SPIN-SUM CORRECTION TO INVERSE COMPTON SCATTERING CROSS SECTION IN STRONG BACKGROUND MAGNETIC FIELD

BY

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Dedicated to

My lovely Mother

(Emahoy Workinesh Tekle)

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Abstract

Astrophysical models need calculations in cross section. It is believed that inverse Compton scattering in strong magnetic fields near the surfaces of Neutron stars is one of the possible mechanisms responsible for X-ray and γ-ray emissions. Spin sum is one fundamental variable in calculations of scattering cross sections involving charged fermions in magnetic field. In this thesis we explicitly calculate the spin-sum of the solutions of Dirac equation and using it we calculate spin-sum corrected total cross of the inverse Compton scattering in the presence of uniform and strong magnetic field.

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General Introduction

When a star runs out of fuel its evolution is nearly finished. White dwarfs, neutron stars (NSs) and black holes are the remnants of dying stars. Stars like the sun die slowly but quietly and become white dwarfs. More massive stars explode and become neutron stars and black holes. Such stellar explosions, though marking the death of one star, may trigger the birth of another. Moreover, the explosion of dying star blasts into space elements vital to human life such us carbon and iron, elements that eventually become incorporated in to new stars, planets and ultimately people.

A neutron star forms when a massive star's iron core collapses and triggers a supernova explosion. The collapse compresses the core's protons and electrons together to make neutrons, forming a ball of neutrons with a radius of about 10 kilometers but which may contain up to about 2 to 3 solar masses. NSs have powerful magnetic field. As the NS rotates, its magnetic field sweeps radiations across the sky. A pulsating radio source is thought to be associated with a rapidly rotating NS. Neutron stars can exist as a binary system, system where the two stars orbit close to each other. Unusually strong magnetic field can also exist in *magnetar* neutron stars. Pulsating X-*ray sources, X-ray bursters and* $\boldsymbol{\gamma}$- *ray burster* are events related with neutron stars. On strong magnetic fields (10^{12}-10^{13}G) of magnetospheres of NSs the interaction of electrons and protons is greatly altered, because the electrons are restricted to moving perpendicular to the field in *Landau* orbital's.

Calculation of elementary particle decays and scattering cross sections in presence of magnetic field become more important after it was understood that the NS cores can sustain magnetic fields of the order of 10^{13} G or more. We assume these field to be uniform so that Dirac equation can be solved exactly. Once Dirac equation is solved exactly then we can calculate scattering cross sections in the presence of the field. Inverse Compton scattering (ICS) in strong magnetic field has been analyzed by Goldreich et al. (1972) and Blandford et al. (1976). Herold (1979) calculated the general differential cross section for Compton scattering in a strong magnetic field. His result is for electron rest frame (ERF) under the assumption that the initial and final states of electrons are in the lowest Landau level.

His expression of cross section has been widely used in astrophysical calculations, because it is assumed that the ICS is talking place near the surfaces of NSs.

However, in actual calculations the cross section in the laboratory frame (LF) is required. Therefore, this thesis aims at a derivation of an exact LF version of Herold's expression of cross section in the ERF. The advantage of this cross section is that it can be used directly in astrophysical calculations for both relativistic and non-relativistic cases.

The first chapter of this thesis focuses on the physics of neutron stars (their discovery, magnetic field and rays they produce). In the second chapter we calculate spin-sum of Dirac equation in strong magnetic field for positive and negative energy solutions (both spin up and down states considered). Finally, after finding Feynman propagator of an electron and scattering amplitude, quantum mechanical, magnetic and relativistic calculation of total differential cross section of inverse Compton scattering corrected for spin sum will be calculated.

Chapter 1- Neutron stars

1.1 Introduction

Almost all the light we see from objects in the night sky is due to thermonuclear fusion reactions. Such reactions are what makes stars shine, and in turn make any surrounding nebulae glow. Even the light we see from the moon and planets – which is simply reflected sunlight – can be traced back to thermonuclear reaction in the sun's core.

By contrast, exotic objects like Crab Nebula in Taurus is undergoing no thermonuclear fusion at all. Despite this, the object is so energetic that it is emitting copious amounts X-rays. At the objects very center is the source of all this dynamic activity: a *neutron star*, the leftover core of what was once a supergiant star.

Before 1967 neutron stars were considered pure speculation. But today we know of more than 1,300 of these exotic stellar corpuses, and hundreds of thousand more may be strewn across the Milky Way Galaxy. Neutron stars have powerful magnetic fields that are billions of times stronger than the sun's filed. As the NS rotates, its magnetic field sweeps beams of radiation across the sky. We detect these beams as pulsating radio signals.

Unlikely normal stars neutron stars actually have solid surfaces that can shift and fracture in a "starquake". When such a quake occurs on the surface of a NS with an unusually strong magnetic filed, it release a truly colossal burst of radiation that for a fraction of a second outshine an entire galaxy of stars. NS's in close binary system also display outrageous behavior, including bouts of explosive fusion that yields a vast outpouring of x-rays.

A massive star in a supernova explosion, leave behind both a supernova remnant and bizarre object called *neutron star*. Under very high pressure, a proton and an electron can combine to from a neutron (as well as a neutrino) through the reaction

$$P+e \rightarrow n+\upsilon \quad ,$$

the neutrinos escaping the star.

