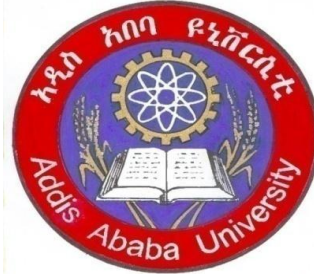


ADDIS ABABA UNIVERSITY
ADDIS ABABA INSTITUTE OF TECHNOLOGY
SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING



**Comparative Study on Fundamental Period of Reinforced
Concrete Buildings**

A thesis submitted to the school of Graduate Studies in Partial fulfillment of
the Requirements for the Degree of Master of Science in Civil Engineering
(Structures)

By

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This is to certify that the thesis prepared by Utino Worabo, entitled: *Comparative Study on Fundamental Period of Reinforced Concrete Buildings*: submitted in partial fulfillment of the requirements for the Degree of Master of Science in Civil Engineering (Structures) complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

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ABSTRACT

Fundamental natural period of vibration T of the building is an important parameter for evaluation of seismic base shear. Empirical equations given in current building codes like ESEN 1998-1:2015, EC8 (EN 1998-1: 2004), Applied Technological Council of 1978 (ATC3-06, 1978), American Society of Civil Engineers (ASCE 7: 2010), Indian seismic code (IS 1893:2002) and other current building codes for the calculation of the fundamental period of a framed structure, primarily as a function of height.

The intent of this study is to understand the dynamic behaviors of buildings with different configurations under seismic loads. It is a well-known fact that the first fundamental period is directly affected by not only the framing type but also the number of floors and the in-plane aspect ratio, regularity, story stiffness, story height and building masses. Therefore, the number of floors and the in-plane aspect ratio, regularity, story stiffness, and story height are used in the parametric studies by selecting 3D actual buildings. For this purpose, eleven sets of floor levels, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13 and 14 stories or 9m-45m height buildings were studied in evaluating the fundamental period.

In order to study the influence of other parameters such as number and width of bays, height and number of story, irregularity and story stiffness were investigated in this study. To investigate these parameters, 39 actual reinforced concrete buildings were modeled and analyzed by Modal analysis and Rayleigh's method using computer software ETABS 2015. Periods of building frames were tabulated and plotted against the pre-listed parameters and finally conclusions were drawn from the results obtained from analysis.

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LIST OF NOTATIONS

A_B = Area of base of structure, m^2 .

A_c = Total effective area of the shear walls in the first storey of the building in m^2

A_i = Area of shear wall "i" in m^2 .

C_t = Constants related to building period.

D = Length of the lateral load - resisting system, in m.

D_i = Length of shear wall "i" in m.

D = Lateral displacement of the top of the building in meters due to the gravity loads applied in the horizontal direction.

E_c = Modulus of elasticity of concrete

e_o = Structural eccentricity

F_i = Lateral applied force at the i^{th} story

G = Shear modulus

g = Acceleration due to gravity

H = Building height in m

h_i = Height of shear wall "i" in m

I = Section moment of inertia

k = Shape factor to account for nonuniform distribution of shear stresses

k_j = Stiffness of j^{th} story

l_s = Radius of gyration of the floor mass in plan

m = Effective seismic mass

MDOF = Multi Degree of Freedom

MMPRFM = Modal Mass Participation Ratio of Fundamental Mode

MRF = Moment-Resisting Frame

R = Torsional radius

RC = Reinforced concrete

$R_{z,i}$ ($F_{Y,i} = 1$) = Rotation of the storey i due to static load $F_{Y,i} = 1$ in Y direction

$R_{z,i}$ ($F_{X,i} = 1$) = Rotation due to load $F_{X,i} = 1$ in X direction,

$R_{z,i}$ ($M = 1$) = Rotation due to torsional moment about the vertical axis

$R_{z,i}$ ($M_{T,i} = 1$) = Rotation of the storey i about the vertical axis due to unit moment

SDOF = Single Degree of Freedom

T = vibration period of building along the direction of motion

T_a = Approximate fundamental period

T_n = Natural period of vibration

TX = Translation mode of vibration along X direction

TY = Translation mode of vibration along Y direction

t_w = Thickness of shear wall

$U_{X,i}(F_{TX,i} = 1)$ = Displacement at storey level i in direction X due to unit force F_{TX}

$U_{Y,i}(F_{TY,i} = 1)$ = Displacement in direction Y due to unit force F_{TY}

ν = Poisson's ratio

W_i = Story weight or self-weight method of the story

X_{CR} and Y_{CR} = Center of stiffness in X and Y direction respectively

X_{CM} and Y_{CM} = Center of mass in X and Y direction respectively

z_o = Amplitude of the generalized coordinate $z(t)$

%ge d/c = Percentage difference

α , β and γ = Regression coefficients

Δ = Lateral story displacement

δ_i = Lateral displacement at the i^{th} story

ϑ = Interstorey drift

λ = Slenderness of building

Γ_1 = the 1st mode participation factor for the irregular building frame under consideration

Γ_n = Mode – participation factor for the n^{th} mode

Γ_{ref} = the 1st mode participation factor for the similar regular building frames

η = Regularity index

ψ = Shape vector that defines the form of deflection

ω_n = Natural vibration frequency

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Chapter One: Introduction

1.1 Background of the Study

The fundamental period of vibration required for the simplified design of reinforced concrete structures has been calculated for many years using a simplified formula relating the period to the height of the building. One of the first formulae of this type was presented almost 30 years ago in ATC3-06 [ATC, 1978] and had the form: $T = C_t H^{3/4}$ Where, C_t was a regression coefficient and H represented the height of the building.

As discussed in Goel and Chopra [1997], the particular form of above equation was theoretically derived by assuming that the equivalent static lateral forces are linearly distributed over the height of the building and the distribution of stiffness with height produces a uniform storey drift under the linearly distributed lateral forces. Furthermore in ATC3-06 [ATC, 1978] the base shear force was assumed to be inversely proportional to $T^{2/3}$.

From the investigation of Goel and Chopra, the period formula in the Uniform Building Code (UBC-1997) and the Structural Engineering Association of California (SEAOC 1996) recommendations were derived from those developed in 1975 as part of ATC-06 project [ATC- 1978] largely based on periods of building “measured” from their motions recorded during the 1971 San Fernando earthquake; however, motions of many more buildings recorded during recent earthquakes, including the 1989 Loma Prieta and 1994 Northridge earthquakes, provided an opportunity to expand greatly the existing database on the fundamental vibration periods of real

buildings were measured by system identification methods applied to the motions of buildings recorded during 1994 Northridge earthquake. These data were combined with similar data from the motions of buildings recorded during the 1971 San Fernando, 1984 Morgan Hill, 1986 Mt. Lewis and Palm Spring, 1987 Whittier, 1989 Loma Prieta, 1990 Upland, and 1991 Seirra Madre earthquakes reported by several investigators.

As Gilles and McClure research, shows the period formulae in the National Building Code of Canada (NBCC 2005) are based on the database comprised of structures in California whose fundamental periods were measured during several earthquakes, from the 1971 San Fernando to the 1994 Northridge earthquakes. However, there was large variability in the data, and the equation fits the data rather poorly. Further, the fundamental period of a structure tends to increase with increasing excitation amplitude (Udwadia and Trifunac 1974); therefore, since each of the earthquakes produced ground motions of significantly different amplitudes at each location, it seems that this data set is somewhat inconsistent. Moreover, Goel and Chopra (1997; 1998) warned that caution should be exercised before using these data in other parts of the world; the reason being that different soil conditions, seismicity, and design and construction practices may affect the measured fundamental periods.

Table 1.1: Code based formulae for fundamental period and their commentaries

SN	Formulae of Fundamental Period	Codes and Commentary
1	$T_1 = C_t H^{3/4}$, in seconds	ESEN 1998-1:2015, EC8, 2001 and IS1893:2000 -for buildings with heights up to 40 m. $C_t = 0.075$ Alternatively, $C_t = 0.075/\sqrt{A_c}$, for structures with concrete or masonry shear wall $A_c = \sum [A_i (0.2 + (L_{wi}/H))^2]$, $L_{wi}/H \leq 0.9$ H =height of building, in m. A_c - total effective area of the shear walls in the first storey of the building, in m ² , A_i - effective cross-sectional area of the shear wall i in the first storey of the building, in m ² . L_{wi} -length of the shear wall i in the first storey in the direction parallel to the applied forces, in m, Other codes like ATC-06, ASCE7:2010, UBC-97, IBC, and USA: 2000 use this formula with varying the value of the regression coefficient C_t .
2	$T_1 = 2 \cdot \sqrt{d}$, in seconds	ESEN 1998-1:2015, EC8, 2001 d , is lateral displacement of the top of the building, in m, due to the gravity loads applied in the horizontal direction.
3	$T_a = \frac{C_t H}{\sqrt{a}}$, in seconds	IS1893:2000, ATC-06 For all other building, including moment-resisting frame buildings with brick infill panels. d - is the dimension of building at its base in the direction under consideration, in m in case of IS1893:2000 and in ft. in case of ATC-06. $C_t = 0.09$ in IS1893:2000 and H , is the height of building, in m. $=0.05$ in ATC-06 and H , is the height of building, in ft.
4	$T = 2\pi \sqrt{\frac{\sum_{i=1}^n W_i \delta_i^2}{g \sum_{i=1}^n W_i \delta_i}}$, in seconds	Rayleigh Equation Where, W_i is the story weight, 'self-weight method of the story' $F_i = W_i$ the lateral applied force at the i^{th} story and are linearly distributed over the height of the building. δ_i -the corresponding lateral displacement. g is acceleration due to gravity
5	$T_a = 0.1N$, in seconds	ASCE 7:2010, IBC The structures not exceeding 12 stories. The storey height to be at least 3 m. N is the number of stories.
6	$T = H(0.02 + 0.01a)$, in seconds	BSLJ, Japan code H is height of building, in m a -the ratio of the total height of steel construction to the height of building $a = 0.0$ for concrete $= 1.0$ for steel

1.2 Statement of the Problem

Fundamental natural period of vibration T of the building is an important parameter for evaluation of seismic base shear. Empirical equations given in Table 1.1 for the calculation of the fundamental period of a framed structure, primarily as a function of height, that does not consider the effect of stiffness of the structure, base dimensions of the structure, strength of material from which a building is being constructed, regularity of structure in both plan and elevation, system of structure, amount of infill and properties of the infill in a single equation.

The most widely used expression of the fundamental period of vibration had the form $T = C_t H^{3/4}$, where C_t is coefficient that has taken in to account the type of the structure as RC moment-resisting frame, steel moment-resisting frame and other buildings, but it does not consider the strength of the materials from which the structure is being built upon. This formulation was theoretically derived using the Rayleigh's method considering the horizontal forces linearly distributed along the height, the mass distribution constant along the height, the linearity of deformed shape and the base shear proportional to $1/T^{2/3}$. Because the assumptions, horizontal forces linearly distributed along the height, the mass distribution constant along the height, the linearity of deformed shape and the base shear may not be valid for all structures.

1.3 Objectives of the Study

1.3.1 General objective of the Study

The main objective of this study is to evaluate the empirical equations provided in current building codes for the estimation of fundamental vibration period of RC buildings.

1.3.2 Specific objectives of the Study

- ✓ To evaluate empirical formulae that estimate fundamental period relative to Modal analysis.
- ✓ To identify the structural parameters that affect the fundamental period of vibration.
- ✓ To show the effect of irregularity of RC framed buildings on fundamental period.

1.4 Significance of the study

Apart from the previous studies the present study may be useful to understand the effect of structural parameters, other than building height or lateral dimension, on fundamental period; to reduce errors while estimating the earthquake forces, that arise from using the approximate formulae suggested in current building codes, which has a great role in determining structural safety and economy in the design life; and it may encourage other researchers to do further study in this particular area.

1.5 Scope of the Study

The scope of this study is to evaluate the accuracy of the period equations provided in the EC8-2004, UBC-94, ESEN 1998-1:2015, ASCE 7:2010, ATC-06, IS1893:2000 and BSLJ codes with respect to the results obtained from the actual reinforced concrete buildings via Modal Analysis or Rayleigh's method. The study is delimited to reinforced concrete multi-storied building frames with total height up to 45m. However, the associated mass and weight is considered from their architectural drawings in the analysis the stiffness of infills is not considered in the present study.

1.6 Limitation of the Study

All buildings modeled in ETABS are assumed to have fixed supports, in which the actual soil-structure interaction is ignored in this study. Also, the effect of basement floors is neglected intentionally in evaluating the fundamental period. The effect of mechanical appendages was not considered. In addition to these omissions, the buildings were built in different areas but all are assumed to be located in the same region, in which the effective ground acceleration is neglected.

1.7 Thesis Organization

Chapter-1 contains background of the study, statement of problem, objective of the study, and scope and limitation of the study. Chapter-2 contains description of the development of code formulae, description of the previous works done on the effects of structural parameters on fundamental period of RC moment resisting building frames by other researchers, and structural configurations and systems for effective earthquake resistance. Chapter-3 describes the methodology employed in this study. It starts with providing an in-depth clarification of the structural modeling, regularity checking as well as methods of structural analysis. A brief description on the study area and source of data is presented. Moreover, a brief outline on the modal analysis technique is presented for better understanding of the results. Chapter-4 begins with a presentation of the variation of fundamental time period with the variation in several parameters of the building. This chapter also explains the effect of each parameter on fundamental period of the building by comparing the differences between the code based period with that of the periods found from modal analysis using ETABS 2015 and Rayleigh's method. And chapter-5 presents significant conclusions based on the findings and the recommendations for the future researches in the area.

Chapter Two: Literature Review

2.1 Introduction

In this chapter, development of commonly used building code formulae, previous studies on fundamental period, and reviews on configurations and systems for effective earthquake resistance are summarized and comments on previous studies are drawn.

The fundamental period of vibration of a building is one of the parameters that have a significant influence on the earthquake induced lateral forces. The period of the structure is the time that is required to complete one cycle. The value of the fundamental period needs to be as accurate as possible in earthquake resistant designs, as lower the value of time period, higher will be the base shear and vice versa, with a special emphasis on designs which are based on either linear static (or lateral force) methods.

The empirical formulae for fundamental period of vibration in all codes provision is used in equivalent static analysis, (ESA), which may be used for any structure of light construction and for all regular structures, with a calculated structural period T . An equivalent static analysis allows the designer to perform a preliminary seismic design based on simplified formulas used to determine the fundamental period, building base shear, and story shear distribution applied statically. ESA consists of a simple approximation to model response spectrum analysis. It only considers the first mode of a structure's lateral response, and presumes that the mode shape for this first mode of response is represented by that of a simple shear beam. For structures having sufficiently low periods of first mode response and regular vertical and horizontal

distribution of stiffness and mass, this procedure approximates modal response spectrum analysis well. However, for longer period structures, higher mode response becomes significant and neglecting these higher modes results significant errors in the estimation of structural response. Also, as the distribution of mass and stiffness in a structure becomes irregular, for example, the presence of torsional conditions or soft story conditions, the assumptions inherent in the procedure with regard to mode shape also become quite approximate, leading to errors, [15]

The ESA approach is supposed to be equivalent to a dynamic analysis as long as the structure meets the following requirements [1], [3], [6] and [7].

- a) The fundamental periods of vibration T_1 in the two main directions less than the following values

$$T_1 \leq \begin{cases} 4T_C \\ \text{or} \\ 2\text{sec} \end{cases} \dots\dots\dots (*)$$

Where, T_C is the period at the end of the constant acceleration part of the design response spectrum.

- b) They meet the criteria of regularity in both plan and elevation.

2.2 Development of Code Formulae

2.2.1 Period-Height Relationships in Seismic Design Codes for Moment

Resisting Frames

The fundamental period of vibration required for the simplified design of reinforced concrete structures has been calculated for many years using a simplified formula

relating the period to the height of the building. One of the first formulae of this type was presented almost 30 years ago in ATC3-06 [ATC, 1978] and had the form [12]:

$$T=C_t H^{3/4} \quad (2.1)$$

where C_t was a regression coefficient and H represented the height of the building in feet.

As discussed in Goel and Chopra [1997], the particular form of Eq. (2.1) was theoretically derived by assuming that the equivalent static lateral forces are linearly distributed over the height of the building and the distribution of stiffness with height produces a uniform storey drift under the linearly distributed lateral forces. Furthermore in ATC3-06 [ATC, 1978] the base shear force was assumed to be inversely proportional to $T^{2/3}$ and thus these two assumptions led to Eq. (2.1), as shown in the workings below. The period of vibration (T) of a single degree of freedom oscillator can be obtained from Eq. (2.2) where m is the mass of the oscillator and k is the stiffness:

$$T = 2\pi \sqrt{\frac{m}{K}} \quad (2.2)$$

The stiffness of the oscillator can be obtained from the base shear (V) divided by the lateral displacement (Δ). From the response spectrum in early design codes, the base shear for the usual range of periods of structures was taken as inversely proportional to the period to the power of two-thirds, with the coefficient of proportionality defined as C_l herein. As mentioned previously, if one assumes that the distribution of stiffness with height produces a uniform storey drift under the linearly distributed lateral forces, then the lateral displacement, Δ , is given by the interstorey drift, ϑ , multiplied by the height, H .

$K = \frac{V}{\Delta}$, but $V \propto \frac{1}{T^{2/3}}$ and the proportionality constant is C_1 . Therefore $V = \frac{C_1}{T^{2/3}}$ and

the stiffness becomes:

$$K = \frac{C_1}{T^{2/3} \vartheta H} \quad (2.3)$$

By replacing Eq. (2.3) in Eq. (2.2), the relationship shown in Eq. (2.1) between period and height can be obtained, as outlined in the workings of Eq. (2.4) to (2.5):

$$T = 2\pi \sqrt{\frac{mT^{2/3}H\vartheta}{C_1}} \quad (2.4)$$

Squaring both sides of Eq. (2.4) gives as:

$$T^2 = (2\pi)^2 \frac{mT^{2/3}H\vartheta}{C_1} \quad (2.5)$$

Re-arranging Equation (2.5) gives as:

$$\frac{T^2}{T^{2/3}} = 4\pi^2 \frac{mH\vartheta}{C_1} \quad (2.5a)$$

$$T^{2-2/3} = 4\pi^2 \frac{mH\vartheta}{C_1} \quad (2.5b)$$

$$T^{4/3} = 4\pi^2 \frac{m\vartheta}{C_1} H \quad (2.5c)$$

$$T = \left[4\pi^2 \frac{m\vartheta}{C_1} \right]^{3/4} H^{3/4} \quad (2.5d)$$

Let, $C_t = \left[4\pi^2 \frac{m\vartheta}{C_1} \right]^{3/4}$ and this leads to Eq. (2.1) as, $T = C_t H^{3/4}$

In ATC3-06 [ATC, 1978], the coefficient C_t in Eq. (2.1) was given equal to 0.025 for reinforced concrete moment resisting frames. This coefficient was identified from a study by Gates and Foth [1978] based on the measured periods of vibration of

reinforced concrete frames during the 1971 San Fernando earthquake. A subsequent re-evaluation by SEAOC-88 [SEAOC, 1988] found that a value of $C_t=0.03$ was more appropriate for reinforced concrete buildings. The coefficient C_t was generally calibrated such that the derived fundamental period would underestimate the period by approximately 10-20% at first yield to obtain a conservative estimate for the base shear [10].

Bertero *et al.* [1988] studied in greater detail the fourteen buildings considered by Gates and Foth [1978] and found that four of the buildings and the longitudinal direction of a fifth building could not be considered as moment-resisting frame (MRF) structures and they thus excluded them from the database. They further identified two buildings with structural damage and two others that had non-structural damage; they discuss how damage leads to stiffness degradation and thus an increase in the period of vibration. Gates and Foth [1978] did not relate the building damage to the period of vibration and thus Bertero *et al.* [1988] reevaluated the time histories of building response for the moment-resisting frame structures and identified times where a sudden increase in the period of vibration took place, which was then correlated to the onset of non-structural and structural damage. The period of vibration at the second increase in period was considered to be the stage when the non-structural components were no longer contributing significantly to the stiffness and interpreted as the period at which the building was essentially vibrating as a bare structural frame. They added a further four buildings to the database and evaluated the bare frame period of vibration for all buildings in a uniform manner at the aforementioned second increase in period. The conclusions of their study was that the formula of Eq. (2.1) with C_t equal to 0.03 does not constitute a reliable estimate of earthquake period of reinforced

concrete moment-resisting frames, and a better fit was found with $C_t = 0.04$ (0.097 with H in meters). For a lower bound estimate of the period, Bertero *et al.* [1988] recommend the use of $C_t = 0.035$ (0.085 with H in meters).

In the buildings used in the Bertero *et al.* [1988] study, the partition walls were generally plaster board/dry walls whilst the outer infill was often constructed with glass curtain walls with spandrels built into the outer frames. Bendimerad *et al.* [1991] found that the participation of these non-structural components on the stiffness of the building was minimal compared to the stiffness of the frame and thus had practically no effect on the building period beyond the first 5 seconds of earthquake motion. Hence, the use of the equation derived from these buildings for the design of “moment resisting frame systems of reinforced concrete in which the frames resist 100 percent of the required seismic force” appears to be justified.

The use of the form of period-height equation shown in Eq. (2.1), along with the SEAOC-88 recommended 0.03 coefficient, has been adopted in many design codes since 1978, for example in UBC-97 [UBC, 1997], in SEAOC-96 [SEAOC, 1996], in NEHRP-94 [FEMA, 1994] and in Eurocode 8 [CEN 1994; 2004]. In Eurocode 8, the C_t coefficient has simply been transformed considering that the height is measured in meters, leading to $C_t = 0.075$. In a similar way that the uses of a 475 year return period to define the seismic actions was adopted in seismic design codes around the world. The period-height equation of Eq. (1) with $C_t = 0.075$ has also spread around the world. Goel and Chopra [1997] used the fundamental period of vibration of buildings measured from their motions recorded during eight California earthquakes from 1971 to 1994 to update the period-height formula in UBC-97. They found that

the best fit lower bound curve (i.e. the mean-1standard deviation) for reinforced concrete frames was given by:

$$T = 0.0466H^{0.9} \quad (2.6)$$

(H in meters)

This period-height formula has recently been included in ASCE 7-05 [2006]. As can be noted from Eq. (7), Goel and Chopra [1997] decided to move away from the 0.75 power regression and found the best-fit regression to be 0.9. Simplified period-height equations can only be applied in Eurocode 8 for buildings up to 40 meters, and thus the period of vibration obtained with such equations will generally be within the range of inverse proportionality between base shear and period (see Figure 2.1).

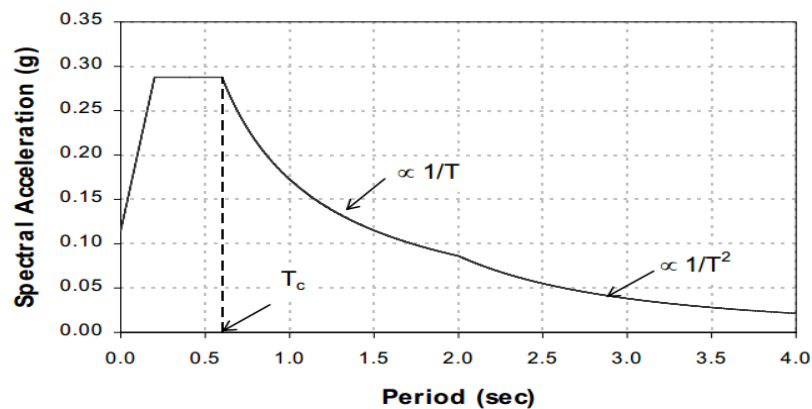


Figure 2.1: Acceleration response spectrum in Eurocode 8 for a peak ground acceleration of 0.1g and site condition

By repeating the calculations in Eq. (2.2) to (2.5) with the base shear force inversely proportional to period, the form of the period-height equation becomes linear: **BSLJ, Japan code.**

$V \propto \frac{1}{T}$ and the proportionality constant is C_1 . Therefore $V = \frac{C_1}{T}$ and the stiffness becomes:

$$K = \frac{V}{\Delta}, \text{ and } K = \frac{C_1}{T \cdot \vartheta H}$$

$$T = 2\pi \sqrt{\frac{mTH\vartheta}{C_1}} \quad (2.7)$$

Squaring both sides of equation (2.7) gives as:

$$T^2 = (2\pi)^2 \frac{mTH\vartheta}{C_1} \quad (2.8)$$

Re-arranging equation (8) gives as:

$$\frac{T^2}{T} = 4\pi^2 \frac{mH\vartheta}{C_1} \quad (2.8a)$$

$$T^{2-1} = 4\pi^2 \frac{mH\vartheta}{C_1} \quad (2.8b)$$

$T = 4\pi^2 \frac{m\vartheta}{C_1} H$ let $C_t = [4\pi^2 \frac{m\vartheta}{C_1}]$ and finally the equation becomes:

$$T = C_t H \rightarrow 0.02H \quad \text{as indicated in Japan code} \quad (2.9)$$

2.2.2 Period-Height Relationships in Seismic Design Codes for Structures with Reinforced Concrete or Masonry Walls

The first period-height relationship for buildings with concrete shear walls had the form presented in Eq. (2.10) and, as with the previous equations, was also calibrated using the measured motions of buildings recorded during the 1971 San Fernando earthquake:

$$T = \frac{C_t H}{\sqrt{D}} \quad (2.10)$$

where D is the dimension of the building at its base in the direction under consideration. With the height and dimension D of the building measured in feet, C_t was proposed in ATC3-06 [ATC, 1978] as 0.05 (which would be 0.09 with these

dimensions measured in meters). This formula comes from the equation of the frequency of vibration of a cantilever (considering shear deformations only), with the thickness of the wall considered to be more or less constant and thus only the width/length of the building is an input parameter, as demonstrated in Eq. (2.11):

$$T = 4 \sqrt{\frac{m}{kG}} \sqrt{\frac{H}{A}} = \frac{aH}{\sqrt{A}} = \frac{aH}{\sqrt{D} t_w} = \frac{a_1 H}{\sqrt{D}} = \frac{C_t H}{\sqrt{D}} \quad (2.11)$$

Where, m is the mass per unit length, G is the shear modulus, κ is the shape factor to account for nonuniform distribution of shear stresses, D is the length of the cantilever and t_w is the thickness. This formula is used in many design codes around the world, but the type of structure to which it is applied varies from code to code; it is noted that the text used in each code to describe the structures to which Eq. (2.10) applies has been maintained. Some codes use this formula specifically for buildings with both frames and shear walls, some use the equation for reinforced concrete MRF with masonry infill panels, but many specify it for use with any building except moment resisting space frames.

