ADDIS ABABA UNIVERSITY

OFFICE OF GRADUATE PROGRAM

FACULTY OF SCIENCE

DEPARTMENT OF STATISTICS

APPLICATION OF GARCH MODELS IN FORECASTING THE VOLATILITY OF EXPORT PRICES

BY

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ABSTRACT

This thesis discusses the GARCH model fitting and volatility forecasting of export prices data. We have chosen to confine our analysis on total export prices, coffee export prices and oil seeds export prices. The implied goal is to fit an appropriate GARCH model, to find out if the mentioned export prices are volatile or not and to forecast the volatility for some future times. ARMA(1,1) is found as the most appropriate model for the conditional mean of total, coffee and oilseeds export prices and it is also found that GARCH(2,1) for modeling volatility of total export prices and coffee export prices, and GARCH(2,2) for modeling volatility of oil seeds export prices as best models. Moreover, the results suggest that the export prices volatility is persistence in all the three cases indicating that past volatility is important in predicting (forecasting) future volatility.

Key Words: export, GARCH, volatility, forecasting, Ethiopia

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

Foreign trade payments have a significance role in the development plans of many developing countries. For most of these countries, export represents an important share of the total development. Trades in the foreign sales are important in forecasting of over all growth of a country. For some developing countries, export trade is such an important factor that an estimate of the foreign exchange earnings represents a first step in the formulation of development plans. As the export of a given country continue to expand, new profit opportunities develop not only to provide inputs for export sector but also to take the advantage of external economic benefits (Hultman, 1967).

For example, the volatility in export prices is an important source of national macroeconomic disturbance mainly due to the importance of exports in the composition of the economy. Generally speaking, price volatility may be derived from country specific factors as well as from various influences emanating from the global market place. In keeping with any open economy devoting a substantial proportion of its resources to export production; prices received for such exports are a crucial determinant of aggregate income and social welfare (Valadkhani, Layton, and Karunaratne, 2005).

The analysis of the effect of export earnings instability on the economic growth of developing countries has long interested economists for several reasons. First of all, from the beginning of the 1960s developing countries have experienced a steady and sustained growth in exports. The outcome of this process, initially encouraged by a growing demand for raw materials to be transformed in industrialized countries, is that today many developing countries find themselves with an exports structure which is highly restricted both in terms of products and in terms of outlets; this high level of exports concentration plus the turbulence and interdependence of world market is considered one of the reasons for the difficulty of promoting growth in a small economy dependant on the production of goods for exports (Aiello, 1999).

Developing countries export mainly primary products, which are characterized by lower price and income elasticity of demand and supply than manufactured products. Additionally, a limited number of commodities (in some cases one or two) constitute total exports, exposing these countries to vagaries of a highly volatile international economic environment. Finally, the direction of trade tends to be in favor of developed countries, and cyclical movements in these countries are promptly transmitted to less developed ones (Akpodje, 2000).

Empirical research on the relationship between growth and export has shown mixed results (positive and negative). After splitting the actual export earnings in to stable and unstable components using a five years moving average, the instability of export earnings is found to have a negative effect on economic growth, while the stable components have a positive effect. Violent and sudden fluctuations in prices, varieties and total amounts of exports, on the assumption that there exists an intimate relation between foreign trade, national income and investment, have a serious adverse impact on the overall growth of the less developed countries (Aggarwal, 1982).

Developing countries often face scarcity of foreign exchange earnings to stimulate economic growth, international trade thus tends to be unstable, which is the only source of foreign exchange for these countries. Moreover, these countries mostly import technology and capital goods. Sub-Saharan countries have similar economic structures low per capital income, largely agrarian economies with very small industrial sectors, relatively low growth rates, and a strictly binding foreign exchange constraint, responses to instability in export earnings are likely to be similar among the nations of sub-Saharan countries (Brempong, 1991).

The African 'export problem' is not simply the general dependence on primary commodity exports, but also the heavy dependence of most countries on a narrow range of primary commodities. In the late 1990s, 39 African countries depended for more than half of their export earnings on just two primary commodities; hence collapse of world commodity prices in 1998 was equivalent to a real income loss of 2.6 percent of gross domestic product (GDP) in 1997-98 for Sub-Saharan Africa (SSA). Commodity prices have not shown any dramatic sign of recovery in recent years. For example, world coffee prices in 2002 were below a third of the level in 1997. Many developing countries are heavily dependent on primary commodities for their export earnings. At least half the countries in sub-Saharan Africa (SSA) and Latin America rely on commodities for over 50% of their exports. There are some countries (mostly in SSA) whose reliance on one, two or three commodities is extreme. Reliance on exports of Renewable Natural Resources (RNR) commodities is also most pronounced in SSA. The aggregate export trend hides important differences when regional and sectoral developments are examined (Overseas Development Institute, 2000).

Asian economies have been by far the best performers: the real value of commodity exports from this region has increased by 60% since 1973. The figures for Latin America and SSA are 5% and minus 50% respectively. The contrasting experiences of Asia and SSA are made even more different by the fact that the Asian economies both increased the value of their commodity exports and reduced the share of commodities in total exports. SSA dependence on commodities has not decreased much and yet revenue from these goods has also sharply reduced as market shares have declined (Overseas Development Institute, 2000).

The strong rise in coffee prices in 1995 in response to news of frost damage in Brazil (the world's largest producer) was aggravated by the fact that stocks were then at low levels. There are two channels through which price fluctuations affect commodity-dependent exporters. The *direct* effect is felt on foreign-exchange earnings (and hence import capacity), reserve levels, the government's fiscal position and producers' incomes. The *indirect* effect works via the exchange rate. For example, during a commodity boom, the balance of payments improves and the exchange rate tends to appreciate. This will affect investment and production decisions, which may well be reversed when the boom subsides. In the face of an adverse price shock, export volumes could be expanded to mitigate the impact on a country's reduced import purchasing power. However, this often cannot be done quickly. It is estimated that, for commodity-dependent exporters, fully 90% of the adjustment to a negative price shock is made via reduced imports, with export expansion accounting for only 10%. Perhaps unsurprisingly, the evidence shows a strong positive correlation between the share of a single commodity in a country's exports and the variability of total export earnings. The need to adjust to irregular price shocks will therefore be intensified for those countries with narrow export bases. For example, Côte d'Ivoire, which is dependent on cocoa and coffee for almost half its exports, experienced negative terms-of-trade shocks in 1981 and in 1985 in excess of 10% of GDP. Empirical studies suggest that negative shocks have a far greater detrimental effect on investment than on consumption (Overseas Development Institute, 2000).

Many developing countries depend on exports of a small number of agricultural commodities, even a single commodity, for a large share of their export revenues. This concentration leaves such countries highly vulnerable to unfavorable market or climatic conditions. A drought or a drop in prices on the international markets can quickly drain their foreign exchange reserves, stifle their ability to pay for essential imports and plunge them into debt (Food and Agriculture Organization-FAO, 2004).

There are some African countries that have had export success, but these are small countries (even relative to Africa) and their success is usually due to specific features. For example, Botswana has managed its diamond resources well and had a steady export performance (although the export/GDP ratio fell from over 50 percent in the early 1990s to almost 30 percent by the end of the decade, while Mauritius has benefited from preferential access to the EU for its sugar and clothing exports (maintaining an export/GDP ratio above 60 percent in the 1990s). Majority of SSA countries, however, are economically small and dependent for their exports on relatively low-value primary commodities (Ackah and Morrissey, 2005).

Oilseeds and wheat grains have witnessed unprecedented volatilities and price fluctuations in the recent past. Extreme volatility in commodity prices, particularly of food commodities, affects producers, consumers, traders, exporters & food procurement agencies of the central and state Government (Pandey, 1990).

Like in most of Sub-Saharan African and other developing countries, the economic problems in Ethiopia is characterized by shortage of foreign exchange as a result of declining exports, particularly primary commodity exports, which are a significant part of the total export earnings of the country. Export earnings increases are associated with increased investment and subsequent increased consumption and output. Coffee contributes an average about 60-66% of the countries export earnings followed since recently by “Chat”. Moreover Coffee prices highly fluctuate from time to time due to internal and external factors. Export earning instability is found to have the expected negative and statistically significant impact on economic development (Gebru, 2004).

In Ethiopia, owing to structural problems and politics that were pursued by different regimes, the performance of the export sector has been less satisfactory. The nation's out put and exports are highly concentrated in agricultural commodities. Primary agricultural products accounted for 80-90% of the merchandise export earnings of Ethiopia during the past four decades. In addition to this, there is geographical concentration of exports that makes the country vulnerable to the economic condition of trading partners. The structure of exports also reveals heavily reliance of the country on raw agricultural product of about 80% of the total export earnings. Manufactured exports, which are crucial for a rapid structure transformation of production is very weak in Ethiopia and the country is one of the least industrialized countries in the world. Since export sector of the country is highly dominated by a few commodities, the fluctuation of earnings in the sector is too much a function of the rise or fall of earnings in these few products. One might, therefore, see that if the earnings from coffee and some other export commodities (like oil seeds, *chat*, pulses, hides and skins) fall in a particular year, there will be a proportional fall in the entire export earnings, which is due to the limited capital flow in the country (World Bank, 2001).

1.2 Statement of the Problem

Heavy and sudden fluctuation in export quantity and prices of exports creates serious problems in national income and investment and then also creates severe adverse impact on the overall growth of less developed countries. Several studies have been conducted on identifying the impact of export earnings fluctuation on economic growth of a country. Most of these researches were performed using cross sectional data and growth functions (models) others used time series data and still growth functions, and some also used Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. Only few researches have been done on the export volatility of Ethiopia. This study tries to use time series data analysis using GARCH model to see the volatility of exports in Ethiopia.

1.3 Objectives

1.3.1 General Objective

The general objective of this study is:

to fit GARCH models, to see whether export prices in Ethiopia are volatile or not and to forecast export prices for some times in the future.

1.3.2 Specific Objectives

The specific objectives are:

☞ to fit and select best ARMA-GARCH models for the total, coffee and oil seeds export prices;

☞ to forecast export prices' volatility for total, coffee and oil seeds export prices for some times in the future;

☞ to determine and discuss the appropriate modeling approach for heteroscedasticity in time series data.

1.4 Research Questions

This research work will seek answers for the following research questions:

Are the Ethiopian export prices volatile or not?

Is it possible to forecast the volatility of Ethiopian export prices?

1.5 Significance of the Study

The results of the research work could be helpful in:

☞ understanding export market volatility;

☞ managing risks with foreign trade performances;

☞ managing, creating and revising effective administrative procedure, rules, and regulations for organizations working on export sector;

☞ helping for policy makers and scholars to revise policies and strategies concerning foreign trade;

☞ helping as a spring board for further studies.

CHAPTER TWO

LITRATURE REVIEW

In a paper presented in a workshop of applied infrastructure - Berlin, Germany, (October 11, 2003), Garcia, Contreras, Akkeren and Garcia used the Gaussian distribution for a time series data; they adhere to the Box-Jenkins modeling approach of parsimony, i.e. using the fewest model parameters as supported by the data, to estimate an ARMA process with conditional-heteroscedastic (GARCH) error components. Their main objective is "predicting day-ahead electricity prices for the Spanish and Californian electricity markets for the year 2000. They have proposed two GARCH models to predict hourly electricity prices in the electricity markets of Spain and California, respectively. Average errors in the Spanish market have been around 7% (depending on the studied week), and around 4% in the Californian market. These errors are very reasonable ones, taking into account the complex nature of price time series. The differences in the models for the Spanish and California markets may reflect different bidding structures. They suggested that, improvements to the models can be gained by including special treatment for weekend data (calendar effect), the inclusion of exogenous variables (demand, water storage, etc.) and the issue of volatility in the price will be addressed.

In a Journal of Economic Perspectives, Engel (2001) pointed out that the goal of GARCH model is to provide a volatility measure—like a standard deviation—that can be used in financial decisions concerning risk analysis, portfolio selection and derivative pricing. The dependent variable could be the return on an asset or portfolio.

The econometric challenge is to specify how the information is used to forecast the mean and variance of the return, conditional on the past information. GARCH model allowed the data to determine the best weights to use in forecasting the variance. This model is also a weighted average of past squared residuals, but it has declining weights that never go completely to zero. The most widely used GARCH specification asserts that the best predictor of the variance in the next period is a weighted average of the long-run average variance, the variance predicted for this period, and the new information in this period that is captured by the most recent squared residual.

ARCH and GARCH models have been applied to a wide range of time series analyses, but applications in finance have been particularly successful. Financial decisions are generally based upon the tradeoff between risk and return; the econometric analysis of risk is therefore an integral part of asset pricing, portfolio optimization, option pricing and risk management. The analysis of ARCH and GARCH models and their many extensions provides a statistical stage on which many theories of asset pricing and portfolio analysis can be exhibited and tested.

Claessen and Mittnik (2002) investigated a range of alternative strategies for predicting volatility in financial markets applying GARCH model to daily returns on the DAX index, the major German stock index. The work of these two scholars investigated the suitability of several forecasting techniques for the volatility of the returns of the German DAX index and examined whether or not the German DAX–index options market is informational efficient. Focusing on the problem of whether or not implied volatility information, derived either from observed option prices or from time series models such as GARCH models, are useful in predicting future return volatility. In summary, they concluded that implied volatility is a biased but highly informative predictor for future volatility. Moreover, implied volatilities are informational efficient relative to other historic volatility information sources.

Guida and Matringe (2004) examined the forecasting performance of GARCH's models using agricultural commodities data. Their data consisted of 3392 daily observations of the NYBOT cocoa, LIFFE cocoa, NYBOT coffee and CAC 40 (Paris) of 13 years period, from the 01 / 01 / 1991 to 31 / 12 /2003. The full sample was split into two parts: an in sample and an out sample. In sample composed of 2870 observations from 01/01/1991 until the 31/12/2001 in order to estimate the parameters their model. An out sample composed of 522 observations from 01/01/2002 until the 31/12/2003, in order to make forecasts. Among the different GARCH models, EGARCH Gaussian, GARCH Gaussian, TARCH Gaussian, and GARCH Gaussian were found to be suitable for NYBOT cocoa, NYBOT coffee, LIFFE cocoa, and CAC 40, respectively.

Valadkhani, Layton and Karunaratne (2005), in their study of Export Price Volatility in Australia tried to show the extent to which Australia's export prices relate to the world prices using quarterly time-series data spanning the period 1969 quarter 4-2002 quarter 3. Using a parsimonious GARCH (1,1), they showed that Australia's export prices are relatively more volatile in both the pre-1975 and post-1975 periods compared to that of other countries.

Usually researchers relate export instability/volatility/ with economic growth of a country. Studies on the relationships of export volatility and economic growth shows different results: some studies find a positive relationship, some other studies find a negative relationship while some other studies find no relationship. McBean (1966), and Knudsen and Parnes (1975) found a positive relationship between export instability and economic growth. They concluded that export earnings instability can lead to a reduction in consumption and in turn, an increase in saving and investment and thus economic growth. The latter two researchers use a transitory index to measure instability and found that marginal propensity to consume out of permanent income is negatively related to export instability using cross section (average data for 1958-1968) of 28 developing countries.

Other earlier studies such as Gyimah-Brempong (1991) used average data for 1960-1986 for 34 sub-Saharan African countries. His cross section study using the production function frameworks found that no matter how export instability is measured, export instability has negative effect on economic growth. He used three different measures of export instability, namely, (a) the coefficient of variation of export earnings (b) the mean of the absolute difference between actual export earnings and its trend values, (c) average of the squares of the ratio of actual export earnings to trend earnings.

A time series analysis by Sinha and Dipendra (1999) using annual data of nine Asian countries showed mixed results such that positively related or negatively related are inconclusive. For India, they found mixed results; for Japan, Malaysia, Philippines and Sri Lanka the evidence suggested a negative relationship between export instability and economic growth; for Korea, Myanmar, Pakistan and Thailand, positive relationship between export instability and economic growth was observed.

Salvatore (2006) confirms that because of widely fluctuating export prices, the export earnings of developing nations are expected to vary significantly from year to year. When export earnings rise, exporters increase their consumption expenditures, investments, and bank deposits. The effects of these are magnified and transmitted to the rest of the economy. This is due to both inelastic and unstable demand and supply.

The Ethiopian export of primary commodities is some how fluctuating even from year to year. Fore example, according to Ethiopian Foreign Trade Annual Report of Ethiopian Customs Authority (2006), export of Ethiopia during the fiscal year 2005/06 increased both in volume and value. The total export of the country was estimated to be Birr 8,834 million experienced a growth rate of 24.7% over a year period which is less than the growth rate of the fiscal year 2004/05 (37.8%). The growth rate during the fiscal year 2005/06 was not uniform across the different commodities. In most of the commodities the value and volume of export increased during 2005/06 fiscal year while export of tea, fruits and vegetables, hides and spices experienced decrease. Commodities such as cotton, flowers, leather and live animals recorded the highest growth rate during 2005/06. Coffee increased by only 9.7% during 2005/06 which is very low compared to 62.8% of 2004/05. The total value of export of live animals during 2004/05 was 113.9 million Birr while that of 2005/05 was 238.9 million Birr, Showing a 109.7% growth rate over a year.

