COULOMB BLOCKADE OSCILLATIONS, TUNNELING AND ELECTRONS TRANSPORT THROUGH QUANTUM DOTS

By

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Abstract

In this thesis, we studied Coulomb blockade oscillations, tunneling and electrons transport through a quantum dot. Coulomb blockade oscillations of the conductance are a manifestation of single electron tunneling through a quantum dot. We focus on the electron transport between the dot and source(drain). The model of the study is the linear conductance capable of describing the basic Physics of electronics states in the quantum dot. Using the master equation analytic expression for the current(I) through the quantum dot was derived and obtain the linear conductance through the dot which is defined as $G = \lim_{V \to 0}(\frac{I}{V})$ in the limit of infinitely of small bias voltage. We will distinguish three temperature regimes, $\frac{e^2}{C} \ll K_B T$, the discreteness of the charge cannot be discerned, $\Delta E \ll K_B T \ll \frac{e^2}{C}$, the classical metallic Coulomb blockade regime many levels are excited by thermal fluctuations, and $K_B T \ll \Delta E \ll \frac{e^2}{C}$, the quantum Coulomb blockade regime, a few levels participate in transport.
Finally, we have performed the numerical computation of electron transport through the quantum dot. We present graphs of the results for using the equations that we derive. The linear conductances are plotted as a function of the gate voltage. The Coulomb blockade oscillations occur as the voltage on a nearby gate electrodes varied. In the valleys, the conductance falls off exponentially as a function of the gate voltage. Figures (4.1-4.8), are the result of the study. At some intermediate temperature, the conductance shows one oscillation but vary for the positive and negative gate voltage. At high temperature, no oscillation conductance and at low temperature conductance oscillates for some appropriately chosen capacitance.

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Chapter 1

Introduction

The numerical computation of electron transport through quantum dots have increasingly important in the field of Physics with the development of supercomputers. From the basic constituents of a system of particles, and their interactions, a computational approach enables to derive the electronic structure and the properties of the system. A system of particles currently considered with attention is the quantum dot. It is an artificial system consisting of several interacting electrons confined to a small regions between layers of metals. The aim of this thesis is to assess the appropriateness of the numerical computation of electrons transport through quantum dots confined by region which is weakly coupled via tunnel barriers to two electron reservoirs and plot the graphs from the derived equations. In this section we discussing quantum dot, Coulomb blockade and single electron tunneling, and some applications of the quantum dot.

1.1 Quantum dots

A quantum dot is a small region typically, a metal material, where a finite number of electrons is confined. The possibility of examining and tuning properties related to quantum mechanical effects and charge quantization motivates the interest in fabricating such small devices. The dot size is in the range of the Fermi wavelength and its capacitance is very small. Therefore, the electrons occupy discrete quantum levels in dots and the level spacing δE is the distance between the adjacent levels. A certain amount of energy, depending on the charging energy E_C, is needed to add an electron to the dot or remove an electron from the dot. These characteristics are very similar to those of atoms. Therefore, quantum dots are referred to as artificial atoms. The challenging advantage of quantum dots is the properties as the spectrum. Example are tunable by an external gate and that source and drain contacts can be attached to it. Therefore transport through the dot can be measured. The tunability of parameters plays an important role for the work in an electron transporting.

Attaching contacts to the dot makes tunneling of electrons from and to the dot possible. This leads to a finite life time of the electrons in the dot resulting in a level broadening Γ, which in devices, we are interested in is smaller than the level spacing. A quantum dot coupled to three terminals. Particle exchange can occur with source and drain contact. The third terminal, coupled electrostatically only is used as a gate electrode to shift the energy spectrum of the dot. A quantum dot attached to leads and to a gate capacitance is the prototype device for a single electron transistor. In the regime where temperature is much smaller than the charging energy but still much larger than the level broadening Γ, transport is governed by single electron process. The quantum dot behave like a transistor in the sense that the current through the dot can be switched on and off by varying the gate voltage applied to the dot [1].

Quantum dots are the most functional and reproducible nanostructures available to researchers. Different synthesis routes create different kinds of quantum dots. They are very small by nature, the smallest objects that we can synthesis on the nano scale. From this fact, they are assimilated to dots, through one quantum dot can be made out of roughly thousands of atoms. All the atoms pool their electrons to "sing with one voice", that is, the electrons are shared and coordinated as if there was only one atomic nucleus setting up an attraction at the centre.

According to the web of science [2], studies of quantum dots have resulted in around 2700 publications since 1987, with increasing number of publications each year exceeding 2500 per year since 2005. Quantum dots research embraces variety of topics in Physics, electrical, and electronic engineering, Chemistry, material science, Biology and medicine, where according to the demand of a specific applications, different types of quantum dots structures are employed.

1.2 Coulomb blockade and single-electron tunneling

The concept of the Coulomb blockade refers to the phenomenon that tunneling through a metallic grain with small capacitance may be inhabited at low temperatures and small applied voltages. The reason is that the addition of a single electron to such a system requires an electrostatic charging energy of the of order $\frac{e^2}{C} \gg k_B T, eV$, where C is its capacitance, T temperature, e charge of electron, k_B is Boltzmann constant and V the applied voltage [3]. Basically, this is the explanation first given by Gorter [4] of an observed [5] anomalous increase of the resistance of thin granular metallic films. A suitable model system to investigate the Coulomb blockade in more detail consists of a confined region (dot) weakly coupled by tunnel barriers to left and right.

An additional gate electrode can be used to control the charge on the dot, which consists of two contributions. The number N of conduction electrons on the dot contributes a charge , Q = -Ne which can change by discrete amounts e only. (Assuming that the tunnel resistance is larger compared to the resistance quantum dot h/e^2, h is the plank constant, so that the number N of electrons on the left right may be treated as a sharply defined classical variable). Nearby external charges, example, those on the gate, induce a displacement charge $C\Phi_{ext}$ is on the dot, which can be varied continuously. Φ_{ext} the part of the electrostatic potential difference between dot and right and left due to the external charges.

The transition from N to N+1 electrons on the dot occurs on reaching a charge imbalance $C\Phi_{ext} - Ne = 2(\frac{1}{2}e)$. The additional electrons causes the charge imbalance to reverse sign, becoming $-\frac{1}{2}$. Tunneling is blocked at low temperatures, except near the points where the charge imbalance jumps from $\frac{1}{2}e$ to $-\frac{1}{2}$e. Here, the electrons can tunnel through the dot one by one. The Coulomb blockade is lifted, and the zero bias conductance exhibits a peak. These are " the Coulomb-blockade oscillations [6], which is the central phenomenon studied in this study.

1.3 Some applications of quantum dots

Exceptional electrical and optical properties make quantum dots attractive components for integration into electronic devices. One significant asset of quantum dots over traditional optoelectronic materials is that they exist in the solid state. Solids tend to be more compact, easily cooled, and allow for direct charge injection. Additionally, quantum dots can interconvert light and electricity in a tunable manner being dependent on crystal size, allowing for easily wavelength selection [7]. As a result, researchers have experimented with quantum dots in lasers, LEDS, photovoltaics and also for new generation of transistors, logic gates with quantum computers as the ultimate goal. Most of these applications are still in early development; however the benefits of the quantum dot computers are evident, and could lead to a complete revolution of the way of building electronic components at atomic scale.

Another application of quantum dots and one of the fastest moving most exciting interfaces of a nanotechnology in the use of (colloidal) quantum dot is biology. Their unique optical properties make them appealing as fluorophones in a variety of biological investigations, in which traditional fluorescent labels based on organic molecules fall short of providing long-term stability and simultaneous detections of multiple signals [8]. The ability to make quantum dots water soluble and target them to specific biomolecules has led to promising applications in cellular labelling, thus improving diagnostic methods (example, tracking cancer cells during metastasis [9] and in developing better drug delivery systems to improve disease therapy [10]). It is even currently studied as neuro electronic interface for converting optical energy into electrical signal responding to the need for prosthetic devices that can repair or replace nerve function [11]. To mention some of the applications, where quantum dots are intensively used in electronic devices, medicine, optics, optical imaging, communications, quantum computations, information and etc [12]. Moreover, one of the interesting applications of quantum dots in the area of biology is to investigate various cellular labelling mechanism (for example in cancer treatment) and developing drug [12]. Generally, the applications of the quantum dots are diverse and all could not be listed here in detail.

1.4 Organization of the thesis

This thesis is concerned with theory of coulomb blockade oscillations, Tunneling and Electrons transport through the quantum dot. The understanding of the physical situations on this is essential. Thus, the arrangement of this thesis is organized as follows. Chapter one : briefly described, introduction to quantum dots, Coulomb blockade and single electron tunneling, and some applications of quantum dots. Chapter two: discussed electron transport through quantum dots and the model of the study, which is weakly coupled to two electron reservoirs. We specialize to the linear-responce regime, and obtain an expression for the conductance. Chapter three: discussed about high and low temperature behavior. Chapter four: presented the result and plotted the graphs of the study. Chapter five: summarized the work of the study.

Chapter 2

Electron transport through quantum dots

This chapter has introduced the basic ideas of electron transport through the quantum dot. Using the master equation analytic expression for the current(I) through the quantum dot will be derived. Then, we shall obtain the linear conductance through the dot which is defined as $G = \frac{I}{V}$ in the limit of infinitely of small bias voltage. Where G is the conductance, I is the current and V is the applied voltage.

2.1 The model of the problem

We consider a quantum dots, which is weakly coupled by tunnel barriers to two electrons reservoirs. A quantum dot has single electron energy levels at $E_p(p = 1, 2, ...)$ is calculated by treating the electron-electron interaction in a mean-field (Hartee) approximation [13]. The levels are labeled in ascending order and measured relative to the bottom of the potential well [14]. In principle, the position of the levels may depend on the number of electrons in the dot. Using figure 2.1, the states in the left and right reservoirs, are occupied according to the Fermi-dirac distributions.

$$f_l(E - E_F) = \frac{1}{1 + exp\frac{E-E_F}{K_B T_l}} \tag{2.1.1}$$

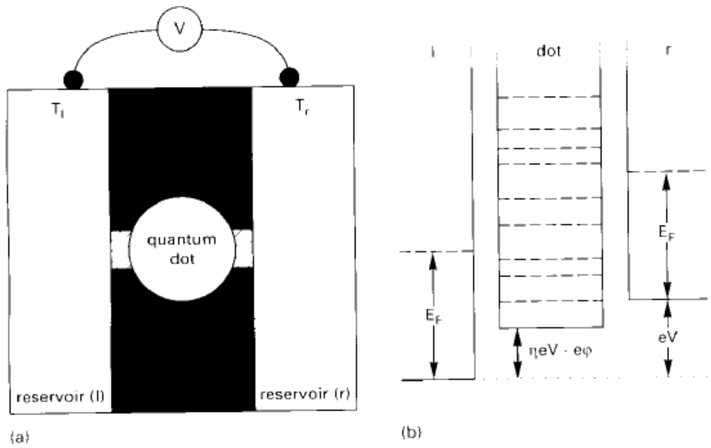

Figure 2.1: (a) Schematic cross section of geometry studied in this paper consisting of a confined region (quantum do) weakly coupled to two electron reservoirs via tunnel barriers (hatched). (b) Profile of the electrostatic potential energy (solid curve) along a line through the tunnel barriers. The Fermi levels in the left and right reservoirs, and the discrete energy levels in the quantum dots are indicated (dashed lines).

$$f_r(E - E_F) = \frac{1}{1 + exp\frac{E-E_F}{K_B T_r}} \tag{2.1.2}$$

Where, E_F is the Fermi-energy, f is the Fermi-Dirac distribution functions, K_B is the Boltzmann's constant , T_l and T_r are the temperatures of left and right reservoirs respectively. A current(I) can be passed through the dot by applying a potential difference V between the left and right. The tunnel rate from level p to the left and right reservoirs is denoted by Γ_p^l and Γ_p^r respectively. Assuming that both kT and ΔE are $\gg h(\Gamma^l + \Gamma^r)$ for all energy levels participation in the conduction. The finite width $h\Gamma = h(\Gamma^l + \Gamma^r)$ of the transmission resonance through the quantum dot that can be disregarded. This assumption allows us to characterize the state of the quantum dot by a set of occupation numbers, one for each energy level. The restriction $kT, \Delta E \gg h\Gamma$ results in the conductance being much smaller than the resistance of the quantum dot (e^2/h), which is a necessary condition for the occurrence of the

Coulomb blockade.

