GRAPHENE AND ITS PHYSICAL PROPERTIES

By
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GRAPHENE AND ITS PHYSICAL PROPERTIES

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Abstract

Graphene is a new two-dimensional material prepared succesfully in experiments several years ago. It is a single layer of carbon atoms arranged in a hexagonal (honeycomb) lattice. The two-dimensional nature of graphene leads to many intersting electronic, thermal, optical transparency, and high mobility properties. One particularly interesting property of graphene is that it exhibits the quantum Hall effect at room temperature. In addition graphene is substantially stronger than steel, and it is very stretchable.

Chapter 1

Introduction

A new two-dimensional (2D) crystalline materials have recently been identified and analyzed. [1] The first material in this new class is graphene, a single atomic layer of carbon. This new material has a number of unique properties, which makes it interesting for both fundamental studies and future applications. The electronic properties of this 2D-material leads to, for instance, an unusual quantum Hall effect. [2,3] Classically when an external magnetic field is applied perpendicularly to a current carrying conductor, the charges experience the Lorentz force and are deflected to one side of the conductor. Then equal but opposite charges accumulated on the opposite side. As long as charge flow, a steady electric potential called the Hall voltage exists and the resistivity of the conductor depends linearly on the magnetic field strength. This is known as the classical Hall effect. In 1980 Klaus Von Klitzing discovered that at low temperature and high magnetic field, the polt of resistivity versus applied magnetic field strength becomes an increasing seires of plateaus. This implied that in quantum mechanics, resistance is quantized in units of h/e^2, where h is the planck's constant and e is the electron charge. The pleteaus correspond to the cases where the resistivity is related to the magnetic

field by integer and some fractional value of a quantity known as the filling factor. Both integer quantum Hall effect and the fractional quantum Hall effect have since been observed in graphene. [4-7]

1.1 Definition of graphene

Graphene is a flat monolayer of carbon atoms tightly packed into two-dimensional (2D) honeycomb lattice and is the basic building block for graphite materials of all other dimensionalities. It can be wrapped up into 0D (zero-dimensional), fullerences, rolled into 1D nano-tubes or stacked into 3D graphite.

Graphene have the following properties:

• Imagine a piece of paper but a million times tinner. This is how thick graphene is.

• Imagine a material stronger than diamond. This is how strong graphene is (in the plane).

• Imagine a material more conducting than copper. This is how conductive graphene is.

• Having such a material in hand, one can easily think of many useful things that can eventally come out.

(From a recently interview with Andre Geim)

1.2 Brief History of graphene

Graphene had already been studied theoretically in 1947 by P.R. Wallace [8] as a text book example for calculations in solid state physics. He predicted the electronic structure and noted the linear dispersion relation. It came as a surprise to the physics community when Andre

Geim, Konstantin Novoselov and their collaborators from the University of Manchester (UK), and the Institute for Microelectronics Technology in Chernogolovka (Russia), presented their results on graphene structures. They published their results in October of 2004 in Science. [1]

1.3 Application of graphene

Graphene can be used for the following applications.

- Electronics

  → Sensors, transistors, interconnects, printable electronics

- Transparent displays.

- Power storage

  → Battery, electrodes, super−capacitors

- Medicine

  → Membranes, sensors

Chapter 2

Crystal Structure And Properties Of Graphene

2.1 Crystal Structure

An ideal crystal is formed by an infinite repetition of identical groups of atoms. Each group of atoms called the basis, is attached to a point on a periodic array called the lattice. In the case of ideal graphene, a two atom basis is attached to a hexagonal lattice and the result is a two-dimensional honeycomb crystal.

