Infinite Magnetic Poles of Pulsars

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By

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Abstract

The recent observations and theory indicates that the presence of infinite non-dipolar pulsars magnetic fields, it has become more and more clear that pulsars posses magnetic field structure, which are much more complicated than the simple assumption of dipolar pulsars magnetic fields. In this work, we analyze the presence of infinite non-dipolar components of the pulsar magnetic field, as derived from a recent model. We derived both the dipolar and infinite non-dipolar components of the field separately. We have approached the problem by applying post-Newtonian approximation, to find the infinite non-dipolar pulsar magnetic field. Among infinite non-dipolar components of pulsar's magnetic field, we have both derived the expression for quadrapole and octupole component of the magnetic field and studied their implication.

Introduction

Observations and theory suggest that complex multipolar magnetic fields prevail near the surface of neutron stars (stellar remnant which support themselves through neutron degeneracy pressure against the gravitational force), and play an important role in the physics of rotation powered pulsar (rotating neutron stars). The complexity of the surface magnetic field is determined by the evolution of magnetic fields in neutron stars. The magnetic field is generated by currents flowing in the thin outer crust of the neutron star, with thickness $\Delta r \ll \mathrm{R_s}$, where $\mathrm{R_s} \sim 10^6$ cm is the neutron star radius. In this case, the superposition of magnetic multi poles of order $\mathrm{L}\sim\frac{R_s}{\Delta r} \geq 10$ dominates the exterior field [1]. Assuming that the evolution of magnetic field in isolated neutron star is due to ohmic decay, the evolution of such high-order multi polar components ($\mathrm{L} \leq 25$) is very similar to that of the dipolar field [2]. The contribution of non-dipolar magnetic components, present in the close vicinity of the neutron star in the form of either multipoles of order higher than the dipole one, or the 'sunspot like' clumps, is necessary. These components are supposed to be strong enough to change the local topology of the magnetic field in the vicinity of pulsar polar caps [3]. For a magnetic dipole aligned with the rotation axis the "polar cap" is circular. We assume the magnetic field line structure of a radio pulsar to be purely dipolar in nature, the non-dipolar stellar magnetic component has been discussed in detail by [1], where it is concluded that "there is no clear evidence either theoretical or observational for large scale non-dipolar field in any pulsar." Even so, since at large distances multipolar components die out faster than the dipole ones [4]. NS are built on a basic assumption that NS matter is neutral composed of mainly heavy nuclei, neutrons, protons, and electrons. The spinning separated charges, which comes as a result of plasma diffusion are the sources for the magnetic fields of pulsars [5]. Magnetic fields allow pulsars to be distinguished from each other and classified into phenomenological very different groups. They are classified as Radio pulsar, X-ray binaries, Magnetars and Thermal X-ray emitters. Binary systems associated with mass transfer onto a neutron star are divided into high-mass and low-mass X-ray binaries (according to the mass of the companion star), with substantially different properties.

It is widely accepted that the magnetic field structure of pulsars may significantly differ from pure dipole. There are different observational hints for the existence of non-dipolar fields, among these are the spectra from pulsars J0218 + 4232 and J2144 - 3933 [6]. There are also other pulsars with non-dipolar fields like the vela pulsar, B1509 - 58 [7].

In this thesis we will see the complex magnetic fields of a compact star called neutron star which is 100 pc away from our plant Earth. Pulsars (rotating neutron stars) are any class of cosmic objects that populate the plane of our galaxy in the Milky way and rapidly rotating by emitting extremely regular pulses of radio waves, with several such objects known to emit pulses of visible light, X-rays and gamma-rays as well [8]. In our work we outline some of the most outstanding questions and emerging science opportunities related to evolution of pulsar's magnetic field compare strength of infinite non-dipolar fields for quadrapole and octupole.

The purpose of this thesis is to show pulsars magnetic fields inherently contain infinite non-dipolar components even though these are of a lesser magnitude.The thesis is outlined as follow: The first Chapter of this thesis contains a general over review on the distinct properties of neutron stars, the structure of neutron stars and their formation.

In Chapter two, some basic points about the pulsars and magnetic field of pulsars discussed. The next two chapters (Chaps. 3 and 4) contain the derivation of simple dipolar component of magnetic field of pulsars and the derivation of infinite non dipolar components of magnetic fields of pulsars. Finally, the results, and conclusion of the work present in Chapter 5.

Chapter 1

General overview on neutron stars

In 1934 Baade and Zwicky proposed the idea of neutron stars, pointing out that they would be at very high density and small radius, and would be much more gravitationally bound than ordinary stars [9]. A neutron star is a type of compact star that can result from the gravitational collapse of a massive star, with a mass > 8 $M_\odot$ after a type-II supernova explosion. They are the densest and smallest stars known to exist in the universe; with radius of only about 11 - 11.5 km (7 miles), they can have a mass of about twice that of the Sun.

Neutron stars are composed almost entirely of neutrons, which are subatomic particles with no electrical charge and slightly with a larger mass than protons. NSs are very hot and are supported against further gravitational collapse by quantum degeneracy pressure due to the phenomenon described by the Pauli exclusion principle, which states that no two neutrons (or any other fermionic particles) can occupy the same space and quantum state simultaneously [10].

NSs have both minimum and maximum mass limits. The maximum mass, which is of purely general relativistic origin is, unknown but lies in the range of $1.4 - 3$ $M_\odot$ and the minimum stable neutron stars mass is about 0.1 $M_\odot$, although a more realistic minimum stems from a neutron star's origin in a supernova [11].

Neutron stars have overall densities of (3.7×10^{17} - 5.9×10^{17}) kg/m^3 which is comparable to the approximate density of an atomic nucleus of 3×10^{17}kg/m^3. The neutron star's density varies from below 1×10^9kg/m^3 in the crust -increasing with depth -to above

8×10^{17}kg/m^3 deeper inside (denser than an atomic nucleus). A neutron star is so dense that one teaspoon (5 milliliters)of its material would have a mass over the Great pyramid of Giza. Because of its small size and high density, a neutron star possesses a surface gravitational field about 2×10^{11} times that of Earth and such stars can also have magnetic field a million times stronger than the strongest magnetic field produced on Earth [10]. Neutron stars may appear in supernova remnants, as isolated objects, or in binary system. When a neutron star is in a binary system, astronomers are able to measure its mass. From a number of such binaries seen with radio or x-ray telescopes, neutron star masses has been found to be about 1.4 $M_\odot$, which is in agreement with what we put as a limit for the mass of neutron star. For binary systems containing an unknown object, this information helps to distinguish whether the object is a neutron star or a black hole since black holes are more massive than neutron stars [12].

In general, compact stars less than 1.4 $M_\odot$ (the Chandrasekhar limit) are called white dwarfs, whereas the compact stars with a mass between 1.4−3 $M_\odot$ (the Tolman-Oppenheimer-Volkoff limit) should be neutron stars, compact stars with more than 10 $M_\odot$ will overcome the degeneracy pressure and gravitational collapse will usually produce a black hole.

1.1 Formation of neutron stars

Neutron stars are created in the aftermath of the gravitational collapse of the core of a massive stars (> 8 $M_\odot$) at the end of its life, which triggers a Type-II supernova explosion. Neutron stars are the final product of stellar evolution [9]. As stars evolves away from the main sequence, subsequent nuclear burning produces an iron-rich core.

When all nuclear fuel in the core has been exhausted, the core must be supported by degeneracy pressure alone. Further deposits of material from shell burning cause the core to exceed the Chandrasekhar limit. Electron -degeneracy pressure overcome and the core collapses further, sending temperatures soaring to over 5×10^9 k.

At these temperatures, photodisintegration (the breaking up of iron nuclei into alpha particles by high-energy gamma ray) occurs. As the temperature climbs even higher, electrons and protons combine to form neutrons via electron capture, releasing a flood of neutrons. When densities reach nuclear density of 4×10^{17} kg/m^3, neutron degeneracy pressure halts the contraction and the in falling outer atmosphere of the star is flung outwards, becoming a Type-II supernova.

The remnant left is called a neutron star. As the core of a massive star is compressed during a Type-II supernova, and collapses into a neutron star, it retains most its angular momentum. Because it has only a tiny fraction of its parent's radius (and therefore its moment of inertia is sharply reduced), a neutron star is formed with very high rotation speed, and then gradually slows down. Neutron stars are known to have rotation periods from about 1.4 ms to 30 s. The neutron star's density also gives it very high surface gravity, with typical values ranging from 10^{12} to 10^{13} m/s^2 (more than 10^{11} times of that of Earth) [10]. One measure of such immense gravity is the fact that neutron stars have an escape velocity ranging from 100,000 km/s to 150,000 km/s, that is, from a third to half the speed of light.