Transport through the dot proceeds by tunneling its discrete electronic states, it will be clear that for small V a net current can flow for a certain values of E_F only $(if \Delta E \gg K_B T)$. In the absence of charging effects, a conductance peak due to resonant tunneling occurs if E_F in the left lines up with one of the energy levels in the dot. This condition is modified by the charging energy. Because the number of N electrons localized in the dot can take an integer values only, a charge imbalance, and hence a potential difference can a rise between the dot and left and right, even if $V = 0$ following the "Orthodox model" of the Coulomb blockade [15]. As indicated in figure 2.2, we consider metallic islands coupled to two electron reservoir left and right via tunnel junctions J_s and J_d and for gate via J_g. J_l has tunnel junction resistance R_l and capacitance C_l. J_r has resistance R_r and capacitance C_r. Then, the bias voltage is

$$\begin{aligned} V_{sd} &= V_s - V_d \\ &= V_l - V_r \\ &= V \end{aligned} \tag{2.1.3}$$

The electrodes are connected to an outer circuit, which controls the difference between μ_l and μ_r using the bias voltage V. The electrostatic potential energy of an electron for the left barrier is :

$$\begin{aligned} U_l &= -e(V_s) \\ &= -eV_l \end{aligned} \tag{2.1.4}$$

The electrostatic potential energy of an electron for the right reservoir is:

$$\begin{aligned} U_r &= -e(V_r) \\ &= U_l + eV \end{aligned} \tag{2.1.5}$$

Therefore, if we take U_l= U_s as the reference potential energy U_r - U_l = eV. The right electron reservoir is lifted in electrostatic energy above that of left by eV. We can therefore take $U_l = 0$ as the reference for potential energy. The electrostatic energy of an electron in right is lifted up above that in the left by eV [16]. The two electron reservoirs left and right are massive (large) metallic materials of the same type so that they have the same Fermi-energy $\in_F^l = \in_F^r = \in_F$ and temperature. The electro chemical potentials $\mu_l = \in_F$ and $\mu_r = \in_F + eV$

$$\mu_r - \mu_l = eV \tag{2.1.6}$$

since we have set that

$$U_r = U_l + eV \tag{2.1.7}$$

Where $U_l = 0$

$$U_r = eV \tag{2.1.8}$$

As the electrons cross the tunnel junctions J_l from quantum dot to left some voltage is dropped across the junction resistance R_l. This voltage dropped across R_l

$$V_{rl} = V_l - V_q \tag{2.1.9}$$

The energy of the left junction resistance is :

$$U_{R_l} = U_l + eV_{qd} \tag{2.1.10}$$

As the electrons cross the tunnel junctions J_r from quantum dot to right some voltage is dropped across the junction resistance R_r. This voltage dropped across R_r is

$$V_{R_r} = V_q - V_r \tag{2.1.11}$$

The energy of the the right junction resistance is :

$$U_{R_r} = -eV_{qd} + eV + U_l \tag{2.1.12}$$

By substituting the value of eV_{qd} from Eq. (2.1.10) into the Eq. (2.1.12), we obtain

$$U_{R_r} + U_{R_l} = eV \tag{2.1.13}$$

The sum of the energy dropped across the left and right junctions is therefore eV. Let

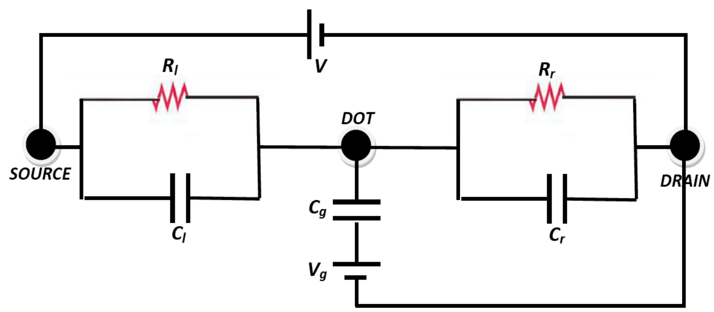

Figure 2.2: Circuit diagram in which the tunnel barriers are represented as a parallel capacitor and resistor. The different gates are represented by a single capacitor C_g. The charging energy in this circuit is $e^2/(C_l + C_r + C_g)$.

η be the fraction of eV dropped across left junction $U(R_l) = \eta eV$ and by substituting into the Eq. (2.1.13), we obtain

$$U(R_r) = eV(1 - \eta) \tag{2.1.14}$$

From

$$U(R_l) = -eV_l + eV_{qd} \tag{2.1.15}$$

$$\eta eV = U_l + eV_{qd} \tag{2.1.16}$$

Since U_l= 0,

$$eV_{qd} = \eta eV \tag{2.1.17}$$

Therefore, the component of the potential energy of eV on quantum dot is ηeV. Moreover, the gate voltage also generate a potential in quantum dot due to high junction resistance between the quantum dot and gate are coupled capacitively (no charge transfer between quantum dot and gate electrode). Let α be fraction dropped across junction between quantum dot and gate electrode. If $\Delta E \gg K_{BT}$, discreteness of the energy levels must be taken into account. The total energy is equal to kinetic energy plus potential energy.

$$E(\{n_i\}, V, V_g) = \sum_{i=1}^{\infty} n_i E_i + U(N) \tag{2.1.18}$$

Where,

$$U(N) = \frac{(Ne)^2}{2C} - Ne\eta V - NeV_g \tag{2.1.19}$$

Where, V_g is externally applied gate voltage and Eq. (2.1.19) is the total energy of N electrons in the quantum dot. The total capacitance $C = C_l + C_r + C_g$ consists of capacitances across the barriers, C_l, C_r, and a capacitance between the dot and gate C_g. The potential energy of the quantum dot

$$\begin{aligned} \Phi(N) &= \frac{Q}{C} + e\eta V - \alpha e V_g \\ &= \frac{Q}{C} + \Phi_{ext} \end{aligned} \tag{2.1.20}$$

Where, $\eta = \frac{C_g}{C}$, and $\alpha = \frac{C_s}{C}$, s = Source(left) , g = gate, d = drain(right) and quantum dot is considered as a transistor. The total number of electrons in the quantum dot is:

$$N = \sum_{i=1}^{\infty} n_i \tag{2.1.21}$$

$$E(\{n_p\} n_p = 0) = E(N+1, n_p = 1) \tag{2.1.22}$$

$$E(N+1, n_p = 1) - E(N, n_p = 0) = E_p + (N + \frac{1}{2})\frac{e^2}{C} - eV_{ext} + \eta eV \tag{2.1.23}$$

Eq. (2.1.23) shows the amount of transition energy of an electron in the left reservoir so as to tunnel to energy level E_p.

$$\begin{aligned} E^{i,l} &= E_p + (N + \frac{1}{2})\frac{e^2}{C} - eV_{ext} + \eta eV \\ &= E_p + U(N+1) - U(N) + \eta eV \end{aligned} \tag{2.1.24}$$

Where $U(N+1)$ - $U(N) = (N+\frac{1}{2})\frac{e^2}{C}$ -eV_{ext} and $E^{i,l} = E(N,N+1)$ is the transition energy.

$$\epsilon_F^r = \epsilon_F + eV \tag{2.1.25}$$

Each one $\epsilon_F^l = \mu(l)$ or $\epsilon_F^r = \mu(r)$, crosses the transition energy, the current $I(r)$ in the quantum dot shows as step. For an electron in the left bath with energy $E^{i,l}$, N electrons in the quantum dot having potential energy U(N) and E_p empty($n_p = 0$). By using Eq. (2.1.22) we obtain

$$\begin{aligned} E^{f,l} &= E_p + \frac{(Ne)^2}{2C} + Ne\eta V + NeV_{ext} - \frac{(N-1)^2}{2C}e^2 \\ &- (N-1)e\eta V - (N-1)eV_{ext} \\ &= E_p + U(N) - U(N-1) + \eta eV \end{aligned} \tag{2.1.26}$$

Where V_{ert} does not depend on source (external due to gate and donors). Where $E^{i,l}$ and $E^{f,l}$ are energies of the initial and final states in the left reservoir. For tunneling from $E^{i,l}$ to $E^{f,qd}$, $E^{i,l}$ must be occupied with the probability that

$$P^l_{occupied} = f(E^{i,l}(N) - E_F) \tag{2.1.27}$$

For tunneling from an initial state $E^{i,l}$ in the left reservoir to a final state P in the quantum dot $E^{f,qd}$ must be empty so that occupation number $n_p = 0$. For tunneling from $E^{i,qd}$ to $E^{f,l}$(unoccupied probability for the left barrier) is given by

$$P^l_{unoccupied} = 1 - f(E^{f,l}(N) - E_F) \tag{2.1.28}$$

But for tunneling from $E^{i,r}$ to $E^{f,qd}$, $E^{i,r}$, must occupied with the probability that

$$P^r_{occupied} = f(E^{i,r}(N) - E_F) \tag{2.1.29}$$

Unoccupied probability for the right reservoir is given by

$$P^r_{unoccupied} = 1 - f(E^{f,r}(N) - E_F) \tag{2.1.30}$$

The initial energy of the right reservoir is given by

$$E^{i,r} = E_p - eV + (N + \frac{1}{2})\frac{e^2}{C} + \eta eV + eV_{ext}$$
$$= E_p + U(N+1) - U(N) - (1-\eta)eV \quad (2.1.31)$$

Similarly, the final energy of the right reservoir is given by

$$E^{f,r} = E_p - eV + \eta eV + (N - \frac{1}{2})\frac{e^2}{C} + eV_{ext}$$
$$= E_p + U(N) - U(N-1) - (1-\eta)eV \quad (2.1.32)$$

If a potential U is applied between the source and drain reservoir, current flows through quantum dot. This current changes the distribution of electrons in the quantum dot. The equilibrium distribution of electrons among the energy levels is changed from its equilibrium distribution. The applied potential difference can now be considered as a perturbation that changes the equilibrium distribution. At T = 0 the position of the conductance peaks as a function of gate voltage can be determined from a consideration of the equilibrium properties of the system only [17]. The temperature dependence of the amplitude and width of the Coulomb-blockade oscillations requires solution of the none equilibrium probability distribution $P(\{n_i\})$ from kinetic equation $\frac{\partial P}{\partial t} = 0$. Beenakker [18] has derived an analytical expression for P in the resonant tunneling regime, which generalizes earlier results by Kulic and Shekhter [19] in the classical regime.

2.2 Linear response conductance

In linear response theory, the response as a linear function of the perturbation.

$$P(\{n_i\}) = P_{eq}(\{n_i\})[1 + \frac{eV}{KT}\Psi(\{n_i\})] \quad (2.2.33)$$

$\Psi(\{n_i\})$ is the correction term due to the application of the perturbation. The equilibrium distribution is shifted by the factor $\Psi(\{n_i\})$. The none equilibrium probability evolves in time according to the master equation. The master equation has gain terms and lose terms. The none equilibrium probability distribution P is a stationary solution of the kinetic equation.

$$\frac{\partial P(\{n_i\})}{\partial t} = -\sum_p P(\{n_i\})\delta_{n_{p,0}}[\Gamma^l_p f(E^{i,l}(N)-E_F)+\Gamma^r_p f(E^{i,r}(N)-E_F)]-$$
$$\sum_p P(\{n_i\})\delta_{n_{p,1}}[\Gamma^l_p(1-f(E^{f,l}(N))-E_F)+\Gamma^r_p(1-f(E^{f,r}(N))-E_F)]$$
$$+\sum_p P(n_1,...,n_{p-1},1,n_{p+1},...,)\delta_{n_{p,0}}\times[\Gamma^l_p(1-f(E^{f,l}(N+1))-E_f)+$$
$$\Gamma^r_p(1-f(E^{f,r}(N+1))-E_F)]+\sum_p P(n_1,...,n_{p-1},0,n_{p+1},...,)\delta_{n_{p,1}}\times$$
$$[\Gamma^l_p f(E^{i,l}(N-1)-E_F)+\Gamma^r_p f(E^{i,r}(N-1)-E_F)]=0 \qquad (2.2.34)$$

This kinetic equation for the stationary is equivalent to the set of detailed balance equations (one for each P = 1, 2,...). The two transition rates from N to N + 1 and from N + 1 to N are equal when detailed balance holds.

$$P_{eq}(n_1,...,n_{p-1},1,n_{p+1},...)[\Gamma^l_p(1-f(E^{f,l}(N+1))-E_F)+\Gamma^r_p(1-f(E^{f,r}(N+1))-E_F)]$$
$$=P_{eq}(n_1,...,n_{p-1},0,n_{p+1},...,)[\Gamma^l_p f(E^{i,l}(N)-E_F)+\Gamma^r_p f(E^{i,r}(N)-E_F)] \qquad (2.2.35)$$

During tunneling energy must be conserved

$$E^{i,r}(N) = E_p + U(N+1) - U(N) - (1-\eta)eV$$
$$= E^{i,l}(N) - eV \qquad (2.2.36)$$

Similarly

$$E^{f,r}(N) = E_p + U(N) - U(N-1) - (1-\eta)eV$$
$$= E^{f,l}(N) - eV \qquad (2.2.37)$$

The (two-terminal) linear response conductance G of the quantum dot is defined as $G = \frac{I}{V}$ in the limit $V \to 0$. To solve the linear response problem we substitute Eq.

