



ADDIS ABABA UNIVERSITY
ADDIS ABABA INSTITUTE OF TECHNOLOGY
SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

**Residual Terrain Model Effect and Gravimetric Terrain
Correction for Gravity Field Modeling over Ethiopia**

A Thesis Submitted to the School of Graduate Studies of Addis Ababa University
in Partial Fulfillment of the Requirements for The Degree of Master of Science in
Geodesy and Geomatics, (Specialization in Geodesy)

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Declaration

I the undersigned certify that this master’s thesis with the title “**Residual Terrain Model Effect and Gravimetric Terrain Correction for Gravity Field Modelling over Ethiopia**” is my work. The research work has not been presented in any other institution for partial fulfillment of the degree. All the materials used in the study with their sources are properly acknowledged.

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Abstract

In this study, residual terrain model effects and terrain corrections were computed at 1.5 arc-minute grid nodes and terrestrial gravity stations and presented in grid maps having 1.5 arc minute spatial resolutions over Ethiopia. 30 arc second SRTM DEM was used in the construction of Residual Terrain Model elevation of d/o 200 and 2190. A smooth surface was observed in the mean elevation surface corresponding to ~100km spatial scale. The mean elevation surface computed from the low pass filter was adopted because it shows better spectral details than the topography spherical harmonic methods. The computations were made with an integration radius of 20km and 200km for inner and outer zones respectively. In Ethiopia, 92.12% of RTM direct effects corresponding to d/o 200 reaches 25mgal and 85.74% of RTM indirect effects are less than 30cm. 98.85% of terrain corrections are less than 15mgal. For reference surface corresponding to ~9km spatial scale, 97.52% of the RTM direct effects are less than 25mgal. 98.26% of indirect effects are less than 5cm. the RTM effects and terrain corrections computed with mean elevation surface corresponding to ~9km spatial scale are not significant. The result of the study can be applicable for accurately interpretation of geoscientific studies.

Keywords: Gravity, RTM Effects, High-frequency, Terrain Corrections,

List of abbreviations

AH- Airy-Heiskanen

BVP- boundary value problems

d/o- degree and order

DTM- Digital Terrain Model

FFT- Fast Fourier Transform

GDEM- Global Digital Elevation Models

GGM- Global Geopotential Models

ICGEM- international center for Global Earth models

PH- Pratt-Hayford

RCR- Remove-Compute-Restore

RTC- residual terrain correction

RTM- Residual Terrain Model

SRTM- Shuttle Radar Topography Mission

TC- Terrain Correction

NASA- National Aeronautics and Space Administration

USGS- United States Geological Survey

Chapter One

1. Introduction

1.1. Background of the study

In classical solutions for boundary value problems (geoid or geopotential determination), we need to choose the mean sea level, more precisely, an equipotential surface close to the mean sea level called the geoid as a boundary. In this case, gravity reductions are needed to the input gravity data: some masses are outside the computation boundary and need to be computationally moved to the inside (Vermeer, 2018).

The classical stoke's formula and/or other formulas which can be used for geoid determination require that the gravity anomalies Δg represent boundary values at the geoid. This implies the gravity g must refer to the geoid and there must be no mass outside the geoid. Since there are masses outside the geoid, gravity reductions are needed before applying the stoke's integral. In gravity reduction, the topographic masses which are located outside the geoid are completely removed or shifted below sea level and the gravity at the topographic surface is lowered to the geoid (Hofmann-Wellenhof & Moritz, 2005). This gravity reduction procedure enables us to create a smooth gravity field, so that interpolation and extrapolation to unobserved areas become easier (Heiskanen & Moritz, 1967).

Based on how they consider the topography in the reduction process there are numerous gravity reduction methods (Bajracharya, 2001). Since the high-frequency components of the measured gravity field are due to the attraction of the topography. The representations of the topographic gravity and potential are needed (Bedada, 2010).

Previously in Ethiopia, the geoid was studied along with the absolute geopotential height system by (Bedada, 2010). This study aims to quantify the effect of terrain on gravimetric measurements and geoid determination and provide an alternative gravimetric terrain correction.

High-resolution Global Gravity Models (GGMs) are a good option for modeling the gravity field, but they are exposed to omission errors containing Short-wavelength gravity field signal that cannot be denoted by a truncated spherical harmonic expansion (Bedada, 2010).

In the solution of the geodetic BVP for the determination of precise geoid and quasi-geoid undulations using the Remove-Compute-Restore (RCR) method before applying the Stokes'/Molodensky's integral, all the high and low frequency signals are removed. The residual geoid/quasi-geoid will be computed. Here instead of using the original mass distribution, we use the reduced mass distribution to obtain the geoid or geopotential solution which gives a slightly different surface from the geoid. The surface is called co-geoid. In order to determine the influence of the removal step, we need a restoration step. The removed signal will be restored to the residual one. This is the concept of the remove-compute-restore approach (forsberg, 1984).

The impact of long-wavelength components of the observed gravity data to the gravity field modeling is computed from Global Gravity Models (GGM). Since the short-wavelength gravity field is mainly originated from the topography of the earth the high-frequency components are computed from the digital elevation model (DEM) (Forsberg, 1984).

During gravity field modeling a problem arises in modeling the mountainous areas which is the short wavelength topographic gravity noise, the need for a terrain reduction that enables us to approximate the isostatic conditions rather than computing the global isostatic reductions will be observed. As first introduced by Forsberg (1984) the residual terrain model (RTM) is done by only considering the high-frequency topographic gravity field since the topography is the main source of the short-wavelength component of the gravity field. In applying the RTM method for RCR approaches we need to choose a smooth mean elevation surface to computationally remove topographic masses above the surface and to fill valleys below. Increasing the resolution of the reference surface will decrease the amount of the Residual Terrain Model effect (Yahaya & Azzab, 2018).

The study focused on modeling the high-frequency components of the gravity field through the RTM technique in Ethiopia. The study constructed RTM elevation from Shuttle Radar Topography Mission Digital Elevation Model data.

1.2. Statement of the problem

Many geoscientific studies have been used gravity measurements for different purposes. The reduction of gravity measurements for different effects depends on the required accuracy of the specific task. Gravity reduction is a challenging task in the modelling of Earth's gravity field (Varga, et al., 2019). Terrain-reduced gravity observations are used in geoid determination (Varga et al., 2019, Bedada, 2010). From the topographic effects in gravity measurements terrain correction is the most significant and sensitive part because it filters variation of topographic effects and density anomalies.

Prior to the availability of modern computers terrain correction was computed by dividing the topography around gravity stations into small Hammer compartments (Hammer, 1939). Because of the enervating nature of the method, it was recently snubbed. Through time with the advancement of modern computers, the use of DEMs in which heights are defined in regular grids for terrain correction become familiar. One of the first of this kind of terrain correction calculation was done by Kane, (1962). Forsberg, (1984) studied the effects of topography in local and regional geoid modeling. (Bajracharya S. , 2003) investigated the different topographic reduction methods, such as Residual Terrain Model, Helmert's condensation method, inversion of Rudzki's method, etc. Bedada, (2010) in his study of the absolute geopotential height system for Ethiopia studied the topographic effects using the combined method of multi-point and vertical line mass by applying a hedgehog algorithm to compute the gravity and potential models of the topographic effects representing the region of Ethiopia and its immediate surroundings.

Apart from hereby presented references and a large number of other studies, the gravimetric terrain correction and residual terrain model (RTM) effects have not been researched enough over Ethiopia. This implies there is a gap in the literature for gravimetric terrain corrections and RTM effects over Ethiopia. This study aims to provide an alternative method to quantify the terrain effects in gravity field modelling over Ethiopia using different resolution SRTM (Shuttle Radar Topographic Mission) GDEM (Global Digital Elevation Model) data.

1.3. Objectives

1.3.1. General objective

The general objective of this study is to provide a residual terrain model and determine the terrain correction on gravity field modelling using high-resolution SRTM GDEM data over Ethiopia.

1.3.2. Specific objectives

- ❖ To compute the mean elevation surfaces of the area
- ❖ To determine residual terrain model (RTM) and compute RTM effects
- ❖ To calculate terrain correction (TC)
- ❖ Investigate the sufficient spatial resolution for good precision of terrain effects
- ❖ To analyze the impact of height discrepancy on RTM effects and terrain corrections

1.4. Research questions

- How can we compute the mean elevation of the given topography which is presented in SRTM GDEM?
- What effects have the RTM on gravity field modelling?
- What are the sufficient spatial resolutions for good precision of terrain effects?
- How can we calculate terrain corrections?
- How does the height discrepancy of gravity stations affect the RTM effects and terrain corrections?

1.5. Significance of the study

The main contribution and significance of this study are modeling the short-wavelength components of the gravity field and terrain correction over Ethiopia. It is an added value in application areas of geodesy and geophysics that requires accurate interpretations. The result of the study can be applicable for geodesists in the determination of gravimetric geoids using the Remove-Compute-Restore approach. In addition to this, the study will help in the refinement of Bouguer anomalies during complete Bouguer reduction. Finally, the study quantifies the characteristics of terrain effects over the computation area specifically over Ethiopia.

1.6. Limitation of the study

This study was performed with all possible efforts in gathering the necessary input and data collection, analysis, and interpretation. However, the study experiences some limitations which affect the processing and analysis of the input data. The main challenge was the capacity of the computer not to handle high-resolution DEMs with large study areas. Due to this the study area was subdivided into 5 degrees by 5-degree different zones.

1.7. Organization of the study

This study is organized into five chapters from which the first chapter presented the introduction part which includes the Background of the study, Statement of the problem, objectives, research questions, and significance of the study.

Chapter two presents detailed reviews of different works of literature about gravity terrain reductions, the different types of gravity reductions such as free air reduction, Bouguer reduction, terrain correction, Helmert's second method of condensation, Residual terrain model, and isostatic reductions. In addition to the theoretical reviews, empirical reviews were presented from different numerical investigated literature.

In chapter three the different materials and software used in the study are reported. The description of the study area is described. The different data and their sources have been presented. Finally, the general workflow and methodology implemented in the study are described in detail.

Chapter four reported the results and discussions of the study. In this chapter, the results of mean elevation surfaces, the influence of integration radiuses, RTM effects and terrain corrections per spatial resolutions, and impact of height discrepancy on terrain effects have been presented and discussed in detail.

Finally, in chapter five the conclusions driven from the study and recommendations posed for further studies are described.

Chapter Two

2. Literature Review

2.1. Theoretical review

2.1.1. The fundamental theory of gravitation

In 1687 Sir Isaac Newton who was an English genius published his main work entitled “mathematical foundations of physic” by which he stated his famous law of gravitation. Newton made fundamental observations about gravitation stating, the force between two attracting bodies is directly proportional to the individual masses, the force is inversely proportional to the square of the distance, and the force is directed along the line connecting the two bodies. This Newton’s fundamental observation on gravitation can mathematically expressed as; (Hofmann-Wellenhof & Moritz, 2005)

$$F_{12} = G \frac{m_1 m_2}{r_{12}^2} \quad (2.1)$$

Where, m_1 and m_2 represents the masses of the two points, r_{12} is the distance between the two points and G is Newton’s gravitational constant having a value of $G = 6.67 * 10^{-11} m^3 kg^{-1} S^{-2}$

Let us assume the two masses to be, one a small body like satellite denoting $m_2=m$ which is the attracted mass, and another a large mass like planet Earth denoting $m_1=M$ which is the attracting mass, we will have: (Vermeer, 2018)

$$F = G \frac{mM}{r^2} \quad (2.2)$$

From Newton’s law of motion, we have:

$$F = ma \text{ which gives } a = G \frac{M}{r^2} \quad (2.3)$$

The term a represents acceleration due to gravity. From the above equation, we can observe that the value $m_2=m$ has disappeared. Showing the observation made by Galileo which can be stated as “all bodies fall equally fast irrespective of their mass”. Considering the attraction of the large body mass like earth to be homogenous mass, Spherical and non-rotating it is easier to deal with the scalar quantity which is potential than the vector components of the gravitational acceleration. the gravitational potential can mathematically be expressed as; (Vermeer, 2018)

$$V = \frac{GM}{r} \quad (2.4)$$

Since the gravitational potential value is a scalar quantity and V is path integral, this made it possible to compute the components of the gravitational acceleration vector by differentiation, for this reason, the potential quantities are computationally flexible. In addition to this, the potential approach can be used to model the size and shape of the Earth with equipotential surfaces by which the value of V is constant.

2.1.2. Gravimetric Geoid determination

The mathematical determination of the physical surface of the Earth was made easier by Geodesists using measured potential and gravity g . The geoid is a surface with constant gravitational potential that corresponds with the mean sea level. Since the Geoid is the mathematical representation of the horizontal level surface of the Earth its determination is crucial in the field of geodesy (Heiskanen & Moritz, 1967).

Every harmonic function in the region where it satisfies Laplace's equation is analytic. A function V harmonic outside a surface S is uniquely determined by its values on S . because of the existence of many mass distributions which have given harmonic function V as exterior potential. It is impossible to determine uniquely the generating masses from the potential which has no unique solution. Stoke's theorem states that there is only one harmonic function V that assumes given boundary values on a surface S , provided that such a harmonic function exists. The assertion that for arbitrarily prescribed boundary values there always exist a harmonic function V that assumes on the surface S . the given boundary value is called Dirichlet's principle. The problem of computing the harmonic functions from its boundary values on the surface S is called Dirichlet's problem or the first boundary value problem of the potential theory (Heiskanen & Moritz, 1967).

