ADDIS ABABA INSTITUTE OF TECHNOLOGY
DEPARTMENT OF CIVIL ENGINEERING

ASSESSMENT OF SHEAR FAILURE FOR EXISTING SIMPLY SUPPORTED BRIDGE GIRDERS

By

Getasew Matebie

A thesis Submitted to the School of Graduate Studies of Addis Ababa University in Partial fulfillment of the degree of Master of Science in Civil Engineering

Advisor

Esayas Gebreyohannes (Dr.)

June 2013

ADDIS ABABA INSTITUTE OF TECHNOLOGY
DEPARTMENT OF CIVIL ENGINEERING
(STRUCTURAL ENGINEERING)

Assessment of Shear Failure for Existing Simply Supported Bridge Girders

By

Getasew Matebie

Addis Ababa Institute of Technology (AAiT)

Approved by Board of Examiners

Esayas Gebreyouhannes (Dr.)	________________	________________
Thesis Advisor	Signature	Date
Adil Zekaria (Dr.Ing.)	________________	________________
Internal Examiner	Signature	Date
Girma Zerayohannes (Dr.Ing.)	________________	________________
External Examiner	Signature	Date
Mr. Dawit Admasu	________________	________________
Chairman	Signature	Date

Declaration

I the undersigned, declare that this thesis is my work and all sources of materials used for this thesis have been duly acknowledged.

Name: Getasew Matebie

Signature:

Place: Addis Ababa University

Date of Submission: June, 2013

Contents

Acknowledgement…………………………………………………………………….i

Abstract…………………………………………………………………………..ii

List of Symbols ……………………………...……………………………………..iii

List of Tables………………………………………………………………….....vi

List of Figures ………………….…………………………………………………viii

Chapter 1. Introduction……………………………………………………….……………1

1.1. Background 1

1.2. Aim and Purpose 1

1.3. Specifications and Delimitations 2

1.4. Methods..2

1.5. Organization...3

Chapter 2. Load Rating for Shear……………………………………………...……....4

2.1. Introduction 4

2.2. Bridge Girder Shear Rating..4

2.3. Shear Rating Methods 6

2.3.1. Introduction 6

2.3.2. Load Factor Rating (LFR) Method 6

2.3.3. Load and Resistance Factor Rating (LRFR) Method....................................8

2.3.4. Comparison of LFR and LRFR..8

Chapter 3. Load and Resistance Factor Rating (LRFR)…………………………………9

3.1. Introduction 9

3.2. General LRFR Equations 9

3.3. The Load and Resistance Factor Load Rating System 13

Chapter 4. Bridge Girder Shear Resistance Determination………………………..16

4.1. Introduction..16

4.2. Shear Resistance Models 16

4.2.1. Introduction 16

4.2.2. 45^0 Truss Model 16

4.2.3. Variable Angle Truss Model 17

4.2.4. Compression Field Theory...18

4.2.5. Modified Compression Field Theory..20

4.3. Shear Resistance using MCFT..29

4.3.1. Shear Resistance Determination using MCFT..29

4.3.2. Determination of β and θ 30

Chapter 5. Analysis of Simply Supported Bridge Girders……………..………………36

5.1. Introduction..36

5.2. Live Load Force Effects..36

5.3. Bridge Girder Loading 40

5.3.1. Dead Loads 40

5.3.2. Vehicular Live Loads..42

5.3.3. Live Load Distribution Factors...44

Chapter 6. LRFR Analysis of Bridge Girders…………………………………...………56

6.1. Introduction..56

6.2. Bridge Girder 1 LRFR Analysis and Rating 56

6.3. Bridge Girder 2 LRFR Analysis and Rating 76

6.4. Bridge Girder 3 LRFR Analysis and Rating 95

6.5. Bridge Girder 4 LRFR Analysis and Rating 109

Chapter 7. Results and Discussion………………………………...…………………129

7.1. Results..129

7.1.1. Results due to Design Vehicular Load..129

7.1.2. Results due to ERA Legal Loads……..130

7.1.3. Results due to ERA Legal Loads……..131

7.2 Discussion...133

Chapter 8. Conclusions and Recommendations……..…………………………………135

8.1. Conclusion..135

8.2. Recommendations...135

References……………………………………………………………..…………...136

Acknowledgements

Thanks to God for each and every success in my life and satisfactory accomplishment of this thesis. He has given me love, courage and strength throughout my school time and the entire life as well.

I would like to express my sincere appreciation to my advisor, Dr. Esayas Gebreyohannes, for his valuable advice, constant guidance, and encouragement during every stage of this research.

I am deeply grateful to my parents, Matebie and Asmare, who have continuously supported me throughout my years of education.

At last but not the least, I would like to express my deepest appreciation to Ato Chalachew Alemu, on behalf of Amhara Rural Roads Authority, who gave me important data for my thesis.

Abstract

Failure of bridges has been one of the major problems in the transportation and socio-economy of Ethiopia. So researches are the main tools that can help investigate the main causes of bridge failures. Failure of bridges may be due to construction or design problems. This thesis addresses whether shear failure of bridge girders is the cause of bridge failures or not.

This study implemented analytical assessment method even though experimental evaluation method can also be performed to rate bridge girders for shear. Analytical assessment method involves only design data during evaluation whereas experimental evaluation takes data directly from bridge girders.

In this study an attempt was made to rate bridge girders for shear by analyzing four simply supported bridge girders which are constructed by Amhara Rural Roads Authority. To achieve this objective the research had a methodology of a task involving literature review; discussion and comparison of rating methods and data collection and analysis for shear capacity evaluation.

LRFR method of evaluation of bridge girders for shear was adopted. LRFR method of evaluation involves the determination of bridge girder shear capacity and determination of shear due to external applied action. Though there have been several methods of determining shear capacity, Modified Compression Field Theory (MCFT) is used in this thesis.

This study showed that 75% of the bridge girders failed to fulfill the requirements of shear capacity for HL-93 design vehicular live load. It also gave that the bridges have shear capacity to resist the shear which can arise from ERA Legal loads.

This study also showed that the average concrete shear capacity, average transverse reinforcement shear capacity and average nominal shear capacity of the four bridge girders determined using Modified Compression Field Theory are 25.53%, 58.88% and 41.60% higher than those obtained from 45^0 Truss Model respectively.

List of Symbols

A = Area of non-composite beam (mm^2)

A_c = area of concrete on the flexural tension side of the member (mm^2)

A_{ps} = area of prestressing steel on the flexural tension side of the member (mm^2)

A_s = area of non-prestressed reinforcing steel on flexural tension side of member (mm^2)

A_v = stirrup area

b_{eff}= effective flange width

b_v = the minimum web width within d_v

C= structural capacity

D = Dead load effect in load factor rating

DC = Dead-load effect due to structural components and attachments in load and resistance factor rating

d_e= the effective depth measured from the extreme compression fiber to the centroid of the tensile force

de=the distance from the face of rail to exterior face of exterior girder(mm)

d_v = the shear depth

DW = Dead-load effect due to wearing surface and utilities

e = eccentricity of a design truck from the center of gravity of the pattern of girders (mm)

EB =modulus of elasticity of beam material (MPa)

ED =modulus of elasticity of deck material (MPa)

e_g = distance between the centers of gravity of the basic beam and deck (mm)

ε_x= longitudinal strain

f_{po} = stress in prestressing steel when the stress in the surrounding concrete

fR = Allowable stress specified in the LRFD code

g= distribution factor

h=girder height

I = Impact factor for live load in Load Factor Rating

I = moment of inertia of beam (mm^4)

IL=influence line

IM = Dynamic load allowance

k_g=longitudinal stiffness parameter

Lcb=curb load

Ld= diaphragm load

LFR=load factor rating

L_g= girder load

LL = Live load effect

$LRFR$= load and resistance factor rating

Lrp=railing and post load

MDC=bending moment due to dead load

MDW=bending moment due to wearing surface

ML= moment due to lane load

M_{tml}= moment due to tandem load

M_{trl}= moment due to truck load

Mu= ultimate moment

Nb = number of beams or girders

NL = number of loaded lanes under consideration

Nu = factored axial force taken as positive if tensile (N)

P = Permanent loads other than dead loads

R = reaction on exterior beam in terms of lanes

RF = Rating Factor

RF_{inv}=inventory rating factor

RF_{LFR}= rating factor value using load factor rating method

RF_{op}=operating rating factor

Rn = Nominal member resistance

S=girder spacing

s=stirrup spacing

s_x=crack spacing parameter

s_θ=crack spacing

U= uniformly distributed lane load

v= average shear stress

V_c=concrete shear capacity

VDC=shear due to dead load

VDW=shear due to wearing surface

V_L= shear due to lane load

V_n=nominal shear resistance

V_p = vertical component of the force in the prestressing strands

V_S=stirrup shear capacity

V_{tml}= shear due to tandem load

V_{trl}=shear due to truck load

V_u=ultimate shear

w=crack width

x = horizontal distance from the center of gravity of the pattern of girders to each girder

X_{ext} = horizontal distance from the center of gravity of the pattern of girders to the exterior girder

α = angle of inclination of transverse reinforcement to longitudinal axis

β=tensile stress factor

θ=the angle of inclination of diagonal compressive stress.

Υ_D= Factor for dead loads (D)

Υ_{DC} = Load factor for structural components and attachments

ΥDW = Load factor for wearing surfaces and utilities

Υ_L = Factor for live load (LL)

Υ_P = Load factor for permanent loads other than dead loads

ϕ =Resistance factor

ϕ_c =Condition factor

ϕ_S =System factor

List of Tables

Table 2-1: Changes from LFR to LRFR……………………………………………………………8

Table 3-1: LRFD Resistance Factors……………………………………………………………10

Table 3-2: Recommended Condition Factor Values…………………………………………....11

Table 3-3: Recommend System Factor Values……………………………...……………….11

Table 3-4: Limit States and Load Factors for LRFR……………………………………………...12

Table 3-5: Generalized Live Load Factors for Legal Loads in LRFR………………………12

Table 3-6: Permit Load Factors for LRFR………………………………………………………..13

Table 3-7: Dynamic Load Allowance Values…………………………………………………….13

Table 4-1: Equations of modified compression field theory……………………………...28

Table 4-2: Values of θ and β for Sections with Transverse Reinforcement………………..33

Table 4-3: Values of θ and β for Sections without Transverse Reinforcement……………35

Table 5-1: recommended unit weights of materials……………………………………...41

Table 5-2: Multiple Presence Factor……………………………………………………..42

Table 5-3: Distribution of Live Load per Lane for Moment in Interior Beam……………..46

Table 5-4: Distribution of Live Loads per Lane for Moment in Exterior
Longitudinal Beams…………………………………………………………..……….47

Table 5-5: Reduction of Load Distribution Factors for Moment in Skewed
Longitudinal Beams on Supports……………………………………………………….50

Table 5-6: Distribution of Live Load per Lane for Shear in Interior Beams……………...51

Table 5-7: Distribution of Live Load per Lane for Shear in Exterior Beams…………….52

Table 5-8: Correction Factors for Load Distribution Factors for Support
Shear of the Obtuse Corner………………………………………………………….….53

Table 5-9: Lever Rule Equations for Exterior Girders…………………………………...53

Table 5-10: Lever Rule Equations for Interior Girders……………………………….……..54

Table 5-11: Correction Factor for Analysis……………………………………..…..55

List of Figures

Fig. 2-1: Flow Chart for Rating Procedure……………………………………………….…5

Fig. 2-2: ERA Legal loads……………………………………………………….……….7

Fig. 3-1: LRFR Flow Chart for Non-Permit Loads…..15

Fig. 4-1: Parallel chord truss Model……………………………………….……………17

Fig. 4-2: Equilibrium Conditions for Variable-Angle Truss Model …………………………17

Fig. 4-3: Compression field theory ……….…………………………………………..19

Fig. 4-4: Concrete average stress-strain relationship in compression ...………………………22

Fig. 4-5: Concrete average stress-strain relationship in tension..….23

Fig. 4-6: Reinforcing Steel Stress-Strain Relationship in Tension ……………………………24

Fig. 4-7: Mohr circle of stress for cracked reinforced concrete….……………………………25

Fig. 4-8: Free body diagram of a uniform shear element in the crack direction for average stresses and local stresses at the cracks …………………………………….…………………26

Fig. 4-9: Mohr circle of strains for reinforced concrete ....…27

Fig. 4-10: Determination of strain ε_x for non-pre-stressed beam …………………………….31

Fig. 4-11: Flexural tension side of a beam ……………….……………………………..32

Fig. 4-12: crack spacing parameter determination………………….……………………...34

Fig. 5-1: Truck moving to the right……………………………………………………...36

Fig. 5-2: IL for Shear Force……………………………………………………………..36

Fig. 5-3: IL for Bending Moment……………………………………………………….37

Fig. 5-4: Truck moving to the Left……………………………………………..……….38

Fig. 5-5: IL for Shear Force……………………………………………………………..38

Fig. 5-6: IL for Bending Moment……………………………………………………….38

Fig. 5-7: Tandem load……………………………………………………………………...39

Fig. 5-8: IL for Shear Force……………………………………………………………..39

Fig. 5-9: IL for Bending Moment………………………………………………………....40

Fig.5-10: Characteristic of the Design Truck…………………………………………...….43

Fig. 5-11: Design Tandem Load……………………………………………………...…44

Fig. 5-12: Common Deck Superstructures……………………………………..………….50

Fig. 7-1: Operating Rating Factors for Bridge Girders…………………………………...128

Fig. 7-2: Inventory Rating Factors for Bridge Girders…………………………………...128

Fig. 7-3: Rating Factors due to Legal Loading Type 3 for Bridge Girders…………………129

Fig. 7-4: Rating Factors due to Legal Loading Type 3-2 for Bridge Girders………………129

Fig. 7-5: Rating Factors due to Legal Loading Type 3-3 for Bridge Girders………………130

Fig. 7-6: Comparison of Concrete Shear Capacity Obtained from 45-Degree
Truss Model and MCFT…………………………………………………………………...130

Fig. 7-7: Comparison of Stirrup Shear Capacity Obtained from 45-Degree
Truss Model and MCFT…………………………………………………………………...131

Fig. 7-8: Comparison of Nominal Shear Capacity Obtained from 45-Degree
Truss Model and MCFT…………………………………………………………………....131

Fig. 7-9: Comparison of Average Shear Capacity Obtained from 45-Degree
Truss Model and MCFT…………………………………………………………………....132

Chapter 1. Introduction

1.1. Background

Bridge evaluation is critical to the safety of the nation's transportation system. Because of increase in vehicular traffic and the effects of aging on our nation's transportation system, the deterioration of highway bridges is inevitable. So bridge evaluation is of much importance in the determination or prevention of overstressing a bridge component. Over loading of a structural member may be a result of a reduction in live load capacity due to deterioration, an increase in dead loads, outdated designs, all of which may render a bridge structurally deficient. Evaluation (also called load rating) analysis of bridges is performed to determine the live load that structures can safely carry.

Bridge girder evaluation for shear involves the determination of the shear capacity of a bridge girder to withstand the live load that a bridge may experience. Shear evaluation has been a very difficult task for many years due to challenges in the determination of shear capacity which is one of the most important components in the rating process.

Even though the behavior of reinforced concrete in shear has been studied for many years, the problem of determining the shear strength of reinforced concrete beams has remained open to discussion.

There are generally three reasons for carrying out a shear strength assessment of a bridge girder: first it may have been decided that the bridge should facilitate heavier traffic loads, second the structure can have suffered from serious damage or deterioration, third there may have been a change in design codes, setting higher demands on the bridge.

This thesis mainly focuses on evaluation of shear for bridge girders because of change in shear design philosophies and new bridge evaluation approach. Therefore this new design and evaluation philosophies will be used when evaluating the chosen bridge girders.

1.2. Aim and Purpose

The overall purpose of this master's thesis is to contribute to the work on improving the methods for structural assessment of bridge girders for shear by providing a more simplified and structured assessment procedure, by evaluating, and thereby facilitating, the use of more accurate models and tools for analysis.

Specific objectives of the thesis are outlined below

- Discuss the behavior of reinforced concrete in shear and shear resistance modeling and determination methods
- Identify the influencing parameters in concrete shear capacity and evaluation
- Identify and apply the reliable method of shear evaluation and capacity determination
- Evaluate the shear capacities of existing bridge girders
- Identify most appropriate methods of bridge girder analysis

1.3. Specifications and Limitations

The target group for this thesis is mainly people working with management or assessment of bridges. However, the thesis does include an introductory description of the underlying theories on which the presented study is based, partly to make the thesis interesting for a broader range of possible readers.

The conducted study implements analytical assessment method even though experimental or load testing method of evaluation can be performed to determine shear capacity. Evaluation of bridge for loading involves assessment of different components of the bridge for different loading conditions. But this thesis considers shear evaluation for bridge girders only. However, many of the underlying theories discussed in this thesis can be applied for other components of the bridge.

Due to the restricted scope of the study, it also only involves limited number of bridge types. The focus is also on simply supported reinforced concrete bridge girders which are the most popular in Ethiopia.

1.4. Methods

In order to meet the objectives of this thesis, design documents are collected from road authority, recently recommended theory of determining shear capacity, Modified Compression Field Theory (MCFT) and recently adopted method of evaluation: Load and Resistance Factor Rating (LRFR) are used in the assessment of the bridge girders for shear.

The assessment was preceded and supported by a continuous literature study. Apart from dealing with the shear behavior of reinforced concrete and how it is modeled, this state-of-the art was especially aimed at understanding how different bridge conditions are considered in the evaluation of bridge girder shear capacity.

Analytical method is used for the evaluation of the bridge girders.

Bridge girder analyses are performed by considering dead and live loading codes, AASHTO LRFD Specifications and ERA Bridge Design Manual, provisions.

1.5. Organization

The subsequent parts of this thesis are organized as follows:

Chapter 2 gives a description about shear rating, rating methods and compares the rating methods. Chapter 3 describes Load and Resistance Factor Rating Method (LRFR) in detail. It includes introduction about LRFR, its importance when compared to other rating methods, general LRFR equations, and the rating system and its applicability. Chapter 4 discusses concrete cracking, shear response of reinforced concrete bridge girders, shear resistance modelling and determination methods and finally presents shear resistance determination using MCFT. Chapter 5 explains loading; live load and dead loads, modelling principles, and Chapter 6 is analysis of selected bridge girders and rating them according to LRFR method. Chapter 7 gives the results of the research and discussion. Finally Chapter 8 concludes the findings and gives recommendations for further work.

Chapter 2. Load Rating for Shear

2.1. Introduction

Bridge design and load rating are similar in overall approach; however, they differ in one fundamental aspect. In design, engineers contend with greater uncertainty in the amount of loading the structure will experience over its service life. With load rating for an existing structure, on the other hand, bridge engineers face uncertainty in the amount of structural resistance (Jaramilla, B., and S. Huo., 2005). In general, load rating involves determining the safe carrying capacity.

2.2. Bridge Girder Shear Rating

The evaluation of a bridge girder for shear is based on the simple principle that the available shear capacity of a girder to carry loads must exceed the required shear capacity to support the applied loadings. To perform an evaluation, therefore, it is necessary to know the physical conditions of the bridge girder and the applied loadings. The present condition can be gathered from recent field inspection reports. The condition and extent of deterioration of structural components of the bridge should be considered in the computation of the dead load and live load effects for the capacity.

Rating procedure flow chart is given by ERA Bridge Design Manual, 2002 as shown in Fig.2-1.

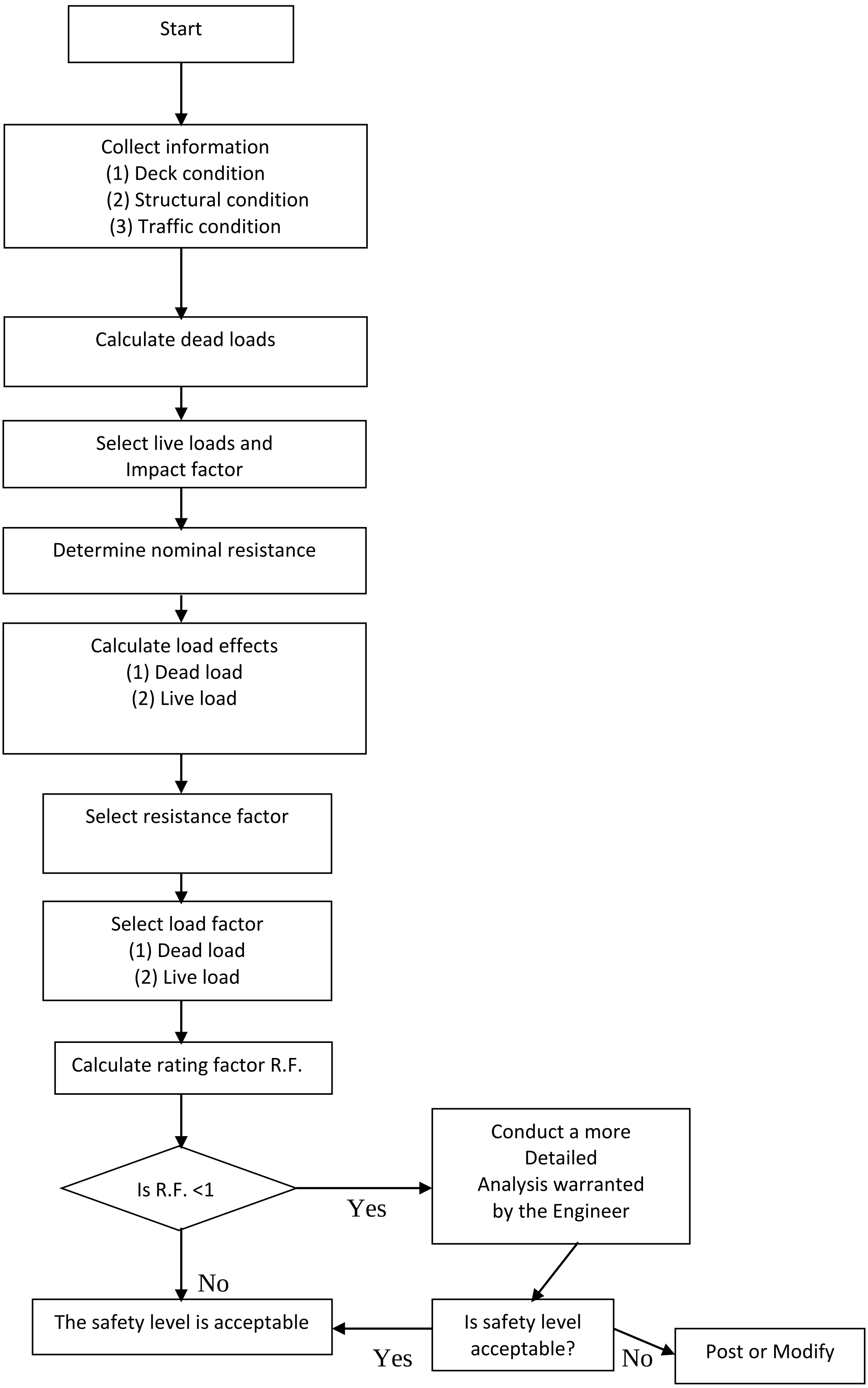


Fig. 2-1 Flow Chart for Rating Procedure (Adopted From ERA Bridge Design Manual, 2002)

2.3. Shear Rating Methods

2.3.1. Introduction

Load rating methods include analysis and, in some cases, load testing or a combination of methods. Of this, rating by structural analysis is by far the most common and most economical method. The most common methods which are used for rating of members are Load Factor Rating (LFR) and Load and Resistance Factor Rating (LRFR) methods which are discussed in the next sections.

2.3.2. Load Factor Rating (LFR) Method

The basic concept of the load factor rating (LFR) methodology is to analyze a structure at its ultimate load level under multiples of the actual dead and live loads. The load factors used to accomplish this are based on engineering judgment and not on statistical studies or probability of failure. The factors were developed assuming normal traffic and overload conditions.

The LFR Method specifies two levels of capacity ratings: inventory and operating. The inventory rating represents the magnitude of load that a bridge can safely sustain for an indefinite period of time, whereas the operating rating refers to the absolute maximum load that may be permitted on a bridge. Further, the inventory load rating accommodates live loads that a bridge can carry for an indefinite period, while the operating load rating refers to live loads that could potentially shorten the bridge life if applied on a routine basis (Jaramilla, B., and S. Huo., 2005).

The rating equation used in the LFR Method is as follows:

$$RF_{LFR} = \frac{[\emptyset Rn - \Upsilon_D D]}{[\Upsilon_L LL(1+I)]} \qquad \text{Eq.[2-1]}$$

Where,

RF = Rating factor

$\emptyset Rn$ = Capacity of the member

ΥD = Factor for dead loads (D)

D = Dead load effect

ΥL = Factor for live load (LL)

LL = Live load effect

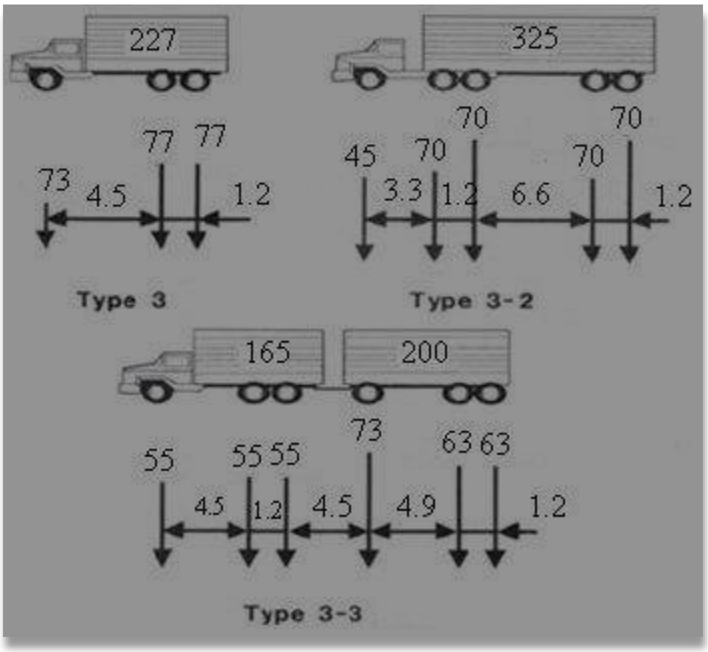
I = Impact factor for live load

The live load factors differ between the inventory and operating ratings; YL is 2.17 for inventory and 1.3 for operating. The dead load factor,YD is fixed at a value of 1.3. Furthermore, the live load effects are based on the AASHTO HS-20 design load. Capacity ($\emptyset Rn$) is calculated as outlined in the ERA Bridge Design Manual. The structural layout and materials determine the capacity.

The load factor rating impact factor (I) is added to all live loads to account for the speed, vibration, and momentum of vehicular traffic. The AASHTO specifications for bridge design define the impact factor as follows:

$$I = \frac{50}{L+125} \leq 0.3 \text{ where, } L\text{=span length} \qquad \text{Eq. [2-2]}$$

After determining the rating factor for shear, the bridge owner then multiplies the rating factor by the weight of the live load truck to yield the bridge girder rating.



Indicated Concentration Loads are Axle Loads in KN

All Dimensions are in m.

Fig. 2-2 ERA Legal Loads (ERA Bridge Design Manual, 2002)

2.3.3. Load and Resistance Factor Rating (LRFR) Method

As with load factor rating, load ratings on both operating and inventory levels can be calculated using the LRFR method. In addition, LRFR uses limit states for strength, service, and fatigue to ensure safety and serviceability in the load rating. Strength is the principal limit state and, therefore, is the main determinant for bridge posting, closing, and repair. Both service and fatigue limit states can be applied selectively to bridges (Jaramilla, B., and S. Huo., 2005). LRFR rating method is discussed in detail in section 3.

2.3.4. Comparison of LFR and LRFR

There are a number of differences between LRFR and LFR. One of the primary objectives in the LRFR is to utilize load models that provide a uniform relationship in the load effects to the actual legal traffic loading over all span lengths. Some areas where differences exist between LRFR and LFR are summarized below.

- Three load rating procedures targeted to specific needs
- Calibrated load and resistance factors
- Site-specific load factors for rating
- System factor
- Condition factor
- Live load models for evaluation
- Live load distribution factors
- Impact factor (dynamic load allowance)
- Load factors for overweight permits
- Serviceability checks
- Member resistances

The differences in the evaluation factors involved in load ratings by LRFR and LFR can be grouped as shown in Table 2-1

Category	LRFR	LFR
Member Resistance	ϕRn by LRFD	ϕRn by LFD
Distribution Factors	LRFD formulas	"S over" formulas
Dead Load Factors	$\gamma DC\ and\ \gamma DW$	γD
Live Load Factor	Calibrated γL	γL
Condition and System Factors	$\phi c\ \phi s$	Not applicable
Dynamic Load Allowance	May be tied to riding surface Conditions	Span length dependent

Table 2-1 Changes from LFR to LRFR (Adopted from Lichtenstein, 2001)

Chapter 3. Load and Resistance Factor Rating (LRFR) Method

3.1. Introduction

The load and resistance factor rating methodology, LRFR, was developed under the NCHRP (National Cooperative Highway Research Program) project 12-46 to be a rating methodology consistent in philosophy with the AASHTO LRFD Bridge Design Specifications in its use of reliability-based limit states. The goal of the design philosophy in the AASHTO LRFD was to achieve a more uniform level of reliability in bridge design. With the introduction of the AASHTO MCE LRFR (2003), the new methodology of rating provided a systematic and flexible approach to bridge rating based on reliability. The LRFR rating philosophy allows for a realistic assessment of a bridge's actual safe load capacity (Murdock, M., 2009).

The LRFR methodology is comprised of three distinct procedures:

• Design load rating (first level evaluation)
• Legal load rating (second level evaluation)
• Permit load rating (third level evaluation)

The results of each procedure serve specific uses and also guide the need for further evaluations to verify bridge safety or serviceability as discussed in section 3.3.

In rating bridge girders for shear using LRFR method, first the girders are checked for operating level design load rating. If RF<1, then Legal load rating is required. Permit vehicle design check for both standard and for single trip is used only when RF<1 for legal loading.

3.2. General LRFR Equations

In evaluation of bridge girders for shear, there is special interest in the live load and in the capacity of a member relative to the live load.

The following general expression is used in determining the load rating of bridge girders for shear.

$$\boldsymbol{RF} = \frac{[C - \gamma_{DC}DC - \gamma_{DW}DW + \gamma_{P}P]}{\gamma_{L}LL(1+IM)} \quad \text{Eq. [3-1]}$$

For the Strength Limit States:

$C = \phi_C \phi_S \phi R_n$

Where the following lower limit shall apply:

$\phi_C \phi_S \geq 0.85$

For the Service Limit States:

$C = fR$

Where:

RF = Rating Factor

C = Capacity

fR = Allowable stress specified in the LRFD code

R_n = Nominal member resistance (as- inspected)

DC = Dead-load effect due to structural components and attachments

DW = Dead-load effect due to wearing surface and utilities

P = Permanent loads other than dead loads

IM = Dynamic load allowance

Υ_{DC} = Load factor for structural components and attachments

Υ_{DW} = Load factor for wearing surfaces and utilities

Υ_P = Load factor for permanent loads other than dead loads

= 1.0

Υ_L =Evaluation live load factor

ϕ_C =Condition factor

ϕ_S =System factor

ϕ =Resistance factor

3.2.1. Resistance Factors

The resistance factor, ϕ is a reduction factor applied to the nominal resistance of a new member to consider the uncertainties associated with its resistance due to deterioration. The dimensional variations of the structure, differences in material properties, current condition and future deterioration, and the inaccuracies in the theory for calculating resistance are taken in account by applying this factor.

Stress Type	ϕ
Flexure and Tension of Reinforced Concrete	0.9
Shear of Normal Weight Concrete	0.9
Flexure and Tension of Pre-stressed Concrete	1.0
Flexure and Shear of Steel Members	1.0
Flexure of Timber Members	0.85
Shear of Timber Members	0.75

Table 3-1 LRFD Resistance Factors

3.2.2. LRFR Condition Factors

The condition factor, ϕ_C is a reduction to a member's resistance to account for the added uncertainties caused by the deterioration a member has experienced and that it is likely to experience between inspections.

Structural condition of member	Φ_C
Good or satisfactory	1
Fair	0.95
Poor	0.85

Table 3-2 Recommended Condition Factor Values

3.2.3. LRFR System Factors

The system factor,ϕ_S is a multiplier applied to the nominal resistance of a member to account for the redundancy of the full superstructure system (Murdock, M., 2009). A bridge's redundancy is the capacity of the structural system to carry loads after one or more of its structural members has been damaged or has failed. The load modifiers relating to ductility and redundancy are incorporated into the system factor (New Mexico, 2005). The recommended values for the system factor are shown in Table 3-3.

Super structure type	System Factor for Flexure	System Factor for Shear
Welded Members in Two-Girder/Truss/Arch Bridges	0.85	1.00
Riveted Members in Two-Girder/Truss/Arch Bridges	0.90	1.00
Multiple Eye Bar Members in Truss Bridges	0.90	1.00
Three-Girder Bridges with Girder Spacing $\leq$6ft	0.85	1.00
Four-Girder Bridges with Girder Spacing $\leq$4ft	0.95	1.00
All Other Girder Bridges and Slab Bridges	1.0	1.00
Floor Beams With Spacing > 12ft. and Non-Continuous Stringers	0.85	1.00
Redundant Stringer Subsystems Between Floor Beams	1.0	1.00

Table 3-3 System Factor Values

When checking shear under the strength limit states, a system factor of 1.0 is recommended for all superstructure types.

3.2.4. Limit States and Load Factors

Strength I is used for the ultimate capacity of structural members and is the primary limit state utilized for bridge girder evaluation for shear. Service limit states are utilized to limit stresses, deformations, and crack widths under regular service conditions and are often considered optional in load rating calculations.

The load factors for the Design Load Rating, shall be taken as shown in Table 3-4. The load factors for the Legal Load Rating shall be taken as shown in Table 3-4 and Table 3-5. The load factors for the Permit Load Rating shall be taken as shown in Table 3-4 and Table 3-6.

Bridge Type	Limit State	Dead Load DC	Dead Load DW	Design Load		Legal Load	Permit Load
				Inventory	Operating		
				LL	LL	LL	LL
Steel	Strength I	1.25	1.50	1.75	1.35	Table 3-5	Table 3-6
	Service II	1.00	1.00	1.30	1.00	1.30	1.00
Reinforced Concrete	Strength I	1.25	1.50	1.75	1.35	Table 3-5	Table 3-6
	Service I	1.00	1.00	--	--	--	1.00
Prestressed Concrete	Strength I	1.25	1.50	1.75	1.35	Table 3-5	Table 3-6
	Service III	1.00	1.00	0.80	--	1.00	--
	Service I	1.00	1.00	--	--	--	1.00
Timber	Strength I	1.25	1.50	1.75	1.35	Table 3-5	Table 3-6

Table 3-4 Limit States and Load Factors for LRFR

Traffic volume	Limit State	Live Load Factor
Unknown	STRENGTH	1.80
ADTT > 5000	STRENGTH	1.80
ADTT = 1000	STRENGTH	1.60
ADTT < 100	STRENGTH	1.40

* For ADTT between 100 and 5000 interpolate the load factor.

Table 3-5 Generalized Live Load Factors for Legal Loads in LRFR

Permit Type	Loading Condition	Distribution Factor	Live Load Factor
Annual	Mixed with Normal Traffic	Two or more lanes	1.30
Single Trip	Mixed with Normal Traffic	One Lane	1.50
Single Trip	Escorted with no other vehicles on the bridge	One Lane	1.15

Table 3-6 Permit Load Factors for LRFR (WDOT, 2011)

3.2.5. Dynamic Load Allowance

Dynamic load allowance is used to increase the static effects of the truck loads due to the dynamic effects of moving vehicles.