(2.2.35) into the Eq. (2.2.33), we obtain

$$\begin{aligned} P_{eq}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)[1 + \frac{eV}{KT}\Psi(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)] \\ \times[\Gamma_p^l(1 - f(E^{f,l}(N+1)) - E_F) + \Gamma_p^r(1 - f(E^{f,r}(N+1) - E_F))] \\ = P_{eq}(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)[1 + \frac{eV}{KT}\Psi(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)] \\ \times[\Gamma_p^l f(E^{i,l}(N) - E_F) + \Gamma_p^r f(E^{i,r}(N) - E_F)] \end{aligned} \tag{2.2.38}$$

By rearranging Eq. (2.2.38), it becomes

$$\begin{aligned} P_{eq}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)[1 + \frac{eV}{KT}\Psi(n_1, ..., n_{p-1}, 1, n_{p+1}, ...)] \\ [\Gamma_p^l + \Gamma_p^r - (\Gamma_p^l f(E^{f,l}(N+1) - E_F)) + \Gamma_p^r f(E^{f,r}(N+1) - E_F)] \\ = P_{eq}(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)[1 + \frac{eV}{KT}\Psi(n_1, ..., n_{p-1}, 0, n_{p+1}, ...)] \\ [\Gamma_p^l f(E^{i,l}(N) - E_F) + \Gamma_p^r f(E^{i,r}(N) - E_F)] \end{aligned} \tag{2.2.39}$$

Upon equating zeroth order terms on both sides of applied voltage from Eq. (2.2.39) , we have

$$\begin{aligned} P_{eq}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)[\Gamma_p^l(1 - f(E^{f,l}(N+1) - E_F)) + \\ \Gamma_p^r(1 - f(E^{f,r}(N+1) - E_F))] \\ = P_{eq}(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)[\Gamma_p^l f(E^{i,l}(N) - E_F) + \Gamma_p^r f(E^{i,r}(N) - E_F)] \end{aligned} \tag{2.2.40}$$

Similarly, equating first order terms of V from Eq. (2.2.39), we obtain

$$\begin{aligned} P_{eq}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)\frac{eV}{KT}\Psi(n_1, ..., n_{p-1}, 1, n_{p+1}, ...) \\ \times[\Gamma_p^l(1 - f(E^{f,l}(N+1) - E_F)) + \Gamma_p^r(1 - f(E^{f,r}(N+1) - E_F))] \\ = \frac{eV}{KT}P_{eq}(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)\Psi(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,) \\ \times[\Gamma_p^l f(E^{i,l}(N) - E_F) + \Gamma_p^r f(E^{i,r}(N) - E_F)] \end{aligned} \tag{2.2.41}$$

But,

$$f(E^{f,l}(N+1)-E_F) = f(E_p + U(N+1) - U(N) + \eta eV - E_F)$$
$$= f(\varepsilon + \eta eV) \qquad (2.2.42)$$

Where $\varepsilon = E_p + U(\tilde{N}+1)$ - $U(\tilde{N})$-E_F and $\tilde{N} = \sum_i n_i(i \neq p)$. Similarly:

$$f(E^{f,r}(N+1)-E_F) = f(E_p + U(N+1) - U(N) - (1-\eta)eV) - E_F$$
$$= f(\varepsilon - (1-\eta)eV) \qquad (2.2.43)$$

Similarly; $f(E^{i,l}(N) - E_F) = f(\varepsilon + \eta eV)$ and $f(E^{i,r}(N) - E_F) = f(\varepsilon - (1-\eta)eV)$. By inserting Eqs. (2.2.42) and (2.2.43) into the Eq. (2.2.41), we obtain

$$P_{eq}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)[1 + \frac{eV}{KT}\Psi(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)]$$
$$[\Gamma_p^l(1 - f(\varepsilon + \eta eV)) + \Gamma_p^r(1 - f(\varepsilon - (1-\eta)eV))]$$
$$= P_{eq}(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)[1 + \frac{eV}{KT}\Psi(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)]$$
$$[\Gamma_p^l f(\varepsilon + \eta eV) + \Gamma_p^r f(\varepsilon - (1-\eta)eV)] \qquad (2.2.44)$$

We can expand the Fermi functions using Taylor series and we take linear terms.

$$f(\varepsilon + \eta eV) = f(\varepsilon) + \eta eV\frac{\partial f}{\partial \varepsilon} \qquad (2.2.45)$$

$$f(\varepsilon - (1-\eta)eV) = f(\varepsilon) - (1-\eta)eV\frac{\partial f}{\partial \varepsilon} \qquad (2.2.46)$$

From Eq. (2.2.44) $\Gamma_p^l[1 - f(\varepsilon + \eta eV)] + \Gamma_p^r[1 - f(\varepsilon - (1-\eta)eV)] = (\Gamma_p^l + \Gamma_p^r)(1 - f(\varepsilon)) + eV\frac{\partial f}{\partial \varepsilon}[(1-\eta)\Gamma_p^r - \eta\Gamma_p^l$ And $\Gamma_p^l f(\varepsilon + \eta eV) + \Gamma_p^r f(\varepsilon - (1-\eta)eV) = (\Gamma_p^l + \Gamma_p^r)f(\varepsilon) + eV\frac{\partial f}{\partial \varepsilon}[\eta\Gamma_p^l - (1-\eta)\Gamma_p^r]$. Therefore, Eq. (2.2.44) becomes

$$P_{eq}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)[1 + \frac{eV}{KT}\Psi(n_1, ..., n_{p-1}, 1, n_{p+1}, ..,)]$$
$$\{(\Gamma_p^l + \Gamma_p^r)(1 - f(\varepsilon)) + eV\frac{\partial f}{\partial \varepsilon}((1-\eta)\Gamma_p^r - \eta\Gamma_p^l)\}$$
$$= P_{eq}(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)[1 + \frac{eV}{KT}\Psi(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)]$$
$$(\Gamma_p^l + \Gamma_p^r)f(\epsilon) + eV\frac{\partial f}{\partial \epsilon}(\eta\Gamma_p^l - (1-\eta)\Gamma_p^r) \qquad (2.2.47)$$

Then, Eq. (2.2.47) can be written as:

$$
\begin{aligned}
&P_{eq}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)(\Gamma_p^l + \Gamma_p^r)(1 - f(\varepsilon) + eV[P_{eq}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,) \\
&(\frac{\partial}{\partial \epsilon}(1-\eta)\Gamma_p^r - \eta\Gamma_p^l) + (\Gamma_p^l + \Gamma_p^r)(1 - f(\epsilon))\frac{\Psi}{KT}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)]+ \\
&(eV)^2\frac{\partial}{\partial \epsilon}[(1-\eta)\Gamma_p^r - \eta\Gamma_p^l]\frac{\Psi}{KT}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)P_{eq}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,) \\
&= P_{eq}(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)(\Gamma_p^l + \Gamma_p^r)f(\epsilon) + eV[P_{eq}(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,) \\
&\frac{\partial}{\partial \epsilon}[\eta\Gamma_p^l - (1-\eta)\Gamma_p^r](\Gamma_p^l + \Gamma_p^r)f(\epsilon)\frac{\Psi}{KT}(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)] + (eV)^2\frac{\partial}{\partial \epsilon} \\
&[\eta\Gamma_p^l - (1-\eta)\Gamma_p^r]\frac{\Psi}{KT}(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)\ P_{eq}(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)
\end{aligned}
\tag{2.2.48}
$$

Equating zeroth order terms of V from Eq. (2.2.48), we obtain

$$
\begin{aligned}
&P_{eq}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)(\Gamma_p^l + \Gamma_p^r)(1 - f(\varepsilon)) \\
&= P_{eq}(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)\ (\Gamma_p^l + \Gamma_p^r)f(\varepsilon)
\end{aligned}
\tag{2.2.49}
$$

By simplifying Eq. (2.2.49), we obtain

$$
P_{eq}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)(1 - f(\varepsilon)) = P_{eq}(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)f(\varepsilon) \tag{2.2.50}
$$

As can be seen from the above equation

$$
\begin{aligned}
1 - f(\varepsilon) &= 1 - \frac{1}{1 + e^{\beta\varepsilon}} \\
&= f(\varepsilon)e^{\beta\varepsilon}
\end{aligned}
\tag{2.2.51}
$$

On inserting Eq. (2.2.51) into the Eq. (2.2.50), we obtain:

$$
P_{eq}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)f(\varepsilon)e^{\beta\varepsilon} = P_{eq}(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)f(\varepsilon) \tag{2.2.52}
$$

By simplifying Eq. (2.2.52), we get

$$
P_{eq}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,) = P_{eq}(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)e^{-\beta\varepsilon} \tag{2.2.53}
$$

Similarly, equating first order terms of V from Eq. (2.2.48), we obtain

$$P_{eq}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)\{\frac{\partial f}{\partial \varepsilon}[(1-\eta)\Gamma_p^r - \eta\Gamma_p^l] + (\Gamma_p^l + \Gamma_p^r)(1 - f(\varepsilon))\frac{1}{KT}\Psi(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)\}$$
$$= P_{eq}(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)\{\frac{\partial f}{\partial \varepsilon}[\eta\Gamma_p^l - (1-\eta)\Gamma_p^r] + (\Gamma_p^l + \Gamma_p^r)f(\varepsilon) \frac{1}{KT}\Psi(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)\} \quad (2.2.54)$$

By simplifying Eq. (2.2.54) and rearranging we obtain

$$e^{-\beta\varepsilon}\{\frac{\partial f}{\partial \varepsilon}[(1-\eta)\Gamma_p^r - \eta\Gamma_p^l] + (\Gamma_p^l + \Gamma_p^r)f(\varepsilon)e^{\beta\varepsilon}\frac{1}{KT}\Psi(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)\}$$
$$= \frac{\partial f}{\partial \varepsilon}[\eta\Gamma_p^l - (1-\eta)\Gamma_p^r] \ + (\Gamma_p^l + \Gamma_p^r)f(\varepsilon) \ \frac{1}{KT}\Psi(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,) \quad (2.2.55)$$

In View of Eq. (2.2.55)

$$\frac{1}{KT}(\Gamma_p^l + \Gamma_p^r)f(\varepsilon)[\Psi(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,) - \Psi(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)]$$
$$= \frac{\partial f}{\partial \varepsilon}[\eta\Gamma_p^l - (1-\eta)\Gamma_p^r] \ [1 + e^{-\beta\varepsilon}] \quad (2.2.56)$$

By arranging Eq. (2.2.56), we obtain

$$\Psi(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,) = \Psi(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,) + \frac{KT}{f}\frac{\partial f}{\partial \varepsilon}(1 + e^{-\beta\varepsilon})[\frac{\eta(\Gamma_p^l + \Gamma_p^r) - \Gamma_p^r}{\Gamma_p^l + \Gamma_p^r}] \quad (2.2.57)$$

By simplifying eq. (2.2.57), we get

$$\Psi(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,) = \Psi(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,) + \frac{KT}{f}\frac{\partial f}{\partial \varepsilon}(1 + e^{-\beta\varepsilon})[\eta - \frac{\Gamma_p^r}{\Gamma_p^l + \Gamma_p^r}] \quad (2.2.58)$$

But,

$$f'(\varepsilon) = -\beta f^2(\varepsilon)e^{\beta\varepsilon} \quad (2.2.59)$$

$$KTf'(\varepsilon) = -f^2(\varepsilon)e^{\beta\varepsilon} \tag{2.2.60}$$

$$\begin{aligned} KTf'(\varepsilon)(1+e^{-\beta\varepsilon}) &= -f^2(\varepsilon)(1+e^{\beta\varepsilon}) \\ &= -f(\varepsilon) \end{aligned} \tag{2.2.61}$$

By substituting Eq. (2.2.61) into the Eq. (2.2.58) and simplifying yields

$$\Psi(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,) = \Psi(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,) + \frac{\Gamma_p^r}{\Gamma_p^l + \Gamma_p^r} - \eta \tag{2.2.62}$$

The solution is the total sum over each occupation number.

$$\Psi(\{n_i\}) = \sum_i n_i(\frac{\Gamma_p^r}{\Gamma_p^l + \Gamma_p^r} - \eta) + constant \tag{2.2.63}$$

Now if $\Gamma_p^l = \Gamma_p^r$, then $\frac{\Gamma_p^r}{\Gamma_p^l+\Gamma_p^r} = \frac{1}{2}$, (for identical tunnel barriers), and if $\eta = \frac{1}{2}$, the first order non-equilibrium correction Ψ to P_{eq} is zero.

$$P(\{n_i\}) = P_{eq}(\{n_i\})\{1 + \frac{eV}{KT}[constant + \sum_{i=1}^{\infty} n_i(\frac{\Gamma_p^r}{\Gamma_p^l + \Gamma_p^r} - \eta)]\} \tag{2.2.64}$$

The stationary current through the left barrier equals the right barrier, and is given by:

$$I = -e\sum_{p=1}^{\infty}\sum_{\{n_i\}}\Gamma_p^l P(\{n_i\})\{\delta_{n_{p,0}} f(E^{i,l}(N) - E_F) - \delta_{n_{p,1}}[1 - f(E^{f,l}(N) - E_F)]\} \tag{2.2.65}$$

The first term gives the probability to find an electron with energy E on the left side and a corresponding empty state on the right side. The second summation is over all realization of occupation numbers $\{n_1, n_2, ...\} \equiv \{n_i\}$ of the energy levels in the quantum dot each with stationary probability $P(\{n_i\})$.

$$I = -e\sum_p\sum_{\{n_i\}}\Gamma_p^l P(\{n_i\})\ \{\delta_{n_{p,0}} f(\varepsilon + \eta eV) - \delta_{n_{p,1}}(1 - f(\varepsilon + \eta eV))\} \tag{2.2.66}$$

Linearizing f , f($\varepsilon + \eta eV$)= f(ε) + $\eta eV\frac{\partial f}{\partial \varepsilon}$ and Eq. (2.2.66) becomes

$$I = -e\sum_p\sum_{\{n_i\}}\Gamma_p^l P(\{n_i\})\{(\delta_{n_{p,0}} + \delta_{n_{p,1}})f + \eta eV\frac{\partial f}{\partial \varepsilon}(\delta_{n_{p,0}} + \delta_{n_{p,1}}) - \delta_{n_{p,1}}\} \tag{2.2.67}$$

Then, we obtain the expression for the current as follows.

$$I = -e\sum_p \sum_{\{n_i\}} \Gamma_p^l P(\{n_i\})[f + \eta eV\frac{\partial f}{\partial \varepsilon} - \delta_{n_{p,1}}] \tag{2.2.68}$$

The current(I) through the quantum dot for first order in V is obtained by linearizing after substitution P($\{n_i\}$).

$$I = -e\sum_p \sum_{\{n_i\}} \Gamma_p^l P_{eq}(\{n_i\})[1+\frac{eV}{KT}\Psi(\{n_i\})]\{\delta_{n_{p,0}}(f+\eta eV\frac{\partial f}{\partial \varepsilon})-\delta_{n_{p,1}}(1-f-\eta eV\frac{\partial f}{\partial \varepsilon})\} \tag{2.2.69}$$

By arranging Eq. (2.2.69), we get

$$I = -\frac{e^2V}{KT}\sum_p \sum_{\{n_i\}} \Gamma_p^l P_{eq}(\{n_i\})\{(\delta_{n_{p,0}} + \delta_{n_{p,1}})\eta KT f^2(\varepsilon)e^{\beta\varepsilon} + \Psi(\{n_i\})\delta_{n_{p,0}}f(\varepsilon) - \Psi(\{n_i\})\delta_{n_{p,1},}f(\varepsilon)e^{\beta\varepsilon}\} \tag{2.2.70}$$

