purely elastic and reversible, i.e. for a line contact $p_{\max}/\tau_c = 0.31$ or expressed in the load intensity $\tau_c/p_{\max} = 1/0.31$. This response is rare to occur in wheel-rail contact zones.
- Elastic shakedown behavior, in which contact load exceeds the elastic limit and plastic deformation takes place during running in. However, after some cycles,

residual stresses or strain hardening shakedown the material response to perfectly elastic state.

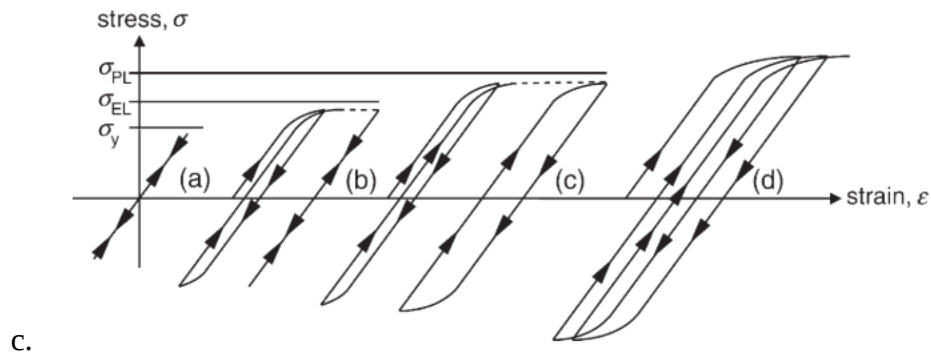


Figure 3-13 Illustration of different material responses to cyclic loading: (a) depicts perfectly elastic; (b) elastic shakedown; (c) plastic shakedown; (d) ratchetting [8]

- d. Plastic shakedown behavior, in which the steady state is a closed elastic plastic loop, with no net strain (plastic deformation) accumulation.
- e. Ratcheting behavior, the steady state is an open elastic plastic loop, there is an accumulation of a net strain at every cycle until the material ductility is exhausted.

In the analysis of the wheel-rail contact the material models should be defined properly to get accurate results that represent the real scenario. Among this is the material property after yielding had occurred.

Elasticity. Elastic properties are applied for the entire global model and for the regions outside the vicinity of contact area in the submodel.

Plasticity. Multiaxial Cyclic plastic deformation occurs in rolling contact fatigues like the wheel-rail contact. Regions in vicinity of contact are assumed to undergo plastic deformation. The plastic zone is taken to extend 15 mm along the depth of the wheel [15] (the red section shown in Figure 3-14).

The basic elements of plasticity theory consist of:

Yield function: determines when plastic flow initiates.

Flow rule: relates the applied stress increments to the resulting plastic strain increments once plastic flow has initiated. Constitutive equations are used to relate the stresses and the plastic strains.

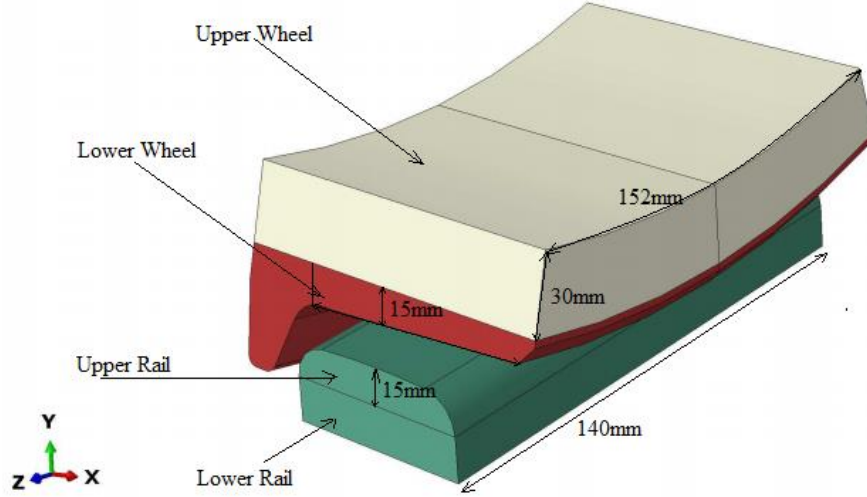


Figure 3-14 Elastic and plastic behavior assignment for wheel sections

Hardening rule: describes the change in the yield criterion as a function of plastic strains. One possible hardening rule is the *isotropic rule*, which assumes that strain hardening corresponds to an enlargement of the yield surface (i.e., an increase in S_y) without change of shape or position in the stress space. Another is the *kinematic rule*, which assumes that strain hardening shifts the yield surface without changing its size or shape [31].

Different constitutive models can be used to accurately capture the effect of cyclic loading in rolling contact stress analysis, such as linear kinematic hardening model [14] or non-linear kinematic hardening model [15]. Non-Linear isotropic/kinematic hardening model (combined) proposed by Chaboche and Lemaitre is adopted in this study due to its ability to reproduce phenomenon like ratcheting and cyclic hardening [4], [15], [16].

$$\dot{\alpha}_k = C_k \frac{1}{\sigma_o} (\sigma - \alpha) \dot{\bar{\epsilon}}^p - \gamma_k \alpha_k \dot{\bar{\epsilon}}^p, \quad (k = 1, 2, 3) \quad 3-3$$

$$\sigma_y = \sigma_o + Q(1 - e^{-R\bar{\epsilon}^p}) \quad 3-4$$

Where α is the back-stress tensor which designates the translation of the yield surface in the stress space; σ is the stress tensor; $\dot{\bar{\epsilon}}^p$ is the plastic strain rate; $\bar{\epsilon}^p$ is the plastic strain; σ_o is the initial yield stress at zero plastic strain; σ_y is the yield stress; C_k, γ_k, R, Q are material parameters calibrated from uniaxial test data.

The material models for both the wheel and the rail were inserted to the analysis according to Table 3-7. They are taken from [26], [31], [32] for **R8T** wheel.

Table 3-7 Material parameter for the R8T wheel

Parameters	$E(GPa)$	ν	$\sigma_o(MPa)$	R	$Q(MPa)$	C_k	γ_k
Values	206	0.29	543	0.47	22.8	6490	0.81

Since there is a considerable increase in temperature at contact region between the wheel and the rail, it is tried to incorporate the effect of the temperature on the elastic modulus of the material. The temperature in the wheel/rail contact region can be as high as 200°C [32][33].

Material Behaviors

Density

Elastic

General Mechanical Thermal Electrical/Magnetic Other

Elastic

Type: Isotropic

☒ Use temperature-dependent data

Number of field variables: 0

Moduli time scale (for viscoelasticity): Long-term

☐ No compression

☐ No tension

Data

	Young's Modulus	Poisson's Ratio	Temp
1	206000	0.3	200

Figure 3-15 Elastic Data on Abaqus 2020

Material Behaviors

Density
Elastic
Plastic
Cyclic Hardening

General Mechanical Thermal Electrical/Magnetic Other

Plastic

Hardening: Combined

Data type: Parameters

Number of backstresses: 1

☒ Use temperature-dependent data

Number of field variables: 0

Data

	Yield Stress At Zero Plastic Strain	Kinematic Hard Parameter C1	Gamma 1	Temp
1	543	6490	0.81	200

Figure 3-16 Plastic data on Abaqus 2020

Suboption Editor

Cyclic Hardening

☐ Use temperature-dependent data

Number of field variables: 0

☒ Use parameters

Data

	Equiv Stress	Q-infinity	Hardening Param b
1	543	22.8	0.47

OK Cancel

Figure 3-17 Cyclic Hardening

3.3 Rolling Contact Fatigue Model

Two types of approaches may be used in analyses of fatigue crack initiation: (1) the defect-tolerant approaches, and (2) the total-life approaches. The defect-tolerant approaches rely on the resistance to crack growth in materials that are inherently flawed by small cracks. These approaches use fracture mechanics to calculate the number of cycles required to propagate a crack to a critical size.

The total-life approaches estimate the resistance to fatigue crack initiation based on nominally defect-free materials and components. These approaches attempt to analyze the total fatigue life to failure (crack initiation); they are classified into stress-based, strain-based and energy-based approaches. The stress-based approach is mostly applied to low cyclic stress ranges that are designed against fatigue crack initiation (high-cycle fatigue failures)[25]. The strain-based approach, also called the strain-life approach, involves the fatigue life prediction of crack initiation according to which the strain range characterizes the fatigue life.

In this paper strain-based approach is adopted because the stresses between the wheel and the rail are high enough to cause plastic deformations (strains). As a result, strains are more likely govern failures due to fatigue.

3.3.1 Critical Plane Approaches

Critical plane models are found more suitable than bulk fatigue damage models as they showed better consistency with experiments and can predict the location and orientation of damage (crack initiation).

Limitation of many critical plane-based criterion reviewed in previous sections lies in handling of compressive mean stresses and handling of non-proportional loading where the maximum normal stress and the maximum shear stress in the cycle does not coincide.

However, among these models critical plane model proposed by Jiang and Sehitogelu and Liu accounted for these limitations and showed good agreement with rolling contact fatigue experimental results [4], [8], [22]–[26], [32]. Besides, these models can be used for both elastic shakedown (HCF) and plastic shakedown (LCF) material responses [10]. It was not possible to implement the Liu's criteria in this paper because it uses large

number of material parameters only available for few materials and require experiments to be determined.

3.3.2 Jiang-Sehitogelu Criterion

Jiang and Sehitogelu defined the damage parameter as, [23].

$$FP = \langle \sigma_{max} \rangle \frac{\Delta \epsilon}{2} + J \Delta \tau \Delta \gamma \quad 3-5$$

The critical plane is a plane with maximum fatigue parameter. After the critical plane is determined the number of cycles to crack initiation is obtained from as shown in chapter 2:

$$FP_{max} = \begin{cases} \frac{(\sigma'_f)^2}{E} (2N_f)^{2b} + \sigma'_f \epsilon'_f (2N_f)^{b+c} \\ \frac{(\tau'_f)^2}{G} (2N_f)^{2b} + \tau'_f \gamma'_f (2N_f)^{b+c} \end{cases} \quad 3-6$$

The first form is used for mode-1 (tensile dominated) crack and the latter is for mode-2 (shear dominated) crack initiation.

3.3.3 Critical Plane Search (CPS)

Proportional loading. In this type of loading the maximum normal stress and the maximum shear stress coincide with each other. Besides, when loading is proportional, the principal stresses do not change direction during the loading cycle, making it easy to calculate the maximum shear stress amplitude [35].

Non-proportional loading. In this type of loading the maximum shear stress and the maximum normal stress do not coincide. When the shear stress reaches maximum, the normal stress reaches its minimum and vice versa [35]. As a result, the principal directions of the alternating stresses are not fixed, but change orientation during this type of loading, making it difficult to calculate the maximum shear stress amplitude. We can see from stress history curve in Figure 3-18 that the loading is non-proportional in our wheel- rail interaction problem.

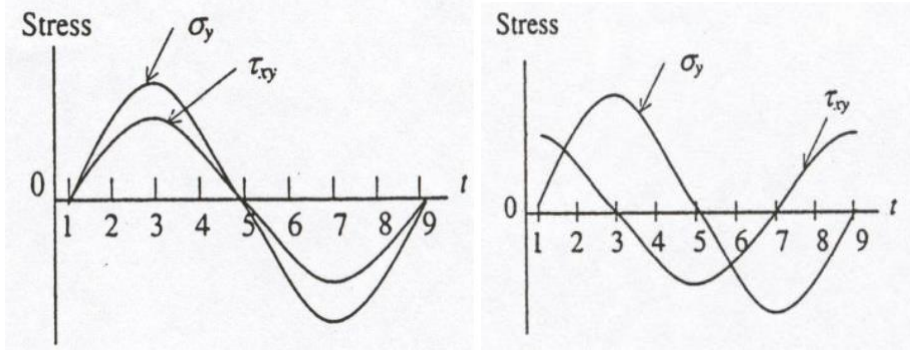


Figure 3-18 Stress and strain history curve for proportional (left) and non-proportional (right) loading

According to Jiang and Sehitogelu the crack plane with orientation θ is defined by:

$$\max_{\theta} \left[\langle \sigma_{max} \rangle \frac{\Delta \epsilon}{2} + J \Delta \tau \Delta \gamma \right] \quad 3-7$$

The critical plane θ is the plane with maximum fatigue parameter, FP max. Therefore, every angle θ is searched for with in a range $[0, \pi]$ using θ increment of 1° . The most critical point at depth d suffering from maximum von-mises stresses is taken for calculations.

