

ADDIS ABABA UNIVERSITY
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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING



**STRUCTURAL STABILITY ANALYSIS OF CORRUGATED
REINFORCED CONCRETE SHELL STRUCTURE**

A Thesis in Structural Engineering

By

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A thesis submitted to the School of Graduate Studies in partial fulfillment for the requirements
for the Degree of Master of Science in Civil Engineering (Structures)

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UNDERTAKING

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ABSTRACT

The structural stability performance of shell structure is advanced. Corrugated shells are among the most unique shell structures available. These shells are shells of translation made up of a series of barrels shells with opposite Gaussian curvature connected edge to edge. The structural stability and performance of these types of shell structures in variable geometry and support condition must be precisely identified to build them. Many researchers have studied the structural performance of different shell structures. Studies on corrugated shell structures are few.

The structural performance of a corrugated reinforced concrete shells with various curvatures and support position is the focus of this study. This research has been started by modeling individual barrels and corrugated multi-span shells by using SAP2000 software. The shells are analyzed by using SAP2000 software using various parameters. The result gain from SAP2000 is validated by using ANSYS software. Then the results of the SAP2000 analysis have been collated and graphically presented. Finally, the behavior and performance of the shell structures have been discussed based on the numerical results. According to the research shortening the span length and radius increases the stability of cylindrical shell structures, in addition, increasing the thickness and number of corrugations increases the global stiffness of corrugated shell structures.

Key words: - Stability, Corrugation, Cylindrical shell,

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NOMENCLATURE

R	Radius (m)
L	Length (m)
T	Thickness of the shell (mm)
E	Young's modulus of concrete (kN/m ²)
N _x	Membrane normal force along x-axis
N _y	Membrane normal force along Y-axis
N _{xy}	Membrane shear force along xy-axis
Q _x	Force (out –of-plane) along x-axis
Q _y	Shear force (out –of-plane) along y-axis
σ _x	Normal stress along x- axis
σ _y	Normal stress along y-axis
σ _{xy}	Shear stress along xy-axis
σ _{yx}	Shear stress along yx-axis
M _x	bending moment along X-axis
M _y	bending moment along Y-axis
M _{xy}	bending moment along XY-axis
M _{yx}	bending moment along YX-axis
F _x	Load along X – axis
F _y	load along y – axis

F_z	load along z-axis
σ_x	Normal stress along x – axis
σ_y	Normal stress along y – axis
σ_{yx}	Shear stress along yx- axis
σ_{xy}	Shear stress along yx – axis
σ_{yz}	Shear stress along yz- axis
γ_{xy}	strain in the xy – axis
ϵ_x	strain in the x- direction
ϵ_y	strain in the y – direction
ν	poisons ratio
τ	pure shear force

CHAPTER ONE

INTRODUCTION

1.1 Background

The structural behaviors of shells are characterized by higher mechanical efficiency when they are compared to flat plates. Rather than the mass of the shell, the configuration or arrangement of shells determines their properties. The way the shell structures supports the external load differs significantly from how the flat plates does. These structures are meant to transport their loads mostly by membrane action, but bending stress is created by horizontal forces such as earthquakes [1]. Wave propagation necessitates the presence of heavy mass in order to have a greater effect on the structure. The damage caused by an earthquake in shell structure is very modest due to the relative lightweight nature of the shell construction [2]. Cylindrical shell roofs are popular because they contain a lot of open space without central support.

Construction of shell structures was started in ancient time. Historical shell roof structures can be seen in old historical buildings such as Istanbul's Hagias Sophia and Rome's Pantheon. Following World War II, shell buildings became popular as long-span concrete constructions. Arched constructions were also built in ancient times in some ancient towns. The dead load stress causes purely compressive bending moment on the surface of the shell structure. An adequate moment resistance must be provided to resist the bending moment by thickening the arc ring. Using steel or other materials is preferable than thickening the arc ring because it is very expensive. Ribbing the concrete is the other method to resist the moment; however, it is not safe for structural efficiency [3]. The general geometry of shell structures is critical to their ability to withstand earthquakes, and shell structures, due to their light weight and great geometric stiffness, withstand seismic loads exceedingly well [4].

In the construction industry, covering a vast floor space with as few supporting members as possible is a very desirable issue. Most interesting buildings, both old and modern, are constructed to meet this desire. Over all geometries, one of the most essential parameters that affect the structural behavior of such a shell is present [5]. Many investigations have revealed that the

geometry of shell structures has a significant impact on their structural behavior. Structural performance of corrugated reinforced shell structures was the focus of this research. The effective thickness of the shell cross section is increased by corrugation. It also increases the cross-section's bending capacity [6].

Shell structures are fascinating because of their high strength-to-weight ratio. These structures are both aesthetically pleasing and structurally sound. Shell structures perform well during earthquakes and are quite effective for covering large areas [7].

The approach of analysis and design must be efficient in order for these structures to perform efficiently. The final outcome of the researched result of this analytical study of shell structures was discussed and concluded at the end of the investigation.

1.2 Statement of the problem

To construct corrugated shells structures, we have to know about structural behavior of these shells. There are many studies in structural behavior of shell structures having geometrical shapes like spherical, cylindrical, hyperbolic shells. But researches in corrugated reinforced shell structures are few. This research had studied the structural performance of corrugated reinforced concrete shell structures having positive and negative Gaussian curvature with variable geometrical parameters. This can be used for supporting Guide line for Design and construction of reinforced corrugated shells structures.

1.3 Objective

1.3.1 General objective

The main objective of this research is to study combined effect corrugation of shell structure on structural behavior of corrugated shell structure in variables parameters.

1.3.2 Specific objective

- Determine structural behavior such as force, moment, stress, and deflection effect using theoretical and finite element methods.
- Asses the performance of corrugated shell and compere its result with single barrels
- Examine the impact of various variables on the structural behavior of a corrugated shell

1.3 Scope

The purpose of this thesis is to investigate the structural performance of corrugated reinforced concrete shell structures using a variety of parameters. This study was conducted with the help of commercially available software SAP2000 and Excel. It is carried out by choosing and assuming various characteristics such as thickness, length, radius, support position. Single barrels, corrugated cylindrical shells and corrugated conical shells were included in this study. The applied loads are self-weight and imposed load. The purpose of this thesis is to investigate properties of combined effect of corrugated shell structures with various geometry and to assess how the corrugation improves the performance and stability of shell structure.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

Roof structures made of asbestos were mostly employed in the early 1970s. Asbestos is a soft gray fibers form of numerous naturally occurring minerals that can be found in metamorphic rocks, primarily in mountainous places. Because asbestos does not burn, it is often used to protect buildings from fire and heat loss [8]. Although asbestos is preferable structure for roofing, it is no longer in use because it causes diseases like lung cancer. This disease is caused by breezing in asbestos fibers. As a result of this other ruffing structures are preferred.



Figure 2. 1Asbestos cement corrugated roof

The corrugated roof structure concept was then used to design a reinforced concrete corrugated shell structure. They are commonly used on the roofs of churches, museums, and meeting halls. (Irina Viner Usmanova Rhythmic Gymnastic Center) is a notable example of recent corrugated cylindrical shell from Russia [9]. It is located in the Moscow Luzhiniki Olympic complex and it

was constructed by steel and concrete composite. It was finished in 2019. Its roof is made up of corrugated shell structure that is done by connecting single barrels by with varying internal angles and Gaussian curvature. The construction started by constructing shell having larger internal angle, then it followed by connecting similar barrels with smaller internal angle. The internal angle of the subsequent cylinders continues to decrease, and the cylinders become flattened. The structure received good astatic value from this fetcher.

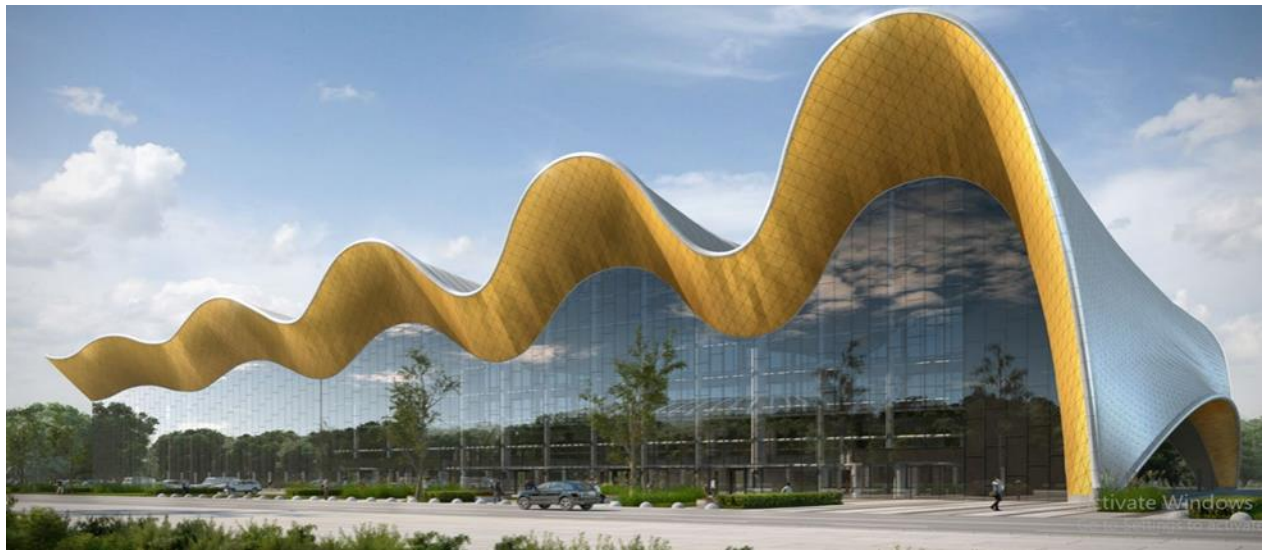


Figure 2. 2 The Luzhniki Complex's Irina Viner-USmanova Rhythmic Gymnastics Center [9].

Timoshenko and Woinowsky-Krieger S.[2] stated that Shell structures behave differently than other roof covering structures due to the presence of significant shear resistance capacity. They have remarkable strength and a greater degree of freedom in structure layout, shape. Their architectural appearance makes them more resistance to collapse even if their supporting structure does. When using membrane theory to analyze thin shells, bending stresses are said to be minimal. In general, shells are susceptible to in plane loading, as well as force and couple normal to the surface, which is referred to as lateral loading [10].

B. Mohraz and W.C Schnobrich [11] investigated the use of a discrete element system to analyze shallow shell structures. They create a discrete model to analyze and numerically calculate the consequences of simply supported and uniformly loaded elliptic parabolic shells, which are then

represented diagrammatically. They also looked at a loaded and rectangular plate. A cylindrical shell, and a hyperbolic parabolic circumscribed by surface characteristics lines that varied uniformly and sinusoidal.

Julio Cesar Molina, Holmer Savastano Jr and Juliano Fiorelli [12], presented numerical results of stresses distribution in corrugated sheets under bending with a certain load. The longitudinal and transverse behavior of the sheets is discussed in terms of stresses, as well as the influence of sheet span on longitudinal stresses.

The typical form of shell constructions supports imposed external load. They are significantly stronger and stiffer than other structural forms because they have spatial curvature. As a result, shells are frequently considered to as the best resistant structures. Shell structures have a substantially higher strength-to-weight ratio than other structures of similar span and size [5].

The structural needs or geometrical condition dictate the structure with cuts of various shapes. Many industrial structures, such as aircrafts, warships, submarines, nuclear reactors, and so on, rely on the stress concentration of cylindrical shells [14]. Strain distribution around the un-stiffened and stiffened subjected to axial compression was investigated. the experimental results were compared to numerical analysis. According to the findings the stiffened one outperforms than un-stiffened one in terms of structural performance.

The advantages of using shell structures in engineering are as follows:

- Load-carrying behavior efficiency
- High level of structural strength and integrity
- High strength-to-weight ratio
- Space containment and high rigidity

2.2 Classification of shell structures

Shell structures are classified into a number of categories based on a variety of characteristics. Shells are classified into two groups depending on how they were made. These are shells of revolution and shells of translation (cylindrical shells). Shells of revolution are which

rotate around a symmetry axis. But, the plane of curve of shells of translation moves parallel to the symmetry axis.

In other words, there are two types of surfaces of shells: developable and non-developable. Shell structures that can be grown into flat form without cutting or stretching their center surface are known as developable shells. Cylindrical shells are good example of these shells. Shells that cannot be developed into a plan without cutting are known as non-developable shells. Spherical and hyperbolic shells are good example of these shells.

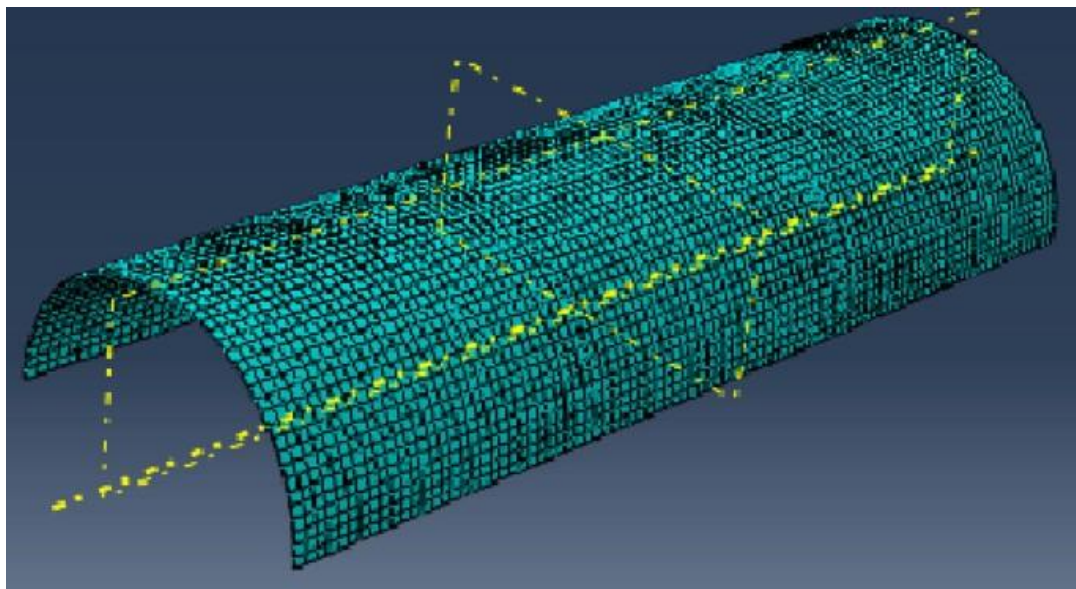


Figure 2. 3 Developable surface

Cylindrical shell structures are developable type of shell structure those are primarily employed in roof structures. Various buildings that serve various purposes such as production, terminal, and maintenance hangar.

Cylindrical shells are classified in two groups based on their dimension. These are long shells and short shells. Long shells are ideal for factory roofs, whereas short shells are popular for air craft hangers.

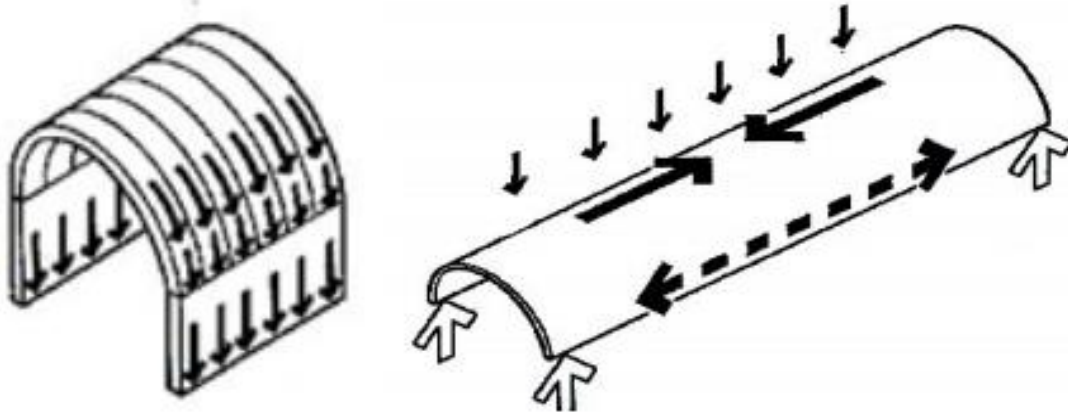


Figure 2. 4 Types of shells (short and long cylindrical shell) [3]

The following symbols describe the dimensions of the shells

L = Span (the length of the shell) h = Rise above the edges

B = Chord width t = Thickness

r = Radius of shell

(Radius = R and $r = \frac{1}{2}$ span)

There are two methods to identify long and short shells.

Method 1: By this method cylindrical shell structures can be classified in three categories.

1. $L/r > 3$ - Long barrel shells. These act as curved beams. They can be analyzed by the "beam method".
2. $L/r < 0.5$ - Short barrel shells. These act more like arches. They can be analyzed as shell structures.
3. Intermediate types- $L/r > 0.5$ but < 3 .

Method 2: cylindrical shell structures are classified into the following two classes [3].

And separate tables have been derived for each. (This criterion has been used in the study.)

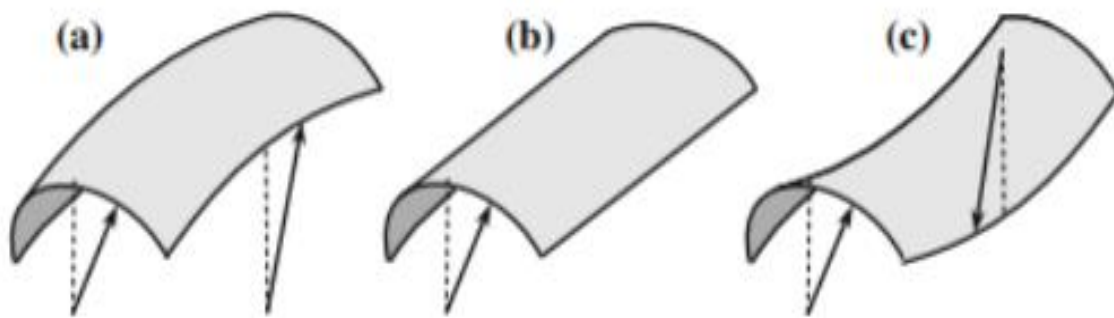
1. Shells with $r/L < 0.6$ or $L/r > 1.66$ (Type I shells). This means that the length of the shell is at least more than approximately 1.7 times the radius of the shell.

In such cases, we can use ASCE Manual No. 31 for its analysis.

2. Shells with $r/L > 0.6$ or $L < 1.6r$ (Type II shells or short shells)

This classification is based on the fact that the length of the arc in type 2 shells is so great that the influence of forces applied to one edge will not affect the other. In type 1 shells, on the other hand, the influence of forces applied to one edge will also be felt on the other edge.