Once a neutron star had formed in response to tremendous pressures, it would be able to resist any further compression. The neutrons themselves would provide counterbalancing outward pressure, *degenerate neutron pressure.*

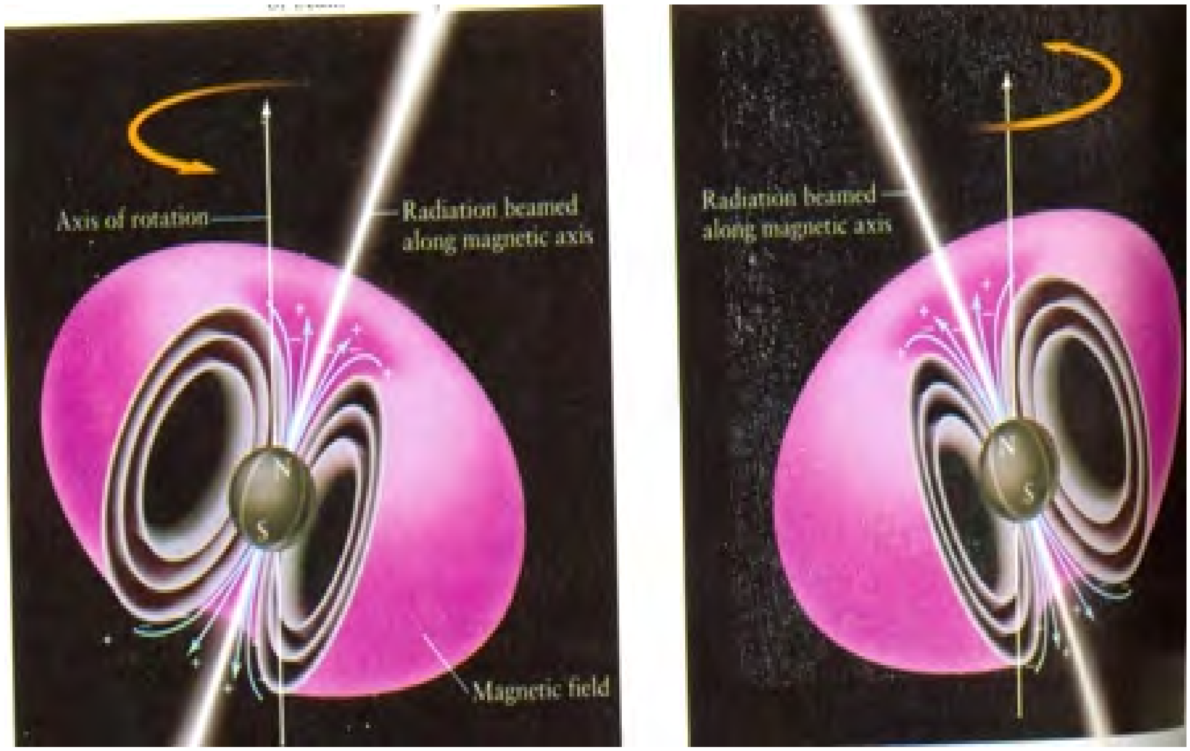
1.2 Pulsars and the Discovery of Neutron stars

A pulsar is a pulsating radio source thought to be associated with rapidly rotating neutron stars. One type of star that appears to pulsate is an eclipsing binary star system which appears dimmer when one star in the binary system passes in front of the other.

However in order to very in brightness with a typical pulsar period of 1 second or less, an eclipsing binary would have to have an equally short orbital period. At the time of its discovery it was the Crab pulsar recognized as the fastest pulsar known to astronomers. Its period is 0.0333 seconds, which means that it flashes 1/0.0333=30 times each second.

The magnetic field of a NS makes it possible for the star to radiate pulses of energy towards our telescopes on Earth. The following two figures show a rotating, magnetized NS. Charged particles are accelerated near a magnetized neutron stars magnetic poles, producing two oppositely directed beams of radiations. If the stars magnetic axis is tilted at an angle from the axis of rotation, the beams sweeps around the sky as the star rotates. If the earth happens to lie in the path of the beams, we detect radiation that appears to pulse (a) on and (b) off.

Earth



(a) (b)

Fig 1 A rotating magnetized neutron star [17]

A radio telescope detects regular pulses of radiation, with the one pulse being received for each rotation of the neutron star. Since 1968 radio astronomers have discovered more than 1600 pulsars scattered across the sky, and it is estimated that perhaps 10^5 more are strewn around the disk of the Milky Way Galaxy.

Each one is presumed to be the neutron star corpse of an extinct, massive stars. Radio telescopes have detected pulsars with a wide verity of periods, from a relatively sluggish 8.51 second to a blindingly fast 0.001396 second. In each case the pulse period is thought to be the same as the rotation period of the NS.

The primary sources of pulsars are thought to be massive stars that become core-collapse supernovae, i.e. in many cases pulsars are found within the remnant of all the supernova.

A beam of electrons whirled along a circular path within the cyclotron, held in orbit by powerful magnets, scientists notified that it emitted a strange light. It is now known that this light, called *synchrotron radiation*, is emitted whenever high- speed electrons move along curved paths through a magnetic field. Synchrotron radiation has a continuous spectrum, but one that is distinctively different from the continuous spectrum emitted by a heated blackbody. This difference helped astronomers confirm that the Crab Nebula (that surround Crab pulsar) emits primary synchrotron radiation, not black body radiation. Thus, the Crab Nebula must contain quite a large number of relativistic electrons spiraling in an extensive magnetic field.

