REVIEW OF SPIN DEPENDENT HALL EFFECT

By

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The undersigned hereby certify that they have read and recommend to the program of graduate studies for acceptance a graduate project entitled "**Review of Spin Dependent Hall Effect**" by **Addis Defar** in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN PHYSICS**.

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Abstract

Hall effects, in general, are transport phenomena, in which an applied field on the particles results in a motion perpendicular to the field. In traditional Hall effect and its quantum versions charges are transported by the action of a Lorentz force. The spin Hall effect(SHE) and its quantum versions are a relativistic spin orbit coupling phenomenon allow to create and manipulate the spin, and generate a spin current. It is of particular importance to the development of transistor like devices. The spin orbit interaction responsible for the SHE is also expected to cause the inverse process of the SHE. The inverse spin Hall effect (ISHE) is a process that converts a spin current into an electric current. The quantum spin Hall effect allows for the existence of an unusual type of material called a topological insulator without external magnetic field which conducts electricity on the surface but not through the bulk of the material and my finding shows the plot of resistivity versus quantized magnetic field is plateau.

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Addis Defar
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Chapter 1

Introduction

1.1 Background of the Study

The new development of technology aims at utilizing the spin degree of freedom for electronic applications because old electronics devices consume high power, they are slow speed, volatile in nature etc. This can be solved by replacing or modifying the charge based electronics with a new field of spin based electronics called Spintronics. Through the spintronics researches, great theoretical and experimental interests have been focused such as the generation of spin currents from charge currents via the spin-orbit interaction. One interesting effects is spin Hall Effect (SHE). The (SHE)is widely recognized phenomenon that converts charge current to the spin current requiring neither external magnetic fields nor ferromagnets or the SHE couples a spin current with a charge current in a solid, and thus is crucial for the integration of spin current technology into conventional electronics based on a charge current. The spin-orbit interaction responsible for the SHE is also expected to cause the inverse process of the SHE: the inverse spin-Hall effect (ISHE), a process that converts a spin current into an electric current. The Spin hall effect which is widely used phenomena explores in order to produce some technological applications,such as replacing ferromagnetic metals with spin injector, spin polarized electron through thin films. For example, in giant magneto-resistance and tunnel magneto-resistance. Both effects are wave like properties and current occurs when electrons move through a barrier that they classically shouldn't be able to move through a barrier.

The history of hall effect began in 1879 by Edwin Hall while he was a doctoral candidate at Johns Hopkins University. Edwin Hall found that when no magnetic field is present current distribution is uniform and no potential difference is seen across the output. When a magnetic field is applied perpendicular to an electrical conductor carrying an electric current a Lorentz force is exerted on the current. This force disturbs the current distribution, resulting in a potential difference (voltage) across the output [1,2]. The appearance of a transverse voltage or electric field E_y in a metal or semiconductor in response to a longitudinal electric current J_x and perpendicular magnetic field B is known as Classical Hall Effect (ordinary Hall effect). In nonmagnetic materials, this transverse voltage arises from a deflection of charge carriers by the Lorentz force, resulting a Hall resistivity $\rho = \frac{E_y}{J_x}$ that is proportional to the magnetic field for weak fields. The ordinary Hall effect has been widely used to identify the carrier type, and measure the density and mobility of the carriers in conducting materials, and also provides a direct method to measure magnetic fields [1,2].

Shortly after the discovery of the ordinary Hall effect in 1881, Hall [2] performed similar measurements on the Ferromagnetic metal (FM) materials nickel (Ni) and cobalt (Co). He found that the Hall resistance shows an unusually large slope at low magnetic field. The unusual Hall effect in FM materials was later called the anomalous hall effect (AHE). In fact, contrary to the original ordinary Hall effect where the effect is proportional to external magnetic field, in AHE the Hall voltage is approximately proportional to the magnetization. The anomalous Hall effect can be either an extrinsic (disorder-related) effect due to spin dependent scattering of the charge carrier or an intrinsic effect which can be described in terms of the Berry phase effect in the crystal momentum space. The ordinary Hall effect and the AHE have been the only two members of the Hall family for about one century.

The spin Hall effect (SHE), another basic member of Hall family in addition to ordinary Hall effect and AHE, was first predicted by Dyakonov et al in 1971 [2]. Unlike the ordinary Hall effect where the positive and negative charge carriers accumulate at opposite edges of the sample, in SHE, spin-up and spin-down carriers accumulate at the opposite edges of the sample resulting from spin-orbit coupling, in which an electrical current, flowing through a sample, can lead to spin transport in a perpendicular direction and to a spin accumulation at lateral boundaries. These effects, which do not require an applied magnetic field, nor a ferromagnetic material, can originate in a variety of intrinsic and extrinsic spin-orbit coupling mechanisms, and depend on the geometry, dimensions, impurity scattering and the band structure of the system. The SHE belongs to the same family as the anomalous Hall effect known for a long time in ferromagnetic which also originates from spin-orbit interaction and the generation of a Hall current in a ferromagnetic material-whose theory had been developed by Luttinger and Karplus , Nozieres and Lewiner back in the 1960s, but differs from it in one essential respect: it does not require magnetic fields and/or ferromagnetism; in other words, it does not require broken time-reversal symmetry.

The discovery of the integer quantum Hall effect opened a new phase in the study of the various forms of the Hall effect [2]. In 1980, von Klitzing, Dorda and Pepper discovered experimentally that, in a two dimensional electron gas produced at a semiconductor hetero-junction subjected to a strong magnetic field, the longitudinal conductance vanishes while quantum plateaus appear in the Hall conductance at values $\nu\frac{e^2}{h}$. The pre factor is an integer $\nu =$ (1,2,3...) , known as the filling factor ,e is the electronic charge and h is the planks constant.

In 1982, Tsui, Stormer and Gossard observed that, in a sample with higher mobility, the quantum plateaus appear at filling factors ν with rational fractions ($\nu = \frac{1}{3}, \frac{2}{3}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}$..) known as the fractional quantum Hall effect, this effect relies fundamentally on the electron-electron interaction as well as the Landau-level quantization.[2]

The quantized anomalous Hall effect can be realized in a ferromagnetic insulator with a strong spin-orbit coupling. The anomalous Hall effect persists in an insulating regime. The anomalous Hall conductance can be expressed in terms of the integral of the Berry curvature over the momentum space or the Chern number for fully filled bands. The Haldane model produces non-zero Chern numbers for an electron band without the presence of a magnetic field or Landau levels. According to the bulk-edge correspondence, the quantized Hall conductance originates from the dissipation less transport of topologically protected edge states. There have been extensive investigations on this topic. One of the promising schemes is based on a magnetically doped topological insulator thin film, where an interplay between the strong spin-orbit coupling in the surface states and magnetic exchange coupling gives rise to a band gap opening to form chiral edge states. In 2013, the experimental observation of the quantum anomalous Hall effect was reported in a Cr-doped $(Bi, Sb)_2Te_3$ ultra-thin film by a group led by Chan in Beijing [2].

One year after the experimental observation of the SHE, in 2005, the quantized version of the SHE, the so-called the quantum spin Hall effect (QSHE), was first theoretically proposed in graphene by Kane. Kane's QSHE model can be considered as two copies of the Haldane model, where the spin-up electrons exhibit a chiral QHE while the spin down electrons exhibit an anti-chiral QHE. In reality, the QSHE was observed in the HgTe/CdTe quantum well structure by Molenkamp in 2007 after the prediction of Zhang in 2006. The QSHE in HgTe/CdTe quantum wells can also be regarded as 2D TI [2], see Fig 1.1.

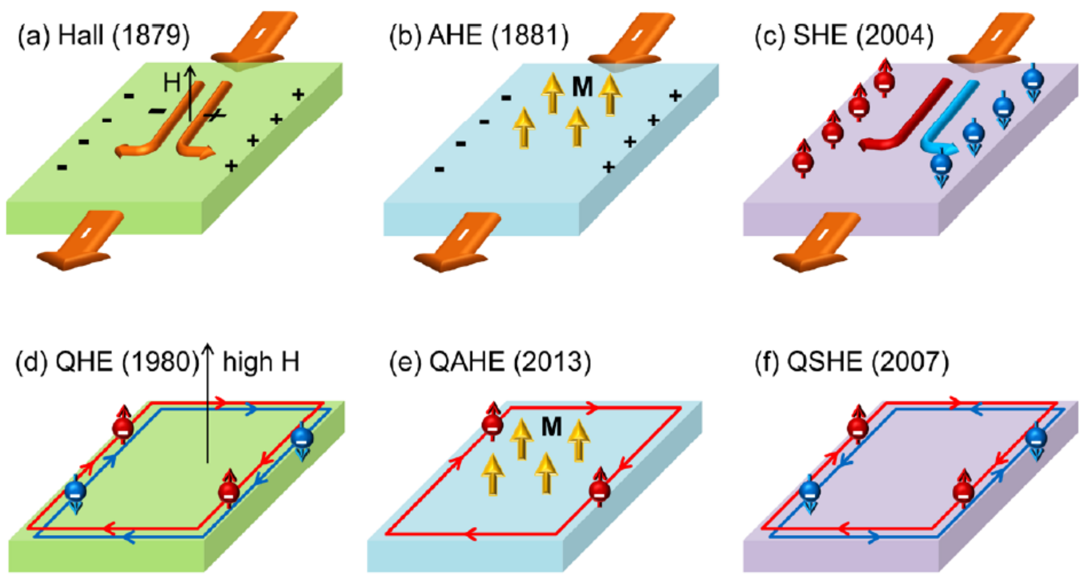

Figure 1.1: Family of Hall effect.