A report from National Coffer (1998 E.C.) revealed that Ethiopia's export performance in the past three consecutive years has shown a great leap in foreign exchange revenue. In 1996 budget year the country earned 819 million USD revenue, that is, an increase of 37% (222.5 million USD) in revenue from the previous fiscal year. The 1998 Budget year revenue stood at 1,008.6 million USD, exceeding by 189.6 million USD that of the 1997 Ethiopian budget year. The 1998 budget year export trade performance, when viewed in relation to that of the past three consecutive years, it has shown an increase both in variety and quantity of export items. Similarly, export destinations are also on the rise. In the aforementioned budget year 793,511 tons of products were exported to more than 100 destinations.

Gemech and Struthers (2007) presented the results of empirical investigation on the effects of market reform programmes and their impacts on the volatility of coffee prices in Ethiopia. The study covers the period from 1982 to the end of 2001, though its main focus is on the period after the beginning of the reforms in 1992. Using the ARCH modeling techniques, they argued that there is evidence that Ethiopia experienced a significant increase in coffee price volatility after the adoption of the market-oriented reform programmes. The econometric evidence presented in the paper, suggested that the market reform programmes were significant cause of the increased volatility after 1992. The authors also suggested a number of possible policy responses to the price volatility, including strengthening the role of smallholder producer associations; and the possibility of adopting appropriate risk management strategies. As a conclusion they stated that, the fact that Ethiopia is a price-taker in global market and is therefore prone to external shocks in coffee prices over which it has little control or influence means that the country will continue to be highly vulnerable to global shock.

CHAPTER THREE

METHODOLOGY

3.1 Univariate Time Series Analysis

At first sight it would not seem very sensible for economists to pay much attention to univariate analysis. After all, economic theory is rich in suggestions for relationships between variables. Thus, attempting to explain and forecast a series using only information about the history of that series would appear to be an inefficient procedure, because it ignores the optional information in related series. There are two possible rationales. The first is that a priori information about the possible relationships between series may not be well founded. In such case a purely statistical model relating current to previous values may be a useful summary device and may be used to generate reliable short-term forecasts. Alternatively, if theoretical speculations about the economic structure are well founded, it can be shown that one manifestation of that structure yields Autoregressive Moving Average (ARMA) equations for each of the dependent variable in the structure (Johnston and DiNardo, 1997).

For forecasting purposes, a simple model that describes the behavior of a variable (or a set of variables) in terms of past values, without the benefit of a well-developed theory, may well prove quite satisfactory. Researchers have observed that the large simultaneous-equations macroeconomic models constructed in the 1960s frequently have poorer forecasting performances than fairly simple, univariate time-series models based on just a few parameters and compact specifications. It is just this observation that raised to prominence of the univariate time-series forecasting models pioneered by Box and Jenkins (1976).

The theories of linear time series usually include stationarity, autocorrelation function, modeling, and estimation/forecasting (Tsay, 2005).

This chapter first discusses some characteristics of financial time series; secondly it provides the modeling of conditional mean of the time series using Box and Jenkins approach and finally the modeling of the conditional variance component of the time series using GARCH/ARCH models.

3.1.1 Financial Time Series and Their Characteristics

Financial time series analysis is concerned with the theory and practice of asset valuation over time. Most financial studies involve returns, instead of prices, of assets.The main reason for using returns instead of prices is that return series are easier to handle than price returns because the former have more attractive statistical properties.

Financial data have some peculiar behaviors that are not usually observed in other non-financial types of data. The following are among the main and common characteristics that distinguish return series from prices series Rossi (2004):

1. Asset prices are generally non stationary. Returns are usually stationary;
2. Return series usually show no or little autocorrelation;
3. Serial independence between the squared values of the series is often rejected;
4. Volatility of the return series appears to be clustered;
5. Normality has to be rejected in favor of some thick-tailed distribution; etc.

Statisticians assume stationarity for a financial variable under study. This is not because they like the word stationary, rather the assumption enables them to estimate parameters globally using the entire available data set. As it is mentioned above and econometricians agreed so far, return series are usually found to be stationary. As a matter of this fact, it is important to formulate stationary series for financial prices before going to further analysis of the price data.

Let $p_t,\ t = 1,2,3, \ldots,$ be a time series of prices of a financial asset, e.g. daily quotes on a share, stock index, currency exchange rate or a commodity prices, etc. Instead of analyzing p_t, which often displays unit-root behavior and thus cannot be modeled as stationary, we often analyze log-returns on p_t (Fryzlewicz, 2007), i.e. the series

$$y_t = \log p_t - \log p_{t-1} = \log \frac{p_t}{p_{t-1}} = \log \left[1 + \frac{p_t - p_{t-1}}{p_{t-1}}\right]$$

Applying Taylor's expansion procedure to the above equation, we can see that y_t is almost equivalent to the relative return $\frac{p_t - p_{t-1}}{p_{t-1}}$. But the log-return is usually preferable instead of relative return. The reason of typically considering log-returns instead of relative returns is to use the additive property of log-returns, which is not shared by relative returns. The series y_t, log-return series, displays many of the typical facts in financial log-return series.

3.1.2 Stationarity

The foundation of time series analysis is stationarity. A time series $\{y_t\}$ is said to be $strictly\, stationary$ if the joint distribution of $(y_{t_1}, \cdots, y_{t_k})$ is identical to that of $(y_{t_1+t}, \cdots, y_{t_k+t})$ for all t, where k is an arbitrary positive integer and $(t_1, \cdots, t_k)$ is a collection of k positive integers. In other words, strict stationarity requires that the joint distribution of $(y_{t_1}, \cdots, y_{t_k})$ is invariant under time shift. This is a very strong condition that is hard to verify empirically. A weaker version of stationarity is often assumed. A time series $\{y_t\}$ is weakly stationary if both the mean of y_t and the covariance between y_t and y_{t-l} are time invariant, where l is an arbitrary integer. More specifically, $\{y_t\}$ is weakly stationary if (a) $E(y_t) = \mu$, which is a constant and (b) $Cov(y_t, y_{t-l}) = \gamma_l$, which only depends on l. In practice, suppose that we have observed n data points $\{y_t | t = 1, \cdots, n\}$. The weak stationarity implies that the time plot of the data would show that the n values fluctuate with constant variation around a fixed level. In application, weak stationarity enables one to make inferences concerning future observations (e.g., prediction).

The covariance $\gamma_l = Cov(y_t, y_{t-l})$ is called the $lag - l$ auto-covariance of y_t. It has two important properties: (a) $\gamma_0 = Var(y_t)$ and (b) $\gamma_{-l} = \gamma_l$. In the finance literature, it is common to assume that an asset return series is weakly stationary. This assumption can be checked empirically provided that a sufficient number of historical returns are available.

If a time series is not stationary, it is necessary to look for possible transformations that might induce stationarity. There are three methods of testing stationarity: graphical analysis, correlogram test and unit root test.

i. Graphical Analysis of the Series

Before pursuing formal tests, it is always advisable to plot the time series under study. Such a plot gives an initial clue about the likely nature of the time series. For instance, if a line-graph of a time series shows an upward trend, suggests perhaps that the mean of the data has been changing. This may be a clue for the series is not stationary. Such an intuitive feel is the starting point of more formal tests of stationarity.

ii. Correlogram Test

One simple test of stationaity is based on the so called $autocorrelation\ function\ (ACF)$. The ACF at lag k, denoted by ρ_k is defined as

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \frac{covariance\ at\ lag\ k}{variance}$$

Since both covariance and variance are measured in the same units of measurement, ρ_k is unit less, or pure, number. It lies between -1 and 1. If we plot ρ_k against k, the graph we obtain is known as the $population\ correlogram.$

In practice it is only possible to compute the $sample\ autocorrelation\ function\ (SACF)$, $\widehat{\rho_k}$.To compute this, first computing sample covariance at lag k, $\widehat{\gamma_k}$, and the sample variance, $\widehat{\gamma_0}$ is must.

Therefore, the SACF at lag k is

$$\widehat{\rho_k} = \frac{\widehat{\gamma_k}}{\widehat{\gamma_0}}$$

A plot of $\widehat{\rho_k}$ against k is known as the $sample\ correlogram.$

For a purely white noise process the autocorrelations at various lags hover around zero. Thus, if the correlogram of an actual (economic) time series resembles the correlogram of a white noise time series, we can say that the time series is probably stationary.

Financial applications often require to test jointly that several autocorrelations of y_t are zero. Box and Pierce (1970) propose the $portmanteau\ statistics\ (Q - statistic)$. The Q-statistic is defined as

$$Q = n \sum_{k=1}^{m} \widehat{\rho_k}^2$$

where $n = sample$ size and $m = lag\ length$. The null hypothesis for this test is $all\ the\ \rho_k up\ to\ certain\ lags\ are\ simultanously\ equal\ to\ zero$. In large samples, the q-statistic is approximately distributed as the $chi - square$ distribution with degree of freedom m. In application, if the computed Q exceeds the critical Q value from the chi-square distribution at the chosen level of significance, rejecting the null hypothesis is inevitable.

Ljung and Box (1978) modify the above Q-statistics as $LB\ statistic(Ljung - Box\ Q - statistics)$ which is defined as

$$LB = n(n+2)\sum_{k=1}^{m}\left(\frac{\widehat{\rho_k}^2}{n-k}\right) \sim \chi^2_{(m)}$$

Although in large samples both Q and LB statistics follow the chi-square distribution with m degrees of freedom, LB statistic has been found to have better performance in small samples than the Q statistic.

iii. Unit Root Test

A test for stationarity (or nonstationarity) that has become widely popular over the past several years is the $unit\ root\ test$. The Augmented Dickey-Fuller (ADF) test is discussed below. To illustrate the use of Dickey-Fuller tests, consider first an Autoregressive of order 1 (AR(1)) process:

$$y_t = \omega + \alpha_1 y_{t-1} + \varepsilon_t$$

where ω and α_1 are parameters and ε_t is assumed to be white noise. y is a stationary series if $-1 < \alpha_1 < 1$. If $\alpha_1 = 1$, y is a nonstationary series (a random walk with drift); if the process is started at some point, the variance of y increases steadily with time and goes to infinity. If the absolute value of α_1 is greater than one, the series is explosive. Therefore, the hypothesis of a stationary series can be evaluated by testing whether the absolute value of α_1 is strictly less than one. The augmented Dickey-Fuller (ADF) test takes the unit root as the null hypothesis H_0: α_1 $= 1$. Since explosive series do not make much economic sense, this null hypothesis is tested against the one-sided alternative $H_1 : \alpha_1 < 1$.

The test is carried out by estimating an equation with y_{t-1} subtracted from both sides of the equation:

$$\Delta y_t = \omega + \delta y_{t-1} + \varepsilon_t$$

where $\delta = \alpha_1 - 1$, Δ is the first difference operator, and the null and alternative hypotheses are

$$H_0: \delta = 0,\ H_1 : \delta < 0.$$

While it may appear that the test can be carried out by performing a t-test on the estimated δ, the t-statistic under the null hypothesis of a unit root does not have the conventional t-distribution. Dickey and Fuller (1979) showed that the distribution under the null hypothesis is nonstandard, and simulated the critical values for selected sample sizes. More recently, MacKinnon (1991) has implemented a much larger set of simulations than those tabulated by Dickey and Fuller. In addition, MacKinnon estimates the response surface using the simulation results, permitting the calculation of Dickey-Fuller critical values for any sample size and for any number of right-hand variables. The simple unit root test described above is valid only if the series is an AR(1) process. If the series is correlated at higher order lags, the assumption of white noise disturbances is violated. The ADF test uses different methods to control for higher-order serial correlation in the series. It makes a parametric correction for higher-order correlation by assuming that the y series follows an AR(p) process and adjusting the test methodology.

The ADF approach controls for higher-order correlation by adding lagged difference terms of the dependent variable y to the right-hand side of the regression:

$$\Delta y_t = \omega + \delta y_{t-1} + \gamma_1 \Delta y_{t-1} + \gamma_2 \Delta y_{t-2} + \cdots + \gamma_{p-1} \Delta y_{t-p+1} + \varepsilon_t$$

This augmented specification is then used to test:

$$H_0: \delta = 0,\ H_1 : \delta < 0$$

in this regression. An important result obtained by Fuller is that the asymptotic distribution of the t-statistic on δ is independent of the number of lagged first differences included in the ADF regression. Moreover, while the parametric assumption that y follows an autoregressive (AR) process may seem restrictive, Said and Dickey (1984) demonstrate that the ADF test remains valid even when the series has a moving average (MA) component, provided that enough lagged difference terms are augmented to the regression.

Researchers usually face two practical issues in performing the ADF test. First, they have to specify the number of lagged first difference terms to add to the test regression (selecting zero yields the Dickey-Fuller (DF) test; choosing numbers greater than zero generate ADF tests). The usual (though not particularly useful) advice is to include lags sufficient to remove any serial correlation in the residuals. Second, they have the choice of including a constant, a constant and a linear time trend, or neither in the test regression. The choice here is important since the asymptotic distribution of the t-statistic under the null hypothesis depends on your assumptions regarding these deterministic terms.

To solve the two problems, one approach would be to run the test with both a constant and a linear trend since the other two cases are just special cases of this more general specification. However, including irrelevant regressors in the regression reduces the power of the test, possibly concluding that there is a unit root when, in fact, there is none. If the series seems to contain a trend one should include both a constant and trend in the test regression. If the series does not exhibit any trend and has a nonzero mean, you should only include a constant in the regression, while if the series seems to be fluctuating around a zero mean, you should include neither a constant nor a trend in the test regression. The null hypothesis of a unit root is rejected against the one-sided alternative if the t-statistic is less than (lies to the left of) the critical value.

3.1.3 Autocorrelation Function (ACF)

Consider a weakly stationary log return series y_t.When the linear dependence between y_t and its past values y_{t-l} is of interest, the concept of correlation (relation between two random variables) is generalized to autocorrelation. The correlation coefficient between y_t and y_{t-l} is called the $lag-l\ autocorrelation$ of y_t and is commonly denoted by ρ_l, which under the weak stationarity assumption is a function of l only. Specifically, we define

$$\rho_l = \frac{Cov(y_t, y_{t-l})}{\sqrt{Var(y_t)Var(y_{t-l})}} = \frac{Cov(y_t, y_{t-l})}{Var(y_t)} = \frac{\gamma_l}{\gamma_0}$$

where the property $Var(y_t) = Var(y_{t-l})$ for a weakly stationary series is used. From the definition, we have $\rho_0 = 1, \rho_l = \rho_{-l}, and -1 \leq \rho_l \leq 1$. In addition; a weakly stationary series y_t is not serially correlated if and only if $\rho_l = 0$ for all $l > 0$.

For a given sample of log returns $\{y_t\}_{t=1}^{n}$, let $\bar{y}$ be the sample mean (i.e., $\bar{y} = {}^{(\sum_{t=1}^{n} y_t)}/_{n}$).

The $lag-1$ sample autocorrelation of y_t is

$$\widehat{\rho_1} = \frac{\sum_{t=2}^{n}(y_t - \bar{y})(y_{t-1} - \bar{y})}{\sum_{t=1}^{n}(y_t - \bar{y})^2}$$

In general, the $lag-l$ sample autocorrelation of y_t is defined as

$$\widehat{\rho_l} = \frac{\sum_{t=l+1}^{n}(y_t - \bar{y})(y_{t-l} - \bar{y})}{\sum_{t=1}^{n}(y_t - \bar{y})^2}, \qquad 0 \leq l \leq n-1$$

3.1.4 Univariate Time Series Model

Treating an asset return (e.g., log return,y_t, of a price) as a collection of random variables over time, we have a time series $\{ y_t\}$. Linear time series analysis provides a natural framework to study the dynamic structure of such a series.

A time series model will typically describe the path of a variable y_t in terms of contemporaneous (and perhaps lagged) factors, x_t, disturbances (innovations), ε_t, and its own past lags, y_{t-1}, $y_{t-1}, y_{t-2}, \cdots$. Thus, the general expression for the time series model is

$$y_t = f(x_t, y_{t-1}, y_{t-2}, \ldots, \varepsilon_t)$$

It is common to assume that innovations are generated independently from one period to the next, with the familiar assumptions

$$E(\varepsilon_t) = 0$$

$$Var(\varepsilon_t) = \sigma_\varepsilon^2$$

and

$$cov(\varepsilon_t, \varepsilon_s) = 0$$

for all t and t ≠ s

In the current context, this distribution of ε_t is said to be covariance stationary or weakly stationary.

