Addis Ababa University

Addis Ababa Institute of Technology

School of Civil and Environmental Engineering

Parametric Study of Reinforced Concrete Deep Beams with Rectangular Web Openings Subjected To a Static Monotonic Loading

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A Thesis Submitted to The School of Civil and Environmental Engineering Addis Ababa University in Partial Fulfillment for The Requirement of Degree of Master of Science

Submitted by

by

Pinel Getu

Advisor: **Dr.Ing. Adil Zekaria**

July 2020

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Undertaking

I, the undersigned, declare that this dissertation work, to the best of my knowledge and belief, is my original work, it has not been presented for a degree in this or any other universities and all sources of material for this dissertation has been fully acknowledged

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Date of submission

This is to certify the above declaration made by the candidate is correct to the best of my knowledge

Dr.-Ing Adil Zekaria

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Abstract

Transverse openings in beams are usually present in building, they are provided for the purpose of allowing the passage of service lines such as sanitary pipes, electric cable ducts, air conditioning ducts and telephone lines. The presence of web openings in beams tends to reduce the flexural and shear strength of the beams. Effect of such openings may even be more dangerous in reinforced concrete deep beams, because deeps beams like transfer girders where the column is directly supported on them, are in general subjected to a very large concentrated load. Since different building codes do not give any provisions regarding RC deep beams with transverse web openings and the effect of parameters like degree of interruption of natural load path, openings size, Aspect ratio of opening sides and compressive strength of concrete on the behavior of such types of beams have not been investigated thoroughly. This research attempts to study the effects of the above-listed parameters using a finite element software VECTOR. In the first portion, the finite element model of a Test beam is validated and verified using the results of an experimental test. Results shows that the software accurately predicts the maximum deflection and failure load of the test beams. The error obtained for both cases where under 5%, which is does not exceed the minimum specified tolerance. After the finite element model was validated, a total 73 of numerical simulation were conducted in an attempt to investigate each of the parameters mentioned earlier. The output parameters that was interest to this research was failure load of the test beams, maximum mid span deflection of the beams, cracking load and the maximum crack width. Results showed that, when openings are placed within the shear span of the beam, as the degree of interruption of the natural load path and opening size increases, the failure load (shear strength) of the beams drastically decreased and strength reduction as high as 95% were observed. However, when openings were placed within the flexural region, failure load of the beams were almost similar to the failure loads of the solid beams without web openings but a considerable increase in deflection as high as 39% was obtained. In addition, the magnitude of cracking load decreased when the size of openings located within the shear span increased. Furthermore, results indicate that, increasing the concrete compressive strength will only be effective in counteracting the strength loss due to web openings when the openings are small (i.e. opening area < 8.59% of the shear span area). For large openings, increasing the concrete grade will have insignificant effect on failure load (shear strength of the beam).

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List of Symbols and Abbreviations

L/D	Length to depth ratio
[B]	strain displacement matrix
[k]	Element stiffness matrix
{d}	nodal displacement matrix
{ε}	nodal strain matrix
ΔΘ	lag angle
A_c	Area of concrete
A_{cs}	Area of compression strut
A_{nz}	Area of nodal zone
A_{si}	Total area of shear reinforcement
A_{ss}	Area tension ties
A_v	area of shear reinforcement
bw, b	web width of the beam
d	effective depth of the beam
d_v	the distance for top compression fiber to center of compression reinforcement
E_c	Elastic modulus of concrete
Φ	resistance factor for material
f_c	28 days, cylindrical compressive strength of the beam
f_{cd}	design strength of concrete
f_{ce}	effective compressive strength of strut
f_{cr}	cracking compressive stress of concrete
f_{cx}	stress in concrete in x direction
f_{cy}	stress in concrete in y direction

F_{nn}	Nominal strength of nodes
F_{nt}	Nominal strength of ties
f_p	peak compressive stress in concrete
f_s	Tensile strength of the ties
f_{scr}	stress in the steel in the cracked region
f_{vy}	yield strength of web reinforcement
f_y	yield strength reinforcement
γ_{xy}	shear strain
γ_{xys}	shear strain due to shear slip in y direction
Ly/Lx	Aspect ratio
ρ_s, ρ_x, ρ_y	tensile reinforcement ratio
ρ_{sk}	skin reinforcement ratio
$\sigma_{Rd,max}$	Compressive strength of strut
υ	factor for accounting cracking in strut
Wo	with web opening
α_i	the angle between the web reinforcement and the strut
β_d	factor for accounting compression softening in concrete
β_n, k_1, k_2, k_3	factor for different types of nodal zones
β_s	factor for different types of struts
ε_1	Principal tensile strain
ε_2	Principal compressive strain
ε_{cr}	cracking strain in concrete
ε_{cx}	stain concrete in x direction
ε_{cy}	strain in concrete in y direction

ε_p	peak compressive strain corresponding to peak compressive stress in concrete
ε_{sx}	stain in steel in x direction
ε_{sy}	stain in steel in y direction
ε_{xs}	strain due to shear slip in x direction
ε_y	yield strain of steel
ε_{ys}	strain due to shear slip in y direction
θ_{nx} ,θ_{ny}	angles between the normal to the cracks and the reinforcements
θ_ε	orientation of the principal strain plane
θ_σ	orientation of the principal stress plane

1 Introduction

1.1 Background

Reinforced concrete Deep beams can be found in different reinforced concrete structures; they have a wide variety application as a load bearing structural members. Deep beams usually occur as a transfer girder in buildings, directly transferring column axial load to other columns sharing the load. Pier caps are also deep beams that transfer the load from the bearings to the piers of the bridges. In addition to the above application, deep beams can also be seen as a pile cap transferring column loads to foundation piles. In general deep beams can be classified as beams that have a considerable depth compared to their span length or in other word beams that have very low span to depth ratio’s (Macgregor & Bartlett, 2000). However, there are no clear demarcation lines defining the zone of deep beams based on span to depth ratio’s, several design codes recommend different span to depth ratio’s for deep beams. ACI code considers beams with span to depth ratio less than 4 as deep beams, the Euro code 2 suggests a ratio of 3 and the Canadian code (CSA-A23.3-04) and Japanese code (JSCE guidelines for concrete) suggests limiting span to depth ratio of less than 2 for simply supported RC deep beams.

Deep beams differ from ordinary slender beams in the way they resist load when they are subjected to a loading. In deep beams subjected to a loading, a significant amount of load is transferred to the support by compression strut’s created between the load and support reaction While slender beams resist the applied load by a combination of flexural stresses and shear stresses (Macgregor & Bartlett, 2000).In recent design practices reinforced concrete beams and frames are generally analyzed based on the assumption that plane section remain plane before and after loading, Hence elastic linear analysis methods are adopted for determination of internal stresses and deformations (Kong, 2002). However, elementary Bernoulli beam theory are not applicable in case of deep beams even under linear elastic assumptions because neither the stress nor the strain distribution is linear when deep beams are subjected to loading. Hence, most codes suggest other types of analysis like the strut-and-tie method and nonlinear finite element method for the analysis of reinforced concrete deep beams.

In construction of modern buildings, many pipes and ducts are necessary to accommodate essential utilities like water supply, water drainage, air conditioning, electricity, computer network and telephone. Most of the time this pipes and ducts are placed underneath the soffit of the beams which leads to the creation of dead spaces in each floor and reducing the floor height of a building. An alternative way to avoid this problem is to allow these pipes and ducts to pass through transverse openings in the floor beams, this arrangement of service's leads to a higher floor height in a building, which creates an open space and comfortable environment (Mansur & Kiang-Hwee, 1999). Since, most design codes offer little or no guidelines regarding reinforced concrete deep beams with web openings. In general, most engineers tend to permit embedment of small pipes or ducts. However, when large openings are present, particularly in reinforced concrete members, they show a general reluctance to deal with such kinds of problems in a scientific manner and they make decisions based on their experience, which can lead to devastating effects, the extreme case may be failure.

Reinforced concrete deep beams with web openings have been the subject of many studies within the past few decades. Different experimental test has been conducted and their response have been studied on both slender and deep beams. While reviewing different researches based on this particular topic a major gap that was observed in both experimental and numerical investigations were , only a few parameters like opening shapes, opening size and opening locations with small number of test samples ware studied. But the behavior of reinforced concrete deep beams with web openings like the ultimate failure load, maximum deflection and the load that causes the first crack is affected by several additional parameters like compressive strength of the concrete, Span to depth ratio (L/H) of beams, type and position of the loading. For rectangular web openings, the effect of aspect ratio of the opening sides of the opening (Ly/Lx) is one important parameter that has not been studied in detail. Henceforth, most research show that opening within the shear zone that intercepts the natural load path between the applied load and center of support , has the most damaging effect in terms of strength reduction (Senthil, et al., 2018). However, these researches fail to investigate the effect of degree of interruption of the natural path by the openings on the overall strength reduction effect. Therefore, identifying the effects of such parameters on the overall behavior of RC deep beams with web openings is a daunting task that needs to be studied in detail.

On this research a numerical investigation of RC deep beams with rectangular web openings is conducted. The study parameters to be investigated are L/D ratios', concrete compressive strength, aspect ratio of openings sides, Degree of interruption, opening location and opening sizes. The finite element software package selected to conduct the research is VECTOR. Even though most scientific researchers agree on the fact that the most reliable way of conducting a research is through experimental investigation, several researches have shown that Finite element analysis using the appropriate models and by applying correct constitutive relation of materials and properly validating the numerical model with experimental test results, Numerical investigation can produce a quite reliable result as well (Demir, et al., 2016). FEM is capable of handling difficult analysis problems like modeling irregularly shaped geometry, modeling composite structures and considering nonlinearity behaviors (Logen, 2010).There are several finite element software packages that are capable of performing nonlinear analysis considering both material and geometric non-linearity. There are some advantages in performing a numerical investigation over experimental investigation like decreased cost for investigation, capability of performing test that are true in scale, decreased requirement of human resources and decreasing time consumption. Software’s like VETOR2D/3D, ABAQUS, ANSYS, ATENA, ADINA and DIANA are readily available for users especially since recently the ease and affordability of getting access to personal computers has grown immensely with in the last decade.

The governing mode of failure of deep beams with web openings is usually shear mode of failure characterized by diagonal cracking at failure therefore shear strength of such type of beams have been the main parameter investigated in the past researchers. Flexural failure rarely occurs in reinforced concrete deep beams with openings. This may occur if the beams are heavily reinforced with diagonal web reinforcement at the corner points of the opening and when the longitudinal reinforcement bars are very much below the balanced reinforcement ratio (Kong, 2002).the cracking load, the maximum crack width and deflections of the beam are also important parameters that are investigated in several researches because most design codes require that, at service load level, there should not be excessive cracking that affects the durability of the structures that leads to corrosion of reinforcement bars. Another SLS requirement of codes is preventing excessive deflection which can impair the appearance (Bhat, et al., 2014). The effect of the above parameters along with additional parameters is discussed it the next chapters.

1.2 Statement of the problem

Web openings in reinforced concrete beams are usually unavoidable in most building floor systems; these openings are most of the time used to pass utility lines such as drainage pipes, AC ducts, electrical cable ducts, telephone lines and computer network lines. The presences of these openings in general affect shear strength and flexural strength of reinforced concrete beams. In addition, depending on the size of the openings, deflections at service level load may increase significantly. The effect of openings may even be more dangerous in reinforced concrete deep beams, since deep beams are usually used as a transfer girder directly supporting the load of columns and as bridge pier caps supporting the load transferred from the bearings. Therefore, they are subjected to very large concentrated loads. However, different design codes provide little or no guidelines to deal with RC deep beams with web openings. For these reason most practicing civil engineers when faced with such types of beams, they tend to decide based on their past experience and not with scientific approaches.

There are several parameters that may affect the behavior of reinforced concrete deep beams with web openings like, opening shape, opening size, opening location, aspect ratio(Ly/Lx) of rectangular opening sizes, position of loading, compressive strength of concrete, span to depth ratio (L/H) of beams and degree of interruption of natural load path by the openings . Even though in the past few researches have been conducted on this particular area there are some major knowledge gaps noticed from reviewing this researches. Most researchers suggest that the most critical opening location is within the shear zone that interrupts the natural load path connecting the applied load the support reaction but fail to discuss on the effects of the degree of interruption on the overall behavior beams with web openings. In addition, for rectangular web openings the effect of the aspect ratios of the opening sides on the overall performance of such types of beams has not been investigated in detail. The effect of opening size has been investigated in a more or less detailed way but several researches suggest different demarcation lines on the division of small openings and large openings. Small openings sizes are defined by several researchers as opening that are small enough that the behavior of such beams are almost identical to solid beams without any web openings. Furthermore, the effect of the above mentioned parameters on the shear strength of beams, deflection, cracking load and crack width have not been studied in detail.

1.3 Research questions

The following research question regarding the statement of the problem can be formulated on what could be the effects of different parameters on cracking load, shear strength, crack width and mid-span deflection of RC deep beams containing rectangular web openings and subjected to a two-point monotonic loading. The following question are expected to be countered by this research:

- What is the effect of variable degree of interruptions of the natural load path connecting the applied load and support reaction?
- What is the effect of varied opening sizes within the shear zone and flexural zone?
- What is the effect of variable aspect ratio of openings sides with in the shear span on the beam?
- What is the effect of increasing concrete compressive strength in counteracting strength loss due to web openings?

1.4 Objective of the research

1.4.1 General objectives

The objective of this research is to determine the effect of the above listed parameters (i.e. opening location , opening size , aspect ratio of opening sides and compressive strength of concrete) on the behavior of reinforced concrete deep beams with rectangular web openings subjected to a two-point monotonic loading.

1.4.2 Specific objectives

The specific objective of the research is to determine the influence of different parameters on cracking load, shear strength, crack width and mid span deflection of RC deep beams with rectangular web openings subjected to a two-point static monotonic loading. The parameters are listed below: -

- Variation of Degree of interruption of the natural load path (i.e. $L_{ntercept}/l_{path}$) connecting the applied load and the support reaction for rectangular openings located in the shear span (i.e. the zone between the applied load and support reaction),
- Variable Opening sizes within the shear span and flexural zone,
- Variable Aspect ratios of opening sides (i.e. Ly/Lx),

- Different Concrete compressive strength,
- Variable Span to depth ratio of the beam (i.e. L/D)

1.5 Scope and limitations of the research

A numerical investigation using finite element software package is performed on simply supported reinforced concrete deep beams with web openings that are rectangular in shapes. The basic assumptions of the research are:

- Perfect bond (no slip) between reinforcement and concrete is assumed when modeling the beams.
- The longitudinal reinforcement is assumed to be tied effectively such that local and anchorage failures are neglected.
- The behavior of such types of beams under monotonic static loading is considered.
- Only Rectangular web (transverse) openings are considered.
- Immediate response of the beams subjected to static monotonic loading is studied, long terms effect such as creep are not considered.
- Test specimen of RC deep beams with rectangular web openings having an ordinary longitudinal and web reinforcement are considered (strengthened openings with diagonal reinforcement near the corner points of the openings are not considered).

Since circumstances involving web openings are versatile and depend on several factors like position of loading, continuity of boundary conditions and material property, Hence, the findings of these research cannot be directly applied to other type conditions with different assumptions.

1.6 Significance of the research

Most design codes offer little or no guidelines regarding the analysis and design of reinforced concrete deep beams with web openings. For these main reason a limited technical knowledge is available for practicing engineers when faced with such types beams and in return most decisions are made from site experiences and rule of thumbs. Even though a few researches have been conducted on RC deep beams with web openings, there are still important parameters that has not been investigated in detail. Therefore, this research attempts to fill the knowledge gaps observed from past studies. The outcome of this research will provide practicing engineers in the field with additional technical knowledge when dealing with RC deep beams containing rectangular web openings. This research will attempt to review and check the different demarcation lines on the division of small openings and large opening suggested by different researchers based on their effect in shear strength reduction. This research will also attempt to give a detailed insight on the effects of, degree of interruption of the natural load by the openings located in the shear span, aspect ratios of opening sides, span to depth ratios of the beam and compressive strength of the beams on the behavior of RC deep beams with rectangular web openings.

In general, the findings of this research will equip practicing and research engineers with additional knowledge on the effects of different parameters listed in the previous sections, on the behavior of RC deep beams with rectangular web openings. The advancement of knowledge in this particular area of interest will lead to a better understanding and decision making when dealing with such types of beams.

1.7 Outline and organization of the research

Chapter 1 gives the introduction inclining background, statement of the problem, research objective and significance. **Chapter 2** focuses on literature survey of past experimental and numerical investigation on similar to the research study. **Chapter 3** views the methodology for validation of FE model and modeling process of the test beams. **Chapter 4** shows and discusses the results obtained from the numerical analysis using VECTOR. **chapter 5** gives the conclusion and recommendation of the research.

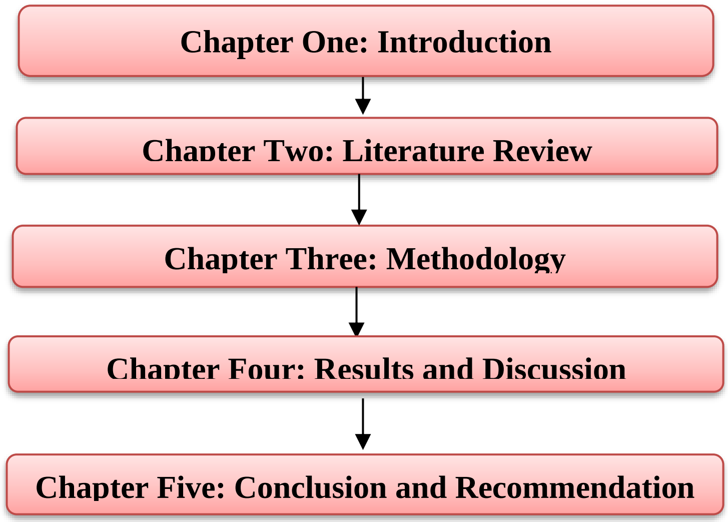


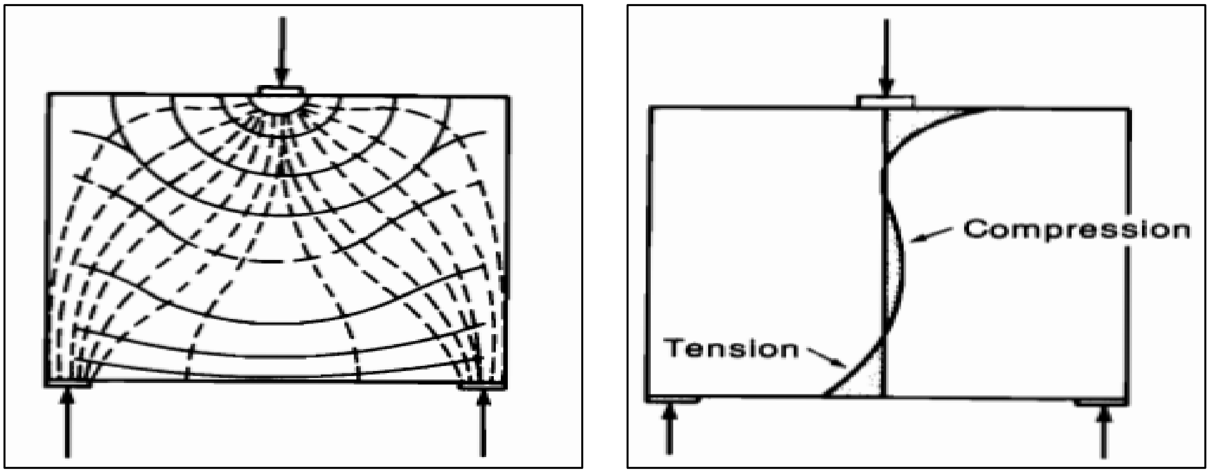
Figure 1-1 Organization of the thesis

2 Literature review

2.1 Behavior of reinforced concrete deep beams under loading

A deep beam can be defined as a beam having a depth comparable to its span length (Kong, 2002). It can also be defined as a beam in which significant amount of load is carried to the support by compression struts created between the applied load and the support reaction (Macgregor & Bartlett, 2000). Deep beams differ from slender beams by the way they resist and respond when subjected to a loading. Slender beams mainly resist load by combination of flexural stresses formed in the longitudinal direction and shear stresses near support while deep beams resist loading by compression strut formed at some angle with the horizontal and tension tie holding the compression struts together. Reinforced concrete deep beams have wide real world application in Tall buildings, offshore structures and foundations (Kong, 2002).Deep beams are usually used as a transfer girder’s in tall buildings and as a pier cap in bridges supporting the loads transferred from bearings. Deep beams are also used as a pile cap to distribute the loads of the superstructure to the respective piles.

St, Venant’s principle states that the localized effect of disturbance induced as a result of concentrated load, openings, or change in geometry of members etc dies out at approximately one-member depth. A deep beam can be considered as a beam which its entire section is under disturbed state. Elastic analysis of deep beams using elastic finite element analysis to classical theory can be applied to determine the stresses in concrete; however, in reinforced concrete structures at approximately one third of the ultimate load cracking will occur (Nelison, et al., 2010). Once cracking has occurred there is a significant redistribution of internal forces and the results of elastic analysis are no longer valid but the results are of interest because they show the distribution of stresses which cause cracking and hence give guidance to the flow of forces after cracking (Macgregor & Bartlett, 2000). The stress trajectories of a simply supported deep beam subjected to a concentrated loading can be seen from figure 2-1. it can be seen that the dashed lines showing the principal compressive stresses connect the applied load and the support reaction and the principal tensile stresses radiate from the applied load and the support reactions. It is possible to represent the internal forces in such type of beams using a statically determinate truss (Nelison, et al., 2010), this type of analysis is referred to as the strut-and-tie method and it is suggested by several codes as a tool for analysis RC deep beams.



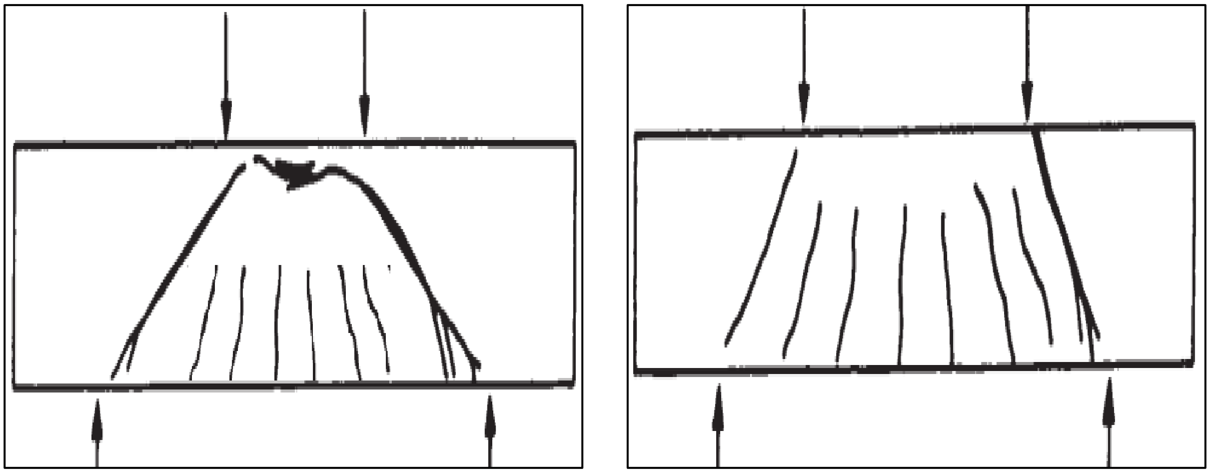
(a) (b)

Figure 2-1(a) Stress trajectories (b) Distribution of elastic stresses at the mid span

In general, the behavior of RC deep beams subjected to a loading needs to be seen separately for beams containing web reinforcement and beams without web reinforcement. The main sources of shear strength are the portion of the concrete below the neutral axis, in the absence of web or shear reinforcement, aggregate interlock and dowel action of the longitudinal reinforcement contribute to the overall shear resistance.

2.1.1 Deep beams without web reinforcements

In general, two types of failures can be observed for reinforced concrete deep beams without web reinforcement and subjected to a two-point loading. For beams with shear span to depth ratios (a/d) between 1.5 and 2. Such type of beams, it can be seen from figure 2-2a that the failure can be characterized by a deep inclined crack which appears to have formed within the shear span independent of the flexural cracks (Kong, 2002).The inclined crack starts at the bottom face of the beams close to the support and extends towards the top face of the beam at the zone of the load point ,eventually causing failure of the compression zone in the middle zone of .Figure 2-2a also indicates that the inclined crack penetrates the compressive zone in the middle span significantly deeper than any flexural crack that develop at the mid span of the beam. Compressive forces developed in the horizontal and inclined portions of the path but for the failure type indicated in figure 2.2a the most critical position that is highly likely to fail first is the horizontal compression strut (Kong, 2002).Failure often occurs in such regions under the combined action of compressive and tensile stresses developed when the structure is loaded.



(a) (b)

Figure 2-2 Type of failure for deep beams with a/d ratio of (a) between 1.5 and 2 (b) 1 *(Kong, 2002)*

The second mode of failure of RC deep beams shown in figure 2-2b usually occurs on beams having shear span to depth ratios of 1 or less. The inclined crack the characterizes such types of failure almost coincides with the line joining the load point and support. The crack usually initiates within the beam web located halfway between the load point and the support at a load level that is much lower than the beam load carrying capacity and the crack propagates simultaneously towards the loading and support point (Kong, 2002).Finally the collapse of the beam occurs as a result of sudden extension of the inclined cracks towards the top and bottom faces of the beam in the regions of the load point and support, respectively within the shear span. This type of failure is commonly known as Diagonal-Splitting failure.

The shape of stress trajectories of deep beams subjected to a two-point loading can be seen in figure 2-3 below. It indicates that they form a barrel shaped region with its larger cross-section located halfway between the load point and the support and tensile stresses exist at right angles of these compressive stress trajectories. Comparing the above shown mode of failure the failure occurs at the inclined compression struts for the second mode of failure while failure occurs on the horizontal compression strut for the second mode of failure. On the other hand, the mode of failure shown in figure 2-2a cannot be categorized as shear failure for certain but the second mode of failure shown in figure 2-2b is a classical diagonal-tension shear failure.

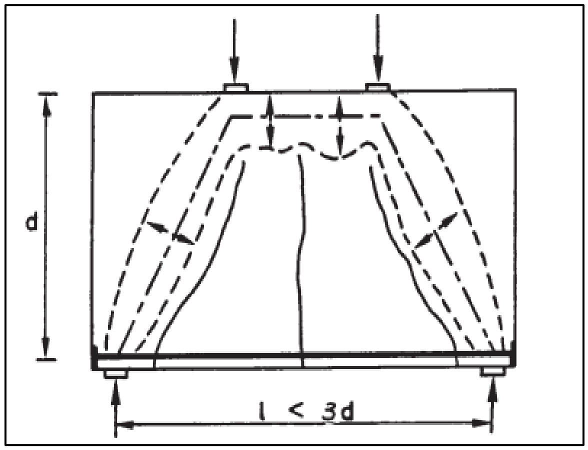


Figure 2-3 Shape of Compressive Stress trajectories for Deep beams subjected to a two point loading

2.1.2 deep beams with web reinforcement

As discussed in the previous section, the failure type shown in figure 2-2a, the failure is associated with large reduction of the size of the compression zone of the cross-section coinciding with the tip of the main inclined crack. Such a reduction in size will lead to the development of tensile stresses within the compressive zone. Hence failure will when the strength of concrete under combined actions of compressive and tensile stresses exceed (Kong, 2002).for beams with web reinforcements the extension of the inclined crack to the compression zone will be prevented by the transverse reinforcement. However, the amount of reinforcement required to sustain the tensile stresses is difficult to determine because the tensile stresses are difficult to calculate. Some researchers suggest that the presence of transverse reinforcement only within the shear span is enough, however the presence of transverse reinforcement beyond the critical section is essential because the inclined crack is most likely to extend into the compression zone, the section through the tip of the inclined crack has the smallest compression zone and at that location concrete will reach the minimum volume.

For deep beams having a type of mode of failure similar to the one shown in figure 2-2b, the presence of the conventional vertical and horizontal delays the cracking process but only gives a small increase in load carrying capacity of the beam. However, the presence of web reinforcement prevents any instability failures caused due to out of plane actions related to heterogeneous nature of concert (Kong, 2002).

2.2 Codes and standard guidelines for RC deep beams

Several design codes have incorporated provisions for analysis and design of solid reinforced concrete deep beams. In general, most design codes agree on the fact that conventional elastic beam theories do not apply for such kinds of beams. Stress and strain distribution are not linear in Deep beams subjected to a loading. However, they recommend different limits of span to depth ratio for classification of Deep beams from slender beams. Most Design codes suggest the strut-and-tie method for analysis and design of deep beams at the ultimate limit state, however they do not provide a detailed and separate methods of analysis to determine the response of such type of beams at service load level. A detailed literature survey of five different design codes is presented in the following sections below.

2.2.1 ACI code

ACI 318-11 design code defines deep beams as beams having the ratios of clear span(Ln) to the overall depth (D) less than 4 or distance of less than 2h respectively for simply supported and regions subjected to concentrated loads. The code suggest that deep beams shall be designed either by accounting a non-linear distribution of strains or by strut-and –tie method. The general provisions and recommendations of the code are listed as follows

2.2.1.1 Provisions for crack control

11.7.2, Deep beams shall be proportioned such that the maximum shear force Vu is less than or equal to

$$\Phi 0.63\sqrt{fc'}bwd \qquad (2.1)$$

Where Φ resistance factor of material taken as 0.9 for concrete, fc' is characteristic cylinder compressive strength of concrete, bw is width of the web and d is effective depth of the beam.

For the purpose of crack control, the total distributed reinforcement along the two side faces of the beam shall not be less than

- The area of shear reinforcement perpendicular to the longitudinal axis of the beam, A_v shall not be less than $0.0025bwS_2$, where S_2 is the lesser of d/5 and 300mm.
- The area of shear reinforcement parallel to the longitudinal axis of the beam, $A_{vh,}$ shall not be less than $0.0025bwS_2$, where S_2 it the lesser of d/5 and 300mm.

2.2.1.2 Procedure for strut-and- tie modeling

The strut-and-tie model consists of three members, the resultant of a fan shaped compression field known as strut, a tension member which represents the longitudinal reinforcement known as a tie and the point in the STM mode where the axis of the struts and ties meet know as a node. The general rules for strut and tie modeling are listed below

- Design of structural concrete members or D-regions should be done by modeling the members or regions as an idealized truss and the truss model shall contain strut, ties and nodes.
- The strut-and-tie model shall be in equilibrium with the applied loads and the reactions
- In determining the geometry of the truss, the dimensions of the struts, ties and nodal zones shall be taken in to account.
- Ties are allowed to cross the struts but struts shall cross or overlap at nodes only
- The angle θ between the axis of any strut and any tie entering in to a single node shall not be less than 25 degrees.
- Design of struts, ties and nodal zones shall be based on:

  $\Phi Fn \geq Fu$ (2.2)

  Where Fu is the factors force acting in the strut, ties and nodes zones; Fn is the nominal strength of the strut, ties and nodal zone.

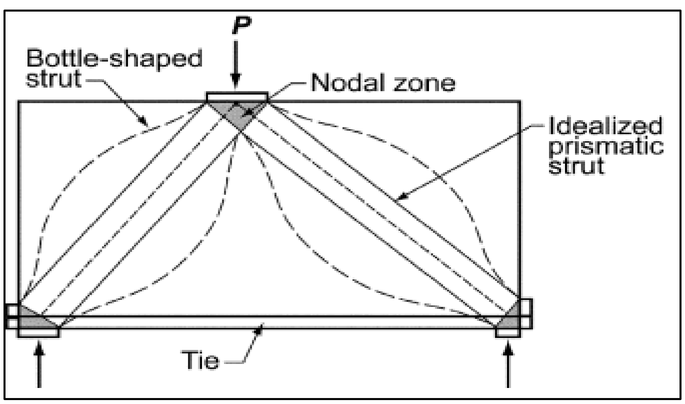


Figure 2-4 Description of Strut-and-Tie model

2.2.1.3 Strength of the STM components

a. Struts

The nominal strength of a strut without longitudinal reinforcement, Fns shall be taken as the smaller value of

$F_{ns} = f_{ce} A_{cs}$ (2.3)

(a) The effective compressive strength of the concrete in the strut given in equation 2.4

(b) The effective compressive strength of the concert in the nodal zone given in equation 2.

The effective compressive strength of the concrete, fce, in the strut is given as

$f_{ce}=0.85\beta s\ fc'$ (2.4)

the value of βs for different types of struts is given in table 2.1 below

Table 2-1 Values of βs

Type of strut	βs
Strut of uniform cross-section	1.0
For strut having width at midsection greater than width at the nodes (a) With reinforcement (b) Without reinforcement	0.75 0.60λ,where λ 1.0 for normal weight concrete
For struts in tension member	0.4
For all other cases	0.6λ

For struts having variable width at the midsection and at the nodes that are crossed by transverse and longitudinal web reinforcement, in order to apply a value βs given in table 2-1, they must fulfill the following requirement:

- For fc' not greater than 40Mpa

$$\sum \frac{A_{si}}{b_s S_i} \sin a_i \geq 0.003 \quad (2.5)$$

Where A_{si} is the total area of surface reinforcement at spacing Si in the ith-layer of reinforcement crossing a strut at angle α_i to the axis of the strut.