Shells are divided into three groups based on the surface's Gaussian curvature. The first is Synoclastic, which has a positive Gaussian curvature and can carry weight in plane meridional and circumferential stress. Spheres, elliptical paraboloids, and other elliptical paraboloids are common examples. Anticlastic is the second. This sort of negative Gaussian curvature has both major curvatures with the same sign. Mono clastice is the other. These types have a cylindrical shell with zero Gaussian curvature, which is a common example of monolothical curvature.



(a) Positive Gaussian curvature (b) zero Gaussian curvature (c) Negative Gaussian curvature

Figure 2. 5 Types of shells (classification of according to Gaussian curvature)

2.3 Barrels

Barrels are cylindrical structures they can be done by wood, metal, or concrete with a flat end and curving side. They can be classified in three different groups: long barrels, short barrels and intermediate barrels. Positive and negative Gaussian curvature can be seen in them. These barrels can interconnect each other edge to edge to form corrugated shell structure.

2.4 Corrugation

Corrugation alters the mechanical properties of shells significantly. It improves the global stiffness and flexibility of the shell. Corrugation applied in a circular direction to increase the load bearing capacity of longitudinally loaded cylindrical shells. Because of its exceptional qualities, corrugated cylindrical shells are frequently employed in recent building designs [15]. Corrugation increases the effective thickness of the shell cross section. This allows a load to be resisted in compression and improves overall stability. There are naturally appeared corrugated barrels as shown in the figure 2.6.



Figure 2. 6 Natural corrugated shells

Corrugated cylindrical shells have a higher astatic value than other shells and have distinct mechanical properties. The flexibility and bending rigidity of the shell are improved by corrugation. Corrugated shell morphologies have up to 79 percent better lateral capacity before cracking than non-folded reinforced shells [16]. Corrugated plates are commonly used in modern building and structure due to their rigidity and higher global stiffens.

Because of their shape, corrugated shell structures effectively support imposed external forces. These shells are significantly stronger and stiffer than other structural forms due to their spatial

curvature. Corrugated shell structure's strength to weight ratio is typically substantially higher than that of other structural systems with the same span and total dimensions. [12]

For various geometric parameters, the current work gives design curves for reinforced concrete corrugated shells structures. The analysis is based on a reinforced shell that is subjected to a uniformly distributed load that remains constant throughout its length and curvature. After thorough verification of the results obtained from finite element analysis, design charts are suggested for easy solution of shell constants. Stress resultants drawn at closer intervals of can be useful for describing reinforcement layout in RC shells. Stress resultants plotted at closer intervals of can be used for detailing of reinforcement arrangement in RC shells. The interaction of axial force and bending moment yield on shells under evenly distributed loads shows compression failure, which leads to concrete crushing.

The increasing complexity of the stress distribution makes it more difficult to comprehend the structural behavior of corrugated shell structures. They are far more sensitive to minor errors than other structures [11]. As a result, the design must be more precise. The structural characteristics and stability performance must be known before the structural design can be completed. Many scholars looked at structural property including buckling load and free vibration in a cylindrical shell with an edge beam and constant support.

2.5 Application of corrugated shells

Corrugated shells are highly required shell structures in the modern construction industry. They mostly utilized for museums, churches and conference halls. They are frequently subjected to a combination of compressive stress and external pressure, thus they must be constructed to accommodate stretch.

2.6 Load carrying capacity of cylindrical shell

The bending and stretching mechanisms are used to handle the loads that are applied to the shell. The most difficult problem in shell structure theory is finding a simple way to describe the interaction between these two actions. The first is Membrane force: - (F_x , F_y , and F_{xy}) forces that are constant in the plane of the plate throughout the thickness of the element. This force is expressed as a force divided by the length of a unit. This load causes the shell to swell or contract as a membrane, but no bending or local curvature changes occur. The other is bending force, which is a typical stress that occurs when a substantial load is applied to a portion of the shell, causing it to bend. This force consists of bending moment twisting couples [3].

Shell structures use membrane action to carry external loads. We can write three equations of equilibrium along the x , y , and z axes, as well as three equations of moments with regard to the x , y , and z axes, for a general situation in three dimensions. There are six equilibrium equations in all, however there are more than six resultants to consider. As a result, Shells have an internal structural indeterminacy (i.e., they can't be analyzed using static equations of equilibrium). When N_x and N_y are modest in compared to their critical values for lateral buckling of the shell, their effect on bending is negligible [7]. When tangential shear force creates tension in the direction of rising X value, it is termed Positive. The Hooke's law shows the relationship between static parameters and strain curvature.

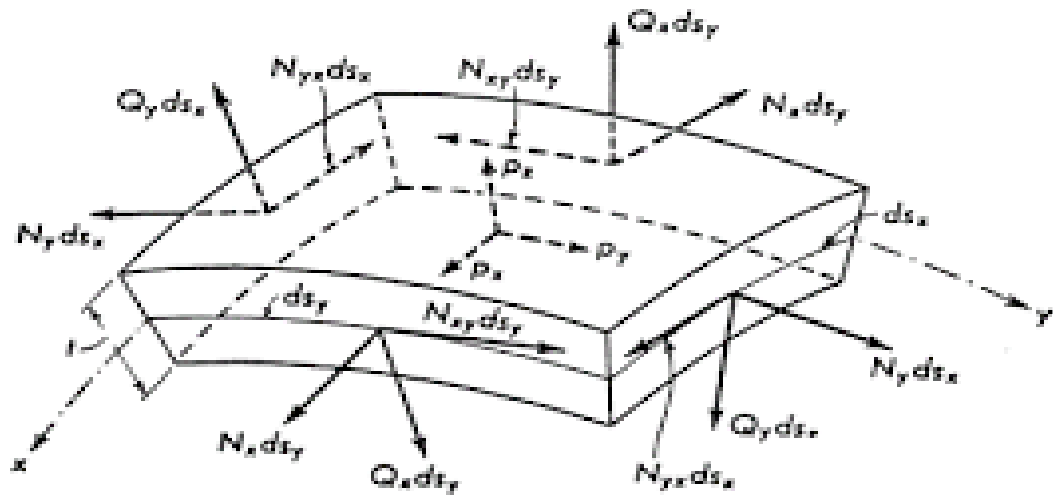


Figure 2. 7 Membrane and stress resultant [7]

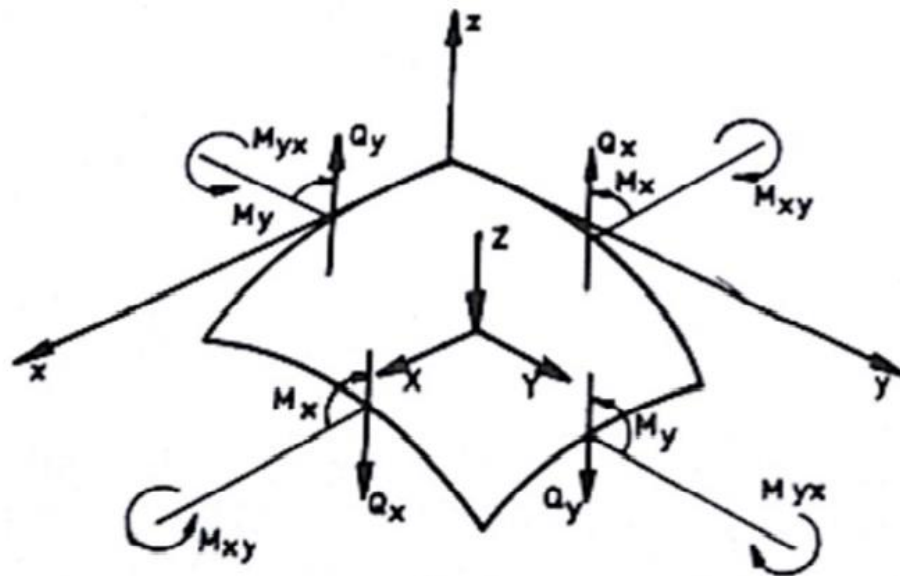


Figure 2. 8 Load, shear, resultant, bending, twisting [7]

Important relations to analyze shell structures

$$\varepsilon_x = \frac{\partial U_x}{\partial x}, \varepsilon_\varphi = \frac{1}{R} \left\{ \frac{RU\varphi}{\partial \varphi} - Ur \right\}, \gamma_{x\varphi} = \frac{1}{R} \frac{\partial U_x}{\partial \varphi} - \frac{\partial V_\varphi}{\partial x} \quad (2.1)$$

$$\chi = \frac{\partial^2 U_x}{\partial x^2}, \phi_\varphi = \frac{1}{R^2} \left[\frac{\partial U_\varphi}{\partial \varphi} + \frac{\partial^2 Ur}{\partial \varphi^2} \right], \phi_{x\varphi} = \frac{1}{R} \left\{ \frac{RU\varphi}{\partial x} - \frac{\partial^2 U_\varphi}{\partial x \partial \varphi} \right\} \quad (2.2)$$

$$N_x = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \sigma_x \left(1 + \frac{z}{r_y} \right) dz, \quad N_y = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \sigma_y \left(1 + \frac{z}{r_x} \right) dz \quad (2.3)$$

$$N_{xy} = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \sigma_{xy} \left(1 + \frac{z}{r_y} \right) dz, \quad N_{yx} = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \sigma_{yx} \left(1 + \frac{z}{r_x} \right) dz \quad (2.4)$$

$$Q_x = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \sigma_{xy} \left(1 + \frac{z}{r_y} \right) dz, \quad Q_y = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \sigma_{yx} \left(1 + \frac{z}{r_x} \right) dz \quad (2.5)$$

$$M_x = \int_{-\frac{t}{2}}^{+\frac{t}{2}} z \sigma_x \left(1 + \frac{z}{r_y} \right) dz, \quad M_y = \int_{-\frac{t}{2}}^{+\frac{t}{2}} z \sigma_y \left(1 + \frac{z}{r_x} \right) dz \quad (2.6)$$

$$M_{xy} = - \int_{-\frac{t}{2}}^{+\frac{t}{2}} z \sigma_{xy} \left(1 + \frac{z}{r_y} \right) dz, \quad M_{yx} = \int_{-\frac{t}{2}}^{+\frac{t}{2}} z \sigma_y \left(1 + \frac{z}{r_x} \right) dz \quad (2.7)$$

2.6.1 Membrane analysis of cylindrical shells

The membrane force is a forcing on the shell's mid surface that causes the shell to expand or contract without bending or changing its local curvature. Cylindrical shell membrane analysis is simple. This behavior is concerned with the membrane-like behavior of shells carrying loads out of plane. They transfer loads in plane member forces using geometry. Different methods and equations have been provided for bending analysis of this shell. Many pieces of software are now available for computer analysis of these shells. In plane forces (plane stress) are the primary cause of membrane action in a shell, however flexural deformation can also create secondary forces. Shells are analogous to a cable that resists loads through tensile stresses, while the flat plane acts like a beam with bending and shear stresses.

There are only three types of membrane forces: direct forces T_x and T_y (which can be tension or compression) and reversible forces R_x and R_y (which can be tension or compression) (shear force).

The three statics equations can readily be used to derive them. The formulas for membrane displacements can also be derived from elasticity theory.

The most efficient structural behavior of thin shells is achieved when they function as membranes. The membrane can withstand applied loads in the tangent plane that are just compression and tension.

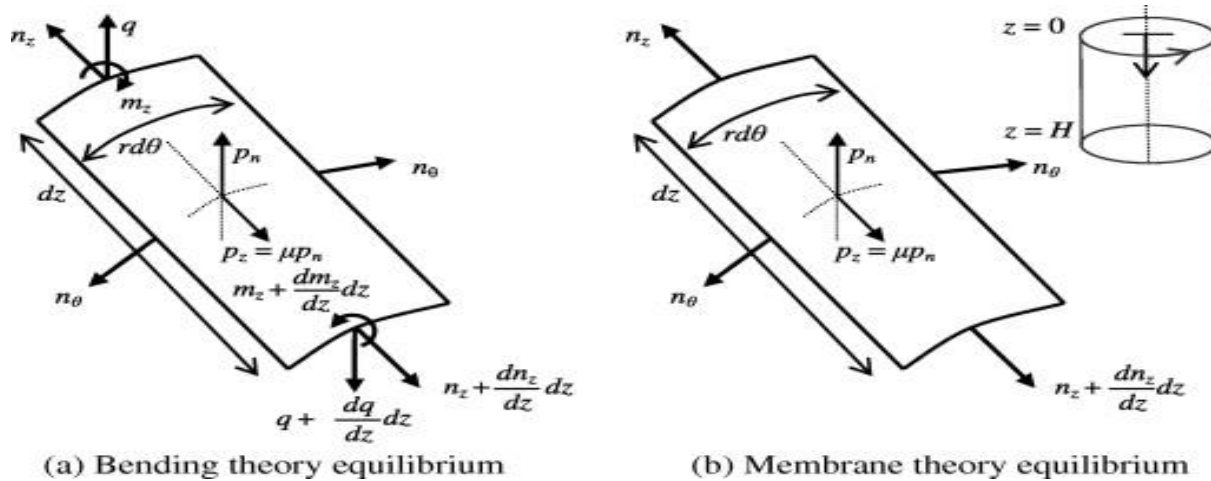


Figure 2. 9 Arbitrary elements in bending and membrane theory equilibrium.[8]

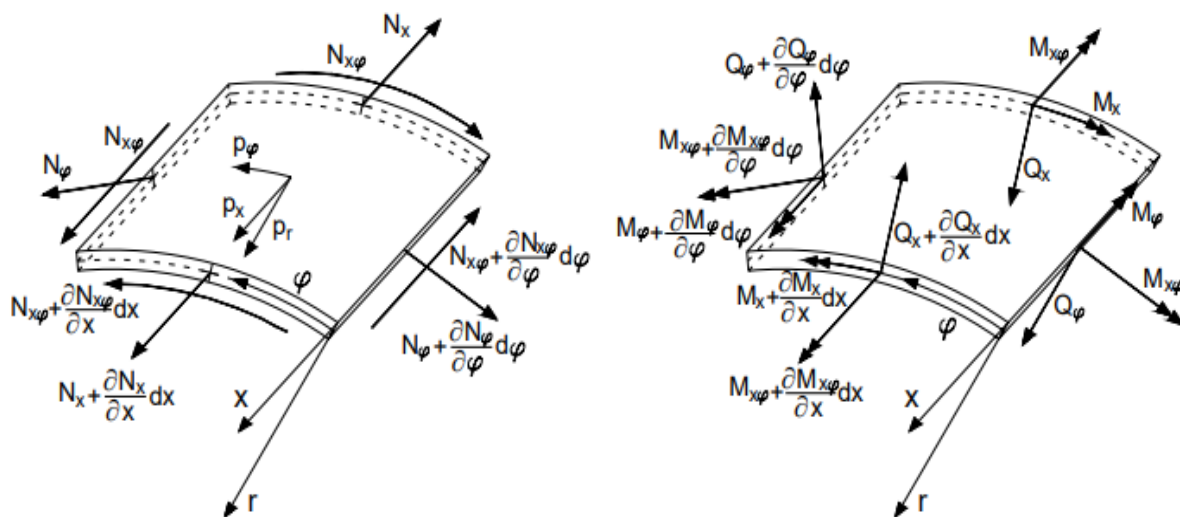


Figure 2. 10 Stress, Bending, twisting and shear component in portion of shell [8]

Force equation relation is discussed as follows according to Hooke's law

$$\frac{(\partial N_x)}{\partial x} + \frac{(\partial N_{\theta x})}{R \partial \theta} + X = 0 \text{ ----- along x direction} \quad (2.8)$$

$$\frac{(\partial N_{\theta})}{\partial \theta} + \frac{(\partial N_{\theta x})}{\partial x} + Y = 0 \text{ -----along } \theta \text{ direction} \quad (2.9)$$

$$\frac{N_{\theta}}{R} + \frac{\partial Q_{\theta}}{R \partial \theta} + \frac{\partial Q_x}{\partial x} + Z = 0 \text{ -----along Z direction} \quad (2.10)$$

From the above equilibrium equation we get the following equation

Vertical load form top --- $N_{\phi} \sin \phi (2\pi R \sin \phi)$

$$N_{\theta} = \frac{wR(1-\cos\phi)}{\sin 2\phi} + \frac{w}{2\pi R \sin 2\phi} = \frac{wR}{2} \text{ (with } w = 0) \quad (2.11)$$

$$N_{\phi} = \frac{wR}{1+\cos\phi} + \frac{w}{2\pi R \sin 2\phi} \quad (2.12)$$

This is the meridional, force per unit length and its value increase ϕ value (depth)

$$N_{\theta} = W r \cos \phi - N_{\phi} \quad (2.13)$$

The strain displacement is written as follows'

$$Y_x \Theta_o = \frac{(\partial U)}{R \partial \theta} + \frac{(\partial v)}{R x} \quad (2.14)$$

$$\epsilon_{\theta o} = \frac{(\partial V)}{R \partial \theta} + \frac{w}{R} \quad (2.15)$$

$$\epsilon_{\theta o} = \frac{(\partial U)}{R x} \quad (2.16)$$

From the above equation we get the following displacement equation. Interims of stress results

$$U = \frac{1}{Eh} \int (N_x - \nu N_{\theta}) dx + F_3(\theta) \quad (2.17)$$

$$V = \frac{1}{Gh} \int (N_{\theta}) dx - \frac{1}{R} \int \frac{(\partial U)}{R \partial \theta} dx + F_4(\theta) \quad (2.18)$$

$$V'' = R \left[\frac{1}{Gh} (N_{\theta} - \nu N_x) - \frac{\partial V}{R \partial \theta} \right] \quad (2.19)$$

2.6.2 Bending analysis of cylindrical shell

Only in areas of the shell where the membrane stresses are insufficient to carry the applied loads does this phenomenon occur. Inadequate membrane behavior causes the bending moment to occur in these areas. Due to the presence of border members that cannot be in equilibrium, the boundary conditions of a cylindrical shell need a change of the membrane forces.

At the shell's edge, bending analysis is impossible. To neutralize the effect, we must apply an equal and opposite force at the boundary. For long shells, we use horizontal and vertical loads, whereas for short shells, we use Transverse Load. As a result, the shell will bend, and we must rely on the bending hypothesis to solve the problem.