Based on Dunkerley's method, the fundamental period of a cantilever, considering flexural and shear deformations [11] is

$$T = \sqrt{T_F^2 + T_S^2} \quad (2.12)$$

in which T_F and T_S are the fundamental periods of pure-flexural and pure-shear cantilevers, respectively. For uniform cantilevers T_F and T_S are given by Chopra, [4]

$$T_F = (2\pi/3.516) \sqrt{\frac{m}{EI}} H^2 \quad (2.13)$$

From 2.11 selected concrete shear walls buildings and the fundamental period of buildings "measured" from their motions recorded during eight California earthquakes, starting with the 1971 San Fernando earthquake and ending with the 1994 Northridge earthquake Goel and Chopra in 1998 arrived at,

$$T_S = 4 \sqrt{\frac{m}{kG}} \frac{1}{\sqrt{A}} H \quad (2.14)$$

In (2.13) and (2.14), m = mass per unit height; E = the modulus of elasticity; G = shear modulus; I = the section moment of inertia; A = section area; and k is the shape factor to account for nonuniform distribution of shear stresses (=5/6 for rectangular sections). Combining (2.7) - (2.9) and recognizing that $G = E / 2(1 + 1\nu)$, where the Poisson's ratio $\nu = 0.2$ for concrete, leads to

$$T = 4 \sqrt{\frac{m}{kG}} \frac{1}{\sqrt{A_c}} H \quad (2.15)$$

With,

$$A_c = \frac{A}{[1 + 0.83(\frac{H}{D})^2]} \quad (2.16)$$

where D is the plan dimension of the cantilever in the direction under consideration. Comparing (2.10) and (2.11) with (2.14) reveals that the fundamental period of a cantilever considering flexural and shear deformations may be computed by replacing the area A in (2.14) with the equivalent shear area A_c , given by (2.16). The period T from (2.15) normalized by T_F is plotted in figure 2.2 against the ratio H/D on a logarithmic scale.

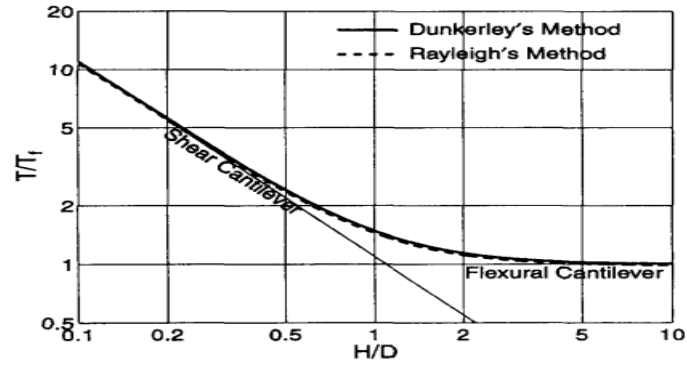


Figure 2.2: Fundamental period of cantilever beam [Chopra, 1998]

Also shown is the period of a pure-shear cantilever and of a pure-flexural cantilever. Eq. (2.10) approaches the period of a pure-shear cantilever (2.14) as H/D becomes small and the period of a pure-flexural cantilever (2.13) for large values of H/D . For all practical purposes, the contribution of flexure can be neglected for shear walls with $H/D < 0.2$, whereas the contribution of shear can be neglected for shear walls with $H/D > 5$; the resulting error is less than 2%. However, both shear and flexural deformations should be included for shear walls with $0.2 \leq H/D \leq 5$. Goel and Chopra in 1997 considered a class of symmetric-plan buildings symmetric in the lateral direction considered with a lateral-force resisting system comprised of a number of uncoupled (i.e., without coupling beams) shear walls connected through rigid floor diaphragms. Assuming that the stiffness properties of each wall are uniform over its height, the equivalent shear area, A_c , is given by a generalized equation of (2.17),

$$A_c = \sum_{i=1}^{NW} \left(\frac{H}{H_i}\right)^2 \frac{A_i}{[1+0.83\left(\frac{H_i}{D_i}\right)^2]} \quad (2.17)$$

where A_i , H_i and D_i are the area, height, and dimension in the direction under consideration of the i^{th} shear wall; and NW = the number of shear walls. With also defined, (2.10) is valid for a system of shear walls of different height. Eq. (2.10) is now expressed in a form convenient for buildings

$$T = 40 \sqrt{\frac{m}{kG}} \frac{1}{\sqrt{\tilde{A}_c}} H \quad (2.18)$$

where ρ = average mass density, defined as the total building mass ($=mH$) divided by the total building volume ($=A_B H$, A_B is the building plan area), i.e., $\rho = m/A_B$; and \tilde{A}_c = the equivalent shear area expressed as a percentage of A_B , i.e.,

$$\tilde{A}_c = 100 \frac{A_c}{A_B} = A_e \quad (2.19)$$

$$T = \frac{C_1}{\sqrt{A_e}} H^\beta \quad (2.20)$$

In Eqn. (2.20) Goel and Chopra proposed $C_1 = 0.0063$ and $\beta = 3/4$, for structures with concrete or masonry shear wall.

$$A_c = \sum_{i=1}^{NW} [A_i (0.2 + \left(\frac{L_{wi}}{H}\right)^2)], \quad \text{with} \quad L_{wi}/H \leq 0.9$$

H = the height of the building, in m, above the base;

A_c - total effective area of the shear walls in the first story of the building, in m^2 ,

A_i -effective cross-sectional area of the shear wall i in the first story of the building, in m^2 .

L_{wi} -length of the shear wall i in the 1st story in the direction parallel to the applied forces, in m.

The Building Code like UBC-97, ASCE 7:2010, and IBC, specified that the period of a multi-story framed building can be estimated by dividing the total number of stories by ten in which the structures not exceeding 12 stories and the story height to be at least 3m.

$$T=0.1N \quad (2.21)$$

where N is the number of stories.

As specified in EC8-2004, and EBCS-8 EN 1998-1:2013, the estimation of T_1 , in seconds, may be calculated by using the following equation, which is developed from Rayleigh's method.

$$T_1 = 2\sqrt{d} \quad (2.22)$$

where, d is the lateral displacement of the top of the building, in m, due to the gravity loads applied in the horizontal direction [4] and [5].

2.2.3 Natural Vibration Period by Rayleigh's Method

Based on the principle of conservation of energy, Rayleigh's method was published in 1873. The application of this method for the system with lumped masses may illustrate as follows [4]:

As an illustration of such a system, consider the shear building as shown in figure below vibrating freely in simple harmonic motion,

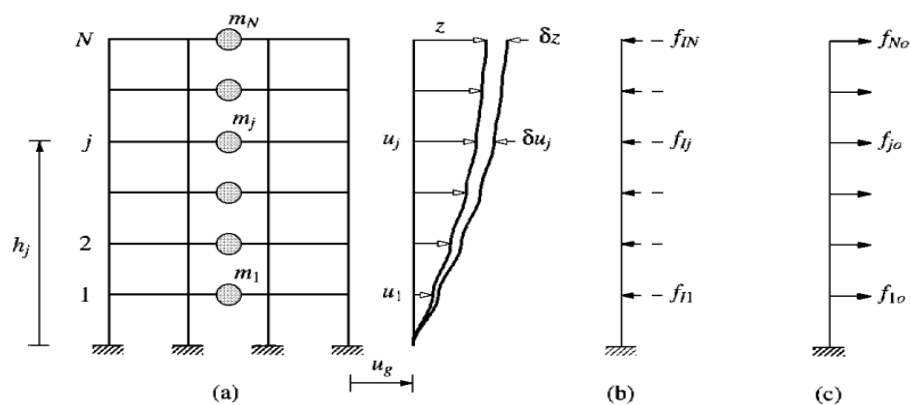


Figure 2.3: (a) Building displacements and virtual displacements; (b) Inertia forces; (c) Equivalent static forces.

$$u(t) = z_o \sin \omega_n t^{\psi} \quad (2.23)$$

Where the vector ψ is an assumed shape vector that defines the form of deflection, z_o is the amplitude of the generalized coordinate $z(t)$, and the natural vibration frequency ω_n is to be determined to find the natural vibration period. The velocity of lumped masses of the system are given by the vector

$$\dot{u}(t) = \omega_n z_o \cos \omega_n t' \psi \quad (2.24)$$

The maximum potential energy of the system over a vibration cycle is equal to its strain energy associated with the maximum displacements, $u_o = (u_{10} \ u_{20} \ \dots \ u_{No})$:

$$E_{so} = \sum_{j=1}^N \frac{1}{2} k_j (u_j - u_{j-1})^2 \quad (2.25)$$

The maximum kinetic energy of the system over a vibration cycle is associated with the maximum velocity \dot{u}_{jo} :

$$E_{ko} = \sum_{j=1}^N \frac{1}{2} m_j \dot{u}_{jo}^2 \quad (2.26)$$

From Eqs. (2.23) and (2.24), $u_{jo} = z_o \psi_j$ and $\dot{u}_{jo} = \omega_n z_o \dot{\psi}_j$ by setting $\sin \omega_n t'$ and $\cos \omega_n t'$ as one respectively. Substituting these in Eqs (2.25) and (2.26) and equating E_{so} to E_{ko} gives:

$$\omega_n^2 = \frac{\sum_{j=1}^N K_j Z_o^2 (\psi_j - \psi_{j-1})^2}{\sum_{j=1}^N m_j Z_o^2 \psi_j^2} \quad (2.27)$$

Where k_j is the stiffness of j^{th} story and can be given by $K_j = \frac{f_j}{\Delta_j}$ and $\Delta_j = z_o(\psi_j - \psi_{j-1})$. Substituting these in Eq.(2.27) and multiplying both numerator and denominator by acceleration due to gravity, g :

$$\omega_n^2 = \frac{g \sum_{j=1}^N \frac{f_j}{\Delta_j} Z_o^2 (\psi_j - \psi_{j-1})^2}{\sum_{j=1}^N g m_j Z_o^2 \psi_j^2} = \frac{g \sum_{j=1}^N \frac{f_j}{z_o(\psi_j - \psi_{j-1})} Z_o^2 (\psi_j - \psi_{j-1})^2}{\sum_{j=1}^N g m_j Z_o^2 \psi_j^2} = \frac{g \sum_{j=1}^N f_j Z_o (\psi_j - \psi_{j-1})}{\sum_{j=1}^N W_j Z_o^2 \psi_j^2}$$

$$\omega_n^2 = \frac{g \sum_{j=1}^N F_j \delta_j}{\sum_{j=1}^N W_j \delta_j^2} \quad (2.28)$$

$$\omega_n = \sqrt{\frac{g \sum_{j=1}^N F_j \delta_j}{\sum_{j=1}^N W_j \delta_j^2}} \text{ and } T = 2\pi / \omega_n \text{ from these relationships the equation of natural}$$

period of vibration becomes:

$$T = 2\pi \sqrt{\frac{\sum_{j=1}^N W_j \delta_j^2}{g \sum_{j=1}^N F_j \delta_j}} \quad (2.29)$$

Where, W_i is the story weight, ‘self-weight method of the story’, F_i the lateral applied force at the i^{th} story and are linearly distributed over the height of the building, δ_i the corresponding lateral displacement, and g is acceleration due to gravity.

But as proofed in [4], distributing lateral forces equal to floor weights, $w = mg = F_i$, estimates very close to the exact value, and better than using linear shape function; and the equation becomes:

$$T = 2\pi \sqrt{\frac{\sum_{j=1}^N W_j \delta_j^2}{g \sum_{j=1}^N W_j \delta_j}} \quad (2.30)$$

2.3 Previous Researches on Fundamental Period of RC Building

Structures

V.M Agrawal *et al.* [2015], proposed an innovative method of quantifying irregularity in stepped building frames that accounts for dynamic characteristics i.e. mass and stiffness. The study emphasized a new approach for quantifying the irregularity in stepped building. It accounts for properties associated with mass and stiffness distribution in the frame. Based on free vibration analysis of 78 stepped frames with varying irregularity and height, this study proposes a correction factor to the empirical

code formula for fundamental period, to render it applicable for stepped buildings the study is based on measure of vertical irregularity, called ‘regularity index’, accounting for the changes in mass and stiffness along the height of the building as a ratio of Γ_1 and Γ_{ref} , Where, Γ_1 is the 1st mode participation factor for the setback building frame under consideration and Γ_{ref} is the 1st mode participation factor for the similar regular building frame without steps. The regularity index is mathematically expressed as follows [14]:

$$\eta = \frac{\Gamma_1}{\Gamma_{1,ref}} \quad (2.31)$$

Their study concluded that regularity index increases with increase in number of stories, and the rate of increase being stiffer when the number of stories per step increases. The study also suggested that, for any given number of stories, the regularity index is least when the number of stories per step is largest. Thus the proposed irregularity index is able to capture effectively the irregularity caused due to various geometrical stepped configurations. In continuation of the study an improvement of code based empirical formula for estimating the fundamental period to render it applicable for stepped building was proposed. Thus a correction factor k was introduced for the empirical formula of IS 1893:2002 and modified it, as shown:

$$T = 0.075h^{0.75} \times k \quad (2.32)$$

$$k = \frac{T}{T_{ref}} = [1 - 2(1-\eta)(2\eta - 1)] \quad (2.33)$$

Where, h = overall building height in meter. This index is reported to be performing better than other measures. However this index is developed using two-dimensional building frames and not validated for actual buildings. Also, application of this index for three dimensional building is limited as 1st mode vibration for a setback building

and 1st mode vibration for the similar regular building may not always be in identical direction.

S. Varadharaja *et al.* [2014], In quantifying the dynamic response parameters like natural frequency of the vibration and mass participation factor can be considered, as these parameters effectively represent the change of mass and stiffness due to presence of setback irregularity. Furthermore, these parameters have a dominant effect on the fundamental period of vibration. To determine the parameter that has the most significant impact on the fundamental period of vibration, the sensitivity analysis was carried out. In general, the sensitivity represents the impact of an input parameter on an output parameter. Using sensitivity analysis, the parameter with the most significant effect on the fundamental period of vibration was determined. To obtain sensitivity, the standard deviation of an input parameter is divided by the standard deviation of the output parameter. The input parameter with the highest sensitivity has the greatest influence on the output parameter. In the study, the dynamic response parameters like natural frequency of the vibration and participation factor are treated as the input parameters, and the fundamental time period is treated as the output parameter [13].

As seen from the sensitivity analysis the fundamental frequency of vibration (ω) has the larger impact on the fundamental period of vibration as compared to the mass participation factor (p). Therefore, based on these results, an irregularity index (η_{ir}) has been proposed to quantify the vertical setback irregularity as shown below in Eq. (2.34).

$$\eta_{ir} = \frac{\omega_i}{\omega_r} \tag{2.34}$$

where, ω_i and ω_r are the modal combinations of frequency of vibration of the irregular and regular building frames. The approximate values of these two factors can be obtained by eigenvalue analysis using Eq. (2.35) as mentioned below

$$[K - \omega^2 M] = 0 \quad (2.35)$$

Where K, M, ω are the Stiffness matrix, Mass matrix and Natural frequency of vibration of the building respectively. The matrix operations to determine the natural frequency of vibration were performed using MatLab v 8.2 software.

The reduction in the mass reduces the fundamental period of vibration, and the stiffness reduction increases it. This variation in the fundamental period of vibration affects the base shear. To obtain the modified time period, the correction factor λ' , which is the ratio of T_i/T_r has been obtained for 305 building frames with different geometries, different bays and with different storey height by time history analysis.

Based on the polynomial fit, the correction factor is proposed as

$$\lambda' = \frac{T_i}{T_r} = 4.4032\eta_{ir}^2 - 10.583\eta_{ir} + 7.2936 \quad (2.36)$$

And the modified time period can be expressed as

$$T = 0.075\lambda' h^{0.75} \quad (2.37)$$

Matias *et al.* [2016], Lack of consideration of other structural parameters makes this code formula wholly approximate. According to modal analysis technique, however, frequency and hence time period of a structure are functions of its stiffness and mass. It is anticipated, therefore, the fundamental time period of the structure would be influenced in addition to height by the width and number of bays, size of columns and

beams, strength and density of concrete of frames, as these would affect the stiffness and mass.

In the study comparison of the fundamental time period by approximate formula, $T_1 = C_t H^{3/4}$, and the modal analysis was carried. A two dimensional dynamic analysis of the models is carried out using SAP 2000 v14 with materials of C-25 grade of concrete and S-300 grade of reinforcing steel for 200 frame models with varying heights and widths used in the study [9].

Different building frame geometries and other parameters were taken from different literatures. They are listed as follows;

- | | |
|-------------------------|-----------------------|
| l –Beam Span (m) | s – Number of Stories |
| p – Number of Bays | h– Column Height (m) |
| m_s – Story Mass (Kg) | |

The values or data under each parameter are determined through direct observation and questionnaire.

To develop new empirical formula, the SPSS v20 for IBM software was used to come up with a best fit to the modal analysis outputs using multiple linear regressions. The mathematical model used for the regression takes the following form,

$$T_1 = C + b_1 l + b_2 h + b_3 s + b_4 m_s + b_5 p \quad (2.38)$$

The values for the constant C and the regression coefficients (b1, b2... & b5) were calculated using the SPSS software.

In the study the model was assumed as linear and values for the regression coefficients (b1, b2 ... & b5) in Eq. (2.38) were found as, C = -0.874, b1= 0.016, b2=

0.595, $b_3=0.017$, $b_4=0.008$ & $b_5= -0.110$, and finally the improved empirical formula introduced as:

$$T_1 = -0.863 + (0.016 \mathbf{l}) + (0.595 \mathbf{h}) + (0.002 \mathbf{s}) + (0.008 \mathbf{m}_s) + (-0.110 \mathbf{p}) \quad (2.39)$$

2.4 Comments on Previous Researches

Investigating the effect of different parameters that affect the fundamental period of vibration is interesting to modify the conservative use of single parameter is being used in empirical formulae provided in current building codes. However, developing those investigations the following comments should be considered.

1. V.M Agrawal [14], the building models were not actual, simply varying the width of bays as 5m, 6m and 7m in both orthogonal directions and uniform story height of 3m as well as taking constant cross section of beams and columns throughout in the building system. The important point in the study was modification of existing empirical formula with introducing correction factor. Introducing that correction factor is interesting, however it was developed from 2D models of building frames and it is difficult to understand for practicing engineers.

2. Similar to the investigation of V.M Agrawal, S.Varadharajan [13] used constant cross section of beams and columns as well as uniform story height with varying number and width of bays. In addition to these variation they introduced setback for building frames at different level is good introduction to investigate the effect of irregularity on fundamental period, however introducing setbacks in only one direction lead to misunderstanding to all setbacks in building have negative impact. From their final conclusion, the number of bays does not affect the response of building significantly. This is biased conclusion because as the number of bays

increase the number of defense lines increase, the story stiffness also increase and vice versa for seismic resistance.

3. Matias [9], to consider actual building data were collected through direct site observation and conduction interview with designed questionnaire, but in the study uniform cross section of structural elements and story height with varying number and width of bays were considered. In addition to these 2D models were considered for all buildings which may not indicate the actual dynamic behavior of buildings. Finally the researcher came up with new empirical formulae which consider more parameters that affect the fundamental period as, $T_1 = -0.863 + (0.016 l) + (0.595 h) + (0.002 s) + (0.008 ms) + (-0.110 p)$, but it is not actual because the constant provided in the formula indicates building with zero height means if there is no building the fundamental period becomes (-0.863 seconds), which is not logical.

2.5 Structural Configurations and Systems for Effective Earthquake

Resistance

Irregularities are commonly associated with geometrical properties, such as size and shape. Criteria to identify irregularities exist and it is often possible to estimate them. Torsion increases as a function of the eccentricity between centers of mass C_M and rigidity C_R [3] and [5].

2.5.1 Plan Regularity

Structures with regular plan configurations are compact, i.e. described by polygonal convex lines. Square, rectangular and circular shapes are compact. Square or rectangular configurations with minor re-entrant corners can still be considered regular. Large re-entrant corners creating crucifix forms give rise to irregular

configurations. The dynamic response of the wings (also termed ‘multi - mass structures’) generally differs from that of the structure as a whole. Multi- mass structures are highly vulnerable at connections between wings. Relative displacements cause severe damage at the intersection of various blocks; torsional effects are likely to occur. Other plan configurations with geometrical symmetry, e.g. I- and H- shapes, are also irregular because of the response of the wings.

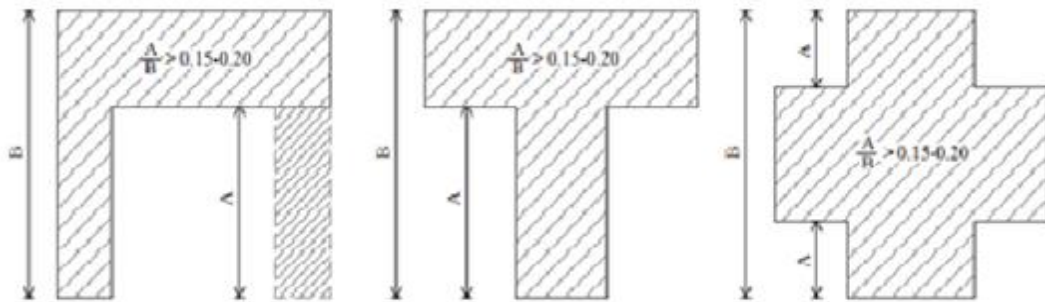


Figure 2.4: Typical limits for plan irregularities (adapted from FEMA 450, 2004)

Plan irregularities depend upon the size of setbacks, i.e. re-entrant corners and edge recesses. Limitations for the setbacks can be expressed as a function of their geometry. For example, for L- , T - and X- sections, the following limitation can be used:

$$\frac{A}{B} = 0.15 - 0.20 \quad (2.40)$$

where, A and B are the length and the depth of the re-entrance, respectively, as shown in Figure 2.4. Eq. (2.40) provides the limitation included in seismic design recommendations in the USA (e.g. FEMA 450, 2004). Alternatively, regularity in plan may be assumed if, for each setback, the area between the outline of the floor and a convex polygonal line enveloping the floor does not exceed 5% of the total area. This criterion is adopted in European design practice (e.g. Eurocode 8, 2004, ESEN

1998-1:2015). A building structure may have a symmetrical geometric shape without re-entrant corners and wings but can still be classified as irregular in plan, since the distribution of mass or vertical seismic resisting elements may be asymmetric. Torsional effects due to earthquake motions can occur even when static centers of mass C_M and resistance C_R coincide. For example, ground-motion waves acting at an angle to the building axis also cause torsion, as may crack and yield in a non-symmetrical fashion. Additionally, these effects can magnify torsion due to eccentricity between the static centers. Generally speaking, buildings having an eccentricity between the static center of mass and the static center of resistance in excess of 30% of the building dimension perpendicular to the direction of the seismic force are considered irregular.

Structures with symmetric and compact shapes but employing plan discontinuity for lateral resisting systems are not regular. Typical examples are three-sided buildings that experience high torsional effects under earthquake loading. Architectural reasons generally impose arrangements of plan layout with steel or reinforced concrete (RC) frames and walls located along three sides of the perimeter (Figure 2.5). In commercial buildings, the necessity for large openings for shop windows on the facade may lead to the use of three-sided buildings of this type. Continuity in plan between lateral resisting systems is essential for clear and continuous load paths.

Core - type buildings with the vertical seismic-force - resisting system concentrated near the center tend to behave poorly during earthquakes. Better performance has been observed when vertical components are distributed near the perimeter of the building. This may, however, cause instability due to torsion. Eccentric locations of rigid cores for external lifts and stairwells also generate undesirable torsional effects

(Figure 2.6). For example, external access towers, which are meant to be used during seismic events, often fail in their function because they experienced large rotations or collapse.

Diaphragm action is another requirement for plan regularity. Relative stiffness and strength of floors and bracing systems are critical for earthquake response. Floor systems with high stiffness and strength ensure adequate distribution of seismic actions among vertical structural elements. Where discontinuities in the lateral force resistance path exist, the structure is no longer regular. Significant differences in stiffness between portions of diaphragms may cause a change in the distribution of seismic forces to the vertical components and create torsional forces.

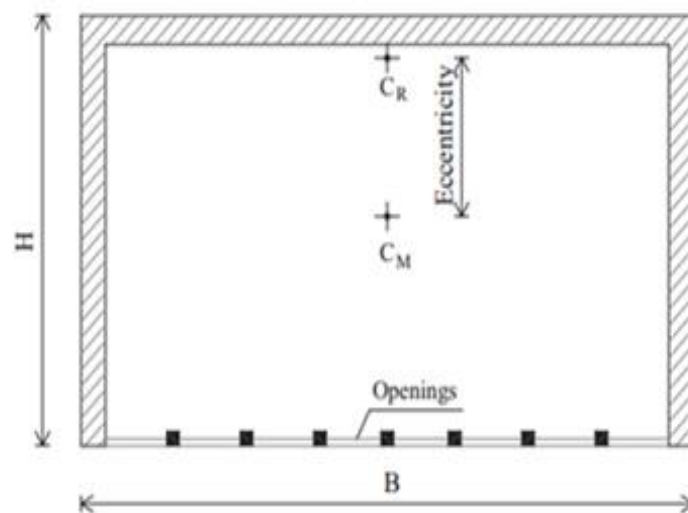


Figure 2.5: Irregularities due to plan discontinuity for lateral resisting systems
Key: C_M =center of mass; C_R = center of rigidity

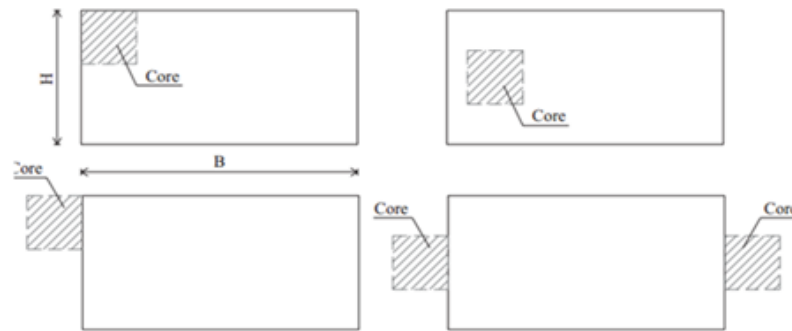


Figure 2.6: Plan irregularities due to unfavourable core location

Building structures with large aspect ratios in plan are susceptible to incoherent earthquake motion (also referred to as ‘out - of - phase effects’). Different foundation materials may generate amplification of the dynamic response in different parts of the building. The higher the aspect ratio, the higher the likelihood of incoherence effects, these effects depend on whether foundation systems, as well as superstructures, are continuous or not. The probability of having similar live loads in large structures is inversely proportional to the size of the structure. Therefore, the plan aspect ratio should be not greater than 2–3. Alternatively, the structure may be subdivided into independently responding parts by using seismic joints. Separation joints should be large enough to accommodate lateral displacements between adjacent buildings and to avoid pounding. As a rule of thumb, the separation(s) can be assumed as 1/100 of the maximum height (H) of the adjacent structures, in meters. Separation joints can help to mitigate unfavourable seismic effects on multi- mass structures. Debris from severely damaged or partially collapsed upper stories can also fall in separation joints. These should be sealed, where possible, to prevent such occurrences.