A univariate time-series model describes the behavior of a variable in terms of its own past values and disturbance term. Thus, the general expression for the univariate time series model is

$y_t = f(y_{t-1}, y_{t-2}, \ldots, \varepsilon_t)$

To make the above equation operational, three things must be specified: the functional form of y_t, $f(\,)$, the number of lags, and a structure for the disturbance term.

3.1.4.1 Autoregressive (AR) Models

A more general p^{th}-order autoregressive or AR(p) process would be written as

$y_t = \alpha_0 + \alpha_1\, y_{t-1} + \alpha_2\, y_{t-2} + \cdots + \alpha_p y_{t-p} + \varepsilon_t$

where $\alpha_i{}'s$ are coefficients, p is a non-negative integer and $\{\varepsilon_t\}$ is assumed to be a white noise with mean zero and variance σ_ε^2.

3.1.4.1.1 Properties of AR Models

For effective use of AR models, it pays to study the basic properties. The main property results given below are generalized results obtained from the detail discussion of properties of AR(1) and AR(2) models (Tsay, 2005).

The mean of a stationary AR(p) series is

$$E(y_t) = \frac{\alpha_0}{1 - \alpha_1 - \cdots - \alpha_p}$$

provided that the denominator is not zero. Corresponding to the p^{th} order difference equation of the ACF of a stationary AR(p) series, $(1 - \alpha_1 B - \alpha_2 B^2 - \cdots - \alpha_p B^p)\rho_l = 0, for\ l > 1$, where B is the back shift operator such that $B\rho_l = \rho_{l-1}$, $B^2\rho_l = \rho_{l-2} \cdots$, the associated polynomial equation of the model is

$$1 - \alpha_1 x - \alpha_2 x^2 - \cdots - \alpha_p x^p = 0.$$

which is referred to as the $characteristic\ equation$ of the model. If all the solutions of this equation are greater than one in modulus, then the series y_t is stationary. Again, inverses of the solutions are the $characteristic\ roots\ of\ the\ model.$ Thus, stationarity requires that all the characteristic roots are less than one in modulus. As stated above, for a stationary AR(p) series, the ACF satisfies the p^{th} order difference equation

$$\left(1 - \alpha_1 B - \alpha_2 B^2 - \cdots - \alpha_p B^p\right)\rho_l = 0, for\ l > 1.$$

A plot of the ACF of a stationary AR(p) model would then show a mixture of damping sine and cosine patterns and exponential decays depending on the nature of its characteristic roots.

3.1.4.1.2 Identifying AR Models

In application, the order p of an AR model is unknown and must be specified empirically. Two general approaches (Partial Autocorrelation Function (PACF) and Information Criterion Functions) are used to determine p.

i. PACF

The PACF of a stationary time series is a function of its ACF and is a useful tool for determining the order p of an AR model. A simple, yet effective way to introduce PACF is to consider the following AR models in consecutive orders:

$$y_t = \alpha_{0,1} + \alpha_{1,1} y_{t-1} + e_{1t},$$

$$y_t = \alpha_{0,2} + \alpha_{1,2} y_{t-1} + \alpha_{2,2} y_{t-2} + e_{2t},$$

$$y_t = \alpha_{0,3} + \alpha_{1,3} y_{t-1} + \alpha_{2,3} y_{t-2} + \alpha_{3,3} y_{t-3} + e_{3t},$$

$$y_t = \alpha_{0,4} + \alpha_{1,4} y_{t-1} + \alpha_{2,4} y_{t-2} + \alpha_{3,4} y_{t-3} + \alpha_{4,4} y_{t-4} + e_{4t},$$

where $\alpha_{0,j}, \alpha_{i,j}$ and $\{e_{jt}\}$ are, respectively, the constant term, the coefficient of y_{t-i}, and the error term of an AR(j) model. These models are in the form of linear regression model and can be estimated by least squares method. The estimates $\widehat{\alpha_{1,1}}, \widehat{\alpha_{2,2}}, \widehat{\alpha_{3,3}}, \widehat{\alpha_{4,4}}, \cdots$, respectively, are called the $lag-1, lag-2, lag-3, lag-4, \cdots$ sample PACF of y_t. For an AR(p) model, the PACF cuts of after p such that its order is p.

ii. Information Criteria

There are several information criteria available to determine the order p of an AR process. All of them are likelihood based. Here, emphasis is given to two of them: Akaike information criterion (AIC) and Schwarz (Bayesian) information criterion (BIC).

AIC is defined as

$$AIC = -\frac{2}{n}\ln(likelihood) + \frac{2}{n}(number\ of\ parameters)$$

where the likelihood function is evaluated at the maximum likelihood estimates and n is the sample size. For a Gaussian AR(l) model, AIC reduces to

$$AIC(l) = \ln(\tilde{\sigma}_l^2) + \frac{2l}{n}$$

where $\tilde{\sigma}_l^2$ is the maximum likelihood estimate of σ_ε^2, which is the variance of ε_t, and n is the sample size.

Similarly, BIC is defined as

$$BIC = -\frac{2}{n}\ln(likelihood) + \frac{\ln(\mathrm{n})}{n}(number\ of\ parameters)$$

For a Gaussian AR(l) model the BIC is given by

$$BIC(l) = \ln(\tilde{\sigma}_l^2) + \frac{l \times ln(n)}{n}$$

Note: **Selection Rule**

To use AIC or BIC to select an AR model in practice, one computes $AIC(l)$ or $BIC(l)$ for $l = 0, \cdots, p$, where p is a pre-specified positive integer, and selects the order k that has the minimum AIC or BIC value, respectively.

3.1.4.1.3 Parameter Estimation in AR Models

For a specified AR model, the conditional least squares method, which starts with the $(p+1)th$ observation, is often used to estimate the parameters. We have

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \cdots + \alpha_p y_{t-p} + \varepsilon_t, \qquad t = p+1, \cdots, n.$$

Denote the estimate of α_i by $\widehat{\alpha_i}$. The fitted model is

$$\widehat{y_t} = \widehat{\alpha_0} + \widehat{\alpha_1} y_{t-1} + \cdots + \widehat{\alpha_p} y_{t-p}$$

and the associated residual is

$$\widehat{\varepsilon_t} = y_t - \widehat{y_t}.$$

The series $\{\widehat{\varepsilon_t}\}$ is called the residual series, from which we obtain

$$\hat{\sigma}_\varepsilon^2 = \frac{\sum_{t=p+1}^{n} \widehat{\varepsilon_t}^2}{n-2p-1}.$$

If the conditional likelihood method is used, the estimate of α_i remain unchanged, but the estimate of σ_ε^2 becomes $\tilde{\sigma}_\varepsilon^2 = \hat{\sigma}_\varepsilon^2 \times {(n-2p-1)}/{(n-p)}$.

3.1.4.1.4 Forecasting Using AR Models

Forecasting is an important application of time series analysis. Suppose we are at time index h and interested in forecasting y_{h+l}, where $l \geq 1$. The time index h is called the $forecast\ origin$ and the positive integer l is the $forecast\ horizon$. Let $\hat{y}_h(l)$ be the forecast of y_{h+l} using the minimum squared error loss function and F_h be the collection of information available at the forecast origin h. Then, the forecast $\hat{y}_h(l)$ is chosen such that

$$E\{[\, y_{h+l} - \hat{y}_h(l)]^2/F_h\} \leq \min_g E[(\, y_{h+l} - g)^2/F_h]$$

where g is a function of the information available at time h (inclusive), that is, a function of F_h. We referred to $\hat{y}_h(l)$ as the $l-step\ ahead\ \ forecast\ of\ \ y_t$ at the forecast origin h.

In general, we have

$$y_{h+l} = \alpha_0 + \alpha_1 y_{h+l-1} + \cdots + \alpha_p y_{h+l-p} + \varepsilon_{h+l}.$$

The l-step ahead forecast based on the minimum squared error loss function is the conditional expectation of y_{h+l} given F_h, which can be obtained

$$\hat{y}_h(l) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \hat{y}_h(l-i),$$

where it is understood that $\hat{y}_h(i) = y_{h+i}$ if $i \leq 0$. This forecast can be computed recursively using forecasts $\hat{y}_h(i)$ for $i = 1, \ldots, l-1$. The l-step ahead forecast error is $e_h(l) = y_{h+l} - \hat{y}_h(l)$. It can be shown that for a stationary AR(p) model, $\hat{y}_h(l)$ converges to $E(y_t)$ as $l \to \infty$, meaning that for such a series long-term point forecast approaches its unconditional mean. This property is referred to as the $mean\ reversion$ in the finance literature.

3.1.4.2 Moving-Average (MA) Models

A more general q^{th}-order moving average or MA(q) process would be written as

$$y_t = c_0 + \varepsilon_t - \beta_1 \varepsilon_{t-1} - \beta_2 \varepsilon_{t-2} - \cdots - \beta_q \varepsilon_{t-q}$$

where q is a non-negative integer, β_i's are coefficients with c_0 the constant term, and $\{\varepsilon_t\}$ is assumed to be a white noise with mean zero and variance σ_ε^2.

3.1.4.2.1 Properties of MA models

Again, as in the case of AR models, properties of MA models are generalized from the detail discussion of MA(1) and MA(2) models (Tsay, 2005).

MA models are always weakly stationary because they are finite linear combinations of a white noise sequence for which the first two moments are time-invariant.

The mean of a stationary MA(q) series is

$$E(y_t) = c_0$$

The variance of an MA(q) model is

$$Var(y_t) = \left(1 + \beta_1^2 + \cdots + \beta_q^2\right)\sigma_\varepsilon^2$$

For an MA(q) model, the lag-q ACF is not zero, but $\rho_l = 0$ for $l > q$.

3.1.4.2.2 Identifying MA Models

The ACF is useful in identifying the order of an MA model. For a time series y_t with ACF ρ_l, if $\rho_q \neq 0$, but $\rho_l = 0$ for $l > q$, then y_t follows an MA(q) model.

3.1.4.2.3 Parameter Estimation and Forecasting in MA Models

Maximum likelihood estimation is commonly used to estimate MA models. Forecasting using MA models is analogous to an AR model except here the model is an MA model and its point forecasts go to the mean of the series quickly after the first q steps, where q is the order of the MA model under consideration.

3.1.4.3 Autoregressive Moving Average (ARMA) Models

In some applications, the AR or MA models become cumbersome because one may need a high-order model with many parameters to adequately describe the dynamic structure of the data. To overcome this difficulty, the ARMA models are introduced.

An extremely general model that encompasses equations AR and MA models is ARMA(p, q) model:

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \varepsilon_t - \sum_{i=1}^{q} \beta_i \varepsilon_{t-i}$$

where $\{\varepsilon_t\}$a white noise series and p and q are non-negative integers. Using the back shift operator, the model can be written as

$$\left(1 - \alpha_1 B - \alpha_2 B^2 - \cdots - \alpha_p B^p\right) y_t = \alpha_0 + (1 - \beta_1 B - \beta_2 B^2 - \cdots - \beta_q B^q)\varepsilon_t$$

The polynomial $1 - \alpha_1 B - \alpha_2 B^2 - \cdots - \alpha_p B^p$ is the AR polynomial of the model. Similarly, $1 - \beta_1 B - \beta_2 B^2 - \cdots - \beta_q B^q$ is the MA polynomial of the model. We require that there are no common factors between the AR and MA polynomials; otherwise the order (p, q) of the model can be reduced. Like a pure AR model, the AR polynomial introduces the characteristic equation of an ARMA model. If all the solutions of the characteristic equation are less than one in absolute value, then the ARMA model is weakly stationary. In this case, the unconditional mean of the model is

$$E(y_t) = \frac{\alpha_0}{1 - \alpha_1 - \cdots - \alpha_p}$$

Note that in most applications the lower order ARMA models are applicable and the parameters are estimated using maximum likelihood method.

3.1.4.3.1 Forecasting Using ARMA Models

The l-step ahead forecast is obtained as

$$\hat{y}_h(l) = E(y_{h+l} \backslash F_h) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \hat{y}_h(l-i) - \sum_{i=1}^{q} \beta_i \, \varepsilon_h(l-i),$$

where it is understood that$\hat{y}_h(l-i) = y_{h+l-i}$ if $l - i \leq 0$ and $\varepsilon_h(l-i) = 0$ if $l - i > 0$ and $\varepsilon_h(l-i) = \varepsilon_{h+l-i}$ if $l - i \leq 0$. The associated forecast error is

$$e_h(l) = y_{h+l} - \hat{y}_h(l).$$

Table 3.1

Summary of Correlation patterns

Process	ACF	$PACF$
AR(p)	Infinite: damps out	Finite: cuts off after lag p
MA(q)	Finite: cuts off after lag q	Infinite: damps out
ARMA(p, q)	Infinite: damps out	Infinite: damps out

3.1.4.4 (G)ARCH Models

Autoregressive Conditional Heteroscedasticity (ARCH) models are specifically designed to model and forecast conditional variances. ARCH models were introduced by Engle (1982) and generalized as GARCH (Generalized ARCH) by Bollersler (1986). These models are widely used in various branches of econometrics, especially in financial time series analysis. See, for example, Bollerslev, Engle, and Nelson (1994) among others. After the development of the ARCH model by Engle (1982), the application of the ARCH/GARCH model(s) is widely exploited by statisticians and econometricians. GARCH models have been developed to account for empirical inferences in financial data.

GARCH concerns understanding and modeling of volatility. Taking into account excess kurtosis and volatility clustering, it provides more accurate forecasts of variances of asset returns than any other model; as a result it is preferable to use GARCH models in financial returns to get better forecasting results of the asset return's variances.

GARCH stands for Generalized Autoregressive Conditional Heteroscedasticity. Loosely speaking, it can be thought of heteroscedasticity as time-varying variance (i.e., volatility). Conditional implies a dependence on the observations of the immediate past, and autoregressive describes a feedback mechanism that incorporates past observations into the present. GARCH, then, is a mechanism that includes past variances in the explanation of future variances. More specifically, GARCH is a time series modeling technique that uses past variances and past variance forecasts to forecast future. Whenever a time series is said to have GARCH effects, the series is heteroscedastic, i.e., its variances vary with time. If its variances remain constant with time, the series is homoscedastic. GARCH modeling, which builds on advances in the understanding and modeling of volatility in the last decade, takes into account excess kurtosis (i.e. fat tail behavior) and volatility clustering, two important characteristics of monetary time series. It provides accurate forecasts of variances and co-variances of asset returns through its ability to model time-varying conditional variances. As a consequence, it is possible to apply GARCH models to such diverse fields as risk management, portfolio management and asset allocation, option pricing, foreign exchange, and the term structure of interest rates.

It is possible to find highly significant GARCH effects in equity markets, not only for individual stocks, but also for stock portfolios and indices, and equity futures markets as well. These effects are important in such areas as value-at-risk (VaR) and other risk management applications that concern the efficient allocation of capital. One can use GARCH models to examine the relationship between long- and short-term interest rates. As the uncertainty for interest rates over various horizons changes through time, applying GARCH models in the analysis of time-varying risk premiums is possible. Foreign exchange markets, which couple highly persistent periods of volatility with significant fat tail behavior, are particularly well suited for GARCH modeling.

The great workhorse of applied econometrics is the least squares model. This is a natural choice, because statisticians are typically called upon to determine how much one variable will change in response to a change in some other variable. Increasingly, however, they are being asked to forecast and analyze the size of the errors of the model. In this case, the questions are about volatility, and the standard tools have become the ARCH/GARCH models. The basic version of the least squares model assumes homoscedasticity. The standard warning is that in the presence of heteroscedasticity, the regression coefficients for ordinary least squares regressions are still unbiased, but the standard errors and confidence intervals estimated by conventional procedures will be too narrow, giving a false sense of precision. Instead of considering this as a problem to be corrected, ARCH and GARCH models treat heteroscedasticity as a variance to be modeled. As a result, not only are the deficiencies of least squares corrected, but a prediction is computed for the variance of each error term. This prediction turns out often to be of interest, particularly in the applications of monetary time series analysis.

Heteroscedasticity has usually been applied only to cross-section models, not to time series models. However, in a seminalpaper, Engel (1982) suggested that heteroscedasticity might also occur in time series context. Sometimes the natural question facing statisticians is the accuracy of the predictions of the model. In this case, the key issue is the variance of the error terms and what makes them large. This question often arises in financial applications where the dependent variable is the return on an asset or portfolio and the variance of the return represents the risk level of those returns. These are time series applications, for which heteroscedasticity is usually an issue. Even a simple look at financial data suggests that some time periods are riskier than others; that is, the expected value of the magnitude of error terms at some times is greater than at others. Moreover, these risky times are not scattered randomly across quarterly or annual data. Instead, there is a degree of autocorrelation in the riskiness of financial returns. Time series analysts, look at plots of daily returns and observe that the amplitude of the returns vary over time which is described as “volatility clustering.” The ARCH and GARCH models are designed to deal with just this set of issues. They have become widespread tools for dealing with time series heteroscedastic models. The goal of such models is to provide a *volatility measure*—like a standard deviation—that can be used in financial decisions concerning risk analysis (Engel, 1982).