The following figure presents the geometrical principle of the geoid. From the figure, The ellipsoidal height can be expressed as follows: (Bajracharya S., 2003)

$$h_e = h + N \quad (2.5)$$

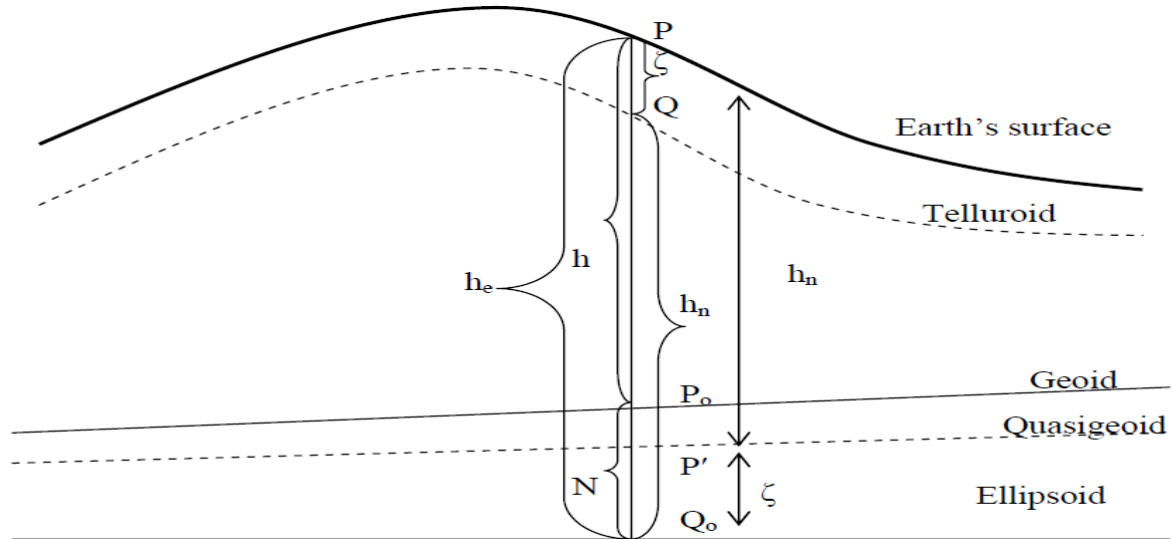


Figure 2.1: geometrical representation of Geoid and quasigeoid (Bajracharya S., 2003)

Where, orthometric height (P_oP): h

ellipsoidal height (Q_oP): h_e

geoidal undulation (P_oQ_o): N

normal height ($P'P = Q_oQ$): h_n

height anomaly ($Q_oP' = PQ$): ζ

2.1.3. The Remove-Compute-Restore concept

The solution to the boundary value problems in the spherical approximation enables geodesists to have a mathematical representation for the size and shape of the earth. The anomalous potential equation includes information to better approximate the anomalous potential with mathematical models and can be expressed as:

$$T = T_m + T_r \quad (2.7)$$

Where T represents the anomalous potential, T_m is modeled anomalous potential and T_r is residual anomalous potential. In the constructed potential model there exist two types of models that contribute to T_m . The first one is T_{Gm} which is the global effects accounts from the medium to long-wavelength components of the gravity field of topography and the second one is the local models which accounts for the short wavelengths of the topography. The computation of a gravitational potential consists of a surface integral of gravity disturbances or anomalies on a smooth regular surface. To use gravity data for any geodetic and geophysical applications, the

gravity data must be reliable and representative. It is recommended to smoothen the measured gravity data by which the complete field is recoverable from it. The Remove-Compute-Restore approach in Geoid determination is a type of smoothing technique by which the high and low frequency signals are removed. The residual geoid/quasi-geoid will be computed. Here instead of using the original mass distribution, we use the reduced mass distribution to obtain the geoid or geopotential solution which gives a slightly different surface from the geoid. The surface is called co-geoid. In order to determine the influence of the removal step, we need a restoration step. The removed signal will be restored to the residual one. This is the concept of the remove-compute-restore approach (forsberg, 1984).

2.1.4. Gravimetric terrain reductions

In gravity field modeling the solution for the Boundary Value Problems, the function has to be known on the boundary. This requires the gravity field to be known at the geoid for any geophysical applications, particularly for geoid modeling/determination (Hieiskanen & Moritz, 1967). In the solution of BVP using Stoke's formula, an assumption was made which states that there should be no mass outside the boundary/geoid. This assumption will bring the necessity of topographic gravity reduction. The reductions of gravity-field-related quantities (e.g., gravity anomalies and disturbances, geoid heights, deflections of the vertical, gradients of the disturbing potential) for the effect of topographic and/or bathymetric masses play a crucial role in geodetic applications (Bajracharya S., 2003).

These influence of topographic masses located above the geoid and masses which are enclosed by the geoid to the gravity field which is the direct and indirect effects are computed either by the removal of the influence of layer of uniform thickness or by the removal of the influence of irregularities of the topography: i.e. the influence of the masses enclosed between the topographic surface and the flat surface of the plate (layer) (Vaníček, 1976). There are various gravimetric topographic reduction methods used to removing the effect of topography above the geoid and masses enclosed within the geoid. (Tziavos et al., 2010).

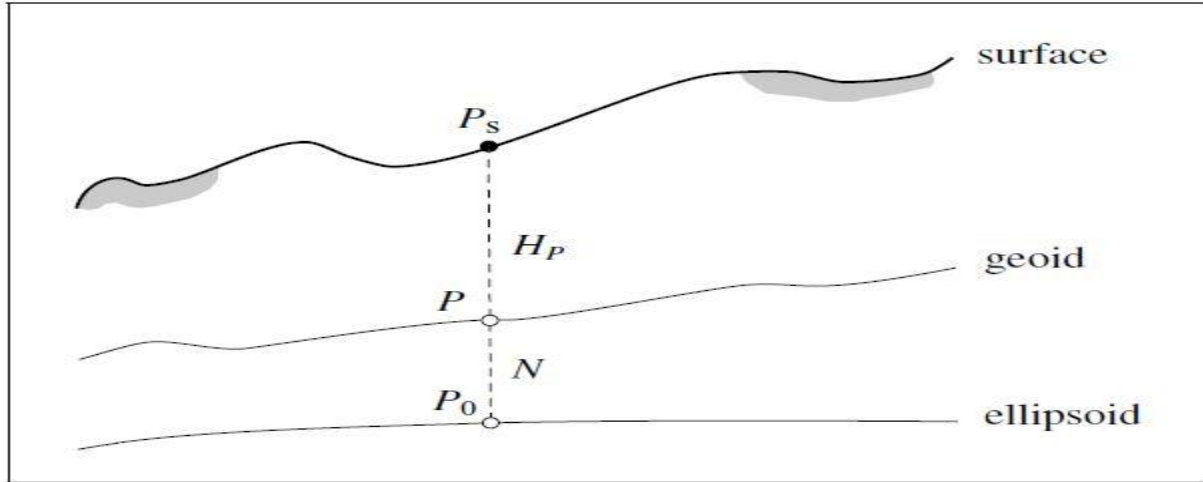


Figure 2.2: Reduction of gravity from surface point P_s down to the geoid point P . The ellipsoid is the set of approximate points P_o .

2.1.4.1. Free air reduction.

As we have seen the necessities of gravity reduction earlier, we will begin with the first and simplest assumption which is to neglect the topographic masses located between the topographic surface and the geoid. By neglecting the masses between the surface point P_s and its projection on the geoid P gravity has to be downward continued along the plumb line through P_s and P over the orthometric height of P_s (i.e., H_p which is the height above the geoid). The type of reduction which neglects all masses of topography is called free air reduction. The vertical gradient of gravity $\partial g / \partial H$ can be used for theoretically correct reduction of gravity to the geoid. If g_{p_s} is observed value at the surface of the earth, then the value g^{FA} at the geoid may be obtained as Taylor expansion; (Hofmann-Wellenhof & Moritz, 2005)

$$g^{FA} = g_{p_s} - \frac{\partial g}{\partial H} * H_p \quad (2.8)$$

Where, H is the height between P_s , the gravity station above the geoid, and P is the corresponding point on the geoid. In linear approximation, we have; (Hofmann-Wellenhof & Moritz, 2005)

$$g^{FA} = g_{p_s} + F \quad (2.9)$$

Where

$$F = -\frac{\partial g}{\partial H} * H_p \quad (2.10)$$

For many practical purposes, it is sufficient to use instead of $\partial g/\partial H$ the normal gradient of gravity (associated with the ellipsoidal height h) $\partial\gamma/\partial h$, obtaining: (Hofmann-Wellenhof & Moritz, 2005)

$$F = -\frac{\partial g}{\partial H} * Hp = +0.3086H[mgal] \quad (2.11)$$

In the computation of the free-air gravity field, the assumption we made was neglecting the gravitational attraction of the topography. Neglecting the gravitational attraction of the topography will appear in the reduced gravity field g_{fa} as a large correlation with the original topography since gravity by itself contains the attractive effect of topographic masses which is unwanted for geoid computations. This is the reason why free-air gravity anomaly data are the usual input data to apply all topographic reduction methods to these quantities. In geodesy, particularly in precise geoid computation, the topography must be considered differently (Vaníček, 1976).

2.1.4.2. Bouguer reduction

Pierre Bouguer (1698 – 1758), who led an expedition of the French Academy of Sciences doing a grade measurement in Peru, noticed the correlation between gravity and the topography of the Andes. And he insisted on topographic reduction by using plates “Bouguer plates” around the computation area (Vermeer, 2018).

To compute the correction for the attraction of the Bouguer plate let’s assume the area around the gravity station P to be completely flat and horizontal, and let the topographic mass above the geoid and below the earth’s surface lies in a thin plate of constant density ρ and thickness/height H. Then the attraction A of the Bouguer plate is obtained by: (Hofmann-Wellenhof & Moritz, 2005)

$$A = 2\pi G\rho H \quad (2.12)$$

By considering crustal density as 2670kg/m^3 we will have $A= 0.1119\text{mgal/m}^*\text{H}$, where H is expressed in meters. Here we are subtracting the attraction of the Bouguer plate from the observed gravity which is called incomplete Bouguer reduction. By adding free air reduction, we will get the complete Bouguer reduction.

$$g_B = g - AB + F \quad (2.13)$$

$$g_B = g + 0.1967 * H \quad (2.14)$$

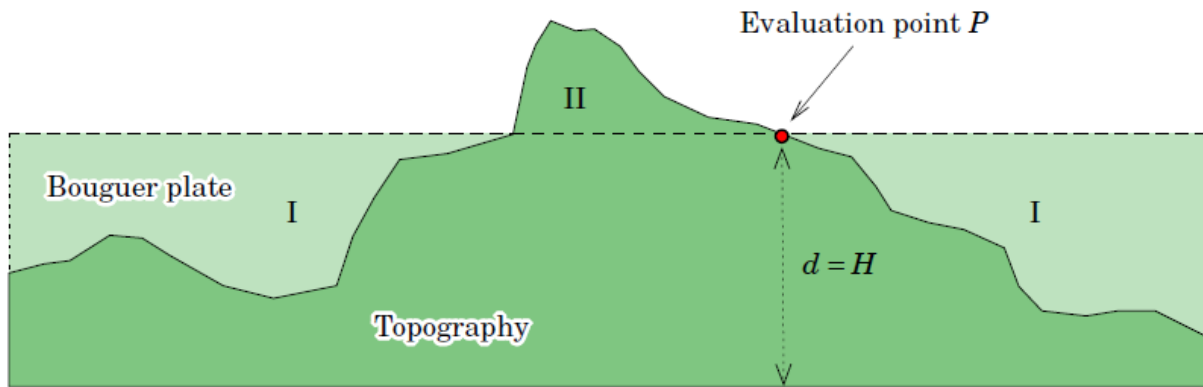


Figure 2.3: Bouguer plate as an approximation to the topography (Vermeer, 2018)

In the Bouguer reduction, all topographic points above the computation point are neglected and points below the computation point are overcompensated and the Bouguer reduction makes a systematic error. (Vermeer, 2018)

2.1.4.3. Terrain correction.

The Bouguer reduction does not consider the excess and deficit of the actual topography this simple procedure in the Bouguer reduction can be refined by taking into account the deviation of the real topographic surface from the Bouguer plate known as terrain correction. The figure below illustrates how we can consider the deviation of the actual topography to the Bouguer plate. At A the mass surplus Δm_+ , which attracts upward is removed, causing g at P to increase. At B the mass deficiency Δm_- is made up, causing g at P to increase again. The terrain correction is always positive. (Heiskanen & Moritz, 1967)

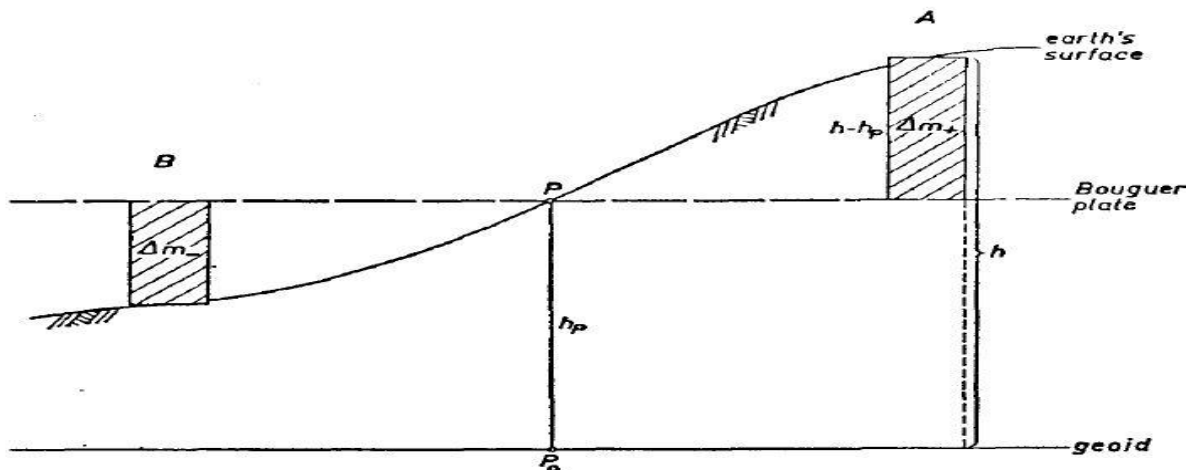


Figure 2.4: terrain correction (Heiskanen & Moritz, 1967)

The classical terrain corrections tc which are used for the refinement of Bouguer reduction given can be expressed as:

$$tc = G\rho \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^h \frac{z-hp}{((x-xp)^2+(y-yp)^2+(z-zp)^2)^{3/2}} dx dy dz \quad (2.15)$$

For the computation of the tc integrals of type equation (2.15), numerous computational approaches have been developed based on conventional methods, in the past before modern computers were made available terrain correction was computed using hammer zones (Hammer, 1939). But recently with the availability of modern computers and Digital Elevation Models (DEMs), many authors have developed methods for computing the integration of terrain correction by implementing a model of rectangular prisms of mass elements (Bajracharya, (2003); Forsberg (1984); Tsoulis, (2001)). Terrain correction calculation using the rectangular prism formula is very time-consuming and rigorous. Recently a new formulation for residual tc (RTC) was studied by (Biagi L, 2001). The calculation of terrain correction is done quickly in the frequency domain using fast Fourier FFT, having the tc convolution integral expanded in the form of Taylor series (Bajracharya S., 2003).

Now a days, various resolution of Digital Elevation Models are available across the globe. The large availability with detailed resolutions of digital terrain models (DTMs) made tc computations using the Fast Fourier Transform method the most effective tool to hold the large number of height data. The tc computation at the immediate vicinity of the computation point is very challenging due to the high contribution of the near zone since the gravity effect decreases as the distance from the computation point increases (Bedada, 2010). If the computation was done from the computation point out to fixed distance the topographic effect will converge near the source mass as we can see from (Heck & Seitz, 2007; Kiamehr, 2006; Makhloof & Ilk, 2008).

2.1.4.4. Residual Terrain Model

Forsberg, (1984) introduced the gravity reduction method by considering the short-wavelength components of the gravity field using the residual terrain model (RTM) approach. In this method, the topographical mass irregularities which are located above the mean elevation or the reference surface are computationally removed and filled below this surface. The reference surface can be determined either by spherical harmonic expansion or low pass filtering of the global digital elevation models. The residual terrain model (RTM) mass reduction method is commonly used in quasi-geoid determination. Since the mean elevation surface in this method adequately represents

the long wavelengths it possesses the advantage of not considering twice the long wavelengths for geoid/quasi-geoid determination. The RTM effect on gravity or direct RTM effect for this reduction method can be approximated by (Forsberg, 1984).

$$\delta A_{RTM} = 2\pi G\rho(h - h_{ref}) - tc \quad (2.16)$$

Where, h_{ref} is elevation of reference surface and h represent the topographic height, and G is the gravitational constant and ρ the crustal density. The classical terrain corrections tc can be expressed as:

$$tc = G\rho \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{h_{ref}}^h \frac{z-hp}{((x-xp)^2+(y-yp)^2+(z-zp)^2)^{3/2}} dx dy dz \quad (2.17)$$

Where, h_{ref} is the elevation of reference surface and h represent the topographic heights.