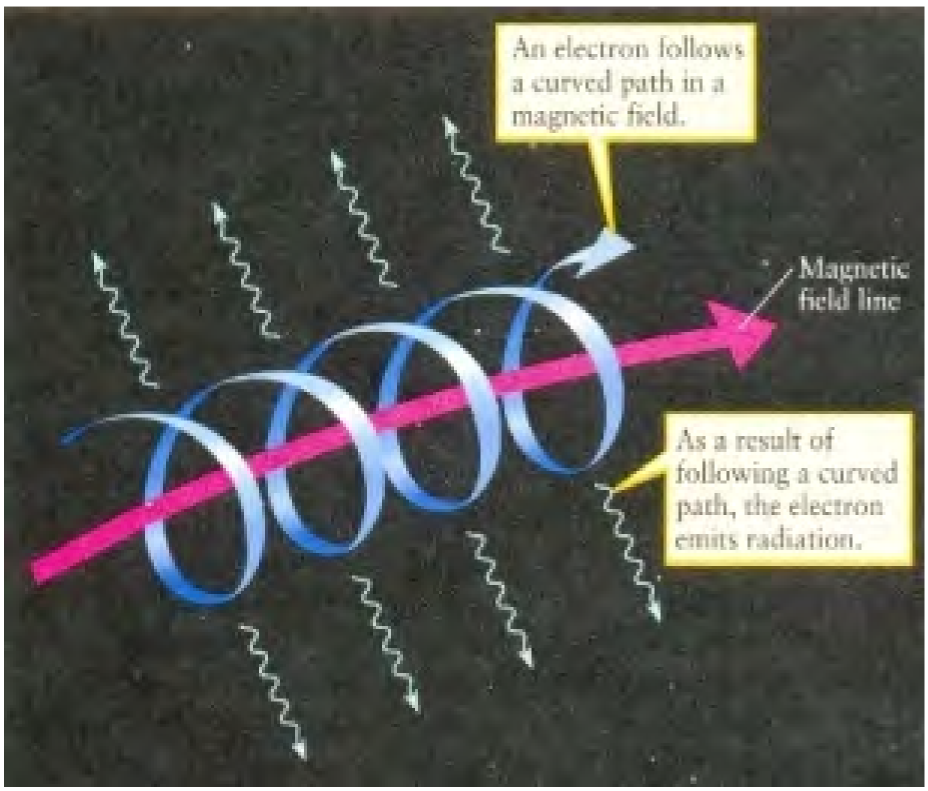


Fig 2 **Synchrotron radiation :** It is emitted by electrons that move at high speed in a magnetic field. [17]

1.2.1 Millisecond pulsars

In 1982, radio astronomers discovered a pulsar whose period is only 1.558 ms. (one millisecond). This remarkable pulsar called PSR 1937 + 21 after its coordinate in the sky, is a neutron star spinning 642 times per second – about 3 times faster than the blades of a kitchen blender. Such incredibly rapid rotation is a clue to this pulsar's unusual history.

PSR 1937+21 is not the only pulsar with unusually rapid rotation. Since 1982, astronomers have discovered more than 180 very fast pulsars, which are now called *millisecond pulsars*. All have periods between 1 and 10 ms, which means that these neutron stars are spinning at rates of 100 to 1000 rotations per second. (As of 2007 the fastest known pulsar is PSRJ1748-2446ad, which rotates 716 times a second). An important clue to understand millisecond pulsars is that the majority are in close binary systems, with only a small separation between the spinning neutron star and its companion.

Few millisecond pulsars, like PSR 1937 + 21, are not members of close binaries. It may be that solitary millisecond pulsars were once part of close binary systems, but the companion stars have been eroded away by the high – energy particles emitted by the pulsar after it was spun up.

It is also possible that both stars in a close binary system might evolve into neutron stars, forming a pair of pulsars. Several such *binary pulsars* are known. Because the two pulsars are so massive and move around each other in tight orbits, they lose energy through of process called *gravitational radiation*. As a result, the two neutron stars spiral towards each other and eventually collide. Some of the neutrons are ejected into space, where many of them decay into protons and combine to form various heavy elements. Some of them seed the interstellar medium with building blocks of planets.

1.3 Magnetic field of Neutron stars

Magnetic fields are found in various scale and positions in our universe. Neutrons stars have powerful magnetic fields that are billions of times stronger than sun's field. The small size of NSs implies that they have intense magnetic field. Every star possesses some magnetic field, but the strength of this field is quite low. The magnetic field, which is bounded to the star's ionized gases, becomes concentrated on to an area 10^{10} times smaller than before the collapse, and thus the field strength increases by a factor of $10^{10.}$

A stars magnetic field splits one of its spectral lines in to two or more lines whose spacing reveals the fields strength. As an example, the earth's magnetic field (which is responsible for Auroras and Van Allen belts) has a strength of about 0.5G. The sun has field strength of about 1G.The field at the surface of a refrigerator magnet is about 100G, and the field used in a scanner is about 15,000 G.

The magnetic splitting of lines in the spectra of certain white dwarfs corresponds to field strength's in excess of 10^6 G, comparable to the strongest fields ever produced in a laboratory on Earth. But the magnetic field surrounding typical neutron star are a million times stronger still, on the order of 1 trillion gauss (10^{12}G.) Some NSs may have field as large as 10^{15} G.

The combination of a powerful magnetic field and rapid rotation would act like a giant electric generator, creating very strong electric field near the neutron star's surface. Indeed, these fields are so intense that part of their energy is used to create electrons and positrons out of nothing in a process called *pair production*. The powerful electric fields push these charged particles out from the neutron star's surface and into its curved magnetic field. As the particles spiral along the curved field, they are accelerated and emit energy in the form of electro magnetic radiation. The result is that two narrow beams of radiation pour out of the neutron stars north and south magnetic regions.

