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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

COST OPTIMIZATION OF RC PLANAR FRAMES ON THE BASIS OF DIRECT DISPLACEMENT-BASED SEISMIC DESIGN METHODOLOGY

A Thesis in Structural Engineering

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A Thesis

Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science

The undersigned have examined the thesis entitled '**Cost Optimization of RC Planar Frames on the Basis of Direct Displacement-Based Design Methodology**' presented by **ABABI MULUGETA**, a candidate for the degree of **Master of Science** and hereby certify that it is worthy of acceptance.

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UNDERTAKING

I certify that research work titled “Cost Optimization of RC Planar Frames on the Basis of Direct Displacement-Based Design Methodology” is my own work. The work has not presented elsewhere for assessment. Where material has used from other sources, it has properly acknowledged / referred.

Ababi Mulugeta

ABSTRACT

The study is to come up with an optimal solution for the design of reinforced concrete planar frames using the principle of direct displacement-based design. Firstly, the fundamentals of displacement-based design introduced with its application to concrete frame design. After that, the design process shaped in a way that result to cost effective design of two-dimensional frames. The problem solved using MATLAB GlobalSearch Algorithm.

Keywords: Direct Displacement-Based Seismic Design, Optimization, GlobalSearch Algorithm

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CHAPTER 1 INTRODUCTION

1.1 Background

In designing building structures for seismic effects, two systems called *moment-resisting frames and structural walls* are mainly used. The analysis and design of both systems based its conceptual framework on strength driven theories for a long period and still does.

The core point of these theories is that a structure can be designed using an initially assumed structural geometry, dictated by other non-seismic actions, and its associated strength to determine structural responses that will be refined to an acceptable level iteratively.

However, this design procedure has problems that need improvements mainly due to strength and stiffness interdependency. That is why the seismic design philosophy is moving to a more robust concept called displacement-based design.

In displacement-based design, the structure is made to achieve a predefined level of deformation under a predefined level of seismic intensity, which makes it a more suitable tool for performance-based design.

The main advantage of the concept is that it gives more dependable design inputs with a lesser computational demand. In addition, since the design method eliminates most of the force-based design deficiencies, it is going to be the next dominant tool for seismic effect design.

Therefore, the effort to furnish an efficient seismic design solution should focus on further improvement of the ideology. One of the streams working on the improvement is optimization of the method employing different optimization algorithms. This paper is an attempt to do the optimization for reinforced concrete planar frames.

1.2 Statement of the Problem

In an attempt to overcome the shortcomings of force based seismic design, researchers develop a well-equipped design concept, which will make the determination of the responses of a structure more reliable, called displacement based design.

Though this design method still needs further researches, it is becoming clear that the method is going to overtake other design philosophies and be a part of design codes. That is what makes this research to focus on the design philosophy and tries to do optimization based on it since structural engineering profession aims to erect safe, durable and economical structures.

1.3 Objectives of the Study

1.3.1 General Objective:

The basic drive of the study is getting an optimal design solution for reinforced concrete frames and setting forth a good framework for further extension of the work for other seismic structures.

1.3.2 Specific Objective:

The specific objective of the research is providing a tool that can automate the optimum seismic design of reinforced concrete 2-D frames utilizing displacement based design methodology.

1.4 Limitations

Due to time restraint, the study addresses only planar reinforced concrete frames with rectangular member's cross-section .The reinforcement detailing aspect of frame member's is not covered. In addition, it is assumed that construction cost of frames is constant along the height of the frame.

CHAPTER 2 LITREATURES REVIEW

2.1 Limitations of Force-Based Seismic Design

As pointed by Priestley, 1993 [1], the myth in strength based seismic design arises from the lack of the method in addressing the interdependency of strength and stiffness.

The inherent problems in force-based seismic design presented in Priestley, 2007 [2] as follows:

2.1.1 Initial Stiffness Estimation

In force-based seismic design, initial stiffness is used to estimate the period(s) of the structure and hence the seismic forces. The assumption is that if the dimensions of the structure are known, the stiffness can be estimated directly with sufficient accuracy.

However, this is unacceptably coarse assumption especially for concrete and masonry structures as stiffness is proportional to strength, which is unknown until the design is finalized. Therefore, conventional design allocating strength based on an assumed stiffness is inappropriate.

This can be improved by an iterative procedure, where the stiffness modified after a preliminary design has been used, and a new estimate of the strength based on this stiffness is determined. However, that is time consuming and rarely adopted.

2.1.2 Use of Constant Behavior Factor

Most seismic design codes specify constant behavior or force reduction factor to consider for ductility capacity for a given structural type and material but this does not account for the fact that ductility capacity is also a function of structural geometry within the structural type.

Furthermore, for frame buildings, code drift limit will restrict actual ductility level to values that are lower than that corresponding to the code behavior-factor, and hence the specified behavior factors are irrelevant to design. This essentially requires an extra level of iteration in a conventional design using initial stiffness, since strength, and hence stiffness cannot be fully defined until the behavior factor appropriate for the code drift limit can be determined.

2.1.3 Displacement-Equivalence Rules for Initial Stiffness Design

In initial-stiffness based design, displacement of the designed inelastic structure is related to the elastic displacement of the initial-stiffness model by a displacement equivalence rule.

The equivalence model implies that the equal displacement approximation applies for period greater than about 0.5 sec, with equal acceleration applying at T=0. At intermediate periods the equal acceleration, which implies inelastic displacements that are larger than the elastic displacement are adopted.

This equivalence rule or variant of it have been widely applied in initial-stiffness design, though some more recent models imply that in moderate period range inelastic displacement are less than elastic displacement.

In inelastic time-history analysis, it is common to specify a level of elastic damping to represent the initial stage of response, before hysteretic damping is activated. This is normally specified as percentage, typically 5%, of critical damping.

There are a number of ways to define this damping but the principal difference is whether the damping force is related to the initial-stiffness or tangent stiffness. In initial-stiffness elastic damping, the damping force is always related to the initial stiffness and the instantaneous velocity, while in tangent-stiffness damping, the damping force reduces when the stiffness reduces because of onset of inelastic response. Mostly, development of displacement-equivalence rules have assumed initial stiffness damping.

2.1.4 Distribution of Required Strength

In force-based seismic design, the base shear is distributed using initial stiffness estimate of different structural elements. This can have undesirable effects. First, there will be a difference in ductility demand, which makes the usage of appropriate force reduction factor problematic. Second, allocating more strength for elastically stiffer members increase their stiffness even further (with respect to the other members) as section stiffness is proportional to strength. Therefore, a redesign should be carried out with improved estimates of member stiffness, which would result in still higher percentage of total seismic resisting force being allocated to that member which increases their vulnerability to shear failure and under-utilization of the other members.

2.2 Direct Displacement-Based Seismic Design

2.2.1 Basic Essence

As described in Priestley, 2003 [3] , direct displacement-based design is a method for designing to achieve, rather than bounded by, a given performance limit states.

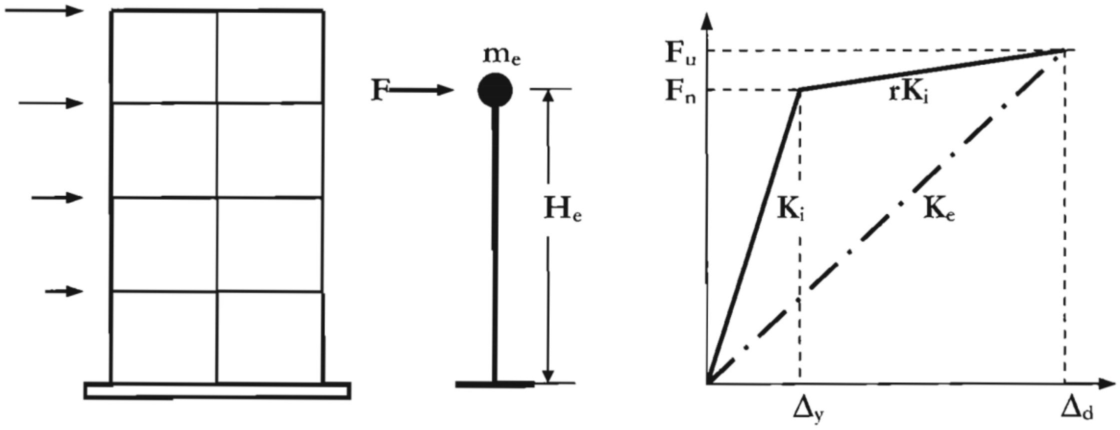
It characterizes the structure by effective secant stiffness rather than initial stiffness and a level of equivalent viscous damping that represent the combined effect of elastic and hysteretic damping rather than 5% elastic damping.

The advantages of these are, as noted by Dzakic, Kraus, Moric [4] :

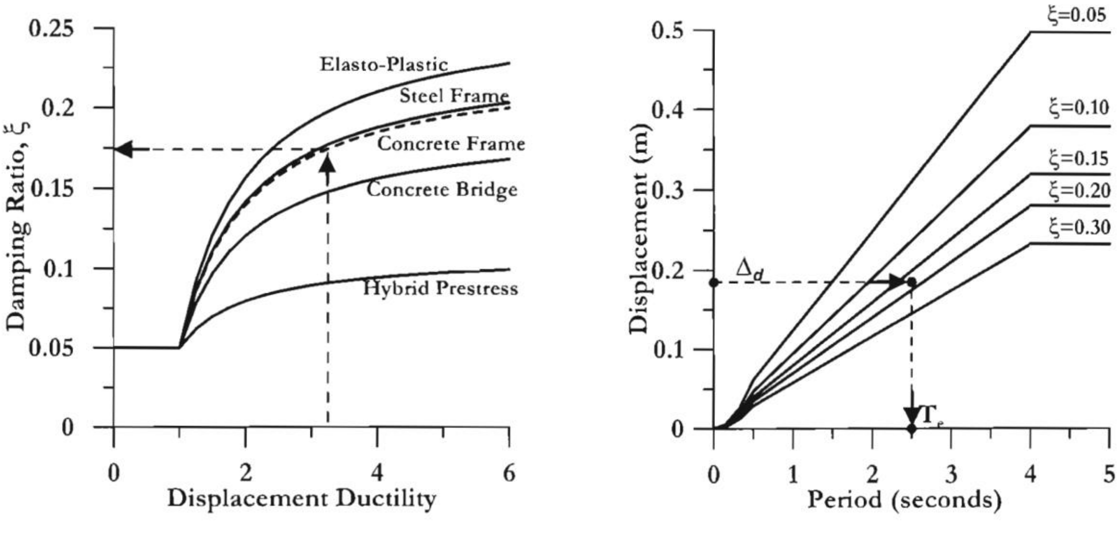
- Limits states are not to be checked, rather used as an input data
- It provides uniformity of risk in structural design
- It eliminates the problems in force-based seismic design
- Ductility, one of the basic and most important parameter in seismic design, is directly used in design
- As the analysis is relatively close to reality, due to the above-mentioned reasons, the base shear calculated by the method is reasonable and significantly lower than the force based design.

2.2.2 Basic Formulation

The displacement-based design method uses a single degree of freedom representation of the design structure:



(a) SDOF Simulation (b) Effective Stiffness K_e



(c) Equivalent damping vs. ductility (d) Design Displacement Spectra

Figure 2-1 : Fundamentals of Direct Displacement - Based Design [5]

2.2.2.1 SDOF Representation

As stated in Decanini, Mollaioli, Mora, [6] the theoretical starting point for the substitute structure method is very similar to the modal analysis of frame structures. In fact, a time-independent global deflection shape is assumed for the lateral displacement of the building stories. In this way, the displacement vector of the frame at time, t is expressed as:

$$\{u(t)\} = \{\Phi\}x(t) \qquad 2\text{-}1$$

where $\{\Phi\}$ is the vector that define the deflection shape and $x(t)$ expresses the time-dependent amplitude of motion.

This assumption allows reducing the MDOF equation of motion:

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + \{R\{u(t)\}\} = [M]\{l\}\ddot{u}_g(t) \qquad 2\text{-}2$$

to the equivalent SDOF equation

$$M_{eq}\ddot{x}(t) + C_{eq}\dot{x}(t) + R_{eq}(x(t)) = L_{eq}\ddot{u}_g(t) \qquad 2\text{-}3$$

where $M_{eq} = \{\Phi\}^T[M]\{\Phi\}; C_{eq} = \{\Phi\}^T[C]\{\Phi\}; R_{eq} = \{\Phi\}^T\{R(t)\} \,\&\, L_{eq} = \{\Phi\}^T[M]\{l\}$

According to Priestley, Calvi, Kowalsky [5], in displacement-based design the secant stiffness, K_e at maximum displacement and an equivalent viscous damping, ξ_{eq} are used to characterize both the elastic and inelastic properties of the structure.

From a single degree of freedom oscillator period equation, the effective stiffness of the substitute structure is:

$$K_e = \frac{4\pi^2 m_e}{T_e^2} \qquad 2\text{-}4$$

where m_e is the effective mass of the structure participating in the fundamental mode of vibration and T_e is effective period.

Thus, the lateral design force or base shear

$$F = V_{base} = K_e \Delta_d \tag{2-5}$$

where Δ_d is the maximum displacement.

Once the design base shear is found, it is going to be distributed between the mass elements of the real structure. Then, analysis of the structure under the distributed seismic force follows and the design of the structural elements will be carried out using capacity design concept tailored to the method and the structural system.

2.2.3 Procedural Steps

The general steps taken in direct displacement-based design is summarized as follows, Priestley, Grant, Blandon [7]:

- Assume the geometry of the structure
- Determine drift based design displacement
- Calculate yield displacement and ductility
- Determine equivalent viscous damping
- Determine effective period from displacement spectra
- Determine effective stiffness from SDOF equation
- Determine design base shear
- Distribute base shear and analyze structure

The list of equations that are used in the analysis and design of concrete frame by direct displacement-based design are available in Appendix-B.

CHAPTER 3 ANALYSIS AND DESIGN OF RC PLANAR FRAMES

3.1 Analysis Method

There are two possible methods for determining the required moment capacities at potential hinge locations for frames designed by DDBD.

1. Analysis Based on Relative Stiffness of Members
2. Analysis Based on Equilibrium Conditions

However, a major problem with the relative stiffness approach is that the member stiffness depends on flexural strengths, which will be unknown until the structural analysis is completed. Hence, an iterative process will be needed to determine the require beam moment capacities.

So, considering the inevitable approximate nature of structural analysis procedure, a simplified analysis procedures based on simple equilibrium is adopted.

3.1.1 Issues to be noted in Equilibrium Analysis of Frames

To ensure drifts to be within a design limit, it is important that the vertical distribution of beam shears follow the seismic demand. This can be achieved by allocating the total beam shear to beams, in proportion to the storey shears in the level below the beam under consideration. For irregular frames with different bay widths, each bay will be considered separately with the total base shear force allocated to the different bays in proportion to the chosen contribution to the overturning moment

In determining columns shear, the total storey shear force is divided between columns, normally in the ratio 1:2 for external/internal columns.

Column moments are determined from consideration of joint equilibrium starting from column base and working up to the top level. However, such analysis results in to columns contra-flexure points that are close to mid-height of columns; thus a simpler design approach of assuming central point of contra-flexure in each column is acceptable and it is used here.

3.2 Design of Frame Members

The design follows the concept of strong column weak beam design concept. The principle is that to avoid a progressive collapse of a structure; beams should form plastic hinges prior to columns.

3.2.1 Capacity Design

To ensure that plastic hinges only occur where inelastic rotations intended and to ensure that undesirable mode of inelastic deformation, such as shear failure do not develop, the dependable strength of plastic hinge locations and actions is set to be higher than the force levels at these locations, corresponding to the maximum feasible strength developed at the plastic hinges. This is in recognition that in ductile design, it is the actual strength, rather than the conservative design strength, that will developed in the design level earthquake.

The general requirement for the capacity protection is

$$\phi_S S_D \geq S_R = \phi^o \omega S_E \tag{3-1}$$

where S_E is the value of the design action being capacity protected corresponding to the design lateral force distribution found from DDBD process; ϕ^o is the ratio of over-strength moment to the required capacity of the plastic hinges; ω is the amplification of the action being considered due to higher mode effect; S_D is the design strength of the capacity protected action; and ϕ_S is the strength reduction factor relating the dependable and design strengths of the action.

3.2.2 Considerations in Design

3.2.2.1 For Beams

There is no consideration of dynamic amplification for beam shears or moments, and thus only moment strength factor needs considering. Although this is correct for the beam response induced because of seismic inertial response of the structure as a whole, there may be dynamic amplification of the gravity load responses because of amplification for vertical acceleration.

The beams plastic hinges are designed for the larger of the factored gravity moments and seismic moments ignoring gravity moments. The reason why the gravity and seismic moments should not combined is that, this combination increases the required section strength and reduces the response drift levels below the target values.

For regions between the beam plastic hinges, design moments are found from the combination of reduced gravity loads applicable for seismic design combination, and over-strength moment capacity at the beam hinges. Since the beam-moments cannot exceed the over-strength values at the plastic hinge locations, the design moment within the beam span is found by adding the gravity moment corresponding to simple support to the seismic moments in the span. To account for elastic vertical response of the beam to vertical ground acceleration, the gravity moments will be amplified by 30%. [5]

Subsequent to building response to an earthquake, softening of beam plastic hinges might result in a redistribution of gravity moments, with a reduction at the beam-ends, and a corresponding increase at the beam mid-span. However, this effect is generally not considered in design.

Seismic shears corresponding to beam plastic hinging are constant along the beam. When combined with reduced gravity shears applicable for seismic load combinations, it will be prudent to consider the effect of beam vertical response on design shear force as with the beam flexural design.

As direct displacement-based design is based on the required strength at maximum displacement demand, a less conservative assumption about material strength and strength reduction factors are required. Thus, for intended plastic hinge locations, it is recommended that design values of material properties exceed the characteristics material strengths in accordance with [8]:

$$f'_{ce} = 1.3 f'_c \qquad \text{3-2}$$

$$f_{ye} = 1.1 f_y \qquad \text{3-3}$$

where f'_{ce} and f_{ye} are effective characteristic strength of concrete and steel respectively while f_y and f'_c are nominal characteristic strengths.

3.2.2.2 For Columns

For columns a more conservative design approach than that for beams is appropriate since the columns should remain essentially elastic, except at column base, & possibly immediately below the roof beam.

Consequently, column moments needed amplification above the values corresponding to the design lateral seismic force to allow for potential over-strength capacity at beam plastic hinges. The over-strengthening results from material strength exceeding the values specified for the design and dynamic amplification of column moments from higher mode effects.

The seismic axial force will be close to zero for interior columns hence the required column reinforcement can be determined based on the assumption of probable gravity loads. For exterior columns, however, seismic beam shear will generate significant axial forces, either tensile or compressive, which should be considered when determining the required column flexural reinforcement except for intended column-base plastic hinges.

3.2.3 Resistance of Concrete Sections

The design strengths of frame members are computed according to Eurocode standards.

3.3 Performance Limit States Addressed by DDBD

Three different performance levels are presented in Calvi, Sullivan [8] to use as direct displacement based design reference. These are level 1, level 2 and level 3 which correspond to a serviceability limit state, a damage control limit state and a collapse-prevention limit state respectively.

- ✓ Serviceability Limit State: only insignificant damage expected and any necessary repairs can be carried out without affecting normal operations.
- ✓ Damage-Control Limit State: damage should be economically repairable
- ✓ Collapse Prevention Limit State: no collapse is required

The following tables summarized the maximum drift and material strain limits for different type of structures:

Table 3-1: Structural Drift Limits [8]

Drift Limit	LS1	LS2	LS3
Buildings with brittle non structural elements	0.004	0.025	*No limit*
Buildings with ductile non structural elements	0.007	0.025	*No limit*
Building with non-structural elements detailed to sustain building displacements	0.010	0.025	*No limit*
Framed Timber walls	0.010	0.020	0.030
RC Bridge Piers	θ_Y	0.030	0.040
Isolated bridges	$2/3*\vartheta_Y$	$2/3*\vartheta_Y$	ϑ_Y

Table 3-2 : Material Strain Limits for RC Structures [8]

Material	LS1	LS2	LS3
Concrete comp. strain	0.004	$\varepsilon_{c,dc}<0.02$	$1.5*\varepsilon_{c,dc}$
Rebar tension strain	0.015	$0.6\varepsilon_{su}<0.05$	$0.9\varepsilon_{su}<0.08$

CHAPTER 4 COST OPTIMIZATION OF RC PLANAR FRAMES

4.1 Problem Formulation

The optimization objective is to minimize structural cost of an RC planar frame using section depth of its members and reinforcement areas as a design variable while the shear, flexure, axial strength and geometrical considerations taken as constraints.

Therefore, mathematically the optimization problem can generally expressed as:

$$\textit{Minimize}, \text{ Cost of frame, } \quad C = C_{beams} + C_{columns} \tag{4-1}$$

$$C_{beams} = \sum_{beams} \left(u_{conc}(bhl)_b\right) + \left(u_{steel}\rho_{steel}\left(A_{s,l}l_l + A_{s,v}l_v\right)_b\right) + \left(u_{form}\left((2h+b)l\right)_b\right) \tag{4-2}$$

$$C_{columns} = \sum_{columns} \left(u_{conc}(bhl)_b\right) + \left(u_{steel}\rho_{steel}\left(A_{s,l}l_l + A_{s,v}l_v\right)_c\right) + \left(u_{form}\left(2(h+b)l\right)_c\right) \tag{4-3}$$

where u_{conc}, u_{steel} *&* u_{form} *are unit cost of concrete, steel, and formwork respectively and* ρ_{steel} *is a unit density of steel.*

While the constraints are as follows:

For beams: For columns:

$$\begin{array}{ll}
\left(M_{Ed} - M_{Rd}\right)_{beams} \le 0 & \left(\dfrac{N_{Ed}}{N_{Rd}} + \dfrac{M_{Ed}}{M_{Rd}} - 1\right)_{columns} \le 0 \\
\left(V_{Ed} - V_{Rd}\right)_{beams} \le 0 & \left(V_{Ed} - V_{Rd}\right)_{columns} \le 0 \\
\left(A_{sl,\min} - A_{s,l}\right)_{beams} \le 0 & \left(A_{sl,\min} - A_{s,l}\right)_{columns} \le 0 \\
\left(A_{s,l} - A_{s,l\max}\right)_{beams} \le 0 & \left(A_{s,l} - A_{s,l\max}\right)_{columnss} \le 0 \\
\left(A_{s,v\min} - A_{s,v}\right)_{beams} \le 0 & \left(A_{s,v\min} - A_{s,v}\right)_{columns} \le 0
\end{array} \tag{4-4}$$

In the above formulation, the design variables and known parameters are as presented in Table 4-1 and Table 4-2 respectively:

Table 4-1: Design Variables

Design Variables	**Members**	**Designation**
Depth	Beams	h_b
	Columns	h_c
Longitudinal Reinforcement	Beams	$(A_{s,l})_b$
	Columns	$(A_{s,l})_c$
Shear Reinforcement	Beams	$(A_{s,v})_b$
	Columns	$(A_{s,v})_c$

Table 4-2: Parameters taken as Known for the Optimization

Parameters	Members/Material	Designation
Width	Beams	b_b
	Columns	b_c
Member Length	Beams	L_b
	Columns	L_c
Longitudinal Reinforcement Length	Beams	$l_{l,b}$
	Columns	$l_{l,c}$
Shear Reinforcement Length	Beams	$l_{v,b}$
	Columns	$l_{v,c}$
Unit Cost	Concrete	u_{conc}
	Steel	u_{steel}
	Formwork	u_{form}

4.1.1 Mathematical Formulation

The optimization problem reformulated here in mathematical terms. All parameters in a braced bracket are matrices while those without it are scalar constants. The symbol $.*$ is used to represent element wise matrix multiplication

Table 4-3 summarizes the assignations taken in mathematical representation of the objective functions.

Table 4-3: Representation of Objective Function Parameters in Mathematical Terms

Design Variables		Assignment
Beams	Depth, h_b	$[x_1]$
	Longitudinal Reinforcement, $(A_{s,l})_b$	$[x_2]$
	Shear Reinforcement, $(A_{s,v})_b$	$[x_3]$
Columns	Depth, h_c	$[x_4]$
	Longitudinal Reinforcement, $(A_{s,l})_c$	$[x_5]$
	Shear Reinforcement, $(A_{s,v})_c$	$[x_6]$
Known Parameters		Assignment
Beams	$u_{conc}[b_b].^{*}[L_b]$	$[k_1]$
	$u_{steel}\rho_{steel}[l_{l,b}]$	$[k_2]$
	$u_{steel}\rho_{steel}[l_{v,b}]$	$[k_3]$
	$u_{form}[L_b]$	$[k_4]$
Columns	$u_{conc}[b_c].^{*}[L_c]$	$[k_5]$
	$u_{steel}\rho_{steel}[l_{l,c}]$	$[k_6]$
	$u_{steel}\rho_{steel}[l_{v,c}]$	$[k_7]$
	$2u_{form}[L_c]$	$[k_8]$

The problem formulation uses organization of constraint functions as expressed in Appendix-C. Therefore, for constants used in the formulation or any detail related to the constraint functions, the Appendix can be consulted.

The parameters in the constraints of external and internal columns used similar form of assignation. The differentiation is made as internal and external in the coding phase.

The optimization problem is formulated as:

$$Minimize\ \ C(x) = Sum\begin{pmatrix} [k_1].^{*}[x_1] + [k_2].^{*}[x_2] + [k_3].^{*}[x_3] + [k_4].^{*}(2[x_1] + [b_b]) + \\ [k_5].^{*}[x_4] + [k_6].^{*}[x_5] + [k_7].^{*}[x_6] + [k_8].^{*}([x_4] + [b_c]) \end{pmatrix} \qquad \text{4-5}$$

Subjected to:

- Beams flexural constraint:
    - At plastic hinges:

$$\begin{cases} \left(\left(\dfrac{[L_B - x_4]}{2}\right).*\left(\dfrac{[T]}{Sum(V_s)}\right).*\left((0.1 + 0.9k_{9_n})k_{10}k_{11}\left(\dfrac{0.07}{0.02 + \xi_{eq}}\right)\right) - \left([k_{16}].*([x_1 - k_{15}])^2 + (0.9 f_{ye}[x_1].*[x_2])\right)\right) \le 0, i = n \\ \left(\left(\dfrac{[L_B - x_4]}{2}\right).*\left(\dfrac{[T]}{Sum(V_s)}\right).*\left(\sum_{k=i}^{n} 0.9k_{10_k} k_{11}k_{12}\left(\dfrac{0.07}{0.02 + \xi_{eq}}\right)\right) - \left([k_{16}].*([x_1 - k_{15}])^2 + (0.9 f_{ye}[x_1].*[x_2])\right)\right) \le 0, i \in (1; n-1) \end{cases} \qquad \text{4-6}$$

-
    - At span:

$$\begin{cases} \left([k_{17}].*\left(\dfrac{[T]}{Sum(V_s)}\right).*\left((0.1 + 0.9k_{9_n})k_{10}k_{11}\left(\dfrac{0.07}{0.02 + \xi_{eq}}\right)\right) + [k_{18}] - \left([k_{16}].*([x_1 - k_{15}])^2 + (0.9 f_{ye}[x_1].*[x_2])\right)\right) \le 0, i = n \\ \left([k_{17}].*\left(\dfrac{[T]}{Sum(V_s)}\right).*\left(\sum_{k=i}^{n} 0.9k_{9_k} k_{10}k_{11}\left(\dfrac{0.07}{0.02 + \xi_{eq}}\right)\right) + [k_{18}] - \left([k_{16}].*([x_1 - k_{15}])^2 + (0.9 f_{ye}[x_1].*[x_2])\right)\right) \le 0, i \in (1; n-1) \end{cases} \qquad \text{4-7}$$

- Beams shear constraint:

$$\begin{cases} \left(\left(\dfrac{\phi^{o}[T]}{Sum(V_s)}\right).*\left((0.1 + 0.9k_{9_n})k_{10}k_{11}\left(\dfrac{0.07}{0.02 + \xi_{eq}}\right)\right) + [k_{24}] - V_{Rd}\right) \le 0, i = n \\ \left(\left(\dfrac{[L_B - x_2]}{2}\right).*\left(\dfrac{[T]}{Sum(V_s)}\right).*\left(\sum_{k=i}^{n} 0.9k_{9_k} k_{10}k_{11}\left(\dfrac{0.07}{0.02 + \xi_{eq}}\right)\right) + [k_{24}] - V_{Rd}\right) \le 0, i \in (1, n-1) \end{cases} \qquad \text{4-8}$$

where $$V_{Rd} = \max\begin{Bmatrix} \min\left(1 + \sqrt{\dfrac{200}{[x_1] - k_{15}}}, 2\right)[k_{19}].*[x_1]^{2/3}.*[x_2]^{1/3} \\ \min\left(1 + \sqrt{\dfrac{200}{[x_1] - k_{15}}}, 2\right)[k_{20}].*[x_1] \end{Bmatrix} + \frac{(4[x_3] - k_{21}).*[k_{22}].*[x_3].*[x_1]\cot\theta}{k_{21}} \qquad \text{4-9}$$

and $$\theta = \sin^{-1}\left[\frac{[k_{23}].*[V_{Ed}]}{[x_1]}\right] \qquad \text{4-10}$$

- Beams reinforcement bounds:
    - Longitudinal reinforcement:

$$[k_{25}].*[x_1]-[x_2]\leq 0 \tag{4-11}$$

$$[x_2]-[k_{26}].*[x_1]\leq 0 \tag{4-12}$$

    - Shear reinforcement:

$$\frac{k_{23}[L_b]}{4[x_4]-k_{23}}-\min\begin{cases}\left[\dfrac{x_1}{4}\right]\\ k_{27}\\ 225\\ k_{28}\end{cases}\leq 0 \tag{4-13}$$

- Exterior columns flexural constraint:

$$\begin{cases}\dfrac{T+P_G}{[k_{29}].*[x_4]}+\left(\dfrac{\left[\dfrac{L_B}{2}\right].*\left(\dfrac{[T]}{Sum(V_s)}\right).*(0.1+0.9k_{9_n})k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)}{[k_{29}].*[x_4]^2}\right)-1\leq 0, i=n\\[2ex] \dfrac{T+P_G}{[k_{29}].*[x_4]}+\left(\dfrac{\left[\dfrac{L_B}{4}\right].*\left(\dfrac{[T]}{Sum(V_s)}\right).*\sum_{k=i}^{n}0.9k_{9_k}k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)}{[k_{29}].*[x_4]^2}\right)-1\leq 0, i\in(2,n-1)\\[2ex] \dfrac{P_G}{[k_{29}].*[x_4]}+\left(\dfrac{k_{31}k_{13}k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)}{[k_{29}].*[x_4]^2}\right)-1\leq 0, i=1\end{cases} \tag{4-14}$$

- Exterior columns shear constraints:

$$\begin{cases}\left(\left(\phi^o[k_{31}].*(0.1+0.9k_{9_n})k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)+0.1\mu\sum_{i=1}^{n}0.9k_{9_i}k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)\right)\right)-V_{Rd}, i=n\\ \left(\left(\phi^o[k_{31}].*\sum_{k=i}^{n}0.9k_{9_k}k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)+0.1\mu\sum_{i=1}^{n}0.9k_{9_i}k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)\right)\right)-V_{Rd}, i\in(1,n-1)\end{cases} \tag{4-15}$$

where

$$V_{Rd}=\left\{\max\begin{cases}\min\left(1+\sqrt{\dfrac{200}{[x_2]-k_{34}}},2\right)[k_{32}].*[x_4]^{2/3}.*[x_5]^{1/3}\\ \min\left(1+\sqrt{\dfrac{200}{[x_2]-k_{30}}},2\right)[k_{33}].*[x_4]\end{cases}\right\}+\left\{\left(\frac{0.9[x_4]}{H_{cl}}N_{Ed}\right)+\frac{(4[x_6]-k_{35}).*[k_{36}].*[x_6].*[x_4]\cot\theta}{k_{35}}\right\} \tag{4-16}$$

- Interior columns flexural Constraint:

$$\begin{cases} \dfrac{P_G}{[k_{29}].*[x_4]} + \left(\dfrac{[L_B]_{average}.*\left(\dfrac{[T]}{Sum(V_s)}\right).*(0.1+0.9k_{9_n})k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)}{[k_{29}].*[x_4]^2} \right) - 1 \leq 0, i = n \\ \dfrac{P_G}{[k_{29}].*[x_4]} + \left(\dfrac{\left[\dfrac{L_B}{4}\right]_{average}.*\left(\dfrac{[T]}{Sum(V_s)}\right).*\sum_{k=i}^{n} 0.9k_{9_k} k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)}{[k_{29}].*[x_4]^2} \right) - 1 \leq 0, i \in (2, n-1) \\ \dfrac{P_G}{[k_{29}].*[x_4]} + \left(\dfrac{k_{31}k_{13}k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)}{[k_{29}].*[x_4]^2} \right) - 1 \leq 0, i = 1 \end{cases} \qquad \textbf{4-17}$$

- Interior columns shear constraints:

$$\begin{cases} 2\left(\phi^o[k_{31}].*(0.1+0.9k_{9_n})k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right) + 0.1\mu\sum_{i=1}^{n} 0.9k_{9_i} k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)\right) - V_{Rd}, i = n \\ 2\left(\phi^o[k_{31}].*\sum_{k=i}^{n} 0.9k_{9_k} k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right) + 0.1\mu\sum_{i=1}^{n} 0.9k_{9_i} k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)\right) - V_{Rd}, i \in (1, n-1) \end{cases} \qquad 4\text{-}18$$

where V_{Rd} is calculated using Eqn. (4.17)

- Columns reinforcement bounds:
  - Longitudinal reinforcement:

$$[k_{38}].*[x_4] - [x_5] \leq 0 \qquad 4\text{-}19$$

$$[x_5] - 4[k_{38}].*[x_4] \leq 0 \qquad 4\text{-}20$$

  - Shear reinforcement:

$$\frac{k_{35}[L_c]}{4[x_6] - k_{35}} - \min\begin{cases}[k_{39}] \\ 175 \\ k_{40}\end{cases} \leq 0 \qquad 4\text{-}21$$

4.2 Algorithm Selection

As it can be inferred from the formulation, the optimization problem is of the type non – linear constrained optimization problem.

Therefore, the following sub-section briefly describes constrained minimization problems according to G.Luenberger , Yinyue Ye [9].

4.2.1 Brief Notes on Non – Linear Constrained Minimization

The general mathematical programing problem can be stated as:

$$Minimize\ f(x)$$

$$Subjected\ \ to\ \begin{cases} h_i(x) = 0, i = 1,2,...m \\ g_j(x) \leq 0,\ j = 1,2,...p \\ x \in S \end{cases} \qquad 4\text{-}22$$

In the formulation *x* is an *n* – dimensional vector of unknowns and f , h_i and g_j are real valued functions of the variables $x_1, x_2, ..., x_n$. The set S is a subset of *n* – dimensional space. The function *f* is the objective function of the problem and h_i and g_j are constraints.

To make a problem smooth in some suitable sense, additional assumptions may have been taken; for instance, the functions in the problem are usually required to be continuous, or perhaps to have continuous derivatives.

An inequality constraint $g_j(x) \leq 0$ said to be active at a feasible point *x* if $g_j(x)$ =0 and inactive at *x* if $g_j(x) < 0$. By convention, we refer to any equality constraint $h_i(x) = 0$ as active at any feasible point.

4.2.1.1 First-Order Necessary Conditions

Let x^* be a point satisfying the constraints

$$h(x^*) = 0 \text{ And } g(x^*) \le 0$$

In addition, let J be the set of indices j for which $g_j(x^*) = 0$. Then x^* is said to be a regular point of the constraints if the gradient vectors $\nabla h_i(x^*), \nabla g_j(x^*)$,$1 \le i \le m$, $j \in J$ are linearly independent.

Karush-Kuhn-Tucker Conditions:

Let x^* be a relative minimum point for the problem

$$Minimize\ f(x)$$

$$Subjected\ \ to\ \begin{cases} h(x) = 0 \\ g(x) \le 0 \end{cases}$$

Suppose x^* is a regular point for the constraints. Then there is a vector $\lambda \in \mathrm{E}^m$ and a vector $\mu \in \mathrm{E}^p$ with $\mu \ge 0$ such that

$$\nabla f(x^*) + \lambda \nabla h(x^*) + \mu \nabla g(x^*) = 0$$

$$\mu g(x^*) = 0 \qquad \text{4-23}$$

4.2.1.2 Second-Order Necessary Conditions:

Suppose the functions *f, g & h* are continuous to second-order and that x^* is a regular point of the constraints.

If x^* is a relative minimum point for problem, then there is $\lambda \in \mathrm{E}^m$, $\mu \in \mathrm{E}^p$ and $\mu \ge 0$ such that KKT conditions hold and

$L(x^*) = F(x^*) + \lambda H(x^*) + \mu G(x^*)$, is a positive semi definite on the tangent sub-space of the active constraints at x^* where F(x), H(x) and G(x) are second-order derivatives.