The indirect effect is the change in the potential and/or the geoid/quasi-geoid due to the removal or the shifting of masses underlying the gravity reductions. The surface which is computed by Stokes' formula is not the geoid itself rather it is the co-geoid. For every gravity reduction there corresponds a different co-geoid (Hofmann-Wellenhof & Moritz, 2005). The calculation of indirect effect can be done by using Brun's formula as (Yahaya & Azzab, 2018).

$$\zeta_{RTM} = \frac{\delta W}{\gamma} = \frac{G\rho}{\gamma} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{h_{ref}}^h \frac{1}{((x-xp)^2+(y-yp)^2+(z-zp)^2)^{1/2}} dx dy dz \quad (2.19)$$

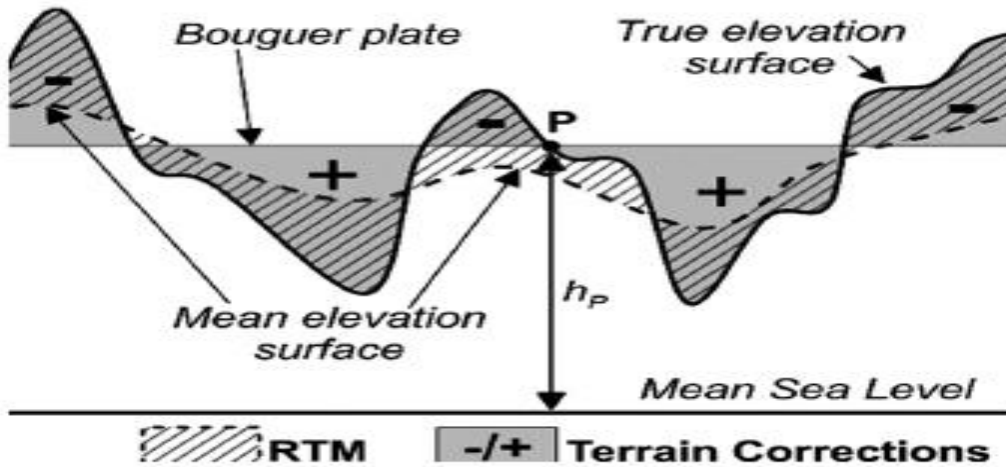


Figure 2.5: Residual Terrain Model (RTM) and terrain corrections (Forsberg, 1984)

2.1.4.5. *Helmert's second method of condensation*

Another well-known and most common gravity reduction method other than the residual terrain model (RTM) approach was proposed by Friedrich Robert Helmert which is known as Helmert's second method of condensation. In this gravity reduction scheme, the topographic masses are shifted vertically downward to mean sea level into a simple mass density layer. The mathematical representation of this topographic reduction method can be expressed as:

$$k = \rho h \quad (2.20)$$

Where κ is the density of the condensation layer, ρ is the crustal density and h represents the elevation of the topography. In this method, the total mass remains unchanged since no mass is being removed and the masses are shifted along the local vertical.

The potential of the condensed masses can be expressed as (Bajracharya S. , 2003)

$$T_{cond} = G \iint_E \frac{k}{s} dE \quad (2.21)$$

The attraction of the regular and irregular parts of the condensed masses can be expressed as (Bajracharya S., 2003):

$$A_{Cond} = \frac{\partial T_c}{\partial h_p} \Big|_{h_p=0} = A_{Cond}^{Reg} + A_{Cond}^{Irreg} = 2\pi G \rho h_p \quad (2.22)$$

Where, A_{cond} = attraction of the condensed mass

A_{Cond}^{Reg} = attraction of the regular part of the condensed masses

A_{Cond}^{Irreg} = attraction of the irregular part of the condensed masses

G = gravitational constant

ρ = density of the Earth's crust

h_p = height of the computation point

Since Helmert's second method of condensation represents the high-frequency gravity signal the Helmet's approach is valuable in the solution of Stoke's/molodensky's boundary value problem.

2.1.4.6. Pratt-Hayford topographic isostatic reduction

The Bouguer reduction does not remove the main irregularities of the gravity field in mountainous areas. The increase in the magnitude of the Bouguer anomalies in mountainous areas would lead us to assume that there is some kind of mass deficiency under the mountains. The assumption of hydrostatic equilibrium by which the gravity field surrounding plumb line deflections in mountainous areas is explained by assuming that under every mountain range there exists a root made from lighter rock species which is originated from the hydrostatic behavior of the Earth's crust over geological time scales is called isostasy (Hieskanen & Moritz 1967; Vermeer, 2018).

In 1854, John Henry Pratt observed that the actual deflections of the vertical which are obtained from astro-geodetic measurements in Himalayas mountain found in India are smaller than the deflection of the vertical that he computed from the visible masses of the mountain. He concluded that some kind of compensation is needed to account for this discrepancy. Pratt proposed a hypothesis which can be stated as “the mountains have grown from beneath the ground like fermenting dough and the density beneath the highest mountains would be the lightest material”.

Later on, in 1910, John Fillmore Hayford studied Pratt's hypothesis and developed mathematical tools needed for computation and since then the method has been named the Pratt-Hayford topographic isostatic reduction. In this method the density under the compensation level is constant. Above the level of compensation, the density of each column of the same cross-section is equal. The principle of the PH isostatic reduction can be illustrated by the following figure:

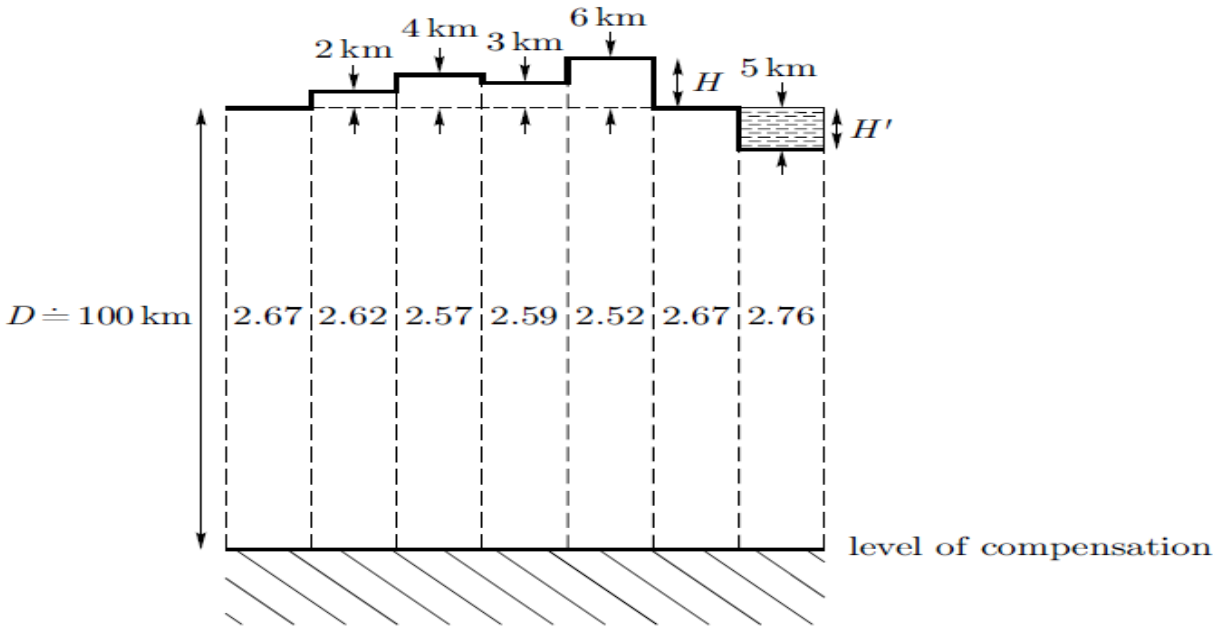


Figure 2.6: Pratt Hayford isostasy model (Wellenhof, and Moritz, 2005)

In this topographic isostatic reduction method, we can relate the density ρ_o of the compensating masses in a column of height D and the density ρ of a column with height equal to the sum of the height of the topography and the compensation depth is then: (Hofmann-Wellenhof & Moritz, 2005)

$$\rho = \frac{D}{D+h} \rho_o \quad (2.23)$$

Where h is the height of the topography. The standard density value ρ_o is 2.67 g/cm³. In equation (2.23) the actual density ρ is smaller than the standard value which shows there exists mass deficiency that can mathematically be expressed as;

$$\Delta\rho = \rho_o - \rho = \frac{h}{D+h} \rho_o \quad (2.24)$$

2.1.4.7. Airy-Heiskanen topographic-isostatic reduction

In 1855, George Biddell Airy proposed another isostatic model in addition to the Pratt-Hyford topographic isostatic model. According to these methods, mountains are floating on fluid lava of higher density, so that the higher the mountain the deeper it sinks which implies there is mass deficit (roots) below mountains and mass surpluses (anti-roots) below oceans.

The AH model is based on the assumptions that the isostatic compensation is complete and local, the density of the mountains is constant and equal to ($\rho_o = 2.67 \text{ g/cm}^3$), the density of Earth's mantle is equal to ($\rho_M = 3.27 \text{ g/cm}^3$) and the normal crust thickness T_o is equal to 30 km (Heiskanen and Moritz 1967). Assuming a constant density of ($\rho_w = 1.027 \text{ g/cm}_3$) for the ocean water, then the condition of floating equilibrium can be written as:

$$(\rho_m - \rho_o)d = \rho_o * H \tag{2.25}$$

For continental cases:

$$(\rho_m - \rho_o)d' = (\rho_o - \rho_w)H' \tag{2.26}$$

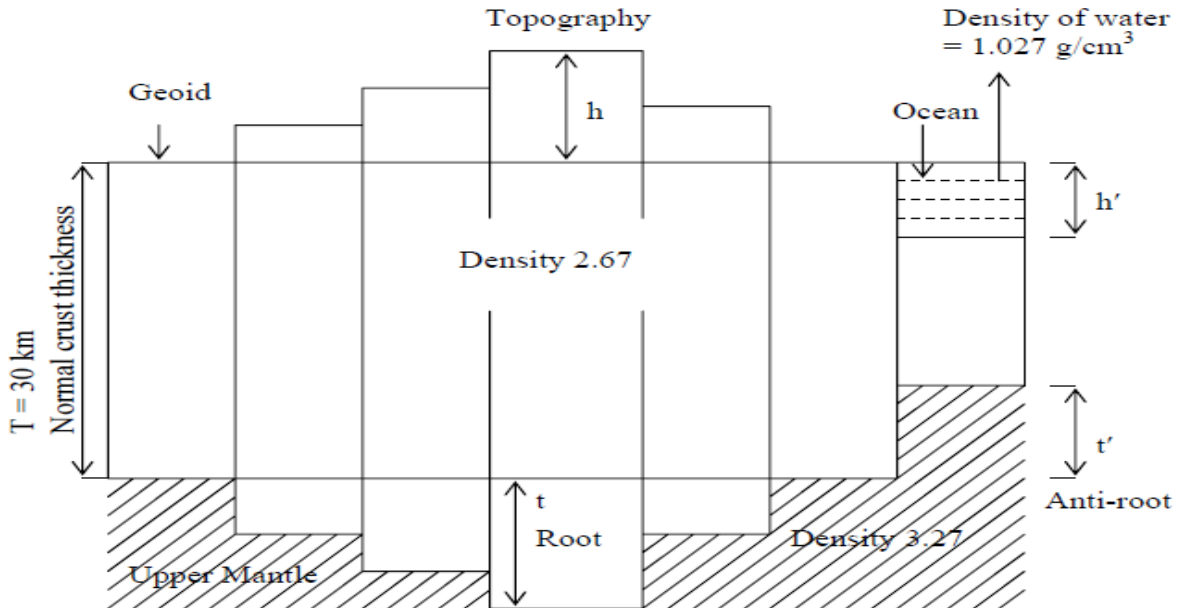


Figure 2.7: The Airy-Heiskanen Model (Bajracharya S. , 2003)

For the oceanic areas d is the thickness of the root, d' is the thickness of the anti-root, H is the height of the topography, and H' is the height of the ocean, i.e., the depth. Given the above-mentioned density values for the crust, the mantle, and ocean water,

$$d = 4.45H \tag{2.27}$$

$$d' = 2.73H' \quad (2.28)$$

The topographic effect due to this AH topographic isostatic scheme at a point P at the surface of the Earth and the corresponding AH reduction is the difference of the gravitational attraction of the topographic masses as described by the available Digital Elevation Models and the compensated masses within the depth of the root.

2.2. Empirical review

The role of topographic computation for regional gravity field modeling or gravitational potential computation is based on the fact that much of the short-wavelength components of the measured gravity field are due to the attraction of the topography. Both geometrical and spherical harmonic representations of the topographic gravity and potential are needed (Bedada, 2010). Prior to the vast availability of modern computers, terrain corrections were calculated by using templates by which the topography around the gravity stations are divided into Hammer zones (Hammer, 1939). Because of the laborious task included in this method and the availability of high-resolution digital elevation data the method was ignored and replaced by new methods using Global digital elevation models. Many authors have devised methods and algorithms for calculation of terrain correction (Forsberg, 1984; Banerjee, 1998; Bedada, 2010; Biagi & Sanso, 2001; Sideris, 1985). Forsberg (1984) introduced the Residual Terrain Model (RTM) technique for the Remove-Compute-Restore approach which is done by considering the short-wavelength components of the gravity field. Hwang et al (2003) developed a method based on Gaussian quadrature to compute terrain correction(TC). Banerjee (1998) reduced gravimetric measurements in the highly demanding areas of the north-western Himalayas. He presented a new computational method. Bedada, (2010) in his study of the absolute geopotential height system for Ethiopia studied the topographic effects using the combined method of multi-point and vertical line mass by applying a hedgehog algorithm to compute the gravity and potential models of the topographic effects representing the region of Ethiopia and its immediate surroundings.

Chapter Three

3. Materials and Methods

3.1. Description of the study area

The study area covers the geographical location of Ethiopia which lies in the range of 3°N – 15°N in latitude and 33°E – 48°E in longitude. It borders Sudan, Eritrea, Djibouti and Somalia, Kenya on west, north, east and south direction respectively. Ethiopia covers 1.14 million km² total surface area and 5328 km length of the border. The study area is presented in the following figure with elevations from Shuttle radar topography mission (SRTM) data with 3 arcs second resolution.