IM should be applied to HL-93 Design Truck or Tandem only but not to Design Lane Load (Sivakumar, B., 2011, ERA Bridge Design Manual, 2002).

Depending on the riding surface and bridge approach conditions, a table for dynamic load allowance is given as shown in Table 3-7.

Riding Surface Condition	IM
Smooth riding surface at approaches, bridge deck, and expansion joints, with no noticeable bumps.	10%
Moderate surface deviations or depressions causing minor bumps.	20%
A rough ride, with significant to severe bumps, or the perception that trucks are being “launched” at the approach to the bridge.	33%

Table 3-7 Dynamic Load Allowance Values

3.3. The Load and Resistance Factor Load Rating System

Live loads to be used in the rating of bridges are selected based upon the purpose and intended use of the rating results. Live load models for load rating are:

Design Load: HL-93 Design Load

Legal Loads: ERA Legal Loads (Type 3, Type 3-2, Type 3-3)

Permit Load: Actual Permit Truck

Design Load Rating

Design load rating is a first-level rating of bridges using the HL-93 loading and LRFD design standards with dimensions and properties for the bridge in its present as-inspected condition. It is a measure of the performance of existing bridges to new bridge design standards. Under this check existing bridge girders will be screened at the design level reliability (Inventory Level) or at a second lower level reliability (Operating Level reliability) for the strength limit state.

Legal Load Rating

This second level evaluation provides a single safe load capacity applicable to legal loads. ERA Bridge Design Manual, 2002 has given Legal Loads as Type 3, Type 3-2 and Type 3-3. Strength is the primary limit state for evaluation. The results of the load rating for legal loads could be used as a basis for decision-making relating to load posting or bridge strengthening even though higher level evaluation other than analytical evaluation is required if rating factor value is less than unity.

Permit Load Rating

Procedures are provided for checking the safety and serviceability of bridges for the issuance of permits for the passage of vehicles above the legally established weight limitations. Calibrated load factors by permit type and traffic conditions at the site have been provided for use in checking the load effects induced by the passage of the overweight truck.

According to ERA Bridge Design Manual, in checking special permits, the actual vehicle weights and dimensions shall be used. If the number of such permits in one year is frequent, then it shall be assumed that two lanes are occupied by such a vehicle. Otherwise, standard vehicles shall be placed in the other lanes.

Fig. 3-1 demonstrates the general procedure for performing a load rating using LRFR.

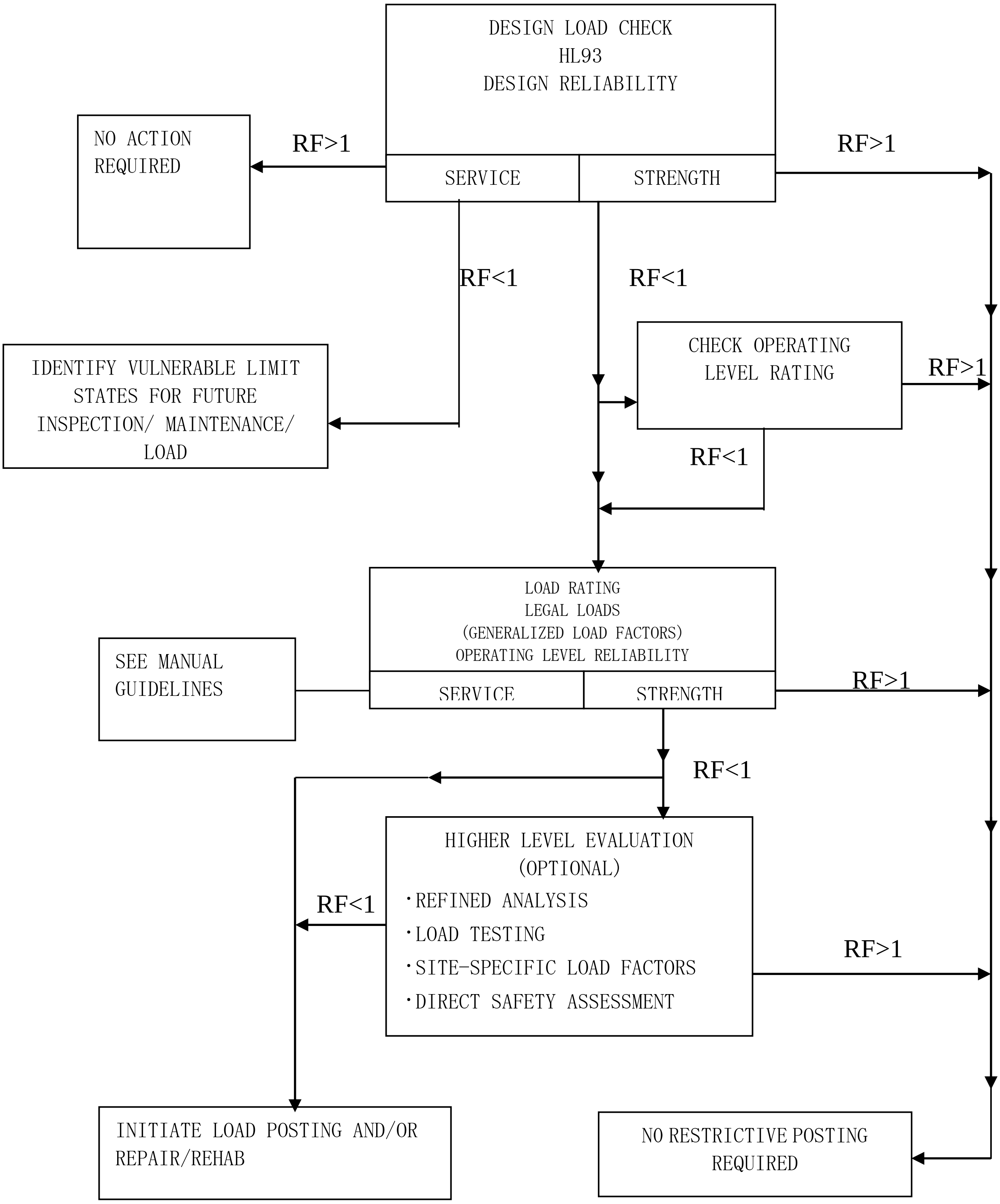


Fig. 3-1 LRFR Flow Chart for Non-Permit Loads (Adopted from Lichtenstein, 2001)

Chapter 4. Girder Shear Resistance Determination

4.1. Introduction

The shear strength of concrete bridge girders needs to be evaluated to determine the shear capacity rating of bridges because of increased traffic loads or deterioration of bridges. Shear strength of concrete bridge girders commonly limits the load capacity ratings of bridges (Esfandiari, A.; Adebar, P., 2009).

The shear strength of structural concrete is a complex phenomenon. Building codes include simplifications that generally result in safe designs. The additional construction costs are justified by the reduced chance of a design error. On the other hand, the consequence of these same simplifications may be greater when a simplified shear design method is used to evaluate existing girders that cannot be made a little stronger. The simplifications may result in unnecessary load restrictions on bridges or unnecessary repairs of bridge girders.

Even though the behavior of reinforced concrete in shear has been studied for more than 100 years (Bentz, E.C., et al., 2006), the problem of determining the shear strength of reinforced concrete girders is still an important issue.

4.2. Shear Resistance Models

4.2.1. Introduction

There have been different models used in the determination of shear capacity. The truss model shown in Fig. 4-1 was widely used to understand the shear behavior of reinforced concrete beams with transverse reinforcement in the early 1900's. After that 45^0 truss model, variable angle truss model, compression field theory, modified compression field theory are some of the models for shear capacity evaluation. These modeling methods are discussed hereafter.

4.2.2. 45^0 Truss Model

In this model, diagonal concrete struts were considered to be the diagonal members of the truss, the stirrups were the vertical members of the truss, the longitudinal reinforcement served as the bottom chord of the truss, and the flexural compression zone served as the top chord of the truss as shown in Fig 4-1. This model was improved by assuming that the diagonal struts extended across more than one stirrup. The tensile stresses in cracked concrete were neglected in this model and diagonal compression stresses were assumed to remain at 45^0 after the concrete cracked (Hawkins, N.M. et al, 2005).

Equilibrium equations for this model can be obtained by assuming an angle of diagonal compression of θ =45° in the equilibrium equations Eq. 4-1, Eq. 4-2, Eq. 4.3.

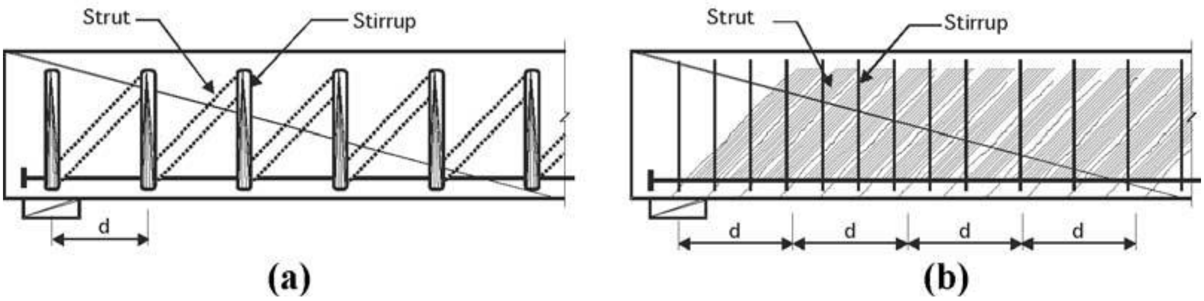
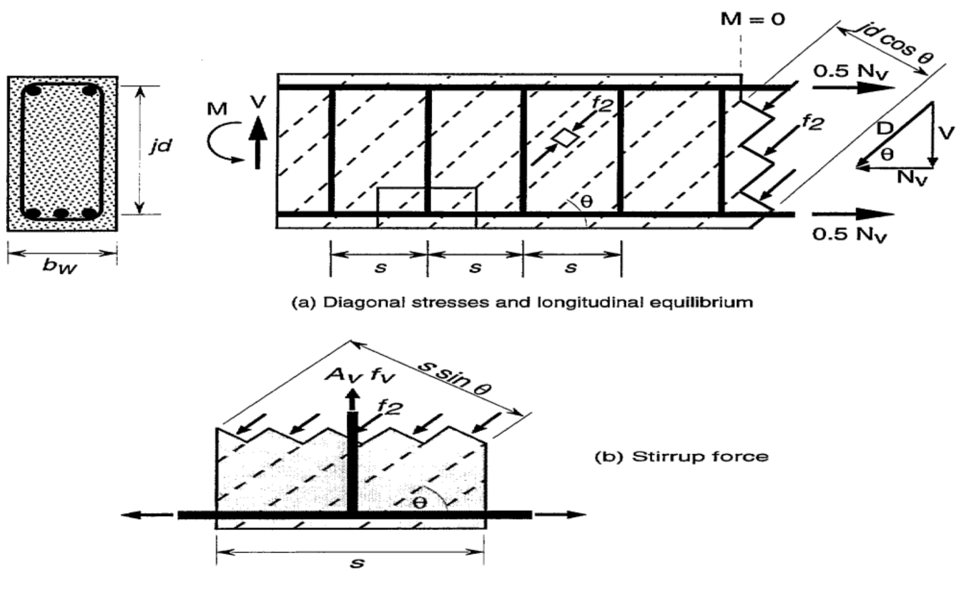


Fig. 4-1 Parallel chord truss Model (adopted from NCHRP Report 549, 2005)

4.2.3. Variable-Angle Truss Model

The variable-angle truss model is a version of the 45° truss model modified by assuming flatter strut angles, θ ≤ 45°. In this model, the three equilibrium equations can be derived in the same manner as for the 45° truss model.



Unknowns: f_2, N_v, f_v, θ

Fig. 4-2 Equilibrium Conditions for Variable-Angle Truss Model

(Adopted from NCHRP Report 549, 2005)

The equilibrium conditions for this model are shown in Fig. 4-2 and Eq. [4-1], Eq. [4-2] and Eq. [4-3]:

$$f_2 = [V(tan\theta + cot\theta)]/[b_w j_d] \quad \text{Eq. [4-1]}$$

$$N_v = Vcot\theta \quad \text{Eq. [4-2]}$$

$$A_v f_v / s = Vtan\theta / jd \quad \text{Eq. [4-3]}$$

However, these three equilibrium equations are not sufficient to solve member forces, because there are four unknowns; the principal compressive stress, f_2; the tension in the longitudinal direction, N_v; the stresses in the shear reinforcement, f_v; and the strut angle or inclination of the principal compressive stresses, θ.

4.2.4. Compression Field Theory

Compression field theory, differing from variable angle truss model, combines Equilibrium equations, Compatibility relations and stress-strain relationships to determine stresses and strains and inclination of principal compressive stresses.

Equilibrium Equations

The compression field theory uses the same approach for equilibrium conditions as described in the variable-angle truss model. Eq. [4-1], Eq. [4-2] and Eq. [4-3] can be expressed respectively in terms of the stresses as shown below. These equilibrium equations can also be derived from Fig. 4-5 (a) and (b):

The vertical component of the diagonal compressive force in the concrete, which is inclined at θ to the longitudinal axis, must equal the applied shear force:

$$f_2 = v(tan\theta + cot\theta) \quad \text{Eq. [4-4]}$$

Where, $v = V/b_w j_d$

Similarly,

$$\rho_x f_{sx} = vcot\theta \quad \text{Eq. [4-5]}$$

$$\rho_y f_{sy} = vtan\theta \quad \text{Eq. [4-6]}$$

where ρ_x and ρ_y are the reinforcement ratios in the longitudinal and transverse directions.

The shear stress v applied to the cracked reinforced concrete causes tensile stresses in the longitudinal reinforcement f_{sx} and the transverse reinforcement f_{sy} and a compressive stress in the cracked concrete f_2 inclined at angle θ to the longitudinal axis.

The CFT model utilizes the deformations for reinforced concrete by assuming that a diagonal compression field carries shear after cracking and assumes that after cracking there will be no tensile stresses in the concrete. It also assumes the angle of inclination of the diagonal stresses coincides with the angle of inclination of the principal compressive strain.

Compatibility Relations

The compatibility conditions used in the compression field theory can be derived from Mohr's circle for strains as shown in Fig. 4-3 (c) and (d).

$$tan^2\theta = (\varepsilon_x - \varepsilon_2)/(\varepsilon_y - \varepsilon_2) \qquad \text{Eq. [4-7]}$$

The principal tensile strain, ε_1, can be derived as;

$$\varepsilon_1 = \varepsilon_x + (\varepsilon_x - \varepsilon_2)cot^2\theta \qquad \text{Eq. [4-8]}$$

The longitudinal strain, ε_x, c an be expressed as;

$$\varepsilon_x = (\varepsilon_1 tan^2\theta + \varepsilon_2)/(1 + tan^2\theta) \qquad \text{Eq. [4-9]}$$

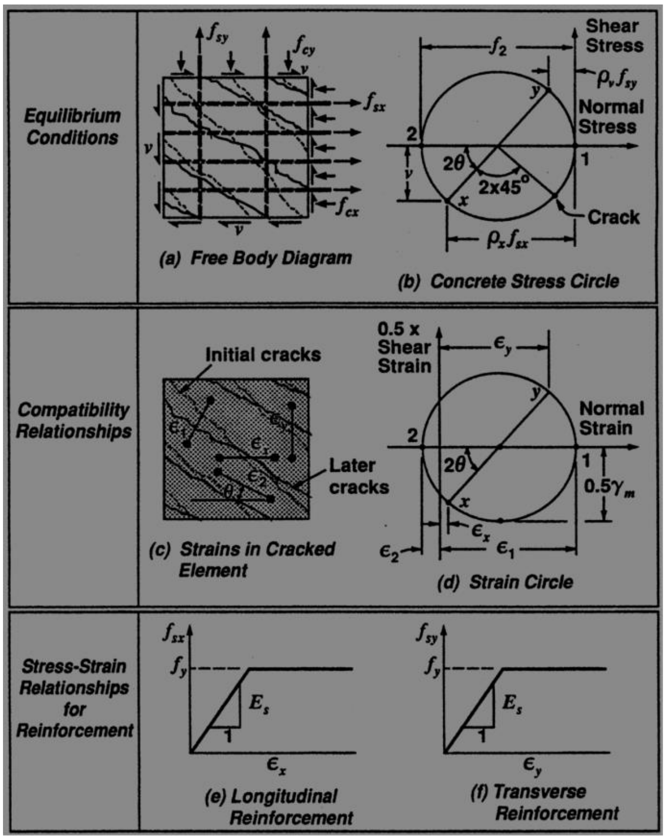


Fig. 4-3 Compression field theory (Adopted from Hawkins, N.M. et al, 2005)

Stress-Strain Relationships

The ability of diagonally cracked concrete to resist compression decreases as the amount of tensile straining increases by considering the maximum compressive stress f_{c2max} that the concrete can resist reduces as the average principal tensile strain ε_1 increases in the following manner (Vecchio and Collins, 1986):

$$f_{c2max} = f_{c'}/[0.8 - 0.34\varepsilon_1/\varepsilon_{c'}] < f_{c'} \qquad \text{Eq. [4-10]}$$

The stress-strain relationships for both longitudinal and transverse reinforcement were assumed as bilinear for the CFT the same as for MCFT discussed subsequently in detail.

Stress-strain relationships for cracked concrete in compression are suggested to be as follows (Vecchio and Collins, 1986):

$$f_2 = [(2\varepsilon_2/\varepsilon_c{}') - (\varepsilon_2/\varepsilon_c{}')^2]f_{c2max} \qquad \text{Eq. [4-11]}$$

Where f_{c2max} is given by Eq. [4-10]

Because the compression theory provides the equilibrium conditions, compatibility conditions, and constitutive relationships for both reinforcement and cracked concrete, it can predict shear behavior for any load level as well as the shear strength of members. However, since the compression field theory neglects the tensile stresses in cracked concrete, it gives conservative results for the shear behavior of members, meaning that it underestimates both the shear stiffness and the shear strength.

4.2.5. Modified Compression Field Theory

Modified Compression Field Theory (MCFT) was developed by Vecchio and Collins (1986) from testing reinforced concrete elements subjected to uniform shear stress (Esfandiari, A., 2009). It is a smeared, rotating crack model where the inclination of diagonal cracks is determined by combining equilibrium requirements, strain compatibility assumptions and empirical average stress – average strain relationships for cracked concrete and reinforcement.

Modified Compression Field Theory is the modification of Compression Field Theory by including the assumption that there is an average tensile stress in concrete after cracking. Compression Field Theory assumes as if there is no tensile stress in concrete after cracking.

In the determination of the point at which the diagonal compressive struts fail and to determine the angle of the struts, MCFT combines equilibrium requirements, strain compatibility assumptions and empirical average stress – average strain relationships for cracked concrete and reinforcement.

Vecchio and Collins, 1986 gives equilibrium requirements, strain compatibility assumptions and empirical average stress – average strain relationships for cracked concrete and reinforcement.

Assumptions

According to Vecchio and Collins, 1986, the following assumptions are taken in the equilibrium, compatibility and material constitutive relations.

- Loads acting on the concrete elements edge planes are assumed to consist of uniform axial stresses and uniform shear stress.
- Deformation of the element is assumed to occur such that the edges remain straight and parallel.
- For each strain state there is only one corresponding stress state, i.e. effect of strain to the transverse directions is neglected.
- Stresses and strains can be considered in terms of average values when taken over areas or distances.
- The concrete and the reinforcing bars are perfectly bonded together at the boundaries of the element.
- The longitudinal and transverse reinforcing bars are uniformly distributed over the element.
- The average shear stress on the plane normal to the reinforcement resisted by the reinforcement is zero.
- The direction of principal strain coincides with the direction of principal average stress

MCFT material constitutive relationships, equilibrium equations, and compatibility relations are explained hereafter.

4.2.5.1. Material Constitutive Relationships

Parabolic stress-strain relationship as shown in Fig. 4-4 is considered for concrete in compression in the principal direction. Cracked concrete subjected to biaxial strains (when concrete is subjected to significant tension strain transverse to the principal compression) has significantly lower principal compressive strength (peak stress) than the uniaxial strength (Esfandiari A, 2009). Vecchio and Collins (1986) showed that the reduction in concrete strength (peak stress) in such cases can be predicted by the following equation:

$$f_{c2max} = f_c'/[0.8 - 0.34\varepsilon_1/\varepsilon_c'] < f_c' \qquad \text{Eq. [4-12]}$$

Where f_{c2max} is concrete peak stress under biaxial strains, ε_1 is concrete principal tensile strain, $f_{c'}$ is concrete peak stress under uniaxial compression and $\varepsilon_{c'}$ is concrete strain corresponding to concrete peak compressive stress. Therefore, a parabolic stress-strain relationship of concrete in the principal compressive direction can be expressed as:

$$f_{c2} = [(2\varepsilon_2/\varepsilon_c') - (\varepsilon_2/\varepsilon_c')2]\, f_{c2max} \qquad \text{Eq. [4-13]}$$

f_{c2} is concrete compressive stress in the principal direction and ε_2 is concrete compressive strain in the principal direction.

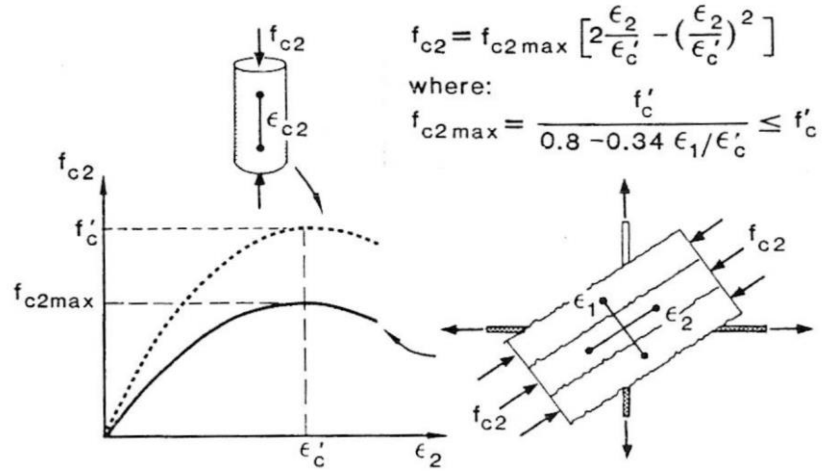


Fig. 4-4 Concrete average stress-strain relationship in compression
(Adopted from Esfandiari, A. 2009, Vecchio and Collins, 1986)

MCFT differs from CFT in the model for principal concrete tension stresses. It accounts for concrete contribution to reinforcing bar stiffness after cracking, which is called tension stiffening and has significant influence on concrete contribution to shear strength of a reinforced concrete element (Esfandiari, A. 2009). In MCFT concrete model, concrete tension stress increases linearly until cracking. After cracking, concrete continues to resist an average tension stress but it reduces as the principal tensile strain increases. Concrete tensile stress-strain relationship in MCFT is shown in Fig. 4-5 and is given by:

$$f_{c1} = E_c . \varepsilon_1 \qquad \varepsilon_1 < \varepsilon_{cr} \qquad \text{Eq. [4-14]}$$

$$f_{c1} = \frac{f_{cr}}{1+\sqrt{500\varepsilon_1}} \qquad \varepsilon_1 > \varepsilon_{cr} \qquad \text{Eq. [4-15]}$$

Where, $f_{cr} = 0.33\sqrt{(f_c')}$

The value of $f_{c1} = \frac{f_{cr}}{1+\sqrt{200\varepsilon_1}}$ in Fig. 4-5 was the original stress-strain relationship proposed by Vecchio and Collins, 1986 but was later modified to be as of Eq. [4-15 (Collins, M.P., et. al 1996).

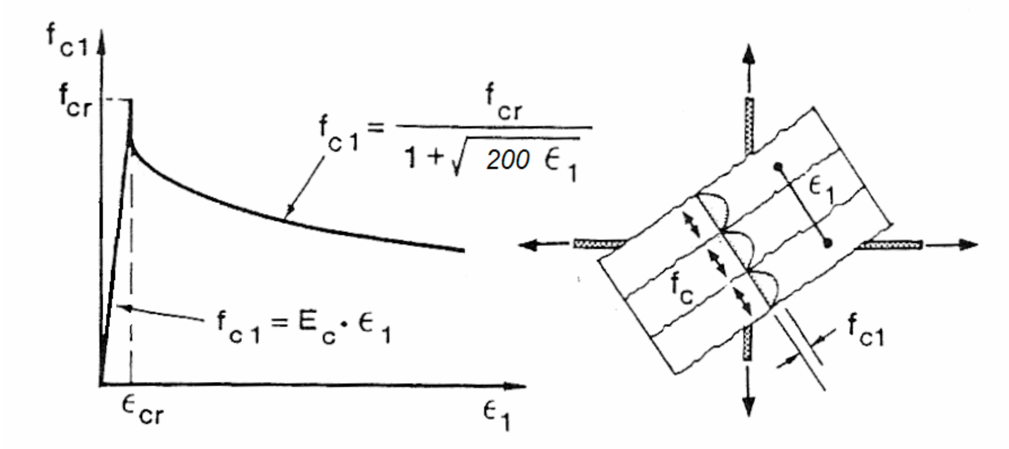


Fig. 4-5 Concrete average stress-strain relationship in tension

(Adopted from Vecchio and Collins, 1986, Esfandiari, A. 2009,)

The stress-strain relationships expressed by Eqs. [4-14] and [4-15] account for average tensile stress in concrete in the principal direction and is valid if aggregate interlock in addition to stress increase in the reinforcing steel at the cracks are capable of equilibrating average stresses as discussed in Equilibrium Equations, next sub-section. Otherwise, tensile stress must be reduced accordingly.

The maximum shear stress that can be resisted by aggregate interlock along a crack is given by:

$$v_{ci} \leq 0.18\sqrt{fc'}/[0.31 + (24w/(a_g + 16))] \quad \text{(in MPa units)} \qquad \text{Eq. [4-16]}$$

where a_g is concrete maximum aggregate size, and w is crack width determined from:

$$w = s_\theta \varepsilon_1 \qquad \text{Eq. [4-17]}$$

in which s_θ is crack spacing and is assumed equal to:

$$s_\theta = 1/[(\,sin\theta/s_x) + (cos\theta/s_z)] \qquad \text{Eq. [4-18]}$$

where s_x is crack control parameter of x-direction reinforcement, and s_z is crack control parameter of z-direction reinforcement. For members with at least minimum amount of reinforcement, crack spacing may be conservatively assumed as $s_\theta = 300$ mm.

Stress-strain relationship for reinforcing steel is bilinear relation as shown in Fig.4.6

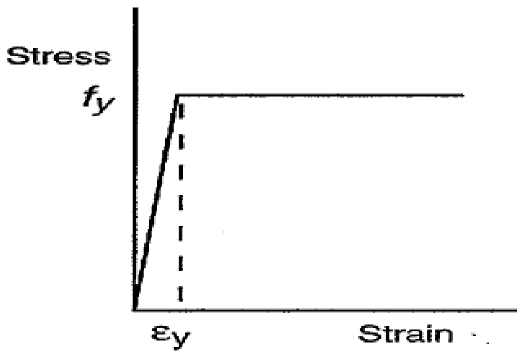


Fig. 4-6 Reinforcing Steel Stress-Strain Relationship in Tension

4.2.5.2. Equilibrium Equations

Equilibrium equations of MCFT, in terms of the average stresses can be expressed by Mohr circles of stresses as shown in Fig. 4-7.

Using Mohr circles of stresses shown in Fig. 4-7, equilibrium equations can be derived as:

$$f_x = f_{cx} + \rho_{sx} f_{sx} \quad \text{(a)}$$

$$f_z = f_{cz} + \rho_{sz} f_{sz} \quad \text{(b)}$$

$$f_{cx} = f_{c1} - v cot\theta_c \quad \text{(c)}$$

$$f_{cz} = f_{c1} - v tan\theta_c \quad \text{(d)}$$

Therefore,

From (a) and (c)

$$f_x = \rho_x f_{sx} + f_{c1} - v cot\theta_c \quad \text{Eq. [4-19]}$$

From (b) and (d)

$$f_z = \rho_z f_{sz} + f_{c1} - v tan\theta_c \quad \text{Eq. [4-20]}$$

We can also derive for the average shear stress v in terms of the principal tensile stress f_{c1} and principal compressive stress f_{c2} as:

$$v = (f_{c1} + f_{c2})/(tan\theta_c + cot\theta_c) \quad \text{Eq. [4-21]}$$

where f_x and f_z are normal stresses in x and z-directions, ρ_x and ρ_z are reinforcement ratios in x and z-directions, f_{sx} and f_{sz} are reinforcement stresses in x and z-directions, and θ_c is concrete angle of principal direction of stresses.

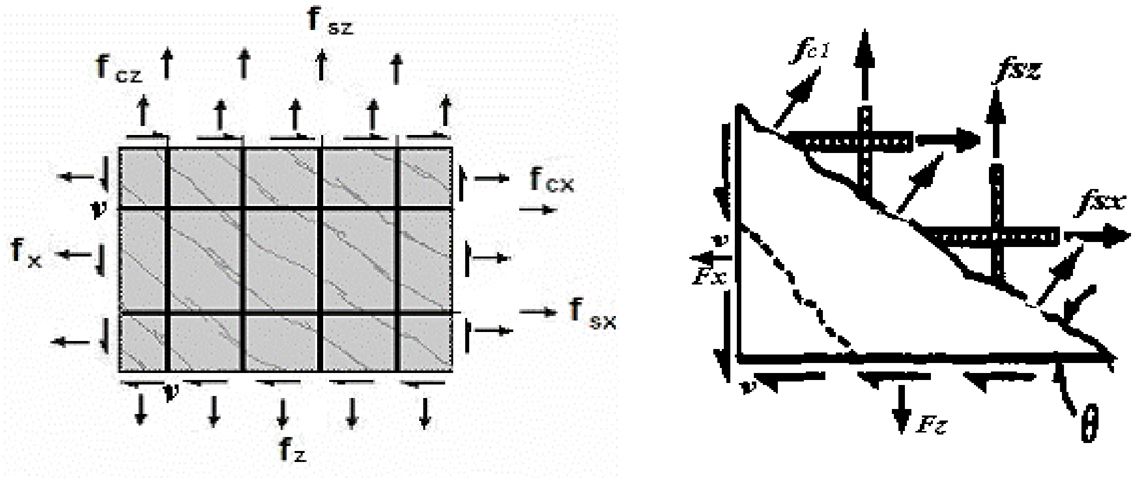
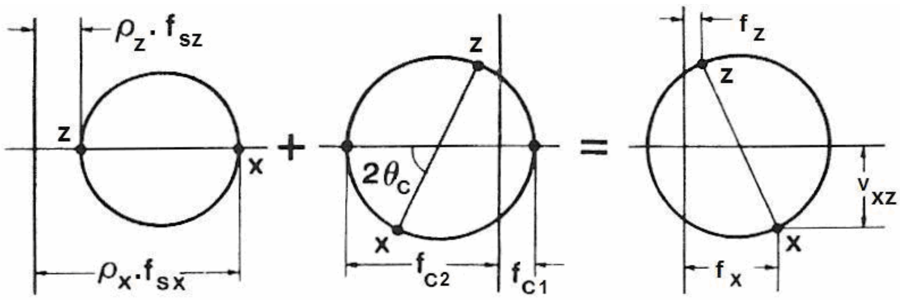


Fig. 4-7a Free Body diagram of Part of an Element



Reinforcement Concrete Reinforced Concrete

Fig. 4-7b Mohr circle of stress for cracked reinforced concrete

(Adopted from Collins M, P, 1986, Esfandiari, A. 2009)

It is also necessary to check stresses at the cracks. Fig. 4-8 compares the free body diagram of a uniform shear element on average and at the cracks. At the cracks, concrete tension stress in the principal direction f_{c1} becomes zero and aggregate interlock stress v_{ci} contributes to equilibrium instead. In addition, reinforcing steel stresses may be higher at the cracks compared to the average stresses. Equilibrium equations at the cracks can be derived from Fig.4-8(d) as:

Longitudinal direction:

$$f_{sxcr}\rho_x(A_{cr}sin\theta_c) - (v_{ci}cos\theta_c)A_{cr} - v(A_{cr}cos\theta_c) = f_x\, A_{cr}sin\theta_c$$

Transverse direction:

$$f_{szcr}\rho_Z(A_{cr}cos\theta_C) + (v_{ci}sin\theta_C)A_{cr} - v(A_{cr}sin\theta_C) = f_Z\, A_{cr}cos\theta_C$$

Therefore;

$$\rho_X f_{sxcr} = (f_X + vcot\theta_C + v_{ci}cot\theta_C) \quad \text{Eq. [4-22]}$$

$$\rho_Z f_{szcr} = (f_Z + vtan\theta_C - v_{ci}tan\theta_C) \quad \text{Eq. [4-23]}$$

where v_{ci} is stress along the cracks due to aggregate interlock, and f_{sxcr} and f_{szcr} are reinforcement stresses at the cracks in x and z-directions, respectively.

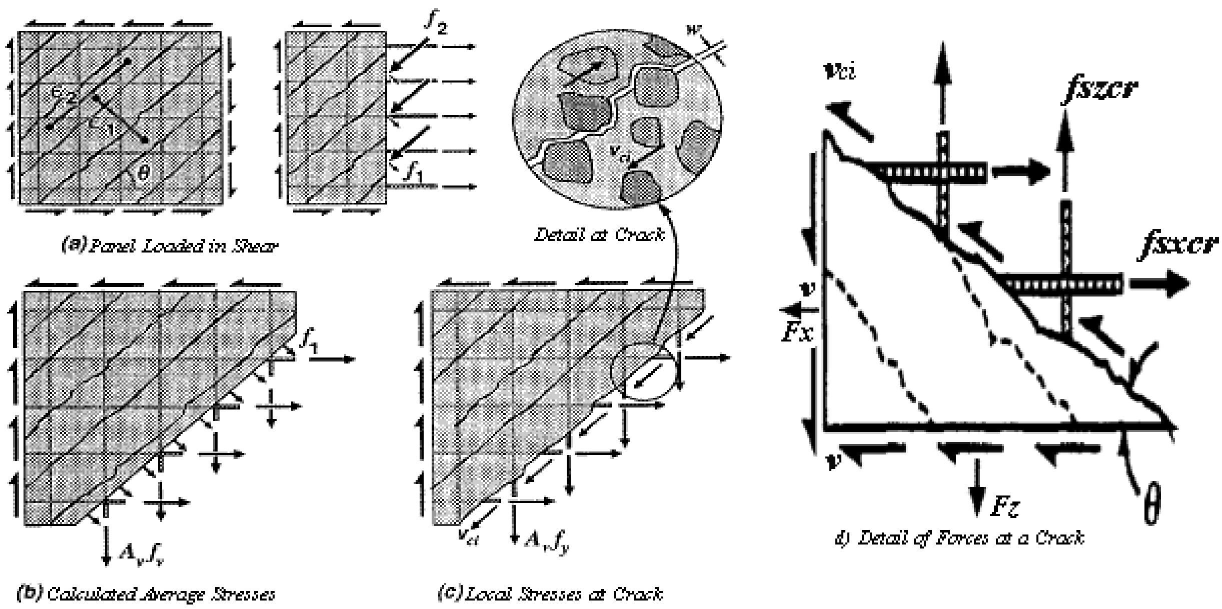


Fig. 4-8 free body diagram of a uniform shear element in the crack direction for average stresses and local stresses at the cracks (Collins, M.P., et. al 1996).