In a volatility forecasting setting of the ARCH type, a number of recent empirical studies look at the relevance of the additional information provided by the lagged implied volatility and assess how it improves on the information given by the past squared returns when it is included in the conditional variance equation. Focus should be given to the analysis of the nearby future contracts for which implied volatility is directly available (Giot, 2002).

For financial return series from real life, the volatility clustering property and time varying variation, or heteroscedasticity, have been observed. However, traditional time series models cannot explain these properties well; but the (G)ARCH models explain these properties better.

If a linear time series model (such as ARMA) is to be fitted to y_t, the estimated parameters will be nearly zero due to the lack of autocorrelation property, which clearly will not be good. Even though there is less/no autocorrelation property among the $y's$, there is a proper way of handling this problem-using GARCH model.

3.1.4.4.1 Testing for (G)ARCH

i. Simple Test

The simplest approach to test for GARCH effect is to examine the squares of residuals. This procedure has the following steps:

Step 1. Fit a mean equation and obtain the residuals $\{e_t\}$.

Step 2. Compute the OLS regression,

$$e_t^2 = \widehat{\omega} + \widehat{\beta_1} e_{t-1}^2 + \widehat{\beta_2} e_{t-2}^2 + \cdots + \widehat{\beta_q} e_{t-q}^2.$$

Step 3.Test the joint significance of

$$\widehat{\beta_1}, \widehat{\beta_2}, \dots, \widehat{\beta_q}$$

If these coefficients are significantly different from zero, the null hypothesis of conditionally homoscedastic disturbances is rejected in favor of the alternative hypothesis ARCH/GARCH disturbances.

ii. Lagrange multiplier (LM) Test

This is a Lagrange multiplier (LM) test for autoregressive conditional heteroscedasticity (ARCH) in the residuals. To test the null hypothesis that there is no ARCH up to order q in the residuals, i.e., $H_0: \beta_1 = \beta_2 = \cdots \beta_q = 0$, we run the regression

$$e_t^2 = \omega + \beta_1 e_{t-1}^2 + \beta_2 e_{t-2}^2 + \cdots + \beta_q e_{t-q}^2 + \mathrm{v_t}$$

where e is a residual and $\mathrm{v_t}$ is error term. This is a regression of the squared residuals on a constant and lagged squared residuals up to order q. The $Obs * R - squared$ statistic is Engle's LM test statistic, computed as the number of observations (n) times the R^2 from the test regression. The LM test statistic is asymptotically distributed$\chi^2(q)$ under quite general conditions, that is, $n * R^2 \sim\chi^2(q)$. ARCH will be accepted if the $p - value$ for LM test is found to be smaller than the significance level.

3.1.4.4.2 Formulating (G)ARCH Models

i. The ARCH Model

The first model that provides a systematic frame work for volatility modeling is Autoregressive conditional heteroscedasticity (ARCH) model. In general, a univariate model of an observed time series y_t could be expressed as

$$y_t = f(t-1, x) + \varepsilon_t \tag{3.1}$$

where

- $f(t-1, x)$ represents the deterministic component of the current return as a function of any information known at time $t-1$, including past innovations $\{\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots\}$, past observations $\{y_{t-1}, y_{t-2}, \ldots\}$, and any other relevant explanatory time series data, x.

- ε_t is the random component. It represents the innovation in the mean of y_t. Note that you can also interpret the random disturbance, or shock, ε_t, as the single-period-ahead forecast error.

(G)ARCH models are designed to model and forecast conditional variances. The variance of the dependent variable is modeled as a function of past values of the variance of dependent variable and white noise. In developing ARCH or GARCH model, two distinct model specifications need be considered: one for the conditional mean and other for the conditional variance.

Let $E_{t-1}[.]$ denotes the conditional expectation when the conditioning set is composed by the past values of the process along with other information available at time $t-1$ (denoted by ψ_{t-1}):

$$E_{t-1}[.] \equiv E\left[. | \psi_{t-1}\right].$$

Analogously for the conditional variance we have,

$$Var_{t-1}[.] \equiv Var\left[. | \psi_{t-1}\right].$$

According to Bollerslev, Engle, and Nelson (1994), the disturbance process $\{\varepsilon_t\}$ in the linear regression model $y_t = x_t'\beta + \varepsilon_t$, where x_t includes lagged dependent variables, follows an ARCH regression model if

$$E_{t-1}[\varepsilon_t] = 0 \quad t = 1,2,\ldots$$

and the conditional variance

$$\sigma_t^2 \equiv Var_{t-1}[\varepsilon_t] = E_{t-1}[\varepsilon_t^2] \quad t = 1,2,3,\ldots.$$

$$\varepsilon_t | \psi_{t-1} \sim N(0, \sigma_t^2)$$

Let $\{y_t\}$ denote the stochastic process of interest with conditional mean

$$\mu_t \equiv E_{t-1}[y_t] \qquad\qquad t = 1,2,3,\ldots$$

Define the $\{\varepsilon_t\}$ process by

$$\varepsilon_t = y_t - \mu_t.$$

It follows from the above equations, that the standardized process

$$z_t = \frac{\varepsilon_t}{\sigma_t}, \qquad t = 1,2,3, \dots$$

will have conditional mean zero and a time invariant conditional variance of unity.

Hence, it is possible to think of ε_t as generated by

$$\varepsilon_t = z_t \sigma_t$$

For $z_t \sim N(0.1)$ and independent of σ_t, ε_t has the following properties:

$$Var_{t-1}[\varepsilon_t] = E_{t-1}[\sigma_t^2]$$

$$E_{t-1}[\varepsilon_t] = 0$$
$$Cov_{t-1}[\varepsilon_t, \varepsilon_{t+1}] = 0$$

From the above properties, ε_t satisfies the conditions for heteroscedastic disturbance. Hence it is conditionally heteroscedastic.

The ARCH(q) regression model is a linear function of past squared disturbances,

$$\sigma_t^2 = \omega + \sum_1^q \alpha_i \varepsilon_{t-i}^2 \tag{3.2}$$

In this model to assure a positive conditional variance the parameters have to satisfy the following constraints: $\omega > 0 \; and \; \alpha_i \geq 0$ for all $i = 1,2, \dots q$

ii. The GARCH Model

The underlying regression is the usual one as in the case of equation (3.1) above. Conditioned on information set at time $t-1$, GARCH(p,q) process is defined as follow,

$$\sigma_t^2 = \omega + \sum_1^p \alpha_i \sigma_{t-i}^2 + \sum_1^q \beta_j \, \varepsilon_{t-j}^2 \tag{3.3}$$

with the constraints $\omega > 0$, $\alpha_i \geq 0$ for all $i = 1,2,\ldots p$ and $\beta_j \geq 0$ for all $j = 1,2,3,\ldots q$

Note that the main problem with an ARCH model is that it requires a large number of lags to catch the nature of the volatility; this can be a problem, as it is difficult to decide how many lags to include, as a result it produces a non-parsimonious model where the non-negativity constraint(s) could be failed. The GARCH model is usually much more parsimonious and often a GARCH(1,1) model is sufficient, this is because the GARCH model incorporates much of the information that a much larger ARCH model with large numbers of lags would contain.

Basic Properties of GARCH Model

i. Uniqueness and Stationarity

According to Bougerol and Picard (1992), a necessary and sufficient condition for the GARCH equation to have a unique and stationary solution is

$$\sum_{1}^{p} \alpha_i + \sum_{1}^{q} \beta_j < 1$$

ii. Mean Zero

In any model in which σ_t is measurable with respect to ψ_{t-1}(which is the case in GARCH model specified prior), the mean of ε_t is zero. That is,

$$E[\varepsilon_t] = 0$$

iii. Lack of Serial correlation

In GARCH even though it is conditional, heteroscedasticity is inevitable. Hence, the auto-covariance of any pair of elements from the series is expected to be zero resulting in lack of serial correlation. For $k > 0$, ε_t is not correlated with ε_{t+k}:

$$E[\varepsilon_t \, \varepsilon_{t+k}] = 0.$$

iv. Unconditional variance

In order to compute $E[\varepsilon_t^2]$, it is useful to consider an alternative representation of ε_t^2. First define the sequence,

$$\sigma_t^2 = \varepsilon_t^2 - v_t \tag{3.4}$$

It is possible to show that v_t is a series with mean zero and has no serial correlation. The main point of this definition is to treat v_t as a “white noise" sequence for further manipulation. From equation (3.4), it is possible to write

$$\varepsilon_t^2 = \sigma_t^2 + v_t$$

$$= \omega + \sum_{1}^{p} \alpha_i \sigma_{t-i}^2 + \sum_{1}^{q} \beta_j \varepsilon_{t-j}^2 + v_t$$

If we denote $r = max(p,q)$, $\alpha_i = 0$ for $i > p$ and $\beta_j = 0$ for $j > q$, then the above can be written as

$$\varepsilon_t^2 = \omega + \sum_1^r \left[\alpha_j + \beta_j\right] \varepsilon_{t-j}^2 - \sum_1^p \alpha_i v_{t-i} + v_t$$

In other words, ε_t^2 is an ARMA(r,p) process with a white noise innovation. Using stationarity (which implies $E[\varepsilon_t^2] = E\left[\varepsilon_{t-j}^2\right]$), the unconditional variance will be,

$$E[\varepsilon_t^2] = \omega + E[\varepsilon_t^2] \sum_{1}^{r} \left[\alpha_j + \beta_j\right]$$

which gives

$$E[\varepsilon_t^2] = \frac{\omega}{1 - \sum_1^r \left[\alpha_j + \beta_j\right]}$$

3.1.4.4.3 Order Determination and Estimation of (G)ARCH Models

If an ARCH model is found to be significant, one can use the PACF of ε_t^2 to determine the ARCH order. This is due to the expectation that ε_t^2 (an unbiased estimate of σ_t^2) is linearly related to

$\varepsilon_{t-1}^2, \varepsilon_{t-2}^2, \dots, \varepsilon_{t-q}^2$ in a manner similar to that of AR(q) model. Specifying the order of a GARCH model is not easy; only lower order GARCH models are used in most applications.

Estimation is difficult for any thing other than low values of p and q. In practice the most frequent application is the GARCH(1,1) model,

$$\sigma_t^2 = \omega + \alpha_1 \sigma_{t-1}^2 + \beta_1 \varepsilon_{t-1}^2$$

Substituting successively for the lagged variance on the right side equation the above equation gives,

$$\sigma_t^2 = \frac{\omega}{(1-\alpha_1) + \beta_1(\varepsilon_{t-1}^2 + \alpha_1^2 \varepsilon_{t-2}^2 + \alpha_1^3 \varepsilon_{t-3}^2 + \cdots)}$$

The current variance now depends on all previous squared disturbances; and provided α_1 is a positive fraction, the weights decline exponentially. The asymptotically efficient estimator is obtained by maximum likelihood (ML) estimation method.

Under the normality assumption, the likelihood function of an ARCH(q) model is

$$f(\varepsilon_1, \cdots, \varepsilon_n | \boldsymbol{\alpha}) = \prod_{t=q+1}^{n} \frac{1}{\sqrt{2\pi\sigma_t^2}} exp\left(-\frac{\varepsilon_t^2}{2\sigma_t^2}\right) \times f\left(\varepsilon_1, \cdots, \varepsilon_q | \boldsymbol{\alpha}\right)$$

where $\boldsymbol{\alpha} = \left(\alpha_0, \alpha_1, \cdots, \alpha_q\right)'$ and $f\left(\varepsilon_1, \cdots, \varepsilon_q | \boldsymbol{\alpha}\right)$ is the joint probability density function of $\varepsilon_1, \cdots, \varepsilon_q$. Since the exact form of $f\left(\varepsilon_1, \cdots, \varepsilon_q | \boldsymbol{\alpha}\right)$ is complicated, it is commonly dropped from the prior likelihood function, especially when the sample size is sufficiently large. This results in using the conditional likelihood function

$$f\left(\varepsilon_{q+1}, \cdots, \varepsilon_n | \alpha_0, \alpha_1, \cdots, \alpha_q\right) = \prod_{t=q+1}^{n} \frac{1}{\sqrt{2\pi\sigma_t^2}} exp\left(-\frac{\varepsilon_t^2}{2\sigma_t^2}\right)$$

where σ_t^2 can be evaluated recursively. We refer to estimates obtained by maximizing the prior likelihood function as the conditional maximum likelihood estimates (MLEs) under normality.

Maximizing the conditional likelihood function is equivalent to maximizing its logarithm, which is easier to handle. Also note that the conditional maximum likelihood estimation method used in the case of ARCH model continues to apply for GARCH models.

3.1.4.4.4 Model Checking

For a properly specified (G)ARCH model, the standardized residuals

$$z_t = \frac{\varepsilon_t}{\sigma_t} \qquad\qquad t = 1,2,3,\dots$$

form a sequence of identically and independently distributed (iid) random variables. Therefore, one can check the adequacy of a fitted (G)ARCH model by examining the series $\{z_t\}$. In particular, the Ljung-Box Q-statistics of z_t can be used to check the adequacy of the mean equation and that of ${z_t}^2$ (which is also an iid) can be used to test the validity of the volatility equation.

3.1.4.4.5 Measuring Volatility

The GARCH (variance) equation is the conditional variance because it is ahead forecast variance based on the past information. $\beta_i\,(i = 1,2,\cdots,q)$, which are the coefficients of the lags of the squared residuals from the mean equation (coefficients in the ARCH terms, usually coefficients in an ARMA model), give the information about volatility observed in the previous periods. The volatility clustering is shown by the size and significance of $\beta_i\,(i = 1,2,\cdots,q)$. $\alpha_i(i = 1,2,\cdots,p)$, which are the coefficients of the lags of the conditional variance (coefficients in the GARCH terms), give the information about the previous periods' forecast variance. The sum, $\sum\alpha_i + \sum\beta_i$, measures the persistence of volatility. Any shock to volatility is permanent if $\sum\alpha_i + \sum\beta_i = 1$, that is, past volatility is significant in predicting future volatility. Volatility is explosive if $\sum\alpha_i + \sum\beta_i > 1$, meaning, a shock to volatility in one period will lead to even greater volatility in the next period. If $\sum\alpha_i + \sum\beta_i < 1$, volatility is neither permanent nor explosive and past volatility prediction is not as such important (Sinah and Dipendra, 2007).

3.1.4.4.6 Volatility Forecasting

Suppose we are at time index h and interested in forecasting σ_{h+l}^2, where $l \geq 1$. Let $\sigma_h^2(l)$ be the forecast of σ_{h+l}^2 and F_h be the collection of information available at the forecast origin h. Then, the forecast $\sigma_h^2(l)$ is chosen such that

$$E\{[\sigma_{h+l}^2 - \sigma_h^2(l)]^2/F_h\} \leq \min_g E[(\sigma_{h+l}^2 - g)^2/F_h]$$

where g is a function of the information available at time h (inclusive), that is, a function of F_h. We referred to $\sigma_h^2(l)$ as the $l - step\ ahead\ \ forecast\ of\ \sigma_t^2$ at the forecast origin h.

ARCH(q)

Consider an ARCH(q) model as in equation (3.2). At the forecast origin h, the 1-step ahead forecast of $\sigma_{h+1}^2$is

$$\sigma_h^2(1) = \omega + \alpha_1\varepsilon_1^2 + \cdots + \alpha_q\varepsilon_{h+1-q}^2$$

The 2-step ahead forecast of $\sigma_{h+2}^2$is

$$\sigma_h^2(2) = \omega + \alpha_1\sigma_h^2(1) + \alpha_2\varepsilon_h^2 + \cdots + \alpha_q\varepsilon_{h+2-q}^2$$

In general the l-step ahead forecast for σ_{h+l}^2 is

$$\sigma_h^2(l) = \omega + \sum_{i=1}^{q} \alpha_i\sigma_l^2\,(l-i)$$

where $\sigma_h^2(l-i) = \varepsilon_{h+l-i}^2$ if $l - i \leq 0$.