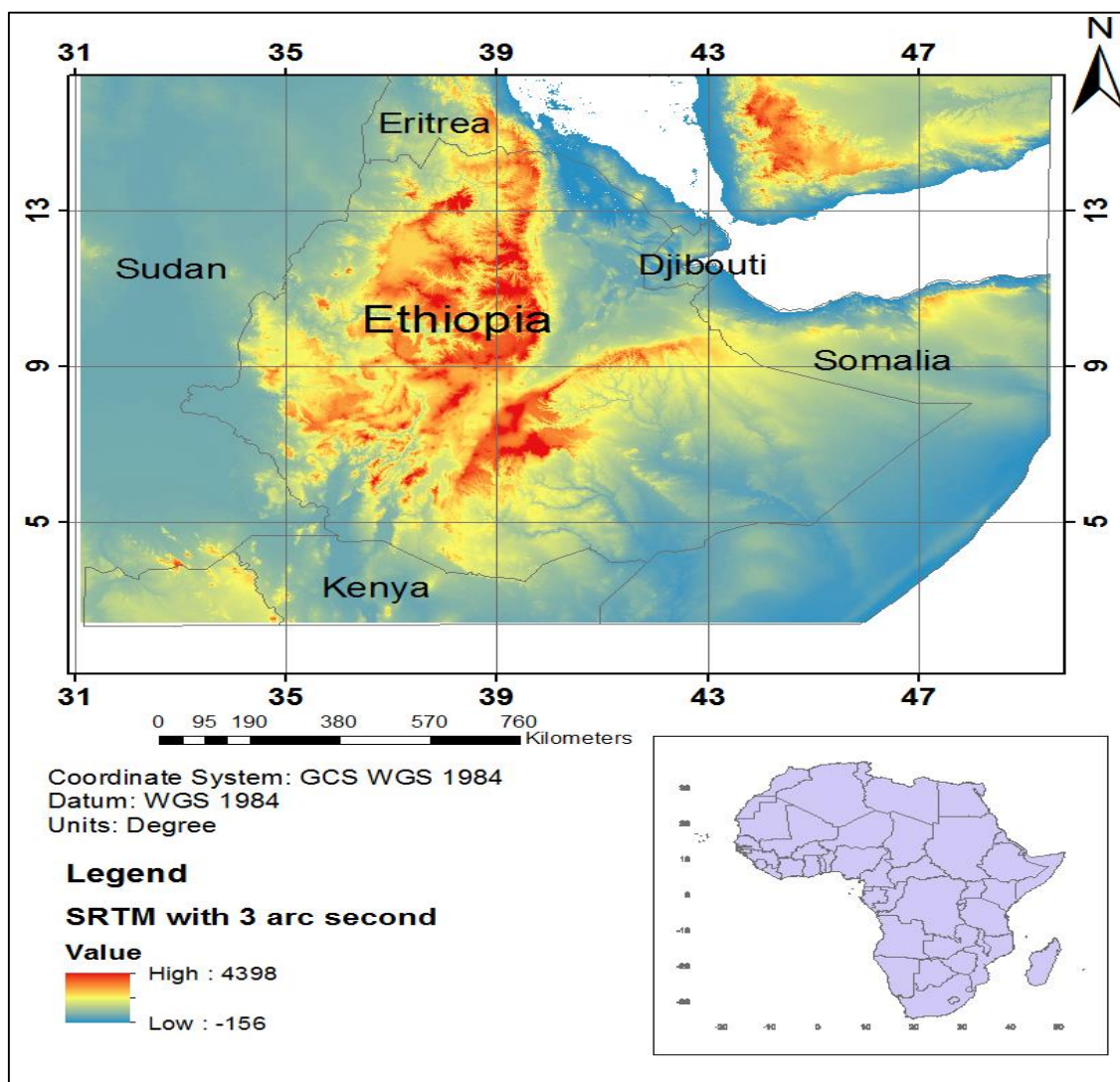


Figure 3.1: location map of Ethiopia

The topography of Ethiopia consists of various configuration. Thus, highland, moderately elevated and low land are revealed the entire territory of Ethiopia. The central high plateau lies surrounding east African great rift in northern and southern direction. The plateau differs from 1500 to 3000m above mean sea level. The highest plateau found in the northern part of Ethiopia having elevation value of 4620 meters above mean sea level and known as Ras-dashen. The lowest point is found at the eastern margin of the region called Denakil depression and is a part of a rift system placed 115 meters below mean sea level. Several river-basin are in the region that originated from the most highland area and flow to lowlands like Abay Basin, Awash Basin, Genale basin, Omo basin, and Tekeze basin. The topography of Ethiopia shows a highly diversified terrain configuration.

3.2. Materials used in the study

The materials used in this study are presented in the following table along with their purpose:

Table 1: software and their use

No	Materials and software	Purpose
1	Gravsoft software	For TC calculation using module TC and mean elevation computation of SRTM GDEM using TCGRID and SELECT
2	math lab software	For plotting of the output
3	GIS	To generate map and layout
4	Microsoft excel	Statistical computation
6	ICGEM calculation service	To calculate the mean elevation of DTM2006.0
7	QGIS	To extract DEM height values for the study area
8	Global mapper software	To convert the DEM height grid into Gravsoft format

3.3. Data and data sources

3.3.1. Terrestrial Gravity Data

The terrestrial gravity data covers from 3° to 15° North and 33° to 48° East. The study used 22,762 terrestrial gravity data points distributed over Ethiopia. The data intended to cover the whole area of Ethiopia. Even if it covers the vast areas of the country, it was not evenly distributed in some parts of the country. The location and elevation of terrestrial gravity points were collected with the help of GPS. The elevation values in the terrestrial data refer to the ellipsoid. The distribution of the terrestrial gravity points was presented in the following figure.

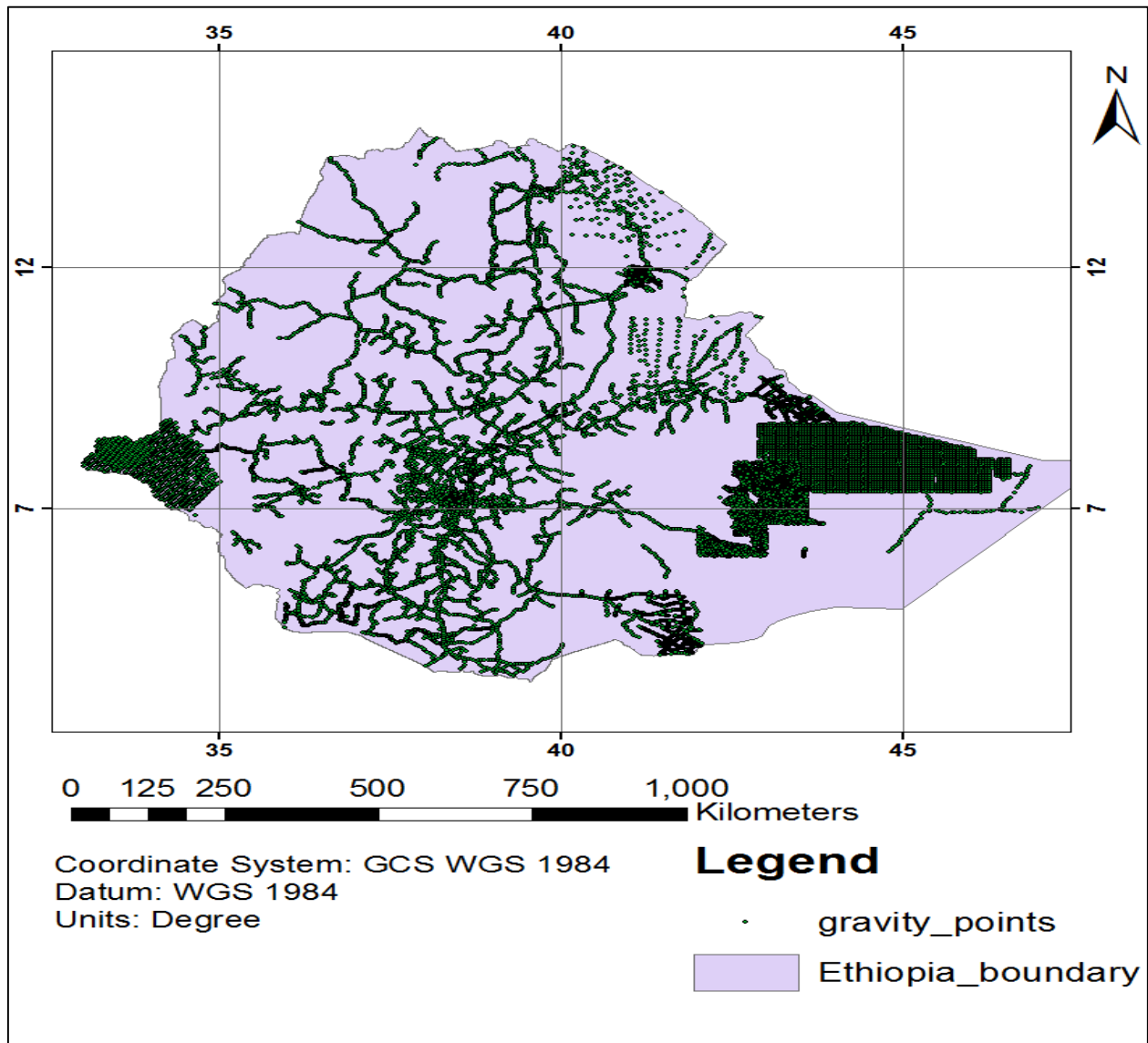


Figure 3.2: distribution of terrestrial gravity data points

3.3.2. Shuttle Radar Topographic Mission (SRTM) elevation data

The main input data for the computation of terrain correction and residual terrain model is SRTM GDEM data. Digital Elevation Models gives information about the configuration of the topographic surface of the Earth. For this reason, DEMs are used in different application areas of geoscience (Varga, et al., 2019). The term Digital Elevation Model may refer to the terrain configuration of the Earth with Digital Terrain Models or the terrain configuration and natural and artificial features with Digital Surface Models (Wilson and Gallant 2000).

Depending on the well representation and description of the actual topography by Digital Elevation Models the qualities of DEMs can be defined by the vertical accuracy. The accuracy of digital elevation models varies from place to place due to the topographic nature of the area. The vertical accuracy of DEM is usually predicted by using independent data of higher accuracy, like leveling benchmarks (Varga, et al., 2019).

The Shuttle Radar Topographic mission (SRTM) Digital Elevation Model (DEM) data was provided by the National Aeronautics and Space Administration (NASA) and covers about 80% of the Earth. Currently the united states geological survey (USGS) have distributed the Shuttle Radar Topographic Mission (SRTM) data free of charge and made it available for download from the national map seamless data distribution system or the USGS flip site (Jarvis, et al. 2008). In this study, SRTM DEM with a spatial resolution of 3 arc seconds and 30 arc seconds was used. And also The DTM2006.0 was used for the construction of mean elevation surfaces using spherical harmonic method.

3.4. Methods

3.4.1. Computation of Mean elevation surfaces and Coarse DEM

The residual terrain model (RTM) terrain reduction method considers only the high frequency components of the topographic gravity field by selecting a smooth surface that could represent the mean elevation of the area. This reference surface can be defined by low-pass filtering of Digital Elevation models or from spherical harmonic expansion of the topographic surface of the globe.

The mean elevation surface defined by the low-pass filter can mathematically be expressed in plane approximation as; (Forsberg, 1984)

$$h_{ref}(p) = \frac{D}{2\pi} \int_{\pi} \frac{h(Q)}{[r^2 + D^2]^{3/2}} d\pi_Q, \quad r = dist(P, Q) \quad (3.1)$$

Where h_{ref} is the mean elevation at computation point P, D is the depth of the earth's crust, $h(Q)$ height of point Q.

According to (Seeber, 2008) the GGM maximum expansion degree of truncation l_{max} can be related with the shortest resolvable wavelength λ .

$$\lambda \text{ (km)} = \frac{40,000}{l_{max}} \quad (3.2)$$

These can be translated/formed as the spatial resolution of $\Delta x = \lambda/2$. The Global Geopotential Models are available in spherical harmonic expansion degrees truncated from short to medium and long wavelengths. Global Geopotential Models with spherical harmonic expansion degrees truncated to degree and order (d/o) of 2190 e.g. EGM2008 (Pavlis et al., 2012), GECO (Gilardoni et al., 2015), and EIGEN-6C4 (Förste, et al., 2014) corresponds to minimum wavelengths λ of about 10 arc minutes (~18km) which equates to a spatial resolution Δx of 5 arc minutes (~9km). recently, 1 up to 2 cm geoid precision and 1 up to 2mgal gravity anomaly precision was expected at spatial gravity missions with d/o 200 or ~100km spatial resolution.

3.4.2. Residual Terrain Model and gravimetric terrain Corrections

Among the aforementioned terrain reduction methods, the study utilizes the residual terrain model (RTM) technique. As briefly stated in the theoretical review section the residual terrain model (RTM) effect on gravity or direct RTM effect for this reduction method can be approximated by the difference of Bouguer plates by which the first one is calculated with the actual elevation and the second one with the height of the mean elevation surface minus the classical terrain correction (Forsberg, 1984).

$$\Delta g_{RTM} = 2\pi G\rho(h - h_{ref}) - tc \quad (3.3)$$

Where, h is the actual height of computation point and h_{ref} represents mean elevations of the computation point, and G is the gravitational constant and ρ is crustal density. The integral equation for the classical terrain correction computation can be expressed as:

$$T_c = G\rho \int_{x1}^{x2} \int_{y1}^{y2} \int_{z1}^h \frac{z-hp}{((x-xp)^2+(y-yp)^2+(z-zp)^2)^{3/2}} dx dy dz \quad (3.4)$$

Where, h_{ref} represent the height of the mean elevation surface and h represent the height of the topography.

The indirect effect is the change in the potential and/or the geoid/quasi-geoid due to the removal or the shifting of masses underlying the gravity reductions. The surface which is computed by Stokes' formula is not the geoid itself rather it is the co-geoid for every gravity reduction there corresponds a different co-geoid. (Hofmann-Wellenhof & Moritz, 2005). The indirect effect can be calculated by using Brun's formula as:

$$\zeta_{RTM} = \frac{\delta W}{\gamma} = \frac{G\rho}{\gamma} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{h_{ref}}^h \frac{1}{((x-xp)^2+(y-yp)^2+(z-zp)^2)^{1/2}} dx dy dz \quad (3.6)$$

Practical implementation of terrain correction calculation is implemented by dividing the topographic surface around the computation point in to two different zones. Depending on how far it is from the computation point the area is divided in to the near zone and the far zone, that is controlled by integration radiuses $R1$ and $R2$. Two kinds of DEMs are usually used in the calculation of terrain corrections. In the calculation of the near zone Digital Elevation Models with high resolution (fine DEM) are used and the far zone is calculated by using DEMs with lower resolutions (coarse DEM). (Forsberg, 1984).