In view of Eq. (2.2.51) $e^{\beta\varepsilon} = \frac{1-f}{f}$; $KTf'(\varepsilon) = f(1-f)$ so that Eq. (2.2.70) becomes

$$I = -\frac{e^2V}{KT}\sum_p \sum_{\{n_i\}} \Gamma_p^l P_{eq}(\{n_i\})e^{\beta\varepsilon}, f(\varepsilon)\{(\delta_{n_{p,0}} + \delta_{n_{p,1}})f - \eta + \Psi(\{n_i\})\delta_{n_{p,0}}e^{-\beta\varepsilon} - \Psi(\{n_i\})\delta_{n_{p,1}}\} \tag{2.2.71}$$

Where, $(P_{eq}(\{n_i\})\delta_{n_{p,0}} + P_{eq}(\{n_i\})\delta_{n_{p,1}})\eta KTf'(\varepsilon)\} = P_{eq}(\{n_i\})\delta_{n_{p,0}}\eta f(\varepsilon)$. In views of Eq. (2.2.53) we have:

$$\begin{aligned} &P_{eq}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...)\Psi(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,)e^{-\beta\varepsilon} \\ &\quad = P_{eq}(\{n_i\})\delta_{n_{p,0}}\Psi(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,) \end{aligned} \tag{2.2.72}$$

By substituting Eq. (2.2.72) into Eq. (2.2.71) and after rearrangement we get:

$$I = -\frac{e^2V}{KT}\sum_p \sum_{\{n_i\}} \Gamma_p^l P_{eq}(\{n_i\})\delta_{n_{p,0}}f(\varepsilon)[\eta + \Psi(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,) - \Psi(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,)] \tag{2.2.73}$$

In view of Eq. (2.2.62) :

$$\eta + \Psi(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,) - \Psi(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,) = \frac{\Gamma_p^r}{\Gamma_p^l + \Gamma_p^r} \tag{2.2.74}$$

Inserting Eq. (2.2.74) into the Eq. (2.2.73), we obtain

$$I = \frac{e^2 V}{KT} \sum_p \sum_{\{n_i\}} \frac{\Gamma_p^l \Gamma_p^r}{\Gamma_p^l + \Gamma_p^r} P_{eq}(\{n_i\}) \delta_{n_{p,0}} f(\varepsilon) \tag{2.2.75}$$

Now, we are ready to calculate the conductance through the quantum dot. The conductance is defined as:

$$G = \lim_{V \to 0} (\frac{I}{V}) \tag{2.2.76}$$

By substituting Eq. (2.2.76) into the Eq. (2.2.75), we get

$$G = \frac{e^2}{KT} \sum_p \sum_{\{n_i\}} \frac{\Gamma_p^l \Gamma_p^r}{\Gamma_p^l + \Gamma_p^r} P_{eq}(\{n_i\}) \delta_{n_{p,0}} f(\varepsilon) \tag{2.2.77}$$

Using Eq. (2.2.53), $P(n_1, ..., n_{p-1}, 0, n_{p+1}, ...,) = P(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,) e^{\beta\varepsilon}$, so that Eq. (2.2.77) can be written as:

$$\begin{aligned} G &= \frac{e^2}{KT} \sum_p \sum_{\{n_i\}} \frac{\Gamma_p^l \Gamma_p^r}{\Gamma_p^l + \Gamma_p^r} P_{eq}(\{n_i\}) \delta_{n_{p,0}} f(E_p + U(N+1) - U(N) - E_F) \\ &= \frac{e^2}{KT} \sum_p \sum_{\{n_i\}} \frac{\Gamma_p^l \Gamma_p^r}{\Gamma_p^l + \Gamma_p^r} P_{eq}(n_1, ..., n_{p-1}, 1, n_{p+1}, ...,) f(\varepsilon) e^{\beta\varepsilon} \end{aligned} \tag{2.2.78}$$

In view of Eq. (2.2.51), $1 - f = f(\varepsilon) e^{\beta\varepsilon}$, the conductance becomes:

$$G = \frac{e^2}{KT} \sum_p \sum_{N=0}^{\infty} \frac{\Gamma_p^l \Gamma_p^r}{\Gamma_p^l + \Gamma_p^r} P_{eq}(N) F_{eq}(E_p|N)[1 - f(E_p + U(N) - U(N-1)) - E_F] \tag{2.2.79}$$

The probability to find N electrons in the quantum dot in equilibrium with the reservoirs (at $T = T_l = T_r$)is given by the Grand Canonical distribution function.

$$P_{eq}(\{n_i\}) = Cexp\{-\beta[\sum n_i E_i + U(N) - NE_F]\} \tag{2.2.80}$$

The distribution of electrons among the discrete energy levels is:

$$\begin{aligned} P_{eq}(N) &= \sum_{\{n_i\}} P_{eq}(\{n_i\})\delta_N \sum_i n_i \\ &= C\sum_{\{n_i\}} e^{-\beta[\sum_i n_iE_i+U(N)-NE_F]}\delta_N, \sum_i n_i \\ &= Ce^{-\beta(U(N)-NE_F)} \sum_{\{n_i\}} e^{-\beta\sum_i n_iE_i}\delta N, \sum_i n_i \end{aligned} \tag{2.2.81}$$

Where, $F(N) = -K_BTlnZ(N) =$ -$K_BTln[\sum_{\{n_i\}} e^{-\beta\sum_i n_iE_i}\delta N, \sum_i n_i] = e^{-\beta F(N)}$. Then, Eq. (2.2.81) becomes

$$P_{eq}(N) = Ce^{-\beta(F(N)+U(N)-NE_F)} \tag{2.2.82}$$

Where, C $= \sum_N e^{-\beta\Omega(N)}$. The free energy of the internal dot of the quantum dot is defined as

$$F(N) = \sum_i n_iE_i - TS(N) \tag{2.2.83}$$

The Landau or grand potential is

$$\begin{aligned} \Omega(N) &= F(N) + U(N) - NE_F \\ &= \sum_i n_iE_i - TS(N) + U(N) - NE_F \end{aligned} \tag{2.2.84}$$

$\Omega(N) = F(N) + U(N) - NE_F$, Ω is the thermodynamics potential of the quantum dot. If $\frac{e^2}{C} \gg$ kT, the effect of the charging energy must also be taken into account. For a given N

$$G = \frac{e^2}{KT}\sum_{p=1}^{\infty}\sum_{N=1}^{\infty}\frac{\Gamma_p^l\Gamma_p^r}{\Gamma_p^l+\Gamma_p^r}P_{eq}(N)F_{eq}(E_p|N)[1-f(E_p+U(N)-U(N-1)-E_F)] \tag{2.2.85}$$

This distribution functions expresses the tunneling of an electron from an initial state P in the dot to a final state in the reservoir requires an occupied initial state and empty final state. In this classical limit, one may approximate;

$$\begin{aligned} P_{eq}(N) &= \frac{e^{-\beta(F(N)+U(N)-NE_F)}}{Z} \\ &= \frac{exp\{-\beta\Omega(N)\}}{\sum_{N=0} e^{-\beta\Omega(N)}} \end{aligned} \tag{2.2.86}$$

The conditional probability that the quantum dot energy level E_p is occupied given that the quantum dot contains N electrons is :

$$\begin{aligned} F_{eq}(E_p|N) &= \frac{1}{P_{eq}(N)} \sum_{\{n_i\}} P_{eq}(\{n_i\})\delta_{n_{p,1}}\delta N, \sum n_i \\ &= \frac{\sum_N e^{-\beta\Omega(N)}}{e^{-\beta\Omega(N)}} \frac{1}{Z} \sum_{\{n_i\}} e^{-\beta[\sum_i n_i E_i + U(N) - NE_F]} \delta_{n_{p,1}}, \delta N, \sum n_i \\ &= e^{\beta F(N)} \sum_{\{n_i\}} e^{-\beta \sum_i n_i E_i} \delta_{n_{p,1}} \delta N, \sum n_i \end{aligned} \tag{2.2.87}$$

Where $Z = \sum_N e^{-\beta\Omega(N)}$, and $U(N) - NE_F = \Omega(N) - F(N)$ Now, from Eq. (2.2.85), we can check that $\sum_N P_{eq}(N)F_{eq}(E_p|N) = \sum_{\{n_i\}} P_{eq}(\{n_i\})\delta_{n_{p,1}}$

$$\begin{aligned} \sum_N P_{eq}(N)F_{eq}(E_p|N) &= \frac{\sum_{\{n_i\}} \sum_N e^{-\beta\Omega(N)} e^{-\beta\sum_i n_i E_i} e^{\beta F(N)}}{\sum_N e^{-\beta\Omega(N)}} \\ &= \sum_{\{n_i\}} \frac{\sum_N e^{-\beta[\sum_i n_i E_i + U(N) - NE_F]} \delta_{n_{p,1}}, \delta N, \sum n_i}{\sum_N e^{-\beta\Omega(N)}} \\ &= \sum_{\{n_i\}} \frac{e^{-\beta(\sum_i n_i E_i + U(N) - NE_F)} \delta_{n_{p,1}}}{\sum_N e^{-\beta\Omega(N)}} \\ &= \sum_{\{n_i\}} P_{eq}(\{n_i\})\delta_{n_{p,1}} \end{aligned} \tag{2.2.88}$$

Then, the conductance becomes:

$$G = \frac{e^2}{K_B T} \sum_{p=1}^{\infty} \sum_{N=1}^{\infty} \frac{\Gamma_p^l \Gamma_p^r}{\Gamma_p^l + \Gamma_p^r} P_{eq}(N) F_{eq}(E_p|N)(1 - f(E_p + U(N) - U(N-1) - E_F)) \tag{2.2.89}$$

Chapter 3

High and low temperature behavoir

3.1 High temperature behavior

We will now discuss some limiting cases of the general result Eq. (2.2.89). We consider the quantum dot in the high temperature limit $K_BT \gg \Delta E$. The high temperature limit of interest for comparision with the low temperature. Classically:

$$E = TS - PV + \mu N + U(N) \tag{3.1.1}$$

The internal energy (E) can be a function of entropy S, Volume, V and the number of particles N: if $P = 0$, negligible,

$$\begin{aligned} E &= U(N) + \sum_i n_i E_i \\ &= TS \ + \mu N + U(N) \end{aligned} \tag{3.1.2}$$

The free energy is

$$\begin{aligned} F &= \sum_i n_i E_i - TS \\ &= \bar{\mu} N \end{aligned} \tag{3.1.3}$$

Now, the thermodynamics potential of the quantum dot is:

$$\begin{aligned}\Omega &= F(N) + U(N) - NE_F \\ &= U(N) + N(\bar{\mu} - E_F)\end{aligned} \tag{3.1.4}$$

Therefore, $P_{eq}(N)$ takes its classical form

$$\begin{aligned}P_{eq}(N) &= P_{classical(N)} \\ &= \frac{e^{-\beta(U(N)+N(\bar{\mu}-E_F))}}{\sum_N e^{-\beta(U(N)+N(\bar{\mu}-E_F))}}\end{aligned} \tag{3.1.5}$$

$\bar{\mu}$ is the chemical potential of the quantum dot in equilibrium. The summation over P in Eq. (2.2.89) may be replaced by integration over E, multiplied by the density of states $\rho(E)$ in the quantum dot. If $KT \ll \bar{\mu}, E_F$, we disregard the energy dependence of density of states and of the tunnel rates. The conductance becomes

$$G = \frac{e^2}{KT}\sum_{N=1}^{\infty}\int_0^{\infty}\frac{\Gamma^l(E)\Gamma^r(E)}{\Gamma^l(E)+\Gamma^r(E)}\rho(E)f(E-\bar{\mu})(1-f(E-\bar{\mu}+\Delta))P_{classical(N)}dE \tag{3.1.6}$$

Where, we have used $\Delta = U(N) - U(N-1) + \bar{\mu} - E_F$ and $f(E-\mu)(1-f(E+U(N)-U(N-1)-E_F)) = f(E-\mu)(1-f(E-\mu+\Delta))$. $\sum_p \to \int \rho(E)dE$. Only states near E $\approx \bar{\mu}$ contribute to conduction as $K_BT \ll \bar{\mu}, E_F$ and then, $\rho(E) \to \rho(\bar{\mu})$, $\Gamma^l(E) \to \Gamma^l(\bar{\mu})$, and $\Gamma^r(E) \to \Gamma^r(\bar{\mu})$. We can write Eq. (3.1.6) as follows;

$$G = \frac{e^2\rho(\bar{\mu})}{KT}\frac{\Gamma^l(\bar{\mu})\Gamma^r(\bar{\mu})}{\Gamma^l(\bar{\mu})+\Gamma^r(\bar{\mu})}\sum_{N=1}^{\infty}P_{classical(N)}\int_0^{\infty}f(E-\bar{\mu})(1-f(E-\bar{\mu}+\Delta))dE \tag{3.1.7}$$

In the classical form one may approximate $F_{eq}(E_p|N)$ by the Fermi-Dirac distribution

$$F_{eq}(E_p|N) = f(E_p - \mu(N)) \tag{3.1.8}$$

If KT $\gg \Delta E$, the chemical potential is determined from

$$N = \sum_{P=1}^{\infty} f(E_p - \mu(N)) \tag{3.1.9}$$

Where $f(E_p - \mu(N)) = \frac{1}{1+e^{\beta(E_p-\mu(N))}}$. If KT $\gg \Delta E$, but KT $\ll \mu, E_F$, one may in general disregard the energy dependence of the density of states and tunnel rates.Therefore,

$$G = \frac{e^2\rho}{KT}\frac{\Gamma^l\Gamma^r}{\Gamma^l+\Gamma^r}\sum_{N=1}^{\infty}P_{classical(N)}\int_0^{\infty} f(E-\mu)(1-f(E-\mu+\Delta))dE \tag{3.1.10}$$