1.1.1 Motivation

My motivation for this project is based on the problem of conventional electronic devices. In conventional electronics,for example, in our computer chips, information is transported by electrical charge this means electrons or other charge carriers have to be moved from one place to another to store or transfer information. This consumes energy and if power is turned off data may be lost. Moreover, large amount of transistors integrated in the electronic devices may be another challenges. As science and technology advances, the issue of getting small devices that can be portable and easily managed with low cost of energy becomes crucial. Among different mechanisms to design such machines or devices is to use the spin property of electrons, and spin dependent quantum Hall effect is one of them.

1.1.2 Objectives of the Study

The main objective of this project work is to review the basic principle of spin dependent quantum Hall effect. The specific objectives are to discuss the classical Hall effect,quantum Hall effect, integral and fractional Hall effect and spin dependent Hall effect. Specifically, the governing mathematical equation will be addressed.

1.1.3 Outline

Chapter 1 provide the general principle of Hall effect and the objective of the project. In chapter 2, we discuss the classical Hall effect, the origin of Hall field,quantum Hall effect, integral and fractional quantum Hall Effect and Quantized Hall Conductance,anomalous Hall Effect and Quantized Hall conductance, anomalous Hall effect and its application. Chapter 3 presents the spin dependent Hall effect and its application. Different numerical analysis are presented. Chapter 4 presents discussion and conclusion.

Chapter 2

The Classical Hall Effect

If a magnetic field is applied in a current carrying conductor or semiconductor in a direction perpendicular to that of the flow of current (that is in z-direction) as shown in Fig 2.1, an electric field is produced in it that exert a force in the positive y-direction. The field developed across the conductor is Hall field and the corresponding potential difference is called Hall voltage and its value is found to be depend on the magnetic field strength, nature of the material and the applied current. This effect is observed in 1879 by Edwin Hall [1,3,4]. He measured the voltage that arises from the deflected motion of charged particles in solids under externally applied electric and magnetic fields. Consider a two-dimensional sample subjected to a perpendicular magnetic field $\vec{B} = (0, 0, B_z)$. Charged particles passing through the sample are deflected by the Lorentz force and accumulate near the boundary. As a result, the charge accumulation along the boundary produces an electric field $\vec{E} = (E_x, 0, 0)$. When the electric and magnetic forces are balanced, the Lorentz force on a moving charged particle is zero:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = 0 \tag{2.0.1}$$

Mathematically we can express it as $eE = evB \Rightarrow$

$$E = Bv, \tag{2.0.2}$$

where $\vec{E}$ the hall electric field developed, $\vec{B}$ the applied magnetic field where $\vec{v}$ is the velocity of the particle and q is the charge of the particle. The voltage difference between the two boundaries is

$$V_H = E_y W, \tag{2.0.3}$$

W being the width of the sample and the electric current through the sample is

$$I = \rho_e v W, \tag{2.0.4}$$

with $\rho_e = nq$ being the density of the charge carriers. The ratio of the voltage to the electric current is known as the Hall resistance.

$$R_H = \frac{V_H}{I} = \frac{B}{nq}, \tag{2.0.5}$$

which is linear in the magnetic field B. In practice, the Hall effect is used to measure the sign of charge carriers q,that is the particle or hole like charge carrier, and the density of charge carriers ρ_e in solids. It can also be used to measure the magnetic field[8,9,11] see Fig 2.1.

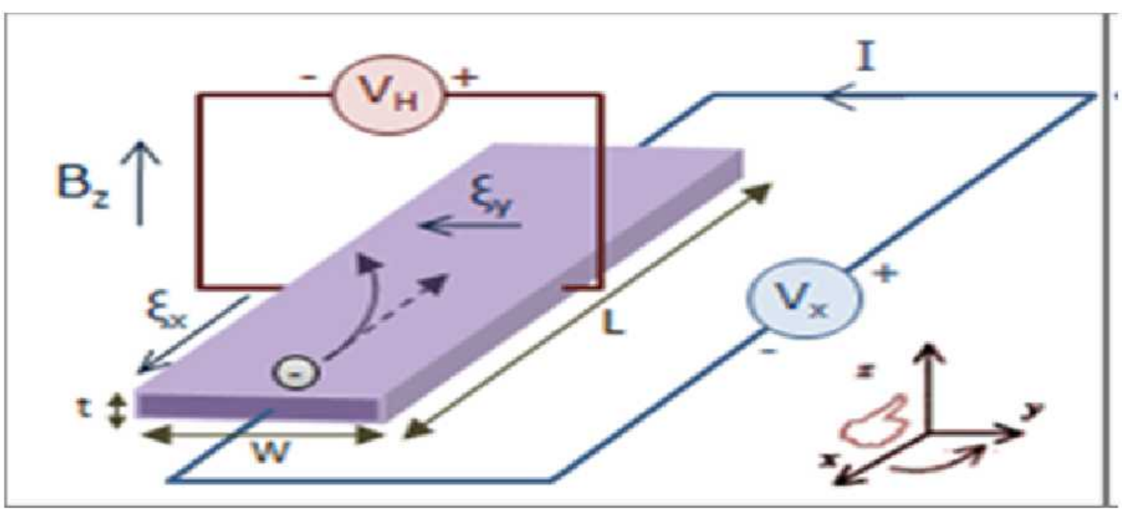

Figure 2.1: Classical Hall effect.

2.1 The Classical Motion in an Electric Field

When a voltage is applied to a conductor or semiconductor,electric current starts flowing through it. In a conductor, the electric current is conducted by free electron where as in semiconductors electric current conducted by free electron and holes.The free electron in conductor or semiconductors always try to flow in a straight line path. However, because of the continues collisions with atoms, free electron slightly change their direction. But if the applied voltage is strong enough, the free electrons forcefully flow the straight path. This happens only if no other forces are applied to it in other direction.If we apply a force such as magnetic force in other direction, the free electron in a conductor or semiconductor change direction. The classical equation of motion of an electron in the presence of magnetic $\vec{B}$ and electric $\vec{E}$ fields can be expressed as [5,10] as:

$$\frac{m^* d\vec{v}}{dt} + \frac{m^*\vec{v}}{\tau} = -e(\vec{E} + \vec{v} \times \vec{B}). \tag{2.1.1}$$

If we suppose that the external forces go back to zero,the state returns to its equilibrium position exponentially with a relaxation time $\frac{dv}{dt} = -\frac{v}{\tau}$, then

$$v = v_0 e^{\frac{-t}{\tau}}. \tag{2.1.2}$$

If the external forces remain constant,the system goes to a stationary state $\frac{dv}{dt} = 0$. Supposing that the external force is due to a homogeneous electric field E, the new stationary velocity, or drift velocity v_d of the charge carrier becomes:

$$\frac{mdv}{dt} + \frac{mv}{\tau} = F_e \Rightarrow 0 + \frac{mv_d}{\tau} = qE,$$

then

$$v_d = \frac{q\tau E}{m^*} = \mu E, \tag{2.1.3}$$

where $\mu = \frac{q\tau}{m^*}$ is the mobility.

2.2 The classical motion in a magnetic field

The Hall effect arises from the fact that a magnetic field causes charged particles to move in circles.Let's recall the basics. The equation of motion for a particle of mass m and charge -e in a magnetic field is

$$\frac{md\vec{v}}{dt} = -e(\vec{v} \times \vec{B}), \tag{2.2.1}$$

with

$$\frac{d\vec{v}}{dt} = \frac{e}{m}(\vec{v} \times \vec{B}) \Rightarrow \frac{d\vec{v}}{dt} = \vec{v} \times \vec{\omega_B},$$

where

$$\omega_B = \frac{eB}{\gamma m}.$$

The motion described by is circular motion perpendicular to $\vec{B}$ and a uniform translational motion parallel to $\vec{B}$. The solution for the velocity is

$$v(t) = v_{\parallel}\varepsilon_3 + \omega_B a(\varepsilon_1 - i\varepsilon_2)e^{-i\omega_B t}, \tag{2.2.2}$$

where ε_3 is unit vector parallel to the field, $\varepsilon_1$and $\varepsilon_2$are other orthogonal unit vectors,$v_{\parallel}$ is the velocity component along the field,and a is the gyration radius. The displacement of the particle is

$$x(t) = x_0 + v_{\parallel}t\varepsilon_3 + ia(\varepsilon_1 - i\varepsilon_2)e^{-i\omega_B t}. \tag{2.2.3}$$