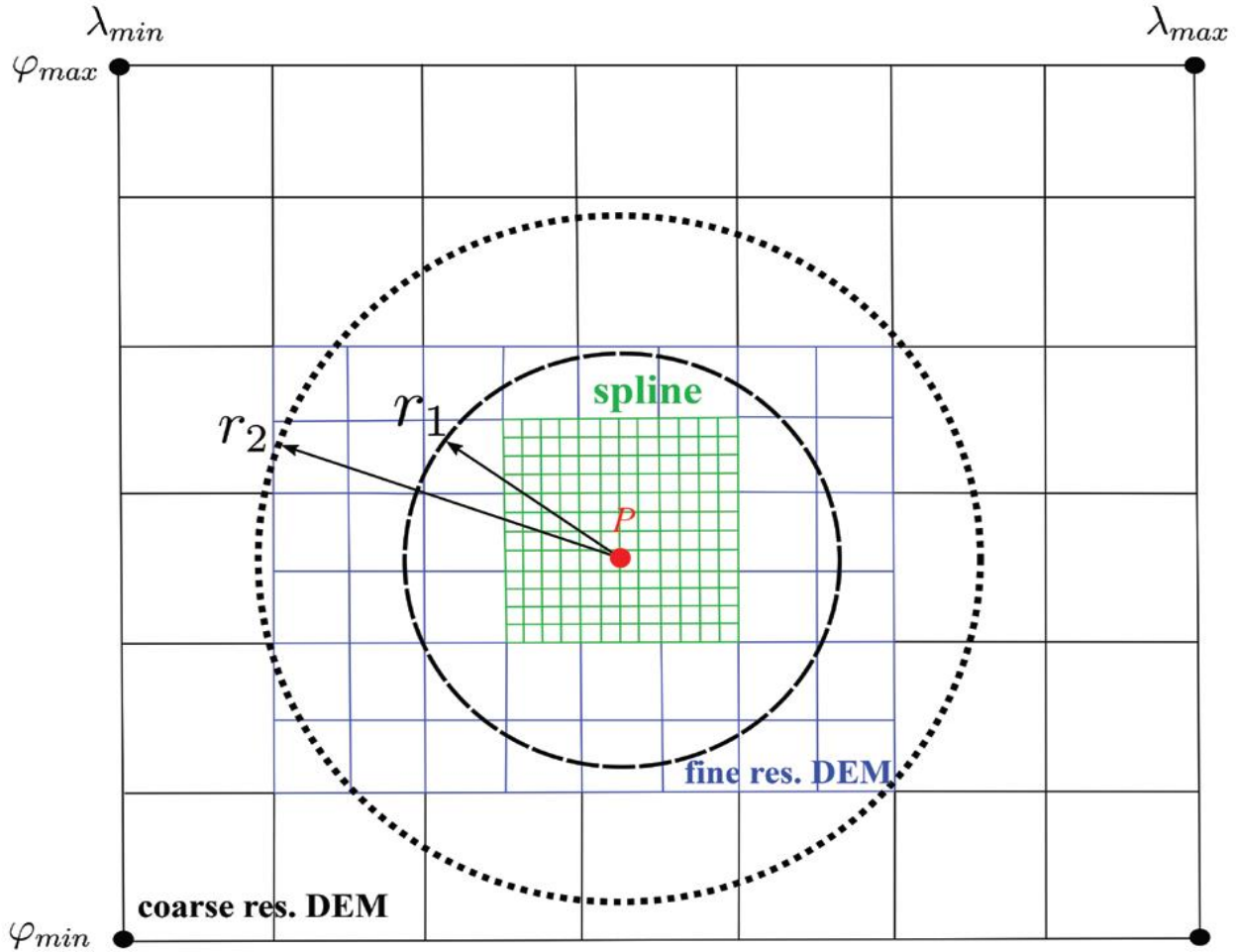


Figure 3.3: practical implementation of terrain correction calculation in the near and far zones (Varga et al 2019)

3.4.3. Integration radius

Numerous studies have been performed on the identification and choice of integration radius R_1 and R_2 based on the required amount of accuracy. (Nowell, 1999) outlined different works of literature on the methods for the choice of the integration radiuses. Many authors recommended the value of R_1 and R_2 to be used. For the choice of R_1 values up to 20km and R_2 values up to 200km are can be used as a standard. (Hwang et al 2003, Varga et al 2015, Yahaya & Azzab 2018) recommended that a value 0f 20km for R_1 and 200km for R_2 is sufficient for 0.1mgal precision.

The study used the mean elevation surface corresponding to 100km spatial scale in order to observe the pattern of the value of the integration radiuses and to obtain reasonably stable values two test points have been chosen in the study, one in a mountainous area and one in the low-lying region.

The method outlined in Hirt et al (2010) was used in this study to find the value of R2. By fixing R1 to be 10km, and R2 to be varying up to 300km. by fixing R2 to be 200km and R1 varying from 5km to 55km the study analyzed the differences of terrain corrections and indirect effects using the 55km as a reference.

3.4.4. GDEM spatial resolutions and terrain effects

Digital elevation models (DEMs) are made available recently with high resolution of spatial scale. In this section, the study tried to investigate DEMs with different resolutions in order to recommend sufficient spatial resolution for an improved and acceptable level of precision for RTM and terrain effects in Ethiopia. Two test areas, one in a low lying region near Dallol depression located in the afar region (14° to 15°East and 40° to 41° North) and another in Ras-Dashen mountain located in Amhara region (13° to 14°East and 38° to 39° North) have been chosen for computing RTM effects and terrain corrections at grid nodes with 3 arcs second spatial resolution, and resampled the version to 7.5,15,30 and 60 arc seconds spatial resolutions of SRTM data. The study analyzed the differences of RTM effects and terrain corrections from different spatial resolutions by setting 3 arcs seconds as a reference.

$$\Delta R = R_{3s} - R_{other} \quad (3.7)$$

Where ΔT represents the difference of RTM effects and terrain corrections, R_{3s} stands for RTM effects and terrain corrections computed from 3 arcs seconds resolution and R_{other} stands for the RTM effects and terrain corrections computed from the 7.5, 15, 30, and 60 arc second resolutions.

3.4.5. Calculation of Residual Terrain Model effects and terrain corrections at grid nodes and gravity stations

Using 30 arcs second SRTM GDEM data the RTM direct and indirect effects and terrain corrections for each gravity station and grid node have been computed with the mean elevation surface corresponding to ~100km and ~9km spatial scale. The study area was divided into $5^\circ \times 5^\circ$ small zones due to computer limitations. The statistics and grid maps that show the distribution of RTM effects terrain corrections in Ethiopia were presented.

3.4.6. Effect of height differences on Residual Terrain Model effects and terrain corrections

The effect of the difference of height values between the gravity stations and height values which are extracted from SRTM Digital Elevation Model data on the accuracy of Residual Terrain Model effects and terrain corrections have been analyzed by preparing an alternative database. To make the two elevation sources at the same datum the geoid undulation was subtracted from the ellipsoidal normal height. The alternative database was created by replacing the heights of gravity stations with height values extracted from SRTM GDEM. The results of the recomputed terrain effects with mean elevation surface corresponding to ~100km spatial scale are compared with the original database by a simple difference

$$\Delta X = X_{original} - X_{alternative} \quad (3.8)$$

Where X represents, elevation value H , RTM direct effect Δg_{RTM} , RTM indirect effect ζ_{RTM} and terrain correction t_c . the differences were analyzed by basic statistics and Pearson correlation test at 5% confidence level.

3.5. Data analysis

The computation of terrain effects was performed using the TC module distributed within the geoid modeling programming package GRAVSOFIT (Geodetic Gravity Field Modelling Programs) (Forsberg & Tscherning, 2008). Modules SELECT and TCGRID were used to calculate the mean elevation surface by low pass filtering of Shuttle Radar Topography Mission Global Digital Elevation Model data, by using the procedures obtained from the GRAVSOFIT tutorials. The SRTM was resampled to 0.1° and applied a 9×9 moving average window in construction of the mean elevation surfaces corresponding to degree and order (d/o) 200 or ~100km spatial scale. The SRTM was resampled to 1 arc minute and applied a 5×5 moving average window in construction of the mean elevation surfaces corresponding to degree and order (d/o) 2190 or ~9km spatial scale. For the DTM 2006.0 (Pavlis, et al. 2007) which was provided by the EGM2008 development team the mean elevation surface corresponding to degree and order, 200 and 2190 were calculated using the calculation service of the ICGEM flip site. The results of the low-pass filter and spherical harmonic methods were compared by simple differences and cross-section profiles.

To minimize the edge effect, the study area was extended by 1° in all directions, covering from 2° to 16° latitude and 32° to 49° longitude. The Residual Terrain Model direct and indirect effects and

terrain corrections were calculated for the entire study area by using mean reference surfaces corresponding to ~100 and ~9km spatial scale. And the study used 30 arcs second SRTM GDEM. In addition to this, sufficient spatial resolution for good precision of terrain effects in Ethiopia was investigated. The Residual Terrain Model effects and terrain corrections were calculated in two test areas using different spatial resolution of Shuttle Radar Topography Mission Global Digital Elevation Model. The test areas were one in low-lying areas located around the Dallol area of the Afar region and another in mountainous areas located around the Ras-Dashen Mountains. By setting 3 arcs second as reference the difference of terrain effects by spatial resolution were analyzed. Finally, the study analyzed the impact of height discrepancy between the height values of the gravity stations and DEM extracted heights on the accuracy of RTM effects and terrain corrections. Because the height value of the gravity stations refers to the ellipsoid and the SRTM height refers to the geoid, the Geoid undulation for each gravity station were subtracted from the normal height

3.6. Workflow

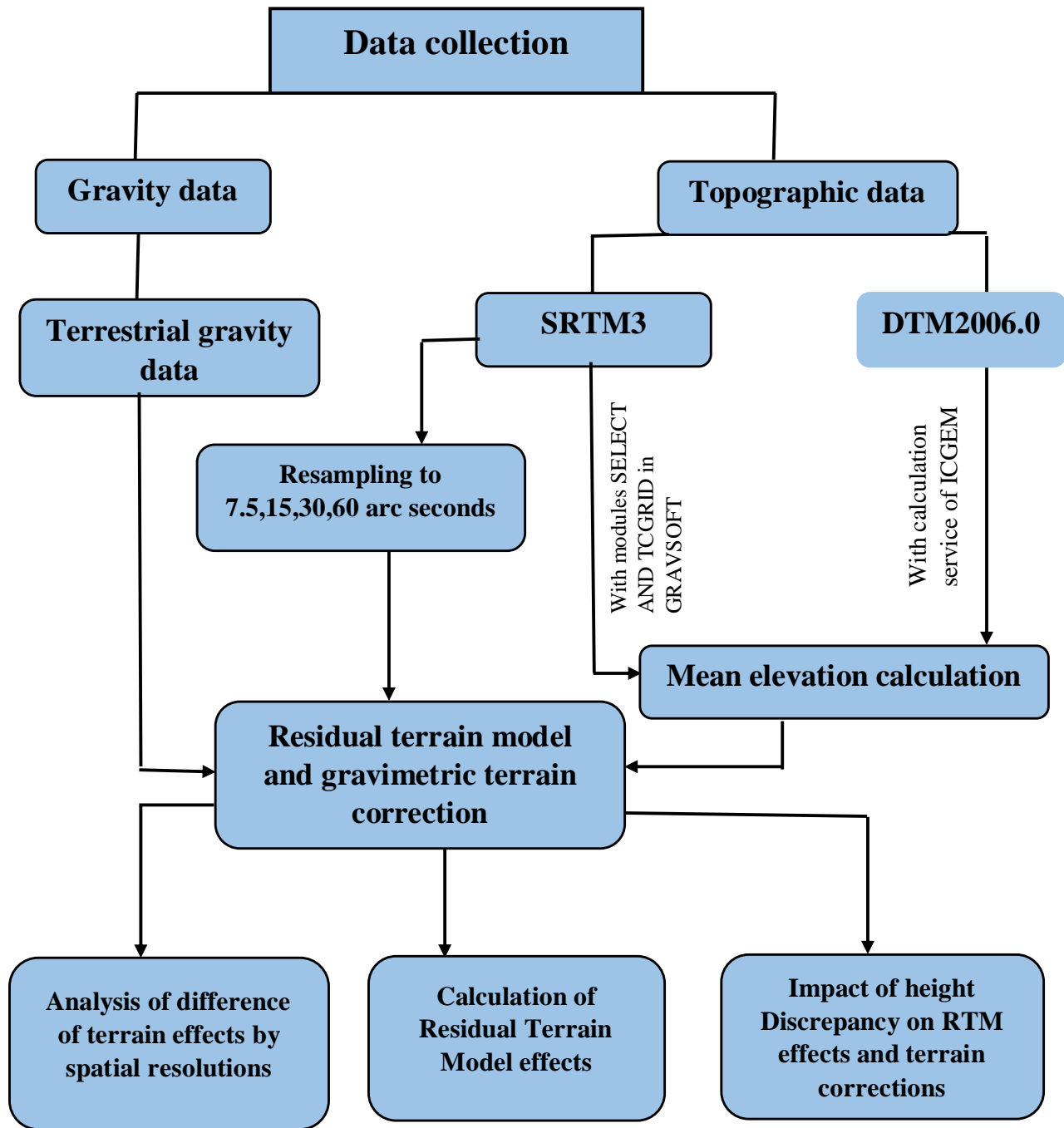


Figure 3.3: work flow chart

Chapter Four

4. Result and Discussion

4.1. Mean elevation surfaces

As the study described in the method section, the output from the computation of mean elevation surfaces can be used as a reference height value in the calculation of RTM and terrain effects. Figure 4a shows the mean elevation surface which represents a spatial scale of ~100km or in degree and order (d/o) of 200 computed from the low-pass filter, and figure 4b, shows the difference of mean elevation surfaces computed by a low-pass filter and mean elevation surfaces derived from the topographic spherical harmonic model.