The normal and shear stresses and strain at an arbitrary plane θ is defined using stress transformation equations as [36], [37]:

3.3.3.1 Two-dimensional CPS

In the consideration location of the plane is identified using a single angle θ in the two-dimensional Y-Z plane. Two-dimensional stress and strain transformation equations (3-8 to 3-11) are used [37] to determine the crack angle. The depth d is determined by exploring all the possible nodes in the radial section of the wheel for maximum fatigue parameter[14]. Some papers assume the depth, d of maximum damage is where von-mises stress is maximum[15]. In this paper, it is proven that both approaches yield the same result.

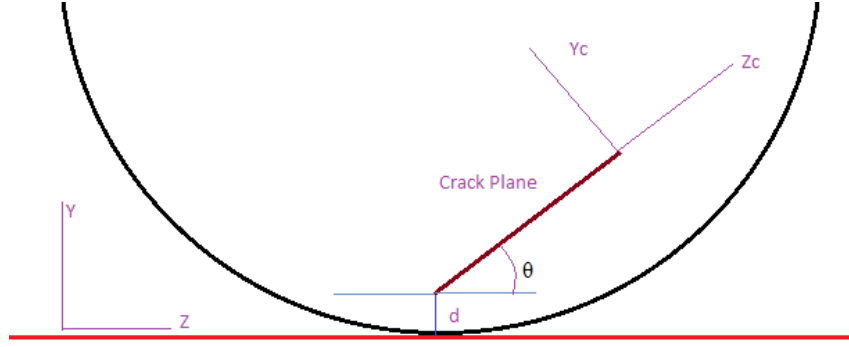


Figure 3-19 Location of critical plane

$$\sigma_{\theta} = \frac{\sigma_z + \sigma_y}{2} + \frac{\sigma_z - \sigma_y}{2} \cos(2\theta) + \tau_{yz} \sin(2\theta) \quad 3-8$$

$$\tau_{\theta} = \frac{\sigma_z - \sigma_y}{2} \sin(2\theta) - \tau_{yz} \cos(2\theta) \quad 3-9$$

$$\epsilon_{\theta} = \frac{\epsilon_z + \epsilon_y}{2} + \frac{\epsilon_z - \epsilon_y}{2} \cos(2\theta) + \frac{\gamma_{yz}}{2} \sin(2\theta) \quad 3-10$$

$$\gamma_{\theta} = -(\epsilon_z - \epsilon_y) \sin(2\theta) + \gamma_{yz} \cos(2\theta) \quad 3-11$$

3.3.3.2 Three-dimensional CPS

The location of the critical plane can be defined by two spherical coordinates θ and φ and the stress and strain on an arbitrary plane (with new coordinate X Y Z) can be calculated using three-dimensional stress and strain transformation equations below [37]:

Table 3-8 Direction cosines

	x	y	z
X	$l_1 = \cos \varphi \sin \theta$	$m_1 = \sin \varphi \cos \theta$	$n_1 = \cos \theta$
Y	$l_2 = -\sin \varphi$	$m_2 = \cos \varphi$	$n_2 = 0$
Z	$l_3 = -\cos \varphi \cos \theta$	$m_3 = -\sin \varphi \cos \theta$	$n_3 = \sin \theta$

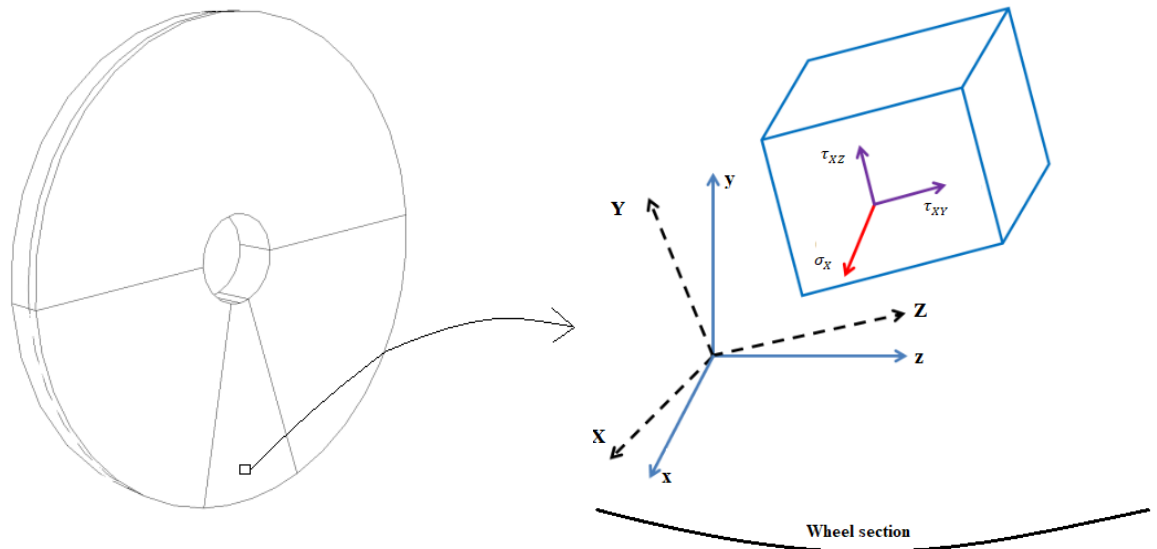


Figure 3-20 A stress element and its stress state in the wheel at a rotated coordinate axis

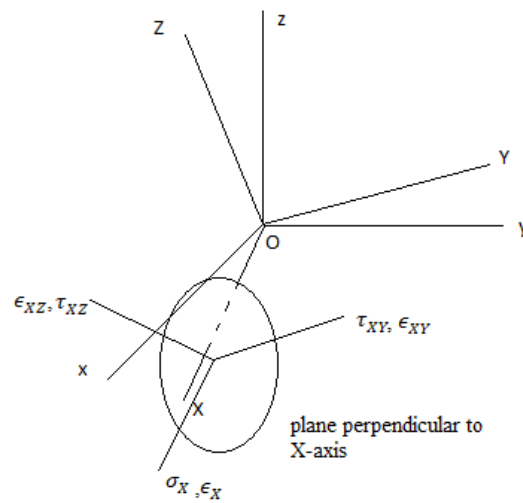


Figure 3-21 A stress state at an arbitrary plane

Stress transformation equations**Normal stress,**

$$\sigma_X = l_1^2 \sigma_x + m_1^2 \sigma_y + n_1^2 \sigma_z + 2m_1 n_1 \tau_{yz} + 2l_1 n_1 \tau_{xz} + 2l_1 m_1 \tau_{xy} \quad 3-12$$

$$\sigma_Y = l_1^2 \sigma_x + m_1^2 \sigma_y + n_1^2 \sigma_z + 2m_1 n_1 \tau_{yz} + 2l_1 n_1 \tau_{xz} + 2l_1 m_1 \tau_{xy} \quad 3-13$$

$$\sigma_Z = l_1^2 \sigma_x + m_1^2 \sigma_y + n_1^2 \sigma_z + 2m_1 n_1 \tau_{yz} + 2l_1 n_1 \tau_{xz} + 2l_1 m_1 \tau_{xy} \quad 3-14$$

Shear stress,

$$\tau_{XZ} = l_1 l_3 \sigma_x + m_1 m_3 \sigma_y + n_1 n_3 \sigma_z + (m_1 n_3 + m_3 n_1) \tau_{yz} + (l_1 n_3 + l_3 n_1) \sigma_x + (l_1 m_3 + l_3 m_1) \tau_{xy} \quad 3-15$$

$$\tau_{XY} = l_1 l_2 \sigma_x + m_1 m_2 \sigma_y + n_1 n_2 \sigma_z + (m_1 n_2 + m_2 n_1) \tau_{yz} + (l_1 n_2 + l_2 n_1) \sigma_x + (l_1 m_2 + l_2 m_1) \tau_{xy} \quad 3-16$$

$$\tau_{YZ} = l_2 l_3 \sigma_x + m_2 m_3 \sigma_y + n_2 n_3 \sigma_z + (m_2 n_3 + m_3 n_2) \tau_{yz} + (l_2 n_3 + l_3 n_2) \sigma_x + (l_2 m_3 + l_3 m_2) \tau_{xy} \quad 3-17$$

Strain transformation equations**Normal strains,**

$$\epsilon_X = l_1^2 \epsilon_x + m_1^2 \epsilon_y + n_1^2 \epsilon_z + 2m_1 n_1 \epsilon_{yz} + 2l_1 n_1 \epsilon_{xz} + 2l_1 m_1 \epsilon_{xy} \quad 3-18$$

$$\epsilon_Y = l_2^2 \epsilon_x + m_2^2 \epsilon_y + n_2^2 \epsilon_z + 2m_2 n_2 \epsilon_{yz} + 2l_2 n_2 \epsilon_{xz} + 2l_2 m_2 \epsilon_{xy} \quad 3-19$$

$$\epsilon_Z = l_3^2 \epsilon_x + m_3^2 \epsilon_y + n_3^2 \epsilon_z + 2m_3 n_3 \epsilon_{yz} + 2l_3 n_3 \epsilon_{xz} + 2l_3 m_3 \epsilon_{xy} \quad 3-20$$

Shear strains,

$$\epsilon_{xz} = l_1 l_3 \epsilon_x + m_1 m_3 \epsilon_y + n_1 n_3 \epsilon_z + (m_1 n_3 + m_3 n_1) \epsilon_{yz} + (l_1 n_3 + l_3 n_1) \epsilon_x + (l_1 m_3 + l_3 m_1) \epsilon_{xy} \quad 3-21$$

$$\epsilon_{xy} = l_1 l_2 \epsilon_x + m_1 m_2 \epsilon_y + n_1 n_2 \epsilon_z + (m_1 n_2 + m_2 n_1) \epsilon_{yz} + (l_1 n_2 + l_2 n_1) \epsilon_x + (l_1 m_2 + l_2 m_1) \epsilon_{xy} \quad 3-22$$

$$\epsilon_{yz} = l_2 l_3 \epsilon_x + m_2 m_3 \epsilon_y + n_2 n_3 \epsilon_z + (m_2 n_3 + m_3 n_2) \epsilon_{yz} + (l_2 n_3 + l_3 n_2) \epsilon_x + (l_2 m_3 + l_3 m_2) \epsilon_{xy} \quad 3-23$$

For a point at wheel the load varies with time due the rotation of the wheel. Therefore, the stress state at a point can be expressed as the Cauchy stress and strain tensor (stress and strain history output), as shown below:

$$\sigma(t) = \begin{bmatrix} \sigma_x(t) & \tau_{xy}(t) & \tau_{xz}(t) \\ \tau_{xy}(t) & \sigma_y(t) & \tau_{yz}(t) \\ \tau_{xz}(t) & \tau_{yz}(t) & \sigma_z(t) \end{bmatrix} \quad 3-24$$

And similarly, for the strain,

$$\epsilon(t) = \begin{bmatrix} \epsilon_x(t) & \epsilon_{xy}(t) & \epsilon_{xz}(t) \\ \epsilon_{xy}(t) & \epsilon_y(t) & \epsilon_{yz}(t) \\ \epsilon_{xz}(t) & \epsilon_{yz}(t) & \epsilon_z(t) \end{bmatrix} \quad 3-25$$

The discrete values of history outputs, $\sigma(t)$ and $\epsilon(t)$, can be obtained from the finite element model analysis at each load step at a point in the wheel which has suffered the maximum von-mises stress [15]. Hence, the stress and strain transformations are evaluated at each time (load step). Alternatively, for easy input to the algorithm, the stress and strain history results can be approximated using trigonometric functions as follows while the wheel is interacting with the rail [20].

$$\sigma_x(t) = \sigma_m + \sigma_a \cos(\omega t) \quad 3-26$$

$$\tau_{xy}(t) = \tau_m + \tau_a \sin(\omega t) \quad 3-27$$

σ_m and τ_m are mean stresses and σ_a and τ_a are stress amplitudes.