Graphene consists of a single layer of carbon atoms, arranged in a honeycomb lattice (Fig. 2.1(c)). It has been studied theoretically for a long time as building block of graphitic materials in other dimensions. [8] The most famous allotrophe of graphene is graphite, consisting of stacked graphene layers held together only by weak Van der Waals forces. 1-dimensional carbon nanotubes and 0-dimensional fullerenes can be described as rolled up graphene sheets. But in contrast to these materials, until its discovery in 2004 by Novoselov and Geim et al. [5],

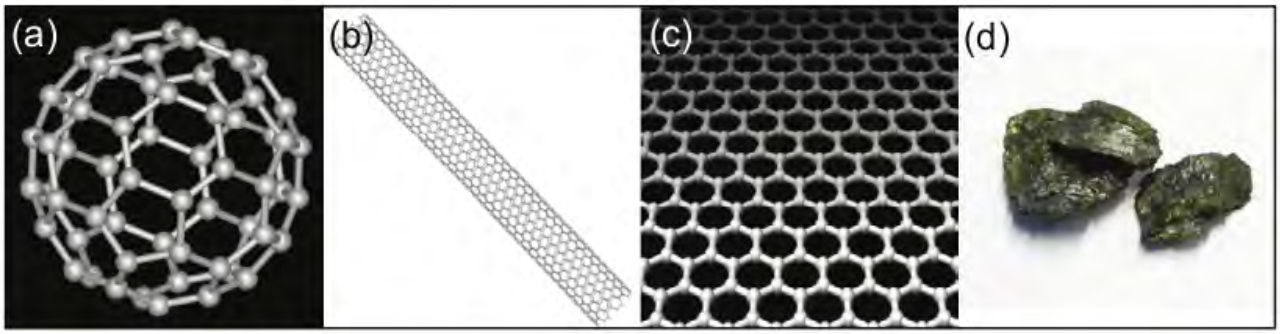

Figure 2.1: Graphitic materials in different dimensions: (a) 0-dimensional fullerene, (b) 1-dimensional carbon nanotube, (c) 2-dimensional graphene, (d) 3-dimensional graphite

Graphene was expected to be thermodynamically unstable [9] and therefore not to exist in real life. The bonds in carbon atoms in graphene are in sp^2 hybridization. The sp^2 orbitals form the three strong covalent bonds to the nearest neighbors. These strong bonds give graphene its extraordinary mechanical strength, making it possible to have stable free-standing graphene sheets, being only one atom thick [10]. The remaining p-electron per atom is delocalized over the whole graphene molecule, and responsible for the electric conductivity. As shown below.

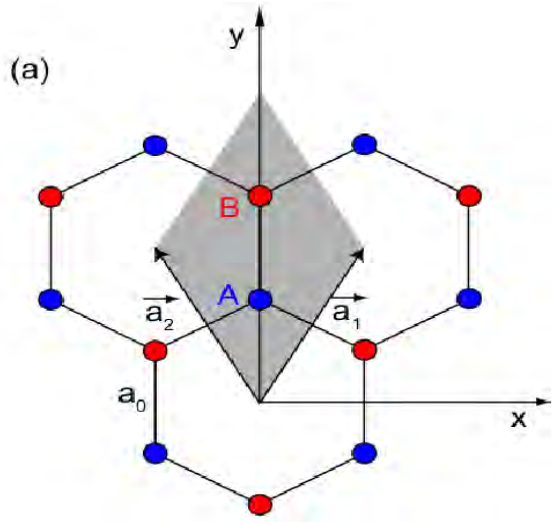

Figure 2.2: Lattice structure of graphene. The atoms belonging to the two sublattices A and B are represented by blue (A) and red (B) circles, the lines between the circles indicate the chemical bonds. a_0 is the nearest-neigbour distance. The unit cell is depicted in grey together with the primitive lattice vectors $\vec{a}_{1,2}$