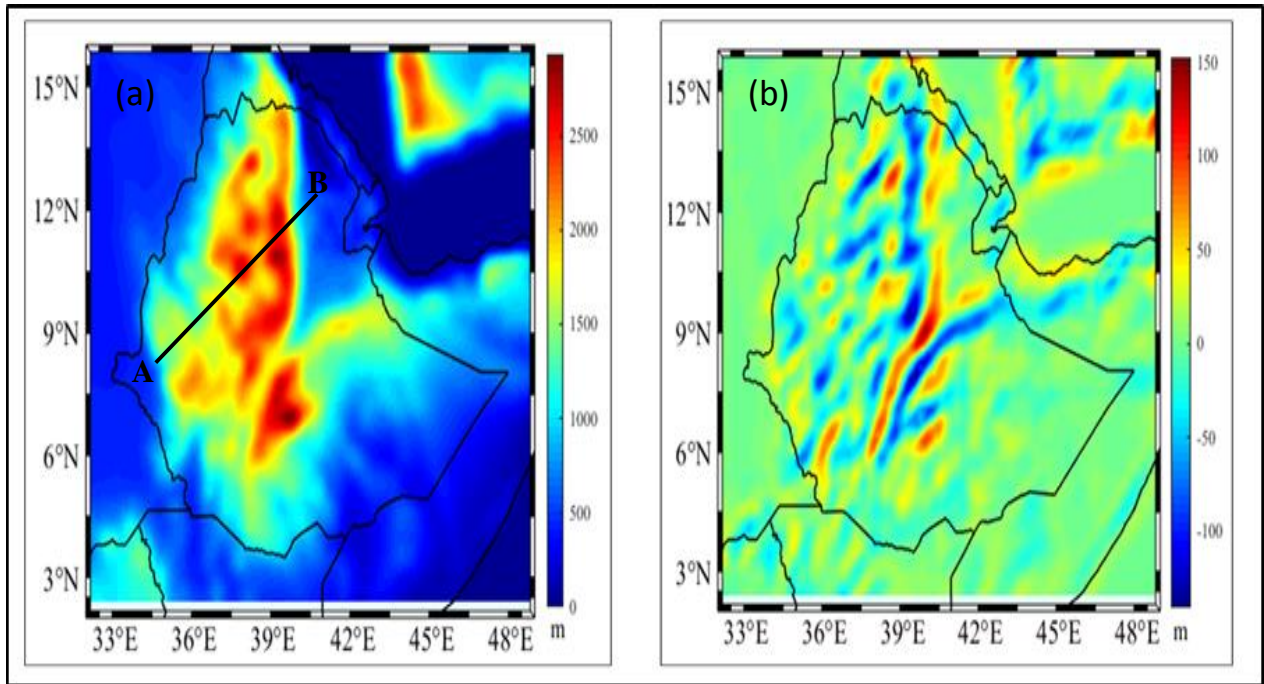


Figure 4.a, b: Mean elevation surface obtained with low-pass filter of SRTM and their differences with mean elevation surface obtained from spherical harmonic expansion in meters: a – 100km spatial scale; b – Difference for 100km spatial scale

Figure 4c, shows the mean elevation surface which represents a spatial scale of ~9km or in degree and order (d/o) of 2190 computed from the low-pass filter and figure 4d, shows the difference of mean elevation surfaces computed by a low-pass filter and mean elevation surfaces derived from topographic spherical harmonic model corresponding to d/o 2190.

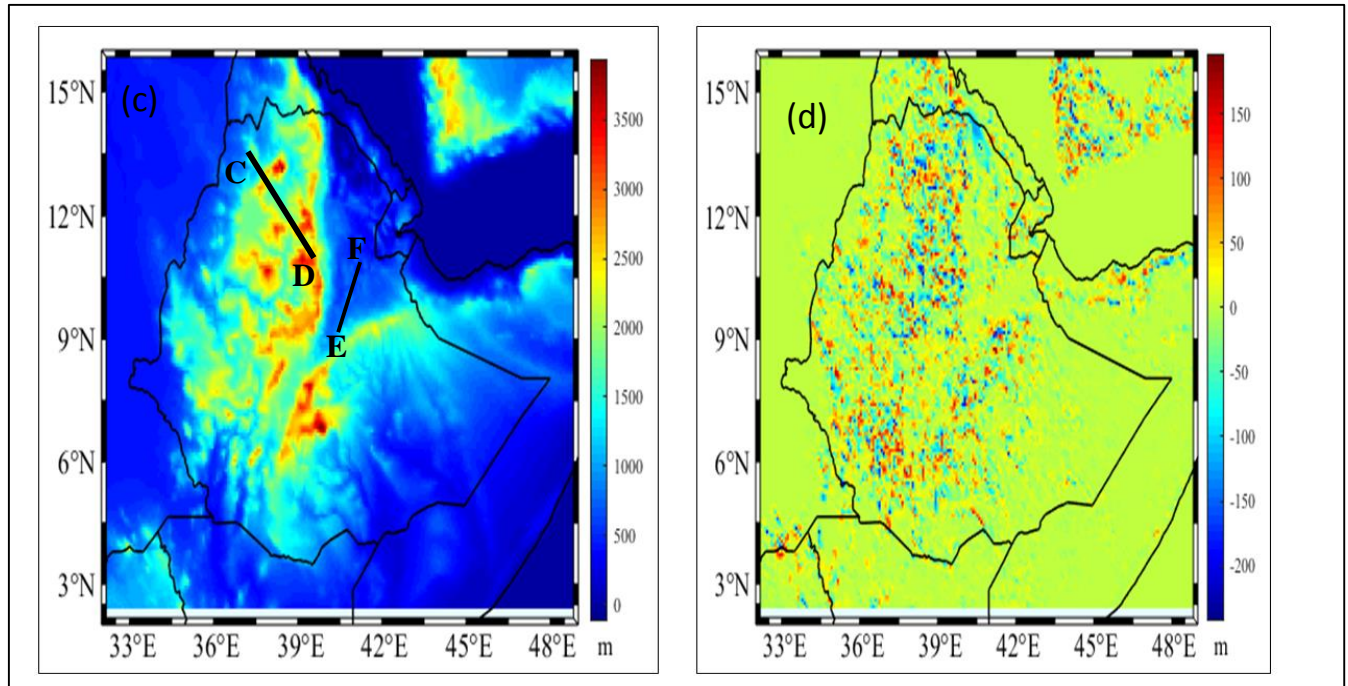


Figure 4.c,d: Mean elevation surface obtained with low-pass filter of SRTM and their differences with mean elevation surfaces obtained from spherical harmonic in meters: a – 9km spatial scale; b – Difference 9km spatial scale

Figure 4e, shows the cross-section profile of sample points A—B taken from the mean elevation surface corresponding to ~100km spatial scale (cf. figure 4a) and figure 4f, and figure 4g, show the cross-section profile of sample points which represent a low lying region and mountainous area respectively taken from the mean elevation surface of d/o 2190 (cf. figure 4c).

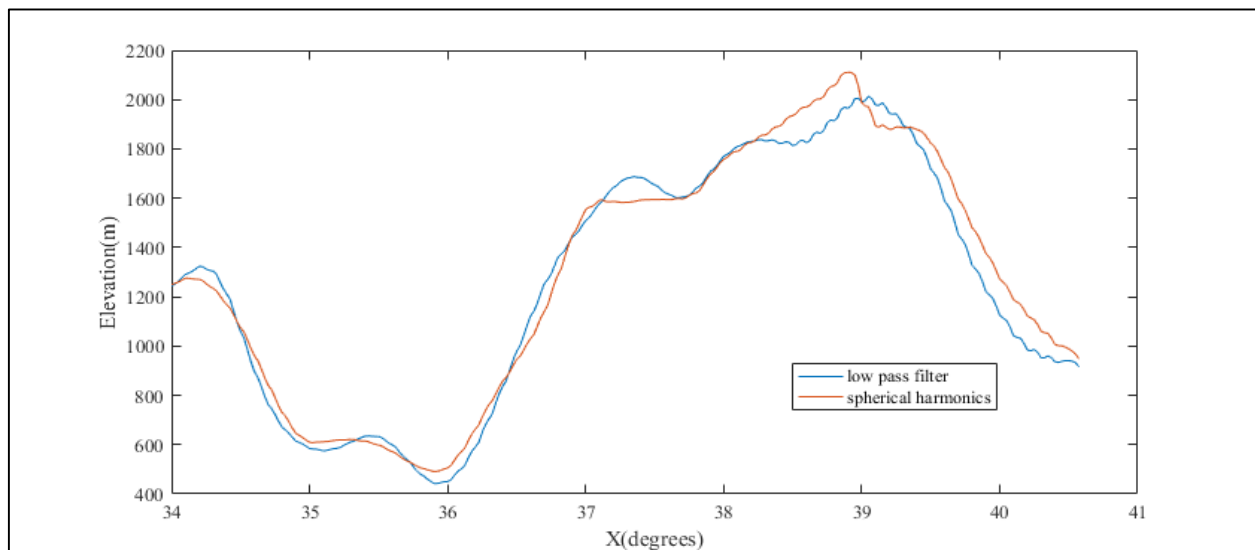


Figure 4. e: cross-section profiles A—B (cf. figure 4a) taken from mean elevation surface corresponding to ~100km spatial scale

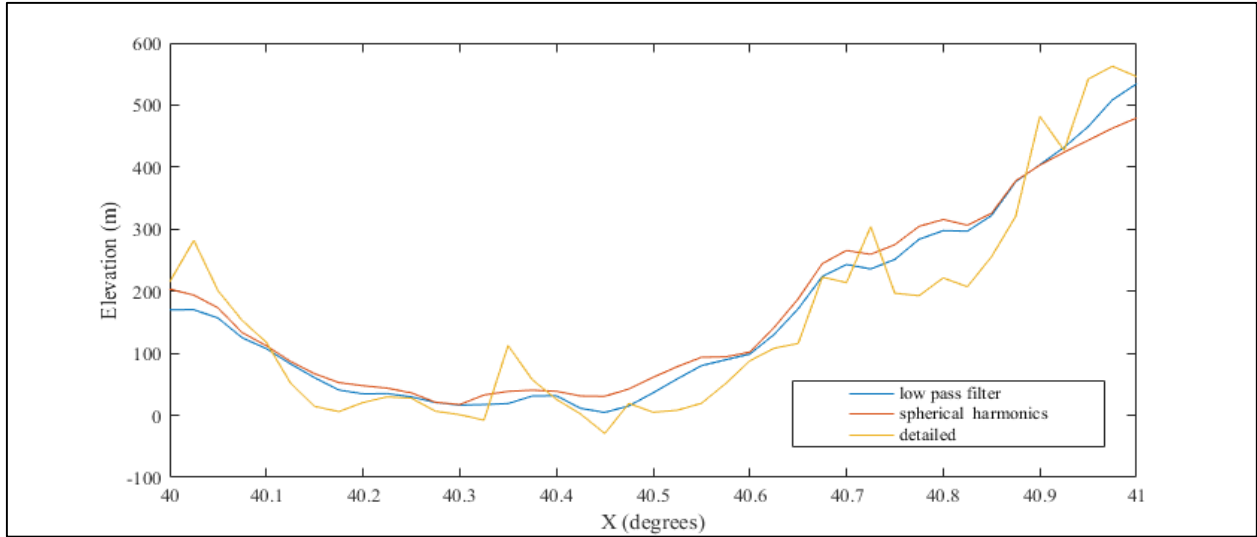


Figure 4. f: cross-section profile E—F (cf. figure 4c) located in a low lying region, taken from mean elevation surface corresponding to ~9km spatial scale

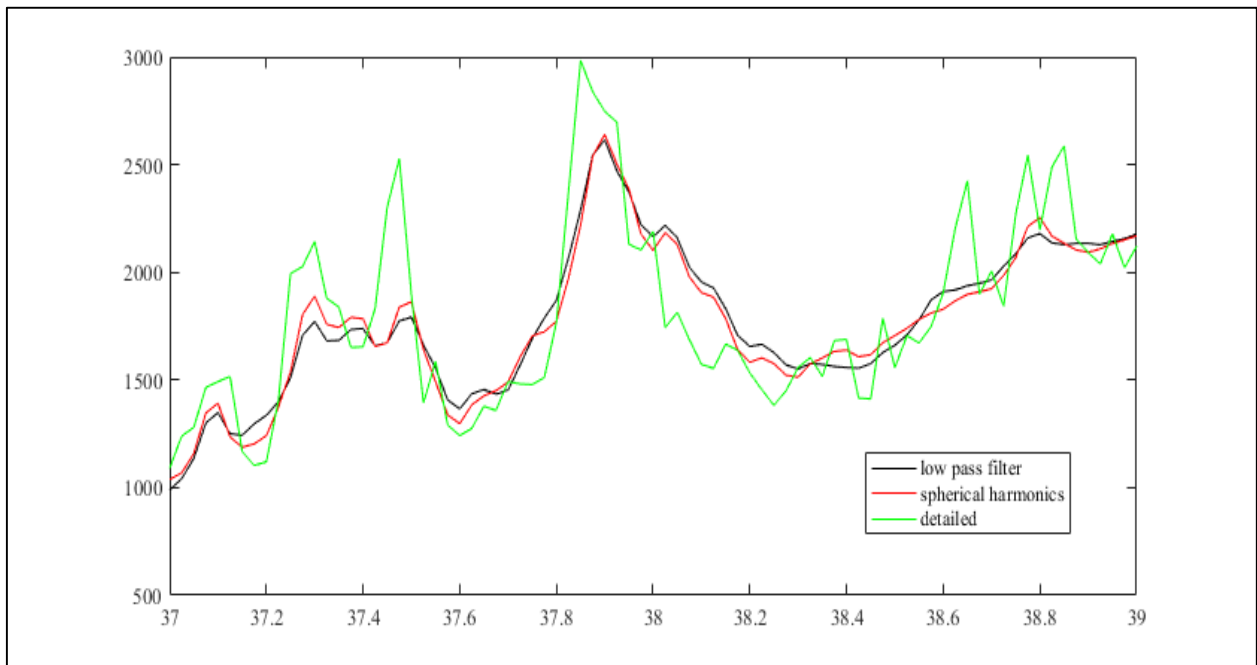


Figure 4. g: cross-section profile C—D (cf. figure 4c) located around Ras-Dashen Mountain, taken from mean elevation surface corresponding to ~ 9km spatial scale

The descriptive statistics of the mean elevation surfaces computed by the low-pass filter method and the mean elevation surfaces derived from the topography spherical harmonic model and their differences have been presented in table 2.

Table 2: The statistical description of reference surface and their discrepancy in meters

Spatial scale	method	Min	MAX	Mean	STD	RMS
100(km)	Low Pass Filter	0.00	2948.36	810.7	637.87	1031.55
	Spherical Harmonics	0.00	2763.31	810.59	625.65	1023.95
	Low Pass-Spherical	229.963	239.38	0.89	52.71	53.32
9(km)	Low Pass Filter	0.00	3990.43	810.76	677.13	1056.33
	Spherical harmonics	0.00	4070	810.14	682.3	1059.17
	difference	249.701	198.82	0.77	9.95	9.97

As it can be seen from figures 4a and 4c, it is obvious that the mean elevation surface corresponding to 100 km spatial scale or degree and order (d/o) of 200 is smoother. In table 2, the difference of mean elevation surfaces computed by a low-pass filter and those derived from spherical harmonic model reaches an absolute value of 250m, with root mean square (RMS) error of 53.32m for mean elevation surface corresponding to d/o of 200 and 9.97m for mean elevation surface corresponding to d/o of 2190.

Cross-section profiles A-B in figure 4e (cf. figure 4a) which were taken from the mean elevation surfaces corresponding to d/o 200 shows that the mean elevation surface computed by the low-pass filter method gives more spectral details than the spherical harmonics method. Cross-section profiles C-D on figure 4g and cross-section profiles E-F on figure 4f (cf. figure 4c) which were taken from the mean elevation surfaces corresponding to d/o of 2190 are located in the low lying regions and mountainous areas respectively. In addition to the previous cross-section profile for the mean elevation surfaces corresponding to d/o of 200, the later cross-section profiles show that the low-pass filter solution presents more spectral details for the mean elevation surfaces corresponding to d/o of 2190. Since the reference surface computed by the low-pass filter method presented more spectral details the study used the mean elevation surface computed by the low-pass filter to better construct RTM reference surfaces by considering local terrain characteristics.