GARCH(p, q)

Forecasts of a GARCH model can be obtained similar to that of an ARCH model except here we include the lags of σ_t^2. For illustration consider the GARCH(1,1) model of equation (3.3) and assume the forecast origin is h. For 1-step ahead forecast, we have

$$\sigma_{h+1}^2 = \omega + \alpha_1\sigma_h^2 + \beta_1\varepsilon_h^2$$

where ε_h and σ_h are known at the time index h. Therefore, the 1-step ahead forecast is

$$\sigma_h^2(1) = \omega + \alpha_1 \sigma_h^2 + \beta_1 \varepsilon_h^2$$

The 2-step ahead forecast at the fore cast origin h is also

$$\sigma_h^2(2) = \omega + (\alpha_1 + \beta_1)\sigma_h^2(1)$$

In general, we have

$$\sigma_h^2(l) = \omega + (\alpha_1 + \beta_1)\sigma_h^2(l-1), l > 1$$

3.1.4.5 Measuring the Accuracy of Forecasting Models

The accuracy of a forecasting model depends on how close the forecast values ($\hat{y}_t$) are to the actual values (y_t). In practice, the difference between the actual and the forecast values is defined as the forecast error,

$$e_t = y_t - \hat{y}_t.$$

If the model is doing a good job in forecasting the actual data, the forecast error will be relatively small. The sum of all $e_t's$ should be equal to or near 0, showing that e_t for each time period is purely random fluctuation around $\widehat{y_t}$ (Cynor and Kirkpatrick, 1994).The theoretical random errors (ε_t), which are inherent in the data generating process, are unobservable. The forecast random error, or residual, or the sum of the random errors ($\sum_{t=1}^{n} e_t$), is observable from the calculation $e_t = y_t - \widehat{y_t}$. The forecast random error is assumed to be zero and has a mean of zero.

Because of the mathematical entity in the definition of forecast error, forecasters usually measure a model's accuracy by looking at various quantitative measures. Most frequently, these measures employ the absolute values of the errors $|e_t|$ or the square of the errors ${e_t}^2$.

As a general rule, the smaller the sum of the absolute errors $\sum_{t=1}^{n} |e_t|$ or the sum of the squared errors $\sum_{t=1}^{n} {e_t}^2$, the more accurate the fit of the model.

The following statistical summary measures of a model's forecast accuracy are defined using the absolute errors:

- The mean absolute error

$$MAE = \frac{\sum_{t=1}^{n} |e_t|}{n}$$

- The mean of the absolute percentage error

$$MAPE = \frac{\sum_{t=1}^{n} \frac{|e_t|}{y_t}}{n}$$

where e_t is the forecast error in time period t;

y_t is the actual value in time period t;

n is the number of forecast observations in the estimation period.

The following statistical summary measures of a model's forecast accuracy are defined using the squared errors:

- The mean square error

$$MSE = \frac{\sum_{t=1}^{n} {e_t}^2}{n}$$

- The root mean square error (standard error)

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n} {e_t}^2}{n}}$$

where e_t is the forecast error in time period t;

n is the number of forecast observations in the estimation period.

Because these statistics give a measure of the forecasting error, the decision of which one to use depends on the make up of the data. If there are only one or two large errors, these will be magnified by using MSE and $RMSE$ (since all the errors are squared); thus the MAE should be used. When all the errors are similar in magnitude, the statistic used is the MSE.

Theil’s inequality coefficient is another statistical measure of forecast accuracy. One specification of Theil’s inequality coefficient compares the accuracy of a forecast model to that of a naive model, which simply uses the actual value for the last time period (y_t) as a forecast for $\widehat{y_{t+1}}$. That is, $\widehat{y_{t+1}} = y_t$ for each time period. The formula is

$$U = \frac{RMSE\ of\ the\ forecasting\ model}{RMSE\ of\ the\ naive\ model}$$

According to Caynor and Kirkpatrick (1994), $U > 1.0$ indicates the forecast model is worse than the naive model; a value of U less than 1.0 indicates the forecast model is better than the naive model. Finally the accuracy could also be demonstrated by different proportions, such as Bias Proportion, and Variance Proportion. The bias proportion tells us how far the mean of the forecast is from the mean of the actual series. But, the variance proportion tells us how far the variation of the forecast is from the variation of the actual series.

The ARMA models for the conditional mean and The (G)ARCH models for the conditional variance are applied to the analysis of export prices data in the next chapter.

CHAPTER FOUR

DATA ANALYSIS

4.1 Data

The data set consists of daily (excluding days for which no export transaction) export prices of Ethiopia from January 1, 1997 to May 30, 2008 of three main export items, namely, daily total of export prices of *any export item* ($\boldsymbol{Total\ Export}$), daily total of export prices of *coffee* ($\boldsymbol{Coffee\ Export}$), and daily total of export prices of *oil seeds* ($\boldsymbol{Oil\ Seeds\ Export}$) obtained from Ethiopian Customs Authority. The total number of daily observations for the three export items, in the above order, is 3114, 2567, and 1305, respectively. The full samples are split into two parts: in sample data, in order to estimate the parameters of models and out sample data, in order to make forecasts. In sample data for total export is composed of 2779 observations from 01/01/1997 until 30/6/2007. Out sample data for total export is composed of 335 observations from 01/07/2007 until 31/05/2008. In sample data for coffee export is composed of 2223 observations from 01/01/1997 until 30/6/2007. Out sample data for coffee export is composed of 344 observations from 01/01/1997 until 30/6/2007. In sample data for oil seeds export is composed of 1159 observations from 01/01/1997 until 30/6/2007. Out sample data for oil seeds export is composed of 146 observations from 01/01/1997 until 30/6/2007. The data analysis is carried out using EViews 3.1 software. Plots of original export prices are displayed in APPENDIX, A.

4.2 Test of Stationarity and Features of Log Return Series

The raw data in the three export groups are real export prices which in many cases are expected to be non-stationary. Thus, the first logical step is to check stationarity of export prices using unit root test. In this test the null hypothesis of unit root is rejected if the t-statistic is less than (lies to the left of) the critical value. Table 4.1 summarizes the unit root test results of the real export prices. But as can be seen in Table 4.1, all the three t-statistics are greater than all the three significant levels' ($1\%, 5\%, and\ 10\%$) critical values. Hence the null hypothesis of unit root would not be rejected, that is there is a unit root problem in each of the data indicating that each export price series is non-stationary.

Table 4.1. Unit Root Test Results for Export prices

$Export\ Prices$	$t\ Statistic$	1% $Critical\ Value$	5% $Critical\ Value$	10% $Critical\ Value$
$Total$	-0.8170	-3.4355	-2.8630	-2.5676
$Coffee$	-0.8664	-3.4360	-2.8632	-2.5677
$Oil\ Seeds$	-1.0105	-3.4382	-2.8642	-2.5682

If a time series data is non-stationary, it is necessary to look for possible transformations that might induce stationarity. In practice, econometricians usually transform financial prices in to return forms. This is because often return series are found to be stationary such that analysis is possible. The log return series is obtained by

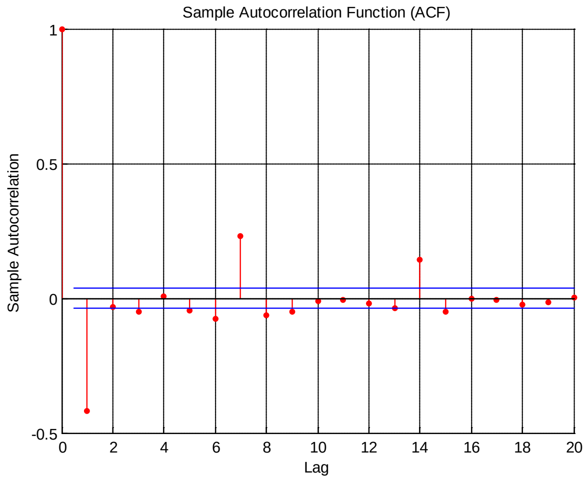
$$y_t = \log \frac{p_t}{p_{t-1}}$$

where y_t is the log return series of the real raw data of export, p_t is the raw real export series and log is the natural logarithm. Plots of log return export series are displayed in APPENDIX, B. Table 4.2 summarizes the unit root test of the log return series for each of the export item. The table shows that all the t-statistics are less than the critical values at all levels of significance. These indicate that the null hypothesis of unit root would be rejected in all of the three cases. Hence the log return series are stationary for each of the export items.

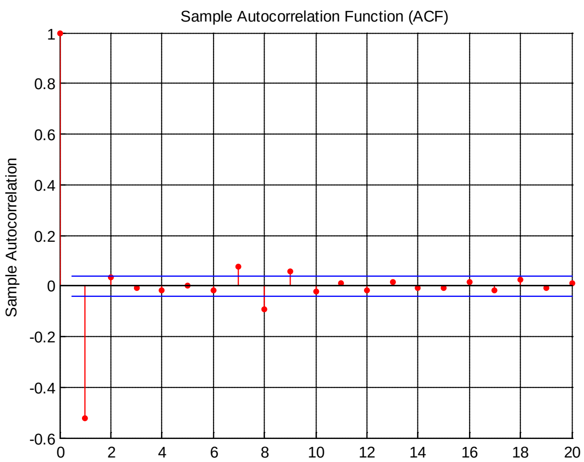
Table 4.2. Stationarity Test Statistics for Export log Returns

$Export\ Prices$	$t\ Statistic$	1% $Critical\ Value$	5% $Critical\ Value$	10% $Critical\ Value$
$Total$	-43.38776	-3.4355	-2.8630	-2.5676
$Coffee$	-40.5264	-3.4359	-2.8632	-2.5677
$Oil\ Seeds$	-27.38902	-3.4382	-2.8642	-2.5682

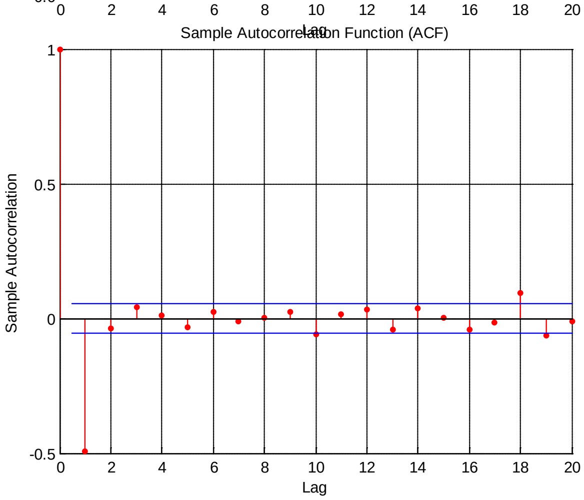
Another important feature of return series is that usually the series show no or little autocorrelation. Figure 4.1 (a, b, c) shows the plots of the sample autocorrelation functions (ACFs) of the three return series for lags 1 to 20. A sample ACF has the approximate upper and lower confidence bounds. The three sample ACFs suggest that the serial correlations of daily export price log returns are very small except at a few lags. Most of the sample ACFs are within or near their two standard-error limits (the two blue horizontal lines) indicating that autocorrelations that fall inside the limits are not significantly different from zero.



a. Sample ACF of Total Log Return Export Prices



b. Sample ACF of Coffee Log Return Export Prices



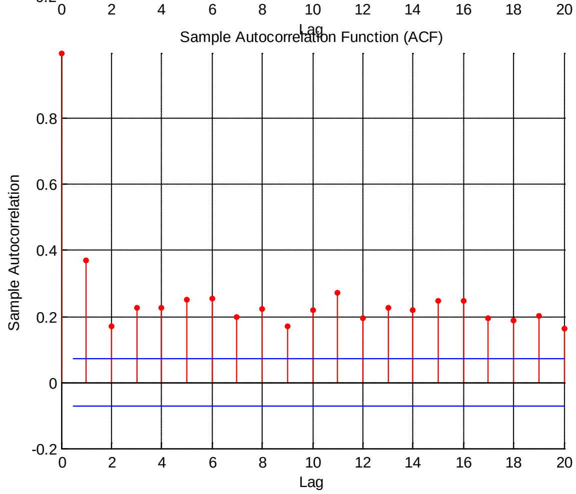
c. Sample ACF of Oil seeds Log Return Export Prices

Figure 4.1 Sample autocorrelation functions of daily (a) log returns of total export prices, (b) log returns of coffee export prices and (c) log returns of oil seeds export prices. In each plot, the two blue horizontal lines denote two standard-error limits of the sample ACFs.

Further more, squared log return series are usually strongly auto-correlated. Figure 4.2 (a, b, c) represents the plots of the sample ACFs of the squared log returns of the three export prices. The three sample ACFs suggest that the serial correlations of daily export price squared log returns are very large. Most of the sample ACFs are out of their two standard-error limits. In particular, for the cases of coffee and oil seeds the sample ACFs are all out of their respective standard-error limits. This indicates that the sample ACFs of the squared log returns are significantly different from zero. This illustrates the degree of persistence in variance and implies that (G)ARCH models may be used for modeling the variance.

a. Sample ACF of Total Squared Log Return Export Prices

b. Sample ACF of Coffee Squared Log Return Export Prices



c. Sample ACF of Oil Seeds Squared Log Return export Prices

Figure 4.2 Sample autocorrelation functions of daily (a) squared log returns of total export prices, (b) squared log returns of coffee export prices and (c) squared log returns of oil seeds export prices. In each plot, the two blue horizontal lines denote two standard-error limits of the sample ACFs.

It is also typical property of financial log-return series to exhibit skewness and kurtosis. Table 4.3 presents general summary statistics of the three different log return series. The table shows that skewness and kurtosis are clearly observed in the three log return series. All the kurtosis exceed 3, which is the kurtosis value of the normal distribution, indicating the log return series data is peaked relative to the normal distribution. All the skewness values are different from 0, which is the skewness value of the normal distribution, showing the distribution of the log return series is not symmetric around its mean compared to the distribution of the normal distribution. Positive skewness means that the distribution has a long right tail and negative skewness implies that the distribution has a long left tail. The Jarque-Bera test of normality indicate to reject ($p - value\ = 0.0000$) the normality of the log return series in the three export items.

Table 4.3 General Summary Statistics

$Export\ Items$	$Mean$	$Median$	$Max.$	$Min.$	$Std.ve.$	$Skewness$	$Kurtosis$	$Jarque - Bera$	$P - Value$
$Total$	0.0005	0.0032	8.1935	-7.1494	0.8151	0.2375	13.6630	14781.9	0.0000
$Coffee$	-0.0002	0.0010	6.6831	-6.4444	0.7933	-0.0020	13.973	12878.1	0.0000
$Oil\ Seeds$	0.0006	-0.0006	3.7475	-3.8710	0.7614	-0.0285	8.1399	1436.66	0.0000

4.3 Volatility Modeling

The next step is to select the best model for the volatility of the log return series. Building a volatility model for the log return series consists of three steps:

1. Specifying a mean equation;
2. Using the residuals of the mean equation to test for ARCH effects;
3. Specifying a volatility model if ARCH effects are statistically significant and carrying out a joint estimation of the mean and volatility equations;

4.3.1 Specification of Mean Equation

To specify a mean equation of the series, it is better to compare some AR(p), MA(q) and ARMA(p, q) models and take the best one.

For an AR(p) series, the sample partial autocorrelation function (PACF) cuts off at lag p. Table 4.4 gives the first 20 lags of a sample PACF for each of the log return series of daily export prices. The first 13, 8 and 17 lags sample PACFs of total, coffee and oil seeds, respectively, appear to be relatively large. This suggests that AR(13), AR(8) and AR(17) models could be identified. Graphs of the sample PACFs are shown in APPENDIX, C.

Table 4.4. Sample PACFs of the Daily Log Returns of Total, Coffee and Oil Seeds Export Prices

p	$PACF\ (Total)$	$PAC\ F\ (Coffee)$	$PACF\ (Oil\ Seeds)$
1	-0.417	-0.522	-0.494
2	-0.251	-0.328	-0.371
3	-0.222	-0.238	-0.253
4	-0.175	-0.212	-0.166
5	-0.210	-0.193	-0.155
6	-0.327	-0.209	-0.110
7	-0.036	-0.085	-0.101
8	-0.020	-0.145	-0.086
9	-0.077	-0.098	-0.031
10	-0.078	-0.089	-0.092
11	-0.099	-0.061	-0.107
12	-0.118	-0.069	-0.065
13	-0.167	-0.048	-0.098
14	-0.058	-0.049	-0.050
15	-0.080	-0.056	-0.019
16	-0.055	-0.049	-0.058
17	-0.033	-0.067	-0.121
18	-0.055	-0.047	-0.004
19	-0.068	-0.032	-0.009
20	-0.056	-0.005	-0.038

For an MA(q) model the sample ACF cuts off after lag q. From Figure 4.1, it is seen that the total log return series has significant ACFs at lags $1, 3, 5, 6, 7, 8, 9, 14$ and 15. Therefore, the total log return series follows MA(15) model. In a similar manner, the coffee log return series follows MA(9) model with significant ACFs at lags $1, 7, 8$ and 9; and the oil seeds log return series has also MA(19) with significant ACFs at lags $1, 10, 18$ and 19.