4.2.1.3 Second-Order Sufficiency Conditions:

Let $\mathrm{f, g \, \& \, h} \in \mathrm{C}^2$. Sufficient conditions that a point x^* satisfying constraints be a strict relative minimum point of the problem is that, there exist $\lambda \in \mathrm{E}^m$, $\mu \in \mathrm{E}^p$ such that

$$\mu \geq 0$$

$$\mu g(x^*) = 0$$

$$\nabla f(x^*) + \lambda \nabla h(x^*) + \mu \nabla g(x^*) = 0 \qquad \text{4-24}$$

And the Hessian matrix

$L(x^*) = F(x^*) + \lambda H(x^*) + \mu G(x^*)$, is a positive definite on the sub-space

$M' = \{y : \nabla h(x^*) y = 0, \nabla g_j(x^*) y = 0\}$, for all $j \in J$

, where $J = \{j : g_j(x^*) = 0, \mu_j > 0\}$

One of the method utilizing optimality conditions directly is interior-point algorithm; therefore, it used here in the optimization process for local optimality. Interior-point methods are linear or non-linear programming method applicable to problems of the form expressed in Eqn. (4.22). They achieves optimization by going through the middle of feasible set defined by the problem rather than around the surface.

The approach is described in Appendix-D in a brief manner. Its implementation here is done using MATLAB optimization solver called *fmincon.*

In using interior-point method, a careful selection of starting point is necessary. Therefore, that is discussed in the next section.

4.2.2 Selection of Feasible Initial Point

Many non-linear solvers are able to find a feasible point if given an initial point that is close to feasibility, but may fail if the initial point is far from feasibility. If no external information is available to guide the initial-point placement, various heuristic can be used.

According to Chinneck [10], a widely applied heuristic called a standard heuristic is as follows:

- If the variable is doubly bounded: set at mid-point
- If the variable is singly bounded: set on the bound
- If the variable is unbounded in both direction: set at zero

However, the standard heuristic may cause numerical errors as it sets many variables to zero and as many variables may have similar bounds. To improve these, Ibrahim and Chinneck [11] develop a simple modification to the standard heuristic that superimpose a random perturbation Δ on the initial values proposed above.

The randomized standard heuristic operate as follows:

- If the variable is doubly bounded: set at mid-point + Δ
- If the variable has a single lower bound : set at bound + Δ
- If the variable has a single upper bound : set at bound – Δ
- If the variable is unbounded in both direction: set at zero + Δ

Δ is a uniformly distributed random number between 0 and 1 (or suitably smaller if the bounds on the variable define a smaller range).

The initial point supplied to a non-linear solver may originate from knowledge of the model, from a previous solution to a similar model, or may be generated by an initial point heuristic. It may even be generated randomly by a naïve modeler.

It is certainly possible to pass this initial point directly to the solver, but better results can be obtained if the initial point is instead passed to an inexpensive point improvement algorithm first.

The point output by the point improvement algorithm is then finally passed to the full-scale, accurate, computationally expensive solver.

The Constraint Consensus Methods, Chinneck [12] are point improvement algorithms that are effective at moving a point that is far from feasibility to a point that is near to feasibility.

4.2.2.1 Basic Constraint Consensus Method

Constraint Consensus algorithms are a variation of projection algorithm, which forms some form of projection for each violated constraint most commonly a projection in the gradient or anti-gradient direction.

The main idea is to use gradient of the violated constraint at the current infeasible point to calculate the closest point that satisfy the constraint, called the orthogonal projection of the violated point.

The vector showing how to move from the current infeasible point to the orthogonal projection on to an individual violated constraint is sometimes called the *feasibility vector,* and denoted by f_{vi} for the i^{th} constraint c_i.

The feasibility vector is given by:

$$f_{vi} = \frac{v_i d_i \nabla c_i(x)}{\|\nabla c_i(x)\|^2} \qquad \text{4-25}$$

where,

- $\nabla c_i(x)$ is the gradient of the constraint, and $\|\nabla c_i(x)\|$ is its length
- v_i is the constraint violation $|c_i(x) - b_i|$, or zero for satisfied constraints
- d_i is +1 if it is necessary to increase c(x) to satisfy the constraint and -1 if it is necessary to decrease c(x) to satisfy the constraint.

In the Basic Constraint Consensus Method, component wise averaging of feasibility vectors form a consensus vector, which improves the infeasible point iteratively

4.2.2.2 Improved Constraint Consensus Methods

Most of the Constraint Consensus Variants tend to produce output points that are on or very close to the limiting values of inequality constraints. This can be a problem for solvers that use a barrier method as such algorithms prefer a starting point that is very close to satisfying the equality constraints but which over satisfies the inequality constraints. Thus, the need for improvements arise.

4.2.2.3 Procedures in SUM Consensus Method

To provide points better suited for barrier methods, Smith, Chinneck and Aitken [13] develops a technique called the SUM method. As its name suggests, the SUM method forms the consensus vector by summing all the feasibility vector at the current iterate, while the feasibility vector being same with the basic constraint consensus method.

- Compute constraint violation at starting point, which is set from knowledge of the model

$$v_i = \max\{0, c_i(x)\} \qquad 4\text{-}26$$

- Calculate the feasibility vector from Eqn. (4.25)
- Sum up the feasibility vectors to form consensus vector, CV.
- Update the initial point using the consensus vector

$$x_{i+1} = x_i + CV$$

- Iterate to acceptable constraint violation

Here, to get an initial feasible point, a starting point is selected guided by experience and knowledge of the model and then that passed through a SUM Consensus algorithm to improve feasibility.

4.2.3 MATLAB *fmincon* Local Optimization Solver

In MATLAB, there is a solver called *fmincon*, which find a local minimum of a constrained nonlinear function using four optimization algorithms; that are Interior-point method, Sequential Quadratic Programming, Active Set method and Trust Region Reflective method.

It solves a problem of the form:

$$
\begin{aligned}
&\min_{x} f(x) \quad {}_{subject \quad to} \\
&c(x) \leq 0 \\
&ceq(x) = 0 \\
&A. \quad x \leq b \\
&Aeq. \quad x = beq \\
&lb \leq x \leq ub
\end{aligned}
$$

where x, b, beq, lb, and ub are vectors, A and Aeq are matrices, c(x) and ceq(x) are functions that return vectors, and f(x) is a function that return a scalar. f (x), c(x), and ceq(x) can be nonlinear function.

So, in this thesis it used as a local optimum solver employing interior-point method within the global optimization tool in MATLAB, $GlobalSearch$ algorithm.

4.2.4 MATLAB *GlobalSearch* Algorithm

GlobalSearch is a MATLAB optimization tool that finds global minima by starting *fmincon* solver from multiple start points.

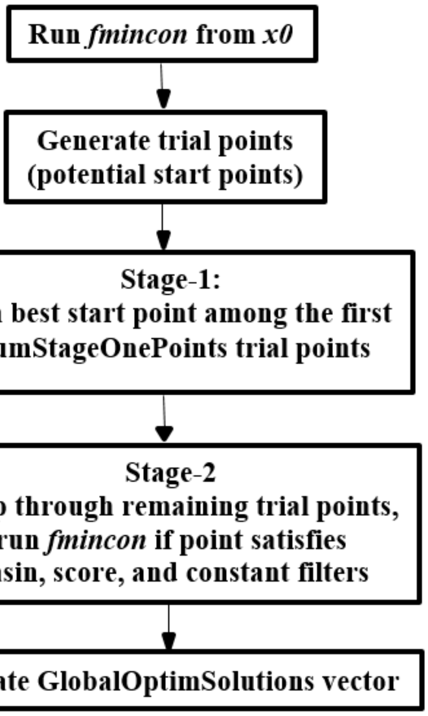


Figure 4-1: Steps in *GlobalSearch* Algorithm [10]

4.3 Solution to the Optimization Problem

A program is written on MATLAB to implement the points discussed up to here. The general pseudo-code for GlobalSearch algorithm and the flow chart for the optimization is given below. The full source code of the program is given in Appendix – A.

4.3.1 Pseudo-code for GlobalSearch Optimization

```
gs=GlobalSearch;
opts =optimoptions (@fmincon,'Algorithm','interior-point');
problem = createOptimProblem('fmincon','x0',x_start,...
'objective', @objective_function, 'nonlcon',...
@constraint_function, 'options', opts);
  [x, fval]= run (gs, problem);
```

4.3.2 Flow Chart for the Optimization

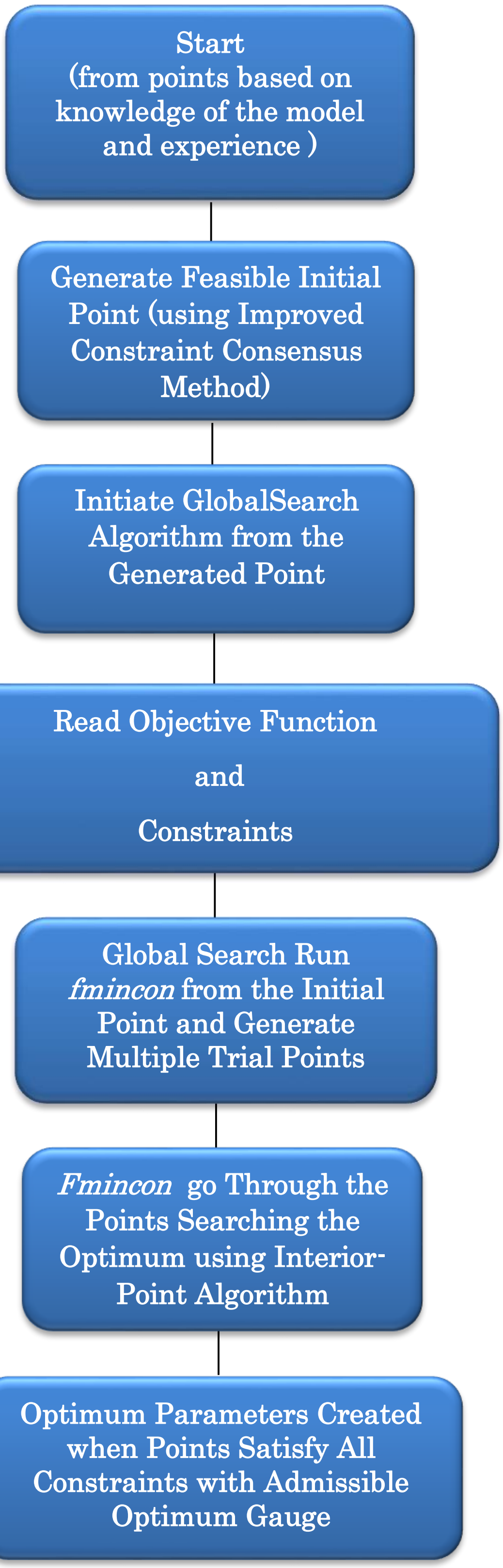


Figure 4-2: Flow Chart for the Optimization

CHAPTER 5 NUMERICAL EXAMPLES AND RESULTS

5.1 Sample Frames Design

5.1.1 Sample Frames

Three frames are taken as sample frames for comparison and verification of the optimization against response spectrum analysis and design by ETABS. A 2-storey 2-bays, 6-storey 4-bays and 12-storey 3-bays reinforced concrete two-dimensional frames with properties listed in Table 3-1, Table 3-2 and Table 3-3.

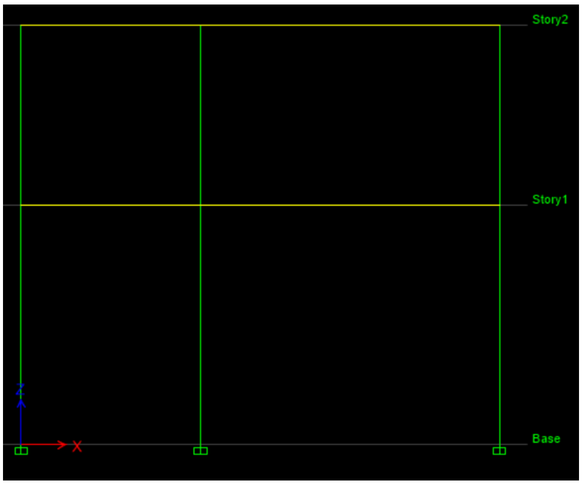


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Figure 5-1: Sample Frame – I

Bay Length		
1st Bay	3	m.
2nd Bay	5	m.
Storey Height		
1st Storey	4	m.
2nd Storey	3	m.
Significant Mass		
1st Storey	15	tonnes
2nd Storey	10	tonnes

Table 5-1: Sample Frame – I Properties

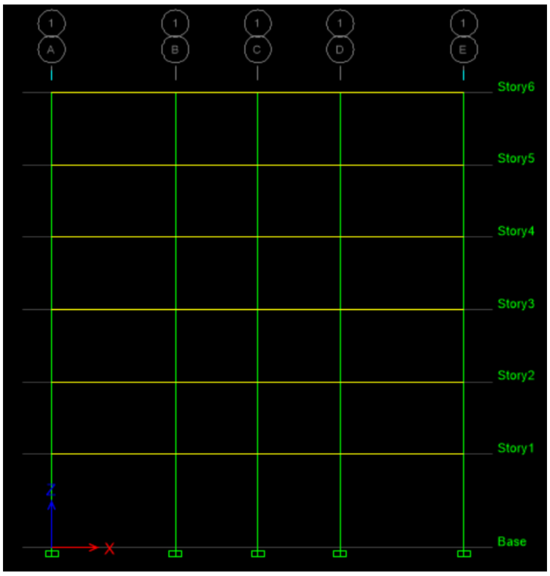


Figure 5-2: Sample Frame – II

Bay Length		
Exterior Bays	6	m.
Interior Bays	4	m.
Storey Height		
1st Storey	4.5	m.
Others Storey	3.5	m.
Significant Mass		
1st Storey	40	tonnes
2nd-5th Storey	35	tonnes
6th Storey	40	tonnes

Table 5-2: Sample Frame – II Properties

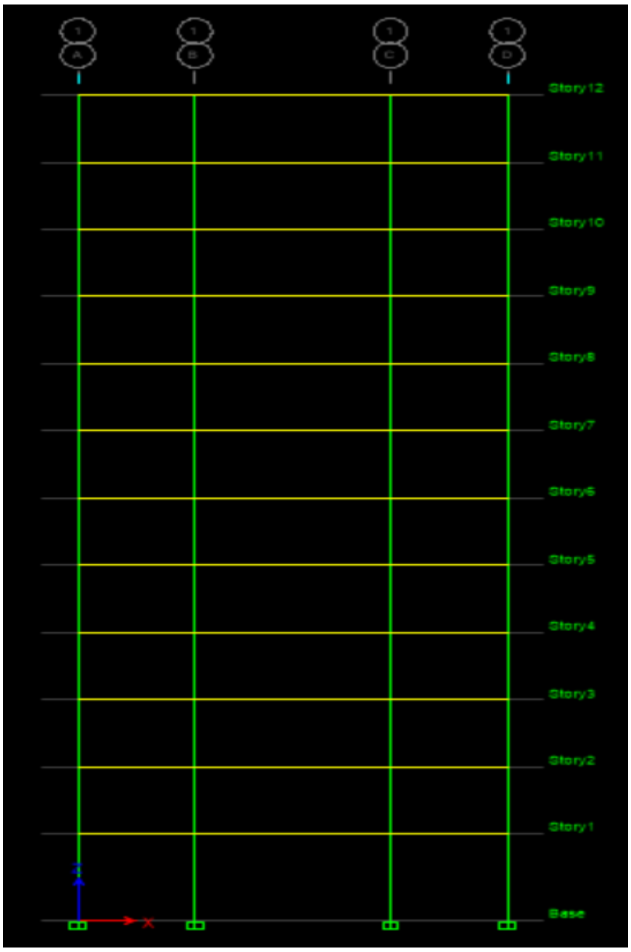


Figure 5-3: Sample Frame - III

Bay Length		
Exterior bays	4.5	m.
Interior bays	7.5	m.
Storey Height		
1st-Storey	4.5	m.
Others-Storey	3.5	m.
Significant Mass		
1st-Storey	65	Tonnes
2nd - 11th Storey	60	Tonnes
12th Storey	70	Tonnes

Table 5-3: Sample Frame – III Properties

All frames are assumed to be located in a seismic region with peak ground acceleration of 0.4g. The elastic 5% damping displacement spectrum is shown in Figure 5-1.

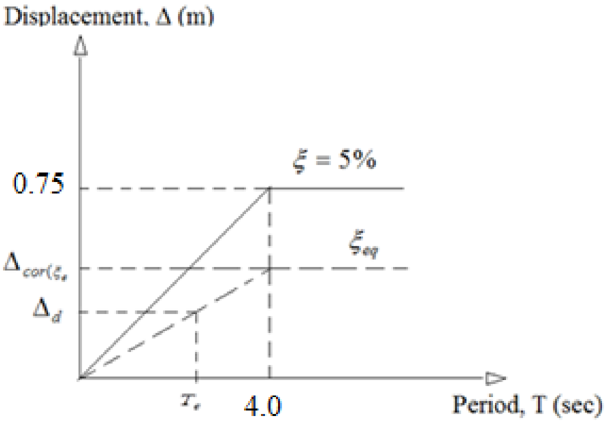


Figure 5-4: Displacement Spectrum

A probable axial load of 150KN on each exterior columns and 300KN on each interior columns are considered.

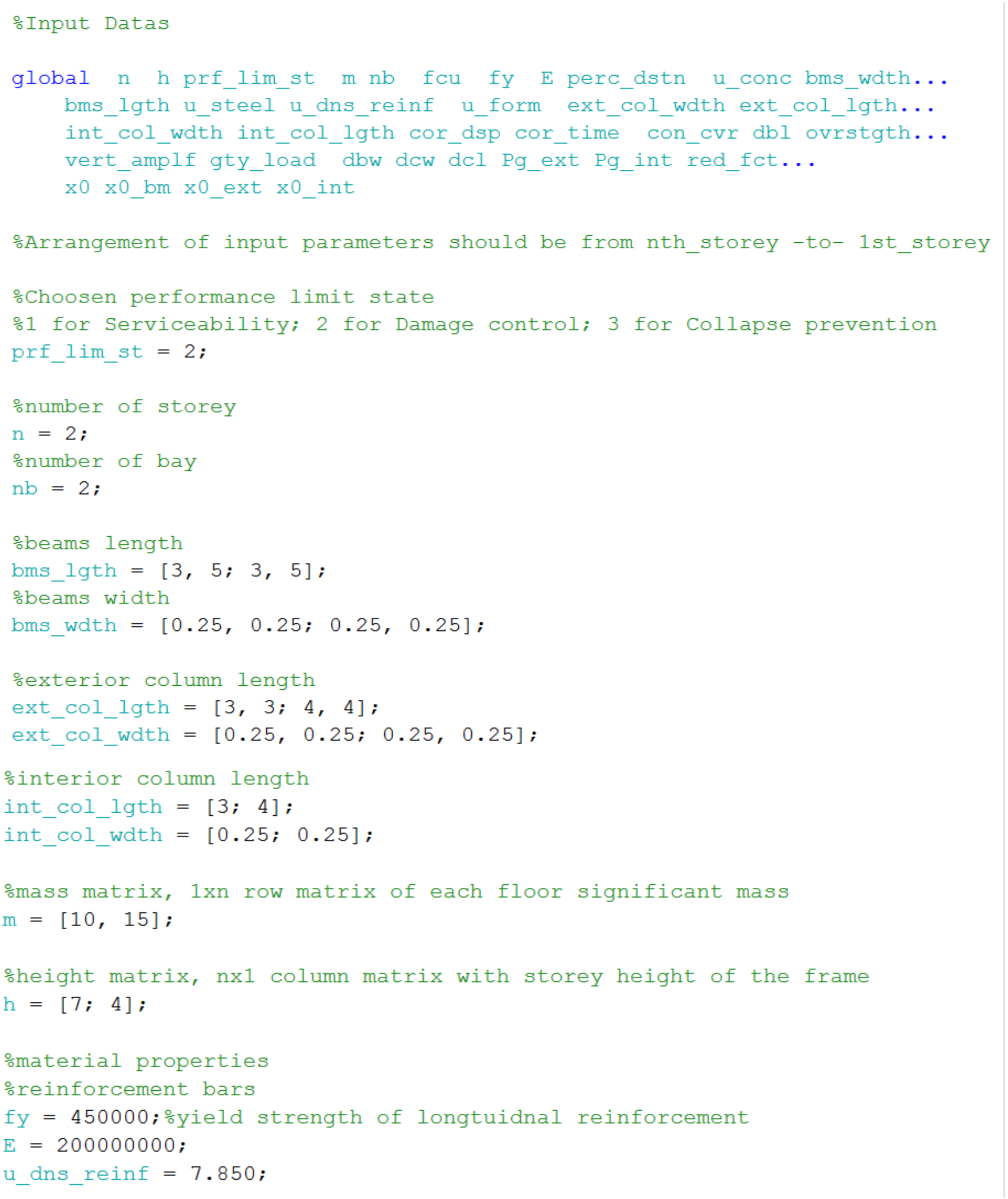
5.1.2 Optimized Design

Sample frames inputs are prepared based on information's about the frames and then used by the optimization program to create optimum design parameters.

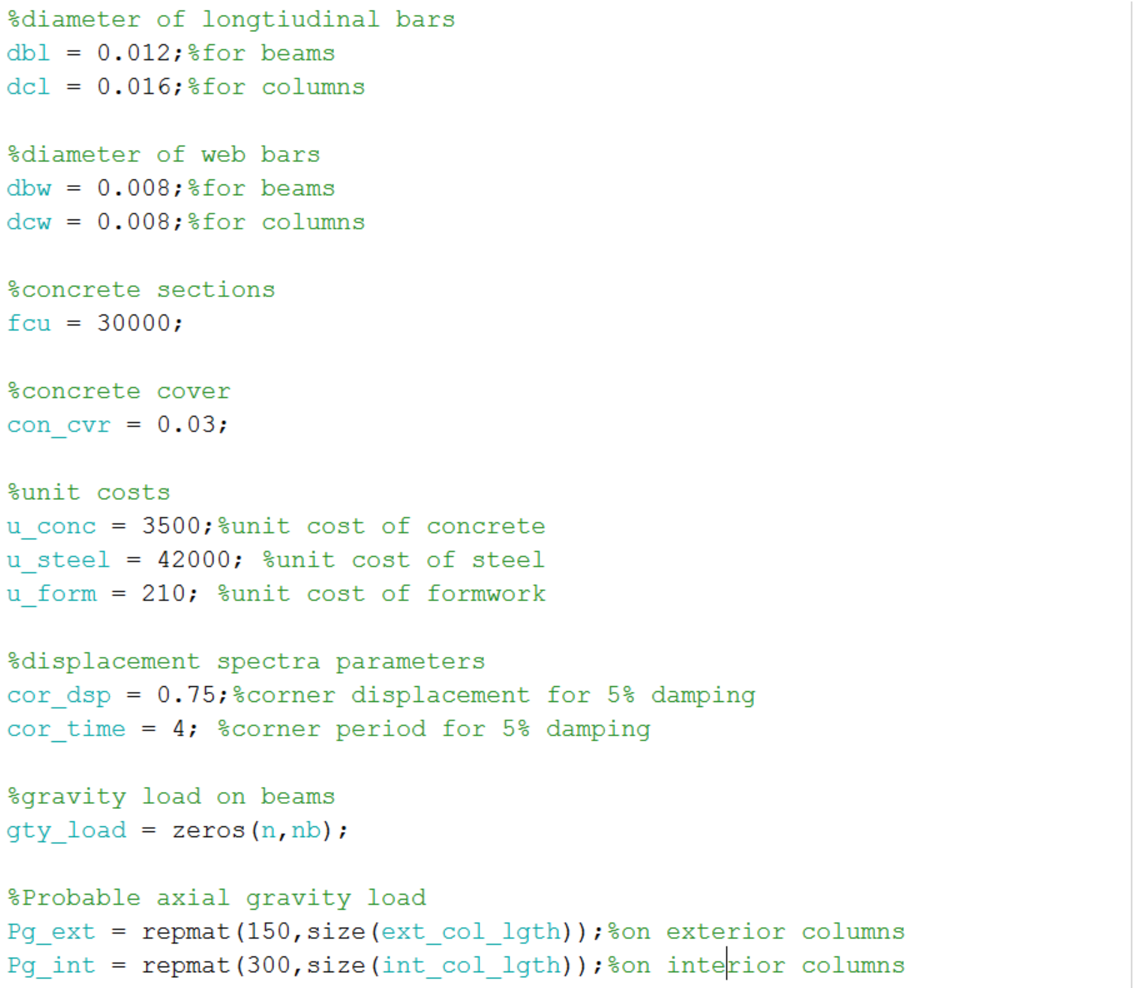
The results of the program are presented below, in order of nth to first storey. All measurement units are in consistent with KNm.

5.1.2.1 Inputs

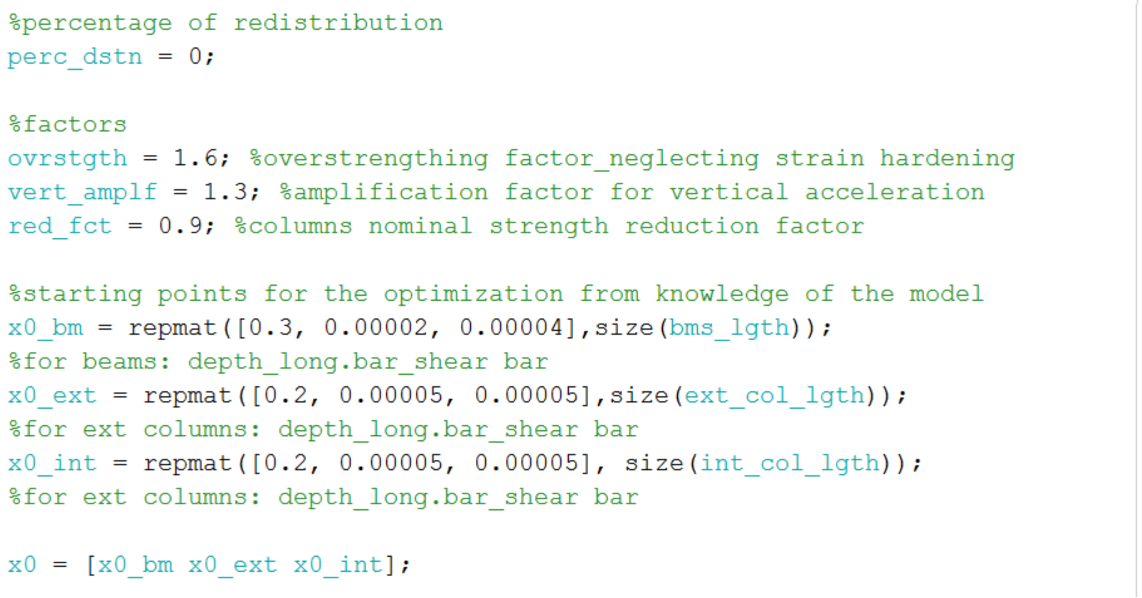
I. Sample Frame – I



```
%Input Datas

global  n  h prf_lim_st  m nb  fcu  fy  E perc_dstn  u_conc bms_wdth...
    bms_lgth u_steel u_dns_reinf  u_form  ext_col_wdth ext_col_lgth...
    int_col_wdth int_col_lgth cor_dsp cor_time  con_cvr dbl ovrstgth...
    vert_amplf gty_load  dbw dcw dcl Pg_ext Pg_int red_fct...
    x0 x0_bm x0_ext x0_int

%Arrangement of input parameters should be from nth_storey -to- 1st_storey

%Choosen performance limit state
%1 for Serviceability; 2 for Damage control; 3 for Collapse prevention
prf_lim_st = 2;

%number of storey
n = 2;
%number of bay
nb = 2;

%beams length
bms_lgth = [3, 5; 3, 5];
%beams width
bms_wdth = [0.25, 0.25; 0.25, 0.25];

%exterior column length
ext_col_lgth = [3, 3; 4, 4];
ext_col_wdth = [0.25, 0.25; 0.25, 0.25];
%interior column length
int_col_lgth = [3; 4];
int_col_wdth = [0.25; 0.25];

%mass matrix, 1xn row matrix of each floor significant mass
m = [10, 15];

%height matrix, nx1 column matrix with storey height of the frame
h = [7; 4];

%material properties
%reinforcement bars
fy = 450000;%yield strength of longtuidnal reinforcement
E = 200000000;
u_dns_reinf = 7.850;
```

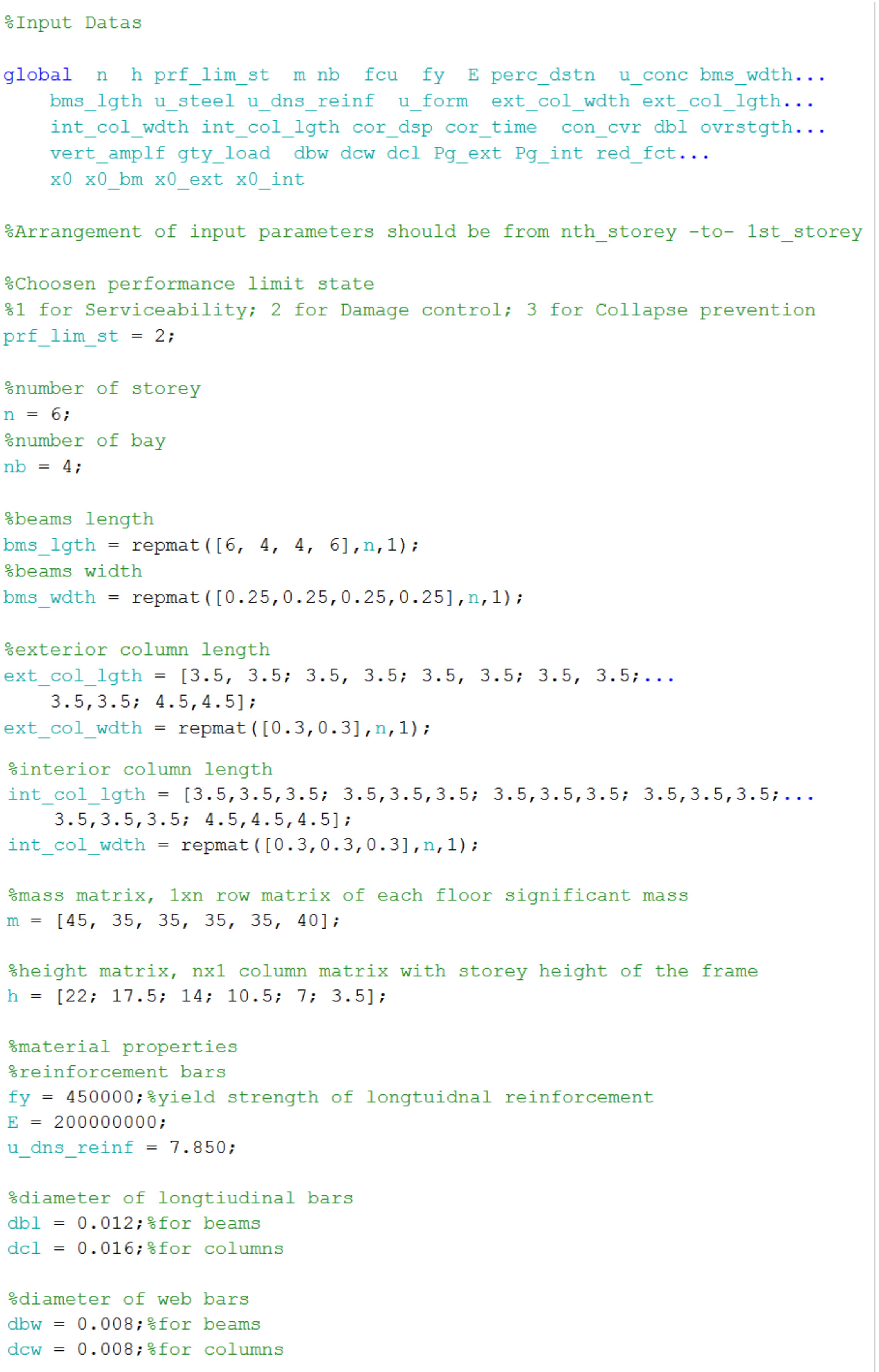


```
%diameter of longtiudinal bars
dbl  = 0.012;%for beams
dcl  = 0.016;%for columns

%diameter of web bars
dbw  = 0.008;%for beams
dcw  = 0.008;%for columns

%concrete sections
fcu  = 30000;

%concrete cover
con_cvr = 0.03;

%unit costs
u_conc  = 3500;%unit cost of concrete
u_steel  = 42000; %unit cost of steel
u_form  = 210; %unit cost of formwork

%displacement spectra parameters
cor_dsp  = 0.75;%corner displacement for 5% damping
cor_time  = 4; %corner period for 5% damping

%gravity load on beams
gty_load  = zeros(n,nb);

%Probable axial gravity load
Pg_ext  = repmat(150,size(ext_col_lgth));%on exterior columns
Pg_int  = repmat(300,size(int_col_lgth));%on interior columns
```

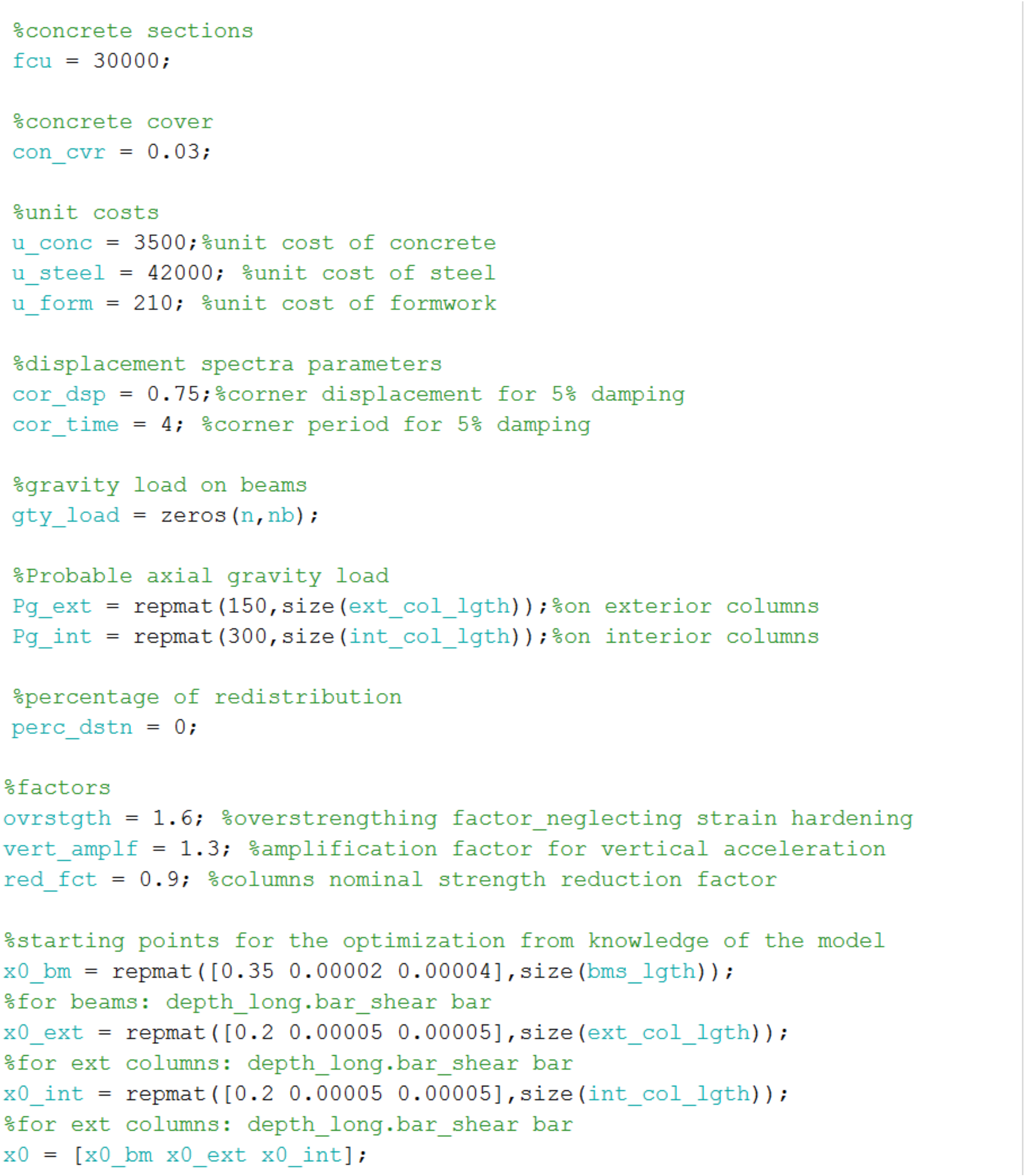


```
%percentage of redistribution
perc_dstn  = 0;

%factors
ovrstgth  = 1.6; %overstrengthing factor_neglecting strain hardening
vert_amplf  = 1.3; %amplification factor for vertical acceleration
red_fct  = 0.9; %columns nominal strength reduction factor

%starting points for the optimization from knowledge of the model
x0_bm  = repmat([0.3, 0.00002, 0.00004],size(bms_lgth));
%for beams: depth_long.bar_shear bar
x0_ext  = repmat([0.2, 0.00005, 0.00005],size(ext_col_lgth));
%for ext columns: depth_long.bar_shear bar
x0_int  = repmat([0.2, 0.00005, 0.00005], size(int_col_lgth));
%for ext columns: depth_long.bar_shear bar

x0  = [x0_bm x0_ext x0_int];
```