Irregularities in plan arise when vertical elements of the lateral force- resisting system are not parallel to or symmetric with major orthogonal axes. Shapes with sharp

corners are unsuitable for seismic resistance because of the high probability of torsional forces under earthquake motions. In addition, different relative stiffnesses between narrower and wider perimeter sides aggravate torsional effects.

Discontinuities in horizontal and vertical lateral resistant systems are an additional source of irregularity in plan. Out-of-plane offsets of vertical elements, for example, may impose significant demands on structural components of earthquake-resistant structures. Extensive damage may be caused by these offsets; they should not be employed in seismic areas.

2.5.2 Elevation Regularity

For a building to be categorized as being regular in elevation, it shall satisfy all the conditions listed in the following paragraphs [3] and [7].

1. All lateral load resisting systems, such as cores, structural walls, or frames, shall run without interruption from their foundations to the top of the building or, if setbacks at different heights are present, to the top of the relevant zone of the building.
2. Both the lateral stiffness and the mass of the individual stories shall remain constant or reduce gradually, without abrupt changes, from the base to the top of a particular building.
 - a. ***Stiffness Irregularities, Soft storey:*** A soft storey is one in which the lateral stiffness is less than 70% of that in the storey above or less than 80% of the average lateral stiffness of the three stories above.
 - b. **Mass Irregularity:** Mass irregularities are considered to exist where the effective mass of any storey is more than 150% of effective mass of an adjacent storey.

3. When setbacks are present, the following additional conditions apply:
 - a. for gradual setbacks preserving axial symmetry, the setback at any floor shall be not greater than 20 % of the previous plan dimension in the direction of the setback
 - b. for a single setback within the lower 15 % of the total height of the main structural system, the setback shall be not greater than 50 % of the previous plan dimension
 - c. if the setbacks do not preserve symmetry, in each face the sum of the setbacks at all stories shall be not greater than 30 % of the plan dimension at the ground floor above the foundation or above the top of a rigid basement, and the individual setbacks shall be not greater than 10% of the previous plan dimension.

Chapter Three: Structural Modeling and Analysis

3.1 Introduction

Modeling a building involves the modeling and assemblage of its various load-carrying elements. The model must ideally represent the mass distribution, strength, stiffness and deformability. Structural modeling is a tool to establish three mathematical models, including (1) a structural model consisting of three basic components: structural members or components, joints (nodes, connecting edges or surfaces), and boundary conditions (supports and foundations); (2) a material model; and (3) a load model. For designing a new structure, connection details and support conditions shall be made as close to the computational models as possible. Since this study is for evaluation of an existing structure, structures are modeled as close to the actual as-built structural conditions.

Structural analysis is a process to analyze a structural system to predict its responses and behaviors by using physical laws and mathematical equations. The main objective of structural analysis is to determine internal forces, stresses and deformations of structures under various load effects.

3.2 Modeling in ETABS

3.2.1 Material Properties

Different types of materials are used for building structural members such as concrete, steel, prestressing tendons, timbers, etc. In this study, only concrete structure is carried out. The material properties that are used for an elastic analysis are: modulus of elasticity, shear modulus, Poisson's ratio, coefficient of thermal expansion, the

mass density and the weight density. The strength of concrete and reinforcing steel for respective buildings is used from their structural drawings. In all buildings the concrete grade used in slabs, beams and staircases, and in some low-rise buildings columns was C-20/25 and for columns and shear walls concrete grade of C-25/30 was used. The modulus of elasticity of concrete for all structural members is given in EC2 as $E_c = 22\left(\frac{f_c}{10}\right)^{0.3} GPa$, where f_c is the characteristic compressive strength of concrete. The poisson's ratio for concrete is assumed to be 0.2. In the self-weight calculation, the unit weight of the reinforced concrete is selected to be $25kN /m^3$. Since the focus of this study is not on the design part of structural members, material properties of rebars are assigned to their respective members of building from their structural drawings. As referred from the structural drawings of the respective buildings the steel grade of S-420 and S-400 for $\phi \geq 12$ and S-300 for $\phi \leq 12$ respectively was used. The modulus of elasticity of reinforcement steel is 200Gpa.

3.2.2 Structural Elements

The three-dimensional analytical models were developed for analysis using ETABS 2015 Software. The cross-section and actual location of the structural elements were referred from the structural drawings for each building. The beam-column connections were assumed to be rigid. The supports were fully fixed against rotation at the base by considering the supporting ground as stiff soil.

The effect of cracking of concrete was ignored and the gross stiffness of structural elements was considered. The mass was lumped at each story level, including the mass of infill walls, when present.

The buildings, floors (including the roof) play a very important role in the overall seismic behaviour of the structure. They act as horizontal diaphragms that collect and

transmit the inertia forces to the vertical structural systems and ensure that those systems act together in resisting the horizontal seismic action. In ETABS, modeling, there are two options that exist in defining the diaphragms. These options are rigid and semi-rigid. In this study, rigid diaphragm is selected for all floor diaphragms. Although, in reality, building floors are expected to have openings in different sizes which would adversely impact the rigid diaphragm assumption, it is assumed in this study that this assumption would not cause negative impact in the period calculations.

3.2.3 Non-Structural Elements

Non-structural elements of buildings (e.g. parapets, gables, curtain walls, partitions, railings) that might cause risks to occupancy or affect the main structure of the building or services were considered while modeling. In the present study the partition walls and outer infill walls were constructed with glass curtain and HCB walls built into the frames and on floors as observed from the architectural drawings of each building. Since these walls are non-load bearings and have no longer significant contribution to the stiffness of the building frames. Therefore, the participation of these non-structural components on the stiffness of the building is minimal compared to the stiffness of the frame and can spalling-off in short period under seismic excitation; thus buildings were essentially vibrating as a bare structural frame.

The frames with identical properties were lumped together, and the lumped frames were connected by rigid links to impose equal horizontal displacement at each floor level, without any moment connection between the frames. The gap element used to separate the lumped frames taken to be stiffer than the surrounding elements.

3.2.4 Loading

In the seismic analysis, the self-weight of the building is a key component. The self-weight of the structural members is automatically included by the program and therefore, they are not calculated as an additional load. The live loads and the super imposed dead loads, however, are assigned as a pressure load to each floor and the super imposed wall loads on beams are assigned as line load. The magnitude of the live loads for all the models is considered as the intended purpose of the building floors. The focus of this study is on the determination of the fundamental period and the lateral deflections; no load combination is selected in the ETABS analyses. In other words, only load cases are considered in evaluating the periods and deflections. For Modal analysis the mass source is considered as Eurocode and taken as permanent load plus 30% of variable load (live load).

3.3 Structural Regularity

Regularity of the structure (in elevation and in plan) influences the required structural model (planar or spatial), the required method of analysis (EN 1998-1:2004) [7]. Regularity in plan may influence the magnitude of the seismic action. For a structure regular in plan and in elevation, the simplest approach can be applied, i.e. a planar model can be used and a lateral force method can be performed, Whereas, for a structure irregular in plan and elevation the standard (i.e. spatial) model and the standard (i.e. modal response spectrum) analysis can be used.

3.3.1 Plan Regularity Checking

In general, the regularity in plan can be checked when the structural model is defined. The criteria for regularity in plan are described as below:

1. With respect to the lateral stiffness and mass distribution, the building structure shall be approximately symmetrical in plan with respect to two orthogonal axes.
2. The plan configuration shall be compact, i.e., each floor shall be delimited by a polygonal convex line. If in plan set-backs (re-entrant corners or edge recesses) exist, regularity in plan may still be considered as being satisfied, provided that these setbacks do not affect the floor in-plan stiffness and that, for each set-back, the area between the outline of the floor and a convex polygonal line enveloping the floor does not exceed 5 % of the floor area.
3. The slenderness $\lambda = L_{\max}/L_{\min}$ of the building in plan shall be not higher than 4, where L_{\max} and L_{\min} are respectively the larger and smaller in plan dimension of the building, measured in orthogonal directions.
4. At each level and for each direction of analysis x and y, the structural eccentricity e_o and the torsional radius r shall be in accordance with the two conditions below:
 - a. The structural eccentricity shall be smaller than 30% of the torsional radius ($e_{ox} \leq 0.30r_x, e_{oy} \leq 0.30r_y$) and
 - b. The torsional radius shall be larger than the radius of the gyration of the floor mass in plan ($r_x \geq l_s, r_y \geq l_s$).

In this study, these criteria of plan regularity and elevation regularity presented in chapter 2 were checked for all building structures to categorize buildings as regular or irregular as follows.

The building as shown in figure 3.1 is symmetric in both orthogonal axis and it has symmetric plan set-back of $A/B = 0.75\text{m}/16\text{m} = 0.046785$, which is 4.68% and is less than 15% as presented in chapter 2. The slenderness of the building is smaller than 4.0. It amounts to $\lambda = 1.5459$, (16m/10.35m) in all storey level. Other two conditions,

the structural eccentricity is smaller than 30% of the torsional radius and the relation of torsional radius and the radius of the gyration of the floor mass are also fulfilled at each storey level in both horizontal directions (Table 3.1). Determination of the structural eccentricity, the torsional radius and the radius of the gyration are carried out as EN 1998-1:2004 [7]. Building is categorized as being regular in plan in both directions.

Table 3.1: Criteria for regularity in plan (All quantities are in (m))

Level	Direction X				Direction Y			
	$ e_{ox} $	$0.3r_x$	r_x	l_s	$ e_{oy} $	$0.3r_y$	r_y	l_s
Roof	0.0085	2.04343	6.81144	5.501	0.316	1.93005	6.43349	5.501
Level 5	0.1145	2.03948	6.79828	5.501	0.077	1.93167	6.43890	5.501
Level 4	0.001	2.03588	6.78627	5.501	0.141	1.93405	6.44682	5.501
Level 3	0.0009	2.03112	6.77041	5.501	0.139	1.93601	6.45337	5.501
Level 2	0.0009	2.02392	6.74639	5.501	0.135	1.93828	6.46092	5.501
Level 1	0.0009	2.00824	6.69414	5.501	0.146	1.94006	6.46688	5.501
Level 0	0.0009	1.95738	6.52459	5.501	0.143	1.88206	6.27355	5.501

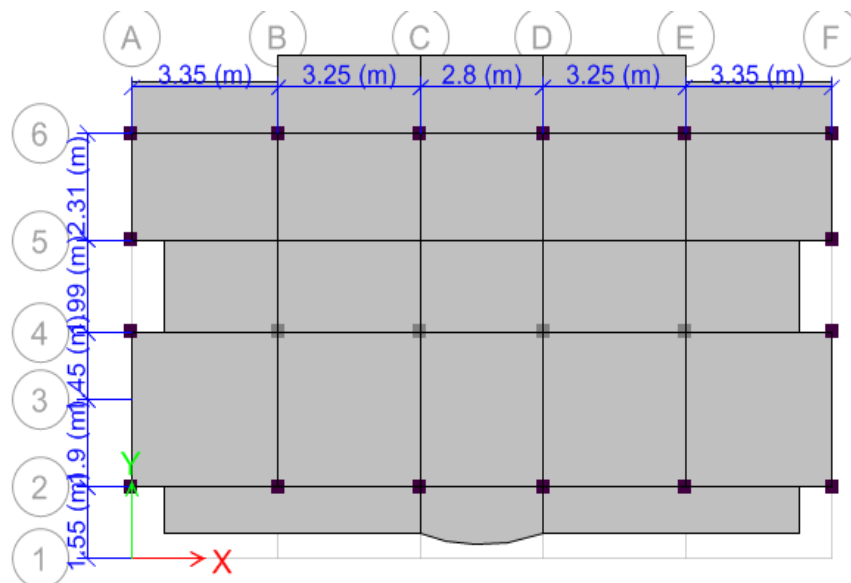


Figure 3.1:- Structural layout of ABY G+5 building

3.3.1.1 Determination of the structural eccentricity (e_{ox} and e_{oy})

The structural eccentricity in each of the two orthogonal directions (e_{ox} and e_{oy}) represents the distance between the center of stiffness (X_{CR} , Y_{CR}) and the center of mass (X_{CM} , Y_{CM}). In general, it has to be calculated for each level. The center of mass and stiffness can be determined from spatial structural model using ETAB software.

Table 3.2: Coordinates of the center of mass (X_{CM} , Y_{CM}), structural eccentricities (e_{ox} and e_{oy}) and the coordinates of the center of stiffness (X_{CR} , Y_{CR})

Level	X_{CM} (m)	Y_{CM} (m)	X_{CR} (m)	Y_{CR} (m)	e_{ox} (m)	e_{oy} (m)	r_x (m)	r_y (m)	$0.3r_x$ (m)	$0.3r_y$ (m)
Roof	7.986	5.672	7.994	5.356	0.0085	0.316	6.811	6.43	2.043	1.93
Level 5	7.885	5.434	8.000	5.357	0.1145	0.077	6.798	6.44	2.039	1.932
Level 4	8.001	5.5	8.000	5.359	0.001	0.141	6.786	6.45	2.036	1.934
Level 3	8.001	5.5	8.000	5.361	0.0009	0.139	6.77	6.45	2.031	1.936
Level 2	8.001	5.5	8.000	5.365	0.0009	0.135	6.746	6.46	2.024	1.938
Level 1	8.001	5.522	8.000	5.376	0.0009	0.146	6.694	6.47	2.008	1.94
Level 0	7.894	5.265	-	-	-	-	6.525	6.27	1.958	1.882

3.3.1.2 Determination of the torsional radius (r_x and r_y)

The torsional radius r_x (r_y) is defined as the square root of the ratio of the torsional stiffness (K_M) to the lateral stiffness in one direction K_{Fy} (K_{Fx})

$$r_{x,i} = \sqrt{\frac{K_{M,i}}{K_{Fy,i}}}, \text{ and } r_{y,i} = \sqrt{\frac{K_{M,i}}{K_{Fx,i}}} \quad (3.2)$$

The procedure for the determination of the torsional and lateral stiffness is similar to that for the determination of structural eccentricity. Three static load cases are defined for each storey level, and loads are represented by F_{Tx} , F_{Ty} and M_T , respectively. The forces and moment are applied in the center of stiffness (in the case of the determination of the structural eccentricity, forces and moment were applied in center

of mass). The torsional and lateral stiffness for both directions are calculated as follows

$$K_{M,i} = \frac{1}{R_{z,i}(M_{T,i}=1)}, \quad K_{F_{X,i}} = \frac{1}{U_{X,i}(F_{T_{X,i}}=1)}, \quad K_{F_{Y,i}} = \frac{1}{U_{Y,i}(F_{T_{Y,i}}=1)} \quad (3.3)$$

where $R_{z,i}(M_{T,i}=1)$ is the rotation of the storey i due to unit moment, $U_{x,i}(F_{T_{X,i}}=1)$ is the displacement at storey level i in direction X due to unit force $F_{T_{X,i}}$ and $U_{Y,i}(F_{T_{Y,i}}=1)$ is the displacement in direction Y due to unit force $F_{T_{Y,i}}$.

Table 3.3: The displacements (U_x , U_y) and rotation (R_z) due to $F_{T_x} = 10^3$ kN, $F_{T_y} = 10^3$ kN and $M_T = 10^3$ kNm, the torsional (K_M) and lateral stiffness in both directions (K_{F_x} , K_{F_y}), and torsional radius (r_x , r_y).

Level	$U_x(F)$ (m)	$U_y(F_Y)$ (m)	$R_z(M_T)$ (rad)	K_{F_x} (kN/m)	K_{F_y} (kN/m)	K_{M_T} (kN/m)	r_x (m)	r_y (m)
Roof	0.558	0.626	0.0135	1791.79	1598.47	74161.97	6.811	6.433
Level 5	0.539	0.600	0.0130	1856.67	1665.56	76976.37	6.798	6.439
Level 4	0.484	0.536	0.0116	2066.54	1864.98	85888.52	6.786	6.447
Level 3	0.402	0.443	0.0097	2485.71	2258.36	103519.7	6.770	6.453
Level 2	0.295	0.321	0.0071	3395.59	3114.30	141743.4	6.746	6.461
Level 1	0.161	0.172	0.0038	6218.91	5803.83	260078	6.694	6.467
Level 0	0.017	0.011	0.0003	1020418	94339.6	4016064	6.525	6.274

3.3.1.3 Determination of the radius of gyration of the floor mass in plan (l_s)

For checking the criteria for regularity in plan, the radius of the gyration of the floor mass (l_s) is also needed. It is defined as the square root of the ratio of the polar moment of inertia of the floor mass in plan to the floor mass. In the case of the rectangular floor area with dimensions l and b and with uniformly distributed mass over the floor, l_s is equal to

$$l_s = \sqrt{\frac{(l^2+b^2)}{12}} \quad (3.4)$$

3.3.2 Elevation Regularity Checking

For a building shown in figure 3.1, ABY G+5; all lateral load resisting systems, such as structural walls, or frames, run without interruption from their foundations to the top of the building and there is no setbacks at different heights. Also both the lateral stiffness and the mass of the individual stories changes gradually, without abrupt changes, from the base to the top of the building as shown in Table 3.4. Therefore a building is categorized as regular in in both plan and elevation directions.

Table 3.4: Vertical Irregularity Checking for a selected building, (ABY 6 storied)

Story	Story Height (m)	Story Mass (Kg)	Story Stiffness, (kN/m)		Stiffness Irregularity				Mass Irregularity
					Difference of story stiffness to above story (%)		Difference of average stiffness of three stories above (%)		Difference of story mass to the above story (%)
Roof	19.6	35714	94380	64371	-	-	-	-	-
Level 5	17	136942	67237	53675	40.37	19.93	-	-	73.92
Level 4	13.6	141281	68107	56554	1.28	5.09	18.65	4.37	3.07
Level 3	10.2	141281	69084	58783	1.41	3.79	10.84	0.99	0.00
Level 2	6.8	141281	70479	61606	1.98	4.58	3.32	8.55	0.00
Level 1	3.4	140598	90298	84178	21.95	26.82	23.34	29.93	0.49
Level 0	0	94262	0	0	-	-	-	-	49.16

The building as shown in figure 3.2 is not symmetric due to unsymmetrical configuration of shear wall, unsymmetrical re-entrance of corners in both orthogonal direction and the maximum plan set-back of $A/B = 5.67m/17.01m = 0.3333$ above ground level, is 33.33% and it is greater than 15%-20% as presented in chapter 2.

The slenderness of the building is smaller than 4.0. It amounts to $\lambda = 1.031$, (17.01m/16.5m), in all storey levels. The structural eccentricity is less than 30% of the torsional radius at all storey levels along 'X' direction (see Table 3.4). Since at least one criterion obeys irregularity, therefore the building ATO is irregular in plan.

Similarly for all buildings the plan regularity criteria were carried out and the buildings were identified as regular and irregular.

Table 3.5: Criteria for regularity in plan (All quantities are in (m))

Level	Direction X				Direction Y			
	$ e_{ox} $	$0.3r_x$	r_x	ls	$ e_{oy} $	$0.3r_y$	r_y	Ls
Roof	0.4034	2.5863	8.6209	7.2529	0.5660	2.5683	8.5611	7.2529
Level 8	0.4289	2.5386	8.4620	7.2529	0.6157	2.5154	8.3846	7.2529
Level 7	0.4803	2.4486	8.1619	7.2529	0.6973	2.4103	8.0342	7.2529
Level 6	0.5138	2.3724	7.9079	7.2529	0.7582	2.3225	7.7417	7.2529
Level 5	0.5425	2.2906	7.6353	7.2529	0.8196	2.2312	7.4373	7.2529
Level 4	0.5645	2.2018	7.3392	7.2529	0.8837	2.1759	7.2529	7.2529
Level 3	0.5731	2.1759	7.2529	7.2529	0.9504	2.1759	7.2529	7.2529
Level 2	0.5534	2.1759	7.2529	7.2529	1.0198	2.1759	7.2529	7.2529
Level 1	0.4441	2.1759	7.2529	7.2529	1.0757	2.1759	7.2529	7.2529
Level 0	-	2.1759	7.2529	7.2529	-	2.1759	7.2529	7.2529

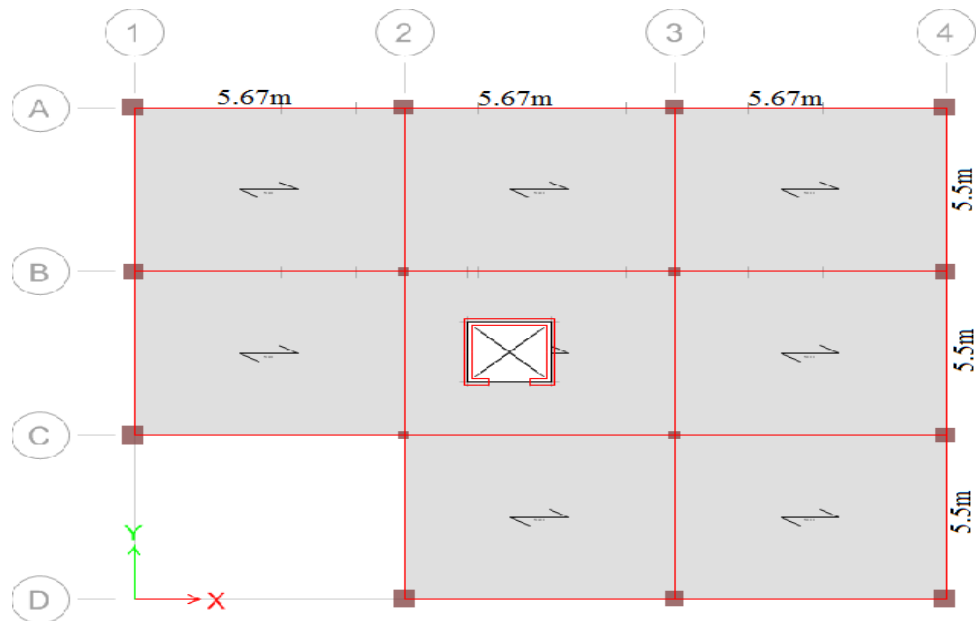


Figure 3.2:- Structural layout of TAO G+9 building

3.4 Linear Dynamic Analysis

Symmetrical buildings with uniform mass and stiffness distribution behave in a fairly predictable manner, whereas buildings that are asymmetrical or with areas of discontinuity or irregularity do not. For such buildings, dynamic analysis is used to determine significant response characteristics such as (1) the effect of the structure's dynamic characteristics on the vertical distribution of lateral forces; (2) the increase in dynamic loads due to Torsional motions; and (3) the influence of higher modes, resulting in an increase in story shears and deformations.

Static method specified in building codes are based on single-mode response with simple corrections for including higher mode effects. While appropriate for simple regular structures, the simplified procedures do not take into account the full range of seismic behaviour of complex structures. Therefore, dynamic analysis is the preferred method for the design of buildings with unusual or irregular geometry. Two methods of dynamic analyses are permitted: (1) elastic response spectrum analysis and (2)

elastic or inelastic time history analysis. The response spectrum analysis is the preferred method because it is easier to use. The time history procedure is used if it is important to represent inelastic response characteristics or to incorporate time dependent effects when computing the structure's dynamic response. Buildings are analyzed as multi degree of freedom (MDOF) systems by lumping storey masses at intervals along the length of a vertically cantilevered pole. During vibration, each mass will deflect in one direction or another. For higher modes of vibration, some masses may move in opposite direction. Or all masses may simultaneously deflect in the same direction as in the fundamental mode. An idealized MDOF system has a number of modes equal to the number of masses. Each mode has its own natural period of vibration with a unique mode shaped by a line connecting the deflected masses. When ground motion is applied to the base of the multi mass system, the deflected shape of the system is a combination of all mode shapes, but modes having periods near predominant periods of the base motion will be excited more than the other modes. Each mode of the multi mass system can be represented by an equivalent single mass system having generalized values M and K for mass and stiffness, respectively. The generalized values represent the equivalent combined effects of story masses $m_1, m_2 \dots m_N$ and $k_1, k_2 \dots k_N$. This concept, shown in Fig. 3.3, provides a computational basis for using response spectra based on single mass systems for analyzing multi-storied building, we can use the response spectra of a single degree of freedom (SDOF) system for computing the deflected shape, story acceleration, forces, and overturning moments. Each predominant mode is analyzed separately and the results are combined statically to compute the multimode response.

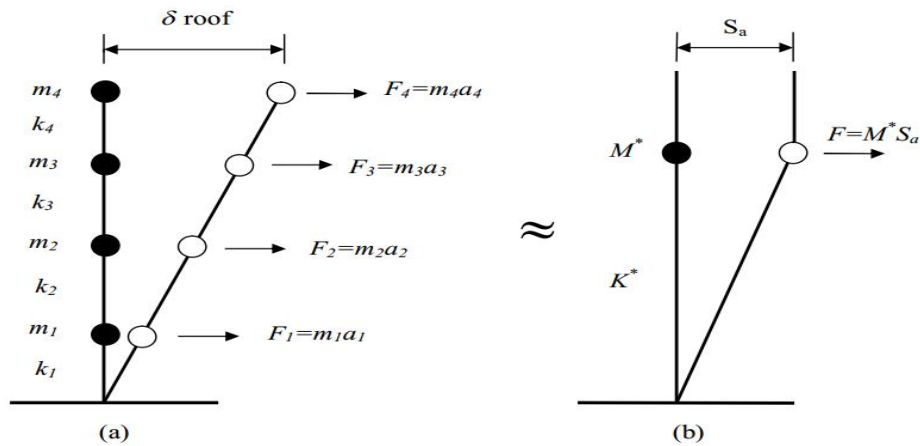


Figure 3.3: Representation of a multi-mass system by a single mass system: (a) fundamental mode of a multi-mass system and (b) equivalent single mass system.

Buildings with symmetrical shape, stiffness, and mass distribution and with vertical continuity and uniformity behave in a fairly predictable manner, whereas when buildings are eccentric or have areas of discontinuity or irregularity; the behavioral characteristics are very complex. The predominant response of the building may be skewed from the apparent principal axes of the building. The resulting torsional response as well as the coupling or interaction of the two translational directions of response must be considered by using a 3D model for the analysis. For a building that is regular and essentially symmetrical, a 2D model is generally sufficient. When the floor plan aspect ratio (length to width) of the building is large, torsion response may be predominant, thus requiring a 3D analysis in an otherwise symmetrical and regular building. For most building, inelastic response can be expected to occur during a major earthquake, implying that an inelastic analysis is more proper for design.