The same approach is used for the strain history outputs too.

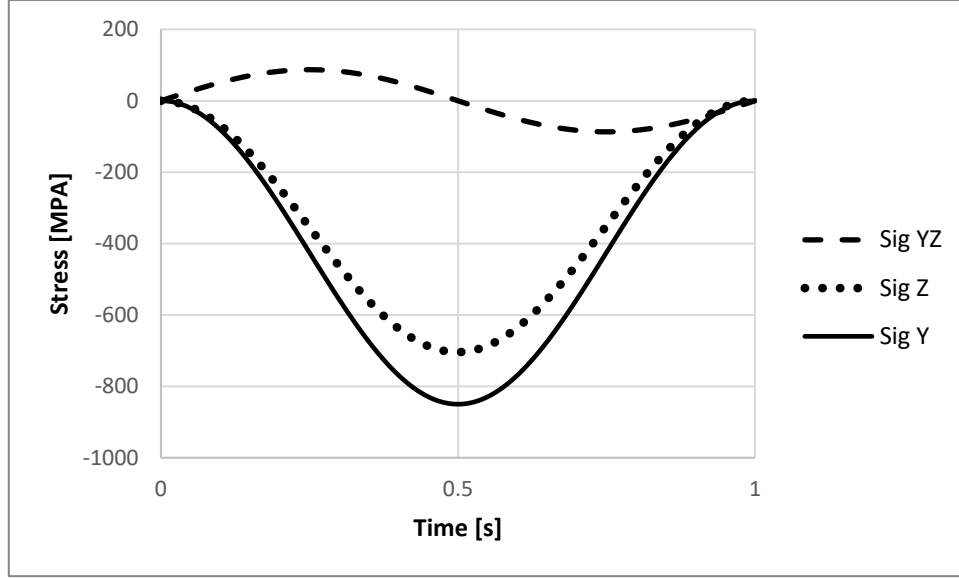


Figure 3-22 Sinusoidal approximation for the non-proportional loading

Recalling the Jiang and Sehitoglu fatigue damage criterion, the damage parameter was defined as, [23].

$$FP = \langle \sigma_{max} \rangle \frac{\Delta \epsilon}{2} + J \Delta \tau \Delta \gamma \quad 3-28$$

$\Delta \epsilon$ is the normal strain range, $\Delta \epsilon = \epsilon(t)_{max} - \epsilon(t)_{min}$

$\Delta \gamma$ is the shear strain range, $\Delta \gamma = \gamma(t)_{max} - \gamma(t)_{min}$

$\Delta \tau$ is the shear stress range, $\Delta \tau = \tau(t)_{max} - \tau(t)_{min}$

$$\Rightarrow \langle \sigma_{max} \rangle = 0.5(|\sigma_{max}| + \sigma_{max})$$

Where, σ_{max} is the maximum normal stress.

The crack plane (the critical plane) is the plane where the damage parameter, FP is maximum. To find this plane every plane should be checked with 1° angle increments. However, this process is highly time demanding if tried manually. Therefore, a MATLAB algorithm (Appendix B and C) that checks every plane within the given increment is developed. The algorithm implements a *nested for-loop* mechanism to iterate every value at every plane.

The algorithm eventually determines the maximum fatigue parameter (FP_{max}) and its orientation. Then FP_{max} is used to calculate the fatigue life N_f according to equation

$$FP_{max} = \begin{cases} \frac{(\sigma'_f)^2}{E} (2N_f)^{2b} + \sigma'_f \epsilon'_f (2N_f)^{b+c} \\ \frac{(\tau'_f)^2}{G} (2N_f)^{2b} + \tau'_f \gamma'_f (2N_f)^{b+c} \end{cases} \quad 3-29$$

3.3.4 Fatigue life analysis of the wheel

The fatigue analysis is made as a post-process using the results from the FE analysis. In the post-process, the most critical material points and plane for crack initiation (the crack plane) are identified. The crack plane is determined by using a numerical algorithm developed using MATLAB together with a theoretical model for fatigue crack initiation, which gives the most critical material plane with respect to fatigue damage. When the crack plane has been identified, the fatigue life to crack initiation is computed, and the results are compared with previous literatures.

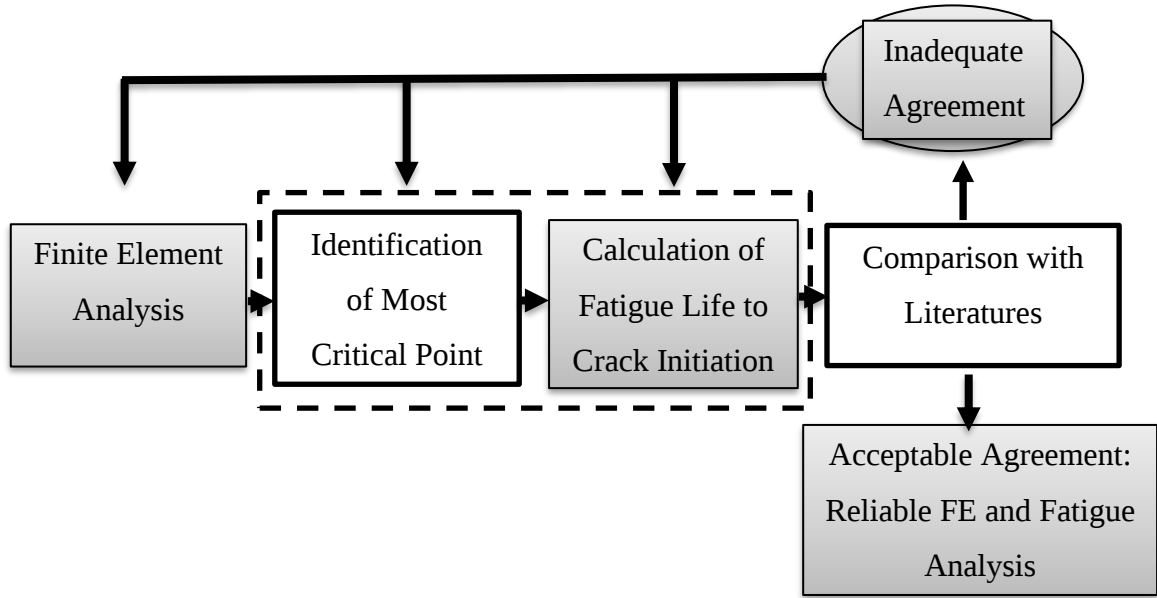


Figure 3-23 A flow diagram presenting a strategy for fatigue life prediction of crack initiation.

3.3.4.1 Calculation of Fatigue Life

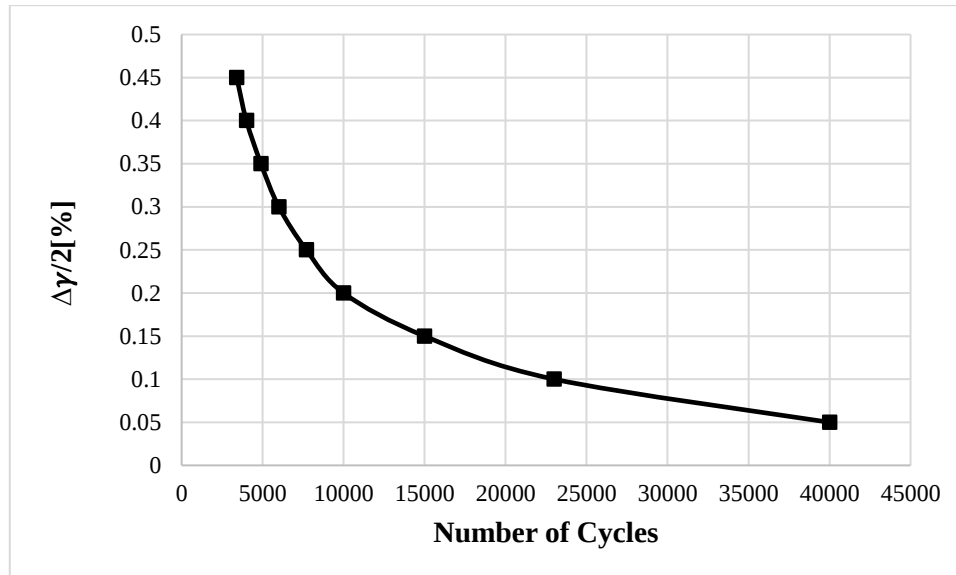
Since all the normal stresses are in compression, the shear cracking (Mode II) governs the fatigue parameter and the number of cycles to crack initiation should be evaluated using the shear form equation.

$$FP_{max} = \frac{(\tau'_f)^2}{G} (2N_f)^{2b} + \tau'_f \gamma'_f (2N_f)^{b+c} \quad 3-30$$

Table 3-9 Mechanical properties of the wheel steel [20]

Parameters	G (GPa)	<i>b</i>	<i>c</i>	τ'_f (Mpa)	γ'_f (%)	<i>J</i>
Value	80	-0.089	-0.559	468	15.45	0.2

The S-N curve for the life prediction was plotted as follows using shear strain amplitude. The variable S represents the combination of shear strain amplitude.

**Figure 3-24 S-N Curve**

CHAPTER 4 RESULT AND DISCUSSION

4.1 Numerical Example

In this section, a specific example simulating the working conditions of R8T wheel was investigated using the procedures and tools developed by the author.

Table 4-1 Working conditions for the numerical example

Parameters		Design value
Train velocity,	V	70 km/hr
Vertical load,	F	160 kN
Friction coefficient,	μ	0.3
Wheel diameter,	2Rw1	660 mm
Wheel radius of curvature,	Rw2	470 mm
Rail radius of curvature,	Rr2	300 mm

4.1.1 Contact pressure distribution

Contact pressure distribution along rolling and lateral directions of the wheel

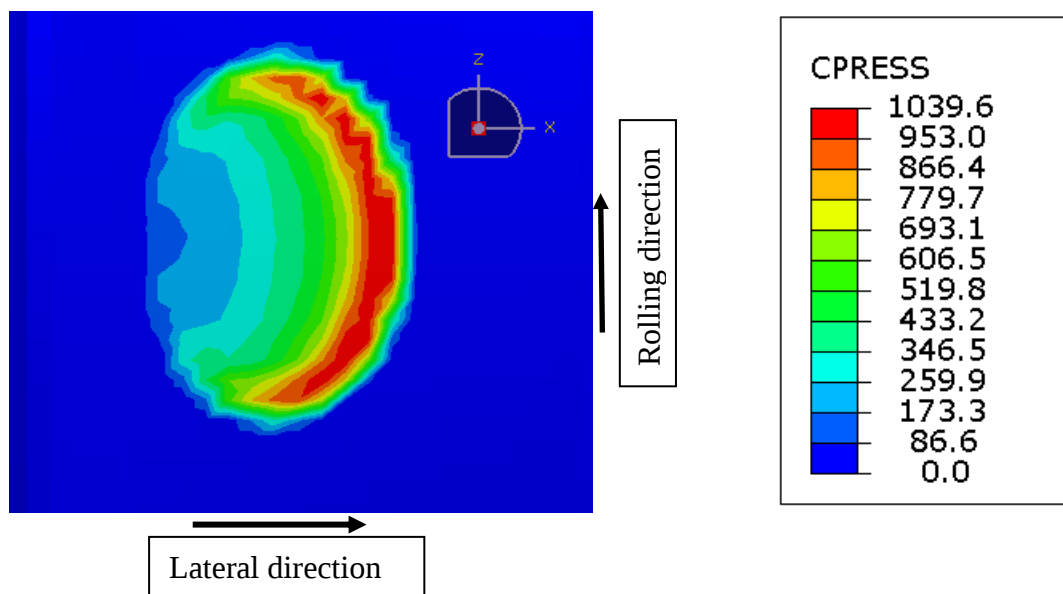


Figure 4-1 Contact Pressure distribution

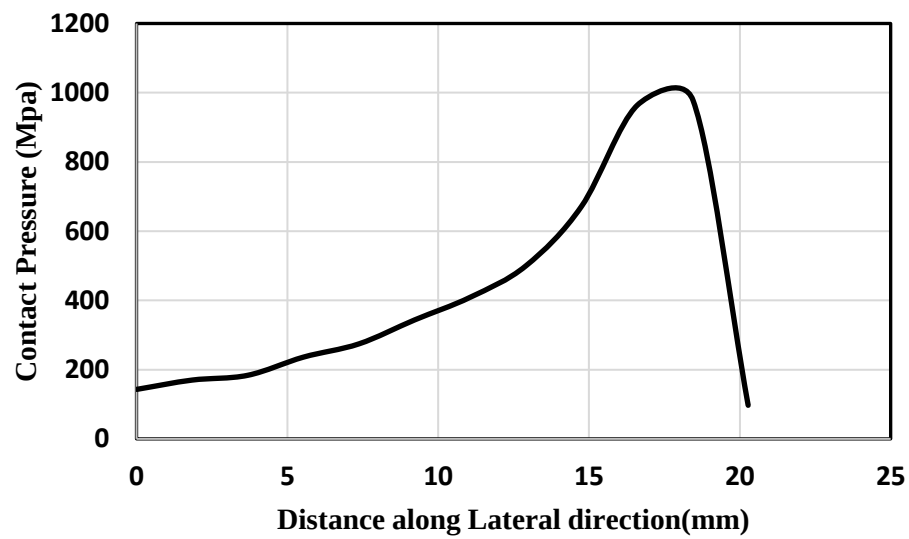


Figure 4-2 Contact pressure distribution on wheel surface along rolling direction