II. Sample Frame - II

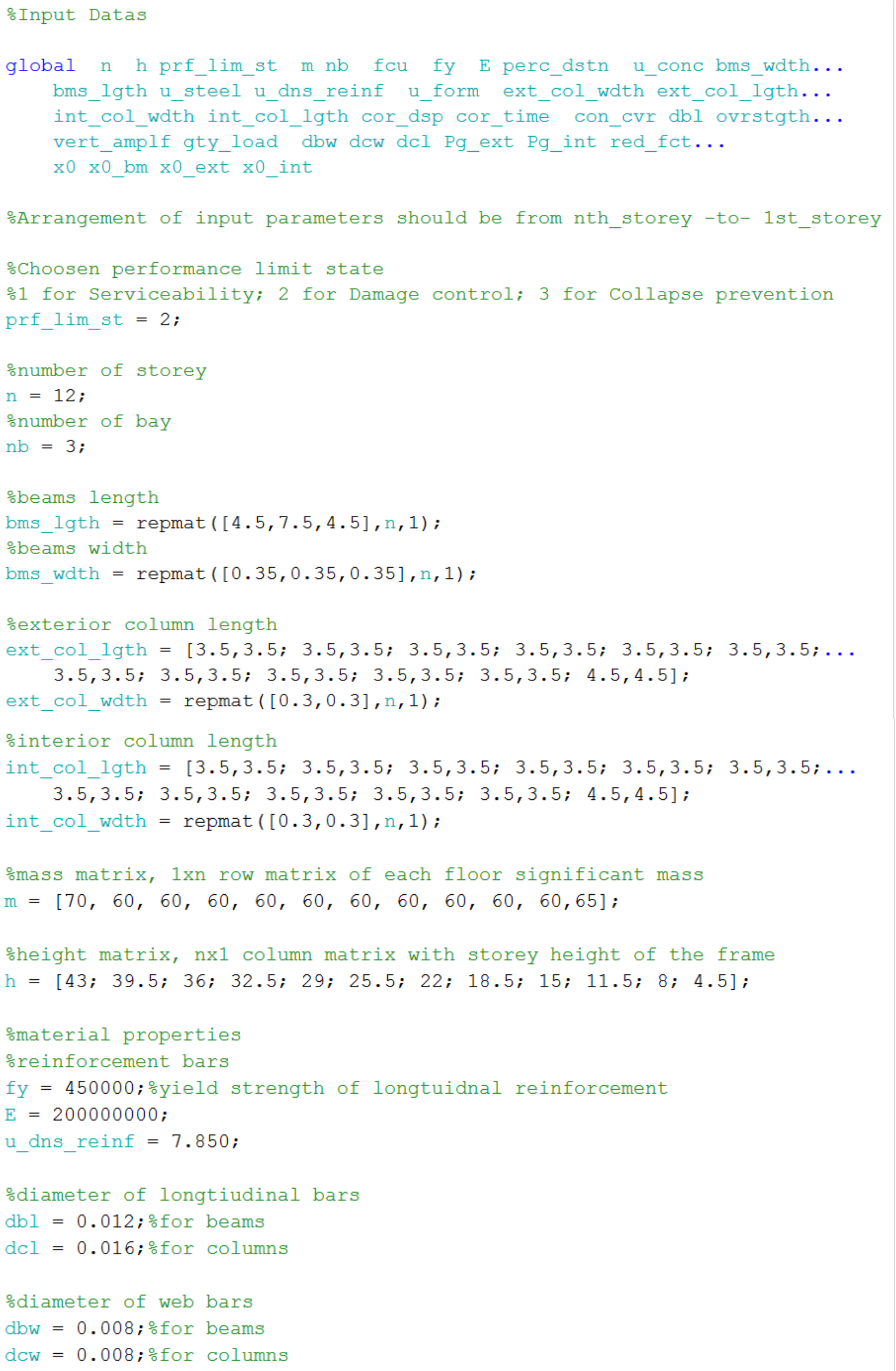


```
%Input Datas

global  n  h prf_lim_st  m nb  fcu  fy  E perc_dstn  u_conc bms_wdth...
    bms_lgth u_steel u_dns_reinf  u_form  ext_col_wdth ext_col_lgth...
    int_col_wdth int_col_lgth cor_dsp cor_time  con_cvr dbl ovrstgth...
    vert_amplf gty_load  dbw dcw dcl Pg_ext Pg_int red_fct...
    x0 x0_bm x0_ext x0_int

%Arrangement of input parameters should be from nth_storey -to- 1st_storey

%Choosen performance limit state
%1 for Serviceability; 2 for Damage control; 3 for Collapse prevention
prf_lim_st = 2;

%number of storey
n = 6;
%number of bay
nb = 4;

%beams length
bms_lgth = repmat([6,  4,  4,  6],n,1);
%beams width
bms_wdth = repmat([0.25,0.25,0.25,0.25],n,1);

%exterior column length
ext_col_lgth = [3.5,  3.5; 3.5,  3.5; 3.5,  3.5; 3.5,  3.5;...
    3.5,3.5; 4.5,4.5];
ext_col_wdth = repmat([0.3,0.3],n,1);

%interior column length
int_col_lgth = [3.5,3.5,3.5; 3.5,3.5,3.5; 3.5,3.5,3.5; 3.5,3.5,3.5;...
    3.5,3.5,3.5; 4.5,4.5,4.5];
int_col_wdth = repmat([0.3,0.3,0.3],n,1);

%mass matrix, 1xn row matrix of each floor significant mass
m = [45, 35, 35, 35, 35, 40];

%height matrix, nx1 column matrix with storey height of the frame
h = [22; 17.5; 14; 10.5; 7; 3.5];

%material properties
%reinforcement bars
fy = 450000;%yield strength of longtuidnal reinforcement
E = 200000000;
u_dns_reinf = 7.850;

%diameter of longtiudinal bars
dbl = 0.012;%for beams
dcl = 0.016;%for columns

%diameter of web bars
dbw = 0.008;%for beams
dcw = 0.008;%for columns
```

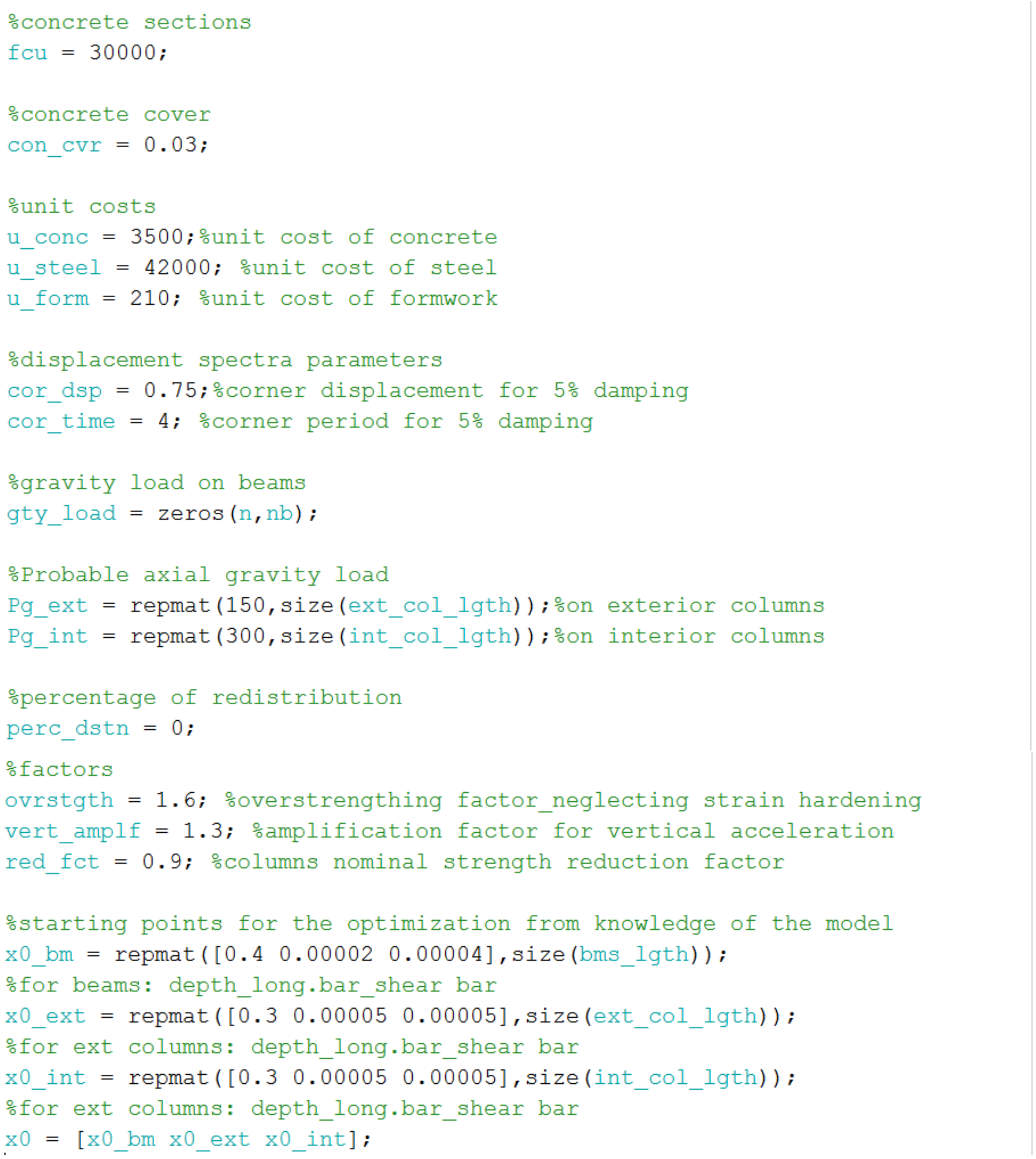


```
%concrete sections
fcu = 30000;

%concrete cover
con_cvr = 0.03;

%unit costs
u_conc = 3500;%unit cost of concrete
u_steel = 42000; %unit cost of steel
u_form = 210; %unit cost of formwork

%displacement spectra parameters
cor_dsp = 0.75;%corner displacement for 5% damping
cor_time = 4; %corner period for 5% damping

%gravity load on beams
gty_load = zeros(n,nb);

%Probable axial gravity load
Pg_ext = repmat(150,size(ext_col_lgth));%on exterior columns
Pg_int = repmat(300,size(int_col_lgth));%on interior columns

%percentage of redistribution
perc_dstn = 0;

%factors
ovrstgth = 1.6; %overstrengthing factor_neglecting strain hardening
vert_amplf = 1.3; %amplification factor for vertical acceleration
red_fct = 0.9; %columns nominal strength reduction factor

%starting points for the optimization from knowledge of the model
x0_bm = repmat([0.35 0.00002 0.00004],size(bms_lgth));
%for beams: depth_long.bar_shear bar
x0_ext = repmat([0.2 0.00005 0.00005],size(ext_col_lgth));
%for ext columns: depth_long.bar_shear bar
x0_int = repmat([0.2 0.00005 0.00005],size(int_col_lgth));
%for ext columns: depth_long.bar_shear bar
x0 = [x0_bm x0_ext x0_int];
```

III. Sample Frame-III



```
%Input Datas

global  n  h prf_lim_st  m nb  fcu  fy  E perc_dstn  u_conc bms_wdth...
    bms_lgth u_steel u_dns_reinf  u_form  ext_col_wdth ext_col_lgth...
    int_col_wdth int_col_lgth cor_dsp cor_time  con_cvr dbl ovrstgth...
    vert_amplf gty_load  dbw dcw dcl Pg_ext Pg_int red_fct...
    x0 x0_bm x0_ext x0_int

%Arrangement of input parameters should be from nth_storey -to- 1st_storey

%Choosen performance limit state
%1 for Serviceability; 2 for Damage control; 3 for Collapse prevention
prf_lim_st = 2;

%number of storey
n = 12;
%number of bay
nb = 3;

%beams length
bms_lgth = repmat([4.5,7.5,4.5],n,1);
%beams width
bms_wdth = repmat([0.35,0.35,0.35],n,1);

%exterior column length
ext_col_lgth = [3.5,3.5; 3.5,3.5; 3.5,3.5; 3.5,3.5; 3.5,3.5; 3.5,3.5;...
    3.5,3.5; 3.5,3.5; 3.5,3.5; 3.5,3.5; 3.5,3.5; 4.5,4.5];
ext_col_wdth = repmat([0.3,0.3],n,1);

%interior column length
int_col_lgth = [3.5,3.5; 3.5,3.5; 3.5,3.5; 3.5,3.5; 3.5,3.5; 3.5,3.5;...
    3.5,3.5; 3.5,3.5; 3.5,3.5; 3.5,3.5; 3.5,3.5; 4.5,4.5];
int_col_wdth = repmat([0.3,0.3],n,1);

%mass matrix, 1xn row matrix of each floor significant mass
m = [70, 60, 60, 60, 60, 60, 60, 60, 60, 60, 60,65];

%height matrix, nx1 column matrix with storey height of the frame
h = [43; 39.5; 36; 32.5; 29; 25.5; 22; 18.5; 15; 11.5; 8; 4.5];

%material properties
%reinforcement bars
fy = 450000;%yield strength of longtuidnal reinforcement
E = 200000000;
u_dns_reinf = 7.850;

%diameter of longtiudinal bars
dbl = 0.012;%for beams
dcl = 0.016;%for columns

%diameter of web bars
dbw = 0.008;%for beams
dcw = 0.008;%for columns
```



```
%concrete sections
fcu = 30000;

%concrete cover
con_cvr = 0.03;

%unit costs
u_conc = 3500;%unit cost of concrete
u_steel = 42000; %unit cost of steel
u_form = 210; %unit cost of formwork

%displacement spectra parameters
cor_dsp = 0.75;%corner displacement for 5% damping
cor_time = 4; %corner period for 5% damping

%gravity load on beams
gty_load = zeros(n,nb);

%Probable axial gravity load
Pg_ext = repmat(150,size(ext_col_lgth));%on exterior columns
Pg_int = repmat(300,size(int_col_lgth));%on interior columns

%percentage of redistribution
perc_dstn = 0;
%factors
ovrstgth = 1.6; %overstrengthing factor_neglecting strain hardening
vert_amplf = 1.3; %amplification factor for vertical acceleration
red_fct = 0.9; %columns nominal strength reduction factor

%starting points for the optimization from knowledge of the model
x0_bm = repmat([0.4 0.00002 0.00004],size(bms_lgth));
%for beams: depth_long.bar_shear bar
x0_ext = repmat([0.3 0.00005 0.00005],size(ext_col_lgth));
%for ext columns: depth_long.bar_shear bar
x0_int = repmat([0.3 0.00005 0.00005],size(int_col_lgth));
%for ext columns: depth_long.bar_shear bar
x0 = [x0_bm x0_ext x0_int];
```

5.1.2.2 Outputs

I. Sample Frame – I

Table 5-4: Optimized Design Output for Sample Frame - I

Design Parameters	Storey	Assumed Initial Infeasible Points	Optimized Result	
Beams		**All Span**	**Span1**	**Span2**
Depth (m)	2	0.30	0.30	0.30
	1	0.30	0.30	0.30
Longitudinal Reinforcement (m^2)	2	0.00002	0.00053	0.00053
	1	0.00002	0.001097	0.001097
Shear Reinforcement (m^2)	2	0.00004	0.002061	0.003401
	1	0.00004	0.003436	0.003401
Exterior Columns		**Both**	**1st**	**2nd**
Depth (m)	2	0.20	0.25	0.20
	1	0.20	0.25	0.25
Longitudinal Reinforcement (m^2)	2	0.00005	0.000512	0.0005
	1	0.00005	0.000541	0.000524
Shear Reinforcement (m^2)	2	0.00005	0.001738	0.001762
	1	0.00005	0.005042	0.005131
Interior Column				
Depth (m)	2	0.20	0.30	
	1	0.20	0.30	
Longitudinal Reinforcement (m^2)	2	0.00005	0.000714	
	1	0.00005	0.000736	
Shear Reinforcement (m^2)	2	0.00005	0.004567	
	1	0.00005	0.012859	

II. Sample Frame – II

Table 5-5: Optimized Design Output for Sample Frame – II

Design Parameters	Storey	Assumed Initial Infeasible Points	Optimized Result	
Beams		All Span	Span 1 & 4	Span 2 & 3
Depth (m)	6	0.35	0.35	
	5	0.35	0.35	
	4	0.35	0.35	
	3	0.35	0.35	
	2	0.35	0.35	
	1	0.35	0.35	
Longitudinal Reinforcement (m2)	6	0.00002	0.000556	
	5	0.00002	0.000987	
	4	0.00002	0.001349	
	3	0.00002	0.001633	
	2	0.00002	0.001830	
	1	0.00002	0.001948	
Shear Reinforcement (m2)	6	0.00004	0.003497	0.002348
	5	0.00004	0.003497	0.002348
	4	0.00004	0.003497	0.003758
	3	0.00004	0.003497	0.005401
	2	0.00004	0.004006	0.006833
	1	0.00004	0.004502	0.007671
Exterior Columns				
Depth (m)	6	0.20	0.25	
	5	0.20	0.25	
	4	0.20	0.25	
	3	0.20	0.30	
	2	0.20	0.30	
	1	0.20	0.30	
Longitudinal Reinforcement (m2)	6	0.00005	0.00069	
	5	0.00005	0.000647	
	4	0.00005	0.000728	
	3	0.00005	0.000796	
	2	0.00005	0.000850	
	1	0.00005	0.000772	
Shear Reinforcement (m2)	6	0.00005	0.00455	
	5	0.00005	0.006545	
	4	0.00005	0.008642	
	3	0.00005	0.010580	
	2	0.00005	0.011534	
	1	0.00005	0.014472	

Continued…

Interior Columns		**All**	**1st & 3rd**	**2nd**
Depth (m)	6	0.20	0.30	0.30
	5	0.20	0.30	0.30
	4	0.20	0.35	0.30
	3	0.20	0.35	0.35
	2	0.20	0.40	0.40
	1	0.20	0.35	0.35
Longitudinal Reinforcement (m2)	6	0.00005	0.000853	0.000838
	5	0.00005	0.000789	0.000775
	4	0.00005	0.000910	0.000894
	3	0.00005	0.001010	0.000992
	2	0.00005	0.001089	0.001069
	1	0.00005	0.000957	0.000957
Shear Reinforcement (m2)	6	0.00005	0.010741	
	5	0.00005	0.013556	
	4	0.00005	0.015908	
	3	0.00005	0.017722	
	2	0.00005	0.018901	
	1	0.00005	0.024749	

Table 5-6: Optimized Design Output for Sample Frame - III

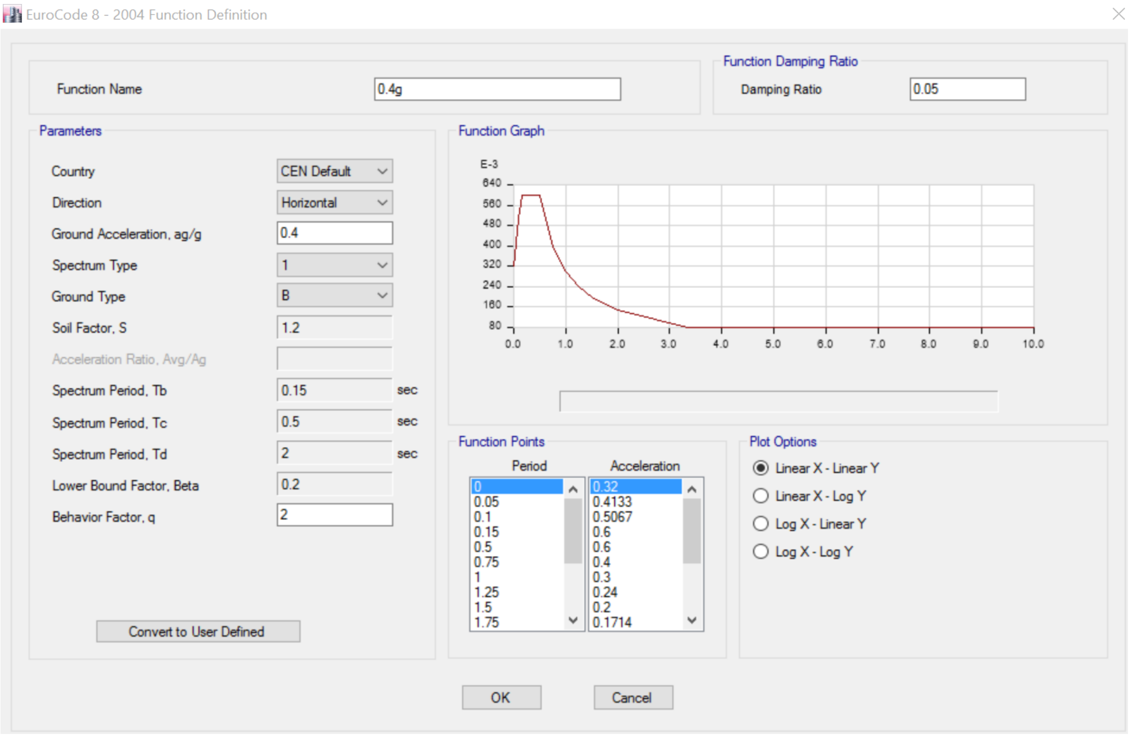
Design Parameters	Storey	Assumed Initial Infeasible Points	Optimized Result	
Beams		**All Span**	**Span 1 & 3**	**Span 2**
Depth (m)	12	0.40	0.40	0.40
	11	0.40	0.40	0.40
	10	0.40	0.40	0.40
	9	0.40	0.40	0.40
	8	0.40	0.40	0.40
	7	0.40	0.40	0.40
	6	0.40	0.40	0.40
	5	0.40	0.40	0.40
	4	0.40	0.40	0.40
	3	0.40	0.40	0.40
	2	0.40	0.40	0.40
	1	0.40	0.40	0.40
Longitudinal Reinforcement (m^2)	12	0.00002	0.000236	0.000236
	11	0.00002	0.000592	0.000592
	10	0.00002	0.000926	0.000926
	9	0.00002	0.001235	0.001235
	8	0.00002	0.001517	0.001517
	7	0.00002	0.001771	0.001771
	6	0.00002	0.001996	0.001996
	5	0.00002	0.002189	0.002189
	4	0.00002	0.002350	0.002350
	3	0.00002	0.002475	0.002475
	2	0.00002	0.002565	0.002565
	1	0.00002	0.002620	0.002620
Shear Reinforcement (m^2)	12	0.00002	0.002406	0.003977
	11	0.00002	0.002406	0.003977
	10	0.00002	0.002406	0.003977
	9	0.00002	0.002450	0.003977
	8	0.00002	0.003355	0.003977
	7	0.00002	0.004324	0.003977
	6	0.00002	0.005315	0.003977
	5	0.00002	0.006282	0.003977
	4	0.00002	0.007172	0.003977
	3	0.00002	0.007930	0.004261
	2	0.00002	0.008505	0.004533
	1	0.00002	0.008878	0.004708

Continued…			
Exterior Columns			
Depth (m)	12	0.30	0.40
	11	0.30	0.40
	10	0.30	0.40
	9	0.30	0.40
	8	0.30	0.45
	7	0.30	0.45
	6	0.30	0.45
	5	0.30	0.50
	4	0.30	0.50
	3	0.30	0.50
	2	0.30	0.50
	1	0.30	0.50
Longitudinal Reinforcement (m^2)	12	0.00002	0.001126
	11	0.00002	0.001079
	10	0.00002	0.001136
	9	0.00002	0.001189
	8	0.00002	0.001238
	7	0.00002	0.001283
	6	0.00002	0.001325
	5	0.00002	0.001363
	4	0.00002	0.001397
	3	0.00002	0.001427
	2	0.00002	0.001452
	1	0.00002	0.001472
Shear Reinforcement (m^2)	12	0.00004	0.008901
	11	0.00004	0.010222
	10	0.00004	0.011450
	9	0.00004	0.012578
	8	0.00004	0.013597
	7	0.00004	0.014497
	6	0.00004	0.015264
	5	0.00004	0.015881
	4	0.00004	0.016320
	3	0.00004	0.016526
	2	0.00004	0.016358
	1	0.00004	0.019582

Continued…			
Interior Columns			
Depth (m)	12	0.30	0.30
	11	0.30	0.30
	10	0.30	0.30
	9	0.30	0.35
	8	0.30	0.35
	7	0.30	0.40
	6	0.30	0.40
	5	0.30	0.45
	4	0.30	0.45
	3	0.30	0.50
	2	0.30	0.50
	1	0.30	0.50
Longitudinal Reinforcement (m^2)	12	0.00005	0.0009
	11	0.00005	0.0009
	10	0.00005	0.0009
	9	0.00005	0.000971
	8	0.00005	0.001058
	7	0.00005	0.001137
	6	0.00005	0.001208
	5	0.00005	0.001271
	4	0.00005	0.001327
	3	0.00005	0.001376
	2	0.00005	0.001417
	1	0.00005	0.001406
Shear Reinforcement (m^2)	12	0.00005	0.012017
	11	0.00005	0.014709
	10	0.00005	0.017225
	9	0.00005	0.019953
	8	0.00005	0.021680
	7	0.00005	0.023592
	6	0.00005	0.025276
	5	0.00005	0.026716
	4	0.00005	0.027895
	3	0.00005	0.028792
	2	0.00005	0.029365
	1	0.00005	0.037899

ETABS Design

As direct displacement-based design is a method for seismic design, a force based seismic design with modal response spectrum analysis by ETABS is used for comparison purpose. Definition of response spectrum function and seismic load cases are shown below. The others input data are taken similar with the optimized design.



EuroCode 8 - 2004 Function Definition

Function Name: 0.4g

Function Damping Ratio

Damping Ratio: 0.05

Parameters

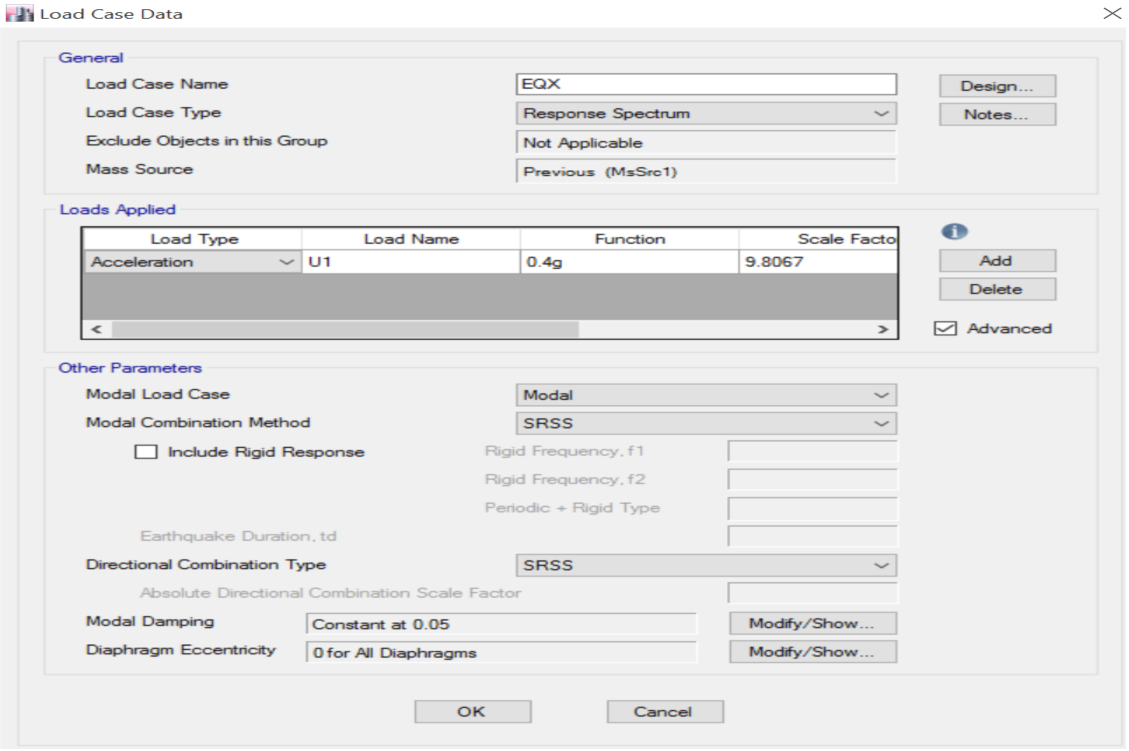
Country: CEN Default
Direction: Horizontal
Ground Acceleration, ag/g: 0.4
Spectrum Type: 1
Ground Type: B
Soil Factor, S: 1.2
Acceleration Ratio, Avg/Ag:
Spectrum Period, Tb: 0.15 sec
Spectrum Period, Tc: 0.5 sec
Spectrum Period, Td: 2 sec
Lower Bound Factor, Beta: 0.2
Behavior Factor, q: 2

Function Points

Period	Acceleration
0	0.32
0.05	0.4133
0.1	0.5067
0.15	0.6
0.5	0.6
0.75	0.4
1	0.3
1.25	0.24
1.5	0.2
1.75	0.1714

Plot Options

● Linear X - Linear Y
○ Linear X - Log Y
○ Log X - Linear Y
○ Log X - Log Y



Load Case Data

General

Load Case Name: EQX
Load Case Type: Response Spectrum
Exclude Objects in this Group: Not Applicable
Mass Source: Previous (MsSrc1)

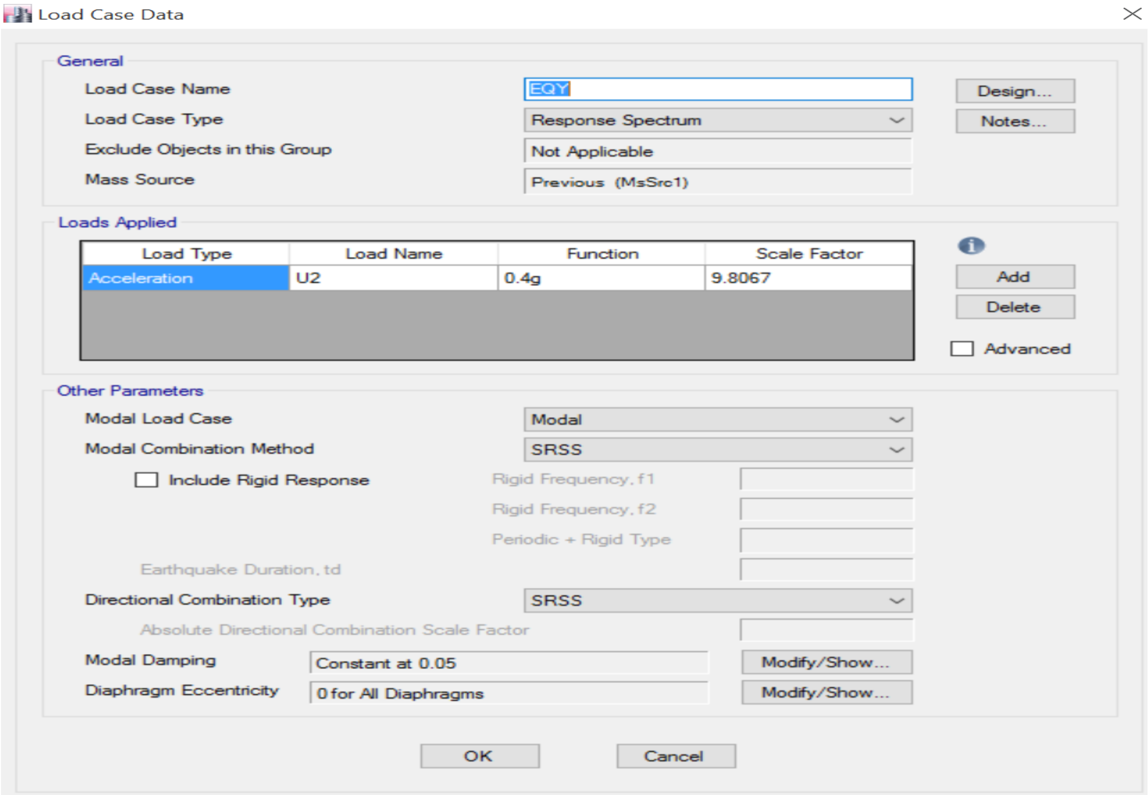
Loads Applied

Load Type	Load Name	Function	Scale Facto
Acceleration	U1	0.4g	9.8067

☑ Advanced

Other Parameters

Modal Load Case: Modal
Modal Combination Method: SRSS
☐ Include Rigid Response
Rigid Frequency, f1
Rigid Frequency, f2
Periodic + Rigid Type
Earthquake Duration, td
Directional Combination Type: SRSS
Absolute Directional Combination Scale Factor
Modal Damping: Constant at 0.05
Diaphragm Eccentricity: 0 for All Diaphragms



General

Load Case Name: EQY

Load Case Type: Response Spectrum

Exclude Objects in this Group: Not Applicable

Mass Source: Previous (MsSrc1)

Loads Applied

Load Type	Load Name	Function	Scale Factor
Acceleration	U2	0.4g	9.8067

☐ Advanced

Other Parameters

Modal Load Case: Modal

Modal Combination Method: SRSS

☐ Include Rigid Response

Rigid Frequency, f1

Rigid Frequency, f2

Periodic + Rigid Type

Earthquake Duration, td

Directional Combination Type: SRSS

Absolute Directional Combination Scale Factor

Modal Damping: Constant at 0.05

Diaphragm Eccentricity: 0 for All Diaphragms

The significant masses of the structure is changed to distributed line loads and applied to the beams as dead loads. Thus, in the seismic analysis the dead load pattern is used as a source of masses.