4.2. Integration radius

In this section, the choice for integration radius R1 and R2 have been analyzed by computing the RTM indirect effects and terrain corrections as a function of R1 and R2 by which the acceptable value of R1 and R2 can be obtained with a sufficient amount of precision. The computation was made for the test points in the low-lying regions and mountainous areas. Figure 5a shows the terrain correction and Residual Terrain Model indirect effect and terrain corrections with respect to calculation radius R2 for the test points which are located in the low-lying region around the Dallol area in the Afar region. In the near 40km radius around the test point the more significant values have been observed with an amplitude of 2-10cm of indirect effect and 0.1-0.5mgal of terrain correction at wavelengths of 10-25km (cf. figure 5a). the values are stable beyond R2= 100km. The difference of RTM indirect effect and terrain correction with integration radius R1 of 55km as a reference value has been presented in figure 5b. the computations were made by setting R2=200km with varying R1. From the figure, we can observe that the influence of R1 on RTM indirect effect and terrain correction have vital values in the near 10km radius around the test point with amplitude of 1-4mm for terrain correction and 0.001-0.004mgal of Residual Terrain Model indirect effect. R1=10km is enough for 0.001mgal and 1mm precision in the test point located in the low lying region (cf. figure 5b).

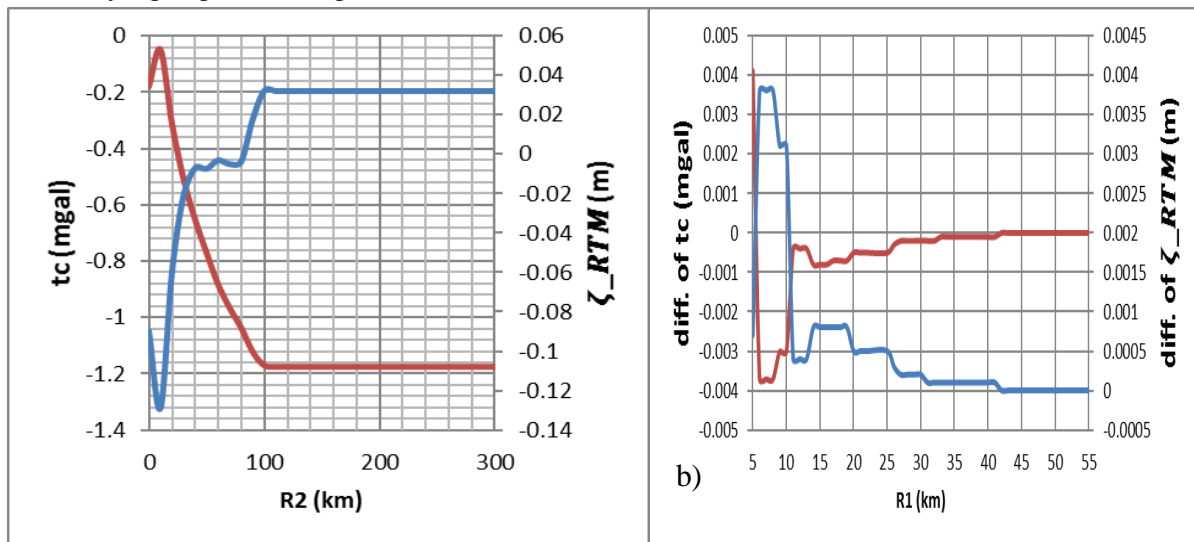


Figure 5. a, b) a), RTM indirect effect and terrain correction for the test point in low lying region as a function of R2 for R1=15km; b), the difference of RTM indirect effect and terrain correction as a function of R1 for R2= 200km. redline = terrain correction, blue line= RTM indirect effect

The RTM indirect effect and terrain correction as a function of integration radius R_2 for the test point located in the mountainous area have been presented in figure 5c and the difference of Residual Terrain Model indirect effects and terrain corrections with respect to calculation radius R_1 for the test point have been presented in figure 5d. the most significant values are observed at about 75km radius surrounding of the test point. The amplitudes are 1.5-3 meters of RTM indirect effect and 0.4-8mgal of terrain corrections at wavelengths of 10-30km. the RTM indirect effect and terrain correction values are stable beyond $R_2= 100$ km (cf. figure 5c). in figure 5d, the RTM indirect effect and terrain correction difference values at $R_1=5$ km are 2.5cm and 0.02mgal respectively. $R_1=20$ km is sufficient for 0.001mgal and 4mm precisions in the mountainous area. To have the best precision, all the computations have been made at $R_1=20$ km and $R_2= 200$ km.

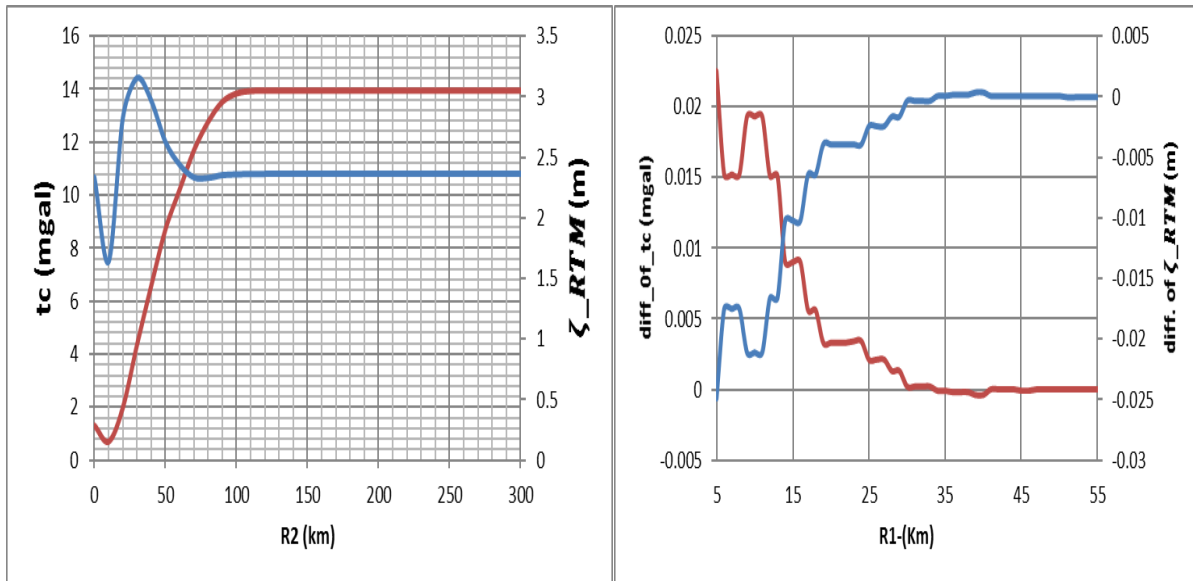


Figure 5. c, d: c), RTM indirect effect and terrain correction for the test point located in a mountainous area as a function of R_2 for $R_1=15$ km; d), the difference of RTM indirect effect and terrain correction as a function of R_1 for $R_2= 200$ km. redline = terrain correction

4.3. RTM terrain effects and DEM spatial resolutions

In order to analyze the differences of RTM effects and terrain corrections by DEM spatial resolutions two test areas have been chosen, one in the low-lying area around Dallol depression located in the Afar region and another in the mountainous area around Ras-Dashen mountain located in the Amhara region. The grids have been resampled to 1.5 arc minutes of resolution to have 1681 grid nodes for each test area. All statistical values increases as the spatial resolution of GDEMs become lower for both test areas.

Table 3, presents the descriptive statistics of terrain corrections and their differences by GDEM spatial resolutions. From table 3 we can observe that the absolute value of terrain correction differences is lower than 0.5mgal for both low lying (Dallol) and mountainous (Ras-Dashen) test areas. The RMS difference varies from 0.01 to 0.08mgal. DEMs with lower spatial resolutions are sufficient for good accuracy.

Table 3: The statistical description of terrain corrections and their differences per SRTM resolutions in mgal

Resolutions	Dallol area					Ras-Dashen mountain				
	MIN	MAX	MEAN	STD	RMS	MIN	MAX	MEAN	STD	RMS
R3''	0	4.79	0.78	0.67	1.03	0	14.24	2.45	2.38	3.41
R3''-R7.5''	-0.11	0.24	-0.02	0.06	0.07	-0.02	0.03	0.01	0.01	0.01
R3''-R15''	-0.09	0.15	-0.01	0.03	0.04	-0.08	0.06	0.00	0.02	0.02
R3''-R30''	-0.33	0.09	0.01	0.04	0.04	-0.1	0.11	0.00	0.03	0.03
R3''-R60''	-0.3	0.13	0.03	0.07	0.08	-0.12	0.08	-0.01	0.01	0.06

The descriptive statistics of RTM indirect effects and their differences by GDEM spatial resolution are presented in table 4. From the table, we can observe that the absolute values of the difference of RTM indirect effects in the low-lying Dallol area reach up to 8cm and in Ras-Dashen mountain the difference reaches 12cm. the mean, standard deviation, and RMS difference values are the same for all resolutions in the Dallol area. In Ras-Dashen mountain the difference values of the RMS are in the range from 1cm to 3cm. the influence of DEM spatial resolutions in the low lying region of the test area is negligible whereas in Ras-Dashen mountain the accuracy of RTM indirect effects can be improved by 3cm with DEM of 3 arcs second spatial resolution as a reference and the resampled DEM resolutions from 7.5 arcs-second to 60 arc second.

Table 4: Statistical description of residual terrain model indirect effects and their differences per SRTM resolutions in meters

Resolutions	Dallol area					Ras Dashen mountain				
	MIN	MAX	MEAN	STD	RMS	MIN	MAX	MEAN	STD	RMS
R3''	-0.27	0.83	0.09	0.25	0.27	-1.08	2.41	-0.03	0.79	0.81
R3''-R7.5''	-0.01	0.02	0.00	0.01	0.01	-0.02	0.02	0.00	0.00	0.01
R3''-R15''	-0.02	0.03	0.00	0.01	0.01	-0.04	0.07	0.00	0.02	0.02
R3''-R30''	-0.05	0.03	0.00	0.01	0.01	-0.11	0.07	0.00	0.02	0.03
R3''-R60''	-0.08	0.06	0.00	0.01	0.01	-0.12	0.08	-0.01	0.03	0.03

The absolute difference value ranges of RTM direct effects in the Dallol area varies from 3.64-24.37mgal and in Ras-Dashen mountain the value reaches 79.37mgal for lower DEM resolutions (table 5). The RMS difference values of the RTM direct effect are in a range from 1.05-4.15mgal for the low lying test area. In Ras-Dashen mountain the mean difference values vary from -0.58mgal to -3.15mgal and the RMS of the differences are in a range from 8.46-9.86mgal. for both test areas when computing the RTM direct effects with DEMs having lower resolutions the difference value ranges exceeds 10mgals requiring to use of high-resolution DEMs in the computation of RTM direct effects.

Table 5: The statistical description of residual terrain model direct effects and their differences Per SRTM resolution in mgal

Resolution	Dallol area					Ras Dashen mountain				
	MIN	MAX	MEAN	STD	RMS	MIN	MAX	MEAN	STD	RMS
R3''	-30.83	91.59	-3.9	25.94	26.04	-72.44	159.66	5.71	41.56	41.62
R3''-R7.5''	-6.7	3.64	-0.18	1.05	1.13	-36.38	54.25	-0.58	8.44	8.46
R3''-R15''	-11.7	5.24	-0.42	1.5	1.56	-39.77	63.32	-1.78	8.89	8.53
R3''-R30''	-18.4	11.46	-0.72	2.73	2.82	-43.65	72.55	-2.35	9.15	9.21
R3''-R60''	-24.37	19.32	-0.9	4.05	4.15	-58.36	79.37	-3.15	9.88	9.86

4.4. RTM effects and Terrain corrections of gravity stations and grid nodes

Table 6 presents the descriptive statistics of RTM effects and terrain corrections with reference surface corresponding to ~100km spatial scale or degree and order (d/o) 200 of 22,738 gravity stations. The study focused on DEM grid data.

Table 6: statistics of terrain corrections and RTM effects of gravity stations

Quantities	Min	Max	Mean	STDev	RMS
Station height (m)	308.7	1112.6	789	169.38	807.03
Δg_{RTM} (mgal)	-51.61	23.32	-1.01	8.58	8.74
tc (mgal)	0.00	5.74	0.87	1.25	1.52
ζ_{RTM} (m)	-0.26	0.37	0.08	0.11	0.12

The descriptive statistics of RTM effects and terrain corrections at 375,922 grid nodes have been presented in table 7. The RTM effects which are computed with reference surface corresponding to ~100km spatial scale are more vital than that of RTM effects computed with mean elevation surface of ~9km spatial scale. The RTM direct effects computed with ~9km spatial scale of reference surface lies in a range from -138.65 up to 76.85mgal whereas the direct effect for ~100km spatial scale varies from -83.13 to 159.69mgals. the RTM indirect effects with reference surface of ~9km spatial scale is not more than 20cm on other hand the RTM indirect effects corresponding to ~100km spatial scale reach 1.75 meters. The terrain correction reaches 44.31mgal.

Table 7: statistics of RTM effects and terrain corrections of grids by mean elevation surfaces

d/o	Computed effects	MIN	MAX	MEAN	STD
2190	Δg_{RTM} (mgal)	-138.65	76.855	-1.54	9.21
	ζ_{RTM} (m)	-0.13	0.205	0.0004	0.017
200	Δg_{RTM} (mgal)	-83.13	159.69	2.31	16.82
	ζ_{RTM} (m)	-0.77	1.74	-0.0012	0.18
	tc (mgal)	0	44.31	2.67	3.34

Figure 6a and 6b show the grid maps of the RTM direct and indirect effects computed with mean elevation surface corresponding to ~100km spatial scale over Ethiopia and its immediate surroundings. The point values were converted into a raster with the resampled resolution having 1.5 arc minute spatial resolution. 92.12% of the RTM direct effects corresponding to ~100km spatial scale are less than 25mgal and 5.53% of the direct RTM effect values are between 25mgal and 50mgal. In addition to this, for the RTM indirect effects corresponding to the aforementioned spatial scale of the reference surface, 85.74% of the values are less than 30cm and 11.30% were lies in a range from 30cm to 90cm.