To evaluate the integral in the Eq. (3.1.10), we make a change of variable. Let $y = \beta(E-\mu) \to dE = \frac{dy}{\beta}$, $x = \Delta\beta = \frac{\Delta}{K_BT}$ and from this follows, then Eq. (3.1.10) becomes:

$$G = \frac{e^2\rho}{KT}\frac{\Gamma^l\Gamma^r}{\Gamma^l+\Gamma^r}\sum_{N=1}^{\infty}P_{classical(N)}KT\int_{-\beta\mu}^{\infty} f(y)(1-f(y+x))dy \tag{3.1.11}$$

We considered the limit of integration $-\beta\mu \to -\infty \to \beta\mu \to \infty$ and since $\beta\mu \gg 1, -\beta\mu \to -\infty$, $\int_{-\infty}^{\infty} f(y)(1-f(y+x))dy = \beta x(1-e^{-\beta x})^{-1} = \beta g(x)$. Then Eq. (3.1.11) can be written as:

$$G = \frac{e^2\rho}{KT}\frac{\Gamma^l\Gamma^r}{\Gamma^l+\Gamma^r}\sum_{N=1}^{\infty}P_{classical(N)}\Delta(N)(1-e^{-\beta\Delta(N)})^{-1} \tag{3.1.12}$$

Where, Γ and ρ are evaluated at energy $\bar{\mu}$ we used that $\mu(N) \approx constant = \bar{\mu}$ for all N which $P_{classical(N)}$ differs appreciably from zero. Where $g(\Delta(N)) = \Delta(N)(1 - e^{-\beta\Delta(N)})^{-1}$. Therefore, for $\Delta E \ll K_BT \ll \bar{\mu}, E_F$

$$G = \frac{e^2\rho(\bar{\mu})}{KT}\frac{\Gamma^l_{(\bar{\mu})}\Gamma^r_{(\bar{\mu})}}{\Gamma^l_{(\bar{\mu})}+\Gamma^r_{(\bar{\mu})}}\sum_{N=1}^{\infty}P_{classical(N)}g(\Delta(N)) \tag{3.1.13}$$

The normalized conductance of Eq. (3.1.13) is

$$\frac{G}{G_\infty} = \sum_{N=1}^{\infty}P_{classican}(N)g(\Delta(N)) \tag{3.1.14}$$

Where, $G_\infty = \frac{e^2\rho}{K_BT}\frac{\Gamma^l\Gamma^r}{\Gamma^l+\Gamma^r}$. Now we check that from Eq. (3.1.13) $g(\Delta(N))$ is equal to K_BT.

$$U(N) = \frac{N^2e^2}{2C} - Ne\Phi_{ert} \tag{3.1.15}$$

$$U(N) - U(N-1) = \frac{e^2}{2C}[N^2 - N^2 + 2N - 1] + e\Phi_{ert}$$
$$= \frac{e^2}{C}(N - \frac{1}{2}) + e\Phi_{ert} \quad (3.1.16)$$

$U(N) - U(N-1) = \Delta(N)$. So that

$$\Delta(N) = \frac{e^2}{C}(N - \frac{1}{2}) + e\Phi_{ert} + \bar{\mu} - E_F \quad (3.1.17)$$

If in addition to KT$\gg \Delta E$ and KT $\gg \frac{e^2}{C}$ (while still KT $\ll \bar{\mu}, E_F$), then the effect of the charging energy may be ignored as well.

$$g(\Delta(N)) = \frac{\Delta(N)}{1 - e^{-\beta\Delta(N)}}$$
$$= \frac{\frac{e^2}{C}(N - \frac{1}{2}) + e\Phi_{ert} + \mu - E_F}{1 - e^{\frac{e^2}{CKT}[(N-\frac{1}{2}) + \frac{C}{e}\Phi_{ert} + \frac{C}{e^2}(\mu - E_F)]}} \quad (3.1.18)$$

We can expand the exponential term in the denominator using Taylor series and Eq. (3.1.18) becomes

$$g(\Delta(N)) = \frac{\frac{e^2}{C}(N - \frac{1}{2}) + e\Phi_{ert} + \mu - E_F}{1 - (1 - \beta[\frac{e^2}{C}(N - \frac{1}{2}) + e\Phi_{ert} + \mu - E_F] + -)} \quad (3.1.19)$$

Neglecting higher order terms

$$g(\Delta(N)) = \beta^{-1}[\frac{\frac{e^2}{C}(N - \frac{1}{2}) + e\Phi_{ert} + \mu - E_F}{\frac{e^2}{C}(N - \frac{1}{2}) + e\Phi_{ert} + \mu - E_F}]$$
$$= K_B T \quad (3.1.20)$$

By substituting Eq. (3.1.20) into the Eq. (3.1.13), we obtain the expression for conductance as follows.

$$G = e^2\rho(\tilde{\mu})\frac{\Gamma^l(\tilde{\mu})\Gamma^r(\tilde{\mu})}{\Gamma^l(\tilde{\mu}) + \Gamma^r(\tilde{\mu})}\sum_{N=1}^{\infty} P_{classical(N)} \quad (3.1.21)$$

By normalization

$$G = e^2\rho(\tilde{\mu})\frac{\Gamma^l(\tilde{\mu})\Gamma^r(\tilde{\mu})}{\Gamma^l(\tilde{\mu}) + \Gamma^r(\tilde{\mu})}$$
$$= G_\infty, if \Delta E, \frac{e^2}{C} \ll KT \ll \mu, E_F \quad (3.1.22)$$

The conductance of the individual barriers and the quantum dot in the high temperature limit $K_B T \gg e^2/C, \Delta E$ neither the discreteness of the energy levels nor the charging are important. The conductance then does not exhibit oscillations as a function of gate voltage. The conductance of the quantum dot in the high temperature limit is the two tunnel barriers in series [20]. $G^l = e^2\rho\Gamma^l$ and $G^r = e^2\rho\Gamma^r$ are conductance of left and right tunnel barriers respectively. The high temperature limit is of interest for comparision with the low temperature.

$$\begin{aligned} G &= \frac{e^2\rho\Gamma^l\Gamma^r}{\Gamma^l+\Gamma^r} \\ &= \frac{G^l G^r}{G^l+G^r} if \Delta E, \frac{e^2}{C} \ll K_B T \ll E_F, \bar{\mu} \end{aligned} \tag{3.1.23}$$

$\frac{1}{G^l} = R_l$ is the resistance of left tunnel barrier. $R_l = \frac{1}{e^2\rho\Gamma^l}$, $R_r = \frac{1}{e^2\rho\Gamma^r}$ tunnel resistance of right reservoir. If we did not take $\beta\mu \to \infty$, then we can expand f(y + x) in Eq. (3.1.11) using Taylor series as follows.

$$f(y+x) = f(y) + xf'(y) + \frac{x^2}{2}f''(y) + - \tag{3.1.24}$$

Let $f'(y) = -f^2 e^y$ then, $f''(y) = -[2ff' + f^2]e^y$. The right hand side Eq. (3.1.24) can be written as:

$$\begin{aligned} \int_{-\beta\mu}^{\infty} f(y)[1 - f - xf' - \frac{x^2}{2}f'' + ...]dy = \int_{-\beta\mu}^{\infty} f(y)(1-f(y))dy - x\int_{-\beta\mu}^{\infty} f(y)f'(y)dy - \\ \frac{x^2}{2}\int_{-\beta\mu}^{\infty} dy f f'' \end{aligned} \tag{3.1.25}$$

Where, $\frac{df}{dy} = -f^2 e^y = -f(y)(1-f(y))$. Let we solve the right hand side of Eq. (3.1.25) separately:

$$\begin{aligned} \int_{-\beta\mu}^{\infty} f(y)(1-f(y))dy &= -\int_{-\beta\mu}^{\infty} \frac{df(y)}{dy}dy \\ &= -f(y)|_{-\beta\mu}^{\infty} \\ &= -[\frac{1}{1+e^{\infty}} - \frac{1}{1+e^{-\beta\mu}}] \\ &= \frac{1}{1+e^{-\beta\mu}} \end{aligned} \tag{3.1.26}$$

$$\begin{aligned}\int_{-\beta\mu}^{\infty} xdyf(y)f'(y) &= x\int_{-\beta\mu}^{\infty}\frac{d}{dy}(f(y)^2)\\ &= \frac{xf(y)^2}{2}\int_{-\beta\mu}^{\infty}\\ &= \frac{x}{2}[\frac{1}{(1+e^{\infty})^2}-\frac{1}{(1+e^{-\beta\mu})^2}]\\ &= -\frac{x}{2(1+e^{-\beta\mu})^2} \end{aligned} \tag{3.1.27}$$

$$\int_{-\beta\mu}^{\infty}\frac{x^2}{2}dyf(y)f''(y) =? \tag{3.1.28}$$

To evaluate the integral in the Eq. (3.1.28) we make a change of variables. Let $f' = -f^2e^y$ and $f'' = -(2ff' + f^2)e^y = f'(y) - 2ff'e^y$.

$$\begin{aligned} ff'' &= ff' - 2f^2f'e^y\\ &= ff' + 2f'^2 \end{aligned} \tag{3.1.29}$$

To obtain the expression of f'^2 :

$$Let\frac{d}{dy}(f(y)e^y) = (f' + f)e^y \tag{3.1.30}$$

Then, from Eq. (3.1.30), we obtain f' as follows.

$$f' = e^{-y}\frac{d}{dy}(fe^y) - f \tag{3.1.31}$$

Now, by multiplying both side of Eq. (3.1.31) by f', we get

$$f'^2 = f'e^{-y}\frac{d}{dy}(fe^y) - ff' \tag{3.1.32}$$

In inserting Eq. (3.1.32) into the Eq. (3.1.29), we obtain:

$$\begin{aligned} ff'' &= ff' - 2ff' + 2f'e^{-y}\frac{d}{dy}(fe^y)\\ &= -ff' + 2f'e^{-y}\frac{d}{dy}(fe^y) \end{aligned} \tag{3.1.33}$$

In inserting Eq. (3.1.33) into the Eq. (3.1.28), we get

$$\int_{-\beta\mu}^{\infty} f(y)f''(y)dy = -\int_{-\beta\mu}^{\infty} ff'dy + 2\int_{-\beta\mu}^{\infty} f'e^{-y}\frac{d}{dy}(fe^{y})dy$$
$$= [2ff' - \frac{f^2}{2}]|_{-\beta\mu}^{\infty} - 2\int_{-\beta\mu}^{\infty} ff''dy + 2\int_{-\beta\mu}^{\infty} ff'dy \qquad (3.1.34)$$

By integrating by part, where, $3\int_{-\beta\mu}^{\infty}\int ff''dy = [2ff' - \frac{f^2}{2} + f^2]|_{-\beta\mu}^{\infty}$ and finally, Eq. (3.1.34) becomes

$$\int_{-\beta\mu}^{\infty} f(y)f''(y)dy = \frac{2}{3}ff'|_{-\beta\mu}^{\infty} + \frac{f^2}{6}|_{-\beta\mu}^{\infty}$$
$$= (\frac{2}{3}f^3e^y + \frac{f^2}{6})|_{-\beta\mu}^{\infty}$$
$$= \frac{2}{3}(\frac{e^{-\beta\mu}}{(1+e^{-\beta\mu})^3}) - \frac{1}{6(1+e^{-\beta\mu})^2} \qquad (3.1.35)$$

So that on taking into account Eqs. (3.1.35), (3.1.27), (3.1.26) and substituting into the the right hand side of Eq. (3.1.25), we get

$$\int_{-\beta\mu}^{\infty} dyf(y)(1-f(y+x)) = \frac{1}{1+e^{-\beta\mu}} + \frac{x}{2(1+e^{-\beta\mu})^2} + \frac{x^2}{12(1+e^{-\beta\mu})^2} - \frac{x^2e^{-\beta\mu}}{3(1+e^{-\beta\mu})^3}$$
$$= \frac{1}{1+e^{-\beta\mu}}[1 + \frac{x^2+6x}{12(1+e^{-\beta\mu})} - \frac{x^2e^{-\beta\mu}}{3(1+e^{-\beta\mu})^2}] \qquad (3.1.36)$$

In inserting Eq. (3.1.36) into the Eq. (3.1.10), we obtain the expression for conductance.

$$G = \frac{e^2\rho(\mu)\Gamma^l(\mu)\Gamma^r(\mu)}{\Gamma^l(\mu)+\Gamma^r(\mu)(1+e^{-\beta\mu})}\sum_{N=1}^{\infty} P_{classical(N)}$$
$$[1 + \frac{\Delta(\Delta+6KT)}{12(KT)^2(1+e^{-\beta\mu})^2} - \frac{\Delta^2e^{-\beta\mu}}{3(KT)^2(1+e^{-\beta\mu})^2}] \qquad (3.1.37)$$

From Eq. (3.1.37), let we first calculate Δ, $P_{classical(N)}$, $< N >$, $< N^2 >$, and $< \Delta^2 >$

$$\Delta = U(N) - U(N-1) + \mu - E_F$$
$$= (N - \frac{1}{2})U_c + \bar{\mu} - E_F$$
$$= NU_c + \bar{\mu} - E_F - \frac{U_c}{2}$$
$$= U_c[N + a] \qquad (3.1.38)$$

$$P_{classical(N)} = Ce^{-\beta(U(N)+N(\bar{\mu}-E_F))}$$
$$= Ce^{-\beta(\frac{1}{2}U_cN^2-(E_F+eV_{ert}-\mu)N)}$$
$$= Ce^{-(N-a)^2/2\sigma^2} \tag{3.1.39}$$