5.1.2.3 Sample Frame –I Outputs

Table 5-7: ETABS Design Result for Sample Frame – I Beams

Story	Design Section	As Top (m^2)	As Bottom (m^2)	As Total (m^2)	At Shear (m^2/m)	At Shear (m^2)
Story2	B25x25	0.000332	0.000228	0.00056	0.00036	0.00108
Story2	B25x25	0.000692	0.000411	0.001103	0.00038	0.0019
Story1	B30x30	0.000618	0.000507	0.001125	0.00069	0.00207
Story1	B30x30	0.001377	0.000919	0.002296	0.00074	0.0037

Table 5-8: ETABS Design Result for Sample Frame – I Columns

Story	Design Section	As (m^2)	At V Minor (m^2)	At V Minor (m^2)
Story2	C30x30	0.0009	0.00041	0.00123
Story2	C40x40	0.0016	0.00047	0.00141
Story2	C30x30	0.0009	0.00041	0.00123
Story1	C40x40	0.0016	0.00037	0.00148
Story1	C50x50	0.0025	0.00046	0.00184
Story1	C40x40	0.0016	0.00037	0.00148

5.1.2.4 Sample Frame – II Output

Table 5-9: ETABS Design Result for Sample – II Beams

Story	Label	Design Section	As Top (m^2)	As Bottom (m^2)	As Total (m^2)	At Shear (m^2/m)	At Shear (m^2)
Story6	B1, B4	B30x30	0.002787	0.001882	0.00467	0.00094	0.00564
Story6	B2, B3	B30x30	0.001645	0.001085	0.00273	0.00088	0.00352
Story5	B1, B4	B30x30	0.002334	0.001583	0.00392	0.00074	0.00444
Story5	B2, B3	B30x30	0.00116	0.000923	0.00208	0.00071	0.00284
Story4	B1, B4	B40x30	0.001509	0.001038	0.00255	0.00073	0.00438
Story4	B2, B3	B40x30	0.001079	0.001038	0.00212	0.00076	0.00304
Story3	B1, B4	B40x30	0.001488	0.001006	0.00249	0.00075	0.0045
Story3	B2, B3	B40x30	0.001072	0.000595	0.00167	0.00078	0.00312
Story2	B1, B4	B45x45	0.001481	0.000898	0.00238	0.00066	0.00396
Story2	B2, B3	B45x45	0.001182	0.000586	0.00177	0.00075	0.003
Story1	B1, B4	B45x45	0.001434	0.001056	0.00249	0.00072	0.00432
Story1	B2, B3	B45x45	0.001065	0.000623	0.00169	0.00077	0.00308

Table 5-10: ETABS Design Result for Sample – II Exterior Columns

Story	Design Section	As (m^2)	At V Minor (m^2/m)	At V Minor (m^2)
Story6	C40x40	0.0016	0.00045	0.00158
Story5	C45x45	0.002025	0.00053	0.00186
Story4	C50x50	0.0025	0.00064	0.00224
Story3	C60x60	0.0036	0.00093	0.00326
Story2	C60x60	0.0036	0.00102	0.00357
Story1	C70x70	0.0049	0.00102	0.00459

Table 5-11: ETABS Design Result for Sample – II Interior Columns

Story	Design Section	As (m^2)	At V Minor (m^2/m)	At V Minor (m^2)
Story6	C50x50	0.0025	0.00055	0.00193
Story5	C60x60	0.0036	0.0009	0.00315
Story4	C60x60	0.0036	0.00104	0.00364
Story3	C70x70	0.0049	0.00141	0.00494
Story2	C70x70	0.0049	0.00153	0.00536
Story1	C80x80	0.0064	0.00153	0.00689

5.1.2.5 Sample Frame – III Outputs

Table 5-12 : ETABS Design Result for Sample Frame – III Beams

Story	Label	Design Section	As Top (m^2)	As Bottom (m^2)	As Total (m^2)	At Shear(m^2/m)	At Shear (m^2)
Story12	B1, B3	B40x30	0.001949	0.001495	0.003444	0.00234	0.01053
Story12	B2	B40x30	0.003481	0.002378	0.005859	0.00257	0.019275
Story11	B1, B3	B40x30	0.001994	0.001379	0.003373	0.00147	0.006615
Story11	B2	B40x30	0.0033	0.002366	0.005666	0.00151	0.011325
Story10	B1, B3	B40x30	0.0017	0.001268	0.002968	0.00138	0.00621
Story10	B2	B40x30	0.003135	0.002203	0.005338	0.00151	0.011325
Story9	B1, B3	B40x30	0.001767	0.00131	0.003077	0.00138	0.00621
Story9	B2	B40x30	0.003187	0.002258	0.005445	0.00151	0.011325
Story8	B1, B3	B40x30	0.001554	0.001271	0.002825	0.0013	0.00585
Story8	B2	B40x30	0.002996	0.002208	0.005204	0.00145	0.010875
Story7	B1, B3	B40x30	0.001577	0.001292	0.002869	0.0013	0.00585
Story7	B2	B40x30	0.003035	0.002244	0.005279	0.00145	0.010875
Story6	B1, B3	B40x30	0.00155	0.001284	0.002834	0.00129	0.005805
Story6	B2	B40x30	0.003029	0.002239	0.005268	0.00145	0.010875
Story5	B1, B3	B40x30	0.00153	0.001284	0.002814	0.00128	0.00576
Story5	B2	B40x30	0.002873	0.002224	0.005097	0.00139	0.010425
Story4	B1, B3	B40x40	0.001524	0.001291	0.002815	0.00127	0.005715
Story4	B2	B40x40	0.002889	0.002238	0.005127	0.00139	0.010425
Story3	B1, B3	B40x40	0.001335	0.001271	0.002606	0.00119	0.005355
Story3	B2	B40x40	0.002872	0.002222	0.005094	0.00139	0.010425
Story2	B1, B3	B50x50	0.001287	0.001243	0.00253	0.00109	0.004905
Story2	B2	B50x50	0.002806	0.002198	0.005004	0.00115	0.008625
Story1	B1, B3	B50x50	0.001248	0.001348	0.002596	0.00115	0.005175
Story1	B2	B50x50	0.00293	0.002467	0.005397	0.00123	0.009225

Table 5-13: ETABS Design Result for Sample – III Exterior Columns

Story	Design Section	As (m^2)	At V Minor (m^2/m)	At V Minor (m^2)
Story12	C50x50	0.0025	0.00054	0.00189
Story11	C50x50	0.0025	0.00065	0.002275
Story10	C60x60	0.0036	0.00095	0.003325
Story9	C60x60	0.0036	0.00102	0.00357
Story8	C70x70	0.0049	0.00098	0.00343
Story7	C70x70	0.0049	0.00106	0.00371
Story6	C70x70	0.0049	0.00113	0.003955
Story5	C70x70	0.0049	0.00119	0.004165
Story4	C70x70	0.0049	0.00123	0.004305
Story3	C80X80	0.0064	0.00142	0.00497
Story2	C80X80	0.0064	0.00147	0.005145
Story1	C90x90	0.0081	0.00125	0.005625

Table 5-14: ETABS Design Result for Sample – III Interior Columns

Story	Design Section	As (m^2)	At V Minor (m^2/m)	At V Minor (m^2)
Story12	C60x60	0.0036	0.00086	0.00301
Story11	C70x70	0.0049	0.0009	0.00315
Story10	C70x70	0.0049	0.00107	0.003745
Story9	C70x70	0.0049	0.0012	0.0042
Story8	C80X80	0.0064	0.00143	0.005005
Story7	C80X80	0.0064	0.00152	0.00532
Story6	C80X80	0.0064	0.0016	0.0056
Story5	C90x90	0.0081	0.00186	0.00651
Story4	C90x90	0.0081	0.00194	0.00679
Story3	C90x90	0.0081	0.00197	0.006895
Story2	C100x100	0.01	0.0023	0.00805
Story1	C110x100	0.011	0.00189	0.008505

5.2 Comparison

The optimized design achieved around 10% decrease in total cost of the 2-storey frame and approximately 20% for the 6-storey and 12-storey frames while maintaining all constraints violations to a precision of $1x10^{-6}$. That means the design code requirements are met and the frames are safe to withstand the loads triggered by the given seismic condition.

The optimization does not show that much improvement in beams section size; that is because the beam sections are tested and improved prior to beginning the optimization, as the program has to start from positive viscous damping.

The program has an average run time of two minutes for Sample – I frame, six and half minutes for Sample - II and fifteen minutes for Sample - III.

CHAPTER 6 CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

In this paper, an optimal seismic design method for reinforced concrete two-dimensional frames is developed based on direct displacement-based design.

To measure the efficiency of the method, three sample frames are designed. Comparing the design in this study with the result from ETABS, it is found to be economical without compromising strength as it searches for optimal design parameters while making internal response of the structure confirming to the requirements set by design codes.

The obtained result shows that the optimization is more effective for higher frames. That is because it is based on displacement-based design, which is basically for seismic design and in smaller frames seismic effects are lesser.

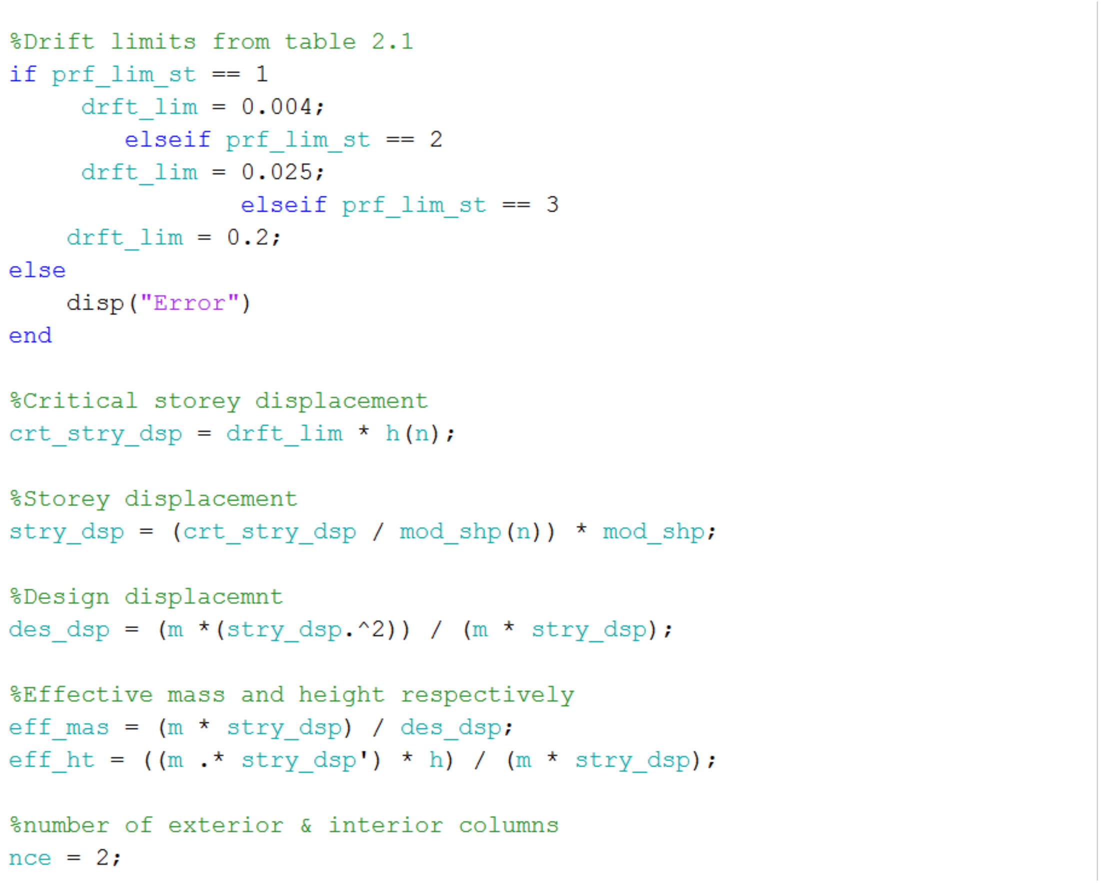
The method developed in this study can also be used for performance-based design since it starts with a predefined performance level as an input.

6.2 Recommendations

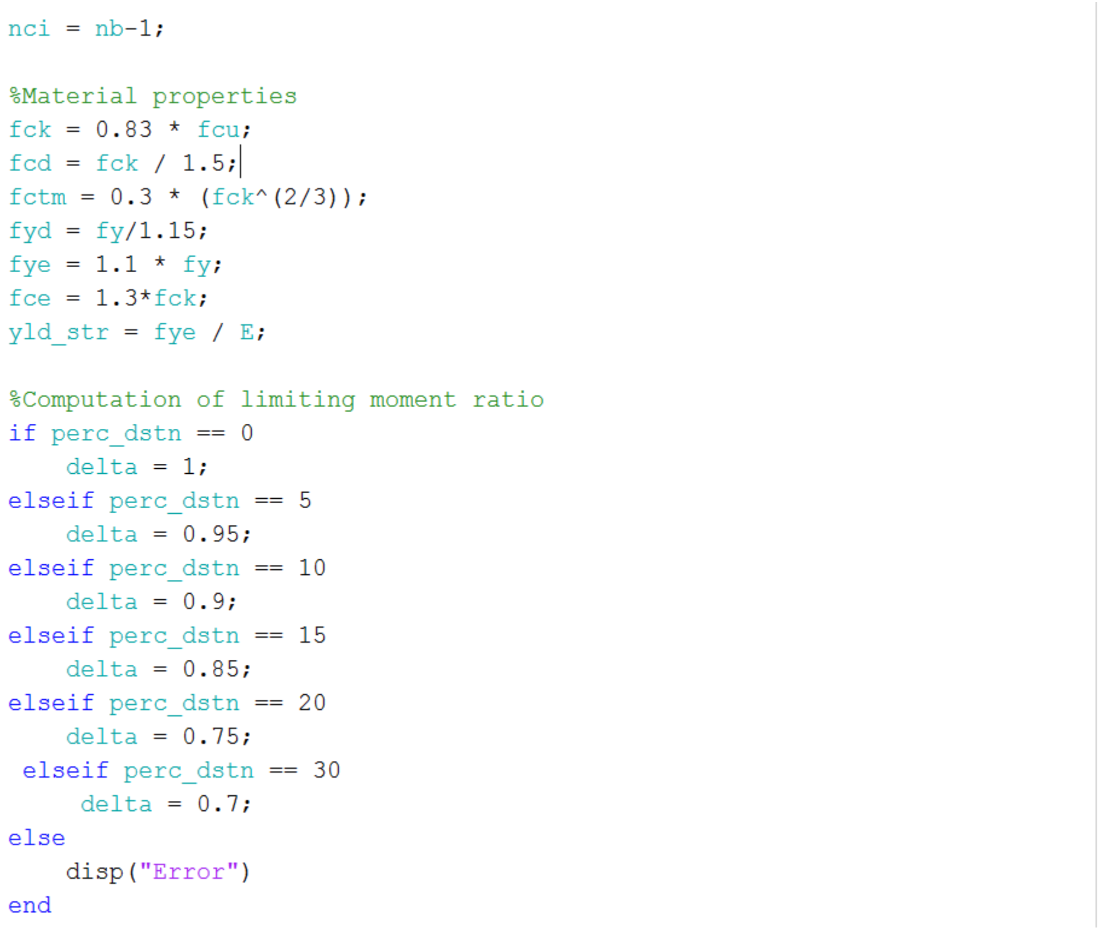
Implementing some modifications, like consideration of torsion and design of slabs, the study can be extended to be applicable to three-dimensional frames. Design of other structures such as structural walls, dual wall-frame systems and bridges can also further studied.

APPENDIX A: SOURCE-CODE FOR THE OPTIMIZATION

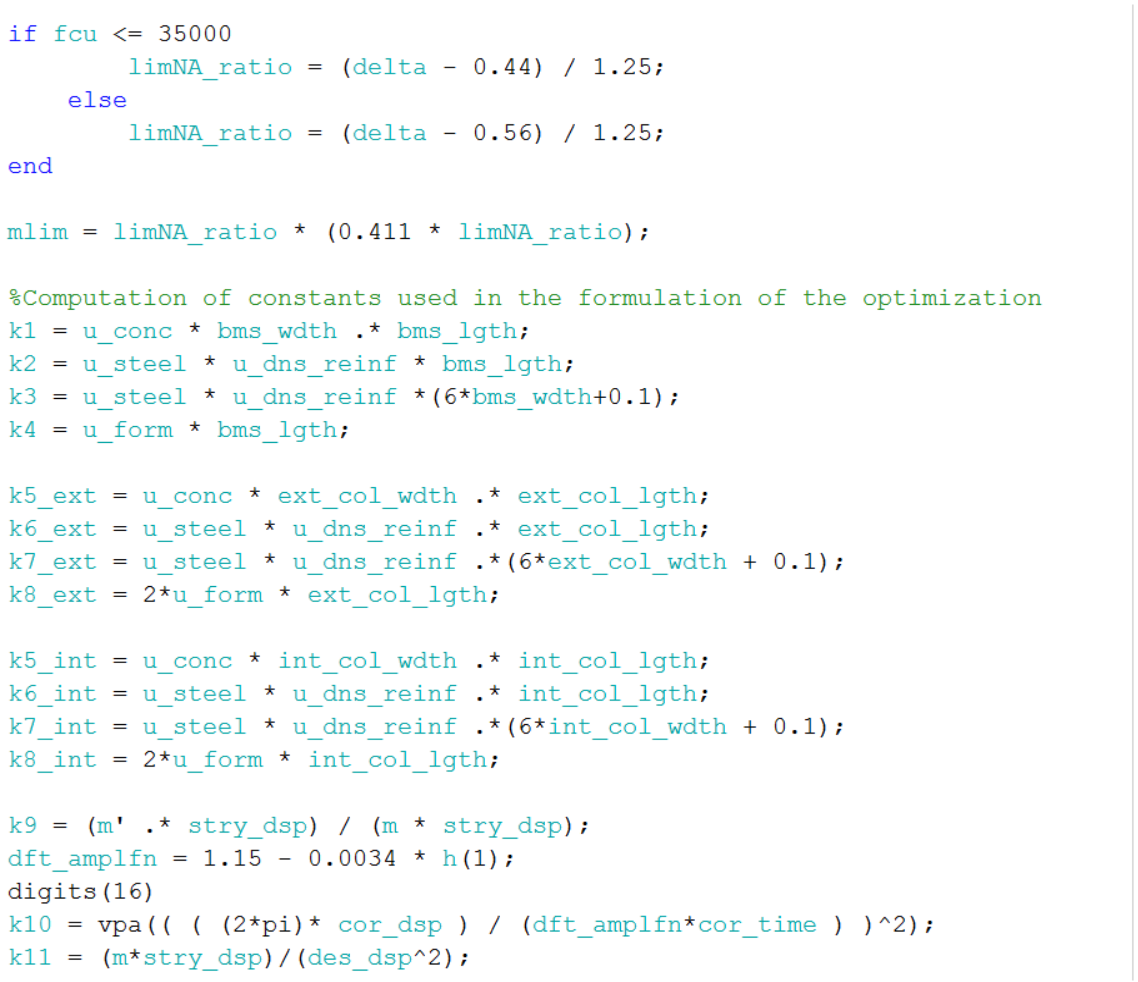
```
%Optimization Tool for 2D RC Frames Based on DDBD

%Indexing of matrix element is in reverse order to be consistent with input

global  n mod_shp h prf_lim_st drft_lim crt_stry_dsp stry_dsp des_dsp m...
    eff_mas eff_ht nce nci nb fck fcu fcd fctm fyd fy fye fce yld_str E...
    perc_dstn delta limNA_ratio mlim k1 u_conc bms_wdth  bms_lgth k2 ...
    u_steel u_dns_reinf k3 k4 u_form k5_ext ext_col_wdth ext_col_lgth...
    k6_ext k7_ext k8_ext k5_int int_col_wdth int_col_lgth k6_int k7_int...
    k8_int k9 dft_amplfn k10 cor_dsp cor_time k11 k12 k13 k15 con_cvr...
    dbl k16 k17 ovrstgth k18 vert_amplf gty_load k19 k20 k21 dbw k22 k23...
    k24 k25 k27 k28 k29_ext k31 k32_ext k33_ext k34 k35 dcw k36_ext...
    k37_ext k38_ext k39_ext k29_int k32_int k33_int k36_int k37_int...
    k38_int k39_int k40 dcl fun lb opts r s t x_start ...
    Pg_ext Pg_int red_fct des_drft yld_drft...
    dsp_dcty k26 vis_damp eqv_vis_damp evd_red_fct Vbase Ft F T ...
    ovr_trn_mom Vs Vb_R Vb theta_bm  coteta_bm Vwd_bm ko_bm k_bm Vrdc_bm...
    ovrstgth_dtlty_p ovrstgth_dtlty wf k30 ko_ext k_ext Vrdc_ext...
    theta_ext coteta_ext Vwd_ext T_ext Vb_ext bms_lgth_ext M_des_ext ...
    ko_int k_int Vrdc_int theta_int coteta_int Vwd_int Vb_int...
    bms_lgth_int M_des_int  v1 v2 v3 v4 v5 v6 v7 v8 v9 v10 v12...
    v13 v14 v15 v16 fv1_x1 fv1_x2 fv1_x3 fv2_x1 fv2_x2 fv2_x3 fv3_x1...
    fv3_x2 fv3_x3 fv4_x1 fv4_x2 fv4_x3 fv5_x1 fv5_x2 fv5_x3 fv_x1 fv_x2...
    fv_x3 fv6_x4 fv6_x5 fv6_x6 fv7_x4 fv7_x5 fv7_x6 fv8_x4 fv8_x5 fv8_x6...
    fv9_x4 fv9_x5 fv9_x6 fv10_x4 fv10_x5 fv10_x6 fv_x4 fv_x5 fv_x6...
    fv12_x7 fv12_x8 fv12_x9 fv13_x7 fv13_x8 fv13_x9 fv14_x7 fv14_x8...
    fv14_x9 fv15_x7 fv15_x8 fv15_x9 fv16_x7 fv16_x8 fv16_x9 fv_x7 fv_x8...
    fv_x9 c1 c2 c3 c4 c5 c6 c7 c8 c9 c10 c12 c13 c14 c15 c16 grad1x1...
    grad1x2 grad1x3 grad2x1 grad2x2 grad2x3 grad3x1 grad3x2 grad3x3...
    grad4x1 grad4x2 grad4x3 grad5x1 grad5x2 grad5x3  grad6x4 grad6x5...
    grad6x6 grad7x4 grad7x5 grad7x6  grad8x4 grad8x5 grad8x6 grad9x4...
    grad9x5 grad9x6 grad10x4 grad10x5 grad10x6 grad12x7 grad12x8...
    grad12x9 grad13x7 grad13x8 grad13x9 grad14x7 grad14x8 grad14x9...
    grad15x7 grad15x8 grad15x9 grad16x7 grad16x8 grad16x9  grad1  grad2...
    grad3 grad4 grad5 grad6 kx grad7 grad8 grad9 grad10 grad12 grad13...
    grad14 grad15 grad16  v v_bm v_ext v_int consensus_vct_bm f...
    mag1 mag2 mag3 mag4 mag5 mag6 mag7 mag8 mag9 mag10 mag12 mag13 mag14...
    mag15 mag16 consensus_vct_ext consensus_vct_int consensus_vct x0

run(uigetfile("*.m",""," Pick the data-file "))

%Equations used here are avaliable in the Appendices of the thesis

%Calculation of mode shape
if n > 4
    mod_shp = (1/(3*h(1)))*((4*h)-((1/h(1))*(h.^2)));
        else
     mod_shp = (1/h(1))* h;
end
```



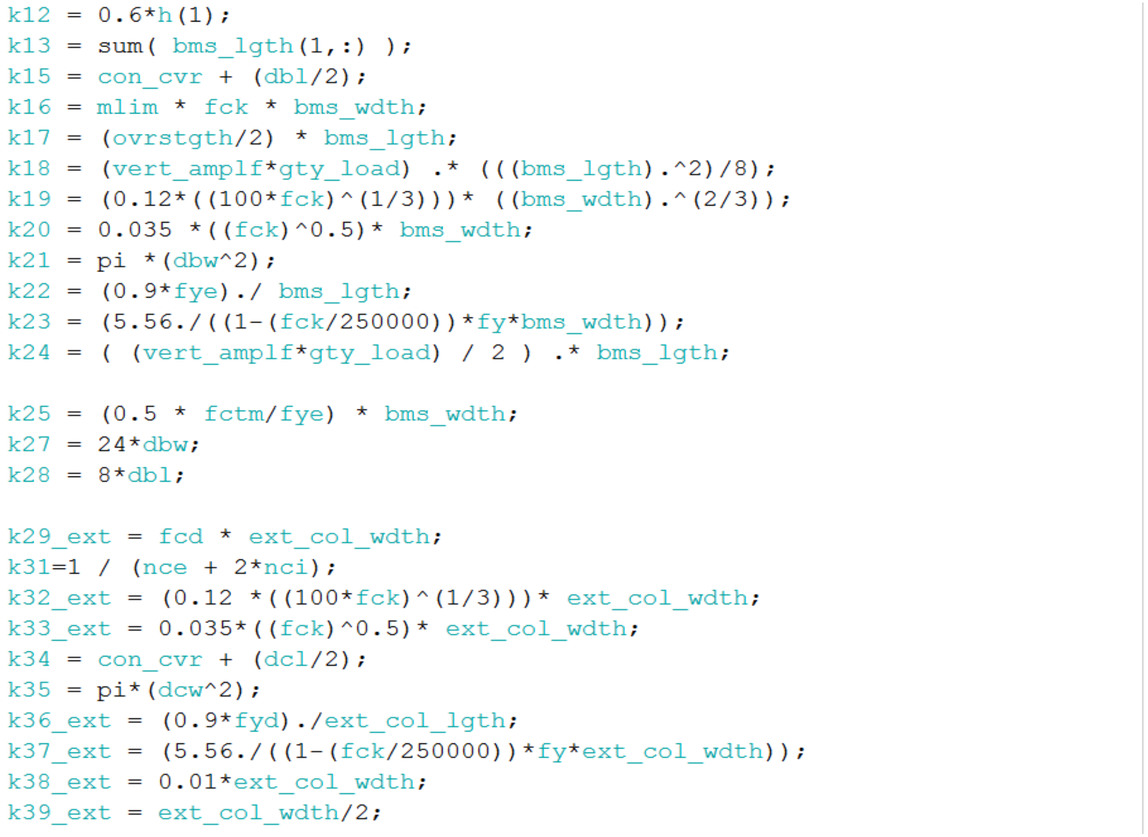
```
%Drift limits from table 2.1
if prf_lim_st == 1
     drft_lim = 0.004;
        elseif prf_lim_st == 2
     drft_lim = 0.025;
                elseif prf_lim_st == 3
    drft_lim = 0.2;
else
    disp("Error")
end

%Critical storey displacement
crt_stry_dsp = drft_lim * h(n);

%Storey displacement
stry_dsp = (crt_stry_dsp / mod_shp(n)) * mod_shp;

%Design displacemnt
des_dsp = (m *(stry_dsp.^2)) / (m * stry_dsp);

%Effective mass and height respectively
eff_mas = (m * stry_dsp) / des_dsp;
eff_ht = ((m .* stry_dsp') * h) / (m * stry_dsp);

%number of exterior & interior columns
nce = 2;
```



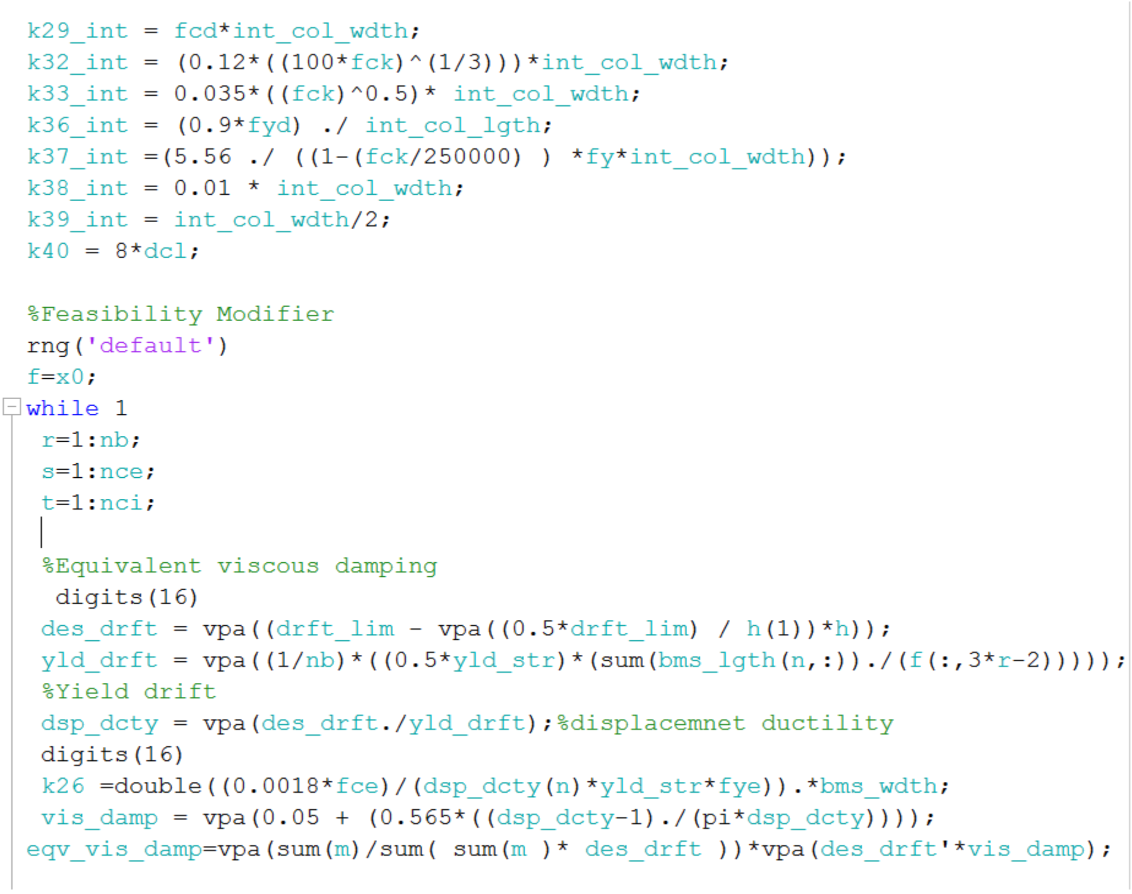
```
nci = nb-1;

%Material properties
fck = 0.83 * fcu;
fcd = fck / 1.5;
fctm = 0.3 * (fck^(2/3));
fyd = fy/1.15;
fye = 1.1 * fy;
fce = 1.3*fck;
yld_str = fye / E;

%Computation of limiting moment ratio
if perc_dstn == 0
    delta = 1;
elseif perc_dstn == 5
    delta = 0.95;
elseif perc_dstn == 10
    delta = 0.9;
elseif perc_dstn == 15
    delta = 0.85;
elseif perc_dstn == 20
    delta = 0.75;
 elseif perc_dstn == 30
     delta = 0.7;
else
    disp("Error")
end
```



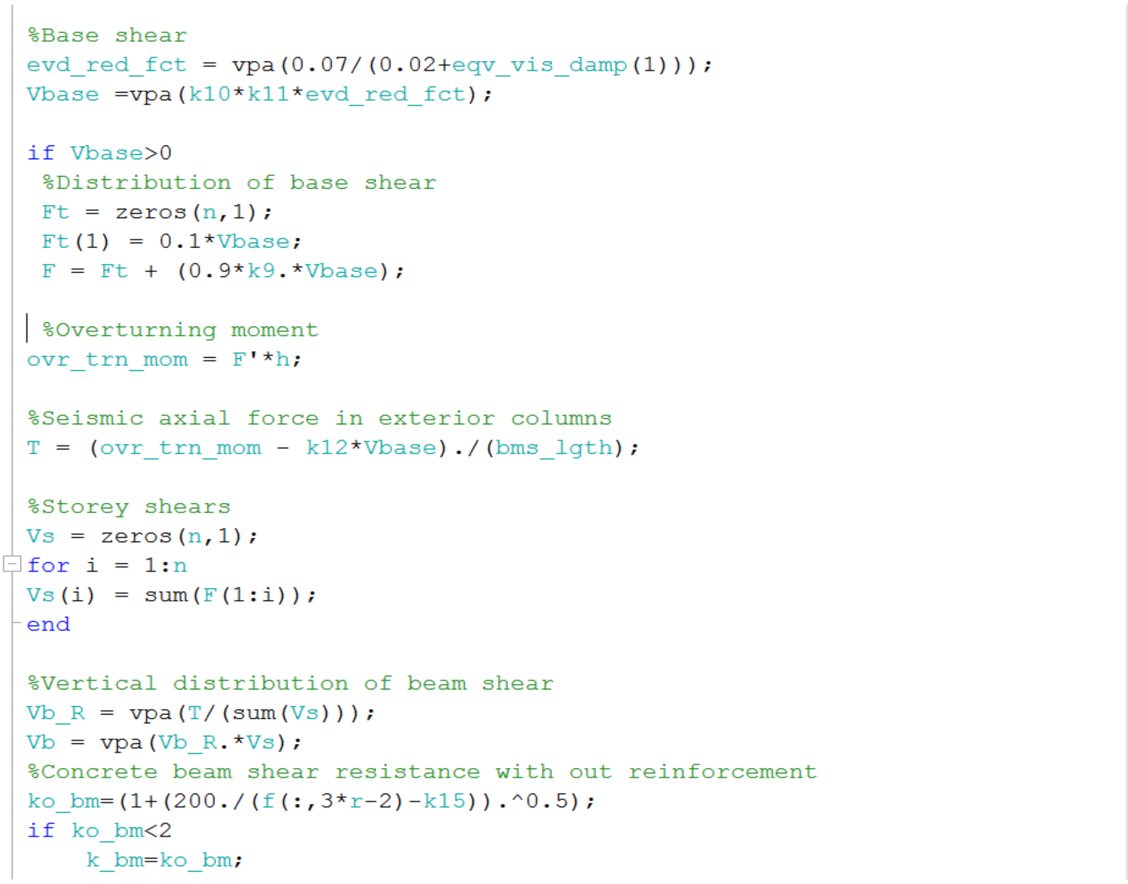
```
if fcu <= 35000
        limNA_ratio = (delta - 0.44) / 1.25;
    else
        limNA_ratio = (delta - 0.56) / 1.25;
end

mlim = limNA_ratio * (0.411 * limNA_ratio);

%Computation of constants used in the formulation of the optimization
k1 = u_conc * bms_wdth .* bms_lgth;
k2 = u_steel * u_dns_reinf * bms_lgth;
k3 = u_steel * u_dns_reinf *(6*bms_wdth+0.1);
k4 = u_form * bms_lgth;

k5_ext = u_conc * ext_col_wdth .* ext_col_lgth;
k6_ext = u_steel * u_dns_reinf .* ext_col_lgth;
k7_ext = u_steel * u_dns_reinf .*(6*ext_col_wdth + 0.1);
k8_ext = 2*u_form * ext_col_lgth;

k5_int = u_conc * int_col_wdth .* int_col_lgth;
k6_int = u_steel * u_dns_reinf .* int_col_lgth;
k7_int = u_steel * u_dns_reinf .*(6*int_col_wdth + 0.1);
k8_int = 2*u_form * int_col_lgth;

k9 = (m' .* stry_dsp) / (m * stry_dsp);
dft_amplfn = 1.15 - 0.0034 * h(1);
digits(16)
k10 = vpa(( ( (2*pi)* cor_dsp ) / (dft_amplfn*cor_time ) )^2);
k11 = (m*stry_dsp)/(des_dsp^2);
```



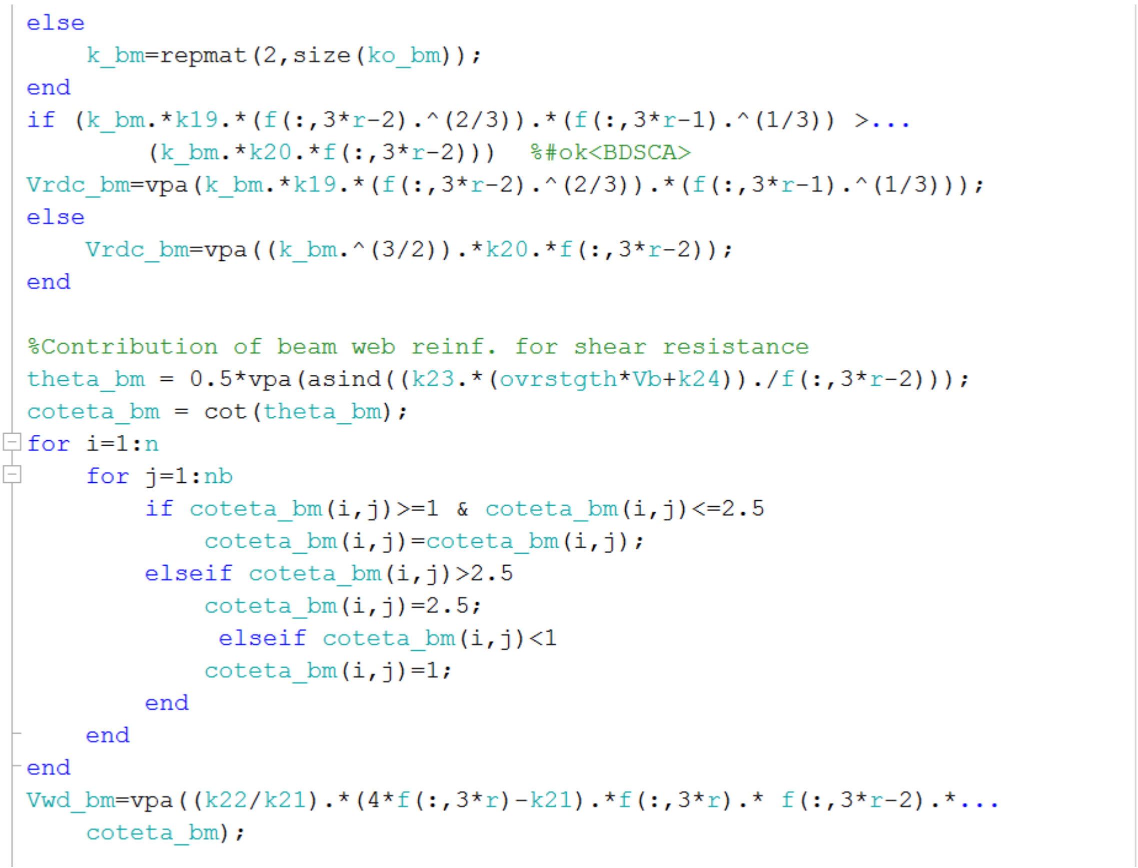
```
k12 = 0.6*h(1);
k13 = sum( bms_lgth(1,:) );
k15 = con_cvr + (dbl/2);
k16 = mlim * fck * bms_wdth;
k17 = (ovrstgth/2) * bms_lgth;
k18 = (vert_amplf*gty_load) .* (((bms_lgth).^2)/8);
k19 = (0.12*((100*fck)^(1/3)))* ((bms_wdth).^(2/3));
k20 = 0.035 *((fck)^0.5)* bms_wdth;
k21 = pi *(dbw^2);
k22 = (0.9*fye)./ bms_lgth;
k23 = (5.56./((1-(fck/250000))*fy*bms_wdth));
k24 = ( (vert_amplf*gty_load) / 2 ) .* bms_lgth;

k25 = (0.5 * fctm/fye) * bms_wdth;
k27 = 24*dbw;
k28 = 8*dbl;

k29_ext = fcd * ext_col_wdth;
k31=1 / (nce + 2*nci);
k32_ext = (0.12 *((100*fck)^(1/3)))* ext_col_wdth;
k33_ext = 0.035*((fck)^0.5)* ext_col_wdth;
k34 = con_cvr + (dcl/2);
k35 = pi*(dcw^2);
k36_ext = (0.9*fyd)./ext_col_lgth;
k37_ext = (5.56./((1-(fck/250000))*fy*ext_col_wdth));
k38_ext = 0.01*ext_col_wdth;
k39_ext = ext_col_wdth/2;
```



```
k29_int = fcd*int_col_wdth;
k32_int = (0.12*((100*fck)^(1/3)))*int_col_wdth;
k33_int = 0.035*((fck)^0.5)* int_col_wdth;
k36_int = (0.9*fyd) ./ int_col_lgth;
k37_int =(5.56 ./ ((1-(fck/250000) ) *fy*int_col_wdth));
k38_int = 0.01 * int_col_wdth;
k39_int = int_col_wdth/2;
k40 = 8*dcl;

%Feasibility Modifier
rng('default')
f=x0;
while 1
 r=1:nb;
 s=1:nce;
 t=1:nci;

 %Equivalent viscous damping
  digits(16)
 des_drft = vpa((drft_lim - vpa((0.5*drft_lim) / h(1))*h));
 yld_drft = vpa((1/nb)*((0.5*yld_str)*(sum(bms_lgth(n,:))./(f(:,3*r-2)))));
 %Yield drift
 dsp_dcty = vpa(des_drft./yld_drft);%displacemnet ductility
 digits(16)
 k26 =double((0.0018*fce)/(dsp_dcty(n)*yld_str*fye)).*bms_wdth;
 vis_damp = vpa(0.05 + (0.565*((dsp_dcty-1)./(pi*dsp_dcty))));
eqv_vis_damp=vpa(sum(m)/sum( sum(m )* des_drft ))*vpa(des_drft'*vis_damp);
```