The conditions that need to be checked in order to determine whether the average tensile concrete stress can be equilibrated at the cracks or the average tensile stress in concrete needs to be reduced can be given as:

$$v_{ci2} = [\rho_X(f_{sxcr} - f_{sX}) - \rho_Z(f_{szcr} - f_{sZ})]sin\theta_C cos\theta_C \quad \text{Eq. [4-24]}$$

$$f_{c1} < \rho_X(f_{sxcr} - f_{sX})cos^2\theta_C + \rho_Z(f_{szcr} - f_{sZ})\, sin^2\theta_C \quad \text{Eq. [4-25]}$$

$$f_{c1} < \rho_X(f_{sxcr} - f_{sX}) + min(v_{cimax}, v_{ci2})cot\theta_C \quad \text{Eq. [4-26]}$$

$$f_{c1} < \rho_Z(f_{szcr} - f_{sZ})\,) + min(v_{cimax}, v_{ci2})tan\theta_C \quad \text{Eq. [4-27]}$$

In the equations above, v_{ci2} is the shear stress on cracks required to achieve biaxial yielding of reinforcement and v_{cimax} is the aggregate interlock capacity determined from Eq. [4-16].

4.2.5.3. Compatibility Equations

MCFT assumes the average strains over a length containing a number cracks satisfy requirements of continuous materials despite cracks in concrete represent discontinuities. So like all other continuous materials, compatibility in reinforced concrete is expressed by Mohr circle of strains as shown in Fig. 4-9.

Since we started by assuming the reinforcement is anchored to the concrete, compatibility requires that any deformation experience by the concrete is equal to the deformation of the reinforcement. That is, $\varepsilon_{SX} = \varepsilon_{CX} = \varepsilon_{X}$, and $\varepsilon_{SZ} = \varepsilon_{CZ} = \varepsilon_{Z}$; where, ε_{SX}, ε_{SZ} are longitudinal and transverse strains in the reinforcement, ε_{CX} and ε_{CZ} are longitudinal and transverse strains in the concrete.

Other important compatibility equations from Mohr's circle of Fig. 4-9 are:

$$tan^2\theta = (\varepsilon_X + \varepsilon_2)/(\varepsilon_Z + \varepsilon_2) \qquad \text{Eq. [4-28]}$$

Similarly from Mohr's circle of Fig. 4-8,

$$(\varepsilon_1 + \varepsilon_2)/2 - (\varepsilon_X + \varepsilon_2) = (\varepsilon_Z - \varepsilon_2) - (\varepsilon_1 + \varepsilon_2)/2$$

$$\varepsilon_1 = \varepsilon_X + \varepsilon_Z + \varepsilon_2 \qquad \text{Eq. [4-29]}$$

$$\gamma_{XZ} = 2(\varepsilon_X + \varepsilon_2)cot\theta \qquad \text{Eq. [4-30]}$$

$$\varepsilon_1 = \varepsilon_X(1 + cot^2\theta) + \varepsilon_2\, cot^2\theta \qquad \text{Eq. [4-31]}$$

where ε_1 and ε_2 are strains in principal directions, ε_X and ε_Z are strains in x and z directions, γ_{XZ} is shear strain, and θ is principal compression strain direction to x-axis.

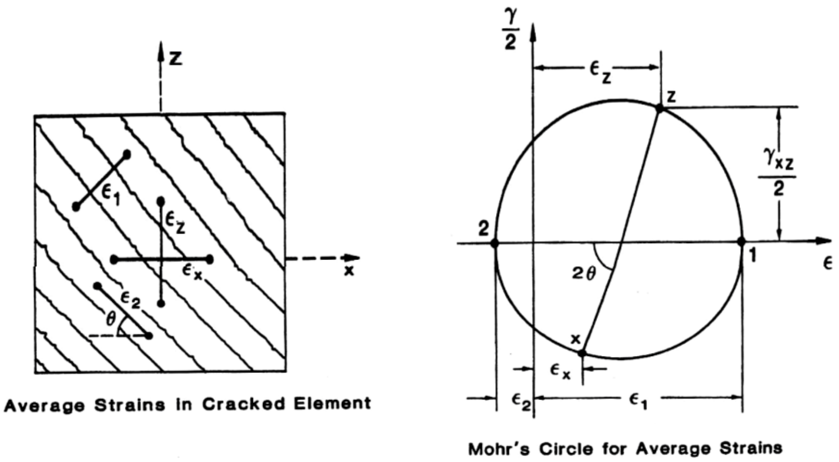


Fig. 4-9 Mohr circle of strains for reinforced concrete

(Adopted from Collins M, P, 1986, Esfandiari, A. 2009,) The material constitutive relations, equilibrium equations and compatibility equations are summarized as shown in Table 4-1.

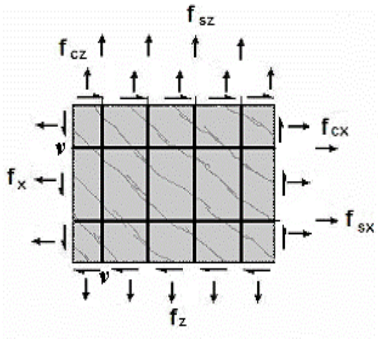
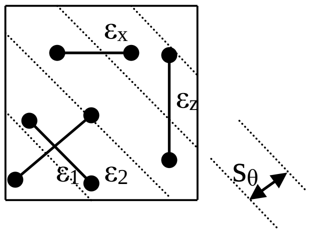
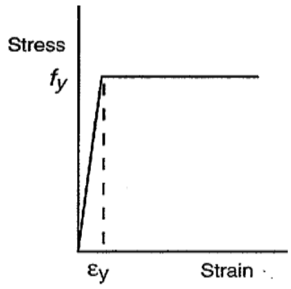
 Equilibrium: Average Stresses: $f_x = \rho_x f_{sx} + f_{c1} - v\cot\theta$ (1) $f_z = \rho_z f_{sz} + f_{c1} - v\tan\theta$ (2) $v = (f_{c1} + f_{c2})/(\tan\theta + \cot\theta)$ (3) Stresses at cracks: $\rho_x f_{sxcr} = (f_x + v\cot\theta + v_{ci}\cot\theta)$ (4) $\rho_z f_{szcr} = (f_z + v\tan\theta - v_{ci}\tan\theta)$ (5)	 Geometric Conditions: Average Strains: $\tan^2\theta = (\varepsilon_x + \varepsilon_2)/(\varepsilon_z + \varepsilon_2)$ (6) $\varepsilon_1 = \varepsilon_x + \varepsilon_z + \varepsilon_2$ (7) $\gamma_{xz} = 2(\varepsilon_x + \varepsilon_2)\cot\theta$ (8) Crack widths: $w = s_\theta \varepsilon_1$ (9) $s_\theta = 1/[(\sin\theta/s_x) + (\cos\theta/s_z)]$ (10)	 Stress-Strain Relationships Reinforcement: $f_{sx} = E_s \varepsilon_x < f_{yx}$ (11) $f_{sz} = E_s \varepsilon_z < f_{yz}$ (12) Concrete: $f_2 = f_c'/[(0.8+170\varepsilon_1)((2\varepsilon_2/\varepsilon_c')-(\varepsilon_2/\varepsilon_c')^2)]$ (13) $f_1 = 0.33\sqrt{f_c'}/(1 + \sqrt{500\varepsilon_1})$ Mpa (14) Shear Stress on Crack: $v_{ci} < 0.18\sqrt{f_c'}/[0.31 + (24w/(a_g + 16))]$ Mpa,mm (15)

Table 4-1 Equations of modified compression field theory (Bentz, E. C. et. al., 2006)

4.3. Shear Resistance using MCFT

4.3.1. Shear Resistance Determination using MCFT

The nominal shear resistance V_n of a concrete member shall be separated into a component, V_c which relies on tensile stresses in the concrete, a component, Vs which relies on tensile stresses in the transverse reinforcement, and a component, V_p, which is the vertical component of the prestressing force.

The nominal shear resistance, Vn of the beam can be obtained using:

For prestressed concrete members,

$$V_n = V_c + V_s + V_p < 0.25fc' \, b_v \, d_v + V_p \qquad \text{Eq. [4-32]}$$

Reinforced concrete members

$$V_n = V_c + V_s < 0.25fc' \, b_v \, d_v \qquad \text{Eq. [4-33]}$$

and

$$Vc = 0.083\beta \sqrt{f_c'} \, b_v d_v \ \ (SI\ Units) \qquad \text{Eq. [4-34]}$$

$$V_s = A_v f_y d_v \, (cot\theta + cot\alpha) \, sin\alpha / s \ (SI\ Unit) \qquad \text{Eq. [4-35]}$$

Where:

Vc = contribution of the concrete

V_s = contribution of the stirrups

V_p = vertical component of the force in the prestressing strands (N)

b_v = the minimum web width within d_v

d_v = the shear depth = $d_e - a/2 > 0.9d_e$ or $0.72h$

s = stirrup spacing (assuming that the stirrups spacing is uniform along the entire zone)

A_v = stirrup area

α = angle of inclination of transverse reinforcement to longitudinal axis (DEG)

θ = angle of inclination of diagonal compressive stresses (DEG)

The factor β (tensile stress factor), which is a function of the average principal tensile strain ε_1 in the cracked concrete, θ and s_x, is the indicator of the ability of the cracked concrete to transmit shear (Collins M, P. et. al., 1996).

$$\beta = 0.33cot\theta/(1 + \sqrt{500}) < 0.18/[0.3 + (24w/(a + 16)] \qquad \text{Eq. [4-36]}$$

Where:

w =the crack width, taken as the crack spacing times the principal tensile strain ε_1.

The factors β and θ are unknown and must be determined.

4.3.2. Determination of β and θ

4.3.2.1. For sections with at least minimum amount of transverse steel (stirrups)

To check minimum amount of transverse reinforcement, use:

$$A_v f_y / b_W s > 0.06\sqrt{fc'} \quad \text{(SI Units)}$$

For sections with at least the minimum amount of transverse steel (stirrups), a value of θ is assumed. Next, the average shear stress, carried by the concrete and stirrups, is found:

$$v = (V_u - \varphi Vp)/(\varphi\, b_v\, d_v) \qquad \text{Eq. [4-37]}$$

The crack widths that govern aggregate interlock capacity can be related to the longitudinal strain at mid-depth, ε_x. This strain can be calculated by determining the average of the longitudinal strains in the flexural tension, ε_{xt}, and flexural compression, ε_{xc}, flanges of a member. Because ε_{xc} will typically be small in comparison to ε_{xt}, due to the large stiffness of concrete in compression, it is a reasonable approximation to assume that ε_x is simply $\varepsilon_{xt}/2$.

ERA has also given recommendations in evaluating the longitudinal strain, ε_x using the same principle as above:

For pre-stressed member

$$\varepsilon_x = [(M_u/d_v) + (0.5N_u) + (0.5(VU - VP)\cot\theta) - (A_{PS} f_{PO})]/(E_S A_S + E_p A_{PS}) < 0.002 \qquad \text{Eq. [4-38]}$$

For reinforced concrete member as can be seen in Fig.4-10, ε_x can be approximated as

$$\varepsilon_x = [(M_u/d_v) + (0.5N_u) + (0.5V_U \cot\theta)]/(E_S A_S) < 0.002 \qquad \text{Eq. [4-39]}$$

Where:

ε_x = longitudinal strain at mid-depth

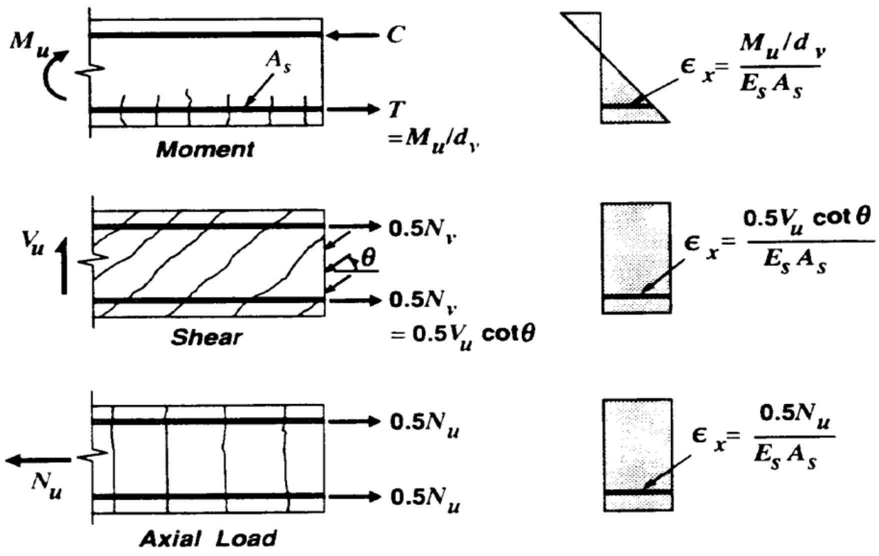


Fig.4-10 Determination of strain ε_x for non-pre-stressed beam
(Adopted from Collins, M.P., et. al 1996)

Fig. 4-10 above shows how the longitudinal strain; ε_X for non-pre-stressed beams can be approximated.

If the value ε_x, calculated from Eq. [4-46], is negative, it shall be multiplied by the factor, $F\varepsilon$ taken as:

$$F\varepsilon = [(E_S A_S) + (E_p A_{pS})]/[(E_C A_C) + (E_S A_S) + (E_p A_{pS})] \qquad \text{Eq. [4-40]}$$

Where: φ = resistance factor for shear specified in Table 3-1.

A_C = area of concrete on the flexural tension side of the member as shown in Fig. 4-11 (mm^2)

A_{pS} = area of prestressing steel on the flexural tension side of the member, reduced for any lack of full development at the section under investigation (mm^2)

N_u = factored axial force taken as positive if tensile (N)

V_u = factored shear force (N)

A_S = area of non-prestressed reinforcing steel on flexural tension side of member, reduced for any lack of full development at the section under investigation (mm^2)

M_u = factored moment (Nmm)

f_{po} = stress in prestressing steel when the stress in the surrounding concrete is 0.0(MPa)

These equations assume cracked section and are only for beams with at least the minimum amount of transverse reinforcement (stirrups).

In Eq. [4-38],

M_u / d_v , is the tensile force in the reinforcing steel due to the moment.

d_v is shear depth = $d - a/2$

N_u, is any applied axial force (not prestressing force). It is assumed that ½ of the axial load is taken by the steel. If the load is compressive, N_u is negative.

$(V_u - V_p)\ cot\theta$, is the axial force component of strut force as shown in the force triangle. Half the force is assumed to be taken by the tensile steel, the other half in the uncracked block.

$A_{ps}f_{po}$ -corrects for the strain in the prestressing steel due to prestressing.

f_{po} -stress in the prestressing steel, usually taken as $0.7f_{pu}$.

But in this thesis, only reinforced concrete bridge girders are used. So there is no need to consider the effect of prestressing steel in the determination of shear capacity.

The flexural tension side of a beam is taken as shown in Fig.4-11. In all the equations for shear which require a value of the area of the longitudinal tensile steel, A_s or A_{ps}, only the steel on the flexural tension side counts. Tensile steel on the flexural compression side (the ½h on the flexural compression side) or compression steel is not considered for shear strength.

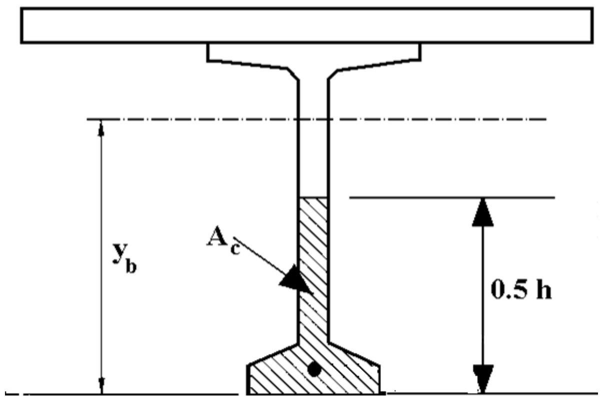


Fig. 4-11 Flexural tension side of a beam

The ERA Tables use v/fc' and ε_x to find β and θ (ERA Bridge Design Manual, 2002).

So once the values of v/fc' and ε_x are calculated, using the table in the ERA Bridge Design Manual, 2002 θ and β can be determined by iteration. If the value of θ is close to the original assumption, use the β given. If not, use the table value of θ as the next estimate and repeat the calculations of ε_x.

After determining the value of β and θ, V_c, V_s and V_n can be calculated using Eq. [4-33], Eq. [4-34] and Eq. [4-35].

v/fc'	$\varepsilon_x * 1000$										
	−0.2	−0.15	−0.1	0	0.125	0.25	0.5	0.75	1	1.5	2
≤0.05	27.0	27.0	27.0	27.0	27.0	28.5	29.0	33.0	36.0	41.0	43.0
	6.78	6.17	5.63	4.88	3.99	3.49	2.51	2.37	2.23	1.95	1.72
0.075	27.0	27.0	27.0	27.0	27.0	27.5	30.0	33.5	36.0	40.0	42.0
	6.78	6.17	5.63	4.88	3.65	3.01	2.47	2.33	2.16	1.90	1.65
0.1	23.5	23.5	23.5	23.5	24.0	26.5	30.5	34.0	36.0	38.0	39.0
	6.50	5.87	5.31	3.26	2.61	2.54	2.41	2.28	2.09	1.72	1.45
0.125	20.0	21.0	22.0	23.5	26.0	28.0	31.5	34.0	36.0	37.0	38.0
	2.71	2.71	2.71	2.60	2.57	2.50	2.37	2.18	2.01	1.60	1.35
0.15	22.0	22.5	23.5	25.0	27.0	29.0	32.0	34.0	36.0	36.5	37.0
	2.66	2.61	2.61	2.55	2.50	2.45	2.28	2.06	1.93	1.50	1.24
0.175	23.5	24.0	25.0	26.5	28.0	30.0	32.5	34.0	35.0	35.5	36.0
	2.59	2.58	2.54	2.50	2.41	2.39	2.20	1.95	1.74	1.35	1.11
0.2	25.0	25.5	26.5	27.5	29.0	31.0	33.0	34.0	34.5	35.0	36.0
	2.55	2.49	2.48	2.45	2.37	2.33	2.10	1.82	1.58	1.21	1.00
0.225	26.5	27.0	27.5	29.0	30.5	32.0	33.0	34.0	34.5	36.5	39.0
	2.45	2.44	2.43	2.37	2.33	2.27	1.92	1.67	1.43	1.18	1.14
0.25	28.0	28.5	29.0	30.0	31.0	32.0	33.0	34.0	35.5	38.5	41.5
	2.36	2.36	2.32	2.30	2.28	2.01	1.64	1.52	1.40	1.30	1.25

Table 4-2 Values of θ and β for Sections with Transverse Reinforcement (ERA Bridge Design Manual, 2002)

4.3.2.2. For sections without transverse reinforcement (stirrups)

For members without transverse reinforcement, β and θ values calculated from the MCFT are given as functions of ε_x, and the crack spacing s_x. ε_x is taken as the largest calculated longitudinal strain which occurs within the web of the member when the section is subjected to M_u , N_u , and V_u .

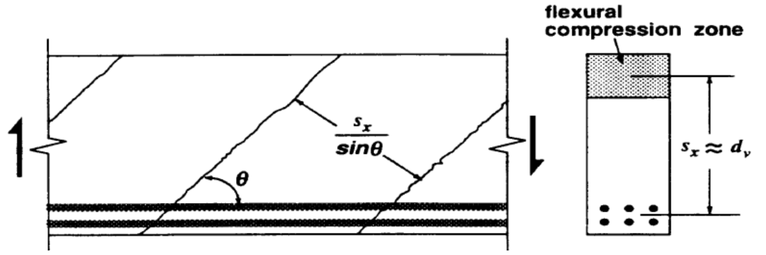
$\varepsilon_x = [(M_u/d_v) + (0.5N_u) + (0.5(V_u - V_p)cot\theta) - (A_{ps}f_{po})]/2(E_sA_s + E_pA_{ps})$ Eq. [4-41]

β and θ can be found from Table 4-2 or Table 4-3. In these tables, β and θ are given as functions of the strain ε_x, the shear stress v and the crack spacing parameter s_x.

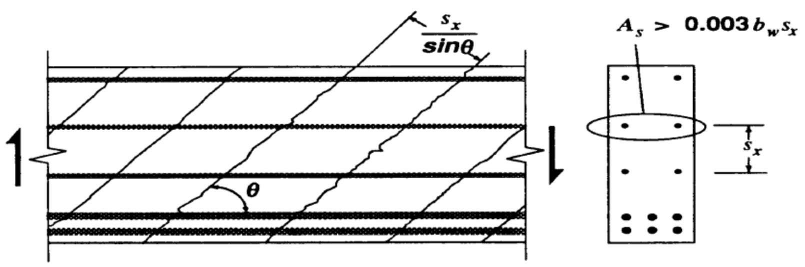
The values in these tables were calculated assuming a diagonal crack spacing of 305mm and a maximum aggregate size of 19mm because it is believed that these values are appropriate for the full range of beams containing stirrups (Collins, M.P., et. al 1996). The values in Table 4-3 can also be used for other aggregate sizes by using an equivalent spacing parameter s_{xe} such that:

$$s_{xe} = s_x * [35/(a + 16)] \quad \text{Eq. [4-42]}$$

The crack spacing parameter s_x, used in Table 4-3, shall be taken as the lesser of either d_v or the maximum distance between layers of longitudinal crack control reinforcement. The area of the reinforcement in each layer shall be $\geq 0.003b_vs_x$.



a) Member without stirrups and with concentrated longitudinal reinforcement



b) Member without stirrups but with well distributed longitudinal reinforcement

Fig. 4-12 crack spacing parameter determination

(Adopted from Collins, M.P., et. al 1996, ERA Bridge Design Manual, 2002)

s_x	$\varepsilon_x * 1000$								
	−0.2	-0.1	0	0.25	0.5	0.75	1	1.5	2
≤ 130	26.0 6.90	26.0 5.70	27.0 4.94	29.0 3.78	31.0 3.19	33.0 2.82	34.0 2.56	36.0 2.19	38.0 1.93
250	27.0 6.77	28.0 5.53	30.0 4.65	34.0 3.45	37.0 2.83	39.0 2.46	40.0 2.19	43.0 1.87	45.0 1.65
380	27.0 6.57	30.0 5.42	32.0 4.47	37.0 3.21	40.0 2.59	43.0 2.23	45.0 1.98	48.0 1.65	50.0 1.45
630	28.0 6.24	31.0 5.36	35.0 4.19	41.0 2.85	45.0 2.26	48.0 1.92	51.0 1.69	54.0 1.40	57.0 1.18
1270	31.0 5.62	33.0 5.24	38.0 3.83	48.0 2.39	53.0 1.82	57.0 1.50	59.0 1.27	63.0 1.00	66.0 0.83
2500	35.0 4.78	35.0 4.78	42.0 3.47	55.0 1.88	62.0 1.35	66.0 1.06	69.0 0.87	72.0 0.65	75.0 0.52
5000	42.0 3.83	42.0 3.83	47.0 3.11	64.0 1.39	71.0 0.90	74.0 0.66	77.0 0.53	80.0 0.37	82.0 0.28

Table 4-3 Values of θ and β for Sections without Transverse Reinforcement (ERA Bridge Design Manual, 2002)

Chapter 5. Analysis of Simply Supported Bridge Girders

5.1. Introduction

Analysis of girders for shear rating involves the determination of shear capacity, shear due to the externally applied loads. By using the methods and rating procedures which are discussed in the previous sections, permanent loads shear effect and live load shear effects determination process is discussed in this section. Bridge girder external shear effects come from permanent loads and live loads. The permanent load shear depends on material properties, geometry of the bridge girders. The live loads are moving loads where their effects depend on the positions of the loads on the bridge girders. So effects of the moving loads are determined at the critical section and where the loads result in higher effects. Since live loads are applied on the bridges analysis of bridge girders involves distribution of load effects into bridge girders.

5.2. Live Load Force Effects

Shear force and moment for truck and lane loads

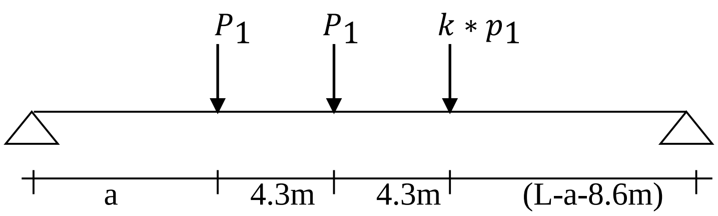


Fig. 5-1 Truck Moving Truck to the right (Abraham Gebre, 2006)

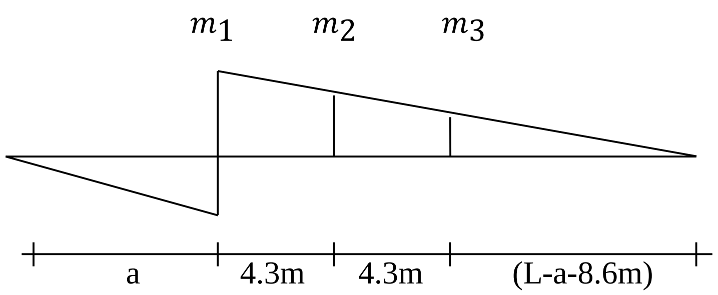


Fig. 5-2 IL for Shear Force (Abraham Gebre, 2006)

Influence line (IL) coefficients for shear force

$m_1 = (L - a)/L$

$m_2 = (L - a - 4.3)/L$

$m_3 = (L - a - 8.6)/L$

$K = P_2/P_1$

$V_{trl} = (P_1 * m_1 + P_1 * m_2 + K * P_1 * m_3)$

$V_L = 0.5 * (L - a) * m_1 * U$

$Vtrl = (P_1/L) * ((L - a) + (L - a - 4.3) + K * (L - a - 8.6))$ Eq. [5-1a]

$V_L = 0.5 * (L - a)^2 * U/L$ Eq. [5-1b]

Where:

V_{trl} = shear force at a distance **a** due to truck load.

V_L = shear force at a distance **a** due to lane load.

U= uniformly distributed lane load

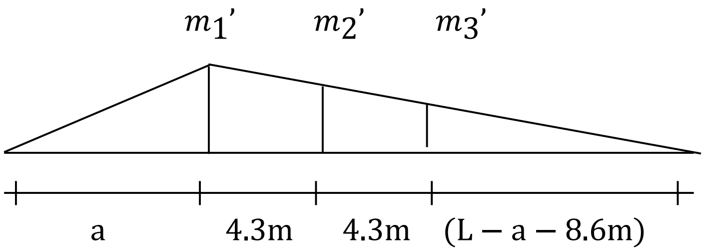


Fig. 5-3 IL for Bending Moment (Abraham Gebre, 2006)

Influence line coefficients for bending moment

$m_1' = a(L - a)/L$

$m_2' = a(L - a - 4.3)/L$

$m_3' = a(L - a - 8.6)/L$

$M_{trl} = (P_1 * m_1' + P_1 * m_2' + K * P_1 * m_3')$

$ML = 0.5 * (L - a) * m1' * U$

$M_{trl} = P_1 * (a/L) * ((L - a) + (L - a - 4.3) + K * (L - a - 8.6))$ Eq. [5-2a]

$ML = 0.5 * a (L - a)^2 U/L$ Eq. [5-2b]

Where:

M_{trl} = Moment at a distance **a** due to truck load.

ML = Bending Moment at a distance **a** due to lane load.

If the value in bracket is negative, then it is taken as zero. It implies that the wheel is out of the span.

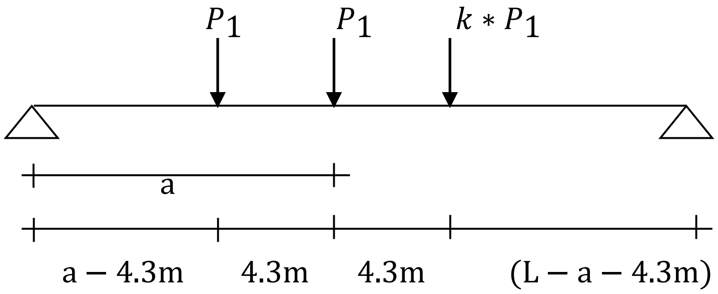


Fig. 5-4 Truck moving to the Left (Abraham Gebre, 2006)

Influence Lines for Shear Force

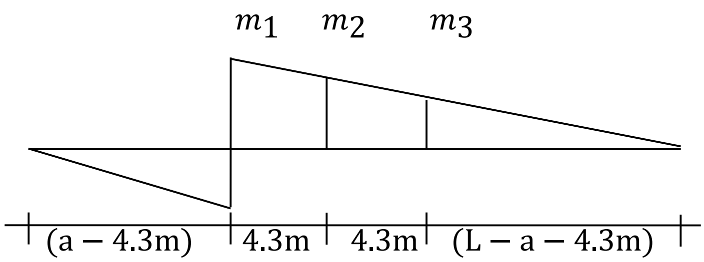


Fig. 5-5 IL for Shear Force (Abraham Gebre, 2006)

Influence line coefficients for Shear Force

$m_1 = (L - a)/L$

$m_2 = a(L - a - 4.3)/L$

$m_3 = (a - 4.3)(L - a)/L$

$K = P_2/P_1$

$V_{trl} = (P_1 * m_1 + P_1 * m_2 + K * P_1 * m_3)$ Eq. [5-3a]

$VL = 0.5 * L * m_1 * U$ Eq. [5-3b]

Where:

V_{trl} = shear force at a distance **a** due to truck load.

VL = shear force at a distance **a** due to lane load.

U = uniformly distributed lane load

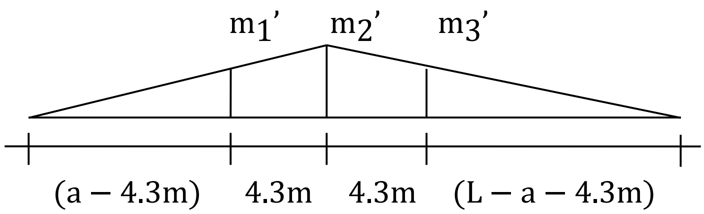


Fig. 5-6 IL for Bending Moment (Abraham Gebre, 2006)

Influence line coefficients for bending moment

$m_1' = a(a - 4.3)(L - a + 4.3)/L$

$m_2' = a(L - a)/L$

$m_3' = a(L - a - 4.3)/L$

$M_{trl} = (P_1 * m_1' + P_1 * m_2' + K * P_1 * m_3')$ Eq. [5-4a]

$ML = 0.5 * L * m_1' * U$ Eq. [5-4b]

Where:

M_{trl} = Moment at a distance **a** due to truck load.

ML = Bending Moment at a distance **a** due to lane load.

If the value in bracket is negative, then it is taken as zero.

Shear Force and Bending Moment for Tandem and Lane Loads

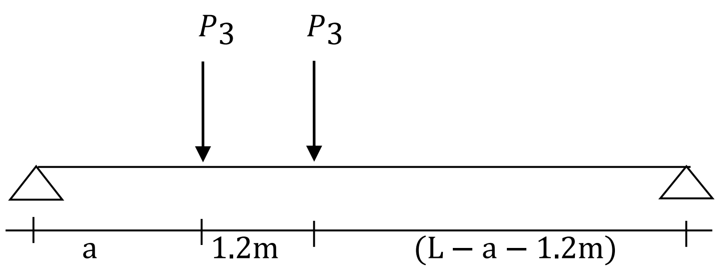


Fig. 5-7 Tandem load (Abraham Gebre, 2006)

Influence Lines for Shear Force

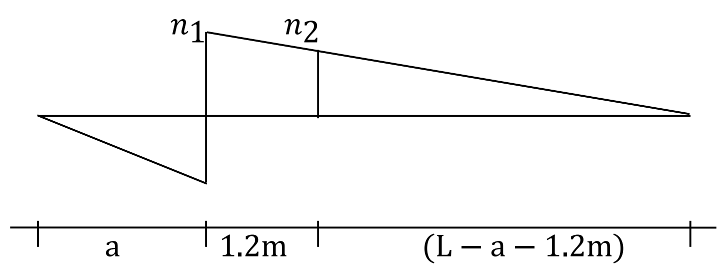


Fig. 5-8 IL for Shear Force (Abraham Gebre, 2006)

Influence line coefficients for Shear Force

$n_1 = (L - a)/L$

$n_2 = (L - a - 1.2)/L$

$V_{tml} = (P_3 * n_1 + P_3 * n_2$

$VL = 0.5 * (L - a) * n_1 * U$

$V_{tml} = (P/L) * ((L - a) + (L - a - 1.2))$ Eq. [5-5a]

$VL = 0.5 * (L - a)^2 U/L$ Eq. [5-5b]

Where:

- V_{tml} is shear force at a distance **a** due to tandem load.

- VL is shear force at a distance **a** due to lane load.

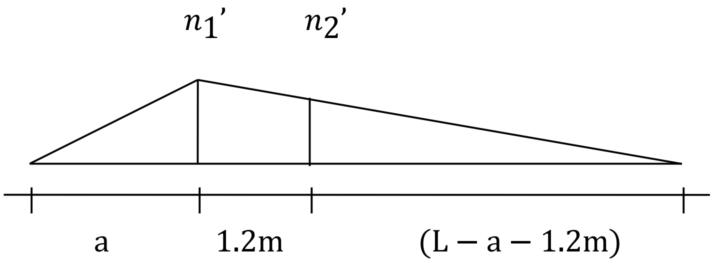


Fig. 5-9 IL for Bending Moment (Abraham Gebre, 2006)

Influence line coefficients for bending moment

$n_1' = a(L-a)/L$

$n_2' = a(L-a-1.2)/L$

$M_{tml} = (P_3 * n_1' + P_3 * n_2')$

$ML = 0.5 * (L-a) * n_1' * U$

$M_{tml} = P_3 * (a/L) * ((L-a) + (L-a-1.2))$ Eq. [5-6a]

$ML = 0.5 * a(L-a)^2 U/L$ Eq. [5-6b]

Where:

- M_{tml} is Moment at a distance **a** due to tandem load.
- ML is Moment at a distance **a** due to lane load.

If the value in bracket is negative, then it is taken as zero. It implies that the wheel is out of the span.

L= C/C length of the bridge into consideration.

5.3. Bridge Girder Loading

5.3.1. Dead Loads

Dead load includes the weight of all components of the structure, appurtenances and utilities attached thereto, earth cover, wearing surface, future overlays, and planned widening.

The dead load shall be estimated from data available from the inspection at the time of analysis. The dead load factor accounts for normal variations of material densities and dimensions. Nominal dimensions and densities shall be used for calculating dead load effects. For overlays, either cores shall be used to establish the true thickness or an additional allowance of 20% should be placed on the nominal overlay thickness indicated at the time of analysis.

The recommended unit weights of materials to be used in computing the dead load should be as in Table 5-1.

MATERIAL	FORCE EFFECT [kN/m^3]
Asphalt surfacing	22.5
Concrete, plain or reinforced (normal weight)	24.0
Steel	79.0
Cast iron	72.0
Timber (treated or untreated)	8.0
Earth (compacted), sand gravel or ballast	18.0

Table 5-1 recommended unit weights of materials

Effective Flange Width

Longitudinal stresses in the flanges are distributed across the flange and the composite deck slab by in plane shear stresses. Therefore, the longitudinal stresses are not uniform. The effective flange width is a reduced width over which the longitudinal stresses are assumed to be uniformly distributed and result in the same force as the non-uniform stress distribution if integrated over the entire width.

The effective flange width may be calculated as:

For Interior Girders

The effective flange width is taken as the least of;

- One-quarter of the effective span length
- 12.0 times the average thickness of the slab plus the greater of the web thickness or one half the width of the top flange of the girder
- The average spacing of adjacent girders

For Exterior Girders

The effective flange width is taken as:

- One half of the effective width of the adjacent interior girder plus the least of
    - One-eighth of the effective span length
    - 6.0 times the average thickness of the slab plus the grater of half the web thickness or one-quarter of the width of the top flange of the basic girder
- The width of the overhang

The effective span length used in calculating the effective flange width may be taken as the actual span length for simply supported spans.
The slab thickness in the analysis is the effective slab thickness ignoring any sacrificial layers (i.e. integral wearing surfaces)

5.3.2. Vehicular Live Load

Number of Design Lanes

Generally, the number of design lanes should be determined by taking the integer part of the ratio $w/3000$, where w is the clear roadway width in mm between curbs and/or barriers. Possible future changes in the physical or functional clear roadway width of the bridge should be considered.