In most applications, lower order ARMA models, say, ARMA$(1,1)$, ARMA$(1,2)$, ARMA$(2,1)$ and ARMA$(2,2)$ are used. Computing different model selection statistics in order to suggest the best of all the ARMA alternatives at the in-sample stage for every series is useful. Here, two statistics, Akaike Information Criterion (AIC) and Bayesian Information or Schwarz Criterion (BIC) are used. The AIC and BIC are ranked relative to the different ARMA models. The two ranks (rank of AIC and rank of BIC) for each model are added to give a single sum. The lower the rank sum is the better the model. Table 4.5 gives the alternative ARMA models together with their AIC and BIC ranks of the three log return series. Hence, considering Table 4.5, the minimum sum of ranks is obtained at ARMA$(1,1)$ for every type of log return series. Therefore the best model for the data under consideration is ARMA$(1,1)$ fore all of the export log return series.

Table 4.5. ARMA model selection for the Daily Log Returns using AIC & BIC of Total, Coffee and Oil Seeds Export Prices

Total export, In-sample Output

$Statistics$	$ARMA(1,1)$	$ARMA(1,2)$	$ARMA(2,1)$	$ARMA(2,2)$
AIC	1.9787 (1)	1.9794 (2)	1.9796 (4)	1.9794 (2)
BIC	1.9829 (1)	1.9858 (2)	1.9860 (3)	1.9879 (4)
$Rank\ Sum$	2	4	7	6

Coffee export, In-sample Output

$Statistics$	$ARMA(1,1)$	$ARMA(1,2)$	$ARMA(2,1)$	$ARMA(2,2)$
AIC	1.6861 (1)	1.6869 (2)	1.6870 (3)	1.6877 (4)
BIC	1.6913 (1)	1.6946 (2)	1.6947 (3)	1.6979 (4)
$Rank\ Sum$	2	4	6	8

Oil Seeds export, In-sample Output

$Statistics$	$ARMA(1,1)$	$ARMA(1,2)$	$ARMA(2,1)$	$ARMA(2,2)$
AIC	1.6959 (1)	1.6971 (2)	1.6980 (3)	1.6991 (4)
BIC	1.7047 (1)	1.7102 (2)	1.7111 (3)	1.7166 (4)
$Rank\ Sum$	2	4	6	8

Next, in order to obtain proper conditional mean equation it is needed to compare the AR, MA and ARMA models chosen so far. As can be seen in Table 4.6, the best in-sample results are achieved by ARMA(1,1) for all export log return prices. Therefore, ARMA(1,1) is the best (having minimum rank sum) conditional mean equation for all the three cases. This result is a more parsimonious representation of the conditional mean equation. Hence, the ARMA(1,1) model given by the following expression is used to model the conditional mean equation of the log return series of export prices:

$$y_t = \mu + \theta_1 y_{t-1} + \varepsilon_t - \rho_1 \varepsilon_{t-1}$$

Table 4.6. Model Selection Using AIC and BIC for the Daily Log Returns of Total, Coffee and Oil Seeds Export Prices

Total export, In-sample Output

$Statistics$	$AR(13)$	$MA(15)$	$ARMA(1,1)$
AIC	2.0278 (3)	2.0081 (2)	1.9787 (1)
BIC	2.0407 (3)	2.0102 (2)	1.9829 (1)
$Rank\ Sum$	6	4	2

Coffee export, In-sample Output

$Statistics$	$AR(8)$	$MA(9)$	$ARMA(1,1)$
AIC	1.7255 (3)	1.6818 (1)	1.6861 (2)
BIC	1.7461 (3)	1.7049 (2)	1.6913 (1)
$Rank\ Sum$	6	3	3

Oil Seeds export, In-sample Output

$Statistics$	$AR(17)$	$MA(19)$	$ARMA(1,1)$
AIC	1.7608 (3)	1.7039 (2)	1.6959 (1)
BIC	1.7915 (3)	1.7867 (2)	1.7047 (1)
$Rank\ Sum$	6	4	2

4.3.2 Test for ARCH Effects

Based on the residuals from the mean equation it is possible to test for the existence of ARCH effect which will allow continuing the analysis using (G)ARCH model. The ARCH LM test discussed in chapter 3 helps to test the hypothesis that there is no ARCH effect up to lag p. If the test provides significant results, then it is possible to conclude that there is an ARCH effect up to lag p. Table 4.7 shows the results of ARCH LM test for the three export items. The last column of Table 4.7 include the p-values that indicate rejection of the null hypothesis that there is no ARCH effect at the first lag at 5% level of significance, i.e, there is an ARCH effect on each of the three cases. The results indicate that the export log return series are volatile and need to be modeled using ARCH or GARCH models.

Table 4.7 ARCH LM Test Summary Statistics

$Export\ Items$	$Obs * R^2$	$t - Statistic$	$Lag(p)$	$Prob.$
$Total$	199.3180	3.9850	1	0.0001
$Coffee$	116.9652	11.066	1	0.0000
$Oil\ Seeds$	75.6167	8.9528	1	0.0000

4.3.3 ARCH or GARCH Model Identification

If an ARCH effect is found to be significant, the PACFs of ε_t^2 are helpful to determine the order of ARCH model. This is due to the expectation that ε_t^2 is linearly related to $\varepsilon_{t-1}^2, \varepsilon_{t-2}^2, \dots, \varepsilon_{t-q}^2$ in a manner similar to that of AR(q) model. The PACFs are presented in Table 4.8. As can be seen in Table 4.8, PACFs of lags 7 from the first column, lag 7 from the second column and lag 3 and 18 from the last column are relatively large. Hence, ARCH(7) for total, ARCH(7) for coffee and ARCH(18) for oil seeds are identified. Graphical representation of the PACFs of ε_t^2 of the three export items are displayed in APPENDIX, D.

Table 4.8. Sample PACFs of Residual Squares of Total, Coffee and Oil Seeds Export Prices

q	$PACF\ (Total)$	$PACF\ (Coffee)$	$PACF\ (Oil\ Seeds)$
1	-0.010	0.000	0.005
2	-0.022	-0.001	-0.008
3	-0.020	0.008	0.058
4	-0.016	0.011	0.022
5	-0.007	0.004	0.042
6	-0.010	0.010	0.010
7	0.053	0.068	0.013
8	-0.015	-0.010	-0.026
9	-0.014	0.022	-0.028
10	-0.018	-0.004	0.011
11	-0.025	0.001	0.005
12	-0.017	-0.002	0.008
13	0.011	0.006	-0.009
14	0.037	-0.006	0.021
15	-0.015	-0.007	-0.017
16	-0.006	-0.003	0.030
17	-0.025	0.004	0.006
18	-0.017	0.004	0.077
19	0.003	0.009	-0.003
20	0.003	0.009	0.007

In practice, for ARCH modeling a large lag q is often needed and this requires that estimating a large number of parameters. To reduce the computational burden, a GARCH model with low lags can help. This results in a more parsimonious representation of the conditional variance process.

The modeling procedure of ARCH models can also be used to build a GARCH model. However, specifying the order of a GARCH model is not easy. Only lower order GARCH models are used in most applications, say, GARCH(1,1), GARCH(1,2), GARCH(2,1), and GARCH(2,2) models. Computing different statistics such Akaike Information Criterion (AIC) and Bayesian information or Schwarz Criterion (BIC) in order to choose the best model based on the in-sample data for every item is helpful. Table 4.9 summarizes these two statistics computed from different GARCH models. Note that the AIC and BIC of the GARCH models are obtained by estimating the mean and variance equations simultaneously. The row ranks indicate the ranking of the various GARCH models based on the two statistics. As a result, the lower the rank sum is the better the model. The best models for the various export items are:

- GARCH(2,1) for total and coffee and
- GARCH(2,2) for oil seeds.

That is, the conditional variance processes are modeled by

$$\sigma_t^2 = \omega + \alpha_1\sigma_{t-1}^2 + \alpha_2\sigma_{t-2}^2 + \beta_1\varepsilon_{t-1}^2 \quad \text{(GARCH(2,1))}$$

$$\sigma_t^2 = \omega + \alpha_1\sigma_{t-1}^2 + \alpha_2\sigma_{t-2}^2 + \beta_1\varepsilon_{t-1}^2 + \beta_2\varepsilon_{t-2}^2 \quad \text{(GARCH(2,2))}$$

Table 4.9 Ranking of GARCH Models Based on AIC and BIC

Total export, In-sample Output

Statistics	*GARCH*(1,1)	*GARCH*(1,2)	*GARCH*(2,1)	*GARCH*(2,2)
AIC	1.8571 (2)	1.8577 (3)	1.8456 (1)	1.8590 (4)
BIC	1.8678 (2)	1.8705 (3)	1.8584 (1)	1.8739 (4)
Rank Sum	4	6	2	8

Coffee export, In-sample

$Statistics$	$GARCH(1,1)$	$GARCH(1,2)$	$GARCH(2,1)$	$GARCH(2,2)$
AIC	1.6844	1.7963	1.6646	1.6667
BIC	1.6973	1.8117	1.6800	1.6846
$Rank\ Sum$	6	8	2	4

Oil Seeds, In-sample

$Statistics$	$GARCH(1,1)$	$GARCH(1,2)$	$GARCH(2,1)$	$GARCH(2,2)$
AIC	1.6990 (4)	1.6980 (3)	1.6625 (2)	1.6458 (1)
BIC	1.7209 (3)	1.7242 (4)	1.6886 (2)	1.6763 (1)
$Rank\ Sum$	7	7	4	2

The variance equation is the conditional variance because it is the one–period ahead forecast variance based on the past information. β_i $(i = 1,2)$, which are the coefficients of the lags of the squared residuals from the mean equation (coefficients in the ARCH terms), give the information about volatility observed in the previous periods. The volatility clustering is shown by the size and significance of β_i $(i = 1,2)$. $\alpha_i (i = 1,2)$, which are the coefficients of the lags of the conditional variance (coefficients in the GARCH terms), give the information about the previous periods' forecast variance. The sum, $\sum \alpha_i + \sum \beta_i$, measures the persistence of volatility. Any shock to volatility is permanent if $\sum \alpha_i + \sum \beta_i = 1$, that is, past volatility is significant in predicting future volatility. Volatility is explosive if $\sum \alpha_i + \sum \beta_i > 1$, meaning, a shock to volatility in one period will lead to even greater volatility in the next period. If $\sum \alpha_i + \sum \beta_i < 1$, volatility is neither permanent nor explosive and past volatility is not significant in predicting future volatility.

4.4 Joint Estimation of Mean and Volatility Equations

The values of the coefficients of the mean and variance equations chosen in the previous section for the three export commodities are shown below in Table 4.10. From Table 4.10, it can be seen that some of the coefficients in the three types of data are not significant at 5% level of significance; in particular, the constant term of the mean equation, μ, is not significant in all cases. Hence, by dropping the constant term, further refinement need to be done.

Table 4.10 Estimated Coefficients for ARMA-GARCH Models

$Export\ Items \rightarrow$	$Total$	$Coffee$	$Oil\ Seeds$
$Mean\ Equation \downarrow$			
μ	0.0002 0.0002 0.4181	−0.0007 0.0011 0.4891	0.0005 0.0014 0.7139
θ_1	0.1281 0.0207 0.0000	−0.0247 0.0300 0.4094	−0.0446 0.0296 0.1317
ρ_1	−0.9861 (0.0032) 0.0000	−0.9386 0.0084 0.0000	−0.9369 0.0110 0.0000
$Variance\ Equation \downarrow$			
ω	0.0009 0.0005 0.1063	0.0200 0.0085 0.0190	0.0911 0.0205 0.0000
α_1	0.6567 0.7335 0.3706	0.5931 0.5538 0.2842	0.3206 0.1389 0.0000
α_2	0.3242 0.7233 0.6540	0.3254 0.5204 0.5318	0.6267 0.1316 0.0000
β_1	0.0171 0.0091 0.0608	0.0194 0.0083 0.0201	0.0201 0.0073 0.0056
β_2			0.0395 0.0086 0.0000

Note: The numbers below the coefficients give standard errors and P-values respectively.

Table 4.11 gives the estimated coefficients after excluding μ, the constant term in the conditional mean equation. Although μ is dropped, there is still insignificant parameter in the mean equation, i.e., AR(1) in the case of coffee. Hence, further refinement is also required.

Table 4.11 Estimated Coefficients for ARMA-GARCH Models without the Constant Term, μ

$Export\ Items \rightarrow$	$Total$	$Coffee$	$Oil\ Seeds$
$Mean\ Equation \downarrow$			
θ_1	0.1243 0.0220 0.0000	−0.0128 0.0290 0.6598	−0.0172 0.0254 0.0408
ρ_1	−0.9832 0.0035 0.0000	−0.9398 0.0082 0.0000	−0.9401 0.0093 0.0000
$Variance\ Equation \downarrow$			
ω	0.0019 0.0005 0.0002	0.0202 0.0025 0.0000	0.4723 0.0100 0.0000
α_1	0.0338 0.0115 0.0032	0.0598 0.0274 0.0292	0.5160 0.0005 0.0000
α_2	0.9297 0.0113 0.0000	0.8496 0.0357 0.0000	0.9979 0.0006 0.0000
β_1	0.0324 0.0027 0.0000	0.0258 0.0033 0.0000	0.0131 0.0012 0.0000
β_2			0.0094 0.0012 0.0000

Note: The numbers below the coefficients give standard errors and P-values respectively.

Table 4.12 Estimated Parameters of ARMA-GARCH Models after dropping AR(1) from Coffee

$Export\ Items \rightarrow$	$Total$	$Coffee$	$Oil\ Seeds$
$Mean\ Equation$			
θ_1	0.1243 0.022 0.0000		−0.0172 0.0254 0.0408
ρ_1	−0.9832 0.0035 0.0000	−0.9412 0.0065 0.0000	−0.9401 0.0093 0.0000
$Variance\ Equation \downarrow$			
ω	0.0019 0.0005 0.0002	0.0241 0.0035 0.0000	0.0472 0.0100 0.0000
α_1	0.0338 0.0115 0.0032	0.0813 0.0147 0.0000	0.0551 0.0005 0.0000
α_2	0.9297 0.0113 0.0000	0.8362 0.0178 0.0000	0.7820 0.0006 0.0000
β_1	0.0324 0.0027 0.0000	0.0049 0.0013 0.0001	0.0131 0.0012 0.0000
β_2			0.0094 0.0012 0.0000

Note: The numbers below the coefficients give standard errors and P-values respectively.

From Table 4.12, it can be seen that all of the coefficients in the three types of data are significant at 5% level of significance. In each case the value of $\sum \alpha_i + \sum \beta_i$ is close to 1. However, it is important to proceed with the Wald test to see whether the restriction $\sum \alpha_i + \sum \beta_i = 1$ is statistically significant or not. The Wald test statistic has Chi-square distribution with one degree of freedom. Table 4.13 shows the Wald test results for the three cases.

Table 4.13 Wald Test on the Restriction$\sum \boldsymbol{\alpha}_i + \sum \boldsymbol{\beta}_i = \mathbf{1}$

$Export\ Item$	$Chi - square$	$P - Value$
$Total$	0.8302	0.3622
$Coffee$	0.9130	0.3393
$Oil\ Seeds$	0.0553	0.8141

As can be seen in Table 4.13, in all the three cases, the null hypothesis $\sum \alpha_i + \sum \beta_i = 1$ can not be rejected at the conventional levels of significance. This result indicates a shock to volatility of exports is permanent for all the three export items. Thus, the past volatility is significant to predict future volatility in each of the three cases.

Next, we examine the adequacy of the models fitted so far. This is done by observing the nature of the sample ACFs of the standardized residuals (for the conditional mean equations) and their squares (for the volatility equations). Since none of the sample ACFs shows any significant serial correlations, i.e., both in the standardized residual and its square are white noise series, then the fitted models appear to be adequate in describing the linear dependence in the conditional mean and volatility equations (see APPENDIX, F).

Additionally, the assumption of conditional normality of residuals is rejected in all the three export items (see APPENDIX E). As a result, to estimate the parameters of ARCH or GARCH models the Quasi-maximum likelihood (QML) method discussed by Bollerslev and Wooldridge (1992) and available in EViews 3.1 is used. Because under Quasi-maximum likelihood method, the parameter estimates are consistent, provided the mean and variance functions are correctly specified.

4.5 Forecasting

The estimated ARMA-(G)ARCH models can be used to compute forecasts of conditional mean and conditional variance. Appendices G and H show the graphical appearances of the actual and forecasted conditional means export log returns and conditional variances of the export log returns.

Performance of forecasts could be evaluated using root mean squared error ($RMSE$) and Theil's inequality coefficient (U) discussed in chapter 3. Table 4.14 summarizes the forecasting accuracy statistics of the constructed models for the three log return series.