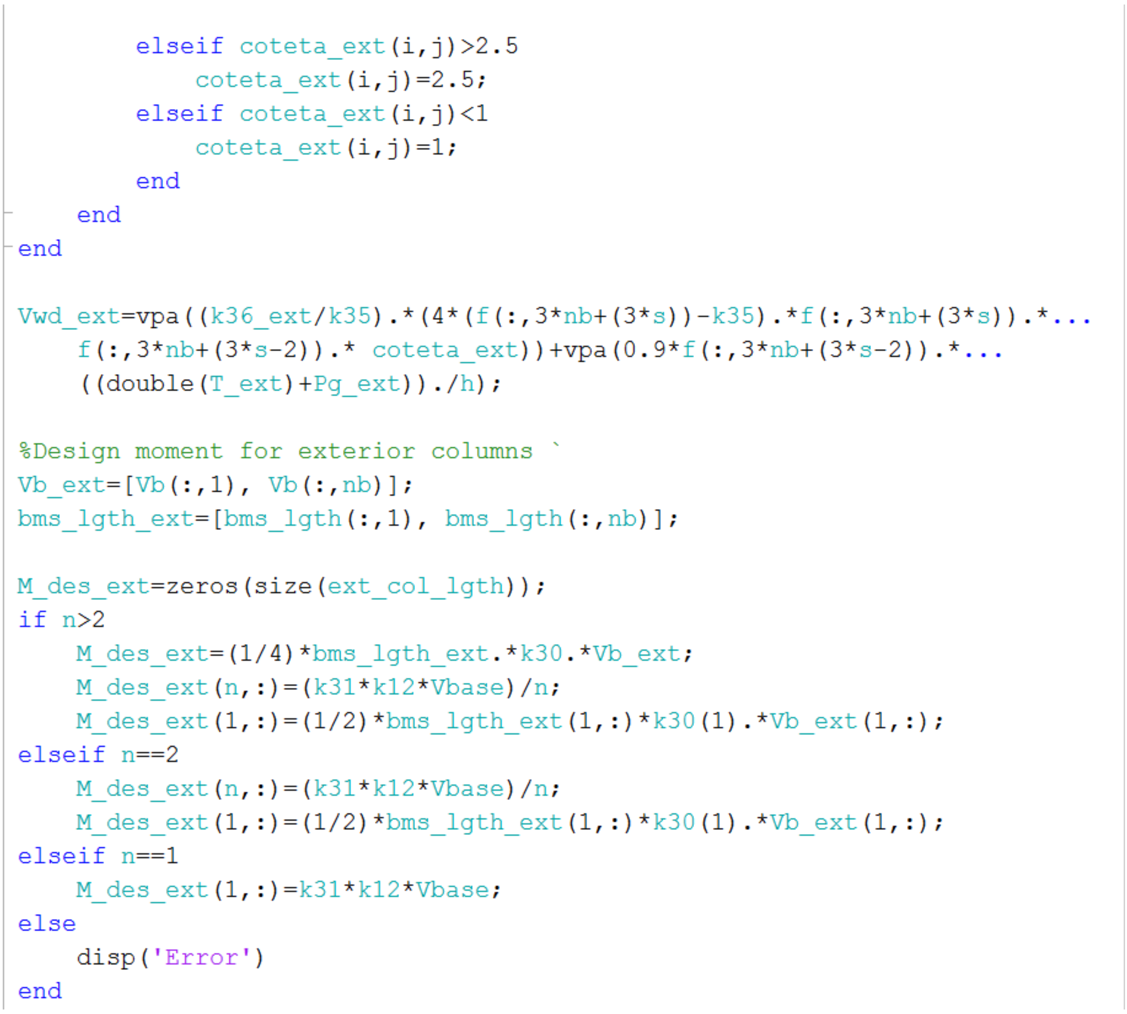
```
%Base shear
evd_red_fct = vpa(0.07/(0.02+eqv_vis_damp(1)));
Vbase =vpa(k10*k11*evd_red_fct);

if Vbase>0
 %Distribution of base shear
 Ft = zeros(n,1);
 Ft(1) = 0.1*Vbase;
 F = Ft + (0.9*k9.*Vbase);

 %Overturning moment
ovr_trn_mom = F'*h;

%Seismic axial force in exterior columns
T = (ovr_trn_mom - k12*Vbase)./(bms_lgth);

%Storey shears
Vs = zeros(n,1);
for i = 1:n
Vs(i) = sum(F(1:i));
end

%Vertical distribution of beam shear
Vb_R = vpa(T/(sum(Vs)));
Vb = vpa(Vb_R.*Vs);
%Concrete beam shear resistance with out reinforcement
ko_bm=(1+(200./(f(:,3*r-2)-k15)).^0.5);
if ko_bm<2
    k_bm=ko_bm;
```



```
else
    k_bm=repmat(2,size(ko_bm));
end
if (k_bm.*k19.*(f(:,3*r-2).^(2/3)).*(f(:,3*r-1).^(1/3)) >...
        (k_bm.*k20.*f(:,3*r-2)))  %#ok<BDSCA>
Vrdc_bm=vpa(k_bm.*k19.*(f(:,3*r-2).^(2/3)).*(f(:,3*r-1).^(1/3)));
else
    Vrdc_bm=vpa((k_bm.^(3/2)).*k20.*f(:,3*r-2));
end

%Contribution of beam web reinf. for shear resistance
theta_bm = 0.5*vpa(asind((k23.*(ovrstgth*Vb+k24))./f(:,3*r-2)));
coteta_bm = cot(theta_bm);
for i=1:n
    for j=1:nb
        if coteta_bm(i,j)>=1 & coteta_bm(i,j)<=2.5
            coteta_bm(i,j)=coteta_bm(i,j);
        elseif coteta_bm(i,j)>2.5
            coteta_bm(i,j)=2.5;
             elseif coteta_bm(i,j)<1
            coteta_bm(i,j)=1;
        end
    end
end
Vwd_bm=vpa((k22/k21).*(4*f(:,3*r)-k21).*f(:,3*r).* f(:,3*r-2).*...
    coteta_bm);
```

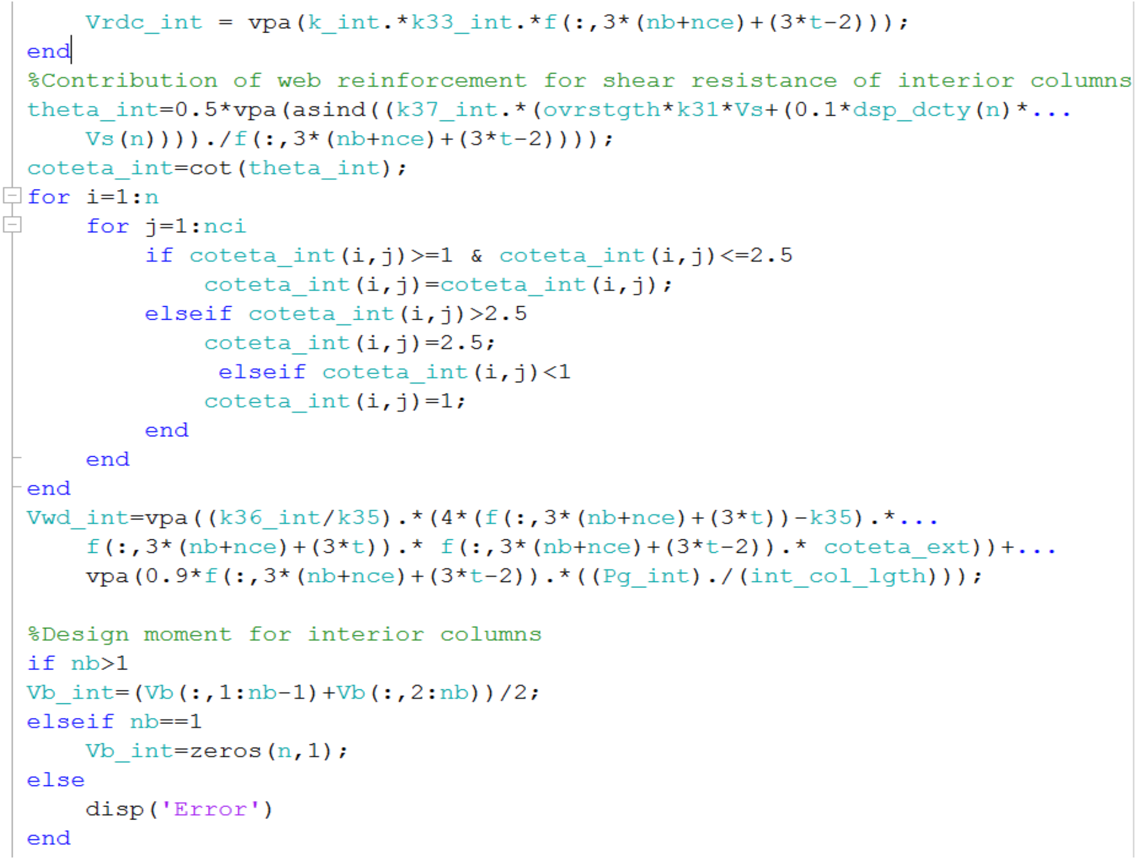
```
%Capacity constraints for beams
c1 =vpa( k17.*Vb+k18)-vpa(k16.*((f(:,3*r-2)-k15).^2))-vpa(0.9*...
    fye*f(:,3*r-2).*f(:,3*r-1));%flexural constraint
c2 = vpa(ovrstgth*Vb+k24)-vpa(Vrdc_bm+Vwd_bm);%shear constrait

%Reinforcement bounds for beams
c3 = vpa(f(:,3*r-1)) - vpa(k26.*f(:,3*r-2));

c4 = vpa(k25.*f(:,3*r-2)) - vpa(f(:,3*r-1));

kx=f(:,3*r-2)/4;
c5 = vpa((k21*bms_lgth)./(4*f(:,3*r)-k21))-vpa(min(min(min(min(kx)),k27),...
    min(0.225,k28)));

%Overstrengthn ductility
ovrstgth_dtlty_p = dsp_dcty/((2^0.5)*ovrstgth);
if vpa(ovrstgth_dtlty_p)> 1
    ovrstgth_dtlty = ovrstgth_dtlty_p;
else
    ovrstgth_dtlty = 1;
end

%Daynamic amplification factor
wf = zeros(size(h));
for i=h
    if h<0.75*h(1)
        wf=1.15+(0.13.*(ovrstgth_dtlty-1));
    else
        wf=1-((h(1)-h).*(1-(1.15+(0.13*(ovrstgth_dtlty-1)))))/(0.25*h(1));
    end
end
```

```
k30 =ovrstgth*wf;

T_ext=[T(:,1), T(:,nb)];

%Concrete shear resistance without reinforcement for exterior columns
ko_ext=(1+(200./(f(:,3*nb+(3*s-2))-k34)).^0.5);
if ko_ext<2
    k_ext=ko_ext;
else
    k_ext=repmat(2,size(ko_ext));
end
digits(16)
if vpa(k_ext.*k32_ext.*(f(:,3*nb+(3*s-2)).^(2/3)).*(f(:,3*nb+(3*s-1)).^...
        (1/3))) > vpa(k_ext.*k33_ext.* f(:,3*nb+(3*s-2)))
Vrdc_ext = vpa(k_ext.*k32_ext.*(f(:,3*nb+(3*s-2)).^(2/3)).*...
    (f(:,3*nb+(3*s-1)).^(1/3)));
else
    Vrdc_ext = vpa((k_ext).^(3/2).*k33_ext.*f(:,3*nb+(3*s-2)));
end

%Contribution of web reinforcement for shear resistance of exterior columns
theta_ext=0.5*vpa(asind((k37_ext.*(ovrstgth*k31*Vs+(0.1*dsp_dcty(n)*...
    Vs(n)))))./f(:,1)));
coteta_ext=cot(theta_ext);
for i=1:n
    for j=1:nce
        if coteta_ext(i,j)>=1 & coteta_ext(i,j)<=2.5
            coteta_ext(i,j)=coteta_ext(i,j);
        elseif coteta_ext(i,j)>2.5
            coteta_ext(i,j)=2.5;
```

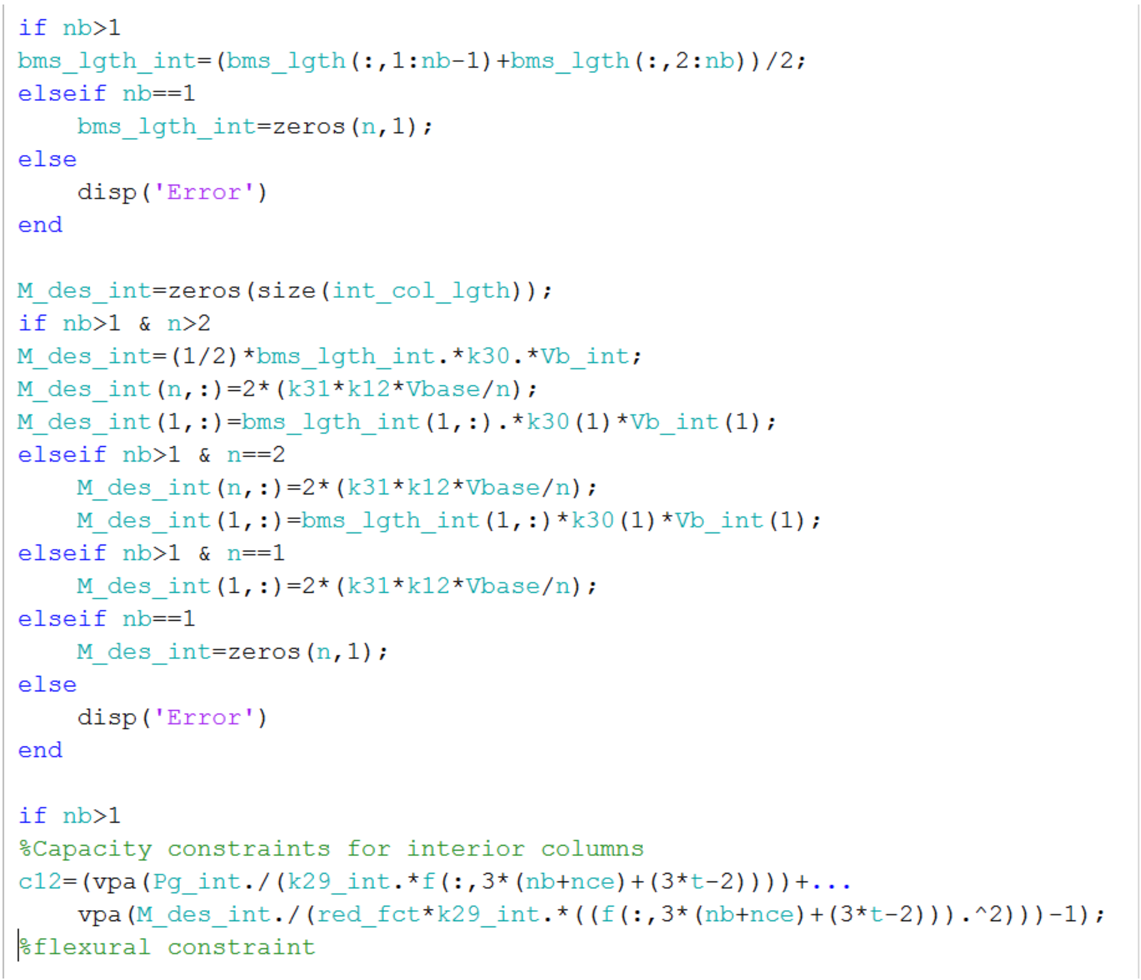


```
        elseif coteta_ext(i,j)>2.5
            coteta_ext(i,j)=2.5;
        elseif coteta_ext(i,j)<1
            coteta_ext(i,j)=1;
        end
    end
end

Vwd_ext=vpa((k36_ext/k35).*(4*(f(:,3*nb+(3*s))-k35).*f(:,3*nb+(3*s)).*...
    f(:,3*nb+(3*s-2)).* coteta_ext))+vpa(0.9*f(:,3*nb+(3*s-2)).*...
    ((double(T_ext)+Pg_ext))./h);

%Design moment for exterior columns `
Vb_ext=[Vb(:,1), Vb(:,nb)];
bms_lgth_ext=[bms_lgth(:,1), bms_lgth(:,nb)];

M_des_ext=zeros(size(ext_col_lgth));
if n>2
    M_des_ext=(1/4)*bms_lgth_ext.*k30.*Vb_ext;
    M_des_ext(n,:)=(k31*k12*Vbase)/n;
    M_des_ext(1,:)=(1/2)*bms_lgth_ext(1,:)*k30(1).*Vb_ext(1,:);
elseif n==2
    M_des_ext(n,:)=(k31*k12*Vbase)/n;
    M_des_ext(1,:)=(1/2)*bms_lgth_ext(1,:)*k30(1).*Vb_ext(1,:);
elseif n==1
    M_des_ext(1,:)=k31*k12*Vbase;
else
    disp('Error')
end
```

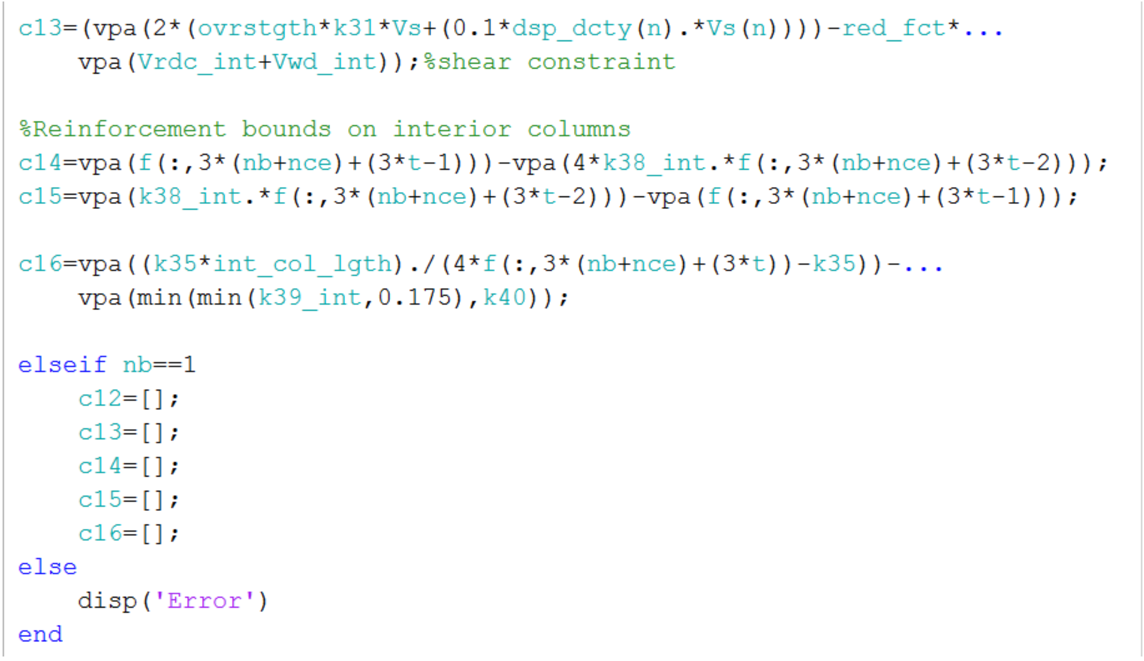
```
%Capacity constraints for exterior columns
c6=(vpa((double(T_ext)+Pg_ext)./(k29_ext.*f(:,3*nb+(3*s-2))))+...
    vpa(M_des_ext./(red_fct*k29_ext.*((f(:,3*nb+(3*s-2))).^2)))-1);
    %flexural constraint

c7=(ovrstgth*k31*Vs+(0.1*dsp_dcty(n)*Vs(n)))-red_fct*vpa(Vrdc_ext+Vwd_ext);
%shear constraint

%Reinforcement bounds for exterior columns
c8=vpa(f(:,3*nb+(3*s-1)))-vpa(4*k38_ext.*f(:,3*nb+(3*s-2)));

c9=vpa(k38_ext.*f(:,3*nb+(3*s-2)))-vpa(f(:,3*nb+(3*s-1)));

c10=vpa(((k35*ext_col_lgth)./(4*f(:,3*nb+(3*s))-k35)))-...
    vpa((min(min(k39_ext,0.175),k40)));

%Concrete shear resistance without reinforcement for interior columns
ko_int=(1+(200./(f(:,3*(nb+nce)+(3*t-2))-k34)).^0.5);
if ko_int<2
    k_int=ko_int;
else
    k_int=repmat(2,size(ko_int));
end
digits(16)
if vpa(k_int.*k32_int.*(f(:,3*(nb+nce)+(3*t-2)).^(2/3)).*...
        (f(:,3*(nb+nce)+(3*t-1)).^(1/3))) > vpa(k_int.*k33_int.*...
        f(:,3*(nb+nce)+(3*t-2)))
Vrdc_int = vpa(k_int.*k32_int.*(f(:,3*(nb+nce)+(3*t-2)).^(2/3)).*...
    (f(:,3*(nb+nce)+(3*t-1)).^(1/3)));
else
```



```
    Vrdc_int = vpa(k_int.*k33_int.*f(:,3*(nb+nce)+(3*t-2)));
end
%Contribution of web reinforcement for shear resistance of interior columns
theta_int=0.5*vpa(asind((k37_int.*(ovrstgth*k31*Vs+(0.1*dsp_dcty(n)*...
    Vs(n))))./f(:,3*(nb+nce)+(3*t-2))));
coteta_int=cot(theta_int);
for i=1:n
    for j=1:nci
        if coteta_int(i,j)>=1 & coteta_int(i,j)<=2.5
            coteta_int(i,j)=coteta_int(i,j);
        elseif coteta_int(i,j)>2.5
            coteta_int(i,j)=2.5;
             elseif coteta_int(i,j)<1
            coteta_int(i,j)=1;
        end
    end
end
Vwd_int=vpa((k36_int/k35).*(4*(f(:,3*(nb+nce)+(3*t))-k35).*...
    f(:,3*(nb+nce)+(3*t)).* f(:,3*(nb+nce)+(3*t-2)).* coteta_ext))+...
    vpa(0.9*f(:,3*(nb+nce)+(3*t-2)).*((Pg_int)./(int_col_lgth)));

%Design moment for interior columns
if nb>1
Vb_int=(Vb(:,1:nb-1)+Vb(:,2:nb))/2;
elseif nb==1
    Vb_int=zeros(n,1);
else
    disp('Error')
end
```



```
if nb>1
bms_lgth_int=(bms_lgth(:,1:nb-1)+bms_lgth(:,2:nb))/2;
elseif nb==1
    bms_lgth_int=zeros(n,1);
else
    disp('Error')
end

M_des_int=zeros(size(int_col_lgth));
if nb>1 & n>2
M_des_int=(1/2)*bms_lgth_int.*k30.*Vb_int;
M_des_int(n,:)=2*(k31*k12*Vbase/n);
M_des_int(1,:)=bms_lgth_int(1,:).*k30(1)*Vb_int(1);
elseif nb>1 & n==2
    M_des_int(n,:)=2*(k31*k12*Vbase/n);
    M_des_int(1,:)=bms_lgth_int(1,:)*k30(1)*Vb_int(1);
elseif nb>1 & n==1
    M_des_int(1,:)=2*(k31*k12*Vbase/n);
elseif nb==1
    M_des_int=zeros(n,1);
else
    disp('Error')
end

if nb>1
%Capacity constraints for interior columns
c12=(vpa(Pg_int./(k29_int.*f(:,3*(nb+nce)+(3*t-2))))+...
    vpa(M_des_int./(red_fct*k29_int.*((f(:,3*(nb+nce)+(3*t-2))).^2)))-1);
%flexural constraint
```



```
c13=(vpa(2*(ovrstgth*k31*Vs+(0.1*dsp_dcty(n).*Vs(n))))-red_fct*...
    vpa(Vrdc_int+Vwd_int));%shear constraint

%Reinforcement bounds on interior columns
c14=vpa(f(:,3*(nb+nce)+(3*t-1)))-vpa(4*k38_int.*f(:,3*(nb+nce)+(3*t-2)));
c15=vpa(k38_int.*f(:,3*(nb+nce)+(3*t-2)))-vpa(f(:,3*(nb+nce)+(3*t-1)));

c16=vpa((k35*int_col_lgth)./(4*f(:,3*(nb+nce)+(3*t))-k35))-...
    vpa(min(min(k39_int,0.175),k40));

elseif nb==1
    c12=[];
    c13=[];
    c14=[];
    c15=[];
    c16=[];
else
    disp('Error')
end
```

```
%Gradient
grad1x1= -vpa(k16.*(2*(f(:,(3*r-2))-k15))+0.9*fye*f(:,(3*r-1)));
grad1x2 = -vpa(0.9*fye*f(:,(3*r-2)));
grad1x3=zeros(n,nb);
grad1=[grad1x1 grad2x2 grad1x3];

if ko_bm<2
    if vpa(k_bm.*k19.*(f(:,(3*r-2)).^(2/3)).*(f(:,(3*r-1)).^(1/3))) >...
        vpa(k_bm.*k20.* f(:,(3*r-2)))
grad2x1 = -(vpa(-((200)^0.5/2)*((f(:,(3*r-2))-k15).^(-3/2)).*k19.*k_bm.*...
    (f(:,(3*r-2)).^(2/3)).*(f(:,(3*r-1)).^(1/3)))+...
    vpa(2/3*k_bm.*k19.*(f(:,(3*r-2)).^(-1/3)).*(f(:,(3*r-1)).^(1/3)))+...
vpa(vpa(k22/k21).*vpa((4*f(:,(3*r))-k21).*f(:,(3*r)).*coteta_bm)));

    else
    grad2x1 = -(vpa(3/2*(k_bm.^(0.5)).*(-((200)^0.5/2)*...
((f(:,(3*r-2))-k15).^(-3/2))).*k20.*f(:,(3*r-2))+((k_bm.^(3/2)).*k20))+...
    vpa(vpa(k22/k21).*vpa((4*f(:,(3*r))-k21).*f(:,(3*r)).*coteta_bm)));
    end
else
    if vpa(k_bm.*k19.*(f(:,(3*r-2)).^(2/3)).*(f(:,(3*r-1)).^(1/3))) >...
        vpa(k_bm.*k20.* f(:,(3*r-2)))
grad2x1 = -(vpa(2/3*k_bm.*k19.*(f(:,(3*r-2)).^(-1/3)).*(f(:,(3*r-1)).^...
    (1/3)))+ vpa(vpa(k22/k21).*vpa((4*f(:,(3*r))-k21).*...
    f(:,(3*r)).*coteta_bm)));
    else
    grad2x1 = -(vpa((k_bm.^(3/2)).*k20)+vpa(vpa(k22/k21).*...
        vpa((4*f(:,(3*r))-k21).*f(:,(3*r)).*coteta_bm)));
    end
end
```

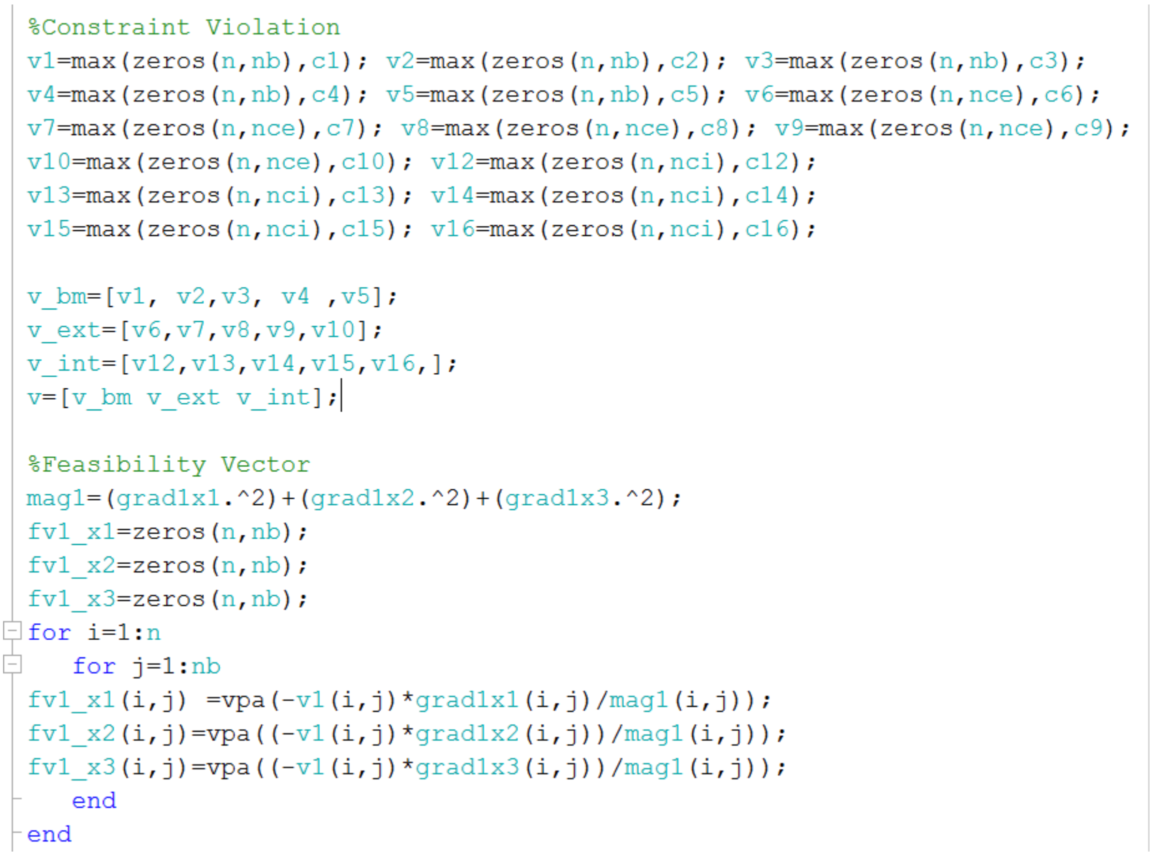
```
if (k_bm.*k19.*(f(:,(3*r-2)).^(2/3)).*(f(:,(3*r-1)).^(1/3))) >...
        (k_bm.*k20.*f(:,(3*r-2))) %#ok<BDSCA>
grad2x2= -(vpa(1/3*k_bm.*k19.*(f(:,(3*r-2)).^(2/3)).*(f(:,(3*r-1)).^...
    (-2/3))));
else
    grad2x2 = zeros(size(c2));
end
grad2x3= -(vpa(vpa(k22/k21).*(8*f(:,(3*r))-k21).*vpa(f(:,(3*r-2)).*...
    coteta_bm)));

grad2=[grad2x1 grad2x2 grad2x3];

grad3x1 = vpa(-k26);
grad3x2 = ones(size(c3));
grad3x3=zeros(n,nb);

grad3=[grad3x1 grad3x2 grad3x3];

grad4x1 = vpa(k25);
grad4x2 = -ones(size(c4));
grad4x3 = zeros(n,nb);

grad4=[grad4x1 grad4x2 grad4x3];

grad5x1 = zeros(n,nb);
grad5x2=zeros(n,nb);
grad5x3 = vpa((-4*k21*bms_lgth)./((4*f(:,(3*r))-k21).^2));

grad5=[grad5x1 grad5x2 grad5x3];
```

```
grad6x4 = -(vpa((double(T_ext)+Pg_ext)./(k29_ext.*...
    (f(:,3*nb+(3*s-2)).^2)))+ vpa(2*M_des_ext./(red_fct*k29_ext.*...
    ((f(:,3*nb+(3*s-2))).^3))));
grad6x5=zeros(n,nce);
grad6x6=zeros(n,nce);

grad6=[grad6x4 grad6x5 grad6x6];

if ko_ext<2
    if vpa(k_ext.*k32_ext.*(f(:,3*nb+(3*s-2)).^(2/3)).*...
            (f(:,3*nb+(3*s-1)).^(1/3))) > vpa(k_ext.*k33_ext.*...
            f(:,3*nb+(3*s-2)))
grad7x4 = -red_fct*(vpa(-((200)^0.5/2)*((f(:,3*nb+(3*s-2))-k34).^...
    (-3/2)).*k32_ext.*k_ext.*(f(:,3*nb+(3*s-2)).^(2/3)).*...
    (f(:,3*nb+(3*s-1)).^(1/3)))+ vpa(2/3*k_ext.*k32_ext.*...
    (f(:,3*nb+(3*s-2)).^(-1/3)).*(f(:,3*nb+(3*s-1)).^(1/3)))+vpa(0.9*...
    ((double(T_ext)+Pg_ext))./(ext_col_lgth))+ vpa(vpa(k36_ext/k35).*...
    vpa((4*f(:,3*nb+(3*s))-k35).*f(:,3*nb+(3*s)).*coteta_ext)));

    else
    grad7x4 = -red_fct*(vpa(3/2*(k_ext.^(0.5)).*(-((200)^0.5/2)*...
        ((f(:,3*nb+(3*s-2))-k34).^(-3/2))).*k33_ext.*f(:,3*nb+(3*s-2))+...
        ((k_ext.^(3/2)).*k33_ext))+vpa(0.9*((double(T_ext)+Pg_ext))./...
        (ext_col_lgth))+ vpa(vpa(k36_ext/k35).*vpa((4*f(:,3*nb+(3*s))-...
        k35).*f(:,3*nb+(3*s)).*coteta_ext)));
    end
else
    if vpa(k_ext.*k32_ext.*(f(:,3*nb+(3*s-2)).^(2/3)).*...
     (f(:,3*nb+(3*s-1)).^(1/3))) >vpa(k_ext.*k33_ext.* f(:,3*nb+(3*s-2)))
grad7x4 =-red_fct*(vpa(2/3*k_ext.*k32_ext.*(f(:,3*nb+(3*s-2)).^(-1/3)).*...
    (f(:,3*nb+(3*s-1)).^(1/3)))+vpa((0.9*((double(T_ext)+Pg_ext)))./...
    (ext_col_lgth))+ vpa(vpa(k36_ext/k35).*vpa((4*f(:,3*nb+(3*s))-k35).*...
    f(:,3*nb+(3*s)).*coteta_ext)));
    else
    grad7x4 = -red_fct*(vpa((k_ext.^(3/2)).*k33_ext)+vpa(0.9*...
        ((double(T_ext)+Pg_ext))./(ext_col_lgth))+vpa(vpa(k36_ext/k35).*...
        vpa((4*f(:,3*nb+(3*s))-k35).*f(:,3*nb+(3*s)).*coteta_ext)));
    end
end

if (k_ext.*k32_ext.*(f(:,3*nb+(3*s-2)).^(2/3)).*(f(:,3*nb+(3*s-1)).^...
        (1/3))) >(k_ext.*k33_ext.*f(:,3*nb+(3*s-2))) %#ok<BDSCA>
grad7x5= -red_fct*(vpa(1/3*k_ext.*k32_ext.*(f(:,3*nb+(3*s-2)).^(2/3)).*...
    (f(:,3*nb+(3*s-1)).^(-2/3))));
else
    grad7x5 = zeros(size(c7));
end
grad7x6= -red_fct*(vpa(vpa(k36_ext/k35).*(8*f(:,3*nb+(3*s))-k35).*...
    vpa(f(:,3*nb+(3*s-2)).*coteta_ext)));

grad7=[grad7x4 grad7x5 grad7x6];
```