In cases where the traffic lanes are less than 3.0 m wide, the number of design lanes shall be equal to the number of traffic lanes, and the width of the design lane shall be taken as the width of the traffic lane.

Multiple Presence of Live Load

Multiple presence factors are used to account for the probability of simultaneous lane occupation by the full HL-93 live load.
The extreme live load force effect shall be determined by considering each possible combination of number of loaded lanes multiplied by the corresponding factor specified in Table 5-2 when lever rule is used but if Approximate Method of Analysis (ERA Bridge Design Manual, 2002) is used, since the application of multiple vehicular loads are taken into consideration in the distribution factors, multiple presence factors are not applied.

Number of Loaded Lanes	1	2	3	>3
Multiple Presence Factor “m”	1.2	1.0	0.85	0.65

Table 5-2 Multiple Presence Factor “m” (ERA Bridge Design Manual, 2002)

The multiple presence factors in Table 5-2 were developed based on an ADTT of 5000 trucks in one direction so the force effect resulting from the appropriate number of lanes shall be reduced for sites with lower ADTT as follows:

- If $100 \leq \text{ADTT} \leq 1000$; 95% of the specified force effect shall be used; and

• If ADTT < 100; 90% of the specified force effect shall be used.

This adjustment is based on the reduced probability of attaining the design event during a 75-year design life with reduced truck volume.

Design Vehicular Live Load

Vehicular live loading on the roadways of bridges or incidental structures, designated HL-93 shall consist of a combination of the:

• Design truck or design tandem and

• Design lane load

Each design lane under consideration shall be occupied by either the design truck or tandem, coincident with the lane load, where applicable. The loads shall be assumed to occupy 3 m transversely within a design lane.

Design Truck

The weights and spacing of axles and wheels for the design truck shall be as specified in Fig. 5-10 and a dynamic load allowance shall be considered.

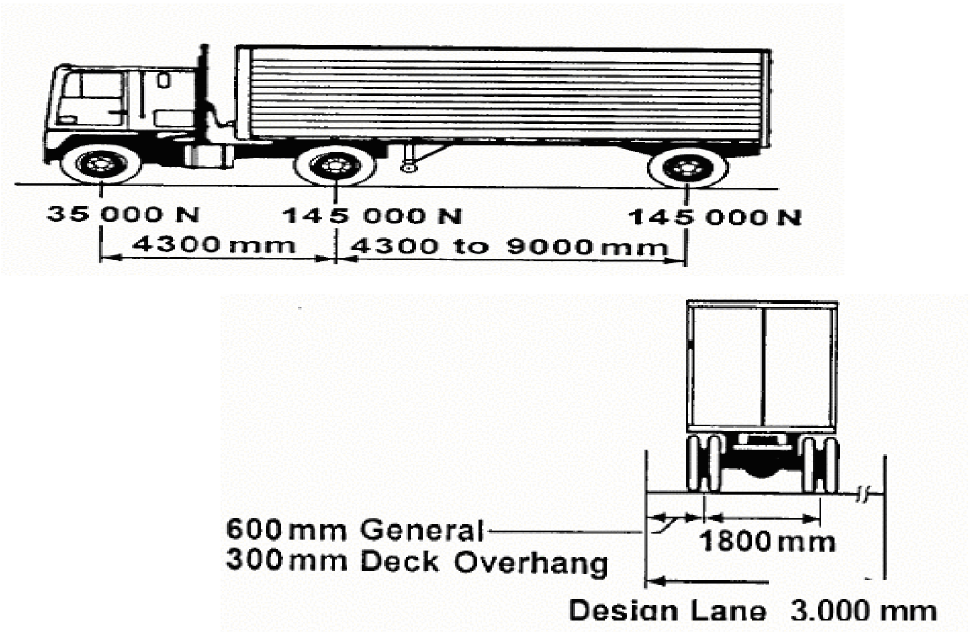


Fig.5-10 Characteristic of the Design Truck (ERA Bridge Design Manual, 2002)

Except on the application of Design Vehicular Live Loads and Fatigue Loads, the spacing between the two 145kN axles shall be varied between 4.3m and 9.0m to produce extreme force effects.

Design Tandem

The design tandem used for Strategic Bridges shall consist of a pair of 110kN axles spaced 1.2 m apart. The transverse spacing of wheels shall be taken as 1.8 m and a dynamic load allowance shall be considered. The spacing and loading is illustrated in Fig. 5-11.

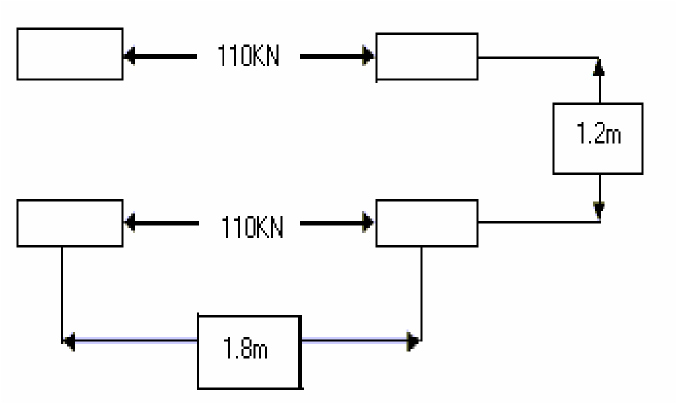


Fig. 5-11 Design Tandem Load (ERA Bridge Design Manual, 2002)

Design Lane Load

The design lane load shall consist of a load of 9.3kN/m uniformly distributed in the longitudinal direction. Transversely, the design lane load shall be assumed to be uniformly distributed over a 3.0-m width. The force effects from the design lane load shall not be subject to a dynamic load allowance (ERA Bridge Design Manual, 2002).

The extreme force effect shall be taken as the larger of the following:

• The effect of the design tandem combined with the effect of the design lane load, or

• The effect of one design truck with the variable axle spacing combined with the effect of the design lane load

5.3.3. Live Load Distribution Factors

ERA Bridge Design Manual, 2002 has given an approximate method for distributing live load for girders that meet the following conditions and any other conditions identified in tables of distribution factors as specified below.

- Width of deck is constant
- Number of beams is not less than four, unless otherwise specified
- Beams are parallel and have approximately the same stiffness

- Unless otherwise specified, the roadway part of the overhang, de, does not exceed 0.9m
- Curvature in plan is less than the limit specified in ERA Bridge Design Manual, 2002.
- Cross-section is consistent with one of the cross-sections shown in Fig. 5-12 below.

In the approximate method of distributing live load effects into girders, distribution factors that can be used for different conditions and effects are given in tables.

Live Load Distribution Factors for Moment

Interior Beams

As of ERA Bridge Design Manual, the live load moment for interior beams shall be determined by applying the lane fractions specified in Table 5-3.

The longitudinal stiffness parameter, Kg, in the table shall be taken as:

$K_g = n\,(I + A\,e_g{}^2)$ in which: $n = EB/\,ED$ Eq. [5-7]

Where:

EB =modulus of elasticity of beam material (MPa)

ED =modulus of elasticity of deck material (MPa)

I = moment of inertia of beam (mm^4)

e_g = distance between the centers of gravity of the basic beam and deck (mm)

A = Area of non-composite beam (mm^2)

The parameters A and I in Eq. [5-7] shall be taken as those of the non-composite beam.

The live load flexural moment for interior beams with concrete decks shall be determined by applying the lane fraction specified in Table 5-3(dimensions in mm).

Type of Beams	Applicable Cross-section from Fig. 5-12	Distribution Factors	Range of Applicability
Concrete Deck on Wood Beams	L	One Design Lane Loaded: $S/3700$ Two or More Design Lanes Loaded: $S/3000$	$S \leq 1800$
Concrete Deck, Filled Grid, or Partially Filled Grid on Steel or Concrete Beams; Concrete T-Beams, T-and Double T-Sections	a, e, k and also i, j if sufficiently connected to act as a unit	One Design Lane Loaded: $0.06+(S/4300)^{0.4}(S/L)^{0.3}(k_g/Lt_s^3)^{0.1}$ Two or More Design Lanes Loaded: $0.075+(S/4300)^{0.6}(S/L)^{0.2}(k_g/Lt_s^3)^{0.1}$	$1100 \leq S \leq 4900$ $110 \leq t_S \leq 300$ $6000 \leq L \leq 73000$ $Nb \geq 4$
		Use lesser of the values obtained from the equation above with Nb = 3 or the lever rule	$Nb = 3$
Multi-cell Concrete Box Beam	D	One Design Lane Loaded: $(1.75+S/1100)(300/L)^{0.35}(1/N_C)^{0.45}$ Two or More Design Lanes Loaded: $(13/N_C)^{0.3}(S/430)(1/L)^{0.25}$	$2100 \leq S \leq 4000$ $18000 \leq L \leq 73000$ $Nc \geq 3$ If Nc>8 use Nc=8
Steel Grids on Steel Beams	A	One Design Lane Loaded: $S/2300$ if $t_g < 100$ mm $S/3050\ if\ t_g \geq 100$ mm Two or More Design Lanes Loaded: $S/2400\ If\ t_g < 100$ mm $S/3050\ If\ t_g \geq 100$ mm	$S \leq 1800$ mm $S \leq 3200$ mm

Table 5-3 Distribution of Live Load per Lane for Moment in Interior Beam

Exterior Beams

The live load flexural moment for exterior beams shall be determined by applying the lane fraction, g, specified in Table 5-4.

Type of Superstructure	Applicable Cross-section from Fig.5-12	One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
Wood Deck on Wood or Steel Beam	a, l	Lever Rule	Lever Rule	N/A
Concrete Deck on Wood Beams	L	Lever Rule	Lever Rule	N/A
Concrete Deck, filled Grid, or Partially Filled Grid on Steel or Concrete Beams: Concrete T-Beams. T and Double-T Sections	a, e, k and also i, j if sufficiently connected to act as a unit	Lever Rule	$g = e\ g_{interior}$ $e = 0.77 + d_e/2800$	$-300 \leq d_e \leq 700$
			Use lesser of the values obtained from the equation above with $Nb = 3$ or the lever rule	$Nb = 3$

Table 5-4 Distribution of Live Loads per Lane for Moment in Exterior Longitudinal Beams

The bridges considered in the development of the equations had interior end diaphragms only, i.e., no interior diaphragms within the spans, and no exterior diaphragms anywhere between boxes. If interior or exterior diaphragms are provided within the span, the transverse load distribution characteristics of the bridge will be improved to some degree. i.e., the distribution factor for the exterior beam shall not be taken to be less than that which would be obtained by assuming that the cross-section deflects and rotates as a rigid cross-section.

Additional investigation is required because the distribution factor for girders in a multi-girder cross-section, Types "a" and "e" in Fig. 5-12, was determined without consideration of diaphragm or cross-frames. The recommended procedure is an interim provision until research provides a better solution (ERA Bridge Design Manual, 2002).

The procedure is as follows:

$NL \quad Nb$

$$R = [NL/Nb] + [X_{ext}\Sigma e/\Sigma x^2] \qquad \text{Eq. [5-8]}$$

Where:

R = reaction on exterior beam in terms of lanes

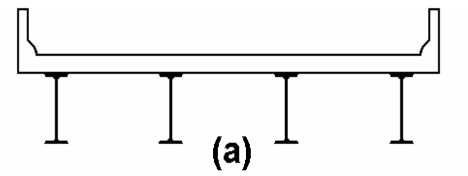
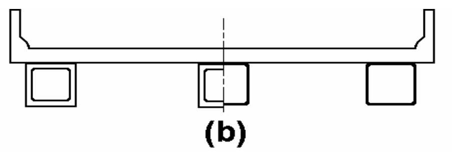
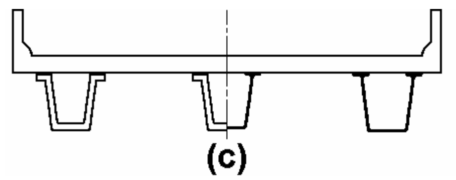
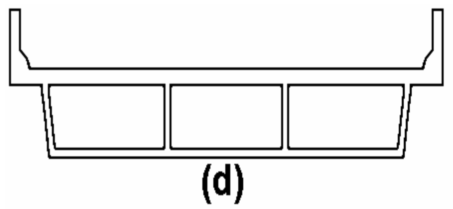
NL = number of loaded lanes under consideration

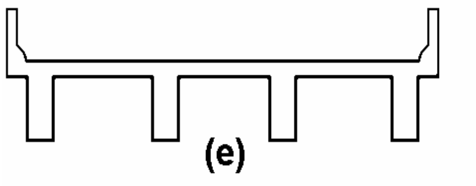
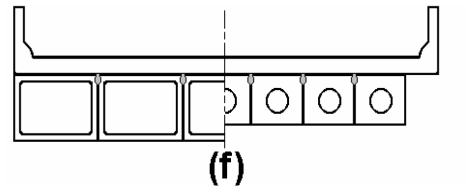
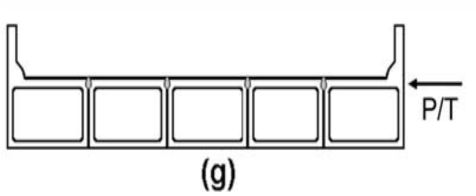
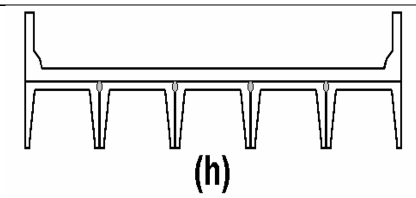
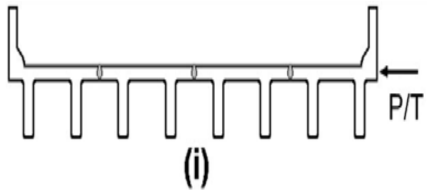
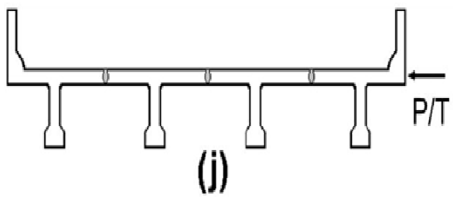
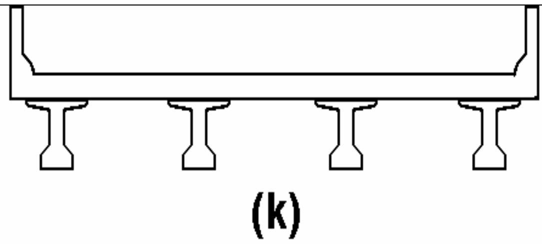
Nb = number of beams or girders

e = eccentricity of a design truck or a design lane load from the center of gravity of the pattern of girders (mm)

x = horizontal distance from the center of gravity of the pattern of girders to each girder (mm)

X_{ext} = horizontal distance from the center of gravity of the pattern of girders to the exterior girder (mm)

Supporting Components	**Type of Deck**	**Typical Cross Section**
Steel Beam	Cast-in-place concrete slab, precast concrete slab, steel grid, glued/spiked panels, stressed wood	
Closed Steel or Precast Concrete Boxes	Cast-in-place concrete slab	
Open Steel or Precast Concrete Boxes	Cast-in-place concrete slab, precast concrete deck slab	
Cast-in-Place Concrete Multicell Box	Monolithic concrete	

Cast-in-Place Concrete Tee Beam	Monolithic concrete	
Precast Solid, Voided or Cellular Concrete Boxes with Shear Keys	Cast-in-place structural concrete overlay	
Precast Solid, Voided, or Cellular Concrete Box with Shear Keys and with or without Transverse Post-Tensioning	Integral concrete	
Precast Concrete Channel Sections with Shear Keys	Cast-in-place structural concrete overlay	
Precast Concrete Double Tee Section with Shear Keys and with or without Transverse Post-Tensioning	Integral concrete	
Precast Concrete Tee Section with Shear Keys and with or without Transverse Post-Tensioning	Integral concrete	
Precast Concrete I or Bulb-Tee Sections	Cast-in-place concrete, precast concrete	

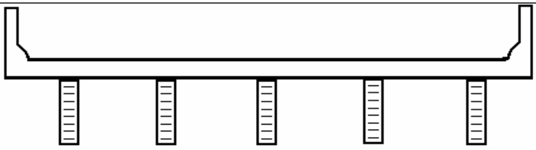
Wood Beams	Cast-in-place concrete or plank, glued/spiked panels or stressed wood	 (l)

Fig. 5-12 Common Deck Superstructures

Skewed Bridges

When the line supports are skewed and the difference between skew angles of two adjacent lines of supports does not exceed 10°, the bending moment in the beams shall be reduced in accordance with Table 5-5.

Accepted reduction factors are not currently available for cases not covered in Table 5-5.

Type of Superstructure	Applicable Cross-section from Fig. 5-12	Any Number of Design Lanes Loaded	Range of Applicability
Concrete Deck, Filled Grid, or Partially Filled Grid on Steel or Concrete Beams, Concrete T-Beams, T or Double T Sections	a, e and k	$1 - c_1 (tan\, \theta)^{1.5}$ $c_1 = 0.25(k_g/Lt_s{}^3)^{0.25}(S/L)^{0.5}$ If $\theta < 30^0$ $then\ c_1 = 0.0$ If $\theta > 60^0$ $use\ \theta = 60^0$	$300 \leq \theta \leq 600$ $1100 \leq S \leq 4900$ $6000 \leq L \leq 3000$ $Nb \geq 4$

Table 5-5 Reduction of Load Distribution Factors for Moment in Longitudinal Beams on Skewed Supports

Live Load Distribution Factors for Shear

Interior Beams

As of ERA Bridge Design Manual, the live load shear for interior beams shall be determined by applying the lane fractions specified in Table 5-6.

The longitudinal stiffness parameter, Kg, in the table shall be taken from Eq. [5-7].

Type of Superstructure	Applicable Cross-section from Fig. 5-12	One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
Concrete Deck on Wood Beams	L	Lever Rule	Lever Rule	N/A
Concrete Deck, Filled Grid, or Partially Filled Grid on Steel or Concrete Beams: Concrete T-Beams. T and Double T Sections	a, e, k and also i, j if sufficiently connected to act as a unit	$0.36 + S/7600$	$0.2+(S/3600)-(S/10700)^{2.0}$	$1100 \leq S \leq 4900$ $6000 \leq L \leq 73000$ $110 \leq t_S \leq 300$ $4x10^9 \leq kg \leq 3x10^{12}$ $Nb \geq 4$
		Lever Rule	Lever Rule	Nb = 3
Multi-cell Concrete Box Beams, Box Sections	D	$(S/2900)^{0.6}(d/L)^{0.1}$	$(S/2200)^{0.9}(d/L)^{0.1}$	$1800 \leq S \leq 4900$ $6000 \leq L \leq 73000$ $890 \leq d \leq 2800$ $Nc \geq 3$

Table 5-6 Distribution of Live Load per Lane for Shear in Interior Beams

Exterior Beams

The live load shear for exterior beams shall be determined by applying the lane fractions specified in Table 5-7. For cases not addressed in Table 5-6 and Table 5-7, the live load distribution to exterior beams shall be determined by using the lever rule.

The additional provisions for exterior beams in beam-slab bridges with cross-frames or diaphragms, specified above, shall apply i.e. in beam-slab bridge cross-sections with diaphragms or cross-frames, the distribution factor for the exterior beam shall not be taken to be less than that which would be obtained by assuming that the cross-section deflects and rotates as a rigid cross-section.

The parameter de shall be taken as positive if the exterior web is inboard of the curb or traffic barrier and negative if it is outboard.

Type of Superstructure	Applicable Cross-section from Fig. 5-12	One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
Wood Deck on Wood or steel Beams	a, l	Lever Rule	Lever Rule	N/A
Beams; Concrete T-beams, T and Double T-Beam	a, e, k and also i, j if sufficiently connected to act as a unit	Lever Rule	$g = e\, g_{\text{interior}}$ $e = 0.6 + de/3000$	$-300 \le de \le 1700$
			Lever Rule	$Nb = 3$
Multi-cell Concrete Box Beams, Box Sections	D	Lever Rule	$g = e\, g_{\text{interior}}$ $e = 0.68 + de/3800$	$-600 \le de \le 1500$
Steel Grid Deck on Steel Beams	A	Lever Rule	Lever Rule	N/A

Table 5-7 Distribution of Live Load per Lane for Shear in Exterior Beams

Skewed Bridges

Shear in the exterior beam at the obtuse corner of the bridge shall be adjusted when the line of support is skewed. The value of the correction factor shall be obtained from Table 5-8. It is applied to the lane fraction specified in Table 5-6 for interior beams and in Table 5-7 for exterior beams.

In determining the end shear in multi-beam bridges, the skew correction at the obtuse corner shall be applied to all the beams.

Type of Superstructure	Applicable Cross-section from Fig. 5-12	Correction Factor	Range of Applicability
Concrete Beams; Concrete T-beams, T- and Double T Section	a, e, k and also i, j if sufficiently connected to act as unit	$1.0+0.20(Lt_s^3/K_g)^{0.3}\tan\theta$	$0^o \le \theta \le 60^o$ $1100 \le S \le 4900$ $6000 \le L \le 73000$ $Nb \ge 4$
Multi-cell Concrete Box Beams, Box sections	D	$1.0+(0.25+L/70d)\tan\theta$	$0^o \le \theta \le 60^o$ $1800 \le S \le 4000$ $6000 \le L \le 73000$ $900 \le d \le 2700$ $Nc \ge 3$

Table 5-8 Correction Factors for Load Distribution Factors for Support Shear of the Obtuse Corner (ERA Bridge Design Manual, 2002)

Lever Rule

Equations were derived in order to simplify the lever rule (Puckett, J. A. et. al., 2007).these equations were derived assuming constant 4ft(1.2m) spacing between multiple vehicles, 2ft(0.6m) spacing between the outer wheel centroid and the curb/barrier and 6ft(1.8m) axle gage. Range of applicability is given in the distribution factor Table 5-9 and Table 5-10. While the limitations are not met, conventional lever rule computations apply.

Number of Loaded Lanes	Distribution Factor	Range of application	Number of Wheels to Beam
1	$(½)+(d_e/2S)$	$d_e+S\le 6$ft $d_e<S$	1
	$1+(d_e/S)-(3/S)$	$(d_e+S)>6$ft	2
2 or more	$1+(d_e/S)-(3/S)$	$(d_e+S)\le 10$ft	2
	$3/2+(3d_e/2S)-(8/S)$	$10<(d_e+S)\le 16$ft	3
	$2+(2d_e/S)-(16/S)$	$16<(d_e+S)\le 20$ft	4

Table 5-9 Lever Rule Equations for Exterior Girders
(Adopted from Puckett, J. A. et. al., 2007)

Number of Loaded Lanes	Distribution Factor	Range of application	Number of Wheels to Beam
1	½	$S \leq 6$ft $de \geq 0$	1
	1-3/*S*	$S > 6$ft $de \geq 0$	2
2 or more	½	$S \leq 4$ft $de \geq 0$	1
	1-2/*S*	$4 < S \leq 6$ft $de \geq 0$	2
	3/2-5/*S*	$6 < S \leq 10$ft $de \geq 0$	3
	2-10/*S*	$10 < S \leq 16$ft $de \geq 0$	4

Table 5-10 Lever Rule Equations for Interior Girders

(Adopted from Puckett, J. A. et. al., 2007)

Note that de is the distance in mm from the face of rail to exterior face of exterior girder. It is positive if the exterior web is in board and vise-versa.

Tables 5-3 and 5-4 above can be used where the tables from approximate method for distribution of live load effect to girders recommend lever rule but we shall take the range of application of the simplified lever rule equations.

Correction Factor for Distribution of Vehicular Loads

The fraction of vehicle load effect transferred to a single member shall be selected in accordance with ERA Bridge Design Manual, 2002. These values represent a possible combination of adverse circumstances.

Loadings shall be placed in positions causing the maximum response. Further, if such a measurement or analysis is made and the expected distribution value is obtained, then this shall be adjusted by the factors shown in the table below. These factors are needed to adjust for the expected bias in distribution factors for different material types.

	Distribution of Loads	Correction Factor		
		Steel	Prestressed	Concrete
1	AASHTO Distribution, Chapter 13	1.00	1.00	1.00
2	Tabulated analysis with simplified assumptions**	1.10	1.05	0.95
3	Refined analysis: finite elements, orthotropic plate, grillage analogy	1.07	1.03	0.90
4	Field measurements	1.03	1.01	0.90
Actual girder distribution shall be multiplied by the appropriate correction factors to obtain the girder distribution for rating.				

Table 5-11 Correction Factor for Analysis*(ERA Bridge Design Manual, 2002)

Since this paper uses the AASHTO distribution method, the correction factors for distribution of vehicular loads shall be taken as unity as given in table 5-11.

Pedestrian live loads, wind loads, temperature effects, earth quake effect and creep and shrinkage do not need to be considered in rating of the bridge girders.

Chapter 6. LRFR Analysis of Bridge Girders

6.1. Introduction

In the previous chapters, methods of rating bridge girders for shear and shear capacity determination methods are discussed. In this chapter four simply supported bridge girders are analyzed and rated for shear. These bridge girders have 25m, 16m, 20m and 12m span lengths with the material properties given in the next sub- sections.

6.2. Bridge Girder 1 LRFR Analysis and Rating

This bridge girder is T-girder having a span length of 25m. It is constructed in Amhara region by Amhara Rural Roads Authority. The bridge has the following properties.

Design Specifications

1. ERA Bridge Design Manual, 2002
2. AASHTO Standard Specifications for Highway Bridges, 1998

Bridge Loadings

Dead Loads

Dead loads are based on the following loadings

Super Structure Loads

Reinforced concrete: 24KN/m^3

Wearing Surface, Asphalt: 22.5KN/m^3

Live Loading

The live loading is AASHTO HL-93

Materials

1. Concrete

Grade of Concrete	28 days cylindrical compressive strength (MPa)	Structural Member
C-30	24	All Structural members unless otherwise noted

2. Reinforcing Steel

AASHTO M31 MGrade	Grade 300(ϕ<20mm)	Grade 420(ϕ ≥20mm)
(Old AASHTO M31 MGrade)	(40)	(60)
Yield(tensile) Strength, min, MPa	300(500)	420(620)

Type of Girder

The type of girder cross-section from those in Table 5-12 is Type e, i.e. Cast –in-place concrete T-beam with monolithic concrete deck.

Input Parameters:

Bridge Geometry

Bridge Width = 7.3m

Number of Girders = 4

Left overhang =1.280m

Girder spacing = 2.11m

Slab Thickness=0.2m

Wearing surface = 0.1m

Left curb width = 0.8m

Right curb width = 0.8m

Curb Height=0.5m

Left skew angle =0^0

Right skew angle =0^0

Load Factors

Resistance Factor (ϕ) =0.9

Condition Factor (ϕ_c) =1.0

System Factor (ϕ_s) = 1.0

Dead Load Factors

Strength I limit State

Dead load Factor (γ_{DC}) =1.25

Dead Load Factor (γ_{DW}) =1.50

Design Live Load Factors

Strength I Limit State

Live Load Factor (γ_{LL}) (Inventory) =1.75

Live Load Factor (γ_{LL}) (Operating) =1.35

Dynamic Load Allowance

Dynamic load allowance=20%

Live Load Distribution Factors

Moment for Interior Bridge Girders

One Design Lane Loaded:

$g_{interior} = 0.06 + (S/4300)^{0.4}(S/L)^{0.3}(k_g//Lt_s{}^3)^{0.1}$

$= 0.06 + (2110/4300)^{0.4}(2110/25000)^{0.3}(k_g/(25000 * 200^3))^{0.1}$

$= 0.06 + (0.752 * 0.476 * 2.185)$

$= 0.842$

Two or More Design Lanes Loaded:

$g_{interior} = 0.075 + (S/4300)^{0.6}(S/L)^{0.2}(k_g//Lt_s{}^3)^{0.1}$

$= 0.075 + (2110/4300)^{0.6}(2110/25000)^{0.2}(k_g/(25000 * 200^3))^{0.1}$

$= 0.075 + (0.652 * 0.61 * 2.185)$

$= 0.944$

Where the value of K_g is as calculated below

$K_g = n\,(I + A\,e_g{}^2)$ in which: $n = EB/ED$

$n = (EB/ED) = 1$,

Effective flange width for interior girders

$beff = min\,\{1/4L, 12ts + max\,(tw, 0.5wf), Savg\}$

$b_{eff} = min\{25m/4, (12 * 0.2m) + max(0.47m, 0.5 * 2.11m), 2.11m\}$

$b_{eff} = 2.11m$

$A = b_{eff} * t_S = 2.11m * 0.2m = 0.422m^2$

$I = bh^3/12 = 0.47 * 1.5^3/12 = 0.132m^4$

$e_g = 0.85m$

$K_g = n\,(I + A\,e_g{}^2)$

$= 1(0.132 + 0.422 * 0.85^2)$

$= 0.437m^4$

Therefore,

Interior girder live load moment distribution factor will be the max of the two values.

$Mgint = max\,(0.842, 0.944)$

$= 0.944$

Moment for Exterior Bridge Girders

One Design Lane Loaded:

Lever Rule

$de = 0.605m = 1.985ft$

$S = 2.11m = 6.923ft$

$de + S = 1.985ft + 6.923ft = 8.908ft$

$de + S > 6ft$

Therefore, the distribution factor lever rule equation will be:

$DF = 1 + (de/S) - (3/S) = 1 + (1.985ft/6.923ft) - (3ft/6.923ft)$

$= 0.853$

When using Lever rule we shall consider multiple presence factor; for single lane loaded the multiple presence factor is 1.2.

$DF = 0.853 * 1.2 = 1.024$

Two or More Design Lanes Loaded:

$g = e * ginterior$

$e = 0.77 + de/2800, de = 605$

$e = 0.77 + 605/2800$

$= 0.986$

$g = 0.986 * ginterior$

$g = 0.986 * 0.944$

$= 0.931$

Therefore,

Exterior girder live load moment distribution factor will be the max of the two values.

$Mgext = max\ (1.024, 0.931)$

$= 1.024$

Check for Diaphragm Contribution

$NL \ \ Nb$

$R = [NL/Nb] + [X_{ext}\sum e/\sum x^2]$

For One Lane Loaded

$NL = 1$

$Nb = 4$

$Xext = 3.17m$

$\sum e = 2.75m$

$\sum x^2 = (x_1{}^2 + x_2{}^2 + x_3{}^2 + x_4{}^2)$

$= [(3.17m)^2 + (1.06m)^2 + (1.06m)^2 + (3.17m)^2]$

$= 22.345m^2$

$NL \ \ Nb$

$R = [NL/Nb] + [X_{ext}\sum e/\sum x^2]$

$= (1/4) + [(3.17m * 2.75m)/22.345m^2]$

$= 0.64$

For Two Lanes Loaded

Only the Values for the terms NL and $\sum e$ vary.

$NL = 2$

$\sum e = -0.13 + 2.75 = 2.62$

$NL \ \ Nb$

$R = [NL/Nb] + [X_{ext}\sum e/\sum x^2]$

$= (2/4) + [(3.17m * 2.62m)/22.345m^2]$

$= 0.872$

The distribution factors obtained previously shall be taken since these values are lower.

Check for Skewness

Skewness angle $\theta=0^0$

Skewness Correction Factor=1

Shear Distribution Factors for Interior Bridge Girders

One Design Lane Loaded:

$DF = 0.36 + S/7600 = 0.36 + 2110/7600$

$= 0.64$

Two or More Design Lanes Loaded:

$DF = 0.2 + (S/3600) + (S/10700)^{2.0} = 0.2 + (2110/3600) + (2110/10700)^{2.0}$

$= 0.83$

Therefore,

Interior girder live load shear distribution factor will be the max of the two values.

$Vgint = max\ (0.64, 0.83)$

$= 0.83$

Shear Distribution factors for Exterior Bridge Girders

One Design Lane Loaded:

Lever Rule

$de\ = 0.605m = 1.985ft$

$S = 2.11m = 6.923ft$

$de + S = 1.985ft + 6.923ft = 8.908ft$

$de + S > 6ft$

Therefore, the distribution factor lever rule equation will be:

$DF = 1 + (de/S) - (3/S) = 1 + (1.985/6.923) - (3/6.923)$

$= 0.853$

Similarly applying multiple presence factor,

$DF = 0.853 * 1.2 = 1.024$

Two or More Design Lanes Loaded:

$g\ =\ e\ ginterior$

$e\ =\ 0.6 + de/3000$

$e = 0.6 + 605/3000 = 0.802$

$g = 0.802 * ginterior = 0.802 * 0.83 = 0.666$

Therefore,

Exterior girder live load shear distribution factor will be the max of the three values.

$Vgext = max\ (1.024, 0.666, 0.872)$

$= 1.024$

Check for Skew-ness

Skewness Correction Factor (for $\theta=0^0$) =1

Therefore,

No correction for skewness

Dead Load Calculation

Interior Girder Loading

Girder Load $(Lg)\ = Ag * \gamma g =\ [(2.11m * 0.2m) + (0.47m * 1.6m)] * 24KN/m^3$

$= 28.18KN/m$

Railing and Post Load(Lrp):

Loads from railings and post can be distributed equally to all girders

Railing Load= $A_r * \gamma_r = (0.2m * 0.4m * 24KN/m^3)$

$= 1.92KN/m$

Post Load

$= ((4 * 0.3m) + (2 * 0.5m)) * ((0.25m * 0.5m) + (0.15m * 0.4m)) * 24KN/m^3$

$= 9.768KN/25m$

$= 0.39KN/m$

Therefore,

Lrp= $1.92KN/m + 0.391KN/m$

$= 2.31KN/m$

Lrp for each girder= $(2.31KN/m)/4 = 0.5775KN/m$

Diaphragm Load (Ld)

Diaphragm Load will be applied on girders as concentrated load on the points where the center lines of the girder and diaphragm meet.

For the two interior diaphragms

$Ld = A_d * \gamma_d * length\ of\ diaphragms\ between\ faces\ of\ girders\ (l_d)$

$= A_d * \gamma_d * l_d$

Height of diaphragm=0.9m

Thickness of diaphragm=0.25m

Length of diaphragm (ld) =1.64m

$Ld = A_d * \gamma_d * l_d$

$= 0.9m * 0.25m * 1.64m * 24KN/m^3$

$= 8.86KN$

For the two end diaphragms

$Lde = A_d * \gamma_d * l_d$

Height of diaphragm=0.6m

Thickness of diaphragm=0.25m

Length of diaphragm (ld)=1.64m

$Lde = 0.6m * 0.25m * 1.64m * 24KN/m^3$

$= 5.91KN$

Curb Load (Lcb) = $A_{cb} * \gamma_{cb} = (0.5m * 0.8m) * 24KN/m^3$

$= 9.6KN/m$

The load will be distributed equally to all the girders

Lcb/Girder= $(9.6KN/m)/4 = 2.4KN/m$

Load from Wearing Surface $(Lw) = A_w * \gamma_w = (0.1m * 2.11m) * 22.5KN/m^3$

$= 4.748KN/m$

Total Interior Girder Load $(Dci) = Lg + Lrp + Lcb$

$= 28.18KN/m + 0.58KN/m + 2.40KN/m$

$= 31.152KN/m$

Total Interior Girder Load $(Dwi) = Lw$

$= 4.748KN/m$

Concentrated Interior Diaphragm Load $(Ld) = 8.86KN$

Concentrated End Diaphragm Load $(Lde) = 5.91KN$

Exterior Girder Loading

Girder Load $(Lg) = A_g * \gamma_g = [(2.11m * 0.2m) + (0.47m * 1.6m)] * 24KN/m^3$

$= 28.18KN/m$

Railing and Post Load(Lrp):

As calculated for interior girders,

Lrp/girder= $0.5775KN/m$

Diaphragm Load(Ld, Lde):

The diaphragm load calculation for the exterior girders is the same as of interior girders. But we will take only half of the load for interior girders.