Where, a = $[\frac{\mu-E_F}{U_c}-\frac{1}{2}]$, $\sigma_{classical} = \sqrt{\frac{K_BT}{U_c}}$, $C = \sum_{N=1}^{\infty} e^{-(N-a)^2/2\sigma^2}$. The average value of $< N >$ is given by:

$$< N >= \frac{\sum_{N=1}^{\infty} Ne^{-(N-a)^2}/2\sigma^2}{\sum_N e^{-(N-a)^2}/2\sigma^2} \tag{3.1.40}$$

Let y = N-a → N = y + a and so that by substituting in the Eq. (3.1.40), we obtain

$$< N > = \frac{\sum_{-a}^{\infty}(y+a)e^{-y^2}/2\sigma^2}{\sum_{-a}^{\infty} e^{-y^2}/2\sigma^2}$$
$$= \sum_{y=-a}^{\infty}(y+a)e^{-cy^2}/\sum_{y=-a}^{\infty} e^{-cy^2}, -a \to -\infty$$
$$= a + \int_{-\infty}^{\infty} ye^{-cy^2}dy/\int_{-\infty}^{\infty} e^{-cy^2}dy$$
$$= a \tag{3.1.41}$$

All others vanishes since it is an integral over an odd function.

$$< N^2 > = \sum_N N^2e^{-(N-a)^2/2\sigma^2}/\sum_N e^{-(N-a)^2}/2\sigma^2$$
$$= \sum_{-a}^{\infty}(y+a)^2e^{-cy^2}dy/\sum_{-a}^{\infty} e^{-cy^2}dy$$
$$= \int_{-\infty}^{\infty}(y+a)^2e^{-cy^2}dy/\int_{-\infty}^{\infty} e^{-cy^2}dy$$
$$= a^2 + \sigma^2 \tag{3.1.42}$$

$$< \Delta^2 > = U_c^2 < N^2 > +2U_c(E_F - \mu + \frac{U_c}{2}) < N > +(E_F - \mu + \frac{U_c}{2})^2$$
$$= U_c^2(a^2+\sigma^2) - 2U_c(E_F - \mu + \frac{U_c}{2})a + (E_F - \mu + \frac{U_c}{2})^2$$
$$= (E_F + eV_{ext} - \mu)^2 + U_cK_BT -$$
$$2(E_F - \mu + \frac{U_c}{2})(E_F + eV_{ext} - \bar{\mu}) + (E_F - \mu + \frac{U_c}{2})^2 \tag{3.1.43}$$

By substituting Eqs. (3.1.41) and (3.1.42) into the Eq. (3.1.43) and simplifying, we obtain

$$< \Delta^2 >= \frac{U_c}{4}(U_c + 4K_BT) + eV_{ext}(eV_{ext} - U_c) \quad (3.1.44)$$

Now, by substituting Eq. (3.1.44) into the Eq. (3.1.37), we get the general expression for conductance as the following.

$$G = G_o[1 + \{\frac{6K_BTeV_{ext} - 3K_BTU_c + U_cK_BT + eV_{ext}(eV_{ext} - U_c) + \frac{U_c^2}{4}}{12(K_BT)^2(1 + e^{-\beta\mu})}\} - \frac{e^2V_{ext}^2 + (-eV_{ext} + \frac{U_c}{4} + K_BT)U_ce^{-\beta\mu}}{3(KT)^2(1 + e^{-\beta\mu})^2}]$$

$$= G_0[1 + \frac{eV_{ext}(6K_BT + eV_{ext} - U_c) - 2U_cK_BT + \frac{U_c^2}{4}}{12(KT)^2(1 + e^{-\beta\mu})} - \frac{(e^2V_{ext}^2 - (eV_{ext} - K_BT - \frac{U_c}{4})U_c)e^{-\beta\mu}}{3(KT)^2(1 + e^{-\beta\mu})^2}]$$

$$= G_0[1 + \frac{e^2V_{ext}^2 + (6K_BT - U_c)eV_{ext} + U_c(2K_B - \frac{U_c}{4})}{12(KT)^2(1 + e^{-\beta\mu})^2} - 3(\frac{e^2V_{ext}^2 - (6K_BT + U_c)eV_{ext} + U_c(2K_BT + \frac{U_c}{4})e^{-\beta\mu}}{12(KT)^2(1 + e^{-\beta\mu})^2})] \quad (3.1.45)$$

By the same procedure and rearranging Eq. (3.1.45) finally we get the following expression of conductance.

$$G = G_\infty[1 + \frac{e^2V_{ext}^2 + 6K_BTeV_{ext} - 2K_BU_c}{12(K_BT)^2}] \quad (3.1.46)$$

Where $G_\infty = \frac{e^2\rho\Gamma^l\Gamma^r}{\Gamma^l + \Gamma^r}$ and $\frac{e^2V_{ext}^2 + 6K_BTeV_{ext} - 2K_BU_c}{12(K_BT)^2}$ is the correction term to the high temperature value.

3.2 Low temperature behavoir

In this section we will discuss the conductance at low temperature, i.e if KT $\ll \Delta E$, then the general equation of conductance becomes:

$$G = \frac{e^2}{KT}\sum_{p=1}^{\infty}\sum_{N=1}^{\infty}\frac{\Gamma_p^l\Gamma_p^r}{\Gamma_p^l + \Gamma_p^r}P_{eq}(N)F_{eq}(E_p|N)(1 - f(E_p + U(N) - U(N-1)) - E_F) \quad (3.2.47)$$

All energy levels E_p are small that G is maximum if 1-f = maximum. 1-f is maximum if f maximum $f = \frac{1}{1+exp(\beta(E_p+U(N)-U(N-1)-E_F)}$. f is minimum if $E_p + U(N) - U(N-1) - E_F$ is maximum. For low temperature, $K_B T \ll \Delta E$, all energy levels P = 1, 2, ..., N are occupied. $F_{eq}(E_p|N)$ = maximum = 1 and $F_{eq}(E_p|N) = 0$ as P > N. The thermodynamics potentials is

$$\Omega(N) = 2\sum_{p=1}^{N/2} E_p + U(N) - NE_F \tag{3.2.48}$$

From Eq. (3.2.48) we can calculate the expression for E_p as follows.

$$\Delta E = E_{p+1} - E_p \tag{3.2.49}$$

By using Eq. (3.2.49) $E_1 = E_1$, $E_2 = E_1 + \Delta E$,..., and $E_n = E_1 + (n-1)\Delta E$. Let K = $2\sum_{p=1}^{N/2} E_p$ and we can find the expression for K(kinetic energy) as follows and where $n = \frac{N}{2}$.

$$\begin{aligned} K &= 2\sum_{p=1}^{N/2} E_p \\ &= 2(E_1 + E_2 + ... + E_n) \\ &= 2nE_1 + 2\sum_{p=1}^{n-1} P\Delta E \end{aligned} \tag{3.2.50}$$

From Eq. (3.2.50) let we say s = $\sum_{p=1}^{n-1} P\Delta E$

$$\begin{aligned} 2s &= \Delta E + 2\Delta E +, ..., +(n-1)\Delta E + (n-1)\Delta E + (n-2)\Delta E +, ..., +\Delta E \\ &= n\Delta E + n\Delta E +, ..., +n\Delta E \\ &= n\Delta E(n-1) \\ &= n(n-1)\Delta E \end{aligned} \tag{3.2.51}$$

Solving for s we get

$$s = \frac{n(n-1)}{2}\Delta E \tag{3.2.52}$$

By substituting Eq. (3.2.52) into the Eq. (3.2.50), we get

$$\begin{aligned} K &= 2nE_1 + 2s \\ &= 2nE_1 + n(n-1)\Delta E \\ &= NE_1 + \frac{N/2(N/2-1)}{2}\Delta E \\ &= NE_1 + N(N-2)\frac{\Delta E}{4} \end{aligned} \tag{3.2.53}$$

The charging energy U(N) of the dot change discretely because of a change in number N of electrons on the dot (through tunneling to or from the source leads), or it changes continuously because of a change in the voltage on the external gate electrode and the charging energy is calculated as follows.

$$\begin{aligned} U &= N(N-1)\frac{U_c}{2} - NeV_{ext} \\ &= \frac{U_c}{2}N^2 - (eV_{ext} + \frac{U_c}{2})N \\ &= \frac{U_c}{2}[N^2 - 2N(\frac{1}{2} + \frac{eV_{ext}}{U_c})] \\ &= \frac{U_c}{2}[(N-N_U)^2 - N_U^2] \end{aligned} \tag{3.2.54}$$

Where, $N_U = \frac{1}{2} + \frac{eV_{ext}}{U_c}$. Finally, inserting Eq. (3.2.53) and Eq. (3.2.54) into the Eq. (3.2.48), we get the expression for $\Omega(N)$ as follows.

$$\begin{aligned} \Omega(N) &= NE_1 + N(N-2)\frac{\Delta E}{4} + N(N-1)\frac{U_c}{2} - NeV_{ext} - NE_F \\ &= [\frac{\Delta E}{4} + \frac{U_c}{2}]N^2 - [\frac{\Delta E}{2} + \frac{U_c}{2}eV_{ext} + E_F - E_1]N \\ &= \frac{1}{2}(U_c + \frac{\Delta E}{2})[N^2 - 2\frac{(E_F + eV_{ext} + \frac{U_c+\Delta E}{2} - E_1)}{U_c + \frac{\Delta E}{2}}]N \end{aligned} \tag{3.2.55}$$

Arranging Eq. (3.2.55), so that it can be written as

$$\Omega(N) = \frac{1}{2}(U_c + \frac{\Delta E}{2})[(N-N_\Omega)^2 - N_\Omega^2] \tag{3.2.56}$$

Where, $N_\Omega = \frac{E_F + eV_{ext} + \frac{U_c + \Delta E}{2} - E_1}{U_c + \frac{\Delta E}{2}}$. $P_{eq}(N)$ is maximum (nonzero) for that value of N that minimizes $\Omega(N)$.

$$\frac{d\Omega}{dN} = \frac{1}{2}(U_c + \frac{\Delta E}{2})((N - N_\Omega)^2 - N_\Omega^2) = 0 \tag{3.2.57}$$

$$N_{min} = N_\Omega = \frac{E_F + eV_{ext} + \frac{U_c + \Delta E}{2} - E_1}{U_c + \frac{\Delta E}{2}} \tag{3.2.58}$$

If $U_c = \frac{e^2}{C} \gg \Delta E$ and $E_1 = 0$, $\Delta E \ll U_c$, eV_{ext}, E_F, then Eq. (3.2.58) reduced it to

$$N_{min} = \frac{eV_{ext} + E_F}{U_c} \tag{3.2.59}$$

$$N_{min} - 1 = \frac{E_{F + eV_{ext} - \frac{U_c}{2}} - E_1}{U_c + \frac{\Delta E}{2}} \tag{3.2.60}$$

$$N_{min} + 1 = \frac{E_F + eV_{ext} - E_1 + \frac{3}{2}U_c + \Delta E}{\frac{\Delta E}{2} + U_c} \tag{3.2.61}$$

$$\Omega(N_{min}) = -\frac{1}{2}(U_c + \frac{\Delta E}{2})N_\Omega^2 \tag{3.2.62}$$

$$\begin{aligned}\Omega(N_{min} - 1) &= -\frac{1}{2}(U_c + \frac{\Delta E}{2})N_\Omega^2 + \frac{1}{2}(U_c + \frac{\Delta E}{2}) \\ &= \Omega(N_\Omega) + \frac{1}{2}(U_c + \frac{\Delta E}{2})\end{aligned} \tag{3.2.63}$$

$$\Omega(N_{min} - 1) = \Omega(N_{min} + 1) \tag{3.2.64}$$

Because $\Omega(N_{min} - 1)$ and $\Omega(N_{min} + 1)$ has horizontally the same value of conductance.

$$\Omega(N_{min}) - \Omega(N_{min-1}) = -\frac{1}{2}(U_c + \frac{\Delta E}{2}) \tag{3.2.65}$$

$$\Omega(N_{min}) - \Omega(N_{min} + 1) = -\frac{1}{2}(U_c + \frac{\Delta E}{2}) \tag{3.2.66}$$

Therefore going from $N_{min} \rightarrow N_{min}$-1 and $N_{min} \rightarrow N_{min} + 1$ increases Ω and as temperature $\rightarrow$ decreases. Therefore, the value of N that minimizes $\Delta(N) = \Omega(N) - \Omega(N-1) = 0$ can have only those particular values that depend on: i, Fermi energy(E_F) ii, $eV_{ext} = \eta eV - \alpha eV_g + eV_{donors}$ varies with V, V_g, V_{donors}, iii, $U_c = \frac{e^2}{C}$.