2.3 The Drude Model and Classical Hall Effect

Ordinary Hall effect [3, 4], discovered more than a century ago, the Lorentz force resulting from a magnetic field B_z applied perpendicular to a two dimensional sample gives rise to an electric field E_y perpendicular to the applied current I_x through the sample. As a result, a transverse resistivity

$$\rho_{xy} = \frac{E_y}{J_x}, \tag{2.3.1}$$

and the longitudinal resistivity

$$\rho_{xx} = \rho_{yy} = \frac{E_{xx}}{J_x}, \tag{2.3.2}$$

where J_x is the current density. The Drude model considers a conductor as a gas of free current-carrying charges. The freely moving carriers suffer randomizing collision events on average every τ seconds. The parameter τ is called the relaxation time [5] and the Drude model shows that the transverse resistivity is linear in the magnetic field:

$$\rho_{xy} = R_0 B_z. \tag{2.3.3}$$

In ferromagnetic systems, an extra contribution to the off-diagonal resistivity was found:

$$\rho_{xy} = R_0 B_z + R_s M. \tag{2.3.4}$$

This anomalous contribution is proportional to the magnetization M, and gives rise to a Hall effect even in the absence of an externally applied magnetic field.
Consider a current I flowing in a sample, perpendicular to an external magnetic field B.The trajectory of an electron under the influence of electric and magnetic fields is described by the equation is

$$m_e \frac{d\vec{v}}{dt} = -e(\vec{E} + \vec{v} \times \vec{B}), \tag{2.3.5}$$

where e is the charge of an electron, m_e its mass,and $\vec{v}$ its velocity. Assuming the electric and magnetic fields are perpendicular to each other ($E = E_x and B = B_z$), the solution to this equation is'cyclotron motion', circular orbit with cyclotron frequency

$$\omega_c = \frac{eB}{m}, \tag{2.3.6}$$

and a linear motion with velocity $v = \frac{E}{B}$. The uniform component of the motion is perpendicular to both applied fields E and B. The electron therefore moves in a spiral trajectory.
If we consider the electron to be inside a crystal rather than completely free, then

most of the effects of the crystal-field potential (the periodic potential of the crystal lattice)can be accounted for simply by modifying the mass of the electron:The free electron mass m_e is replaced with an effective mass m^*. In practice, the crystalline lattice is not perfect, and the scattering of the electron from defects, lattice vibrations(phonons) and other electrons must be taken into account. This can be included by introducing a relaxation time τ,which is a measure of the time in which the charge carrier will be scattered. Drude assumed that the probability that an electron collides with an ion during a time interval dt is simply proportional to $\frac{dt}{\tau}$, where τ is called the collision time or relaxation time. Then Newton's law give equation of motion with the addition of this damping term, the equation of motion becomes:

$$\frac{m^* d\vec{v}}{dt} + \frac{m^*}{\tau} = -e(\vec{E} + \vec{v} \times \vec{B}), \tag{2.3.7}$$

where $\vec{v}$ is the drift velocity and τ is the scattering time. $\vec{B}$ is the magnetic field applied along the z-axis.

Case1:If E and v are assumed to vary with time as $e^{-i\omega t}$. This equation can be expressed in its three components as.

$$\frac{m^* dv_x}{dt} + \frac{m^* v_x}{\tau} = qE_x + qv_y B,$$

$$-m^* i\omega v_x + \frac{m^* v_x}{\tau} = qE_X + qv_y B. \tag{2.3.8}$$

$$\frac{m^* dv_y}{dt} + \frac{m^* v_y}{\tau} = qE_y - qv_x B,$$

$$-m^* i\omega v_y + \frac{m^* v_y}{\tau} = qE_y - qv_x B. \tag{2.3.9}$$

$$\frac{m^* dv_z}{dt} + \frac{m^* v_z}{\tau} = qE_z,$$

$$-m^* i\omega v_z + \frac{m^* v_z}{\tau} = qE_z. \tag{2.3.10}$$

Substituting equation 2.3.8 in to equation2.3.9,and defining $\omega_c = \frac{eB}{m^*}$, we get

$$v_x = \frac{q\tau}{m^*}\left(\frac{E_x(1 - i\omega\tau) + \omega_c \tau E_y}{1 - (\omega^2 - \omega_c^2)\tau^2 - 2i\omega\tau}\right),$$

$$v_y = \frac{q\tau}{m^*}\left(\frac{E_y(1-i\omega\tau)+\omega_c\tau E_x}{1-(\omega^2-\omega_c^2)\tau^2-2i\omega\tau}\right).$$

The solution can be written in matrix form. For example in 2D as

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix},$$

$$J_x = \sigma_{xx}E_x + \sigma_{xy}E_y = nqv_x = \frac{nq^2\tau}{m^*}\left(\frac{E_x(1-i\omega\tau)+\omega_c\tau E_y}{1-(\omega^2-\omega_c^2)\tau^2-2i\omega\tau}\right),$$

$$J_y = \sigma_{yx}E_x + \sigma_{yy}E_y = nqv_y = \frac{nq^2\tau}{m^*}\left(\frac{E_y(1-i\omega\tau)+\omega_c\tau E_x}{1-(\omega^2-\omega_c^2)\tau^2-2i\omega\tau}\right),$$

where

$$\frac{nq^2\tau}{m} = \sigma_0.$$

These leads to the following result.

$$\sigma_{xx} = \sigma_{yy} = \frac{\sigma_0(1-i\omega\tau)}{1-(\omega^2-\omega_c^2)\tau^2-2i\omega\tau)} = \frac{\rho_{xx}}{\rho_{xx}^2+\rho_{xy}^2},$$

$$\sigma_{xy} = -\sigma_{yx} = \frac{\sigma_0\omega_c\tau}{(1-(\omega^2-\omega_c^2)\tau^2-2i\omega\tau)} = \frac{\rho_{xy}}{\rho_{xx}^2+\rho_{xy}^2},$$

$$\sigma_{zz} = \frac{\sigma_0}{1-i\omega\tau},$$

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \frac{\sigma_0}{(1-(\omega^2-\omega_c^2)\tau^2-2i\omega\tau)}\begin{pmatrix} 1-i\omega & \omega_c\tau \\ -\omega_c\tau & 1-i\omega \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix},$$

Hence, the off diagonal elements are zero.

Case2: When $\vec{v}$ and $\vec{E}$ not vary with time. The equation motion can is easily solved in the steady-state $\frac{dv}{dt} = 0$ (where F = 0). The x and y components are:

$$\frac{m^*v_x}{\tau} = qE_x + qv_yB \Rightarrow v_x = \frac{q\tau}{m^*}(E_x + v_yB), \tag{2.3.11}$$

$$\frac{m^*v_y}{\tau} = qE_y - qv_x \Rightarrow v_y = \frac{q\tau}{m^*}(E_y - v_xB). \tag{2.3.12}$$

Substituting eq (2.3.11) in to eq (2.3.12),and defining $\omega_c = \frac{eB}{m^*}$, we get

$$v_x = \frac{q\tau}{m^*}(E_x + \frac{q\tau}{m^*}(E_y - v_xB)B) \Rightarrow v_x + \frac{v_xq^2\tau^2B^2}{m^{*2}} = \frac{q\tau}{m^*}(E_x + \frac{E_yq^2\tau^2B^2}{m^{*2}}),$$

$$v_x = \frac{q\tau}{m^*}(\frac{E_x + \omega_c \tau E_y}{1 + \omega_c^2 \tau^2}). \tag{2.3.13}$$

$$v_y = \frac{q\tau}{m^*}(E_y - \frac{q\tau}{m^*}(E_x + v_y B)B) \Rightarrow v_y + \frac{v_y q^2 \tau^2 B^2}{m^{*2}} = \frac{q\tau}{m^*}(E_y - \frac{E_x q^2 \tau^2 B^2}{m^{*2}}),$$

$$v_y = \frac{q\tau}{m^*}(\frac{E_y - \omega_c \tau E_x}{1 + \omega_c^2 \tau^2}). \tag{2.3.14}$$

This leads to present the current density for 2D as

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix},$$

$$J_x = \sigma_{xx} E_x + \sigma_{xy} E_y = nqv_x = \frac{nq^2\tau}{m}(\frac{E_x + \omega_c \tau E_y}{1 + \omega_c^2 \tau^2}),$$

$$J_y = \sigma_{yx} E_x + \sigma_{yy} E_y = nqv_y = \frac{nq^2\tau}{m}(\frac{E_y - \omega_c \tau E_x}{1 + \omega_c^2 \tau^2}),$$

where $\frac{nq^2\tau}{m} = \sigma_0 \Rightarrow \sigma_{xx} = \sigma_{yy} = \frac{\sigma_0}{1+\omega_c^2\tau^2} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}, \sigma_{xy} = -\sigma_{yx} = \frac{\sigma_0 \omega_c \tau}{1+\omega_c^2\tau^2} = \frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2},$

$$\Rightarrow \begin{pmatrix} J_x \\ J_y \end{pmatrix} = \frac{\sigma_0}{1 + \omega_c^2 \tau^2} \begin{pmatrix} 1 & \omega_c \tau \\ -\omega_c \tau & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \tag{2.3.15}$$