1.2 The interior structure of neutron stars

The structure of a neutron star is believed to consist of a solid crust, a super fluid interior and an inner core, presumably also solid (neutron solid) and some others [13]. As shown in Fig. 1.1 the interior structure of a neutron star consists of iron, neutron rich nuclei and electrons in the outer crystalline solid crust with thickness 0.1 km, corresponding to densities around 4×10^{11} gcm^{-3}, the neutron drip layer is reached.

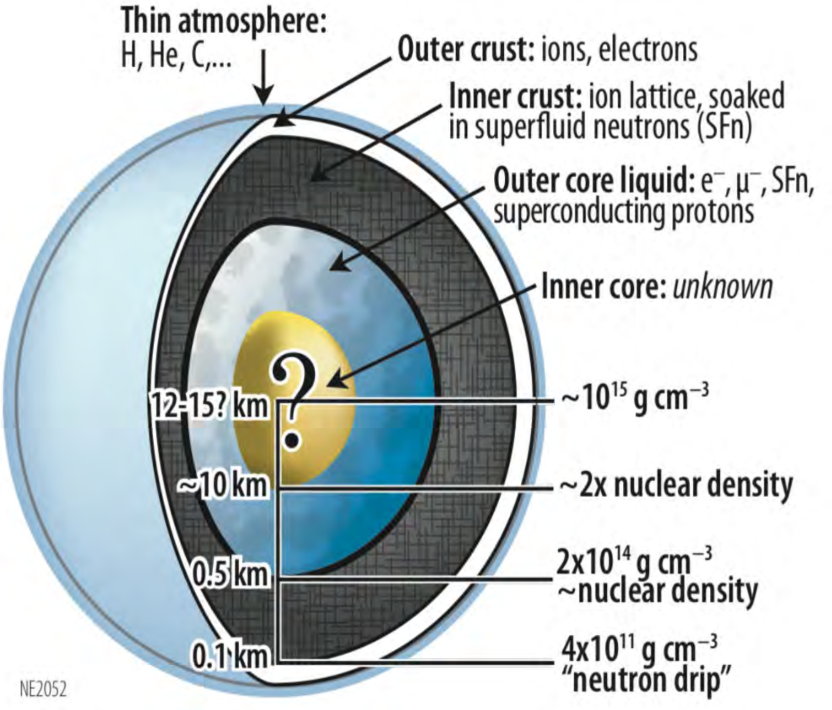

Figure 1.1: The interior structure of neutron star from [14]

The inner crust contain neutron rich nuclei, free super fluid neutrons and electrons and the interior, super fluid neutrons, super fluid protons and electrons with thickness 0.5 km and the corresponding densities around 2×10^{14} gcm^{-3} (approximate to nuclear density). Inside, there is an inner core and outer core. The outer core may contain a neutron superfluid and a proton superconductor with thickness $\sim$ 10 km and the corresponding densities $\sim$ 2×nuclear density.

The composition of the inner core, which likely reaches densities a few times larger than the nuclear equilibrium density, is unknown. The outer layer of a neutron star has a complex structure which depends strongly on the nuclear density. In the inner crust of the star due to high density and pressure, a large fraction of neutron occupy unbounded state [11].

The atmosphere is very thin, with thickness ranging from several mm to cm, depending on the surface temperature. The hotter the surface the thicker the atmosphere. Most of the radiation is emitted from the atmosphere [15].

Chapter 2

Pulsars and their magnetic fields

2.1 Pulsars

Since their discovery by Jocelyn Bell Burnel and Anton Hewish at Cambridge in 1967 [16], pulsars have assumed a central role in astronomy and astrophysics. They offer an opportunity to explore theoretical physics under extreme condition.

Pulsars provide a wealth of information about NS physics, general relativity, the Galactic gravitational potential and magnetic field, the interstellar medium, celestial mechanics, planetary physics and even cosmology.

Pulsars are rapidly rotating highly magnetized NSs. As the name implies pulsars emit electromagnetic radiation in a broad frequency range from radio waves to X-rays and γ-rays, and pulsars pulse because they rotate. The formation of the pulsar begins when the core of a massive star is compressed during a supernova phase which collapses into a neutron star and the neutron star retains most of its angular momentum and since its radius is small (compared to its progenitor's radius), it is formed with very high rotation speed.

Soon after the discovery radio pulsars were identified with rotating magnetized neutron stars. Since then a number of astrophysical objects involving neutron stars have been observed both in radio and X-ray bands. These include millisecond pulsars, binary radio pulsars, high-mass X-ray binaries and low-mass X-ray binaries. Recently, an isolated non pulsating neutron star which radiates thermally was observed by Hubble space Telescope

[17]. Known radio pulsars appear to emit short pulses of radio radiation with pulse periods between 1.4 ms and 8.5 seconds. Even though the word pulsar is a combination of “pulse” and “star”, pulsars are not pulsating stars. Radio pulsars are regularly pulsating sources of radio waves, interpreted as magnetized, rotating neutron stars [18], [19].

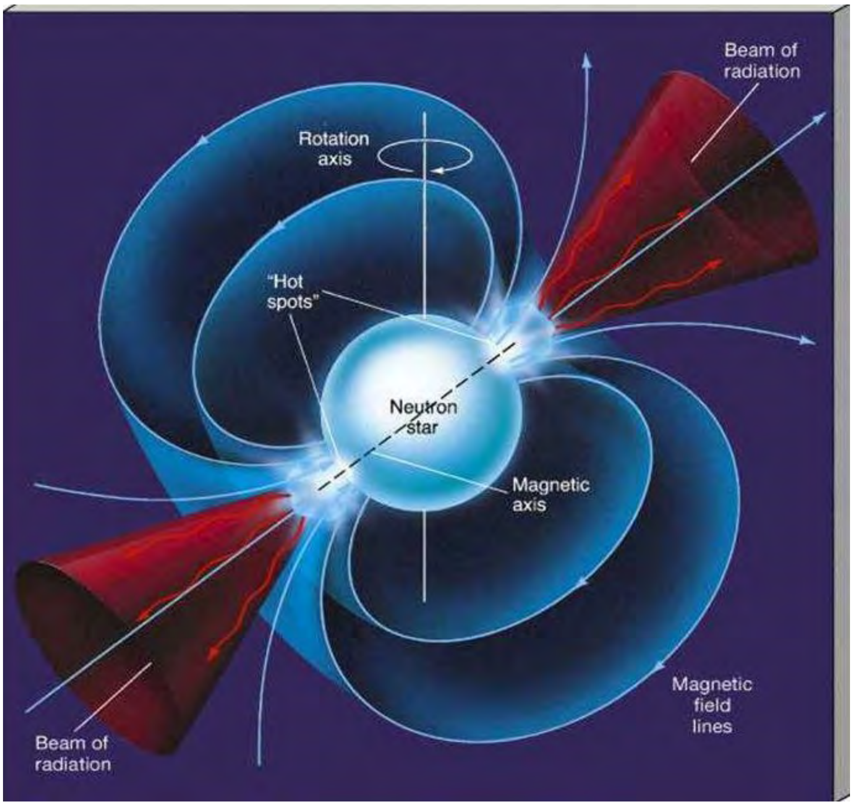

Figure 2.1: Rotating neutron star having a magnetic axis tilted from the rotation axis from [20]

Fig. 2.1 shows pulsars appear to be spinning neutron stars with rotation axes tilted to their magnetic fields. Energetic electrons and light pour out the magnetic poles, and as the star spins the beam of light is swept across the sky like a lighthouse beacon.

Beams of radiation emerging from the poles of a roughly dipolar magnetic field misaligned with respect to the rotation axis appear as pulses every time they sweep the location of the Earth. Some neutron stars emit radio waves that pulse on and off. These stars are called pulsars. Pulsars don’t really turn radio waves on and off they just appear that way to observers on Earth because they are spinning.

What happens is that the radio waves only escape from the North and South poles of the neutron star. If the spin axis tilted with respect to the magnetic poles, the radio waves sweep around like the light beam from a light house. Pulsars (pulsating radio sources) are rapidly spinning neutron stars. The pulsating signal is attributed to the "light house effect," abeam of radiation inclined to the rotation axis, that sweeps the observer's field of view once (or twice) every period. The period of this pulse which can be determined from measurements of the pulsating signal is observed to increase very slowly this combined with the fact that the emission was pulsed, was consistent with emission from an inclined rotator with a strong magnetic field. For the crab pulsar, the implied loss of rotational energy is $\sim 10^{38}$ erg s^{-1}. However, the radio luminosity in its pulses is $\sim 10^{30}$ erg s^{-1}, only a small fraction of this power, implying that different mechanisms are responsible for the radio emission and the slow-down of a pulsar. The latter is attributed to the large amplitude electromagnetic radiation of a rotating as often assumed magnetic dipole.