- The reinforcement required shall be placed in either two orthogonal directions at angles α_1 and α_2 to the axis of the strut or in one direction at angles of α to the axis of the strut

If compression reinforcement is used parallel to the axis of the strut and enclosed on ties or spirals, for such cases the nominal strength of longitudinally reinforced strut is

$$Fns = fceA_{cs} + A_s'fs' \quad (2.6)$$

Where A_s' and fs' are the area of compression reinforcement and compression stress in the strut respectively.

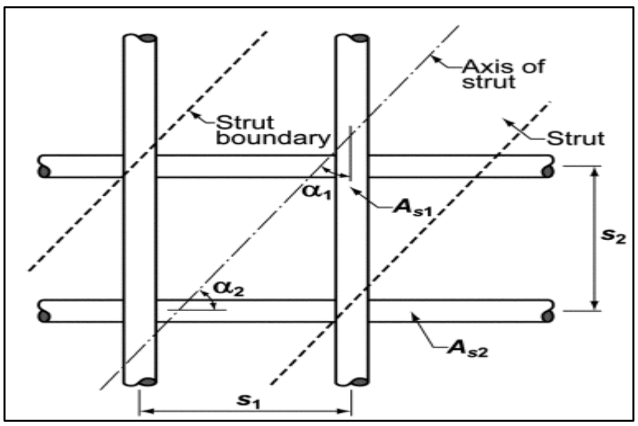


Figure 2-5 Reinforcement's crossing the strut

b. Ties

The nominal strength of ties ties , Fnt is taken as

$$F_{nt}=A_{ts}\, f_y \quad (2.7)$$

Where Ats is the area of longitudinal tension reinforcement and fy is the yield strength

In order to prevent bond slip between the reinforcement and the concrete, Tie reinforcement shall be anchored by a mechanical reinforcement device, post-tensioning anchorage devices, standard hooks or straight bar development as required by the code.

c. Nodal zones

The nominal compression strength of the nodal zones is

$$F_{nn} = f_{ce}\, A_{nz} \quad (2.8)$$

Where A_{nz} is the smaller of

- The area of the face of the nodal zone on which Fu acts, perpendicular of the line of action of Fu

- The area of a section through the nodal zone taken perpendicular to the line of action of the resultant force on the section

The effective compressive stress, force, on the face of a nodal zone to the strut-and-tie forces is given as

$f_{ce}=0.85\beta n\ f_c'$ (2.9)

the values of βn are given in table 2-2 below

Table 2-2 Values of βn

Type of nodal zone	βn
Nodal zones bounded by struts or bearing areas, or both	1.0
Nodal zones anchoring one tie	0.8
Nodal zones anchoring two or more ties	0.6

2.2.2 Euro-code

EN 1992 1-1 defines a deep beam as a member which a span less than 3 times the overall section depth. For controlling of crack width and propagation the code suggests the following provisions

- Deep beams should be provided with an orthogonal reinforcement mesh near each face of the beam with minimum of $A_{s,min}$ given as

$$\text{maxof}\begin{Bmatrix} 0.001Ac \\ 150mm^2/m \end{Bmatrix} \quad (2.10)$$

- The distance between two adjacent bars should not exceed the lesser of twice the deep beam thickness or 300mm
- The reinforcements corresponding to the ties should fully be anchored by bending the bars, by using U-hoops or by anchorage device, unless sufficient length is available between the node and the end of the beam permitting an anchorage length of l_{bd}

2.2.3 Canadian standard

The Canadian standard A23.3-04 defines deep beams as flexural members with span to depth ratio of less than 2.the standard recommends usage of minimum horizontal and vertical reinforcement known as skin reinforcement for beams having a depth greater than 750mm.The total are of reinforcement is given as

$$\rho_{sk} A_{cs} \tag{2.11}$$

where ρ_{sk} is 0.008 and 0.01 for interior and exterior exposure respectively

the standard also suggests, the maximum spacing between the skin reinforcements should not exceed 200mm and such skin reinforcement may be included in strength calculation if strain compatibility analysis is used to determine the stresses in the individual bars.

2.2.4 Other codes and standards

JSCE15 (Japan society of civil engineers) standard specification for concrete classifies deep beams as those beams having span length to the overall depth ratio of less than 2, less than 2.5 and less than 3, for simply supported beams, continuous beams with two spans and continuous beams with three or more span respectively. The standard allows the analysis and the design of such types of by applying the classical beam theory principles like assuming linear strain distribution. The standard only recommends the usage of the strut and tie method for discontinuity regions such as abrupt cross-sectional changes regions, corners and opening zones. The provisions suggested by the standard for design of deep beams are listed below:

- Since the behavior of deep beams are similar to those of tied arches in a way that axial tension reinforcement operates in a same manner as tension tie members in tied arches, therefore in the examination of flexure, axial tension in reinforcement using the maximum flexural moment obtained by ordinary beam theory by the stress-strain relationship shown in figure 2-15
- Shear should, in general, be checked for ULS where design shear capacity obtained from the contribution of concrete and shear reinforcement

For analysis of deep beams, the CEB-FIP model code 1990 recommends using either Linear elastic analysis, analysis by statically admissible stress fields consisting of struts-and-ties or non-linear analysis considering geometric imperfections, cracks and settlements of supporting members at boundaries. CEB-FIP code however does not suggest any limits for deep beams like the pervious codes and standards discussed above. If strut-and-tie method is used for design of deep beams, CEB-FIP code gives the following provisions:

- The tensile resistance of concrete should not be relied upon in analysis

- For struts with concentered nodes at both ends the transverse reinforcement between the nodes may be designed for 40% of the strut force
- The strut and tie should be balanced in the node region and that strength criteria for nodes and anchorage should be fulfilled
- The struts should be ordinated in alignment with compressive stress-trajectories and should be placed in the respective center of gravity of these stresses
- Ties should also be oriented by following the principle tensile stress- trajectories however their orientation is also influenced by the practicality of the reinforcement layout

2.3 Behavior of reinforced concrete deep beams with web openings

2.3.1 Effect of web openings

Opening in reinforced concrete beams are unavoidable for reasons discussed in the above sections. openings, depending on the effect they in the general behavior of the beam, they can be classified as small opening or large openings. Openings are considered to be small when the beam type behavior is maintained, in other words, if the beam behaves almost exactly the same as the solid beam without any openings in terms of strength, deflection and crack formation. Some researches show that opening depths less than 40% of the overall depth of the beam can be considered as small (Mansur & Kiang-Hwee, 1999).The provisions of openings however ,produces discontinuities and disturbances in normal distributions of stresses, thus leading to stress concentrations and early cracking around the opening regions. therefore, special types of reinforcement and strengthening methods should be employed to contain the width and propagations of cracks to a tolerable limit and premature failure of the beam (Mansur & Kiang-Hwee, 1999).

Consider a beam shown in figure 2-6, the openings are placed below the neutral axis around the mid span of the beam. It is clear to see that the provisions of opening will not alter the load carrying capacity of the beam because concrete is assumed to be cracked anyway in flexure and the tensile resistance of concrete below the NA is neglected, although some tensile resistance of concrete near the longitudinal bar may be present because of tension-stiffening effect. But in general the ultimate strength of the beam will not be affected by the presence of the opening. Now, consider opening the intercepts the failure plane as shown in figure 2-7 below, regardless of whether the opening is located below the NA or above there will be a considerable decrease in the shear strength of the beam. Since shear is resisted by a combination shear in the compression zone, shear transferred in the inclined crack region by aggregate interlock, by dowel action of the longitudinal reinforcement and number shear reinforcement intercepting the crack as shown in figure 2-9 below. Therefore, the presence of any opening will decrease the contribution of the shear transferred by aggregate interlock and in addition, it will decrease the number of shear reinforcements intercepting the crack, thus reducing the overall shear strength of the beam. The conventional shear resistance capacity form was adjusted to take account openings and was proposed by (Mansur, 1998).It may be observed that the stirrups available to resists shear across the failure plane are those by the sides of openings within a distance (dv-d),where dv is the distance between the top and bottom longitudinal bars, in a similar manner the net depth for calculating the shear resistance capacity of the concrete alone is replaced by (d-d0).the modified concrete shear resistance (Vc) and shear resistance provided by the web reinforcements (Vs) are given as:

$$Vc = \frac{1}{6}\sqrt{fc'}bw(d - d_0) \tag{2.12}$$

$$Vc = \frac{Avfyv}{s}(d - d_v) \tag{2.13}$$

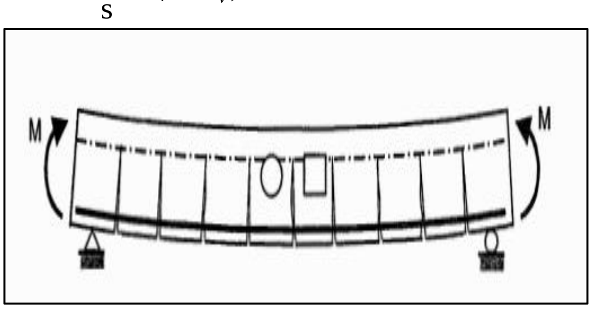


Figure 2-6 beams with web openings below NA

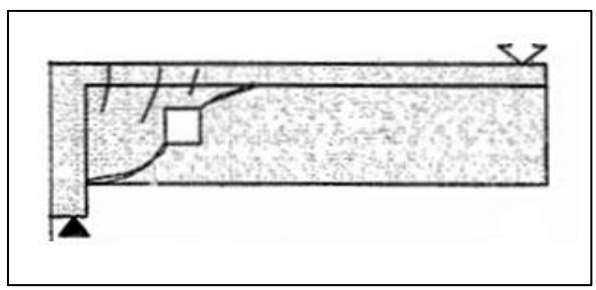


Figure 2-7 beams with openings near support

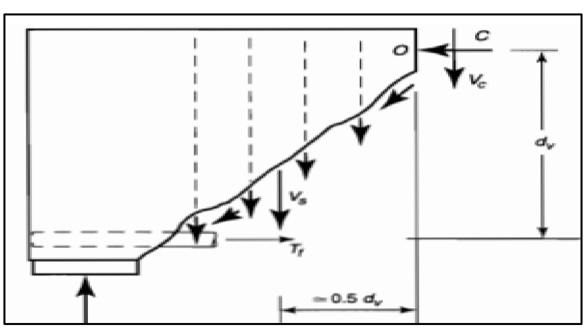


Figure 2-8 Shear resistance of solid RC section

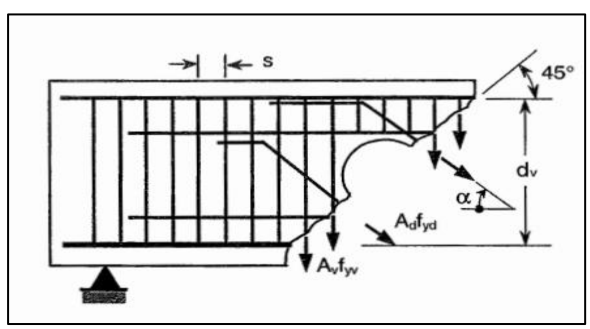


Figure 2-9 Shear resistance of RC section with. Opening

The above formulas given in equation 2.21 and 2.22 are only applicable for slender beams ,In reinforced concrete deep beams classical elastic theory is not applicable because stress pattern in non-linear and strain distribution is also non-linear (Kong, 2002).Some building codes eventually incorporated some provisions regarding analysis and design of RC deep beams, however researches on deep beams with web openings are still limited and most national codes do not provide any guidelines for such type of beams. There are several parameters that affect the; behavior of deep beams with web openings such as:

- Span to depth ratio
- Geometric and cross-sectional and material properties
- Amount and location of longitudinal reinforcement s
- Type and location of reinforcements
- Shear span to depth ratio
- Type and position of loading
- Size shape and location of loading

In general, the strain distribution of deep beams at the mid span before cracking, shows non-linear strain distributions and more than one neutral axis. The number of neutral axis decreases as the load increases and finally at the ultimate loading stage only one neural axis is present. Generally, in deep beams having rectangular web openings subjected to a loading, the first crack usually appears in the support bearing regions and opening corners at load levels of 36-55% of the ultimate load, this initial cracks tend to propagate in there forward diagonal direction slowly. For loading ranges about 50-97% of the ultimate failure load ,typical diagonal cracks longer than the initial ones suddenly emerge within in the shear zone above and below the openings, these critical diagonal cracks instantaneously propagate both ways towards the bearing regions and opening corner’s and eventually the presence of such type of cracks will lead to the failure of the structure (Kong, 2002).

The maximum crack width at failure is observed to be greater when the opening center is located at the center of the shear zone than any other position. The strength of the beam will increase when openings are located away from what is known as the loaded quadrant shown in figure 2-10 below. Again for openings located completely outside the shear region ,the beam with web opening may be assumed to be a solid beam without any web opening (Kong, 2002).The load-deflection characteristic is one important tool to determine the behavior of beams with web openings, it can be observed form different experimental investigation that the flexibility of the beam decreases as the location the opening is moved away from the support to the interior of the beam. On the other hand, the observed flexural cracks are found to be very few and generally occur in the range of loads about 60-95% of the ultimate load (Kong, 2002).

2.3.2 Effects of web and main longitudinal reinforcements

The main longitudinal bar not only acts as a tension reinforcement in flexure but contributes in a great deal for shear strength of the beam by dowel action and the web reinforcement controls crack width and deflection. The first cracking is generally not influenced by provision of web reinforcements. The inclined type of web reinforcement placed perpendicular to the plane of rapture has been found to be the most effective arrangement for resistance against sliding. (Yang & Chung, 2007). However, the more practical effective web reinforcement is a horizontal web reinforcement with nominal vertical web steel may further increases the strength of the beam.

In general failure will be gradual and slow in beams containing web reinforcement, while it is sudden and brittle in beams without web reinforcement. A vertical web reinforcement placed near the vertical edge of a beam with web opening located in its neighborhood, guards against premature failure due to rotation of corner beam (Kong, 2002). In addition, it was observed that after cracking of the beam the steel strain rapidly increased at the location near the supports and the steel strain the flexural zone remained almost constant (i.e. tension was uniform)

2.3.3 Empirical Analysis and design equation

The empirical equations presented by Kong 2002 were developed from surveillance of the test results. Several hundreds of test specimens with different opening sizes, opening locations and loadings were tested (Kong, 2002).From this finite number of tests, the observed behavior was used to develop design requirements and recommendations. Since the empirical equations are based on a finite number of test results, it can be applicable only when the following conditions are satisfied

- The effect of openings laying inside the region EFGH in webs of deep beams as shown in figure 2-10 will only be considered,

- The size of the opening is limited by a1X ≤ $X_{N}/2$ and a2D ≤ 0.3D, as shown in figure 2-17
- The eccentricities ex and ey of the opening are limited to the maximum of $X_N/4$ and 0.6D/4 in the x and y directions respectively as shown in figure 2-11

Incorporation of the simplified measurements of opening parameters in strength equation for solid deep beams will give the ultimate strength equation for deep beam with web opening fulfilling the above stated requirements.

2.3.3.1 Web openings parameters

a. The typical mode of failure in deep beams is the diagonal mode of failure, however there are other slightly different forms of failure termed as shear-proper, shear flexure and shear compression. This variation of modes of failure are mainly related to the shear span to depth ratio, which can be taken in to consideration by introducing a parameter $\lambda 1$

$$\lambda_1 = \left[1 - \frac{1}{3}\left[\frac{K_1 X_N}{K_2 D}\right]\right] \qquad \text{for} \qquad \frac{K_1 X_N}{K_2 D} \le 1 \tag{2.14}$$

$$\lambda_1 = \frac{2}{3} \qquad \text{for} \qquad \frac{K_1 X_N}{K_2 D} \ge 1 \tag{2.15}$$

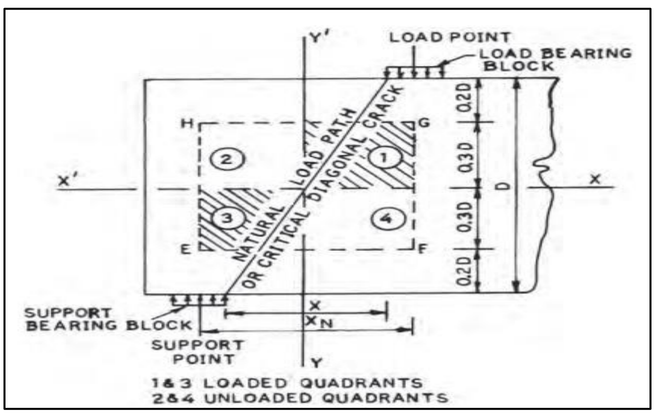


Figure 2-10 practical region for web opening

b. If the opening intercepts the natural load path, the effect of this discrete load path is taken in to account by a parameter is taken account by parameter $\lambda 2$

$$\lambda 2 = 1 - m \tag{2.16}$$

Where m is the ratio of path length intercept to the total path intercept length along the natural load path

c. The combined effect of the size of the opening as well as location of the opening is accounted by incorporating a constant $\lambda 3$

$$\lambda_3 = \left[0.85 \pm 0.3\left(\frac{e_x}{X_{net}}\right)\right]\left[0.85 \pm 0.3\left(\frac{e_y}{Y_{net}}\right)\right] \quad (2.17)$$

Where $e_x \leq X_{N/4}$, $e_y \leq 0.6D/4$,X_{net}= (X_N-a1x) and Y_{net}=(0.6D-a2D)

$\lambda 3$ takes values between 0.5 and 1 depending whether the opening location is in the loaded and unloaded quadrant

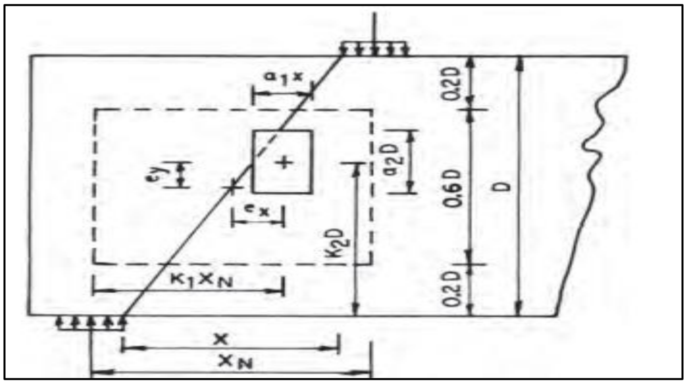


Figure 2-11 typical position of opening intercepting the load path

For opening located partially outside the shear zone, parameters e_x, e_y, K_1 X_N and K_2D may be measured as indicated below

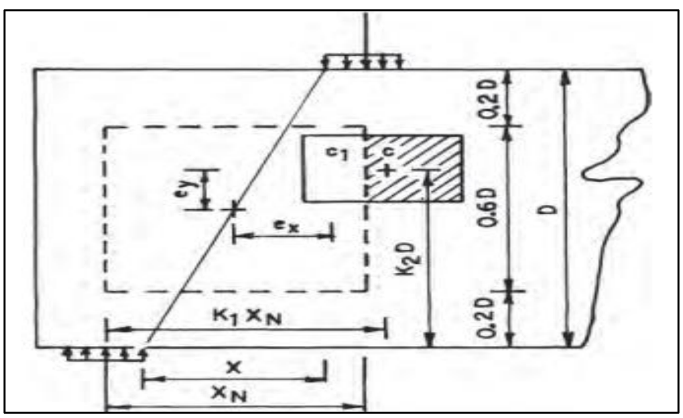


Figure 2-12 typical position of opening partially outside shear zone

For calculating λ_3 ,for openings that lie outside the shear zone as shown in figure 2-12 by the hatched line ,it should be ignored. consequently, the center c1 of openings is to be determined for the remaining part which lies within the domain of shear zone. In addition, if the opening location is outside the shear zone the beam should be treated as solid beam with web opening.

2.3.3.2 Ultimate shear strength

After evaluating the parameters $\lambda 1, \lambda 2$ and $\lambda 3$,the general shear strength of the deep beams with web openings can be calculated by summing up the contributions of concrete(i.e. Pc), longitudinal reinforcement(i.e. Ps) and web reinforcement (i.e. Pw) as shown in the equations below

$$P_u = \frac{Q_u}{2} = P_c\,(\lambda_1).(\lambda_2).(\lambda_3) + \Psi_s\, P_s + \Psi_w\, P_w \qquad (2.18)$$

$$P_c = \frac{cbD}{\sin\beta\cos\beta(\tan\beta + \tan\Phi)} \qquad (2.19)$$

$$P_s = F_s\left[\frac{\tan\beta\tan\Phi - 1}{\tan\beta + \tan\Phi}\right] \qquad (2.20)$$

$$P_w = F_w\left[\frac{\sin\alpha\cos\beta + \cos\alpha}{\left[\dfrac{\tan\beta + \tan\Phi}{\tan\beta\tan\Phi}\right]} - \frac{\cos\alpha}{\left[\dfrac{\tan\beta + \tan\Phi}{1 - \tan\alpha\tan\beta}\right]}\right] \qquad (2.21)$$

Where $F_s = A_s f_{sy}$, $F_w = \sum_i^n A_{wi}.f_{wyi}$, F_{sy} is the yield point stress of tensile, F_{wy} is yield point stress for inclined web steel intercepted by the potential diagonal crack at failure. β it the angle of inclination of the rapture plane with the horizontal , α is the angle of inclination of the inclined web bar with the horizontal ,n is the number of web bars intercepted by the potential diagonal crack. Ψs is 0.65 and Ψw is 0.5.

2.3.3.3 Ultimate flexural strength

The predominate mode of failure of deep beams with web opening is usually shear by diagonal tensile cracking or diagonal compression failure, however flexural failure in such beams may also occur when excessive web reinforcements are provided perpendicular to the plane of rapture (Kong, 2002). The failure mode of the beams may be known from the comparative values of the ultimate load values obtained both for shear and flexure Assuming only one neutral axis exists at failure and a triangular stress distribution exists for the concrete portion in the tensile zone, the following simplified equation was proposed for determination of flexural strength of deep beam (Kong, 2002).

$$\frac{Mfl}{bd^2} = \frac{\rho_s.fsy}{bd^2}(0.86) + \frac{\rho_{wt}\ .f_{wy}.\cos\alpha}{bd^2}(0.52) + 0.033 \qquad (2.22)$$

$\rho s = \frac{A_s}{bd}$ and $\rho s = \sum_i n\frac{A_{wi}}{bd}$ where A_s is area of longitudinal tensile reinforcement and Aw is area of web reinforcement

2.4 Studies on RC deep beams with web openings

2.4.1 Experimental investigations

In the past few years several experimental investigations have been conducted on reinforced concrete deep beams with different type of opening shapes and locations. The experiments involve loading such types of beams to failure and observing the different parameters like load-deflection curve, ultimate failure load and the cracking pattern. When openings are present in reinforced concrete continuous deep beams, reduction of stiffness occurs and it causes a redistribution of internal forces and moments (Mansur, et al., 1991).Tests results conducted on continuous RC deep beams having two and three spans and where the openings are located within the positive (sagging) moment regions and negative moment regions shows that, the final failure load occurs by a formation of plastic hinges at the two opening ends and even though the failure load decreases as the opening location increases there is no change on the failure mode (Mansur, et al., 1991).In addition it was observed for the test result that the location of opening has a very little effect on the load that causes the first crack (Mansur, et al., 1991).

Another important parameter that has been a subject of interest of several researcher's is opening shape. Most common shapes that are present as a web opening are circular, rectangular and square opening shapes. Circular openings usually occur when it is required to pass PVC pipes through while rectangular and square opening usually occur when electrical cable duct pipes and AC duct pipes are required to pass through the reinforced concrete structures. On the research conducted by (AL-SHEIKH & Ahmed, 2014) reinforced beams having circular, square and rectangle opening shapes located at the shear and flexural zone were subjected to a two point loading. The results showed that circular opening showed the least reduction in ultimate strength and the maximum reduction in ultimate strength in RC beams with small openings located in the flexural region were 1.5% while for openings located in the shear zone, a reduction of ultimate strength of beam as high as 10% was observed. Circular openings in general can effectively transmit the load in an arch manner and the diagonal crack observed at failure is well defined compared to other opening shapes (Kong, 2002). In a similar research conducted by (Nair & P.E, 2015) deep beams with circular and rectangular opening shapes with variable location were tested under one point loading and results shows that for the circular opening having a diameter of 100mm and square opening of having sides 100mm ,the ultimate failure load reduction observed was 39% and 60% respectively. Whenever there are rectangular or square web openings present in RC beams subjected to a loading, high stress concentration exists at the corner points especially when the openings intercept the natural load path connecting the applied load and the support reaction thus leading to a higher capacity reduction compared to circular openings.

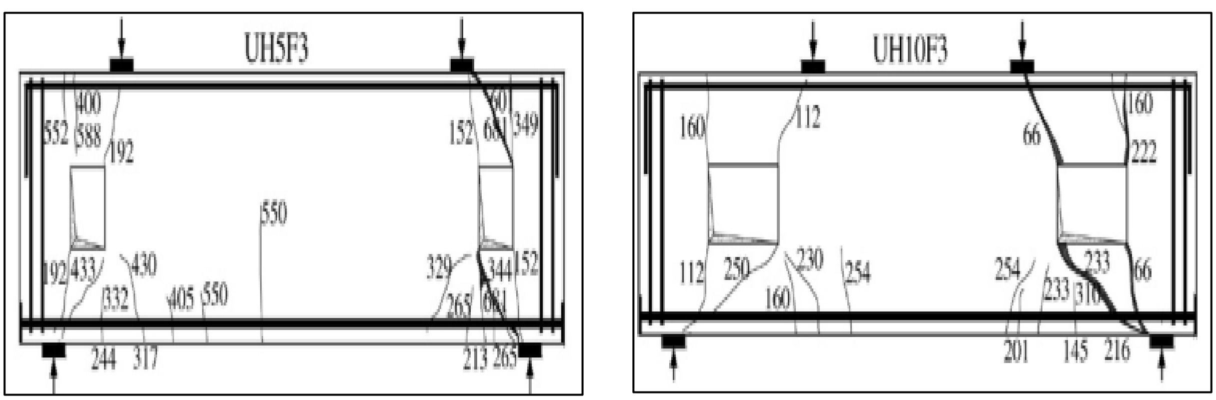
Mode of failure of reinforced concrete deep beams with and without web opening is another important parameter that has been a subject of different researches. Mode of failure of reinforced concrete deep beams is of a great interest to the designer because reinforced concrete structures are designed in such a way that ductile failure is assured to give occupants sufficient warning to vacate the area and to go to safety. Usually flexural failure is ductile while shear failure by diagonal cracking is sudden and brittle failure. Over twenty tests on reinforced concrete deep beams with or without circular opening subjected to a four-point loading where conducted by (Campione & Minafò, 2012). The test results obtained from the experimental investigations showed that the failure mode and the cracking load are mainly affected by the presence and the location of the holes rather than the amount of web reinforcement or section depth. When opening locations are present within the shear span a reduction of ultimate load carrying capacity of 18-30% were observed , openings located within the shear zone reduce the ultimate strength of the beam rather than openings located within the mid-span (Campione & Minafò, 2012). In addition, it was observed that the presence of vertical stirrups increased the load carrying capacity of the beam by 15%.

(a) (b)

Figure 2-13 (a) Specimen without opening (b) specimen with openings with in the shear zone

The mode of failure of reinforced concrete deep beams with circular opening located with in the shear zone can be observed from figure 2-13(b). It can be seen that at failure load the crack propagates from the loading point to the top of the opening and from the support bearing to bottom of the circular opening separately. This type of failure is termed as shear failure by diagonal cracking.

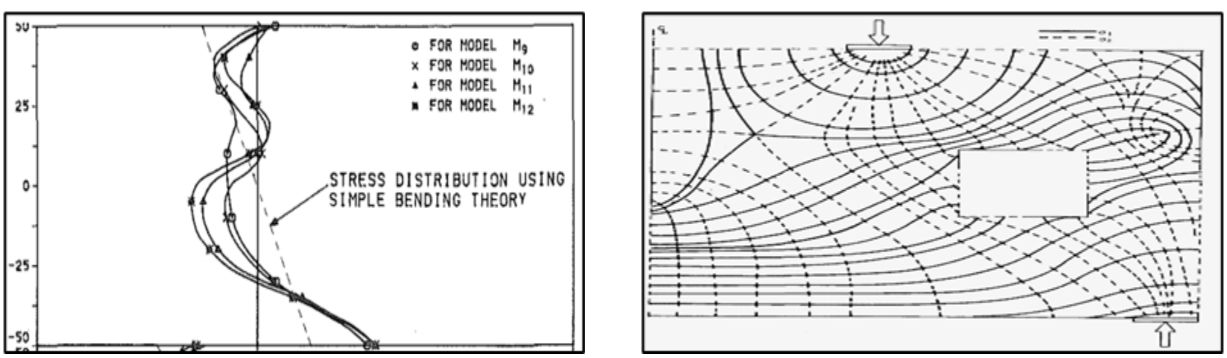
There are several solutions that can be applied to increase the strength of reinforced concrete deep beams with web opening. Currently fiber polymers like carbon and glass are being used to strengthen the web opening so that significant strength reduction is avoided. However, the above methods of strengthening are expensive and are not generally readily available, instead engineer tend to use other economical methods of strengthening like using a higher compressive strength concrete. Nonetheless such solutions are not provided scientifically backed but rather are given as a rule of thumb. (Yang, et al., 2006) attempted to investigate the effect of concrete compressive strength of reinforced concrete deep beams and results from the research showed that effect of concrete strength significantly decreased in concrete deep beams with web opening compared to solid RC deep beams, however the maximum crack width at service level loads decreased with increase in concrete strength in deep beams with web openings (Yang, et al., 2006). Another major parameter that has not been studied in detail is the aspect ratios of opening sides. no clear study has been conducted regarding this particular parameter but some researchers suggest that narrow rectangular web openings are preferred from square and circular openings to minimize strength loss (S & Subha, 2016).



(a) (b)

Figure 2-14 Crack pattern at failure (a) for Rectangular opening with high aspect ratio of opening sides (b) aspect ratio of opening sides close to 1 (Yang, et al., 2006).

Deep beams can be considered as beams, which have a considerable depth compared to their span length, different codes suggest different demarcation lines of span to depth ratio. ACI 318 code classifies beams as deep beams when the span to depth ration is less than 4, EN 1992 classifies deep beams as a beam with span to depth ratio less than 3, the Canadian code A23.04 considers deep beams for those having beams having span to depth ratio of less than 2. Several other codes suggest different limit of span to depth ratio's. However, almost all the different codes agree on Bernoulli beam theory does not apply to such type of beams because when deep beams are subjected to external loading plane section does not remain plane before and after bending and nether stress or strain have a liner distribution across the depth of the beam. For the reason stated above most design codes suggest the strut and tie model for the analysis of Deep beams. In order to fully understand and use the strut and tie model, the stress distribution when subjected to loading must be known first (Kong, 2002). In a research by (Haque, et al., 1986) Comparison of the mid span bending stress distribution (σx) along the depth of the beam was done between the predicted results from the tests and classical beam bending theory and results shows that bending theory makes reasonable estimate of the mid span extreme fiber tension for shallower beams(L/D=2) but for deeper solid beams the classical bending theory highly underestimates the mid span tension and overestimates the compression stress. In addition, several tests on reinforced deep beam having rectangular web openings were tested and the distribution of stresses were observed, results show that the value of diagonal stresses above the openings increase significantly and diagonal stress concentration exist the corner points of opening (Haque, et al., 1986).



(a) (b)

Figure 2-15 a) Horizontal bending stress at mid span of Beam b) Stress trajectories for Deep beams with web openings (Haque, et al., 1986)

2.4.2 Numerical and Analytical investigations

Most scientific researchers agree that experimental studies are the most accurate and reliable methodology to conduct researcher's. While the above statement is true numerical studies has many disadvantages as well. Namely its difficult to perform experimental studies because of the expense and unavailability of testing machines. Experimental investigation are also time consuming and require more finance and human resource compared to numerical investigations using finite element analysis (Demir, et al., 2016). Finite element method is one such versatile numerical method capable of handling versatile difficult problems faced in structural analysis like variable member thickness, modeling irregularly shaped bodies, openings within a member and composite structures made of multiple material types like reinforced concrete structures (Logen, 2010). The use of finite element method software packages for scientific research's has been immensely growing following the ease of access and affordability of personal computers. Since in finite element analysis results are obtained by so solving large number of simultaneous algebraic equations, which in turn makes it nearly impossible to solve it by using manual hand calculations. however, researcher warn FEA package users to understand that the structural behavior is not dictated by the computer program and instead the user should develop a feel for the structures and make use of programs that give numerical results close to the reality (Bhavikatti, 2005).

In the past few decades few researches have been conducted using finite element method regarding reinforced concrete deep beams with web openings. Numerical investigation (S & Subha, 2016) was performed on RC deep beams with circular, rectangular and square opening shapes positioned at the mid span of opening and subjected to one point loading using ANSYS software package. From the numerical test results it was found that the best size of opening for minimum shear stresses was about 1-5% of the beam area and the best position for two rectangular opening are 0.17L and 0.28L from one edges to the side of the opening along the length and 0.29D-0.36D from the top of the beam to the center of the openings. where D is total depth of the beam and L is total length of the beam.