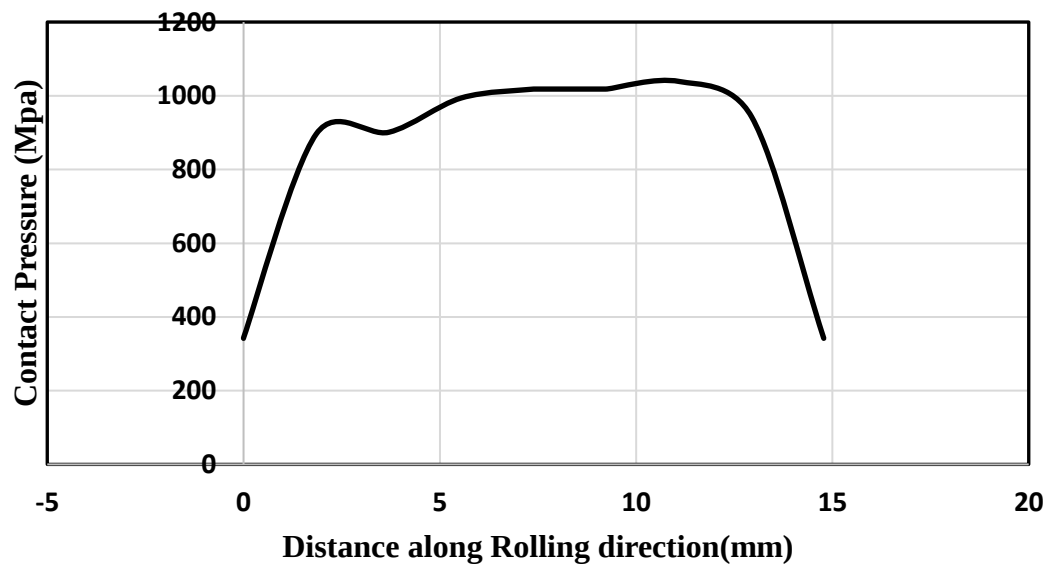


Figure 4-3 Contact pressure distribution on wheel surface along lateral direction

4.1.2 Surface and subsurface stresses

4.1.2.1 Von-mises stress:

Von-mises stress distribution at the wheel tread surface and different cross-sections is depicted as shown in Figure 4-4. The maximum von-mises stress was found on the surface of the wheel.

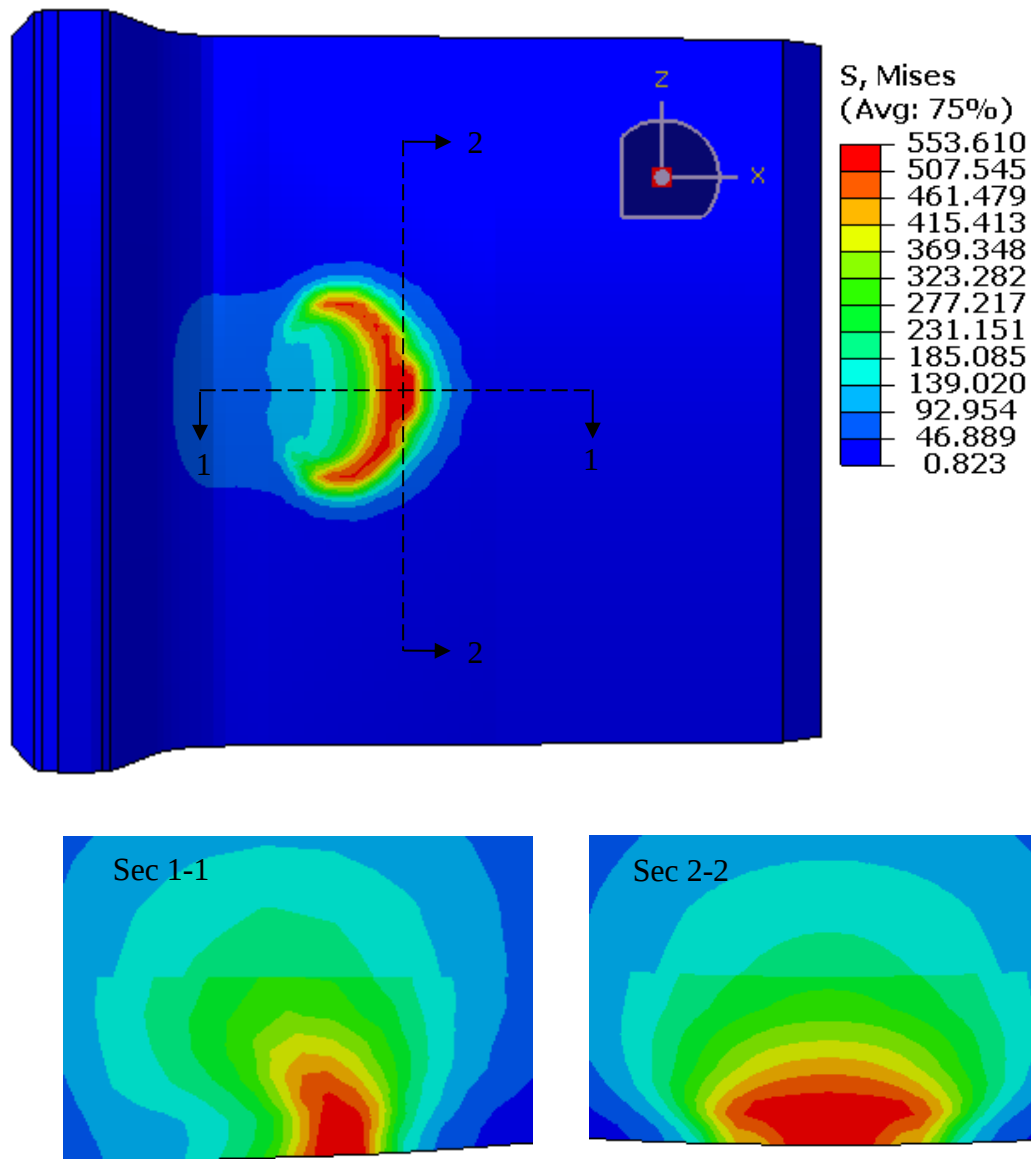


Figure 4-4 Von-Mises stress distribution of wheel from both left and front section

4.1.2.2 Shear stress:

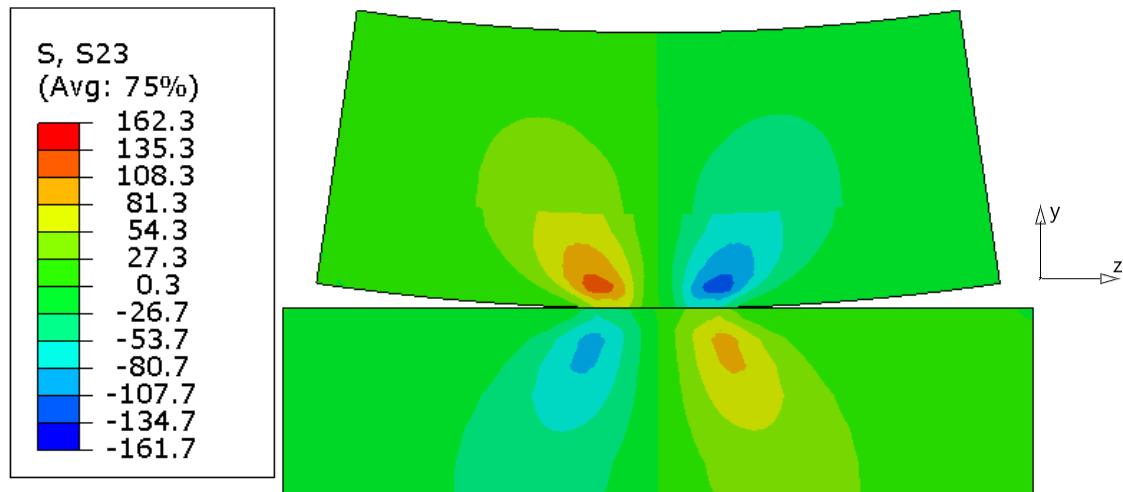


Figure 4-5 Distribution of τ_{yz} (S23)

4.1.2.3 Normal stress:

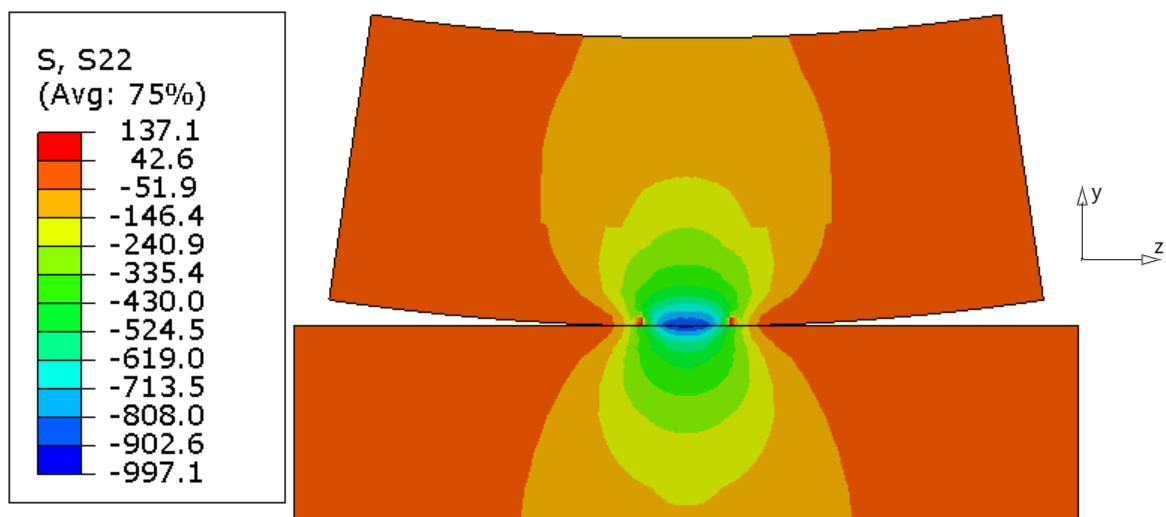


Figure 4-6 Distribution of σ_y (S22)