3.4.1 Modal Analysis

When free vibration is under consideration, the structure is not subjected to any external excitation (force or support motion) and its motion is governed only by the initial conditions. There are occasionally circumstances for which it is necessary to determine the motion of the structure under conditions of free vibration. However, the

analysis of the structure in free motion provides the most important dynamic properties of the structure which are the natural frequencies and the corresponding modal shapes. By considering the fact that the damping levels are usually very small in structural systems, the equation of free vibration can be written as:

$$M\ddot{U} + KU = 0 \quad (3.5)$$

Looking for a solution in the form of $v_i = q(t)\phi_i, i = 1, 2, \dots, N$, where the dependence on time and that on space variables can be separated. Substituting for u , the equation of motion changes to the following form:

$$M\{\phi\}\ddot{q}(t) + K\{\phi\}q(t) = 0 \quad (3.6)$$

This is a set of N simultaneous equations of the type

$$\sum_{j=1}^N m_{ij}\phi_j\ddot{q}(t) + \sum_{j=1}^N k_{ij}\phi_jq(t) = 0; \quad i = 1, 2, \dots, N \quad (3.7)$$

Where the separation of variables leads to:

$$-\frac{\ddot{q}(t)}{q(t)} = \frac{\sum_{j=1}^N k_{ij}\phi_j}{\sum_{j=1}^N m_{ij}\phi_j}; \quad i = 1, 2, \dots, N \quad (3.8)$$

As the terms on either side of this equation is independent of each other, this quantity can hold good only when each of these terms are equal to a positive constant, say ω^2 .

Thus we have,

$$\ddot{q}(t) + \omega^2q(t) = 0 \quad (3.9)$$

$$\sum_{j=1}^N (k_{ij} - \omega^2m_{ij})\phi_j = 0 \quad (3.10)$$

The solution of Eq. 3.6 is $q(t) = \sin(\omega t - \alpha)$ a harmonic of frequency, ω . Hence the motion of all coordinates is harmonic with same frequency and same phase difference, α . The above equation is a set of N simultaneous linear homogenous

equations in unknowns of ϕ_j . The problem of determining constant (ω^2) for which the Eq. 3.11 has a non-trivial solution is known as the *characteristic value* or *Eigenvalue* problem. The Eigenvalue problem may be rewritten, in matrix notations as,

$$(K - \omega^2 M)\{\phi\} = 0 \quad (3.11)$$

A non-trivial solution for the Eq. 3.12 is feasible when only the determinant of the coefficient matrix vanishes, *i.e.*

$$|K - \omega^2 M| = 0 \quad (3.12)$$

Expanding the determinant will give an algebraic equation of the N^{th} degree in the frequency parameter ω^2 , which is known as the characteristic equation, for a system having N degrees of freedom. The roots of characteristic equation are known as the Eigenvalues and the positive square roots of these Eigenvalues are known as the *natural frequencies* (ω_i), ($\omega_1^2, \omega_2^2, \omega_3^2, \dots, \omega_N^2$) of the MDOF system. The mode having the lowest frequency is called the first mode; the next higher frequency is the second mode, etc. The vector made up of the entire set of modal frequencies, arranged in sequence, will be called the *frequency vector* ω :

$$\omega = \left[\begin{array}{c} \omega_1 \\ \omega_2 \\ \omega_3 \\ \dots \\ \omega_N \end{array} \right] \quad (3.13)$$

It is only at these N frequencies that the system admits synchronous motion at all coordinates. For stable structural systems with symmetric and positive stiffness and mass matrices the Eigenvalues will always be real and positive.

Natural period of a structure is its time period of undamped free vibration. The frequencies at which normal mode vibrations are possible for a structure are called the natural frequencies of the structure. The structure is said to be vibrating in its k^{th} normal mode when its frequency is equal to the k^{th} natural frequency. The lowest of the natural frequencies of a structure is called fundamental natural frequency. The associated natural period is called fundamental natural period, (T_1), which is the first (longest) modal time of vibration. The corresponding vibration period for respective mode of the structural system is the reciprocal of its natural frequency and given as:

$$T_n = \frac{2\pi}{\omega_n} \quad (3.14)$$

3.4.1.1 Modal Shapes

When the eigenvalues of vibration have been determined as Eq. (3-14), then for each eigenvalues the resulting synchronous motion has a distinct shape and is known as *natural/normal mode shape* or *eigenvector*. The normal modes are as much a characteristic of the system as the eigenvalues are. They depend on the inertia and stiffness, as reflected by the coefficients m_{ij} and k_{ij} . These shapes correspond to those structural configurations, in which the inertia forces imposed on the structure due to synchronous harmonic vibrations are exactly balanced by the elastic restoring forces within the structural system. These eigenvectors are determined as the non-trivial solution of Eq. 3.12.

Normal modes are special undamped free vibrations in which all points on the structure vibrate harmonically at the same frequency such that all these points reach their individual maximum responses simultaneously. A system is said to be vibrating

in a normal mode when all its masses attain maximum values of displacements and rotations simultaneously, and pass through equilibrium positions simultaneously.

3.4.1.2 Mode Participation Factor

Modal participation factor of mode k of vibration is the amount by which mode k contributes to the overall vibration of the structure under horizontal and vertical earthquake ground motions. Since the amplitude of 95% mode shapes can be scaled arbitrarily, the value of this factor depends on the scaling of mode shapes, [8].

The forced vibration of MDOF system excited by support motion is described by the coupled system of differential equation as:

$$M\ddot{u} + C\dot{u} + Ku = -Ml\ddot{u}_g \quad (3.15)$$

Where \ddot{u}_g denotes ground acceleration, v is the vector of structural displacements relative to the ground displacements, and l is a vector of influence coefficients. The i^{th} element of vector l represents the displacement of i^{th} degree of freedom due to a unit displacement of the base. The nature of this equation is similar to that of standard forced vibration problem. Hence this can be solved using mode-superposition method and the equation can be decoupled as:

$$\ddot{q}_n + 2\xi_n\omega_n\dot{q}_n + \omega_n^2q_n = -\Gamma_n\ddot{u}_g(t) \quad (3.16)$$

Where, $\Gamma_n = \frac{L_n}{M_n}$, is known as the *mode – participation factor* for the n^{th} mode.

$$L_n = \phi_n^T ml \quad \text{and,} \quad M_n = \phi_n^T m\phi_n$$

$$\Gamma_n = \frac{\phi_n^T ml}{\phi_n^T m\phi_n} \quad (3.17)$$

$$l = \sum_{n=1}^N \Gamma_n \phi_n \quad (3.18)$$

3.4.2 Rayleigh Method of Analysis

The dead load W , used for calculating the base shear, includes the total dead load of the structure, the actual weight of partitions, the weight of finishing, and 25% of the floor live load in storage and warehouse occupancies in which its effective seismic weight is more than 5% at the floor level [2], computed by considering half above and below the respective floor. In this study, the seismic weight of a floor was obtained automatically from the ETABS software and the lateral forces were distributed over the height of the building from the self-weight of the floor and applied laterally at the i^{th} story with considering accidental eccentricity of story mass to optimize torsional effects, i.e. $\pm 5\%$; the corresponding lateral displacements at each story were recorded and finally the fundamental period was computed using the Rayleigh's formula.

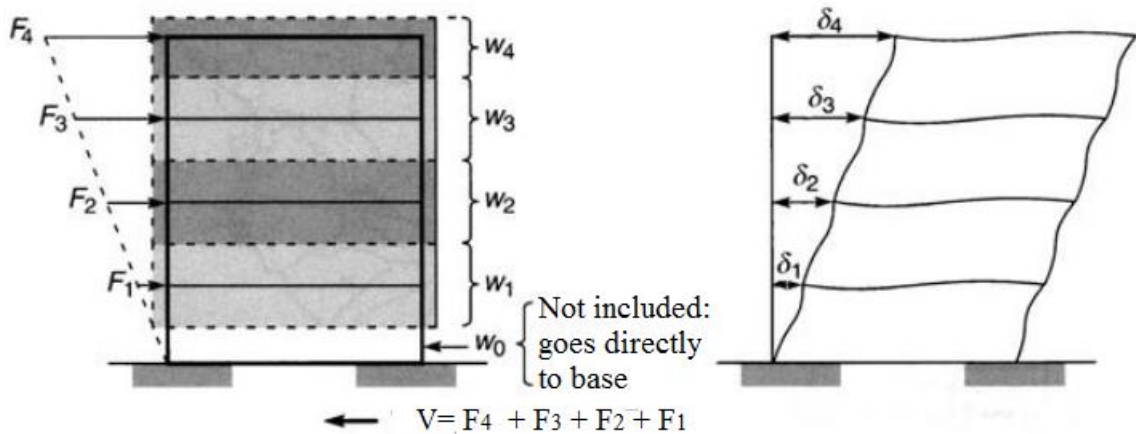


Figure 3.4: Period calculation for MDOF system via Rayleigh formula

$$T = 2\pi \sqrt{\frac{1}{g} \frac{W_4 \delta_4^2 + W_3 \delta_3^2 + W_2 \delta_2^2 + W_1 \delta_1^2}{F_4 \delta_4 + F_3 \delta_3 + F_2 \delta_2 + F_1 \delta_1}}, \quad \text{but } F_i = W_i \quad (3.19)$$

3.4.3 Period - Story Displacement Relation

The period-story displacement relation is expressed in Eq. (2.22); to determine the fundamental period this relation similar procedure was followed as the Rayleigh

formula and the lateral displacement, d , at the top of the building was recorded and then the fundamental period was computed using the above formula.

3.4.4 Period - Building Dimension Relation

For the other empirical formulae expressed in Eqs (2.1, 2.9, 2.10 and 2.21), the corresponding variables like height of the building, number of story and lateral dimension of building at its base in the direction under consideration were recorded and the fundamental period was computed using the corresponding equation.

Chapter Four: Results and Discussions

4.1 Introduction

In this study 39 actual buildings with different base dimensions, width and number of bays, story height and numbers, configuration of shear walls, cross sectional dimensions of structural elements such as columns, beams, slabs and shear walls, and buildings intended for different purposes are modeled and analyzed for linear dynamic behavior using finite element software ETABS 2015(v15.0.0). The results and discussions presented here are focused on fundamental period of RC buildings that support the objectives of the present study. The details of collected buildings and the outline of the analysis procedure followed in this study are discussed in chapter three.

The results of the parametric study are discussed in this section. For each model the first fundamental period is investigated. The mode shapes, story stiffness, modal mass participation ratios and other results are studied in detail to understand the degree of changes in period from model to model. Then, the first fundamental period is investigated further by comparing the results obtained from linear dynamic analysis (i.e. Modal analysis and Rayleigh's method) for individual models to the current building code formulae with their associated limitations. Thus, it is intended to evaluate how good the code formulae with respect to results from the linear dynamic analysis. Finally, the mode shapes of buildings in both orthogonal directions and rotations are studied to understand the effect of various structural configurations.

4.2 Fundamental Time Period for Reinforced Concrete Buildings

The fundamental time periods of all the 39 selected RC buildings were computed using different methods available in literature including current building code based empirical formulae. These methods are explained in Chapter 2 and 3.

The fundamental periods for all the selected buildings as obtained from different methods are tabulated in Tables 4.1-4.3. Table 4.1 presents the results of all buildings i.e. moment-resisting framed buildings, RC framed buildings with concrete shear wall as well as buildings governed by translation and rotation, which is the effect of building irregularity, mode of vibration. As shown in Table 4.1 two of moment-resisting buildings and four of framed buildings with concrete shear wall were being governed by rotation mode of vibration. Table 4.2 presents the results of moment-resisting RC buildings whereas Table 4.3 presents the results of RC framed buildings with concrete shear walls with neglecting buildings whose mode of vibration is governed by rotation. The fundamental periods presented here are calculated as per different current building code empirical equations such as ESEN 1998-1:2015, EC 8-2004, UBC-94, ASCE 7:2010, ATC-06, IS1893:2000 and BSLJ as well as Rayleigh method, and period obtained from Modal analysis.

Table 4.1: Summary of building data, fundamental period, and governing mode and modal mass participation ratio of fundamental mode for reinforced concrete buildings.

SN	Bldg. Code	No. of Story	H(m)	Lmax / Lmin	S.W	Ac (m ²)	T1 Modal	T1 Rayleigh	T1= 2·√d	T1= 3/4 C _H	D(m)	T1 = 0.09H/√D	0.02H	0.1N	T = 0.0063H/√Ac	Governing Mode	MMPREM	Remark
1	SOS	3	9	1.2019	NO	-	0.6	0.56702	0.5654	0.3897	11.538	0.2385	0.18	0.3		TY	82.81%	For buildings 11, 14, 15, 17, 18, 19, 21, 22, 23, 25-33, 35, 36 and 38 the effective cross sectional area of shear wall is less than 0.6m ² , which overestimate s the fundamental Period by increasing the regression coefficient, Ct.
2	MAR	3	9	1.1342	NO	-	0.471	0.45484	0.4429	0.3897	10.31	0.2523	0.18	0.3		TX	86.73%	
3	HHSC	3	9.6	1.3061	NO	-	0.485	0.41747	0.4817	0.409	24.5	0.1746	0.192	0.3		Rotation	19.82%	
4	SZD	3	9	1.9136	NO	-	0.58	0.52998	0.53	0.3897	22	0.1727	0.18	0.3		TY	89.30%	
5	SRA	3	9.6	1.1428	NO	-	0.601	0.58325	0.5616	0.409	26.515	0.1678	0.192	0.3		TX	94.49%	
6	HD	3	9	4.1071	NO	-	0.62	0.5959	0.5917	0.3897	5.6	0.3423	0.18	0.3		TY	90.09%	
7	TVET	3	9	1.3257	NO	-	0.612	0.58124	0.5628	0.3897	23.2	0.1682	0.18	0.3		TY	86.88%	
8	HSC	4	11.4	1.341	NO	-	0.719	0.68345	0.6743	0.4653	26.1	0.2008	0.228	0.4		TX	84.07%	
9	SEB	4	12.8	1.1195	NO	-	0.593	0.59292	0.6407	0.5083	20.543	0.2547	0.257	0.4		Rotation	55.86%	
10	ALE	5	15	1.0742	NO	-	0.806	0.76424	0.763	0.5716	13.348	0.3695	0.3	0.5		TY	87.20%	
11	MU	5	16.3	1.023	YES	0.2128	0.563	0.52371	0.5237	1.3189	34.997	0.248	0.326	0.5	0.55215571	TX	66.89%	
12	PO2	5	14.1	2	NO	-	0.851	0.79273	0.782	0.5457	8.5	0.4353	0.282	0.5		TY	85.10%	
13	YB	5	16.2	1.5094	NO	-	1.174	1.07683	1.0775	0.6056	10.6	0.4478	0.324	0.5		TY	75.13%	
14	KAS	6	18	1.2058	YES	0.1295	1.049	0.95039	0.9291	1.8213	18.245	0.3793	0.36	0.6	0.96137377	TX	78.18%	
15	MC	6	16.4	3.4001	YES	0.4686	0.619	0.59769	0.5915	0.8908	19.33	0.3347	0.327	0.6	0.59643262	TY	71.64%	
16	ABY	6	19.6	1.5094	NO	-	1.289	1.20856	1.1838	0.6986	10.6	0.5418	0.392	0.6		TY	84.36%	
17	HB	7	22.6	1.3219	YES	0.1571	0.796	0.76898	0.753	1.9593	21.15	0.4417	0.451	0.7	0.69286874	TY	76.95%	
18	OGO	7	23.1	1.3793	YES	0.184	1.306	1.20369	1.2002	1.8424	29	0.3861	0.462	0.7	1.1432438	TY	71.04%	
19	AO1	7	22.4	2.6163	YES	0.1744	0.72	0.70369	0.7164	1.849	17.2	0.4861	0.448	0.7	0.65200031	TY	65.88%	
20	AO2	7	22.4	1.1929	NO	-	1.043	0.95499	0.953	0.7722	25.215	0.4015	0.448	0.7		TY	74.41%	
21	JOS	7	21.3	1.026	YES	0.1277	0.719	0.64738	0.628	2.0811	19.7	0.4319	0.426	0.7	0.50182893	TY	64.74%	
22	WON	7	23.5	1.2937	YES	0.1837	1.471	1.33412	1.4674	1.8689	26.05	0.4147	0.47	0.7	1.2549543	TY	71.41%	
23	BG	7	23.4	1.1789	YES	0.3858	0.907	0.8908	0.8805	1.2847	28	0.398	0.468	0.7	0.83658615	TY	74.67%	
24	ABC	8	20	2.7129	NO	-	1.015	0.98414	1.0048	0.709	11.051	0.5412	0.4	0.8		TY	81.42%	
25	KAS1	8	26.5	1.6063	YES	0.2725	1.003	0.9884	0.9671	1.678	35.9	0.3981	0.53	0.8	0.88279758	TX	76.42%	
26	PO1	9	26.2	1.1465	YES	0.1017	1.389	1.36497	1.3595	2.7232	18.48	0.5485	0.524	0.9	1.13636615	TX	69.24%	
27	TAO	9	29.1	1.0882	YES	0.1638	1.631	1.63319	1.5936	2.3219	18.5	0.6089	0.582	0.9	1.48819621	Rotation	9.21%	
28	BIY	9	31.1	1.1414	YES	0.2814	1.271	1.15482	1.1496	1.8598	27.85	0.5295	0.621	0.9	1.06650242	TY	69.33%	
29	NEW1	9	28.8	1.1692	YES	0.544	0.867	0.7872	0.7784	1.2642	23.35	0.5364	0.576	0.9	0.73725786	Rotation	5.40%	
30	NEW2	9	28.8	1.1692	YES	0.544	0.977	0.87356	0.8699	1.2642	23.35	0.5364	0.576	0.9	0.79967746	Rotation	15.11%	
31	NEW3	9	28.8	1.2625	YES	0.3266	0.937	0.90175	0.8964	1.6315	18.66	0.6	0.576	0.9	0.84727464	TY	67.80%	
32	TYP1	9	28.1	1.0223	YES	0.2074	0.798	0.74162	0.7383	2.0074	17.9	0.5967	0.561	0.9	0.70946313	TY	65.00%	
33	DEJ	10	30.4	1.3193	YES	0.0259	1.503	0.90175	1.4748	6.0262	28.15	0.5148	0.607	1	1.35226227	TY	61.94%	
34	KAS2	10	33	1.0415	YES	1.3259	0.793	0.74367	0.7375	0.8968	35.15	0.5009	0.66	1	0.69652519	TX	62.49%	
35	TYP2	10	31.9	1.208	YES	0.2062	1.227	1.13685	1.1343	2.2151	11.3	0.853	0.637	1	1.08054752	TX	66.48%	
36	SOF	11	33.2	1.2525	YES	0.197	1.326	1.20366	1.1979	2.3365	15.25	0.7649	0.664	1.1	1.04705881	TX	63.03%	
37	AE1	13	41.6	1.0796	YES	0.8828	1.321	1.193	1.1788	1.3075	39.691	0.5943	0.832	1.3	1.12588053	TY	66.38%	
38	KAS3	13	41.6	2.5082	YES	0.3976	1.665	1.52993	1.4888	1.9483	18.3	0.8752	0.832	1.3	1.40566764	TY	62.03%	
39	AE2	14	44.8	1.0662	YES	0.8769	1.367	1.2849	1.2758	1.3869	39.691	0.64	0.896	1.4	1.1329097	TY	64.22%	

Note: Table 4.1 presents buildings arranged in ascending order of their number of stories.

4.2.1 Fundamental Period of RC Moment-Resisting Framed Buildings

The results presented in Tables 4.2 is also shown graphically in Figures 4.1 - 4.3 separately as period obtained from rational analysis (Modal analysis and Rayleigh's method), period obtained from empirical formulae and finally to compare results of rational analysis and empirical formulae for better understanding. The fundamental periods, T_1 , of 3 to 8 story reinforced concrete moment-resisting buildings are plotted in bar chart with their specified codes. It can be seen that the difference of mean and standard deviation of fundamental period obtained from Modal analysis is almost similar with that obtained from the Rayleigh's method and $T=2\cdot\sqrt{d}$, which is an empirical formula derived from the Rayleigh's method, as shown in Figure 4.1 and Table 4.2 and 4.3, because in all three cases the structural properties and deformational characteristics of the resisting elements in the substantiated analysis are considered. Moreover, the result obtained from the Rayleigh's method is more similar to that obtained from Modal analysis and then the results from the empirical formula, Eq. (2.22), follows.

Table 4.2: Summery of reinforced concrete moment-resisting framed buildings

SN	Bldg Code	No of Story	H(m)	Lmax/Lmin	$T_{1\text{modal}}$	$T_{1\text{Rayleigh}}$	$T_1=2\cdot\sqrt{d}$	$T_1=C_{RH}^{3/4}$	$T=0.0466H^{0.9}$	$T=0.02H$	Govern Mode	MMPRFM
1	SOS	3	9	1.201875	0.6	0.567022	0.565365	0.3897114	0.336669811	0.18	TY	82.81%
2	MAR	3	9	1.134213	0.471	0.454835	0.442894	0.3897114	0.336669811	0.18	TX	86.73%
3	SZD	3	9	1.913636	0.58	0.529978	0.530029	0.3897114	0.336669811	0.18	TY	89.30%
4	HD	3	9	4.107143	0.62	0.5959	0.591716	0.3897114	0.336669811	0.18	TY	90.09%
5	TVET	3	9	1.325714	0.612	0.581237	0.562756	0.3897114	0.336669811	0.18	TY	86.88%
6	SRA	3	9.6	1.142759	0.601	0.583251	0.56158	0.409039	0.356804256	0.192	TX	94.49%
7	HSC	4	11.4	1.340996	0.719	0.683454	0.674282	0.4653073	0.416485881	0.228	TX	84.07%
8	P02	5	14.1	2	0.851	0.792729	0.78203	0.5457272	0.504293207	0.282	TY	85.10%
9	ALE	5	15	1.074168	0.806	0.764244	0.762984	0.5716493	0.533172879	0.3	TY	87.20%
10	YB	5	16.2	1.509434	1.174	1.076832	1.077493	0.6056163	0.571412096	0.324	TY	75.13%
11	ABY	6	19.6	1.509434	1.289	1.208563	1.183775	0.6986398	0.67829151	0.392	TY	84.36%
12	ABC	8	19.99	2.712877	1.015	0.984138	1.004782	0.7090402	0.690426467	0.3998	TY	81.42%
13	AO2	7	22.4	1.192941	1.043	0.954995	0.952962	0.7722311	0.764907877	0.448	TY	74.41%

Table 4.3: Percentage difference of fundamental period obtained by code based formulae relative to Modal analysis for RC MRF buildings.

SN	Bldg. Code	%ge d/c b/n Modal analysis and Rayleigh method	%ge d/c b/n Modal analysis and $T= 2.\sqrt{d}$	%ge d/c b/n Modal analysis and $T= 0.075H^{3/4}$	%ge d/c b/n Modal analysis and $T= 0.0466H^{0.9}$	%ge d/c b/n Modal analysis and $T= 0.02H$
1	SOS	5.50	5.77	35.05	43.90	70
2	MAR	3.43	5.97	17.26	28.52	61.78
3	SZD	8.62	8.62	32.81	41.95	68.97
4	HD	3.89	4.56	37.14	45.70	70.97
5	TVET	5.03	8.05	36.32	44.99	70.59
6	SRA	2.95	6.56	31.94	40.63	68.05
7	HSC	4.94	6.22	35.28	42.07	68.29
8	P02	6.85	8.11	35.87	40.74	66.86
9	ALE	5.18	5.34	29.08	33.85	62.78
10	YB	8.28	8.22	48.41	51.33	72.40
11	ABY	6.24	8.16	45.80	47.38	69.59
12	ABC	3.041	1.01	30.14	31.98	60.61
13	AO2	8.44	8.63	25.96	26.66	57.05

Table 4.4: Difference of mean and standard deviation for fundamental period obtained from code based formulae relative to Modal analysis for RC MRF buildings.

SN	Bldg. Code	D/c b/n Modal analysis and Rayleigh method	D/c b/n Modal analysis and $T= 2.\sqrt{d}$	D/c b/n Modal analysis and $T= 0.075H^{3/4}$	D/c b/n Modal analysis and $T= 0.0466H^{0.9}$	D/c b/n Modal analysis and $T= 0.02H$
1	SOS	0.033	0.035	0.21	0.263	0.42
2	MAR	0.016	0.028	0.081	0.134	0.291
3	SZD	0.05	0.05	0.19	0.243	0.4
4	HD	0.024	0.028	0.23	0.283	0.44
5	TVET	0.031	0.049	0.222	0.275	0.432
6	SRA	0.018	0.039	0.192	0.244	0.409
7	HSC	0.036	0.045	0.254	0.303	0.491
8	P02	0.058	0.069	0.305	0.347	0.569
9	ALE	0.042	0.043	0.234	0.273	0.506
10	YB	0.097	0.097	0.568	0.603	0.85
11	ABY	0.08	0.105	0.59	0.611	0.897
12	ABC	0.031	0.01	0.306	0.325	0.615
13	AO2	0.088	0.09	0.271	0.278	0.595
Mean		0.046	0.053	0.281	0.322	0.532
S. Deviation		0.027	0.029	0.144	0.136	0.176

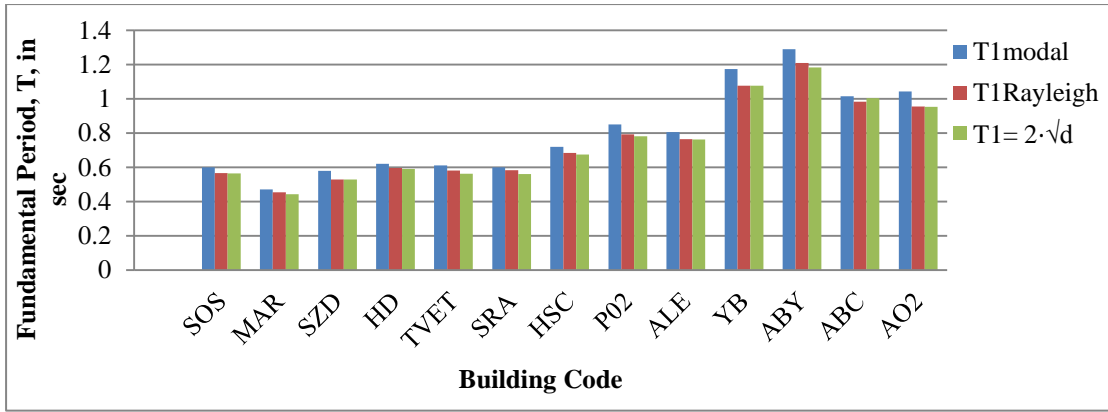


Figure 4.1: Fundamental period from rational analysis Vs building codes for RC moment-resisting framed buildings

As shown in figure 4.2 the fundamental period obtained from the empirical formulae presented in current building codes like ESEN 1998-1:2015, EC 8-2004, UBC-94, ASCE 7:2010, ATC-06 and IS1893:2000 Eqn. (2.1) and the empirical formula derived by Goel and Chopra [1997] (Eqn.2.6) give almost equal results but the empirical formula presented in Japan building code, BSLJ, Eqn. (2.9), underestimates the fundamental period than equation 2.1 and 2.6. As presented in Table 4.4 the difference of period obtained from Modal Analysis and code based empirical formulae, the difference of mean and standard deviation indicates the accuracy of estimating fundamental period decreases from left to right relative to period obtained from Modal Analysis.