For small magnetic field, $\omega_c \tau << 1, \omega_c^2 \tau^2$ can be neglected.The resistance along the same direction as the electric field, the diagonal resistivity (the longitudinal resistivity $\rho = \frac{1}{\sigma}$) is independent of the magnetic field.i.e, $\rho_{xx} = \rho_{yy} = \frac{E_{xx}}{J_x} = \sigma_0^{-1}$. The resistivity perpendicular to the electric field direction the hall resistivity(transverse resistivity) is

$$\rho_{yx} = -\rho_{xy} = Ey/Jx = \sigma_0^{-1} \omega_c \tau = B/en_e \tag{2.3.16}$$

and is proportional to the magnetic field strength. If there is no magnetic field applied, then the off-diagonal terms disappear

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \sigma \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}, \tag{2.3.17}$$

and the tensor Equation can be reduced to the well known form of Ohm'sLaw

$$J = \sigma E = \frac{1}{\rho} E = nev. \tag{2.3.18}$$

2.4 The Hall coefficient in Metal

The hall coefficient is a parameter that measures the magnitude of the Hall Effect in the sample [5,6] is defined as the ratio of the induced electric field to the product of the current density and the applied magnetic field. It is a characteristic of the material from which the conductor is made, since its value depends on the type, number, and properties of the charge carriers that constitute the current.The Hall coefficient is denoted as

$$R_H = \frac{E_y}{J_x B} \tag{2.4.1}$$

where E_y, J_x, and B are the magnitudes of the transverse electric field, the longitudinal current density, and the magnetic field, respectively. To explain Hall coefficient [5,6,7] in conductor consider a sample of block of conductor of length l, width w and thickness t through which electric current I is applied along the x-axis. The electric field E is assumed to be $E = (E_x, E_y, 0)$. Therefore current density is given by $J_x = \frac{I}{wt}$. If the magnetic field is applied along negative z-axis, i.e, B = (0, 0, B), the Lorentz force moves the charge carriers (say electrons) toward the y-direction. This results in accumulation of charge carriers at the top edge of the sample. This set up a transverse electric field E_y in the sample. This develop a potential difference along y-axis is known as Hall voltage V_H and this effect is called Hall Effect. see Fig 2.2.

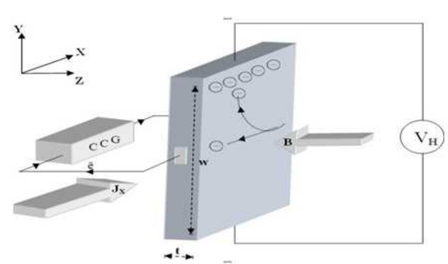

Figure 2.2: Hall effect in metal

In steady state condition, the magnetic force is balanced by the electric force. Mathematically we can express it as $eE = evB$, $I = neAv$, where n is the number density of electrons in the conductor of length l,breadth'w'and thickness t, A is the cross-sectional area the Hall voltage V_H and the Hall coefficient respectively are

$$V_H = -E_y w = \frac{JBw}{nq} \tag{2.4.2}$$

$$R_H = \frac{E_y}{J_X B} = \frac{1}{nq} \tag{2.4.3}$$

2.5 The Hall Coefficient in Semiconductor

The vector equation of motion for a particle of charge q and mass m^* in a solid, undergoing an external force F_e is given by eq(2.3.11)and eq(2.3.12)

$$v_x = \frac{q\tau}{m^*}(E_x + v_y B),$$

and

$$v_y = \frac{q\tau}{m^*}(E_y - v_x B).$$

We now consider an arbitrary number of carrier types with each type being labeled by the integer j; the j^{th} carrier type has effective mass m_j, charge q_j , scattering rate j and number density n_j. For each carrier type, for $\omega_c^2 \ll 1$, implying that terms $\omega_c^2\tau^2$ can be neglected. Equation (2.3.14) therefore becomes

$$v_{yj} = \frac{q_j\tau_j}{m_j^*}(E_y - \frac{q_j B\tau_j, E_x}{m^*}) \tag{2.5.1}$$

The net transverse current must be zero,as there is nowhere for it to go.Therefore

$$\Sigma n_j q_j v_{yj} = 0. \tag{2.5.2}$$

Equation (2.5.2) can be used to derive the Hall coefficient for an arbitrary number of carrier types. We shall use them to treat the simple case of electrons and heavy holes in a semiconductor. The Hall effect represents a steady state of the system.i.e $\frac{dx}{dt} = \frac{dy}{dt} = 0$.eq (2.3.11) and (2.3.12)can be used.

$$\frac{nq_c^2\tau}{m_c^*}(E_y - \frac{q_cB\tau_cE_x}{m_c^*}) + \frac{pq_h^2\tau_h}{m_h^*}(E_y - \frac{q_hB\tau_hE_x}{m_h^*}) = 0,$$

$$\frac{nq_c^2\tau_cE_y}{m_c^*} - \frac{nq_c^3\tau_c^2BE_x}{m_c^*} + \frac{pq_h^2\tau_hE_y}{m_h^*} - \frac{pq_h^3B\tau_h^2E_x}{m_{hh}^{*2}} = 0,$$

$$\frac{nq_c^2\tau_cE_y}{m_c^*} + \frac{pq_h^2\tau_hE_y}{m_h^*} - \frac{nq_c^3\tau_c^2BE_x}{m_c^*} - \frac{pq_h^3B\tau_h^2E_x}{m_h^{*2}} = 0,$$

$$E_y(\frac{nq_c^2\tau_c}{m_c^*} + \frac{pq_h^2\tau_h}{m_h^*}) = BE_x(\frac{nq_c^3\tau_c}{m_c^*} + \frac{pq_h^3\tau_h^2}{m_h^2}).$$

,since q_c=-e and q_h=+e

$$E_y(\frac{n(-e^2)\tau_c}{m_c} + \frac{pe^2\tau_h}{m_h^*}) = BE_x(\frac{n(-e^3)\tau_c}{m_c^*} + \frac{pe^3\tau_h^2}{m_h^2}),$$

$$E_ye(\frac{ne\tau_c}{m_c^*} + \frac{pe\tau_h}{m_h^*}) = BE_xe(\frac{n(-e)^2\tau_c^*}{m_c^*} + \frac{pe^2\tau_h^2}{m_h^{*2}}),$$

$$E_y(\frac{ne\tau_c}{m_c^*} + \frac{pe\tau_h}{m_h^*}) = -BE_x(\frac{n(-e)^2\tau_c^2}{m_c^{*2}} + \frac{pe^2\tau_h^2}{m_h^{*2}}).$$

Defining,from equation (2.1.3) $\mu = \frac{q\tau}{m^*} \Rightarrow \mu_h = \frac{q_h\tau_h}{m_h^*}$ is the hole mobility and $\mu_c = \frac{q_c\tau_c}{m_c^*}$ is the electron mobility

$$E_y(n\mu_c + p\mu_h) = E_xB(p\mu_h^2 - n\mu_c^2),$$

$$E_y = E_xB(\frac{p\mu_h^2 - n\mu_c^2}{n\mu_c + p\mu_h}), \tag{2.5.3}$$

$$J_x = nq_cv_{ex} + pq_hv_{hx},$$

$$J_x = nq_c(\frac{q_c\tau_c}{m_c^*}(E_x + v_yB)) + pq_h(\frac{q_h\tau_h}{m_h^*}(E_x + v_yB)),$$

$$J_x = \frac{nq_c^2\tau_cE_x}{m^*} + \frac{nq_c^2\tau_cv_yB}{m^*} + \frac{pq_h^2\tau_hE_x}{m^*} + \frac{pq_h^2\tau_hv_yB}{m^*},$$

$$J_x = \frac{nq_c^2\tau_cE_x}{m^*} + \frac{pq_h^2\tau_hE_x}{m^*} + \frac{nq_c^2\tau_cv_yB}{m^*} + \frac{pq_h^2\tau_hv_yB}{m^*}.$$

Since the motion of the charge in y- direction negligible, $v_y = 0$ then and using $q_c =$ -e

$$J_x = (\frac{ne^2\tau_c}{m^*} + \frac{pe^2\tau_h}{m^*})E_x$$

$$J_x = e(n\mu_c + p\mu_h)E_x,$$

$$R_H = \frac{E_y}{J_x B} = \frac{1}{e}\left(\frac{p\mu_h^2 - n\mu_c^2}{(n\mu_c + p\mu_h)^2}\right), \tag{2.5.4}$$

where n is the electron concentration and p is the hole concentration, μ_c the electron mobility, μ_h the hole mobility and e the absolute value of the electronic charge. For large applied fields the simpler expression analogous to that for a single carrier type holds.