The following is moment equilibrium equation

$$\frac{(\partial M_{\theta x})}{R \partial \theta} + \frac{(\partial M_x)}{\partial x} - Q_x = 0 \text{ ----- along } x \text{ direction} \quad (2.20)$$

$$\frac{(\partial M_{x\theta})}{\partial x} + \frac{(\partial M_{\theta})}{R \partial \theta} - Q_{\theta} = 0 \text{ ----- along } Y \text{ direction} \quad (2.21)$$

$$\frac{M_{\theta x}}{R} + (N_{x\theta} - N_{\theta x}) = 0 \text{ ----- along } Z \text{ direction} \quad (2.22)$$

From the about equilibrium equation

$$N_x = \frac{Et}{R(1-\nu^2)} \left[R \frac{\partial U_x}{\partial x} + \nu \left(\frac{\partial U_{\phi}}{\partial \phi} - Ur \right) \right] \quad (2.23)$$

$$N_{\theta} = \frac{Et}{R(1-\nu^2)} \left[VR \frac{\partial U_x}{\partial x} + \frac{\partial U_{\phi}}{\partial \phi} - Ur \right] \quad (2.24)$$

$$N_{x\phi} = \frac{Et}{2(1-\nu)} \left[\frac{\partial U_x}{\partial x} + \frac{1}{R} \frac{\partial U_{\phi}}{\partial \phi} \right] \quad (2.25)$$

$$N_x = \frac{Et}{R(1-\nu^2)} \left[R \frac{\partial U_x}{\partial x} + \nu \left(\frac{\partial U_{\phi}}{\partial \phi} - Ur \right) \right] \quad (2.26)$$

$$N_{\phi} = \frac{Et}{R(1-\nu^2)} \left[VR \frac{\partial U_x}{\partial x} + \frac{\partial U_{\phi}}{\partial \phi} - Ur \right] \quad (2.27)$$

$$N_{x\phi} = \frac{Et}{2(1-\nu)} \left[\frac{\partial U_x}{\partial x} + \frac{1}{R} \frac{\partial U_\phi}{\partial \phi} \right] \quad (2.28)$$

$$M_x = \frac{Et^3}{12(1-\nu^2)} \left[\frac{\partial^2 U_x}{\partial x^2} + \frac{V}{R^2} \left(\frac{\partial U_\phi}{\partial \phi} - \frac{\partial^2 U_r}{\partial \phi^2} \right) \right] \quad (2.29)$$

$$M_\phi = \frac{Et^3}{12(1-\nu^2)} \left[V \frac{\partial^2 U_r}{\partial x^2} + \frac{1}{R^2} \left(\frac{\partial U_\phi}{\partial \phi} - \frac{\partial^2 U_r}{\partial \phi^2} \right) \right] \quad (2.30)$$

$$M_{x\phi} = \frac{Et^3}{12(1-\nu)R} \left[\frac{\partial U_x}{\partial x} + \frac{\partial^2 U_\phi}{\partial x \partial \phi} \right] = M_{ex} \quad (2.31)$$

$$\Theta_x = D \left[\frac{\partial^3 W}{\partial x^3} + \frac{\partial^2 w}{R^2 \partial \theta^2 \partial x} \right] \quad (2.32)$$

$$\Theta_\theta = D \left[\frac{\partial^3 W}{R^3 \partial \theta^3} + \frac{\partial^3 w}{\partial \theta \partial x^2} \right] \quad (2.33)$$

2.7 Boundary condition

The general equation of boundary condition must be examined first in order to solve differential equations. One of the benefits of modeling to analyze the behavior of shell structures is their adaptability to diverse boundary conditions. The deflection of a shell is closely related to the vertical force applied to it. Geometry and the boundary condition are expressed using the strain displacement. Because shell structures are indeterminate, employing boundary conditions in shell structure models allows us to derive alternative equations to solve indeterminate structures. The stability of a structure is greatly influenced by the condition of the lateral support. It has the ability to increase or decrease the structure's mobility. This movement could lead to a rise in the defalcation of the sell structure. The following are some cylindrical shell boundary conditions.

2.7.1 Single span cylindrical shell

The cylindrical shell's deflection is determined by the support condition at the edge. This thesis takes into account both free and supported edges.

Free edge: - they have no supporting reaction and moment along the edge will be zero. They are supported only on the four edges of the shell.

Supported edge: - the vertical and horizontal displacement is restricted but there is rotation at the edge

When the edge is simply supported

$$\text{At } \Theta = \pm\Theta_0, N_{\Theta} = M_{\Theta} = \varepsilon x = w = 0$$

When the edge is clamped, the displacement and rotation are entirely limited when the edge is supported monolithically by the beam, and the force acting on the shell is directly transferred to the beam, resulting in the shell components being equal to the beam.

$$u_s = u_b, \quad \beta_s = \beta_b, \quad \Delta w_s = w_b, \quad \Delta H_s = v_b$$

Where U_b = longitudinal displacement of beam

V_b = lateral displacement of beam

W_b = Vertical displacement of beam

β_b = twisting effects of beam

U_s = longitudinal displacement of shell

V_s = lateral displacement of shell

W_s = Vertical displacement of shell

β_s = twisting effects of shell

2.7.2 Multiple span cylindrical shells

The bending stress, normal stress, and circumferential stress, including the twisting effect of the barrel, are identical at the junction point of two barrels in a multiple span cylindrical shell.

Equation of inner shell of free edge

$$\beta_s = \Delta H_s = \Delta F = \frac{\partial N_{x\theta}}{\partial \theta} = 0, \quad \text{where } \Delta F = N_{\theta} \sin \theta + Q_{\theta} \cos \theta \quad (2.34)$$

For multi span shell with edge beam have edge beam in all shell junction and free ends. The vertical and horizontal stress including twisting effect of the two connected shells and beam are equal at the junction pointes.

Equation of inner shell with edge beam

$$u_s = u \quad \beta_s = 0 \quad \Delta w_s = w_b$$

Shell element specified in a suitable coordinate system is used to analyze the buckling of barrels subjected to external loads. The element's governing equation in terms of radial deflection is derived using an operator. For cylindrical shells with varying boundary conditions, the derived eighth order partial differential equation can be used. For example, axial compressive pressures on simply supported cylindrical shells are analyzed using either a one-variable or two-variable form function. The critical stresses found for cylindrical shell buckling are compared to those computed using the finite element program SAP2000.

CHAPTER THREE

MATERIAL AND ANALYSIS METHOD

3.1 Introduction

Analytical study of cylindrical shell by using formula is extensive computation work. The parameters vary and their interaction is complex. However, based on the analyses of some selected reinforced concrete cylindrical shell, the parametric study is done by considering different parameters of shell structure. This category includes the rods of curvature, thickness and span length of the cylindrical shell. In addition, different Gaussian curvature is also considered.

The shell geometry is an important parameter that affects the structural performance of cylindrical shell. These geometrical properties include the length, the radius and thickness of the shell. The first case is carried out by varying the length of the shell by putting the other parameters constant. The second case is carried out by varying the radius of the barrels by keeping the other parameters constant. Then the third case is done by studying structural behavior of the shell by varying the thickness and keeping the other parameters constant.

3.2 Stability influencing parameter

The structural characteristics that influence the stability of the reinforced concrete corrugated shell can be divided into five groups:

- The corrugated shell's geometric shape
- Defined boundaries conations
- Material Characteristics
- The support conditions
- The loading conditions

3.3 Geometry

3.3.1 Geometry dimension selection

There are different sample selection methods for parametric study. In this study the geometric parameter of the cylindrical shell is selected by using Latin Hypercube sampling method LHS Sampling method.

This software involves dividing the cumulative density function in to equal portion and their choosing a random data point in each portion. Combinations are generated by using standard deviation and variance of the application sampling.

The sample combination gain from LHS sampling method is difficult to use directly to modeling the shell structures. They require approximation to simplify modeling of the shell structures. Because of this, the samples shown below are approximated and interpolated samples.

The geometrical property and the procedures followed in the theoretical and numerical analysis of each model are described as follows. The parameters are selected by considering mostly used range of parameters.

The Important parameters of the cylindrical shell are thickness, radius of curvature, length of each barrel and internal angel. The influence of the variation of these parameters in restricted range is considered. For the analysis of internal load, free and Buckling load simply supported bindery condition is considered.

Table 3. 1 parameters used to model the structures

Trial	Length	Radius for single barrel	Radius for corrugated shells	Thickness	Edge beam		Internal Angle	Angle of Inclination
	(m)	(m)	(m)	(m)	Width(m)	Depth (m)		
1	8	8	4.0	0.2	0.2	0.2	15 ^o	10
2	10	8.8	4.4	0.2	0.21	0.2	17.3 ^o	11.6
3	12	9.6	4.8	0.22	0.22	0.22	19.6 ^o	13.26
4	14	10.4	5.2	0.22	0.23	0.22	21.9 ^o	14.92
5	16	11.2	5.6	0.24	0.24	0.24	24.2 ^o	16.58
6	18	12	6.0	0.24	0.25	0.24	26.5 ^o	18.24
7	20	12.8	6.4	0.26	0.26	0.26	28.8 ^o	19.90
8	22	13.6	6.8	0.26	0.27	0.26	31.1 ^o	21.56
9	24	14.4	7.2	0.28	0.28	0.28	33.4 ^o	23.226
10	26	15.2	7.6	0.28	0.29	0.28	35.7 ^o	24.88
11	28	16	8.0	0.30	0.3	0.30	38.0 ^o	26.54
12	30	16.8	8.4	0.30	0.31	0.30	40.3 ^o	28.20
13	32	17.6	8.8	0.32	0.32	0.32	42.6 ^o	29.86
14	34	18.4	9.2	0.32	0.33	0.32	44.9 ^o	31.52
15	36	19.2	9.6	0.34	0.34	0.34	47.2 ^o	33.186
16	38	20	10.0	0.34	0.35	0.34	49.5 ^o	34.84
17	40	20.8	10.4	0.36	0.36	0.36	51.8 ^o	36.506
18	42	21.6	10.8	0.36	0.37	0.36	54.1 ^o	38.166
19	44	22.4	11.2	0.38	0.38	0.38	56.4 ^o	39.82
20	46	23.2	11.6	0.38	0.39	0.38	58.7 ^o	41.48
21	48	24	12.0	0.40	0.4	0.40	61.0 ^o	43.14
22	50	24.8	12.4	0.40	0.41	0.40	63.3 ^o	44.80
23	52	25.6	12.8	0.42	0.42	0.42	65.6 ^o	46.46
24	54	26.4	13.2	0.42	0.43	0.42	67.9 ^o	48.12
25	56	27.2	13.6	0.44	0.44	0.44	70.2 ^o	49.78
26	58	28	14.0	0.44	0.45	0.44	72.5 ^o	51.44
27	60	28.8	14.4	0.46	0.46	0.46	74.8 ^o	53.10
28	62	29.6	14.8	0.46	0.47	0.46	77.1 ^o	54.76
29	64	30.4	15.2	0.48	0.48	0.50	79.4 ^o	56.42
30	66	31.2	15.6	0.48	0.5	0.50	81.7 ^o	60.0

3.3.2 Rise to span ratio

Shells with a higher rise value have more reserved strength, according to membrane theory and they are stronger than shells with a lower rise value [2] and [5]. For shells without edge beam, the decrease of rise to span ratio minimizes the buckling factor. The curvature of the surface increases as the rise to span ratio rises. When the rise is larger, the bending action and the radius of the shell become increased. In this case the membrane action becomes larger and this leads to the decrease of central deflection.

The magnitude of peak tensile stress at the crown of the cylindrical shell becomes smaller with the increase of rise to span ratio. So the rise to span ratio and the tensile stress on the crown of the shell are inversely proportional. [5], [12] & [16].

3.3.3 Thickness

Thickness is important parameter that highly affects the shell stiffness and deflection. Due to the increased rigidity of the shell, deflection decreases as thickness increases. Volumetric displacement variation, on the other hand, increases as thickness increases due to the increase in dead weight. The values are safe when the maximum compressive stress in the shell does not exceed the concrete's permitted compressive strength, and the displacements must be less than the standards' limit [15].

Due to the decrease in the variance of the vertical displacement, the bending moments and shearing forces decrease as the shell thickness increases. The shell stress diminishes as the shell thickness increases.

3.4 Edge beam (support conditions)

Cylindrical shell constructions can be considered with or without an edge (supporting) beam. The structural stability is limited when the shell is without an edge beam. To improve the global stiffness of barrels, edge beams are used based on the result given on table 3.1.

3.5 Material property

Although the effect of material property is less when compared to other characteristics, it is vital to understand the material property that is applied to study the load's effect. For this thesis, the concrete class C 25 is used. The reinforcement has yield strength of 300 mPs for both the shell and the edge beam.

Table 3. 2 Material property of concrete

Material property	Values
Weight per unit	2.5×10^1
Modules of elasticity	2.9×10^7
Poisons ratio	2.0×10^{-1}
Coefficient of thermal expansions	1.0×10^{-5}
Shear modulus	1.2×10^7
Specified concrete compressive strength f_c	2.0×10^4

Table 3. 3 Material property of reinforcement

Material property	Values
Weight per unit volume	7.7×10^1
Modules of elasticity E	2.1×10^8
Poisons ratio ν	3.0×10^{-1}
Coefficient of thermal expansions A	1.2×10^5
Minimum yield stress f_y	2.6×10^5
Minimum yield stress f_u	3.0×10^5
Expected yield F_{ye}	2.6×10^5
Expected Tensile stress F_{ue}	3.0×10^5

3.6 Load determination

The applied load on this shell structure is stated as follows. The first one is the self-wight of the shell structure and the edge beam. The unit weights of concrete are taken from table A-1 concrete material concrete and mortar of ES AM 1991 -1-1 2015 Annex A to be 25kN/m^2 [18]

Self-wight can be obtained by multiplying unit weight by thickness of structures. The other important loading on roof is wind load because it is a roof structure then, other heavy loads are not considered the roof load of those structures are used 0.4 kN/m^2 .

The next process in design is to identify the loads acting on the structure. Load factors are calculated using Euro codes which lead to design values of several kinds of loads. This section describes how and what loads are assumed to act on our structure.

3.6.1 Permanent loads

Permanent loads are referred to as the dead or static loads acting on the structure and are calculated using Euro code EN1991-1-1 [20]

. One of the dead loads is the self-weight of concrete used in shell construction and other finishing installations. In this analysis the self-weight of the shell is variable with variable thickness, length and radius.

3.6.2 Live load

The dead and live loads have been combined according to Ethiopian Building Code of Standard. The equivalent form of live load is the projected areal distribution of the shell. Therefore, the load combination will follow the usual combination of structures.

3.6.3 Loading condition

Cylindrical shell roof structure is mostly subjected to two types of loads, these are the self-weight and wind load. These loads are applied in radial and circumferential loading are considered.

Barrel shell has two Gaussian curvatures. The first one is positive Gaussian curvature, having positive Gaussian curvature having convex shape. It is compression member that subjected only to axial compressive force can fail by yielding and bulking. The second one is negative Gaussian curvature having concave shape. It is a tension member that is subjected to axial tensile force. In this thesis long edge beam is provided at the connection point of the convex and concave barrels for long barrels to increase the global stiffness of the corrugated shells

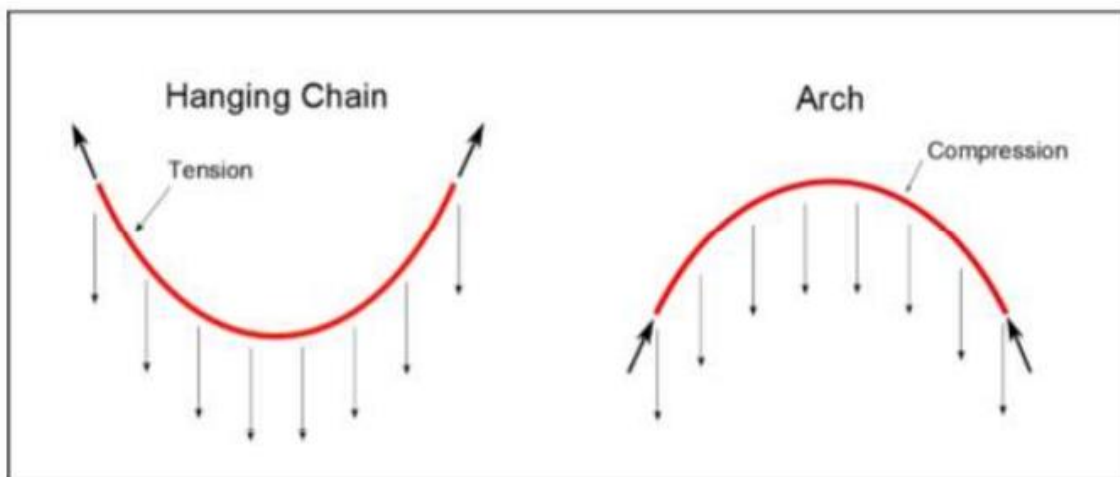


Figure 3. 1 The two curvature directions of barrels

3.6.4 Load combination

Both dead load and live load are applied along vertical down ward direction

Dead load along Θ direction is calculated by

$$w_{\Theta} = w_g \sin \Theta \quad \text{where } w_g \text{ is dead load}$$

Dead load at z direction (w_z) is calculated by $w_z = w_g \cos \Theta$

Live load at Θ direction is calculated by $w_{\Theta} = w_p \sin \Theta \cos \Theta$ where w_p is live load

Live load at z direction (w_z) is given by $w_z = w_p \cos^2 \Theta$

Factors The euro codes are referred again to obtain suitable load combinations and design safety factors for this design. Article A1.3 and A1.4 in the euro code provides design values for the various actions on a structure in ultimate and serviceability limit states. The combination and factors used in this project are mentioned hereafter.

Factored load = $1.35 \times (\text{dead load}) + 1.5 \times (\text{live load})$

3.6.5 Material property

Yang's modules (E) = 30000 MPa

- The mass density of the shell(C) = 2500 kg/m³
- Poises ratio (μ) = 0.2

3.7 Analysis procedure

The analysis process began with the selection of model parameters, and then carried out by modeling a number of shells using the selected parameters on the SAP2000 software. Checking the impact of the variable parameters on the stability of the corrugated shell structure based on the software's output is the final and most important step of the analysis.