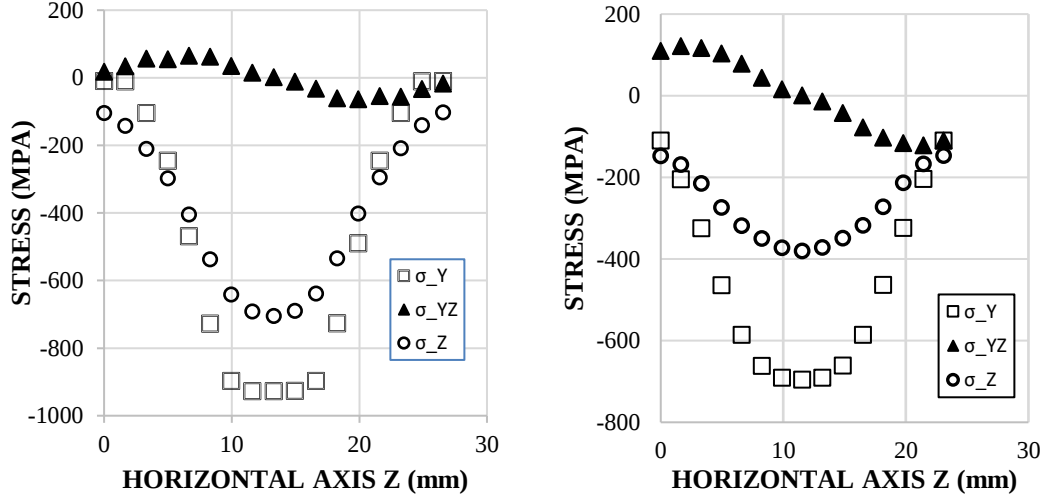


Figure 4-7 Stress plots of the wheel at x=0, y=0 (left) and x=0, y=2.6mm (right)

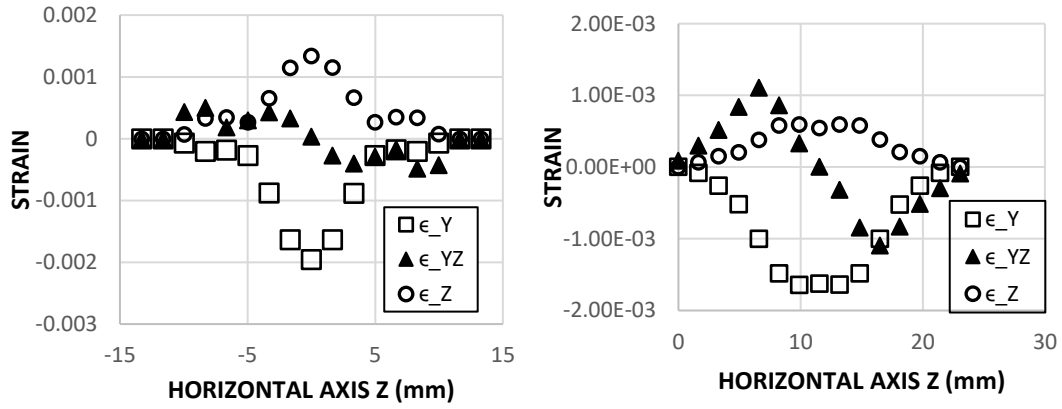


Figure 4-8 Strain plots of the wheel at x=0, y=0 (left) and x=0, y=2.6mm (right)

The stress and strain plots shown in Figure 4-7 and Figure 4-8 are approximated using a sinusoidal response as shown in Figure 3-22. Here it is important to note that the horizontal axis (z-axis) was replaced by the time using the equation below.

$$z/R = \omega t \quad 4-1$$

Where, z is horizontal distance, ω is the angular velocity of the wheel, R is the wheel radius and t is the time.

4.1.3 Investigation of the Crack Orientation

In this section, study of the orientation of the crack including both depth and angle are conducted.

4.1.3.1 Crack Depth

Some studies like El-Sayed et al [15] paper apply the damage theory where depth of crack initiation is assumed to be at a point of maximum von-mises stress. However, the investigation at wheel load of 160kN and 0.3 friction coefficient shows depth of crack initiation (where the fatigue parameter is maximum) is at 2.6mm below the wheel surface even if the location of maximum von-mises stress is at the wheel surface. This due to the fact that failure of the wheel is highly dominated by the shear strain and shear stress and these shear components are higher below the surface as shown in Figure 4-5, Figure 4-7, Figure 4-8 and Figure 4-9.

The maximum fatigue parameter (FP) at various depths in the wheel are also investigated as shown in Figure 4-10. As illustrated in the figure, the fatigue parameter increased as the depth increased until the maximum fatigue parameter is achieved at 2.6 mm. Once the maximum FP is reached it will decrease as we further go deeper.

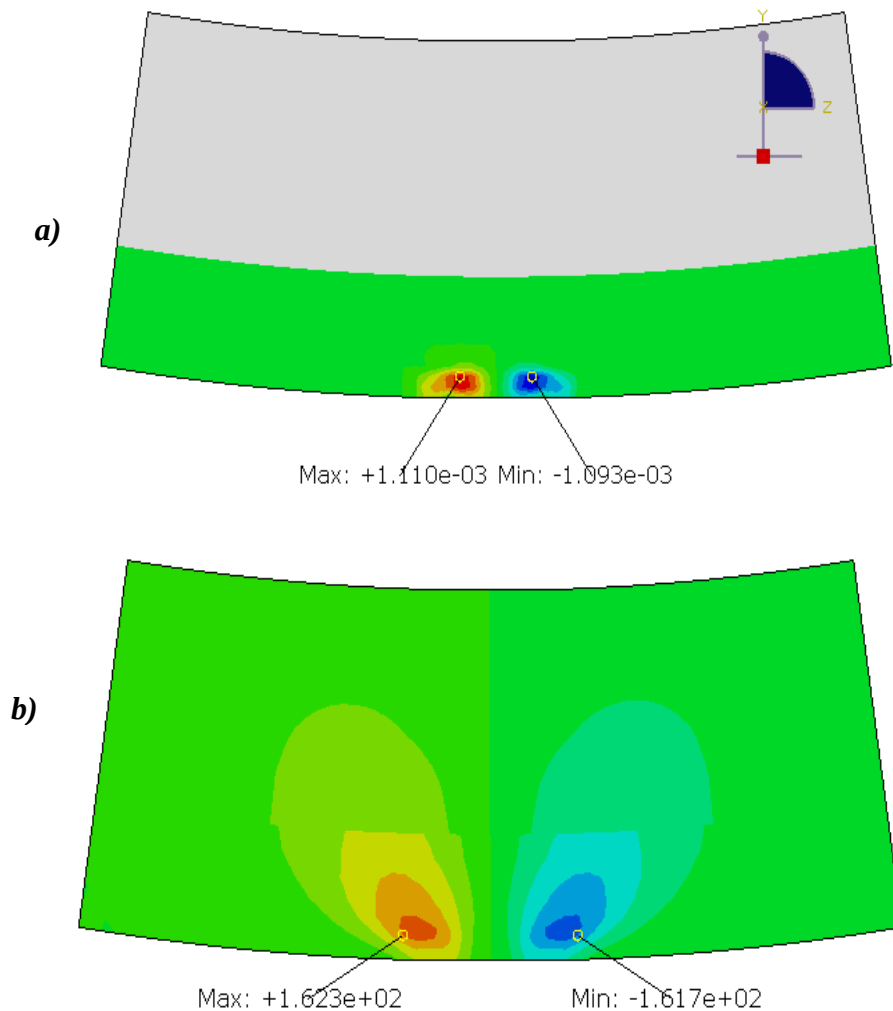
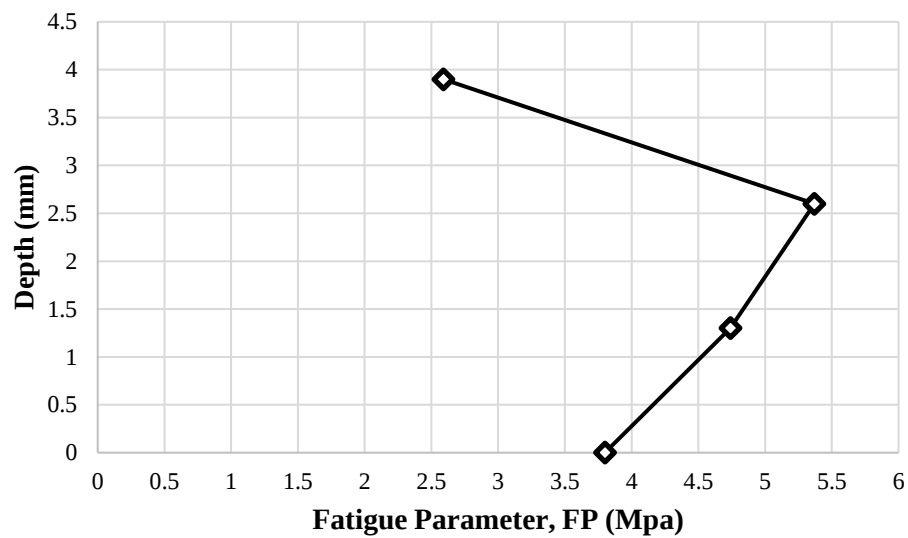


Figure 4-9 Distribution of (a) shear strain, γ_{yz} and (b) shear stress, τ_{yz} **Figure 4-10 An illustration of change in fatigue parameter with wheel depth****Table 4-2 Investigation of fatigue damage along the wheel depth in the vicinity of contact**

No	Node label	Location Depth, y(mm)	Maximum von-mises stress (MPa)	Maximum equivalent plastic strain, PEEQ (%)	Maximum fatigue parameter, FP_{max} (MPa)	Fatigue life N_f (cycles)
1	1144	0	567	0.804	3.80	66,326
2	9945	1.3	553	0.804	4.74	46,154
3	10010	2.6	549	0.520	5.40	37,327
4	9932	3.9	523	0.160	2.59	125,602
5	486	5.8	517	0.023	0.37	>1,000,000

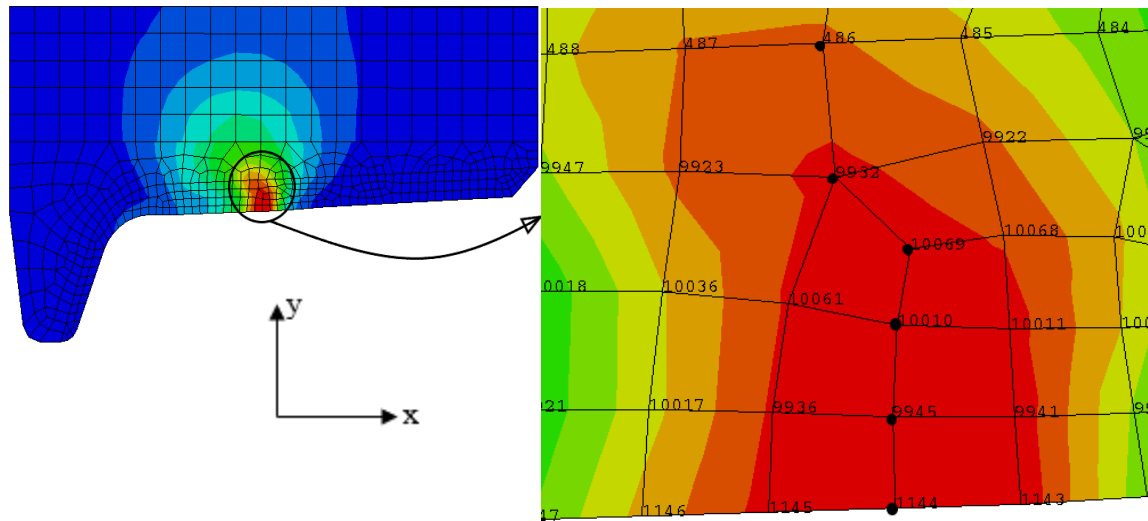


Figure 4-11 Node Labels along the depth

4.1.3.2 Crack Angle

To fully understand crack orientation the angle of crack initiation (nucleation), θ should also be known. This is the angle where the fatigue parameter becomes maximum. The MATLAB algorithm was used to search for the plane θ with maximum fatigue damage parameter (FPmax). The possible angle of crack initiation at four depth location was predicted searching for planes where the fatigue parameter, FP attains its maximum value.