One of the first models of pulsar emission was that of an orthogonal rotator based on the fact that a magnetic dipole rotating in a vacuum losses its rotational energy due to magnetic dipole radiation if the rotational and magnetic axes are misaligned [21].

The crab and vela pulsars in the late fall of 1968, both of which are situated in supernova explosions. The neutron star (Crab pulsar) and the Crab nebula in which the pulsar is embedded are the remnant of the supernova explosions observed by Chinese astronomers in 1054A.D. The Crab pulsar is thus about 940 years. It is the youngest known pulsar. The Crab pulsar is observed optically as a faint star near the center of the Crab nebula. The energy source of the crab nebula might be a rotating neutron star and a simple magnetic model, capable of converting neutron star rotational energy into electromagnetic radiation. Gold predicted a small increase in the period as the pulsar slowly lost rotational energy and after the slowdown of the crab pulsar was discovered. The implied energy loss was roughly the same as the energy required to power the crab nebula [9].

In isolated neutron stars the period decrease because of the loss of rotational energy due to the magnetic dipole radiation and in the monitoring of such systems, sudden spin-ups in the pulse period have been observed [22]. These 'glitches' only persist for a few days, before the pulsar continues its normal spin-down. These changes in the pulse period are believed to be an indication for the existence of a super fluid in the interior of the star, where the 'glitch' occurs when a sudden transfer of angular momentum between the core and the outer crust takes place [23].

There are also ‘accreting pulsars’ which funnel matter from a companion star onto their magnetic polar caps as they rotate. If a neutron star has a binary companion that has filled its Roche Lobe, then it can accrete matter from its companion star. If the neutron star’s magnetic field is strong enough, the accreting material is channeled by it and accumulates at the magnetic poles. There are two classes of such binary stars, high-mass X-ray binaries (HMXB) and low-mass X-ray binaries (LMXB). High-mass X-ray binaries (HMXB) is in which the companion of the neutron star is usually a massive O and B star with the mass $\sim 10\ \mathrm{M}_{\odot} - 40\ \mathrm{M}_{\odot}$. The neutron star is strongly magnetized with the magnetic field typical for normal radio pulsars, $\mathrm{B} \sim 10^{12} - 10^{13}$ G. Matter accreted onto the neutron star moves along the magnetic field lines and hits the neutron star surface in the vicinity of the magnetic pole. The companion of the neutron star in low- mass X-ray binaries is a faint star with the mass $\mathrm{M} \leq 2\ \mathrm{M}_{\odot}$. Low-mass X-ray binaries also named X-ray bursters. The X-ray bursts are thought to be produced by a thermonuclear flash of the accreted nuclear fuel. Accreted matter spreads over the surface of the neutron star. This occurs because the magnetic field is low, $\mathrm{B} \leq 10^{10}$ G.

2.2 Magnetic field of pulsars

The magnetic field of a pulsars (a rotating neutron stars) are the strongest magnetic field known in the universe. Magnetic fields of most pulsars are around 10^{11}- 10^{13} G, whereas the typical rotation periods are about one second. However, there exist a handful of known pulsars with considerably smaller rotation periods, which also have much weaker magnetic fields round 10^{8} G. These objects are called millisecond pulsars and they are often found in binary systems [24]. Millisecond pulsars are thought to be neutron stars which have been spun up in such a binary accretion process. Since their magnetic fields are weaker by a factor of 10^{3}- 10^{4} compared to the magnetic fields of ordinary pulsars, presumably the magnetic field of the neutron star also decreases during the accretion phase. Typical the magnetic field strength at neutron star surfaces are of order of 10^{12} G for comparison, the Earth’s magnetic fields is only 1 G. We infer these field strengths from the slow increase in pulse period due to the loss of rotational energy by the emission of magnetic dipole radiation, or more directly from synchrotron lines in X-ray spectra of X-ray pulsars [25].

Magnetic fields play an essential role by accelerating particles, by channeling these particles or causing accretion flows, by producing synchrotron emission or resonant cyclotron scattering, and by providing the main mechanism for angular momentum loss from non-accreting stars. Moreover, evidence is reveals that soft gamma-ray repeaters and anomalous X-ray pulsars are really only slightly distinct types of very strongly magnetized neutron stars ("magnetars") in which the magnetic field is the main energy source for the observed radiation. The magnetic field of the magnetars may be so strong as to reach two orders of magnitude larger than the quantum critical threshold, 10^{14} - 10^{15} G. It is believed that the strong magnetic field of neutron stars originated from the collapse of the core of a supernova with the conservation of magnetic flux. On the other hand, we actually know surprisingly little about neutron star magnetic fields [26].

In particular, most "measurements" of neutron star magnetic fields are indirect inferences, which are put in doubt both by their inconsistency with other observational evidence and with plausible theoretical models for the physics of their surroundings. The plasma density gradient which is inherent to degenerate neutron star matter is observed to lead to large scale plasma diffusion and subsequent charge separation. As a result of this, excess negative charges accumulated on the crust, while, at the same time, almost the same amount of excess positive charges are left behind in the solid core. The surface magnetic field of neutron star is believed to be due to the spinning separated charge, these fields are also found to be temperature dependent, and they decay through neutrino and photon emission [5].

The electron and proton currents is resulting from the charge separation process are modeled by [5]

$$n_i = -D^O(\partial_i n_o - \Gamma^0_{0i} n_o) + \mu n_o g_{\beta i} F^{0\beta} \qquad (2.2.1)$$

where, $n_\alpha = (n_i, n_o)$ is the four particle current and D^o represents the diffusion coefficient which is considered here scalar.

Since NSs normally rotate very fast, the spinning polarization (surface) charge is expected to generate a strong magnetic field. The NS's surface magnetic fields are naturally multipolar which is in agreement with current experimental findings.

The intensity of the dipole is determined not only by the magnitude of polarization (surface) charge which depends strongly on the core plasma density (or central density), but also from the spin frequency [13].

Chapter 3

Dipolar field

Normally, gravitational fields are so weak that a practicing astrophysicist ignore relativistic effects that is why different authors ignore this field. In calculating the magnitude of the separated charges and the resulting dipole fields [5] has ignored gravity. For this case, we assume a uniform spherical surface charge distribution as simple case in magnetostastics. In the absence of gravity we expect to have dipolar surface fields for the assumed symmetry of pulsars. These are the result of this model equation taken at equilibrium and in the absence of gravity.

$$0 = -D^0(\partial n_0 - \mu n_0 g_{\beta i} F^{0\beta}) \tag{3.0.1}$$

In this chapter we will derive the vector potential and magnetic field by ignoring gravity.

3.1 The vector potential

The vector potential of rotating charged pulsar in an observation point say p characterized by the radius vector r can be written, according to [27] which are also the solution for poission equation by taking the time dependent surface current density J(x', t) as:

$$A(\boldsymbol{x}) = \frac{\mu_0}{4\pi} \int \frac{J(\boldsymbol{x'})}{|\boldsymbol{x} - \boldsymbol{x'}|} d^3x^{'} \tag{3.1.1}$$

what is x' in bold is this the four space - time coordinates. Using the spherical harmonic expansion from [27] we can express the denominators

$$\frac{1}{|\boldsymbol{x}-\boldsymbol{x'}|} = 4\pi \sum_{l=0}^{\infty} \sum_{m=-1}^{l} \frac{1}{2l+1} \frac{r^{l} <}{r^{l+1} >} Y^{*}_{lm}(\theta', \phi') Y_{lm}(\theta, \phi) \tag{3.1.2}$$

The vector potential in Eqn. (3.1.1) will become

$$A(\boldsymbol{x}) = \frac{\mu_0}{4\pi} \sum \frac{4\pi}{2l+1} \quad \int J(\boldsymbol{x'}') \frac{r^{l} <}{r^{l+1} >} Y^{*}_{lm}(\theta', \phi') \mathbf{Y}_{lm}(\theta, \phi) d^{3}\boldsymbol{x} \tag{3.1.3}$$

In a uniform spherical charge distribution with surface charge density σ, the current density in the system may be given as:

$$\overrightarrow{J} = \sigma\delta(\overrightarrow{r'} - \overrightarrow{R})\overrightarrow{v}$$

where, $\overrightarrow{v} = \overrightarrow{\omega} \times \overrightarrow{r}$ is the velocity and ω is the angular velocity taking the rotation about the z-axis we obtain v as

$$\overrightarrow{v} = R\omega \sin\theta' \hat{e}_{\phi'} \tag{3.1.4}$$

where, $\hat{e}_{\phi'} = -\sin\phi'\hat{x} + \cos\phi'\hat{y}$

From Eqn. (3.1.4) which clearly shows that J has only ϕ' component as

$$\overrightarrow{J_{\phi}} = \sigma\delta(\overrightarrow{r}' - \overrightarrow{R})\omega R \mathrm{sin}\theta^{'}. \tag{3.1.5}$$

Since the geometry is spherically symmetric we may choose the observation point in the x-z plane ($\phi^{'}$ =0). Since the azimuthal integration in Eqn. (3.1.1) is symmetric in $\phi' = 0$; the x component of the current does not contribute.