$$R_H = \frac{1}{|e|}\left(\frac{p - nb^2}{(p + nb^2)^2}\right),$$

with

$$b = \frac{\mu_c}{\mu_h}.$$

2.6 Quantum Hall Effect

The quantum Hall effect [7,8,9] is a quantum mechanical version of the Hall effect in two dimensions. This effect is very well understood now and can be simply explained in terms of single-particle orbital of an electron in a magnetic field . It is known that the motion of a charged particle in a uniform magnetic field is equivalent to that of a simple harmonic oscillator in quantum mechanics, in which the energy levels are quantized with energy $E_n = (n + \frac{1}{2})\hbar\omega_c$ and $\omega_c = \frac{eB}{m}$ is the cyclotron frequency. The energy levels, called Landau levels, are highly degenerate. In 1980, Klaus von Klitzing who was awarded the Nobel Prize for this work just five years later(discovered experimentally that,in a two dimensional electron gas produced at a semiconductor hetero-junction subjected to a strong magnetic field). He discovered that the Hall-voltage of a 2DEG of a Silicon-MOSFET as a function of the charge carrier density at low temperatures and high magnetic field strength,the plot of resistivity vs applied magnetic field strength becomes an increasing series of plateaus. Rather than a continuous increase of the Hall voltage with decreasing carrier density or increasing magnetic field, as would be expected from the classical Hall effect. This implied that in quantum mechanics, resistance is quantized in units of $\frac{h}{e^2}$. The plateaus corresponded to the cases where the resistivity was related to the magnetic field by integer and some fractional values of a quantity known as the filling factor. These integer and fractional values led to the theory of the integer quantum Hall effect and the fractional quantum Hall effect. Both of these effects have since been observed in graphene, a single layer of carbon atoms in a hexagonal lattice, at room temperature[7,8,9].

2.6.1 Two Dimensional Electron System

A two dimensional electron system is formed in a heterostructure which has layers of GaAs over $Ga_{1-x}Al_xAs$ [8,11]. One method is the MOSFET (metal-oxide-semiconductor field effect transistor). In a MOSFET the electrons are confined to the surface of a semiconductor, by sandwiching an oxide (insulator) in between the semiconductor and a gate. Applying an electric field through the gate, will put a drag on the conductance electrons of the semiconductor, pushing them against the impenetrable oxide-layer, hence creating a plane (two dimensions) of electrons. It was in a silicon MOSFET von Klitzing and collaborators discovered the quantum Hall effect [11].

2.6.2 Landau Level

Another important concept in the explanation of the quantum Hall effect is Landau levels [8,11]. Consider an electron confined to the x-y plane in the presence of a uniform magnetic field in the z-direction. The Hamiltonian is given by

$$\hat{H} = \frac{1}{2m}\left(\vec{p} - \frac{q}{c}\vec{A}\right)^2, \tag{2.6.1}$$

where $\vec{A}$ is the vector potential related to the applied magnetic field by the Maxwell relation $\nabla \times \vec{A} = \vec{B}$, $\vec{p} = \vec{p_x}i + \vec{p_y}j$.

2.6.3 Filling Factor

Another important feature in describing the quantum Hall effect is the filling factor[8]. The electron orbital corresponding to a given n value is localized at $y = k_x l^2$. If the electron is confined to a space of length L_x in the x-direction with the periodic boundary conditions $e^{ik_x.(x+L_x)} = e^{ik_x.x}$then the allowed values of k_x and n_x are $k_x = \frac{2\pi n_x}{L_x} \Rightarrow n_x = \frac{L_x k_x}{2\pi}$, where n_x is the corresponding Landau level quantum number. Counting the number of states in a region of $L_x L_y$ defined by y = 0 and $y = L_y$ (For simplicity, the state at y = 0 has $n_x = 0$ and the state $y = L_y$ corresponds to the wave vector $k_x = \frac{L_y}{l^2}$ gives the total number of states in this region, N_x: $N_x = \frac{L_x L_y}{2\pi l^2}$. The degeneracy of the Landau states G is then

$$G = \frac{N_x}{L_x L_y} = \frac{1}{2\pi l^2} = \frac{B}{\phi_0}, \tag{2.6.2}$$

where is ϕ_0 is the flux quantum $\phi_0 = \frac{hc}{e}$. Therefore, there is one state per flux quantum in each Landau level. The filling factor is the number of occupied Landau levels for electrons in a given magnetic field. It is defined as [8,9]

$$\nu = \frac{\rho}{G} = 2\pi l^2 \rho = \frac{\rho}{B/\phi_0}, \tag{2.6.3}$$

where ρ_0 is the two dimensional density of electrons. Hence, the number of electrons that can exist in a given Landau level increases proportionally with the magnetic field strength so that as the magnetic field strength increases fewer and fewer Landau levels are occupied. Thus, the filling factor is a measure of both the applied magnetic field strength and the number of Landau levels that are occupied in a system.

2.6.4 The Quantum Hall Conductivity(σ_H)

As previously explained the quantum Hall effect occurs when a two-dimensional system of electrons is subject to a uniform and strong magnetic field. It is characterized by quantized values of the Hall conductivity that is Hall effect at low temperature and strong magnetic field. In two-dimensional electron systems, with the increase of magnetic field, the longitudinal conductivity σ_{xx} becomes zero. That means the sample shows insulating property when the Hall conductivity is quantized , and at the same time, the transverse Hall conductance σ_{xy} [8,9,11] exhibits a series of quantized plateaus and given by

$$\sigma_H = \sigma_{xy} = \nu \frac{e^2}{h}. \tag{2.6.4}$$

In a two-dimensional system, the density of electrons [8] ρ can be written as:$\rho = \frac{\nu B}{\phi_0}$. Then, the classical Hall resistance with q = e is

$$R_H = \frac{B}{\rho e} = \frac{h}{\nu e^2}, \tag{2.6.5}$$

where e is the elementary charge and h is Planck's constant. Therefore, the Hall resistance is quantized in units of $\frac{h}{e^2}$ and is inversely proportional to the filling factor. The pre factor ν is known as the "filling factor" [11], and can take on either rational $\nu = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{2}{3}, \frac{3}{5}, \frac{1}{5} \ldots$ values or integer $\nu = 1, 2, 3 \ldots$. The quantum Hall effect is referred to as the integer or fractional quantum Hall effect depending on whether ν is an integer or fraction respectively. When one Landau level is fully filled, the filling factor is $\nu = 1$ and the corresponding Hall conductance is $\frac{e^2}{h}$.

2.6.5 The Integer Quantum Hall Effect

The integer quantum Hall effect is very well understood, and can be simply explained in terms of single-particle orbital's of an electron in a magnetic field [9]. The integral quantum Hall effect (IQHE) was discovered by Klaus von Klitzing in 1980. Von Klitzing was studying the Hall effect of two-dimensional electrons in silicon MOSFET (metal oxide-semiconductor field effect transistor). He found that at low temperatures and high magnetic field, the Hall resistance of the system did not vary linearly with the strength of the magnetic field as predicted by the classical Hall effect. The plot of the resistivity as a function of magnetic field strength exhibited many plateaus which indicated that the Hall resistance is quantized. When the factor takes integer values, the Landau levels are completely filled and the integer quantum Hall effect (IQHE) takes place. It is believed that this phenomenon is related to the transport properties of non interacting electrons. Integer values of the filling factor describes a system of non-interacting electrons where the highest Landau level is completely filled. Once the Landau level is completely filled, a gap exists requiring a finite amount of energy to reach the next degenerate Landau level. However, impurities in the sample create localized potentials that can trap electrons in localized states. Therefore, if the filling is changed slightly, the extra electrons fill the localized states and do not contribute to the current. Thus,in regions where the filling factor has an integer value, there is a plateau in the plot of resistivity vs magnetic field strength where the longitudinal resistance of the sample disappears[2,7,8,9,11].

2.6.6 The Fractional Quantum Hall Effect

The fractional quantum Hall effect is more complicated, as its existence relies fundamentally on electron-electron interactions [7,8,11]. In 1982,the fractional quantum Hall effect (FQHE) was discovered by Horst Stormer and Dan Tsui. By repeating von Klitzing's earlier experiments with cleaner samples in higher magnetic fields, they found that the plateaus in the plot of resistivity vs magnetic field strength also occurred at some fractional values of the filling factor. Because fractional values of the filling factor refers to partially filled Landau states, the plateaus could only be explained in terms of interacting particles. When the factor takes rational values with an odd denominator, the Landau levels are only partially filled: this is the fractional quantum Hall effect (FQHE),which results from the repulsive Coulomb interaction between electrons. Laughlin proposed that the $\nu = \frac{1}{3}$ state is a new type of many-body condensate, which can be described by the Laughlin wave function. The quasi-particles in the condensate carry fractional charge $\frac{e}{3}$ because of their strong Coulomb interaction. The observed Hall conductance plateaus arise from the localization of the fractionally charged quasi-particles in the condensate, and the fractional quantum Hall effect can be regarded as the integer quantum Hall effect of these quasi-particles. In 1988, Jainendra K. Jain proposed that the quasi-particles, called composite fermions, can be regarded as a combination of an electron charge and quantum magnetic flux. This picture is applicable to all the quantum plateaus observed in the fractional quantum Hall effect, which is now well accepted in terms of a topological quantum phase of composite fermions that breaks the time-reversal symmetry. In 1988, Duncan Haldane proposed that the integer quantum Hall effect can be realized in a lattice system of spinless electrons in a periodic magnetic flux. Though the total magnetic flux is zero, electrons are driven to form a conducting edge channel by the periodic magnetic flux. As there is no pure magnetic field, the quantum Hall conductance originates from the electron band structure for the lattice instead of the discrete Landau levels create in a strong magnetic field. Thus, this is a version of the quantized anomalous Hall effect in the absence of an external field or Landau levels. Furthermore, it was found that the role of the periodic magnetic flux can be replaced by the spin-orbit coupling[7,8,9,11].