For the two interior diaphragms

$$Ld = (A_d * \gamma_d * l_d)/2$$

$$= (0.9m * 0.25m * 1.64m * 24KN/m^3)/2$$

$$= 4.43KN$$

For the two end diaphragms

$$Ld = (A_d * \gamma_d * ld)/2$$

$$= (0.6m * 0.25m * 1.64m * 24KN/m^3)/2$$

$$= 2.955KN$$

Curb Load (Lcb) $= A_{cb} * \gamma_{cb} = (0.5m * 0.8m) * 24KN/m^3$

$= 9.6KN/m$

The load will be distributed equally to all the girders

$\mathrm{Lcb/Girder} = (9.6KN/m)/4 = 2.4KN/m$

Load from Wearing Surface $(Lw) = A_W * \gamma_W = (0.1m * b_{eff}) * 22.5KN/m^3$

Effective Flange Width for Exterior Girders

$$b_{eff} = min\{((b_{eff}(interior)/2) + min(L/8, 6t_S + 0.5t_W)), b_{ov}\}$$

Where b_{ov} is overhang width

$b_{eff} = min\{(2.11m/2) + min(25m/8, (6 * 0.2m) + (0.5 * 0.47m)), b_{ov})$

$b_{eff} = min\,\{(1.05m + 1.435), 1.28m\}$

$b_{eff} = 1.28m$

$Lw\ = A_W *\ \gamma_W =\ (0.1m * 1.28m) * 22.5KN/m^3$

$= 2.88KN/m$

Total Exterior Girder Load $(DCe)\ = Lg + Lrp + Lcb$

$= 28.18KN/m + 0.5775KN/m + 2.4KN/m$

$= 31.152KN/m$

Total Exterior Girder Load $(Dwe)\ = Lw$

$= 2.88KN/m$

Concentrated Exterior Diaphragm Load $(Ld)\ = 4.43KN$

Concentrated End Diaphragm Load $(Lde)\ = 2.955KN$

Interior Girder Dead Load Shear Force and Bending Moment Calculation

Shear Force

$VDC =\ V(DCi) + V(Ld) + V(Lde)$

$= wL/2 - wa - 8.86$

$= (31.152 * 25/2) - (31.152a) - 8.86$

$= 380.54 - 31.152a$

$VDW = V(DWe)$

$=\ (wL/2) - wa$

$= (4.748KN/m * 25m/2) - (4.748KN/m * a)$

$= 59.35 - 4.748a$

Bending Moment

$MDC =\ M(DCi) + M(Ld) + M(Lde)$

$= wa(L - a)/2 + P2 * a + 0.25P1$

$= 31.152a(25 - a)$

$= 15.576a(25 - a) + 8.86a + 1.478$

$MDW = M(DWe)$

$= wa * (L - a)/2$

$= 4.748a(25 - a)/2$

$= 2.374a(25 - a)$

Exterior Girder Dead Load Shear Force and Bending Moment Calculation

Shear Force

$VDC = V(DCi) + V(Ld) + V(Lde)$

$= (wL/2) - wa - P2$

$= (31.152KN/m * 25m/2) - (31.152KN/m * a) - 4.43KN$

$= 385.1 - 31.152a$

$VDW = V(DWe)$

$= (wL/2) - wa$

$= (2.88KN/m * 25/2) - (2.88KN/m * a)$

$= 36 - 2.88a$

Bending Moment

$MDC = M(DCi) + M(Ld) + M(Lde)$

$= wa(L-a)/2 + P2 * a + 0.25P1$

$= (31.152KN/m * a(25m - a)/2) + (4.43KN * a) + (0.25 * 2.96KN)$

$= 15.576a(25 - a) + 4.43a + 0.74$

$MDW = M(DWe)$

$= wa(L-a)/2$

$= (2.88KN/m * a) * (25m - a)/2$

$= 1.44a(25 - a)$

Girder Live loading

Truck and Lane Load Shear Force at a Section

From Eq. [5-1a] & [5-1b]

$V_{trl} = (P_1/L) * ((L-a) + (L - a - 4.3m) + K * (L - a - 8.6m))$

$= (145KN/m/25m) * ((25m - a) + (25m - a - 4.3m) + 7/29(25m - a - 8.6m)$

$= 288 - 12.998a$

$VL = 0.5 * (L-a)^2 * U/L$

$= 0.5 * (25m - a)^2 * (3KN/m)/25m$

$= 3(25 - a)^2/50$

Truck and Lane Load Bending Moment at a Section

From Eq. [5-2a] & [5-2b]

$M_{trl} = P_1 * (a/L) * ((L-a) + (L - a - 4.3m) + K * (L - a - 8.6m))$

$= 145 * (a/25m) * ((25 - a) + (25m - a - 4.3m) + 7/29(25m - a - 8.6m))$

$= 5.8a(49.658 - 2.241a)$

$= 288a - 12.998a^2$

$ML = 0.5 * a\ (L - a)^2 U/L$

$= 0.5 * a(25m - a)^2 * (3KN/m)/25m$

$= 3a(25 - a)^2/50$

Tandem and Lane Load Shear Force at a Section

From Eq. [5-5a] & [5-5b]

$Vtml = (P_3/L) * ((L - a) + (L - a - 1.2))$

$= (110KN/25m) * ((25m - a) + (25m - a - 1.2))$

$= 4.4(48.8 - 2a)$

$= 214.72 - 8.8a$

$VL = 0.5 * (L - a)^2 U/L$

$= 0.5 * (25m - a)^2 * (3KN/m)/25m$

$= 3(25 - a)^2/50$

Tandem and Lane Load Bending Moment at a Section

From Eq. [5-6a] & [5-6b]

$M_{tml} = P_3 * (a/L) * ((L - a) + (L - a - 1.2))$

$= 110KN * (a/25m) * ((25m - a) + (25m - a - 1.2m))$

$= 214.72a - 8.8a^2$

$ML = 0.5 * a(L - a)^2 U/L$

$= 0.5 * a(25m - a)^2 * (3KN/m)/25m$

$= 3a(25 - a)^2/50$

The live load interior and exterior girder shear force and bending moment shall be multiplied by the distribution factors obtained as in live load distribution factors and the dynamic load allowance.

Note: We do not apply dynamic load allowance for lane loading.

Interior Girder

Shear

$V_{trl} = (288 - 12.998a) * 0.83 * 1.2$

$= 286.848 - 12.946a$

$V_{tml} = (214.72 - 8.8a) * 0.83 * 1.2$

$= 213.861 - 8.765a$

$VL = [3(25 - a)^2/50] * 0.83$

$= 0.05(25 - a)^2$

Moment

$M_{trl} = (288a - 12.998a^2) * 0.944 * 1.2$

$= 326.246a - 14.724a^2$

$M_{tml} = (214.72a - 8.8a^2) * 0.944 * 1.2$

$= 243.235a - 9.969a^2$

$ML = [3a(25 - a)^2/50] * 0.944$

$= 0.0566a(25 - a)^2$

Exterior Girder

Shear

$V_{trl} = (288 - 12.998a) * 1.024 * 1.2$

$= 353.894 - 15.972a$

$V_{tml} = (214.72 - 8.8a) * 1.024 * 1.2$

$= 263.848 - 10.813a$

$VL = [3(25 - a)^2/50] * 1.024$

$= 0.0614(25 - a)^2$

Moment

$M_{trl} = (288a - 12.998a^2) * 1.024 * 1.2$

$= 353.894a - 15.972a^2$

$Mtml = (214.72a - 8.8a^2) * 1.024 * 1.2$

$= 263.848a - 10.813a^2$

$ML = [3a(25 - a)^2/50] * 1.024$

$= 0.0614(25 - a)^2$

Rating of Interior Girder

Determination of angle θ

Shear at Critical Section

The critical section near the supports is the greater of $0.5\, d_v \cot\theta$ and d_v,

$$d_v = d_e - a/2 \geq 0.9d_e, 0.72h$$

Where,

$d_e = h - y_b$, is the effective depth measured from the extreme compression fiber to the centroid of the tensile force

θ is the angle of inclination of diagonal compressive stress.

$h = 1.7m, yb = 0.29m$

$de = h – yb = 1.7m - 0.29m = 1.41m$

$d_v = d_e - a/2 \geq 0.9d_e ,0.72h$

$= 1.41m - 0.01m > 0.9 * 1.41m, 0.72 * 1.7m$

$= 1.4m$

The calculation procedure assumes a value for θ and solves to verify the assumption. Iteration is used until a solution is found.

Assume $\theta = 40^0$ degrees.

Dead Load Moments at distance a:

$MDC = 15.576a(25 - a) + 8.86a + 1.478$

$MDW = 2.374a(25 - a)$

Dead Load shear at distance a:

$VDC = 380.54 - 31.152a$

$VDW = 59.35 - 4.748a$

Live Load Moment at distance a:

Applying impact factor and distribution factor, the live loads are

$M_{trl} = 326.246a - 14.724a^2$

$M_{tml} = 243.235a - 9.969a^2$

$ML\ (No\ dynamic\ allowance) = 0.053(25 - a)^2$

$MTL = Max\ [(Mtrl + ML), (Mtml + ML)]$

Live Load Shear at distance a:

$Vtrl = 286.848 - 12.946a$

$Vtml = 213.861 - 8.765a$

$VL(No\ dynamic\ allowance) = 0.05(25 - a)^2$

$VTL = Max\ [(Vtrl + VL), (Vtml + VL)]$

$Mu = 1.25MDC + 1.5MDW + 1.75MTL$

$Vu = 1.25VDC + 1.5VDW + 1.75VTL$

Check minimum amount of transverse reinforcement using,

$$Avfy/bws > 0.06\sqrt{fc'} \quad (SI\ Units)$$

$(\pi * 12^2/4) * 300/470 * 150 > 0.06\sqrt{24}$

$0.481 > 0.294$

Minimum amount of transverse reinforcement condition is satisfied; determine average shear stress, v.

$v = V_u/\phi b_v d_v$

$\varepsilon_x = [(M_u/d_v) + (0.5N_u) + (0.5VUcot\ \theta)]/ 2(E_S A_S) < 0.002$

$E_S = 200000Mpa = 200 * 106KN/m^2$

$A_S = 314mm^2 * 14 = 4.396 * 10^{-3}m^2$

If ε_x is negative, it shall be multiplied by the following factor:

$F\varepsilon = (E_S A_S)/ [(E_C A_C) + (E_S A_S)]$

The values are positive, no need of multiplication by the factor.

From Table 4-3 with s_x and$1000\varepsilon_x$;

After a number of iterations, It is found that $\theta = 37.194^0$ and $\beta = 2.102$

The critical shear section is at ‘a’ distance from face of support, $Max\ \{d_v, 0.5d_v cot\theta\}$

From iteration, $a = 1.846m$

The shear effects are recalculated based on the controlling section

Recalculated shear values at Critical Section

Shear due to dead load $(VDC) = 315.243KN$

Shear due to dead load $(VDW) = 49.398KN$

Shear due to live load $(VTL) = 285.942KN$

Calculate Shear Strength Capacity

a) **Using Modified Compression Theory (MCFT)**

$V_C(ERA) = 0.083\beta\sqrt{f_c'}\ b_v d_v, V_C\ (SI\ Units)$

$b_v = 0.37m, d_v = 1.40m\ and\ f_C' = 24Mpa$

$V_C = 0.083 * 2.102\sqrt{24} * 370 * 1400$

$= 442.737$

$V_S = A_v f_y d_v cot\ \theta/s$

Stirrup spacing$(s) = 140mm$

$A_v = \pi D^2/4$

$= \pi * 122/4$

$= 113.04mm2$

$Vs = 113.04 * 300 * 1400 * cot(37.194^0)/140mm$

$= 447.177KN$

$V_n = V_C + V_S < 0.25f_C' b_v d_v$

$V_c + V_s = 442.737KN + 447.177KN$

$= 889.914KN$

$0.25f_c'b_vd_v = 0.25 * 24000KN/m^2 * 0.37m * 1.4m$

$= 3108KN$

Therefore,

$R_n = V_n = 889.914KN$

b) Using 45^0 Truss Model

$Vc = 0.1667\sqrt{f_c'}\ b_wd$

$= 0.1667 * \sqrt{24} * 370 * 1400$

$= 423.03KN$

$V_S = A_vf_yd/s$

$= 113.04 * 300 * 1400/140$

$= 339.12KN$

$Vn = Vc + Vs = 423.03KN + 339.12KN$

$= 762.15KN$

LRFR Load Rating Results

Strength – Shear

At Critical Section:

Shear force capacity $(R_n) = 889.914KN$

Shear due to dead load $(VDC) = 315.243KN$

Shear due to dead load $(VDW) = 49.398KN$

Shear due to live load $(VTL) = 285.942KN$

Inventory:

$RF_{Inv} = [\varphi c\varphi s\varphi\ R_n - \Upsilon_{DC}DC - \Upsilon_{DW}DW]/\Upsilon_L LL(1 + IM)$

$= [(0.9 * 1.0 * 1.0 * 889.914) - (1.25 * 315.24) - (1.5 * 49.40)]/(1.75 * 285.94)$

$= 0.665$

Operating:

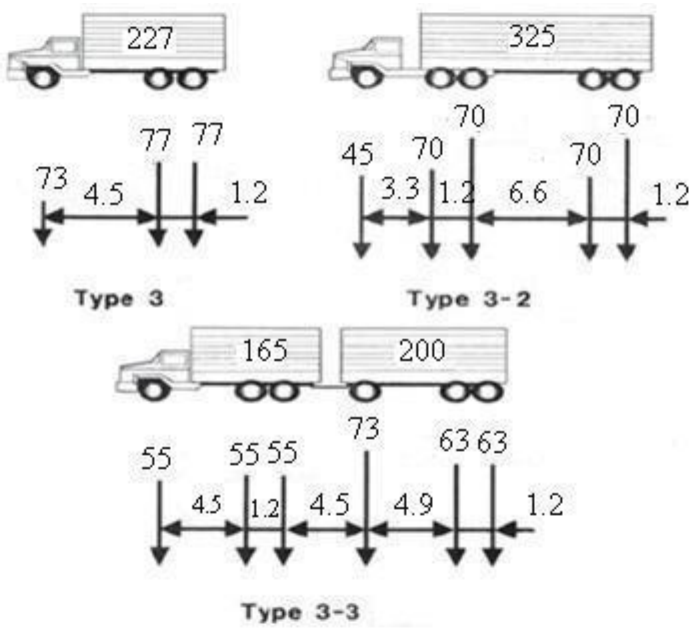
$RF_{Op} = [\varphi c\varphi s\varphi\ R_n - \Upsilon_{DC}DC - \Upsilon_{DW}DW]/\Upsilon_L LL(1 + IM)$

$= [(0.9 * 1.0 * 1.0 * 889.914) - (1.25 * 315.24) - (1.5 * 49.40)]/(1.35 * 285.94)$

$= 0.862$

The inventory and operating level rating factors are less than unity. So we need to check for the legal loads.

Rating of Bridge Girder for ERA Legal Loads



ERA Legal Loads

Loading for Type 3 Legal Load

Shear Force and Bending Moment Determination

Shear Force

$m_1 = (L - a)/L = (25 - a)/25$

$m_2 = (L - a - 1.2)/25 = (23.8 - a)/25$

$m_3 = (L - a - 5.7)/25 = (19.3 - a)/25$

$V_{T3} = (p_1 m_1 + p_2 m_2 + p_3 m_3)$

$= 77(25 - a)/25 + 77(23.8 - a)/25 + 73(19.3 - a)/25$

$= (1925 - 77a + 1832.6 - 77a + 1408.9 - 73a)/25$

$= 206.66 - 9.08a$

By applying live load distribution factor and dynamic load allowance,

$V_{T3} = (206.66 - 9.08a) * 1.2 * 0.83$

$= 205.833 - 9.044a$

Bending Moment

$M_{T3} = 206.66a - 9.08a^2$

By applying live load distribution factor and dynamic load allowance,

$M_{T3} = (206.66a - 9.08a^2) * 1.2 * 0.944$

$= 234.104a - 10.286a^2$

In the same manner as before,

The angle is found as $\theta = 33.297^0$ and $\beta = 2.34$

Recalculated shear values at Critical Section

Shear due to dead load $(VDC) = 306.308KN$

Shear due to dead load $(VDW) = 48.036KN$

Shear due to live load $(VTL) = 209.859KN$

Calculate Shear Strength Capacity

$V_C(ERA) = 0.083\beta\sqrt{f_c'}\, b_v d_v, V_C\ (SI\ Units)$

$b_v = 0.37m, d_v = 1.40m,\ f_C' = 24Mpa$

$V_C = 0.083 * 2.34\sqrt{24} * 370 * 1400$

$= 492.867KN$

$V_S = A_v f_y d_v cot\ \theta/s$

Stirrup spacing$(s) = 140mm$

$A_v = \pi D^2/4$

$= \pi * 12^2/4$

$= 113.04mm^2$

$V_S = 113.04 * 300 * 1400 * cot(33.297^0)/140mm$

$= 516.65KN$

$V_n = V_C + V_S < 0.25 f_C' b_v d_v$

$Vc + Vs = 520.248KN + 516.65KN$

$= 1036.898KN$

$0.25 f_C' b_v d_v = 0.25 * 24000KN/m2 * 0.37m * 1.4m$

$= 3108KN$

Therefore,

$R_n = V_n = 1036.898KN$

LRFR Shear Rating Result

At Critical Section:

Shear force capacity $(R_n) = 1036.898KN$

Shear due to dead load $(VDC) = 306.308KN$

Shear due to dead load $(VDW) = 48.036KN$

Shear due to live load $(VTL) = 209.859KN$

$RF = [\varphi c \varphi s \varphi\ R_n - \Upsilon_{DC} DC - \Upsilon_{DW} DW]/\Upsilon_L LL(1+IM)$

$= [(0.9 * 1.0 * 1.0 * 1036.898) - (1.25 * 306.31) - (1.5 * 48.04)]/(1.65 * 209.86)$

$= 1.381$

Loading for Type 3-2 Legal Load

Shear Force and Bending Moment Determination

Shear Force

$m_1 = (L-a)/L = (25-a)/25$

$m_2 = (L-a-1.2)/L = (23.8-a)/25$

$m_3 = (L-a-7.8)/L = (17.2-a)/25$

$m_4 = (L-a-9.0)/L = (16.0-a)/25$

$m_5 = (L-a-12.3)/L = (12.7-a)/25$

$V_{T3\text{-}2} = (p_1 m_1 + p_2 m_2 + p_3 m_3 + p_4 m_4 + p_5 m_5)$

$=70(25-a)/25+70(23.8-a)/25+70(17.2-a)/25+70(16-a)/25+45(12.7-a)/25$

$= (1750-70a+1666-70a+1204-70a+1120-70a+571.5-45a)/25$

$=252.46-13a$

By applying live load distribution factor and dynamic load allowance,

$V_{T3\text{-}2} = (252.46 - 13a) * 1.2 * 0.83$

$= 251.450 - 12.948a$

Bending Moment

$M_{T3\text{-}2} = 252.46a - 13a^2$

Similarly,

$M_{T3\text{-}2} = (252.46a - 13a^2) * 1.2 * 0.944$

$= 285.987a - 14.726a^2$

In the same manner as before,

The angle is found as $\theta = 33.539^0$ and $\beta = 2.333$

Recalculated shear values at Critical Section

Shear due to dead load (VDC) $= 306.915KN$

Shear due to dead load (VDW) $= 48.129KN$

Shear due to live load (VTL) $= 246.469KN$

Calculate Shear Strength Capacity

$V_c(ERA) = 0.083\beta\sqrt{f_c'}\ b_v d_v, V_c\ (SI\ Units)$

$b_v = 0.37m, d_v = 1.40m, fc' = 24Mpa$

$V_c = 0.083 * 2.333\sqrt{24} * 370 * 1400$

$= 491.393KN$

$V_S = A_v f_y d_v cot\,\theta/s$

Stirrup spacing$(s) = 140mm$

$A_v = \pi D^2/4$

$= \pi * 12^2/4$

$= 113.04mm^2$

$V_S = 113.04 * 300 * 1400 * cot(33.539^0)/140$

$= 511.928KN$

$V_n = V_c + V_S < 0.25f_c' b_v d_v$

$V_c + V_S = 491.393KN + 511.928KN$

$= 1003.321KN$

$0.25f_c' b_v d_v = 0.25 * 24000KN/m^2 * 0.37m * 1.4m$

$= 3108KN$

Therefore,

$R_n = V_n = 1003.321KN$

LRFR Load Rating Result

At Critical Section:

Shear force capacity $(Rn) = 1003.321KN$

Shear due to dead load $(VDC) = 306.915KN$

Shear due to dead load $(VDW) = 48.129KN$

Shear due to live load $(VTL) = 246.469$

$RF = [\varphi c \varphi s \varphi\, R_n - \Upsilon_{DC}DC - \Upsilon_{DW}DW]/\Upsilon_L LL(1 + IM)$

$= [(0.9 * 1.0 * 1.0 * 1003.32) - (1.25 * 306.92) - (1.5 * 48.13)]/(1.65 * 246.47)$

$= 1.10$

Loading for Type 3-3 Legal Load

Shear Force and Bending Moment Determination

Shear Force

$m_1 = (L - a)/L = (25 - a)/25$

$m_2 = (L - a - 1.2)/L = (23.8 - a)/25$

$m_3 = (L - a - 6.1)/L = (18.9 - a)/25$

$m_4 = (L - a - 10.6)/L = (14.4 - a)/25$

$m_5 = (L - a - 11.8)/L = (13.2 - a)/25$

$m_6 = (L - a - 16.3)/L = (8.7 - a)/25$

$V_{T3\text{-}3} = (p_1m_1 + p_2m_2 + p_3m_3 + p_4m_4 + p_5m_5 + p_6m_6)$

=63(25-a)/25+63(23.8-a)/25+73(18.9-a)/25+55(14.4-a)/25+55(13.2-a)/25+55(8.7-a)/25

$= 258.024 - 14.56a$

Bending Moment

$M_{T3\text{-}3} = 258.024a - 14.56a^2$

Applying live load distribution factor and dynamic load allowance;

$M_{T3\text{-}3} = (258.024a - 14.56a^2) * 1.2 * 0.944$

$= 292.290a - 16.494a^2$

$V_{T3\text{-}3} = (258.024 - 14.56a) * 1.2 * 0.83$

$= 256.992 - 14.502a$

In the same manner as before,

The angle is found as $\theta = 33.58^0$ and $\beta = 2.332$

Recalculated shear values at Critical Section

Shear due to dead load $(VDC) = 312.872KN$

Shear due to dead load $(VDW) = 48.143KN$

Shear due to live load $(VTL) = 248.391KN$

Calculate Shear Strength Capacity

$V_c(ERA) = 0.083\beta\sqrt{f_c'}\, b_v d_v, V_c\ (SI\ Units)$

$b_v = 0.37m, d_v = 1.40m, f_c' = 24Mpa$

$V_c = 0.083 * 2.332\sqrt{24} * 370 * 1400$

$= 491.182KN$

$V_s = A_v f_y d_v cot\,\theta/s$

Stirrup spacing$(s) = 140mm$

$Av = \pi D^2/4$

$= (\pi * 12^2/4)$

$= 113.04mm^2$

$Vs = 113.04 * 300 * 1400 * cot(33.578^0)/140$

$= 511.174KN$

$V_n = V_c + V_s < 0.25 f_c' b_v d_v$

$V_c + V_s = 491.182KN + 511.174KN$

$= 1002.356KN$

$0.25 f_c' b_v d_v = 0.25 * 24000KN/m^2 * 0.37m * 1.4m$

$= 3108KN$

Therefore,

$R_n = V_n = 1002.356KN$

LRFR Load Rating Result

At Critical Section:

Shear force capacity $(R_n) = 1002.356KN$

Shear due to dead load $(VDC) = 312.872KN$

Shear due to dead load $(VDW) = 48.143KN$

Shear due to live load $(VTL) = 248.391KN$

$RF = [\phi_c \phi_s \phi\, R_n - \Upsilon_{DC} DC - \Upsilon_{DW} DW]/\Upsilon_L LL(1 + IM)$

$= [(0.9 * 1.0 * 1.0 * 1002.36) - (1.25 * 312.87) - (1.5 * 48.14)]/(1.65 * 248.39)$

$= 1.07$

The design load rating results above shows that it does not fulfil the criteria for design loading but it almost fulfils the requirement to resist the ERA legal loading.

6.3. Bridge Girder 2 LRFR Analysis and Rating

Similar to the above rated bridge girder, this bridge girder is constructed by Amhara Roads Authority and also this one is T-girder with span length of 16m. It has the same material property as the above bridge girder.

Design Specifications

1. ERA Bridge Design Manual 2002
2. AASHTO Standard Specifications for Highway Bridges, 1998

Bridge Loadings

Dead Loads

Dead loads are based on the following loadings

Super Structure Loads

Reinforced concrete: 24KN/m^3

Wearing Surface, Asphalt: 22.5KN/m^3

Live Loading

The live loading is AASHTO HL-93

Type of Girder

The type of girder cross-section from those in Table 5-12 is Type e, i.e. Cast –in-place concrete T-beam with monolithic concrete deck.

Input Parameters:

Bridge Geometry

Bridge Width = 7.3m

Number of Girders = 4

Left overhang = 1.15m

Girder spacing = 2.2m

Slab Thickness= 0.2m

Wearing surface = 0.1m

Girder web width= 0.38m

Girder height= 1.2m

Left curb width = 0.8m

Right curb width = 0.8m

Curb Height= 0.5m

Left skew angle = 0^0

Right skew angle = 0^0

Load Factors

Resistance Factor (ϕ) = 0.9

Condition Factor (ϕ_C) = 1.0

System Factor (ϕ_S) = 1.0

Dead Load Factors

Strength I limit State

Dead load Factor (γ_{DC}) = 1.25

Dead Load Factor (γ_{DW}) = 1.50

Design Live Load Factors

Strength I Limit State

Live Load Factor (γ_{LL}) (Inventory) = 1.75

Live Load Factor (γ_{LL}) (Operating) = 1.35

Dynamic Load Allowance

Dynamic load allowance = 20%

Live Load Distribution Factors

Moment for Interior Bridge Girders

One Design Lane Loaded:

$g_{interior} = 0.06 + (S/4300)^{0.4}(S/L)^{0.3}(kg//Lt_s{}^3)^{0.1}$

$= 0.06 + (2200/4300)^{0.4}(2200/16000)^{0.3}(kg/(16000 * 2003))^{0.1}$

$= 0.537$

Two or More Design Lanes Loaded:

$g_{interior} = 0.075 + (S/4300)^{0.6}(S/L)^{0.2}(k_g//Lt_s{}^3)^{0.1}$

$= 0.075 + (2110/4300)^{0.6}(2110/25000)^{0.2}(k_g/(25000 * 200^3))^{0.1}$

$= 0.505$

Where the value of Kg is as calculated below

$K_g = n\,(I + A\,e_g{}^2)$ in which: $n = EB/\,ED = 1$

Effective flange width for interior girders

$b_{eff} = min\,\{1/4L, 12ts + max\,(t_W, 0.5w_f), S_{avg}\}$

$b_{eff} = min\{16m/4, (12 * 0.2m) + max(0.38m, 0.5 * 2.2m), 2.2m\}$

$b_{eff} = 2.2m$

$A = b_{eff} * t_s = 2.2m * 0.2m = 0.44m^2$

$I = bh^3/12 = 0.38 * 1.0^3/12 = 0.032m^4$

$e_g = 0.6m$

$K_g = n\,(I + A\,e_g{}^2)$

$= 1(0.032 + 0.44 * 0.60^2)$

$= 0.1904m^4$

Therefore,

Interior girder live load moment distribution factor will be the max of the two values.

$M_{gint} = max\,(0.537, 0.505)$

$= 0.537$

Moment for Exterior Bridge Girders

One Design Lane Loaded:

Lever Rule

$d_e = 0.61m = (0.61/0.3048)ft = 2.00ft$

$S = 2.2m = 7.218ft$

$d_e + S = 2.00ft + 7.218ft = 9.218ft$

$d_e + S > 6ft$

Therefore, the distribution factor lever rule equation will be:

$DF = 1 + (d_e/S) - (3/S) = 1 + (2.00ft/7.218ft) - (3ft/7.218ft)$

$= 0.861$

When using Lever rule we shall consider multiple presence factor; for single lane loaded the multiple presence factor is 1.2.

$DF = 0.861 * 1.2 = 1.03$

Two or More Design Lanes Loaded:

$g = e * g_{interior}$

$e = 0.77 + de/2800, de = 610$

$e = 0.77 + 610/2800$

$= 0.988$

$g = 0.988 * 0.537$

$= 0.531$

Therefore,

Exterior girder live load moment distribution factor will be the max of the two values.

$M_{gext} = max\,(1.03, 0.531)$

$= 1.03$

Check for Diaphragm Contribution

$NL \;\; Nb$

$R = [NL/Nb] + [Xext\sum e/\sum x^2]$

For One Lane Loaded

$NL = 1, Nb = 4, Xext = 3.30m, \sum e = 2.45m$

$\sum x^2 = (x_1{}^2 + x_2{}^2 + x_3{}^2 + x_4{}^2)$

$= [(3.30m)^2 + (1.10m)^2 + (1.10m)^2 + (3.30m)^2]$

$= 24.20m^2$

$NL \; Nb$

$R = [NL/Nb] + [X_{ext}\sum e/\sum x^2]$

$= (1/4) + [(3.30m * 2.45m)/24.2m^2]$

$= 0.584$

For Two Lanes Loaded

Only the Values for the terms NL and $\sum e$ vary.

$NL = 2$

$\sum e = -1.25 + 2.45 = 1.2$

$NL \; Nb$

$R = [NL/Nb] + [X_{ext}\sum e/\sum x^2]$

$= (2/4) + [(3.30m * 1.20m)/24.20m^2]$

$= 0.663$

Check for Skew-ness

Skewness angle $\theta = 0^0$

Skewness Correction Factor= 1

Shear for Interior Bridge Girders

One Design Lane Loaded:

$DF = 0.36 + S/7600 = 0.36 + 2200/7600$

$= 0.65$

Two or More Design Lanes Loaded:

$DF = 0.2 + (S/3600) - (S/10700)^{2.0}$

$= 0.2 + (2200/3600) - (2200/10700)^{2.0}$

$= 0.769$

Therefore,

Interior girder live load shear distribution factor will be the max of the two values.

$V_{gint} = max\,(0.65, 0.769)$

$= 0.769$

Shear for Exterior Bridge Girders

One Design Lane Loaded:

Lever Rule

$d_e = 0.61m = (0.61/0.3048)ft = 2.0ft$

$S = 2.11m = 7.218ft$

$d_e + S = 2.0ft + 7.218ft = 9.218ft$

$d_e + S > 6ft$

Therefore, the distribution factor lever rule equation will be:

$DF = 1 + (d_e/S) - (3/S) = 1 + (2.0/7.218) - (3/7.218)$

$= 0.861$

Similarly applying multiple presence factor,

$DF = 0.861 * 1.2 = 1.03$

Two or More Design Lanes Loaded:

$g = e * g_{interior}$

$e = 0.6 + d_e/3000$

$e = 0.6 + 610/3000 = 0.803$

$g = 0.803 * g_{interior} = 0.803 * 0.853 = 0.685$

Therefore,

Exterior girder live load shear distribution factor will be the max of the three values.

$V_{gext} = max\ (1.03, 0.769, 0.663)$

$= 1.03$

Check for Skewness

Skewness Correction Factor (for $\theta = 0^0$) $= 1$

Therefore,

No correction for skewness

Dead Load Calculation

Interior Girder Loading

Girder Load $(Lg) = A_g * \gamma_g = [(2.20m * 0.2m) + (0.38m * 1.0m)] * 24KN/m^3$

$= 19.680KN/m$

Railing and Post Load(Lrp):

Loads from railings and post can be distributed equally to all girders

Railing Load$= A_r * \gamma_r = 0.2m * 0.4m * 24KN/m^3$

$= 1.92KN/m$

Post Load

$= ((4 * 0.3m) + (2 * 0.5m)) * ((0.25m * 0.5m) + (0.15m * 0.4m)) * 24KN/m^3$

$= 9.768KN/25m$

$= 0.39KN/m$

Therefore,

$\text{Lrp} = 1.92KN/m + 0.391KN/m$

$= 2.31KN/m$

Lrp for each girder = $(2.31KN/m)/4 = 0.578KN/m$

Diaphragm Load (Ld)

Diaphragm Load will be applied on girders as concentrated load on the points where the center lines of the girder and diaphragm meet.

For the three diaphragms

$$Ld = A_d * \gamma_d * length\ of\ diaphragms\ between\ faces\ of\ girders\ (l_d)$$

$$= A_d * \gamma_d * l_d$$

Height of diaphragm = 0.9m

Thickness of diaphragm = 0.25m

Length of diaphragm (ld) = 1.82m

$$Ld = A_d * \gamma_d * l_d$$

$$= 0.9m * 0.25m * 1.82m * 24KN/m^3$$

$$= 9.828KN$$

Curb Load (Lcb) $= A_{cb} * \gamma_{cb} = (0.5m * 0.8m) * 24KN/m^3$

$= 9.6KN/m$

The load will be distributed equally to all the girders

Lcb/Girder = $9.6KN/m/4 = 2.4KN/m$

Load from Wearing Surface $(Lw) = A_w * \gamma_w = (0.1m * 2.20m) * 22.5KN/m^3$

$= 4.95KN/m$

Total Interior Girder Load $(Dci) = Lg + Lrp + Lcb$

$= 19.680KN/m + 0.578KN/m + 2.4KN/m$

$= 22.658KN$

Total Interior Girder Load $(Dwi) = Lw$

$= 4.95KN/m$

Concentrated Diaphragm Load $(Ld) = 9.828KN$

Exterior Girder Loading

Girder Load $(Lg) = A_g * \gamma_g = [(2.20m * 0.2m) + (0.38m * 1.0m)] * 24KN/m^3$

$= 19.680KN/m$

Railing and Post Load (Lrp):

As calculated for interior girders,

Lrp/girder = 0.578KN/m

Diaphragm Load (Ld):

The diaphragm load calculation for the exterior girders is the same as of interior girders. But we will take only half of the load for interior girders.