The force and displacement of the shell stress is calculated by the following formula in addition to the software.

$$\omega = \omega_0 + \omega_1$$

ω = total displacement

ω_0 = homogeneous solution

ω_1 = particular solution

$$\omega_0 = \sum_{m=1,3}^{\infty} [(A_1 \cos b\theta + A_2 \sin b\theta) e^{a\theta} + (A_3 \cos b_1\theta + A_4 \sin b_1\theta) e^{a_1\theta} + (A_5 \cos b\theta + A_6 \sin b\theta) e^{-a\theta} + (A_7 \cos b_1\theta + A_8 \sin b_1\theta) e^{-a_1\theta}] \cos \alpha_m x \quad (3.1)$$

$$\omega_1 = \sum_{m=1,3}^{\infty} (C E_m \cos \alpha_m x \cos \theta) \quad (3.2)$$

General formula for force on the shell

$$F = \sum_{m=1,3}^{\infty} M F [((A_1 \alpha_1 - A_2 \alpha_2) \cos b\theta - (A_1 \alpha_2 - A_2 \alpha_1) \sin b\theta) e^{a\theta} + ((A_3 \alpha_3 - A_4 \alpha_4) \cos b_1\theta - (A_3 \alpha_4 - A_4 \alpha_3) \sin b_1\theta) e^{a\theta} \pm ((A_5 \alpha_1 - A_6 \alpha_2) \cos b\theta + (A_5 \alpha_2 - A_6 \alpha_1) \sin b\theta) e^{-a\theta} \pm ((A_7 \alpha_3 - A_8 \alpha_4) \cos b_1\theta + (A_7 \alpha_4 - A_8 \alpha_3) \sin b_1\theta) e^{-a\theta}]$$

$$\lambda = (\sqrt[3]{3(1-v^2)}) (\sqrt{R} \alpha_m) \left(\sqrt[4]{\frac{R}{h}} \right) \quad j = \sqrt{\frac{\sqrt{(1+(1+\varepsilon)^2)} + (1+\varepsilon)}{2}} \quad (3.4a)$$

$$\varepsilon = \frac{1}{\sqrt[4]{3(1-v^2)}} \left(\sqrt{\frac{h}{R}} \right) (R \alpha_m) \quad k = \sqrt{\frac{\sqrt{(1+(1+\varepsilon)^2)} - (1+\varepsilon)}{2}} \quad (3.4b)$$

$$j_1 = \sqrt{\frac{\sqrt{(1+(1-\varepsilon)^2)} - (1-\varepsilon)}{2}} \quad k_1 = \sqrt{\frac{\sqrt{(1+(1+\varepsilon)^2)} + (1+\varepsilon)}{2}} \quad (3.4c)$$

$$a = \lambda j, \quad a_1 = \lambda j_1, \quad b = \lambda k, \quad b_1 = \lambda k_1$$

$$\alpha_m = m\pi/L, \quad E_m = (-1)^{(m-1)/2} * \frac{4w}{m\pi} \quad (3.5)$$

W= combined load

(The combination of total dead load and equivalent live load combined according to EBCS 1995)

m= positive integer varied from one to infinity

R= radius of the shell, h = thickness of the shell, D = flexural rigidity

ν = Poisson's ratio L = length of the shell

The analysis had been carried out using the methodologies listed below

- Sample selection by using LHS sampling
- Modeling of the cylindrical shell structure by using SAP 2000
- Validation of the result gated with other software
- Providing varies geometry (different length, different radius, different thickness)
- Analysis of force, moment, stress and deformation by using various parameters
- Discusses on the result gain from the SAP 2000 software of all effect

3.8 Validation of analysis

Among the available finite element software SAP2000 is the preferred of this thesis since it is familiar and it has predefined templates. The results are validated using hand calculations and ANSYS software.

Table 3. 4 The sample parameter used for verification of the analysis

Parameters	Samples			
	Sample 1	Sample 2	Sample 3	Sample 4
Shell radius	3m	4m	4m	4m
Shell length	10m	10m	12m	12m
Shell thickness	0.1cm	0.1cm	0.1cm	0.2cm
Concrete grade	C25	C25	C25	C25

3.8.1 Analytical analysis

Analysis of semicircular reinforced concrete shell roof resisted on wall

Shell data -Radius $r = 6\text{m}$

Length $L = 12\text{m}$

$L/r > 1.6$ -----long shell

DL = 3.5 KN/m²

LL = 0.5 KN/m²

Membrane analysis with non-furrier loading

1 Calculate Maximum value of T_x at crown at $x=L/2$

$$T_x = \frac{PL^2}{r} \left(\frac{x}{L}\right) \left(1 - \frac{x}{L}\right) * \text{Coff}$$

Were

T_x = Direct force (tension or compression) in the longitudinal direction

T_ϕ = Direct force in the ϕ (transverse direction)

S = Tangential shear force considered +ve when it produces tension in the direction of increasing value of X and ϕ

M_ϕ = Bending moment in radial force is taking as +ve when it produces tension inside the shell

$$T_x \text{ due to dead load (DL)} = \frac{1}{4} (39.9) * 2.5 * 1 = 24.94 \text{ KN (comp)}$$

$$T_x \text{ due to Live load (LL)} = \frac{1}{4} (39.9) * 0.5 * 1.5 = 7.48 \text{ KN (comp)}$$

Total compression = 24.94 KN/m (thickness 100 mm)

$$\text{Max stress} = \frac{24.94 * 1000}{1000 * 100} = 0.249 \text{ N/mm}^2 \quad (\text{Thickness of shell} = 100\text{mm})$$

2. Calculate T_x at edge at $x = L/2$

Due to DL = 0

Due to LL = $\frac{1}{2}(39.9 * 0.5 * 1.5) = 14.9\text{KN}$

$$\text{Stress} = \frac{14.9 * 1000}{1000 * 100} = 0.12 \text{ N/mm}^2$$

Max tension = 0.149 N/mm² (which is small)

3.8.2 Numerical analysis by SAP 2000

The numerical analysis is performed by modeling a cylindrical shell with the above characteristics in SAP 2000 software and obtaining the results of the analysis. The analysis results were tabulated for validation. Stress distribution and maximum deflection on single cylindrical shell model on SAP2000 is described as follows.

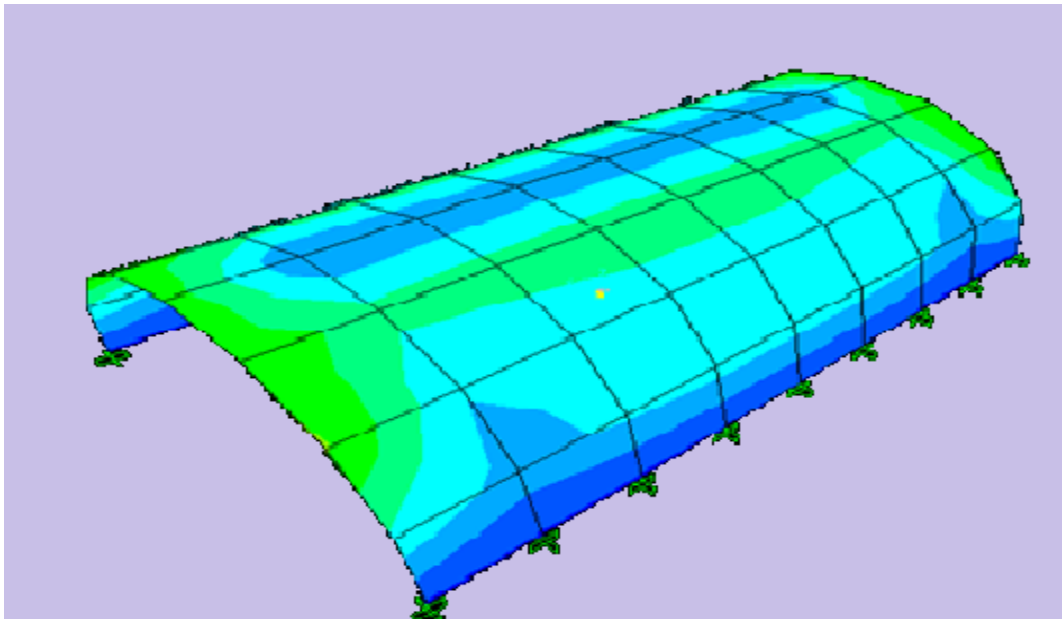


Figure 3. 2 Stress distribution in single Barrel from SAP 2000

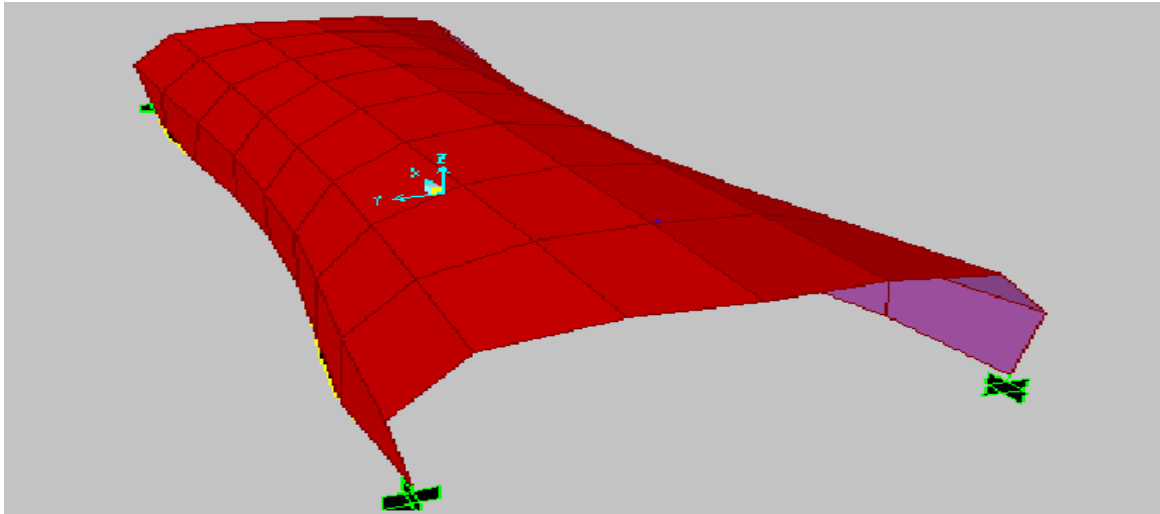


Figure 3. 3 Maximum deflections in single barrel from SAP 2000

3.8.3 Numerical analysis by using ANSYS software

ANSYS software solves problems include static/dynamic, structural analysis, heat transfer, and fluid problems, as well as acoustic and electromagnetic problems. In this thesis it had been used for validation of results of SAP 2000. Circumferential stress distribution and maximum deflection on single cylindrical shell model on ANSYS software is described as follows.

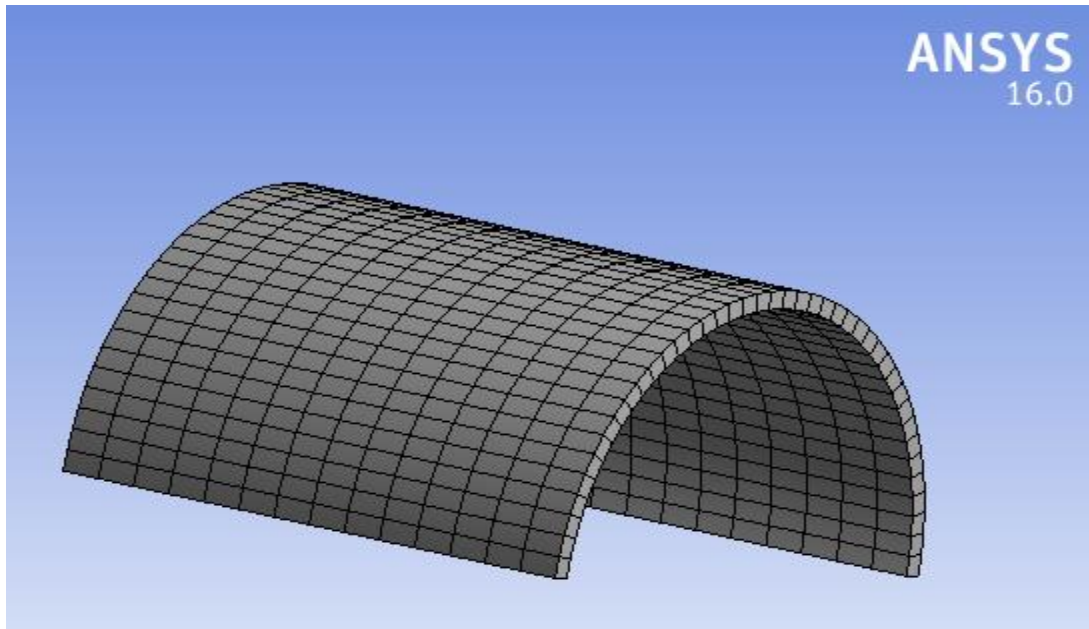


Figure 3. 4 3D model done on ANSYS

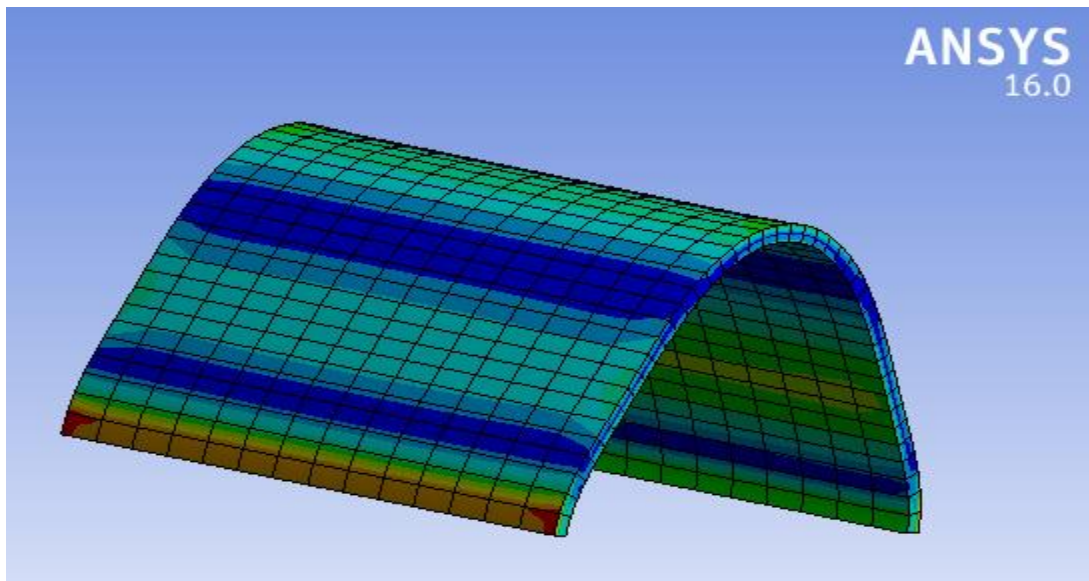


Figure 3. 5 Equivalent stress distributions

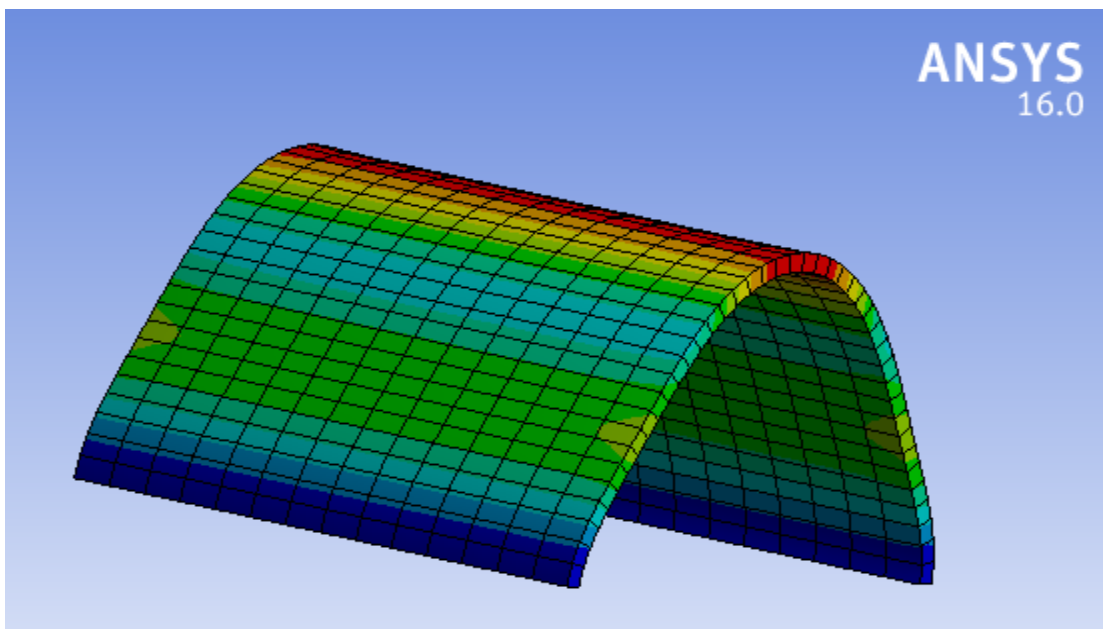


Figure 3. 6 Maximum deflection

3.8.4 Comparison of analysis result

The result from numerical analysis and of the above five sample are compared with analytical result to verify the result of SAP2000 as shown in the table 3.4.

Table 3. 5 Comparison of analysis

Sample	Result from SAP 2000		Result from ANSYS		Difference	
	Maximum stress (kN/m)	Maximum deflection(m)	Maximum stress(kN/m)	Maximum deflection (m)	Maximum stress (kN/m)	Maximum Deflection(m)
Sample 1	334.16	0.364	364.78	0.4038	298.4	0.0398
Sample 2	1062.5	1.16	1164.67	1.2497	975.87	0.0897
Sample 3	281.6	0.352	352.76	0.4324	704.34	0.0804
Sample 4	472.2	0.53	532.45	0.5579	578.8	0.0279

CHAPTER FOUR

RESULT AND DISCUSSIONS

4.1 Shell geometry of single cylindrical shell

The shell geometry is a critical factor in the structural performance of a cylindrical shell. The length, radius, and thickness of the shell are all geometrical features. The analysis of single cylindrical shell and corrugated barrels are carried out for one hundred eighty samples for each parameter by modeling with SAP 2000. The combined sample by using LHS method is presented on the appendix. The maximum shell stress and maximum deflection is selected among the result of single barrels and corrugated cylindrical shell. The analysis results are stated below.

4.1.1 Results from variable length of the shell

Stability of shell is described by deflection and the shell stress. In this section, the effect of variable length on the stability of reinforced concrete cylindrical shell is described depending on the circumferential stress and maximum deflection at the crown and at the edge of the shell. As shown in the analysis result on the Figures 4.2, shell stress at crown of the shell increase in 39.18% and the circumferential stress increase by 25 % at edge of the shell as the length of the single cylindrical shell increase from 7m to 65 m. Thirty sample are used for this parameter. The maximum deflection at crown of cylindrical shell increases in 65.71% and the maximum deflection at edge in 60 % when the length of the shell increases from 7m to 65m as shown on the figure 4.3. This result shows that the deflection of the shell increases, when the length of the shell increase. As a result the stability of the shell decreases with the increase of the deflection. The shortest shell is more stable than the longest shell, where other parameters are constant.