Table 4.14. Summary of Forecasting Accuracy Statistics

$Export\ Item$	MAE	$RMSE\ of$ the $forecasting$ $model\ (RMSE_1)$	$RMSE\ of$ the $naive$ $model\ (RMSE_2)$	U	$Bias$ $Prop.$	$Var.$ $Prop.$
$Total$	0.3285	0.4521	0.7274	0.6215	0.0000	0.2346
$Coffee$	0.3000	0.2346	0.4027	0.5826	0.0000	0.2135
$Oil\ Seeds$	0.4276	0.5295	0.8624	0.6139	0.0000	0.3345

Table 4.14 indicates that the forecasting model has a relatively smaller $RMSE$ than the naive model and the lower the Theil's Inequality Coefficient (U) is an indicator of better performance of the forecasting model than the naïve model in all the three cases. These two forecast accuracy statistics indicate better forecasting ability of the fitted models.

Moreover, all the bias proportions are zero indicating that the mean of the forecasts does a job of tracking the mean of the dependent variable. Similarly, the variance proportions are also small enough to tell the variations in the actual series are not highly far from the variations of the forecasts.

The forecasts for the conditional mean of export log returns are displayed in APPENDIX, G; where as the forecasts for the conditional variance or volatility of export log returns are displayed in APPENDIX, H. It can be seen that GARCH model forecasts capture the pattern of volatility. In the long run forecasts the conditional variances seems to converge to the unconditional variances 0.4634, 0.3106 and 0.3362 for total, coffee and oil seeds log return export prices, respectively.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

The main objective of this study is to fit GARCH models in order to see whether export earning in Ethiopia is volatile or not and to forecast export earning for some times in the future for some selected export prices (total export prices, coffee export prices and oil seeds export prices), and also to measure the volatility of export prices of these three export prices.

Knowledge of volatility is of crucial importance in many areas. For traders in foreign exchange markets, in particular for exporters, studying the volatility of asset prices is useful for making some sort of decisions. This is because high volatility could mean huge losses or gains which create greater uncertainty in financial planning. Thus, in volatile markets it is difficult for companies to raise capital in the capital markets.

The base of financial time series data study is founded on the level of stationarity of the data. Often, raw data of item prices are non-stationary, which is also the case in this study. That is the export prices are found to be non-stationary. Hence, in order to make these non-stationary financial time series data stationary, log transformation is applied on the return series. As a result, the log return export prices are found to be stationary (Table 4.2).

Although being stationary is the basic assumption that should be satisfied by a time series data under study, there are some basic characteristics to be fulfilled with the analysis of time series data. One main characteristic, the distribution financial log return series show high skewness and kurtosis compared to the normal distribution, which is an indication of presence of volatility in the time series data (Table 4.3). Another characteristic of a stationary time series data is the behavior of autocorrelation in that data. Usually, log return series show no or little autocorrelation but the squares of the log return series exhibit high autocorrelation. Having autocorrelation in the squared log returns helps to use the concept of heteroscedasticity embedded in a (G)ARCH model. As can be seen from figure 4.2, squared log return series of export prices are found strongly auto correlated which is an indication of heteroscedasticity in log returns of export prices.

In order to properly account heteroscedasticity in log returns export prices, ARCH or GARCH models are considered which require two model specifications, one for the conditional mean and one for the conditional variance. Among the various alternatives like AR(p), MA(q) and ARMA(p,q) to model the conditional mean using the AIC and BIC model selection procedures, ARMA(1,1) is selected as a mean equation for the total, coffee, and oil seeds export items. Similarly, the conditional variance is modeled using ARCH(q) and GARCH(p,q) models and it is found that the variance equation for total and coffee export be GARCH(2,1) and for oil seeds export be GARCH(2,2).

In all the three cases the coefficients of the ARCH terms are significant at 5% level of significance implying there is clustering of volatility of growth of exports. That is, large changes in log returns of export prices are likely to be followed by further large changes. Similarly, the significance of the coefficients of the GARCH terms in each case at the 5% level of significance indicates that the present conditional variance is highly dependent on its past variances (Table 4.12).

In addition, the sum of the coefficients of the ARCH and GARCH terms is close to one in each export item case. This is also supported by Wald test. These results indicate that the volatility shocks are permanent or persistence in each of the three export items. Thus, past volatility is important to predict future volatility and the forecasts of the conditional variances converge to the steady state quite slowly.

The results from this study indicate that there was high volatility in real export prices over the sample period for the three data groups under consideration. The reasons for export prices volatility could be adverse. Broadly these reasons may be divided in to two: internal and external impacts. Internal impacts are policy-induced, institutional and structural obstacles and so on. The external impact is mainly due to excessive exposure to external shocks.

In regard to internal impacts, policy makers and government administrative bodies can play active role in designing and implementing appropriate policies, and removing or reducing institutional and structural obstacles related to the production, transportation and export of agricultural commodities. Moreover, implementing the provision of suitable insurance or other facilities for small-scale individual producers and exporters may be among the possible solutions practiced by the government. Similarly, organizations or individuals engaged in the export sector can effectively use the information on market volatility to manage risks with foreign trade performances and also to create and revise effective administrative procedures.

Where as related to external impacts, it is not possible to reduce external exposure without abandoning international trade; the only course of action is to improve the country's capacity to moderate the impact of external shocks. Furthermore, from this study, it is noted that further studies can be held to see the impact of volatility on over all economic growth of the country.

In general, modeling and forecasting volatility is important to analyze the risk of holding export commodities, to obtain more accurate confidence intervals by modeling the time varying variance of the errors as confidence intervals may be time varying, and to obtain more efficient estimators by handling properly heteroscedasticity in the errors.

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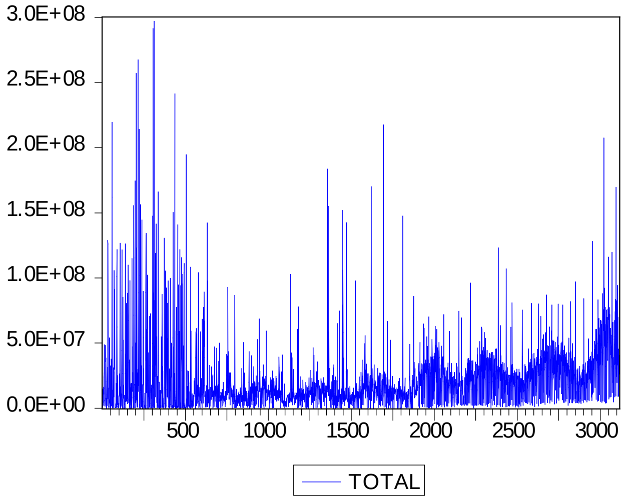
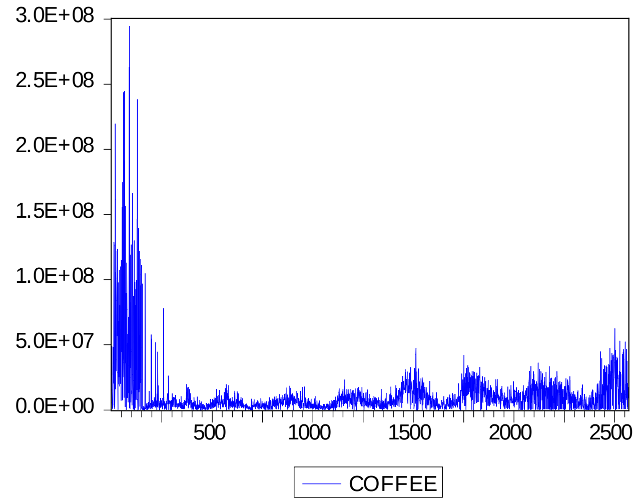
APPENDICES

APPENDIX A, Plot of the Original Export Prices Data

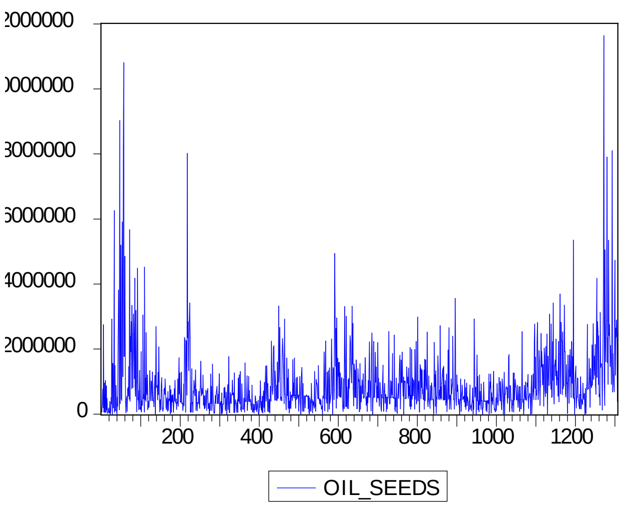
a. Total Original Export Price



b. Coffee Original Export Prices

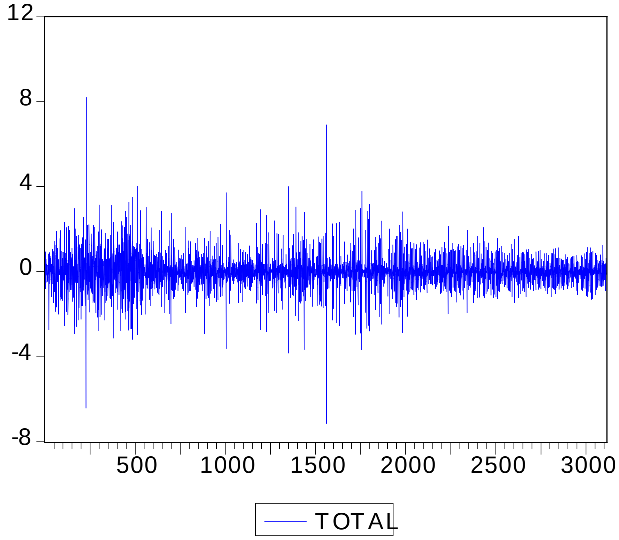


c. Oil Seeds Original Export Prices

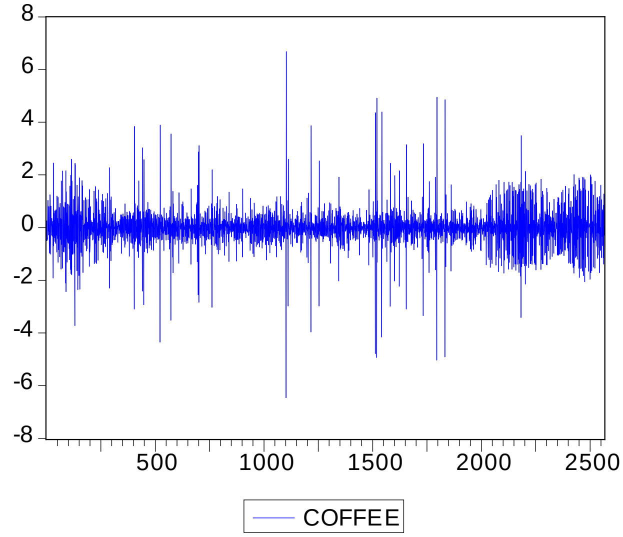


APPENDIX B, Volatility Clustering of Log Return Export Prices

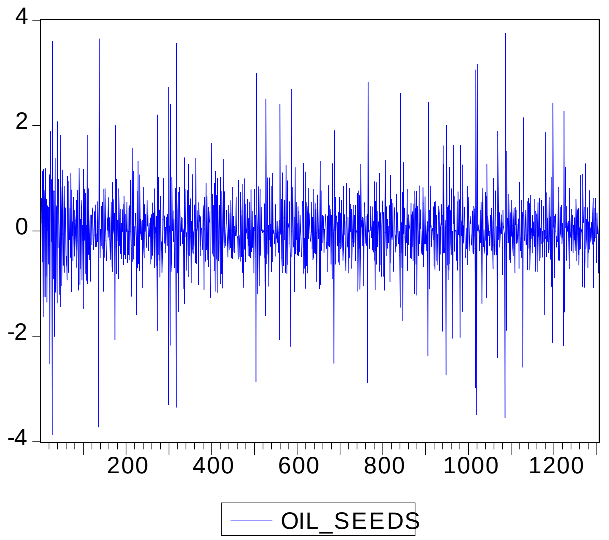
a. Total Log Return Export Prices



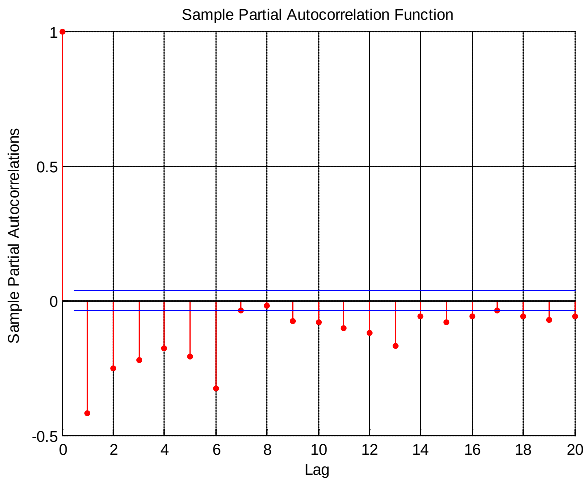
b. Coffee Log Return Export Prices



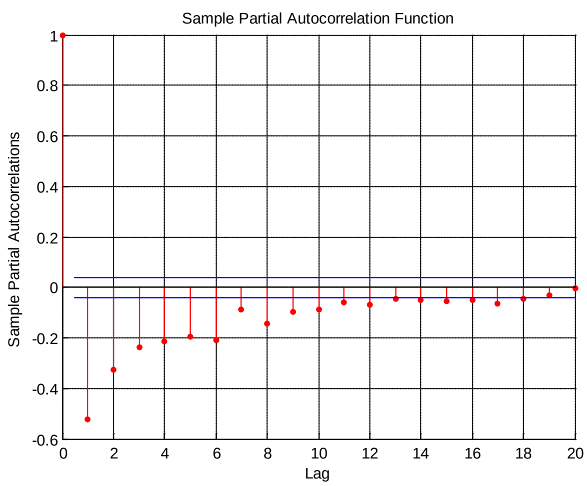
c. Oil Seeds Log Return Export Prices



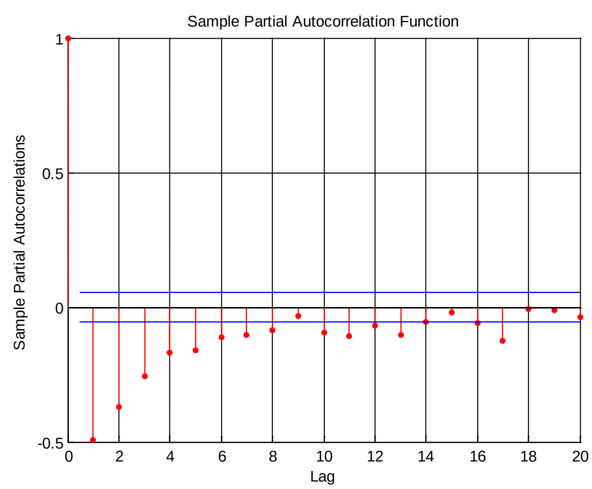
APPENDIX C, Graph of the Sample PACFs of Log Return Export Prices



a. Plot of the Sample PACFs of Total Log Return Export Prices Series

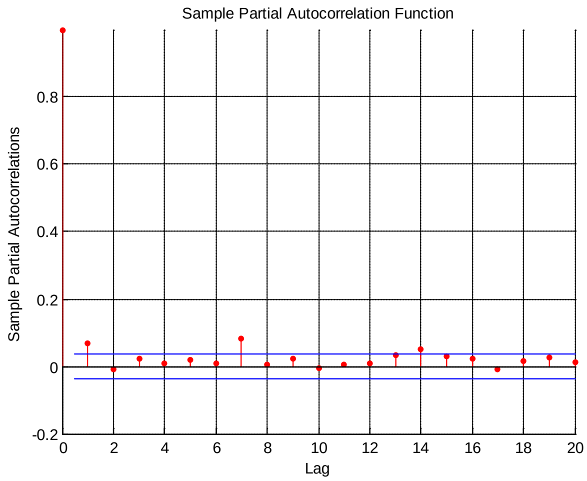


b. Plot of the Sample PACFs of Coffee Log Return Export Prices Series

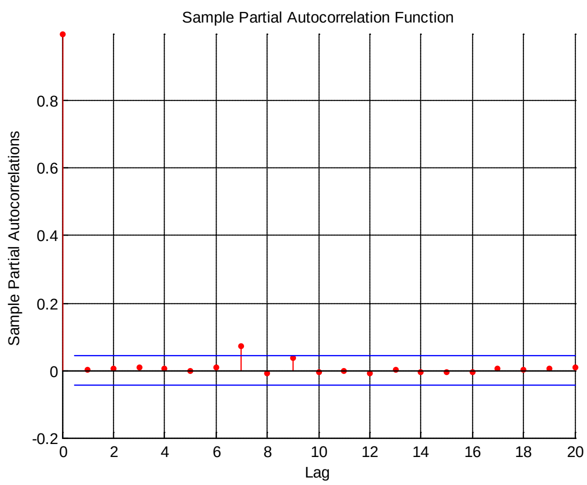


c. Plot of the Sample PACFs of Oil Seeds Log Return Export Prices Series

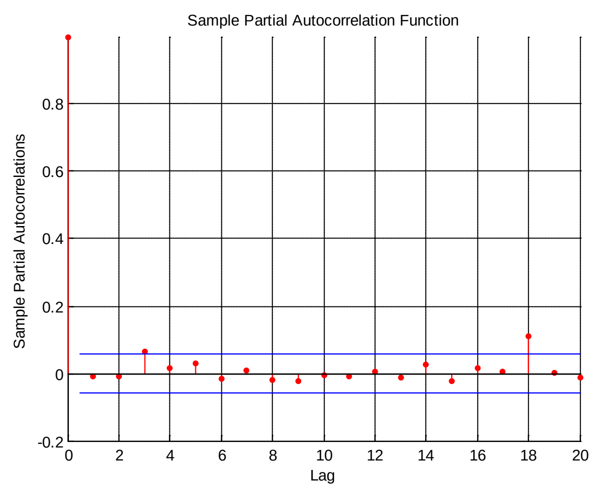
APPENDIX D, Graph of the Sample PACFs of Residual Squares of Log Return Export Prices