This leaves only the y component, which is Aϕ [27]. The vector potential becomes

$$A\phi = \frac{\mu_0 4\pi\sigma\omega R}{4\pi} \int \sum \frac{1}{2l+1} \frac{r^{l} <}{r^{l+1} >} Y^{*}_{lm}(\theta', \phi') Y_{lm}(\theta, \phi) \delta(\mathbf{r}' - \mathbf{R}) \sin\theta \cos\phi \; d^{3}x', \tag{3.1.6}$$

where, $d^{3}x = r^{'2} dr' \sin\theta' d\theta' d\phi'$.

$$\mathrm{A}\phi = \frac{\mu_0 4\pi\sigma\omega R}{4\pi}\int\sum\frac{1}{2l+1}\frac{r^l <}{r^{l+1} >}Y^*_{lm}\,(\theta',\phi')Y_{lm}(\theta,\phi)$$
$$\times\ \delta(\mathbf{r}'-\mathbf{R})\mathrm{sin}\theta\mathrm{cos}\phi'\ \ r'^2dr'\mathrm{sin}\theta\ 'd\theta'd\phi'$$

But $\sin\theta^{'}\cos\phi^{'} = \sqrt{\frac{8\pi}{3}}Y_{l1}(\theta',\phi')$, with $\phi^{'} = 0$ [27]. Then A_ϕ will be

$$A\phi = \frac{\mu_0 4\pi\sigma\omega R}{4\pi}\int\sum\frac{1}{2l+1}\frac{r^l <}{r^{l+1} >}Y^*_{lm}(\theta',\phi')Y_{lm}(\theta,\phi)\delta(\mathbf{r}'-\mathbf{R})-\sqrt{\frac{8\pi}{3}}Y_{l1}(\theta',\phi')r^{'2}dr'\sin\theta'd\theta'd\phi' \tag{3.1.7}$$

From the normalization and orthogonality condition [27].

$$\int d\phi\int\mathrm{sin}\theta d\theta\ Y^*_{lm}(\theta',\phi')Y_{lm}(\theta,\phi) = \delta_{\ l'l}\delta_{m'm}$$

The integration contributes only for $l = 1$ and $m = 1$

The θ equation for P(θ) is customarily expressed interms of $x = \cos\theta$, instead of θ itself. Then it takes the form.

$$\frac{d}{dx}\left[(1-x^2)\frac{dp}{dx}\right] + \left[l(l+1) - \frac{m^2}{1-x^2}\right]p = 0 \tag{3.1.8}$$

This equation is called the generalized Legendre equation, and its solutions are the associated legendre functions. The legendre differential equation with $m^2 = 0$

$$\frac{d}{dx}\left[(1-x^2)\frac{dp}{dx}\right] + \left[l(l+1)\right]p = 0 \tag{3.1.9}$$

The generalization of P$_\mathrm{l}(\cos\theta)$, namely, the solution of Eqn. (3.1.8) with l and m both arbitrary. In essentially the same manner as for the ordinary legendre functions it can be shown that to have finite solutions on the interval $-1 \leq x \leq 1$, the parameter l must be zero or a positive integer and the integer m can take on only the values $-l, -(l-1), ..., 0, ..., (l-1), l$. The solution having these properties is called an associated legendre function $p_l^m(x)$. For positive m it is defined by the formula [27]

$$p_l^m(x) = (-1)^m(1-x^2)^{\frac{m}{2}\frac{d^m p_l(x)}{dx^m}} \tag{3.1.10}$$

From Eqn. (3.1.10) we have the relation.

$$P_l^1(\cos\theta\ ) = -[1-\cos\theta^2]^{\frac{1}{2}}\frac{dp\ (\cos\theta\ )}{d(\cos\theta)}$$

$$p_l^1 = -(\sin^2\theta)1/2\tfrac{dp\ \ (\cos\theta\ \ )}{d(\cos\theta)} = -\sin\theta$$

Therefore, the vector potential in Eqn. (3.1.7) may be written as

$$A\phi = -\frac{\mu_0 Q\omega}{4\pi R}\int\sum_{lm}\frac{1}{2l+1}\frac{r^l <}{r^{l+1} >}Y_{lm}^*(\theta^{'},\phi^{'})Y_{lm}(\theta,\phi)\delta(\mathbf{r'}{-}\mathbf{R})p_l^1(\cos\theta)\cos\phi^{'}r^{'2}dr'd\phi^{'}d(\cos\theta^{'}) \tag{3.1.11}$$

the constant $\dfrac{4\pi}{\mu_0} = \mathrm{c}$ and $\mathrm{Q} = 4\pi\mathrm{R}^2\sigma$

Finally the vector potential associated with the spinning crust containing the separated negative charge has only a ϕ - component which can be written as:

$$A\phi{=}-\frac{|Q|\,\omega}{Rc}\int\sum_{lm}\frac{1}{2l+1}\frac{r^l <}{r^{l+1} >}Y_{lm}^*(\theta^{'},\phi^{'})Y_{lm}(\theta,\phi)\delta(\mathbf{r'}{-}\mathbf{R})p_l{}^1(\cos\theta)\mathrm{cos}\phi\,{}^{'}r^{'2}dr'd\phi^{'}d(\cos\theta^{'}). \tag{3.1.12}$$

3.1.1 The vector potential outside the pulsar's surface

The vector potential outside the sphere crust can be derive from Eqn. (3.1.12) by substituting $r_< \equiv R$ and $r_> \equiv r$ as

where, $r_< \equiv R$ and $r_> \equiv r$ to represent the interior and exterior regions of the pulsar, respectively.

$$A_{\phi,out} = -\frac{|Q|\,\omega}{Rc}\int\sum_{lm}\frac{1}{2l+1}\frac{R}{r^2}Y_{lm}^*(\theta',\phi')Y_{lm}(\theta,\phi)\delta(\mathbf{r}-\mathbf{R})\sin{}^2\theta^{'}cos\phi' r^{'2}dr'd\theta' d\phi' \tag{3.1.13}$$

Having performed the integral over the delta function gives

$$\int\delta(\mathbf{r}-\mathbf{R})r^{'2}dr' = R^2 \tag{3.1.14}$$

Substituting Eqn. (3.1.14) into Eqn. (3.1.13) gives

$$A_{\phi,out} = -\frac{|Q|\,\omega R^2}{Rc} - \sqrt{\frac{8\pi}{3}}\sum \frac{1}{2l+1}\frac{R}{r^2}Y_{lm}(\theta,\phi)Y^*_{lm}(\theta',\phi')Y_{l1}(\theta',\phi')\sin\theta d\theta' d\phi' \quad (3.1.15)$$

$$A_{\phi,out} = -\frac{|Q|\,\omega R^2}{Rc} - \sqrt{\frac{8\pi}{3}}\frac{1}{2+1}\frac{R}{r^2}(-\sqrt{\frac{3}{8\pi}})\sin\theta e^{i\phi}.$$

Finally the vector potential outside the pulsar's surface will be:

$$A_{\phi,out} = -\frac{|Q|\,\omega}{3c}\left(\frac{R}{r}\right)^2\sin\theta. \quad (3.1.16)$$

3.2 Derivation of dipolar part (component) of magnetic field

In this section we will derive the dipolar component of the pulsar magnetic field. The magnetic field derived from the curl of the vector potential as

$$\overrightarrow{B} = \overrightarrow{\nabla}\times\overrightarrow{A} \quad (3.2.1)$$

Which can be written as

$$\overrightarrow{\nabla}\times\overrightarrow{A} = \hat{e}_r\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A\phi) - \frac{\partial A\theta}{\partial\phi}\right] + \hat{e}\theta\left[\frac{1}{r\sin\theta}\frac{\partial Ar}{\partial\phi} - \frac{1}{r}\frac{\partial(rA\phi)}{\partial r}\right] + \hat{e}\phi\frac{1}{r}\left[\frac{\partial(rA\theta)}{\partial r} - \frac{\partial Ar}{\partial\theta}\right]. \quad (3.2.2)$$

Since we have only ϕ-component of the vector potential, then the magnetic field will be

$$\overrightarrow{B} = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\ A\phi)\hat{e}_r - \frac{1}{r}(\frac{\partial(rA\phi)}{\partial r}\hat{e}\theta \quad (3.2.3)$$