2.7 Anomalous Hall Effect

Hall found that the Hall resistivity and the hall voltage in ferromagnetic metals acquires an extra term which depends on the magnetization of the samples [13]. Hall measured the resistances in ferromagnetic as well as paramagnetic metals under various magnetic fields and observed that it could have an additional contribution other than the term linear in the magnetic field. That contribution can depend on the magnetization M in a ferromagnetic metal, and hence the Hall effect can persist even in the absence of an external magnetic field and call it anomalous hall effect [13]. has been recognized as a useful tool for measuring the magnetic hysteresis M(H) loops of perpendicular magnetic recording media (PMRM), ferromagnetic/semiconductorhetero structures and diluted-magnetic-semiconductors [12]. An empirical relation describes this effect [13]:

$$\rho_{xy} = \rho_{OH} + \rho_{AH} = R_{OB} + \mu_0 R_s M.$$

Generally, the anomalous Hall effect [13] can be either an extrinsic (disorder-related) effect or intrinsic effect. The extrinsic effect is due to spin dependent scattering of the charge carrier. The intrinsic effect is due to the spin-dependent band structure of the conduction electrons and can be described in terms of the Berry phase effect in the crystal momentum space. This effect originates from spin orbit coupling. The two extrinsic spin-orbit coupling mechanisms are: skew scattering and side jump. Skew scattering which takes in consideration the influence of asymmetric scattering of electrons from impurities due to the effective spin-orbit interaction of the electron and impurity. Side-jumps in which the electron velocity is detected in opposite directions by the electric fields experienced when approaching or leaving an impurity which gives rise to two scattering mechanisms.(AHE)is another means of charge transport in ferromagnetic material by the interaction of moving electron with their spin[14]. The AHE [13,14] produces voltage difference in magnetic materials transverse to an applied current(I) and a magnetization(M) perpendicular to the current, as similar to the normal Hall effect by external magnetic field in non-magnetic materials. The anomalous hall effect.

$$V_H = \frac{\mu_0 R_s I}{t}(M \cos\theta), \tag{2.7.1}$$

where $t = filmthickness$, and the angle θ is the angle between the magnetization(M)and the normal. The AHE depends on the perpendicular component of M, and produces an electric field perpendicular to M_z and the current density.

2.8 The Quantum Anomalous Hall Effect

The quantum anomalous Hall effect (QAHE), the last member of Hall family, was predicted to exhibit quantized Hall conductivity $\sigma_{yx} = \frac{e^2}{h}$ without any external magnetic field. The (QAHE) shares a similar physical phenomenon with the integer quantum Hall effect(IQHE),whereas its physical origin relies on the intrinsic topological inverted band structure and ferromagnetism. Since the QAHE does not require external energy input in the form of magnetic field,it is believed that this effect has unique potential for applications in future electronic devices with low-power consumption.

2.9 Application of Hall Effect

In more than a hundred years of their history, Hall effect devices have been used to demonstrate the basic laws of physics, to study details of carrier transport in solids, to detect the presence of a magnet and as measuring devices for magnetic fields. The Hall voltage of a Hall plate can be regarded as signal carrying information. The Hall voltage can give us information about the magnetic induction if we know the material properties,device geometry and biasing condition. Alternatively, we may control the biasing conditions and magnetic induction of a Hall device with a known geometry. From the measured Hall voltage, some important properties of the material the device is made of may be deduced. So, the Hall device can be applied as a means of characterizing material or either as magnetic sensors or as material analysis tools [15].

2.9.1 Hall Effect Sensor

A Hall effect sensor is a transducer that varies its output voltage in response to magnetic field. In a hall effect sensor a thin strip of metal has a current applied along it, in the presence of a magnetic field the electrons are deflected towards one edge of the metal strip, producing a voltage gradient across the short-side of the strip (perpendicular to the current). Inductive sensors are just a coil of wire, in the presence of a changing magnetic field a current will be induced in the coil, producing a voltage at its output. Hall effect sensors have the advantage that they can detect static (non-changing) magnetic fields. In its simplest form, the sensor operates as an analog transducer, directly returning a voltage. With a known magnetic field, its distance from the Hall plate can be determined. Using groups of sensors, the relative position of the magnet can be deduced [1]. Today, the Hall effect micro sensors are mostly used as key elements in contact less sensors for linear position, angular position, velocity, rotation, electrical current, and so on. Most currently produced Hall magnetic sensors are discrete elements, but an ever-increasing portion comes in the form of integrated circuits. Integrated Hall magnetic sensors incorporate electronic circuits for biasing, offset reduction, compensation of temperature effects, signal amplification, and more [15]. Hall sensors are commonly used to time the speed of wheels and shafts, such as for internal combustion engine ignition timing, tacho meters and anti-lock braking systems. They are used in brush less DC electric motors to detect the position of the permanent magnet [1].

Chapter 3

Spin Dependent Hall Effect

3.1 Introduction

The spin version of the Hall effect was first proposed by Russian physicists Dyakonov and Perel in 1971 and more recently by Hirsch 1999 [25]. Hirsch used subtle physical reasoning based on the anomalous Hall effect to predict both the spin hall effect (SHE) and the inverse spin hall effect (ISHE)[20,25]. The spin hall effect(SHE)is widely recognized phenomena that converts charge current in to spin current requiring neither external magnetic field nor ferromagnetic material. When un polarized charge current injected in to the sample such as paramagnetic metal. Charge circulates or accumulate at lateral boundaries and a transverse spin current induced giving rise to a "spin Hall voltage". But there is no hall voltage since the number of spin up and spin down electrons are exactly the same. This implies that it can be used to convert charge current into a spin current and this is called spin Hall effect(SHE)[20]. Similarly, when a spin current injected , spin up and the spin down electrons flow and a transverse charge current induced or a transverse charge imbalance will be generated, giving rise to a Hall voltage,in the absence of charge current and magnetic field.This is inverse spin hall effect (ISHE) [20].

The spin Hall effect (SHE) of light which is an analogue of the SHE in electronic systems, is a promising candidate for investigating the SHE in semiconductor spintronics/valleytronics, high-energy physics and condensed matter physics, owing to

their similar topological nature in the spin-orbit interaction. The SHE of light exhibits unique potential for exploring the physical properties of nano structures, such as determining the optical thickness, and the material properties of metallic and magnetic thin films and even atomically thin two dimensional materials.
The SHE and AHE are essentially the same and originate from the same mechanisms. They both are spin-dependent phenomenon and have spin imbalance in transverse direction. The only difference is that in a ferromagnetic metal,the magnetization creates an imbalance in the population between the spin-up and spin-down electrons that consequently leads to the anomalous Hall effect, while the Hall resistance vanishes in the absence of an external magnetic field and magnetization in a paramagnetic metal, the AHE has transverse Hall voltage due to the spin polarization of the ferromagnetic materials, while there is no transverse Hall voltage in SHE for non-magnetic metals. SHE is also somewhat similar to the normal Hall effect, where charges of opposite signs accumulate at the sample boundaries due to the action of the Lorentz force in magnetic field.

3.2 Spin Polarized Transport without Ferromagnetic Material

One of the present obstacles in spintronics device is the lack of high efficiency spin injection and filtering methods. Of several ways to obtain spin current without external ferromagnetic materials, one is the spin-Hall effect. Another effect, which demonstrates spin polarized charge transport, is the quantum Hall effect(QHE) and its family.for example,for odd filling factors(ν), and especially for $\nu = 1$, the current is partially or fully spin polarized. In order to create QHE states, the device has to be subjected to a high external magnetic field [17].

3.3 The Spin Hall effect, Spin Hall Current and the Spin Hall Angle

In analogy to the conventional Hall effect, the spin Hall effect(SHE) has been proposed to occur in paramagnetic systems as a result of spin-orbit interaction, and refers to the generation of a pure spin current transverse to an applied electric field even in the absence of applied magnetic fields. A pure spin current can be thought of as a combination of a current of spin-up electrons in one direction and a current of spin-down electrons in the opposite direction, resulting in a flow of spin angular momentum with no net charge current[28]. The spin Hall current is induced by the external electric field according to the equation.

$$j_i^j = \sigma_s \epsilon_{ijk} E_k. \tag{3.3.1}$$

σ_s is the spin hall conductance, $j_i^j$is the spin current in the i-the component of the spin along the direction of j, ϵ_{ijk} is the totally unit antisymmetric tensor in three dimensions, E_k is the electric field.