It depends on as the gate voltage is varied V_{ext} varies and thus N_{min} assumes different values and the conductance becomes peaked [21]. As temperature $\rightarrow$ decreases, conductance is suppressed.

$$\begin{aligned} P(N) &= Ce^{-\frac{(U_c+\Delta E)}{2K_BT}[(N-N_{min})^2-N_{min}^2]} \\ &= Cexp\{\frac{-(N-N_{min})^2}{2\sigma^2}\} \end{aligned} \tag{3.2.67}$$

In Eq. (3.2.67) the expression of C is given by

$$C = \sum_{N=1}^{\infty} e^{-(N-N_{min})^2/2\sigma^2} \tag{3.2.68}$$

Using Eq. (3.2.67) into the Eq. (3.2.48) we get

$$\frac{G}{G_0} = \frac{\sum_{N=1}^{\infty} e^{-\frac{(N-N_{min})^2}{2\sigma^2}} \sum_{P=1}^{N} \frac{1}{1+e^{-\beta\Delta}}}{\sum_{N=1}^{\infty} e^{-\frac{(N-N_{min})^2}{2\sigma^2}}} \tag{3.2.69}$$

Where $\Delta = NE_c + P\Delta E - (E_F + eVg + +E_c)$ and $G_0 = \frac{e^2\Gamma_p^l\Gamma_r}{KT(\Gamma_p^l+\Gamma_r)}$. As temperature $\rightarrow$ decreases, the width of the Gaussian distribution $\sigma_N \rightarrow$ decreases, and thus P(N) becomes sharply peaked around $N = N_{min}$. $P_{eq}(N)$ is sharply peaked around N = a. Where, $\sigma = \sqrt{\frac{K_BT}{\Delta E+U_c}}$. As temperature decreases σ decreases. If $\sigma < 1$ then any number $N \neq N_{min}$ has zero probability. $\Delta(N)$ is minimum at N = N_{min}. In the low temperature regime, $KT \ll \Delta E$, the term with P= N = N_{min} gives the dominant contribution to the sum over P and N in the conductance.

$$\begin{aligned} P_{eq}(N_{min}) &= \frac{e^{-\beta\Omega(N_{min})}}{e^{-\beta\Omega(N_{min})} + e^{-\beta\Omega(N_{min}-1)}} \\ &= f(\Delta_{min}) if KT \ll \Delta E \end{aligned} \tag{3.2.70}$$

$\Delta_{min} = \Omega(N_{min}) - \Omega(N_{min}-1) = E_{Nmin} + U(N_{min}) - U(N_{min}-1)$ and $\Omega(N_{min}) = \sum_{p=1}^{N_{min}} E_p + U(N_{min}) - U(N_{min}-1) - E_F$. Moreover $F_{eq}(E_p|N) = 1$ in this limit. In the low temperature limit: By substituting Eq. (3.2.70) into the Eq. (3.2.47), we have the expression for conductance:

$$G = \frac{e^2}{KT}\frac{\Gamma^l_{N_{min}}\Gamma^r_{N_{min}}}{\Gamma^l_{N_{min}} + \Gamma^r_{N_{min}}} f(\Delta_{min})(1 - f(\Delta_{min})) \tag{3.2.71}$$

Eq. (3.2.71) can be changed into the cosine hyperbolic functions and $f(1-f)$ can be written as

$$\begin{aligned} f(1-f) &= \frac{e^x}{(2e^x(\frac{e^{\frac{-x}{2}}+e^{\frac{x}{2}}}{2}))^2} \\ &= \frac{1}{4Cosh^2 x} \end{aligned} \tag{3.2.72}$$

Where, $Coshx = \frac{e^x+e^{-x}}{2}$ and $f(\Delta) = \frac{1}{1+e^{\beta\Delta}}$. By substituting Eq. (3.2.72) into the Eq. (3.2.71), we obtain

$$G = \frac{e^2\Gamma^l\Gamma^r}{4K_BT(\Gamma^l + \Gamma^r)}\frac{1}{Cosh^2(\beta\Delta_{min})} \tag{3.2.73}$$

Then, Eq. (3.2.73) can be arranged as

$$\frac{G}{G_{max}} = \frac{1}{Cosh^2(\frac{\Delta_{min}}{KT})} \tag{3.2.74}$$

Where, $G_{max} = \frac{e^2\Gamma^l\Gamma^r}{4K_BT(\Gamma^l+\Gamma^r)}$ is the amplitude. and $x = \beta\Delta$. For quantum dot, P = N + 1 is vacant for release P = N occupied and thus N $\rightarrow$ N + 1 and N +1 $\rightarrow$ N, N $\rightarrow$ N -1, N-1 $\rightarrow$ N.

3.3 The periodicity of the Coulomb-blockade oscillations

The charging energy takes the form.

$$U(N) = \frac{(Ne)^2}{2c} - Ne\Phi_{ext} \tag{3.3.75}$$

A metallic dot has a very energy level separation at the Fermi level, so that the change in Fermi leads may be neglected. The ground state of such corresponds to the minimum value of U(N) and which requires the occupation sequence $N \rightarrow N-1 \rightarrow N \rightarrow N-1, ...,$ is blocked at low temperatures, except when U(N) = U(N-1). The conductance thus exhibits peaks periodic in Φ_{ext}, located at

$$\Phi_{ext} = (N - \frac{1}{2})\frac{e}{c} \tag{3.3.76}$$

A peak in low temperature conductance occurs for some integer N.

$$E_N + U(N) - U(N-1) = E_F \tag{3.3.77}$$

By substituting Eq. (3.3.77) into the Eq. (3.3.76) gives as the condition for a conductance peak.

$$E_N + (N - \frac{1}{2})\frac{e^2}{C} = E_F + e\Phi_{ext} \tag{3.3.78}$$

$$\begin{aligned} E_N^* &= E_N + (N - \frac{1}{2})\frac{e^2}{C} \\ &= E_F + e\Phi_{ext} \end{aligned} \tag{3.3.79}$$

The normalized energy level spacing is enhanced above the bare level spacing (ΔE) by the charging energy $\frac{e^2}{C}$.

$$\Delta E^* = \Delta E + \frac{e^2}{C} \tag{3.3.80}$$

If $\frac{e^2}{C} \ll \Delta E$, Eq. (3.3.79) is the usual condition for resonant tunneling and if $\frac{e^2}{C} \gg \Delta E$, Eq. (3.3.79), describes the periodicity of classical Coulomb blockade oscillation in the conductance versus gate voltage. The electrostatic potential difference between gate and 2DEG reservoir. We have

$$\Phi_{ext} = \Phi_{donor} + \alpha\Phi_{gate} \tag{3.3.81}$$

Eqs. (3.3.79) and (3.3.81) imply in this case a periodicity amount ΔE, while E_F stay constant. Then, the periodicity of the gate voltage is given by:

$$\Delta V_{gate} = \frac{e}{\alpha C}[1 + \frac{\Delta E}{e^2/C}] \tag{3.3.82}$$

Where, $\alpha = \frac{C_{gate}}{C_{gate} + C_{dot}}$.

Chapter 4

Numerical results and discussion

4.1 Numerical computation of electron transport through quantum dots

In this section, we perform the numerical computation of electron transport through the quantum dots. We present graphs of the results of using the equations that we have derived. As can be seen from Eq.(2.2.89) the conductance cannot be evaluated analytically but can be evaluated numerically. The usefulness of numerical computation is more recognized and today it is used in many domains of research and development. we have described the model of the quantum dot analysed by listing its essential parameters and by writing the physical laws that rule its behaviour.

We have solved Eq. (3.1.14), numerically for different values of thermal energy and plotted normalized conductance as a function of gate voltage is shown in fig 4.1. At high temperature the thermal energy is strong enough to overcome the repulsive of Coulomb force. The alternative pattern of peaks and suppressed regions conductance as a function of gate voltage is observed in the dot. When $\mu_s \approx \mu_d$, the number of electrons on the dot fluctuate by one and a peak in conductance is observed. Each peak occurs when an electrochemical potential potential of the dot fall into the bias window. in the valleys between the peaks the number of electrons on the dot is fixed

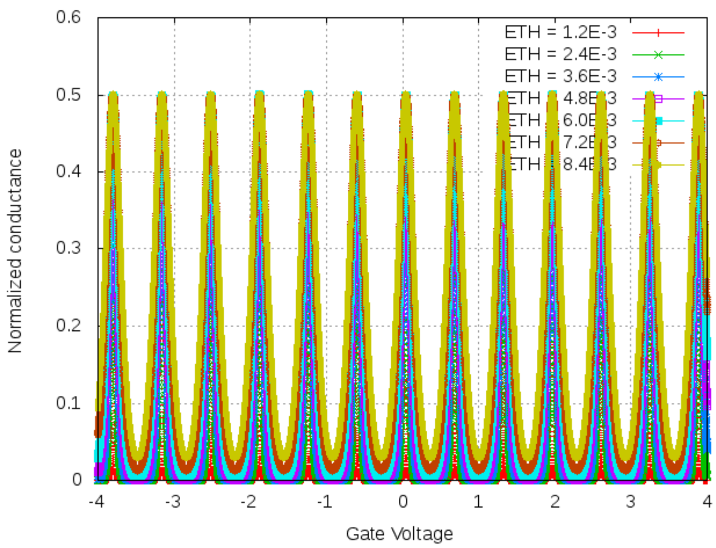

Figure 4.1: Coulomb blockade peaks normalized conductance versus gate voltage plotted graph of Eq. (3.1.14)

because of Coulomb blockade. We conclude that at high temperature the conductance of the quantum dot is constant.

The interaction potential energy of electrons plotted from Eq. (3.2.54) is shown below in fig 4.2. As N increases the potential is negative and its magnitude increases and finally become large negative around ∼ -47eV, and constant at N = 20 to N = 30. After that, potential energy becomes less negative and finally become zero for N = 80 and U = 0. After this $U > 0$, there is repulsion between electron and quantum dot is not stable. With increasing gate voltage, the number of electrons on the dot in its ground state increases. The potential energies are parabolic and no crossing of energies lines. We conclude that, the interaction potential energies of electrons form upward parabola with the same value of Fermi energy, average level spacing, charging energy but different value of gate voltage.

Fig (4.3), shows the graph of Eq. (3.2.67) with the same charging energy, Fermi

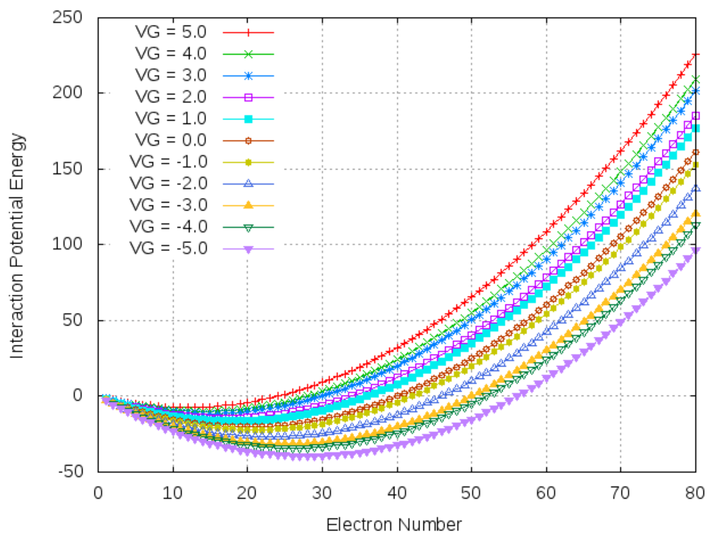

Figure 4.2: Interaction potential energy curves versus electron number in the quantum dot and plotted graph of Eq. (3.2.54)

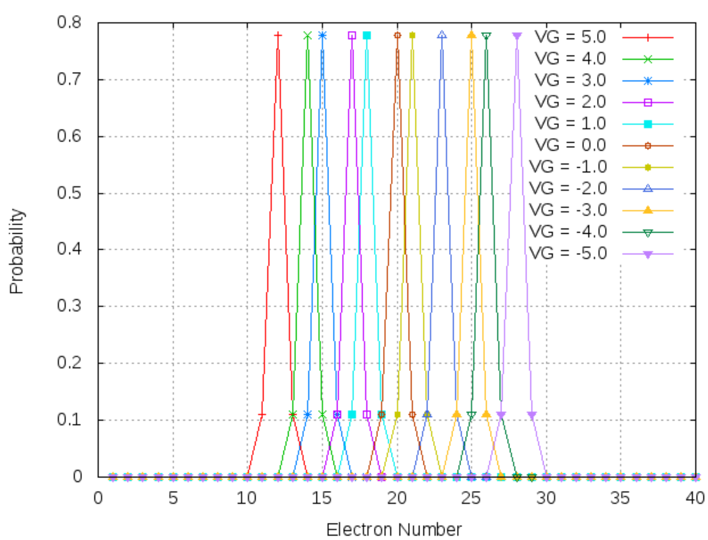

Figure 4.3: Probability density versus disappearing oscillation except between N =10 and N = 30 with increasing number of electron electron number and plotted graph of Eq. (3.2.67)

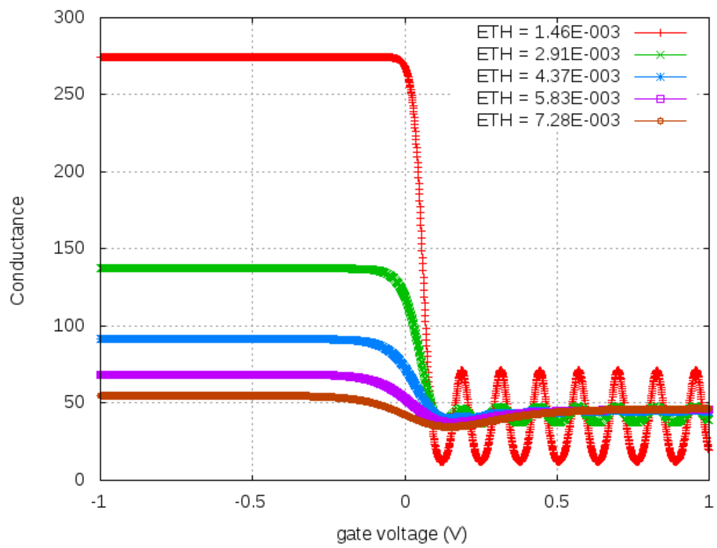

Figure 4.4: Normalized conductance versus gate voltage and plotted graph of Eq. (3.2.69).

energy, chemical potential, thermal energy, average level spacing but different value of gate voltage and external voltage. The total capacitance $C_T = 1.59 \times 10^{-19}$F and thermal energy 2.50×10^{-2}eV. The probability is zero for $N < 10$ and $N > 30$. The probability is not affected by the number of electrons in the dot. The distributions are well described by a Gaussian fit. The width of the distributions are narrower from top to bottom and the fluctuations are measured in units of the classical charging energy E_c, temperature, and average level spacing. The width of the distribution (standard deviation in the unit of the orbital mean level spacing).