3.2.1 Magnetic field outside pulsar's surface

The magnetic field outside the pulsar's surface can be calculated by substituting Eqn. (3.1.16) into Eqn. (3.2.3) as follow

$$\overrightarrow{B}_{out} = \overrightarrow{\nabla} \times \overrightarrow{A}\phi_{,out}$$

$$\begin{aligned}\overrightarrow{B}_{out} &= \tfrac{1}{r\sin\theta}\left[\tfrac{\partial}{\partial\theta}(\sin\theta A\phi, out)\right] e_r + \left[-\tfrac{1}{r}\tfrac{\partial(rA\phi,out)}{\partial r}\right] \\ &= \tfrac{1}{r\sin\theta}\left[\tfrac{\partial}{\partial\theta}(-\sin\theta\tfrac{|Q|\omega}{3c}(\tfrac{R}{r})^2\sin\theta\right] e_{\hat{r}} + \left[\tfrac{-1}{r}\tfrac{\partial}{\partial r}(r - \tfrac{|Q|\omega}{3c}\ (\tfrac{R}{r})^2\sin\theta)\right] e\hat{\theta}, \\ &= \left[-2\cos\theta\tfrac{|Q|\omega}{3rc}(\tfrac{R}{r})^2\right]\hat{e}_r + \left[-\tfrac{|Q|\omega}{3cr}\ (\tfrac{R}{r})^2\sin\theta\right]\hat{e}_\theta\end{aligned}$$

$$= -\frac{|Q|\,\omega}{3c}(\frac{R^2}{r^3})\left[2\cos\theta\hat{e}_r + \sin\theta\hat{e}_\theta\right]. \tag{3.2.4}$$

But we have, $\hat{e}_r = \sin\theta\hat{j} + \cos\theta\hat{k}$

$$\hat{e}_\theta = \cos\theta\hat{j} - \sin\theta\hat{k} \tag{3.2.5}$$

Substituting Eqn. (3.2.5) into (3.2.4) using trigonometric identity we get

$$2\cos\theta\hat{e}_r + \sin\theta\hat{e}_\theta = 3\cos\theta\hat{e}_r - \hat{k}$$

Then, the magnetic field outside the pulsar's surface will be

$$\overrightarrow{B}_{out} = -\frac{|Q|\,\omega}{3c}\left(\frac{R^2}{r^3}\right)\left[3\cos\theta\hat{e}_r - \hat{k}\right]. \tag{3.2.6}$$

$$= -\tfrac{|Q|\omega}{3c}\tfrac{R^2}{r^3}\left[3(\hat{k}.\hat{r})\hat{r} - \hat{k}\right].$$

where, θ is the zenith angle and $\hat{e}_r$ and $\hat{k}$ are unit vectors along the radial direction and the spin axis respectively.

Eqn. (3.2.6) clearly shows that pulsar magnetic fields generated by the spinning separated charges are dipolar.

3.3 Flux conservation

The magnetic field of neutron stars are among the strongest known magnetic field in the universe. The magnitude of the magnetic field can be estimated by considering the conservation of the magnetic flux during the collapse of the late type star, which is similar to the magnetic field in a closed loop [28]. We consider that an ordinary star has magnetic field of 10^4 G and rotate. Pulsar (rotating neutron stars) are formed in supernova explosion in which massive stars die explosively and here the field are scaled up by conservation laws of magnetic flux. In the process the residual magnetic field of the star gets conserved in the neutron star which is formed after the explosion. As a result the intensity of the magnetic field of the progenitor star will intensify inside the neutron star to a high level. We demonstrate this as follows; Let us assume that the progenitor star is as big as our Sun whose surface magnetic field is known to be 10^4 G the magnetic flux corresponding to this will be:

$$\phi_s = 4\pi r^2 \times 10^4 \ \text{Gcm}^2 \tag{3.3.1}$$

where, $r_s = 7 \times 10^2$ km $= 7 \times 10^{10}$ cm is radius of the Sun.

$$\phi_s = 4\pi \times (7 \times 10^{10})^2 \times 10^4 \ \text{Gcm}^2,$$

$$= 6.15 \times 10^{26} \ \text{Gcm}^2 \tag{3.3.2}$$

Flux conservation is that pulsar magnetic fields are remnants of the field of their progenitors.

By the principle of flux conservation we expect;

$$\phi_s = \mathrm{B_{Ns}} \ \mathrm{A_{Ns}},$$

where, $\mathrm{B_{Ns}}$ and $\mathrm{A_{Ns}}$ are the magnetic field and area of neutron star respectively.

$$4\pi r_s^2 \times 10^4 \ \text{G} = \ \mathrm{B_{Ns}} \times 4\pi \mathrm{R}_*^2 \tag{3.3.3}$$

where, R_* is the radius of the neutron star which known to be of the order $R_* = 10$ km $=$ 10^6 cm

$$B_{Ns} = \frac{4\pi r_s^2 \times 10^4 Gcm^2}{4\pi R_*^2} = 0.49 \times 10^{14} \ \text{G}, \tag{3.3.4}$$

$$\mathrm{B_{Ns}} = 5 \times 10^{13} \ \text{G}$$

When a dying star collapses to form a pulsar magnetic flux conservation results in the formation of regions of extremely strong magnetic field near the pulsar.

From Eqn. (3.2.6) the component of the dipolar magnetic field.

$$\overrightarrow{B}_r = 2\tfrac{|Q|\omega}{3c}\left(\tfrac{R^2}{r^3}\right)\cos\theta\hat{e}_r \quad \text{and}$$

$$\overrightarrow{B}_\theta = \frac{|Q|\,\omega}{3c}\left(\frac{R^2}{r^3}\right)\sin\theta\hat{e}_\theta \tag{3.3.5}$$

When we consider the maximum intensity of the radial part of the dipolar magnetic field, we obtain

$$\overrightarrow{B}_r = 2\frac{|Q|\,\omega}{3c}\left(\frac{R^2}{r^3}\right)\cos\theta\hat{e}_r \tag{3.3.6}$$

But we assume that the magnitude of magnetic field that we have derived from the magnetic flux conservation is dominated by dipolar, so we can relate the dipolar magnetic field that we have derived in Eqn. (3.2.6) with magnetic flux conservation in Eqn. (3.3.4)

$$5\times10^{13}G = 2\frac{|Q|\,\omega}{3c}\left(\frac{R^2}{r^3}\right) \tag{3.3.7}$$

Take another assumption that the field point is settled around the surface of the pulsar, this lead to make an approximation $r \approx R$, then the equation will be

$$5\times10^{13}\ \mathrm{G} = 2\frac{|\mathrm{Q}|\,\omega}{3\mathrm{c}}\left(\frac{1}{\mathrm{R}}\right) \tag{3.3.8}$$

where, $\mathrm{R} = 10^6$ cm which is the radius of neutron star and substituting this in Eqn. (3.3.8) we get

$$\tfrac{5\times10^{13}\times3\times10^{6}Gcm}{2} = \tfrac{|Q|\omega}{c}$$

$$7.5\times10^{19}\ \mathrm{Gcm} = \frac{|\mathrm{Q}|\,\omega}{\mathrm{c}} \tag{3.3.9}$$

Substituting Eqn. (3.3.9) in to (3.3.6) we get the radial part of the dipolar magnetic field as

$$\overrightarrow{B}_r = 2\left[7.5\times10^{19}\frac{R^2}{3r^3}\right]\cos\theta\hat{e}_r\ \mathrm{Gcm} \tag{3.3.10}$$

and the angular component of the magnetic field will be

$$\overrightarrow{B}_\theta = 7.5 \times 10^{19} \frac{R^2}{3r^3} \sin\theta \hat{e}_\theta \text{ Gcm} \tag{3.3.11}$$

From Eqn. (3.3.10) and (3.3.11) taking the maximum value of $\cos\theta$ and $\sin\theta$ from this what we observed that the magnitude of magnetic field of the pulsars is estimated to 6×10^{13} G, this magnetic field is the strongest magnetic field.

3.4 Summary

The main summary of this chapter is both the vector potential and the magnetic field for the dipolar component outside the pulsar's surface have been derived by assuming a uniform spherical surface charge distribution as simple magnetostatics in the absence of gravity. Pulsars (rotating neutron stars) are formed in supernova explosion in which massive stars die explosively during this the magnetic flux is conserved so one can estimate the magnitude of the magnetic field by considering the conservation of magnetic flux during the collapse of the late type star.

Chapter 4

Infinite non-dipolar fields

All current pulsar radiation data show that NS magnetic fields are complex. So far there was no definite explanation to this mystery in current pulsar Astrophysics. The current model predicts that pulsar magnetic fields are predominantly dipolar in agreement with observational findings [18], [21].