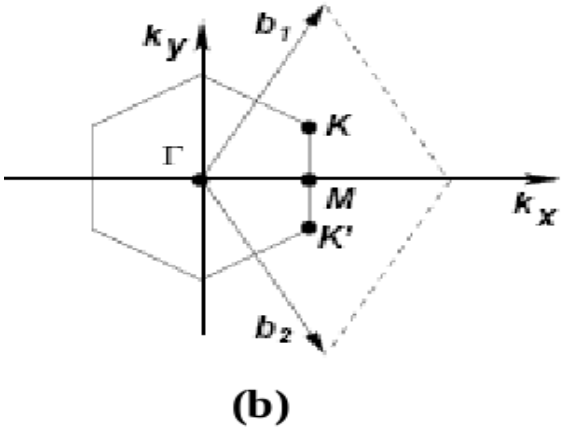

Figure 2.3: First Brillouin zone in the reciprocal lattice of graphene, with indication of the high-symmetry points.

Fig.2.2:(a) let us name the primitive vectors in real space as $\vec{a}_1$ and $\vec{a}_2$ and in reciprocal lattice as $\vec{b}_1$ and $\vec{b}_2$. The real space lattice voctors and the corresponding reciprocal ones are

$$\vec{a}_{1,2} = \frac{\sqrt{3}a}{2}(\sqrt{3}, \pm 1), \tag{2.1}$$

$$\vec{b}_{1,2} = \frac{2\pi}{\sqrt{3}a}(\frac{1}{\sqrt{3}}, \pm 1). \tag{2.2}$$

where a = 0.142 nm is the distance between the atoms. The first Brillouin zone is a hexagon, where the corners form two inequivalent groups of K points, traditionally labelled K and K' and are given by:

$$K = \frac{2\pi}{3a}(1, \frac{1}{\sqrt{3}}) \tag{2.3}$$

and

$$K' = \frac{2\pi}{3a}(1, \frac{-1}{\sqrt{3}}). \tag{2.4}$$

2.2 Properties of Graphene

Graphene is a single layer of carbon atoms arranged in a hexagonal lattice. The two-dimensional nature of graphene leads to many interesting electronic, thermal, and optical properties. One particularly interesting property of graphene is that it exhibits the quantum Hall effect at room temperature.

2.2.1 Density of graphene

The unit hexagonal cell of graphene contains two carbon atoms and has an area of 0.052 nm^2. We can thus calculate its density as being 0.77 mg/m^2. A hypothetical hammock measuring $1m^2$ made from graphene would thus weigh 0.77 mg.

2.2.2 Optical transparency of graphene

The high frequency conductivity for Dirac fermions in graphene has been stated to be a constant equal to $\frac{\pi e^2}{2h}$, from the infrared through the visible range of the spectrum. The optical transmittance T and reflectance R are then T = 97.7% and R = 2.3 % . The expression of T and R in terms of fundamental constants that do not directly involve material parameters is stated to be a result of the structure and electronic properties of graphene.

2.2.3 Strength of graphene

Graphene has a breaking strength of 42 N/m. For a hypothetical steel film of the same thickness as graphene (which can be taken to be 3.35 A^0, i.e. the layer thickness in graphite), this would give a 2D breaking strength of (0.084 - 0.40) N/m. Thus graphene is more than 100 times stronger than the strongest steel.

2.2.4 Electrical conductivity of graphene

The sheet conductivity of a 2D material is given by $\sigma = en\mu$. The mobility is theoretically limited to $\mu = 2 \times 10^5 \ \frac{cm^2}{Vs}$ by acoustic phonons at a carrier density of n = $10^{12} \ cm^{-2}$. Using the layer thickness we get a bulk conductivity of $0.96 \times 10^6 \ (\Omega cm)^{-1}$ for graphene. This is somewhat higher than the conductivity of copper, which is $0.60 \times 10^6 \ (\Omega cm)^{-1}$.

2.2.5 Thermal properties

Since the carrier density of non-doped graphene is relatively low, the electronic contribution to thermal conductivity (Wiede-mann-Franz law) is negligible. The thermal conductivity (κ) of graphene is thus dominated by phonon transport, namely diffusive conduction at high temperature and ballistic conduction at sufficiently low temperature. [11] Based on the Green-Kubo approach, it has a $\kappa \propto \frac{1}{T}$ dependence for defect-free graphene as temperature T increases beyond about 100 K. At room temperature thermal conductivity of about 6000 $\frac{W}{mK}$ for a suspended monolayer graphene was predicted, and this value was stated to be much higher than that of graphitic carbon.