According to Kebede L.W. (2002), [10], high magnetic fields can be generated by an acquired angular momentum from the basic field. Surface fields by spinning separated charges are found to be dipolar.

The stronger the magnetic field of neutron stars, the greater the rate at which it emits energy in the form of radiation and hence the more rapidly it will slow down. From the slow down rate of the Crab pulsar, astronomers calculate that its magnetic field must be about $5x10^{12}$ G, which is very close to the previous estimate of 10^{12}G.The calculations of the cross sections and decay rates in presence of a magnetic field uses a bench mark value of the background field:

$$\mathbf{B_c = m^2/e = 4.414x10^{13}\ G,}$$

Where m is the electron mass and e is the charge of the proton. B_c is some times called the '*critical field*' although nothing critical happens at this field strength. The only condition which defines the critical field is that when the magnitude of the magnetic field reaches the value B_c , the electron cyclotron frequency equals its rest mass in the natural units. If the magnitude of the magnetic field is above the critical field the electronic wave functions get considerably modified. It is assumed henceforth that while discussing scattering the magnitude of the external fields will be around $B_{c.}$

Of all pulsars, three pulsars in question appear to have the strongest magnetic fields of any object in the universe, far stronger than those of "ordinary" pulsars.

1.3.1 Magnetars

A *magnetar* is an unusual type of Neutron star with extraordinary strong magnetic field that can be about 10^{15} G (about 1,000 times stronger than that of the 10^{12} G field of an ordinary NS). This field is produced by convection inside the NS when it first formed.

The magnetic field of an ordinary star like the sun is produced by the motion of its electrically conducting gas, or plasma. The same is true for a supergiant stars, i.e. when thermonuclear reactions cease, it begins to collapse, the stars magnetic field moves with the infalling material and becomes greatly amplified. This amplified magnetic field remains with the resulting NS, which typically ends up with a field strength of about 10^{12} G.

However, Neutron stars can generate an additional magnetic field as it forms. For about the first 10 seconds after a NS forms from the core of a collapsing supergiant, it has an extremely high internal temperature-about 10^{11} K, as compared to a relatively frigid $1.55x10^{7}$ K in the present - day sun's core – and there is a furious convection of the NS material.

This material includes electrically charged protons and electrons, and the convective motion produce a magnetic field. If the newly formed NS spins relatively slowly, so that the time it takes a convection cell to rise and full (about 0.005 s) is less than the time required for the NS to make a complete rotation, the magnetic fields caused by different convection cells remain uncorrelated and do not contribute much to the NS's magnetism.

This seems to be what happened for the vast majority of NSs. But, if a brand new NS has a rotation period of less than 0.005 s, its rapid rotation causes the convective motion in different parts of the NS to organize together. When the NS has completely formed, it is left with a magnetic field that can be as strong as 10^{15} G. The magnetism of such a star could pull the keys out of your pocket and demagnetize your credit cards at a distance of 200,000 km, or half the distance from the Earth to the Moon.

The magnetic field of such magnetars exerts tremendous stresses on the electrically conducting material at the star's surface. From time to time these stresses can cause a localized fracture of the surface called a *starquake*. A starquake on the surface of a magnetar releases magnetic energy in the form of an intense blast of X-rays and gamma rays. The theoretical model of a magnetar turns out to be excellent description of the three pulsars that produced the energetic blasts. Each of these pulsars emit moderate gamma-ray bursts at seemingly random intervals, which is why they are also called *soft gamma repeaters*. This is the origin of the acronym "SGR'' in the pulsars designations (The adjective "soft" means that the gamma rays emitted are of relatively low energy).

In general, because magnetars represent such an extreme state of matter, with magnetic effects stronger than those found any where else in the universe, astronomers are studying them with great interest. Even NSs that lack the intense magnetic field of a magnetar can display exotic behavior. As we have seen in the previous section of this chapter, this happens when a NS is one member of a *binary star* system in which the two stars orbit close to each other.

1.4 Neutron stars

A *Neutron star* is a very compact, dense star composed almost entirely of packed degenerate neutrons. NSs have a mass ranging from 1.4- 3 times that of the sun's mass and a radius of about 10 km. Because neutron stars are very small, they should rotate (spin) rapidly.

The radiation from pulsars gives us information about the surroundings of neutron stars, as well as how they rotate. The models for NSs are very different. For one thing, NSs are thought to have a solid crust on their surface. Furthermore, the interior of NS is a sea of densely packed, degenerate neutrons, with properties quite unlike those of ordinary gases or even degenerate electrons. Detailed calculations strongly suggest that degenerate neutron matter can flow with out any friction whatsoever, a phenomenon called *super fluidity***.** Within a NS, elongated, friction-free whirlpools of super fluid neutrons may form. Their interaction with the star's crust may be the cause of sudden changes in a pulsar's rotation. In addition to the general slowing down of pulsars, astronomers have found that they sometimes exhibit a sudden speed up, sometimes called a *glitch***.** Current opinion among NS theorists is that glitches are cased by the super fluid neutrons within the NS.

In addition to super fluidity, models of the internal structure of a NS strongly suggest that the protons in the core can move around without experiencing any electrical resistance whatsoever. This phenomenon is called *superconductivity*. This property occurs on the Earth with certain substances at low temperatures. Once electric current starts moving in superconducting material, it keeps moving forever. We may have been surprised on the existence of protons in a neutron star. While neutron stars are made up of predominantly of neutrons, some protons and electrons must be scattered through the stars interior. Indeed, a pulsars magnetic field must be anchored to the neutron star by charged particles. Neutrons are electrically neutral, so without the protons and electrons in its interior, a neutron star would rapidly lose its magnetic field. The following figure shows a model of internal structure of a NS.