a. Plot of the Sample PACFs of Total Residual Square Series



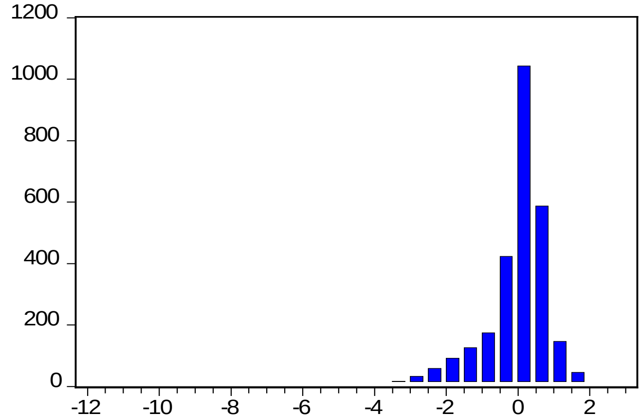
b. Plot of the Sample PACFs of Coffee Residual Square Series



c. Plot of the Sample PACFs of Oil Seeds Residual Square Series

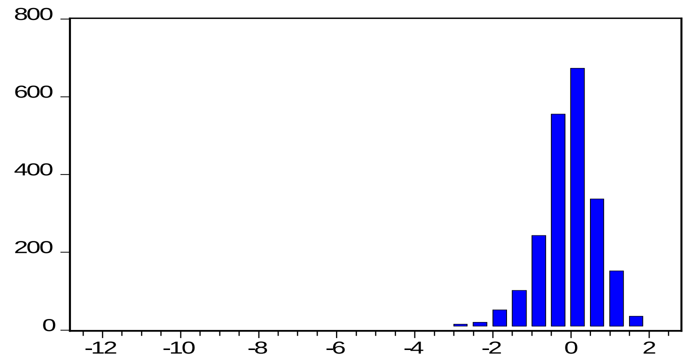
APPENDIX E, Normality Test of Residuals

a. Histogram, Normality Check of Total



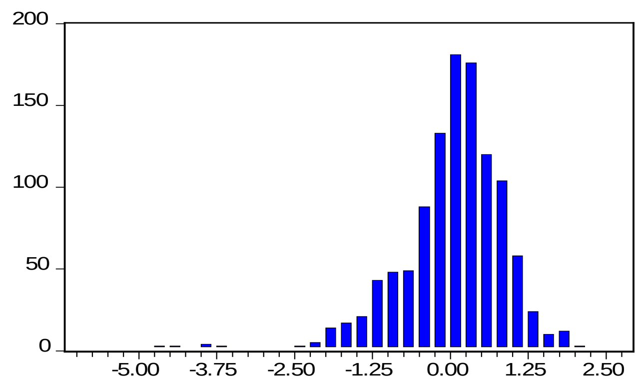
Series: Residuals	
Skewness	-2.397980
Kurtosis	16.12677
Jarque-Bera	22607.50
Probability	0.000000

b. Histogram, Normality Check of Coffee



Series: Residuals	
Skewness	-3.074483
Kurtosis	25.48236
Jarque-Bera	50297.42
Probability	0.000000

c. Histogram, Normality Check of Oil Seeds



Series: Residuals	
Skewness	-1.887491
Kurtosis	9.829712
Jarque-Bera	2889.997
Probability	0.000000

APPENDIX F, Tables of the Sample ACs of standardized Residuals and Their Squares of Log Return Series

Total Log Return Export Prices

$Standardized\ Residual$				$Standardize\ Residual\ Square$		
Lag	AC	$Q\ Statistics$	$P-Value$	AC	$Q\ Statistics$	$P-Value$
1	-0.001	0.0008		-0.010	0.2735	0.601
2	-0.024	1.5589		-0.022	1.5865	0.452
3	-0.017	2.3365	0.126	-0.020	2.6839	0.443
4	-0.017	3.1231	0.210	-0.016	3.3563	0.500
5	-0.002	3.1344	0.371	-0.004	3.4106	0.637
6	-0.013	3.5884	0.465	-0.009	3.6121	0.729
7	0.061	13.913	0.016	0.053	11.504	0.118
8	-0.015	14.581	0.024	-0.015	12.151	0.145
9	-0.011	14.913	0.037	-0.015	12.790	0.172
10	-0.019	15.937	0.043	-0.018	13.673	0.188
11	-0.021	17.225	0.045	-0.025	15.432	0.164
12	-0.015	17.813	0.058	-0.015	16.060	0.189
13	0.015	18.470	0.071	0.013	16.537	0.221
14	0.039	22.677	0.031	0.042	21.354	0.093
15	-0.013	23.142	0.040	-0.016	22.113	0.105
16	-0.009	23.366	0.055	-0.008	22.296	0.134
17	-0.024	25.027	0.050	-0.026	24.162	0.115
18	-0.018	25.930	0.055	-0.018	25.050	0.124
19	0.009	26.148	0.072	0.005	25.113	0.157
20	0.004	26.201	0.095	0.006	25.224	0.193

Coffee Log Return Export Prices

$Standardized\ Residual$				$Standardize\ Residual\ Square$		
Lag	AC	$Q\ Statistics$	$P-Value$	AC	$Q\ Statistics$	$P-Value$
1	-0.009	0.1623	0.687	0.000	0.0005	0.983
2	-0.013	0.5608	0.755	-0.001	0.0017	0.999
3	-0.027	2.2197	0.528	0.008	0.1323	0.988
4	-0.040	5.8011	0.215	0.011	0.4047	0.982
5	-0.015	6.2896	0.279	0.004	0.4384	0.994
6	0.036	9.1493	0.165	0.010	0.6620	0.995
7	0.065	18.610	0.060	0.068	11.083	0.135
8	-0.034	21.260	0.056	-0.009	11.284	0.186
9	0.048	26.345	0.052	0.022	12.384	0.193
10	-0.005	26.392	0.053	-0.003	12.401	0.259

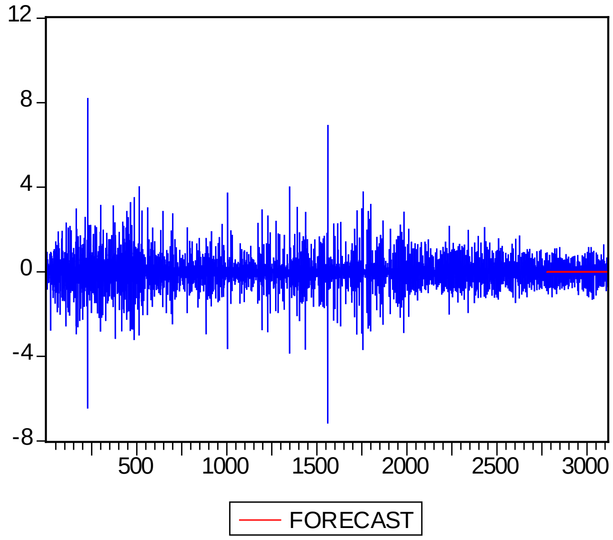
11	0.013	26.743	0.055	0.003	12.418	0.333
12	-0.005	26.790	0.058	-0.002	12.424	0.412
13	-0.009	26.970	0.063	0.007	12.544	0.484
14	-0.001	26.971	0.069	-0.001	12.547	0.562
15	-0.006	27.042	0.078	-0.008	12.691	0.626
16	0.012	27.376	0.057	0.000	12.691	0.695
17	-0.013	27.735	0.058	0.004	12.723	0.755
18	0.044	32.008	0.082	0.004	12.762	0.805
19	0.009	32.207	0.090	0.008	12.920	0.843
20	0.006	32.301	0.092	0.010	13.126	0.872

Oil Seeds Log Return Export Prices

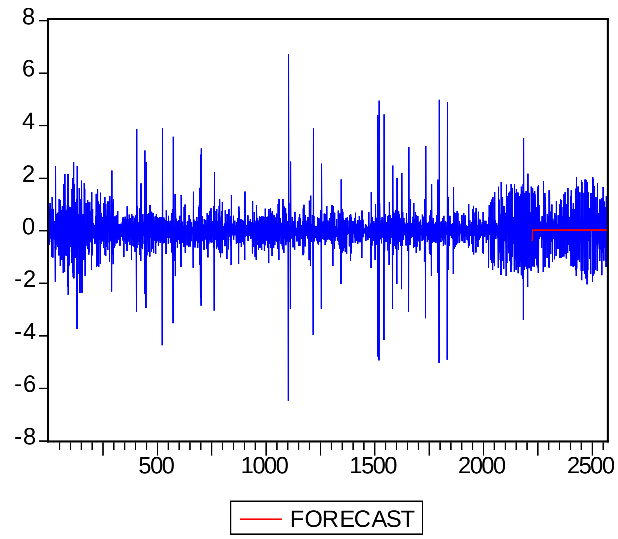
$Standardized\ Residual$				$Standardize\ Residual\ Square$		
Lag	AC	$Q\ Statistics$	$P-Value$	AC	$Q\ Statistics$	$P-Value$
1	0.014	0.2422	0.623	0.005	0.0339	0.854
2	-0.015	0.5074	0.776	-0.008	0.1060	0.948
3	0.063	5.0905	0.165	0.058	4.0195	0.259
4	0.038	6.7856	0.148	0.023	4.6146	0.329
5	-0.006	6.8223	0.234	0.041	6.5727	0.254
6	0.002	6.8275	0.337	0.014	6.7927	0.340
7	-0.003	6.8351	0.446	0.015	7.0451	0.424
8	-0.011	6.9792	0.539	-0.021	7.5474	0.479
9	-0.020	7.4511	0.590	-0.025	8.2988	0.504
10	-0.072	13.555	0.194	0.015	8.5597	0.574
11	-0.010	13.675	0.252	0.005	8.5853	0.660
12	0.021	14.216	0.287	0.005	8.6126	0.736
13	-0.017	14.571	0.335	-0.011	8.7447	0.792
14	0.026	15.375	0.353	0.019	9.1802	0.819
15	0.002	15.381	0.424	-0.016	9.4817	0.851
16	-0.042	17.502	0.354	0.029	10.444	0.842
17	0.005	17.534	0.419	0.011	10.591	0.877
18	0.064	22.320	0.218	0.074	17.125	0.515
19	-0.029	23.295	0.225	0.001	17.125	0.581
20	-0.014	23.534	0.263	0.006	17.162	0.642

APPENDIX G, Forecast of Mean of Log Return Export Prices

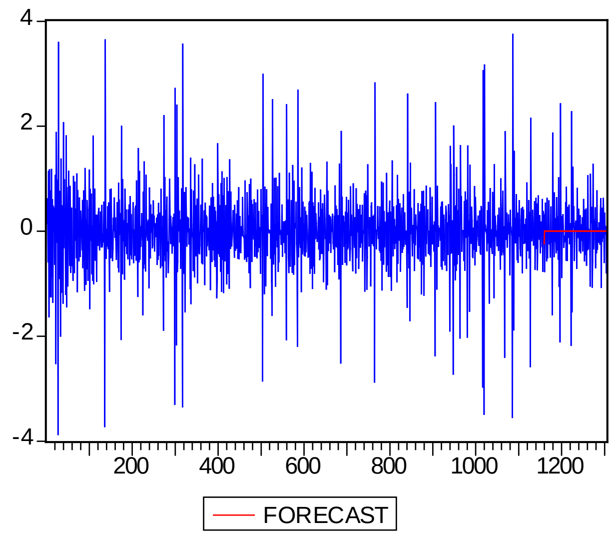
a. Forecast of Mean, Total Log Return Export Prices



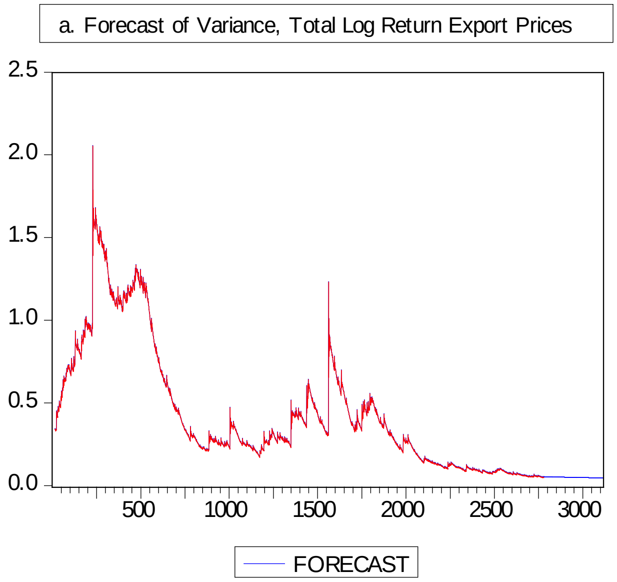
b. Forecast of Mean, Coffee Export Prices

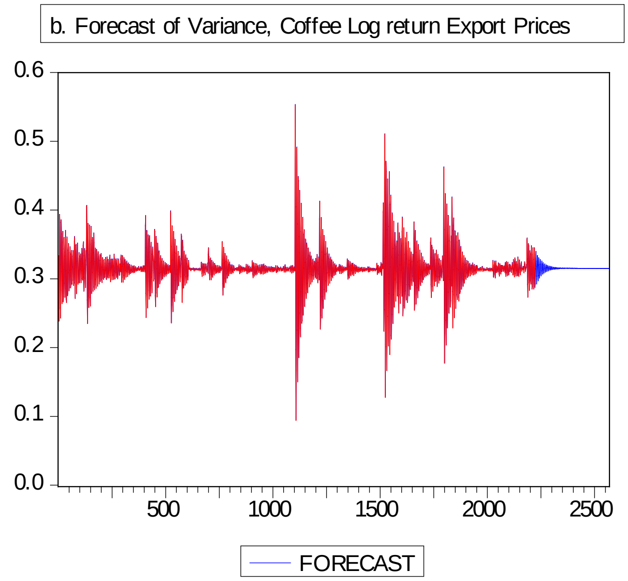


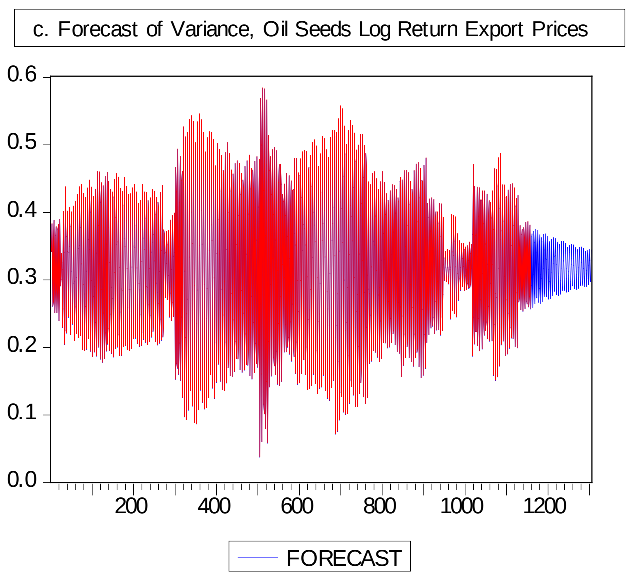
c. Forecast of Mean, Oil Seeds Log Return Export Prices



APPENDIX H, Forecasts of Conditional Variances of Log Return Export prices







DECLARATION

I, the undersigned, declare that the thesis is my original work, has not been presented for degrees in any University and all sources of materials used for thesis have been duly acknowledged.

Name: Amare Terefe

Signature:

Place: Faculty of Science, Addis Ababa University

Date: February, 2009

The thesis has been submitted for examination with my approval as a University advisor.

Fentaw Abegaz (PhD)