2.2.6 Giant mobility and lowest resistivity

Graphene has a very high electron mobility at room temperature, with values of $15,000$ $\frac{cm^2}{Vs}$ and it can be increased upto 2×10^5 $\frac{cm^2}{Vs}$ at a carrier density of 10^{14} cm^{-2}. The corresponding resistivity of the graphene sheet would be 10^{-6} $\Omega.cm$, less than the resistivity of silver, the lowest resistivity substance known at room temperature.

Chapter 3

The Hall Effect

3.1 Classical Hall Effect

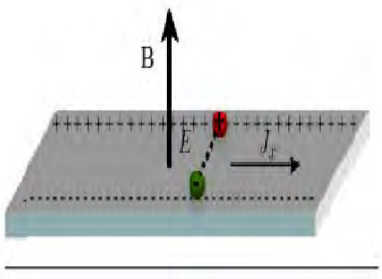

Figure 3.1: Physical picture of classical quantum Hall effect

The Hall effect describes how current flowing through a sample will be affected by the application of a magnetic field. Consider a two-dimensional conducting plate in an applied electric field, E. According to Ohm's law, the current flowing through the plate, I, is proportional to the applied voltage, V, and is inversely proportional to the resistance of the plate, R. This is equivalent to

$$\mathbf{J} = \sigma \mathbf{E} \tag{3.1}$$

where σ is the conductivity of the plate, and J = qρ**v** is the current density for particles of charge q and ρ is carrier density moving with a velocity **v**. Ohm's law states that current will flow in the same direction as the applied electric field. In 1879, Edwin Hall discovered that in the presence of an applied magnetic field, the current in the plate will actually flow in a direction perpendicular to the applied electric field. Consider the conducting plate in the presence of an applied magnetic field perpendicular to the xy-plane, $B = B_z$. Then the charges flowing through the plate are subject to the Lorenz force:

$$\mathbf{F_{Lorenz}} = q\left[\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}\right] \tag{3.2}$$

If the applied electric field is in the y-direction, $E = E_y$, then the particle will move with velocity **v** given by

$$cE = -v \times B \rightarrow v = \frac{E_y}{Bc} \tag{3.3}$$

Then from Ohm's law, the conductivity of the plate, or Hall conductivity, is given by

$$\sigma_H = \frac{J}{E} = \frac{q\rho\mathbf{v}}{E_y} = \frac{\rho cq}{B} \tag{3.4}$$

Similarly, the Hall resistance $R_H = \frac{1}{\sigma}$ is defined as:

$$R_H = \frac{B}{\rho qc} \tag{3.5}$$

The Hall resistivity in equation (3.5) above is proportionl to the magnetic field B and inversely proportional to the carrier density, ρ. This effect is known as the **Classical Hall effect**.

3.2 Landau Levels

The important concept in the explanation of the quantum Hall effect is Landau Levels. Consider an electron confined to the xy-plane in the presence of a uniform magnetic field in the z-direction.

If we choose the Landau gauge of the vector potential **A**, to be

$$A_x = -B_y,$$

$$A_y = A_z = 0 \tag{3.6}$$

then Schrodinger's equation for electrons in a homogeneous magnetic field is

$$\hat{H} = \frac{1}{2m}(\hat{p} - \frac{e}{c}\hat{A})^2 \tag{3.7}$$

This can be rewritten as

$$\hat{H}\Psi = \frac{1}{2m}\left[\left(\hat{p}_x + e\vec{B}y\right)^2 + p_y^2 + p_z^2 - 2m\mu s_z\vec{B}\right]\Psi = E\Psi, \tag{3.8}$$