The model also predicts the presence of an unlimited number of poles along with the main dipole component making NS surface magnetic fields naturally complex. This has not yet been convincingly achieved by any of the other models that are currently in use even though it is widely accepted by many that the magnetic field structure near the surface of neutron stars significantly differs from the pure star centered dipole structure. Current models require the introduction of new ingredients to explain complicated energy dependent light curves from pulsars. In this regared, non-dipolar components in the magnetic field structure of NSs are expected to provide a promising way of extending current emission model. In the following sections, it is show on that the non-dipolar components, according to this model, are realized by including the rotation of the frame by way of general covarient differentiation of the current equation which given in Eqn. (4.0.1) stated below.

$$0 = D^0 \Gamma^0{}_{0i} + \mu n_0 g_{\beta i} F^{0\beta} \tag{4.0.1}$$

where, Γ^0_{oi} is the affine connection and $F^{0\beta}$ is electromagnetic field tensor and n_0 is the temporal the four particle current.

In this chapter we will derive the infinite non-dipolar components of pulsar magnetic fields resulting from the rapidly spinning of pulsar with angular frequency ω, by using post-Newtonian approximation. The non-dipolar field is derived by using equation which is responsible for the infinite non-dipolar component given in Eqn. (4.0.1).

Einstein's field equations are not exactly solvable. Existing solutions are generally approximate. The Schwarzchild solution is one such solution derived under the approximation of spatial isotropy and time independence. One other approximate solution is developed by expanding the metric tensors in powers of $\dfrac{GM}{r} \sim v^2$ where, V, M, and r denote the typical velocity, mass and separation of particles in the system under consideration, respectively. This approach is what is called Post-Newtonian approximation. Under this approximation the metric tensor is expanded as [29]:

$$
\begin{aligned}
g_{00} &= -1 + \overset{2}{g}_{00} + \overset{4}{g}_{00} + - - -, \\
g_{ij} &= \delta_{ij} + \overset{2}{g}_{ij} + \overset{4}{g}_{ij} + - -, \\
g_{i0} &= \overset{3}{g}_{i0} + \overset{5}{g}_{i0} + - - -.
\end{aligned} \tag{4.0.2}
$$

The inverse of the metric tensor is given by

$$
\begin{aligned}
g^{00} &= -1 + \overset{2}{g}00 + \overset{4}{g}00 + - - -, \\
g^{ij} &= \delta_{ij} + \overset{2}{g}ij + \overset{4}{g}ij + - - -, \\
g^{i0} &= \overset{3}{g}i0 + \overset{5}{g}i0 + - - .
\end{aligned} \tag{4.0.3}
$$

where the symbol $\overset{N}{g}\,\alpha\beta$ denoting the terms in $g^{\alpha\beta}$ of order v^N where, v is the typical velocity of the particles under consideration

The affine connection $(\Gamma^{\mu}_{\mu\lambda})$ may now be obtained from the familiar formula [29]

$$\Gamma^{\mu}{}_{\mu\lambda} = \frac{1}{2} g^{\mu\rho} \left\{ \frac{\partial g_{\rho} \upsilon}{\partial x^{\lambda}} + \frac{\partial g_{\rho} \lambda}{\partial x^{\mu}} - \frac{\partial g_{\upsilon\lambda}}{\partial x^{\rho}} \right\} \qquad \text{then follow}$$

$$\Gamma^{\mu}_{\mu\lambda} = \frac{1}{2} g^{\mu\rho} \frac{\partial g_{\rho\mu}}{\partial x^{\lambda}}. \tag{4.0.4}$$

From this we have

$$\Gamma^{0}_{0i} = \frac{1}{2} g^{0\rho} \frac{\partial g_{\rho 0}}{\partial x^{i}}. \tag{4.0.5}$$

This implies

$$\Gamma^0{}_{0i} = \frac{1}{2} g^{00} \frac{\partial g_{00}}{\partial x^i} + \frac{1}{2} g^{0j} \frac{\partial g_{0j}}{\partial x^i} \ ,$$

$$= \frac{1}{2} g_{00} \frac{\partial g 00}{\partial x^i} - \frac{1}{2} g_{0j} \frac{\partial g^{j0}}{\partial x^i}. \tag{4.0.6}$$

Substituting Eqn. (4.0.6) into (4.0.1) we can get

$$D^0 \left(\frac{1}{2} \mathrm{g}^{00} \frac{\partial g^{00}}{\partial x^i} + \frac{1}{2} \mathrm{g}^{\mathrm{j}0} \frac{\partial g_{j0}}{\partial x^i} \right) n_0 + \mu n_0 \left(g_{00} F^{00} + g^{ki} F^{ki} \right) = 0 \tag{4.0.7}$$

From Eqn. (4.0.2), (4.0.3) and (4.0.7) we will get

$$\frac{D^0}{2} \left[\left(-1 + \overset{2}{g}_{00} + \overset{4}{g}_{00} + \dots \right) \partial i \left(-1 + \overset{2}{g} 00 + \ \dots \right) - \left(\overset{3}{g}_{i0} + \overset{5}{g}_{i0} + \dots \right) \partial i (\overset{3}{g} j 0 \qquad + \dots) \right] n_0$$
$$+ \mu n_0 (\delta_{ki} + g^2_{ki} + - - -) F^{0K}. \tag{4.0.8}$$

By approximating the metric tensors to their first term we get

$$\frac{D^0}{2} \left[(-1) \, \partial i \, (-1) - \overset{3}{g}_{0j} \partial i \overset{3}{g} 0 j \right] n_0 + \mu n_0 (\delta_{ki} F^{ok}) = 0, \tag{4.0.9}$$

which results

$$\frac{D^0}{2} (\overset{3}{g}_{0j} \partial i \overset{3}{g} 0 j) n_0 + \mu n_0 F^{0i} = 0. \tag{4.0.10}$$

The electromagnetic field tensors that results the electric field from Eqn. (4.0.10) we will get

$$F_{0i} = \frac{D^0}{2\mu} (\overset{3}{g}_{0j} \partial_i \overset{3}{g} 0 j),$$
$$\overset{3}{g}_{0j} = \overset{3}{g} 0 j = \zeta^i = \delta_{ik} \zeta^i, \qquad \text{then}$$

$$F_{0i} = \frac{D^0}{2\mu} \zeta^i \frac{\partial \zeta^i}{\partial x^i} \tag{4.0.11}$$

ζ_i is a vector potential which is given by [29]

$$\zeta_i(x,t) = -4G \int \frac{\overset{1}{T} i 0 (x', t)}{|x - x'|} d^3 x' \tag{4.0.12}$$

where, $\overset{1}{T}i0(x', t)$ is momentum density.

The electromagnetic field tensors F_{0i} represent the electric field E_i from Eqn. (4.0.11)

$$E = \frac{3D}{2\mu}\zeta^i \ \frac{\partial\zeta^i}{\partial x^i}, (D^0 \to D). \tag{4.0.13}$$

4.1 Derivation of the charge density

From the gauss's law we have [27]

$$\nabla . E = \frac{\rho}{\varepsilon_0} \tag{4.1.1}$$

where, ρ and E the surface charge density and the electric field respectively

$$\rho = \varepsilon_0 \nabla . E$$

From Eqn. (4.0.13)

$$\rho = \varepsilon_0 \partial i E_i$$

$$\rho = \varepsilon_0 \partial i \left[\frac{3D}{2\mu}\zeta^i \frac{\partial\zeta^i}{\partial x^i}\right]$$

$$\rho = \frac{\varepsilon_0 3D}{2\mu}\left[\left(\frac{\partial\zeta^i}{\partial x^i}\right)^2 + \zeta_i \frac{\partial^2\zeta^i}{\partial x^{i2}}\right]. \tag{4.1.2}$$

The harmonic coordinate condition

$$g^{\mu\upsilon}\Gamma^{\lambda}_{\mu\upsilon} = 0, \tag{4.1.3}$$

where, $g^{\mu\upsilon}$ is metric tensor and $\Gamma^{\lambda}_{\mu\upsilon}$ is affine connection.

We are now ready to make use of the Einstein field equations, which we take in the form [29]

$$R_{\mu\upsilon} = -8\pi G[T_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}T^{\lambda}_{\lambda}], \tag{4.1.4}$$

where, $R_{\mu\upsilon}$ is Ricci tensor.