For the diaphragms

$$Ld = (A_d * \gamma_d * l_d)/2$$
$$= (0.9m * 0.25m * 1.82m * 24KN/m^3)/2$$
$$= 4.914KN$$

Curb Load (Lcb) $= A_{cb} * \gamma_{cb} = (0.5m * 0.8m) * 24KN/m^3$

$= 9.6KN/m$

The load will be distributed equally to all the girders

Lcb/Girder $= (9.6KN/m)/4 = 2.4KN/m$

Load from Wearing Surface (Lw) $= A_W * \gamma_W = (0.1m * b_{eff}) * 22.5KN/m^3$

Effective Flange Width for Exterior Girders

$b_{eff} = min\{((beff(interior)/2) + min(L/8, 6t_s + 0.5t_w)), b_{ov}\}$

Where b_{ov} is overhang width

$b_{eff} = min\{(2.20m/2) + min(16m/8, (6 * 0.2m) + (0.5 * 0.38m)), b_{ov})$

$b_{eff} = min\{(1.10m + 1.39m), 1.15m\}$

$b_{eff} = 1.15m$

$Lw = A_W * \gamma_W = (0.1m * 1.15m) * 22.5KN/m^3$

$= 2.59KN/m$

Total Exterior Girder Load (DCe) $= Lg + Lrp + Lcb$

$= 19.680KN/m + 0.578KN/m + 2.4KN/m$

$= 22.658KN/m$

Total Exterior Girder Load (Dwe) $= Lw$

$= 2.59KN/m$

Concentrated Diaphragm Load (Ld) $= 4.914KN$

Interior Girder Dead Load Shear Force and Bending Moment Calculation

Shear Force

$VDC = V(DCi) + V(Ld)$

$= wL/2 - wa + 0.5P$

$= (22.658 * 16m/2) - (22.658KN/m * a) + 0.5 * 9.83KN$

$= 186.179 - 22.658a$

$VDW = V(DWe)$

$= (wL/2) - wa$

$= (4.95KN/m * 16m/2) - (4.95KN/m * a)$

$= 39.60 - 4.95a$

Bending Moment

$MDC = M(DCi) + M(Ld)$

$= wa(L - a)/2 + 0.5P * a$

$= ((22.658KN/m * a/2) * (16m - a)) + (0.5 * 9.828KN * a)$

$= 11.329a(16 - a) + 4.914a$

$MDW = M(DWe)$

$= wa(L - a)/2$

$= (4.95KN/m * a) * (16m - a)/2$

$= 2.475a(16 - a)$

Exterior Girder Dead Load Shear Force and Bending Moment Calculation

Shear Force

$VDC = V(DCi) + V(Ld)$

$= wL/2 - wa + 0.5P$

$= (22.658KN/m * 16m/2) - (22.658KN/m * a) + (0.5 * 4.914KN)$

$= 183.721 - 22.658a$

$VDW = V(DWe)$

$= wL/2 - wa$

$= (2.59KN/m * 16m/2) - (2.59KN/m * a)$

$= 20.72 - 2.59a$

Bending Moment

$MDC = M(DCi) + M(Ld)$

$= wa/2(L - a) + 0.5P * a$

$= (22.658KN/m * a(16m - a)/2) + (0.5 * 4.914KN * a)$

$= 11.329a(16 - a) + 2.457a$

$MDW = M(DWe)$

$= wa(L - a)/2$

$= (2.59KN/m * a) * (16m - a)/2$

$= 1.295a(16 - a)$

Girder Live loading

Truck and Lane Load Shear Force at a Section

From Eq. [5-1a] & [5-1b]

$V_{trl} = (P_1/L) * ((L-a) + (L-a-4.3m) + K * (L-a-8.6m))$

$= (145KN/m/16m) * ((16m-a) + (16m-a-4.3m) + 7/29(16m-a-8.6m)$

$= 267.217 - 20.31a$

$VL = 0.5 * (L-a)^2 * U/L$

$= 0.5 * (16m-a)^2 * (3KN/m)/16m$

$= 0.09375(16-a)^2$

Truck and Lane Load Bending Moment at a Section

From Eq. [5-2a] & [5-2b]

$M_{trl} = P_1 * (a/L) * ((L-a) + (L-a-4.3m) + K * (L-a-8.6m))$

$= 145 * (a/16m) * ((16-a) + (16m-a-4.3m) + 7/29(16m-a-8.6m))$

$= 267.217a - 20.31a^2$

$ML = 0.5 * a\,(L-a)^2 U/L$

$= 0.5 * a(25m-a)^2 * (3KN/m)/25m$

$= 0.09375a(16-a)^2$

Tandem and Lane Load Shear Force at a Section

From Eq. [5-5a] & [5-5b]

$V_{tml} = (P_3/L) * ((L-a) + (L-a-1.2))$

$= (110KN/16m) * ((16m-a) + (16m-a-1.2))$

$= 211.75 - 13.75a$

$VL = 0.5 * (L-a)^2 U/L$

$= 0.5 * (16m-a)^2 * (3KN/m)/16m$

$= 0.09375(16-a)^2$

Tandem and Lane Load Bending Moment at a Section

From Eq. [5-6a] & [5-6b]

$M_{tml} = P_3 * (a/L) * ((L-a) + (L-a-1.2))$

$= 110KN * (a/16m) * ((16m-a) + (16m-a-1.2m))$

$= 211.75a - 13.75a^2$

$ML = 0.5 * a(L-a)^2 U/L$

$= 0.5 * a(16m-a)^2 * (3KN/m)/16m$

$= 0.09375a(16 - \text{a})^2$

The live load interior and exterior girder shear force and bending moment shall be multiplied by the distribution factors obtained in live load distribution factors calculation and the dynamic load allowance.

Note: We do not apply dynamic load allowance for lane loading.

Interior Girder

Shear

$Vtrl = (267.217 - 20.31a) * 0.853 * 1.2$

$= 273.523 - 20.789a$

$Vtml = (211.75 - 13.75a) * 0.853 * 1.2$

$= 216.747 - 14.075a$

$VL = 0.09375(16 - a)^2 * 0.853$

$= 0.08(16 - a)^2$

Moment

$Mtrl = (267.217a - 20.31a^2) * 0.537 * 1.2$

$= 172.195a - 13.088a^2$

$Mtml = (211.75a - 13.75a^2) * 0.537 * 1.2$

$= 135.445a - 8.861a^2$

$ML = 0.09375a(16 - a)^2 * 0.537$

$= 0.05(16 - a)^2$

Exterior Girder

Shear

$V_{trl} = (267.217 - 20.31a) * 0.861 * 1.2$

$= 276.089 - 20.984a$

$V_{tml} = (211.75 - 13.75a) * 0.861 * 1.2$

$= 218.78 - 14.207a$

$VL = 0.09375(16 - a)^2 * 0.861$

$= 0.081(16 - a)^2$

Moment

$Mtrl = (267.217a - 20.31a^2) * 1.03 * 1.2$

$= 330.280a - 25.103a^2$

$Mtml = (211.75a - 13.75a^2) * 1.03 * 1.2$

$= 261.723a - 16.995a^2$

$ML = 0.09375a(16 - a)^2 * 1.03 * 1.2$

$= 0.116(16 - a)^2$

Rating of Interior Girder

Determination of angle θ

Shear at Critical Section

The critical section near the supports is the greater of $0.5\ d_v\ cot\theta$ and d_v.

$d_v = d_e - a/2 \geq 0.9d_e, 0.72h$

Where,

d_e is the effective depth measured from the extreme compression fiber to the centroid of the tensile force

θ is the angle of inclination of diagonal compressive stress.

$d_e = h - y_b, h = 1.20m, y_b = 0.214m$

$d_e = h - y_b = 1.20m - 0.214m = 0.986m$

$d_v = d_e - a/2 \geq 0.9d_e, 0.72h$

$= 0.986m - 0.01m > 0.9 * 0.986m, 0.72 * 1.2m$

$= 0.976m$

The calculation procedure assumes a value for θ and solves to verify the assumption. Iteration is used until a solution is found.

Assume $\theta = 40^0$ degrees.

Dead Load Moments at distance a:

$MDC = 11.329a(16 - a) + 4.914a$

$MDW = 2.475a(16 - a)$

Dead Load shear at distance a:

$VDC = 186.179 - 22.658a$

$VDW = 39.60 - 4.95a$

Live Load Moment at distance a:

Applying impact factor and distribution factor, the live load effects are

$Mtrl = 172.195a - 13.088a^2$

$Mtml = 135.445a - 8.861a^2$

$ML\ (No\ dynamic\ allowance) = 0.05(16 - a)^2$

$MTL = Max\ [(Mtrl + ML), (Mtml + ML)]$

Live Load Shear at distance a:

$Vtrl = 273.523 - 20.789a$

$Vtml = 216.747 - 14.075a$

$VL(No\ dynamic\ allowance) = 0.08(16 - a)^2$

$VTL\ =\ Max\ [(Vtrl + VL), (Vtml + VL)]$

$Mu = 1.25MDC + 1.5MDW + 1.75MTL$

$Vu = 1.25VDC + 1.5VDW + 1.75VTL$

Check minimum amount of transverse reinforcement using,

$A_v f_y / b_W s\ > 0.06\sqrt{fc'}$ $(SI\ Units)$

$(\pi * 12^2/4) * 300/370 * 150 > 0.06\sqrt{24}$

$0.6110 > 0.3286$

The condition is satisfied; calculate the average shear stress v.

$v = V_u/\phi b_v d_v$

$\varepsilon_x\ =\ [(M_u/d_v) + (0.5N_u) + (0.5VUcot\ \theta)]/\ 2(E_S A_S\ ) < 0.002$

$E_S = 200000Mpa = 200 * 106KN/m^2$

$A_S\ =\ 314mm^2 * 10\ =\ 3140mm^2\ =\ 3.14 * 10^{-3}m^2$

If ε_x is negative, it shall be multiplied by the following factor:

$F\varepsilon =\ (E_S A_S)/\ [(E_C A_C) + (E_S A_S)]$

The values are positive, no need of multiplication by the factor.

From Table 4-2 or Table 4-3 with $v/\ fc'$ and $1000\varepsilon_x$

After a number of iterations, It is found that $\theta\ =\ 34.412^0$ and $\beta\ =\ 2.365$

The critical shear section is at 'a' distance from face of support, from iteration

$a = 1.426m$

The shear effects are recalculated based on the controlling section

Recalculated shear values at Critical Section

Shear due to dead load $(VDC)\ = 148.211KN$

Shear due to dead load $(VDW)\ = 31.305KN$

Shear due to live load $(VTL)\ = 292.3557KN$

Calculate Shear Strength Capacity

a) Using Modified Compression Field Theory

$V_c(ERA) = 0.083\beta\sqrt{f_c'}\ b_v d_v, Vc\ (SI\ Units)$

$b_v = 0.27m, d_v = 0.976m,\ \ f_c' = 24Mpa$

$V_c(ERA) = 0.083\beta\sqrt{f_c'}\, b_v d_v$

$= 0.083 * 2.365\sqrt{24} * 270 * 976$

$= 253.413KN$

$V_S\ =\ A_v f_y d_v cot\ \theta/s$

Stirrup spacing(s) $= 120mm$

$Av =\ \pi D^2/4$

$= 113.04mm^2$

$Vs =\ 113.04 * 300 * 976 * cot(34.4120)/120$

$=\ 402.904KN$

$V_n\ =\ V_C\ +\ V_S\ <\ 0.25f_c' b_v d_v$

$V_C\ +\ V_S =\ 253.413KN + 402.904KN$

$= 656.317KN$

$0.25f_c' b_v d_v = 0.25 * 24000KN/m^2 * 0.27m * 0.929m$

$= 1504.98KN$

Therefore,

$R_n =\ V_n = 656.317KN$

b) Using 45^0 Truss Model

$Vc\ = 0.1667\sqrt{f_c'}\ b_W d$

$= 0.1667 * \sqrt{24} * 270 * 976$

$= 215.206KN$

$V_S\ = A_v f_y d/s$

$= 113.04 * 300 * 976/120$

$= 275.818KN$

$Vn = Vc + Vs = 215.206KN + 275.818KN$

$= 491.024KN$

LRFR Load Rating Results

At Critical Section:

Shear force capacity (R_n) $=\ 656.317KN$

Shear due to dead load (VDC) $= 148.211KN$

Shear due to dead load (VDW) $= 31.305KN$

Shear due to live load $(VTL) = 292.3557KN$

Inventory:

$RF_{Inv} = [\phi_c\phi_s\phi R_n - \Upsilon_{DC}DC - \Upsilon_{DW}DW]/\Upsilon_L LL(1 + IM)$

$= [(0.9 * 1.0 * 1.0 * 656.317) - (1.25 * 148.21) - (1.5 * 31.31)]/(1.75 * 292.36)$

$= 0.700$

Operating:

$RF_{op} = [\phi_c\phi_s\phi R_n - \Upsilon_{DC}DC - \Upsilon_{DW}DW]/\Upsilon_L LL(1 + IM)$

$= [(0.9 * 1.0 * 1.0 * 656.317) - (1.25 * 148.21) - (1.5 * 31.31)]/(1.35 * 292.36)$

$= 0.907$

Loading for Type 3 Legal Load

Shear Force and Bending Moment Determination

Shear Force

$m_1 = (L - a)/L = (16 - a)/16$

$m_2 = (L - a - 1.2)/16 = (14.8 - a)/16$

$m_3 = (L - a - 5.7)/16 = (10.3 - a)/16$

$V_{T3} = (p_1m_1 + p_2m_2 + p_3m_3)$

$= 77(16 - a)/16 + 77(14.8 - a)/16 + 73(10.3 - a)/16$

$= (1232 - 77a + 1139.6 - 77a + 751.9 - 73a)/16$

$= 195.219 - 14.188a$

By applying live load distribution factor and dynamic load allowance,

$V_{T3} = (195.219 - 14.188a) * 1.2 * 0.853$

$= 199.826 - 14.523a$

Bending Moment

$M_{T3} = 195.219a - 14.523a^2$

By applying live load distribution factor and dynamic load allowance,

$M_{T3} = (195.219a - 14.523a^2) * 1.2 * 0.537$

$= 125.799a - 9.143a^2$

In the same manner as before,

The angle is found as $\theta = 30.159^0$ and $\beta = 2.455$

Recalculated shear values at Critical Section

Shear due to dead load $(VDC) = 142.432KN$

Shear due to dead load $(VDW) = 30.0428KN$

Shear due to live load $(VTL) = 198.395KN$

Calculate Shear Strength Capacity

$V_C(ERA) = 0.083\beta\sqrt{f_c'}\ b_v d_v, V_C\ (SI\ Units)$

$b_v = 0.27m,\ \ d_v = 0.976m,\ \ f_c' = 24Mpa$

$Vc = 0.083 * 2.455\sqrt{24} * 270 * 976$

$= 263.056KN$

$V_S = A_v f_y d_v cot\,\theta/s$

Stirrup spacing(s) $= 120$mm

$A_v = \pi D^2/4$

$= \pi * 12^2/4$

$= 113.04mm^2$

$Vs = 113.04 * 300 * 976 * cot(30.159^0)/120$

$= 474.975KN$

$V_n = V_C + V_S < 0.25 f_C' b_v d_v$

$V_C + V_S = 263.056KN + 474.975KN$

$= 738.031KN$

$0.25 f_C' b_v d_v = 0.25 * 24000KN/m^2 * 0.27m * 976m$

$= 1504.98KN$

Therefore,

$R_n = V_n = 738.031KN$

LRFR Load Rating Result

At Critical Section:

Shear force capacity $(Rn) = 738.031KN$

Shear due to dead load $(VDC) = 142.432KN$

Shear due to dead load $(VDW) = 30.043KN$

Shear due to live load $(VTL) = 198.395KN$

$RF = [\phi_C\phi_S\phi R_n - \Upsilon_{DC}DC - \Upsilon_{DW}DW]/\Upsilon_L LL(1 + IM)$

$= [(0.9 * 1.0 * 1.0 * 738.03) - (1.25 * 142.43) - (1.5 * 30.04)]/(1.65 * 198.40)$

$= 1.348$

Loading for Type 3-2 Legal Load

Shear Force and Bending Moment Determination

Shear Force

$m_1 = (L - a)/L = (16 - a)/16$

$m_2 = (L - a - 1.2)/L = (14.8 - a)/16$

$m_3 = (L - a - 7.8)/L = (8.2 - a)/16$

$m_4 = (L - a - 9.0)/L = (7.0 - a)/16$

$m_5 = (L - a - 12.3)/L = (3.7 - a)/16$

$V_{T3\text{-}2} = (p_1 m_1 + p_2 m_2 + p_3 m_3 + p_4 m_4 + p_5 m_5)$

$= 70(16 - a)/16 + 70(14.8 - a)/16 + 70(8.2 - a)/16 + 70(7 - a)/16 + 45(3.7 - a)/16$

$= 211.656 - 20.313a$

By applying live load distribution factor and dynamic load allowance,

$V_{T3\text{-}2} = (211.656 - 20.313a) * 1.2 * 0.853$

$= 216.651 - 20.792a$

Bending Moment

$M_{T3\text{-}2} = 211.656a - 20.313a^2$

Similarly,

$M_{T3\text{-}2} = (211.656a - 20.313a^2) * 1.2 * 0.537$

$= 139.610a - 13.090a^2$

In the same manner as before,

The angle is found as θ $= 30.216^0$ and β $= 2.451$

Recalculated shear values at Critical Section

Shear due to dead load (VDC) $= 142.520KN$

Shear due to dead load (VDW) $= 30.062KN$

Shear due to live load (VTL) $= 203.206KN$

Calculate Shear Strength Capacity

$V_c(ERA) = 0.083\beta\sqrt{f_c'}\, bVdV, Vc\ (SI\ Units)$

$b_v = 0.27m,\ \ d_v = 0.976m,\ \ f_c' = 24Mpa$

$V_c = 0.083 * 2.451\sqrt{24} * 270 * 976$

$= 262.628KN$

$V_s = A_v f_y d_v \cot\theta/s$

Stirrup spacing(s) $= 120mm$

$Av = \pi D^2/4$

$= \pi * 12^2/4$

$= 113.04mm^2$

$Vs = 113.04 * 300 * 976 * cot(30.2160)/120$

$= 473.889KN$

$V_n = V_c + V_s < 0.25f_c'b_vd_v$

$V_c + V_s = 262.628KN + 473.889KN$

$= 736.517KN$

$0.25f_c'b_vd_v = 0.25 * 24000KN/m^2 * 0.27m * 0.976m$

$= 1504.98KN$

Therefore,

$R_n = V_n = 736.517KN$

LRFR Load Rating Result

At Critical Section:

Shear force capacity (R_n) $= 736.517KN$

Shear due to dead load (VDC) $= 142.520KN$

Shear due to dead load (VDW) $= 30.062KN$

Shear due to live load (VTL) $= 203.206KN$

$RF = [\phi_c\phi_s\phi\ R_n - \Upsilon_{DC}DC - \Upsilon_{DW}DW]/\Upsilon_L LL(1 + IM)$

$= [(0.9 * 1.0 * 1.0 * 736.517) - (1.25 * 142.52) - (1.5 * 30.06)]/(1.65 * 203.21)$

$= 1.311$

Loading for Type 3-3 Legal Load

Shear Force and Bending Moment Determination

Shear Force

$m_1 = (L - a)/L = (16 - a)/16$

$m_2 = (L - a - 1.2)/L = (14.8 - a)/16$

$m_3 = (L - a - 6.1)/L = (9.9 - a)/16$

$m_4 = (L - a - 10.6)/L = (5.4 - a)/16$

$m_5 = (L - a - 11.8)/L = (4.2 - a)/25$

$m_6 = (L - a - 16.3)/L = (-0.3 - a)/25$

m_6 is negative shows that it is out of the bridge girder. So it is not considered.

$V_{T3\text{-}3} = (p_1m_1 + p_2m_2 + p_3m_3 + p_4m_4 + p_5m_5)$

$= 63(16 - a)/16 + 63(14.8 - a)/16 + 73(9.9 - a)/16 + 55(5.4 - a)/16 + 55(4.2 - a)/16$

$= (1008 - 63a + 932.4 - 63a + 722.7 - 73a + 297 - 55a + 231 - 55a)/16$

$= 199.444 - 19.313a$

Bending Moment

$M_{T3\text{-}3} = 199.444a - 19.313a^2$

Applying live load distribution factor and dynamic load allowance;

$M_{T3\text{-}3} = (199.444a - 19.313a^2) * 1.2 * 0.537$

$= 128.528a - 12.445a^2$

$V_{T3\text{-}3} = (199.444 - 19.313a) * 1.2 * 0.853$

$= 204.151 - 19.769a$

In the same manner as before,

The angle is found as $\theta = 30.11^0$ and $\beta = 2.460$

Recalculated shear values at Critical Section

Shear due to dead load $(VDC) = 142.35KN$

Shear due to dead load $(VDW) = 30.025KN$

Shear due to live load $(VTL) = 192.512KN$

Calculate Shear Strength Capacity

$V_C(ERA) = 0.083\beta\sqrt{f_c'}\, b_v d_v, V_C\ (SI\ Units)$

$b_v = 0.27m, d_v = 0.976m, f_c' = 24Mpa$

$Vc(ERA) = 0.083 * 2.46\sqrt{24} * 270 * 976$

$= 263.592KN$

$V_S = A_v f_y d_v cot\ \theta/s$

Stirrup spacing$(s) = 120mm$

$A_v = \pi D^2/4$

$= \pi * 12^2/4$

$= 113.04mm^2$

$V_S = 113.04 * 300 * 976 * cot(30.105^0)/120$

$= 476.006KN$

$V_n = V_C + V_S < 0.25 f_c' b_v d_v$

$V_C + V_S = 263.592KN + 476.006KN$

$= 739.598KN$

$0.25 f_c' b_v d_v = 0.25 * 24000KN/m^2 * 0.27m * 0.976m$

$= 1504.98KN$

Therefore,

$R_n = V_n = 739.598KN$

LRFR Load Rating Result

At Critical Section:

Shear force capacity $(R_n) = 739.598KN$

Shear due to dead load $(VDC) = 142.35KN$

Shear due to dead load $(VDW) = 30.025KN$

Shear due to live load $(VTL) = 192.512KN$

$RF = [\Phi_c \Phi_s \Phi\, R_n - \Upsilon_{DC} DC - \Upsilon_{DW} DW] / \Upsilon_L LL(1 + IM)$

$= [(0.9 * 1.0 * 1.0 * 739.598) - (1.25 * 142.35) - (1.5 * 30.03)]/(1.65 * 192.51)$

$= 1.394$

The rating factors obtained by rating the bridge girder for the legal loading shows they fulfil the requirement for the capacity for shear.

6.4. Bridge Girder 3 LRFR Analysis and Rating

This bridge girder is a T-girder with a span length of 20m. Similar to the previous girders, this bridge girder has the same material properties.

Design Specifications

1. ERA Bridge Design Manual 2002
2. AASHTO Standard Specifications for Highway Bridges, 1998

Bridge Loadings

Dead Loads

Dead loads are based on the following loadings

Super Structure Loads

Reinforced concrete: 24KN/m^3

Wearing Surface, Asphalt: 22.5KN/m^3

Live Loading

The live loading is AASHTO HL-93

Type of Girder

The type of girder cross-section from those in Table 5-12 is Type e, i.e. Cast –in-place concrete T-beam with monolithic concrete deck.

Input Parameters:

Bridge Geometry

Bridge Width = 7.3m

Number of Girders = 4

Left overhang =1.3m

Girder spacing = 2.1m

Slab Thickness=0.2m

Wearing surface = 0.1m

Girder web width=0.40m

Girder height=1.6m

Left curb width = 0.8m

Right curb width = 0.8m

Curb Height=0.45m

Left skew angle =0^0

Right skew angle =0^0

Load Factors

Resistance Factor $(\phi) = 0.9$

Condition Factor $(\phi_C) = 1.0$

System Factor $(\phi_S) = 1.0$

Dead Load Factors

Strength I limit State

Dead load Factor $(\gamma_{DC}) = 1.25$

Dead Load Factor $(\gamma_{DW}) = 1.50$

Design Live Load Factors

Strength I Limit State

Live Load Factor (γ_{LL}) $(Inventory) = 1.75$

Live Load Factor (γ_{LL}) $(Operating) = 1.35$

Dynamic Load Allowance

Dynamic load allowance$(IM) = 20\%$

Live Load Distribution Factors

Moment for Interior Bridge Girders

One Design Lane Loaded:

$g_{interior} = 0.06 + (S/4300)^{0.4}(S/L)^{0.3}(k_g/Lt_s3)^{0.1}$

$= 0.06 + (2100/4300)^{0.4}(2100/20500)^{0.3}(k_g/(20500 * 200^3))^{0.1}$

$= 0.06 + (0.751 * 0.505 * 1.071)$

$= 0.466$

Two or More Design Lanes Loaded:

$g_{interior} = 0.075 + (S/4300)^{0.6}(S/L)^{0.2}(k_g//Lts3)^{0.1}$

$= 0.075 + (2100/4300)^{0.6}(2100/20500)^{0.2}(k_g/(20500 * 200^3))^{0.1}$

$= 0.075 + (0.651 * 0.634 * 1.071)$

$= 0.517$

Where the value of K_g is as calculated below

$K_g = n\,(I + A\,e_g{}^2)$ *in which*: $n = EB/\,ED$

$n = (\,EB/ED) = 1,$

Effective flange width for interior girders

$b_{eff} = min\,\{1/4L, 12t_s + max\,(t_w, 0.5w_f), S_{avg}\}$

$b_{eff} = min\{20.5m/4, (12 * 0.2m) + max(0.40m, 0.5 * 2.10m), 2.10m\}$

$b_{eff} = min(5.125, 2.4m + max(0.40m, 1.05m), 2.10m\}$

$b_{eff} = 2.10m$

$A = b_{eff} * t_s = 2.10m * 0.2m = 0.42m^2$

$I = bh^3/12 = 0.40 * 1.4^3/12 = 0.091m^4$

$e_g = 0.75m$

$K_g = 1(0.091 + 0.42 * 0.752)$

$= 0.327m^4$

Therefore,

Interior girder live load moment distribution factor will be the max of the two values.

$Mgint = max\,(0.466, 0.517)$

$= 0.517$

Moment for Exterior Bridge Girders

One Design Lane Loaded:

Lever Rule

$d_e = 0.90m = 2.953ft$

$S = 2.11m = 6.89ft$

$d_e + S = 2.953ft + 6.89ft = 9.843ft$

$d_e + S > 6ft$

Therefore, the distribution factor lever rule equation will be:

$DF = 1 + (d_e/S) - (3/S) = 1 + (2.953ft/6.89ft) - (3ft/6.89ft)$

$= 0.993$

When using Lever rule we shall consider multiple presence factor; for single lane loaded the multiple presence factor is 1.2.

$DF = 0.993 * 1.2 = 1.192$

Two or More Design Lanes Loaded:

$g = e * g_{interior}$

$e = 0.77 + d_e/2800, d_e = 900$

$e = 0.77 + 900/2800$

$= 1.091$

$g = 1.091 * g_{interior}$

$g = 1.091 * 0.517$

$= 0.564$

Therefore,

Exterior girder live load moment distribution factor will be the max of the two values.

$M_{gext} = max\ (1.192, 0.564)$

$= 1.192$

Check for Diaphragm Contribution

$NL \quad Nb$

$R = [NL/Nb] + [X_{ext}\sum e/\sum x^2]$

For One Lane Loaded

$NL = 1$

$Nb = 4$

$X_{ext} = 3.15m$

$\sum e = 2.72m$

$\sum x^2 = (x_1{}^2 + x_2{}^2 + x_3{}^2 + x_4{}^2)$

$= [(3.15m)^2 + (1.04m)^2 + (1.04m)^2 + (3.15m)^2]$

$= 22.008m^2$

$R = (1/4) + [(3.15m * 2.72m)/22.008m^2]$

$= 0.64$

For Two Lanes Loaded

Only the Values for the terms NL and $\sum e$ vary.

$NL = 2$

$\sum e = -0.13 + 2.72 = 2.59$

$R = (2/4) + [(3.15m * 2.59m)/22.008m^2]$

$= 0.761$

The distribution factors obtained previously shall be taken since these values are lower.

Check for Skewness

Skewness angle $\theta = 0^0$

Skewness Correction Factor=1

Shear Distribution Factors for Interior Bridge Girders

One Design Lane Loaded:

$DF = 0.36 + S/7600 = 0.36 + 2100/7600$

$= 0.636$

Two or More Design Lanes Loaded:

$DF = 0.2 + (S/3600) - (S/10700)^{2.0} = 0.2 + (2100/3600) - (2100/10700)^{2.0}$

$= 0.745$

Therefore,

Interior girder live load shear distribution factor will be the max of the two values.

$Vgint = max\,(0.636, 0.745)$

$= 0.745$

Shear Distribution factors for Exterior Bridge Girders

One Design Lane Loaded:

Lever Rule

$d_e = 0.90m = 2.953ft$

$S = 2.10m = 6.89ft$

$d_e + S = 2.953ft + 6.89ft = 9.843ft$

$d_e + S > 6ft$

Therefore, the distribution factor lever rule equation will be:

$DF = 1 + (d_e/S) - (3/S) = 1 + (2.953ft/6.89ft) - (3ft/6.89ft)$

$= 0.993$

Similarly applying multiple presence factor,

$DF = 0.993 * 1.2 = 1.192$

Two or More Design Lanes Loaded:

$g = e * g_{interior}$

$e = 0.6 + d_e/3000$

$e = 0.6 + 900/3000 = 0.90$

$g = 0.90 * g_{interior} = 0.90 * 0.745 = 0.671$

Therefore,

Exterior girder live load shear distribution factor will be the max of the three values.

$V_{gext} = max\ (1.192, 0.671, 0.761)$

$= 1.192$

Check for Skewness

Skewness Correction Factor (for $\theta=0^0$) =1

Therefore,

No correction for skewness

Dead Load Calculation

Interior Girder Loading

Girder Load(Lg) $= A_g * \gamma_g = [(2.10m * 0.2m) + (0.40m * 1.6m)] * 24KN/m^3$

=25.44KN/m

Railing and Post Load(Lrp):

Loads from railings and post can be distributed equally to all girders

Railing Load= $A_r * \gamma_r = (0.2m * 0.4m * 24KN/m^3)$

$= 1.92KN/m$

Post Load

$= ((4 * 0.3m) + (2 * 0.5m)) * ((0.25m * 0.5m) + (0.15m * 0.4m)) * 24KN/m^3$

$= 9.768KN/20.5m$

$= 0.476KN/m$

Therefore,

$Lrp = 1.92KN/m + 0.476KN/m$

$= 2.396KN/m$

Lrp for each girder= $(2.396KN/m)/4 = 0.599KN/m$

Diaphragm Load (Ld)

Diaphragm Load will be applied on girders as concentrated load on the points where the center lines of the girder and diaphragm meet.

For the interior and two end diaphragms

$$Ld = A_d * \gamma_d * length\ of\ diaphragms\ between\ faces\ of\ girders\ (l_d)$$

$$= A_d * \gamma_d * l_d$$

$$\text{Height of diaphragm} = 1.1\text{m}$$

$$\text{Thickness of diaphragm} = 0.25\text{m}$$

$$\text{Length of diaphragm } (ld) = 1.70\text{m}$$

$$Ld = A_d * \gamma_d * l_d$$

$$= 1.1m * 0.25m * 1.70m * 24KN/m^3$$

$$= 11.22KN$$

Curb Load (Lcb) $= A_{cb} * \gamma_{cb} = (0.45m * 0.8m) * 24KN/m^3$

$= 8.64KN/m$

The load will be distributed equally to all the girders

Lcb/Girder= $8.64KN/m/4 = 2.16KN/m$

Load from Wearing Surface (Lw) $= A_w * \gamma_w = (0.1m * 2.10m) * 22.5KN/m^3$

$= 4.725KN/m$

Total Interior Girder Load (Dci) $= Lg + Lrp + Lcb$

$= 25.44KN/m + 0.599KN/m + 2.16KN/m$

$= 28.199KN/m$

Total Interior Girder Load (Dwi) $= Lw$

$= 4.725KN/m$

Concentrated Interior and End Diaphragm Load (Ld) $= 11.22KN$

Exterior Girder Loading

Girder Load (Lg) $= A_g * \gamma_g = [(1.30m * 0.2m) + (0.40m * 1.6m)] * 24KN/m^3$

$= 21.6KN/m$

Railing and Post Load (Lrp):

As calculated for interior girders,

Lrp/girder= $0.599KN/m$

Diaphragm Load(Ld):

The diaphragm load calculation for the exterior girders is the same as of interior girders. But we will take only half of the load for interior girders.

For the interior and two exterior diaphragms

$$Ld = (A_d * \gamma_d * l_d)/2$$
$$= 11.22/2$$
$$= 5.61KN$$

Curb Load $(Lcb) = A_{cb} * \gamma_{cb} = (0.45m * 0.8m) * 24KN/m^3$

$$= 8.64KN/m$$

The load will be distributed equally to all the girders

Lcb/Girder= $(8.64KN/m)/4 = 2.16KN/m$

Load from Wearing Surface $(Lw) = A_W * \gamma_W = (0.1m * b_{eff}) * 22.5KN/m^3$

Effective Flange Width for Exterior Girders

$b_{eff} = min\{((b_{eff}(interior)/2) + min(L/8, 6t_S + 0.5t_W)), b_{ov}\}$

Where b_{ov} is overhang width

$b_{eff} = min\{(2.10m/2) + min(20.5m/8, (6 * 0.2m) + (0.5 * 0.40m)), b_{ov})$

$b_{eff} = min\{(1.05m + 1.4), 1.30m\}$

$b_{eff} = 1.30m$

$Lw = A_W * \gamma_W = (0.1m * 1.30m) * 22.5KN/m^3$

$$= 2.925KN/m$$

Total Exterior Girder Load $(DCe) = Lg + Lrp + Lcb$

$$= 21.6KN/m + 0.599KN/m + 2.16KN/m$$
$$= 24.359KN/m$$

Total Exterior Girder Load $(Dwe) = Lw$

$$= 2.363KN/m$$

Concentrated Exterior Diaphragm Load $(Ld) = 5.61KN$

Interior Girder Dead Load Shear Force and Bending Moment Calculation

Shear Force

$VDC = V(DCi) + V(Ld)$

$$= wL/2 - wa + 5.61$$
$$= (28.199 * 20.5/2) - (28.199a) + 5.61$$
$$= 294.65 - 28.2a$$

$VDW = V(DWi)$

$= (wL/2) - wa$

$= (4.725KN/m * 20.5m/2) - (4.725KN/m * a)$

$= 48.431 - 4.725a$

Bending Moment

$MDC = M(DCi) + M(Ld)$

$= wa(L - a)/2 + 0.5 * Ld * a$

$= 28.2a(20.5 - a)/2 + 5.61a/2$

$= 14.1a(20.5 - a) + 2.81a$

$MDW = M(DWi)$

$= wa * (L - a)/2$

$= 4.725a(20.5 - a)/2$

$= 2.363a(20.5 - a)$

Exterior Girder Dead Load Shear Force and Bending Moment Calculation

Shear Force

$VDC = V(DCe) + V(Ld)$

$= (wL/2) - wa + 2.81$

$= (24.359KN/m * 20.5m/2) - (24.359KN/m * a) + 2.81KN$

$= 252.49 - 24.36a$

$VDW = V(DWe)$

$= (wL/2) - wa$

$= (2.363KN/m * 20.5/2) - (2.363KN/m * a)$

$= 24.22 - 2.36a$

Bending Moment

$MDC = M(DCe) + M(Ld)$

$= wa(L - a)/2 + 0.5 * Ld * a$

$= (24.359KN/m * a(20.5m - a)/2) + 2.81a$

$= 12.18a(20.5 - a) + 2.81a$

$MDW = M(DWe)$

$= wa(L - a)/2$

$= (2.363KN/m * a) * (20.5m - a)/2$

$= 1.18a(20.5 - a)$

Girder Live loading

Truck and Lane Load Shear Force at a Section

From Eq. [5-1a] & [5-1b]

$Vtrl = (P1/L) * ((L - a) + (L - a - 4.3m) + K * (L - a - 8.6m))$

$= (145KN/m/20.5m) * ((20.5m - a) + (20.5m - a - 4.3m) + 7/29(20.5m - a - 8.6m)$

$= 279.76 - 15.85a$

$VL = 0.5 * (L - a)^2 * U/L$

$= 0.5 * (20.5m - a)2 * (3.1KN/m)/20.5m$

$= 3.1(20.5 - a)^2/41$

Truck and Lane Load Bending Moment at a Section

From Eq. [5-2a] & [5-2b]

$Mtrl = P1 * (a/L) * ((L - a) + (L - a - 4.3m) + K * (L - a - 8.6m))$

$= 145 * (a/20.5m) * ((20.5 - a) + (20.5m - a - 4.3m) + 7/29(20.5m - a - 8.6m))$

$= 279.76a - 15.85a^2$

$ML = 0.5 * a\,(L - a)^2 U/L$

$= 0.5 * a(20.5m - a)^2 * (3.1KN/m)/20.5m$

$= 3.1a(20.5 - a)^2/41$

Tandem and Lane Load Shear Force at a Section

From Eq. [5-5a] & [5-5b]

$Vtml = (P_3/L) * ((L - a) + (L - a - 1.2))$

$= (110KN/20.5m) * ((20.5m - a) + (20.5m - a - 1.2))$

$= 213.57 - 10.73a$

$VL = 0.5 * (L - a)^2 U/L$

$= 0.5 * (20.5m - a)^2 * (3.1KN/m)/20.5m$

$= 3.1(20.5 - a)^2/41$

Tandem and Lane Load Bending Moment at a Section

From Eq. [5-6a] & [5-6b]

$Mtml = P_3 * (a/L) * ((L - a) + (L - a - 1.2))$

$= 110KN * (a/20.5m) * ((20.5m - a) + (20.5m - a - 1.2m))$

$= 213.57a - 10.73a^2$

$ML = 0.5 * a(L - a)^2 U/L$

$= 0.5 * a(20.5m - a)^2 * (3.1KN/m)/20.5m$

$= 3.1a(20.5 - a)^2/41$

Interior Girder

Shear

$Vtrl = (279.76 - 15.85a) * 1.2 * 0.822$

$= 275.955 - 15.634a$

$Vtml = (213.57 - 10.73a) * 0.822 * 1.2$

$= 210.665 - 10.58a$

$VL = [3.1(20.5 - a)^2/41] * 0.822$

$= 0.062(20.5 - a)^2$

Moment

$Mtrl = (279.76a - 15.85a^2) * 0.517 * 1.2$

$= 173.56a - 9.83a^2$

$Mtml = (213.57a - 10.73a^2) * 0.517 * 1.2$

$= 132.5a - 6.6a^2$

$ML = (0.062a(20.5 - a)^2) * 0.517$

$= 0.032a(20.5 - a)^2$

Exterior Girder

Shear

$Vtrl = (279.76 - 15.85a) * 1.2 * 1.192$

$= 400.17 - 22.67a$

$Vtml = (213.57 - 10.73a) * 1.192 * 1.2$

$= 305.49 - 15.35a$

$VL = [3.1(20.5 - a)^2/41] * 1.192$

$= 0.09(20.5 - a)^2$

Moment

$Mtrl = (279.76a - 15.85a^2) * 1.192 * 1.2$

$= 400.17a - 22.67a^2$

$Mtml = (213.57a - 10.73a^2) * 1.192 * 1.2$

$= 305.49a - 15.35a^2$

$ML = (0.062a(20.5 - a)^2) * 1.192$

$= 0.09a(20.5 - a)^2$

Rating of Interior Girder

Determination of angle θ

Determine the Critical Section for Shear

The critical section near the supports is the greater of $0.5\ d_v\ cot\ \theta$ and d_v

$d_v\ =\ d_e - a/2\ \geq 0.9d_e\ ,0.72h$

Where,

d_e is the effective depth measured from the extreme compression fiber to the centroid of the tensile force

θ is the angle of inclination of diagonal compressive stress.