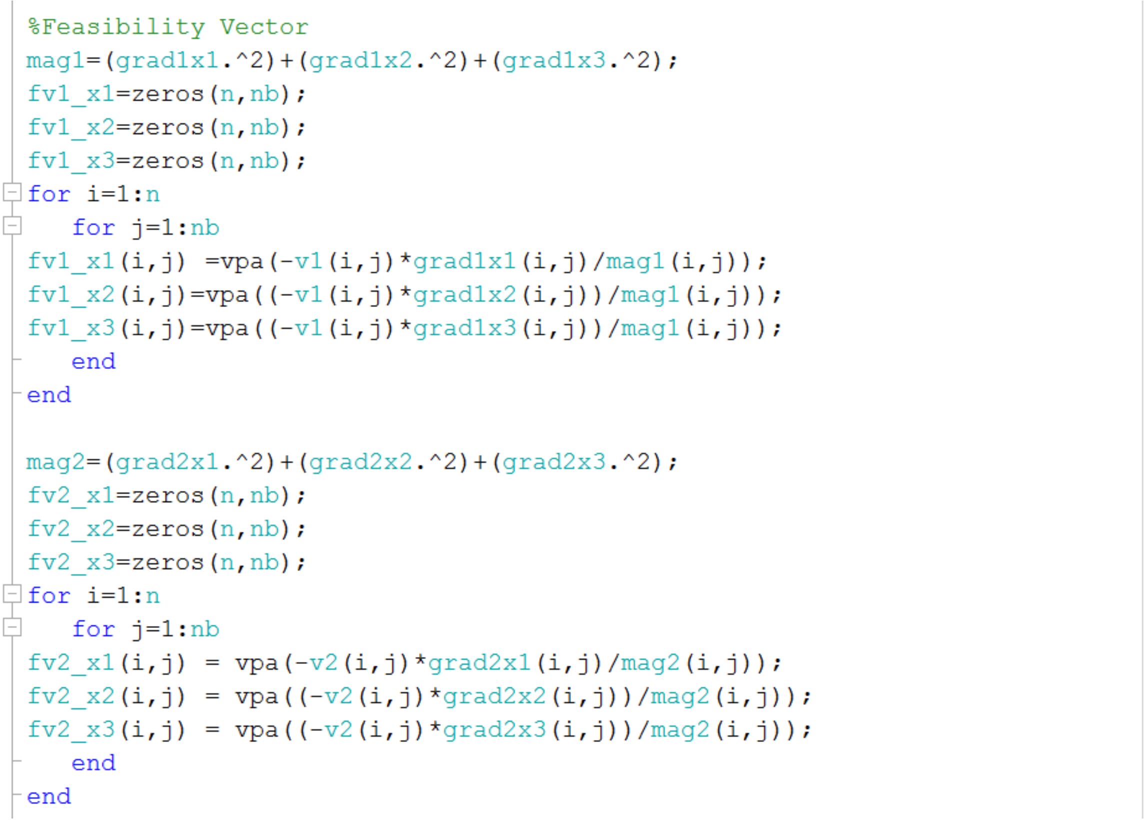
```
grad8x4=vpa(-4*k38_ext);
grad8x5 = ones(size(c8));
grad8x6=zeros(n,nce);
grad8=[grad8x4 grad8x5 grad8x6];

grad9x4 = vpa(k38_ext);
grad9x5 = -ones(size(c8));
grad9x6 = zeros(n,nce);

grad9=[grad9x4 grad9x5 grad9x6];

grad10x4=zeros(n,nce);
grad10x5=zeros(n,nce);
grad10x6 = vpa((-4*k35*ext_col_lgth)./((4*f(:,3*nb+(3*s))-k35).^2));

grad10=[grad10x4 grad10x5 grad10x6];


if nb>1

grad12x7=-(vpa(Pg_int./(k29_int.*((f(:,3*(nb+nce)+(3*t-2))).^2)))+...
    (2*M_des_int./(red_fct*k29_int.*...
    ((f(:,3*(nb+nce)+(3*t-2))).^3))));
grad12x8=zeros(n,nci);
grad12x9=zeros(n,nci);

grad12=[grad12x7 grad12x8 grad12x9];
```

```
if nb>1

grad12x7=-(vpa(Pg_int./(k29_int.*((f(:,3*(nb+nce)+(3*t-2))).^2)))+...
    (2*M_des_int./(red_fct*k29_int.*...
    ((f(:,3*(nb+nce)+(3*t-2))).^3))));
grad12x8=zeros(n,nci);
grad12x9=zeros(n,nci);

grad12=[grad12x7 grad12x8 grad12x9];

if ko_int<2
    if vpa(k_int.*k32_int.*(f(:,3*(nb+nce)+(3*t-2)).^(2/3)).*...
            (f(:,3*(nb+nce)+(3*t-1)).^(1/3))) > vpa(k_int.*k33_int.*...
            f(:,3*(nb+nce)+(3*t-2)))
grad13x7 = -red_fct*(vpa(-((200)^0.5/2)*((f(:,3*(nb+nce)+(3*t-2))-k15).^...
    (-3/2)).*k32_int.*(f(:,3*(nb+nce)+(3*t-2)).^(2/3)).*...
    (f(:,3*(nb+nce)+(3*t-1)).^(1/3)))+ vpa(2/3*k_int.*k32_int.*...
  (f(:,3*(nb+nce)+(3*t-2)).^(-1/3)).*(f(:,3*(nb+nce)+(3*t-1)).^(1/3)))+...
    vpa(0.9*((Pg_int)./(int_col_lgth)))+ vpa(vpa(k36_int/k35).*...
   vpa((4*f(:,3*(nb+nce)+(3*t))-k35).*f(:,3*(nb+nce)+(3*t)).*coteta_int)));
    else
    grad13x7 = -red_fct*(vpa(3/2*(k_int.^(0.5)).*(-((200)^0.5/2)*...
        ((f(:,3*(nb+nce)+(3*t-2))-k34).^(-3/2))).*k33_int.*...
        f(:,3*(nb+nce)+(3*t-2))+((k_int.^(3/2)).*k33_int))+vpa(0.9*...
        ((Pg_int)./(int_col_lgth)))+...
    vpa(vpa(k36_int/k35).*vpa((4*f(:,3*(nb+nce)+(3*t))-k35).*...
    f(:,3*(nb+nce)+(3*t)).*coteta_int)));
    end
```

```
else
   if vpa(k_int.*k32_int.*(f(:,3*(nb+nce)+(3*t-2)).^(2/3)).*...
           (f(:,3*(nb+nce)+(3*t-1)).^(1/3))) > vpa(k_int.*k33_int.*...
           f(:,3*(nb+nce)+(3*t-2)))
grad13x7 = -red_fct*(vpa(2/3*k_int.*k32_int.*(f(:,3*(nb+nce)+(3*t-2)).^...
    (-1/3)).*(f(:,3*(nb+nce)+(3*t-1)).^(1/3)))+vpa(0.9*...
    ((Pg_int)./(int_col_lgth)))+vpa(vpa(k36_int/k35).*...
  vpa((4*f(:,3*(nb+nce)+(3*t))-k35).*f(:,3*(nb+nce)+(3*t)).*coteta_int)));
    else
    grad13x7 = -red_fct*(vpa((k_int.^(3/2)).*k33_int)+vpa(0.9*...
        ((Pg_int)./(int_col_lgth)))+vpa(vpa(k36_int/k35).*...
  vpa((4*f(:,3*(nb+nce)+(3*t))-k35).*f(:,3*(nb+nce)+(3*t)).*coteta_int)));
   end
end

if (k_int.*k32_int.*(f(:,3*(nb+nce)+(3*t-2)).^(2/3)).*...
        (f(:,3*(nb+nce)+(3*t-1)).^(1/3))) > (k_int.*k33_int.*...
        f(:,3*(nb+nce)+(3*t-2))) %#ok<BDSCA>
grad13x8= -red_fct*(vpa(1/3*k_int.*k32_int.*(f(:,3*(nb+nce)+(3*t-2)).^...
    (2/3)).*(f(:,3*(nb+nce)+(3*t-1)).^(-2/3))));
else
    grad13x8 = zeros(size(c13));
end
grad13x9=-red_fct*(vpa(vpa(k36_int/k35).*(8*f(:,3*(nb+nce)+(3*t))-k35).*...
    vpa(f(:,3*(nb+nce)+(3*t-2)).*coteta_int)));

grad13=[grad13x7 grad13x8 grad13x9];

grad14x7 = vpa(-4*k38_int);
grad14x8 = ones(size(c14));
grad14x9=zeros(n,nci);

grad14=[grad14x7 grad14x8 grad14x9];

grad15x7 = vpa(k38_int);
grad15x8 = -ones(size(c15));
grad15x9=zeros(n,nci);
grad15=[grad15x7 grad15x8 grad15x9];

grad16x7=zeros(n,nci);
grad16x8=zeros(n,nci);
grad16x9=vpa((-4*k35*int_col_lgth)./((4*f(:,3*(nb+nce)+(3*t))-k35).^2));
grad16=[grad16x7 grad16x8 grad16x9];

elseif nb==1

    grad12=[];
    grad13=[];
    grad14=[];
    grad15=[];
    grad16=[];
else
    disp('Error')
end
```

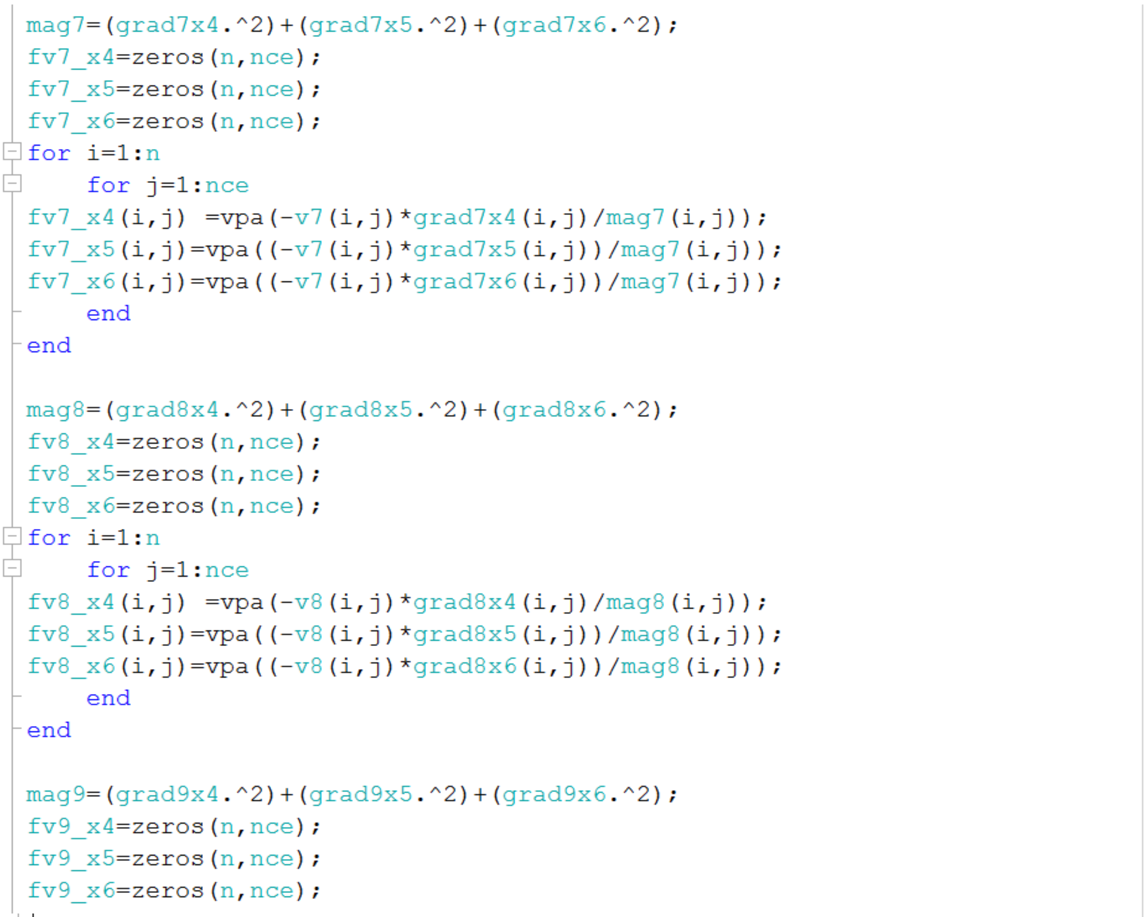


```
%Constraint Violation
v1=max(zeros(n,nb),c1); v2=max(zeros(n,nb),c2); v3=max(zeros(n,nb),c3);
v4=max(zeros(n,nb),c4); v5=max(zeros(n,nb),c5); v6=max(zeros(n,nce),c6);
v7=max(zeros(n,nce),c7); v8=max(zeros(n,nce),c8); v9=max(zeros(n,nce),c9);
v10=max(zeros(n,nce),c10); v12=max(zeros(n,nci),c12);
v13=max(zeros(n,nci),c13); v14=max(zeros(n,nci),c14);
v15=max(zeros(n,nci),c15); v16=max(zeros(n,nci),c16);

v_bm=[v1, v2,v3, v4 ,v5];
v_ext=[v6,v7,v8,v9,v10];
v_int=[v12,v13,v14,v15,v16,];
v=[v_bm v_ext v_int];

%Feasibility Vector
mag1=(grad1x1.^2)+(grad1x2.^2)+(grad1x3.^2);
fv1_x1=zeros(n,nb);
fv1_x2=zeros(n,nb);
fv1_x3=zeros(n,nb);
for i=1:n
   for j=1:nb
fv1_x1(i,j) =vpa(-v1(i,j)*grad1x1(i,j)/mag1(i,j));
fv1_x2(i,j)=vpa((-v1(i,j)*grad1x2(i,j))/mag1(i,j));
fv1_x3(i,j)=vpa((-v1(i,j)*grad1x3(i,j))/mag1(i,j));
   end
end
```

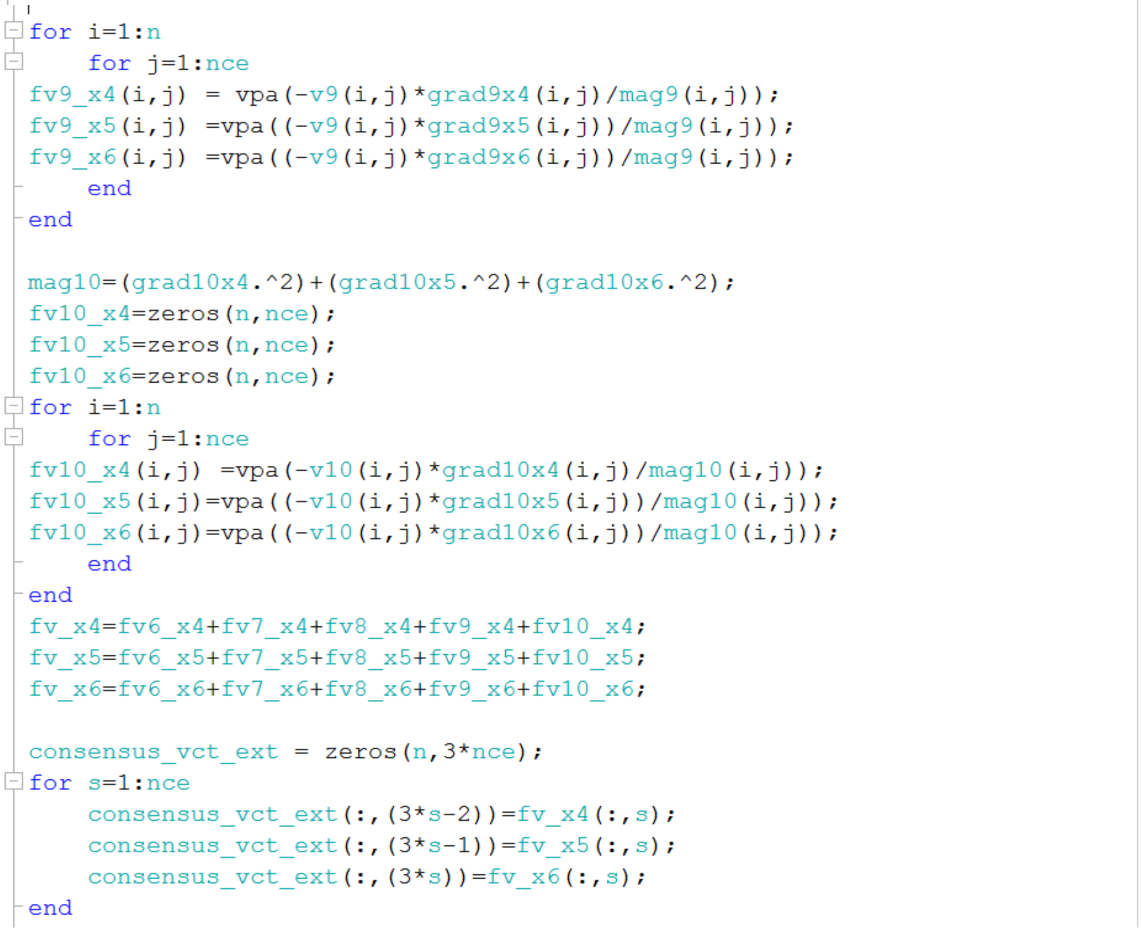


```
%Feasibility Vector
mag1=(grad1x1.^2)+(grad1x2.^2)+(grad1x3.^2);
fv1_x1=zeros(n,nb);
fv1_x2=zeros(n,nb);
fv1_x3=zeros(n,nb);
for i=1:n
   for j=1:nb
fv1_x1(i,j) =vpa(-v1(i,j)*grad1x1(i,j)/mag1(i,j));
fv1_x2(i,j)=vpa((-v1(i,j)*grad1x2(i,j))/mag1(i,j));
fv1_x3(i,j)=vpa((-v1(i,j)*grad1x3(i,j))/mag1(i,j));
   end
end

mag2=(grad2x1.^2)+(grad2x2.^2)+(grad2x3.^2);
fv2_x1=zeros(n,nb);
fv2_x2=zeros(n,nb);
fv2_x3=zeros(n,nb);
for i=1:n
   for j=1:nb
fv2_x1(i,j) = vpa(-v2(i,j)*grad2x1(i,j)/mag2(i,j));
fv2_x2(i,j) = vpa((-v2(i,j)*grad2x2(i,j))/mag2(i,j));
fv2_x3(i,j) = vpa((-v2(i,j)*grad2x3(i,j))/mag2(i,j));
   end
end
```

```
mag3=(grad3x1.^2)+(grad3x2.^2)+(grad3x3.^2);
fv3_x1=zeros(n,nb);
fv3_x2=zeros(n,nb);
fv3_x3=zeros(n,nb);
for i=1:n
   for j=1:nb
fv3_x1(i,j) = vpa(-v3(i,j)*grad3x1(i,j)/mag3(i,j));
fv3_x2(i,j)= vpa((-v3(i,j)*grad3x2(i,j))/mag3(i,j));
fv3_x3(i,j)= vpa((-v3(i,j)*grad3x3(i,j))/mag3(i,j));
    end
end

mag4=(grad4x1.^2)+(grad4x2.^2)+(grad4x3.^2);
fv4_x1=zeros(n,nb);
fv4_x2=zeros(n,nb);
fv4_x3=zeros(n,nb);
for i=1:n
   for j=1:nb
fv4_x1(i,j) = vpa(-v4(i,j)*grad4x1(i,j)/mag4(i,j));
fv4_x2(i,j)= vpa((-v4(i,j)*grad4x2(i,j))/mag4(i,j));
fv4_x3(i,j)= vpa((-v4(i,j)*grad4x3(i,j))/mag4(i,j));
    end
end

mag5=(grad5x1.^2)+(grad5x2.^2)+(grad5x3.^2);
fv5_x1=zeros(n,nb);
fv5_x2=zeros(n,nb);
fv5_x3=zeros(n,nb);
for i=1:n
    for j=1:nb
fv5_x1(i,j) = vpa(-v5(i,j)*grad5x1(i,j)/mag5(i,j));
fv5_x2(i,j)= vpa((-v5(i,j)*grad5x2(i,j))/mag5(i,j));
fv5_x3(i,j)=vpa((-v5(i,j)*grad5x3(i,j))/mag5(i,j));
    end
end

fv_x1=fv1_x1+fv2_x1+fv3_x1+fv4_x1+fv5_x1;
fv_x2=fv1_x2+fv2_x2+fv3_x2+fv4_x2+fv5_x2;
fv_x3=fv1_x3+fv2_x3+fv3_x3+fv4_x3+fv5_x3;

consensus_vct_bm = zeros(n,3*nb);
for r=1:nb
    consensus_vct_bm(:,(3*r-2))=fv_x1(:,r);
    consensus_vct_bm(:,(3*r-1))=fv_x2(:,r);
    consensus_vct_bm(:,(3*r))=fv_x3(:,r);
end

mag6=(grad6x4.^2)+(grad6x5.^2)+(grad6x6.^2);
fv6_x4=zeros(n,nce);
fv6_x5=zeros(n,nce);
fv6_x6=zeros(n,nce);
for i=1:n
   for j=1:nce
fv6_x4(i,j) =vpa(-v6(i,j)*grad6x4(i,j)/mag6(i,j));
fv6_x5(i,j)=vpa((-v6(i,j)*grad6x5(i,j))/mag6(i,j));
fv6_x6(i,j)=vpa((-v6(i,j)*grad6x6(i,j))./mag6(i,j));
    end
end
```

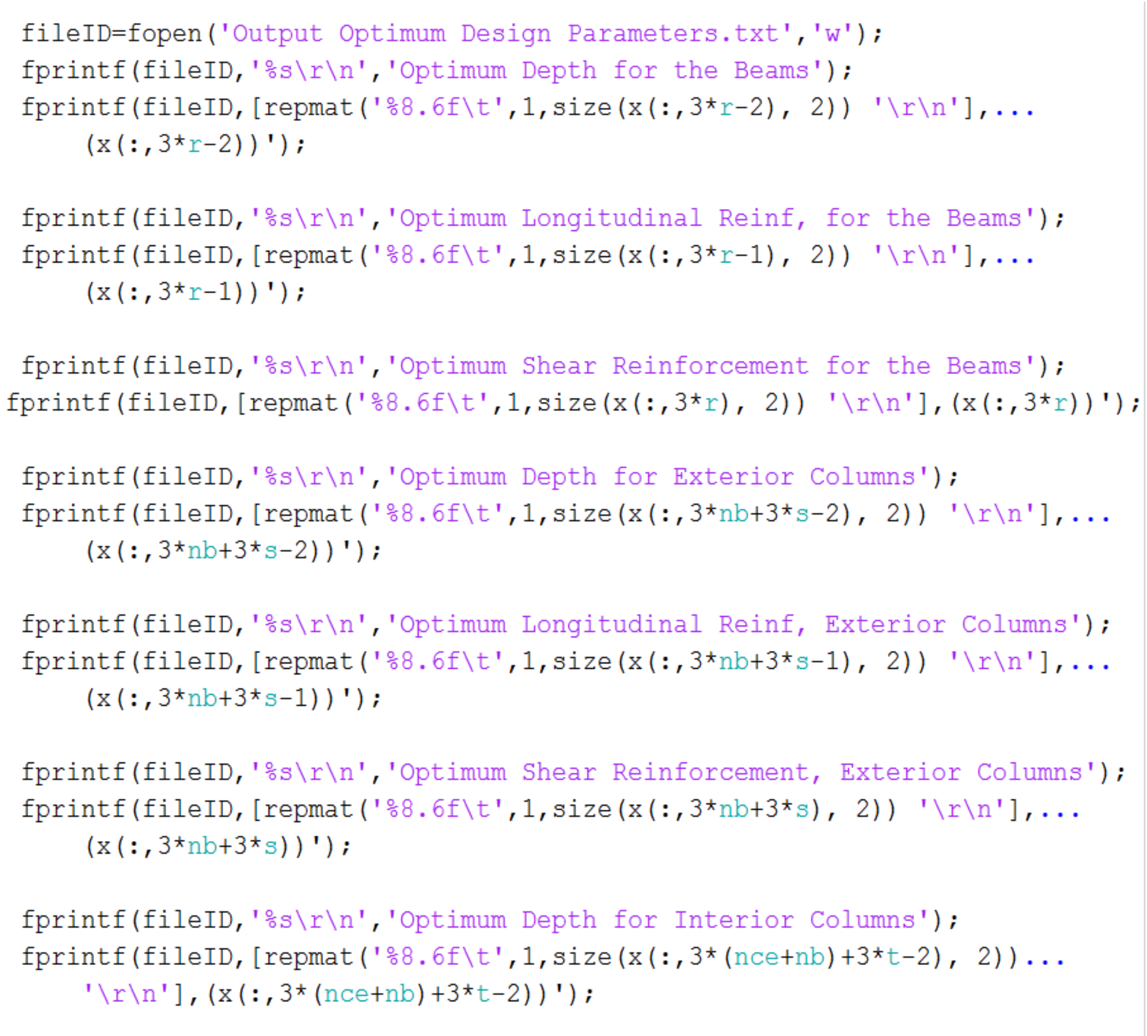


```
mag7=(grad7x4.^2)+(grad7x5.^2)+(grad7x6.^2);
fv7_x4=zeros(n,nce);
fv7_x5=zeros(n,nce);
fv7_x6=zeros(n,nce);
for i=1:n
    for j=1:nce
fv7_x4(i,j)  =vpa(-v7(i,j)*grad7x4(i,j)/mag7(i,j));
fv7_x5(i,j)=vpa((-v7(i,j)*grad7x5(i,j))/mag7(i,j));
fv7_x6(i,j)=vpa((-v7(i,j)*grad7x6(i,j))/mag7(i,j));
    end
end

mag8=(grad8x4.^2)+(grad8x5.^2)+(grad8x6.^2);
fv8_x4=zeros(n,nce);
fv8_x5=zeros(n,nce);
fv8_x6=zeros(n,nce);
for i=1:n
    for j=1:nce
fv8_x4(i,j)  =vpa(-v8(i,j)*grad8x4(i,j)/mag8(i,j));
fv8_x5(i,j)=vpa((-v8(i,j)*grad8x5(i,j))/mag8(i,j));
fv8_x6(i,j)=vpa((-v8(i,j)*grad8x6(i,j))/mag8(i,j));
    end
end

mag9=(grad9x4.^2)+(grad9x5.^2)+(grad9x6.^2);
fv9_x4=zeros(n,nce);
fv9_x5=zeros(n,nce);
fv9_x6=zeros(n,nce);
```

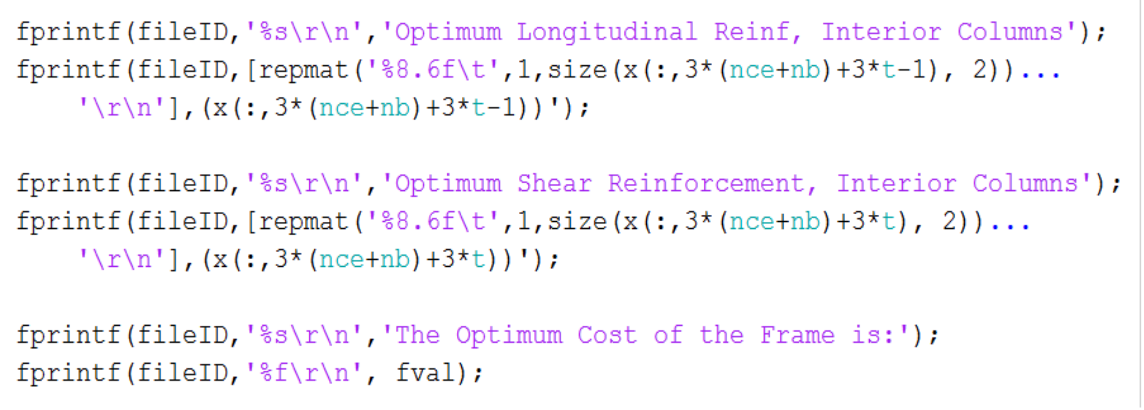


```
for i=1:n
    for j=1:nce
fv9_x4(i,j)  = vpa(-v9(i,j)*grad9x4(i,j)/mag9(i,j));
fv9_x5(i,j)  =vpa((-v9(i,j)*grad9x5(i,j))/mag9(i,j));
fv9_x6(i,j)  =vpa((-v9(i,j)*grad9x6(i,j))/mag9(i,j));
    end
end

mag10=(grad10x4.^2)+(grad10x5.^2)+(grad10x6.^2);
fv10_x4=zeros(n,nce);
fv10_x5=zeros(n,nce);
fv10_x6=zeros(n,nce);
for i=1:n
    for j=1:nce
fv10_x4(i,j)  =vpa(-v10(i,j)*grad10x4(i,j)/mag10(i,j));
fv10_x5(i,j)=vpa((-v10(i,j)*grad10x5(i,j))/mag10(i,j));
fv10_x6(i,j)=vpa((-v10(i,j)*grad10x6(i,j))/mag10(i,j));
    end
end
fv_x4=fv6_x4+fv7_x4+fv8_x4+fv9_x4+fv10_x4;
fv_x5=fv6_x5+fv7_x5+fv8_x5+fv9_x5+fv10_x5;
fv_x6=fv6_x6+fv7_x6+fv8_x6+fv9_x6+fv10_x6;

consensus_vct_ext = zeros(n,3*nce);
for s=1:nce
    consensus_vct_ext(:,(3*s-2))=fv_x4(:,s);
    consensus_vct_ext(:,(3*s-1))=fv_x5(:,s);
    consensus_vct_ext(:,(3*s))=fv_x6(:,s);
end
```

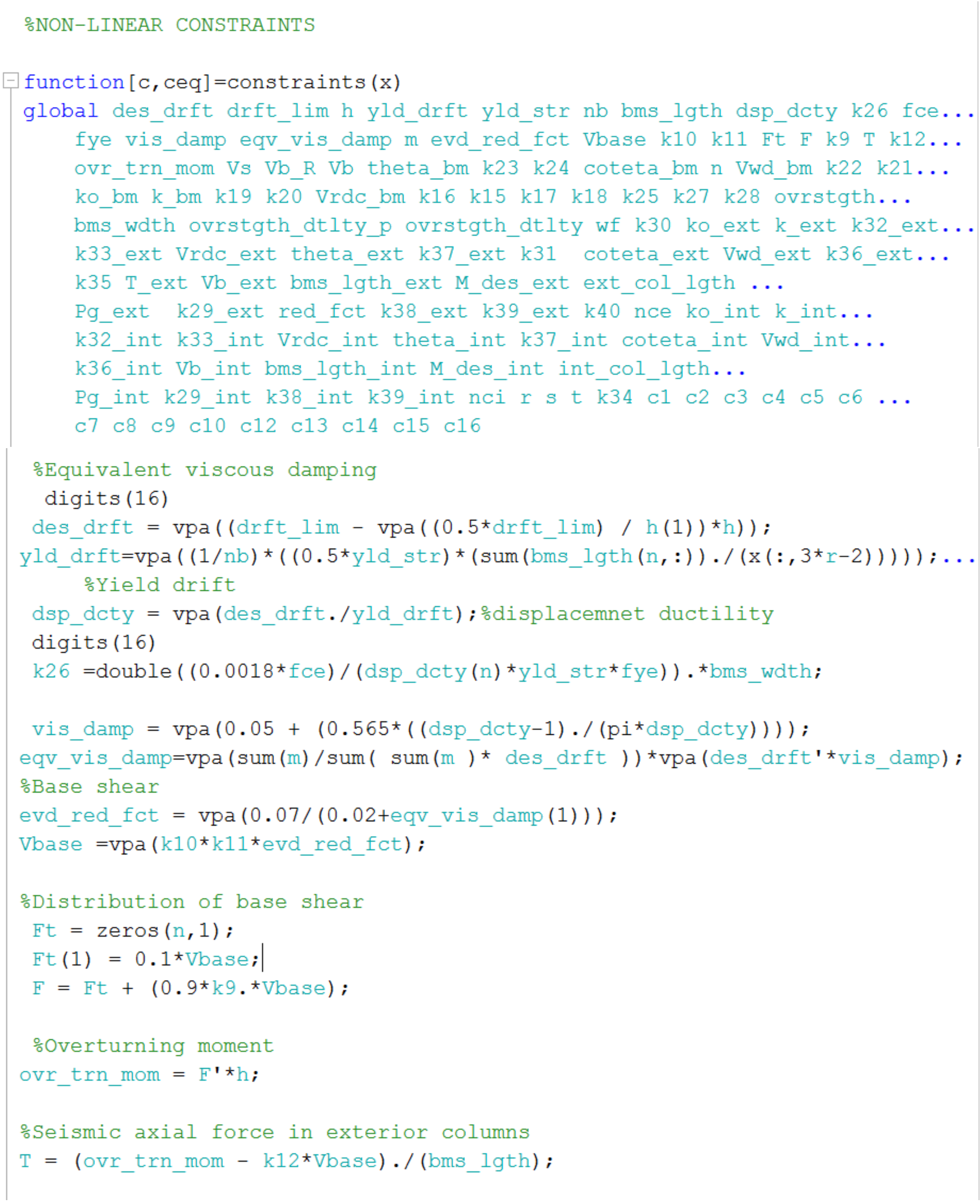
```
 consensus_vct=[consensus_vct_bm consensus_vct_ext consensus_vct_int];

 for i=1:n
     for j=1:(3*(nb+nce+nci))
         if(f(i,j)+consensus_vct(i,j))>0
             f(i,j)=f(i,j)+consensus_vct(i,j);
         else
             f(i,j)=0;
         end
     end
 end
     if all(v <=1e-6)
         break
     end
  else
       disp('Negative viscous damping, please revise sections')
      break
 end
 end
 x_start=f;

%FRAME OPTIMIZATION
r=1:nb;
s=1:nce;
t=1:nci;

rng('default')
gs = GlobalSearch('StartPointsToRun','bounds-ineqs',...
    'NumTrialPoints',20,'NumStageOnePoints',10);

opts = optimoptions(@fmincon,'Algorithm','interior-point',...
   'FiniteDifferenceStepSize',1e-3,'ConstraintTolerance',1e-6,...
'OptimalityTolerance',1e6,'MaxFunEvals',Inf);

fun = @(x)sum(sum(k1.*x(:,3*r-2)+k2.*x(:,3*r-1)+k3.*x(:,3*r)+k4.*...
    ((2*x(:,3*r-2)+ bms_wdth))))+sum(sum(k5_ext.*x(:,3*nb+3*s-2)+...
    k6_ext.*x(:,3*nb+3*s-1)+ k7_ext.*x(:,3*nb+3*s)+ k8_ext.*...
    (x(:,3*nb+3*s-2)+ext_col_wdth)))+ sum(sum(k5_int.*...
    x(:,3*(nb+nce)+3*t-2)+k6_int.*x(:,3*(nb+nce)+3*t-1)+ k7_int.*...
    x(:,3*(nb+nce)+3*t)+ k8_int.*(x(:,3*(nb+nce)+3*t-2)+int_col_wdth)));
    %Objective function
lb=[repmat([0 0 0],size(bms_lgth)) repmat([0 0 0],size(ext_col_lgth))...
    repmat([0 0 0],size(int_col_lgth))];

problem = createOptimProblem('fmincon','x0', x_start,'objective',fun,...
    'lb',lb,'nonlcon' , @constraints, 'options', opts);

[x, fval ]= run(gs , problem);
```