Most building codes suggest analytical method of analysis like the strut and tie method for solid RC deep beams but they offer little or no information on how to analyzes RC deep beams with openings. In a numerical research (Mohamed, et al., 2014) the behavior of reinforced concrete deep beams with rectangular web openings using a FEA software package ABAQUS was studied. Both opening locations within the shear span and flexure zone was considered. The research findings show that when openings intercept the natural load path ultimate strength reduction as high as 35% was observed. However, when openings do not intercept the natural load path connecting the loading point and the support reaction only 6%-8% strength reduction in ultimate failure load were observed. Furthermore, the opening size for which only 10% strength reduction occurs is found to be opening size that is less than 20% of the total depth. one way of checking the accuracy of the result obtain from FEA simulation of RC deep beams with web openings is to compare the results to identical experimental tests previously and generally the results obtained from the numerical simulation are in good agreement with the experimental test results in terms of failure load and maximum deflection obtained (Mohammad & Ibrahem, 2007). Further more distributions of stresses and deformation of RC deep beams with web openings can be fairly estimated using nonlinear FEA and in general maximum deviations of 6% was obtained between the actual and the predicted ultimate failure load and a deviation of maximum 20% has been obtained for actual and predicted maximum mid span deflection (Senthil, et al., 2018).

Analytical and empirical methods can also be used with a certain degrees of accuracy to analyze reinforced concrete deep beams with web openings. The most popular method used for analysis and design of RC deep beams is the Strut and tie method, however design codes offer little or no guidance towards the use of STM for application in RC deep beams with web openings. Recently researchers have been trying to adopt STM to RC deep beams with web openings. In a research (Zechmann & Matamoros, 2002) a strut and tie model was proposed for RC deep beams with web openings and compared with experimental test result of ultimate failure load. Results showed that the STM applies better for beams with web reinforcements than those without web reinforcement and the ultimate load predicted for beams with web reinforcement was 91% of the tested ultimate load. In addition to the STM for analytical analysis of reinforced concrete deep beams with web openings, empirical equations have also been proposed to determine the strength reduction factor. After the strength reduction factor is determined it is then multiplied to the ultimate failure load of the solid RC deep beam (without web openings) to determine the ultimate failure load of the beams with web opening (Kong, 2002).

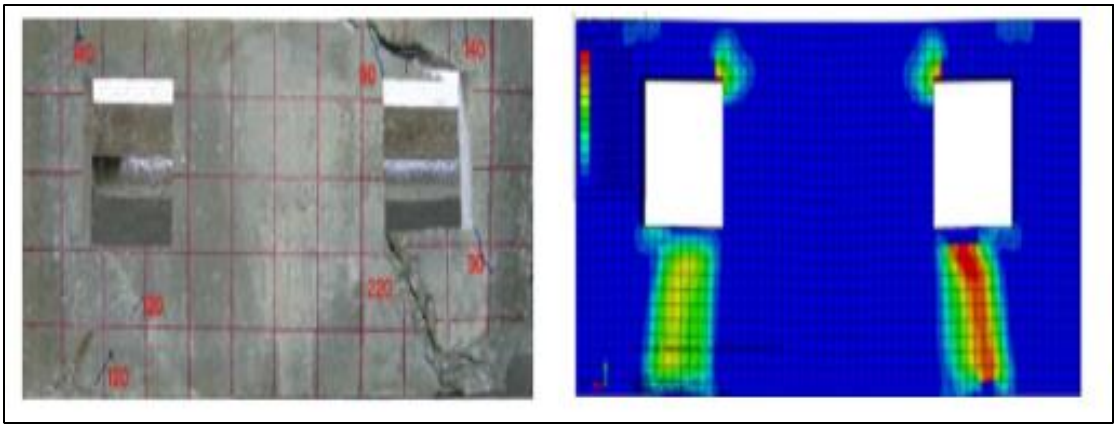


Figure 2-16 Deformed profile of experiment and profile (Senthil, et al., 2018)

2.5 Nonlinear finite element procedure

2.5.1 Incremental force method

Among the several practically available methods for solving nonlinear systems of equation, the Newton-Raphson method is the fastest and easy to implement in finite element analysis (Kim, 2015).This numerical model assumes an initial estimate ,u^0 and finds its increment Δu, so that the new estimate , $u^0 + \Delta u$ is close to the solution to Eq 2.44.

In order to determine the increment, the nonlinear equations are approximated by linear ones. This pattern is repeated until the original nonlinear equations are satisfied. Suppose an approximate solution of at the ith iteration is known the next iteration can be approximated by the following equation as follows:

$$P(u^{i+1}) \approx P(u^i) + K^i_T (u^i).\Delta u^i = f \qquad (2.23)$$

Where $K^i_T = \left(\frac{\partial P}{\partial u}\right)^i$ is the jacobian matrix at the ith iteration, commonly known as tangent stiffness matrix and Δu^i is the solution increment.

The aim is to calculate Δu^i and iteratively update the solution, u^{i+1}. After rearranging the terms, a system of linearized systems of equation can be obtained as follows

$$K^i_T \,.\, \Delta u^i = f - P(u^i) \qquad (2.24)$$

The term on the right side is known as the residual after solving the displacement increment Δu^i a new approximate solution can be obtained as follows:

$$u^{i+1} = u^i + \Delta u^i \tag{2.25}$$

usually the initial estimate is set to the undeformed shape of the structure (i.e. all displacements are initially set to zero). In nonlinear structures this assumption is true even if the relationship between load and displacement is nonlinear. When the applied load is small, the expected displacement is also so small, therefore the Newton-Raphson method convergence quickly. However, when the applied loads are large and the resulting displacement are also large a convergence difficulty occurs (Kim, 2015).The concept of the incremental force method is to apply the load in increments. Within each load, the procedure is the same as a typical Newton-Raphson method. The next load increment is applied after the solution cross ponding to the pervious load increment has converged. The converged solution at each increment is then used as an initial estimate of the next increment (Kim, 2015).

2.5.2 Convergence Check

The main purpose of nonlinear finite element analysis is to satisfy equilibrium condition stated in equation 2.45, which is equivalent to making the vector of residuals as close to zero as possible. The iteration stops when the convergence criteria given in Eq 2.70 say (0.001) is met. In that case the iteration converges and the solution is the current displacement. However, in some cases the iteration may not ever converge no matter how many numbers of iteration is done. In order to prevent an infinite loop of the convergence iteration, the program usually stops when the iteration counter reaches the maximum allowed number of iteration set by the user, then the program stops with an error message. In order to avoid such type of error the forced can be further halved and the convergence is tried again, which is known as the bisection method

$$\text{conv} = \frac{\sum_{j=1}^{n} (R_j^{\,i+1})^2}{1 + \sum_{j=1}^{n} (f_j)^2} \tag{2.26}$$

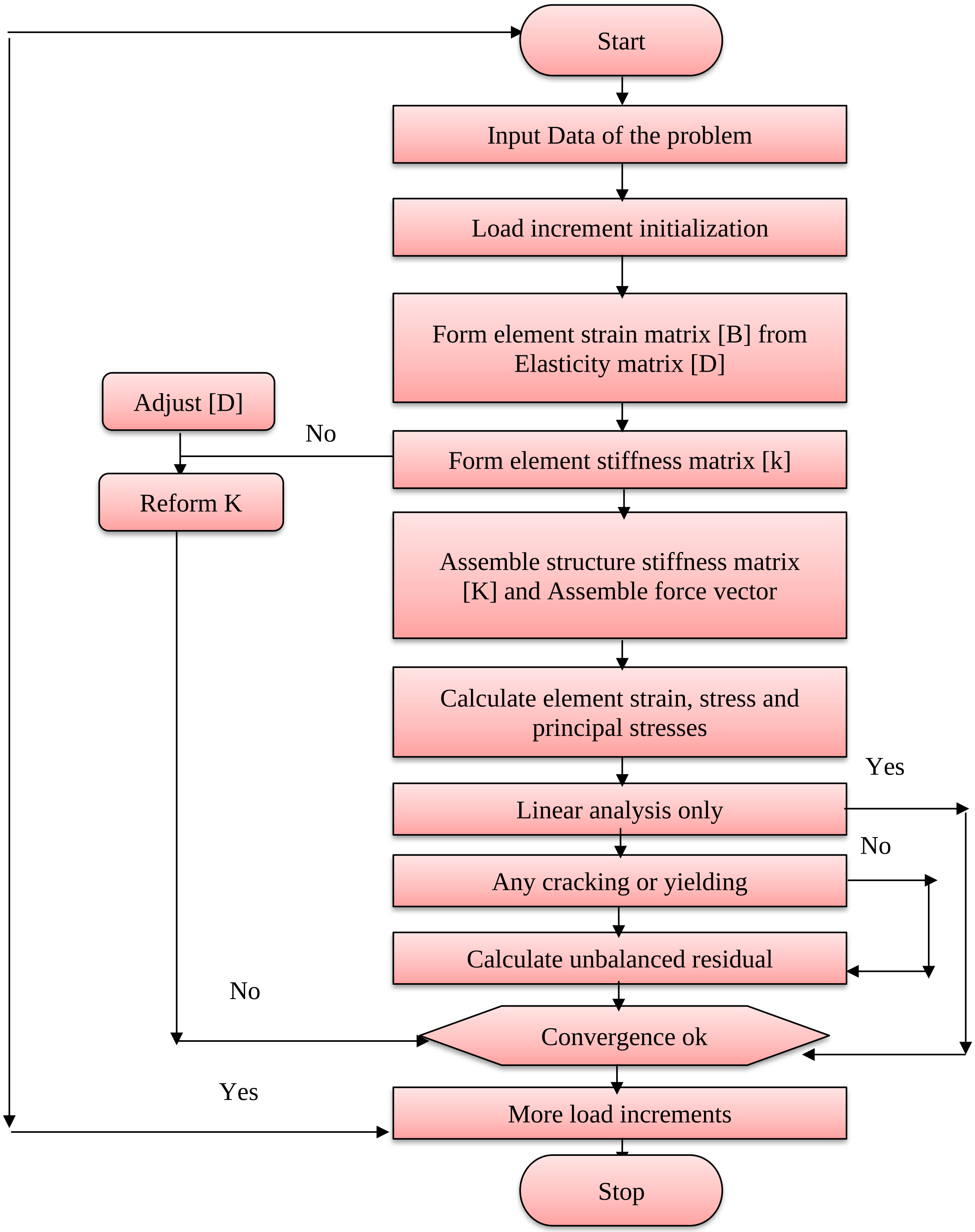


Figure 2-17 Flow chart for Nonlinear analysis

3 Methodology

The behavior of RC deep beams with transverse rectangular web openings is investigated through numerical analysis in VECTOR. Finite element analysis software VECTOR has a capability of considering material and geometric nonlinearities when performing analysis. In addition, the software can also consider tension stiffing and tension-softening behaviors that usually occur in reinforced concrete structure, thus making it a suitable tool to conduct this research. For the purpose of validating and verification, identical numerical simulation of two RC deep beams with and without web openings subjected to a two point static loading obtained from the experimental research (A.Jasim, et al., 2018) was conducted using VECTOR. After the finite element model validation is conducted, Numerical simulation of 73 RC deep beams was conducted, for the aim of identifying the effects of the aforementioned parameters on the behavior of such types of beams. The parameters studied were degree of interaction of the natural load path by the openings, openings size, aspect ratio of opening sides and compressive strength of concrete for different span to depth ratio of beams.

3.1 Modeling in VECTOR2D

3.1.1 Geometry modeling

Geometric modeling of RC deep beam is done by producing a separate regions for concrete, reinforcement, web opening (void), support and loading plates. Using the mesh and structure option in VECOTR 2D, first the region of the concrete is defined by inserting the xy coordinates of the four corners of the concrete region. After defining the coordinates, a hybrid rectangular plane element is assigned to that region. The rectangular plane element automatically changes in two or more triangular elements when boundary shapes are difficult to be modeled using rectangular elements, thus the term hybrid is assigned. The geometry of the reinforcement is defined using a line by proving the initial and end coordinates and assigning a truss (bar) element to it. Since longitudinal bars mainly take axial forces only, truss element are suitable to accurately model longitudinal flexural bars. In a similar manner the regions of the support and loading plates are defined by inserting the coordinates of the corner points the four plates (i.e. two loading plates and two support plates) and a rectangular plane element is assigned to them. The geometry of the web openings are modeled by defining a coordinates of the corner points of the void within the concrete region. The voids give a signal to the program to not assign any elements within that region. After the geometry of all the different types of elements are defined the appropriate types of material properties are assigned to each element's types.

Mesh generation is accomplished by using the auto-mesh generation built in tool in the software. The inputs for mesh generation are element size in x and y and maximum aspect ratio. If the given region is difficult to model with the selected element sizes or aspect ratios, the program adjusts the element size and aspect ratio's until suitable model conforming with the geometry is obtained. In order to assure compatibility condition at the interface between concrete and reinforcement, the nodes of rectangular plane elements are set to coincide with the nodes of the truss elements. In addition, since finite element method only assures compatibility at nodal points, additional constraint is assigned to assure compatibility concrete and reinforcement element edges. Provision of such constraints gives a more accurate predication of the response of RC structures.

3.1.2 Material modeling

3.1.2.1 Concrete

a. Compression behavior model

In describing the behavior of concrete subjected to uniaxial compression, a separate concrete model is assigned for the pre-peak and post-peak behavior of concrete. For numerical test specimens having a compressive strength of less than or equal to 40Mpa, the congested (parabola) is selected. For numerical test models having compressive strength of concrete greeter than 40Mpa Popovics (high strength) model is selected. For post peak response, the modified park and kent model is selected for all types of concrete compressive strength of concrete. The following equation describes the pre and post-peak response of concrete under uniaxial compression.

Pre-peak response

For $f_{ck} \leq$ 40Mpa, Hongetsed (parabola) model

$$f_{ci} = -f_p \left\{ \left(\frac{e_{ci}}{e_p} \right) - \left(\frac{e_{ci}}{e_p} \right)^2 \right\} < 0 \quad \text{for} \quad e_{ci} < 0 \qquad (3.1)$$

For fck > 40Mpa, popovics model

$$f_{ci} = -f_p \left(\frac{e_{ci}}{e_p} \right) \frac{n}{n - 1 + \left(\frac{e_{ci}}{e_p} \right)^{nk}} \quad \text{for} \quad e_{ci} < 0 \qquad 3.2)$$

Where

$$n = 0.8 + \frac{f_p}{17} \qquad (3.3)$$

$$k = \begin{cases} 1.0 & \text{for} \quad e_p < e_{ci} < 0 \\ 0.67 + \frac{f_p'}{62} \, {}^{3}1.0 & \text{for} \quad e_p < e_p < 0 \end{cases} \qquad (3.4)$$

Post-peak response

Modified park and Kent model

$$f_{ci}^{\,b} = -[fp + Zmfp(\varepsilon_{ci} - \varepsilon_p)] < 0 \quad \text{or} \quad 0.2f_p \quad \text{for} \quad \varepsilon_{ci} < \varepsilon_p < 0 \qquad (3.5)$$

$$Z_m = \frac{0.5}{\frac{3 + 0.29|f_c'|}{145|f_c'| - 1000}\left(\frac{\varepsilon_o}{-0.002}\right) + \left(\frac{|f_{lat}|}{170}\right) + \varepsilon_p} \qquad (3.6)$$

f_{lat} is summation of principal stresses, acting transversally to the direction under consideration

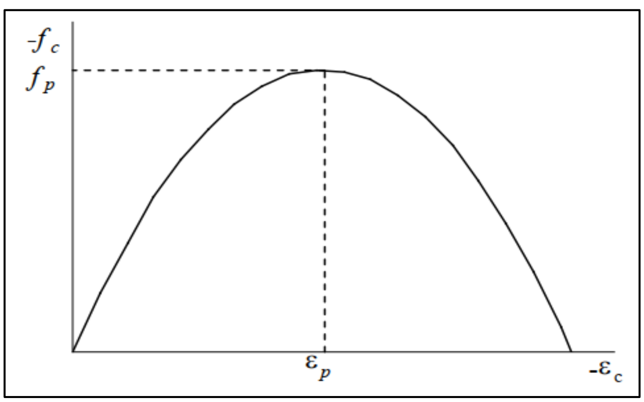


Figure 3-1 Hongeted (parabola)

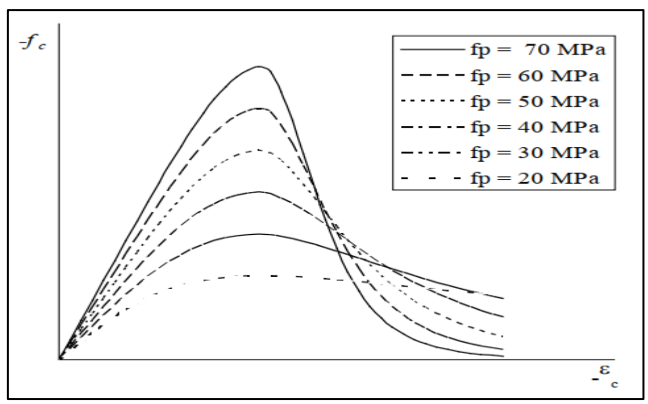


Figure 3-2 Popovics model

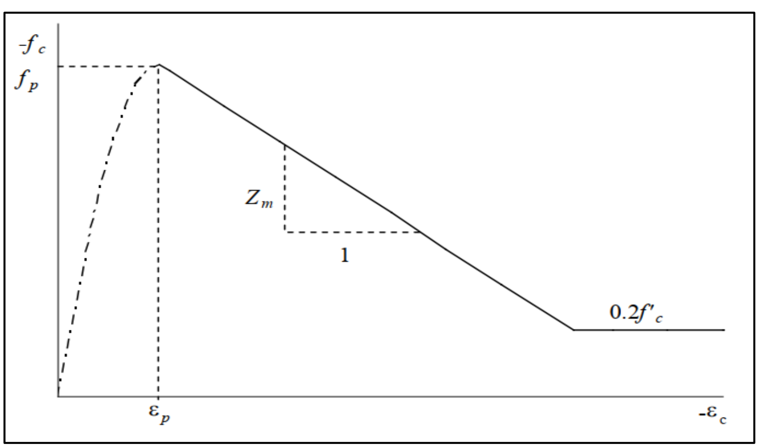


Figure 3-3 Modified park and Kent concrete model

b. Compression softening

Compression softening in cracked concrete is the reduction of compressive strength and stiffness relative to the uniaxial compressive strength, due to coexisting transverse cracking. The strength and strain softened model selected is Vecchio 1992-A ($\varepsilon 1/\varepsilon 2$).The following equation describe the compression softening model.

$$f_p = \beta_d f'_c \tag{3.7}$$

$$\varepsilon_p = \beta_d \varepsilon_o \tag{3.8}$$

$$\beta_d = \frac{1}{1+C_s.C_d} \leq 1 \tag{3.9}$$

$$C_d = \begin{cases} 0 & \text{if} \quad r < 0.28 \\ 0.35(r-0.28)^{0.8} & \text{if} \quad r > 0.28 \end{cases} \tag{3.10}$$

$$r = \frac{-\varepsilon_{c1}}{\varepsilon_{c2}} \leq 400 \tag{3.11}$$

$$C_d = \begin{cases} 0 \quad \text{if shear slip is not considered} \\ 0.55 \, \text{if shear slip is considered} \end{cases} \tag{3.12}$$

c. Tensile behavior model

The behavior of concrete subjected to uniaxial tensile stresses can be modeled by describing the tensile softening behavior for concrete located at a distance of 0.75db away from the longitudinal reinforcement and by describing the tension stiffing behavior for concrete elements located within the region of 0.75d from the longitudinal reinforcement. A bilinear model is selected for tension softening behavior and modified bentz model is selected to model tension stiffing effect.

Prior to cracking, the response of concrete is assumed to linear-elastic, as follows:

$$f_{c1} = E_c \varepsilon_{c1} \quad \text{for } 0 < \varepsilon_{c1} < \varepsilon_{cr} \tag{3.13}$$

Where,

$$\varepsilon_{cr} = \frac{f_{cr}}{E_c} \tag{3.14}$$

$$E_c = 4500\sqrt{f_c'} \tag{3.15}$$

$f_{cr}=0.33\sqrt{f_c'}$ (3.16)

ε_{cr} Is the cracking strain, E_c is elastic modulus of concrete, $\varepsilon c1$ is the principal strain, and fcr is the cracking stress of the concrete determined by the cracking criteria model

Tension stiffing model (modified benz)

$$f_{c1} = \frac{f_t'}{1+\sqrt{c_t \varepsilon_{c1}}} \quad \text{for} \quad \varepsilon_{c1} > \varepsilon_t' \tag{3.17}$$

Where,

$$c_d = 3.6\, t_d.m \tag{3.18}$$

$$\frac{1}{m} = \sum_{i=1}^{n} \frac{4\rho_i}{d_{bi}}.\left|\cos(\theta - \alpha_i)\right| \tag{3.19}$$

ρi is the reinforcement ratio, d_{bi} is the rebar diameter , θ is the inclination of principal direction , α_i is the inclination of reinforcement and td is equal to 0.6

Tension softening model (bilinear model)

$$f_{c1} = \begin{cases} f_{cr}\left(1-0.8\left(\frac{\varepsilon_{c1}-\varepsilon_{cr}}{\varepsilon_{ch3}-\varepsilon_{cr}}\right)\right) & \text{for} \quad \varepsilon_{c1} < \varepsilon_{ch3} \\ 0.2f_{cr}\left(1-\left(\frac{\varepsilon_{c1}-\varepsilon_{ch3}}{\varepsilon_{ch4}-\varepsilon_{ch3}}\right)\right) & \text{for} \quad \varepsilon_{ch3} \le \varepsilon_{c1} < \varepsilon_{ch4} \end{cases} \tag{3.20}$$

Where,

$\varepsilon_{ch2} = 0.64\,\varepsilon_{ch} + \varepsilon_{cr}$

(3.21) $\varepsilon_{ch3} = 6.8\,\varepsilon_{ch} + \varepsilon_{cr}$

(3.22)

$\varepsilon_{ch} = \frac{G_f}{L_{ref} f_{cr}}$; Characteristics strain using Gf from (Bazant, 2002) (3.21)

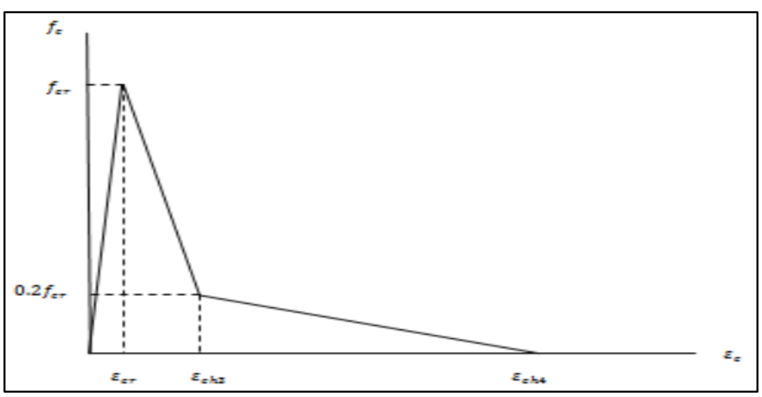


Figure 3-4 Modified Benz Tension softening model

3.1.2.2 Reinforcement and steel plate material model

Elastic perfectly plastic stress strain relationship is selected as a material model for the reinforcement and steel for the loading and support plates. The ultimate strain at failure is taken as 0.01.yeild strain is calculated by dividing the yield strength of the steel or reinforcement by elastic modules of steel which is taken as 200Gpa.

Shear and skin reinforcements use the same stress strain relationship stated above, however they are modeled as smeared reinforcement considered by modifying the concrete behavior.

3.2 Numerical model for validation of software result

Two Finite element models of the experimental test specimen conducted by (A.Jasim, et al., 2018) were created for the purpose of validation and verification of numerical simulation. The description of the test specimen is shown in table 3-1.The first test specimen is a solid reinforced concrete deep beam used as a control beam having a span to depth ration of 3, while the second model contains two symmetrical square openings (200mmx200mm) located with a the shear span. Both test specimens were subjected to a two point loading with shear span to depth ratio of 0.9(i.e. a is 450mm and D is 500mm). The cylindrical compressive strength of concrete of both of the test beams was observed to be 27Mpa.In addition; both beams have 3∅16 bottom reinforcement, similar web and skin reinforcements. The test specimens where subjected to a two point loading through 1.5cm thick loading and support plates. The experimental test specimen data's are expressed in table 3-1 below as follows:

Table 3-1 Experimental test specimen data

f_{ck}, cylindrical compressive strength	27MPa
f_{yd}, yield strength flexural reinforcement	624MPa
$f_{yd}s$, yield strength stirrup and skin reinforcement	569MPa
f_{ydp}, yield strength of steel and loading plates	400MPa
Length of the beam	1500mm
Depth of the beam	500mm
Width of the beam	150mm
Flexural reinforcement	3∅16mm
Stirrup spacing	∅6mm c/c 90mm
Skin reinforcement	∅6mm c/c 86mm

The material and geometric model are conducted according to the models discussed in section 3.1.2. Two mesh sizes (i.e. 50mm and 25mm) were used in the numerical model, since the accuracy finite element analysis is mesh sensitive, an attempt is made to determine the suitable element mesh size. In addition, the maximum iteration and convergence criteria is set to be equal to 100 and 1.00001 respectively. 1.2kN/load step was selected for the load increment factor the two point loading, furthermore self-weight of the beam is also considered as a gravitational body forces having a density of 2450kg/m^3 .Restraint at the boundary was provided by assigning a pin and roller support at the center of the support plates located at opposite ends. The pin support restraints translation in both x and y direction while the roller support prevents translation in the y direction only. The flexural reinforcement is modeled using a discrete bar element, however the web and skin reinforcements are considered by a reinforcement's ratios both in x and y direction. The calculations of reinforcement ratios are shown as follows.

Shear reinforcement

$$\rho_v = \frac{A_s}{A_c} = \frac{2 \times n \times a_s}{L \times W} = \frac{2 \times \frac{1450}{90} \times 27.3}{1450 \times 100} = 0.60\%$$

Skin reinforcement

$$\rho_h = \frac{A_s}{A_c} = \frac{2 \times n \times a_s}{L \times W} = \frac{2 \times \frac{450}{86} \times 27.3}{450 \times 100} = 0.605\%$$

Where ρ_v and ρ_h are reinforcement ratios of stirrup and skin reinforcement respectively in percent.

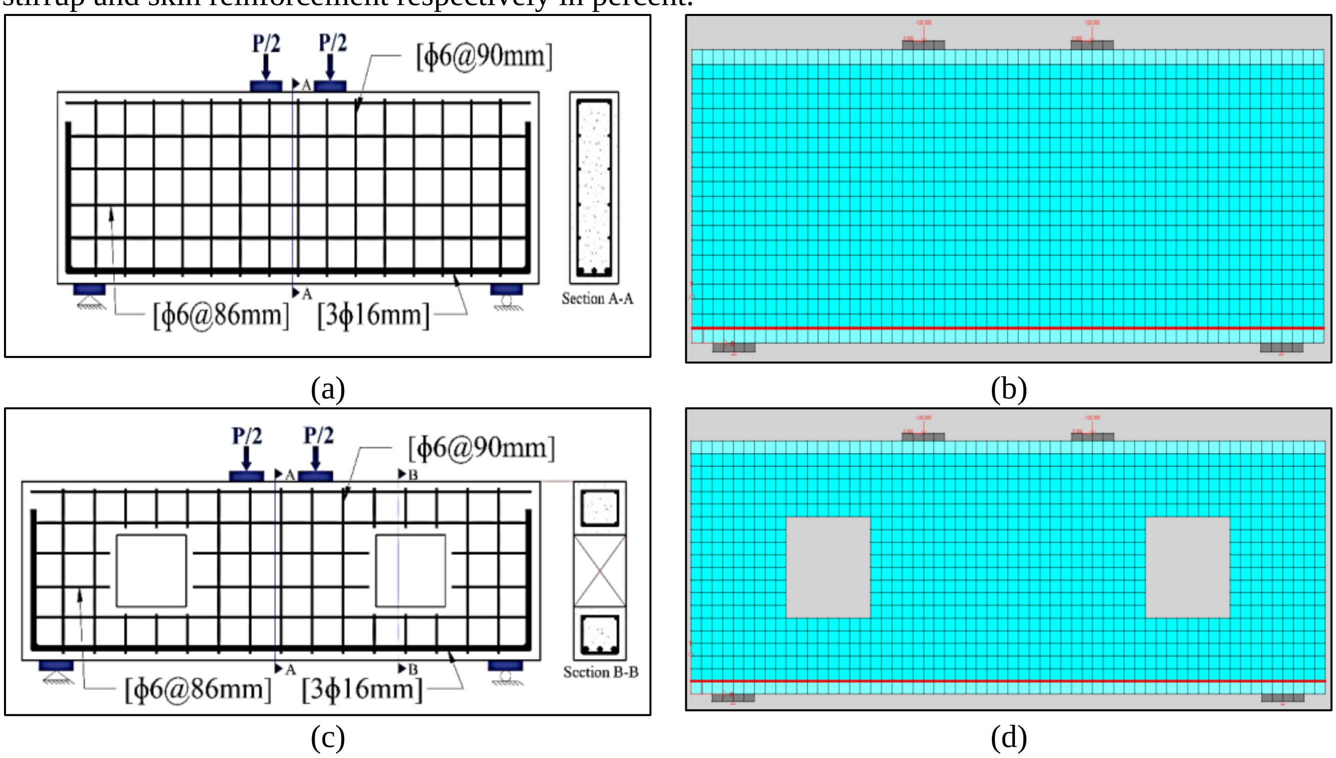
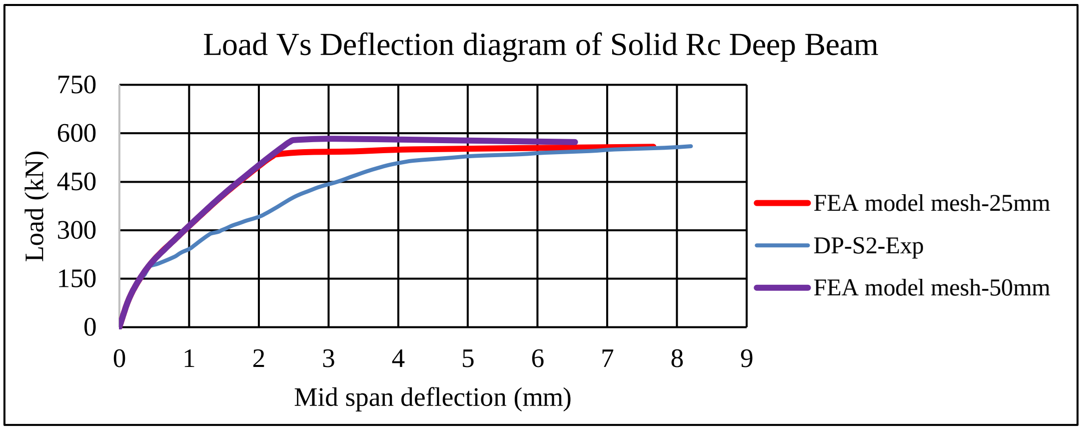
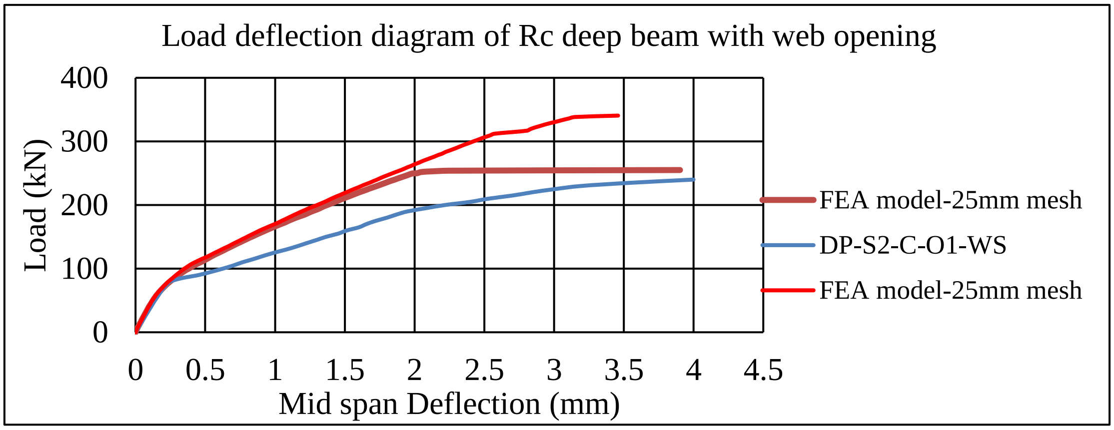


Figure 3-5 Experimental test specimens and Numerical models (a) Solid RC beam (b) Numerical model of solid RC deep beam (c) beam with web opening (d) Numerical model beam with web opening

The concrete cover is modeled as plain concrete while the interior concrete is modeled as a concrete with smeared reinforcements by assigning the reinforcement ratios calculated earlier through the material definition input tab in VECTOR2D. There are several other input parameters that are required to be given as an input but since the VECTOR2D manual recommends to use the default value as much as possible no other input changes have been made in the program. Other input parameters like concrete dilation, cracking criteria (i.e. Mohr coulomb stress), crack width check, confinement strength and crack slip calculation method (walreven) have been assigned a default value recommended by the software manual. The above figure 3-5 shows both the experimental test beams and equivalent numerical models. The load deflection diagram of both the experimental test and finite element analysis for both the solid beam and beams with web openings are shown in figure 3-6 below.