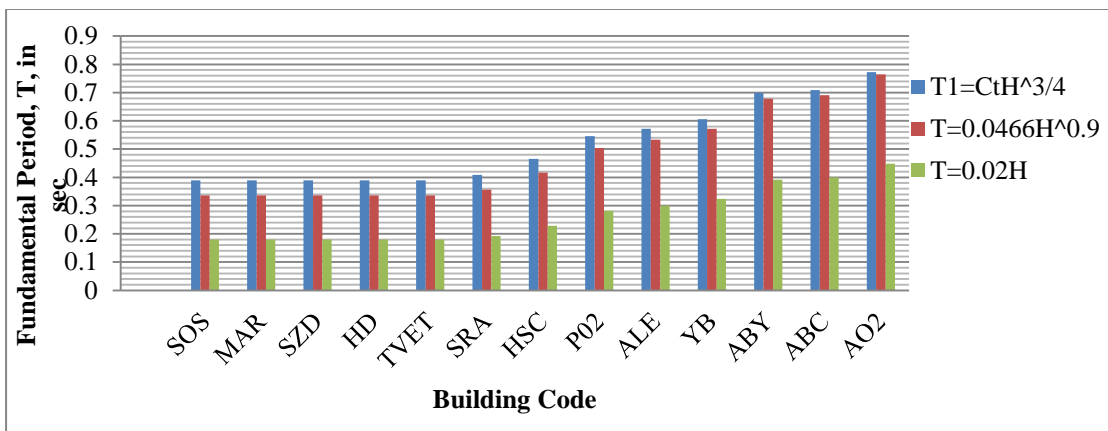


Figure 4.2: fundamental period estimated from empirical formulae Vs building codes for RC moment-resisting framed buildings

However, the rational analysis considers the structural properties and deformational characteristics of the resisting elements in the analysis [2], for new structures these calculations cannot be carried out until the system is designed. Therefore, the code-based empirical formulae have the advantage that no structural analyses are required for their use, and they are thus suitable for preliminary design purposes.

As shown in the figure 4.3, and Tables 4.2 and 4.3 that the results obtained from code empirical formulae give the lower fundamental period than that obtained from Modal analysis for moment-resisting framed buildings. Also comparing the accuracy of fundamental period obtained from Modal analysis and empirical formulae, the results obtained from an equation 2.1 is better than that obtained from equations 2.6 and 2.9. Therefore, using empirical formula, in current code of ESEN 1998-1:2015, EC 8-2004, UBC-94, ASCE 7:2010, ATC-06 and IS1893:2000 Eqn. (2.1), is fairly good for preliminary design of moment-resisting RC framed buildings and using Eqn. 2.9 underestimates the fundamental period and consequently increases the base shear.

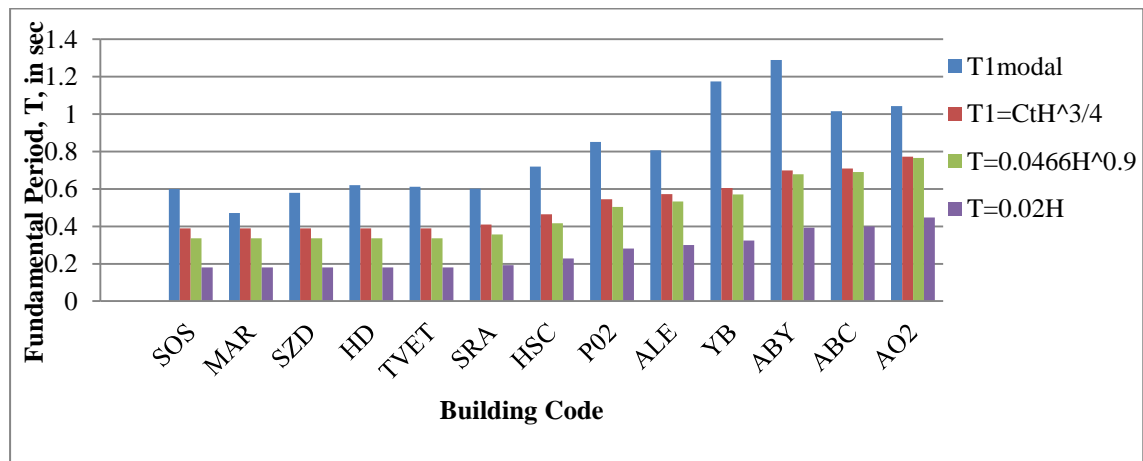


Figure 4.3: Comparison of fundamental period estimated from empirical formulae and Modal analysis for moment- resisting framed buildings

4.2.2 Fundamental Period of RC Frame with Concrete Shear Wall Buildings

Table 4.5: Summary of reinforced concrete buildings with concrete shear wall

SN	Bldg. Code	No. of Story	H(m)	Lmax/Lmin	Ac (m ²)	T1 Modal	T1 Rayleigh	T1= 2·√d	T1= $\frac{3/4}{C_H}$	D(m)	T1 = $\frac{0.09H\sqrt{D}}$	0.02H	0.1N	T = $\frac{0.0063H\sqrt{A_e}}$	Governing Mode	MMPREM	Remark
1	KAS	6	18	1.206	0.129	1.049	0.95039	0.929	1.821	18.25	0.37927	0.36	0.6	0.9613738	TX	78.18%	For buildings 1, 3-14, 16, 17 and 19 the effective cross sectional area of shear wall is less than 0.6m ² , which overestimate s the fundamental period by increasing regression coefficient, Ct.
2	MC	6	16.4	3.4	0.469	0.619	0.59769	0.591	0.891	19.33	0.33469	0.327	0.6	0.5964326	TY	71.64%	
3	HB	7	22.6	1.322	0.157	0.796	0.76898	0.753	1.959	21.15	0.44169	0.451	0.7	0.6928687	TY	76.95%	
4	OGO	7	23.1	1.379	0.184	1.306	1.20369	1.2	1.842	29	0.38606	0.462	0.7	1.1432438	TY	71.04%	
5	AO1	7	22.4	2.616	0.174	0.72	0.70369	0.716	1.849	17.2	0.4861	0.448	0.7	0.6520003	TY	65.88%	
6	JOS	7	21.3	1.026	0.128	0.719	0.64738	0.628	2.081	19.7	0.43191	0.426	0.7	0.5018289	TY	64.74%	
7	WON	7	23.5	1.294	0.184	1.471	1.33412	1.467	1.869	26.05	0.41474	0.47	0.7	1.2549543	TY	71.41%	
8	BG	7	23.4	1.179	0.386	0.907	0.8908	0.88	1.285	28	0.398	0.468	0.7	0.8365862	TY	74.67%	
9	KAS1	8	26.5	1.606	0.273	1.003	0.9884	0.967	1.678	35.9	0.39805	0.53	0.8	0.8827976	TX	76.42%	
10	PO1	9	26.2	1.147	0.102	1.389	1.36497	1.36	2.723	18.48	0.54852	0.524	0.9	1.1363661	TX	69.24%	
11	BIY	9	31.1	1.141	0.281	1.271	1.15482	1.15	1.86	27.85	0.52953	0.621	0.9	1.0665024	TY	69.33%	
12	NEW3	9	28.8	1.263	0.327	0.937	0.90175	0.896	1.632	18.66	0.60004	0.576	0.9	0.8472746	TY	67.80%	
13	TYP1	9	28.1	1.022	0.207	0.798	0.74162	0.738	2.007	17.9	0.59669	0.561	0.9	0.7094631	TY	65.00%	
14	DEJ	10	30.4	1.319	0.026	1.503	0.90175	1.475	6.026	28.15	0.51483	0.607	1	1.3522623	TY	61.94%	
15	KAS2	10	33	1.042	1.326	0.793	0.74367	0.738	0.897	35.15	0.50095	0.66	1	0.6965252	TX	62.49%	
16	TYP2	10	31.9	1.208	0.206	1.227	1.13685	1.134	2.215	11.3	0.853	0.637	1	1.0805475	TX	66.48%	
17	SOF	11	33.2	1.252	0.197	1.326	1.20366	1.198	2.336	15.25	0.76492	0.664	1.1	1.0470588	TX	63.03%	
18	AE1	13	41.6	1.08	0.883	1.321	1.193	1.179	1.308	39.69	0.59428	0.832	1.3	1.1258805	TY	66.38%	
19	KAS3	13	41.6	2.508	0.398	1.665	1.52993	1.489	1.948	18.3	0.87521	0.832	1.3	1.4056676	TY	62.03%	
20	AE2	14	44.8	1.066	0.877	1.367	1.2849	1.276	1.387	39.69	0.63999	0.896	1.4	1.1329097	TY	64.22%	

Table 4.6: Percentage difference of fundamental period obtained by code based formulae relative to Modal analysis for RC framed with concrete shear wall buildings

S N	Bldg. Code	%ge d/c b/n Modal analysis and Rayleig h	%ge d/c b/n Modal analysis and $T=2\sqrt{d}$	%ge d/c b/n Modal analysis and $T=\frac{0.075}{\sqrt{A_c}}H^{3/4}$	%ge d/c b/n Modal analysis and $T=0.9\frac{H}{\sqrt{D}}$	%ge d/c b/n Modal analysis and $T=0.05H^{3/4}$	%ge d/c b/n Modal analysis and $T=0.02H$	%ge d/c b/n Modal analysis and $T=\frac{0.0063}{\sqrt{A_e}}\frac{H}{\sqrt{D}}$
1	MC	3.44	4.45	43.92	34.32	45.93	47.17	3.65
2	KAS	6.73	8.82	78.74	57.12	62.78	64.67	5.66
3	JOS	9.96	12.65	189.45	31.05	39.93	40.75	30.20
4	AO1	2.27	0.50	156.81	28.50	32.49	37.78	9.44
5	HB	3.39	5.40	146.15	34.96	44.51	43.29	12.96
6	OGO	7.83	8.10	41.07	59.66	70.44	64.62	12.46
7	BG	1.79	2.92	41.64	41.35	56.12	48.40	7.76
8	WON	9.31	0.25	27.05	63.70	71.81	68.02	14.69
9	P01	1.73	2.12	96.05	58.31	60.51	62.28	18.19
10	KAS1	1.46	3.58	67.30	41.78	60.31	47.16	11.98
11	TYP1	7.07	7.49	151.55	23.63	25.23	29.70	11.09
12	NEW3	3.76	4.33	74.12	33.66	35.96	38.53	9.58
13	DEJ	0.77	1.88	300.94	56.98	65.75	59.61	10.03
14	BIY	9.14	9.55	46.32	48.25	58.34	51.14	16.09
15	TYP2	7.35	7.56	80.53	45.35	30.48	48.07	11.94
16	KAS2	6.22	6.99	13.09	13.19	36.83	16.77	12.17
17	SOF	9.23	9.66	76.21	47.86	42.31	49.94	21.04
18	AE1	9.69	10.77	1.02	38.00	55.01	37.02	14.77
19	KAS3	8.11	10.58	17.01	50.81	47.44	50.03	15.58
20	AE2	6.01	6.67	1.45	36.66	53.18	34.46	17.12

Table 4.7: Difference of mean and standard deviation for fundamental period obtained from code based formulae relative to Modal analysis for RC framed with concrete shear wall buildings

SN	Bldg. Code	D/c b/n MA and RM	D/c b/n MA and $T = 2\sqrt{d}$	D/c b/n MA and $T = \frac{0.075}{\sqrt{A_c}} H^{3/4}$	D/c b/n MA and $T = 0.09 \frac{H}{\sqrt{D}}$	D/c b/n MA and $T = 0.05H^{3/4}$	D/c MA and $T = 0.02H$	D/n MA and $T = \frac{0.0063}{\sqrt{A_e}} H$
1	MC	0.021	0.028	0.272	0.212	0.284	0.292	0.023
2	KAS	0.069	0.09	0.802	0.582	0.64	0.659	0.058
3	JOS	0.072	0.091	1.362	0.223	0.287	0.293	0.217
4	AO1	0.016	0.004	1.129	0.205	0.234	0.272	0.068
5	HB	0.027	0.043	1.163	0.278	0.354	0.3446	0.103
6	OGO	0.102	0.106	0.536	0.779	0.92	0.844	0.163
7	BG	0.016	0.027	0.378	0.375	0.509	0.439	0.07
8	WON	0.137	0.004	0.398	0.937	1.056	1.0006	0.216
9	P01	0.024	0.029	1.334	0.81	0.84	0.865	0.253
10	KAS1	0.015	0.036	0.675	0.419	0.605	0.473	0.12
11	TYP1	0.056	0.06	1.209	0.189	0.201	0.237	0.089
12	NEW3	0.035	0.041	0.695	0.315	0.337	0.361	0.09
13	DEJ	0.012	0.028	4.523	0.856	0.988	0.896	0.151
14	BIY	0.116	0.121	0.589	0.613	0.741	0.65	0.204
15	TYP2	0.09	0.093	0.988	0.556	0.374	0.5898	0.146
16	KAS2	0.049	0.055	0.104	0.105	0.292	0.133	0.096
17	SOF	0.122	0.128	1.01	0.635	0.561	0.6622	0.279
18	AE1	0.128	0.142	0.013	0.502	0.727	0.489	0.195
19	KAS3	0.135	0.176	0.283	0.846	0.79	0.833	0.259
20	AE2	0.082	0.091	0.02	0.501	0.727	0.471	0.234
Mean		0.066	0.07	0.874	0.497	0.573	0.5402	0.152
S. Deviation		0.045	0.048	0.962	0.257	0.262	0.2524	0.077

As shown in the Table 4.5, however, the height of building is greater its fundamental period obtained from rational analysis is being smaller than a building that has smaller height than it. That is why the rational analysis considers the structural properties and deformational characteristics of the resisting elements in the analysis. Also, it can be observed that the difference of mean and standard deviation of the fundamental period obtained from rational analysis (Modal analysis and Rayleigh's method), and Eq. (2.22) is almost similar. Comparing the accuracy with Modal analysis, Rayleigh's method is closer and then Eq. (2.22) follows as presented in Table 4.7 and figure 4.4.

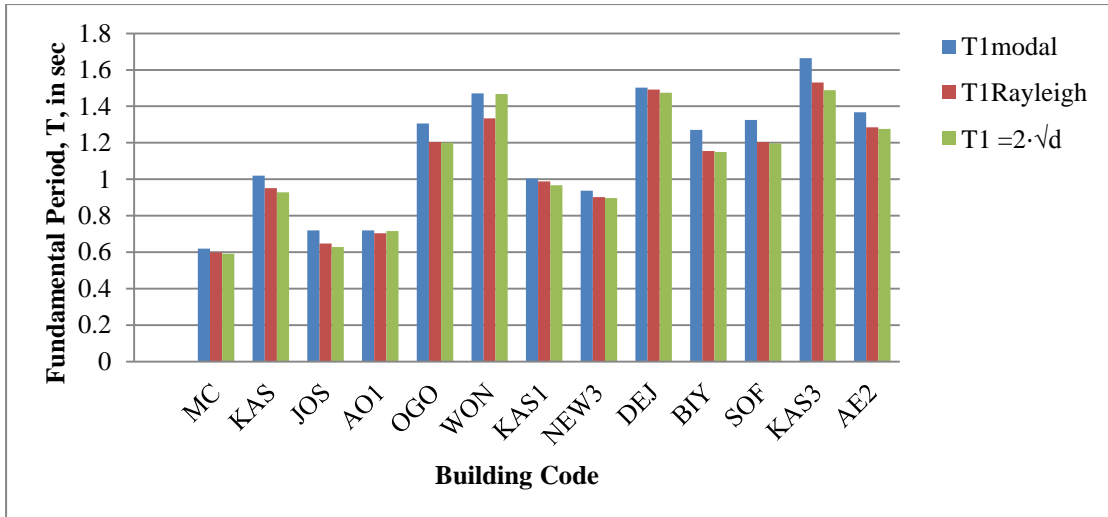


Figure 4.4: Fundamental period from rational analysis Vs building codes for RC buildings with concrete shear wall

In figure 4.5, the empirical formulae those consist only one building parameter, which is height simply follow a smooth line. The empirical formulae those contain more than one variable, however, the estimation is conservative their result follow almost the same pattern as that obtained from rational analysis.

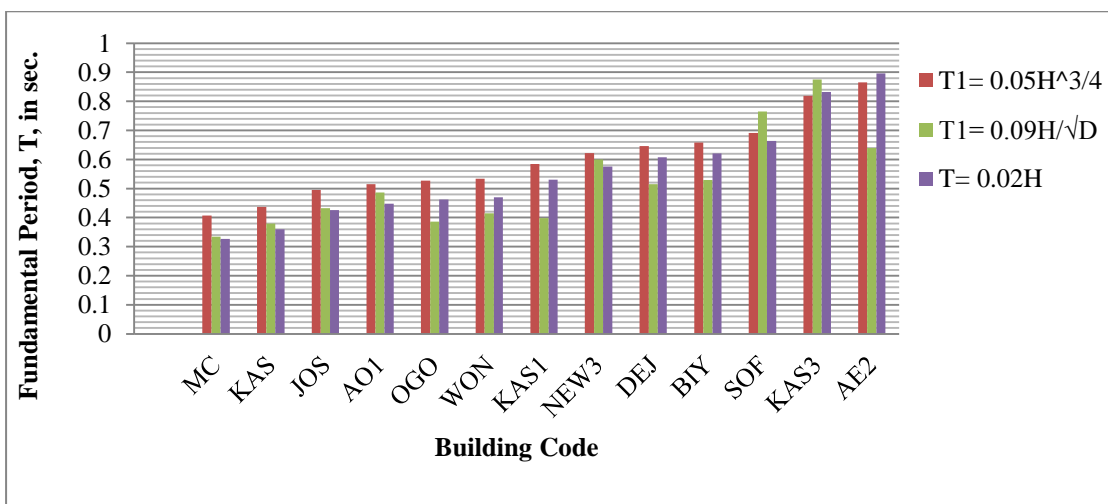


Figure 4.5: Fundamental period estimated from empirical formulae Vs building codes for RC buildings with concrete shear wall

When we come to the period obtained from the empirical formula has a form $T = 0.075H^{3/4}/\sqrt{A_c}$ is mainly depends on building height and effective cross sectional area

of shear wall. From Table 4.3 effective area of shear wall, A_c , of building MC is almost 3.62 times that of KAS and JOS, and its fundamental period is smaller than that of the two buildings. However, effective area of shear wall, A_c , of KAS and JOS is almost equal and their fundamental period obtained from the above empirical formula are exaggerated as compared with that obtained from rational analysis as shown in Table 4.5 and 4.6; That is why the effective area shear wall, A_c , of these buildings is being equal to 0.129498m^2 and the coefficient $0.075/\sqrt{A_c} = 0.2084$, which causes the fundamental period even greater than the fundamental period obtained from the same buildings without shear wall as $T = 0.075H^{3/4}$ and that obtained from Modal analysis as shown in Table 4.5. Therefore, using the empirical formula for a building with small effective cross sectional area of shear wall overestimates and buildings with large effective cross sectional area of shear wall underestimates the fundamental period as compared with Modal analysis; but as shown in figure 4.6 the empirical formula proposed by Goel and Chopra, Eqn.2.20, estimates the satisfactory results. Also figure 4.6 shows that the code empirical formulae give the lower fundamental period than that obtained from Modal analysis for moment-resisting frame with concrete shear wall buildings. Furthermore, as presented in Table 4.7 the difference of period obtained from Modal Analysis and code based empirical formulae, the difference of mean and standard deviation indicates estimating fundamental period using Rayleigh's method is more the accurate relative to Modal Analysis and then Eqn.2.22 follows because they are rational methods. The empirical formula proposed by Goel and Chopra, Eqn.2.20, estimates more accurate fundamental period relative to Modal Analysis than other empirical formulae.

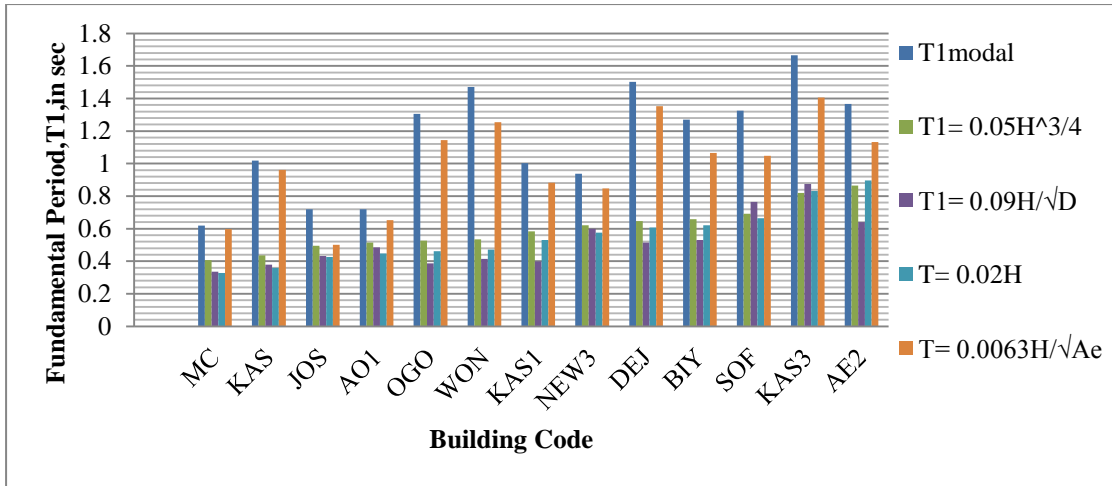


Figure 4.6: Comparison of fundamental period estimated from empirical formulae and Modal analysis for moment-resisting framed buildings with concrete shear wall

4.3 Parameters Affecting Fundamental Period of RC Buildings

One of the main objectives of the present study is to identify the structural parameters that can affect fundamental period of reinforced concrete buildings. It is, therefore, required to know the important parameters which control the fundamental period of reinforced concrete buildings. This section analyses the fundamental period computed using the rational methods (i.e. Modal analysis and Rayleigh method) and code based empirical formulae against different possible parameters. Although the results of all the selected buildings are considered for analysis, results of 39 building are presented in Table 4.1, 4.2 and 4.5 for convenience.

4.3.1 Effect of Number and Width of Bays

Results of the analysis are showed in Table 4.4, in which values of the fundamental period of vibration are tabulated with building height, number of stories, and linear dynamic analysis i.e. Modal analysis, and Rayleigh’s method. From Table 4.8, it can be observed that the value of fundamental period of vibration corresponding to first mode of vibration obtained from dynamic analysis varies from building to building

depending on different structural parameters but, the period obtained from a specified code based empirical formulae are similar for all buildings with different width and number of bays and the same number of story and building height, i.e. 9m.

Table 4.8: Effect of bay number and width on fundamental period of vibration

Bldg. Code	No. of Bays	Average Width of Bays	H (m)	T1 Modal	T1 Rayleigh	T1= $2\cdot\sqrt{d}$	T1= $CtH^{3/4}$	T= $0.0466H^{0.9}$	T= $0.02H$	T= $0.1N$
MAR	5	4.33	9	0.471	0.435	0.482	0.390	0.337	0.18	0.3
SOS	3	4.725	9	0.6	0.543	0.639	0.390	0.337	0.18	0.3
TVET	4	4.9	9	0.612	0.578	0.619	0.390	0.337	0.18	0.3
SZD	4	5.5	9	0.58	0.530	0.594	0.390	0.337	0.18	0.3
HD	1	5.9	9	0.62	0.596	0.663	0.390	0.337	0.18	0.3

As shown in figure 4.7, the period obtained from dynamic analysis decreases with an increase in number of bays along the direction of motion. However, with an increase in number of bays transverse to the direction of motion the period increases, which can be realized from Table 4.8, Appendix A as well as Appendix B for buildings TVET and SZD. Code based empirical formulae on other hand yields invariant (remain unchanged) of period than that obtained by dynamic analysis. The difference widens with decrease in number of bays along the direction of motion or with increase in number of bays transverse to direction of motion. Hence code based formulae lead to conservative evaluation of base shear as it is inversely proportional to $T^{2/3}$.

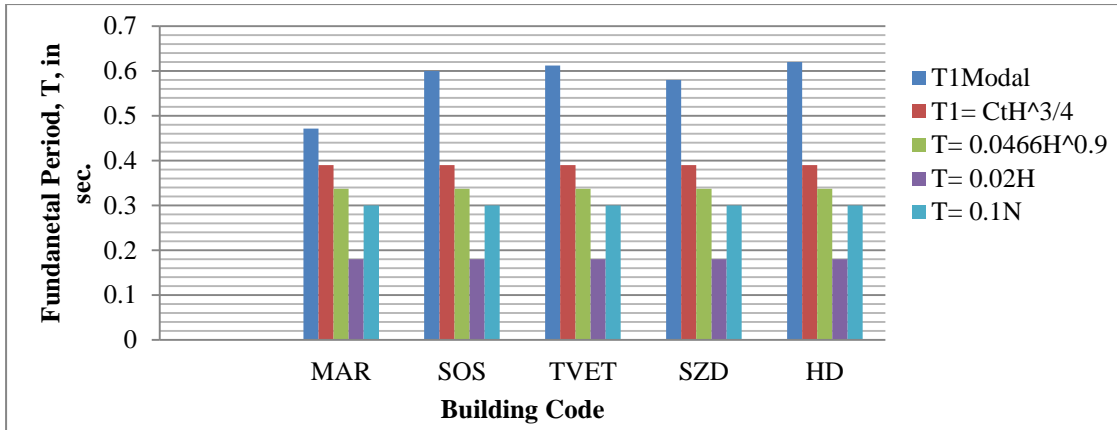


Figure 4.7: Effect of number and width of bays on fundamental vibration period along the direction of motion with the same building height.

The fundamental period obtained through dynamic analysis show that the period of structure increases as the bay width increase as shown in figure 4.7 and Table 4.8. The code based empirical formulae, however, yield fundamental period invariant of width of bays.