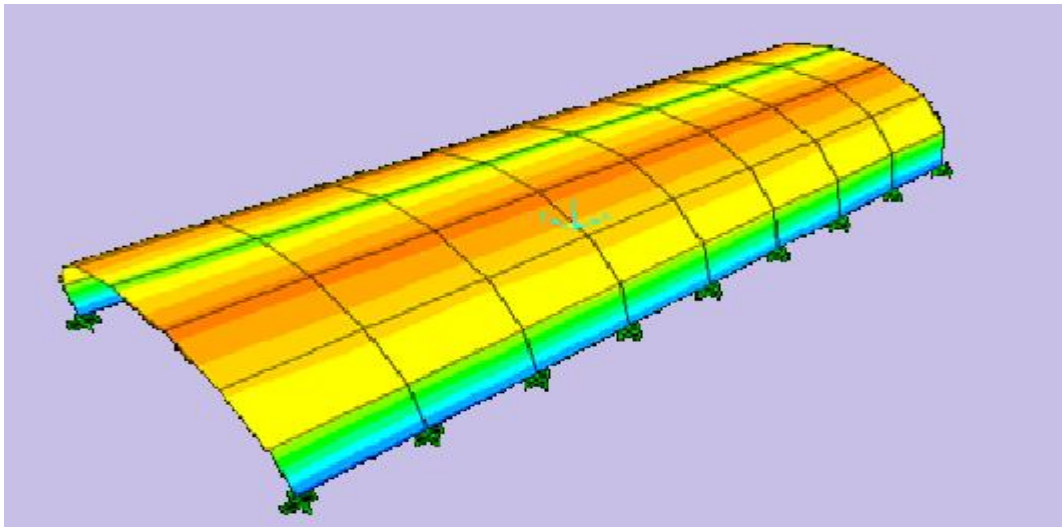


Figure 4 1 Stress distribution of cylindrical shell

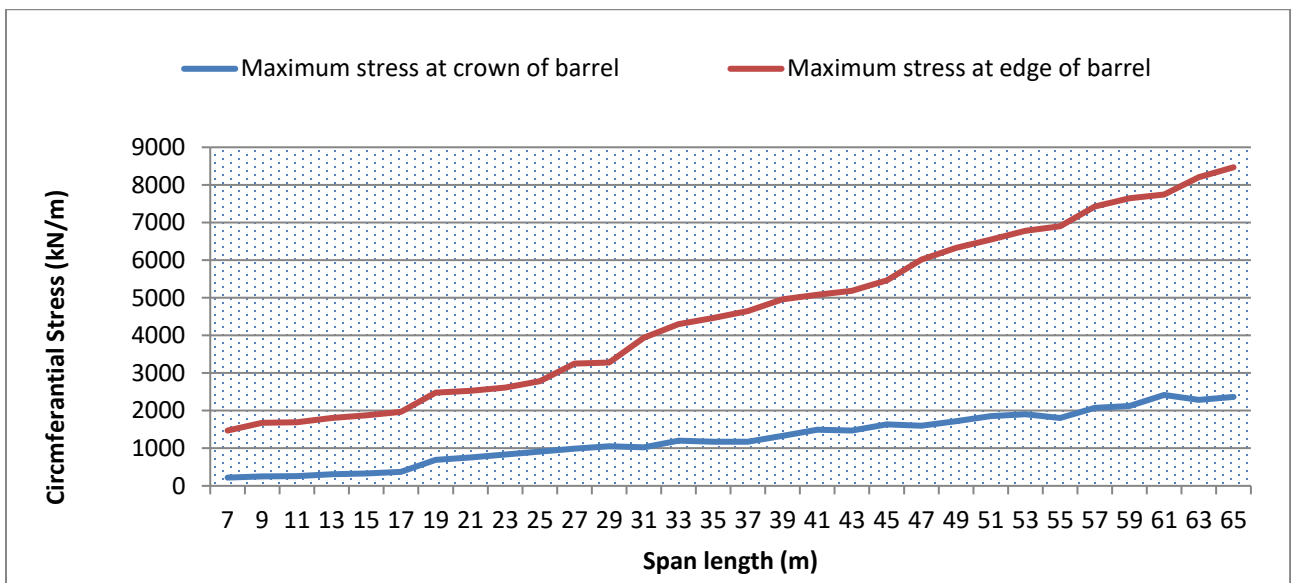


Figure 4 2 Maximum stresses on the shell with variable length

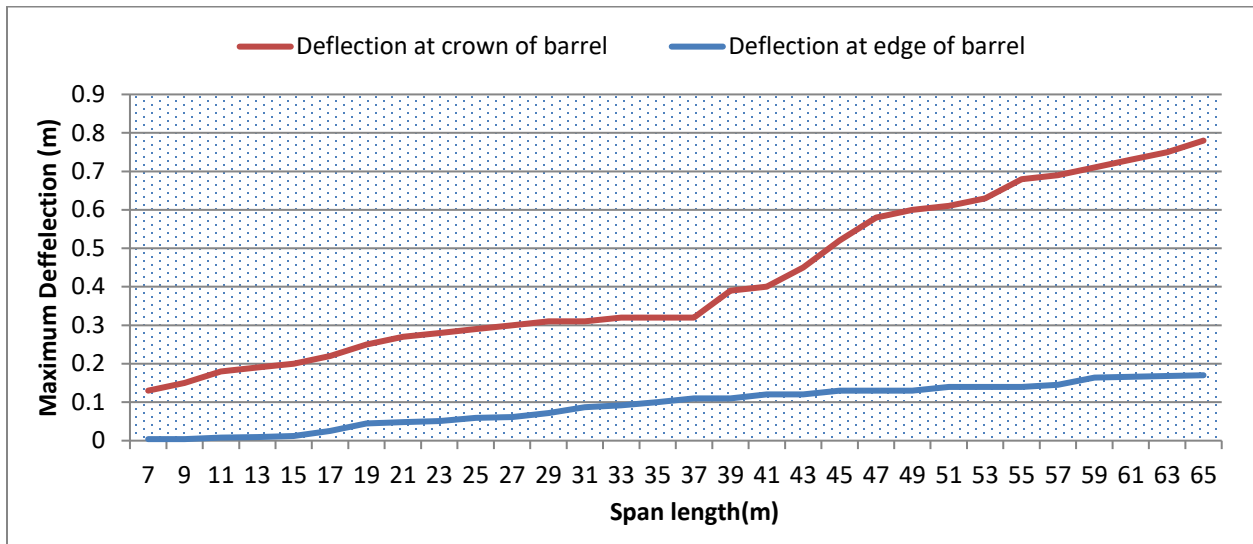


Figure 4 3 Maximum deflections of the shell with variable length

4.1.2 Result from variable radius of the shell

In this section, the effect of radius on the performance of reinforced concrete cylindrical shell described depending on the circumferential stress and deflection. As shown in the analysis result on figure 4.4 indicate, the maximum stress at crown of the barrel and edge of the barrel increase in 45.83% and 83.33% respectively as the radius of single cylindrical shell increase from 8m to 31.2m. Thirty numbers of samples are used for this parameter. As shown on figure 4.5, the maximum deflection at crown of the barrel also increases in 51.85% and maximum deflection at edge 45% as the radius of the shell increase from 8 m to 31.2m. When radius of cylindrical shell increase, the deflection of the also increase. As the deflection increase the stability of the shell become minimized.

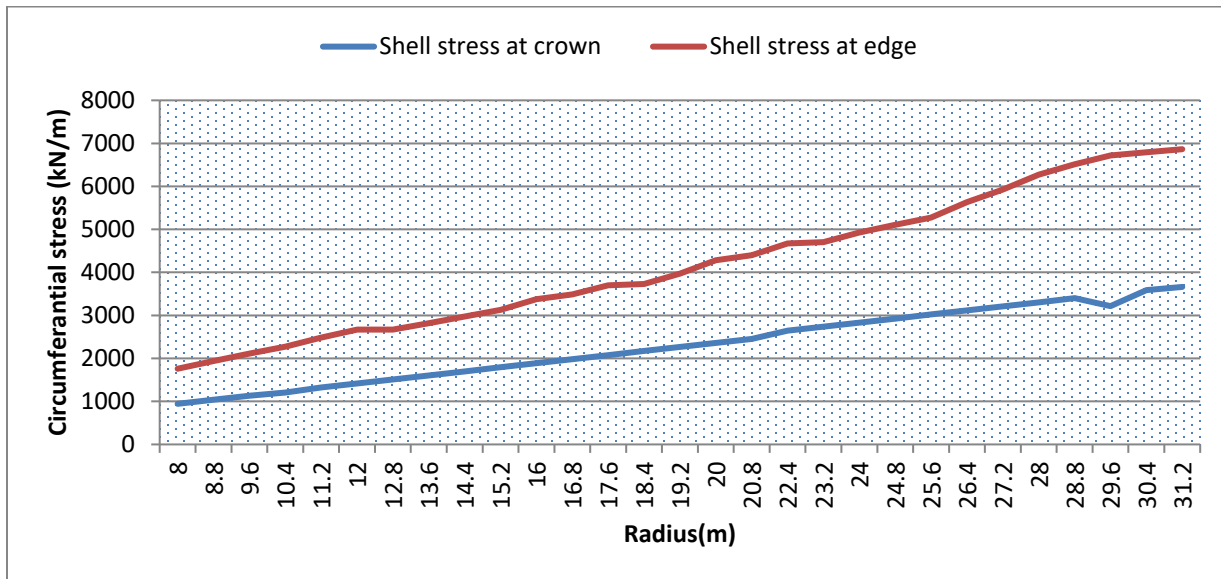


Figure 4 4 Maximum stresses on single barrel with variable radius

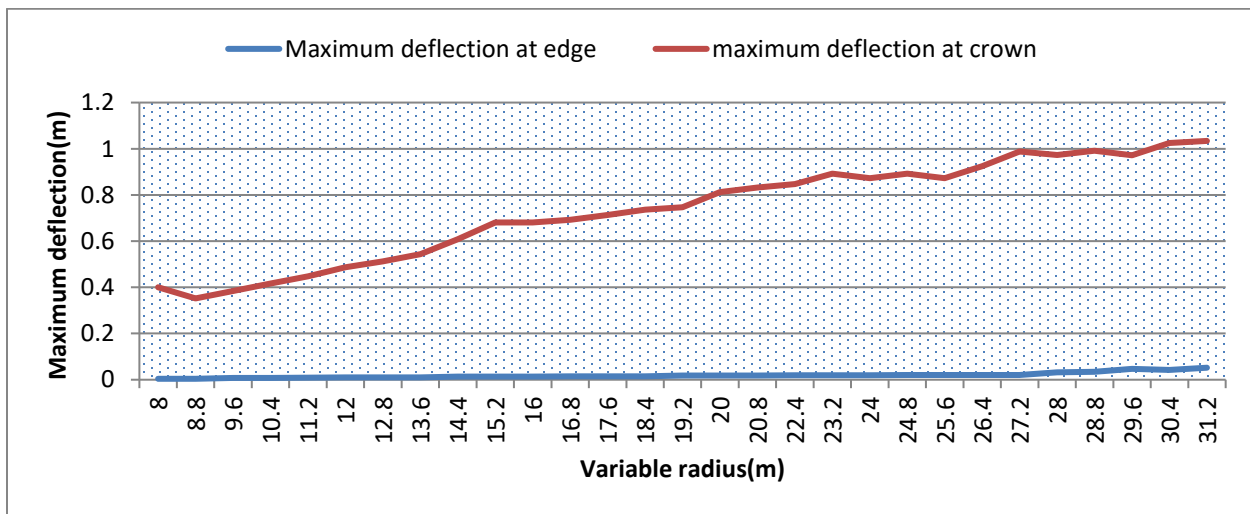


Figure 4 5 Maximum deflection barrel with variable radius

4.1.3 Result from variable shell thickness

In this section the effect of thickness on the performance of reinforced concrete cylindrical shell is described depending on the maximum shell stress and the maximum defalcation of single cylindrical shell. Fifteen numbers of samples are used from 0.20m to 0.48m. As shown on figure 4.6 the circumferential stress at crown of the barrel increases by 8.33%. Similarly, stresses at edge of the barrel also increase by 24.4% as the thickness of cylindrical shell increases. This is because

the increasing of the self-weight of the shell will increase due to the increase of the thickness. On the other hand, the maximum deflection at the crown of the barrel will decrease in 89% at crown and 56% at edge of barrel. In as the thickness of the cylindrical shell increases from 0.20m and 0.48m as shown figure 4.7. As the thickness of the shell increases the deflection become smaller and the stability of the cylindrical shell become increased. Although the higher thickness resists deflection, it can increase the self-Wight and affect the stability of the shell. Because of the above risen, the shell thickness must be in optimum amount.

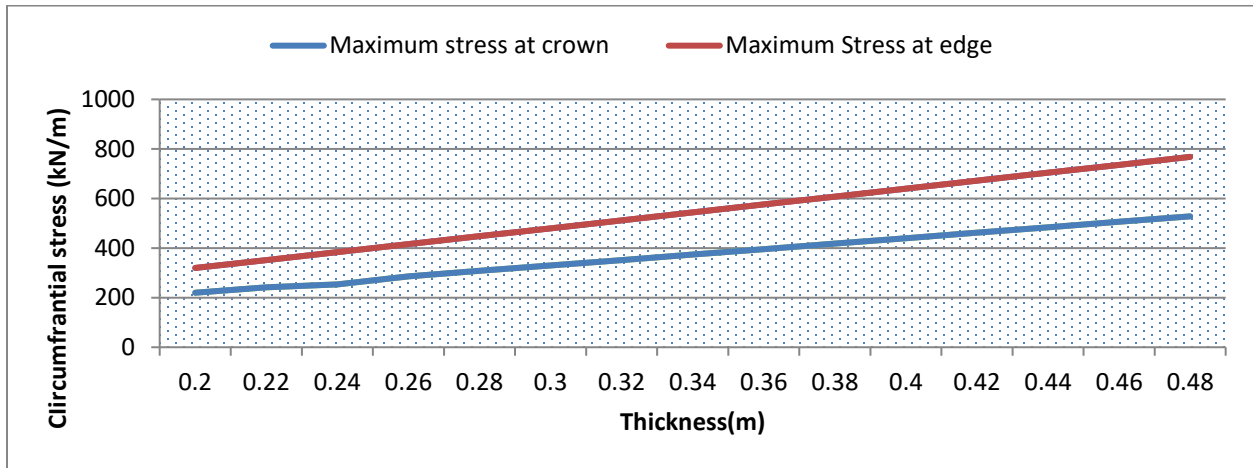


Figure 4 6 Maximum stress of single barrel variable thickness

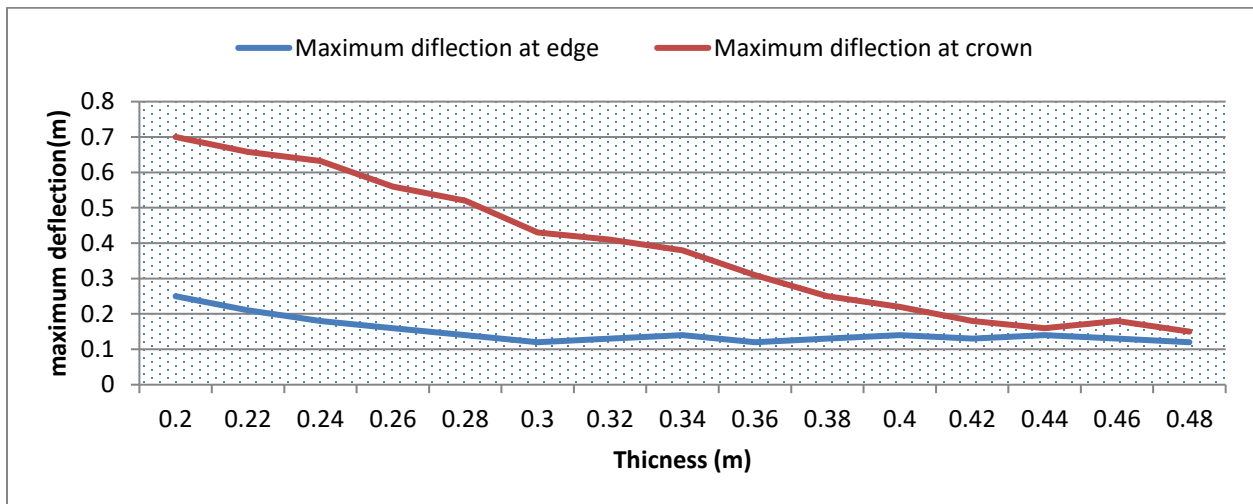


Figure 4. 7 Maximum deflection of the shell with variable thickness

4.1.4 Result in variable internal angel

Internal angle represent that the angle of expansion of the cylindrical shell. When the internal angle approaches to 90° the shape of cylindrical shell become more curved it acts perfectly as shell structure. When the internal angle approaches to 0° the shape of cylindrical shell becomes flattened it become similar to flat slab. The effect of this internal angle on maximum stress of cylindrical shell is described on the figure 4.8. Fifty samples are used between 15° to 80° . As shown in the figure, when internal angle increases from 15° to 80° , the circumferential stress at crown increase by 80%.The edge stress of barrel increase by 81.13%. Figure 4.9 shows that the maximum deflections of cylindrical shell increase by 92% at crown and increase by 46% at edge of barrel. As the internal angel of the cylindrical shell increases the deflection of the shell also increases, so the stability of the shell decreases.