(a)



(b)

Figure 3-6 Load Vs Deflection diagram of (a) Solid Rc Deep Beam (b) beam with rectangular opening

3.3 Numerical simulation for parametric study

Finite element models are formulated and analyzed for investigating the behavior of deep beams with web openings. The variable parameters considered in the parametric study are opening location, opening size, aspect ratio of opening sides, and compressive strength of concrete while parameters like flexural reinforcement, web reinforcement, skin reinforcement and yield strength of all the reinforcements are held constant and can be seen in table 3-2.In trying to investigate the effect of one parameter, the investigated parameter is varied while all other parameters are held constant. Each finite element models are assigned a unique designation letters as shown in table 3-3. The first number character indicates the width of the opening, after the number character the following three characters containing an alphabet, hyphen and a number (e.g.A-1.5) indicates the aspect ratio of the opening sides (i.e. w /L). The next alphabet letter written as either S or F indicates the location of the opening, where S indicates opening is within the shear span and F indicates that opening location is in the flexural zone. The next alphabet letter ranges form letter P to Letter V; they indicate the degrees of interruption of natural load path by the opening within the shear span. P, S, T, U, V indicate degree of interruption of 28.5% , 20%, 15%, 8%, 0.8% and 0% respectively which are explained in detail in section 3.1.4.1.a. The consecutive three characters containing a letter and two numbers indicate the cubic compressive strength of the concrete (i.e. C25). Finally, the last character containing a combination of letters, numbers and symbols indicate the span to depth ratio of the beam (LD-2.5). For instance, a designation of finite element model 100A-1.5SPC25LD-2.5 means a beam having web opening width of 100mm with 1.5 opening side's aspect ratio, the opening is located within the shear span of a beam and the beam has length to depth ratio of 2.5 and has 25Mpa cubic compressive strength of concrete. There are three control beams with different span to depth ratio, they are designed, by using the strut-and-tie-method according to ACI318M code to withstand a factored load of 130kN, the calculations are clearly shown in Appendix A.

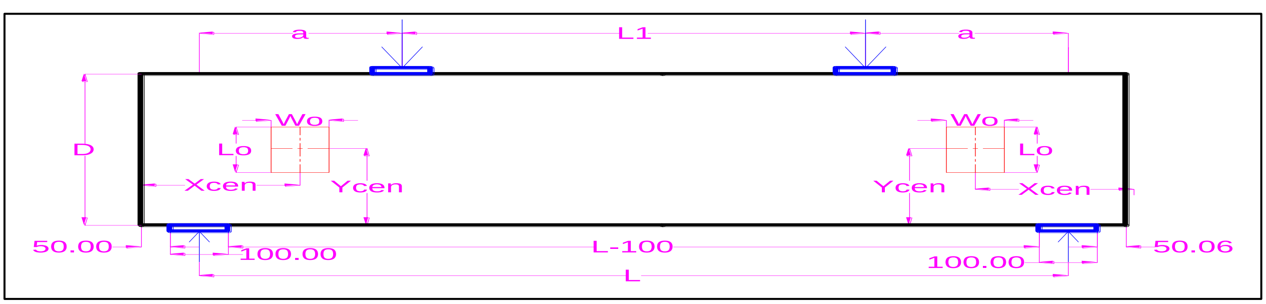


Figure 3-7 Typical Finite element model with web opening within shear span

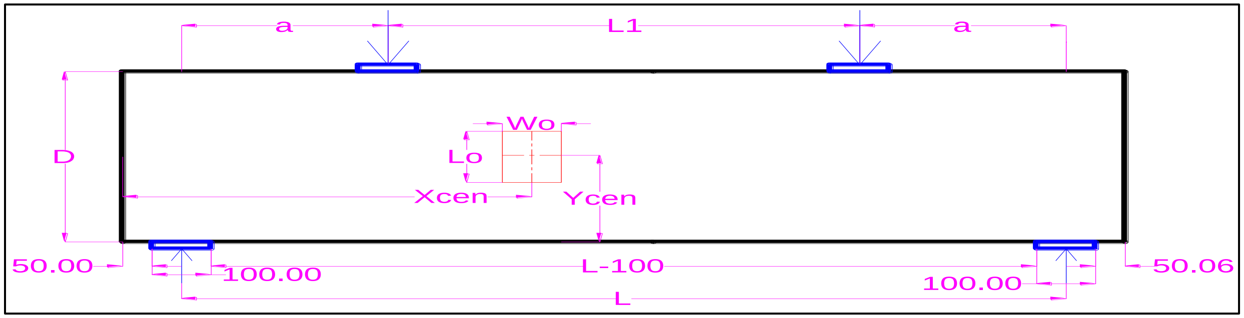


Figure 3-8 Typical Finite element model for opening in the flexural zone (L1)

Table 3-2 Reinforcement Data of FEA models for parametric study

f_{yd}, yield strength flexural reinforcement	400Mpa
f_{yd}, yield strength stirrup and skin reinforcement	400Mpa
F_{yd}, yield strength of steel and loading plates	400Mpa
Bottom Flexural reinforcement	3∅14mm
Stirrup and skin reinforcement spacing	∅8mm c/c 110mm (0.6%) & ∅6mm c/c 100mm (0.375%)

Table 3-3 Finite Element test models detail

No	FEA model Designation	Beam Geometry			Loading position		Web Opening variables					Fcu/fck (Mpa)	Remark
		L	D	L/D	a	L1	Wo	Lo	Xcen	Ycen	Aspect		
1	Solid beam LD-3	1500	500	3	350	800	-	-	-	-	-	C25/20	Control beams
2	Solid beam LD-2.5	1250	500	2.5	350	550	-	-	-	-	-	C25/20	
3	Solid beam LD-2	1000	500	2	350	300	-	-	-	-	-	C25/20	
4	100A-1.5SPC25LD-3	1500	500	3	350	800	100	150	275	250	1.5	C25/20	for investigating the effect degree of interruption Opening location within the shear span
5	100A-1.5SRC25LD-3	1500	500	3	350	800	100	150	273.5	293	1.5	C25/20	
6	100A-1.5SSC25LD-3	1500	500	3	350	800	100	150	280	319	1.5	C25/20	
7	100A-1.5STC25LD-3	1500	500	3	350	800	100	150	273.85	155.7	1.5	C25/20	
8	100A-1.5SUC25LD-3	1500	500	3	350	800	100	150	273.85	127	1.5	C25/20	
9	100A-1.5SVC25LD-3	1500	500	3	350	800	100	150	273.8	395	1.5	C25/20	
10	100A-1.5SPC25LD-2.5	1250	500	2.5	350	550	100	150	275	250	1.5	C25/20	
11	100A-1.5SRC25LD-2.5	1250	500	2.5	350	550	100	150	273.5	293	1.5	C25/20	
12	100A-1.5SSC25LD-2.5	1250	500	2.5	350	550	100	150	280	319	1.5	C25/20	
13	100A-1.5STC25LD-2.5	1250	500	2.5	350	550	100	150	273.85	155.7	1.5	C25/20	

No	FEA model Designation	Beam Geometry			Loading position		Web Opening variables					Fcu/fck (Mpa)	Remark
		L	D	L/D	a	L1	Wo	Lo	Xcen	Ycen	Aspect		
14	100A-1.5SUC25LD-2.5	1250	500	2.5	350	550	100	150	273.85	127	1.5	C25/20	
15	100A-1.5SVC25LD-2.5	1250	500	2.5	350	550	100	150	273.8	395	1.5	C25/20	
16	100A-1.5SPC25LD-2	1000	500	2	350	300	100	150	275	250	1.5	C25/20	
17	100A-1.5SRC25LD-2	1000	500	2	350	300	100	150	273.5	293	1.5	C25/20	
18	100A-1.5SSC25LD-2	1000	500	2	350	300	100	150	280	319	1.5	C25/20	
19	100A-1.5STC25LD-2	1000	500	2	350	300	100	150	273.85	155.7	1.5	C25/20	
20	100A-1.5SUC25LD-2	1000	500	2	350	300	100	150	273.85	127	1.5	C25/20	
21	100A-1.5SVC25LD-2	1000	500	2	350	300	100	150	273.8	395	1.5	C25/20	
22	100A-1.5FPC25LD-2.5	1250	500	2.5	350	550	100	150	550	250	1.5	C25/20	For investigating the effect of Opening location in the flexural zone
23	100A-1.5FRC25LD-2.5	1250	500	2.5	350	550	100	150	610	250	1.5	C25/20	
24	100A-1.5FSC25LD-2.5	1250	500	2.5	350	550	100	150	725	250	1.5	C25/20	
25	100A-1.5FPC25LD-3	1500	500	3	350	800	100	150	550	250	1.5	C25/20	
26	100A-1.5FRC25LD-3	1500	500	3	350	800	100	150	700	250	1.5	C25/20	
27	100A-1.5FSC25LD-3	1500	500	3	350	800	100	150	850	250	1.5	C25/20	
28	50A-1.5SPC25LD-3	1500	500	3	350	800	50	75	275	262	1.5	C25/20	For investigating the effect of opening size in the shear zone
29	150A-1.5SPC25LD-3	1500	500	3	350	800	150	225	230	250.5	1.5	C25/20	
30	200A-1.5SPC25LD-3	1500	500	3	350	800	200	300	275	250	1.5	C25/20	
31	250A-1.5SPC25LD-3	1500	500	3	350	800	250	375	275	249.5	1.5	C25/20	
32	275A-1.5SPC25LD-3	1500	500	3	350	800	275	412	274.5	250	1.5	C25/20	
33	50A-1.5SPC25LD-2.5	1250	500	2.5	350	550	50	75	275	262	1.5	C25/20	
34	150A-1.5SPC25LD-2.5	1250	500	2.5	350	550	150	225	230	250.5	1.5	C25/20	
35	200A-1.5SPC25LD-2.5	1250	500	2.5	350	550	200	300	275	250	1.5	C25/20	
37	250A-1.5SPC25LD-2.5	1250	500	2.5	350	550	250	375	275	249.5	1.5	C25/20	
38	275A-1.5SPC25LD-2.5	1250	500	2.5	350	550	275	412	274.5	250	1.5	C25/20	
39	50A-1.5SPC25LD-2	1000	500	2	350	300	50	75	275	262	1.5	C25/20	
40	150A-1.5SPC25LD-2	1000	500	2	350	300	150	225	230	250.5	1.5	C25/20	
41	200A-1.5SPC25LD-2	1000	500	2	350	300	200	300	275	250	1.5	C25/20	
42	250A-1.5SPC25LD-2	1000	500	2	350	300	250	375	275	249.5	1.5	C25/20	
43	275A-1.5SPC25LD-2	1000	500	2	350	300	275	412	274.5	250	1.5	C25/20	
44	50A-1.5FC25LD-3	1500	500	3	350	800	50	75	850	250	1.5	C25/20	For investigating

No	FEA model Designation	Beam Geometry			Loading position		Web Opening variables					Fcu/fck (Mpa)	Remark
		L	D	L/D	a	L1	Wo	Lo	Xcen	Ycen	Aspect		
45	150A-1.5FC25LD-3	1500	500	3	350	800	150	225	850	250	1.5	C25/20	the effect of opening size located at the center of the beam (single opening)
46	200A-1.5FC25LD-3	1500	500	3	350	800	200	300	850	250	1.5	C25/20	
47	250A-1.5FC25LD-3	1500	500	3	350	800	250	375	850	250	1.5	C25/20	
48	275A-1.5FC25LD-3	1500	500	3	350	800	275	412	850	250	1.5	C25/20	
49	50A-1.5FC25LD-2	1000	500	2	350	300	50	75	600	250	1.5	C25/20	
50	100A-1.5FC25LD-2	1000	500	2	350	300	100	150	600	250	1.5	C25/20	
51	150A-1.5FC25LD-2	1000	500	2	350	300	150	225	600	250	1.5	C25/20	
52	200A-1.5FC25LD-2	1000	500	2	350	300	200	300	600	250	1.5	C25/20	
53	250A-1.5FC25LD-2	1000	500	2	350	300	250	375	600	250	1.5	C25/20	
54	281A-0.25SPC25LD-3	1500	500	3	350	800	281	70.25	275	250	0.25	C25/20	For investigating the effect of opening sides aspect ratio within the shear span
55	200A-0.5SPC25LD-3	1500	500	3	350	800	200	100	287.5	250	0.5	C25/20	
56	141A-1.0SPC25LD-3	1500	500	3	350	800	141	141	275.5	250	1.0	C25/20	
57	116A-1.5SPC25LD-3	1500	500	3	350	800	116	173	275	250	1.5	C25/20	
58	100A-2.0SPC25LD-3	1500	500	3	350	800	100	200	275	250	2.0	C25/20	
59	90A-2.5SPC25LD-3	1500	500	3	350	800	90	225	275	250	2.5	C25/20	
60	282A-0.25SPC25LD-2	1000	500	2	350	300	282	70.5	276	250	0.25	C25/20	
61	200A-0.5SPC25LD-2	1000	500	2	350	300	200	100	275	250	0.5	C25/20	
62	141A-1.0SPC25LD-2	1000	500	2	350	300	141	141	275.5	250	1	C25/20	
63	116A-1.5SPC25LD-2	1000	500	2	350	300	116	173	303	250	1.5	C25/20	
64	100A-2.0SPC25LD-2	1000	500	2	350	300	100	200	275	250	2.0	C25/20	
65	90A-2.5SPC25LD-2	1000	500	2	350	300	90	225	275	250	2.5	C25/20	
66	100A-1.5SPC30LD-2	1000	500	2	350	300	100	150	275	250	1.5	C30/24	For investigating the effect of compressive strength of concrete
67	100A-1.5SPC40LD-2	1000	500	2	350	300	100	150	275	250	1.5	C40/32	
68	100A-1.5SPC50LD-2	1000	500	2	350	300	100	150	275	250	1.5	C50/40	
69	100A-1.5SPC60LD-2	1000	500	2	350	300	100	150	275	250	1.5	C60/48	
70	200A-1.5SPC30LD-2	1000	500	2	350	300	200	300	275	250	1.5	C30/24	
71	200A-1.5SPC40LD-2	1000	500	2	350	300	200	300	275	250	1.5	C40/32	
72	200A-1.5SPC50LD-2	1000	500	2	350	300	200	300	275	250	1.5	C50/40	
73	200A-1.5SPC60LD-2	1000	500	2	350	300	200	300	275	250	1.5	C60/48	

3.3.1. Opening location

a. Shear span

A total of eighteen numerical simulations are conducted to investigate the effect of degree of interruption of the natural load path connecting the reaction points of the support and the load Point by the web openings. The details of the finite element models to investigate this parameter are shown in table 3-3 from row 4 to row 21. Degree of interruption is calculated by dividing the intercepted path by the opening to the total load path connecting the support and the load point (i.e. Lintercept/Lpath) as shown in figure 3-9.while investigating the effect of degree of interruption other parameters like opening size ,compressive strength of concrete and aspect ratios of opening sides are held constant. On all of the 18 finite element models Opening size was 100mmx150mm, the aspect ratio calculated was 1.5 and C25/20 concrete strength were chosen. The degree of interruption is expressed in percentage by the following equation.

$$\text{Degree of interruptions (\%)} = \frac{\text{Lintercept}}{\text{Lpath}} \times 100 \qquad (3.22)$$

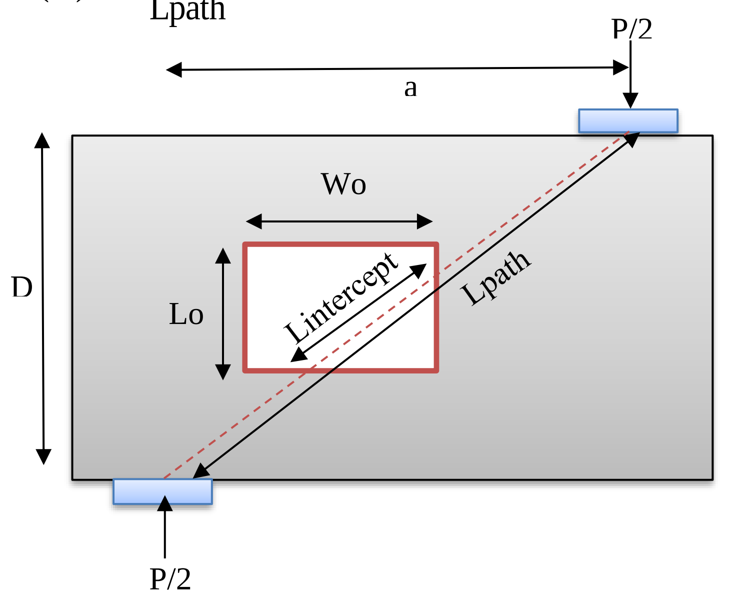
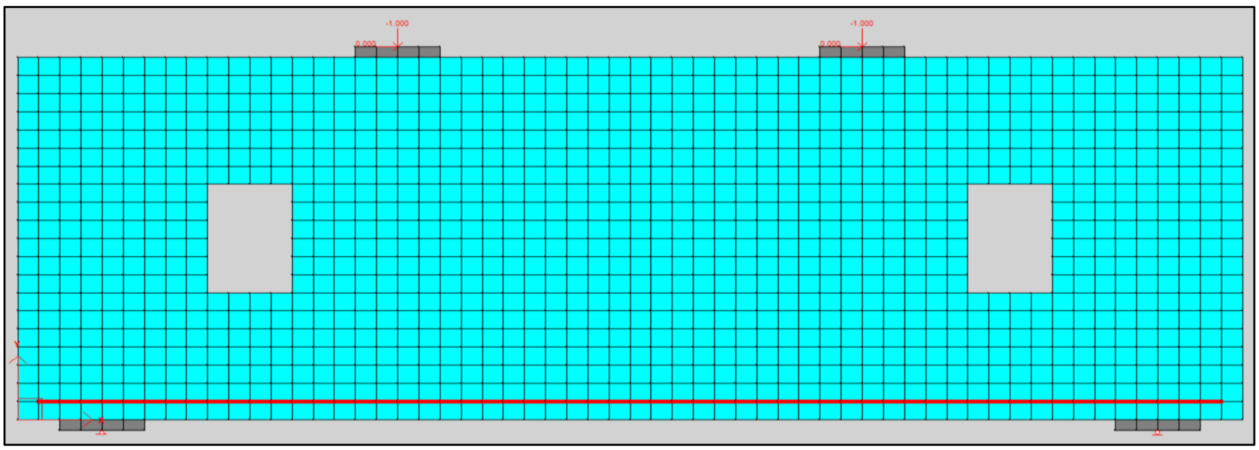


Figure 3-9 shear span section Deep beam with web opening

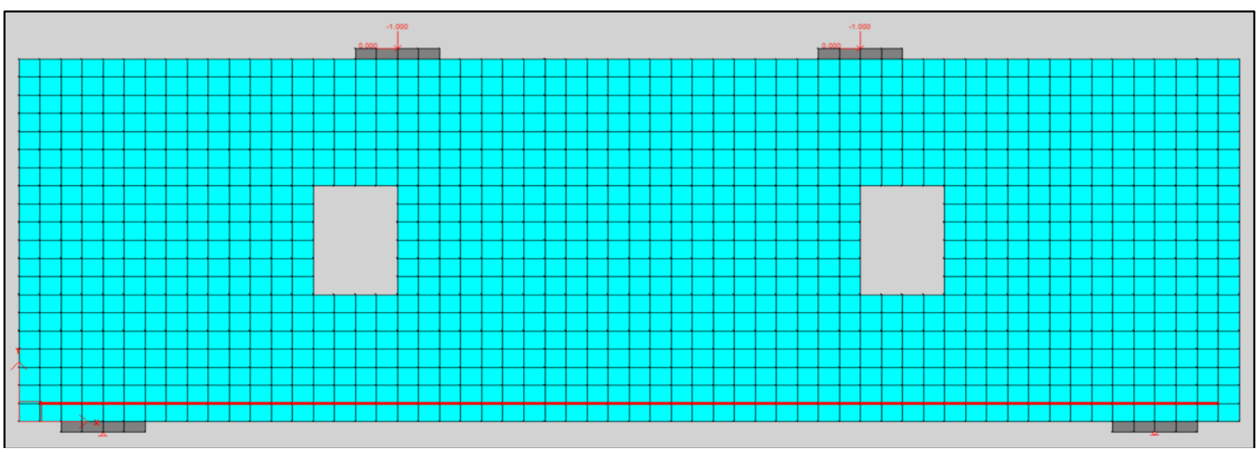
The loading and support plates are provided to avoid premature local failure by distributing the load. The dimensions of these plates are 100mm length, 150mm in width and 15mm thick. The yield strength of the loading and support plate is 400Mpa. The calculation of degree of interruption are shown in table 3-4 and the sample finite element models are show in figure 3-10 below.

Table 3-4 Calculation of degree of interruption

Finite element model	Lpath (mm)	Lintercept (mm)	Lintercept / Lpath (%)
100A-1.5SPC25LD-3/2.5/2	610.33	174.91	28.5%
100A-1.5SRC25LD-3/2.5/2	610.33	135.0	22.0%
100A-1.5SSC25LD-3/2.5/2	610.33	91.43	15.0%
100A-1.5STC25LD-3/2.5/2	610.33	48.04	8.0%
100A-1.5SUC25LD-3/2.5/2	610.33	4.25	0.8%
100A-1.5SVC25LD-3/2.5/2	610.33	0	0%



(a)



(b)

Figure 3-10 Finite element models with degree of interruption (a) 100A-1.5SPC25LD-2.5 (28.5%) (b) 100A-1.5SVC25LD-2.5 (0%)

b. Opening in the mid span (flexural region)

A finite element models Deep beams having three different opening locations in the flexural zones (i.e. between the point loads) were conducted. edge of the opening side coincides with the point of load application for the 1st opening location , the center of the openings coincides with center of the point of the section between the applied load and the beam center point and the 3rd opening location center coincides with the center point of the beam(mid span). Seven numerical simulation was conducted to investigate the effects of web opening location in the flexural zone and the details of these finite element models are given in table 3-3 from row 22-row 27. In a similar manner, parameters like opening size, aspect ratio of opening sides and concrete compressive strength are held constant.

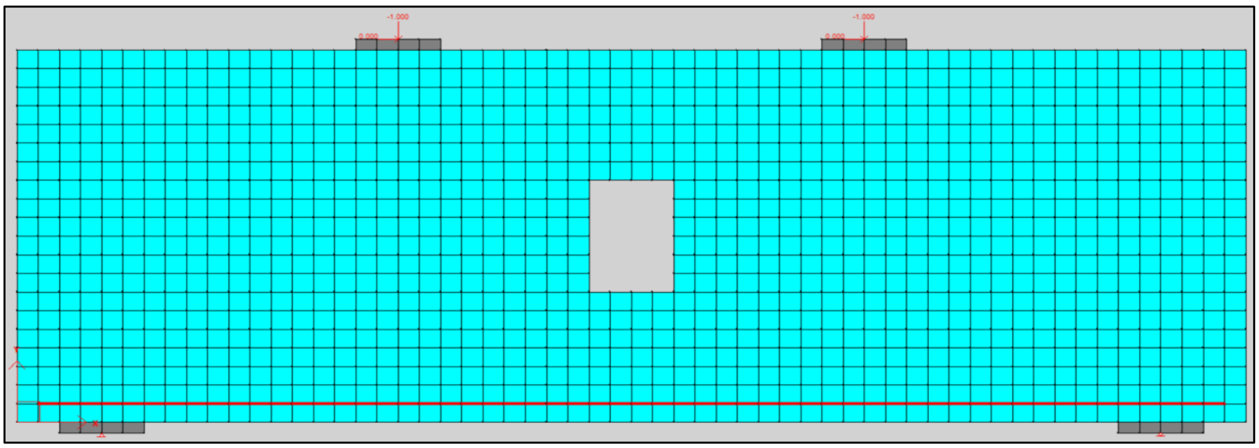


Figure 3-11 Finite element model 100A-1.5FSC25LD-2.5 (opening center coinciding with the center of the mid span)

3.3.2. Opening size

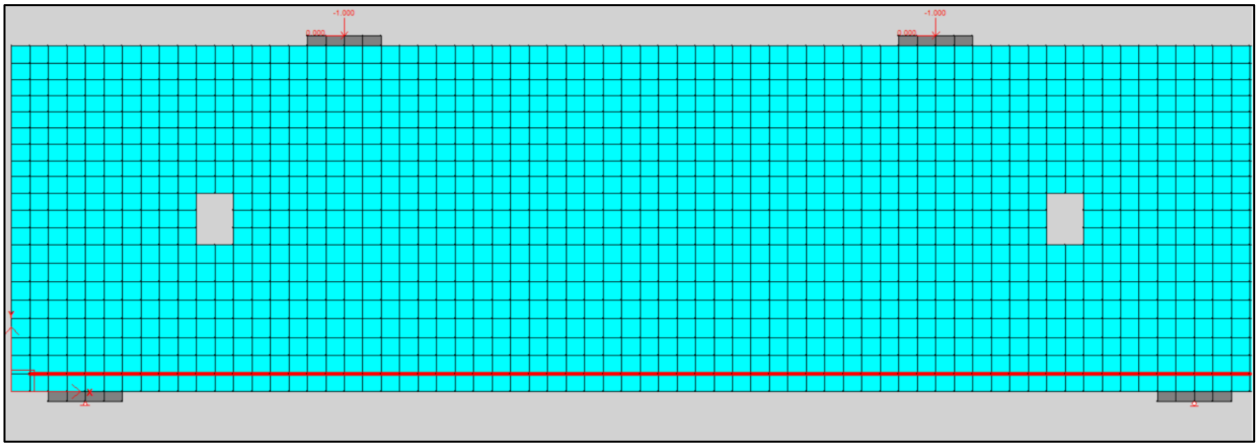
a. Shear span

Sixteen numerical simulation were conducted, in an attempt to investigate the effect of opening sizes located within the shear span of the beam on the behavior of RC deep beams. While investigating the effect of this parameter other parameters like opening location, aspect ratio of opening sides and compressive strength of concrete are kept constant. Opening size is expressed as a ration of opening area to the shear span area as shown in equation 3.25 below. The detail of the finite element models are shown in table 3-3 from row 28-two 43. In addition, the detail calculation of the percentage calculation of opening size can be seen from table 3-3.

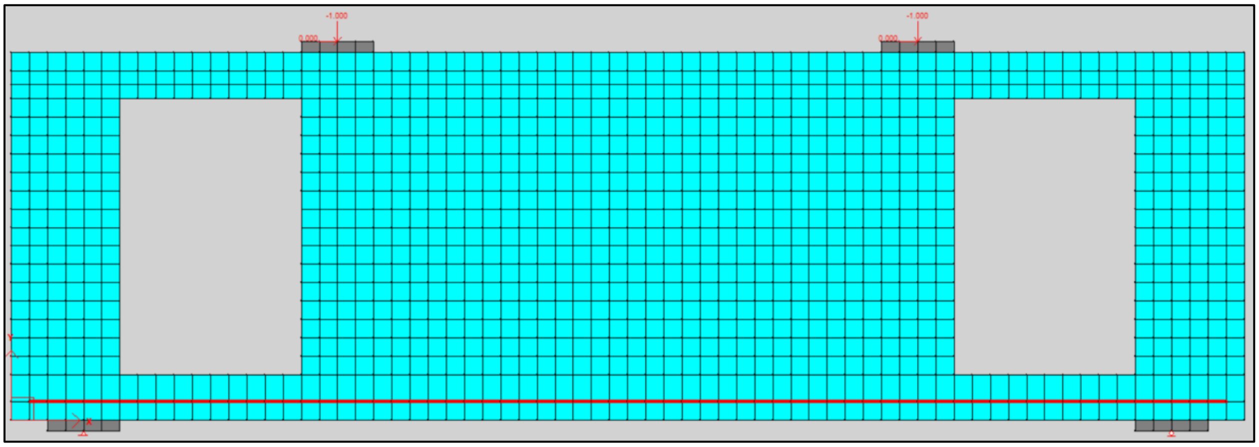
$$\text{Opening size ratio} = \frac{\text{Area of opening size}}{\text{Area of shear span}} \times 100 = \frac{Wo \times Lo}{a \times D} \times 100 \qquad (3.23)$$

Table 3-5 opening size percentage calculation

Finite element model	Opening size		Shear span		$\frac{Wo \times Lo}{a \times D} \times 100$ (%)
	Wo (mm)	Lo (mm)	a (mm)	D (mm)	
50A-1.5SPC25LD-3/2.5/2	50	75	350	500	2.14
100A-1.5SPC25LD-3/2.5/2	100	150	350	500	8.57
150A-1.5SPC25LD-3/2.5/2	150	225	350	500	19.3
200A-1.5SPC25LD-3/2.5/2	200	300	350	500	34.3
250A-1.5SPC25LD-3/2.5/2	250	375	350	500	53.57
275A-1.5SPC25LD-3/2.5/2	275	412	350	500	64.74



(a)



(b)

Figure 3-12 Finite element model with opening size with percentage (a) 50A-1.5SPC25LD-3 (2.14%) (b) 250A-1.5SPC25LD-3 (53.57%)

b. Opening in the mid span (flexural zone)

The effect of opening size located within the flexural zone is investigated by finite element models listed in table 3-3 from row 47 to row 53. The different opening sizes are placed at the center of the mid span of the beams. Others parameters like aspect ratios of opening sides and compressive strength of concrete are kept constant.

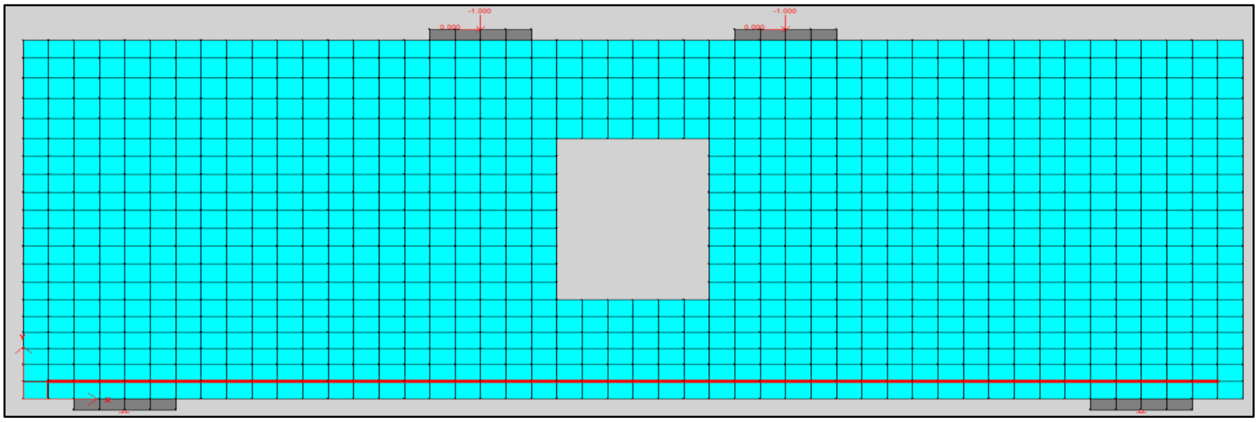
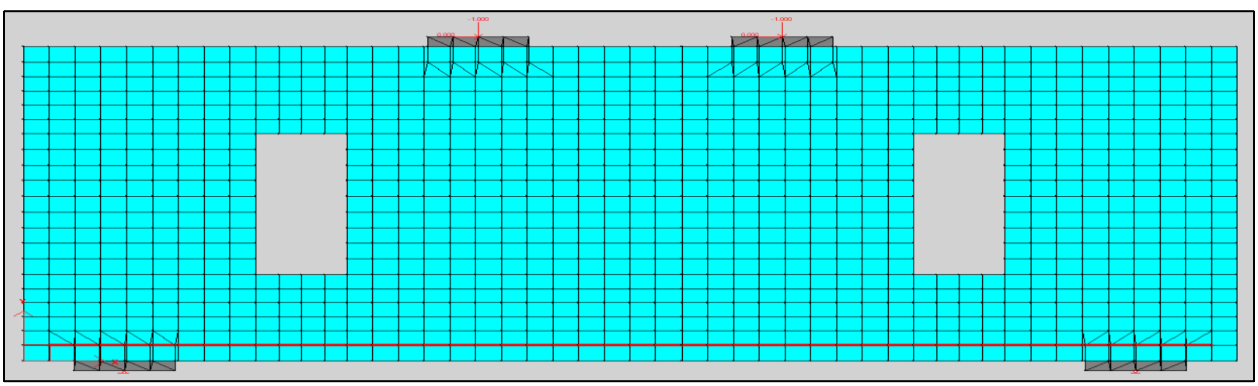


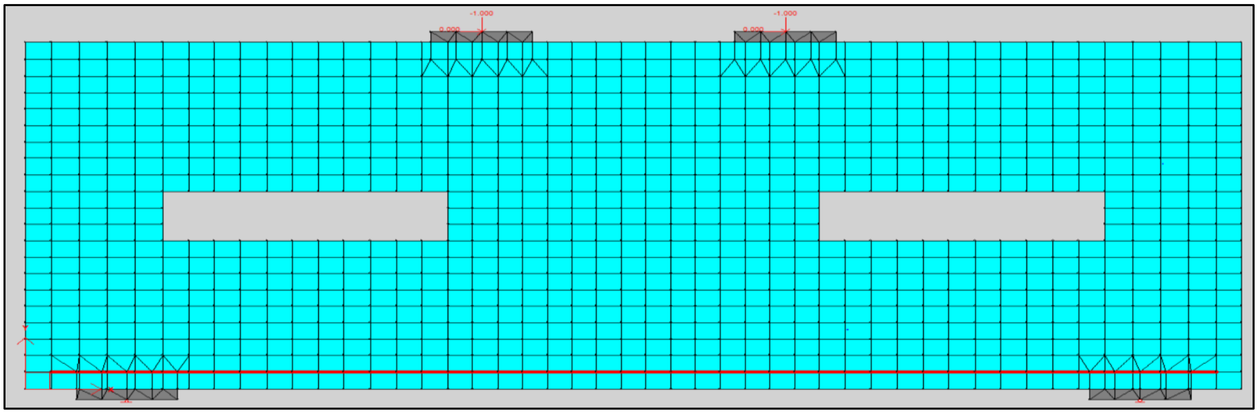
Figure 3-13 Finite element model 150A-1.5FSC25LD-2

3.3.3. Aspect ratio of opening sides

Numerical simulation of RC deep beams having the same opening size area but with different rations of openings sides were conducted in an attempt to investigate the effect of aspect ratio. Twenty-one FE models were created for deep beam with variable length to depth ratios. The FE models can be seen in table 3-3 of row 54 to row 65. Aspect ratio is calculated by dividing the length of the opening by the width of the opening (i.e. L_o/W_o). The locations of the openings were held constant though all the models, the center of the openings coincides with the center of the shear span. five different aspect ratios were considered for beams having different span to depth ratios, the aspect ratios considered are 0.25, 0.5, 1, 1.5 and 2.