4.3.2 Effect of Story Stiffness

At the most beginning in Table 4.5 the three buildings with the height of 16.35m, 18m and 21.3m with codes of MC, KAS and JOS respectively are presented with different fundamental periods. In the case the fundamental period of building KAS with height of 18m is greater than that of building MC with height of 16.35m but it is less than that of building JOS with height of 21.3m. This is because all the three cases the fundamental period depends on story mass and stiffness of the building. As we know that the story stiffness is inversely related with the fundamental period of vibration as $T = 2\pi\sqrt{m/K}$ from modal analysis and similar form with different coefficient of Rayleigh's method. Similarly as the story stiffness increase the story displacement decrease, and consequently the fundamental period decreases as the form $T = 2\sqrt{d}$. As

shown in Table 4.3 the effective cross sectional area of shear wall of a building MC is greater than both of KAS and JOS whereas it is almost equal for buildings KAS and JOS. As we know, shear walls provided in the building to resist lateral forces by increasing the story stiffness. Therefore the fundamental period of building MC is being smaller than these two buildings is valid because its effective cross sectional area of shear wall is almost 3.62 times these two buildings. On the other hand KAS and JOS have almost the same effective cross sectional area of shear wall but KAS is more slender than JOS, therefore the building KAS possess a reduced story stiffness and consequently it possess greater fundamental period than JOS, however, the total height of JOS is slightly greater.

Similarly the last three rows in Table 4.5 are the fundamental periods and other necessary data of the buildings AE1, KAS3 and AE2 respectively. As discussed above, the structural configuration (effective shear wall/s parallel to the direction of maximum story displacement, slenderness) and other parameters can affect the fundamental period of vibration rather relating the fundamental period with only one parameter like building height or/and lateral dimension (D) or number of story of the building.

Table 4.9: Effect of stiffness on fundamental period of vibration

Bldg. Code	No. of Story	H (m)	Lmax/Lmin	Ac (m ²)	T1 Modal	T1 Rayleigh	T1= 2·√d	T1= 0.05H ^{3/4}	T1= 0.09H/√D	T= 0.0063H/√Ae
P01	9	26.2	1.147	0.102	1.340	1.398	1.596	0.579	0.5485	1.1364
TYP1	9	28.0	1.022	0.207	0.798	0.742	0.901	0.609	0.5967	0.7095
NEW3	9	28.8	1.263	0.327	0.937	0.902	1.049	0.622	0.6000	0.8473
BIY	9	31.0	1.141	0.281	1.271	1.155	1.265	0.658	0.5295	1.0665

Further expression from Table 4.9 and figure 4.9, four buildings with identical story number, and different building height, slenderness, $\lambda = L_{max}/L_{min}$, and effective cross-sectional area of shear wall, A_c , on which the story stiffness of a building mainly depends upon in addition to the cross-section of moment-resisting frames of the reinforced concrete building. As shown in Table 4.9 and figure 4.9, however, the height of building P01 is smaller, its slenderness is quite equal with building BIY, but its fundamental period is larger than other buildings due to the flexibility of building as its effective cross-sectional area of shear wall is smaller as 104% - 221% than the other, which leads to the building to be less stiffer than the others and consequently increases the fundamental period of vibration.

On other hand buildings TYP1, NEW3 and BIY as their height arranged in ascending order, their fundamental period obtained from rational analysis and empirical formula proposed by Goel and Chopra as Eq.(20) are nearly the same. However, from Table 4.9 the effective cross-sectional area of shear wall of the building NEW3 is greater than that of building BIY but their fundamental period of vibration is reverse because the slenderness of BIY is smaller than that of NEW3. Further conclusions can be drawn for the effect of slenderness of building reduces the stiffness of building and consequently the fundamental period of vibration increases, which results in reduction in base shear while designing a new building.

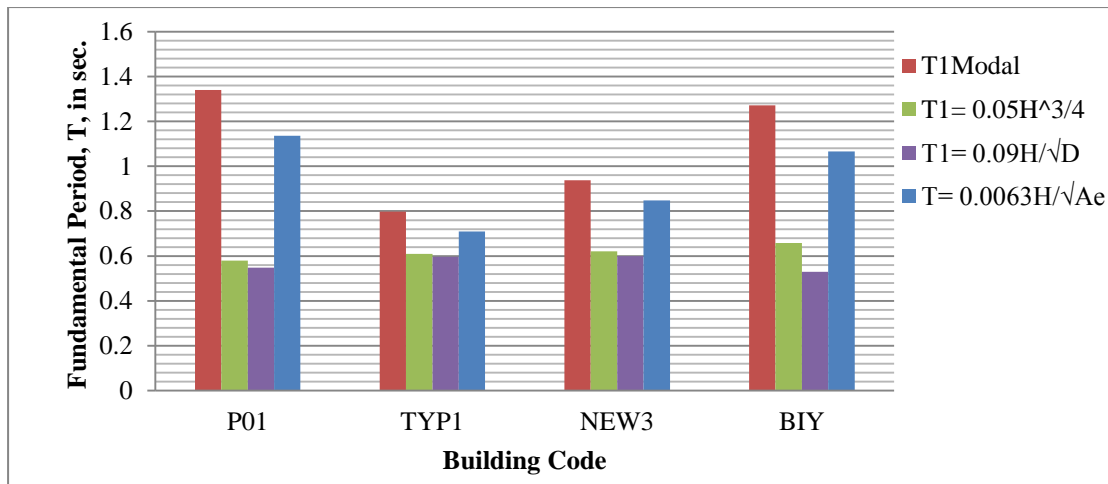


Figure 4.8: Fundamental vibration period versus building codes with the same number of story

4.3.3 Effect of Height and Number of Stories

As per many current building codes, the approximate fundamental period of vibration, T_1 , is a function of height and pan dimension of a building, and in some codes, ASCE 7:2010, IBC, NBCC-95 and FEMA, 1994, it is also a function of number of stories.

The results compiled in Table 4.1, 4.2 and 4.5, the fundamental period of vibration obtained from both dynamic analysis (i.e. Modal analysis and Rayleigh's method) and code based empirical formulae increases as the height and number of stories of building increases, keeping the other structural parameters in mind.

4.3.4 Effect of Irregularity

In the present study the regularity of structure is expressed in detail. The irregularity of building may influence the magnitude of the seismic action through reducing natural period, increasing base shear and concentrating stresses in some area like re-entrant corners. From the Table 4.1 and Appendix A, as a building is being irregular consequently its fundamental period decreases, the mode of vibration is governed by

rotation rather than translation and the modal mass participation factor in fundamental mode is reduced than the regular building with same height.

4.4 Mode Shapes and Modal Mass Participation Factors

Computational modal analysis, which is part of the effort to identify the structural dynamic characteristics along with the 3D model, was performed within the ETABS 2015 software. Periods and corresponding mode shapes during this free vibration were determined for the structure. It is well known that a multistory structure has multiple degrees of freedom (DOFs) and mode shapes that describe the modes of vibration for the structure in terms of relative amplitudes and angles [4] and the mode shapes are typically characterized by structural properties.

A mode shape of any building is the deformed shape associated with its natural period. The vibration of a building at its first natural period is called the first mode shape, and is the most important mode shape since the majority of the buildings mass is used in this case.

The first three modes for a regular building occurs in the translational (x and y directions) and rotational modes. A regular building can be described in terms of the orientation of its lateral resisting system. The system has symmetry in along both X and Y directions, then its first dominant mode shape become pure translation. However, if the lateral resisting system causes asymmetry and disturb other regularity criteria, then not the translational modes but rather the rotational modes become more critical.

According to [2], Modal analysis shall include a sufficient number of modes to obtain a combined modal mass participation of the actual mass in each of the orthogonal

horizontal directions. The total number of modes for 3D frame analysis is three times the total number of stories of the structure. Therefore, overall response of a building is the sum of the responses of all of its modes. As presented in Table 4.8 the Modal analysis include total modal mass participation of all 24 modes of the structure ABC, which has eight stories that were assembled to be satisfactory with the ASCE code requirement. The table enables to identify the dominant direction of vibration for each of the first 24 modes. It is obvious from the table that the first five mode mass participation factors exceed 90 percent of the total mass of the structure in both direction, and the Y-direction is dominant due the less number of defense lines. The difference in the mass participation factors between orthogonal directions can be attributed to the difference in stiffness and mass moment-resisting frame system only because there are no shear walls in both directions for the selected building structure ABC as shown in Appendix B.

Table 4.10: Modal participating mass ratios for a selected structure, (ABC 8 storied)

Mode	Period (sec)	Participation in X-direction	Participation in Y-direction	Summation of Participation in X-direction	Summation of Participation in Y-direction
1	1.015	0	0.8142	0	0.8142
2	0.924	0.8144	0.0000007502	0.8144	0.8142
3	0.889	0.0013	0.0003	0.8157	0.8145
4	0.329	0	0.1132	0.8157	0.9277
5	0.298	0.1127	0	0.9284	0.9277
6	0.287	0.0001	0.0000288	0.9285	0.9277
7	0.186	0	0.0297	0.9285	0.9574
8	0.168	0.0304	0	0.9589	0.9574
9	0.161	0.00003995	0.000008164	0.9589	0.9574
10	0.125	0	0.0196	0.9589	0.977
11	0.114	0.0192	0	0.9781	0.977
12	0.11	0.00001435	0.00000318	0.9781	0.9771
13	0.093	0	0.0087	0.9781	0.9858
14	0.085	0.0083	0	0.9865	0.9858
15	0.083	0.00002772	0.000008715	0.9865	0.9858
16	0.073	0	0.0084	0.9865	0.9942
17	0.068	0.0084	0	0.9949	0.9942
18	0.066	0.000008044	0.00000162	0.9949	0.9942
19	0.063	0	0.0057	0.9949	0.9999
20	0.06	0.0041	0	0.999	0.9999
21	0.058	0	0	0.999	0.9999
22	0.058	0	0.0001	0.999	1
23	0.057	0.001	0	1	1
24	0.055	8.359E-07	0.000000654	1	1
25	0.008	0	0	1	1
26	0.008	0	0	1	1
27	0.008	0	0	1	1

Chapter Five: Conclusions and Recommendations

5.1 Conclusions

Fundamental period of all the selected building models were estimated as per rational analysis (i.e. Modal analysis, Rayleigh's method) and code based empirical equations. The results were critically analyzed and presented in chapter four. The aim of the analyses and discussions were to evaluate code based empirical equations, and to identify structural parameter that can affect the fundamental period of both moment-resisting frame and shear wall-frame system reinforced concrete buildings. Based on the work presented in this thesis following conclusions can be drawn:

1. It is found that the fundamental period of vibration in a RC framed building is not only a function of building height or number of stories. This study shows that buildings with the same overall height or number of stories have different fundamental periods with a significant variation of structural parameters which is not addressed in current building codes.
2. The fundamental period obtained even from rational analysis (Modal analysis and Rayleigh's method) varies for the buildings even have the same height, number of stories and number of bays due to variation of other structural parameters like story stiffness, effective cross-sectional area of shear wall, cross-sectional dimension of beams and columns, story mass and others.
3. Compared with the fundamental period obtained from rational analysis empirical formulae given in current building codes underestimate the fundamental period of vibration, which favours the structure by increasing the base shear.

4. The change in effective cross-section and configurations of shear wall in RC framed building may change the story stiffness of a building. In this study, it is found that as increase in effective cross-sectional area of shear wall in the direction of motion increases the story stiffness and consequently decreases the fundamental period of vibration and vice versa.
5. The empirical formulae for estimation of fundamental period of vibration proposed in current building codes are remain unchanged with structural parameters other than only building height, effective cross-sectional area of shear wall or number of stories. But the rational analysis (Modal analysis and Rayleigh's method) for estimation of fundamental period show that the width and number of bays of RC buildings have significant influence on fundamental period. The fundamental period of vibration increases with increasing bay width and increasing number of bays transverse to the direction of motion and the period decreases with increasing number of bays in the direction of motion.
6. The variation of fundamental period due to structural parameters that obtained from rational analysis is more for taller buildings and comparatively less for shorter buildings. It is also found that variation of fundamental periods calculated from Modal analysis and Rayleigh's method is below 10% for both moment-resisting frame and shear wall-frame system buildings.
7. The empirical formula given in Eqn.2.1 as suggested in many current building codes provides good estimation of fundamental period of vibration for moment-resisting frame buildings than other empirical formulae. Whereas the empirical formula proposed by Goel and Chopra in 1998 as Eqn. (2.20) provides better estimation of fundamental period than those provided in current building codes for RC framed with concrete shear wall.

8. The empirical equation of fundamental period provided in most current code for RC framed with concrete shear wall as $T = C_t H^{3/4}$, where $C_t = 0.075/\sqrt{A_c}$, is effective for the effective cross-sectional area of shear wall, A_c , fall in range of $0.6\text{m}^2 \leq A_c \leq 0.9\text{m}^2$, i.e. for A_c below 0.6m^2 the expression overestimates and above 0.9m^2 the expression underestimates the fundamental period as compared with Modal analysis.

5.2 Recommendations

Since the predicted fundamental period is used to obtain the expected seismic load affecting the structure, a precise estimation of it is important for the safety of the applied procedure in the design steps and consequently in the future performance of the structure after it is constructed. Therefore, it is important to determine the accurate limits for building periods. This limitation varies in current building codes. To achieve this goal, there are some recommendations that need to be taken into account as listed below:

1. Conservativeness of parameter inclusion in empirical formulae should be modified to include more parameters that can accurately define the fundamental period of vibration in current building code standards.
2. The effect of buildings with different floor layouts such as trapezoidal, triangular, circular shapes, and diaphragm discontinuities should be investigated.
3. The strength of construction materials and shape of building varies from project to project. Therefore, it is important to say that depending on complexity of a building and variation of structural parameters the design chart should be developed for fundamental vibration period.
4. The soil-structure interaction model should be developed instead of simply fixing the base of building to supporting ground to consider the actual soil condition.

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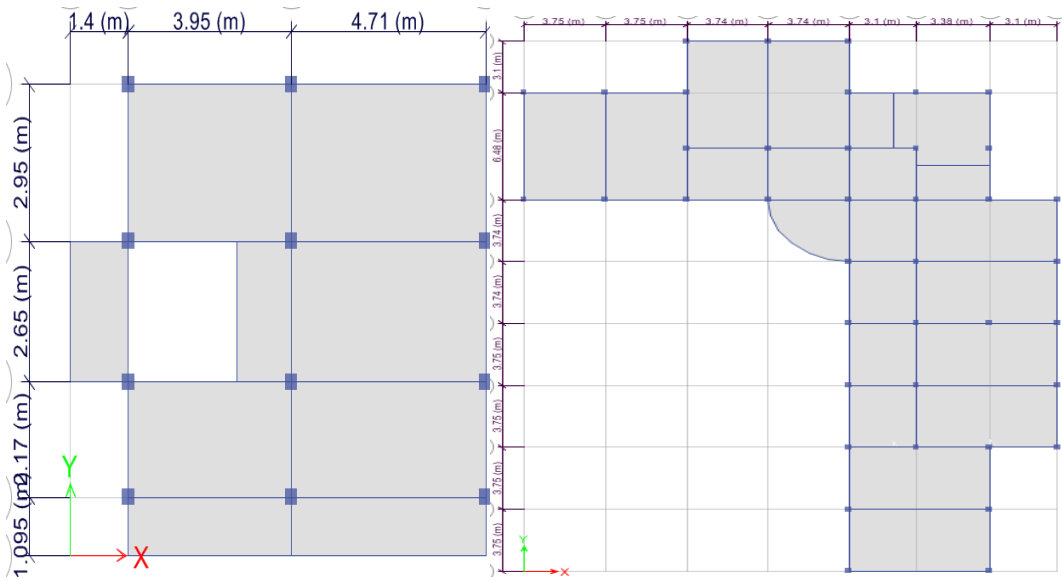
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Appendices

Appendix A: Summary of Existing Building Data

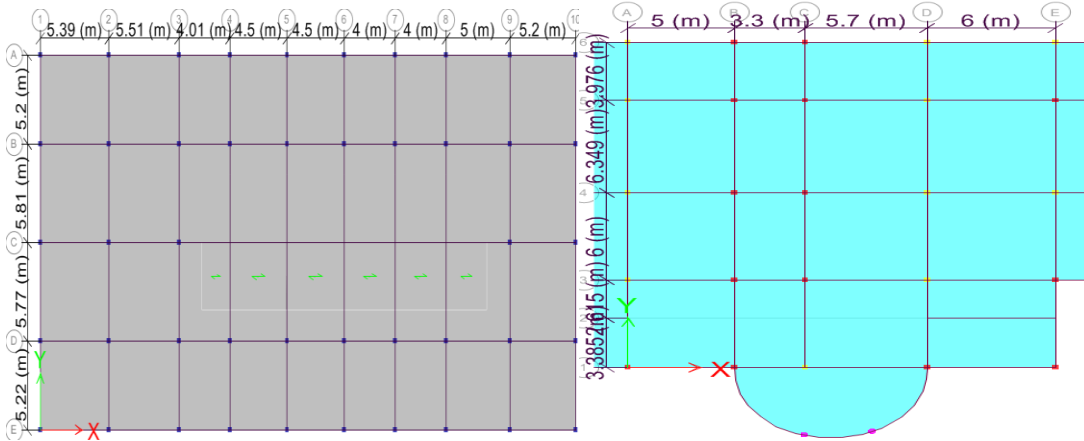
S.No.	Building Code	Building Purpose	No.of Story	Bldg H(m)	Plan Dimension		Bays		Regularity		Shear Wall	Diaph. Discnt	Floor Type	Slab Depth (mm)
					X(m)	Y(m)	X	Y	Plan	Elevation				
1	MAR	Residence	3	9	10.31	9.09	5	6	Regular	Regular	Yes	Yes	Solid	150
2	HHSC	Office	3	9.6	24.5	32	6	8	Irregular	Regular	No	No	Solid	150
3	SZD	Office	3	9	42.1	22	9	4	Regular	Regular	No	Yes	Solid	150
4	SRA	Office	3	9.6	23.2	26.512	4	5	Regular	Regular	No	No	Solid	150
5	HD	Apartment	3	9	23	5.6	6	1	Regular	Regular	No	No	Solid	150
6	TVET	Office	3	9	23.2	17.5	7	4	Regular	Regular	Yes	Yes	Solid	150
7	SOS	Residence	3	9	9.6	11.538	2	3	Regular	Regular	No	Yes	Solid	150
8	SEB	Residence	4	12.8	18.35	20.543	3	5	Irregular	Regular	No	No	Ribbed	300
9	HSC	Office	4	11.4	35	26.1	6	5	Regular	Regular	No	Yes	Ribbed	300
10	Ale	Mixed	5	15	14.338	13.348	3	4	Regular	Regular	No	Yes	Solid	180
11	MU	Mixed	5	16.3	34.997	34.209	9	5	Regular	Regular	Yes	No	Ribbed	300
12	P02	Residence	5	14.1	17	8.5	5	4	Regular	Regular	No	No	Solid	150
13	YB	Hotel	5	16.2	16	10.6	5	6	Regular	Regular	No	No	Solid	150
14	MC	Shopping	6	16.4	65.724	19.33	16	5	Regular	Regular	Yes	No	Ribbed	300
15	KAS	Hotel	6	18	22	18.245	5	3	Irregular	Regular	Yes	No	Ribbed	240
16	ABY	Hotel	6	19.6	16	10.6	5	6	Regular	Regular	No	No	Solid	150
17	HB	Hotel	7	22.6	16	21.15	4	4	Regular	Regular	Yes	No	Ribbed	300
18	OGO	Office	7	23.1	40	29	6	5	Regular	Regular	Yes	No	Ribbed	300
19	AO1	Office	7	22.4	45	17.2	8	3	Regular	Regular	No	No	Solid	160
20	AO2	Office	7	22.4	25.215	30.08	8	5	Regular	Irregular	Yes	Yes	Solid	160
21	JOS	Office	7	21.3	19.2	19.7	6	6	Regular	Regular	Yes	No	Solid	150
22	WON	Hotel	7	23.5	33.7	26.05	7	5	Regular	Regular	Yes	No	Ribbed	300
23	BG	MC	7	23.4	28	23.75	5	6	Regular	Regular	Yes	No	Ribbed	300
24	ABC	Mixed	8	20	29.98	11.051	2	7	Regular	Regular	No	Yes	Solid	150
25	KAS1	Mixed	8	26.5	35.9	22.35	5	4	Regular	Regular	Yes	No	Ribbed	300
26	TAO	Office	9	29.1	17	18.5	3	4	Irregular	Regular	Yes	No	Ribbed	300
27	P01	Hotel	9	26.2	18.48	21.188	2	4	Regular	Regular	No	No	Solid	200
28	BIY	Mixed	9	31.1	24.4	27.85	4	4	Regular	Regular	Yes	Yes	Solid	150&250
29	NEW1	Mixed	9	28.8	27.3	23.35	4	5	Irregular	Regular	Yes	Yes	Solid	150&220
30	NEW2	Mixed	9	28.8	27.3	23.35	4	5	Irregular	Regular	Yes	Yes	Solid	150&220
31	NEW3	Mixed	9	28.8	14.78	18.66	3	4	Regular	Regular	Yes	No	Solid	220
32	TYPE1	Mixed	9	28.1	18.3	17.9	4	4	Regular	Regular	Yes	Yes	Solid	150
33	DEJ	Mixed	10	30.4	21.337	28.15	6	6	Regular	Regular	Yes	Yes	Ribbed	240
34	KAS2	Mixed	10	33	35.15	36.61	6	7	Regular	Regular	Yes	Yes	Ribbed	300
35	TYPE2	Mixed	10	31.9	11.3	13.65	4	4	Regular	Regular	Yes	No	Solid	150
36	SOF	Mixed	11	33.2	15.25	19.1	3	3	Regular	Regular	Yes	Yes	Solid	200
37	AE1	Apartment	13	41.6	42.32	39.2	6	7	Regular	Irregular	Yes	No	Ribbed	300
38	KAS3	Mixed	13	41.6	45.9	18.3	8	3	Regular	Regular	Yes	No	Solid	180
39	AE2 3	Apartment	14	44.8	42.32	39.691	6	7	Regular	Irregular	Yes	No	Ribbed	300

Appendix B: Structural Layout of Selected Buildings



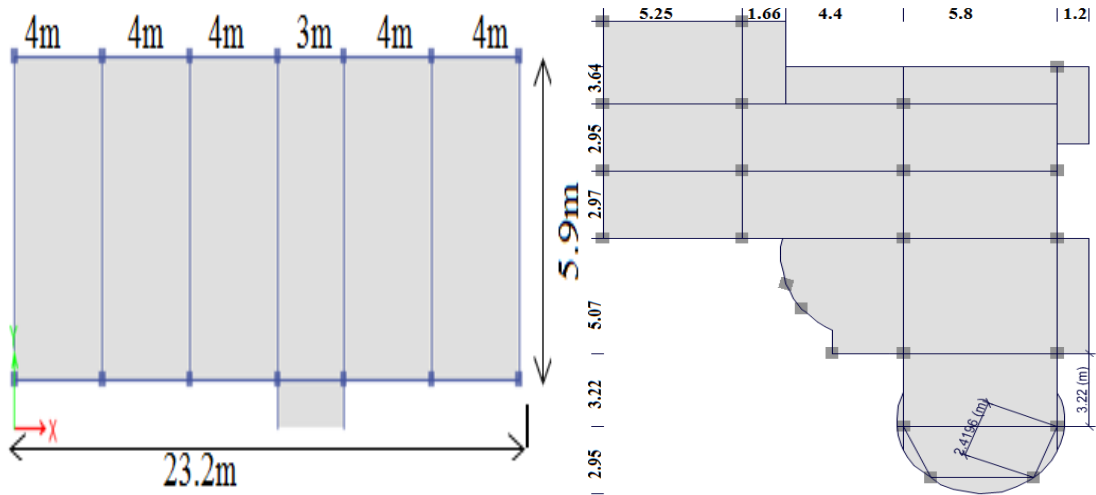
1. MAR 3 storied

2. HHSC 3 storied



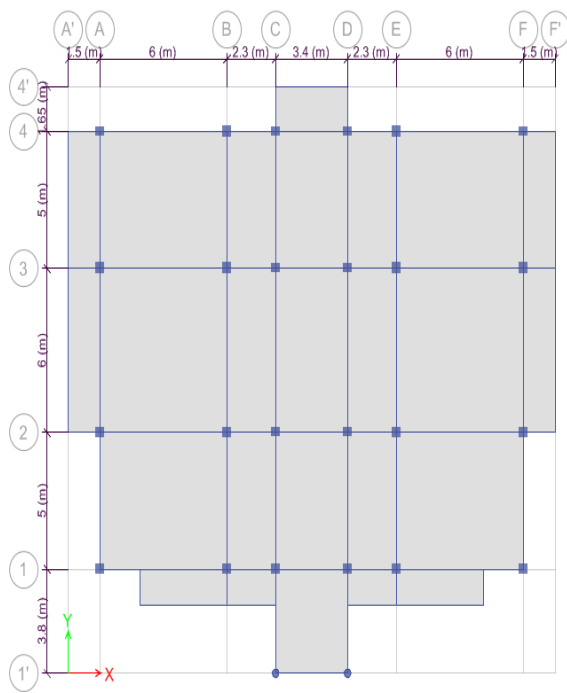
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4. SRA 3