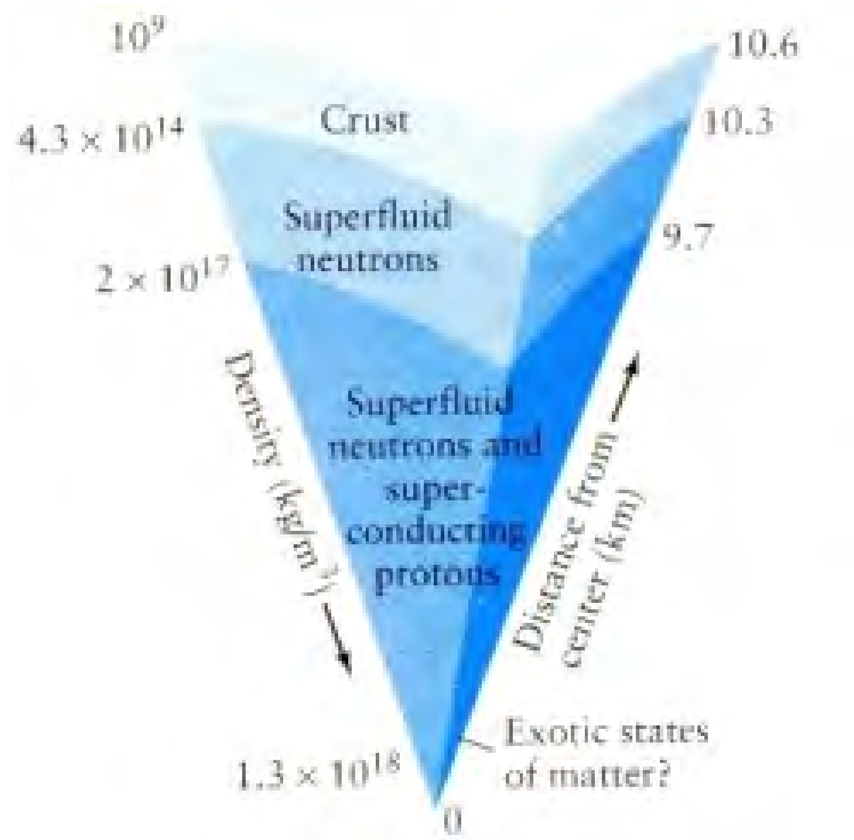


Fig 3 *A model of a neutron star's internal structure [17]*

1.4.1 Atmosphere of neutron stars

While the deep interiors of NSs. remain mysterious, astronomers have recently found evidence that the crusts of neutron stars are surrounded by a highly unusual atmosphere. The evidence come from the NS 1E 1207.4-5209, which emits X-rays and lie at the center of a supernova remnant. Dips on X-ray spectrum graph indicate that they are caused by absorption of certain X-ray wave lengths by an atmosphere around the NS.

The atoms that make up such an atmosphere must be dramatically different from those around us. In the intense magnetic field of a young NS, ordinary spherical atoms are deformed in to elongated cigar shapes and have radically altered spectral lines. So a bizarre property is also seen on the atmosphere of a NS.

1.4.2 Neutron stars in close binary system *(Pulsating X-ray sources)*

Millisecond pulsar are not the only result of having a neutron star in a close binary system with an ordinary star. Even more exotic are *pulsating X-ray sources*, in which material from the companion star is drawn on to the magnetic poles of the neutron star, producing intense hot spots that emit breathtaking amounts of X- rays. As the NS spins and the X-ray beams are directed alternately to ward us and away from us, we see the X-rays flash on and off. Pulsating X-ray sources were discovered in 1971 by the *Uhuru* space craft. The first X-ray source found to emit pulses was Centaurus X-3. The X-ray pulses have a regular period of 4.84s. Similar pulses were discovered from Hercules X-1, which had a period of 1.24s. These two sources are members of binary star systems.

One piece of evidence is that every 2.087 days, Centaurus X-3 appears to turn off for almost 12 hours. Apparently it is a NS in an eclipsing binary system, and it takes nearly 12 hours for the X-ray source to pass behind its companion star. Hercules X-1 too, appears every 1.7 days by its visible companion, the star HZ Hercules. Putting all the pieces together, astronomers now realize that pulsating X-ray sources like Centaurus x-3 and Hercules X-1 are binary systems in which one of the stars is a neutron star. All these binary systems have very short orbital periods, which means that the distance between the two stars is quite small. The NS can therefore capture gases escaping from its ordinary companion. More than 20 such exotic binary systems, also called *X-ray binary pulsars*, are known. A typical rate of mass transfer from the ordinary star to the NS is roughly 10^9 solar masses per year.

Because of its strong gravity, a NS in a pulsating X-ray source easily captures much of the gas escaping from its companion. But like an ordinary star, the NS is rotating rapidly and has a powerful magnetic field inclined to the axis of rotation.(see fig 1 .) As the gas falls toward the NS, its magnetic field funnels the incoming matter down to the stars north and south magnetic polar regions.

The NSs gravity is so strong that the gas is traveling at nearly half the speed of light by the time it crashes on to the stars surface. The violent impact creates spots on both poles with searing temperatures of about 10^8K.

Materials raised to these temperatures emits abundant X-rays, and because so much material is dumped on to these hot spots the X-ray luminosity reaches roughly 10^{31} watts.