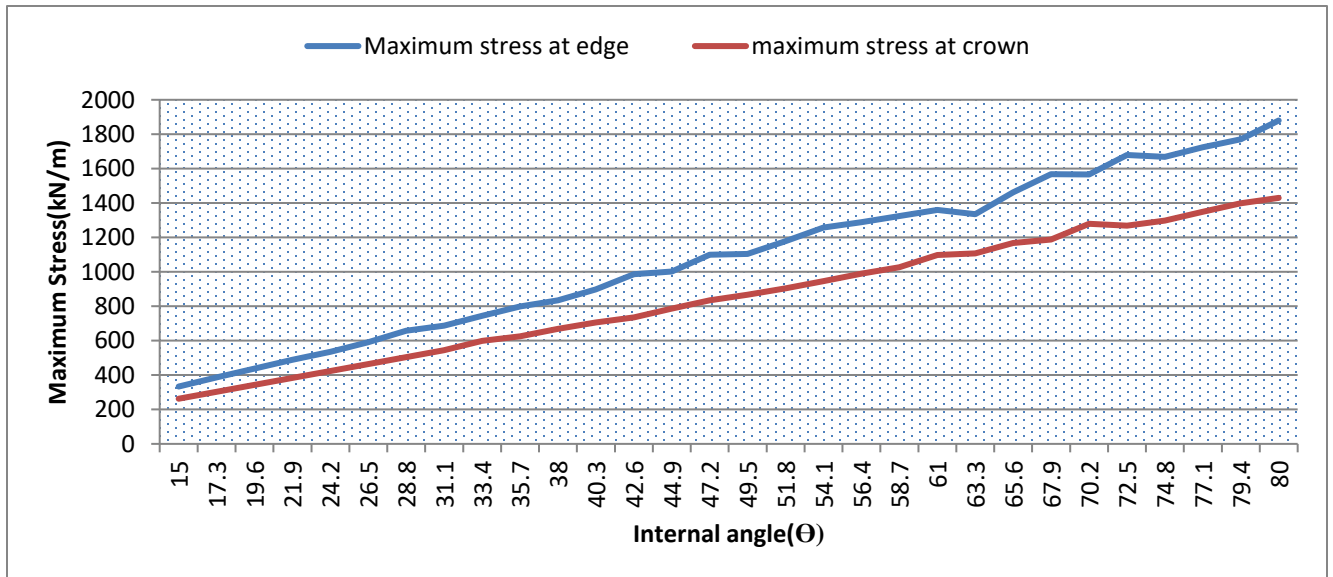


Figure 4 8 Shell stress of single barrel in variable internal angle

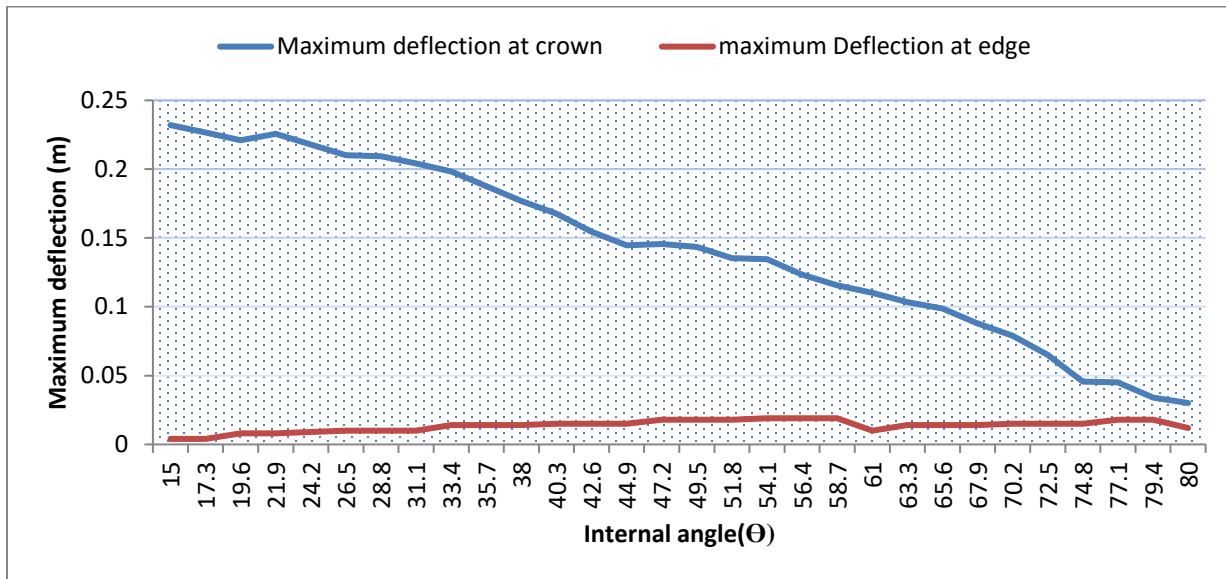


Figure 4 .9 Maximum deflection of the barrel with variable internal angel

4.1.5 Result of variable angle of inclination

The effect variable angel of inclination on performance of barrel is described on figure 4.14. Thirty samples are used for this parameter between 10° to 60°. This graph shows that the circumferential stress decreases by 20.23% at crown of the barrel and it decrease by 57.42% at the edge of barrel, when the angle of inclination increases from 10° to 60°. As shown on the figure 4.15, the maximum deflection of inclined barrel decreases by 53.9% at the upper part of the barrel and the maximum deflection increase by 83.3% at lower part of the shell. When the angel of inclination increase, the maximum deflection also decreases so the shell become more stable.

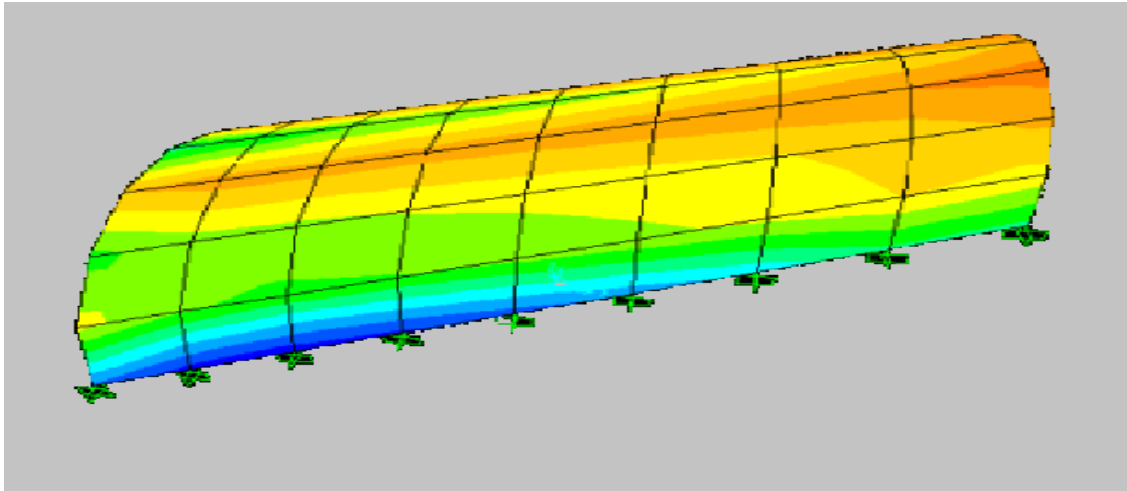


Figure 4.10 Stress distribution in inclined cylindrical shell

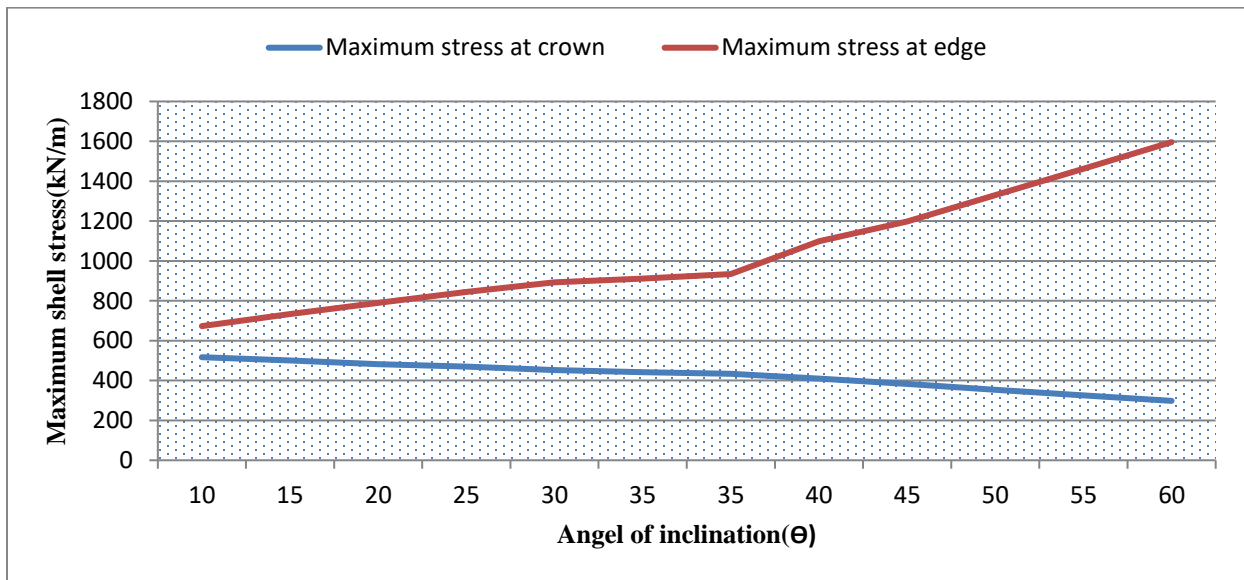


Figure 4 11 Maximum Shell Stress at with variable angle of inclination

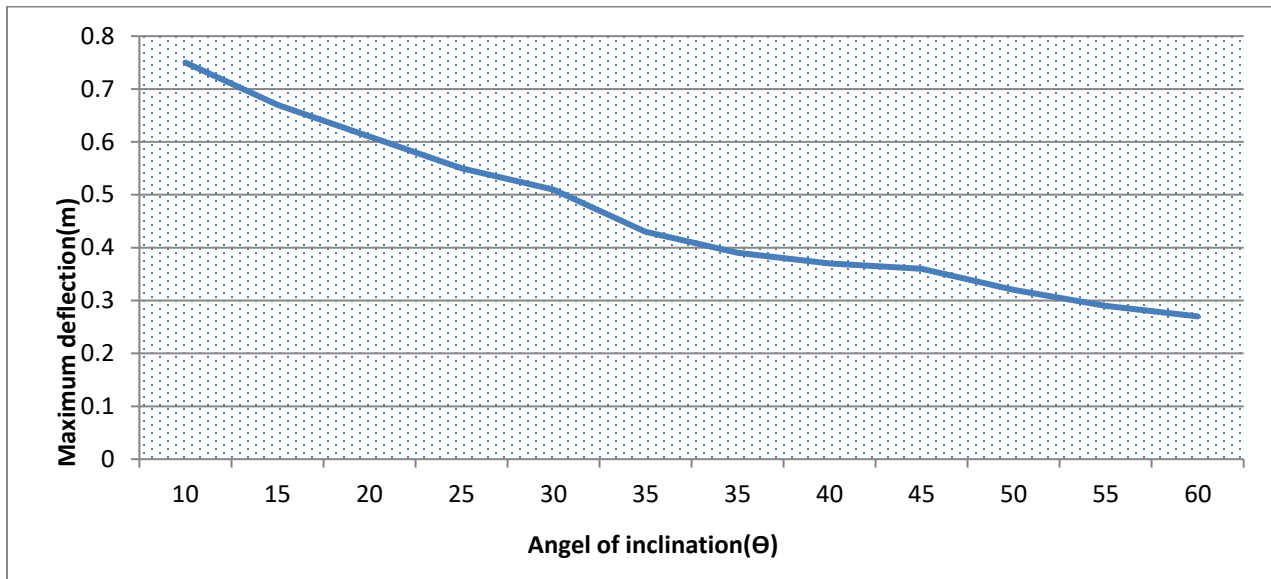


Figure 4 12 Maximum deflection of the barrel of inclination

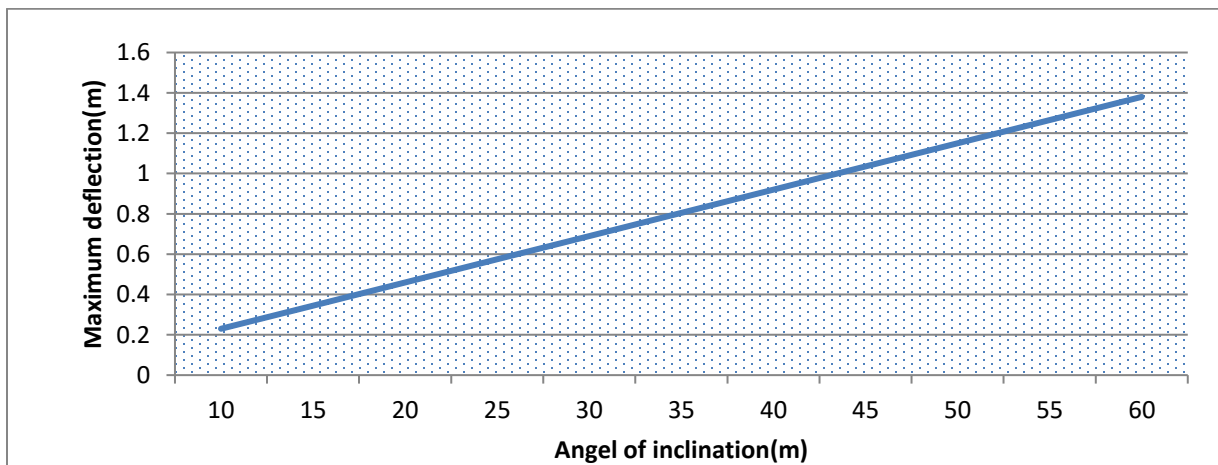


Figure 4 13 Maximum deflection at lower part of barrel

4.1.6 Depth of the edge beam

The support condition is one parameter that can highly affect the structural performance of corrugated shell. Edge beam is used for as a support for single barrels. The effect of variable depth of edge beam is described as follows. As shown in the figure 4.17 the shell stress at the crown of barrel decrease by 23.6 %, as the edge beams depth increase from 0.2m to 0.5m. The shell stress at the edge of barrel increase by 76%, when the depth of the beam increases from 0.2m to 0.5m.

When the depth of the edge beam of cylindrical shell increase the deflection become decreased and the stability of the shell become higher.

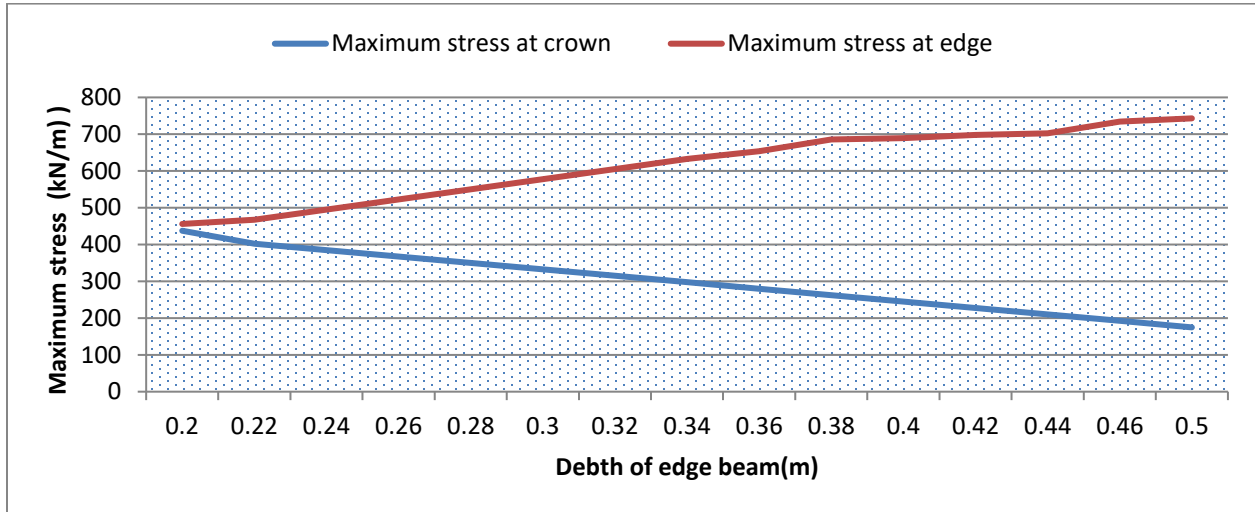


Figure 4 14 Maximum shell stress with variable depth of edge beam

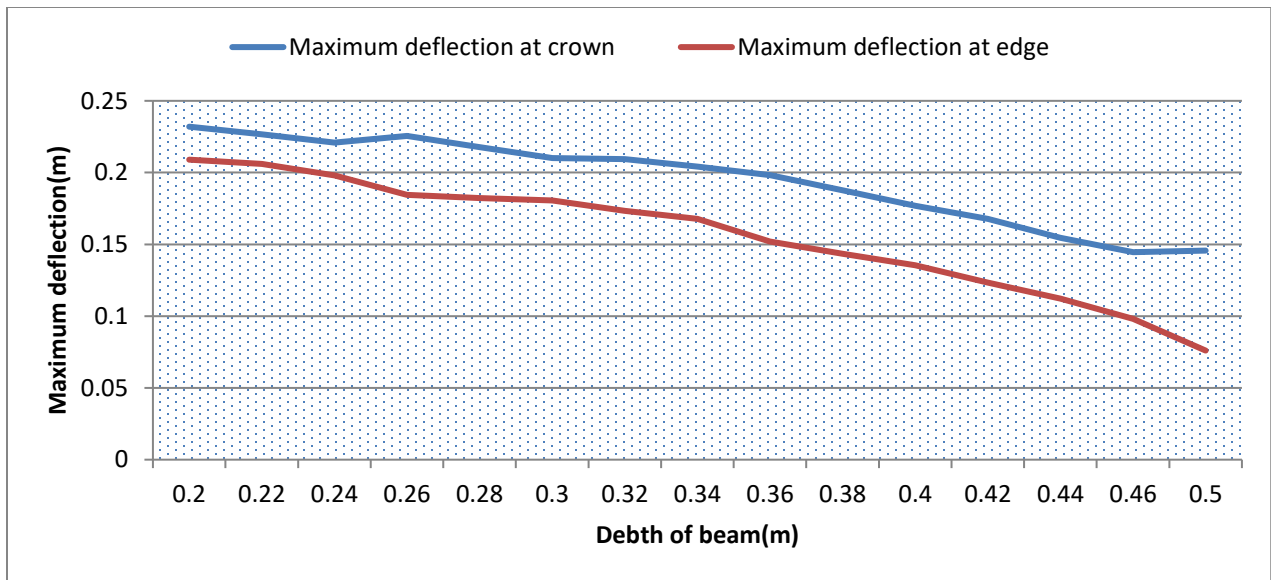


Figure 4 15 Maximum deflection of the barrel with variable depth of edge beam

As shown in the figure 4.18 the maximum deflection at the crown of barrel decrease by 40 %, and it decrease by 60% at crown of barrel, as the edge beams depth increase from 0.2m to 0.5m.

4.2 Shell geometry in corrugated cylindrical shell

In the above section, the parametric study of single barrels was discussed this study is focused on corrugation shells. To study the behavior of corrugated shells, parametric study of multi span cylindrical shells must be known. The following section deals with the effect of the variable parameters in corrugated shell structure. The picture shown below shows stress distribution in corrugated cylindrical shell.