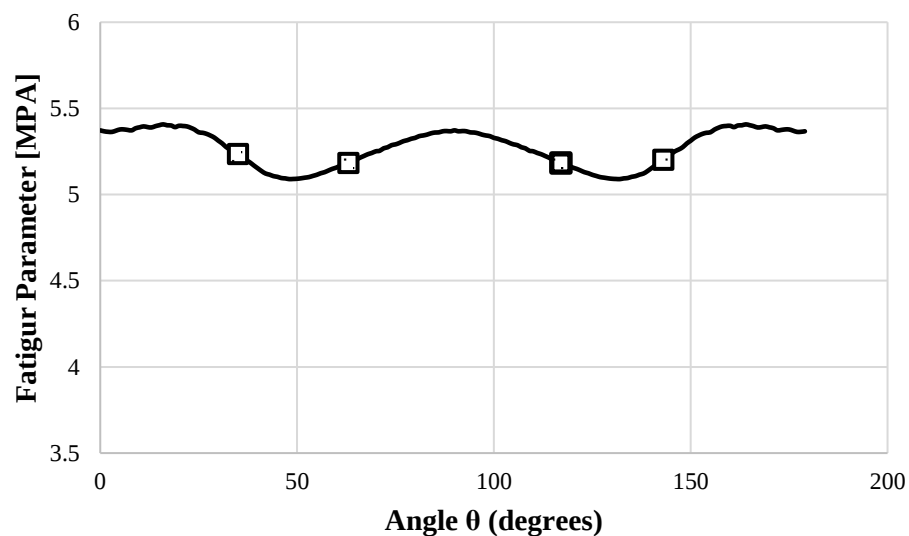
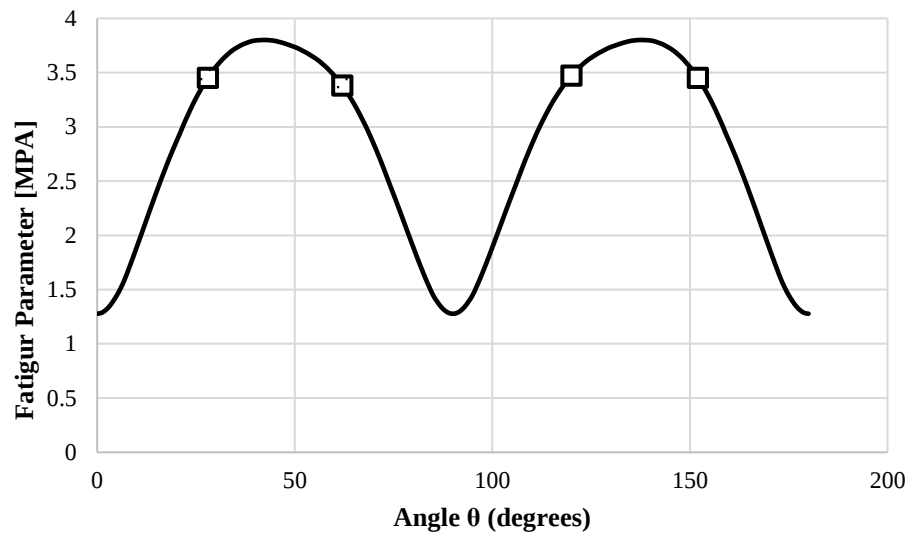


Figure 4-12 An illustration of the search of the critical plane 2.6mm below the surface



**Figure 4-13 An illustration of the search of the critical plane at the surface
(maximum von-mises stress)**

It turned out that the predicted crack angle differs for the two cases considered in Figure 4-12 and Figure 4-13. Taking the range of 5 % from the maximum fatigue damage, the predicted crack angle range for the first case (at 2.6mm depth) is on $(0^{\circ}, 40^{\circ})$, $(62^{\circ}, 123^{\circ})$ whereas for the second case (at the surface) the predicted crack orientation was on the range $(30^{\circ}, 58^{\circ})$ and $(120^{\circ}, 152^{\circ})$.

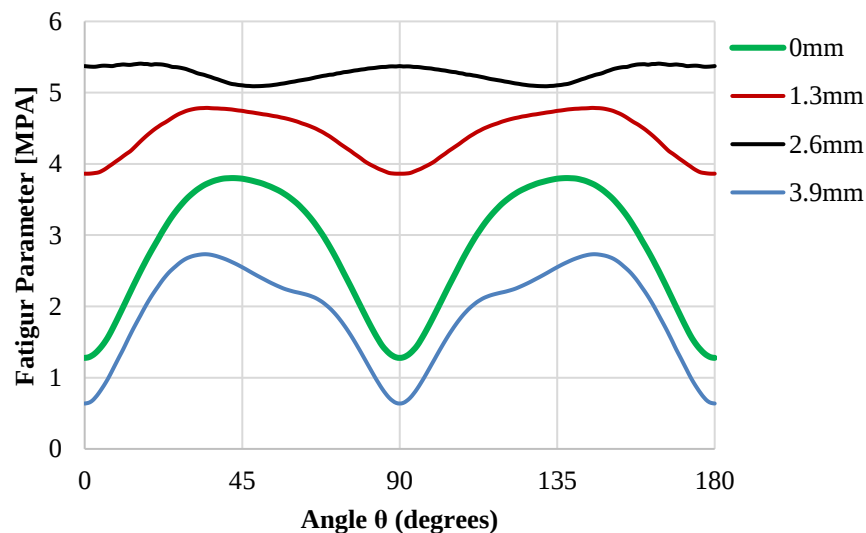


Figure 4-14 An illustration of the search of the critical plane at different depths

For the wheel depth 0mm, 1.3mm, and 3.9mm the fatigue parameter curves have almost similar trend and achieve maximum FP at closer range of angles, whereas the trend at depth of 2.6 mm is different from the others.

4.2 Parametric study

The influence of several factors on the fatigue life of the wheel are studied on this section using finite element and critical plane models proposed above. The effect of the following factors on fatigue life are considered; vertical loads, friction coefficient, wheel diameter.

4.2.1 Vertical loads

The effect of the vertical loads on fatigue life is studied at a straight track with wheel diameter of 660mm and friction coefficient of 0.3. The vertical loads in Table 4-3 were taken for investigation.

Table 4-3 Effect of Vertical load on fatigue life at friction coefficient of 0.3

Loading condition	F (KN)	FP_{MAX} (MPa)	N_f	Log(N_f)
L1	110	2.48	135,141	5.13
L2	140	4.10	58,529	4.76
L3	160	5.40	37,327	4.57
L4	180	6.84	25,466	4.40
L5	200	8.39	18,343	4.26
L6	220	10.15	13,531	4.13

As expected, the fatigue life decreases as the vertical load increases. As shown in Table 3-1 and Figure 4-15, the endurance limit is not achieved under the investigated loads.

4.2.2 Friction coefficient

Three conditions when $\mu = 0.2, 0.3, 0.4$ are taken for investigation of friction effect at wheel load of 160kN and diameter 660mm as shown in Table 4-4 and Figure 4-15. The investigation shows that an increase in coefficient of friction has decreased the fatigue life of the wheel.

The application of boundary and elastohydrodynamic lubrication can lower coefficient of friction between wheel and rail (increasing fatigue life) at a risk of unexpected and invisible subsurface crack initiations.

Additionally, the analysis revealed that the depth of the most damaged element (with maximum fatigue parameter, FPmax) depends on the value of friction coefficients. Figure 4-17 shows for the friction consideration $\mu = 0.2$ and $\mu = 0.3$, the maximum fatigue parameter is found at depth of 2.6mm and 3.9mm below the wheel surface, respectively. For friction coefficient considerations $\mu = 0.4$, maximum fatigue damage (location of crack initiation) is found on the surface of the wheel.

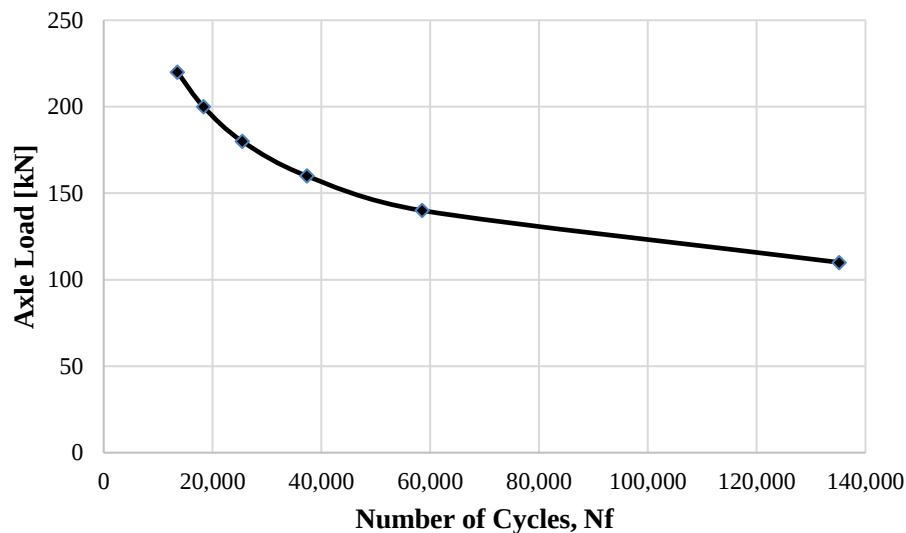


Figure 4-15 An illustration of variation of fatigue life with the vertical axle load

Table 4-4 Effect of Friction on fatigue life at F = 160kN

Friction coefficient, μ	Fatigue Parameter, FPmax	Fatigue life, Nf	Log(Nf)
0.2	4.0	60,954	4.78
0.3	5.4	37,327	4.57
0.4	9.6	14,642	4.16