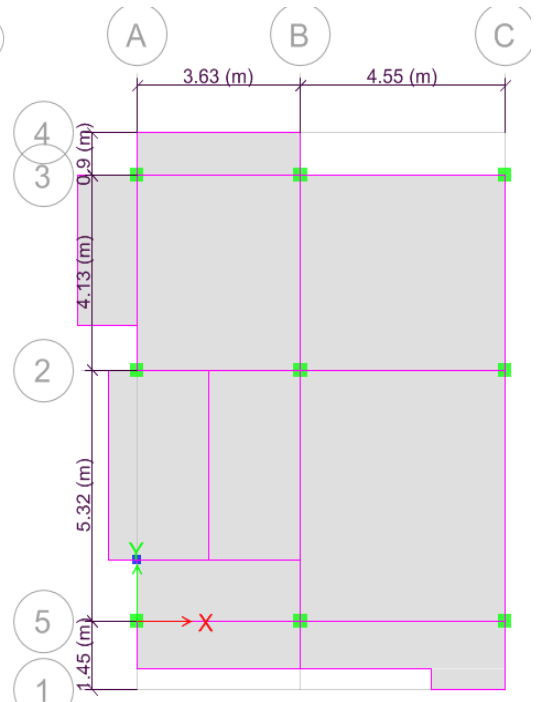


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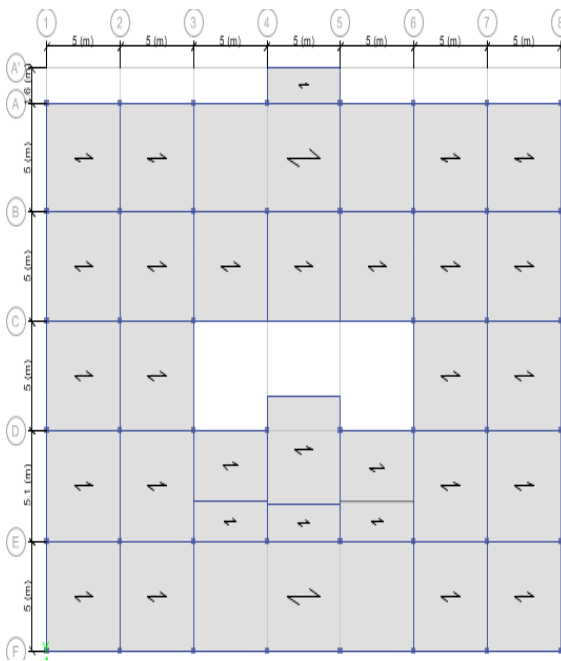
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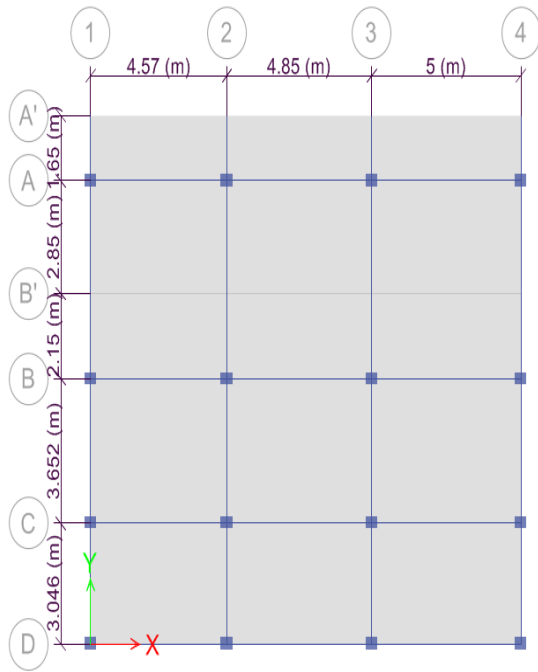
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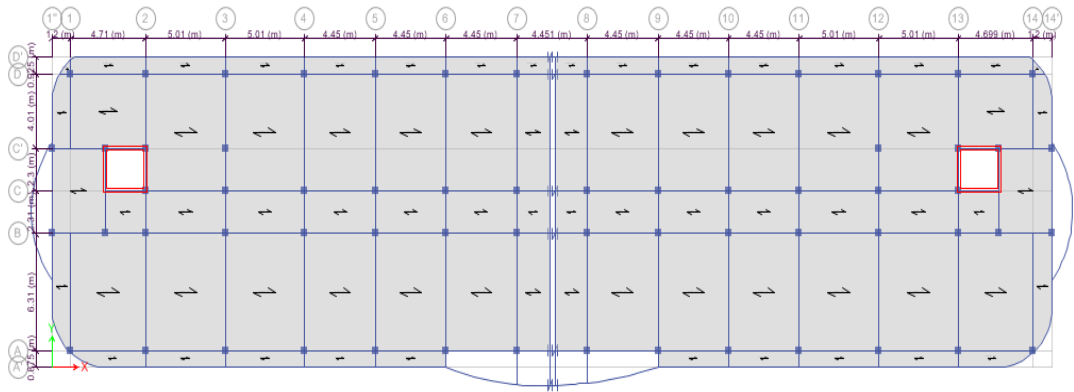
8. SOS 3 storied



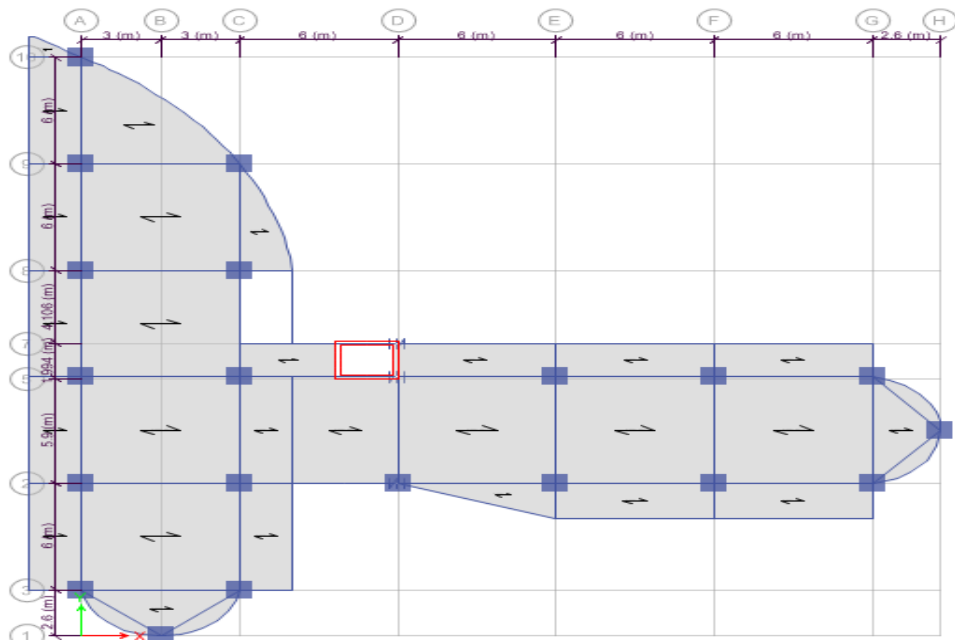
9. HSC 4 storied



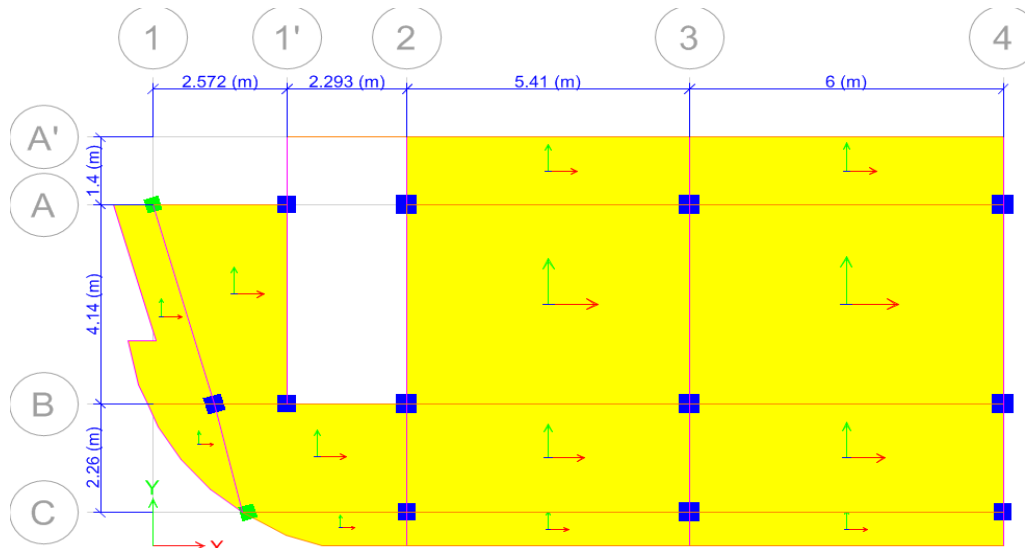
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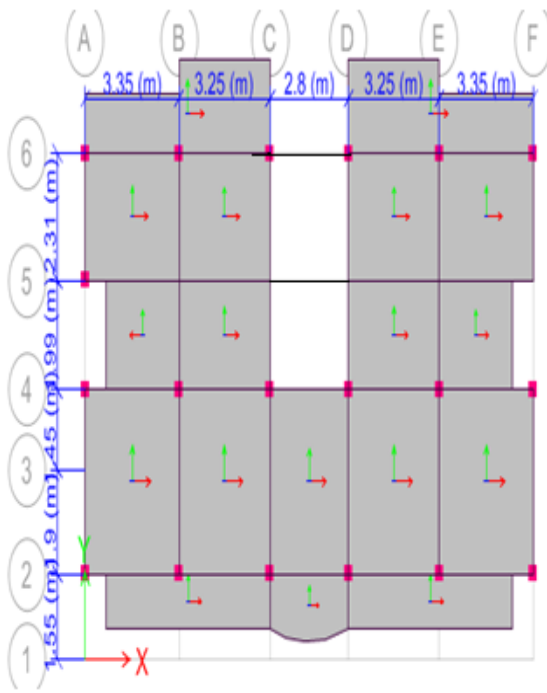
11. MC 5 storied



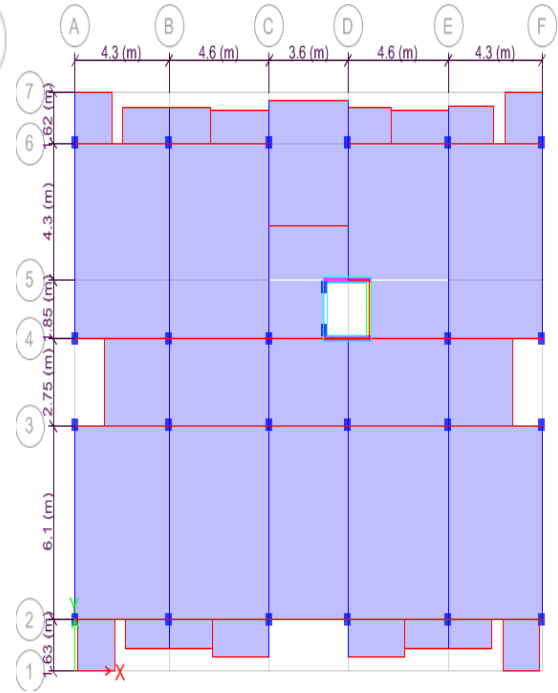
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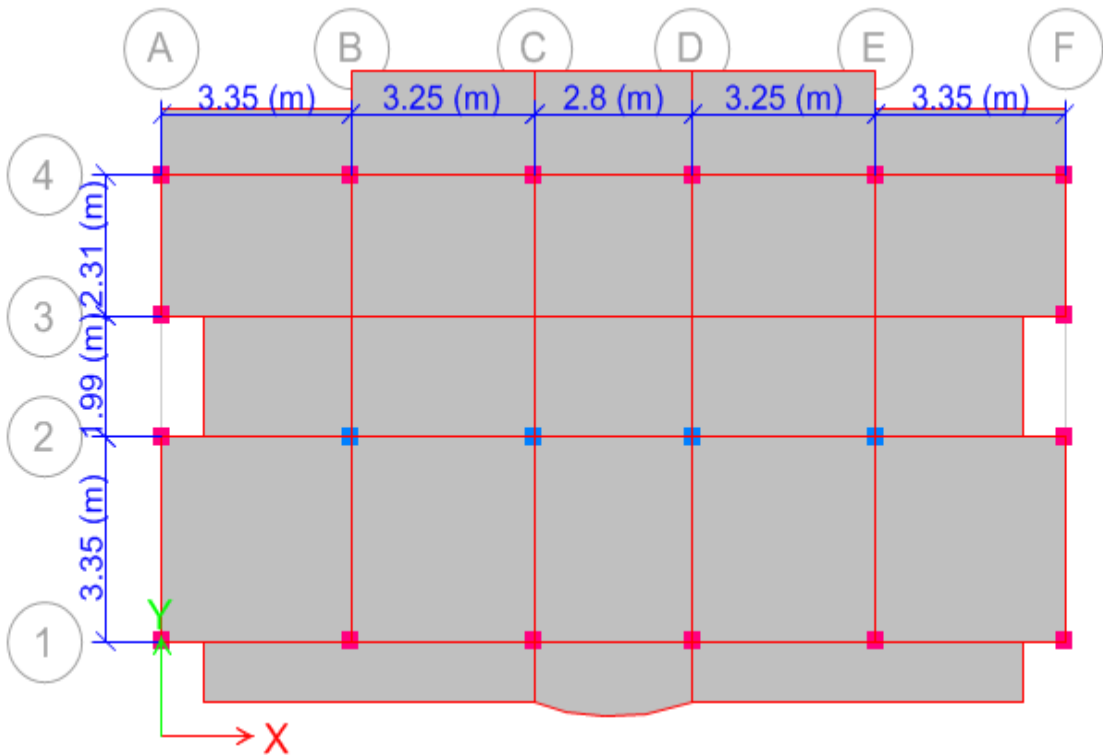
13. P02 5 storied



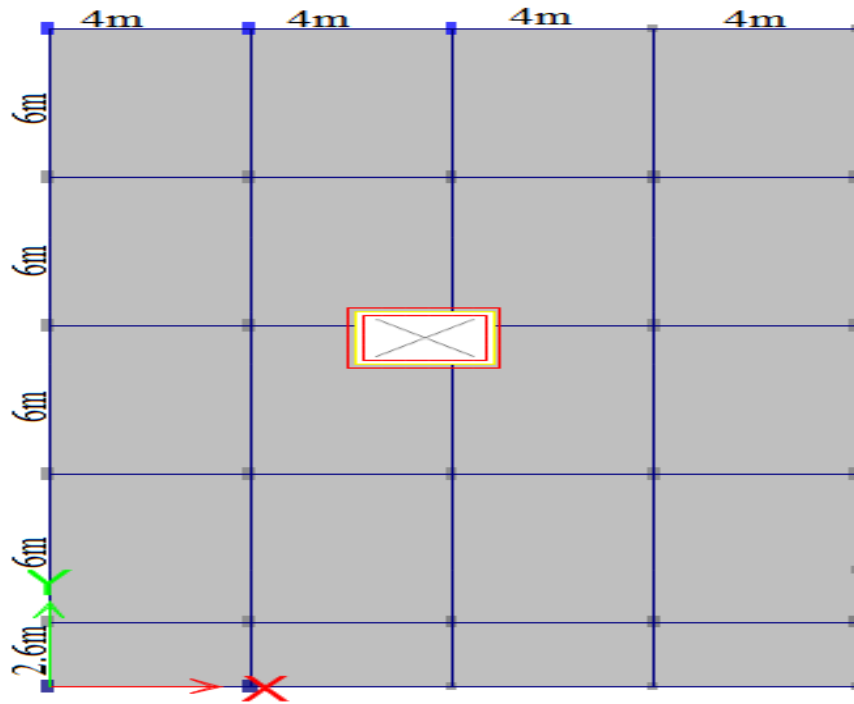
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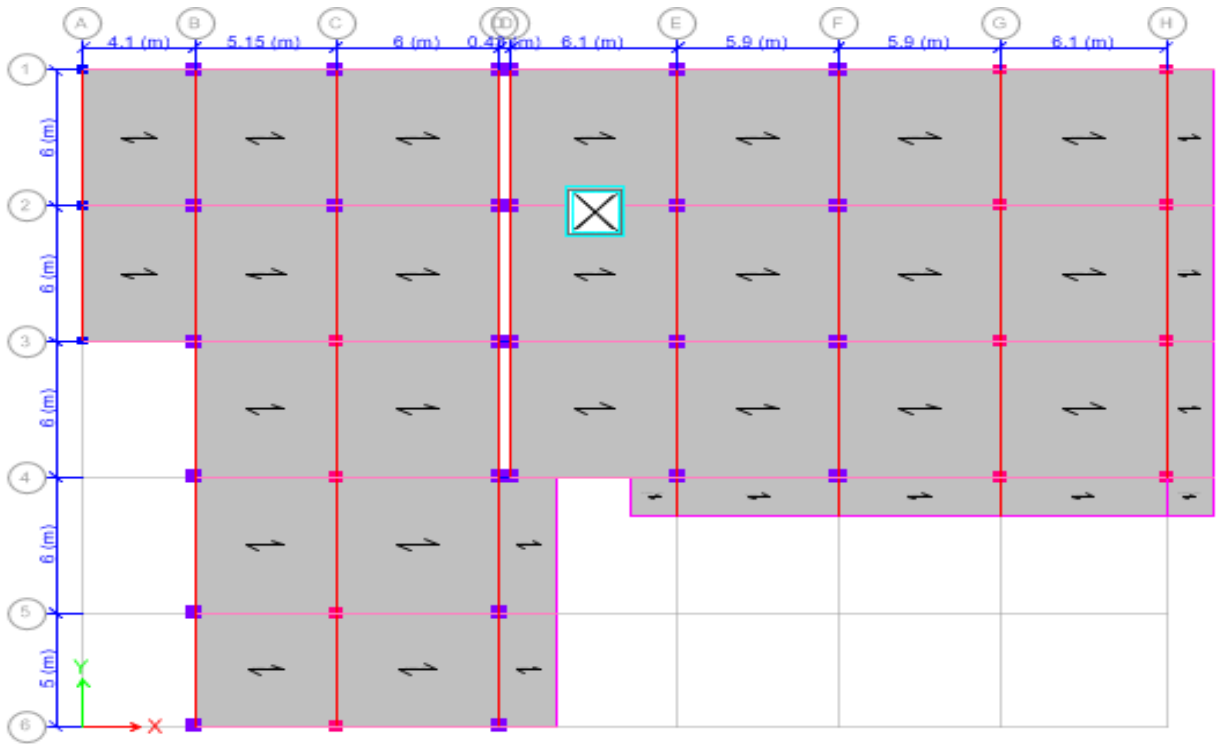
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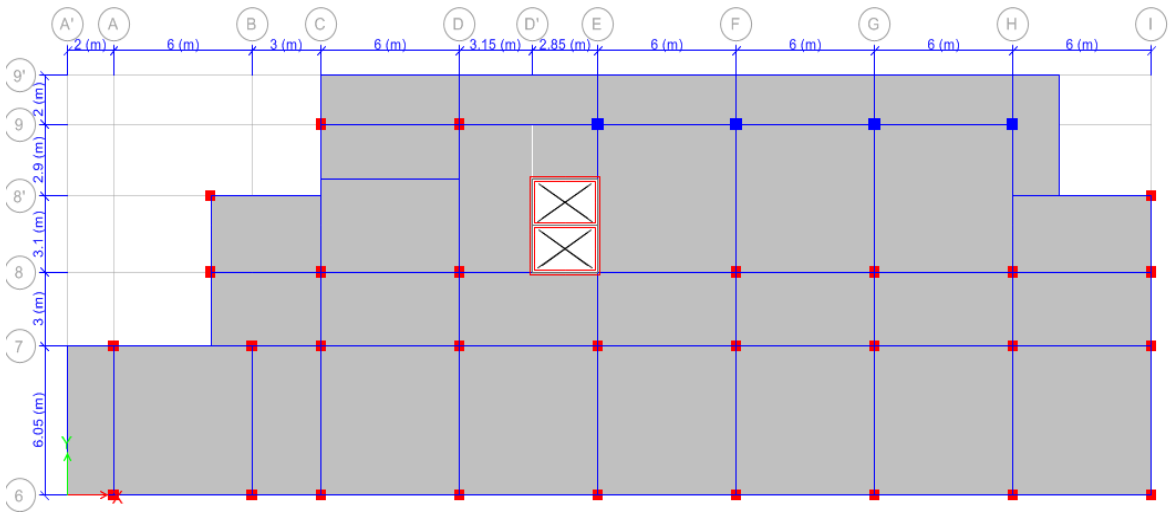
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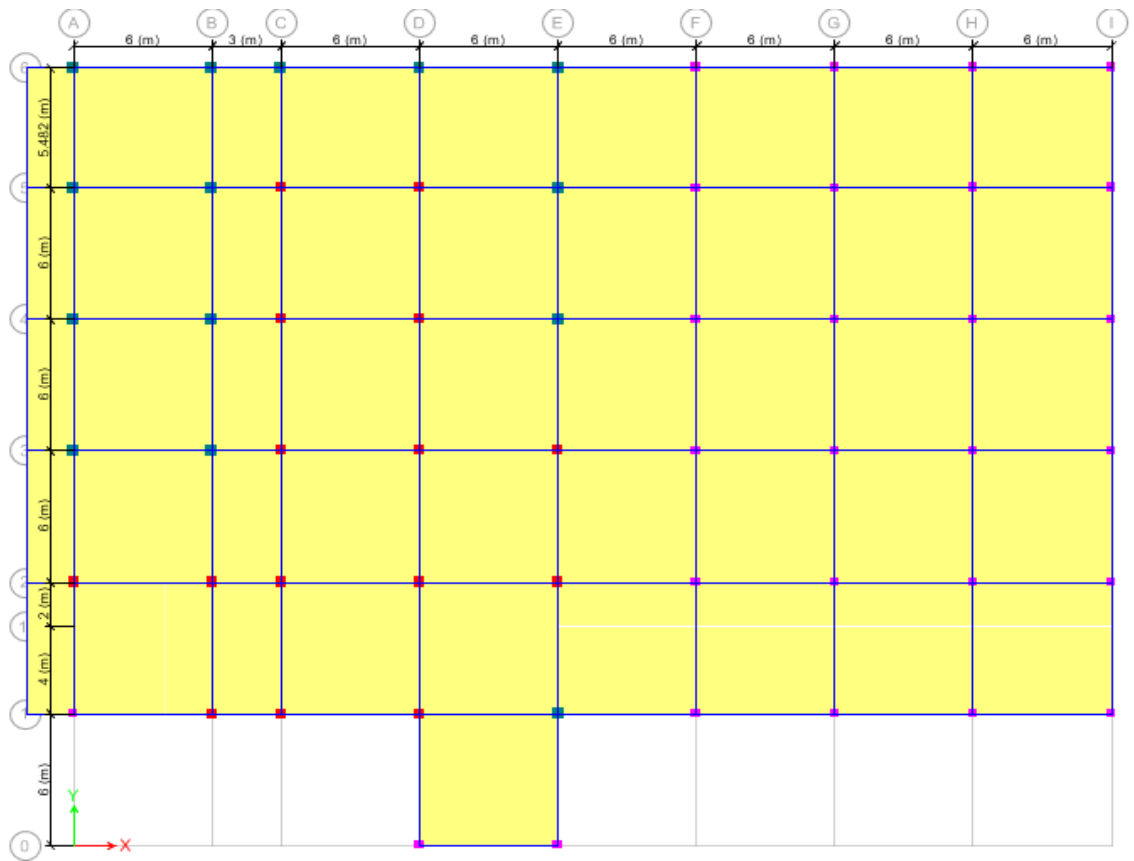
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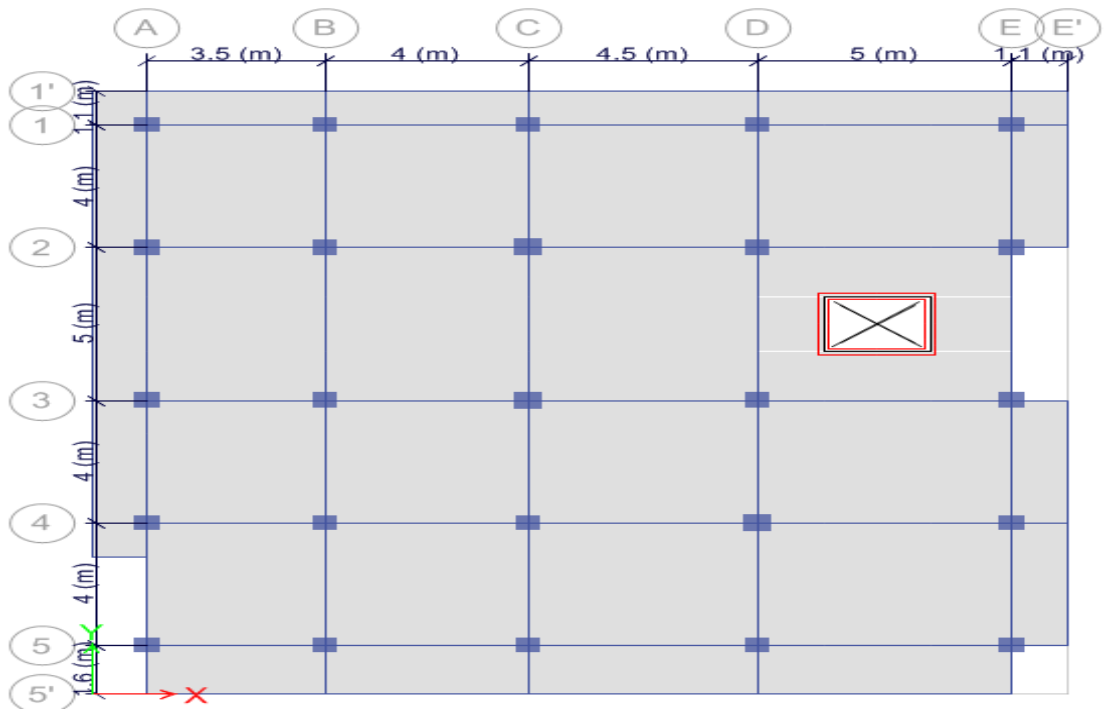
18. OGO 7 storied