From the harmonic coordinate condition in Eqn. (4.1.3) and the expansions of field equation in Eqn. (4.1.4) that is used to give

$$\nabla^2 \overset{2}{g}_{00} = -8\pi G \overset{0}{T}00,$$

$$\partial_i{}^2 \zeta_i = \nabla^2 \overset{3}{g}_{i0} = +16\pi G \overset{1}{T}i0, \tag{4.1.5}$$

and from the first equation of Eqn. (4.1.5) $\overset{2}{g}_{00} = -2\phi$, where, ϕ is the Newtonian scalar potential, defined by poisson's equation [29].

$$\nabla^2 \phi = 4\pi G \overset{0}{T}00. \tag{4.1.6}$$

Also $\overset{2}{g}_{00}$ must vanish at infinity, so the solution Eqn. (4.1.6) is:

$$\phi(x,t) = -G \int \frac{\overset{0}{T}00(\boldsymbol{x'},t)}{|\boldsymbol{x}-\boldsymbol{x'}|} d^3\boldsymbol{x'}. \tag{4.1.7}$$

The coordinate condition above imposes on ϕ and ζ the further relation

$$4\tfrac{\partial\phi}{\partial t} + \nabla\zeta = 0 \text{ or}$$

$$\partial^2{}_i \zeta_i = -\frac{4\partial\phi}{\partial t}. \tag{4.1.8}$$

By substituting Eqn. (4.1.5) and (4.1.8) into (4.1.2) we get the charge density as:

$$\rho = \frac{\varepsilon_0 3D}{2\mu}\left[\left(\frac{-4\partial\phi}{\partial t}\right)^2 + \zeta_i 16\pi G \overset{1}{T}i0\right]$$

$$\rho = \frac{24\varepsilon_0 D}{\mu}\left[\left(\frac{\partial\phi}{\partial t}\right)^2 + \pi G\zeta_i \overset{1}{T}i0\right] \tag{4.1.9}$$

where, ϕ is the scalar potential

However, under the assumption of a stationary pulsar, spinning at a rate of angular frequency ω it is possible to write the momentum density $\overset{1}{T}i0$ as [29]:

$$\overset{1}{T}i0(x',t) = \overset{0}{T}00(r')\left[\omega\left(r'\right) \times x'\right]_I \tag{4.1.10}$$

From Eqn. (4.0.12) and (4.1.10) the vector potential outside the sphere is:

$$\overrightarrow{\zeta}_i = \frac{2G}{r^3}(\overrightarrow{x} \times \overrightarrow{J'}) \tag{4.1.11}$$

where, $\overrightarrow{J'} = \frac{8\pi}{3} \int \omega\left(r'\right) \overset{0}{T}00\left(r'\right)\left(r'\right)^4 dr$

From Eqn. (4.1.10) and (4.1.11) hence, if we further consider a spherical NS structure, then it follows that

$$\begin{aligned}
\zeta_i \overset{1}{T}i0 &= \tfrac{2G}{r^3}(x \times J')\left[(\omega\left(r'\right) \times\ x'\right] \overset{0}{T}00 \\
&= \frac{2G}{r^3}\overset{0}{T}00\left[J'R\sin\theta\left(-\sin\phi\hat{i} + \cos\phi\hat{j}\right)\omega R\sin\theta'(-\sin\phi'\hat{i} + \cos\phi 6'\hat{j}\right] \\
&= -\frac{2G}{r^3}\overset{0}{T}00 J'\omega R^2 \sin\theta\sin\theta'(\sin\phi\sin\phi' + \cos\phi\cos\phi') \\
&= -\frac{2G}{r^3}\overset{0}{T}00 J'\omega R^2 \sin\theta\sin\theta'\cos(\phi - \phi')
\end{aligned}$$

$$\zeta_i \overset{1}{T}i0 \cong -\frac{2G}{r^3}\overset{0}{T}00 J'\omega R^2 \sin\theta\sin\theta' \tag{4.1.12}$$

The expansion of the scalar potential is [29]:

$$\phi(x) = -\frac{GM^0}{r} - \frac{Gx.D^0}{r^3} + 0(\frac{1}{r^3}), \tag{4.1.13}$$

where, $M^0 = \int T^{00} d^3x'$, and noting that M^0 is constant in time.

$$\mathrm{D}^0 = \int xT^{00} d^3x'.$$

Now keeping only the first term in the expansion of the scalar potential $\phi(x)$, that is

$$\phi(x) = -\frac{GM^0}{r} \tag{4.1.14}$$

Finally, we can write

$$\frac{\partial\phi(x)}{\partial t} = 0. \tag{4.1.15}$$

Substituting Eqn. (4.1.12) and (4.1.15) into Eqn. (4.1.9) we obtain the total density of the charges ρ, which are located in the two separate regions discussed in section 2.2, to be

$$\rho = \gamma' J'\omega\frac{R^2}{r^3}\sin\theta\sin\theta'. \tag{4.1.16}$$

where, $\gamma' = \dfrac{48\pi\varepsilon_0 DG^2T^{00}}{\mu}$.

4.2 Derivation of the electric vector potential outside pulsar's surface

Using the charge density in Eqn. (4.1.16) and the general expression for the electric vector potential given in [27]. i.e.

$$A(\boldsymbol{x}) = \frac{\mu_0}{4\pi}\int \frac{J(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}d^3x' \tag{4.2.1}$$

we find the vector potential outside pulsar's surface as

$$A_{\phi,out} = -\frac{\mu_0}{4\pi}\int \frac{\gamma' J'\omega^2 R^3}{r^3}\sin\theta \sin^2\theta' \delta(r'-R)\frac{d^3\boldsymbol{x'}}{|\boldsymbol{x}-\boldsymbol{x'}|} \tag{4.2.2}$$

Using spherical harmonic expansion of Eqn. (3.1.2), the vector potential may be written as

$$A_{\phi,out} = -\frac{\mu_0\gamma' J'\omega^2 R^3}{4\pi r^3}\int \sin\theta \sin^2\theta' \delta\left(r'-R\right) 4\pi \sum_{lm}\frac{1}{2l+1}\frac{r^l <}{r^{l+1} >}Y^*_{lm}\left(\theta',\phi'\right)Y_{lm}(\theta,\phi)$$
$$\times r'^2 dr^{'}\sin\theta' d\theta' d\phi', \tag{4.2.3}$$

where, $r_< \equiv R$ and $r_> \equiv r$ to represent the interior and exterior regions of the pulsar, respectively.

The symmetry of the problem allows us to choose a point of observation on the x-z plane ($\phi = 0$) and having performed the integral over delta function Eqn. (4.2.3) reduces to

$$A_{\phi,out} = -\frac{\mu_0\gamma' J'\omega^2 R^7}{5r^6}\int \sin\theta \sin^2\theta' Y^*_{lm}\left(\theta',\phi'\right)Y_{lm}(\theta,\phi)\sin\theta' d\theta' d\phi' \tag{4.2.4}$$

Using the mathematical relation

$\sin^2\theta' = 4\sqrt{\tfrac{2\pi}{15}}Y_{2,2}(\theta',\phi')$ we have

$$\int \sin^2\theta' Y^*_{lm}(\theta',\phi')\sin\theta' d\theta' d\phi' = 4\sqrt{\frac{2\pi}{15}}\int Y_{22}(\theta',\phi')Y^*_{lm}(\theta',\phi')\sin\theta' d\theta' d\phi' \tag{4.2.5}$$

From orthogonality relation between Y_{22} and Y_{lm} we see that the integration have nonvanishing value only for $l = 2$ and $m = 2$, then from Eqn. (4.2.4) and (4.2.5) the vector potential become reduced to

$$A_{\phi,out} = -4\sqrt{\frac{2\pi}{15}}\frac{\mu_0\gamma' J'\omega^2 R^7}{5r^6}\sin\theta Y_{2,2}(\theta,\phi) \tag{4.2.6}$$

But inserting the value of $Y_{2,2}(\theta,\phi)$ into Eqn. (4.2.6), then finally the vector potential outside pulsar's surface will be

$$A_{\phi,out} = -\frac{\mu_0\gamma' J'\omega^2 R^7}{5r^6}\sin^3\theta. \tag{4.2.7}$$

4.3 Derivation of quadrapole magnetic field of pulsars

The magnetic field is:

$$\overrightarrow{B} = \overrightarrow{\nabla} \times \overrightarrow{A} \tag{4.3.1}$$

Since we have only ϕ component of the vector potential, the magnetic field of the pulsar will be

$$\begin{aligned}\overrightarrow{B}_{out} &= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A\phi)\right]e_{\hat{r}} - \frac{1}{r}\left[\frac{\partial}{\partial r}(rA\phi)\right]e_{\hat{\theta}} \\ &= \tfrac{1}{r\sin\theta}\left[\tfrac{\partial}{\partial\theta} - \sin\theta(\tfrac{\mu_0\gamma' J'\omega^2 R^7}{5r^6}\sin^3\theta)\right]e'_r - \tfrac{1}{r}\left[\tfrac{\partial}{\partial r}(-r\tfrac{\mu_0\gamma' J'\omega^2 R^7}{5r^6}\sin^3\theta)\right]e\theta' \\ &= -\frac{\mu_0\gamma' J'\omega^2 R^7}{5r^7}\left[\left(4\sin^3\theta\cos\theta\right)\hat{e}r + \left(5\sin^3\theta\right)e'\theta\right] \end{aligned} \tag{4.3.2}$$

Substituting Eqn. (3.2.5) into (4.3.2), then we can find

$$\overrightarrow{B}_{out} = -\frac{\mu_0\gamma\text{'} J\text{'}\omega^2 R^7}{5r^7}\left[\left(4\sin^3\theta\cos\theta\hat{j} + 4\sin^2\theta\cos^2\theta\hat{k}\right) + (5\sin^3\theta\cos\theta\hat{j} - 5\sin^4\theta\hat{k}\right]$$

using $\gamma \equiv \frac{\mu_0\gamma'}{5}$

Finally, the magnetic field outside the pulsar's surface can be written as;

$$B_{out} = -\frac{\gamma J'\omega^2 R^7}{r^7}\left[9\sin^3\theta\cos\theta\hat{j} + \left(4\sin^2\theta - 9\sin^4\theta\right)\hat{k}\right]. \tag{4.3.3}$$

Eqn. (4.3.3) is an expression for the quadrapole magnetic field component outside the pulsar's surface.