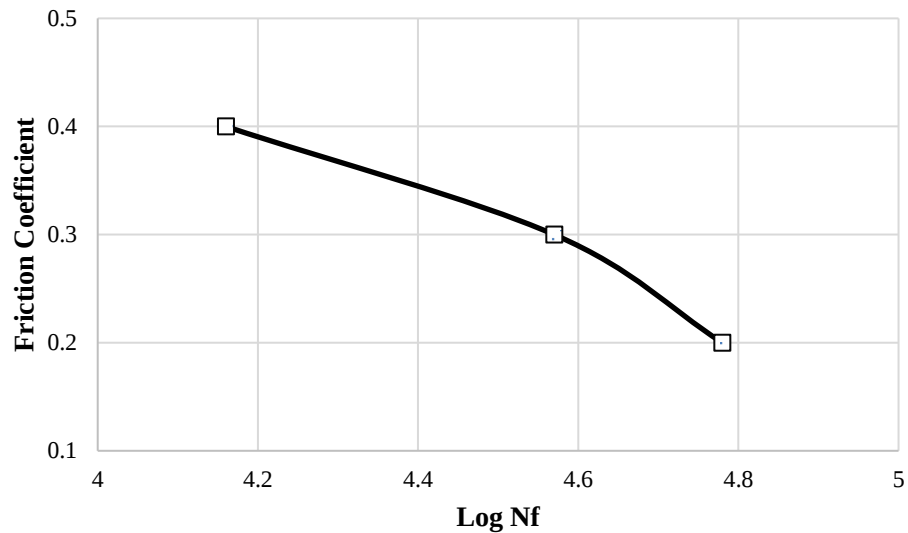


Figure 4-16 Variation of fatigue life with coefficient of friction

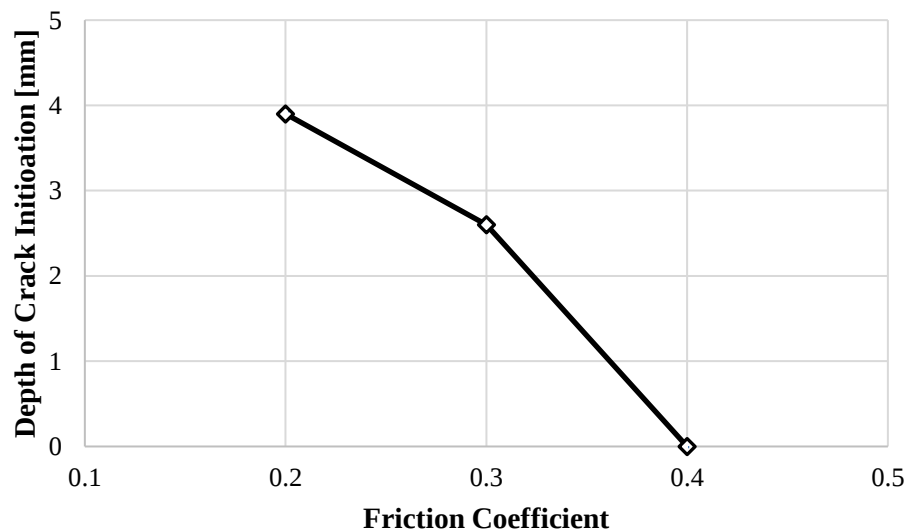


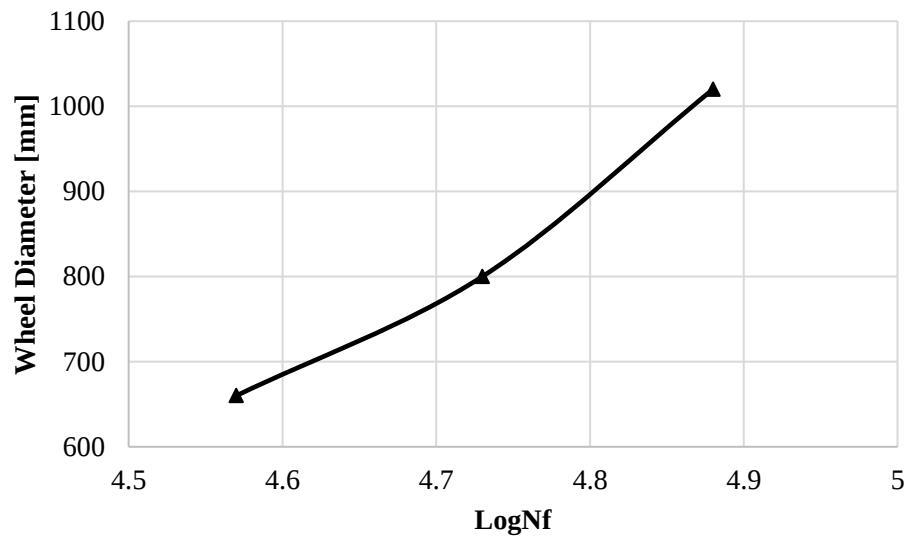
Figure 4-17 Location (Depth) of Crack Initiation at Varying Coefficients of Friction

4.2.3 Wheel diameter

The wheel diameter influences the fatigue damage, because according to the Hertz theory, the radius of the wheel will have effect on contact stress in the wheel. Different set of numerical simulations of wheels with different diameters, from 0.66 m to 1.02 m are used for investigation, as depicted in Figure 4-18. The result has shown that larger wheels can last longer than the smaller ones.

Table 4-5 Effect of wheel diameter on fatigue life

Wheel diameter (mm)	FP _{MAX} (MPa)	N _f (10 ⁵)	Log(N _f)
660	5.4	0.373	4.57
800	4.3	0.541	4.73
1020	3.5	0.759	4.88

**Figure 4-18 Variation of fatigue life with wheel diameter**

4.3 Comparison with Experimental Work

In this section, a comparison between the results of an experimental work done by Jiang et al [26] and results obtained in this paper is made. In the experiment specimens made from steel S640N were investigated under various loading conditions. Among specimens tested those with closer loading path of wheel-rail loading were investigated.

The loading conditions used in those specimens were inserted to the numerical model and fatigue criteria developed to determine the fatigue lives. As shown in Table 4-6, about 34% of the specimens investigated under the prediction tool developed in this paper showed an error below 12% when compared to the experimental findings. The rest 64% of the specimens showed discrepancies with the predicted results.

The result suggests that the fatigue life estimation of the tool developed works best at low fatigue cycles and when the shear strain and shear stress are the major causes for crack initiation. For higher fatigue cycles and failures dominated by stress a numerical model with another set of material model and fatigue criteria should be adopted. Another possible reason for the discrepancy is that even though the specimens used in the test are loaded with similar loading paths, they do not consider the complex geometry in the wheel and rail model.

Table 4-6 Comparison of fatigue life experimental and numerical results

Specimen number	$\Delta\epsilon/2$ (%)	$\Delta\gamma/2$ (%)	$\Delta\sigma/2$ (MPa)	$\Delta\tau/2$ (MPa)	N_f (exp.)	N_f (prediction)	Error (%)
126	0.3	0.52	436	277	1,050	1,188	11.8
138	0.3	0.52	440.2	273.4	767	823	7.3

$\Delta\epsilon/2$ is axial strain amplitude, $\Delta\gamma/2$ is shear strain amplitude, $\Delta\sigma/2$ normal stress amplitude and $\Delta\tau/2$ is shear stress amplitude.

CHAPTER 5 CONCLUSIONS AND RECCOMENDATIONS

In this section, the conclusions from outcome of the research are explained and some recommendations are forwarded for the betterment of the general outcome in future works.

5.1 Conclusion

In this paper, the contact pressure, surface and subsurface stresses and fatigue life to crack initiation of the wheel under different considerations was investigated. Due to its non-proportional loading nature the maximum shear stress and normal stress direction rotates and it is necessary to survey all planes to know the orientation of the critical plane. The survey was effectively done by an algorithm developed using MATLAB. The critical plane is, as discussed before, the plane experiencing the maximum fatigue parameter, FP_{max} .

In design of wheel against fatigue the parameters such as, friction, wheel diameter and vertical load were found very critical in determining the fatigue life of a wheel.

At higher coefficient of friction ($f=0.4$), the maximum damage is likely to occur at the surface due to an increase in the traction force. This facilitates surface crack initiation on the wheel.

At lower coefficient of friction ($f=0.2$ and $f=0.3$), the maximum fatigue damage is most likely to occur below the wheel surface provided that there are no already existing surface defects. So, this promotes the formation of subsurface crack.

It is also shown that the submodelling technique was efficient as it saves a great deal of computation time.

5.2 Recommendation and Future work

The number of cycles simulated was only for two cycles due to the limitation in computational efficiency. This can reduce the accuracy of the results in the numerical analysis. The system can be better simulated using a more efficient computer.

The critical plane search method (CPS) implemented in the paper was an efficient and easy way to predict the location and orientation of critical crack plane. However, the author

understands there is a lot of room for improvement in the algorithm used for the CPS. For instance, it can be made to check all nodes in the submodel for possible damage by getting stress and strain history results for each node in excel format from Abaqus. This can result in more accurate and reliable results at an expense of high computational capacity demand.

Besides, this research can be more complete if the following are considered in future works;

- Effect track curvature and gradient
- Contact at a flange
- Investigation of lateral and spin creepages
- Effect of rail geometry on fatigue analysis of the wheel

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APPENDIX A**Stress transformation equations**

Normal stress transformation,

$$\sigma_X = l_1^2 \sigma_x + m_1^2 \sigma_y + n_1^2 \sigma_z + 2m_1 n_1 \tau_{yz} + 2l_1 n_1 \tau_{xz} + 2l_1 m_1 \tau_{xy}$$

$$\sigma_Y = l_1^2 \sigma_x + m_1^2 \sigma_y + n_1^2 \sigma_z + 2m_1 n_1 \tau_{yz} + 2l_1 n_1 \tau_{xz} + 2l_1 m_1 \tau_{xy}$$

$$\sigma_Z = l_1^2 \sigma_x + m_1^2 \sigma_y + n_1^2 \sigma_z + 2m_1 n_1 \tau_{yz} + 2l_1 n_1 \tau_{xz} + 2l_1 m_1 \tau_{xy}$$

Shear stress transformation,

$$\begin{aligned} \tau_{XZ} = & l_1 l_3 \sigma_x + m_1 m_3 \sigma_y + n_1 n_3 \sigma_z + (m_1 n_3 + m_3 n_1) \tau_{yz} + (l_1 n_3 + l_3 n_1) \sigma_x \\ & + (l_1 m_3 + l_3 m_1) \tau_{xy} \end{aligned}$$

$$\begin{aligned} \tau_{XY} = & l_1 l_2 \sigma_x + m_1 m_2 \sigma_y + n_1 n_2 \sigma_z + (m_1 n_2 + m_2 n_1) \tau_{yz} + (l_1 n_2 + l_2 n_1) \sigma_x \\ & + (l_1 m_2 + l_2 m_1) \tau_{xy} \end{aligned}$$

$$\begin{aligned} \tau_{YZ} = & l_2 l_3 \sigma_x + m_2 m_3 \sigma_y + n_2 n_3 \sigma_z + (m_2 n_3 + m_3 n_2) \tau_{yz} + (l_2 n_3 + l_3 n_2) \sigma_x \\ & + (l_2 m_3 + l_3 m_2) \tau_{xy} \end{aligned}$$

Strain transformation equations

Normal strains,

$$\begin{aligned} \epsilon_X = & l_1^2 \epsilon_x + m_1^2 \epsilon_y + n_1^2 \epsilon_z + 2m_1 n_1 \epsilon_{yz} + 2l_1 n_1 \epsilon_{xz} \\ & + 2l_1 m_1 \epsilon_{xy} \end{aligned}$$

$$\epsilon_Y = l_2^2 \epsilon_x + m_2^2 \epsilon_y + n_2^2 \epsilon_z + 2m_2 n_2 \epsilon_{yz} + 2l_2 n_2 \epsilon_{xz} + 2l_2 m_2 \epsilon_{xy}$$

$$\begin{aligned} \epsilon_Z = & l_3^2 \epsilon_x + m_3^2 \epsilon_y + n_3^2 \epsilon_z + 2m_3 n_3 \epsilon_{yz} + 2l_3 n_3 \epsilon_{xz} \\ & + 2l_3 m_3 \epsilon_{xy} \end{aligned}$$

Shear strains,

$$\epsilon_{XZ} = l_1 l_3 \epsilon_x + m_1 m_3 \epsilon_y + n_1 n_3 \epsilon_z + (m_1 n_3 + m_3 n_1) \epsilon_{yz} + (l_1 n_3 + l_3 n_1) \epsilon_x \\ + (l_1 m_3 + l_3 m_1) \epsilon_{xy}$$

$$\epsilon_{XY} = l_1 l_2 \epsilon_x + m_1 m_2 \epsilon_y + n_1 n_2 \epsilon_z + (m_1 n_2 + m_2 n_1) \epsilon_{yz} + (l_1 n_2 + l_2 n_1) \epsilon_x \\ + (l_1 m_2 + l_2 m_1) \epsilon_{xy}$$

$$\epsilon_{YZ} = l_2 l_3 \epsilon_x + m_2 m_3 \epsilon_y + n_2 n_3 \epsilon_z + (m_2 n_3 + m_3 n_2) \epsilon_{yz} + (l_2 n_3 + l_3 n_2) \epsilon_x \\ + (l_2 m_3 + l_3 m_2) \epsilon_{xy}$$