The SHE efficiency of a material is characterized by the spin Hall angle, defined as the ratio of polarized transverse spin current to longitudinal charge current densities in the steady state or the ratio of the spin hall conductivity and charge conductivity [23].

$$\gamma = \frac{j_\perp}{j_\parallel}. \tag{3.3.2}$$

Similar to the charge accumulation at the sample edges, which causes a Hall voltage in the conventional Hall effect, spin accumulation is expected at the sample edges in the spin Hall effect. One can understand this effect by using the analogy between an electron and a spinning tennis ball, which deviates from its straight path in air in a direction depending on the sense of rotation (the Magnus effect). In the spin Hall effect, because the spin of an electron is coupled to its magnetic moment, if an electric field is placed perpendicular to the direction of current flow, the electrons' spin degree of freedom interacts with the field and also experiences a Lorentz force.

However, since electron spin can point either up or down, the two types of electrons will separate and move two opposite side of the conductor [21,28].see Fig 3.1.

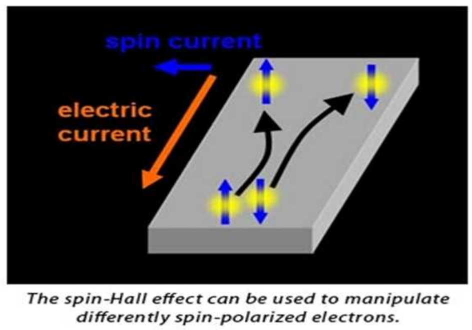

Figure 3.1: Spin hall effect

In a cylindrical wire the spins wind around the surface, like the lines of the magnetic field produced by the current. However, the value of the spin polarization is much greater than the (usually negligible) equilibrium spin polarization in this magnetic field[25].See Fig 3.2.

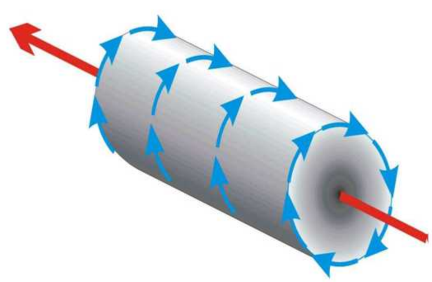

Figure 3.2: Spin hall effect in cylindrical wire

The possible origins of the SHE can be classified into two categories, intrinsic and extrinsic [20,27], depending on the predominant spin-orbit (SO) effects, either on the wave functions of the conduction band, or on the scattering potential of impurities or defects. The intrinsic one depends on the crystalline potential associated with the band structure and is largely independent of scattering, the extrinsic one depends on the potential due to random impurities of the sample. According to the special relativity theory when an electron move in an electric field, it feels a magnetic field in its own frame. The magnetic field couples the electron's spin and creates a spin dependence in band structure. The faster the electron moves the stronger the coupling. The interaction of the electron spin with this magnetic field results in the so-called spin-orbit coupling, which is described by the following general expression [20].

$$\vec{H_{so}} = \frac{\hbar}{4m_0^2c^2}(\nabla V_{so} \times \hat{P}).\vec{\sigma}, \tag{3.3.3}$$

where m_0 is the free electron mass, c is the speed of light and $\vec{\sigma}$ is the vector of Pauli matrices, ∇V_{so} is the electric field , $\hat{P}$ is the momentum operator.

3.4 Intrinsic and Extrinsic Spin Hall Effect

Initially, theorists argued that the spin accumulation was caused through asymmetric scattering of the spin-up and spin-down electrons with in the impurity potentials, hence termed the extrinsic spin Hall effect. The extrinsic spin Hall effect, which is due to the spin orbit scattering by impurities as well as due to the non equilibrium electronic population set up to give the electric current. In the extrinsic SHE, the electrons are deflected by spin orbit terms of the scattering potentials through the skew scattering and side jump mechanisms.

The spin orbit coupling in the electron band structure can produce a transverse spin current even without impurity scattering, hence called the intrinsic spin Hall effect. More importantly,it opens a possible pathway for controlling the spin states of photons and developing next generation photonic spin Hall devices as a fundamental constituent of the emerging spin optics.

The intrinsic spin Hall effect has been theoretically predicted for semiconductors with spin orbit coupled band structures. This is in sharp contrast with the extrinsic spin Hall effect, where the effect arises only from the Mott scattering from the impurity atoms [20,23,27,28].

3.5 Inverse Spin Hall Effect

The inverse spin Hall effect (ISHE) is phenomenon consists in the spin dependent scattering induced by the spin orbit interaction and results in the conversion of a spin current into an electric signal by generating electromotive field E_{ISHE} in the material in which spin polarized charges are flowing. Conversely, Spin Hall effects is the collection of transport phenomena whereby charge currents propagating in nonmagnetic materials are converted to transverse spin currents [29].

The SHE has promoted the development of spintronics by offering an effective approach for generating, manipulating, and detecting spin polarized electrons.

For the Inverse Spin Hall Effect a space dependent spin polarization causes a spin polarized current which induces an electric current which leads to an accumulation of electrical charges of opposite signs on opposing lateral boundaries.This can be measured as a charge imbalance at the edges of the sample.It was first observed in 1984. More recently, the existence of both direct and inverse effects was demonstrated not only in semiconductors but also in metals. E_{ISHE} is given by the following expression[20].

$$E_{ISHE} = \frac{\gamma}{\sigma_c}(\vec{J}_s \times \vec{P}), \tag{3.5.1}$$

where E_{ISHE},the induced electric field,γ and σ_c represent the spin Hall angle and the electrical conductivity of the material, respectively, and $\vec{P}$ is the spin polarization vector defined as $\vec{P} = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow}$, $n_\uparrow$ for spin up electron and $n_\downarrow$ for spin down electron

3.6 The Quantum Spin Hall Effect

The quantum spin Hall effect for electrons allows for the existence of an unusual type of material called a topological insulator which conducts electricity on the surface but not through the bulk of the material. It is a quantum version of the spin Hall effect, and it can be regarded as a combination of quantum anomalous Hall effects for spin up and spin-down electrons with opposite chirality. Generally, it has no charge Hall conductance, but a non-zero spin-Hall conductance. In 2005, Kane and Mele [2] generalized the Haldane model to a graphene lattice of spin$-\frac{1}{2}$ electrons with the spin-orbit coupling. In this material, electrons can propagate ballistically and the carrier density and polarity can be controlled by an external gate. Spin-orbit and hyperfine interactions are extremely weak in graphene and therefore the spin coherence length is expected to be long. These characteristics make graphene appealing for passive spintronic applications. A striking possibility is to modify graphene for active spintronics. This may be achieved via spin-orbit splitting of the band dispersion example, by bringing heavy metallic atoms in close contact to graphene.The strong spin-orbit coupling is introduced to replace the periodic magnetic flux in the Haldane model. This interaction looks like a spin dependent magnetic field to electron spins. The different electron spins experience opposite spin transverse force. As a result, a bilayer spin-dependent Haldane model can be realized in a spin$-\frac{1}{2}$ electron system with the spin-orbit coupling, which exhibits the quantum spin Hall effect. When spin dependent edge states exist around the boundary of the system, electrons with different spins move in opposite directions and form a pair of helical edge states.[17,28]. See Fig 3.3

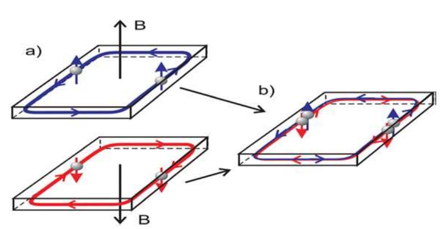

Figure 3.3: Quantum spin hall effect

3.7 The Quantum Spin Hall Conductivity(σ_s)

The quantum spin Hall Effect model can be considered as two copies of the Haldane model;in one of the layer we have spin down electrons in the presence of a down ward magnetic field where as in the other layer we have a spin up electrons in the presence of an up magnetic field. These two layers are placed together. The spin up electrons have positive charge hall conductivity while the spin down electron have negative charge hall conductivity. Hence, the charge hall conductance of the whole system vanishes. However, the spin hall conductance remains finite, as the chiral states are spin-up while the anti-chiral states are spin down as shown in Fig above. The spin hall conductance(σ_s) is hence quantized in units of $2\frac{e^2}{h}\frac{\hbar}{2e}$. The two layers system placed together in each of the layers we have a charge quantum hall effect at the same time, but with opposite hall conductivity. On the plateau the longitudinal hall conductivity $\sigma_{xx} = \sigma_{yy} = 0 \Rightarrow \rho_{xx} = \rho_{yy} = 0$
Similarly the transverse spin conductivity is $\sigma_{xy} = -\sigma_{yx} = \sigma_{s\uparrow} - \sigma_{s\downarrow} = \nu 2\frac{e}{4\pi} \Rightarrow$

$$\rho_{xy} = -\rho_{yx} = \frac{1}{\nu}\left(\frac{2\pi}{e}\right). \tag{3.7.1}$$

The resistivity can be expressed in tensor form[17,30].
$\rho_s = \begin{pmatrix} \rho_{xx} & -\rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \Rightarrow \rho_s = \begin{pmatrix} 0 & -\frac{2\pi}{\nu e} \\ \frac{2\pi}{\nu e} & 0 \end{pmatrix}$. From equation 2.3.16 and 3.7.1 $\rho_{xy} = \frac{B}{ne} = \frac{1}{\nu}(\frac{2\pi}{e})$, $\frac{B}{ne} = \frac{2\pi}{\nu e} \Rightarrow B_i = n\frac{2\pi}{\nu}$, for a sample of given density n,there is a discrete set of magnetic field B_i, $(\nu = 1, 2, 3, ..)$. $\Rightarrow \rho_{xy} = \frac{B_i}{e}$ [11,29]. From the above information the plots of resistivity versus for arbitrary magnetic field is shown in Fig 3.4.