where s_z is the z component of the electron spin, and μ is the magnetic moment of electrons [16]. The Hamiltonian, Eq. (3.8), does not include x and z explicitly. So p_x and p_z are the constant of motion, and we can take the wave function as

$$\Psi(x, y, z) = exp\left[\frac{i}{h}(p_x x + p_z z)\right]\varphi(x) \tag{3.9}$$

and putting it into Eq. (3.8), we obtain

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2}+\frac{m\omega_c^2}{2}(y-y_o)^2\right]\varphi(y)=\left[E+\mu s_z B-\frac{(P_z)^2}{2m}\right]\varphi(y), \tag{3.10}$$

where $\omega_c=\frac{eB}{mc}$, $y_0=\frac{-cp_x}{eB}$, which is just a shifted harmonic oscillator equation. So we can directly obtain the eigen-values of the energy

$$E_n=(n+\frac{1}{2})\hbar\omega_c-\mu S_z B+\frac{P_z^2}{2m}. \tag{3.11}$$

If electrons are confined in two-dimensions, then the last term in Eq. (3.11) will vanish. Furthermore, when the system is in a strong magnetic field, electrons are totally spin-polarized so that the second term is a constant. Thus, Eq. (3.11) gives a series of discrete energy levels known as Landau Levels which will take the form:

$$E_n=(n+\frac{1}{2})\hbar\omega_c. \tag{3.12}$$

The degeneracy factor for each Landau Level is given by the number of center coordinates y_0 within the sample. For a given sample with the area of L_xL_y, the space of center coordinates is

$$\Delta y_0=\frac{c}{eB}\Delta p_x=\frac{c\hbar}{eB}\Delta K_x=\frac{c\hbar}{eB}(\frac{2\pi}{L_x})=\frac{ch}{eBL_x}. \tag{3.13}$$

So the degeneracy factor N_B is

$$N_B=\frac{L_y}{\Delta y_0},$$

and Substitute Δy_o from equation (3.13), we get

$$N_B = \frac{L_y}{\frac{ch}{eBL_x}},$$

$$N_B = \frac{L_x L_y B}{\frac{ch}{e}},$$

$$N_B = \frac{SB}{\frac{ch}{e}},$$

$$N_B = \frac{\Phi}{\Phi_0}, \tag{3.14}$$

where $\Phi = S\mathbf{B} \Rightarrow$ (Φ is a magnetic flux through area (S)) , and

$\Phi_0 = \frac{ch}{e} \Rightarrow$ (Φ_0 is elementary flux).

Additionally, the degeneracy factor per unit area is

$$n_B = \frac{N_B}{S} = \frac{eB}{ch}. \tag{3.15}$$

It should be noted that this degeneracy factor for each Landau Level is independent of semiconductor parameters like effective mass.

3.3 Filling Factor

Another important feature in describing the quantum Hall effect is the filling factor. The filling factor (ν), is the number of occupied Landau Levels for electrons in a given magnetic field. It is defined as:

$$\nu = \frac{number of total electrons}{\frac{number of states}{electron}}.$$

But in terms of the degeneracy of the Landau states ν is

$$\nu = \frac{\rho}{n_B}, \tag{3.16}$$

where ρ is carrier density for two dimensional electron and $n_B = \frac{e\mathbf{B}}{hc}$ is the degeneracy of the Landau states. Then the carrier density becomes:

$$\rho = \nu \frac{eB}{hc}. \tag{3.17}$$

From Ohm's law the Hall conductivity σ is

$$\sigma = \frac{J}{E} = \frac{\rho q \mathbf{v}}{\frac{Bv}{c}} = \frac{\rho q c}{B}. \tag{3.18}$$

Substitute equation (3.17) into equation (3.18), Hall conductivity (σ) is

$$\sigma = \nu \frac{e^2}{h} \tag{3.19}$$

where $\nu = 1, 2, 3...$ Similarlly, the Hall resistivity with $q = e$ is

$$R_H = \frac{1}{\sigma} = \frac{h}{\nu e^2}. \tag{3.20}$$

Therefore, Hall resistance is quantized in units of $\frac{h}{e^2}$ and inversily proportional to the filling factor (ν) .