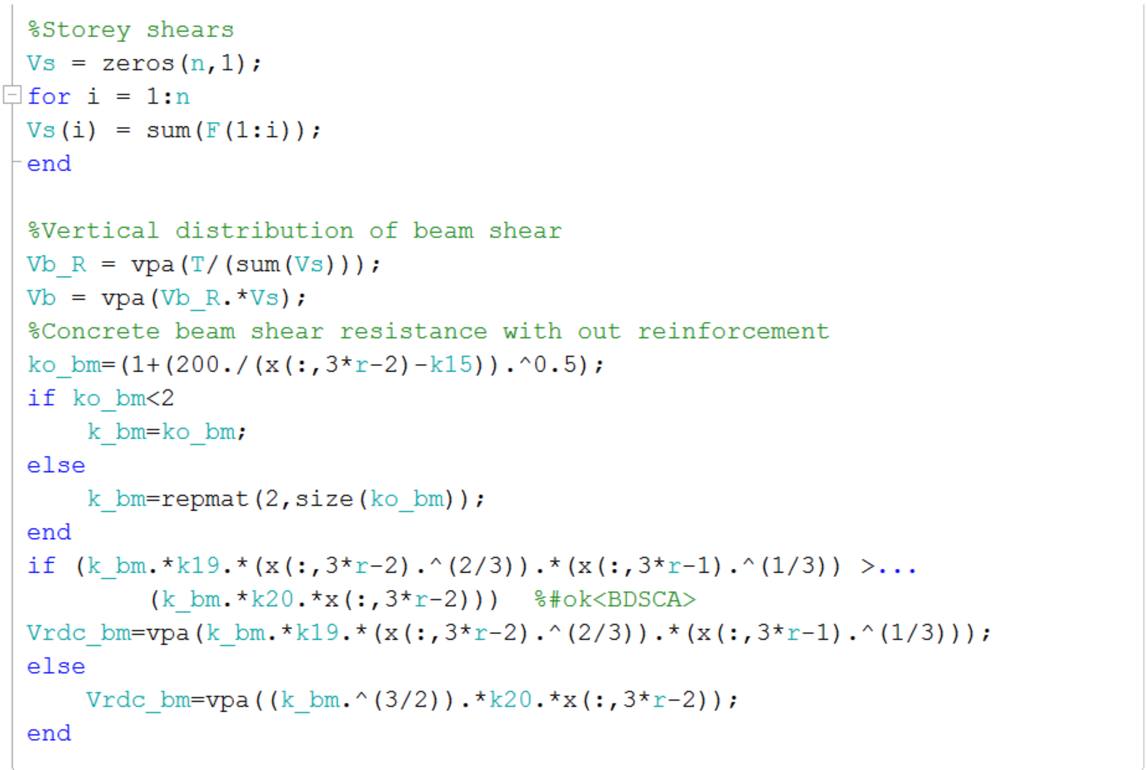
```
fileID=fopen('Output Optimum Design Parameters.txt','w');
fprintf(fileID,'%s\r\n','Optimum Depth for the Beams');
fprintf(fileID,[repmat('%8.6f\t',1,size(x(:,3*r-2), 2)) '\r\n'],...
    (x(:,3*r-2))');

fprintf(fileID,'%s\r\n','Optimum Longitudinal Reinf, for the Beams');
fprintf(fileID,[repmat('%8.6f\t',1,size(x(:,3*r-1), 2)) '\r\n'],...
    (x(:,3*r-1))');

fprintf(fileID,'%s\r\n','Optimum Shear Reinforcement for the Beams');
fprintf(fileID,[repmat('%8.6f\t',1,size(x(:,3*r), 2)) '\r\n'],(x(:,3*r))');

fprintf(fileID,'%s\r\n','Optimum Depth for Exterior Columns');
fprintf(fileID,[repmat('%8.6f\t',1,size(x(:,3*nb+3*s-2), 2)) '\r\n'],...
    (x(:,3*nb+3*s-2))');

fprintf(fileID,'%s\r\n','Optimum Longitudinal Reinf, Exterior Columns');
fprintf(fileID,[repmat('%8.6f\t',1,size(x(:,3*nb+3*s-1), 2)) '\r\n'],...
    (x(:,3*nb+3*s-1))');

fprintf(fileID,'%s\r\n','Optimum Shear Reinforcement, Exterior Columns');
fprintf(fileID,[repmat('%8.6f\t',1,size(x(:,3*nb+3*s), 2)) '\r\n'],...
    (x(:,3*nb+3*s))');

fprintf(fileID,'%s\r\n','Optimum Depth for Interior Columns');
fprintf(fileID,[repmat('%8.6f\t',1,size(x(:,3*(nce+nb)+3*t-2), 2))...
    '\r\n'],(x(:,3*(nce+nb)+3*t-2))');
```



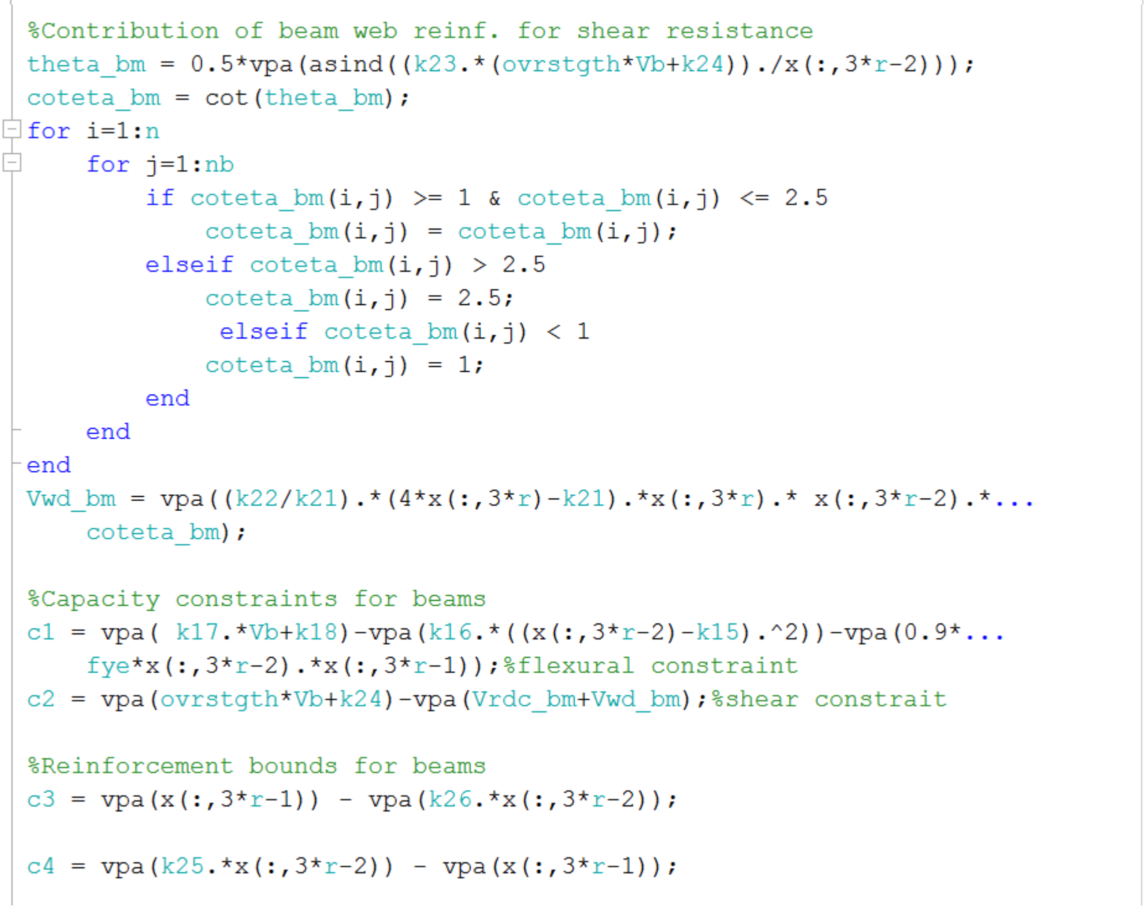
```
fprintf(fileID,'%s\r\n','Optimum Longitudinal Reinf, Interior Columns');
fprintf(fileID,[repmat('%8.6f\t',1,size(x(:,3*(nce+nb)+3*t-1), 2))...
    '\r\n'],(x(:,3*(nce+nb)+3*t-1))');

fprintf(fileID,'%s\r\n','Optimum Shear Reinforcement, Interior Columns');
fprintf(fileID,[repmat('%8.6f\t',1,size(x(:,3*(nce+nb)+3*t), 2))...
    '\r\n'],(x(:,3*(nce+nb)+3*t))');

fprintf(fileID,'%s\r\n','The Optimum Cost of the Frame is:');
fprintf(fileID,'%f\r\n', fval);
```



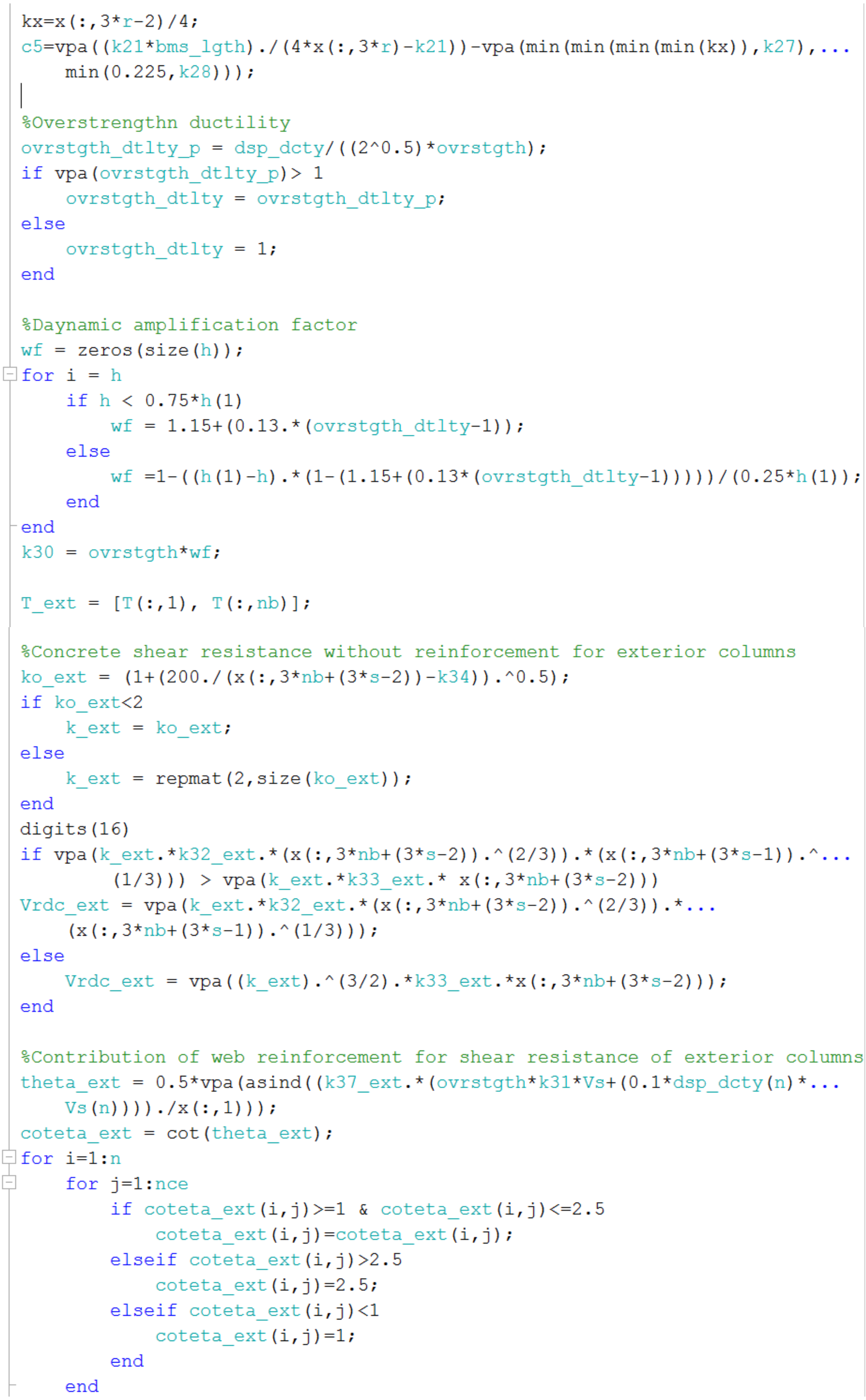
```
%NON-LINEAR CONSTRAINTS

function[c,ceq]=constraints(x)
global des_drft drft_lim h yld_drft yld_str nb bms_lgth dsp_dcty k26 fce...
    fye vis_damp eqv_vis_damp m evd_red_fct Vbase k10 k11 Ft F k9 T k12...
    ovr_trn_mom Vs Vb_R Vb theta_bm k23 k24 coteta_bm n Vwd_bm k22 k21...
    ko_bm k_bm k19 k20 Vrdc_bm k16 k15 k17 k18 k25 k27 k28 ovrstgth...
    bms_wdth ovrstgth_dtlty_p ovrstgth_dtlty wf k30 ko_ext k_ext k32_ext...
    k33_ext Vrdc_ext theta_ext k37_ext k31  coteta_ext Vwd_ext k36_ext...
    k35 T_ext Vb_ext bms_lgth_ext M_des_ext ext_col_lgth ...
    Pg_ext  k29_ext red_fct k38_ext k39_ext k40 nce ko_int k_int...
    k32_int k33_int Vrdc_int theta_int k37_int coteta_int Vwd_int...
    k36_int Vb_int bms_lgth_int M_des_int int_col_lgth...
    Pg_int k29_int k38_int k39_int nci r s t k34 c1 c2 c3 c4 c5 c6 ...
    c7 c8 c9 c10 c12 c13 c14 c15 c16
 %Equivalent viscous damping
  digits(16)
 des_drft = vpa((drft_lim - vpa((0.5*drft_lim) / h(1))*h));
yld_drft=vpa((1/nb)*((0.5*yld_str)*(sum(bms_lgth(n,:))./(x(:,3*r-2)))));...
     %Yield drift
 dsp_dcty = vpa(des_drft./yld_drft);%displacemnet ductility
 digits(16)
 k26 =double((0.0018*fce)/(dsp_dcty(n)*yld_str*fye)).*bms_wdth;

 vis_damp = vpa(0.05 + (0.565*((dsp_dcty-1)./(pi*dsp_dcty))));
eqv_vis_damp=vpa(sum(m)/sum( sum(m )* des_drft ))*vpa(des_drft'*vis_damp);
%Base shear
evd_red_fct = vpa(0.07/(0.02+eqv_vis_damp(1)));
Vbase =vpa(k10*k11*evd_red_fct);

%Distribution of base shear
 Ft = zeros(n,1);
 Ft(1) = 0.1*Vbase;
 F = Ft + (0.9*k9.*Vbase);

 %Overturning moment
ovr_trn_mom = F'*h;

%Seismic axial force in exterior columns
T = (ovr_trn_mom - k12*Vbase)./(bms_lgth);
```



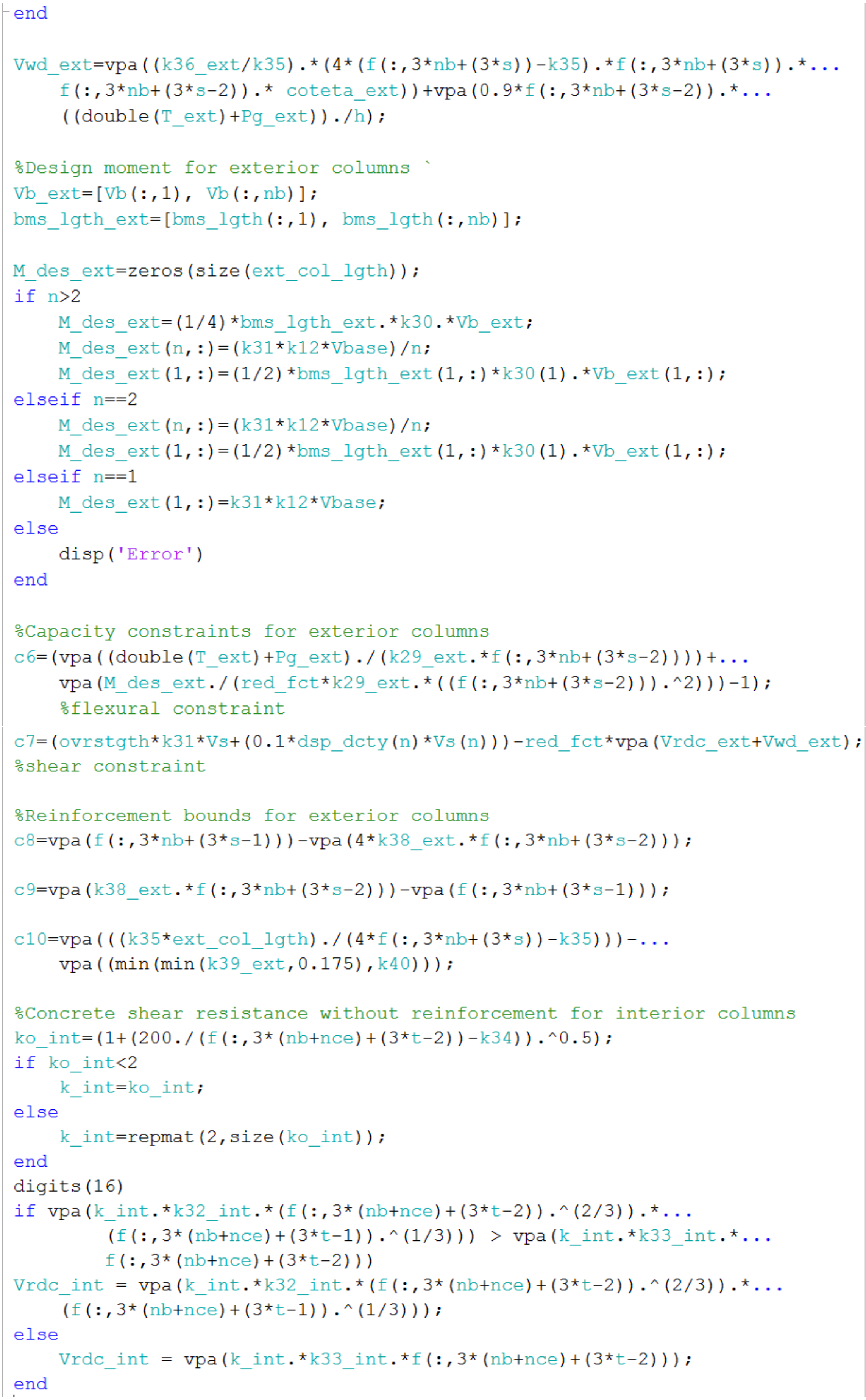
```
%Storey shears
Vs = zeros(n,1);
for i = 1:n
Vs(i) = sum(F(1:i));
end

%Vertical distribution of beam shear
Vb_R = vpa(T/(sum(Vs)));
Vb = vpa(Vb_R.*Vs);
%Concrete beam shear resistance with out reinforcement
ko_bm=(1+(200./(x(:,3*r-2)-k15)).^0.5);
if ko_bm<2
    k_bm=ko_bm;
else
    k_bm=repmat(2,size(ko_bm));
end
if (k_bm.*k19.*(x(:,3*r-2).^(2/3)).*(x(:,3*r-1).^(1/3)) >...
        (k_bm.*k20.*x(:,3*r-2)))  %#ok<BDSCA>
Vrdc_bm=vpa(k_bm.*k19.*(x(:,3*r-2).^(2/3)).*(x(:,3*r-1).^(1/3)));
else
    Vrdc_bm=vpa((k_bm.^(3/2)).*k20.*x(:,3*r-2));
end
```



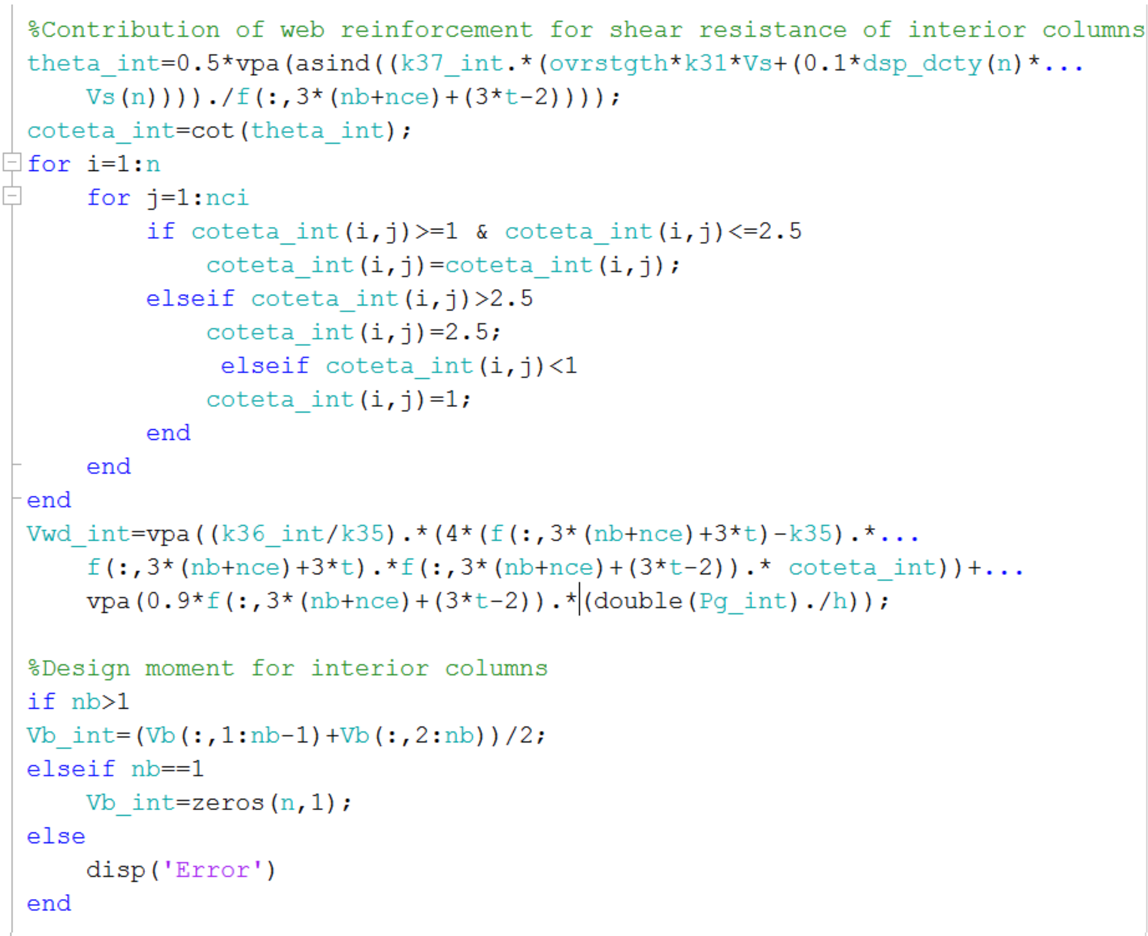
```
%Contribution of beam web reinf. for shear resistance
theta_bm = 0.5*vpa(asind((k23.*(ovrstgth*Vb+k24))./x(:,3*r-2)));
coteta_bm = cot(theta_bm);
for i=1:n
    for j=1:nb
        if coteta_bm(i,j) >= 1 & coteta_bm(i,j) <= 2.5
            coteta_bm(i,j) = coteta_bm(i,j);
        elseif coteta_bm(i,j) > 2.5
            coteta_bm(i,j) = 2.5;
             elseif coteta_bm(i,j) < 1
            coteta_bm(i,j) = 1;
        end
    end
end
Vwd_bm = vpa((k22/k21).*(4*x(:,3*r)-k21).*x(:,3*r).* x(:,3*r-2).*...
    coteta_bm);

%Capacity constraints for beams
c1 = vpa( k17.*Vb+k18)-vpa(k16.*((x(:,3*r-2)-k15).^2))-vpa(0.9*...
    fye*x(:,3*r-2).*x(:,3*r-1));%flexural constraint
c2 = vpa(ovrstgth*Vb+k24)-vpa(Vrdc_bm+Vwd_bm);%shear constrait

%Reinforcement bounds for beams
c3 = vpa(x(:,3*r-1)) - vpa(k26.*x(:,3*r-2));

c4 = vpa(k25.*x(:,3*r-2)) - vpa(x(:,3*r-1));
```



```
kx=x(:,3*r-2)/4;
c5=vpa((k21*bms_lgth)./(4*x(:,3*r)-k21))-vpa(min(min(min(min(kx)),k27),...
    min(0.225,k28)));

%Overstrengthn ductility
ovrstgth_dtlty_p = dsp_dcty/((2^0.5)*ovrstgth);
if vpa(ovrstgth_dtlty_p)> 1
    ovrstgth_dtlty = ovrstgth_dtlty_p;
else
    ovrstgth_dtlty = 1;
end

%Daynamic amplification factor
wf = zeros(size(h));
for i = h
    if h < 0.75*h(1)
        wf = 1.15+(0.13.*(ovrstgth_dtlty-1));
    else
        wf =1-((h(1)-h).*(1-(1.15+(0.13*(ovrstgth_dtlty-1)))))/(0.25*h(1));
    end
end
k30 = ovrstgth*wf;

T_ext = [T(:,1), T(:,nb)];

%Concrete shear resistance without reinforcement for exterior columns
ko_ext = (1+(200./(x(:,3*nb+(3*s-2))-k34)).^0.5);
if ko_ext<2
    k_ext = ko_ext;
else
    k_ext = repmat(2,size(ko_ext));
end
digits(16)
if vpa(k_ext.*k32_ext.*(x(:,3*nb+(3*s-2)).^(2/3)).*(x(:,3*nb+(3*s-1)).^...
        (1/3))) > vpa(k_ext.*k33_ext.* x(:,3*nb+(3*s-2)))
Vrdc_ext = vpa(k_ext.*k32_ext.*(x(:,3*nb+(3*s-2)).^(2/3)).*...
    (x(:,3*nb+(3*s-1)).^(1/3)));
else
    Vrdc_ext = vpa((k_ext).^(3/2).*k33_ext.*x(:,3*nb+(3*s-2)));
end

%Contribution of web reinforcement for shear resistance of exterior columns
theta_ext = 0.5*vpa(asind((k37_ext.*(ovrstgth*k31*Vs+(0.1*dsp_dcty(n)*...
    Vs(n))))./x(:,1)));
coteta_ext = cot(theta_ext);
for i=1:n
    for j=1:nce
        if coteta_ext(i,j)>=1 & coteta_ext(i,j)<=2.5
            coteta_ext(i,j)=coteta_ext(i,j);
        elseif coteta_ext(i,j)>2.5
            coteta_ext(i,j)=2.5;
        elseif coteta_ext(i,j)<1
            coteta_ext(i,j)=1;
        end
    end
```

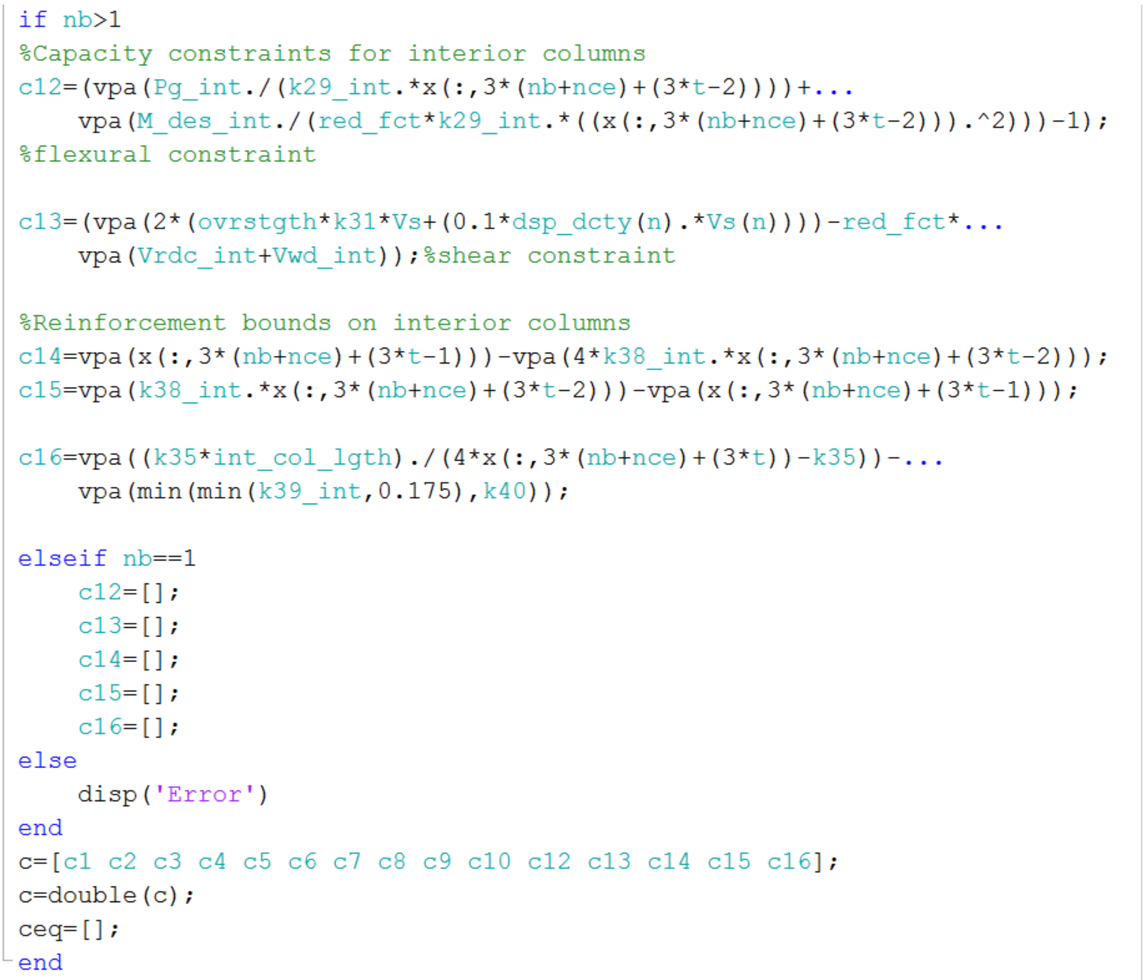


```
end

Vwd_ext=vpa((k36_ext/k35).*(4*(f(:,3*nb+(3*s))-k35).*f(:,3*nb+(3*s)).*...
    f(:,3*nb+(3*s-2)).* coteta_ext))+vpa(0.9*f(:,3*nb+(3*s-2)).*...
    ((double(T_ext)+Pg_ext))./h);

%Design moment for exterior columns `
Vb_ext=[Vb(:,1), Vb(:,nb)];
bms_lgth_ext=[bms_lgth(:,1), bms_lgth(:,nb)];

M_des_ext=zeros(size(ext_col_lgth));
if n>2
    M_des_ext=(1/4)*bms_lgth_ext.*k30.*Vb_ext;
    M_des_ext(n,:)=(k31*k12*Vbase)/n;
    M_des_ext(1,:)=(1/2)*bms_lgth_ext(1,:)*k30(1).*Vb_ext(1,:);
elseif n==2
    M_des_ext(n,:)=(k31*k12*Vbase)/n;
    M_des_ext(1,:)=(1/2)*bms_lgth_ext(1,:)*k30(1).*Vb_ext(1,:);
elseif n==1
    M_des_ext(1,:)=k31*k12*Vbase;
else
    disp('Error')
end

%Capacity constraints for exterior columns
c6=(vpa((double(T_ext)+Pg_ext)./(k29_ext.*f(:,3*nb+(3*s-2))))+...
    vpa(M_des_ext./(red_fct*k29_ext.*((f(:,3*nb+(3*s-2))).^2)))-1);
    %flexural constraint
c7=(ovrstgth*k31*Vs+(0.1*dsp_dcty(n)*Vs(n)))-red_fct*vpa(Vrdc_ext+Vwd_ext);
%shear constraint

%Reinforcement bounds for exterior columns
c8=vpa(f(:,3*nb+(3*s-1)))-vpa(4*k38_ext.*f(:,3*nb+(3*s-2)));

c9=vpa(k38_ext.*f(:,3*nb+(3*s-2)))-vpa(f(:,3*nb+(3*s-1)));

c10=vpa(((k35*ext_col_lgth)./(4*f(:,3*nb+(3*s))-k35)))-...
    vpa((min(min(k39_ext,0.175),k40)));

%Concrete shear resistance without reinforcement for interior columns
ko_int=(1+(200./(f(:,3*(nb+nce)+(3*t-2))-k34)).^0.5);
if ko_int<2
    k_int=ko_int;
else
    k_int=repmat(2,size(ko_int));
end
digits(16)
if vpa(k_int.*k32_int.*(f(:,3*(nb+nce)+(3*t-2)).^(2/3)).*...
        (f(:,3*(nb+nce)+(3*t-1)).^(1/3))) > vpa(k_int.*k33_int.*...
        f(:,3*(nb+nce)+(3*t-2)))
Vrdc_int = vpa(k_int.*k32_int.*(f(:,3*(nb+nce)+(3*t-2)).^(2/3)).*...
    (f(:,3*(nb+nce)+(3*t-1)).^(1/3)));
else
    Vrdc_int = vpa(k_int.*k33_int.*f(:,3*(nb+nce)+(3*t-2)));
end
```



```
%Contribution of web reinforcement for shear resistance of interior columns
theta_int=0.5*vpa(asind((k37_int.*(ovrstgth*k31*Vs+(0.1*dsp_dcty(n)*...
    Vs(n))))./f(:,3*(nb+nce)+(3*t-2))));
coteta_int=cot(theta_int);
for i=1:n
    for j=1:nci
        if coteta_int(i,j)>=1 & coteta_int(i,j)<=2.5
            coteta_int(i,j)=coteta_int(i,j);
        elseif coteta_int(i,j)>2.5
            coteta_int(i,j)=2.5;
             elseif coteta_int(i,j)<1
            coteta_int(i,j)=1;
        end
    end
end
Vwd_int=vpa((k36_int/k35).*(4*(f(:,3*(nb+nce)+3*t)-k35).*...
    f(:,3*(nb+nce)+3*t).*f(:,3*(nb+nce)+(3*t-2)).* coteta_int))+...
    vpa(0.9*f(:,3*(nb+nce)+(3*t-2)).*(double(Pg_int)./h));

%Design moment for interior columns
if nb>1
Vb_int=(Vb(:,1:nb-1)+Vb(:,2:nb))/2;
elseif nb==1
    Vb_int=zeros(n,1);
else
    disp('Error')
end
```

```
if nb>1
bms_lgth_int=(bms_lgth(:,1:nb-1)+bms_lgth(:,2:nb))/2;
elseif nb==1
    bms_lgth_int=zeros(n,1);
else
    disp('Error')
end

M_des_int=zeros(size(int_col_lgth));
if nb>1 & n>2
M_des_int=(1/2)*bms_lgth_int.*k30.*Vb_int;
M_des_int(n,:)=2*(k31*k12*Vbase/n);
M_des_int(1,:)=bms_lgth_int(1,:).*k30(1).*Vb_int(1,:);
elseif nb>1 & n==2
    M_des_int(n,:)=2*(k31*k12*Vbase/n);
    M_des_int(1,:)=bms_lgth_int(1,:)*k30(1).*Vb_int(1,:);
elseif nb>1 & n==1
    M_des_int(1,:)=2*(k31*k12*Vbase/n);
elseif nb==1
    M_des_int=zeros(n,1);
else
    disp('Error')
end
```



```
if nb>1
%Capacity constraints for interior columns
c12=(vpa(Pg_int./(k29_int.*x(:,3*(nb+nce)+(3*t-2))))+...
    vpa(M_des_int./(red_fct*k29_int.*((x(:,3*(nb+nce)+(3*t-2))).^2)))-1);
%flexural constraint

c13=(vpa(2*(ovrstgth*k31*Vs+(0.1*dsp_dcty(n).*Vs(n))))-red_fct*...
    vpa(Vrdc_int+Vwd_int));%shear constraint

%Reinforcement bounds on interior columns
c14=vpa(x(:,3*(nb+nce)+(3*t-1)))-vpa(4*k38_int.*x(:,3*(nb+nce)+(3*t-2)));
c15=vpa(k38_int.*x(:,3*(nb+nce)+(3*t-2)))-vpa(x(:,3*(nb+nce)+(3*t-1)));

c16=vpa((k35*int_col_lgth)./(4*x(:,3*(nb+nce)+(3*t))-k35))-...
    vpa(min(min(k39_int,0.175),k40));

elseif nb==1
    c12=[];
    c13=[];
    c14=[];
    c15=[];
    c16=[];
else
    disp('Error')
end
c=[c1 c2 c3 c4 c5 c6 c7 c8 c9 c10 c12 c13 c14 c15 c16];
c=double(c);
ceq=[];
end
```

APPENDIX B: EQUATIONS IN CONCRETE FRAME DESIGN BY DDBD

Parameters	Equations	Remarks
Design Displacement, Δ_d	$\Delta_d = \dfrac{\sum_{i=1}^{n} m_i \Delta^2{}_i}{\sum_{i=1}^{n} m_i \Delta_i}$	where m_i and Δ_i are masses & displacements of the n significant mass locations respectively.
Design Storey Displacement, Δ_i	$\Delta_i = \delta_i \left(\dfrac{\Delta_c}{\delta_c} \right)$	where δ_i is inelastic mode shape, Δ_c is displacement at critical storey and δ_c is value of mode shape at critical storey.
Normalized Inelastic Mode Shape, δ_i	$\delta_i = \dfrac{H_i}{H_n}, n \leq 4$ $\delta_i = \dfrac{4}{3}\left(\dfrac{H_i}{H_n}\right)\left(1 - \dfrac{H_i}{4H_n}\right), n > 4$	where H is storey height and n is number of storey.
Design Displacement Drift Amplification for Higher Mode	$\Delta_{d,\omega} = \omega_\theta \Delta_d$, $\omega_\theta = 1.15 - 0.0034 H_n$	where ω_θ is drift amplification factor for higher modes.
Critical Storey Displacement :		
- From Drift Limit	$\Delta_c = \theta_c H$	where θ_c is code-specified non-structural drift limit.
- From Strain Limit	$\Delta_c = \Delta_y + \Delta_p$, $\Delta_y = \dfrac{\phi_y (H + L_{sp})^2}{3}$	where ϕ_y is yield curvature capacity, H is column

	$\Delta_p = \phi_p L_p H = (\phi_{ls} - \phi_y) L_p H$	height, L_{sp} is effective additional height representing strain penetration effects $= 0.022 f_{ye} d_{bl}$, f_{ye} and d_{bl} are yield strength *(in MPa)* and diameter of the longitudinal reinforcement respectively, ϕ_p is plastic curvature capacity, $L_p = kL_c + L_{sp} \geq 2L_{sp}$ is plastic hinge length, $k = 0.2\left(\frac{f_u}{f_y} - 1\right) \leq 0.08$ and L_c is length from the critical section to the point of contra-flexure in the member.
Equivalent Mass, m_e	$m_e = \frac{\sum_{i=1}^{n}(m_i \Delta_i)}{\Delta_d}$	where m_i and Δ_i are masses and displacements of the n significant mass locations respectively, Δ_d is design displacement and H is storey height.
Equivalent Height, H_e	$H_e = \frac{\sum_{i=1}^{n}(m_i \Delta_i H_i)}{\sum_{i=1}^{n}(m_i \Delta_i)}$	
Design Displacement Ductility, μ	$\mu = \frac{\Delta_d}{\Delta_y}$	where Δ_y is yield displacement.