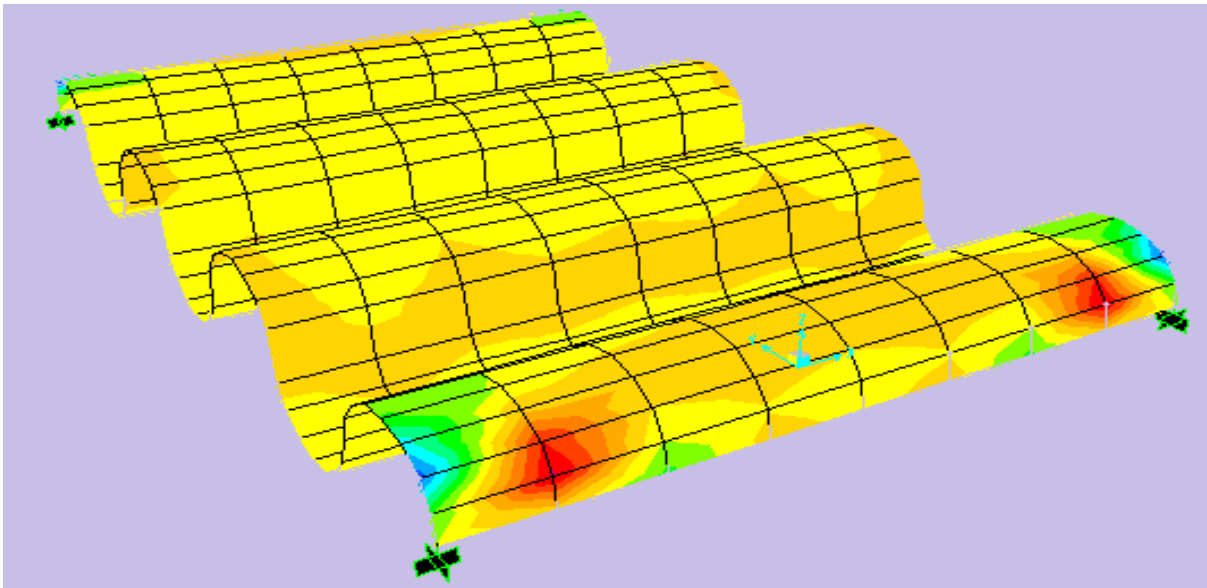


Figure 4 16 Stress distribution in corrugation cylindrical shell

4.2.1 Result in variable span length of corrugated shell

This section deals with the effect of variable length on the performance of reinforced concrete corrugated cylindrical shell. As shown in the figure 4.20 and, the circumferential stress on the upper wave on corrugated cylindrical shell increases in 36.36% and the stress at lower wave increase by 57.45% as the span length of the shell increases from 8m to 65m.

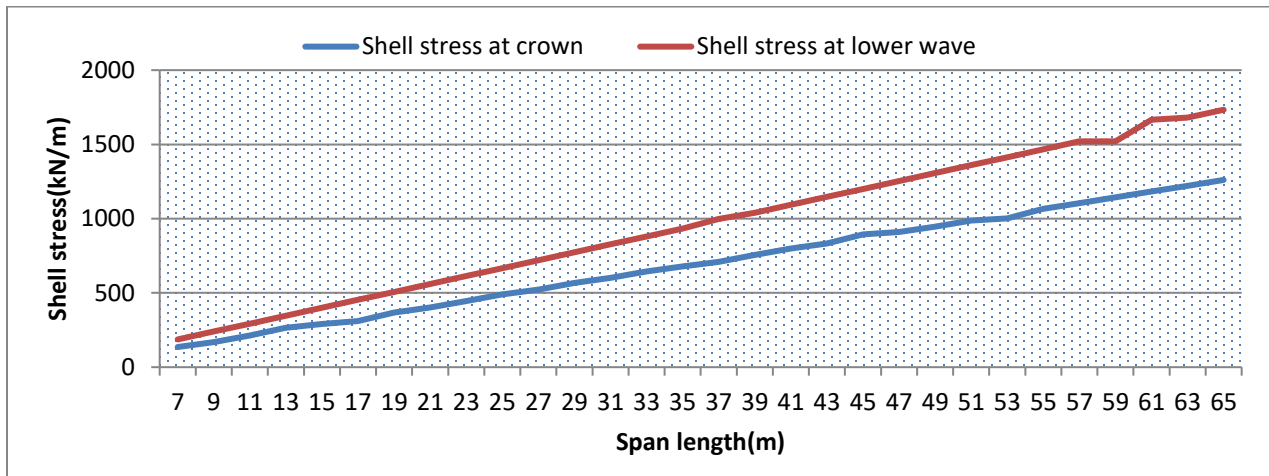


Figure 4 17 Maximum shell stress on the with variable span length

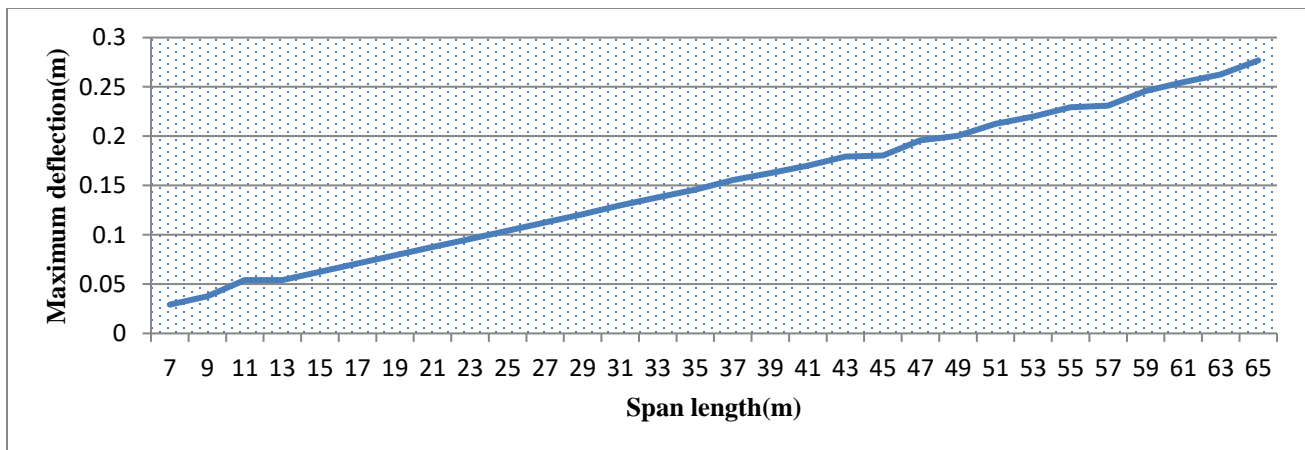


Figure 4 18 Maximum deflection in variable span length corrugated barrels

As shown in the figure 4.21 deflection increases by 93% with the increase of the span length of corrugated cylindrical shell from 8m to 65m. When the length of the shell increase, The deflection of the shell increases. As a result the stability of the shell decreases with the increase of the deflection.

4.2.2 Result in variable radius of corrugated shell

This section deals with the effect of variable radius of barrels on performance of reinforced concrete corrugated cylindrical shell. As shown in the figure 4.22, maximum shell stress on both

upper and lower wave increase by 74.64% and 52.29% respectively when the radius of the barrels increases from 4m to 15.6m.

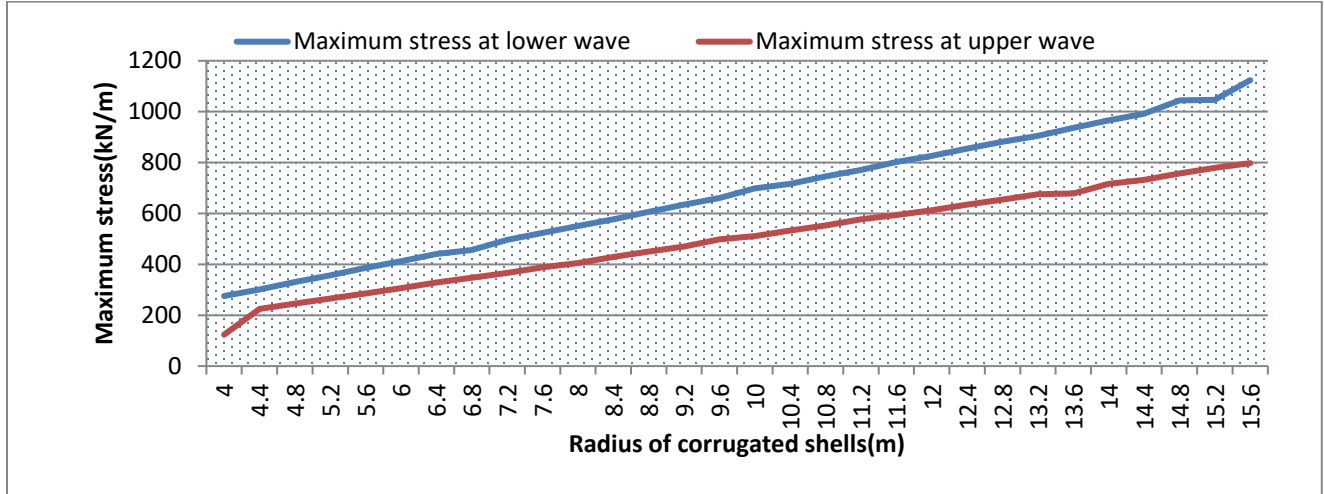


Figure 4 19 Maximum shell stress on the in variable radius

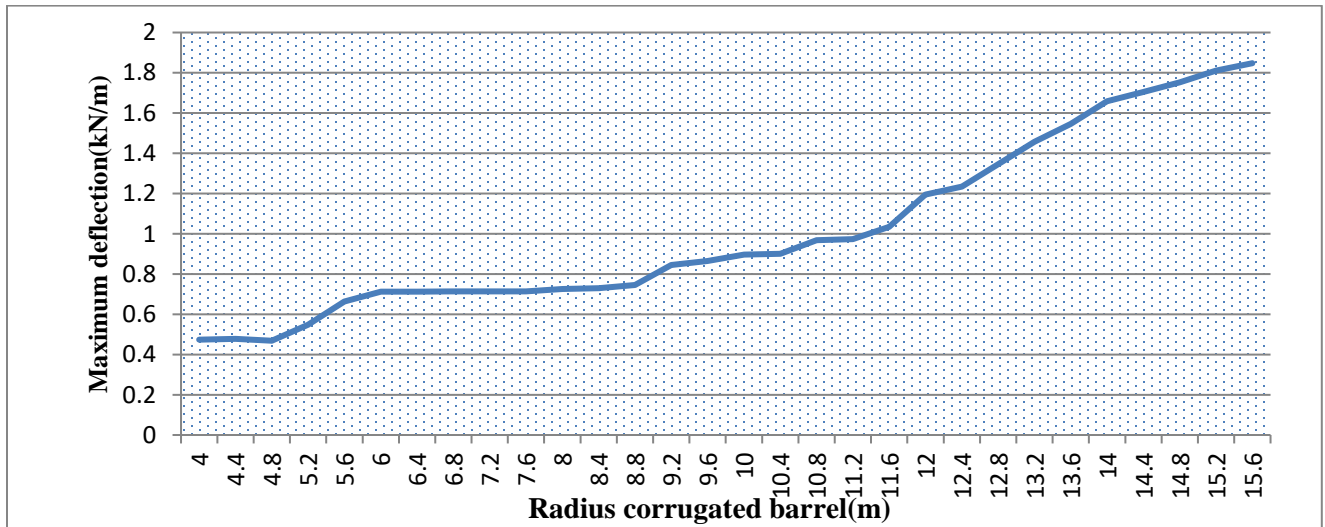


Figure 4 20 Maximum deflection of corrugated cylindrical shell in variable radius

As shown in the figure 4.23. The deflection of the corrugated cylindrical shell structures increase with the increase with 92.85 % when the radius of the shell increase from 4m to 15.6m. Increasing the radius of each barrel of corrugated shell results increment on the deflection of the shell. As the deflection increase the stability of the shell become minimized.

4.2.3. Result in variable thickness of corrugated shell

This section deals with the effect of variable thickness of barrels on the stability of reinforced corrugated shell. As shown on the figure 4.24 the maximum circumferential shell stress on upper and lower wave of corrugated cylindrical shell increase by 53.78% and 60.23% respectively, when the thickness of the shells increases from 0.02m to 0.6m.

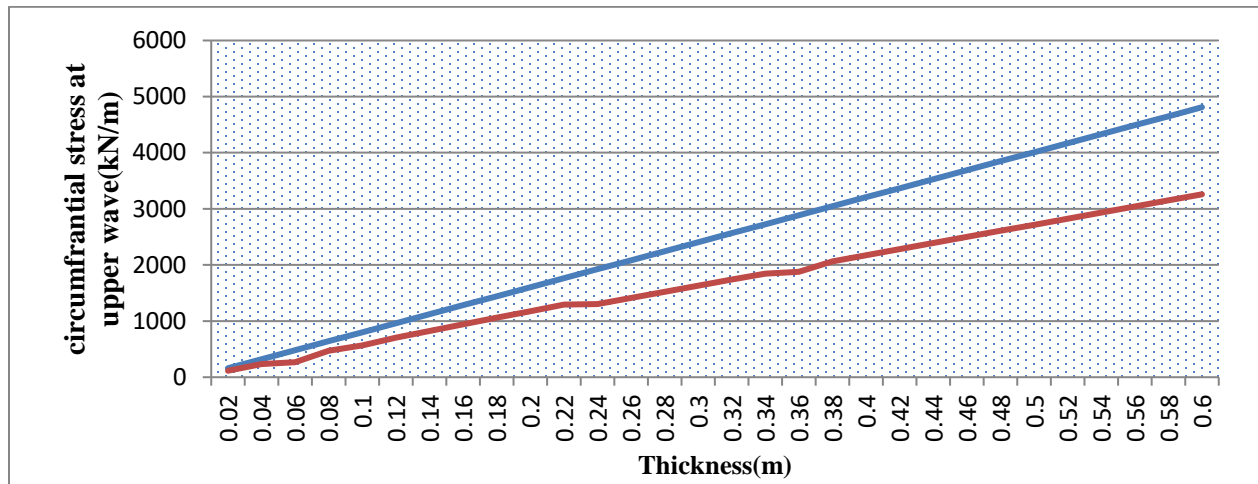


Figure 4 21 Maximum shell stress on the variable thickness

The analysis shows on figure 4.25 that the maximum deflection of the reinforced concrete corrugated cylindrical shell decreases by 83.33% when the thickness of the shell increases from 0.02m to 0.6m. As the thickness of the corrugated shell increases the deflection become smaller and the stability of the cylindrical shell become increased. Even though a thicker material resists deflection, it can add to one's own weight and affect the stability of the shell. To achieve a stable structure, the appropriate thickness must be used.

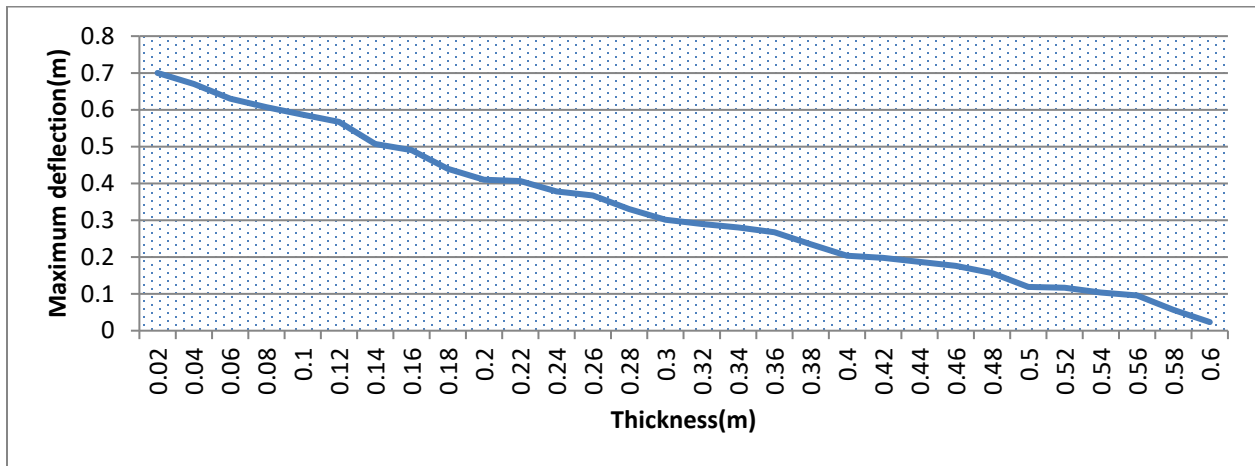


Figure 4 22 Maximum deflection in variable thickness

4.2.4. Result in variable number of lobes of corrugated shell

This section deals with the effect of number of lobes on the structural performance of reinforced concrete corrugated cylindrical shell. As shown on the figure 4.26, the maximum shell stresses decreases by 85.71% on the upper wave of the corrugated shells and the stress decrease by 76.9% on the lower wave of the corrugated shell ,when the number of lobes increases from 3 lobes to 11 lobes. Figure 4.28 show that the maximum circumferential stress in lower wave of corrugated shell decreases when the number of lobes of corrugation.