3.4 Integer Quantum Hall Effect

In 1980, Klitzing et al. found that when the electrons of a sample are well confined in two dimensions (e.g. semiconductor heterostructures) and in the condition of a strong magnetic field and low temperature, the measured Hall resistivity or Hall voltage is no longer inversely proportional to the carrier density (equivalent to a gate voltage applied to the sample) or proportional to the magnetic field B. That is, the Hall resistance is quantized and it is given by (3.20)

$$R_H = \frac{h}{\nu e^2} = \frac{25812.8}{\nu}\Omega \ , \qquad \nu\text{=1,2,3,...}$$

This effect is called integer quantum Hall effect (QHE). Interestingly, the values of the Hall resistance are independent on the materials chosen in the measurements.

Then the experimental results show that when the filling factor is an integer, a quantized Hall resistivity ρ_{xy} is always expected. Besides changing magnetic field, therefore, we can also change carrier density (i.e. changing the gate voltage in experiments) to make the filling

factor be an integer and realize QHE. Moreover, in the quantum Hall regime the longitudinal resistivity ρ_{xx} becomes immeasurably small. Thus, if we write resistivity as a matrix, then it will have the form as

$$\rho = \begin{pmatrix} \rho_{xx} \ , \ \rho_{xy} \\ \\ \rho_{yx} \ , \ \rho_{yy} \end{pmatrix} = \begin{pmatrix} 0 \ , \ \frac{h}{\nu e^2} \\ \\ \frac{-h}{\nu e^2} \ , \ 0 \end{pmatrix}. \tag{3.21}$$

Since conductivity is the inverse of resistivity, we can obtain

$$\sigma_{xy} = \frac{-\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}, \qquad \sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}. \tag{3.22}$$

By considering the spin degeneracy and possible sublattice degeneracy, we add a factor g to the conductivities and finally obtain their values in the quantized Hall regime to be

$$\sigma_{xy} = -\nu \frac{ge^2}{h}, \qquad \sigma_{xx} = 0, \qquad \nu = 0, \pm 1, \pm 2, ... \tag{3.23}$$

The Integer Quantum Hall Effect (IQHE) refers to the scenario where the filling factor, ν, has an integer value. Integer values of the filling factor describes a system of non-interacting electrons where the highest Landau Level is completely filled. Once the Landau Level is completely filled, a gap exists requiring a finite amount of energy to reach the next degenerate Landau Level. However, impurities in the sample create localized potentials that can trap electrons in localized states. Therefore, if the filling factor is changed slightly, the extra electrons fill the localized states and do not contribute to the current.

3.5 Fractional Quantum Hall Effect

In high magnetic field each electron captures an even number (2m) of quantized vertices to become a composite fermion (CF). The dynamics of composite fermions are described by an effective magnetic field given by:

$$B^* = B - 2\pi(2m)\rho/e, \tag{3.24}$$

where ρ is carrier density for two dimensional electron.

The composite particles do not feel the external field B but the effective field B * . Therefore, the FQHE of electrons can be considered as an IQHE of these composite particles [16]. The Hall plateaus at fractional filling factors appear as integer filling factors except for the trivial modification that now each electron carries with it 2m flux quanta.