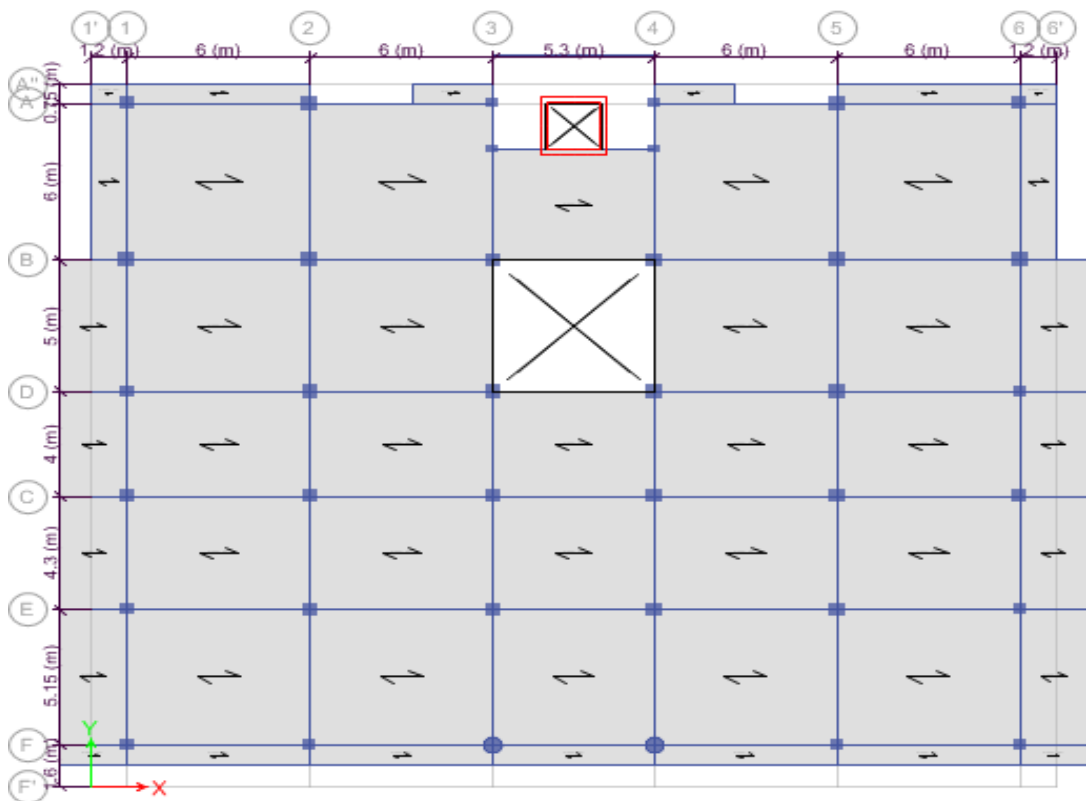
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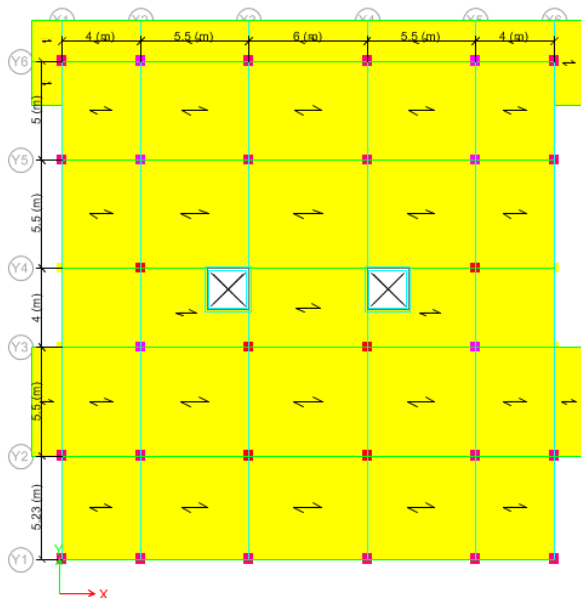
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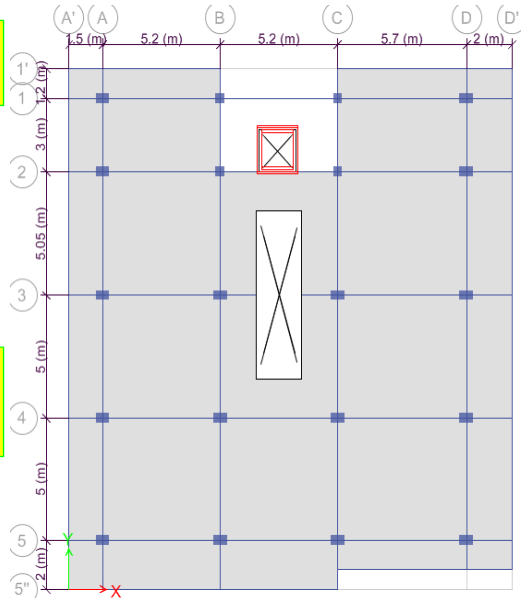
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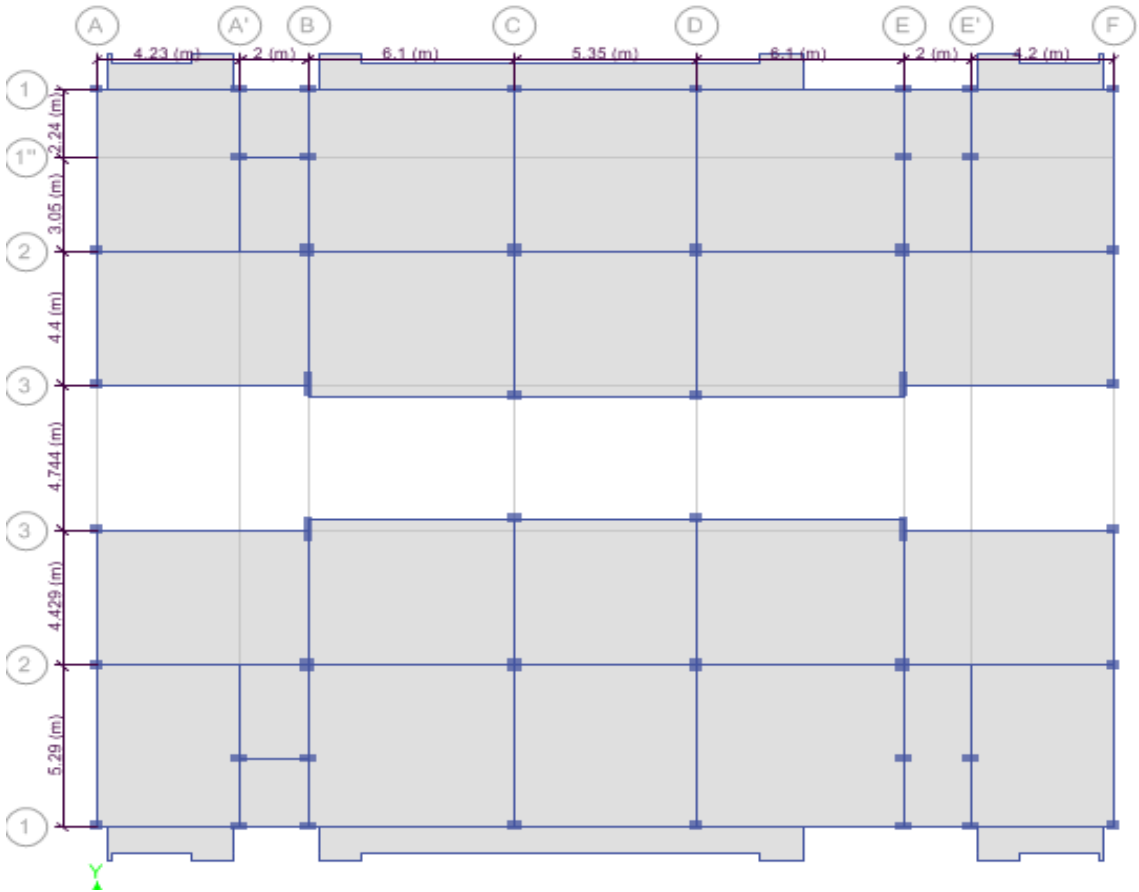
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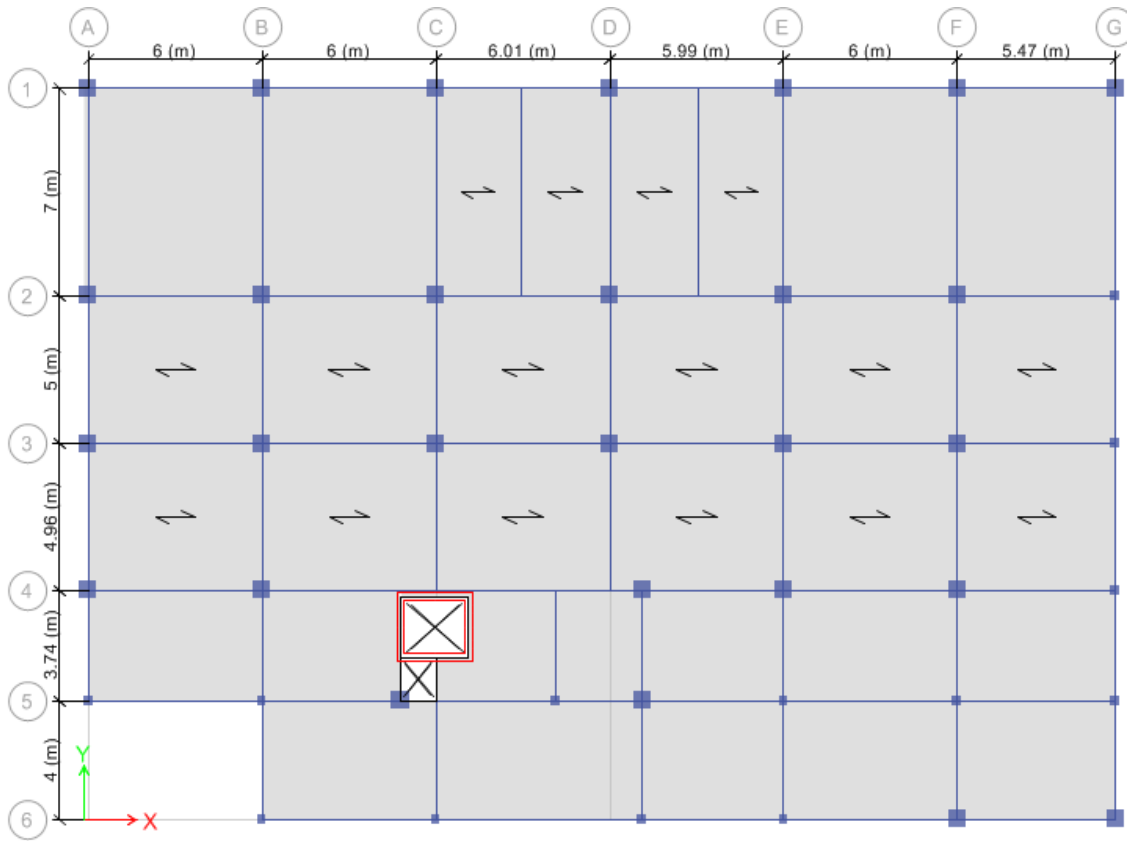
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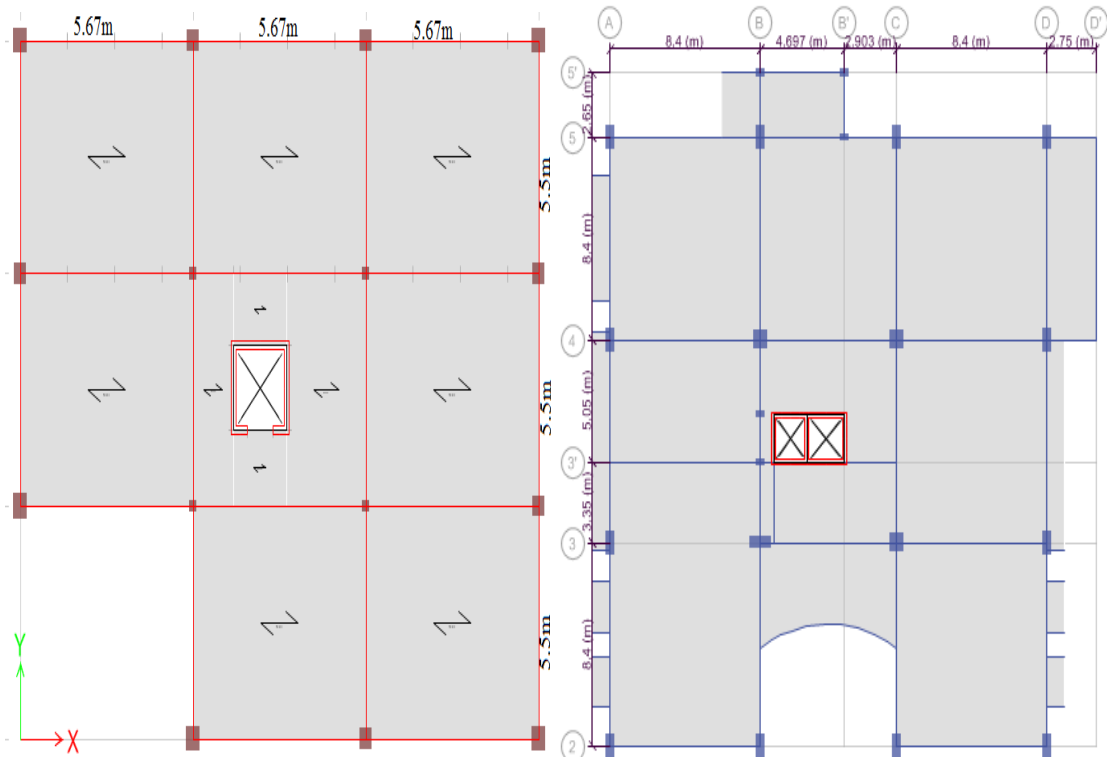
24. P01 8 storied



25. ABC 8 storied

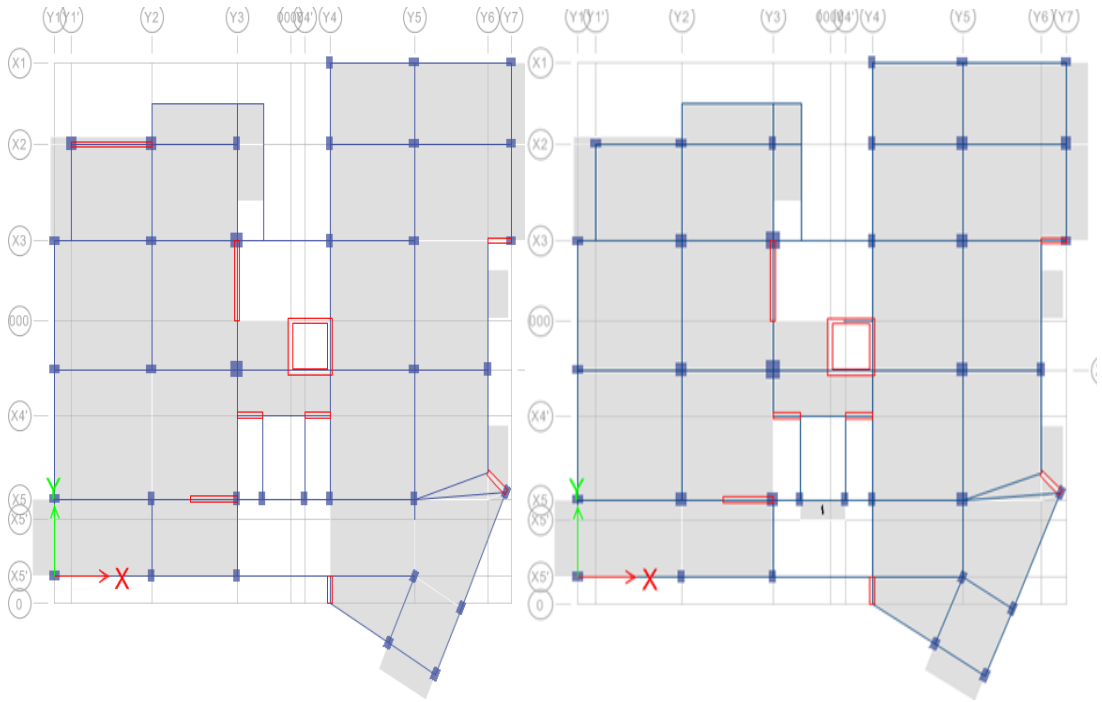


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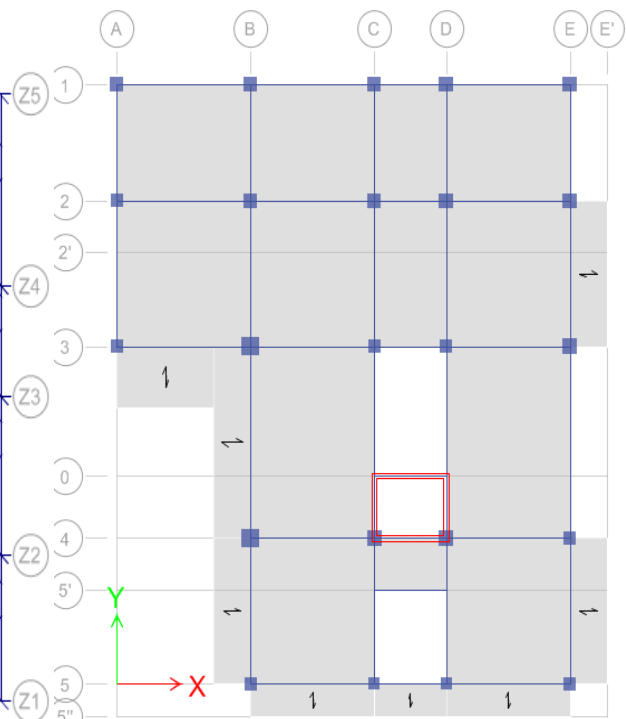
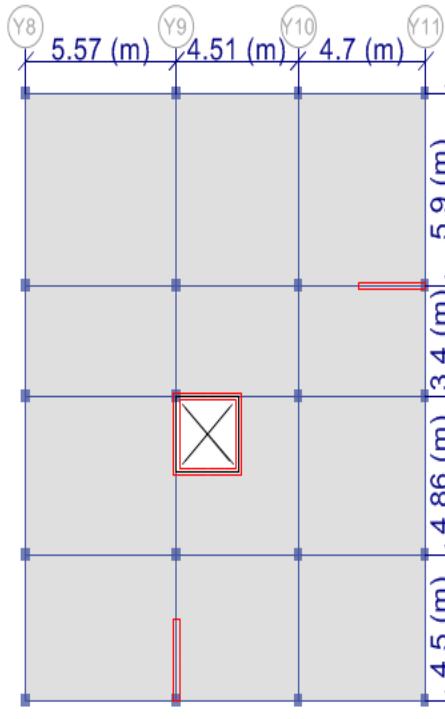
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28. BIY 9 storied



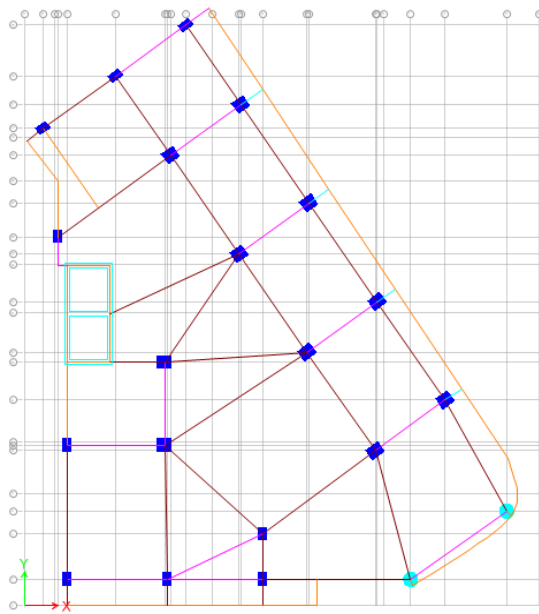
29. NEW1 9 storied

30. NEW2 9 storied

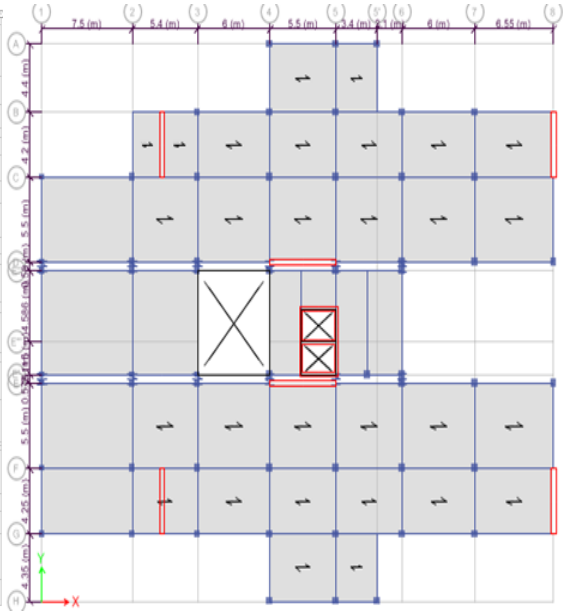


31. NEW3 9 storied

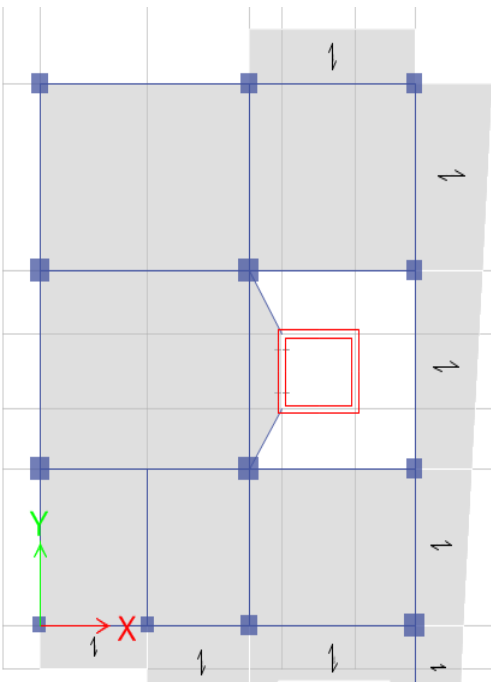
32. TYPE1 9 storied



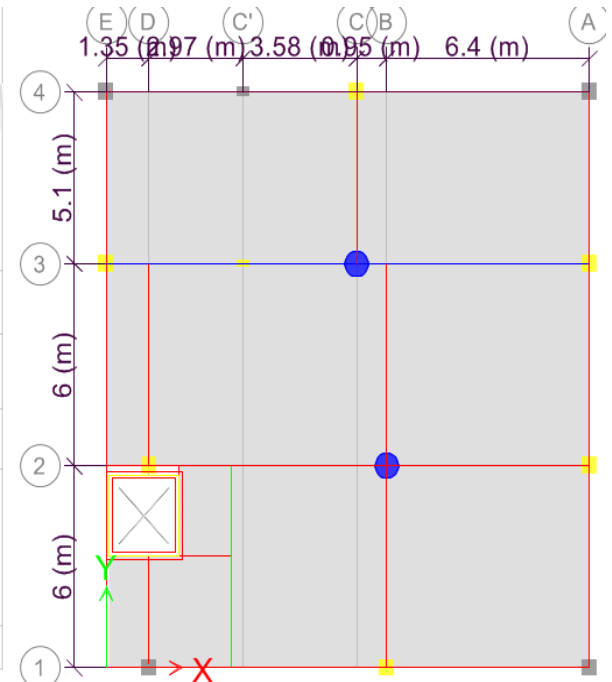
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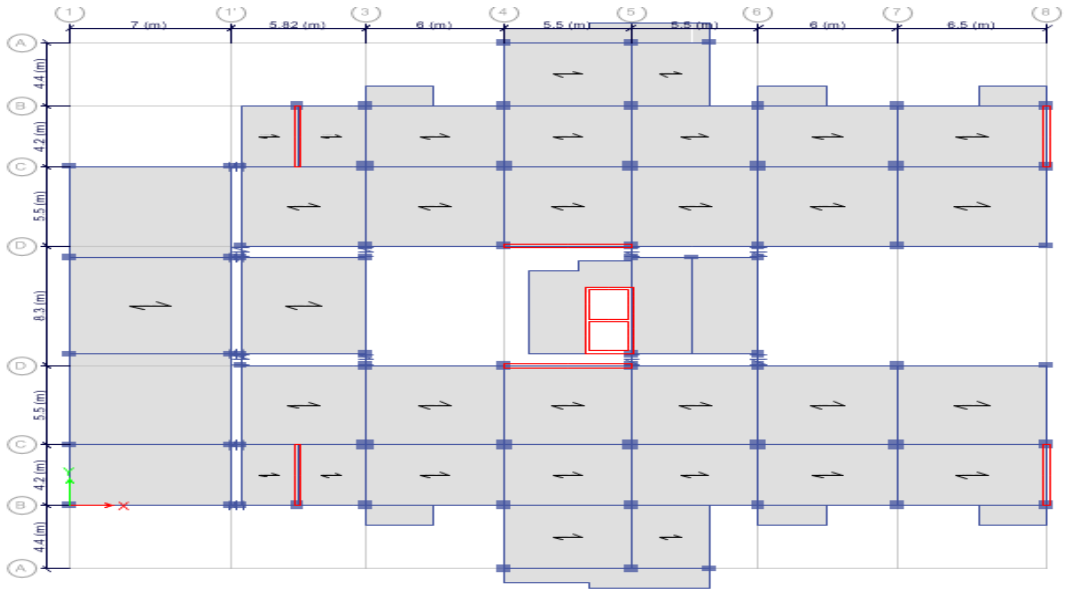
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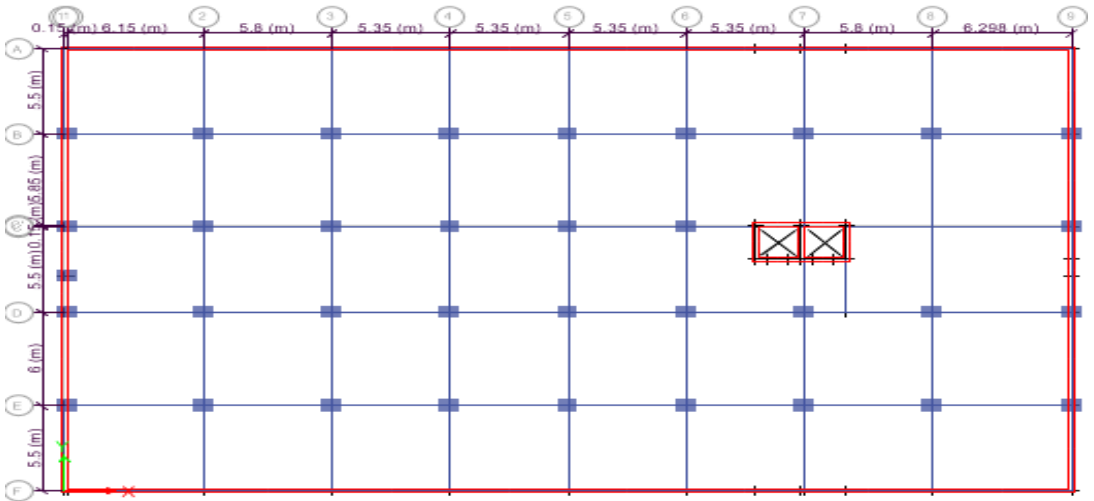
35. TYPE2 10 Storied



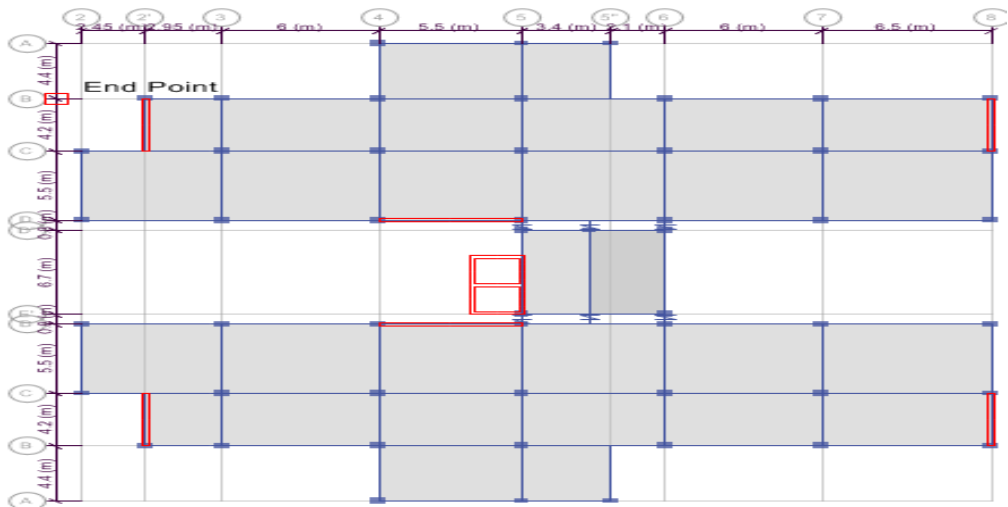
36. SOF 11 Storied



37. AE2 13 Storied



38. KAS3 13 Storied



39. AE2 14 Storied