(a)



(b)

Figure 3-14 Finite element model having aspect ratio of (a) 90A-2.5SPC25LD-2 (2.5) (b) 282A-0.25PC25LD-2 (0.25)

3.3.4. Compressive strength of concrete

In most cases when unexpected opening are present at last minute during construction , the usual remedial measure taken by most site engineers or resident engineers is to increase the concrete grade of that particular section in order to counteract the possible strength loss that may occur due to provision of openings with in structural members such as beams. Hence in order to investigate the effectiveness of such measures, Seven finite element models were conducted to in an attempt to investigate the effect of compressive strength of concrete on the behavior of RC deep beams with web openings. Other parameter like opening location , opening size and aspect ratio of opening sides were kept constant. The details of the FE models are given in table 3-3 of row 66 to row 73. The cubic compressive strength of concrete considered were C25, C30, C40 , C50 and C60 . The material models for concrete cylindrical compressive strength of less than 40Pa and above 40Mpa are given in section 3.1.2.1a in detail.

4 Result and Discussion

In this chapter, the finite element analysis results will be shown and discussed in detail. The first sub-section under this chapter discusses the result obtained from the numerical simulation of test beams for validation and verification of the finite element model. Two Rc deep beams where modeled (i.e. solid Rc deep beam and deep beam with a 200m square opening), the beams were obtained from the experimental research of (A.Jasim, et al., 2018). The tabular load deflection data of the experimental test results are given in Appendix B While the second sub-section under this chapter discusses the Numerical simulation result of finite element models for investigating the effect of the aforementioned parameters on the behavior of RC deep beams with rectangular web openings, in the order described in chapter 3. A total of 73 Finite element models were created and analyzed and the results obtained are discussed in a clear manner in the following sections.

4.1 Validation of Fea models

4.1.1 Solid Rc deep beam

Two finite element models were created to check the proper mesh size in an attempt to correctly predict the response of the reinforced concrete deep beam subjected to a two-point loading. The experimental investigation test of solid RC deep beam by (A.Jasim, et al., 2018) shows the observed failure load of the beam was 560kN with maximum mid span deflection of 8.2mm and the tabular load deflection data of the beam is given in table A-4 located in Appendix B. The first finite element model of the test beam has a maximum square mesh size of 50mm. the analysis result for this mesh size shows that the beam failure load was 572.8kN and has a maximum mid span deflection of 6.53mm. The calculated errors of the results were 2.3% and 21% for failure load and mid span deflection respectively. Since the error of the maximum mid span deflection is greater than 10% , a finer mesh size is required to reduce the error percentage. The second finite element model with mesh size of 25mm gives a better predication of the failure load and mid span deflection of the beam. Form the finite element analysis results, it was observed that the maximum failure load and mid span deflection obtained was 558.2kN and 7.662mm respectively. The following calculated errors are 0.3% and 6.5 % respectively for failure load and mid span deflection. Since both of the calculated errors are below 10%, it can be concluded the finite element model accurately predicts the response of the beam and further refinement of the mesh size will not be necessary. The load deflection diagram of the experimental test results and the two finite element models with two different mesh sizes are shown in figure 3-6a.

Additional output parameters like cracking patter and failure mode were also used to check the accuracy of the finite element model. As expected, the first crack occurred at the mid span bottom fiber of the beam for the solid beam, the cracks propagated to the left and right of this points, to the support points. Failure occurred when the shear cracks near the support propagated to the compression zone and wider shear cracks were observed above the support point. Since the observed strain in the steel at failure was less than the yield strength of the steel, the as expected mode of failure of the beam is shear.

4.1.2 RC deep beam with web openings

In a similar manner, the output variable selected for comparing the results of the finite element model contained a square web opening and the experimental test are the failure load and the maximum mid span deflection at the ultimate failure load. The load deflection diagram of finite element model of deep beam with web openings for meshes sizes of 25mm and 50mmm and the experimental test result of (A.Jasim, et al., 2018) is shown in figure 3-6b. The experimental tests results form (A.Jasim, et al., 2018) shows that maximum failure load of the beam was 240kN and the maximum mid span deflection was 4mm.The load vs deflection tabular data can be seen in table A-5 in Appendix B section. On the other hand, The first finite element model of the beam with openings has maximum mesh sizes of 50mm, the failure load and mid span deflection obtained were 340kN and 3.45mm respectively. The values obtained from the finite element analysis results are far from the exact values especially the failure load .However the finite element model with 25mm mesh sizes accurately predicts the response of the beam in both failure load and mid span deflection. The failure load and maximum mid span deflection obtained from the finite element analysis results were 255kN and 3.9mm respectively. The calculated error for the two output parameters (i.e. failure load and deflection) are 6.3% and 2.5% respectively. It can be seen that the percentage of error for both cases are below 10% and it can be concluded that the finite element model with maximum mesh size of 25mm accurately predict as the response of such type of beams and no further refinement of mesh sizes is required.

Other output parameters like cracking pattern, the location of the first crack and mode of failure of the beam has also been used to check the validity of the finite element analysis result against the experimental test results. From the several experimental tests on deep beams with openings, it is seen that the predominant mode of failure of such type of beams is shear failure by tensile cracking. The crack causing the collapse of the structure stretches from the load point to the corner point of the opening and form the support point to the other corner point of the opening. In general, the crack width of such type of cracks are significantly higher than the crack width observed in the flexural region of the mid span. Another appealing reason for validation the mode of failure of the beams is that, the observed maximum tensile strain in the steel at failure was 0.4mm/mm, which is less than the yield strain of the steel. This indicates that the steel has not yielded at the ultimate failure of the load and the governing mode of failure is in fact shear.

The exact mode of crack propagation, crack width distribution and mode of failure can be observed form finite element model. Furthermore the location of the first crack for beams with web opening are usually at the opposite corner points of the openings rather than at the mid span, as in the case of beams without web openings which the finite element model accurately captures such a response. In addition, the maximum crack width that was observed in the finite element analysis result was at the corner points of the opening, which is usually the case for such type of beams.

4.2 Parametric study

4.2.1 Control beams

The three control beams having variable span to depth rations of 3, 2.5 and 2 and all other parameters were kept constant. The details of Controls beams were given earlier in the methodology chapter. The Control beams were subjected to a two point static monotonic loading until failure, the load deflection diagrams of the beams are shown in figure 4-1 below. Initially the beams were designed according to ACI318 code to with stand a total load of 260kN.However the observed failure load for the three controls beam with span to depth ratios of 3, 2.5 and 2 were 494kN, 494.4kN and 495kN respectively. The difference in the results arise because the beams were designed using STM which is a lower bound analysis method, it means that it the predicted load resistance by such methods always fails below the true load carrying capacity of the structure. In addition material safety factors are applied in code based design, which reduce the load carrying capacity of the structure in order to make a conservative and safe estimation of the strength of a structure.

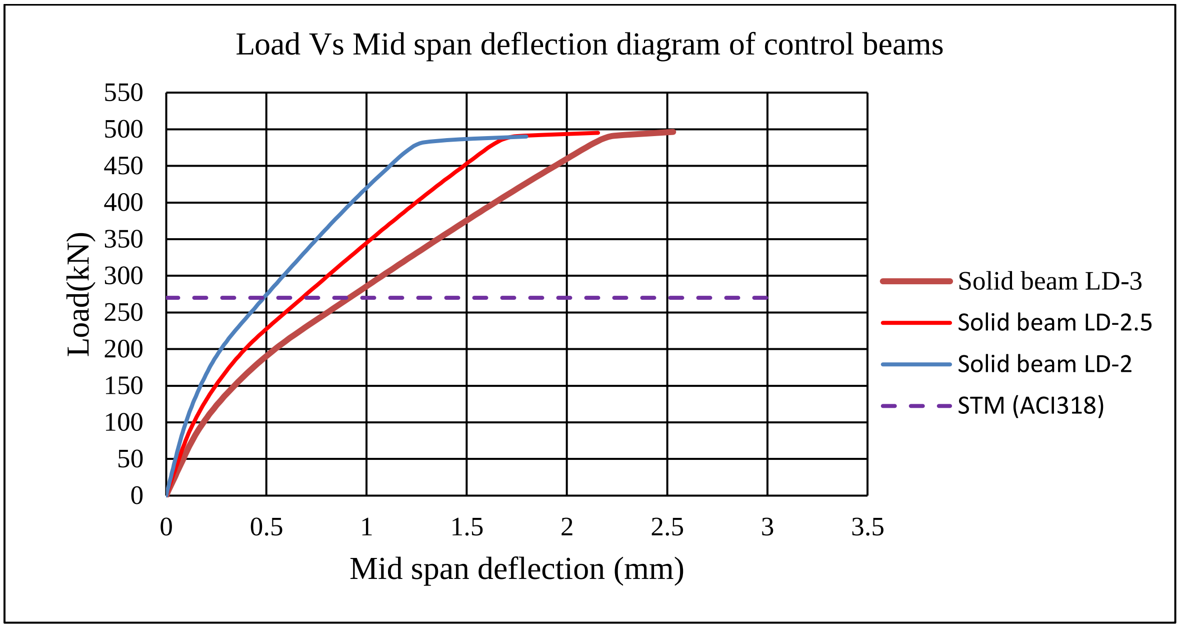


Figure 4-1 Load deflection diagrams of the control beams

The mid span deflection of the beams at the failure load obtained from the finite element analysis were 2.53mm, 2.15mm and 1.79mm for control beam having span to depth ratio of 3, 2.5 and 2 respectively. As expected, since the shear span (i.e. a=350mm) is held content for the three beams, the maximum deflection occurred for the beam having a longer span because deflection has a direct incremental correlation with span length of the beam. Furthermore, the load that causes the first crack for the three control beams was approximately the same, with an average value of 68.0kN. The first crack appeared in the bottom fiber of mid span of the beam, it then propagated to the upper portion of the beam. Since the maximum tensile strain for solid beams subjected to two-point loading occurs at the mid span bottom fiber of the beam the program uses the Mohr-stress criteria for determine the cracking strength of the concrete ,which states that the cracking strength of concrete is a function of principal tensile stress, principal compressive stress and concrete uniaxial compressive strength. The results is consistent to the actual response of such beam in experimental test. Another important output parameter that needs to be considered is the maximum crack width and its location. The finite element analysis result indicates that the maximum crack width is about 1.82mm, 1.7mm and 1.4mm for beams with span to depth ratio of 3, 2.5 and 2 respectively. Location of the maximum crack width is just below the point load in the upper horizontal compression strut of the beam. The program determines the crack width by using empirical equation, which is a function of average tensile strain, and average crack spacing. The results are summarized and given in table 4-1 below.

Table 4-1 FEA analysis results for control beams

Beam designation	L/D ratio	$P_{failure}$ (kN)	P_{crack} (kN)	∆max(mm)	W(mm)
Solid beam LD-3	3	494	66.61	2.53	1.88
Solid beam LD-2.5	2.5	494.4	67.06	2.15	1.7
Solid beam LD-2	2	495	70	1.79	1.4

4.2.2 Opening location

4.2.2.1 Shear span (Degree of interruption)

In an attempt to investigate the effect of degree of interruption of the natural load path by the openings within the shear span, 18 numerical simulation were conducted. Failure load vs degree of interruption of the finite element models is show in figure 4-2 below. It can be seen from the figure that for all of the beams having different span to depth ratios , the failure load decreases as the degree of interruption increases or vice versa. The decrease in strength is expressed as a percentage compared to the failure load of the control beam, a similar trend for all the beams with different span to depth ratios containing web openings. The average maximum decrease in shear strength (b/c. the main mode of failure of beams with web openings is shear) for the three beam classes with span to depth ratios ranging from 3 to 2 is 20.1%, and it occurs when degree of interruption is maximum (i.e. when the opening center is placed at the center of the shear span). As the degree of interruption decrease, the strength loss compared to the beam without web openings decrease as well but the rate of strength gain is small. The main reason for the decrease in failure load as the degree of interruption increases is that ,when deep beams are subjected to in plane loads, the main mechanism in which they resist load is by transferring the load directly to support by in plane stresses which lead to a formation of compression struts between the load point and support. The above compression strut is known as natural load path and the line connecting the loading and support point can approximately represent it. Any interruption of this natural load path by openings can cause significant redistribution stresses that cause high stress concentration especially near the corner pointe of the rectangular openings and eventually leading to significant reduction the shear strength of the beam.

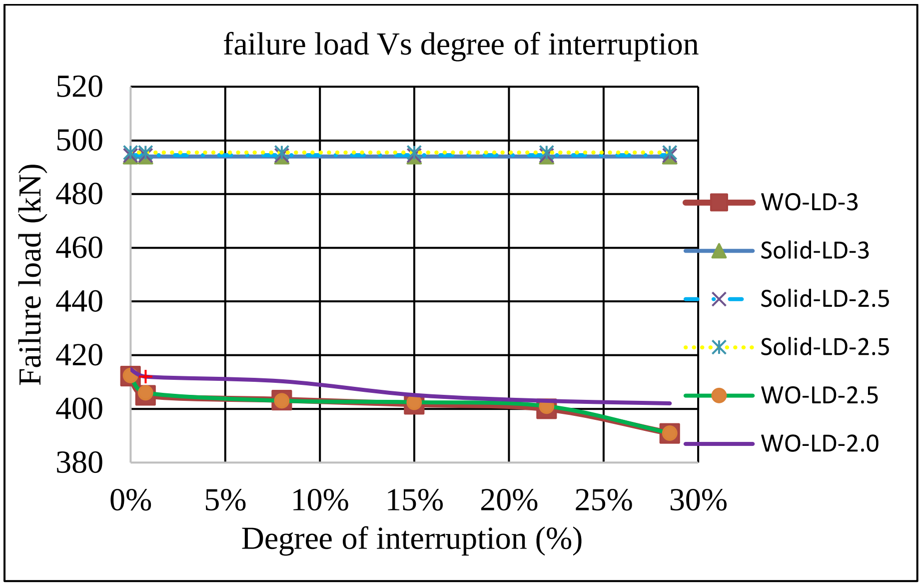


Figure 4-2 Failure load vs degree of interruption diagram

Compared to the web opening having a maximum degree of 28.5%, the strength gain of the beam with 0% degree of interruption is only 4.9%. In average, the rate of increase in strength for 1% decrement in degree of interruption is 0.35%, which indicates that although the shear strength of beams is affected by degree of interruption of the natural load by the web opening, However, its effect is not that significant. As long as the web interrupts, the natural load path there will be significant strength loss in the beam regardless of the value of the degree of interruption. In other words, web openings with small degree of interruption can still cause a significant loss in shear strength of the beam as much as web openings with large degree of interruption. The percentage decrease in strength compared to the solid beam without opening is calculated and given in table A-6 in Appendix B.

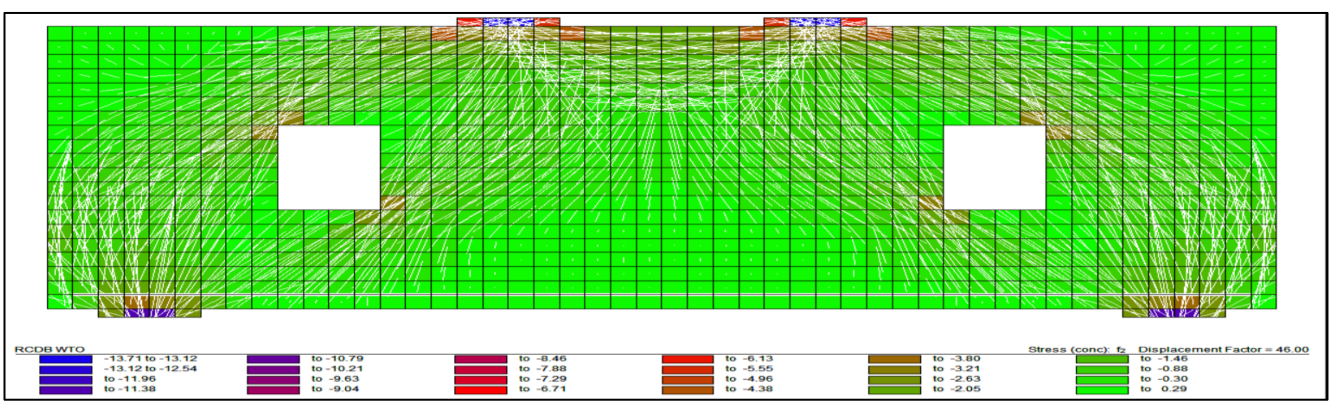


Figure 4-3 principal compressive stress trajectory of model 100A-1.5SPC25LD-2

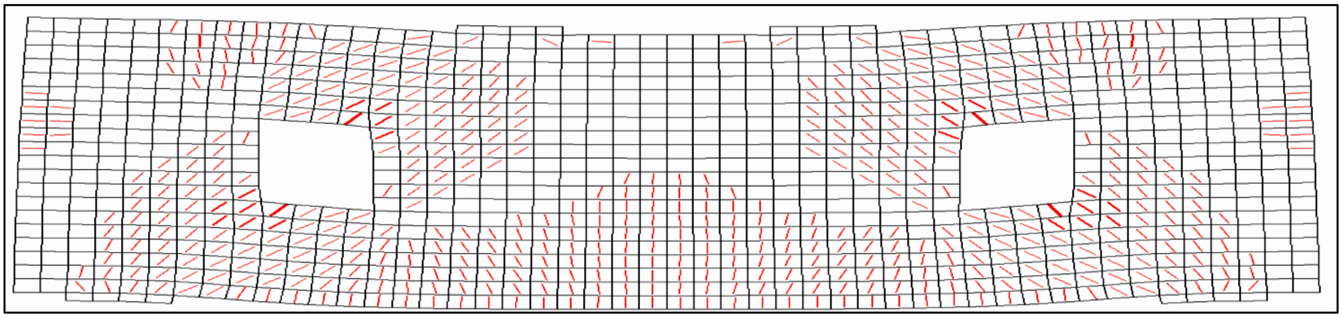


Figure 4-4 Cracking pattern at failure of model 100A-1.5SPC25LD-2

4.2.2.2 Flexural region

Six finite element models were analyzed to investigate the effect of opening location within the flexural zone of the beams. All of the openings provide were located between the loading points, it can be seen for the analysis result that openings located within in the flexural region has insignificant effect on the failure load. figure 4-5 shows that the failure load of the beams with opening located in the flexural region is almost similar to the failure load of beam without web openings (i.e. Solid RC deep beams). The reduction in strength observed in such beams compared to beams without web opening is less than 0.5%. this is mainly because, the openings located in the flexural zone of beams do not intercept any of the compression struts identified for deep beams subjected to two-point loading as show in figure 2-3. The reduction in strength calculated for each of the finite element model is given in table A-7 and it can be seen that the decrease in strength for all of the FEA modes is very small. The variable Lo descries the location of the opening center from the support point and 0.5L describes half the length of the beam (i.e mid span 0.5L/Lo=1).

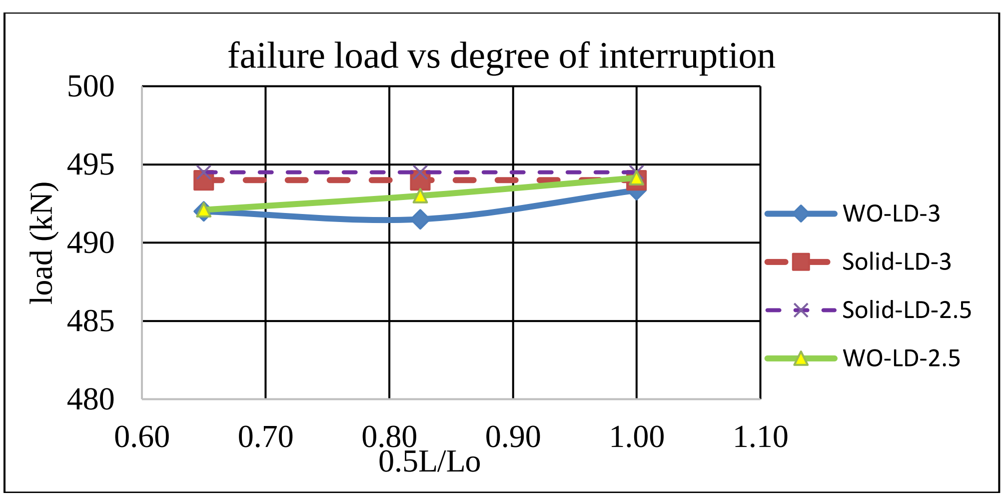


Figure 4-5 Failure load vs opening location in flexural region

The effect of openings location within the flexural region (i.e. located between the two load points) on the mid span defection has been also investigated in this research. Figure 4-6 below shows that there is a considerable increase in deflection in beams with web openings compared to deep beams without web openings, especially when the opening is located at the center of the mid span, increase in deflection about 20% is observed form the analysis result. However for opening located close the load point; an increase in deflection of 3-4.5% was obtained form the finite element analysis results. Thus, it can be summarized that openings located near the boundary of the shear span has little effect on the maximum mid span deflection. In general, the effect of such types of openings is more profound in mid span deflection rather than on failure load. The increase in deflection occurs because of stiffness reduction of the beam around the opening, the opening causes a reduction of moment of inertia of the beam with in that region. Since the moment of inertia of the beam is the most important geometric parameter that has a direct incremental relation with defection in beams. Thus, a decrease in moment of inertia, ultimately results in an increase in vertical displacement of the beam.

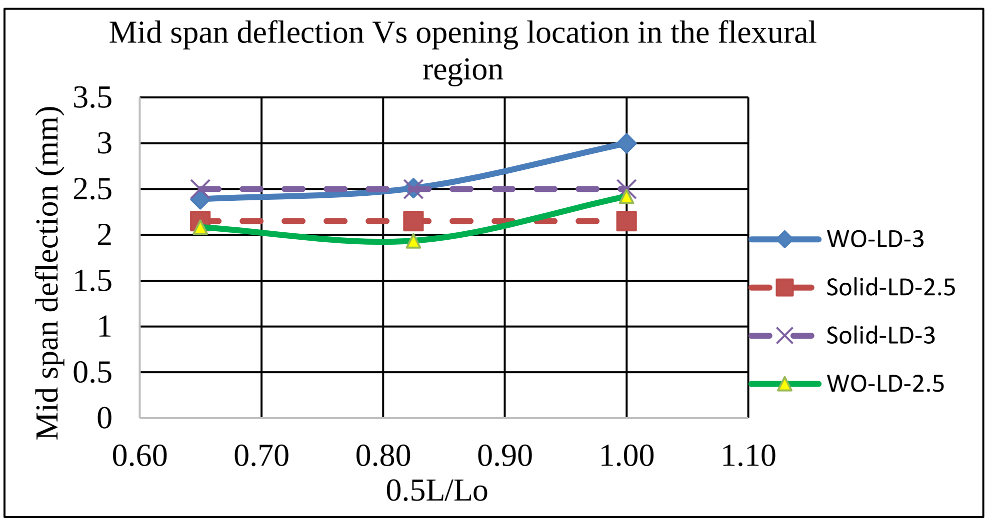


Figure 4-6 Mid span deflection vs opening location in flexural region

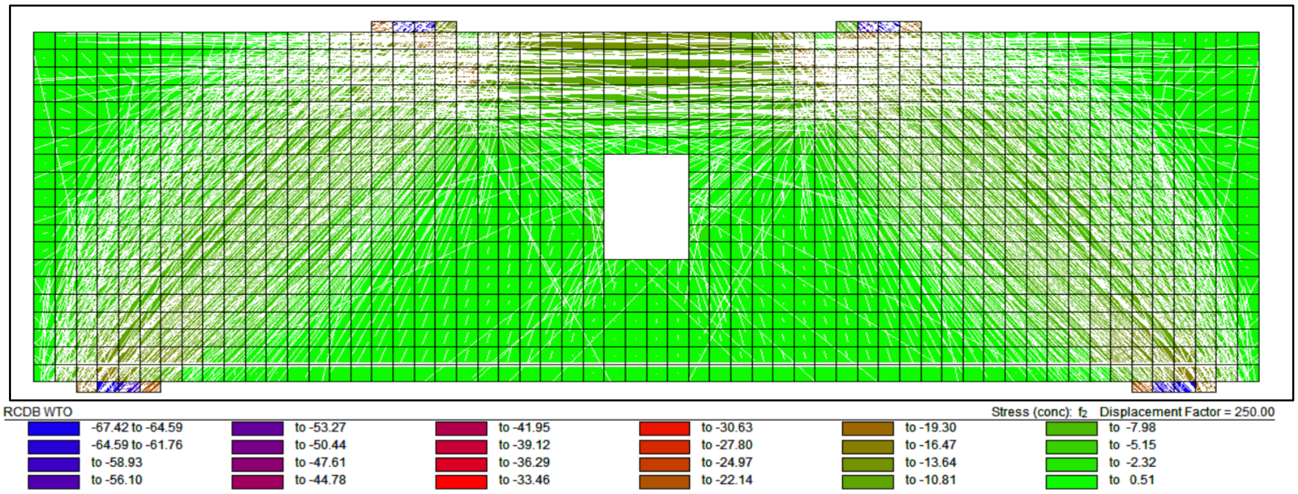


Figure 4-7 principal compressive stress trajectories of model 100A-1.5FSC25LD-2.5

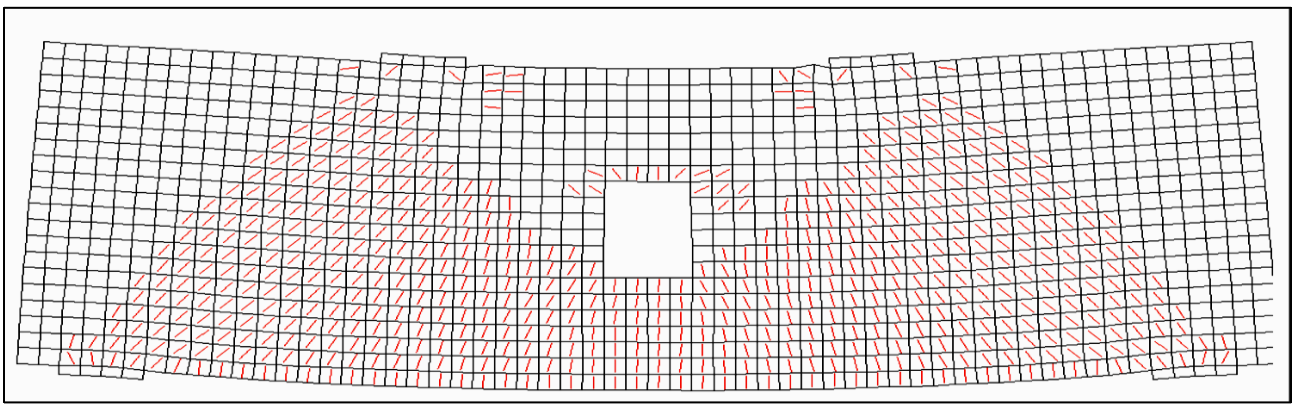


Figure 4-8 Cracking pattern at failure of model 100A-1.5FSC25LD-2.5

4.2.3 Opening size

4.2.3.1 Shear span

Another important parameter that is attempted to be addressed in this research is the effect of opening size on the failure load of beams with web openings located within the shear span of the beam. A total of 12 finite element models have been created and analyzed and the results are given in figure 4-9. It can be observed that as the opening size increase the failure load of the beam falls drastically in the downward direction. For shear opening of greater than 34% of the shear span area , more than 80% reduction in shear strength (failure load) of the beam was observed for beams having span to depth ration of 3 , 2.5 and 3. The maximum strength reduction percentage obtained in average for the three classes of beam was 95.7% which corresponding to a 64.4% opening in shear span. The main reason for the decrease in shear strength of the beam is because when the opening size increase, it interrupts the natural load path further more and more. Which leads to a significant redistribution of internal stresses around the opening sides and causing stress concentration especially at the corner points of the rectangular openings, Ultimately leading to early cracking and failure by diagonal tensile cracking along the line connecting the corner of the rectangular openings and the line connecting the support point to the lower corner point of the opening. Furthermore relatively medium opening sizes interrupt the inclined compression strut only, however as the opening size is wide enough it may also interrupt the horizontal compression strut connecting the two load points, which Leads to an even a more drastic strength loss of the beam. The respective shear strength reduction are calculated and given in table A-8 in Appendix B. Analysis results show that for every 1% increase of opening size within the shear span, a strength loss of 1.8% by average is observed.

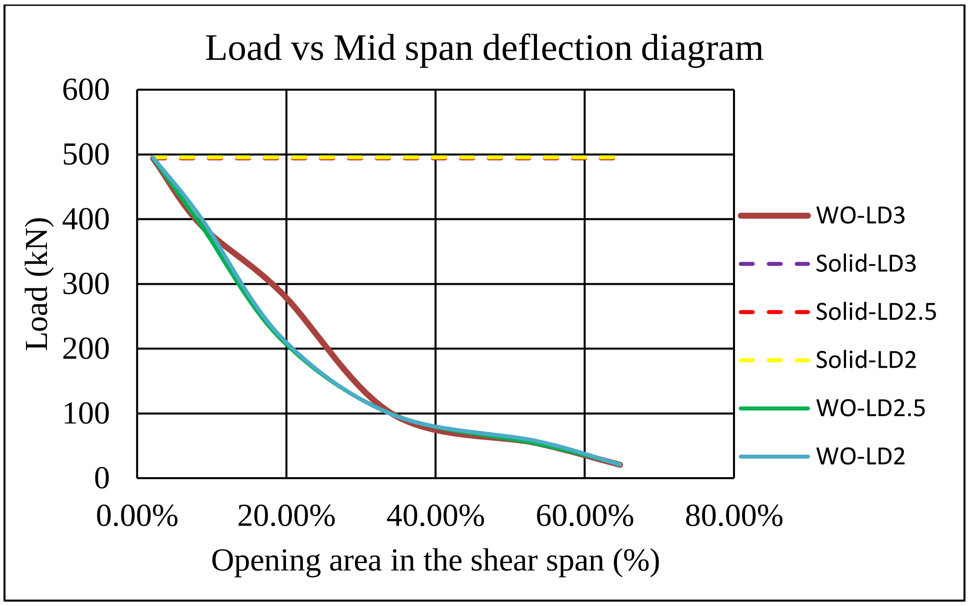


Figure 4-9 Failure load vs opening size in shear span

The 1st crack occurred at the mid span for beams for opening sizes less than or equal 8.57% of the shear span area. For greater opening sizes, the 1st crack occurred at the bottom corner of the opening closest to the support point. The maximum crack width observed for all of the finite element models were at the upper most corner of the opening closest to the load point. The 1st cracking load obtained for opening sizes ranging between 8.57%-64.74% was in average 67.8kN to 9.8kN. The decrease in percentage of magnitude of the cracking load ranges between 0.17% - 85.6% for opening size of ranging between 8.57% - 64.74%. Furthermore, the increase in maximum crack width for the above given range of opening sizes ranges from 25.7% -173.9%.

Since high stress concentration occurs at the corner points of the openings, it creates a large increase in principal tensile strain , which intern is directly related to cracking in concrete. Finally resulting in an increase in the crack width. The maximum crack width and cracking load are given in table A-9.