4.4 The electric vector potential for octupole

The presence of $e^{i\phi'}$ means that only $m = +1$ will be contributed to the sum. Hence, From Eqn. (4.2.3) the electric vector potential outside the pulsar's surface for $l = 3$ and $m = 1$ will be:

$$A_{\phi,out} = \frac{\mu_{0\gamma' J'\omega^2 R^8}}{4\pi r^7}\frac{4\pi}{7}\int_0^{2\pi}\int_0^{\pi} \sin\theta \sin^3\theta' \cos\phi' Y^*_{3,1}(\theta',\phi')Y_{3,1}\left(\theta,\phi\right)d\phi' d\theta' \tag{4.4.1}$$

The spherical harmonics for octupole will have the relation [27].

$$Y^*_{3,1}(\theta',\phi') = -\frac{1}{4}\sqrt{\frac{21}{4\pi}}\sin\theta'(5\cos^2\theta' - 1)e^{i\phi'} \tag{4.4.2}$$

Substituting Eqn. (4.4.2) into (4.4.1) will give:

$$A_{\phi,out} = \frac{\mu_{0\gamma' J'\omega^2 R^8}}{4\pi r^7}\frac{4\pi}{7}\int_0^{2\pi}\int_0^{\pi} \sin\theta \sin^3\theta' \cos\phi' \left[-\frac{1}{4}\sqrt{\frac{21}{4\pi}}\sin\theta'(5\cos^2\theta' - 1)e^{i\phi'}\right] Y_{3,1}(\theta,\phi)d\phi' d\theta' \tag{4.4.3}$$

Having performed the integral over an angle ϕ' and θ' function Eqn. (4.4.3) reduces to

$$A_{\phi,out} = \frac{\mu_{0\gamma' J'\omega^2 R^8}}{r^7}\frac{1}{7}\sin\theta\pi\left[\frac{1}{4}\sqrt{\frac{21}{4\pi}}\right]Y_{3,1}(\theta,\phi) \tag{4.4.4}$$

Using the mathematical relation of the spherical harmonics [27]

$$Y_{3,1}(\theta,\phi) = \frac{1}{4}\sqrt{\frac{21}{4\pi}}\sin\theta(5\cos^2\theta - 1)e^{i\phi} \tag{4.4.5}$$

Substituting Eqn. (4.4.5) into (4.4.4) finally, the vector potential outside the pulsar's surface will have a form:

$$A_{\phi,out} = -\frac{\mu_0\gamma' J'\omega^2 R^8}{r^7}\left[\frac{21}{448}\sin^2\theta(5\cos^2\theta - 1)\right]. \tag{4.4.6}$$

4.5 The octupole magnetic field outside the pulsar's surface.

Magnetic field outside the pulsar's can be obtained by taking curl of $A_{\phi out}$.

$$\overrightarrow{B} = \overrightarrow{\nabla} \times \overrightarrow{A}_{\phi out} \tag{4.5.1}$$

From Eqn. (3.2.3) and (4.4.6) the magnetic field will have a form

$$\begin{aligned}
\overrightarrow{B}_{out} &= \tfrac{1}{r\sin\theta}\tfrac{\partial}{\partial\theta}\left[\sin\theta(-\tfrac{\mu_0\gamma' J'\omega^2 R^8}{r^7}\left[\tfrac{21}{448}\sin^2\theta(5\cos^2\theta - 1)\right])\right]\hat{e}_r \\
&\quad - \tfrac{1}{r}\left[\tfrac{\partial}{\partial r}\mathrm{r} - \tfrac{\mu_0\gamma' J'\omega^2 R^8}{r^7}\left[\tfrac{21}{448}\sin^2\theta(5\cos^2\theta - 1)\right]\right]\hat{e}_\theta, \\
\overrightarrow{B}_{out} &= -\tfrac{21}{448}\tfrac{\mu_0\gamma' J'\omega^2 R^8}{r^8}\left[(25\sin\theta\cos\theta\cos^2\theta - 13\sin\theta\cos\theta)\hat{e}_r - 6\sin^2\theta(5\cos^2\theta - 1)\hat{e}_\theta\right],
\end{aligned}$$

Simplifying the last result gives us

$$\overrightarrow{B}_{out} = -\frac{21}{448}\frac{\mu_0\gamma' J'\omega^2 R^8}{r^8}\left[\sin\theta\cos\theta(25\cos^2\theta - 13)\hat{e}_r - (6\sin^2\theta(5\cos^2\theta - 1))\hat{e}_\theta\right]. \tag{4.5.2}$$

Eqn. (4.5.2) clearly shows that the final expression for octupole magnetic field exterinal regions of the pulsar's body.

4.6 Summary

In this chapter we have derived the total density of charge ρ, which are located in the two separated regions discussed in section 2.2 starting from gauss's law and also the infinite non-dipolar components of pulsar magnetic fields resulting from the rapidly spinning of pulsar with angular frequency ω is obtained, by using post-Newtonian approximation. From this infinite non-dipolar components we considered only the vector potential and the magnetic fields for quadrupole and octupole outside the pulsar's surface.

Chapter 5

Discussion and conclusion

5.1 Discussion

Eqn. (3.2.6) indicate that pulsar magnetic fields generated by the spinning separated charges is dipolar. However, this separately located charges which are also spinning with the pulsar are believed to produce the pulsars magnetic field.

Thus we can decompose the total charge density (both $+ve$ and $-ve$) into two components; i.e. ρ_0 (a background density which is constant), and ρ_1 (a varying component): If we have a total charge density ρ_{total}, this is written as

$$\rho_{total} = \rho_0 + \rho_1 \tag{5.1.1}$$

The ρ_0 component is the charge density which is required to produce the dipolar component of the pulsar's magnetic field, whereas ρ_1 is quite small and depends on the zenith angle of the pulsar surface which determines the infinite non-dipolar field.

The dipolar component varies with angle θ, the magnetic field for the dipolar case is inversely proportional to r^3 i.e $B \propto r^{-3}$. The dipole will rotate to minimize its potential energy, defining the angular dependence in equilibrium.

From Eqn. (4.0.1), the term that contains Γ^0_{0i} is responsible for the electric field from which we derive the charge density ρ_1 and this charge is the source for the infinite non-dipolar component of pulsar magnetic field.

$$B_{non-dipolar} = 0[\theta = 0^0 \text{ and } \theta = 180^0]$$

When the angle θ varies, the non-dipolar component in Eqn. (4.3.3) and (4.5.2) is:

$B_{non-dipolar} \propto r^{-7}$ for quadrapole and

$B_{non-dipolar} \propto r^{-8}$ for octupole

This shows that the infinite non-dipolar components of the pulsars magnetic field decreases enormously with the distance r as compared to the dipolar component. When angle θ, varies the dipolar and infinite non-dipolar components of pulsar magnetic field is different. The infinite non-dipolar components decay faster than the dipolar one, this shows that pulsar's magnetic field has different structure from what we observe.

5.2 conclusion

It can be concluded that Eqn. (4.3.3) and (4.5.2) suggest that the magnetic fields outside the pulsar's surface are among infinite non-dipolar component. The source of the magnetic fields for infinite non-dipolar component of pulsar's magnetic field is the charge density in Eqn. (4.1.16). The magnitude of magnetic field of pulsar is estimated to 6×10^{13} G. This magnitude of the magnetic field is estimated by magnetic flux conservation law. We have derived infinite non-dipolar component of pulsar's magnetic field, among the infinite non-dipolar component of pulsar's magnetic field we have derived the expression for quadrapole and octupole component of the magnetic field.

Moreover, observations and theory indicates that infinite non-dipolar components of pulsars magnetic field cannot be neglected in every pulsar, so that our result also show this.

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Declaration

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

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