For a carrier density ρ , an effective filling factor ν^* for the composite particle can be written as [18]

$$\nu^* = \frac{2\pi\rho}{eB^*}. \tag{3.25}$$

In the lowest Landau Level the electron filling factor is:

$$\nu = \frac{2\pi\rho}{eB}. \tag{3.26}$$

Substituting $\rho = \frac{\nu^* eB^*}{2\pi}$ from Eq. (3.26) to Eq. (3.25), we get

$$B = B^*(1 + 2m\nu^*). \tag{3.27}$$

From Eqs. (3.25), (3.26) and (3.27), we can write

$$\frac{\nu}{\nu^*} = \frac{B^*}{B} = \frac{1}{1+2m\nu^*}.$$

Or,

$$\nu = \frac{\nu^*}{1+2m\nu^*}. \tag{3.28}$$

From the recent results [12-14] about the IQHE in monolayer graphene, the quantized Hall conductivity is found to be

$$\sigma_{xy} = \pm 2(2n+1)\frac{e^2}{h} \equiv \pm 4(n+\frac{1}{2})\frac{e^2}{h}. \tag{3.29}$$

In the case of graphene, the effective filling factor ν^* is associated with the integer quantum Hall effect of composite particles and if we ignore the spin then we can write:

$$\nu^* = (2n+1), \qquad n = 0, 1, 2, 3..... \tag{3.30}$$

Hence, the quantized Hall conductivity in graphene in FQHE can be written as:

$$\sigma_{xy} = \pm\frac{2n+1}{2m(2n+1)+1}\frac{2e^2}{h}. \tag{3.31}$$

where n is an integer and (2m) is an even number. Here, the factor 2 is due to the spin degeneracy. Here the positive values correspond to the Dirac electrons (conduction band) and the negative values correspond to the Dirac holes (valance band) in the monolayer graphene.

3.6 Landau Levels Energy In Graphene

In two dimensions, when electrons are subjected to a perpendicular magnetic field they follow close cyclotron orbits that in quantum mechanics are quantized. The energy of these orbitals has discrete values. These orbitals are called Landau Levels (LLs). The number of electrons per Landau Level is directly proportional to the strength of the applied magnetic field. At strong magnetic field each LL is highly degenerate (i.e. there are so many states which have the same energy) and each LL has so many states that all of the free electrons in the system stay only in some LLs. In case of conventional 2-dimensional electron system (non relativistic case) the Hamiltonian of the system is given by:

$$H = \frac{p^2}{2m}. \tag{3.32}$$

But in case of monolayer graphene (relativistic case)

$$H = v_F(\vec{\sigma} \cdot \vec{P}) = v_F(\sigma_x P_x + \sigma_y P_y) = \hbar v_F \sigma \cdot K, \tag{3.33}$$

where $v_F = 10^6 \ \frac{m}{s}(Fermi\ velocity)$

K =quasi-particles momentum

σ = are the 2D Pauli matrices

$$\sigma_x = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \qquad \sigma_y = \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix}.$$

Therefore, the energy-dispersion becomes

$$E = \hbar v_F k. \tag{3.34}$$

In presence of magnetic field the effective Hamiltonian for nonrelativistic case [21-22] is given by,

$$\vec{H} = \frac{(\vec{p} - e\vec{A})^2}{2m}, \tag{3.35}$$

and for monolayer graphene is given by:

$$H = v_F[\vec{p} - e\vec{A}]\sigma H = v_F \vec{\pi}\sigma H = v_F \begin{bmatrix} 0 \ , & (\pi_x - i\pi_y) \\ (\pi_x + i\pi_y) \ , & 0 \end{bmatrix}. \tag{3.36}$$

Thus from ladder operators, we can write

$$\pi_x = \frac{\hbar}{\sqrt{2}\ell_B}(a^\dagger + a), \tag{3.37}$$

$$\pi_y = \frac{\hbar}{i\sqrt{2}\ell_B}(a^\dagger - a). \tag{3.38}$$

where $\hat{a}^\dagger, \hat{a}$ are operators, substitute Eq. (3.37) and Eq. (3.38) into Eq. (3.36), we get

$$H = v_F \frac{\hbar\sqrt{2}}{\ell_B} \begin{bmatrix} 0, a \\ a^\dagger, 0 \end{bmatrix}$$

. Putting value of $\omega' = \frac{v_F\sqrt{2}}{\ell_B}$

$$H = \omega'\hbar \begin{bmatrix} 0 \ , & a \\ a^\dagger \ , & 0 \end{bmatrix}. \tag{3.39}$$

In perpendicular magnetic field the Hamiltonian generates a discrete Landau Level energy spectrum. The corresponding eigenfunctions can be expressed in terms of the conventional nonrelativistic Landau functions.