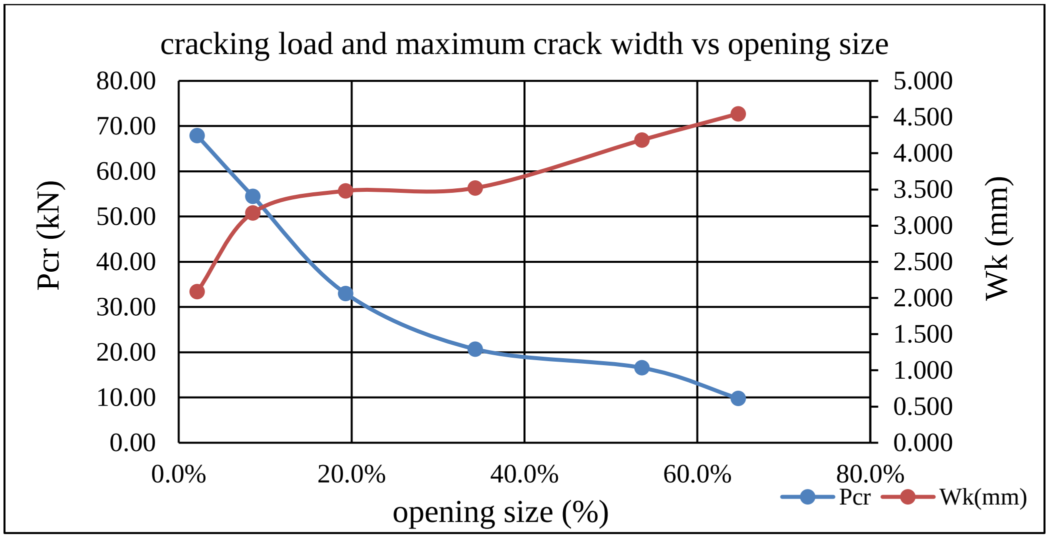


Figure 4-10 cracking load and maximum crack width vs opening size (shear span) diagram

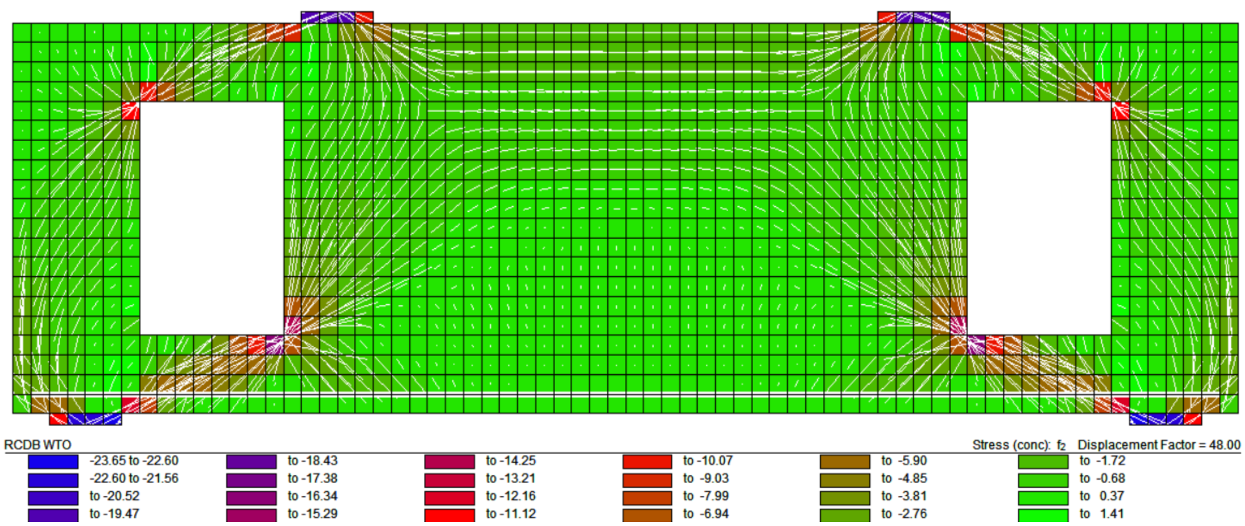


Figure 4-11 principal compressive stress trajectories of model 200A-1.5SPC25LD-2.5

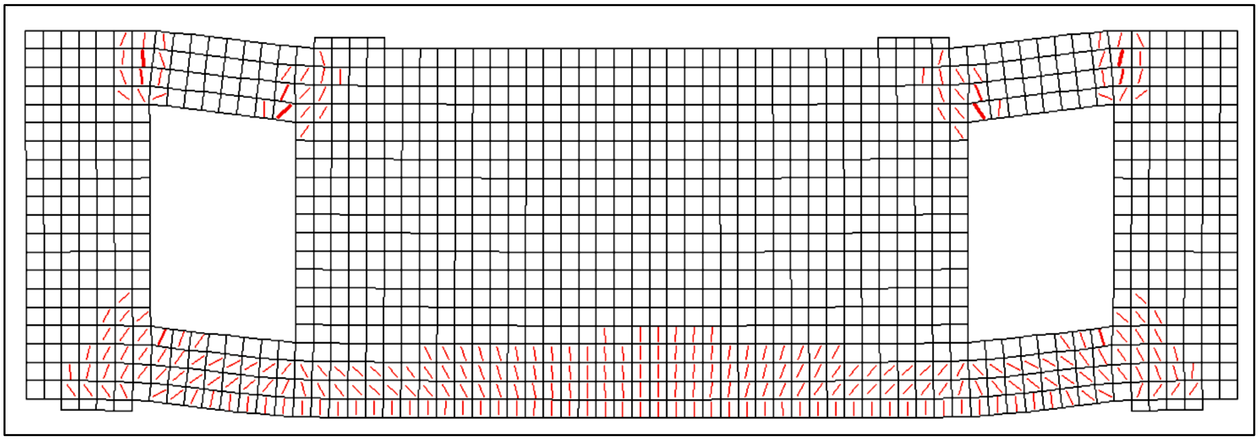


Figure 4-12 Cracking load pattern at failure of model 200A-1.5SPC25LD-2.5

4.2.3.2 Flexural region

As discussed in the previous section, the size of openings located within the shear span has a great deal effect on the shear strength (failure load) of such type of beams. When openings are moved away from the shear span to the mid span of the beam, one can easily estimate that there effects will not be significant in terms of strength reduction of the beam however they might have significant effect on the serviceability limit states of such structures, in terms of deflection and crack width. A total of 10 finite element models have been created and analyzed to investigate these particular effect. As expected openings that are less than or equal to 93,750.00mm2 (250 by 375mm, rectangular openings) show a strength reduction of less than 0.24% and 0.81% for beams with span to depth ratios of 3 and 2 respectively. The highest strength loss in percentage obtained for beams with span to depth ratios of 3 and 2 where 1.48% and 14% respectively. The main reason for insignificant decrease in strength for beam with opening size less than the previously stated area, is that the openings do not intercept any compression struts transferring the applied loads to the respective support points, thus no significant redistribution of stresses causing stress concentration near the opening location will not occur, resulting is a small reduction in strength of the beam. Furthermore a 14% reduction in strength of the beam was observed with opening area of 93,750.00mm2 for beam with span to depth ratio of 2 ,this is due to the interception of the horizontal compression strut by as the opening areas.

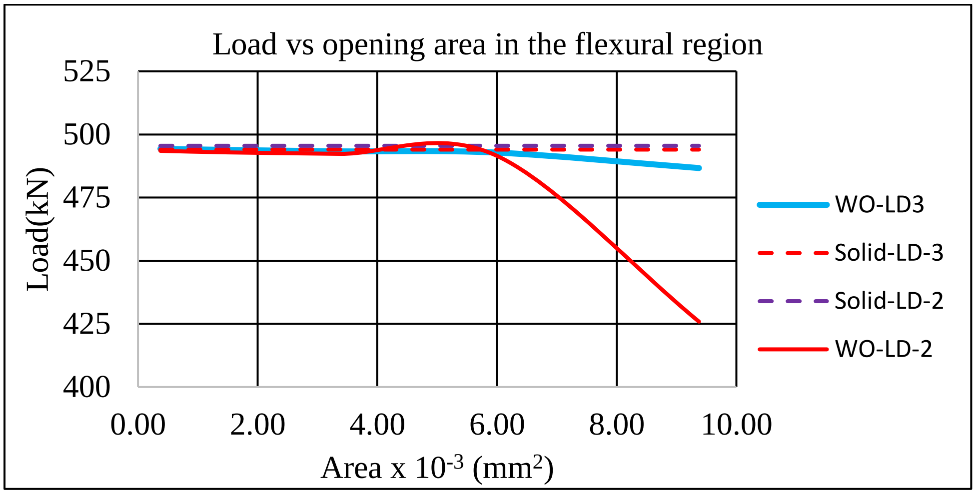


Figure 4-13 Failure load vs opening size in flexural region

The effect of such types of openings in the maximum mid span deflection is quite significant compared to their effect on strength loss. As shown in figure 4-14 the mid span deflection tends to increase when the size of the opening located at the center of the mid span increases. The increase in deflection for opening area of 3,750.00 mm^2 for beams having span to depth ratio of 3 and 2 are 3.44% and 0.56% respectively. In a similar manner, the increase in deflection observed for beams with web opening area of 93,750.00 mm^2 was 13.2% and 176%. The latter almost doubling its magnitude. The maximum increase in mid span deflection occurs at different opening areas for the two beams having different span to depth ratios. The main reason behind this phenomena is , when openings are present in beam web, there will be a deceases in the flexural stiffness of the beam ,and eventually leading to an increase in deflection of the beam, since flexural stiffness (moment of inertia) and deflection are inversely related.

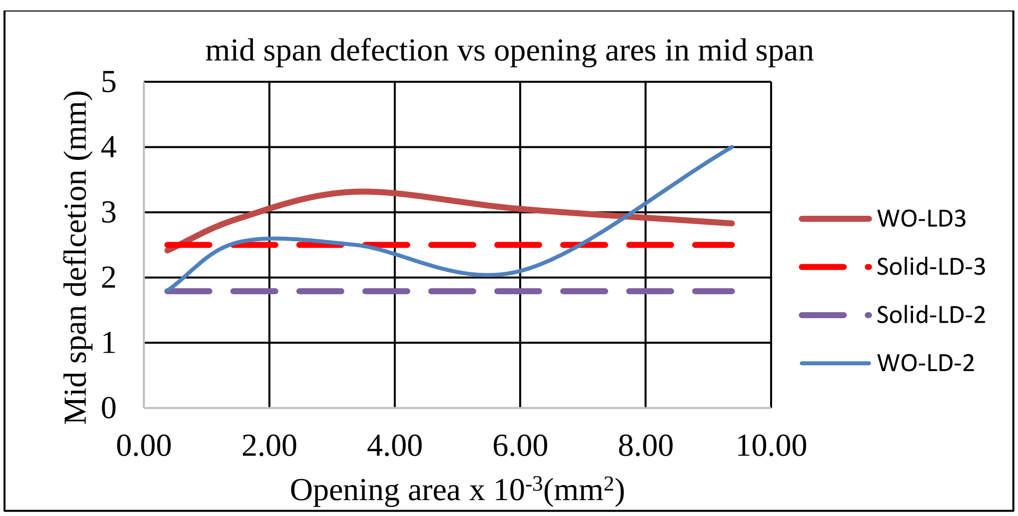


Figure 4-14 mid span deflection vs opening size in flexural region

The cracking load and the location of the first crack for beams with opening located at the center of the mid span are almost identical with the values obtained for the solid beams (without web opening). The location of the first crack is the mid span bottom fiber for all sizes of openings considered. The only slight different in the cracking load occur for beam with web opening of 93,750.00, which decreased by an average of 20%. However, the crack width of such types of beams increased compared to Solid beams without openings. As shown in figure 4-15 below, There is a general incremental direct relationship with the opening area and crack width. The location of the maximum crack width for the different finite element models is found to be either at the point above the upper corner of the opening or the mid-section of the inclined compression strut connecting one of the point loads to the support point.

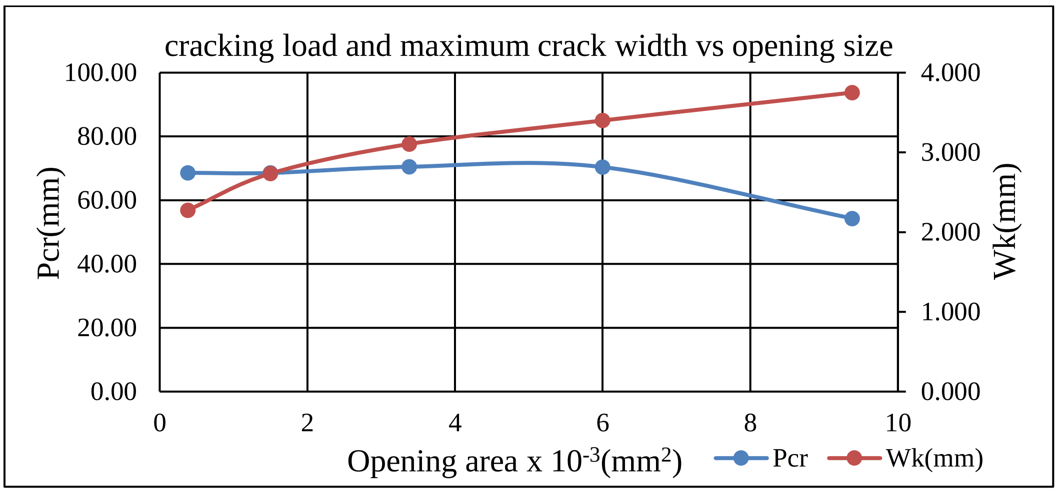


Figure 4-15 cracking load and maximum crack width vs opening size (mid span) diagram

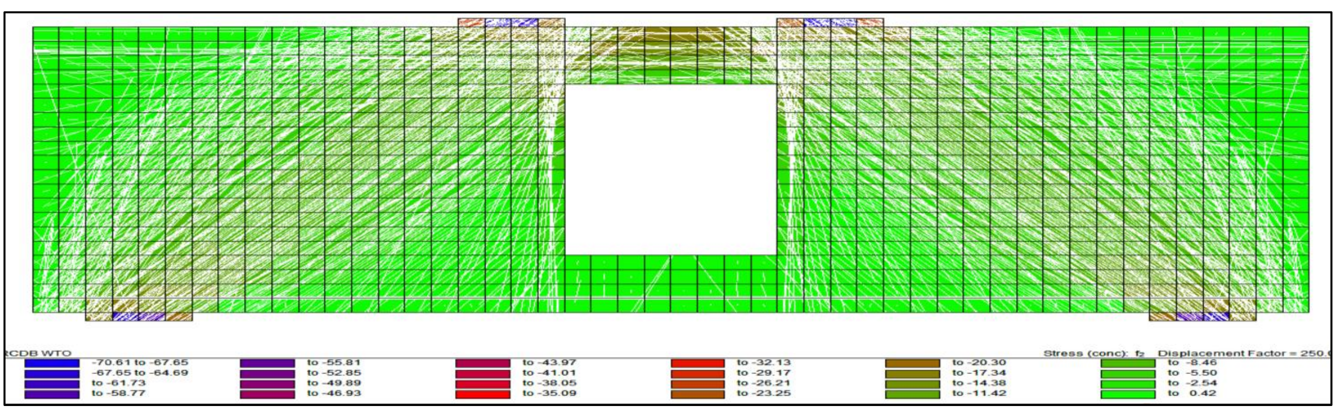


Figure 4-16 Principal compressive stress trajectories of model 200A-1.5FSC25LD-2

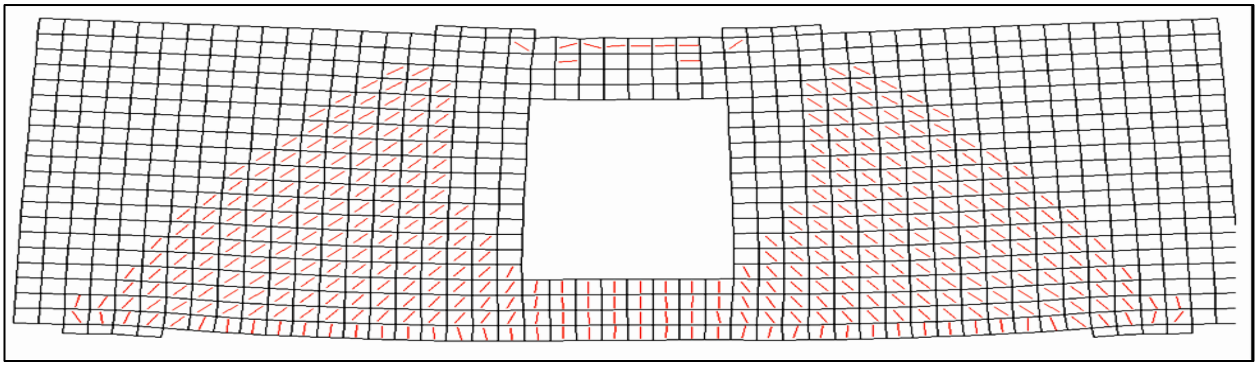


Figure 4-17 cracking patter at failure of model 200A-1.5FSC25LD-2

4.2.4 Aspect ratio of opening sides

Another important parameter that has not been addressed in a detail manner in past researches is the effect of opening aspect ratio on the shear strength beams especially when the opening is located within the shear span of the beam. Despite having the same opening area and same opening location that coincides the center of the opening with the center of the shear span but having different aspect ratios of openings sides , the beams exhibit different properties in terms of shear strength. The finite element analysis results are given in figure 4-18 shows that a direct incremental relationship between aspect ratios of opening sides and shear strength of the beams, thus as the aspect ratio of the opening sides decrease, the shear strength or failure load of the beam also decrease. Form the finite element models, the maximum strength reduction compared to solid beams occurs for aspect ratio of 0.25 which corresponding to an average of 56% strength loss. In addition, the minimum strength reduction for FEM considered occurs for aspect ratio of 2 corresponding to an average strength loss 34.3% is observed and for opening side aspect ratio ranging between 0.5 and 1.5, an average strength reduction range from 48% to 38% .

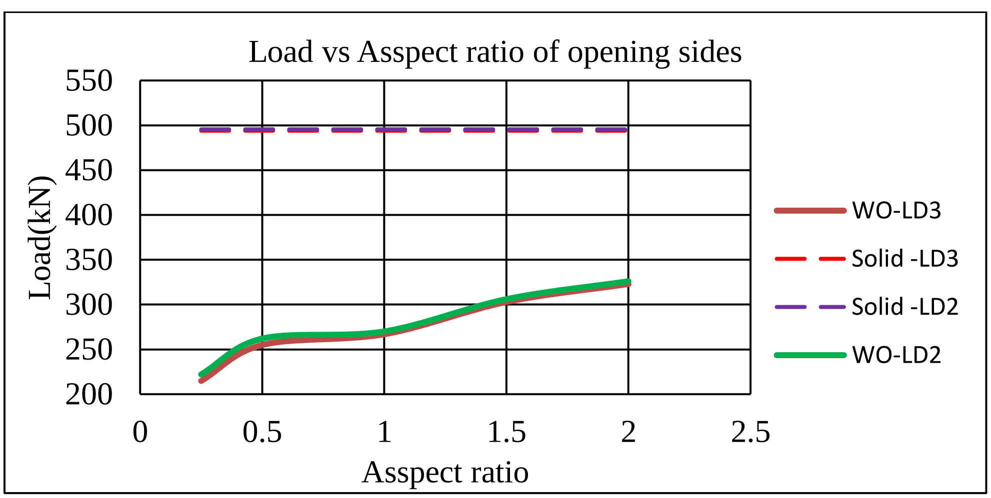
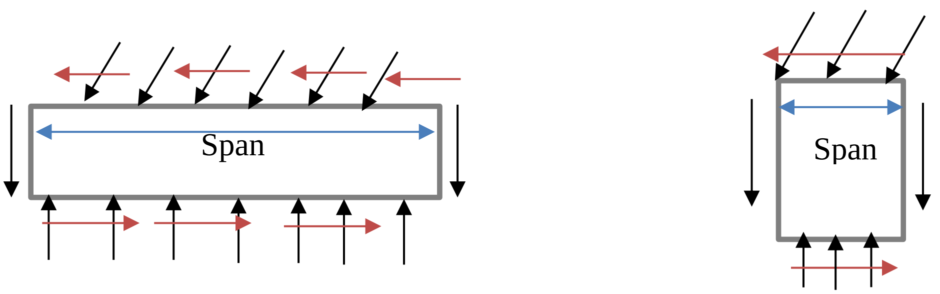


Figure 4-18 failure load vs Aspect ratio of opening sides

The main reason for having a direct incremental/ detrimental relationship between the shear aspect ratio and shear strength of such type of beam can be explained by observing the stress trajectories (compressive and tensile stresses) of beams with web opening shown in figure 2-21.b. The openings intercepts the principal compressive stresses and principal tensile stresses at act transvers to the direction of the compression struts and if we isolate the opening and apply the compressive stresses as a load, the opening can be ideally be represented by a hollow beam supported on its sides and subjected to a compressive load at its top as shown if figure 4-19 below. Openings with lower aspect ratios, have a higher span length in the x direction. So larger span means a larger moment and thus creating a larger tensile stress in the bottom part of the opening, ultimately causing a higher stress concentration at the corner points of the openings. In table A-10 there is a clear discerption of strength reduction of the beam for each finite element model. Opening sides with aspect ratio of less than 1 tend to have a more critical damaging effect on the shear strength of the beam compared for opening sides with aspect ratio of greater than 1.



(a) (b)

Figure 4-19 Idealized Free body diagram of (a) Very low aspect ratio (<< 1 b) high aspect ratio (>>1)

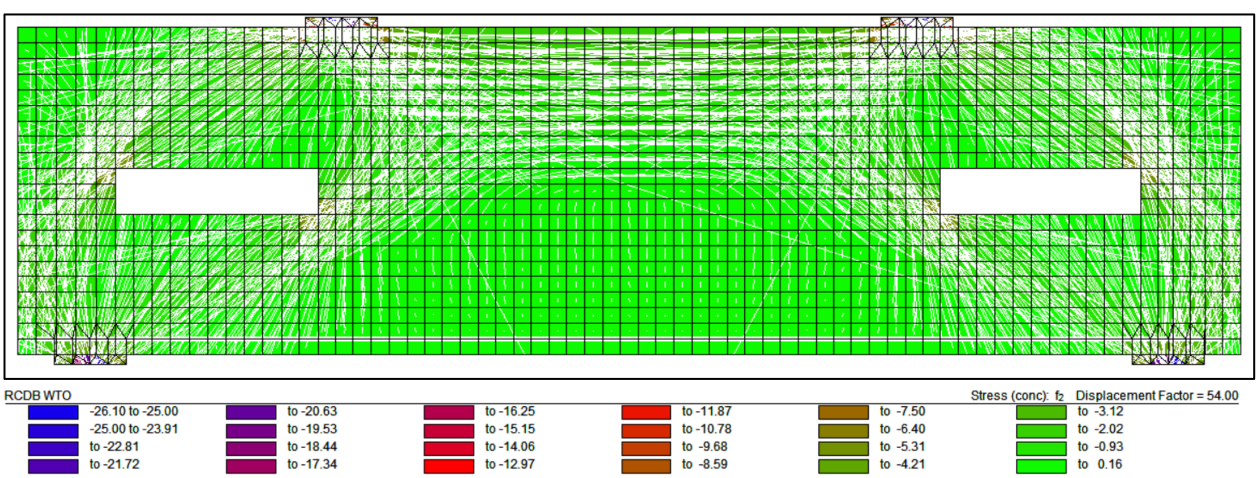


Figure 4-20 Principal compressive stress trajectories of model 281A-0.25SPC25LD-3

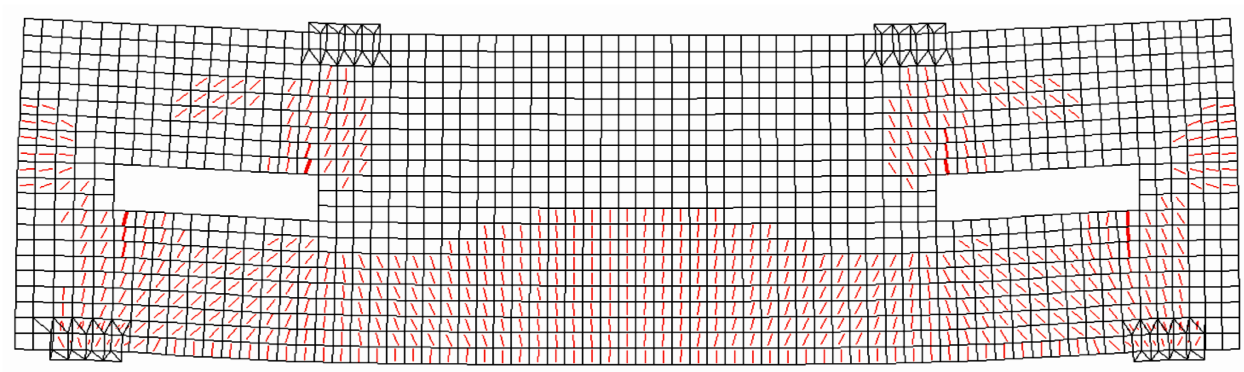


Figure 4-21 cracking patter at failure of model 281A-0.25SPC25LD-2

4.2.5 Compressive strength of concrete

When openings are present in deep beams , it is generally assumed that there will be a decrease in the load carry capacity of the beam, so most of the time the solution provided for counteracting the strength loss is to increase the compressive strength of the beam. This research attempts to investigate the effect of increasing the compressive strength of concrete in the shear strength of the beam. A total of 10 finite element models have been created and analyzed to examine the effectiveness of such type of solutions. From figure 4-22 below , for small openings within the shear span having a total opening area of less than or equal to 15,000mm^2, increasing the compressive strength will lead to a considerable increase in the failure load (shear strength) of the beam. From the finite element models considered, the minimum strength reduction of the beam compared to the solid beam occurs for cubic compressive strength of 60Mpa, which results in 8.36% strength reduction only. On the other hand, the same beam with cubic compressive strength of 25mpa, exhibits a strength reduction of 18.86% , which is much larger, compared to the former. So for beams with small openings within the shear span, increasing the compressive strength of concrete can considerably increase the shear strength or failure load of the beam.

For large openings within the shear span of the beam, the effect of compressive strength is rather different than for small openings. It can be seen form table A-11, increasing the compressive strength of concrete does not have significant effect on the shear strength of the beam. FEA results show that for web opening of 60,000.mm^2 with in the shear span of the beam having maximum degree of interruption, the strength reduction observed for concrete grade of C25 and C60 were 80.28% and 75.28% respectively. Even though there is 140% increase in compressive strength of concrete, there is only a 0.35% increase in the failure load of the beam as the concrete grade changes form C25 to C60. Which clearly shows , for large openings within the shear span causing a considerable decrease in strength to the beam, increasing the compressive strength of concrete does not have significant effect on the strength of the beam.

When large opening within the shear span of the beams are present, there will be early cracking because of tensile stress concentration present around the boundary of the openings. Since the tensile strength of concrete does not increases in a significant manner as the grade of concrete increases, early cracking is unavoidable and cracks will propagate.

In addition, crack width the critical location in the beam will increase as well, finial leading to collapse of the beam. So in general it can be summarized as increasing the concrete grade will only be effective in increasing the strength of the beam for beam with small openings only but for beams containing large openings, its effect is insignificant and other methods of strengthening must be considered.

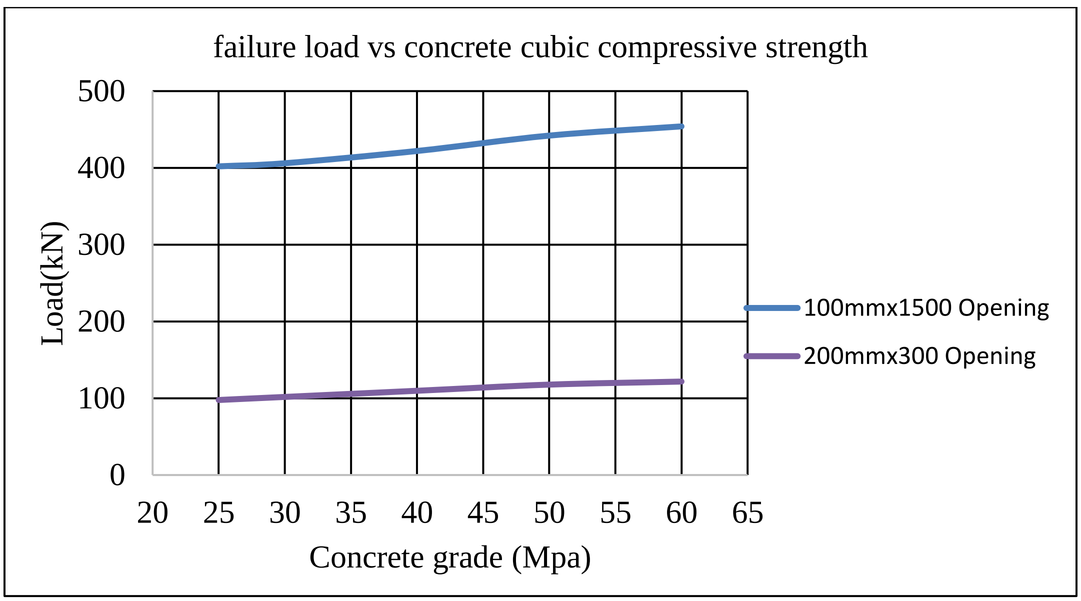


Figure 4-22 failure load vs concrete cubic compressive strength diagram

5 Conclusion and recommendation

In this research, the need for investigating the behavior of reinforced concrete deep beams with web openings was outlined in detail and a special emphasis on rectangular web openings were given. Even though there had been a few researches on this particular topic, some parameters like degree of interruption of the load path by the openings, aspect ratios of opening sides, effects of openings sizes and compressive strength of concrete on the general behavior of such type of beams have not been thoroughly investigated in the past. This research attempts to fill those gaps observed and provide addition technical knowledge. In general, openings in RC structures like beams and slabs are almost unavoidable, and most of the time there are not usually considered in the design stage , thus decisions on where to provide the opening and to allow the maximum openings size to be created is usually decided on site and since standard building codes do not provide any guidelines towards such type of problems. When openings are presents in deep beams the problem gets even bigger, since deep beams are usually subjected to high concentrated loads. This research’s main goal is to provide additional technical knowledge so that it may assist in handling web openings scientifically so that beams are safe and serviceable.

5.1 Conclusion

The following conclusions can be derived from the results obtained for the numerical simulations:

- Opening location
    - ✓ The maximum strength in reduction in RC deep beams occurs when the opening is located at the center of the natural load path (the line connecting the point load and the support point) or when the degree of interruption of the natural load path is maximum for the given opening size.
    - ✓ The failure load or shear strength of beams with transverse openings with in the shear span decreases when the degree of interruption of the natural load path increases. However, As the load as the compression struts are intercepted by openings there will still be a significant strength loss regardless of the magntidue of the degree of interruption (i.e, the rate of loss of strength as the degree of interruption increases is small).

- ✓ Openings located within the flexural region of the beam (i.e. between the two point loads) have insignificant effect on the failure load of the beam regardless where they are placed in the flexural region as long as the openings do not intercept the horizontal compression strut connecting the two point loads.

- Opening size
  - ✓ When the size of openings located at the center of the shear span, increases the failure load (shear strength) of the beam decreases drastically. For percentage of openings sizes, greater than 34% cause a shear strength reduction of more than 80%.
  - ✓ The size of openings located at the center of the mid span has insignificant effect on the failure load (i.e. failure load remains constant even if the opening size increases) of the beam as long as the opening does not intercept the horizontal compression strut connecting the two point loads.
  - ✓ The cracking load of the beam decreases when the size of openings located within the shear span of the beam increases. However the magnitude of the cracking load is the same as the cracking load of solid RC deep beams (i.e. beams without web openings)
  - ✓ The maximum crack width increases when the sizes of openings located within the shear span or mid span of the beam increases and the location of the maximum crack width is the top corner of the opening. The magnitude of the crack width doubles when opening areas within the shear span is greater than 53.57%.
- Aspect ratio of openings sides
  - ✓ The failure load of beams with openings located at the center of the shear span, decreases as the aspect ratios of the opening sides decreases for the same opening area. Openings having aspect ratio of less than 1 have a lower shear strength or failure load compared to openings with aspect ratio of greater than 1 for the same opening area.
- Compressive strength of concrete.
  - ✓ The shear strength of beams with small web openings (< 8.57percentage) located at the center of the shear span increases when the concrete grade increases.

However, for large openings size , increasing grade of concrete will result in a very small increase in strength of the beam that is almost insignificant.

5.2 Recommendation

- For RC deep beams subjected to a Two point loading , the recommended ideal location and size of rectangular opening is when the left edge of the opening side is just to the right of the loading plate and the opening depth is less than 50% of the depth of the beam which is similar to Test specimens 100A-1.5FPC25-LD3/2.5.
- The finding of theses research applies only to simply supported RC deep beams with rectangular web openings. The same idea and methodology can be extended to continuous beam with web opening to investigate the effect of continuity on the behavior of such types of beams.
- The type of loading considered in this research is a static monotonic load , investigating the behavior of RC deep beams with rectangular openings subjected to impact loading is one research area that has not been investigated in detail in the past.
- This research attempted to investigate on how to strengthen beams with web openings by only increasing the concrete grade and observing the response of the beams. However, other method of strengthening deep beams with rectangular web openings such as using fiber polymers (i.e. glass fibers, carbon fibers or bamboo) has not been considered.
- The same research can be extended to investigate the effect of rectangular web openings on fiber reinforced concrete deep beams.
- Since building standards and codes do not provides any guide lines on how to analyze such type of beams, and this research utilized nonlinear finite element analysis tool to investigate the behavior of such type of beams. If other methods analysis for such type of beams like proposing a strut and tie model for RC deep beam with web openings will further simplify and assist the analysis and design of such types of beams.