MATLAB code:

```
% Value of direction cosines

L1 = cos(s*pi/180)*sin(t*pi/180);
m1 = sin(s*pi/180)*cos(t*pi/180);
n1 = cos(t*pi/180);

L2 = -sin(s*pi/180);
m2 = cos(s*pi/180);
n2 = 0;

L3 = -cos(s)*cos(t*pi/180);
m3 = -sin(s)*cos(t*pi/180);
n3 = sin(t*pi/180);

% value of the stress vector
sigma_x1(ti+1,t+1,s+1)=sigma_x.*(L1.^2)+sigma_y.*(m1.^2)
+sigma_z.*(n1.^2)+(2*tau_xy.*L1.*m1) + (2*tau_yz.*m1.*n1)+(2*tau_xz.*n1.*L1);

Tau_x1y1(ti+1,t+1,s+1)=sigma_x.*(L1.*L2) + sigma_y.*(m1.*m2)+sigma_z.*(n1.*n2) +
tau_xy.*(L1.*m2 + L2.*m1) + tau_xz.*((m1.*n2)+ (n1.*m2))+tau_xz.*((n1.*L2)+
(n2.*L1));

Tau_x1z1(ti+1,t+1,s+1)=sigma_x.*(L1.*L3) + sigma_y.*(m1.*m3) + sigma_z.*(n1.*n3)
+ tau_xy.*(L1.*m3 + L3.*m1) + tau_xz.*((m1.*n3)+ (n1.*m3)) + tau_xz.*((n1.*L3)+
(n3.*L1));

% value of the strain vector

e_x1(ti+1,t+1,s+1)=e_x.*(L1.^2) + e_y.*(m1.^2) +e_z.*(n1.^2)+(2*e_xy.*L1.*m1) +
(2*e_yz.*m1.*n1) + (2*e_xz.*n1.*L1);
```

```
e_xly1(ti+1,t+1,s+1) = e_x.*(L1.*L2) + e_y.*(m1.*m2) + e_z.*(n1.*n2) +
e_xy.*(L1.*m2 + L2.*m1) + e_xz.*((m1.*n2)+(n1.*m2)) + e_xz.*((n1.*L2)+(
n2.*L1));

e_xlz1(ti+1,t+1,s+1) = e_x.*(L1.*L3) + e_y.*(m1.*m3) + e_z.*(n1.*n3) +
e_xy.*(L1.*m3 + L3.*m1) + e_xz.*((m1.*n3)+(n1.*m3)) + e_xz.*((n1.*L3)+(
n3.*L1));
```

Variation of Stress and Strain with time

$$\sigma(t) = \begin{bmatrix} \sigma_X(t) & \tau_{XY}(t) & \tau_{XZ}(t) \\ \tau_{XY}(t) & \sigma_Y(t) & \tau_{YZ}(t) \\ \tau_{XZ}(t) & \tau_{YZ}(t) & \sigma_Z(t) \end{bmatrix}$$

$$\epsilon(t) = \begin{bmatrix} \epsilon_X(t) & \epsilon_{XY}(t) & \epsilon_{XZ}(t) \\ \epsilon_{XY}(t) & \epsilon_Y(t) & \epsilon_{YZ}(t) \\ \epsilon_{XZ}(t) & \epsilon_{YZ}(t) & \epsilon_Z(t) \end{bmatrix}$$

MATLAB Code:

```
% Stress amplitudes
amp_x = -443;
amp_y = -1158;
amp_z = -577;
amp_xy = 248;
amp_xz = -73;
amp_yz = 153;
% Strain amplitudes
amp_ex = -0.00224;
amp_ey = -0.00463;
amp_ez = -0.001795;
amp_exy = 0.0052642;
amp_exz = -0.00166;
amp_eyz = 0.00388;
% variation of stresses with time at x,y,z coordinate
format short
sigma_x = amp_x*sin (0.025*pi*ti);
sigma_y= amp_y*sin (0.025*pi*ti);
sigma_z= amp_z*sin (0.025*pi*ti);
tau_xy = amp_xy*sin (0.025*pi*ti);
tau_xz = amp_xz*cos (0.025*pi*ti);
tau_yz = amp_yz*cos (0.025*pi*ti);
```

```
% variation of strains with time at x, y, z coordinate
e_x = amp_ex*sin (0.025*pi*ti);
e_y= amp_ey*sin (0.025*pi*ti);
e_z= amp_ez*sin (0.025*pi*ti);
e_xy = amp_exy*sin (0.025*pi*ti);
e_xz = amp_exz*cos (0.025*pi*ti);
e_yz = amp_eyz*cos (0.025*pi*ti);
```

General outline of the MATLAB algorithm

```
% Variation of stresses and strains with time (History output)

    for ti = 0:10 % time variable

         $\sigma = \sigma(t);$ 
         $\epsilon = \epsilon(t);$ 
        % Value of the stress and strain vector at an arbitrary plane

            for t=0:180;          % angle theta
                for s=0:180;      % angle sai
                     $\sigma = \sigma(\theta, \varphi);$ 
                     $\epsilon = \epsilon(\theta, \varphi);$ 
                end
            end
        end

        end

    % CALCULATION OF THE FATIGUE PARAMETER, FP
    FP = FP ( $\sigma, \epsilon$ );
```

APPENDIX B

MATLAB algorithm for 2D Critical Plane Search

```

clear all;
clc;

for ti = 0:100 % time variable
%   stress and strain amplitudes and mean values
    amp_y = 347;
    mean_y = -347;
    amp_z = 190;
    mean_z = -190;
    amp_yz = 121;
    amp_ey = 0.082;
    mean_ey = -0.082;
    amp_ez = -0.0298;
    mean_ez = 0.0298;
    amp_eyz = 0.111;
%   variation of stresses with time at x,y,z coordinate
    format short

    sigma_y = mean_y + amp_y*cos(0.02*pi*ti);
    sigma_z = mean_z + amp_z*cos(0.02*pi*ti);
    tau_yz = amp_yz*sin(0.02*pi*ti);
%   variation of strains with time at x,y,z coordinate
    format short

    e_y = mean_ey + amp_ey*cos(0.02*pi*ti);
    e_z = mean_ez + amp_ez*cos(0.02*pi*ti);
    e_yz = amp_eyz*sin(0.02*pi*ti);

    for t = 0:180 % Angle theta
        %   Value of the stress vector at time t
        sigma_y1(ti+1,t+1) = (sigma_y + sigma_z)./2 + ((sigma_y - sigma_z)./2).*cos(2.*t.*pi./180) -
        tau_yz.*sin(2.*t.*pi./180);
        tau_y1z1(ti+1,t+1) = ((sigma_y - sigma_z)./2).*sin(2.*t.*pi./180) + tau_yz.*cos(2.*t.*pi./180);
        %   Values of strain tensor

        e_y1(ti+1,t+1) = (e_z + e_y)./2 + ((e_y - e_z)./2).*cos(2.*t.*pi./180) - (e_yz./2).*sin(2.*t.*pi./180);
        e_y1z1(ti+1,t+1) = ((e_y - e_z)./2).*sin(2.*t.*pi./180) + (e_yz./2).*cos(2.*t.*pi./180);
    end
end

%   Calculation of the fatigue parameter, FP
for i=1:181
    r1=max(tau_y1z1(:,i))-min(tau_y1z1(:,i));
    r2=max(e_y1(:,i))-min(e_y1(:,i));
    r3=max(e_y1z1(:,i))-min(e_y1z1(:,i));
    r4=0.5*(abs(max(sigma_y1(:,i)))+max(sigma_y1(:,i)));

    FP(i)=r4.*(r2./2)+0.2.*r1.*r3;
    FDP=permute(FP,[2,1]);
    FP_max=max(FP);
end

```

APPENDIX C

MATLAB algorithm for 3D Critical Plane Search

```

clear all;
clc;

% time variable
for ti = 0:10

% Stress amplitudes
amp_x = -772;
amp_y = -621;
amp_z = -626;
amp_xy = 318;
amp_xz = -4.6;
amp_yz = 6;

% Strain amplitudes
amp_ex = -0.0032;
amp_ey = -0.0039;
amp_ez = -0.00088;
amp_exy = 0.0116;
amp_exz = -0.0001;
amp_eyz = 0.00016;

% variation of stresses with time at x,y,z coordinate
format short
sigma_x = amp_x*sin(0.025*pi*ti);
sigma_y = amp_y*sin(0.025*pi*ti);
sigma_z = amp_z*sin(0.025*pi*ti);
tau_xy = amp_xy*sin(0.025*pi*ti);
tau_xz = amp_xz*cos(0.025*pi*ti);
tau_yz = amp_yz*cos(0.025*pi*ti);

% variation of strains with time at x,y,z coordinate

format short
e_x = amp_ex*sin(0.025*pi*ti);
e_y = amp_ey*sin(0.025*pi*ti);
e_z = amp_ez*sin(0.025*pi*ti);
e_xy = amp_exy*sin(0.025*pi*ti);
e_xz = amp_exz*cos(0.025*pi*ti);
e_yz = amp_eyz*cos(0.025*pi*ti);

for t=0:180;
    for s=0:180;

L1 = cos(s*pi/180)*sin(t*pi/180);
m1 = sin(s*pi/180)*cos(t*pi/180);
n1 = cos(t*pi/180);

L2 = -sin(s*pi/180);
m2 = cos(s*pi/180);
n2 = 0;

L3 = -cos(s)*cos(t*pi/180);
m3 = -sin(s)*cos(t*pi/180);
n3 = sin(t*pi/180);

% value of the stress vector at time t
sigma_xl(ti+1,t+1,s+1) = sigma_x.*(L1.^2) + sigma_y.*(m1.^2) +sigma_z.*(n1.^2)+(2*tau_xy.*L1.*m1) +
(2*tau_yz.*m1.*n1) + (2*tau_xz.*n1.*L1);

```

```

Tau_xly1(ti+1,t+1,s+1) = sigma_x.*(L1.*L2) + sigma_y.*(m1.*m2) + sigma_z.*(n1.*n2) + tau_xy.*(L1.*m2 +
L2.*m1) + tau_xz.*((m1.*n2)+( n1.*m2)) + tau_xz.*((n1.*L2)+ (n2.*L1));
Tau_xlz1(ti+1,t+1,s+1) = sigma_x.*(L1.*L3) + sigma_y.*(m1.*m3) + sigma_z.*(n1.*n3) + tau_xy.*(L1.*m3 +
L3.*m1) + tau_xz.*((m1.*n3)+( n1.*m3)) + tau_xz.*((n1.*L3)+ (n3.*L1));

% Stress Range Calculations
% 2D matrix
z=permute(sigma_x1,[3,2,1]);
s1=sigma_x1(:,:);
s2=Tau_xly1(:,:);
s3=Tau_xlz1(:,:);

r_xly1 = max(Tau_xly1(:)) - min(Tau_xly1(:));
r_xlz1 = max(Tau_xlz1(:)) - min(Tau_xlz1(:));
R = ((r_xly1.^2)+(r_xlz1.^2)).^0.5;

% Maximum Normal stress

Smax = 0.5*(abs(max(sigma_x1(:)))+max(sigma_x1(:)));

% values of strain tensor

e_x1(ti+1,t+1,s+1) = e_x.*(L1.^2) + e_y.*(m1.^2) +e_z.*(n1.^2)+(2*e_xy.*L1.*m1) + (2*e_yz.*m1.*n1) +
(2*e_xz.*n1.*L1);
e_xly1(ti+1,t+1,s+1) = e_x.*(L1.*L2) + e_y.*(m1.*m2) + e_z.*(n1.*n2) + e_xy.*(L1.*m2 + L2.*m1) +
e_xz.*((m1.*n2)+( n1.*m2)) + e_xz.*((n1.*L2)+ (n2.*L1));
e_xlz1(ti+1,t+1,s+1) = e_x.*(L1.*L3) + e_y.*(m1.*m3) + e_z.*(n1.*n3) + e_xy.*(L1.*m3 + L3.*m1) +
e_xz.*((m1.*n3)+( n1.*m3)) + e_xz.*((n1.*L3)+ (n3.*L1));
% re_xly1(ti+1,t+1,s+1) = max(e_xly1(:)) - min(e_xly1(:));

% Strain Range calculation
s4=e_x1(:,:);
s5=e_xly1(:,:);
s6=e_xlz1(:,:);

% CALCULATION OF THE FATIGUE PARAMETER, FP

end
end
end
for i=1:8281
    d1(i)=max(s1(:,i))-min (s1(:,i));
    d2(i)=max(s2(:,i))-min (s2(:,i));
    d3(i)=max(s3(:,i))-min (s3(:,i));
    d4(i)=max(s4(:,i))-min (s4(:,i));
    d5(i)=max(s5(:,i))-min (s5(:,i));
    d6(i)=max(s6(:,i))-min (s6(:,i));
    d11(i)=0.5*(abs(d1(i))+d1(i));
    d23(i)=((d2(i).^2)+(d3(i).^2)).^0.5;
    d56(i)=((d5(i).^2)+(d6(i).^2)).^0.5;
    FP(i)=(d11(i).*d4(i)./2)+(0.2.*d23(i).*d56(i));
    FP_max=max(FP);

end

```