The eigenvalue equation

$$H\Psi_n = E_n\Psi_n, \tag{3.40}$$

where the wave functions $\Psi_n = \begin{bmatrix} u_n \\ \\ \upsilon_n \end{bmatrix}$.

Hence from equation (3.40)

$$\omega'\hbar \begin{bmatrix} 0, a \\ \\ a^+, 0 \end{bmatrix} \begin{bmatrix} u_n \\ \\ \upsilon_n \end{bmatrix} = E_n \begin{bmatrix} u_n \\ \\ \upsilon_n \end{bmatrix}. \tag{3.41}$$

From the equation (3.41), we get

$$\hbar\omega' a\upsilon_n = E_n u_n. \tag{3.42}$$

$$\hbar\omega' a^\dagger u_n = E_n \upsilon_n. \tag{3.43}$$

Multiplying equation (3.42) with (3.43), we get

$$(\hbar\omega')^2 a^\dagger a u_n \upsilon_n = E_n^2 \upsilon_n u_n. \tag{3.44}$$

Solving equation (3.44) we get, the energy eigenvalue as

$$E_n^2 = (\hbar^2\omega'^2)n$$

$$E_n = \pm\frac{\hbar v_F\sqrt{2n}}{\ell_B}$$

$$E_n = \pm\frac{\hbar v_F\sqrt{2n}}{\sqrt{\frac{\hbar}{eB}}}, \text{ (putting the value of } \ell_B)$$

$$\Delta E_n = \pm(\sqrt{2(n+1)\hbar e B v_F^2} - \sqrt{2n\hbar e B v_F^2}) \tag{3.45}$$

This is the expression for LL energy spectrum in case of monolayer graphene. This expression shows that, in contrast to the case of conventional two Dimensional Electron System (2DES), the Landau Levels in monolayer graphene are not equidistant and the largest energy separation is between the zero and the first Landau Level. Here the positive values correspond to the Dirac electrons (conduction band) and the negative values correspond to the Dirac holes (valance band) in the monolayer graphene. There is a single state E_0 at zero energy.

3.7 The Quantum Hall Effect In Graphene

Graphene has a large concentration of charge carriers which keeps the lowest Landau Level completely populated at high magnetic fields. Therefore, any carriers above the lowest Landau Level will not be able to overcome the energy gap, and the quantum Hall effect is observed. The quantum Hall effect has been observed for the integer values of the filling factor $\nu = 0, \pm 1, \pm 2, \pm 6, \pm 10, ...$ as well as the fractional filling factors $\nu = \frac{1}{3}, \frac{2}{3}, \frac{1}{5}$. Because graphene exhibits both the integer and fractional quantum Hall effects, it is ideal for studying QHE, and may be proven useful in the development of quantum computers. [17-20]

Chapter 4

Conclusion

Graphene is a monoatomic layer of graphite with carbon atoms arranged in a two dimensional honeycomb lattice configuration. The electronic structure of graphene can be modelled by two-dimensional massless relativistic fermions. This property gives rise to numerous applications both in applied science and in theoretical physics. Graphene research is one of the fastest growing areas in material science, but it is still a young field. There are many challenges and opportunities for investigation, because graphene is not a standard solid state material. It is a new star in material science. Graphene has some peculiar properties which is not matched with the ordinary metal and semiconductor. So it is necessary to establish a new generalized theory for it. Graphene has potential for serving as an excellent electronic material that can be used in place of silicon for making ultrafast and stable transistors. It is considered as a promising candidate for electronics and spintronics applications. It provides a bridge between condensed matter physics and quantum electrodynamics.

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