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Appendix A

A.1 Strut and Tie model and analysis of the control beams

a Control beam 1

To initially develop the strut-and-tie model, it is convenient to know the depth of the compression block of concrete. Assuming that there is only one layer of Φ16 longitudinal bars provided for positive reinforcement, effective depth of the section can be calculate as:

$d = h - cover - d_{b\ stirrup} - d_{b\ main}$

$= 500 - 25 - 8 - 16 = 451mm$

According to ACI 318 code, $Mu \leq \Phi Mn$

The shear forces and bending moment diagram of the beam are given shown below in figure A-1

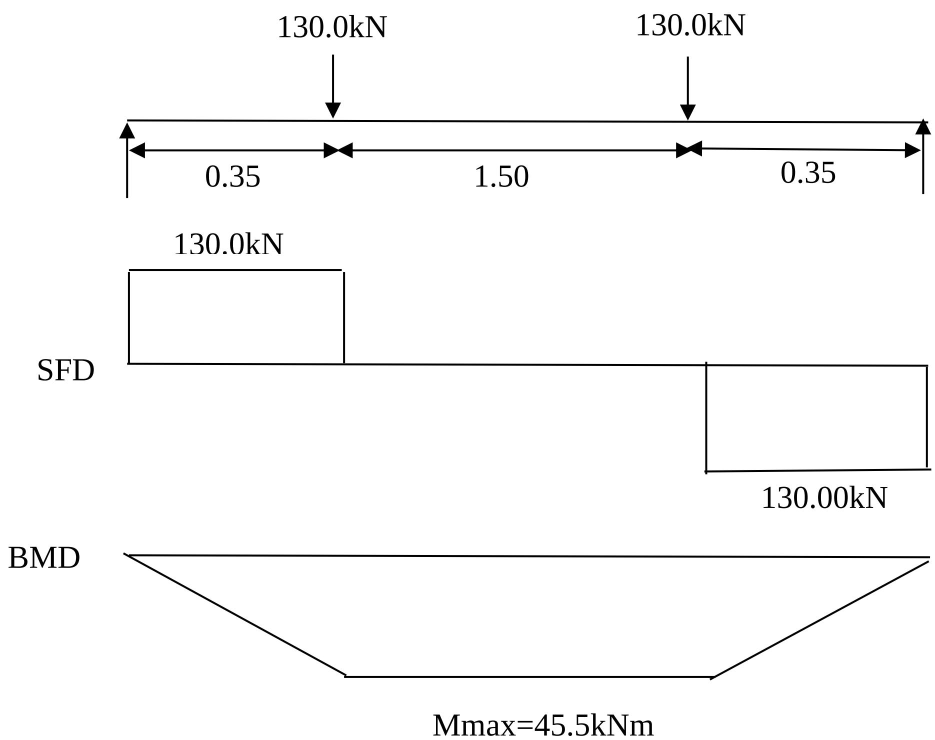


Figure A- 1 Bending moment and shear force diagram of control beam 1

According to ACI 318M building code $M_u \leq \Phi M_n$, where M_u is the applied maximum moment, M_n is nominal section moment resistance capacity shown in equation A-1 and Φ is strength reduction factor taken as 0.9 for flexural strength

$$M_n = \phi A_s F_y (d - \frac{a}{2}) \quad (A.1)$$

$$a = \frac{A_s f_y}{0.85 f_c' b} \quad (A.2)$$

Where As is flexural reinforcement, f_y is yield strength of reinforcement, fc is cylindrical compressive strength of concrete, d is effective depth of the beam and a is depth of the neutral axis multiplied by 0.8

Inserting equation A.2 in Equation A.1 and solving for A_s we will get the following quadratic equation

$$A_s^2 - \frac{1.7 f_c' b d A_s}{f_y} + \frac{1.7 f_c b M_n}{\phi f_y^2} = 0 \quad (A.3)$$

Substituting all the material property and geometrical data in equation A-3 we will get

$$A_s^2 - \frac{1.7 \times 20 \times 150 \times 451 \times A_s}{400} + \frac{1.7 \times 20 \times 150 \times 45.5 \times 10^6}{0.9 \times 400^2} = 0 \quad (A.4)$$

$$A_s^2 - 5750.25 A_s + 1611458.33 = 0$$

(A.5)

Using quadratic equation solution formula

$$A_s = \frac{5750.25 \pm \sqrt{(-5750.25)^2 - 4 \times 1611458.33}}{2} = 295.41 mm^2 \quad (A.6)$$

$$a = \frac{295.41 \times 400}{0.85 \times 20 \times 150} = 43.61 \quad (A.7)$$

$C = \frac{a}{0.8} = \frac{43.61}{0.8} = 54.52 mm$, Take C as 65mm

The strut-and-Tie model for control beam 1 is shown in figure A-2 below

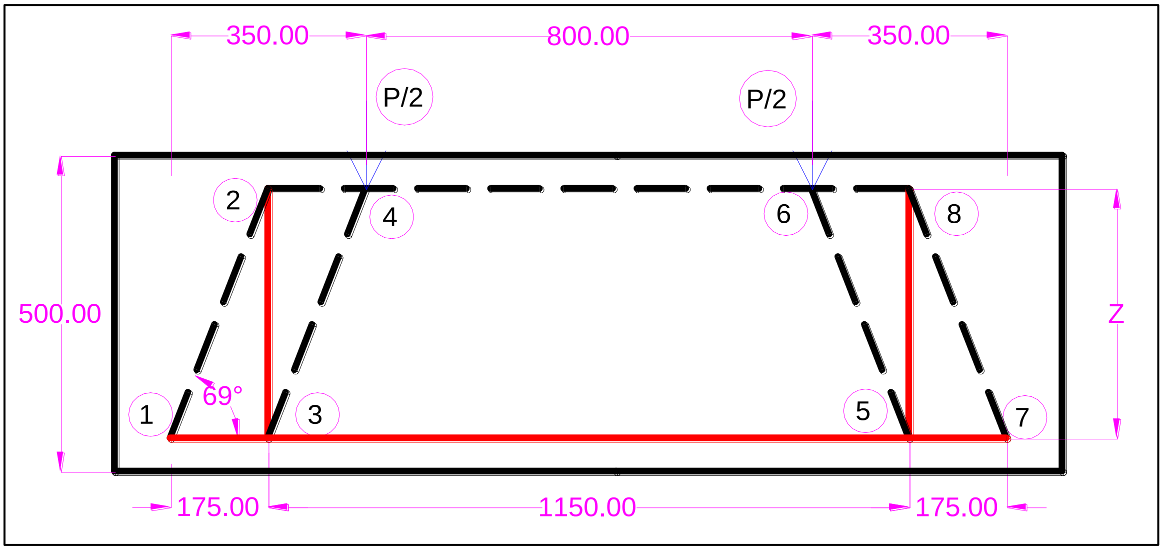


Figure A- 2 Strut-and-Tie model of control beam 1

By applying equilibrium condition at each joint, the member forces of the truss model are calculated and given in table A-1 below

Table A- 1 Member forces of STM of control beam 1

Member	Angle form x-axis(degree)	Force (kN)	Type
1-2	69.00	139.25	C
1-3	0.00	49.90	T
2-4	0.00	49.90	C
2-3	90.00	130.00	T
3-4	69.00	139.25	C
3-5	0.00	99.80	T
4-6	0.00	99.80	C
5-6	69.00	139.25	C
6-8	0.00	49.90	C
5-7	0.00	49.90	T
5-8	90.00	130.00	T
8-7	69.00	139.25	C

After determination of member forces of the truss model, the next step is to verify the strength of the strut and nodal zone. The verification of strut and nodal zones should be checked by comparing the available strut or nodal area with the required, therefore for strut and nodes the wreq will be is calculate using the following equation shown below

$$W_{req} = \frac{F_u}{\phi 0.85 \beta_s f_c ' b} \tag{A.8}$$

Strut 1-2 sample calculation

$$W_{req} = \frac{F_u}{\phi 0.85 \beta_s f_c ' b} = \frac{139.25 \times 10^3}{0.75 \times 0.85 \times 0.8 \times 20 \times 150} = 91.01mm \tag{A.9}$$

Where

Φ=0.75; for STM ,βs=0.8 for strut anchoring one tie and b is width of the beam which is 150mm

The remaining calculation are summarized and can be seen in Table A-2 and Table A-3 below

Table A- 2 Verification of strength of strut

Element	Node i-j	Force (kN)	βs	Wreq(mm)	Wprov(mm)	Check
1	1&2	139.25	0.8	91.01	111.28	Ok
2	2&4	49.90	1	26.09	65.00	Ok
3	3&4	139.25	0.8	91.01	119.27	Ok
4	4&6	99.80	1	52.19	65.00	Ok

Table A- 3 Verification of strength of nodal zones

Node	**Type**	**Force**	**Fu**	β**n**	**Wreq**	**Wprov**	**check**
1	CCT	Rxn	130.00	0.8	84.97	100.00	Ok
		1&2 (C)	139.25	0.8	91.01	111.28	Ok
		1&3 (T)	49.90	0.8	32.62	50.00	Ok
2	CCT	1&2(C)	139.25	0.8	91.01	111.28	Ok
		2&3 (T)	130.00	0.8	84.97	90.32	Ok
		2&4 (C)	49.90	0.8	32.62	65.00	Ok
3	CCTT	1&3(T)	49.90	0.6	43.49	53.53	Ok
		2&3 (T)	130.00	0.6	113.29	200.00	Ok
		3&4 (C)	139.25	0.6	121.35	119.27	Not Ok!!!
		3&5(T)	99.80	0.6	86.98	53.53	Distribution of steel
4	CCCC	P/2	130.00	1.00	67.97	100.00	Ok
		2&4(C)	49.90	1.00	26.09	65.00	Ok
		3&4 (C)	139.25	1.00	72.81	119.27	Ok
		4&6 (C)	99.80	1.00	52.19	65.00	Ok

Verification of the bearing strength at support and loading

The ultimate load and the reaction are (V=R) 130kN

Since the loading plate is 150mmx100mm, the compressive stresses are

$$\sigma_a = \frac{V}{A,plate} = \frac{130\times10^3}{100\times150} = 8.6Mpa \quad (A.9)$$

Since 8.6Mpa ≤ (0.85×20Mpa)   Ok

Steel required in Ties

Once the strength of struts and nodes are verified, the amount of steel is required in ties are calculated

- **Tie (1-3)**

$$As, req = \frac{F_u}{\Phi f_y} = \frac{49.9\times10^3}{0.75\times400} = 166.34mm^2$$

Use Φ 12mm, n= $\frac{As, req}{as} = \frac{166.34}{113.09} = 1.47$ use $2\Phi 12$ bar

- **Tie (2-3)**

$$As, req = \frac{F_u}{\Phi f_y} = \frac{130.0\times10^3}{0.75\times400} = 433.3mm^2$$

Use Φ 8mm , n= n= $\frac{As, req}{as} = \frac{433.33}{50.2} = 9$

Assuming the reinforcement will be distributed within a width of 1000mm

$$s = \frac{1000}{n} = \frac{1000}{9} = 111mm$$

Provide Φ 8mm clc 110mm for the entire section of the beam

- **Tie (3-5)**

$$As, req = \frac{F_u}{\Phi f_y} = \frac{99.9\times10^3}{0.75\times400} = 332.68mm^2$$

Use Φ 14mm, n= $\frac{As, req}{as} = \frac{332.68}{153.9} = 2.16$ use $3\Phi 14$ bar for the entire length of the beam

Reinforcement for bottled shape strut

In Aci 318 Section A.3.3, it is specified that the layers or girds of reinforcement parallel to the plane of the member must cross struts 1-2 and 3-4 .ACI 318 gives the following provision

$$\sum \frac{A_{si}}{2s_i} \sin \gamma i = \rho vi \sin \gamma i \geq 0.003 \quad (A.10)$$

Where

A_{si} is the total reinforcement at spacing s_i in a layer of reinforcement with bars at angle γi to the axis of the strut, suppose only horizontal reinforcement is provided so γi is 69^0 reorienting Equation A.4 and substituting the value of γi and b we will get

$$\frac{2A_{si}}{150si} = \frac{0.003}{\sin 69} \quad (A.11)$$

To control cracking, ACI code puts a limit on the maximum center-to-center spacing of reinforcements give as

$$S = \min \begin{Bmatrix} \frac{D}{5} \\ 305mm \end{Bmatrix} = \min \begin{Bmatrix} \frac{500}{5} = 100mm \\ 305mm \end{Bmatrix} = 100mm \quad (A.12)$$

Substituting si in equation A.6 ,As_i is found to be 24.1mm^2 ,so Φ6 clc 100mm is provided

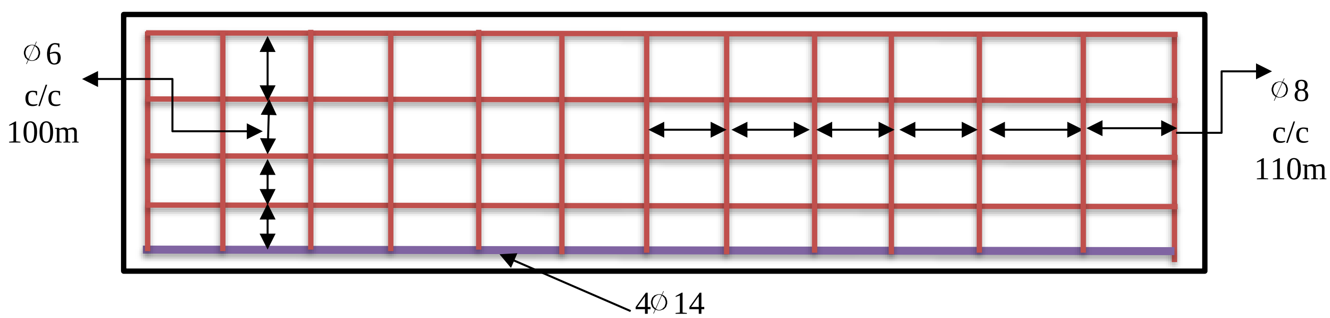


Figure A- 3 Reinforcement arrangement

b Control beam 2&3

Since the shear, span and depth of the beams are the same for all the control beams, thus the bending moment diagram, shear forces diagram and the member forces from the strut and tie model are all the same. Therefore, the required amount of reinforcement in the ties of the STM for all beams will be the same as well. The strut-and-tie models of control beam 2 and 3 are shown in figure A-4 and figure A-5 respectively.

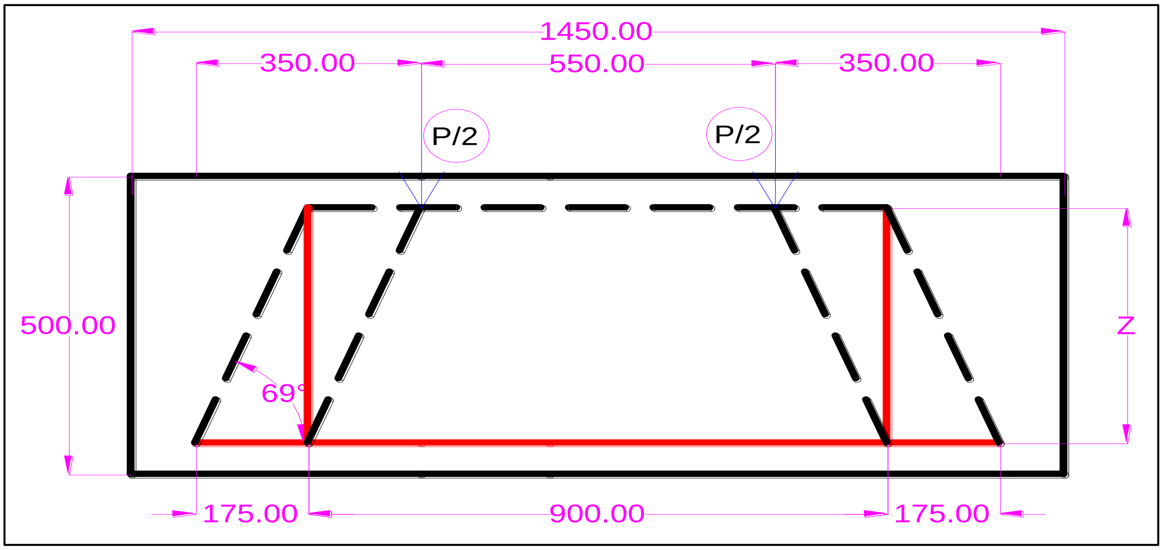


Figure A- 4 Strut and tie model of control beam 2

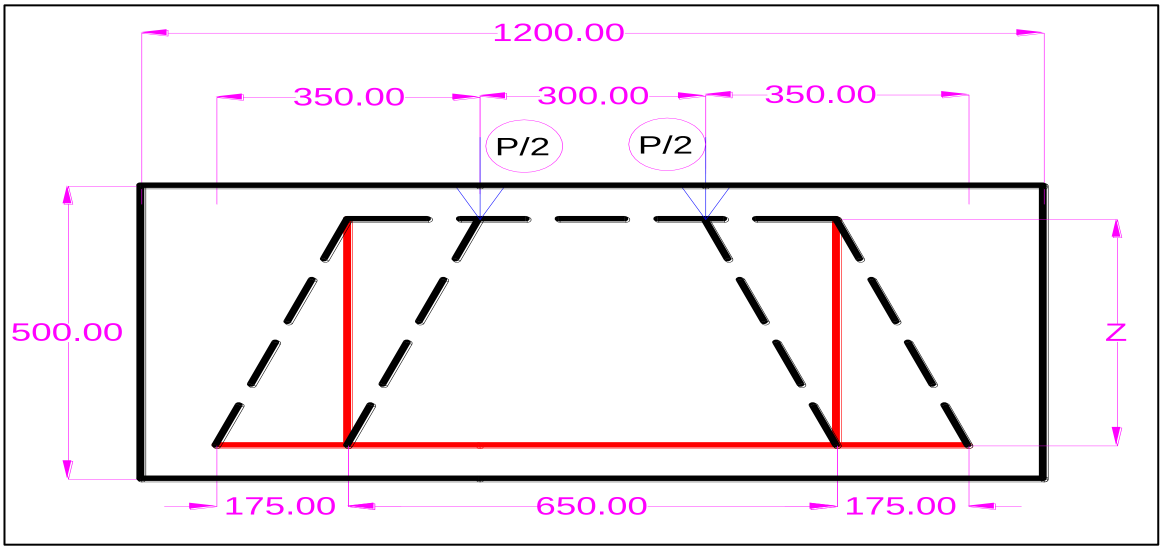


Figure A- 5 Strut and tie model of control beam 3

Appendix B

Table A- 4 Load deflection data of Solid RC deep beam from experimental investigation of (Abbas AbdulMajeed Allawi, 2018)

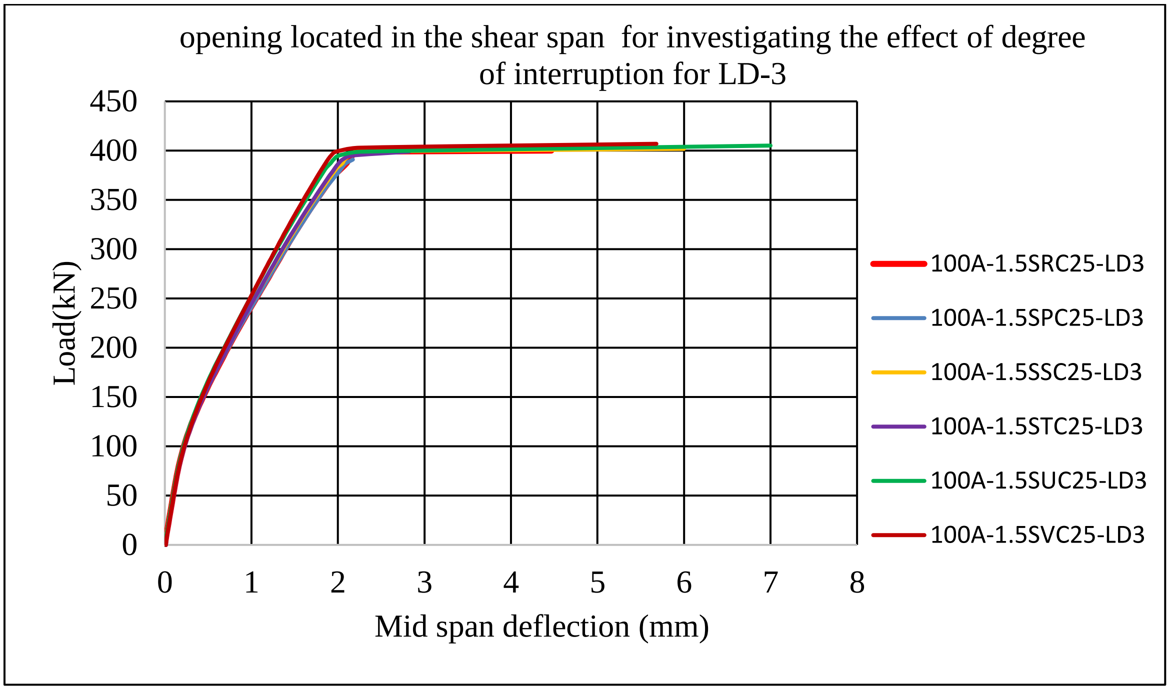
Vertical deflection (mm)	Load (kN)	Vertical deflection (mm)	Load (kN)	Vertical deflection (mm)	Load (kN)
0	0	0.413	185		
0.008	5	0.445	190		
0.016	10	0.534	195		
0.025	15	0.595	200		
0.033	20	0.653	205	2.402	390
0.042	25	0.708	210	2.441	395
0.05	30	0.761	215	2.483	400
0.058	35	0.809	220	2.528	405
0.067	40	0.843	225	2.579	410
0.075	45	0.879	230	2.636	415
0.083	50	0.927	235	2.698	420
0.092	55	0.984	240	2.758	425
0.1	60	1.027	245	2.816	430
0.11	65	1.057	250	2.883	435
0.118	70	1.087	255	2.961	440
0.126	75	1.117	260	3.049	445
0.135	80	1.147	265	3.127	450
0.144	85	1.178	270	3.195	455
0.152	90	1.21	275	3.26	460
0.161	95	1.242	280	3.32	465
0.17	100	1.276	285	3.388	470
0.178	105	1.316	290	3.452	475
0.187	110	1.417	295	3.519	480
0.197	115	1.466	300	3.59	485
0.207	120	1.523	305	3.667	490
0.218	125	1.574	310	3.747	495
0.23	130	1.625	315	3.831	500
0.242	135	1.695	320	3.933	505
0.256	140	1.761	325	4.061	510
0.272	145	1.826	330	4.206	515
0.291	150	1.906	335	4.5	520
0.319	155	1.98	340	4.8	525
0.351	160	2.042	345	5.1	530
0.369	165	2.085	350	5.75	535
0.38	170	2.129	355	6.12	540
0.395	175	2.17	360	6.75	545
0.393	180	2.21	365	7.11	550
0.413	185	2.25	370	7.82	555
0.445	190	2.289	375	8.2	560
0.534	195	2.326	380		
0.595	200	2.364	385		

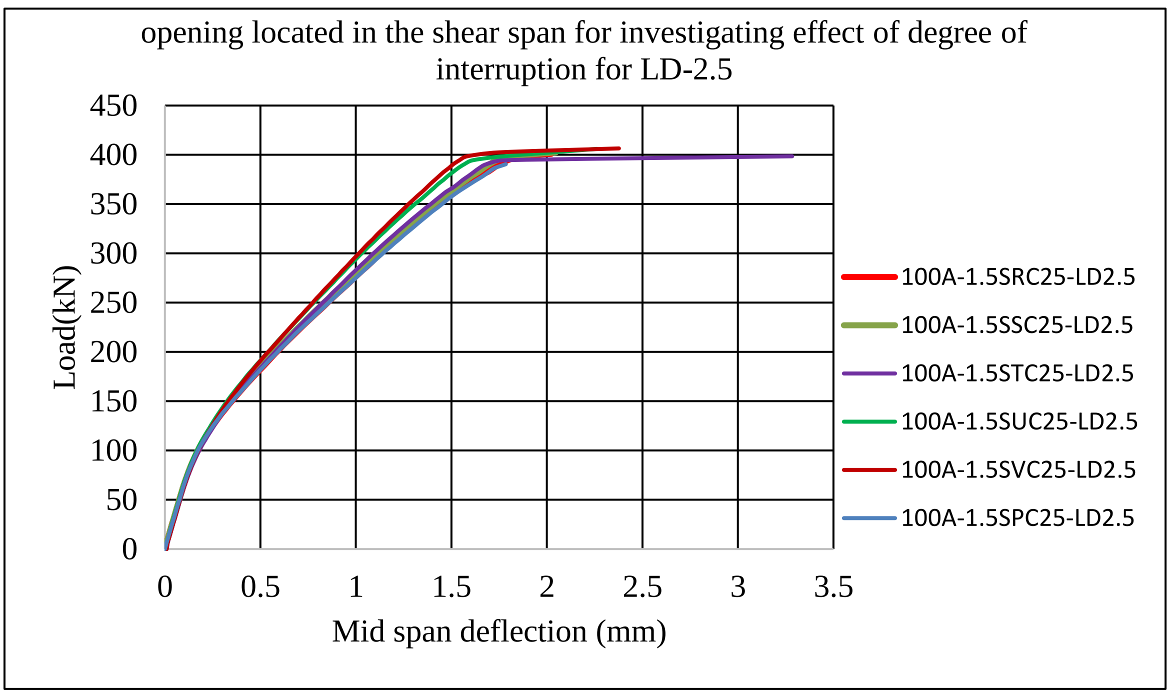
Table A- 5 Load deflection data of RC deep beam with transverse opening from experimental investigation of (A.Jasim, et al., 2018)

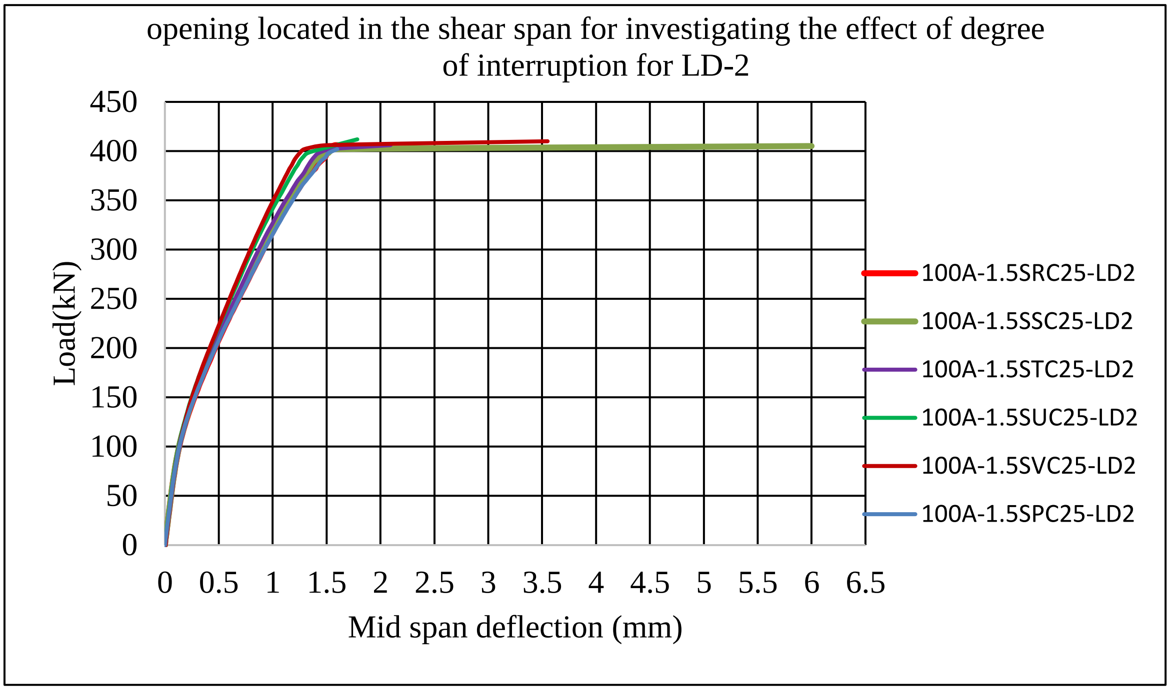
Vertical deflection (mm)	Load (kN)	Vertical deflection (mm)	Load (kN)
0	0	2.704	215
0.014	5	2.842	220
0.028	10	3.003	225
0.042	15	3.196	230
0.056	20	3.55	235
0.069	25	4	240
0.083	30		
0.097	35		
0.111	40		
0.125	45		
0.139	50		
0.155	55		
0.17	60		
0.185	65		
0.204	70		
0.226	75		
0.251	80		
0.33	85		
0.453	90		
0.542	95		
0.626	100		
0.701	105		
0.766	110		
0.844	115		
0.916	120		
0.992	125		
1.079	130		
1.156	135		
1.225	140		
1.297	145		
1.365	150		
1.452	155		
1.514	160		
1.601	165		
1.655	170		
1.72	175		
1.801	180		
1.87	185		
1.95	190		
2.08	195		
2.217	200		
2.4	205		

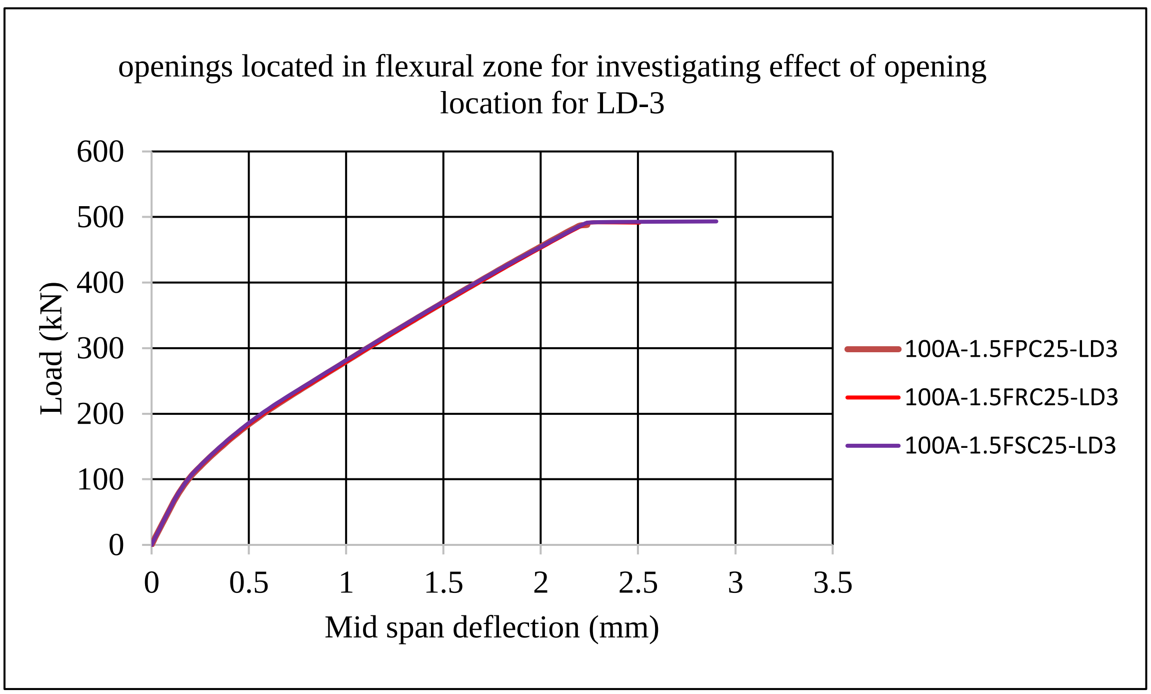
Appendix C

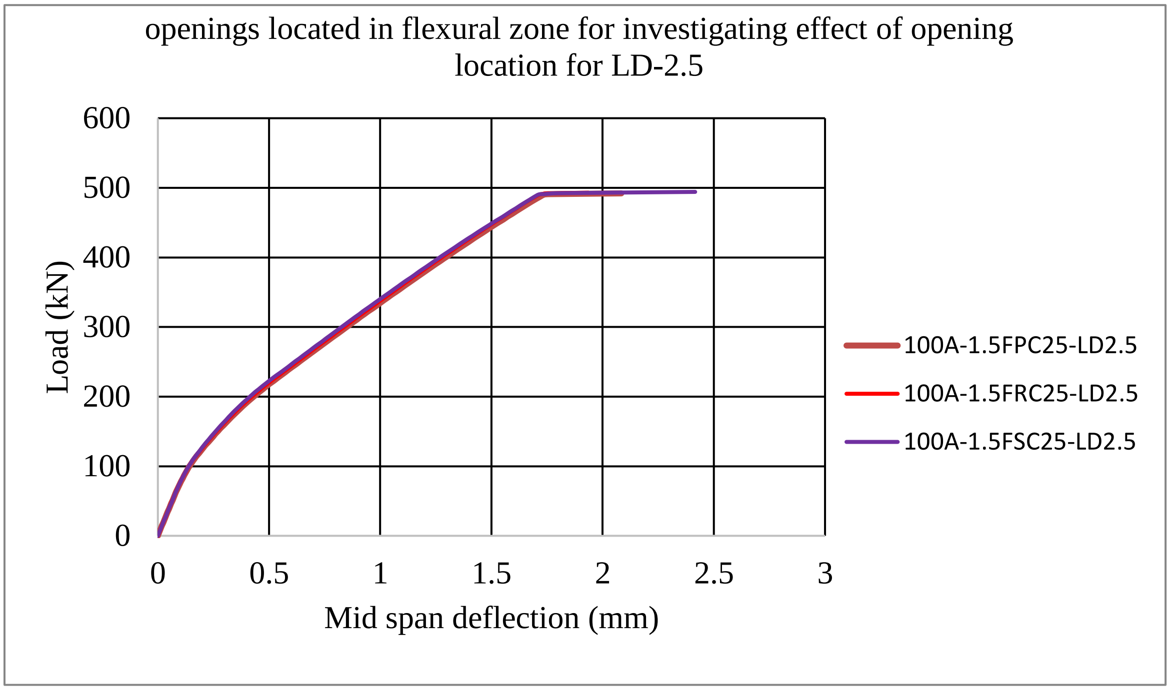
a. Finite element analysis results Load vs Deflection diagrams