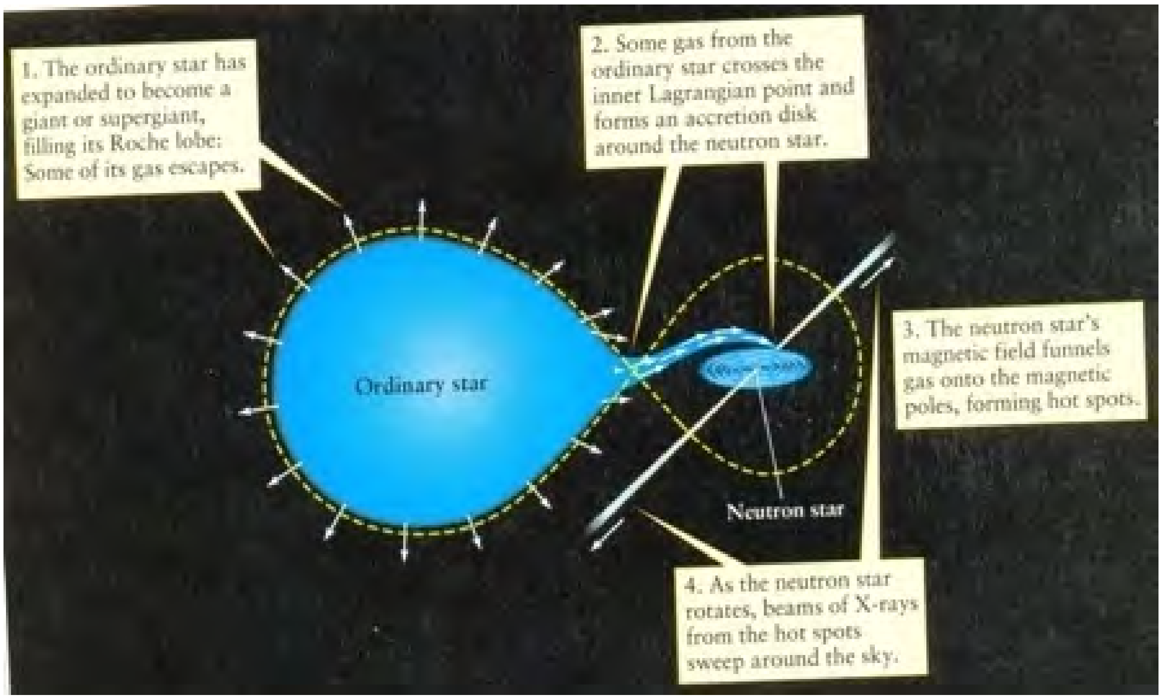


Fig 4 A model of pulsating X-ray source [17]

This is much greater than the X-ray luminosity of isolated pulsars like the Crab, whose surface temperatures are relatively chilly 10^6k or so. It is also nearly 10^5 times greater than the combined luminosity of the sun at all wavelengths. As the NS rotates, the beam of X-rays from its magnetic poles sweep around the sky. If the earth happens to be in the path of one of the two beams, we can observe a pulsating X-ray source.

We detect a pulse once per rotation period, which means that the NS in Hercules X-1 spins at the rate of once every 1.24 seconds. If the orbital plane of the binary system is nearly edge on to our line of sight, as for Centaurus X-3 and Hercules X-1, the pulsating X-ray appear to turn off when the NS is eclipsed by the companion star.

Other pulsating X-ray sources, such as Scorpius X-1, do not undergo this sort of turning off. The orbital planes of these systems are oriented more nearly face-on to our line of sight, so no eclipse occur.

The NSs in most pulsating X-ray sources have rotation periods (and hence pulse periods) of a few seconds. But as time passes, the NS accretes more and more mass from its companion. In the process, it will spin up and may eventually become a millisecond pulsar.

1.4.3 X-ray bursters and neutron stars

Explosive thermonuclear processes on NSs produce novae & busters. These exotic phenomena occur when stellar corpse is part of a close binary system. One example is a *nova* in which a faint star suddenly brightens by a factor of 10^4 to 10^8 over a few days or hours, reaching a peak luminosity of about 10^5 times that of the sun. By contrast, *supernova* has a peak luminosity of about 10^9 times value of the sun's luminosity. Every year two or three novae are seen in our Galaxy, and several dozen more are thought to take place in remote regions of the galaxy that are obscured from our view by interstellar dust. All novae are members of a close binary system containing white dwarf.

A surface explosion similar to a nova also occurs with neutron stars. In 1975 it was discovered that some objects in the sky emit sudden, powerful bursts of X- rays. There is an abrupt increase of the rays followed by a gradual decline. An entire burst typically lasts for only 20s. Unlike pulsating X-ray source, there is a fairly long interval of hours or days between bursts. Sources that behave in this fashion are known as *X-ray busters*. Several dozen X-ray busters have been discovered in our galaxy. This burst was recorded by an earth-orbiting X-ray telescope.

X-ray busters, like novae, are thought to involve close binaries whose stars are engaged in a mass transfer. With a burster however, the stellar corpse is a NS rather than a white dwarf. Gases escaping from the ordinary companion star fall on to the NS.

The X-ray buster's magnetic field is probably not strong enough to funnel the falling material toward the magnetic poles, so the gases are distributed more evenly over the surface of the NS.

The energy released as these gases crash down on to the NS's surface produces the low-level X-rays that are continuously emitted by the burster. The process on X-ray bursters is explosive, since the fuel is compressed so tightly against the star's surface that it becomes degenerate, like the star itself.