We have solved Eq. (3.2.69), numerically for different values of thermal energy and plotted normalized conductance as a function of gate voltage. The plot of Eq. (3.2.69) is shown in fig. 4.4. For negative gate voltage conductance shows no oscillations rather has a constant positive value 274.62. For $ETH = 1.46 \times 10^{-3}$ and negative gate voltage V_g = -0.037V the conductance falls from max to min(G_{max} = 274.62)

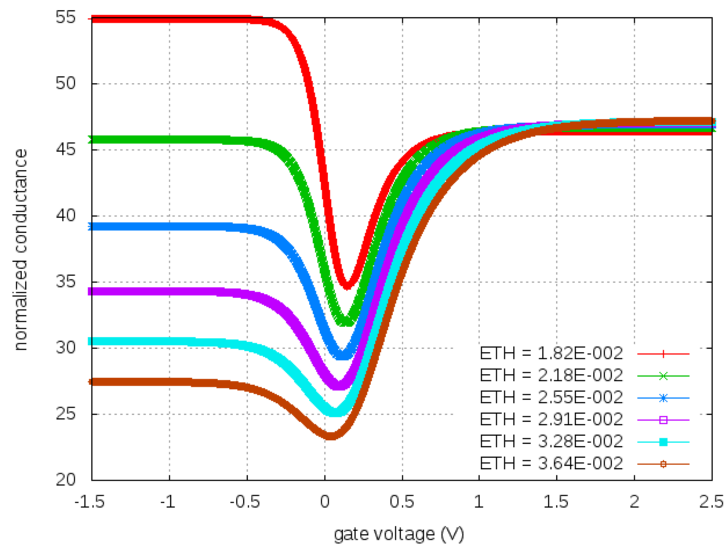

Figure 4.5: Blocking conditions. Normalized conductance versus gate voltage and plotted graph of Eq. (3.2.69).

to $G_{min} = 12.05$ and oscillate between $G_{min} = 12.05$ and $G_{max} = 71.31$. As thermal energy increase, oscillation decrease and the amplitude of the conductance oscillation is decreased $(K_B T \gg \Delta E)$ for $ETH \approx 7.28 \times 10^{-3}$.

At low temperature conductance through the dot is carried by electron tunneling between the dot and leads. Electrons tunneling are for a process classically disallowed. Negative voltages applied to the metallic gates shows narrow tunnel barriers. We conclude that the constant value of conductance in the positive gate voltage is less than the constant value in the negative gate voltage and no oscillation in the negative gate voltage.

The plot of Eq. (3.2.69) for low temperature in fig 4.5. is shown. For both negative and positive gate voltage conductance show no oscillation. Negative voltage $V_g =$ 0.5V the conductance falls G_{max} to G_{min} ($G_{max} = 54.92$ to $G_{min} = 34.69$)for ETH = 1.82E-2, and shows no oscillations. As gate voltage, increases, conductance decrease

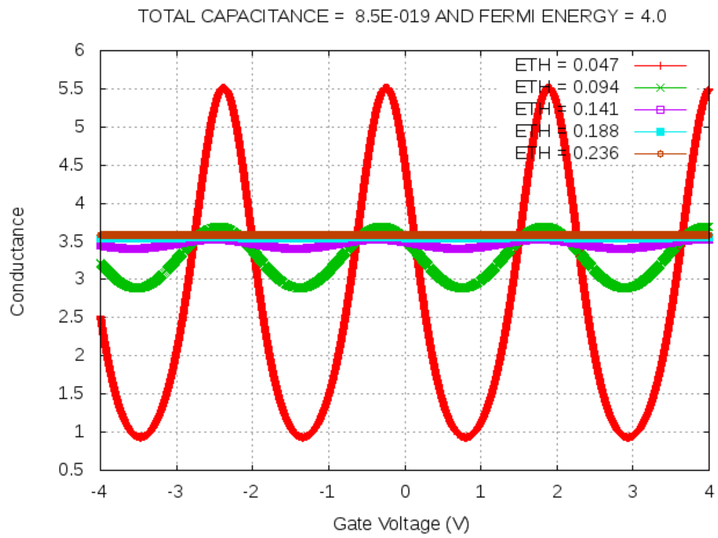

Figure 4.6: Normalized conductance versus gate voltage when $K_BT \ll \Delta E$ and plotted using Eq. (3.2.69).

approach a minimum value in the positive gate voltage($G_{min} = 34.69$) and increases gradually becomes constant(G = 46.70). The conductances blocking condition (G = is minimum) are fulfilled at different gate voltage. As can be seen from the figure, at high temperature, the conductance does not exhibit oscillation as a function of the gate voltage.

At fixed bias voltage, conductance valley(minima) through the dot are shifted to the right in energy proportional to the applied gate voltage. Transport blocked (Coulomb blockade) regions in which no energy lies in the required energy window. We can conclude that the constant value of the conductance in the positive gate voltage is greater than the constant value of the conductance in the negative gate voltage except for $ETH = 1.82 \times 10^{-2}$.

In fig 4.6, we have plotted the normalized conductance as a function of gate voltage. For a different capacitance with the same Fermi energy, the conductance becomes

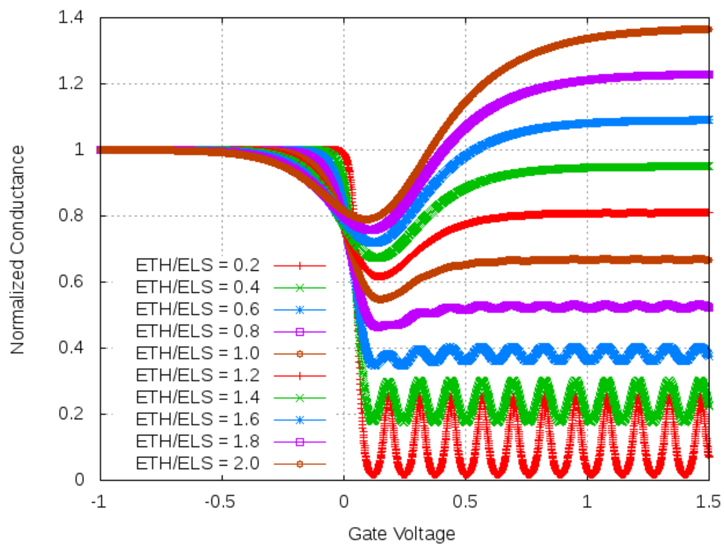

Figure 4.7: Normalized conductance versus gate voltage and plotted graph of Eq. (3.2.69)

oscillatory for both negative and positive gate voltage. For $ETH \approx 0.236$ oscillation disappear, conductance becomes constant($G \approx$) 3.6. The faster oscillations correspond to changes in the dot charge, and the slower oscillations are caused by direct coupling from the gate. The size of the oscillations caused by dot charge increases gradually as the total capacitance of the dot decreases. As ETH increases, amplitude of oscillation decreases. Level spacing, capacitance, temperature, chemical potential affect the oscillation nature of the conductance.

In fig 4.7, the normalized conductance in Eq. (3.2.69) is plotted as a function of gate voltage. For negative gate conductance has constant value and shows no oscillation for $\frac{ETH}{ELS} = 0.2$. For negative gate voltage, V_g = -0.18V the conductance falls from $G_{max} = 1$ to $G_{min} = 0.02$ and shows oscillation in the positive gate voltage. For a positive gate voltage V_g = 0.12V, the conductance increasing to $G_{max} = 0.25$, then oscillates between $G_{max} = 0.25$ and $G_{min} = 0.02$. As the ratio of thermal energy to

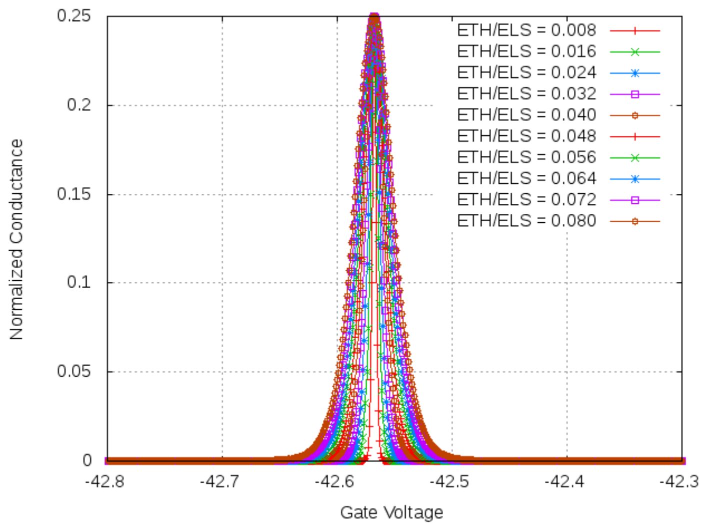

Figure 4.8: The variation of the normalized conductance ($\frac{G}{G_{max}}$) as a function of gate voltage and plotted graph of Eq. (3.2.74)

level spacing increases, the conductance increases and oscillation disappear for $\frac{ETH}{ELS}$ = 1.

The oscillation decrease quickly with increasing temperature and finally the conductance is constant. We conclude that as temperature increase the constant value of conductance in the positive value is greater than the constant value of conductance in the negative value for the ratio of thermal energy to level spacing is equal to($\frac{ETH}{ELS}$ = 1.6, 1.8 and 2) respectively. .

In fig 4.8, we have plotted the normalized conductance in Eq. (3.2.74) for different values of thermal energy. It shows sharp maximum around $V_g \approx =$ -42.6V, and zero a way from this value. As temperature decreases, the width of the Gaussian distribution $\sigma_N \rightarrow$ decreases, and thus P(N) becomes sharply peaked around N = N_{min}. Numerical value of Gaussian width of probability $\approx$ 6.93$\times 10^{-31}$, the total capacitance, C_T = 1.59 $\times 10^{-17}$F, and Fermi energy = 7.04eV. On the Coulomb blockade peak, a single level contributes to transport.

Chapter 5

Summary and Conclusion

We have performed the numerical computation of electron transport through the quantum dots. The conductance cannot be evaluated analytically but can be evaluated numerically. We have presented graphs of the results of using the equations that we have derived. When quantum dot has $N = N_{min}$ current occur when the one electron tunnels either to the right or left reservoir. After that N_{min}-1 electrons are in the quantum dot. Now an electron can tunnel into the quantum dot either from the right or from left. Then $N = N_{min}$ the quantum dot is in the blockade state. The available thermal energy is not enough to overcome the repulsion force of the electrons that are already in the quantum dot.

At high temperature the thermal energy is strong enough to overcome the repulsive of Coulomb force. The alternative pattern of peaks and suppressed regions conductance as a function of gate voltage is observed in the dot. The interaction potential energy of electrons when $U \gg 0$, there is repulsion between electrons and quantum dot is not stable. For the total capacitance $C_T = 1.59 \times 10^{-19}$F and thermal energy 2.50×10^{-2}eV, the probability is zero for $N < 10$ and $N > 30$. Then, we can conclude that the probability is not affected by the number of electrons in the dot.

We have plotted the normalized conductance as a function of the gate voltage for various values of C_s, C_d, C_g, E_c and ΔE. The graphs the normalized conductance shows different behavior, saturation, oscillation depression, and varying amplitude of oscillation and period. As temperature increases amplitude of oscillation decreases and finally becomes constant as gate voltage varies. Coulomb blockade oscillation is clearly observed where the quantum dot is on(conductance different from zero) and off(conductance minimum). For $\Delta E \ll K_BT \ll E_c$ and $K_BT \ll \Delta E \ll E_c$ both show oscillation of conductance. Conductance widely sensitive to the material parameters capacitance(size), temperature(K_BT), level spacing(ΔE). For appropriately chosen values of these parameters conductance shows the three behaviors saturation, oscillation and depression.

Negative gate voltage for $C_T = 1.1 \times 10^{-18}F$ and $E_F = 4$eV. For negative gate voltage conductance shows no oscillations rather has a constant positive value 274.62. For $ETH = 1.46 \times 10^{-3}$ and negative gate voltage V_g = -0.037V the conductance falls from max to min(G_{max} = 274.62) to G_{min} = 12.05 and oscillate between G_{min} = 12.05 and G_{max} = 71.31. As thermal energy increase, oscillation decrease and the amplitude of the conductance oscillation is decreased ($K_BT \gg \Delta E$) for $ETH \approx$ 7.28E-3.Oscillation decreases vanishes so $\frac{G}{G_{max}}$ occur for V_g = constant as thermal energy increases.

For the total capacitance, $C_T = 8.5 \times 10^{-18}$F and $E_F = 4.0$eV. $\frac{G}{G_{max}}$ oscillation for both $V_g < 0$ and $V_g > 0$. Oscillation vanishes as thermal energy increases. As the ratio of thermal energy to level spacing increases, the conductance increases and oscillation disappear for $\frac{ETH}{ELS} = 1$. We conclude that as temperature increase the constant value of conductance in the positive value is greater than the constant value of conductance in the negative value for the ratio of thermal energy to level spacing is equal to($\frac{ETH}{ELS}$ = 1.6, 1.8 and 2) respectively.

$\frac{G}{G_{max}} = \frac{1}{Cosh^2(\frac{\Delta_{min}}{K_BT})}$ does not oscillate. Shows sharp max and around $V_g \approx$ -42.6V, and zero a way from this value. Level spacing, capacitance, temperature, chemical potential affect the oscillation nature of the conductance.

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