$d_e\ =\ h - y_b, h = 1.5m, y_b = 0.20m$

$d_e\ = 1.5m - 0.20m = 1.30m$

$d_v\ =\ d_e - a/2\ \geq 0.9d_e\ ,0.72h$

$= 1.30 - 0.01m > 0.9 * 1.30m, 0.72 * 1.5m$

$= 1.29m$

The calculation procedure assumes a value for θ and solves to verify the assumption.

Iteration is used until a solution is found.

Assume $\theta\ =\ 40^0$ degrees.

Dead Load Moments at distance a:

$MDC\ = 14.1a(20.5 - a) + 2.81a$

$MDW\ = 1.18a(20.5 - a)$

Dead Load shear at distance a:

$VDC\ = 252.49 - 24.36a$

$VDW = 24.22 - 2.36a$

Live Load Moment at distance a:

Applying impact factor and distribution factor, the live loads are

$Mtrl = 173.56a - 9.83a^2$

$Mtml\ = 132.5a - 6.6a^2$

$ML\ (No\ dynamic\ allowance)\ = 0.032a(20.5 - a)^2$

$MTL\ =\ Max\ [(Mtrl + ML), (Mtml + ML)]$

Live Load Shear at distance a:

$Vtrl = 275.955 - 15.634a$

$Vtml = 210.665 - 10.58a$

$VL(No\ dynamic\ allowance) = 0.062(20.5 - a)^2$

$VTL = Max\ [(Vtrl + VL), (Vtml + VL)]$

$Mu = 1.25MDC + 1.5MDW + 1.75MTL$

$Vu = 1.25VDC + 1.5VDW + 1.75VTL$

Check minimum amount of transverse reinforcement using,

$A_v f_y / b_W s > 0.06\sqrt{f_c'}$ $(SI\ Units)$

$(\pi * 12^2/4) * 300/(400 * 150) > 0.06\sqrt{24}$

$0.565 > 0.294$

Minimum amount of transverse reinforcement condition is satisfied; determine average shear stress v.

$v = V_u / \phi b_v d_v$

$\varepsilon_x = [(M_u/d_v) + (0.5N_u) + (0.5VUcot\ \theta)]/\ 2(E_S A_S) < 0.002$

$E_S = 200000Mpa = 200 * 106KN/m^2$

$A_S = 615.752mm^2 * 16 = 9.852 * 10^{-3}m2$

If ε_x is negative, it shall be multiplied by the following factor:

$F\varepsilon = (E_S A_S)/\ [(E_C A_C) + (E_S A_S)]$

The values are positive, no need of multiplication by the factor.

From Table 4-2 with v/f_c' and $1000\varepsilon_x$;

After a number of iterations, it is found that $\theta = 28.259^0$and $\beta = 2.988$,

The critical shear section is at 'a' distance from face of support, $Max\ \{dv, 0.5dvcot\theta\}$

$$a = 2.651m$$

The shear effects are recalculated based on the controlling section

Recalculated shear values at Critical Section

Shear due to dead load $(VDC) = 187.904KN$

Shear due to dead load $(VDW) = 17.963KN$

Shear due to live load $(VTL) = 254.256KN$

Calculate Shear Strength Capacity

a) Using Modified Compression Field Theory

$Vc(ERA) = 0.083\beta\sqrt{f_c'}\ b_v d_v, Vc\ (SI\ Units)$

$b_v = 0.30m,\quad d_v = 1.29m,\quad f_c' = 24Mpa$

$V_c = 0.083 * 2.988\sqrt{24} * 300 * 1290$

$= 506.407KN$

$V_S = A_v f_y d_v cot\ \theta/s$

Stirrup spacing(s) $= 150mm$

$Av = \pi D^2/4$

$= \pi * 12^2/4$

$= 113.04mm^2$

$V_s = 113.04 * 300 * 1290 * cot(28.259^0)/150$

$= 542.569KN$

$V_n = V_c + V_s < 0.25f_c' b_v d_v$

$V_c + V_s = 506.407KN + 542.569KN$

$= 1048.976KN$

$0.25f_c' b_v d_v = 0.25 * 24000KN/m^2 * 0.30m * 1.29m$

$= 2322KN$

Therefore,

$R_n = V_n = 1048.976KN$

b) Using 45^0 Truss Model

$Vc = 0.1667\sqrt{f_c'}\ b_w d$

$= 0.1667 * \sqrt{24} * 300 * 1290$

$= 316.047KN$

$V_s = A_v f_y d/s$

$= 113.04 * 300 * 1290/150$

$= 291.643KN$

$V_n = V_c + V_s = 316.047KN + 291.643KN$

$= 607.690KN$

LRFR Load Rating Result

Strength – Shear

At Critical Section:

Shear force capacity (R_n) = 1048.976KN

Shear due to dead load (VDC) = 187.904KN

Shear due to dead load (VDW) = 17.963KN

Shear due to live load (VTL) = 254.256KN

Inventory:

$RF_{Inv} = [\varphi c \varphi s \varphi\ R_n - \Upsilon_{DC} DC - \Upsilon_{DW} DW]/\Upsilon_L LL(1 + IM)$

$= [(0.9 * 1.0 * 1.0 * 1048.98) - (1.25 * 187.90) - (1.5 * 17.96)]/(1.75 * 254.26)$

$= 1.53$

Operating:

$RF_{Op} = [\varphi c \varphi s \varphi \, R_n - \Upsilon_{DC} DC - \Upsilon_{DW} DW]/\Upsilon_L LL(1 + IM)$

$= [(0.9 * 1.0 * 1.0 * 1048.98) - (1.25 * 187.90) - (1.5 * 17.96)]/(1.35 * 254.26)$

$= 1.98$

The rating factors above are greater than unity, so this shows that the bridge girder shear capacity fulfils the requirement for resisting design loading. So there no need to check for legal loadings.

6.5. Bridge Girder 4 LRFR Analysis and Rating

This is a T-girder with span length of 12m and has the same material properties with the above three girders.

Design Specifications

1. ERA Bridge Design Manual 2002
2. AASHTO Standard Specifications for Highway Bridges, 1998

Bridge Loadings

Dead Loads

Dead loads are based on the following loadings

Super Structure Loads

Reinforced concrete: $24KN/m^3$

Wearing Surface, Asphalt: $22.5KN/m^3$

Live Loading

The live loading is AASHTO HL-93

Type of Girder

The type of girder cross-section from those in Table 5-12 is Type e, i.e. Cast –in-place concrete T-beam with monolithic concrete deck.

Input Parameters:

Bridge Geometry

Bridge Width = 7.3m

Number of Girders = 4

Left overhang =1.3m

Girder spacing = 2.1m

Slab Thickness=0.2m

Wearing surface = 0.1m

Girder web width=0.40m

Girder height=0.9m

Left curb width = 0.75m

Right curb width = 0.75m

Curb Height=0.30m

Left skew angle =0^0

Right skew angle =0^0

Load Factors

Resistance Factor (ϕ) $= 0.9$

Condition Factor (ϕ_c) $= 1.0$

System Factor (ϕ_S) $= 1.0$

Dead Load Factors

Strength I limit State

Dead load Factor (γ_{DC}) $= 1.25$

Dead Load Factor (γ_{DW}) $= 1.50$

Design Live Load Factors

Strength I Limit State

Live Load Factor (γ_{LL}) $(Inventory)$ $= 1.75$

Live Load Factor (γ_{LL}) $(Operating)$ $= 1.35$

Dynamic Load Allowance

Dynamic load allowance$(IM) = 20\%$

Live Load Distribution Factors

Moment for Interior Bridge Girders

One Design Lane Loaded:

$$g_{interior} = 0.06 + (S/4300)^{0.4}(S/L)^{0.3}(k_g/Lt_s{}^3)^{0.1}$$

$$= 0.06 + (2100/4300)^{0.4}(2100/12500)^{0.3}(k_g/(12500 * 200^3))^{0.1}$$

$$= 0.06 + (0.751 * 0.586 * 1.009)$$

$$= 0.504$$

Two or More Design Lanes Loaded:

$ginterior = 0.075 + (S/4300)^{0.6}(S/L)^{0.2}(k_g//Lt_s{}^3)^{0.1}$

$= 0.075 + (2100/4300)^{0.6}(2100/12500)^{0.2}(k_g/(12500 * 200^3))^{0.1}$

$= 0.075 + (0.651 * 0.700 * 1.009)$

$= 0.535$

Where the value of K_g is as calculated below

$K_g = n (I + A e_g{}^2)$ in which: $n = EB/ ED$

$n = (EB/ED) = 1$

Effective flange width for interior girders

$b_{eff} = min \{1/4L, 12t_S + max (t_W, 0.5w_f), S_{avg}\}$

$b_{eff} = min\{12.5m/4, (12 * 0.2m) + max(0.40m, 0.5 * 2.10m), 2.10m\}$

$b_{eff} = min(3.125, 2.4m + max(0.40m, 1.05m), 2.10m\}$

$b_{eff} = 2.10m$

$A = b_{eff} * t_S = 2.10m * 0.2m = 0.42m^2$

$I = bh^3/12 = 0.40 * 0.9^3/12 = 0.0243m^4$

$e_g = 0.45m$

$K_g = n (I + A e_g{}^2)$

$= 1(0.0243 + 0.42 * 0.452)$

$= 0.109m^4$

Therefore,

Interior girder live load moment distribution factor will be the max of the two values.

$Mgint = max (0.504, 0.535)$

$= 0.535$

Moment for Exterior Bridge Girders

One Design Lane Loaded:

Lever Rule

$d_e = 0.85m = 2.79ft$

$S = 2.10m = 6.89ft$

$d_e + S = 2.79ft + 6.89ft = 9.68ft$

$d_e + S > 6ft$

Therefore, the distribution factor lever rule equation will be:

$DF = 1 + (d_e/S) - (3/S) = 1 + (2.79ft/6.89ft) - (3ft/6.89ft)$

$= 0.97$

When using Lever rule we shall consider multiple presence factor; for single lane loaded the multiple presence factor is 1.2.

$DF = 0.97 * 1.2 = 1.16$

Two or More Design Lanes Loaded:

$g = e * g_{interior}$

$e = 0.77 + de/2800, d_e = 850$

$e = 0.77 + 850/2800$

$= 1.074$

$g = 1.074 * g_{interior}$

$g = 1.074 * 0.535$

$= 0.575$

Therefore,

Exterior girder live load moment distribution factor will be the max of the two values.

$Mgext = max\ (1.16, 0.575)$

$= 1.16$

Check for Diaphragm Contribution

$NL\ \ Nb$

$R = [NL/Nb] + [X_{ext} \sum e / \sum x^2]$

For One Lane Loaded

$NL = 1, Nb = 4, X_{ext} = 3.15m, \sum e = 2.72m$

$\sum x^2 = ({x_1}^2 + {x_2}^2 + {x_3}^2 + {x_4}^2)$

$= [(3.15m)^2 + (1.04m)^2 + (1.04m)^2 + (3.15m)^2]$

$= 22.01m^2$

$R = (1/4) + [(3.15m * 2.72m)/22.01m^2]$

$= 0.64$

For Two Lanes Loaded

Only the Values for the terms NL and $\sum e$ vary.

$NL = 2$

$\sum e = -0.13 + 2.72 = 2.59$

$R = (2/4) + [(3.15m * 2.59m)/22.008m^2]$

$= 0.761$

The distribution factors obtained previously shall be taken since these values are lower.

Check for Skewness

Skewness angle $\Theta = 0^0$

Skewness Correction Factor=1

Shear Distribution Factors for Interior Bridge Girders

One Design Lane Loaded:

$DF = 0.36 + S/7600 = 0.36 + 2100/7600$

$= 0.636$

Two or More Design Lanes Loaded:

$DF = 0.2 + (S/3600) - (S/10700)^{2.0} = 0.2 + (2100/3600) - (2100/10700)^{2.0}$

$= 0.745$

Therefore,

Interior girder live load shear distribution factor will be the max of the two values.

$Vgint = max\ (0.636, 0.745)$

$= 0.745$

Shear Distribution factors for Exterior Bridge Girders

One Design Lane Loaded:

Lever Rule

$d_e = 0.85m = 2.79ft$

$S = 2.10m = 6.89ft$

$d_e + S = 2.79ft + 6.89ft = 9.68ft$

$d_e + S > 6ft$

Therefore, the distribution factor lever rule equation will be:

$DF = 1 + (d_e/S) - (3/S) = 1 + (2.79ft/6.89ft) - (3ft/6.89ft)$

$= 0.97$

Similarly applying multiple presence factor,

$DF = 0.97 * 1.2 = 1.164$

Two or More Design Lanes Loaded:

$g = e * g_{interior}$

$e = 0.6 + d_e/3000$

$e = 0.6 + 850/3000 = 0.883$

$g = 0.883 * g_{interior} = 0.883 * 0.745 = 0.658$

$gext = 0.658 * 2 = 1.315$

Therefore,

Exterior girder live load shear distribution factor will be the max of the three values.

$V_{gext} = max\ (1.164, 1.315, 0.761)$

$= 1.315$

Check for Skewness

Skewness Correction Factor (for $\theta = 0^0$) = 1

Therefore,

No correction for skewness

Dead Load Calculation

Interior Girder Loading

Girder Load (Lg) $= A_g * \gamma_g = [(2.10m * 0.2m) + (0.40m * 0.9m)] * 24KN/m^3$

$= 18.72KN/m$

Railing and Post Load (Lrp):

Loads from railings and post can be distributed equally to all girders

Railing Load= $A_r * \gamma_r = (0.2m * 0.4m * 24KN/m^3)$

$= 1.92KN/m$

Post Load= $((4 * 0.3m) + (2 * 0.5m)) * ((0.25m * 0.5m) + (0.15m * 0.4m)) * 24KN/m^3$

$= 9.768KN/12.5m$

$= 0.781KN/m$

Therefore,

$Lrp = 1.92KN/m + 0.781KN/m$

$= 2.7KN/m$

Lrp for each girder = $(2.7KN/m)/4 = 0.675KN/m$

Diaphragm Load (Ld)

Diaphragm Load will be applied on girders as concentrated load on the points where the center lines of the girder and diaphragm meet.

For the interior and two end diaphragms

$Ld = A_d * \gamma_d * length\ of\ diaphragms\ between\ faces\ of\ girders\ (l_d)$

$= A_d * \gamma_d * l_d$

Height of diaphragm = 0.75m

Thickness of diaphragm = 0.30m

Length of diaphragm (l_d) = 1.70m

$Ld = A_d * \gamma_d * l_d$

$= 0.75m * 0.25m * 1.70m * 24KN/m^3$

$= 7.65KN$

Curb Load $(Lcb) = A_{cb} * \gamma_{cb} = (0.3m * 0.75m) * 24KN/m^3$

$= 5.4KN/m$

The load will be distributed equally to all the girders

Lcb/Girder= $5.4KN/m/4 = 1.35KN/m$

Load from Wearing Surface $(Lw) = A_w * \gamma_w = (0.1m * 2.10m) * 22.5KN/m^3$

$= 4.725KN/m$

Total Interior Girder Load $(Dci) = Lg + Lrp + Lcb$

$= 18.72KN/m + 0.675KN/m + 1.35KN/m$

$= 20.745KN/m$

Total Interior Girder Load $(Dwi) = Lw$

$= 4.725KN/m$

Concentrated Interior and End Diaphragm Load $(Ld) = 7.65KN$

Exterior Girder Loading

Girder Load $(Lg) = A_g * \gamma_g = [(1.30m * 0.2m) + (0.40m * 0.9m)] * 24KN/m^3$

$= 14.88KN/m$

Railing and Post Load (Lrp):

As calculated for interior girders,

$Lrp/\text{girder} = 0.675\text{KN/m}$

Diaphragm Load (Ld):

The diaphragm load calculation for the exterior girders is the same as of interior girders. But we will take only half of the load for interior girders.

For the interior and two exterior diaphragms

$Ld = (A_d * \gamma_d * l_d)/2$

$= 7.65/2$

$= 3.825KN$

The load will be distributed equally to all the girders

Curb Load $(Lcb) = 1.35KN/m$

Load from Wearing Surface $(Lw) = A_w * \gamma_w = (0.1m * b_{eff}) * 22.5KN/m^3$

Effective Flange Width for Exterior Girders

$beff = min\{((beff(interior)/2) + min(L/8, 6ts + 0.5tw)), bov\}$

Where bov is overhang width

$beff = min\{(2.10m/2) + min(12.5m/8, (6 * 0.2m) + (0.5 * 0.40m)), bov)$

$beff = min\{(1.05m + 1.4), 1.30m\}$

$beff = 1.30m$

$Lw = Aw * \gamma w = (0.1m * 1.30m) * 22.5KN/m3$

$= 2.925KN/m$

Total Exterior Girder Load $(DCe) = Lg + Lrp + Lcb$

$= 14.88KN/m + 0.675KN/m + 1.35KN/m$

$= 16.905KN/m$

Total Exterior Girder Load $(Dwe) = Lw$

$= 2.925KN/m$

Concentrated Exterior Diaphragm Load $(Ld) = 3.825KN$

Interior Girder Dead Load Shear Force and Bending Moment Calculation

Shear Force

$VDC = V(DCi) + V(Ld)$

$= wL/2 - wa + 3.825$

$= (20.745 * 12.5/2) - (20.745a) + 3.825$

$= 133.481 - 20.745a$

$VDW = V(DWi)$

$= (wL/2) - wa$

$= (4.725KN/m * 12.5m/2) - (4.725KN/m * a)$

$= 29.531 - 4.725a$

Bending Moment

$MDC = M(DCi) + M(Ld)$

$= wa(L - a)/2 + 0.5 * Ld * a$

$= 20.745a(12.5 - a)/2 + 7.65a/2$

$= 10.375a(12.5 - a) + 3.83a$

$MDW = M(DWi)$

$= wa * (L - a)/2$

$= 4.725a(12.5 - a)/2$

$= 2.363a(12.5 - a)$

Exterior Girder Dead Load Shear Force and Bending Moment Calculation

Shear Force

$VDC = V(DCe) + V(Lde)$

$= (wL/2) - wa + 3.825$

$= (16.905KN/m * 12.5m/2) - (16.905KN/m * a) + 3.825KN$

$= 109.48 - 16.91a$

$VDW = V(DWe)$

$= (wL/2) - wa$

$= (2.925KN/m * 12.5/2) - (2.925KN/m * a)$

$= 18.28 - 2.93a$

Bending Moment

$MDC = M(DCe) + M(Ld)$

$= wa(L - a)/2 + 0.5 * Ld * a$

$= (16.91KN/m * a(12.5m - a)/2) + 3.83a$

$= 8.46a(12.5 - a) + 3.83a$

$MDW = M(DWe)$

$= wa(L - a)/2$

$= (2.93KN/m * a) * (12.5m - a)/2$

$= 1.47a(12.5 - a)$

Girder Live loading

Truck and Lane Load Shear Force at a Section

From Eq. [5-1a] & [5-1b]

$Vtrl = (P1/L) * ((L - a) + (L - a - 4.3m) + K * (L - a - 8.6m))$

$= (145KN/m/12.5m) * ((12.5m - a) + (12.5m - a - 4.3m) + 7/29(12.5m - a - 8.6m)$

$= 251.04 - 25.70a$

$VL = 0.5 * (L - a)^2 * U/L$

$= 0.5 * (12.5m - a)^2 * (3.1KN/m)/12.5m$

$= 3.1(12.5 - a)^2/25$

Truck and Lane Load Bending Moment at a Section

From Eq. [5-2a] & [5-2b]

$M_{trl} = P_1 * (a/L) * ((L - a) + (L - a - 4.3m) + K * (L - a - 8.6m))$

$= 145 * (a/12.5m) * ((12.5 - a) + (12.5m - a - 4.3m) + 7/29(12.5m - a - 8.6m))$

$= 251.04a - 25.70a^2$

$ML = 0.5 * a\ (L - a)^2 U/L$

$= 0.5 * a(12.5m - a)^2 * (3.1KN/m)/12.5m$

$= 3.1a(12.5 - a)^2/25$

Tandem and Lane Load Shear Force at a Section

From Eq. [5-5a] & [5-5b]

$V_{tml} = (P_3/L) * ((L - a) + (L - a - 1.2))$

$= (110KN/12.5m) * ((12.5m - a) + (12.5m - a - 1.2))$

$= 8.8(23.8 - 2a)$

$= 209.44 - 17.6a$

$VL = 0.5 * (L - a)^2 U/L$

$= 0.5 * (12.5m - a)^2 * (3.1KN/m)/12.5m$

$= 3.1(12.5 - a)^2/25$

Tandem and Lane Load Bending Moment at a Section

From Eq. [5-6a] & [5-6b]

$M_{tml} = P_3 * (a/L) * ((L - a) + (L - a - 1.2))$

$= 110KN * (a/12.5m) * ((12.5m - a) + (12.5m - a - 1.2m))$

$= 209.44a - 17.6a^2$

$ML = 0.5 * a(L - a)^2 U/L$

$= 0.5 * a(12.5m - a)^2 * (3.1KN/m)/12.5m$

$= 3.1a(12.5 - a)^2/25$

The live load interior and exterior girder shear force and bending moment shall be multiplied by the distribution factors obtained as in live load distribution factors and the dynamic load allowance.

Note: We do not apply dynamic load allowance for lane loading.

Interior Girder

Shear

$V_{trl} = (251.04 - 25.70a) * 1.2 * 0.822$

$= 247.626 - 25.351a$

$V_{tml} = (209.44 - 17.6a) * 0.822 * 1.2$

$= 206.592 - 17.361a$

$VL = [3.1(12.5 - a)2/25] * 0.822$

$= 0.102(12.5 - a)2$

Moment

$M_{trl} = (251.04a - 25.70a^2) * 1.2 * 0.535$

$= 161.17a - 16.50a^2$

$Mtml = (209.44a - 17.6a^2) * 1.2 * 0.535$

$= 134.46a - 11.30a^2$

$ML = 0.102a(12.5 - a)^2 * 0.535$

$= 0.055a(12.5 - a)^2$

Exterior Girder

Shear

$Vtrl = (251.04 - 25.70a) * 1.2 * 1.164$

$= 350.653 - 35.90a$

$Vtml = (209.44 - 17.6a) * 1.164 * 1.2$

$= 292.546 - 24.584a$

$VL = [3.1(12.5 - a)^2/25] * 1.164$

$= 0.144(12.5 - a)^2$

Moment

$M_{trl} = (251.04a - 25.70a^2) * 1.2 * 1.16$

$= 349.45a - 35.774a^2$

$M_{tml} = (209.44a - 17.6a^2) * 1.2 * 1.16$

$= 291.54a - 24.50a^2$

$ML = 0.102a(12.5 - a)^2 * 1.16$

$= 0.118a(12.5 - a)^2$

Rating of Interior Girder

Determination of angle θ

Shear at Critical Section

The critical section near the supports is the greater of $0.5\ d_v \cot\theta$ and d_v,

$d_v = d_e - a/2 \geq 0.9de$,$0.72h$

Where,

d_e is the effective depth measured from the extreme compression fiber to the centroid of the tensile force

θ is the angle of inclination of diagonal compressive stress.

$d_e = h - y_b$

$h = 0.9m$

$y_b = 0.15m$

$d_e \ = \ h - \ y_b = 0.9m - 0.15m = 0.75m$

$d_v \ = \ d_e - a/2 \ \geq 0.9de \ ,0.72h$

$\ = 0.75 - 0.01m > 0.9 * 0.75m, 0.72 * 0.9m$

$= 0.74m$

The calculation procedure assumes a value for θ and solves to verify the assumption. Iteration is used until solution is found.

Assume $\theta \ = \ 40^0$ degrees.

Dead Load Moments at distance a:

$MDC \ = 10.375a(12.5 - a) + 3.83a$

$MDW = 2.363a(12.5 - a)$

Dead Load shear at distance a:

$VDC \ = 133.481 - 20.745a$

$VDW = 18.28 - 2.93a$

Live Load Moment at distance a:

Applying impact factor and distribution factor, the live loads are

$M_{trl} \ = 161.17a - 16.50a^2$

$M_{tml} \ = 134.46a - 11.30a^2$

$ML \ (No \ dynamic \ allowance) = 0.055a(12.5 - a)^2$

$MTL \ = \ Max \ [(Mtrl + ML), (Mtml + ML)]$

Live Load Shear at distance a:

$V_{trl} = \ 247.626 - 25.351a$

$V_{tml} \ = 206.592 - 17.361a$

$VL(No \ dynamic \ allowance) = 0.102(12.5 - a)^2$

$VTL \ = \ Max \ [(Vtrl + VL), (Vtml + VL)]$

$Mu = 1.25MDC + 1.5MDW + 1.75MTL$

$Vu = 1.25VDC + 1.5VDW + 1.75VTL$

Check minimum amount of transverse reinforcement using,

$A_v f_y / b_W s \ > 0.06\sqrt{f_c}' \quad (SI \ Units)$

$(\pi * 14^2/4) * 300/400 * 150 > 0.06\sqrt{24}$

$0.769 > 0.294$

Minimum amount of transverse reinforcement condition is satisfied; determine average shear stress v.

$v = V_u/\Phi b_v d_v$

$\varepsilon_x = [(M_u/d_v) + (0.5N_u) + (0.5V_u cot\,\theta)]/\,2(E_S A_S) < 0.002$

$E_S = 200000Mpa = 200 * 106KN/m^2$

$As = 615.752mm^2 * 12 = 7.389 * 10^{-3}m^2$

If ε_x is negative, it shall be multiplied by the following factor:

$F\varepsilon = (E_S A_S)/\,[(E_C A_C) + (E_S A_S)]$

The values are positive, no need of multiplication by the factor.

From Table 4-2 with v/f_c' and $1000\varepsilon_x$;

After a number of iterations, it is found that $\theta = 29.88^0$ and $\beta = 2.475$,

The critical shear section is at 'a' distance from face of support, $Max\,\{dv, 0.5dvcot\theta\}$

$$a = 2.415m$$

The shear effects are recalculated based on the controlling section

Recalculated shear values at Critical Section

Shear due to dead load (VDC) $= 187.573KN$

Shear due to dead load (VDW) $= 17.931KN$

Shear due to live load (VTL) $= 277.291KN$

Calculate Shear Strength Capacity

a) Using Modified Compression Field Theory

$V_c(ERA) = 0.083\beta\sqrt{f_c'}\,b_v d_v, V_c\ (SI\ Units)$

$b_v = 0.30m,\ \ d_v = 0.74m,\ \ f_c' = 24Mpa$

$V_c = 0.083 * 2.475\sqrt{24} * 300 * 740$

$= 223.414KN$

$V_S = A_v f_y d_v cot\,\theta/s$

Stirrup spacing(s) $= 150mm$

$Av = \pi D^2/4$

$= \pi * 14^2/4$

$= 153.93mm^2$

$Vs = 153.93 * 300 * 740 * cot(29.88^0)/150$

$= 396.505KN$

$V_n = V_c + V_s < 0.25 f_c' b_v d_v$

$V_c + V_s = 223.414KN + 396.505KN$

$= 616.919KN$

$0.25 f_c' b_v d_v = 0.25 * 24000KN/m2 * 0.30m * 0.74m$

$= 1332KN$

Therefore,

$R_n = V_n = 616.919KN$

b) Using 45^0 Truss Model

$V_c = 0.1667 \sqrt{f_c'}\ b_w d$

$= 0.1667 * \sqrt{24} * 300 * 740$

$= 181.298KN$

$V_s = A_v f_y d/s$

$= 153.93 * 300 * 740/150$

$= 227.816KN$

$V_n = V_c + V_s = 181.298KN + 227.816KN$

$= 409.114KN$

LRFR Load Rating Result

Strength – Shear

At Critical Section:

Shear force capacity $(R_n) = 616.919KN$

Shear due to dead load $(VDC) = 187.573KN$

Shear due to dead load $(VDW) = 17.931KN$

Shear due to live load $(VTL) = 277.291KN$

Inventory:

$RF_{Inv} = [\varphi c \varphi s \varphi\ R_n - \Upsilon_{DC} DC - \Upsilon_{DW} DW]/\Upsilon_L LL(1 + IM)$

$= [(0.9 * 1.0 * 1.0 * 616.92) - (1.25 * 187.57) - (1.5 * 17.93)]/(1.75 * 277.29)$

$= 0.606$

Operating:

$RF_{Op} = [\varphi c \varphi s \varphi\ R_n - \Upsilon_{DC} DC - \Upsilon_{DW} DW]/\Upsilon_L LL(1 + IM)$

$= [(0.9 * 1.0 * 1.0 * 616.92) - (1.25 * 187.57) - (1.5 * 17.93)]/(1.35 * 277.29)$

$= 0.785$

Since the rating factor value is less than unity we have to check the bridge girder for ERA Legal Loads.