Note that we should not confuse X-ray bursters with magnetars, which are also NS's that emit powerful bursts of X-rays. An X-ray burster is a member of a binary system that accrets matter from its companion star, then releases nuclear energy when the accreted matter undergoes thermonuclear reactions. By contrast, a burst from a magnetar is a release of magnetic energy stored in its extraordinary powerful field; no thermonuclear reactions are involved, and no accretion from a companion star is required. There is also a huge difference in the strength and duration of the two kinds of burst. An X-ray burster releases about 10^{32} joules in a typical 20-second burst, while a magnetar burst can release more than 10^{39} joules of X-ray and γ-ray radiation in a mere 0.2 second.

One of the great puzzles in modern astronomy has been the nature of an even more remarkable class of events called *gamma ray bursters*. As the name suggests, these objects emit sudden, intense bursts of high-energy gamma rays. We will discuss these in the next section.

1.4.4 Gamma-Ray Bursters and neutron stars

Gamma-ray bursters (GRBs)are incredibly bright flashes of radiation from remarkably distant galaxies, out side the disk of our Galaxy. GRB's may have been produced when one of the most massive stars of these Galaxy's become a supernova.

GRB's were discovered in the late 1960's by the orbiting Vela Satellites, whose detectors noticed flashes of gamma rays coming from random parts of the sky at random intervals. With a new generation of gamma-ray telescopes currently in orbit, new gamma ray bursters are being found at a rate of about one per day. GRBs fall in to two types. *Long-duration gamma-ray bursters*, which are more common, last from about 2 to about 1,000 seconds before fading to invisibility.

The less common *short-duration gamma-ray bursters* last from a few hundreths of a second to about 2 seconds, and tend to emit photons of shorter wavelength and hence higher energy. Unlike X-ray bursters, GRBs of both types appear to emit only one burst in their entire history.

GRBs are seen with roughly equal probability in all parts of the sky. This suggests that they are not in the disk of our galaxy. One idea was that GRBs are relatively close to us and lie in a spherical halo surrounding the Milky Way. Alternatively, GRBS could be strewn across space like galaxies, with some of them billons of light-years away. As of 2007, the most distant GRB yet seen lies within a galaxy some 13 billion (1.3×10^{10}) light years way. To be seen at such immense distance a GRB must release a truly stupendous amount of energy in the form of radiation. A relatively bright long-duration GRB was detected on March 2003, by the orbiting gamma ray telescope HETE-2 (*High Energy Transient Explorer*). This burster denoted GRB 030329 for the date it was observed-in the constellation Leo. It was so intense that its high-energy photons temporarily ionized part of the earth's atmosphere-the only event of its kind ever caused by an object beyond our own galaxy.

Subsequent observations with the *Hubble Space Telescope* revealed an expanding shell of gas at the position of the GRB. A long-duration GRB can not simply be an ordinary core-collapse supernova. It releases about 10^{46} joules of energy, of which only about 0.03% is released as electro magnetic radiation. Most of the remaining energy goes in to neutrinos, while some goes in to accelerating the derbis that expands away from the supernova.

If we assume that a GRB emits light equally in all directions, the inverse square law that relates brightness, luminosity, and distance tells us that the most energetic bursters would have to emit about 3×10^{47} joules of radiation in less than a minute. It would take 100,000 supernovae going off simultaneously to release this much radiation.

The newest tool in the search for answers about GRBs is a NASA satellite called *Swift*, which was launched in 2004.

Chapter 2 - Spin sum to Dirac Solutions in a strong magnetic field

2.1 Introduction

In this section we derive the spin sum of the solutions of the Dirac equation in presence of a magnetic field. We will have two spinors which have two different position coordinates in general and so their spatial dependence is explicitly shown to be different.

But before that we will see positive and negative solutions of Dirac equation. Since calculations heavily depend on the choice of the background guage field giving rise to the magnetic field, it should also be first mentioned.

2.2 Solutions of Dirac equation in a strong magnetic Field

The state of an electron in the magnetic field is described by the solution of the Dirac equation.Charged fermions in a uniform magnetic field can have a guage conditions given by ,

$$\mathrm{A}_0 \rightarrow \mathrm{A}_0$$

$$A \rightarrow A + \nabla A(x) \tag{2.2.1}$$

$$p \rightarrow p + eQ\nabla A(x) \tag{2.2.2}$$

Where A is a scalar function of x , p is canonical momentum, Q is the sign of the fermion charge and e is charge of the proton. The canonical momentum does not remain gauge invariant as in case of the free space. Because

$$\Pi_0 = p_0 - eQA_0 \text{ and} \tag{2.2.3}$$

$$\Pi_i = p_i - eQA_i = p_i + eQ\nabla A(x) - eQ(A_i + \nabla A(X)) \tag{2.2.4}$$

remain gauge invariant under the gauge transformation Eq. (2.2.1) and (2.2.2) we can also say

$$\Pi_\mu = p_\mu - eQA_\mu \tag{2.2.5}$$

is gauge invariant.

Let us Consider the magnetic field to be along the Z-axis and gauge $A(x) = (0, Bx_1, 0)$.