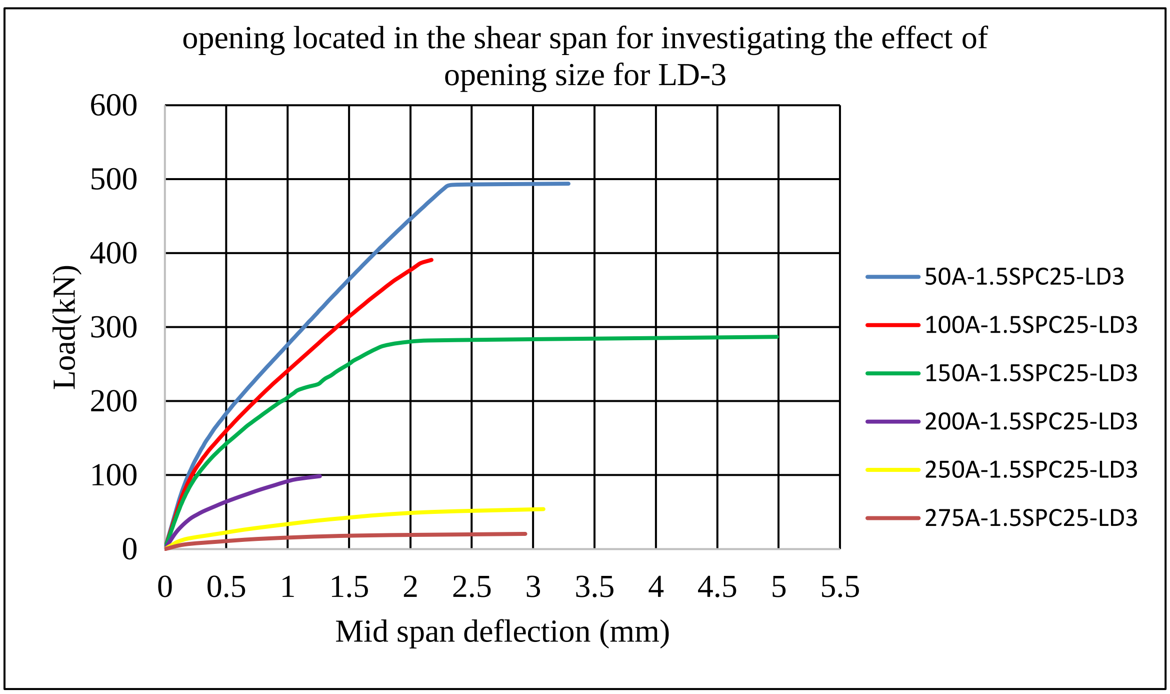


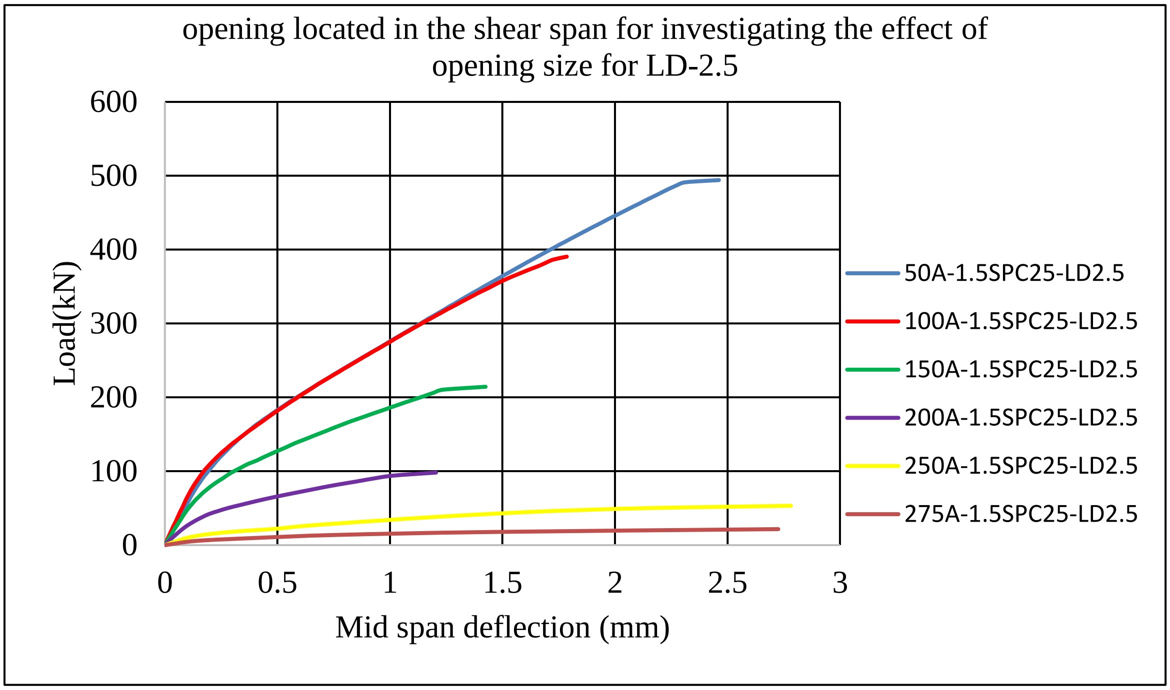


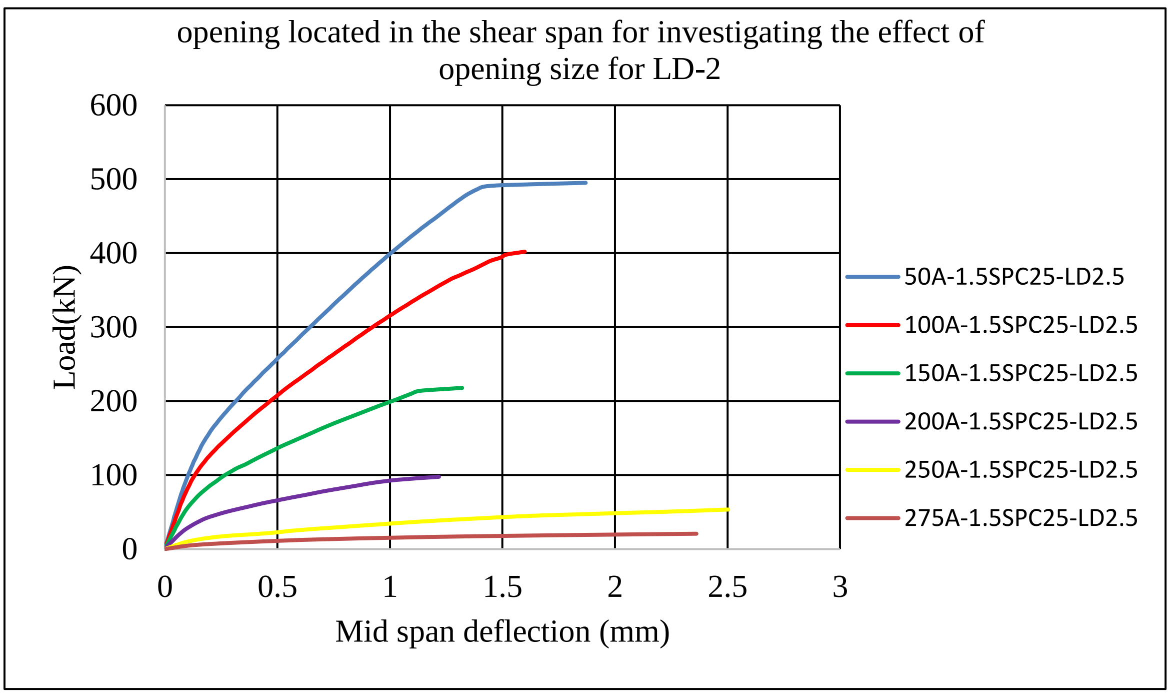

opening located in the shear span for investigating the effect of degree of interruption for LD-2
Load(kN)
450
400
350
300
250
200
150
100
50
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
Mid span deflection (mm)
100A-1.5SRC25-LD2
100A-1.5SSC25-LD2
100A-1.5STC25-LD2
100A-1.5SUC25-LD2
100A-1.5SVC25-LD2
100A-1.5SPC25-LD2

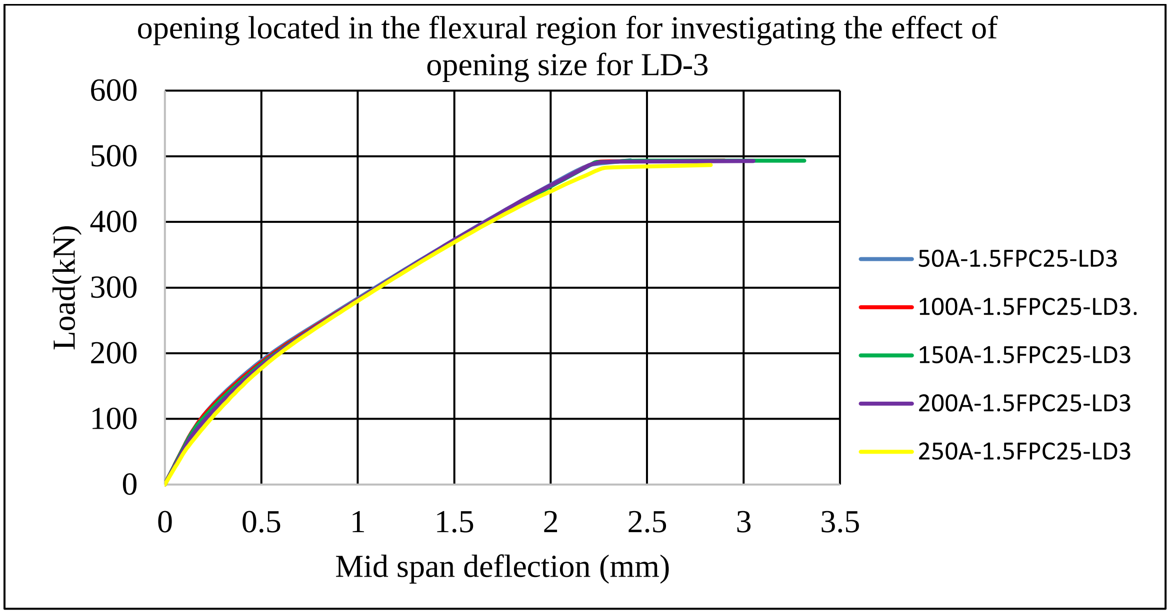

openings located in flexural zone for investigating effect of opening location for LD-3
Load (kN)
600
500
400
300
200
100
0
0
0.5
1
1.5
2
2.5
3
3.5
Mid span deflection (mm)
100A-1.5FPC25-LD3
100A-1.5FRC25-LD3
100A-1.5FSC25-LD3

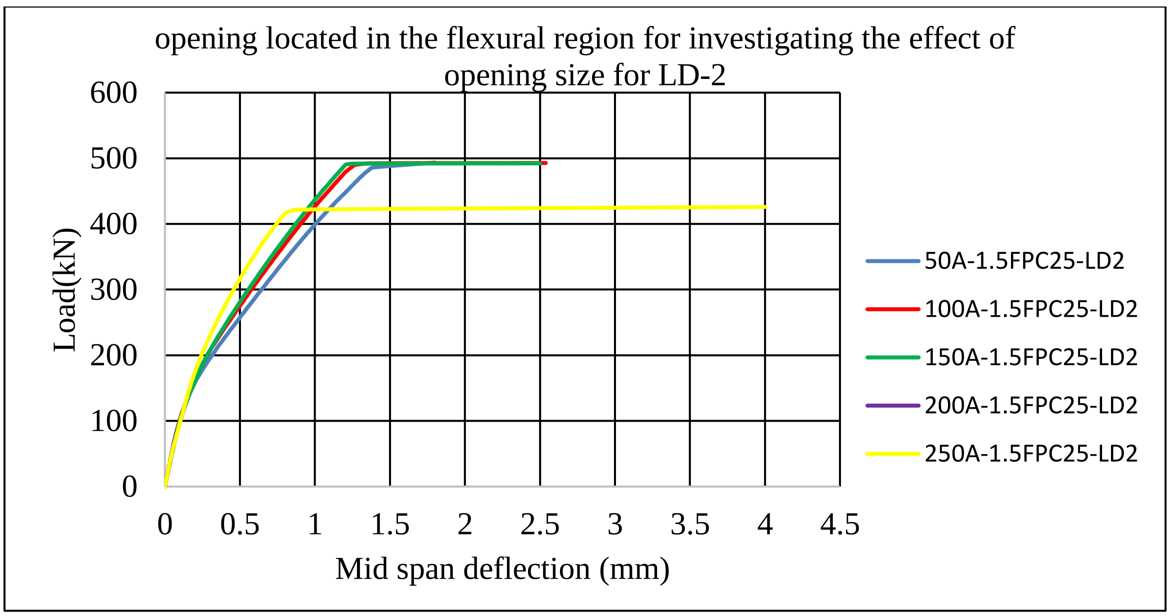

openings located in flexural zone for investigating effect of opening location for LD-2.5
600
500
400
300
200
100
0
Load (kN)
0
0.5
1
1.5
2
2.5
3
Mid span deflection (mm)
100A-1.5FPC25-LD2.5
100A-1.5FRC25-LD2.5
100A-1.5FSC25-LD2.5

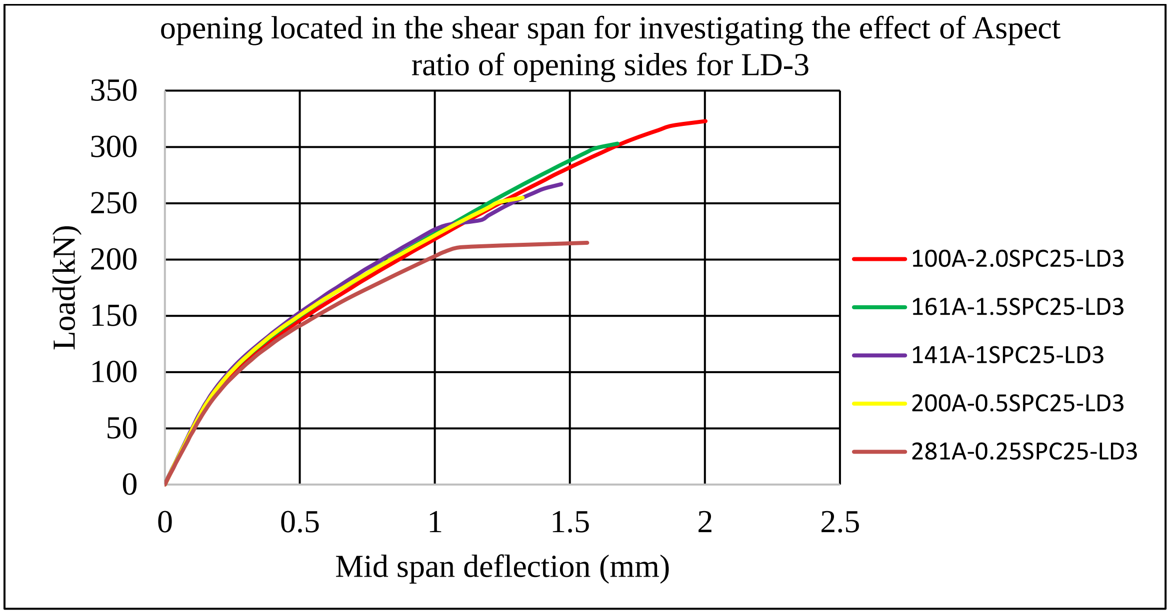

opening located in the shear span for investigating the effect of opening size for LD-3
600
500
400
300
200
100
0
Load(kN)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Mid span deflection (mm)
50A-1.5SPC25-LD3
100A-1.5SPC25-LD3
150A-1.5SPC25-LD3
200A-1.5SPC25-LD3
250A-1.5SPC25-LD3
275A-1.5SPC25-LD3

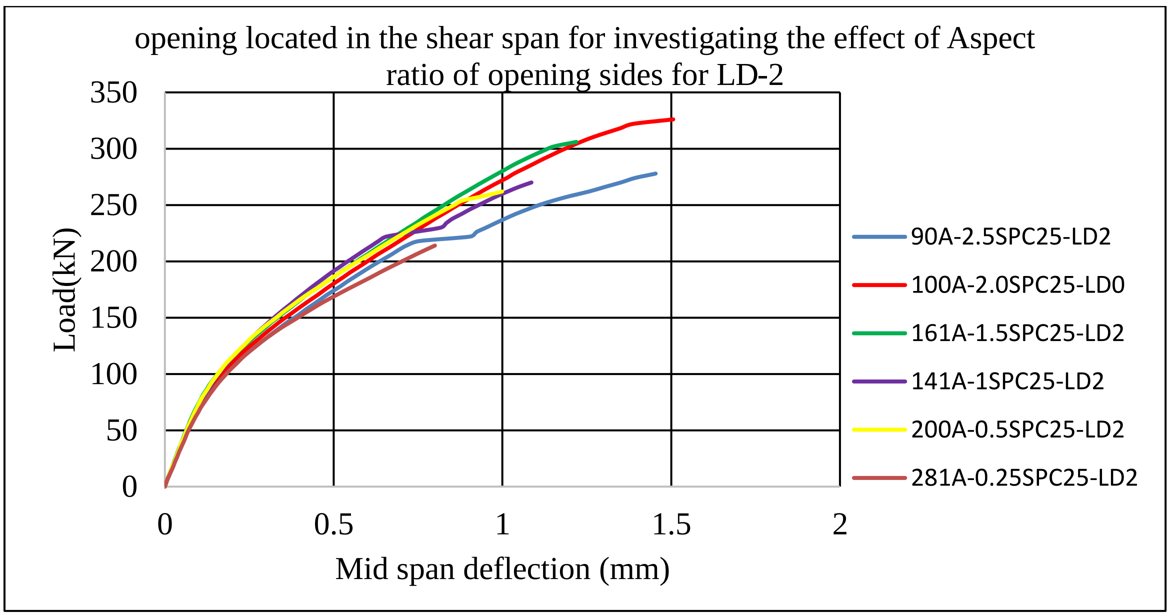

opening located in the shear span for investigating the effect of opening size for LD-2.5
Load(kN)
0
100
200
300
400
500
600
0
0.5
1
1.5
2
2.5
3
Mid span deflection (mm)
50A-1.5SPC25-LD2.5
100A-1.5SPC25-LD2.5
150A-1.5SPC25-LD2.5
200A-1.5SPC25-LD2.5
250A-1.5SPC25-LD2.5
275A-1.5SPC25-LD2.5

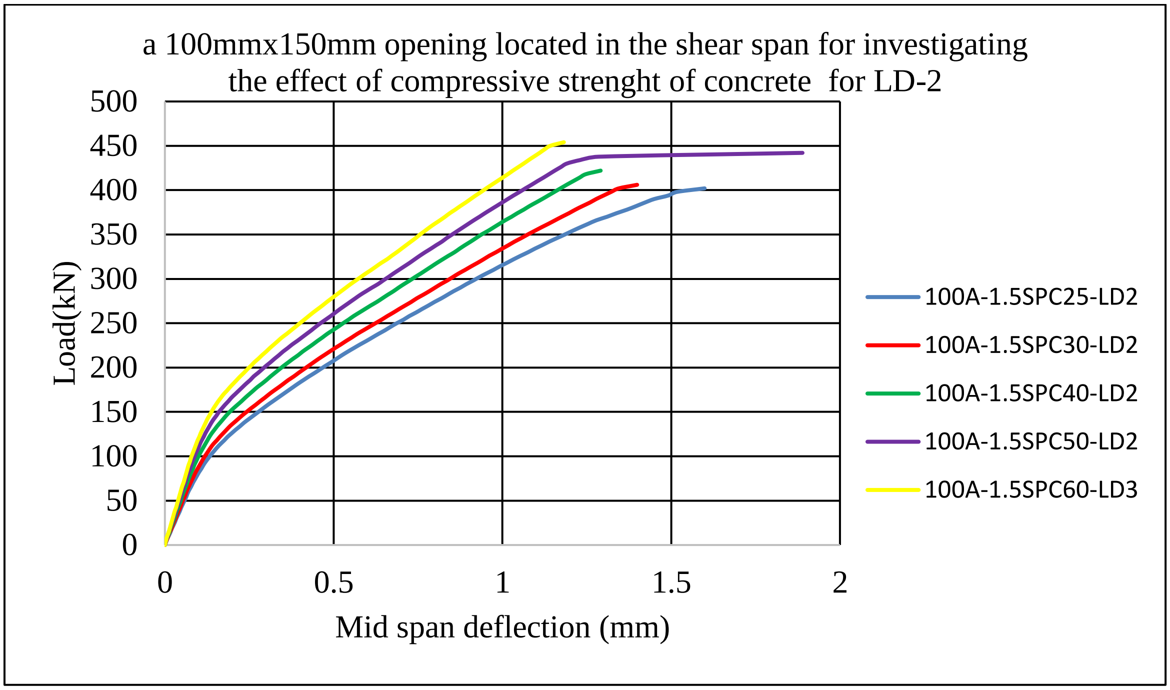

opening located in the shear span for investigating the effect of opening size for LD-2
Load(kN)
0
100
200
300
400
500
600
0
0.5
1
1.5
2
2.5
3
Mid span deflection (mm)
50A-1.5SPC25-LD2.5
100A-1.5SPC25-LD2.5
150A-1.5SPC25-LD2.5
200A-1.5SPC25-LD2.5
250A-1.5SPC25-LD2.5
275A-1.5SPC25-LD2.5

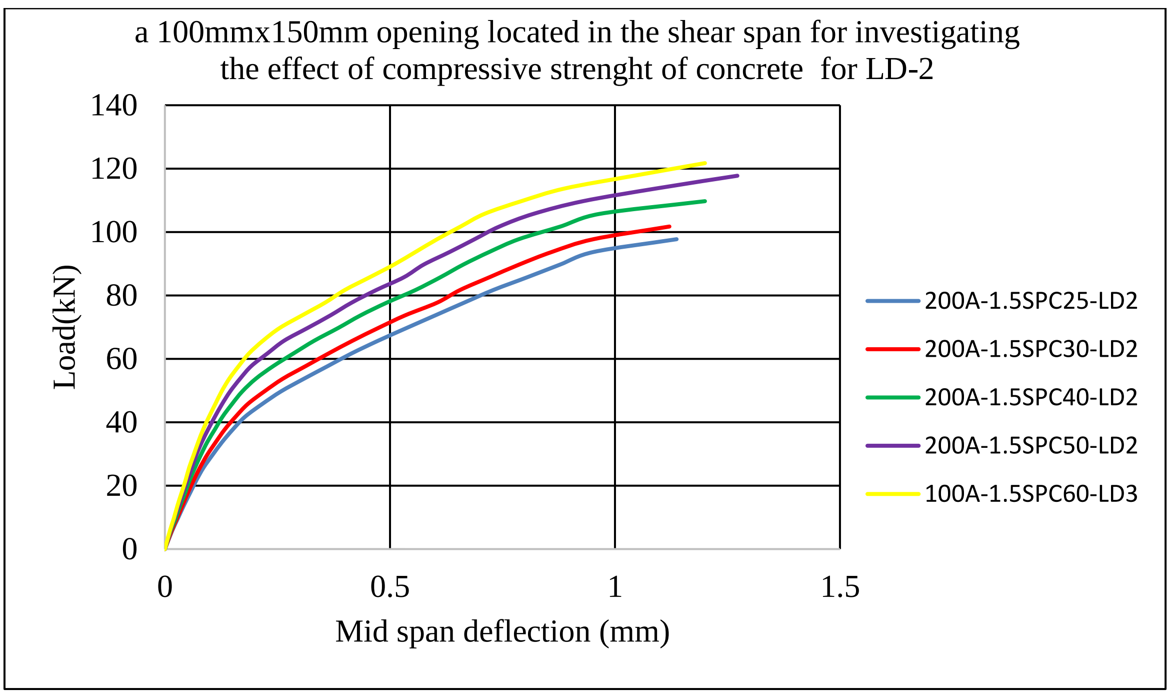

opening located in the flexural region for investigating the effect of opening size for LD-3
Load(kN)
Mid span deflection (mm)
0
100
200
300
400
500
600
0
0.5
1
1.5
2
2.5
3
3.5
50A-1.5FPC25-LD3
100A-1.5FPC25-LD3.
150A-1.5FPC25-LD3
200A-1.5FPC25-LD3
250A-1.5FPC25-LD3


opening located in the flexural region for investigating the effect of opening size for LD-2
Load(kN)
Mid span deflection (mm)
0
100
200
300
400
500
600
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
50A-1.5FPC25-LD2
100A-1.5FPC25-LD2
150A-1.5FPC25-LD2
200A-1.5FPC25-LD2
250A-1.5FPC25-LD2


opening located in the shear span for investigating the effect of Aspect ratio of opening sides for LD-3
Load(kN)
350
300
250
200
150
100
50
0
0
0.5
1
1.5
2
2.5
Mid span deflection (mm)
100A-2.0SPC25-LD3
161A-1.5SPC25-LD3
141A-1SPC25-LD3
200A-0.5SPC25-LD3
281A-0.25SPC25-LD3


opening located in the shear span for investigating the effect of Aspect ratio of opening sides for LD-2
Load(kN)
350
300
250
200
150
100
50
0
0
0.5
1
1.5
2
Mid span deflection (mm)
90A-2.5SPC25-LD2
100A-2.0SPC25-LD0
161A-1.5SPC25-LD2
141A-1SPC25-LD2
200A-0.5SPC25-LD2
281A-0.25SPC25-LD2


a 100mmx150mm opening located in the shear span for investigating the effect of compressive strenght of concrete for LD-2
Load(kN)
0
50
100
150
200
250
300
350
400
450
500
0
0.5
1
1.5
2
Mid span deflection (mm)
100A-1.5SPC25-LD2
100A-1.5SPC30-LD2
100A-1.5SPC40-LD2
100A-1.5SPC50-LD2
100A-1.5SPC60-LD3


a 100mmx150mm opening located in the shear span for investigating the effect of compressive strenght of concrete for LD-2
Load(kN)
0
20
40
60
80
100
120
140
0
0.5
1
1.5
Mid span deflection (mm)
200A-1.5SPC25-LD2
200A-1.5SPC30-LD2
200A-1.5SPC40-LD2
200A-1.5SPC50-LD2
100A-1.5SPC60-LD3

b. Finite element analysis results in tabular format showing increase or decrease in strength and deflection

Table A- 6 Finite element analysis result for investigating the effect of degree of interruption

Model designation	L/D ratio	Degree of interruption (%)	$P_{failure}$ (kN)	Decrease in strength (%)	Average decrease in strength (%)
100A-1.5SPC25LD-3	3	28.5	390.9	20.8	20.16
100A-1.5SRC25LD-3		22	400.05	19.02	18.847
100A-1.5SSC25LD-3		15	401.797	18.66	18.49
100A-1.5STC25LD-3		8	403.362	18.35	17.99
100A-1.5SUC25LD-3		0.8	405.12	17.99	17.56
100A-1.5SVC25LD-3		0	412.3	16.54	16.45
100A-1.5SPC25LD-2.5	2.5	28.5	391	20.95	
100A-1.5SRC25LD-2.5		22	401.04	18.92	
100A-1.5SSC25LD-2.5		15	402.5	18.62	
100A-1.5STC25LD-2.5		8	403.29	18.46	
100A-1.5SUC25LD-2.5		0.8	406.1	17.89	
100A-1.5SVC25LD-2.5		0	412.629	16.57	
100A-1.5SPC25LD-2	2	28.5	402.55	18.71	
100A-1.5SRC25LD-2		22	403.1	18.60	
100A-1.5SSC25LD-2		15	405.14	18.19	
100A-1.5STC25LD-2		8	410.293	17.15	
100A-1.5SUC25LD-2		0.8	412.0	16.80	
100A-1.5SVC25LD-2		0	414.793	16.24	

Table A- 7 Finite element analysis result for investigating the effect of opening location in the flexural region

Model designation	L/D ratio	Opening location (0.5L/Lo)	$P_{failure}$ (kN)	Mid span deflections (mm)	Decrease in strength (%)	Increase in deflection (%)
100A-1.5FPC25LD-3	3	0.65	492	2.39	0.4	4.40
100A-1.5FRC25LD-3		0.82	491.5	2.51	0.51	0.40
100A-1.5FSC25LD-3		1.0	493.35	3.0	0.13	20.0
100A-1.5FPC25LD-2.5	2.5	0.65	492.1	2.084	0.49	3.07
100A-1.5FRC25LD-2.5		0.82	493.045	1.936	0.30	9.95
100A-1.5FSC25LD-2.5		1	494.12	2.42	0.07	12.56

Table A- 8 FEA analysis result for investigating the effect of opening size in the shear span

Model designation	L/D ratio	Opening area ratio (%)	$P_{failure}$ (kN)	Decrease in strength (%)	Pcrack (kN)	Max Crack width (mm)
50A-1.5SPC25LD-3	3	2.14	494	0.06	66.98	2.73
100A-1.5SPC25LD-3		8.57	390.9	20.87	54.9	3.83
150A-1.5SPC25LD-3		19.3	286.931	41.92	38.778	4
200A-1.5SPC25LD-3		34.3	98.589	80.04	26.587	5
250A-1.5SPC25LD-3		53.57	54.016	89.07	22.3	5.5
275A-1.5SPC25LD-3		64.74	20.571	95.84	10.212	5.33
50A-1.5SPC25LD-2.5	2.5	2.14	494.12	0.08	66.54	1.7
100A-1.5SPC25LD-2.5		8.57	391	20.93	54.504	4.43
150A-1.5SPC25LD-2.5		19.3	214.37	56.65	30.37	3.82
200A-1.5SPC25LD-2.5		34.3	98.188	80.14	18.182	2.74
250A-1.5SPC25LD-2.5		53.57	53.238	89.23	13.938	3.3
275A-1.5SPC25LD-2.5		64.74	21.636	95.62	9.812	4.47
50A-1.5SPC25LD-2	2.0	2.14	495.12	0.08	70.134	1.83
100A-1.5SPC25LD-2		8.57	402.55	18.76	54.053	1.27
150A-1.5SPC25LD-2		19.3	217.922	56.02	29.92	2.62
200A-1.5SPC25LD-2		34.3	97.73	80.28	17.32	2.82
250A-1.5SPC25LD-2		53.57	57.31	88.43	13.488	3.75
275A-1.5SPC25LD-2		64.74	20.763	95.81	9.359	3.84

Table A- 9 Finite element analysis result for investigating the effect of opening size in the flexural region

Model designation	L/D ratio	Opening area x 10^{-3} (mm^2)	$P_{failure}$ (kN)	Mid span deflection (mm)	Decrease in strength (%)	Increase in deflection (%)	Pcr In (kN)	Wk In (mm)
50A-1.5FC25LD-3	3	0.38	493.9	2.414	0.02%	3.44%	67.048	1.9
100A-1.5FC25LD-3		1.50	493.7	2.901	0.06%	16.04%	67.009	2.47
150A-1.5FC25LD-3		3.38	493.3	3.315	0.14%	32.60%	66.94	2.71
200A-1.5FC25LD-3		6.00	492.80	3.05	0.24%	22.00%	66.848	3.2
2500A-1.5FC25LD-3		9.38	486.69	2.83	1.48%	13.20%	54.8	3.5
50A-1.5FC25LD-2	2	0.38	493.5	1.8	0.40%	0.56%	70.146	2.65
100A-1.5FC25LD-2		1.50	492.88	2.539	0.53%	41.84%	70.106	3
150A-1.5FC25LD-2		3.38	492.3	2.5	0.65%	39.66%	74.038	3.5
200A-1.5FC25LD-2		6.00	491.50	2.10	0.81%	17.32%	73.94	3.6
250A-1.5FC25LD-2		9.38	425.89	4.00	14.05%	123.46%	53.8	4

Table A- 10 analysis result for investigating the effect of Aspect ratio of opening sides

Model designation	L/D ratio	Aspect ratio	$P_{failure}$ (kN)	Decrease in strength (%)
281A-0.25SPC25LD-3	3	0.25	214.835	56.51%
200A-0.5SPC25LD-3		0.5	254.921	48.40%
141A-1.0SPC25LD-3		1	266.922	45.97%
116A-1.5SPC25LD-3		1.5	302.92	38.68%
100A-2.0SPC25LD-3		2	322.931	34.63%
281A-0.25SPC25LD-2	2	0.38	222.058	55.19%
200A-0.5SPC25LD-2		1.50	262.012	47.12%
141A-1.0SPC25LD-2		3.38	270.021	45.51%
116A-1.5SPC25LD-2		6.00	306.02	38.24%
100A-2.0SPC25LD-2		9.38	326.026	34.20%

Table A- 11 analysis result for investigating the effect of concrete grade on the failure load of beams with web openings of opening sides

Model designation	Opening size	Fcu (Mpa)	$P_{failure}$ (kN)	Decrease in strength (%)
100A-1.5SPC25LD-2	100mmx150mm	25	402.055	18.86%
100A-1.5SPC30LD-2		30	406.055	18.05%
100A-1.5SPC40LD-2		40	422.055	14.82%
100A-1.5SPC50LD-2		50	442.068	10.78%
100A-1.5SPC60LD-2		60	454.058	8.36%
200A-1.5SPC25LD-2	200mmx300mm	25	97.736	80.28%
200A-1.5SPC30LD-2		30	101.732	79.47%
200A-1.5SPC40LD-2		40	109.731	77.85%
200A-1.5SPC50LD-2		50	117.755	76.24%
200A-1.5SPC60LD-2		60	121.734	75.43%