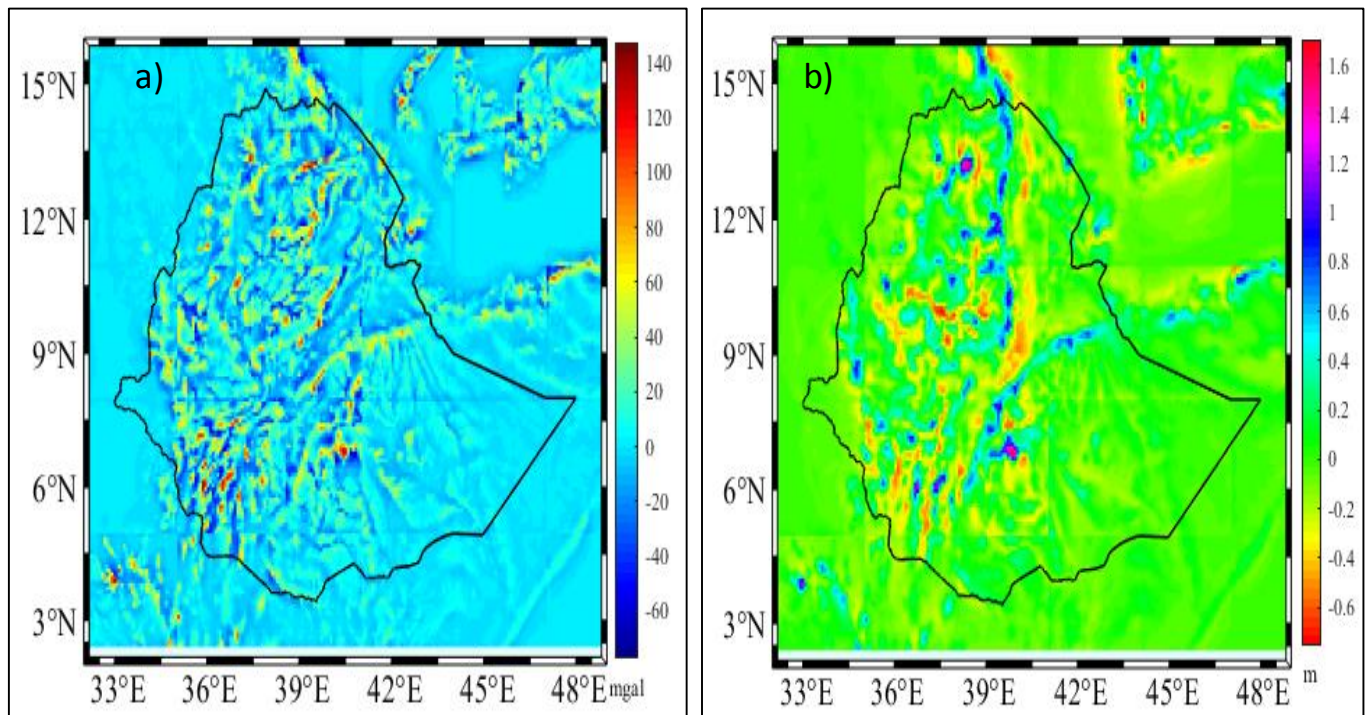


Figure 6.a, b: RTM effects: a) direct effects; b) indirect effects with reference surface corresponding to ~100km spatial scale.

The grid maps of RTM direct effects and indirect effects are presented in figures 6c and 6d respectively which were computed with reference surface corresponding to ~9km spatial scale. 97.51% of the RTM direct effects corresponding to mean elevation surface of d/o 2190 are less than 25mgal and 2.46% of the direct effect values are in a range from 25mgal to 50mgal. For the indirect effects of ~9km spatial scale, 98.26% of the indirect effect values are less than 5cm and 1.56% of the values are between 5cm to 10cm.

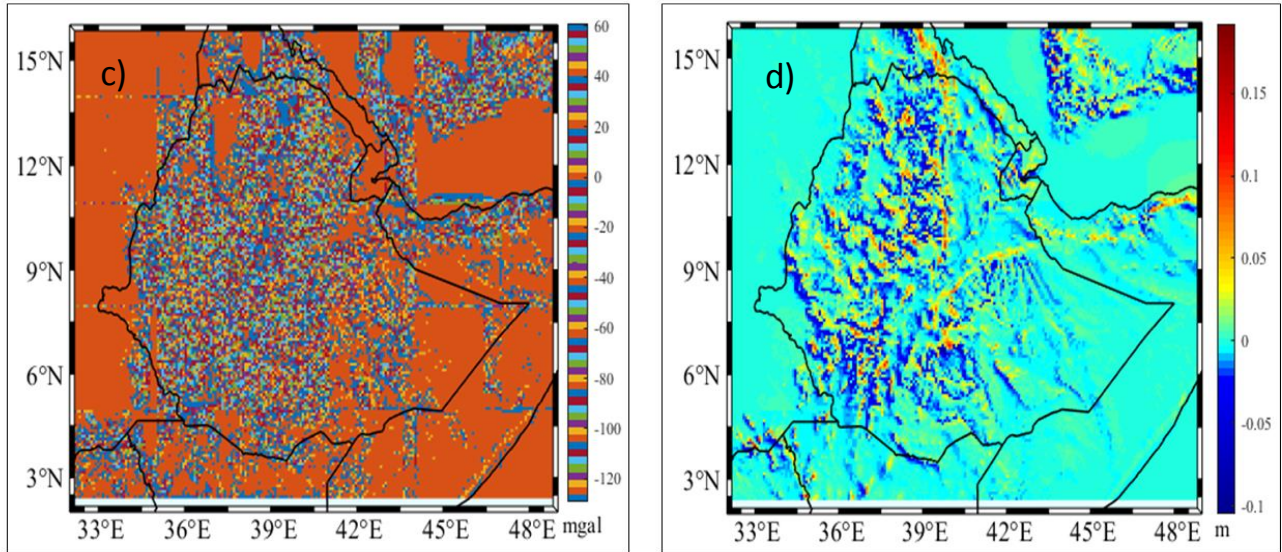


Figure 6.c, d: RTM effects: c) direct effects; d) indirect effects with mean elevation surface corresponding to ~9km spatial scale

As figure 7 shows the terrain correction distribution over Ethiopia high values are observed near mountainous areas implying the correlation of terrain corrections with the topographic configuration of the computation area. Since the RTM effects and terrain corrections computed in reference with the mean elevation surface corresponding to d/o of 2190 or ~9km spatial scale are not significant because of low values of terrain effects, the terrain corrections were computed with mean elevation surface corresponding to ~100km spatial scale. 98.85% of terrain corrections over Ethiopia are less than 15mgal and 1.09% of terrain correction values are between 15mgal and 30mgal which shows high values of terrain corrections implying the topography of the study area has high elevation values and there exists rugged topography in most of the study area.

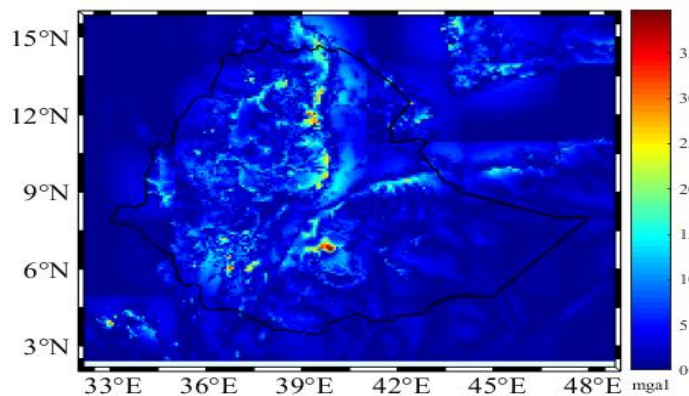


Figure 7: terrain corrections

4.5. Effect of height difference on Residual Terrain Model effects and terrain corrections

The statistical description which describe the difference of Residual Terrain Model effects and terrain corrections computed from the reference surface corresponding to ~100km spatial scale of the original gravity stations from the alternative gravity database are presented in table 8. As the mean values of terrain corrections show from the table, terrain corrections computed from the original database are greater than that of the alternative database. Whereas the RTM effects are overestimated with the alternative database. The difference of terrain corrections and direct RTM effects are highly correlated with the height differences with correlation coefficients of 0.987 and 0.980 respectively for terrain correction and RTM direct effects. For the RTM indirect effects, the influence of height difference between the original database and the alternative database is not weighty. Since height values from DEMs better represent the topographic configurations around the gravity stations, the computed RTM effects and terrain corrections from the alternative database are more suitable for geodetic applications. Therefore, to overcome the impact of height discrepancy between height sources from DEMs and other height sources the study insists to use height values from GDEMs.

Table 8: The statistical description of the different effects for the replaced SRTM heights and their differences with the gravity station heights

	Calculated values	MIN	MAX	MEAN	STD	RMS
Calculated values for the replaced heights	H (SRTM3)	307.67	1117.15	791	168.95	809.31
	tc (mgal)	0.00	5.86	0.66	1.25	1.54
	Δg_{RTM} (mgal)	-35.86	51.67	-1.54	8.11	8.15
	ζ_{RTM} (m)	-0.29	1.42	0.17	0.5	0.56
Difference of values	H (m)	-69.03	132.91	-2.42	11.18	11.44
	tc (mgal)	-1.19	2.02	0.21	0.17	0.17
	Δg_{RTM} (mgal)	-20.37	4.16	-0.88	1.78	1.89
	ζ_{RTM} (m)	-0.11	0.03	-0.01	0.04	0.05

4.6. Discussion

In the process of gravity field modeling, we need to have reliable and representative gravity data. The solution for the Boundary Value Problems is the function has to be known on the boundary. This requires the gravity field to be known at the geoid for any geophysical applications, particularly for geoid modeling/determination (Heiskanen & Moritz, 1967). The influence of topographic masses located above the geoid and masses which are enclosed by the geoid to the gravity field which is the direct and indirect effects are computed either by the removal of the influence of layer of uniform thickness or by the removal of the influence of irregularities of the topography: i.e. the influence of the masses enclosed between the topographic surface and the flat surface of the plate (layer). (Vaniček, 1976). In order to remove or shift the topographic masses above the geoid, the topography should be precisely modeled. This study aims to model the short-wavelength components of the topographic gravity field by using the Residual Terrain Model (RTM) approach. From the study the RTM direct effects computed with the reference surface of ~100km spatial scale have a standard deviation of 16.82 mgal and for the RTM indirect effects a value of 0.18 centimeters were obtained in addition to this a standard deviation value of 3.34mgal was obtained for terrain corrections.

Yahaya & Azzab, 2018 computed the terrain correction and RTM effects over Niger republic. The terrain correction corresponding to ~100km spatial scale has a STdev value of 0.37mgal and the RTM direct effect has a standard deviation value of 4.93mgal. and indirect effect with a standard deviation of 0.11 meters implying low values of RTM effects and terrain corrections due to small changes in topography. From the present study a standard deviation values of 3.34mgal, 16.82mgal and 0.18 meters respectively for terrain correction, RTM direct effect and RTM indirect effects. As compared with the aforementioned study the present study showed high values of terrain effects due to the diversified topographic nature of the study area. (Bajracharya S., 2003) computed the RTM effects of the Canadian Rockies and obtained a Standard deviation of 23mgal for the direct effect. The RTM indirect effect have a standard deviation of 0.46 meters. (Forsberg, 1984) computed the RTM effects and terrain corrections of Colorado mountain and obtained mean and standard deviation values of terrain corrections 6.33mgal and 4.94mgal respectively. For RTM indirect effects a value of 0.82 and 0.65 meters mean and standard deviation respectively. The RTM direct effects have a standard deviation of 43.49mgals respectively. Since the two studies

are done only on mountainous areas high terrain effect values are observed as compared with the present study.

Yahaya & Azzab, (2018) investigated difference of terrain effects per spatial resolution in Niger republic. In low lying area the difference of standard deviation of terrain correction reaches 0.02mgal. and in mountainous area the difference reaches 1.61mgal. from both the present and the above stated studies we can observe that the value of differences increases as the resolution of Digital Elevation Models become lower requiring to use high resolution Digital Elevation Models in computation of terrain corrections and Residual terrain model effects.

Hwang, et al. (2003) computed terrain corrections by developing a method based on Gaussian quadrature in test points located in Taiwan and obtained a standard deviation of 13.58mgal. to conclude what we have discussed here many studies have been done to handle the topographic effects in modeling the gravity field. The need for topographic reduction in gravity field modeling is because the high-frequency components of the gravity field are due to the topographic masses distributed throughout the earth. Among the aforementioned gravity reduction techniques, the residual terrain model (RTM) approach is commonly used in geoid determination. (Bajracharya S., 2003) analyzed the different gravity reduction techniques and showed the RTM method has a very small indirect effect and it provides a smooth surface. The integrals of terrain correction were solved in many studies using different approaches by utilizing digital elevation models. To model, the effect of rugged topography high-resolution DEMs should be used. The influence of the resolution of DEMs in terrain effects is not significant in low-lying regions.

Chapter Five

5. Conclusion and Recommendation

5.1. Conclusion

In this study, the Residual Terrain Model (RTM) and terrain effects were computed and reported in a grid map of 1.5 arc minute resolution over Ethiopia and its immediate surroundings. The SRTM-DEM with spatial resolutions of 30 arc seconds was used to construct RTM surfaces corresponding to ~9km and ~100km spatial scales. The mean elevation surfaces computed with the low-pass filter method and those derived from spherical harmonic models were compared to observe which method will give a smooth reference surface and present more spectral details. The mean elevation surface which was computed with a low-pass filter was adopted rather than the mean elevation surface derived from topographic spherical harmonic models. The integration radiuses for the outer and inner computation zones for the study area were investigated and suggested using integration radius $R_2=200\text{km}$ for the outer zone and integration radius $R_1=20\text{km}$ for the inner zone is sufficient for good precision of RTM effects and terrain corrections. The study investigated RTM effects and terrain corrections per spatial resolution of DEMs in low lying and mountainous test areas and suggested using 3 arcs second DEMs to get good precision of RTM direct and indirect effects and terrain corrections.

The RTM direct and indirect effects and terrain corrections were computed with reference surfaces corresponding to ~100km and ~9km spatial scales. The terrain correction and RTM effect values are correlated with the topography high values are observed in mountainous and rugged areas. In Ethiopia, 92.12% of Residual Terrain Model direct effects for degree and order (d/o) of 200 reaches 25mgal and 85.74% of Residual Terrain Model indirect effects are less than 30cm. 98.85% of terrain corrections are less than 15mgal. For reference surface corresponding to ~9km spatial scale, 97.52% of the RTM direct effects are less than 25mgal. 98.26% of indirect effects are less than 5cm. the RTM effects and terrain corrections computed with mean elevation surface corresponding to ~9km spatial scale are not significant.

To investigate the impact of height discrepancy on RTM effects and terrain corrections between the terrestrial gravity database and DEMs, the RTM direct and indirect effects were computed for both the original gravity database and the alternative replaced DEM height database. The terrain

corrections were overestimated with the original database. The difference of terrain corrections and RTM direct effects are correlated with height differences.

5.2. Recommendations

The main input of the study is to model the high-frequency components of the gravity field and terrain correction by the RTM technique in Ethiopia. Therefore, the outputs obtained from this research work can be applicable in many geodetic and geophysical applications that require accurate interpretations. The result from this study can be used for refinement of Bouguer anomalies, to estimate omission and commission errors, gridding of free air anomalies, and geoid determination. The contribution of this study for future works can be outlined as follows:

- The terrain correction from the study can be used for complete Bouguer reduction for terrestrial gravity stations in Ethiopia.
- The Residual Terrain Model effects of 9km spatial scale can be used to estimate omission errors for high-resolution global geopotential models.
- The Residual Terrain Model effects of 100km spatial scale can be used for gridding of gravity data and gravimetric geoid determination in the Remove-Compute-Restore approach.

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