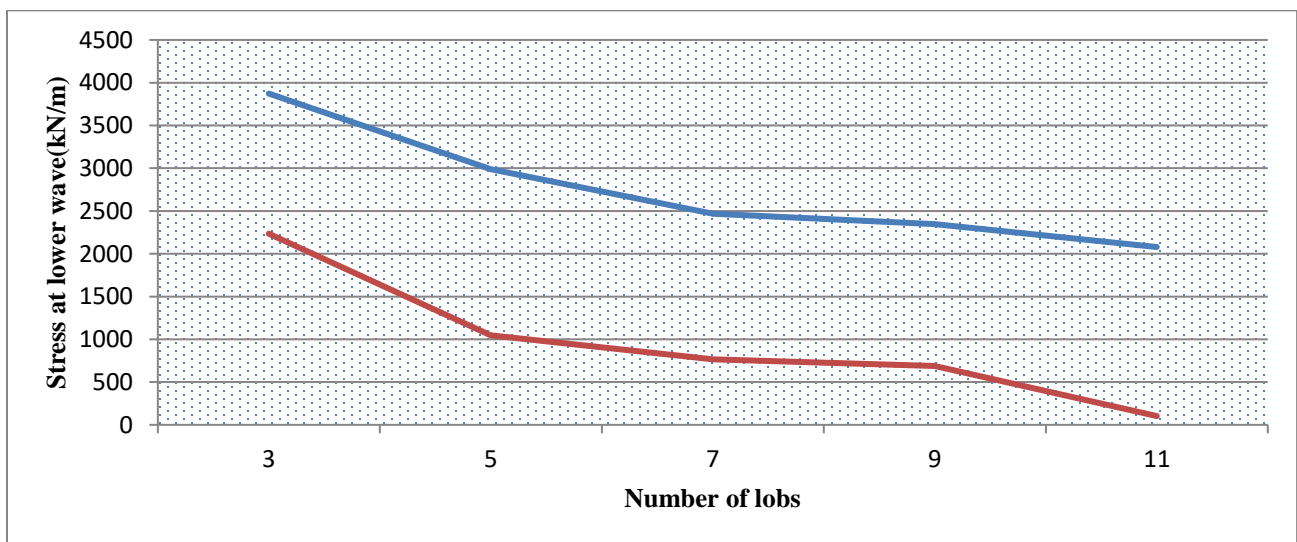


Figure 4 23 Maximum Stress on variable number of lobes

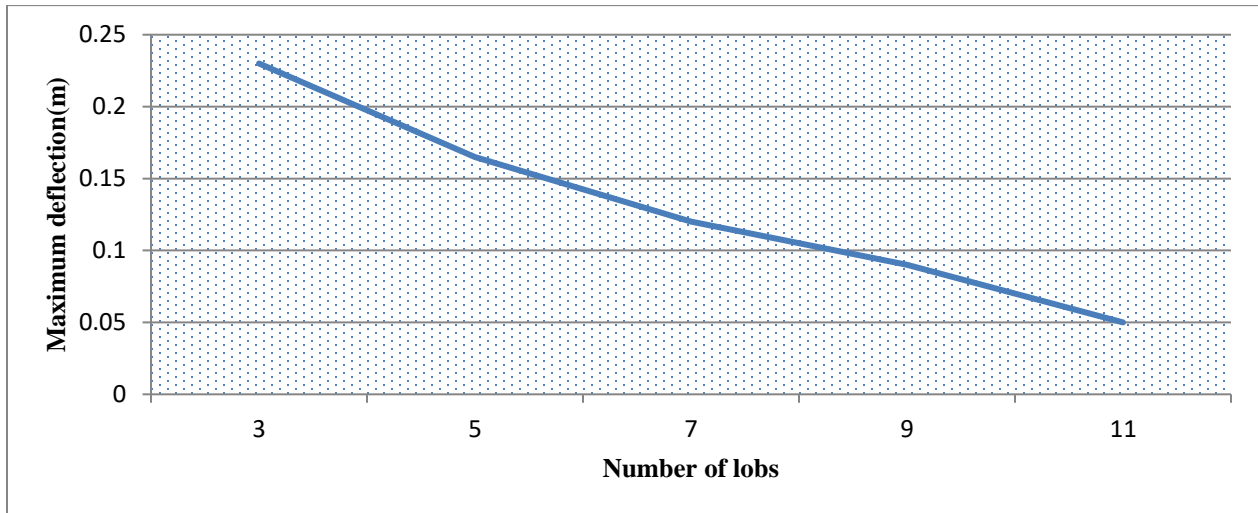


Figure 4 24 Deflection on corrugated cylindrical shell with variable number of lobes

As the number of lobes increases the maximum deflection decreases by 73.46% when the number of lobes increases from 3 lobes to 11 lobes. As the number of the barrels (lobs) increases the maximum deflection decreases. Then the structure will be more stable. To improve the load bearing capacity and global stiffness of corrugated shell, increasing the number of lobes is best method. The corrugation increases the moment of inertia of shell and it makes the shell more stable.

4.2.5. Result in variable angle of inclination of corrugated shell

This section deals with the effect of height of inclination on the structural performance of reinforced concrete corrugated shell. As shown on the figure 4.28, the maximum shell stresses on the upper wave of the corrugated shells increases, by 53.84 % when the angle of inclination increases from 10° to 34°. The maximum stress at lower wave of the corrugated barrels increase by 38.3%, when the angles of inclination increase from 10° to 34°.



Figure 4 25 Shell stress of corrugated barrels with variable Angle of inclination

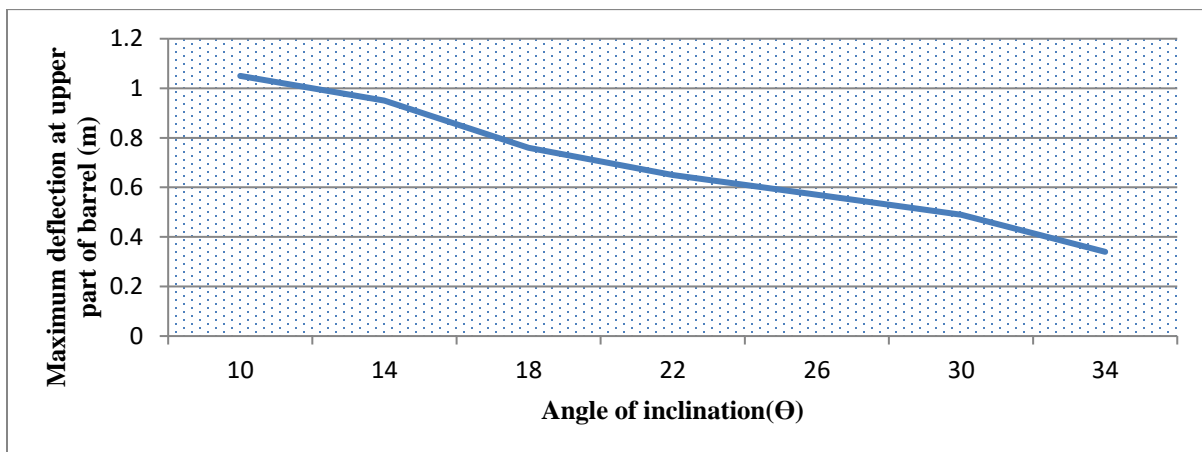


Figure 4 26 Maximum deflection of corrugated barrels with variable inclination

As shown on figures 4.29 maximum deflection of corrugated barrel decreases by 72.72% when the height of inclination increase.

4.2.1. Conical inclined corrugated shells

The following section deals with the effect of different parameters in corrugated barrels. The picture shown below shows stress distribution in corrugated barrel variable parameters. The radius of the barrels is small at upper part of shell and larger at lower side. The barrels are inclined to the center.

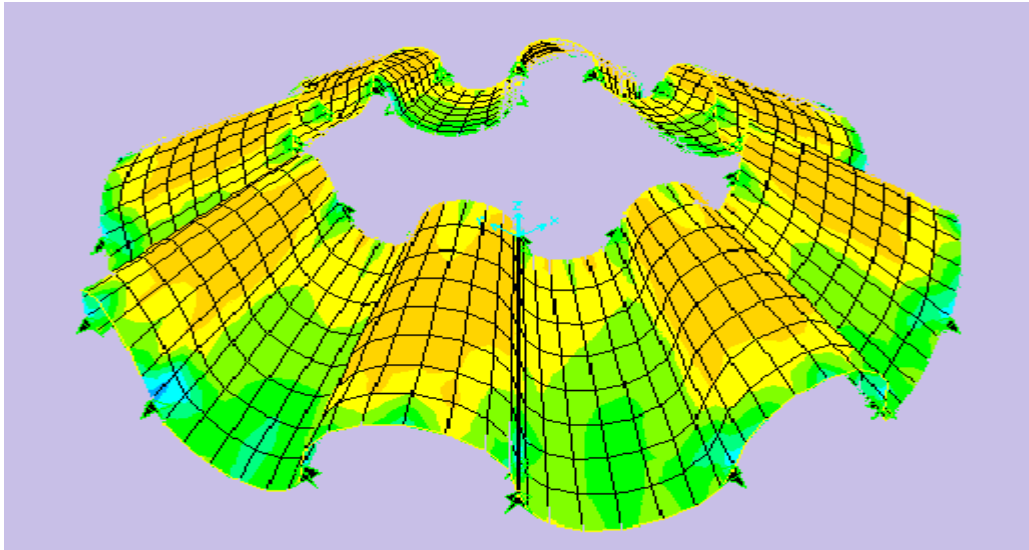


Figure 4 .27 Stress distributions in corrugation barrels with different slop different radius

As shown on the above picture, the corrugated shape is done by connecting number of individual lobes. The single lobe contains two barrels connected at their edge in opposite direction. In this section the effect of increasing the number of barrels on the performance of reinforced concrete cylindrical shell with different support position is described depending on the maximum circumferential stress and maximum deflection.

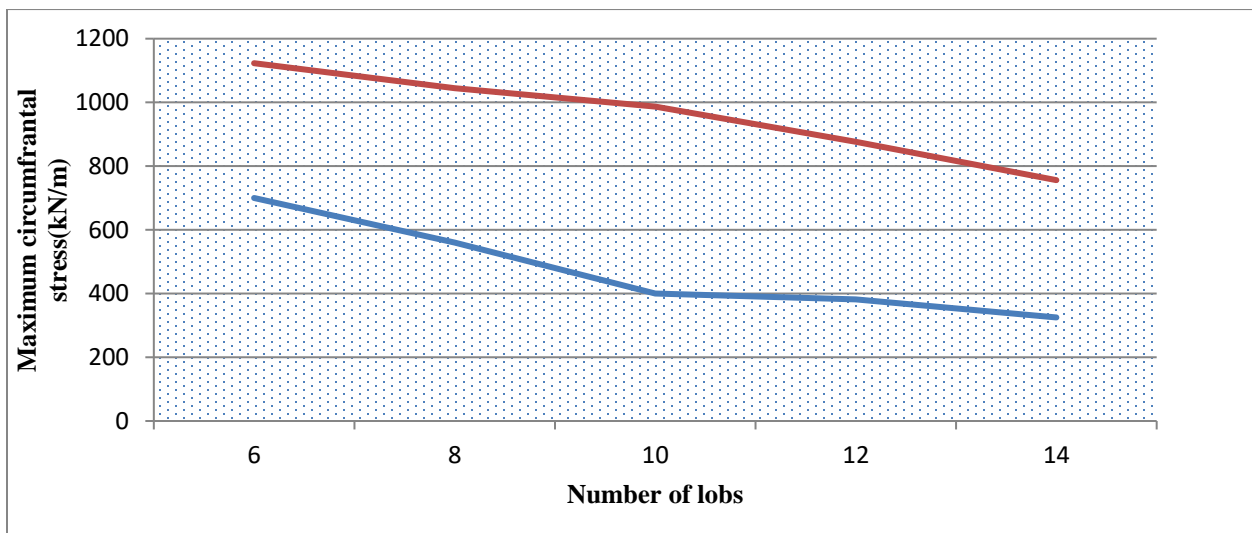


Figure 4 28 Stress on upper wave of corrugated conical shell with many members of lobes

As shown in the figure 4.31, the shell stress distribution on corrugated conical shell increases on the lower part of the shell. To improve the stability of the shell around the lower part of the barrels, increasing the amount of reinforcement is best mechanism.

4.3 Stability Analysis of corrugated shell structures

A structure's ability to resist unwanted movements including deformation, sliding, collapsing, and overturning is referred to as stability. The geometry of the member arrangements and the conditions of the supports both affect stability.

Various methods are used to describe the stability of concrete structures. It can be the ability to restore equilibrium or resistance to immediate change, displacement, or overturning. For instance, a stable structure must also maintain its stability under any potential system of loads. Therefore, when determining whether a structure is stable or not, load kinds and point of applications are not taken into account. A shell structure's geometry will alter under compression if it is unable to match the requirements, which will reduce its ability to withstand loading and make it unstable. Design must take instability into account because it can result in catastrophic failure. Finally, the following parts will talk about a structure's stability.

Stability standards

To determine if a structure is in stable equilibrium under a specific set of loadings, stability criteria must be established.

1. If upon releasing the structure from its virtually displaced state the structure returns to its previous configuration, structure is stable.
2. The structure is in stable equilibrium when small perturbations do not cause large movements like a mechanism.
3. If the structure does not return to its original state following the release of the deflection and buckling. Therefore, structure is in unstable equilibrium when small perturbations produce large movements, and the structure never returns to its original equilibrium position.

4.31 Deflection and Stability

Deflection refers to how much a structural component changes shape in response to a load. Depending on the severity of the load, the component's shape, and the material used to make it, the change may be either a change in distance or an angle that is either apparent or invisible.

- How is deflection related to stability?
 - o When a structure deflects, the total potential energy, which maintains structural stability is highly affected.
 - o Structure deflections act against stability.

A structure has a tendency to lose stiffness, undergo a detectable change in geometry, and become unstable when subjected to a large amount compressive force (or stress). When deflection happens, the shell structure is unable to maintain a stable equilibrium state and loses its ability to support the applied loads.

Making ensuring the shell structure is strong enough to withstand the imposed loading with an adequate load factor against collapse is not sufficient. It is also necessary to make certain that deflections do not become excessive.

Since the geometry of the shell structure has a significant impact on its resistance to deflection, it also has a significant impact on the stability of the shell structure. The geometrical shape of the barrel has a significant impact on how much the deflection of the shell structure increases.

For instance, as barrel length grows, deformation increases as well, as shown in figure (4.21). This indicates that the corrugated shell will be more stable at its optimum span length.

On the other hand, when we consider the influence of varying radius, the deflection likewise increases with the rising radius of the barrel as shown on figure (4.23) causing the structure to be stable at the optimum radius.

The other significant factor that influences the stability of the shell structures is the thickness of the barrels. According to figure (4.24), as the thickness of the barrel grows, the barrel's stiffness

becomes improved and the structure becomes more stable since it is less likely to deform. However, raising the thickness in a greater amount increases the shell's self-weight, which has a detrimental effect on the performance of the structures. To address this problem, it is better to employ more lobes than a single barrel. According to the figure below (4.32), increasing the quantity of corrugated barrels is a better way to mitigate structural deformation than increasing the structure's thickness. The global stiffness of shell structures is improved as the number of corrugations increases and it makes corrugated shell structures less vulnerable to deflection. The following diagram (4.32) illustrates the comparison between the effect of the thickness and number of lobes of corrugated shell structure.

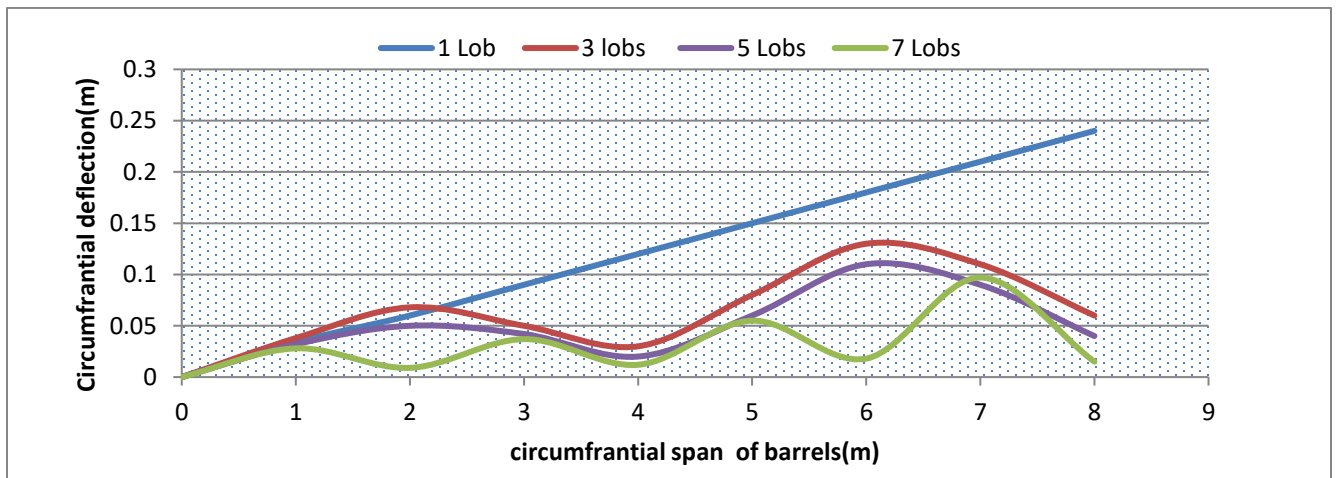


Figure 4 29 Comparison between single barrel and corrugated shell structure

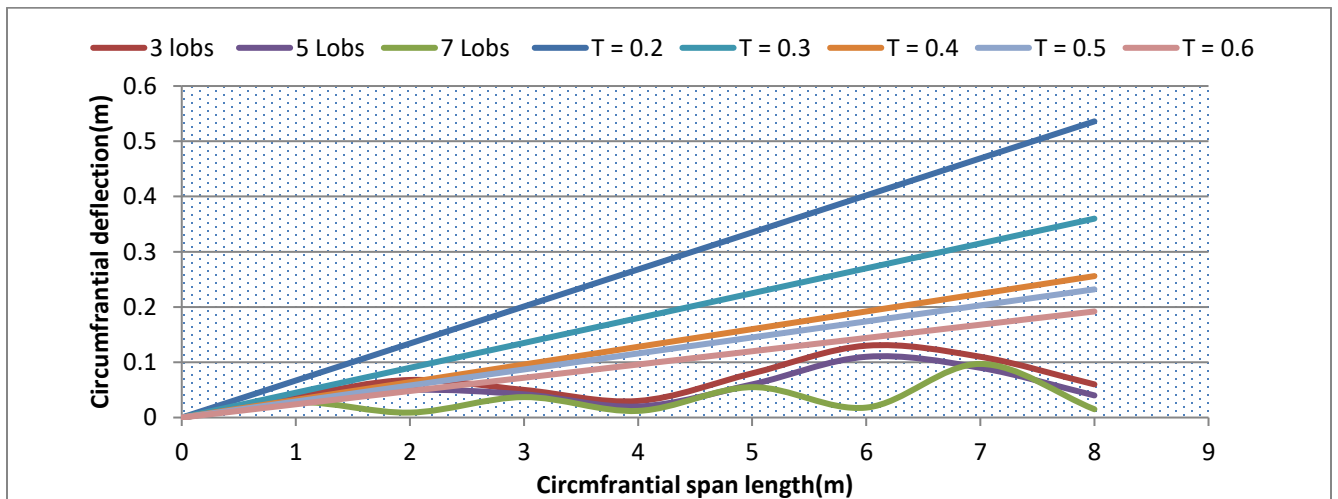


Figure 4 30 Comparison between the effect of thickness and number of lobes

CHAPTER FIVE

CONCLUSIONS AND RECCOMENDATION

5.1 Conclusions

In this thesis the parametric study of corrugated cylindrical shell by using finite element software package SAP2000 , parameters those describe the shell geometry , such as length , radius , thickness of the shell are used to investigate force , moment , stress and deflection. In addition to the geometrical of other parameters such as number of lobs of corrugation, internal angle and the slop of inclination of the barrels are parameters used to analyze the shell structures.

The following conclusions are derived from the findings of this study:

1. The structural stability of a corrugated reinforced cylindrical shell can be enhanced by reducing the length and radius.in addition to this increasing the thickness of the barrels improves the load baring capacity of the shell.
2. The structural performance and stability of a corrugated reinforced cylindrical shell can greatly improve as the number of corrugations rises than increasing the thickness of the shell, because the number of lobes increases the shell's global stiffness.

5.2 Recommendation

As discussed in the above result, the recommended way to improve the stability and structural performance of corrugated cylindrical shell structure is varying the geometry of the shell.

Increasing the number of the barrels is more advisable to achieve required strength and stability effective than increasing the radius and length of corrugated cylindrical shell.

This study focused only in static analysis of reinforced concrete corrugated cylindrical shell structure. It has recommended other researchers can make on dynamic analysis of reinforced cylindrical shell.

In this study SAP2000 software is used for modeling and analysis of corrugated cylindrical shell, also be recommended that to analyze corrugated cylindrical shell roof structure by using other commercial software.

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