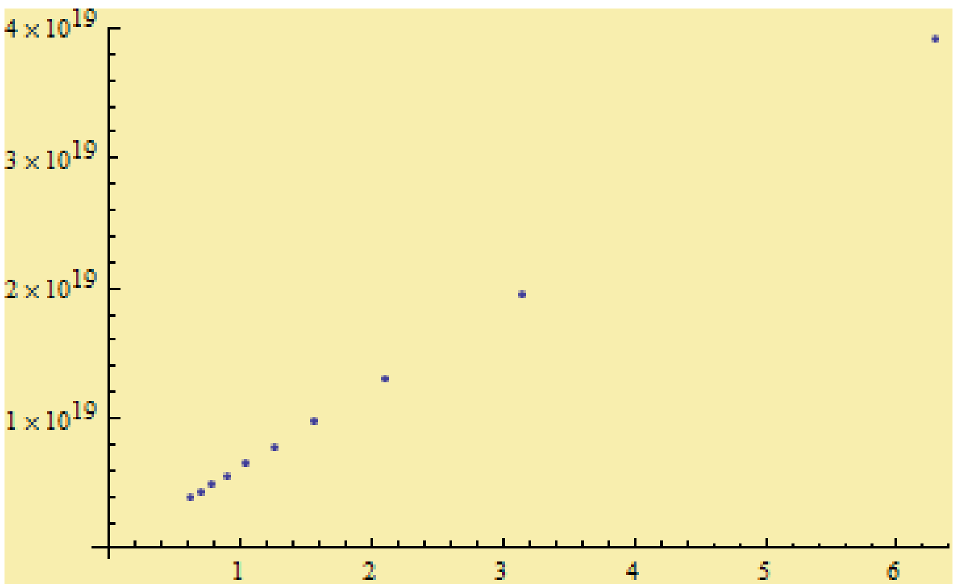

Figure 3.4: Resistivity versus magnetic field for quantum spin Hall effect

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Chapter 4

Discussion

Hall effects, in general,are transport phenomena, in which an applied field on the particles results in a motion perpendicular to the field. In traditional Hall effect and its quantum versions the effect depends on the electrical charge and hence, charges are transported by the action of a Lorentz force. That is if a magnetic field is placed perpendicular to the direction of current flow in a conductor,charges move across the conductor resulting potential difference called hall voltage. The reason for this is the electrons in the current interact with the magnetic field. The drude model considers a conductor as a gas of free current carrying charges. The conductivity and the resistivity can be expressed in tensor form as

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma xy \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

.

If E and v vary with time

$$\sigma_{xx} = \sigma_{xy} = \frac{\sigma_0(1 - i\omega\tau)}{1 - (\omega^2 - \omega_c^2)\tau^2 - 2i\omega\tau)} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2},$$

$$\sigma_{xy} = -\sigma_{yx} = \frac{\sigma_0\omega_c\tau}{(1 - (\omega^2 - \omega_c^2)\tau^2 - 2i\omega\tau)} = \frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2},$$

$$\sigma_{zz} = \frac{\sigma_0}{1 - i\omega\tau},$$

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \frac{\sigma_0}{(1 - (\omega^2 - \omega_c^2)\tau^2 - 2i\omega\tau)} \begin{pmatrix} 1 - i\omega & \omega_c\tau \\ -\omega_c\tau & 1 - i\omega \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$\sigma_{xz} = \sigma_{zx} = \sigma_{yz} = \sigma_{zy} = 0$

If E and v not vary with time

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \frac{\sigma_0}{1+\omega_c^2\tau^2} \begin{pmatrix} 1 & \omega_c\tau \\ -\omega_c\tau & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

The hall coefficient is a parameter that measures the magnitude of the Hall effect in the sample is defined as the ratio of the induced transverse electric field to the product of the longitudinal current density and the applied magnetic field.

$$R_H = \frac{E_y}{J_x B}$$

In metals the hallcoefficient is given by

$$R_H = \frac{1}{ne}$$

in semiconductors the hall coefficient is given by

$$R_H = \frac{E_y}{J_x B} = \frac{1}{e}\left(\frac{p\mu_{hh}^2 - n\mu_c^2}{(n\mu_c + p\mu_h)^2}\right)$$

Hall effect is very useful as a means to measure either the carrier density or the magnetic field, it differentiates between positive charges moving in one direction and negative charges moving in the opposite, the Hall effect offered the first real proof that electric currents in metals are carried by moving electrons, not by protons, to study details of carrier transport in solids, to detect the presence of a magnet and as measuring devices for magnetic fields. The Hall effect micro sensors are mostly used as key elements in contactless sensors for linear position, angular position, velocity, rotation, electrical current.

Anomalous hall effect is another means of charge transport in ferromagnetic material.In anomalous hall effect and its quantum version charges/spins are transported as a result of spin orbit interaction and acquire no external energy in the form of magnetic field to transport charge.

The quantum Hall effect is a quantum mechanical version of the Hall effect. This effect is very well understood now and can be simply explained in terms of single-particle orbital's of an electron in a magnetic field. It exist at low temperature and high magnetic field in two dimensional electron system. The quantum hall conductance and the hall resistance are given by.

$$\sigma_H = \sigma_{xy} = \nu \frac{e^2}{h},$$

$$R_H = \frac{B}{\rho e} = \frac{h}{\nu e^2}.$$

The quantum Hall effect is referred to as the integer or fractional quantum Hall effect depending on whether ν is an integer or fraction respectively. Unlike the traditional hall effect in which the plot of resistivity versus magnetic field linear. In quantum hall effect the plot of the resistivity as a function of magnetic field strength exhibited many plateaus. The integer quantum Hall effect is a phenomena related to transport properties of non- interacting electrons while the fractional quantum Hall effect exist on electron-electron interactions.

The study of spin dependent transport properties of electrons in solid state systems now attracted much attention because manipulation and detection of the spin current and accumulation are key issues for developing the future spintronic devices.

The interaction of the electron spin with this magnetic field results in the so-called spin-orbit coupling.

The spin Hall effect (SHE) is widely recognized phenomenon that converts charge current to the spin current requiring neither external magnetic fields nor ferromagnets as a result of spin orbit coupling. These effects, can originate in a variety of intrinsic and extrinsic spin-orbit coupling mechanisms, and depend on the geometry, dimensions, impurity scattering and the band structure of the system.

The spin-orbit interaction responsible for the SHE is also expected to cause the inverse process of the SHE. The inverse spin-Hall effect(ISHE),a process that converts a spin current into an electric current. The SHE belongs to the same family as the anomalous Hall effect known for a long time in ferromagnetic which also originates from spin- orbit interaction and the generation of a Hall current in a ferromagnetic material, but differs from it in one essential respect: it does not require magnetic fields and/or ferromagnetism; in other words, it does not require broken time-reversal symmetry.

The SHE efficiency of a material is characterized by the spin Hall angle, defined as the ratio of polarized transverse spin current to longitudinal charge current densities in the steady state.

The intrinsic spin hall effect depends on the crystalline potential associated with the band structure and is largely independent of scattering, the extrinsic one depends on the potential due to random impurities of the sample.

The quantum spin Hall effect for electrons allows for the existence of an unusual type of material called a topological insulator which conducts electricity on the surface but not through the bulk of the material.

The transverse spin hall conductivity is $\sigma_{xy} = -\sigma_{yx} = \sigma_{s\uparrow} - \sigma_{s\downarrow} = \nu 2\frac{e}{4\pi} \Rightarrow$

$$\rho_{xy} = -\rho_{yx} = \frac{1}{\nu}(\frac{2\pi}{e}).$$

The conductivity and the resistivity can be expressed in tensor form.

$$\sigma_s = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{yx} & \sigma_{yy} \end{pmatrix}$$

$$\rho_s = \begin{pmatrix} \rho_{xx} & -\rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \Rightarrow \rho_s = \begin{pmatrix} 0 & -\frac{2\pi}{\nu e} \\ \frac{2\pi}{\nu e} & 0 \end{pmatrix}$$

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