Yield Displacement, Δ_y	$\Delta_y = \theta_y H_e$	where θ_y is yield drift.
Yield Drift, θ_y :		
- Equal Bays Width	$\theta_y = 0.5\varepsilon_y \frac{L_b}{h_b}$	where L_{bi} is bay width and h_b is depth of beams.
- Unequal Bays Width	$\theta_y = \frac{\sum_j \theta_{y_{1j}} + \sum_k \theta_{y_{2k}}}{n_b}$, $\theta_{y_i} = 0.5\varepsilon_y \frac{L_{bi}}{h_{bi}}$	
Equivalent Viscous Damping:		
- Constant Beams Depth	$\xi_{eq} = 0.05 + 0.565\left(\frac{\mu - 1}{\mu\pi}\right)$	where μ is displacement ductility.
- Varying Beams Depth	$\xi_{eq} = \frac{\sum_{i=1}^{n}\left(\sum_{k=1}^{m} m_k\right)\theta_i \xi_i}{\sum_{i=1}^{n}\left(\sum_{k=1}^{m} m_k\right)\theta_i}$	where there are ***m*** locations of plastic hinges at each beam level, θ_i is the design drift at level ***i*** $= \left(1.00 - 0.5\frac{H_i}{H_n}\right)\theta_o$ and the damping ξ_i at level ***i*** is based on the drift ductility at that level: $\mu = \frac{\theta_i}{\theta_{yi}}$
Damping Modifier, R_ξ	$R_\xi = \left(\frac{0.07}{0.02 + \xi_{eq}}\right)^{0.5}$	where ξ_{eq} is equivalent viscous damping.
Effective Stiffness, K_e	$K_e = \frac{4\pi^2 m_e}{T_e^2}$	where m_e and T_e are effective mass and period of the substituting structure.
Design Base - Shear Force, V_{base}	$V_{base} = K_e \Delta_d$	where K_e is effective stiffness and Δ_d is design displacement

Distribution of Base- Shear to Floor Levels, F_i	$F_i = F_t + 0.9V_B \frac{m_i\Delta_i}{\sum_{i=1}^{n} m_i\Delta_i}$	where $F_t = 0.1V_B$ at roof level & $F_t = 0$ at all other level
Total Overturning Moment induced by Lateral Force, OTM	$OTM = \sum_{i=1}^{n} F_i H_i$	where F_i is force at each floor level, H_i is storey height and n is number of storeys .
Internal Forces that equilibrate OTM	$OTM = \sum M_{C_j} + T.L_{base}$	where M_{C_j} are column base moments, T=C are seismic axial forces in the exterior columns, and L_{base} is distance between T and C.
Seismic Axial Forces induced by Seismic Beam Shears, $T \,\&\, C$	$T = C = \sum_{i=1}^{n} V_{B_i} = \frac{\left(\sum_{i=1}^{n} F_i H_i - \sum_{j=1}^{m} M_{C_j}\right)}{L_{base}}$	
Seismic Beams Shear, V_{Bi}	$V_{Bi} = T.\frac{V_{Si}}{\sum_{i=1}^{n} V_{Si}}$	where T *is* seismic axial forces in the exterior columns and V_{Si} is storey shear.
Storey Shear, V_{Si}	$V_{Si} = \sum_{k=i}^{n} F_k$	where F_i is force at each floor level
Beams Moment at Column Centerline induced by Lateral Force	$M_{Bi,l} + M_{Bi,r} = V_{B_i}.L_{B_i}$	where V_{Bi} is beam seismic shear , L_{B_i} is beam span between column centerlines, and $M_{Bi,l}$ & $M_{Bi,r}$ are beam moments at column centerlines at left & right end of the beam respectively.
Beams Plastic Hinge Moment at Column Face, M_{Bi}	$M_{Bi} = \left(\frac{M_{Bi,l} + M_{Bi,r}}{2}\right) - \frac{V_{Bi}h_c}{2}$	

Column Shears		
- Exterior Columns, V_{ce}	$V_{ce} = \dfrac{V_s}{n_{ce} + 2n_{ci}}$	where V_S is storey shear, n_{ce} & n_{ci} are number of exterior and interior columns respectively
- Interior Columns, V_{ci}	$V_{ci} = 2V_{ce}$	where V_{ce} *is* exterior column shear
Columns Moment		
- Exterior Columns	$[M_c] = \begin{cases} \dfrac{V_B L_B}{2}, i = n \\ \dfrac{V_B L_B}{4}, i \in (2, n-1) \end{cases}$	where V_B is beam seismic shear and L_B is beam span between column centerlines
- Interior Columns	$[M_c] = \begin{cases} (V_B L_B)_{average}, i = n \\ \dfrac{(V_B L_B)_{average}}{2}, i \in (2, n-1) \end{cases}$	where V_B is average beam seismic shear and L_B is average span of beams that input to the column
Design Forces		
Beams Flexure		
- Plastic Hinges	$M_{B,des} = \phi^o M_{B,i}$	where M_{Bi} is beam plastic hinge moment and ϕ^o is over-strength factor
- Span, at distance ***x*** from the left support	$M_x = M_{E,l}{}^o + \left(M_{E,r}{}^o - M_{E,l}{}^o\right)\dfrac{x}{L_B} + \dfrac{w_G L_B}{2}x - \dfrac{w_G x^2}{2}$	where moments $M_{E,l}{}^o$ and $M_{E,r}{}^o$ are over-strengthen moments at left and right column centerlines, taking due account of signs, and distance x and L_B are measured from the column centerline. The gravity load w_G assumed constant along the span.
Beam Shear, at distance ***x*** from the left support	$V_x = \dfrac{\left(M_{E,r}{}^o - M_{E,l}{}^o\right)}{L_B} + \dfrac{w_G{}^o L_B}{2} - w_G{}^o x$	
Column Flexure	$M_{des} = \phi^o \omega_f M_c$	where M_c is column moment found from analysis,

		ϕ^o is over-strength factor and ω_f is dynamic amplification factor.
Column Shear	$V_{des} = \phi^o V_E + 0.1\mu V_{E,base} \le \frac{M_t^{\ o} + M_b^{\ o}}{H_C}$	where shear demand V_E correspond to the design lateral force distribution found from the DDBD process, $V_{E,base}$ is value of V_E at the base of the column, μ is system and the upper limit of column shear corresponds to development of plastic hinging, at over-strength capacity at the top and bottom of the column, separated by the clear column height H_c.
Column Axial	Exterior Columns: $P_{des} = T + P_G$ Interior Columns: $P_{des} = P_G$	where T is seismic axial force and P_G is probable axial gravity load
Over-strength factor, ϕ^o	$\phi^o = 1.25$, if strain hardening considered $\phi^o = 1.6$, if strain hardening ignored	
Dynamic Amplification Factor, ω_f	$\omega_{f,c} = 1.15 + 0.13(\mu^o - 1)$, from the first storey to ¾ point of the structure height $\omega_{f,t} = 1.00$, at the top of the structure	where $\mu^o = \frac{\mu}{\phi^o} \ge 1$ is reduced ductility

<table>
<tr><td>Strength Reduction Factor, ϕ_f</td><td>$\phi_f = 0.9$</td><td></td></tr>
<tr><td colspan="3">Strengths of Concrete Section</td></tr>
<tr><td colspan="3">Beams Section:</td></tr>
<tr><td>- Flexure</td><td>$M_{rd} = m_{\lim} f_{ce} b h_b^{\ 2}$, concrete section

$M_{rd,l} = 0.9 h_b f_{ye} A_{s,l}$, to be provided by reinforcement</td><td>where $m_{\lim} = \left(\frac{x}{d}\right)_{\lim}\left(0.411\left(\frac{x}{d}\right)_{\lim}\right)$
$$\left(\frac{x}{d}\right)_{\lim} = \begin{cases} \frac{\delta - 0.44}{1.25}, & f_{cu} \leq 35 \\ \frac{\delta - 0.56}{1.25}, & f_{cu} > 35 \end{cases}$$
<table>
<tr><td>% of Distribution</td><td>0</td><td>5</td><td>10</td><td>15</td><td>20</td><td>30</td></tr>
<tr><td>δ</td><td>1</td><td>0.95</td><td>0.9</td><td>0.85</td><td>0.75</td><td>0.7</td></tr>
</table>
, b_b & h_b are beams width and depth respectively, $A_{s,l}$ is longitudinal reinforcement f_{ce} & f_{ye} are amplified characteristic material strength of concrete and steel respectively,</td></tr>
<tr><td>- Shear</td><td>Element without Shear Reinforcement:
$V_{Rd,c} = 0.12k(100\rho f_{ck})^{\frac{1}{3}} b_b h_b \geq 0.035\sqrt{f_{ck}}\, k^{\frac{3}{2}} b_b h_b$</td><td>where $k = 1 + \sqrt{\frac{200}{d}} \leq 2$ and $\rho = \frac{A_{s,l}}{b_w h_b} \leq 0.02$</td></tr>
</table>

	Contribution of Shear Reinforcement: $V_{w,d} = \frac{A_{sw} z f_{ywd} \cot\theta}{S}$	where $z = 0.9h_b$ $\cot\theta = \frac{y}{z}, 1 \le \cot\theta \le 2.5$ $\theta = 0.5\sin^{-1}\left[\frac{5.56[V_{Ed}]}{b_w h_b\left(1-\frac{f_{ck}}{250}\right)f_{ck}}\right]$
Columns Section:		
- Axial	$N_{rd} = f_{cd} b_c h_c$	where b_c & h_c is section dimension of column and f_{cd} is concrete design strength.
- Flexure	$M_{rd} = f_{cd} b_c h_c^2$	
- Shear	Concrete: Similar as that of the beam	
	Contribution of Reinforcement: $V_{Rd,w} = \frac{z}{H_{cl}} N_{Ed} + \rho_w b_c z f_{ywd} \cot\theta$	

APPENDIX C: CONSTRAINTS FUNCTIONS

C-1: Mathematical Representation for Vertical Distribution of Beam Shears

All internal response of the frame are a dependent on the vertical distribution of beam shears, $[V_B]$. Therefore, its evaluation presented here:

The seismic beam shear, $[V_B] = \dfrac{T[V_s]}{Sum(V_s)}$, $\quad V_{s,i} = \sum_{k=i}^{n} F_k$

$$[F] = \begin{cases} (0.1 + 0.9k_{9_n})V_{base} & for \; i = n \\ 0.9[k_9]V_{base} & for \; i \in (1, n-1) \end{cases}$$

where $[k_9] = \dfrac{[m]^*[\Delta]}{[m]*[\Delta]}$ and $V_{base} = K_e \Delta_d = \left(\dfrac{4\pi^2 m_e}{T_e^2} \right) \Delta_d$

$$\Rightarrow V_{base} = \left(\frac{2\pi\Delta_{cor_{5\%}}}{\omega_\theta T_{cor}} \right)^2 \left(\frac{([m]*[\Delta])}{\Delta_d^2} \right) \left(\frac{0.07}{0.02 + \xi_{eq}} \right)$$

Taking, $\left(\dfrac{2\pi\Delta_{cor_{5\%}}}{\omega_\theta T_{cor}} \right)^2 = k_{10}$ and $\left(\dfrac{([m]*[\Delta])}{\Delta_d^2} \right) = k_{11}$

$$V_{base} = k_{10} k_{11} \left(\frac{0.07}{0.02 + \xi_{eq}} \right)$$

$$\Rightarrow [V_s] = \begin{cases} (0.1 + 0.9k_{9_n})k_{10}k_{11} \left(\dfrac{0.07}{0.02 + \xi_{eq}} \right), i = n \\ \displaystyle\sum_{k=i}^{n} 0.9k_{9_k} k_{10}k_{11} \left(\dfrac{0.07}{0.02 + \xi_{eq}} \right), i \in (1, n-1) \end{cases}$$

$$Sum(V_s) = (0.1 + 0.9k_{9_n})k_{10}k_{11} \left(\frac{0.07}{0.02 + \xi_{eq}} \right) + \sum_{i=1}^{n-1} \sum_{k=i}^{n} 0.9k_{9_k} k_{10}k_{11} \left(\frac{0.07}{0.02 + \xi_{eq}} \right)$$

$$OTM = [F]' * [H] = \sum_{i=1}^{n} ([F] .* [H])_i$$

$$\Rightarrow OTM = (0.1 + 0.9k_{9_n})k_{10}k_{11}\left(\frac{0.07}{0.02+\xi_{eq}}\right)H_n + \sum_{i=1}^{n-1} 0.9k_{9_k}k_{10}k_{11}\left(\frac{0.07}{0.02+\xi_{eq}}\right)H_i$$

$$\therefore T = \frac{OTM - k_{12}k_{10}k_{11}\left(\frac{0.07}{0.02+\xi_{eq}}\right)}{k_{13}}$$

where $k_{12} = 0.6H_1$ and $k_{13} = L_{base}$

When the width of bays is varying, T should be calculated for each bay separately.

Thus, T becomes $T = \dfrac{OTM - k_{12}k_{10}k_{11}\left(\frac{0.07}{0.02+\xi_{eq}}\right)}{L_{B,i}}$

$$\Rightarrow [V_B] = \left(\frac{[T]}{Sum(V_s)}\right)\begin{cases} (0.1 + 0.9k_{10_n})k_{11}k_{12}\left(\frac{0.07}{0.02+\xi_{eq}}\right), i = n \\ \sum_{k=i}^{n} 0.9k_{10_k}k_{11}k_{12}\left(\frac{0.07}{0.02+\xi_{eq}}\right), i \in (1; n-1) \end{cases}$$

C-2: Beams Flexural Constraint

The resistance moment of a concrete beam section,

$$M_{Rd} = m_{\lim} f_{ck}[b_b] .* \left[h_b - d_b'\right]^2 + (0.9 f_{ye} h_b A_{s,l})$$

Let $d_b' = c_{cover} + \frac{d_{b,l}}{2} = k_{15}$, where c_{cover} is concrete cover and $d_{b,l}$ is diameter of beam longitudinal reinforcement and $m_{\lim} f_{ck}[b_b] = [k_{16}]$

$$\Rightarrow [M_{Rd}] = [k_{16}] .* ([x_1 - k_{15}])^2 + (0.9 f_{ye}[x_1] .* [x_2])$$

The design moment, $[M_{Ed}]$:

At plastic hinges: $$[M_{Ed}] = [V_B].*\left(\frac{[L_B - x_2]}{2}\right)$$

At span: $$[M_{Ed}] = [k_{17}].*[V_B] + [k_{18}]$$

where $$[k_{17}] = \frac{\phi^o [L_B]}{2} \quad \text{and} \quad k_{18} = \frac{w_G{}^o [L_B]^2}{8}$$

Thus, the flexural constraint for beams are summarized as follows:

At plastic hinges:

$$\begin{cases} \left(\left(\frac{[L_B - x_4]}{2}\right).*\left(\frac{[T]}{Sum(V_s)}\right).*\left((0.1 + 0.9k_{9_n})k_{10}k_{11}\left(\frac{0.07}{0.02 + \xi_{eq}}\right)\right) - \left([k_{16}].*([x_1 - k_{15}])^2 + (0.9 f_{ye}[x_1].*[x_2])\right)\right) \le 0, i = n \\ \left(\left(\frac{[L_B - x_4]}{2}\right).*\left(\frac{[T]}{Sum(V_s)}\right).*\left(\sum_{k=i}^{n} 0.9k_{10_k} k_{11}k_{12}\left(\frac{0.07}{0.02 + \xi_{eq}}\right)\right) - \left([k_{16}].*([x_1 - k_{15}])^2 + (0.9 f_{ye}[x_1].*[x_2])\right)\right) \le 0, i \in (1; n-1) \end{cases}$$

At span:

$$\begin{cases} \left([k_{17}].*\left(\frac{[T]}{Sum(V_s)}\right).*\left((0.1 + 0.9k_{9_n})k_{10}k_{11}\left(\frac{0.07}{0.02 + \xi_{eq}}\right)\right) + [k_{18}] - \left([k_{16}].*([x_1 - k_{15}])^2 + (0.9 f_{ye}[x_1].*[x_2])\right)\right) \le 0, i = n \\ \left([k_{17}].*\left(\frac{[T]}{Sum(V_s)}\right).*\left(\sum_{k=i}^{n} 0.9k_{9_k} k_{10}k_{11}\left(\frac{0.07}{0.02 + \xi_{eq}}\right)\right) + [k_{18}] - \left([k_{16}].*([x_1 - k_{15}])^2 + (0.9 f_{ye}[x_1].*[x_2])\right)\right) \le 0, i \in (1; n-1) \end{cases}$$

C-3 Beams Shear Constraint

The shear resistance of a concrete beam section, $V_{Rd} = V_{Rd,c} + V_{\omega,d}$

$$V_{Rd,c} = \max\begin{cases} (0.12k(100\rho f_{ck})^{1/3})[b_b].*[h_b] \\ 0.035\sqrt{f_{ck}}k^{3/2}[b_b].*[h_b] \end{cases}, \text{ where } k = 1 + \sqrt{\frac{200}{h_b - d_b'}} \text{ and } \rho = \left[\frac{A_{s,l_b}}{b_b h_b}\right]$$

Let $$(0.12(100 f_{ck})^{1/3})[b_b^{1/3}] = [k_{19}] \quad \text{and} \quad 0.035\sqrt{f_{ck}}[b_b] = [k_{20}]$$

$$\Rightarrow V_{Rd,c} = \max\begin{cases} \min\left(1+\sqrt{\dfrac{200}{[x_1]-k_{15}}},2\right)[k_{19}].*[x_1]^{2/3}.*[x_2]^{1/3} \\ \min\left(1+\sqrt{\dfrac{200}{[x_1]-k_{15}}},2\right)[k_{20}].*[x_1] \end{cases}$$

$$V_{\omega,d} = \frac{0.9A_{s,v_b}h_b f_{ywd}\cot\theta}{S_b}, \quad S_b = \frac{\pi d_{b,\omega}{}^2 L_b}{4A_{sv,b} - \pi d_{b,\omega}{}^2}$$

Let $\pi d_{b,\omega}{}^2 = k_{21}$, *where* $d_{b,\omega}$ *is the diameter of beam web-reinforcement* and $\dfrac{0.9 f_{ywd}}{L_b} = [k_{22}]$

$$\Rightarrow V_{\omega,d} = \frac{(4[x_3]-k_{21}).*[k_{22}].*[x_3].*[x_1]\cot\theta}{k_{21}}$$

$$\text{where } \theta = \sin^{-1}\left[\frac{[k_{23}].*[V_{Ed}]}{[x_1]}\right], \ [k_{23}] = \frac{5.56}{\left(1-\dfrac{f_{ck}}{250}\right)f_{yk}[b_b]}$$

Thus, shear resistance for beams section is:

$$V_{Rd} = \max\left\{\begin{matrix} \min\left(1+\sqrt{\dfrac{200}{[x_1]-k_{15}}},2\right)[k_{19}].*[x_1]^{2/3}.*[x_2]^{1/3} \\ \min\left(1+\sqrt{\dfrac{200}{[x_1]-k_{15}}},2\right)[k_{20}].*[x_1] \end{matrix}\right\} + \frac{(4[x_3]-k_{21}).*[k_{22}].*[x_3].*[x_1]\cot\theta}{k_{21}}$$

The design shear force of beams can be calculated as:

$$[V_{Ed}] = \phi^o[V_B] + [k_{24}] \text{ , where } [k_{24}] = \frac{w_G{}^o L_B}{2}$$

Therefore, the shear constraint of beams expressed as:

$$\begin{cases} \left(\left(\dfrac{\phi^o[T]}{Sum(V_s)}\right).*\left((0.1+0.9k_{9_n})k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)\right)+[k_{24}]-V_{Rd}\right) \le 0, i = n \\ \left(\left(\dfrac{[L_B-x_2]}{2}\right).*\left(\dfrac{[T]}{Sum(V_s)}\right).*\left(\displaystyle\sum_{k=i}^{n}0.9k_{9_k}k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)\right)+[k_{24}]-V_{Rd}\right) \le 0, i \in (1,n-1) \end{cases}$$

C-4: Beams Reinforcement Bounds

- Longitudinal Reinforcement:

$$\left(A_{s,l_{\min}} \le A_{s,l} \le A_{s,l_{\max}}\right)_b$$

$$\left(A_{s,l_{\min}}\right)_b = \frac{0.5 f_{ctm}}{f_{yk}} b_b h_b, \qquad f_{ctm} = 0.3\left(f_{ck}\right)^{2/3}$$

$$\left(A_{s,l_{\max}}\right)_b = \frac{0.0018 f_{cd}}{\mu_\phi \varepsilon_{yd} f_{yd}} b_b h_b$$

Let $\frac{0.5 f_{ctm}}{f_{yk}}[b_b] = [k_{25}]$ and $\frac{0.0018 f_{cd}}{\mu_\phi \varepsilon_{yd} f_{yd}}[b_b] = k_{26}$

Then, the bounds on longitudinal reinforcement are:

$$[k_{25}].*[x_1] - [x_2] \le 0$$

$$[x_2] - [k_{26}].*[x_1] \le 0$$

- Shear Reinforcement

$$\left(A_{s,v_{\min}} \le A_{s,v}\right)_b \equiv \left(S \le S_{\max}\right)_b$$

$$S_b = \frac{\pi d_{b,\omega}{}^2 L_b}{4A_{sv,b} - \pi d_{b,\omega}{}^2} = \frac{k_{21}[L_b]}{4[x_3] - k_{21}}$$

$$S_{\max,b} = \min\begin{cases} \left[\dfrac{x_1}{4}\right] \\ 24 d_{b,\omega} = k_{27} \\ 225 \\ 8 d_{b,l} = k_{28} \end{cases}$$

The bound on shear reinforcement of beams is:

$$\frac{k_{23}[L_b]}{4[x_4] - k_{23}} - \min\begin{cases} \left[\dfrac{x_1}{4}\right] \\ k_{27} \\ 225 \\ k_{28} \end{cases} \le 0$$

C-5: Exterior Columns Flexural Constraint

The resistance of concrete column section for axial force and bending moment are $N_{Rd} = [k_{29}].*[x_2]$ and $M_{Rd} = [k_{29}].*[x_2]^2$, respectively where $[k_{29}] = f_{cd}[b_c]$.

The design moment of exterior column section, $M_{Ed} = k_{30}[M_c]$

where $k_{30} = \phi^o \omega_f$ and $[M_c] = \begin{cases} [V_B].*\dfrac{[L_B]}{2}, i = n \\ [V_B].*\dfrac{[L_B]}{4}, i \in (2, n-1) \end{cases}$

For column base, the design moment is, $M_{Ed} = k_{31}k_{12}V_{base}$, where $k_{31} = \dfrac{1}{n_{ce} + 2n_{ci}}$

The design axial load for exterior columns is the total of seismic beam shear and axial gravity load.

The flexural constraint for exterior columns is:

$$\begin{cases} \dfrac{T + P_G}{[k_{29}].*[x_4]} + \left(\dfrac{\left[\dfrac{L_B}{2}\right].*\left(\dfrac{[T]}{Sum(V_s)}\right).*(0.1 + 0.9k_{9_n})k_{10}k_{11}\left(\dfrac{0.07}{0.02 + \xi_{eq}}\right)}{[k_{29}].*[x_4]^2} \right) - 1 \le 0, i = n \\ \dfrac{T + P_G}{[k_{29}].*[x_4]} + \left(\dfrac{\left[\dfrac{L_B}{4}\right].*\left(\dfrac{[T]}{Sum(V_s)}\right).*\sum_{k=i}^{n} 0.9k_{9_k}k_{10}k_{11}\left(\dfrac{0.07}{0.02 + \xi_{eq}}\right)}{[k_{29}].*[x_4]^2} \right) - 1 \le 0, i \in (2, n-1) \\ \dfrac{P_G}{[k_{29}].*[x_4]} + \left(\dfrac{k_{31}k_{13}k_{10}k_{11}\left(\dfrac{0.07}{0.02 + \xi_{eq}}\right)}{[k_{29}].*[x_4]^2} \right) - 1 \le 0, i = 1 \end{cases}$$

C-6: Exterior Columns Shear Constraint

The shear resistance of a concrete column sections, $V_{Rd} = V_{Rd,c} + V_{\omega,d}$

where $V_{Rd,c} = \max\begin{cases} \left(0.12k(100\rho f_{ck})^{1/3}\right)[b_c].*[h_c] \\ 0.035\sqrt{f_{ck}}\,k^{3/2}[b_c].*[h_c] \end{cases}$

$$k = 1 + \sqrt{\frac{200}{h_c - d_c^{'}}} \text{ and } \rho = \left[\frac{A_{s,l_c}}{b_c h_c}\right]$$

Let $\left(0.12(100 f_{ck})^{1/3}\right)\left[b_c^{1/3}\right] = [k_{32}]$, $0.035\sqrt{f_{ck}}[b_c] = [k_{33}]$ and $d_c^{'} = c_{cover} + \frac{d_{c,l}}{2} = k_{34}$,

where c_{cover} is concrete cover and $d_{c,l}$ is diameter of the column longitudinal reinforcement

$$`\Rightarrow V_{Rd,c} = \max\begin{cases} \min\left(1 + \sqrt{\frac{200}{[x_4] - k_{34}}}, 2\right)[k_{32}].*[x_4]^{2/3}.*[x_5]^{1/3} \\ \min\left(1 + \sqrt{\frac{200}{[x_4] - k_{30}}}, 2\right)[k_{33}].*[x_4] \end{cases}$$

$$V_{\omega,d} = \left(\frac{0.9[x_4]}{H_{cl}} N_{Ed}\right) + \frac{0.9A_{s,v_c} h_c f_{ywd} \cot\theta}{S_c}, \quad S_c = \frac{\pi d_{c,\omega}{}^2 L_c}{4A_{sv,c} - \pi d_{c,\omega}{}^2}$$

Let $\pi d_{c,\omega}{}^2 = k_{35}$, *where* $d_{c,\omega}$ *is the diameter of beam web-reinforcement* and $\frac{0.9 f_{ywd}}{L_c} = [k_{36}]$

$$\Rightarrow V_{\omega,d} = \left(\frac{0.9[x_4]}{H_{cl}} N_{Ed}\right) + \frac{(4[x_6] - k_{35}).*[k_{36}].*[x_6].*[x_4]\cot\theta}{k_{35}}$$

where $\theta = \sin^{-1}\left[\frac{[k_{37}].*[V_{Ed}]}{[x_4]}\right]$, $[k_{37}] = \frac{5.56}{\left(1 - \frac{f_{ck}}{250}\right) f_{yk}[b_c]}$

Then, shear resistance for columns is:

$$V_{Rd} = \left\{ \max \left\{ \begin{matrix} \min\left(1+\sqrt{\dfrac{200}{[x_4]-k_{34}}},2\right)[k_{32}].*[x_4]^{2/3}.*[x_5]^{1/3} \\ \min\left(1+\sqrt{\dfrac{200}{[x_4]-k_{30}}},2\right)[k_{33}].*[x_4] \end{matrix} \right. \right\} + \left\{ \left(\frac{0.9[x_4]}{H_{cl}} N_{Ed} \right) + \frac{(4[x_6]-k_{35}).*[k_{36}].*[x_6].*[x_4]\cot\theta}{k_{35}} \right\}$$

The design shear force for exterior columns is:

$$[V_{Ed}] = \phi^o [V_E] + 0.1\mu V_{E,base}$$

where $V_{E,base} = V_{s,base}$ and $[V_E] = [k_{31}].*[V_s]$

Thus, the shear constraint for exterior columns summarized as:

$$\begin{cases} \left(\phi^o [k_{31}].*(0.1+0.9k_{9_n})k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right) + 0.1\mu\sum\limits_{i=1}^{n} 0.9k_{9_i}k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)\right) - V_{Rd}, i = n \\ \left(\phi^o [k_{31}].*\sum\limits_{k=i}^{n} 0.9k_{9_k}k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right) + 0.1\mu\sum\limits_{i=1}^{n} 0.9k_{9_i}k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)\right) - V_{Rd}, i \in (1, n-1) \end{cases}$$

C-7: Interior Columns Flexural Constraint

The computation of flexural strength is as shown in the exterior column section.

The design moment of exterior column section, $M_{Ed} = k_{30}[M_c]$

$$\text{where } [M_c] = \begin{cases} ([V_B].*[L_B])_{average}, i = n \\ \dfrac{([V_B].*[L_B])_{average}}{2} i \in (2, n-1) \end{cases}$$

For column base, the design moment is, $M_{Ed} = 2k_{31}k_{12}V_{base}$, where $k_{31} = \dfrac{1}{n_{ce} + 2n_{ci}}$

The design axial load for interior columns is the probable axial gravity load.

Therefore, for interior columns the flexural constraint is:

$$\begin{cases} \dfrac{P_G}{[k_{29}].*[x_4]} + \left(\dfrac{[L_B]_{average}.*\left(\dfrac{[T]}{Sum(V_s)}\right).*(0.1+0.9k_{9_n})k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)}{[k_{29}].*[x_4]^2} \right) - 1 \leq 0, i = n \\ \dfrac{P_G}{[k_{29}].*[x_4]} + \left(\dfrac{\left[\dfrac{L_B}{4}\right]_{average}.*\left(\dfrac{[T]}{Sum(V_s)}\right).*\sum_{k=i}^{n} 0.9k_{9_k}k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)}{[k_{29}].*[x_4]^2} \right) - 1 \leq 0, i \in (2, n-1) \\ \dfrac{P_G}{[k_{29}].*[x_4]} + \left(\dfrac{k_{31}k_{13}k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)}{[k_{29}].*[x_4]^2} \right) - 1 \leq 0, i = 1 \end{cases}$$

C-8: Interior Columns Shear Constraint

The shear resistance is calculated as that of the interior columns and the design shear force $(V_{Ed})_{\text{int}} = 2(V_{Ed})_{ext}$.

Thus, interior columns shear constraints is:

$$\begin{cases} 2\left(\phi^o[k_{31}].*(0.1+0.9k_{9_n})k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right) + 0.1\mu\sum_{i=1}^{n} 0.9k_{9_i}k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)\right) - V_{Rd}, i = n \\ 2\left(\phi^o[k_{31}].*\sum_{k=i}^{n} 0.9k_{9_k}k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right) + 0.1\mu\sum_{i=1}^{n} 0.9k_{9_i}k_{10}k_{11}\left(\dfrac{0.07}{0.02+\xi_{eq}}\right)\right) - V_{Rd}, i \in (1, n-1) \end{cases}$$

C-9: Columns Reinforcement Bounds

- Longitudinal Reinforcement:

$$\left(A_{s,l_{\min}} \leq A_{s,l} \leq A_{s,l_{\max}}\right)_c$$

where $\left(A_{s,l_{\min}}\right)_c = 0.01[b_c][h_c]$ and $\left(A_{s,l_{\max}}\right)_c = 0.04[b_c][h_c]$

Let, $0.01[b_c] = [k_{38}]$ and then the bounds on longitudinal reinforcement of columns are:

$$[k_{38}].*[x_4] - [x_5] \leq 0$$

$$[x_5] - 4[k_{38}].*[x_4] \leq 0$$

- Shear Reinforcement

$$\left(A_{s,v_{\min}} \le A_{s,v}\right)_c \equiv \left(S \le S_{\max}\right)_c$$

$$S_c = \frac{\pi d_{b,\omega}{}^2 L_c}{4A_{sv,c} - \pi d_{b,\omega}{}^2} = \frac{k_{35}\left[L_c\right]}{4\left[x_6\right] - k_{35}}$$

$$S_{\max,c} = \min\begin{cases}\left[\dfrac{b_c}{2}\right] = \left[k_{39}\right] \\ 175 \\ 8d_{c,l} = k_{40} \\ \end{cases}$$

The bound on shear reinforcement of columns is:

$$\frac{k_{35}\left[L_c\right]}{4\left[x_6\right] - k_{35}} - \min\begin{cases}\left[k_{39}\right] \\ 175 \ \le 0 \\ k_{40}\end{cases}$$

APPENDIX D: INTERIOR-POINT METHOD

As described in A. Geletu [11] , to approach the optimal solution from the interior of the feasible set, interior-point methods have two basic requirements:

- The interior of the feasible sets should not be empty,
- Almost all iterates should remain in (the interior of the) feasible set

The interior of the feasible set said to be non-empty if:

- There is $x^* \in \Re$ such that $h_i(x^*) = 0, \nabla g_j(x^*) \le 0$,$1 \le i \le m$, $j \in J$
- The Mangasarian-Frmomvitz Constraint Qualification (MFCQ) is satisfied at a feasible point x^*

Mangasarian-Frmomvitz Constraint Qualification

Let x^* be a feasible point of the problem, then MFCQ is said to be satisfied at x^* if there is a vector $d \in \Re$, $d \ne 0$, such that

i. $d^{|T} \nabla g_j(x^*) < 0$, $j \in J$

ii. $d^{|T} \nabla h_i(x^*) = 0$, $1 \le i \le m$

MCFQ indicates that, there is α such that

- $x^* + \alpha d > 0$
- $g(x^* + \alpha d) \approx g(x^*) + \alpha d^T \nabla g_j(x^*) < 0$, $j \in J$
- $h_i(x^* + \alpha d) \approx h_i(x^*) + \alpha d^T \nabla h_i(x^*) < 0$, $1 \le i \le m$

These implies that $x^* + \alpha d$ is an interior of the feasible set and the interior of the feasible set is thus not empty.

To force almost all iterates remain in the interior of the feasible set interior point methods use barrier functions.

A barrier function is a function B defined on the interior of a feasible set S such that:

i. B is continuous,

ii. $B(x) \geq 0$,

iii. $B(x) \rightarrow \infty$ as $x \rightarrow$ the boundary of S

A well-known barrier function is the logarithmic barrier function

$$B(x,\mu) = f(x) - \mu(\sum_{1}^{m} \log(g_j(x)) + \sum_{i=1}^{n} \log(x_i)$$

, where μ is known as barrier parameter.

After defining the barrier function, the interior-point method considers the parametric problem below instead of the original non-linear programing problem

$$(NLP)_\mu \qquad \min B(x,\mu)$$

$$S.t.$$

$$h_i(x) = 0,\ 1 \leq i \leq m$$

To find the optimal solution x_μ of $(NLP)_\mu$ for a fixed value of the barrier parameter μ, the necessary optimality condition is to be satisfied for the Lagrange function of $(NLP)_\mu$.

$$L_\mu(x,\mu) = f(x) - \mu(\sum_{j=1}^{m} \log(g_j(x)) + \sum_{i=1}^{n} \log(x_i) - \sum_{i=1}^{m} \lambda_i h_i(x)$$

Thus, the question will be solving the following system

$$h_i(x) = 0$$

$$\nabla f(x) - \mu\left(\sum_{j=1}^{m} \frac{1}{g_j(x)} \nabla g_j(x) + \sum_{i-1}^{n} \frac{1}{x_i} e_i\right) - \sum_{i=1}^{m} \lambda_i \nabla h_i(x) = 0$$

Commonly, this system is solved iteratively using the Newton Method.

Algorithm for Newton Method:

$F_\mu(x,\lambda)=0$, for a fixed μ, where

$$F_\mu(x,\lambda)=\begin{pmatrix} h_i(x) \\ \nabla f(x)-\mu\left(\sum_{j=1}^{m}\frac{1}{g_j(x)}\nabla g_j(x)+\sum_{i-1}^{n}\frac{1}{x_i}e_i\right)-\sum_{i=1}^{m}\lambda_i\nabla h_i(x)\end{pmatrix}$$

Step 0: Choose (x_0,λ_0)

Step k: Find $(\Delta_x^k,\Delta_\lambda^k)=d$ by solving the linear system

$$J_{F_\mu}(x_k,\lambda_k)d=-F_\mu(x_k,\lambda_k)$$

- Determine a step length α_k
- Set $x_{k+1}=x_k+\alpha_k\Delta_x^k$ and $\lambda_{k+1}=x_k+\alpha_k\Delta_\lambda^k$

STOP if convergence achieved; otherwise CONTINUE.

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