Rating of Bridge Girder for ERA Legal Loads

Loading for Type 3 Legal Load

Shear Force and Bending Moment Determination

Shear Force

$m_1 = (L - a)/L = (12.5 - a)/12.5$

$m_2 = (L - a - 1.2)/12.5 = (11.3 - a)/12.5$

$m_3 = (L - a - 5.7)/25 = (6.8 - a)/25$

$V_{T3} = (p_1 m_1 + p_2 m_2 + p_3 m_3)$

$= 77(12.5 - a)/12.5 + 77(11.3 - a)/12.5 + 73(6.8 - a)/12.5$

$= (962.5 - 77a + 870.1 - 77a + 496.4 - 73a)/12.5$

$= 186.32 - 18.16a$

By applying live load distribution factor and dynamic load allowance,

$V_{T3} = (186.32 - 18.16a) * 1.2 * 0.822$

$= 183.786 - 17.913a$

Bending Moment

$M_{T3} = 186.32a - 18.16a^2$

By applying live load distribution factor and dynamic load allowance,

$M_{T3} = (186.32a - 18.16a^2) * 1.2 * 0.535$

$= 119.617a - 11.66a^2$

In the same manner as before,

The angle is found as $\theta = 27.975^0$ and $\beta = 2.863$

Recalculated shear values at Critical Section

Shear due to dead load $(VDC) = 99.375KN$

Shear due to dead load $(VDW) = 13.463KN$

Shear due to live load $(VTL) = 181.61KN$

Calculate Shear Strength Capacity

$V_c(ERA) = 0.083\beta\sqrt{f_c'}\, b_v d_v, V_c\ (SI\ Units)$

$b_v = 0.30m,\ \ d_v = 0.74m,\ \ f_c' = 24Mpa$

$V_c = 0.083 * 2.863\sqrt{24} * 300 * 740$

$= 258.438KN$

$V_S = A_v f_y d_v cot\ \theta/s$

Stirrup spacing$(s) = 150mm$

$A_v = \pi D^2/4$

$= \pi * 14^2/4$

$= 153.938mm^2$

$V_S = 153.938 * 300 * 740 * cot(27.975^0)/150$

$= 428.934KN$

$V_n = V_c + V_S < 0.25f_c{'}b_v d_v$

$V_c + V_S = 258.438KN + 428.934KN$

$= 687.372KN$

$0.25f_c{'}b_v d_v = 0.25 * 24000KN/m^2 * 0.30m * 0.74m$

$= 1332KN$

Therefore,

$R_n = V_n = 687.372KN$

LRFR Load Rating Result

Strength – Shear

At Critical Section:

Shear force capacity $(R_n) = 687.3721KN$

Shear due to dead load $(VDC) = 99.375KN$

Shear due to dead load $(VDW) = 13.463KN$

Shear due to live load $(VTL) = 181.61KN$

$RF = [\varphi c \varphi s \varphi\ R_n - \Upsilon_{DC}DC - \Upsilon_{DW}DW]/\Upsilon_L LL(1 + IM)$

$= [(0.9 * 1.0 * 1.0 * 687.37) - (1.25 * 99.38) - (1.5 * 13.46)]/(1.65 * 181.61)$

$= 1.583$

Loading for Type 3-2 Legal Load

Shear Force and Bending Moment Determination

Shear Force

$m_1 = (L - a)/L = (12.5 - a)/12.5$

$m_2 = (L - a - 1.2)/L = (11.3 - a)/12.5$

$m_3 = (L - a - 7.8)/L = (4.7 - a)/12.5$

$m_4 = (L - a - 9.0)/L = (3.5 - a)/12.5$

$V_{T3\text{-}2} = (p_1m_1 + p_2m_2 + p_3m_3 + p_4m_4)$

$= 70(12.5 - a)/12.5 + 70(11.3 - a)/12.5 + 70(4.7 - a)/12.5 + 70(3.5 - a)/12.5$

$= (875 - 70a + 791 - 70a + 329 - 70a + 245 - 70a)/12.5$

$= 179.2 - 22.4a$

By applying live load distribution factor and dynamic load allowance,

$V_{T3\text{-}2} = (179.2 - 22.4a) * 1.2 * 0.822$

$= 176.763 - 22.095a$

Bending Moment

$M_{T3\text{-}2} = 176.763a - 22.095a^2$

Similarly,

$M_{T3\text{-}2} = (176.763a - 22.095a^2) * 1.2 * 0.535$

$= 113.482a - 14.185a^2$

In the same manner as before,

The angle is found as $\theta = 27.975^0$ and $\beta = 2.863$

Recalculated shear values at Critical Section

Shear due to dead load $(VDC) = 99.375KN$

Shear due to dead load $(VDW) = 13.463KN$

Shear due to live load $(VTL) = 181.61KN$

Calculate Shear Strength Capacity

$V_c(ERA) = 0.083\beta\sqrt{f_c'}\, b_v d_v, V_c\ (SI\ Units)$

$b_v = 0.30m,\ \ d_v = 0.74m,\ \ f_c' = 24Mpa$

$V_c = 0.083 * 2.863\sqrt{24} * 300 * 740$

$= 258.439KN$

$V_s = A_v f_y d_v cot\ \theta / s$

Stirrup spacing$(s) = 150mm$

$Av = \pi D^2/4$

$= \pi * 14^2/4$

$= 153.938mm^2$

$V_s = 153.938 * 300 * 740 * cot(27.975^0)/150mm$

$= 428.934KN$

$V_n = V_c + V_s < 0.25 f_c' b_v d_v$

$V_c + V_s = 258.438KN + 428.934KN$

$= 687.372KN$

$0.25 f_c' b_v d_v = 0.25 * 24000KN/m2 * 0.30m * 0.74m$

$= 1332KN$

Therefore,

$R_n = V_n = 687.372KN$

LRFR Load Rating Result

Strength – Shear

At Critical Section:

Shear force capacity (R_n) $= 687.3721KN$

Shear due to dead load (VDC) $= 99.375KN$

Shear due to dead load (VDW) $= 13.463KN$

Shear due to live load (VTL) $= 181.61KN$

$\text{RF} = [\varphi c \varphi s \varphi \text{ R}_{\text{n}} - \Upsilon_{\text{DC}}\text{DC} - \Upsilon_{\text{DW}}\text{DW}]/\Upsilon_{\text{L}}\text{LL}(1 + \text{IM})$

$= [(0.9 * 1.0 * 1.0 * 687.37) - (1.25 * 99.38) - (1.5 * 13.46)]/(1.65 * 181.61)$

$= 1.583$

Loading for Type 3-3 Legal Load

Shear Force and Bending Moment Determination

Shear Force

$m_1 = (L - a)/L = (12.5 - a)/12.5$

$m_2 = (L - a - 1.2)/L = (11.3 - a)/12.5$

$m_3 = (L - a - 6.1)/L = (6.4 - a)/12.5$

$m_4 = (L - a - 10.6)/L = (1.9 - a)/12.5$

$V_{T3\text{-}3} = (p_1 m_1 + p_2 m_2 + p_3 m_3 + p_4 m_4)$

$= 63(12.5 - a)/12.5 + 63(11.3 - a)/12.5 + 73(6.4 - a)/12.5 + 55(1.9 - a)/12.5$

$= (787.5 - 63a + 711.9 - 63a + 467.2 - 73a + 104.5 - 55a)/12.5$

$= 165.688 - 20.32a$

Bending Moment

$M_{T3\text{-}3} = 165.688a - 20.32a^2$

Applying live load distribution factor and dynamic load allowance;

$M_{T3\text{-}3} = (165.688a - 20.32a^2) * 1.2 * 0.535$

$= 106.372a - 13.045a^2$

$V_{T3\text{-}3} = (165.688 - 20.32a) * 1.2 * 0.822$

$= 163.434 - 20.044a$

In the same manner as before,

The angle is found as $\theta = 28.064^0$ and $\beta = 2.992$

Recalculated shear values at Critical Section

Shear due to dead load $(VDC) = 99.483KN$

Shear due to dead load $(VDW) = 13.478KN$

Shear due to live load $(VTL) = 157.872KN$

Calculate Shear Strength Capacity

$V_C(ERA) = 0.083\beta\sqrt{f_c'}\ b_v\, d_v, V_C\ (SI\ Units)$

$b_v = 0.30m,\ \ d_v = 0.74m,\ \ f_c' = 24Mpa$

$V_C = 0.083 * 2.992\sqrt{24} * 300 * 740$

$= 270.083KN$

$V_S = A_v f_y d_v cot\ \theta/s$

Stirrup spacing$(s) = 150mm$

$A_v = \pi D^2/4$

$= \pi * 14^2/4$

$= 153.938mm^2$

$V_S = 153.938 * 300 * 740 * cot(28.064^0)/150mm$

$= 427.330KN$

$V_n = V_C + V_S < 0.25 f_c' b_v d_v$

$V_C + V_S = 270.083KN + 427.330KN$

$= 697.413KN$

$0.25 f_c' b_v d_v = 0.25 * 24000KN/m^2 * 0.30m * 0.74m$

$= 1332KN$

Therefore,

$R_n = V_n = 697.413KN$

LRFR Load Rating Result

Strength – Shear

At Critical Section:

Shear force capacity $(R_n) = 697.413KN$

Shear due to dead load $(VDC) = 99.483KN$

Shear due to dead load $(VDW) = 13.478KN$

Shear due to live load $(VTL) = 157.872KN$

$$RF = [\varphi c \varphi s \varphi\ R_n - \Upsilon_{DC} DC - \Upsilon_{DW} DW]/\Upsilon_L LL(1 + IM)$$

$$= [(0.9 * 1.0 * 1.0 * 697.413) - (1.25 * 99.483) - (1.5 * 13.478)]/(1.65 * 157.872)$$

$$= 1.855$$

The design load rating factor shows that the bridge girder is unfit for design loading but it fulfils the requirement to resist legal loading.

Chapter 7. Results and Discussions

7.1. Results

7.1.1. LRFR Results Due to Design Vehicular Load

When rating a bridge girder for design vehicular load, the rating factors that should be checked are operating rating factor and inventory rating factor. Both rating factors for the analyzed bridge girders are obtained as given in Fig. 7-1 and Fig. 7-2 below.

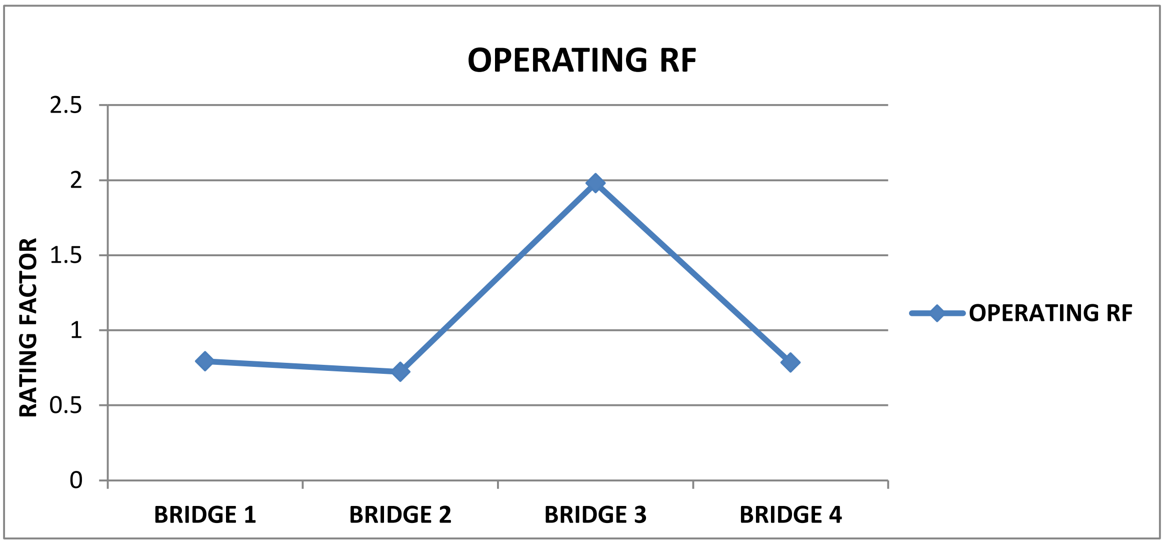


Fig. 7-1 Operating Rating Factors for Bridge Girders

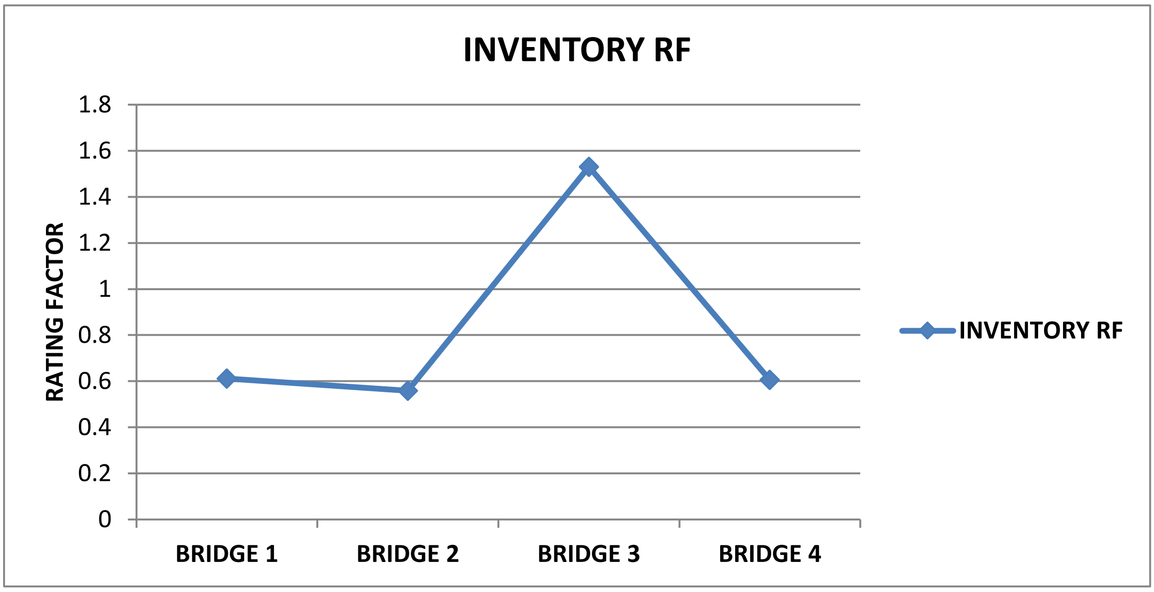


Fig. 7-2 Inventory Rating Factors for Bridge Girders

7.1.2. LRFR Results Due to ERA Legal Loads

As can be seen in Fig. 7-1 and Fig. 7-2, the operating rating factor values are less than unity except that of bridge girder 3. Therefore, it is necessary to analyze the three bridge girders for legal loads to evaluate whether the rating factors are greater than unity or not. Figs. 7-3, 7-4 and 7-5 show the legal load rating factor results obtained for the three bridge girders.

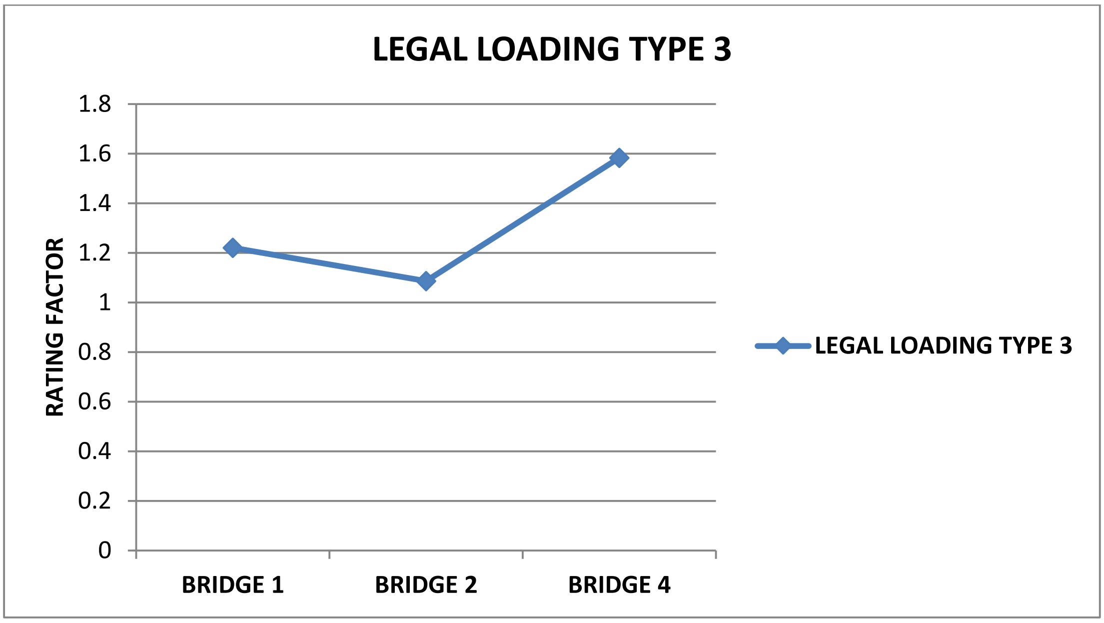


Fig. 7-3 Rating Factors due to Legal Loading Type 3 for Bridge Girders

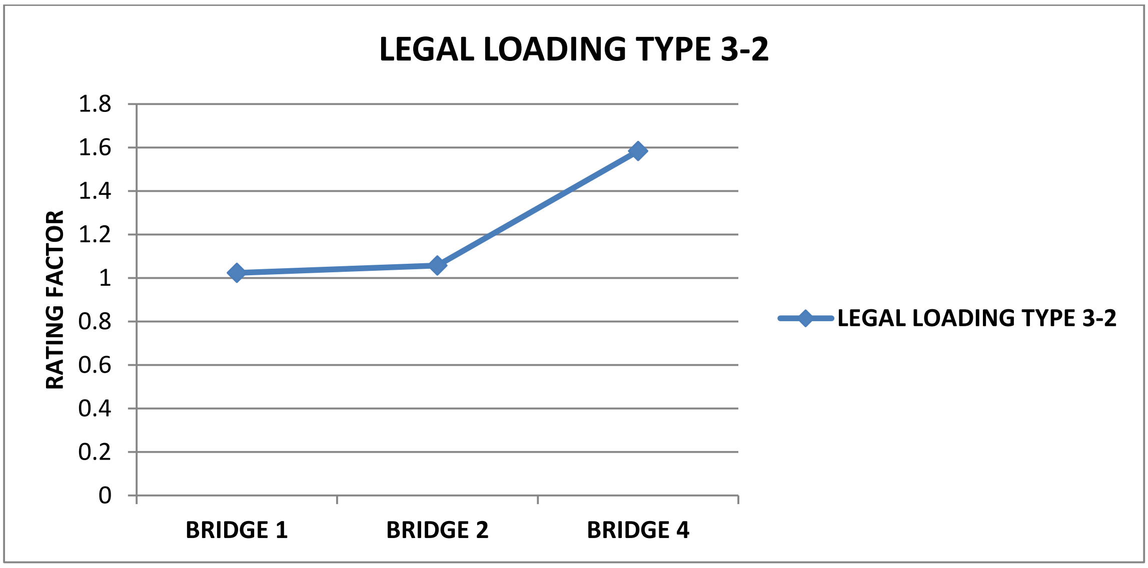


Fig. 7-4 Rating Factors due to Legal Loading Type 3-2 for Bridge Girders

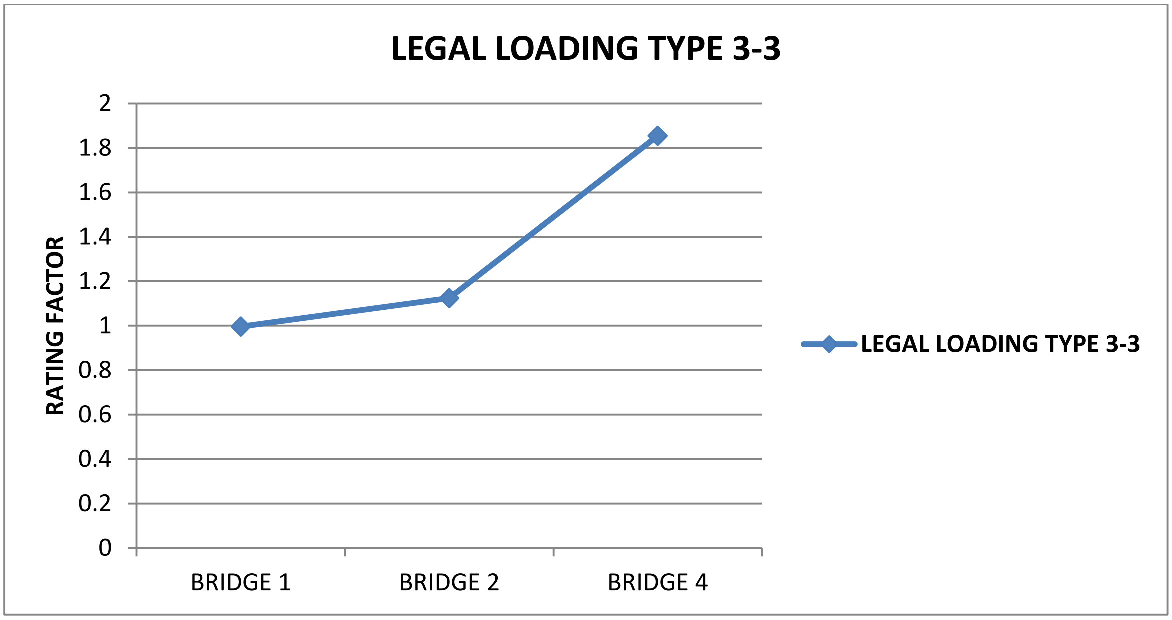


Fig. 7-5 Rating Factors due to Legal Loading Type 3-3 for Bridge Girders

7.1.3. Results of Shear Capacity Comparison b/n 45^0 Truss Model and MCFT Model

In this sub section the four bridge girders' shear capacity from concrete, stirrup and nominal shear capacity and average shear capacity of the four girders obtained from 45-degree truss model are compared with those obtained from Modified Compression Field Theory (MCFT).

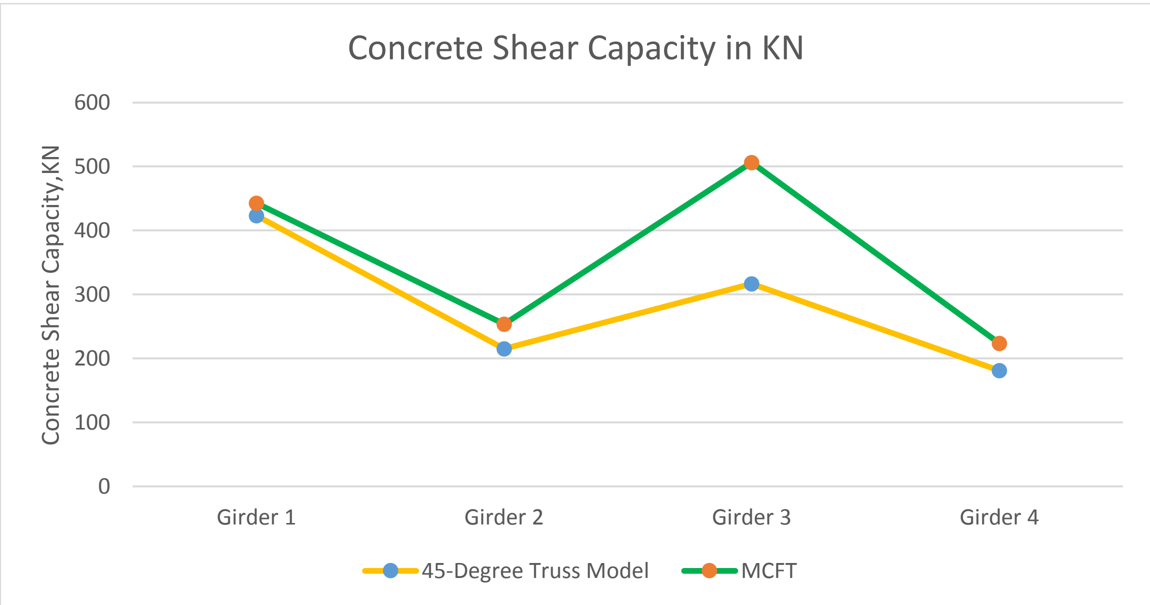


Fig. 7-6 Comparison of Concrete Shear Capacity Obtained from 45-Degree Truss Model and MCFT.

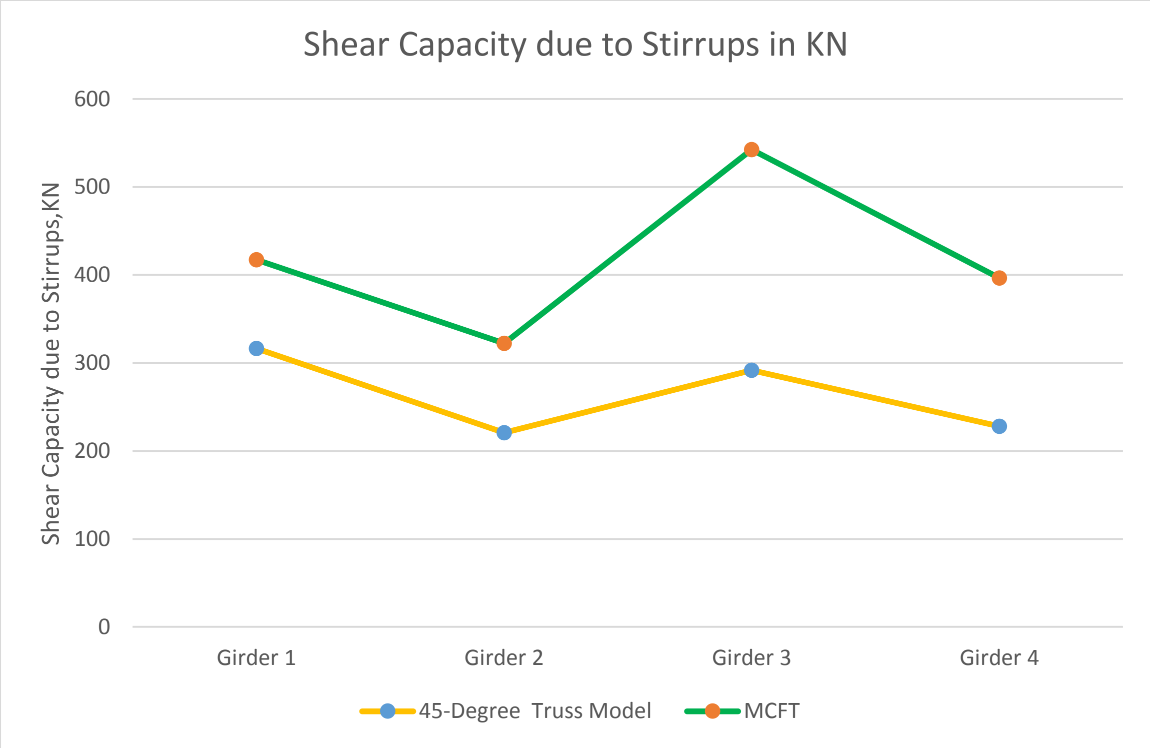


Fig. 7-7 Comparison of Stirrup Shear Capacity Obtained from 45-Degree Truss Model and MCFT.

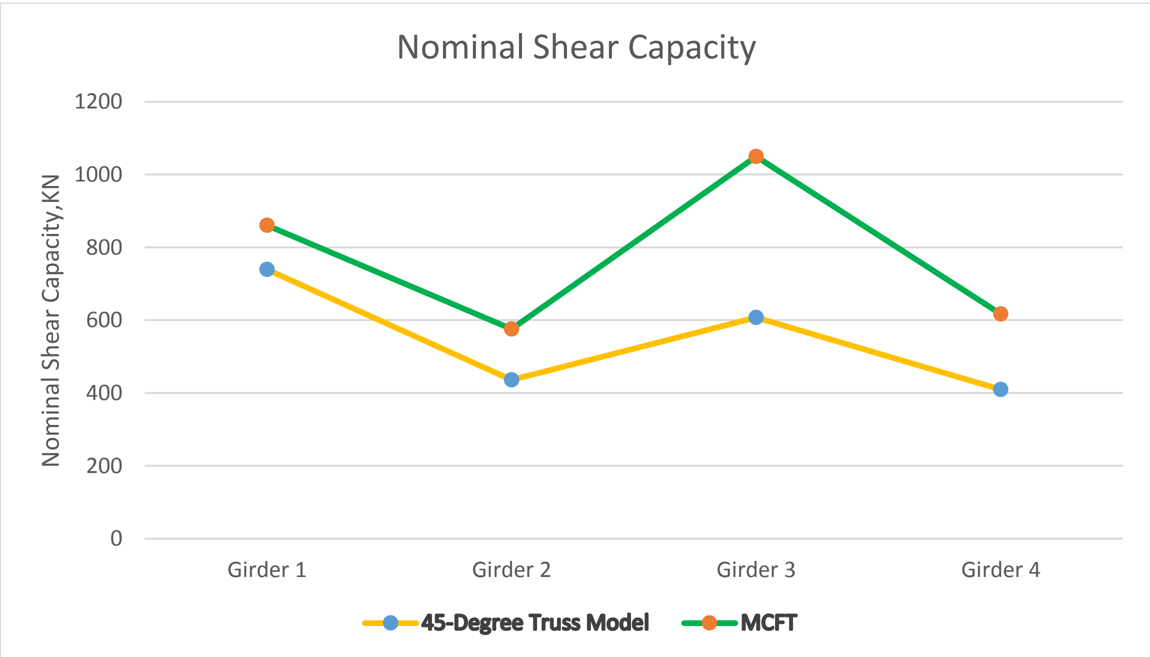


Fig. 7-8 Comparison of Nominal Shear Capacity Obtained from 45-Degree Truss Model and MCFT.

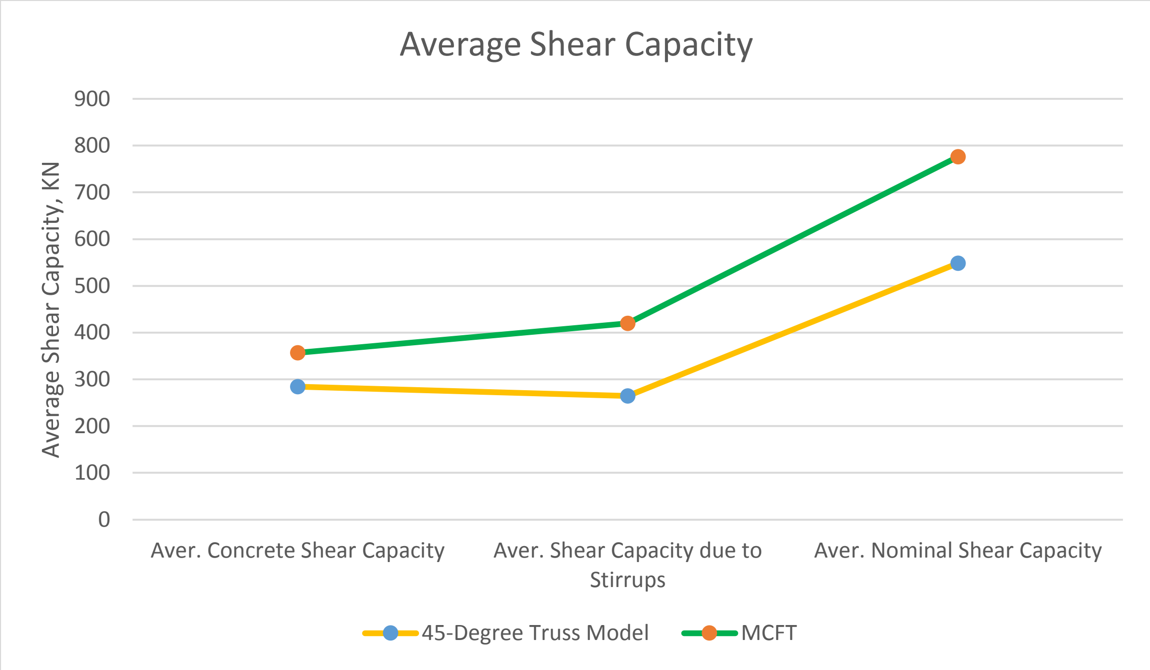


Fig. 7-9 Comparison of Average Shear Capacity Obtained from 45-Degree Truss Model and MCFT.

7.2. Discussion

In the evaluation of shear capacity of bridge girders, the flow chart in Fig. 3-1 shows what procedures we should follow to rate the girders whether they are safe for use or an appropriate action should be taken. First the girders shall be checked for the design vehicular load. The design vehicular load, HL-93 is given in ERA Bridge Design Manual, 2002. LRFR rating factors obtained show that three of the four bridge girders do not have enough shear capacity for the HL-93 design vehicular live load except one bridge girder.

Rating factors due to legal loading were obtained for the three bridge girders which have operating rating values less than unity. These legal load rating factors show that the bridge girders have shear capacity to resist the legal loads. But the bridge girders were rated by using analytical method. So if we use experimental method the result will be different due to the existing condition of bridge girders, construction approach and precision, difference in geometry and material property.

There have been many models for the determination of shear capacity which involves the determination of concrete and reinforcement contributions to the nominal shear capacity of a reinforced concrete. 45-degree truss model was widely used method and recently modified compression field theory is being used in the determination of shear capacity. The question is which method is more reliable and economical method than the other. In this study, results from comparison of the shear capacities from the concrete and transverse reinforcement of the bridge girders are evaluated for the four bridge girders, finally the average shear capacities of the four bridge girders which are obtained by the two modelling methods are compared.

Chapter 8. Conclusions and Recommendations

8.1. Conclusions

From the case study bridge girders, it is obtained that the shear capacities of 75%, i.e three of the four bridge girders, do not fulfill the requirement of LRFD shear design for resisting the shear from HL-93 design vehicular live load. Since this study is analytical considering the design data, the study result shows that bridge girders shall first be rated for shear and checked in accordance with the requirements of newly recommended design standards.

The study also shows that the four bridge girders can support legal loads without any maintenance or closing of the bridge from service due to shear capacity of bridge girders. But since this study considers only design data the rating factor values may also be less than those obtained in this study.

From the comparison of shear capacity of concrete, reinforcement and nominal shear capacity evaluated using 45^0 Truss Model and Modified Compression Field Theory (MCFT), it is obtained that the average concrete shear capacity, average transverse reinforcement shear capacity and average nominal shear capacity of the four bridge girders determined using Modified Compression Field Theory are 25.53%, 58.88% and 41.60% higher than those obtained from 45^0 Truss Model respectively.

8.2. Recommendations

1. From the study, it is obtained that most of the bridge girders' shear design is not complementary with the newly recommended shear design standards, so it is recommended to rate bridge girders for shear design check after bridge girder design.

2. It is important to rate bridge girders for shear continuously in some interval of time because their capacity deteriorates from time to time.

4. In addition to shear rating, it is important to assess bridge girders for other cases.

5. The study shows that reinforced concrete shear capacity obtained from Modified Compression Field Theory is 41.60% greater than from 45^0 Truss Model. So it is better to check the reliability of shear design methods and use the more reliable one in shear design of bridge girders and other structural elements, for instance, beams.

References

1. Abraham Gebre, Master's Thesis (2006), Computer Program for Comparative Study of the Analysis and Design of Slab and T-Girder Bridges, Addis Ababa, AAU
2. Bentz, E. C.; Vecchio, F. J.; and Collins, M. P., "Simplified Modified Compression Field Theory for Calculating Shear Strength of Reinforced Concrete Elements," *ACI Structural Journal*, V. 103, No. 4, July-Aug.2006, pp. 614-624.
3. Collins, M. P.; Mitchell, D.; Adebar, P.; and Vecchio, F. J.(1996), "A General Shear Design Method," *ACI Structural Journal*, V. 93, No. 1, Jan.-Feb. 1996, pp. 36-45.
4. Esfandiari, A.; Adebar, P. (2009), "Shear Strength Evaluation of Concrete Bridge Girders," *ACI Structural Journal*, V. 106, No. 4, Jul.-Aug. 2009, pp. 416-426.
5. Esfandiari, A., "Shear Strength of Structural Concrete Members Using a Uniform Shear Element Approach," PhD thesis, Department of Civil Engineering, University of British Columbia, Vancouver, BC, Canada, Apr. 2009
6. Ethiopian roads Authority(2002), Bridge Design Manual, Addis Ababa
7. Hawkins, N. M.; Kuchma, D. A.; Mast, R. F.; Marsh, M. L.; and Reineck, K. H., 2005 "Simplified Shear Design of Structural Concrete Members Appendices," *NCHRP Report* 549, American Association of State Highway and Transportation Officials, Washington, DC.
8. Hawkins, N. M.; Kuchma, D. A.; Mast, R. F.; Marsh, M. L.; and Reineck, K. H., 2005 "Simplified Shear Design of Structural Concrete Members," *NCHRP Report* 549, American Association of State Highway and Transportation Officials, Washington, DC.
9. Jaramilla, B., and S. Huo. (2005)," Looking to Load and Resistance Factor Rating." *Public Road*, V. 69, No. 1(July–August): pp. 58–65.
10. Lichtenstein Consulting Engineers Inc. (2001). "*NCHRP* Web Document 28 (Project C12-46): Contractor's Final Report: Manual for Condition Evaluation and Load Rating of Highway Bridges Using Load and Resistance Factor Philosophy." 3rdEd.
11. Load Rating of Pre-stressed Concrete Girder Bridges: A comparative Analysis of LFR and LRFR (2005), New Mexico State University, Las Cruces, New Mexico.
12. Murdock, M., Master's Thesis (2009), Comparative Load Rating Study Under LRFR and LFR Methodologies for Alabama Highway Bridges, Auburn, Alabama.
13. O'Malley. C., N. Wang, B. R. Ellingwood and A.-H. Zureick (2009). Condition assessment of existing bridge structures: Report of Task 4- Development of guide

Lines for Condition Assessment, evaluation and Rating of Bridges in Georgia. *GDOT Project Report RP05-01*.
(ftp://ftp.dot.state.ga.us/DOTFTP/Anonymous-Public/Research_Projects)

14. Oregon Department of Transportation(2012), ODOT LRFR Manual, Oregon: ODOT

15. Puckett, J. A., Huo, X. S., Mertz, D., Jablin, M.C., Patrick, M.D., and Peavy, M.D., 2007 "Simplified Live Load Distribution Factor Equations." *NCHRP Report for 12-62*. TRB, National Research Council, Washington, DC.

16. Sivakumar B. (2011), Load and Resistance Factor Rating of Highway Bridges, New York.

17. Theory of Elasticity.(2009), Lecture notes, Civil Engineering Department, Faculty of Technology, AAU

18. Thomas Åhnberg, Master's Thesis (2011),Safety Evaluation of Shear Capacity of Reinforced Concrete Bridges, Göteborg, Sweden, Chalmers University of Technology.

19. Vecchio, F.J., and Collins, M.P., "Predicting the Response of Reinforced Concrete Beams Subjected to Shear Using Modified Compression Field Theory, " *ACI Structural Journal*, V. 85, No. 3, May-June 1988, pp. 258-268.

20. Vecchio, F.J., and Collins, M.P., "The Modified Compression-Field Theory for Reinforced Concrete Elements Subjected to Shear, " *ACI Structural Journal*, V. 83, No. 2, Mar.-Apr. 1986, pp. 219-231.

21. Wisconsin Department of Transportation(WDOT)(2011), " Bridge Rating, " Wisconsin: WDOT