When there is a uniform background magnetic field around a fermion, the generalied momentum is given by

$$p^\mu \to p^\mu - eQA^\mu \tag{2.2.6}$$

And

$$p^0 \to p^0 \tag{2.2.7}$$

$$p \to p - eQA \tag{2.2.8}$$

Considering interactions given by the above equations the standared form of Dirac equation can be written as

$$[i\gamma^0 \partial_0 + \gamma^i \Pi_i - m]\psi = 0 \tag{2.2.9}$$

This gives

$$i\frac{\partial\psi}{\partial t} = \mathcal{H}_B\,\psi \tag{2.2.10}$$

where $\mathcal{H}_B$ is Dirac Hamiltonian in the presence of magnetic field.

$$\mathcal{H}_B = \alpha.\pi + \beta m \tag{2.2.11}$$

The eigen state of Eq. (2.2.10) could be given as

$$\psi = e^{-iEt}\begin{pmatrix}\emptyset\\ \chi\end{pmatrix} \tag{2.2.12}$$

Hence Eq. (2.2.10) can be expanded to

$$\frac{i\,\partial[e^{-iEt}\binom{\emptyset}{\chi}]}{\partial t} = \alpha.(-i\nabla - eQA)\left[e^{-iEt}\begin{pmatrix}\emptyset\\ \chi\end{pmatrix}\right] + m\beta\begin{pmatrix}\emptyset\\ \chi\end{pmatrix}[e^{-iEt}] \tag{2.2.13}$$

$$Ee^{-iEt}\begin{pmatrix}\emptyset\\ \chi\end{pmatrix} = \begin{pmatrix}0 & \sigma.(-\nabla - eQA)\\ \sigma.(-i\nabla - eQA) & 0\end{pmatrix}e^{-iEt}\begin{pmatrix}\emptyset\\ \chi\end{pmatrix} + me^{-iEt}\begin{pmatrix}\emptyset\\ -\chi\end{pmatrix}$$

(2.2.14)

Any arbitrary vector T in three dimensions is given as

$$\sigma.T = \begin{pmatrix} T_z & T_x - iT_y \\ T_x + iT_y & -T_z \end{pmatrix} \tag{2.2.15}$$

Where σ denotes Pauli matrices. Using the above equations, eq (2.2.10) can be written as

$$(E - m)\emptyset = \sigma.(-i\nabla - eQA)\chi \tag{2.2.16}$$

$$(E + m)\chi = \sigma.(-i\nabla - eQA)\emptyset \tag{2.2.17}$$

To get rid of χ we multiply Eq. 16 by $(E + m)$ from left side and using Eq. (2.2.17) we obtain,

$$(E^2 - m^2)\emptyset = (\sigma.(-i\nabla - eQA))^2\emptyset \tag{2.2.18}$$

Using the identity relation

$$(\sigma.a)(\sigma.b) = a.b + i\sigma.(a \times b)\,, \tag{2.2.19}$$

the right hand side of Eq. (2.2.18) becomes

$$\begin{aligned}\left(\sigma.(-i\nabla - eQA)\right)^2\emptyset &= [-i\nabla - eQA][-i\nabla - eQA]\emptyset \\ &\quad + i\sigma.[(\nabla - eQA) \times (-i\nabla - eQA)]\emptyset \\ &= [-\nabla^2 + ieQ(\nabla.A + A.\nabla) + e^2Q^2A^2]\emptyset \\ &\quad + i\sigma.[ieQ(A \times \nabla + \nabla \times A)]\emptyset \end{aligned} \tag{2.2.20}$$

$$\Rightarrow (E^2 - m^2)\emptyset = [-\nabla^2 + (eQB)^2x^2 - eQB\left(2ix\frac{\partial}{\partial y} + \sigma_3\right)]\emptyset \tag{2.2.21}$$

Because the coordinates y and z do not appear in the above equation but through their derivatives, its solution can be written as

$$\emptyset = e^{-ip.X}f(x) \tag{2.2.22}$$

In Eq. (2.2.22) X stands for spatial coordinates and is different from x, which is one of the component of X. It represents the 3 vector X with out its x coordinate. This means

$$p.X = p_y y + p_z z \tag{2.2.23}$$

In Eq. (2.2.22) $f(x)$ represent a 2- component matrix which depends on the x -coordinate and some momentum component. From Eqs. (2.2.21) and (2.2.22) we notice that $f(x)$ is an eigen state of σ_3 with eigen values $s = 1 \; and \; 2$, so that

$$\sigma_3 f_s = s f_s. \tag{2.2.24}$$

Let $$f_1(x) = \begin{pmatrix} F_1^1(x) \\ F_1^2(x) \end{pmatrix}, \quad f_-(\mathrm{y}) = \begin{pmatrix} F_2^1(x) \\ F_2^2(x) \end{pmatrix} \tag{2.2.25}$$

From the above equations we can write two independent solutions in general as:

$$f_1(x) = \begin{pmatrix} F_1(x) \\ 0 \end{pmatrix}, \quad f_2(x) = \begin{pmatrix} 0 \\ F_2(x) \end{pmatrix} \tag{2.2.26}$$

Using Eq. (2.2.26) in 22 then the result to eq.(2.2. 21) , we get the following

For $s = 1$,

LHS of eq. (2.2.21):

$$(E^2 - m^2)e^{ip.X} \begin{pmatrix} F_1 x \\ 0 \end{pmatrix} = \begin{pmatrix} (E^2 - m^2)F_1(x) \\ 0 \end{pmatrix} e^{ip.X} \tag{2.2.27}$$

RHS of eq. (2.2.21):

Since $-\nabla^2 \emptyset = \{p_x^2 + p_z^2\} \begin{pmatrix} F_1(x) \\ 0 \end{pmatrix} e^{ip.X} - \begin{pmatrix} \frac{d^2 F_1(x)}{dx^2} \\ 0 \end{pmatrix} e^{ip.X}$ and (2.2.28)

$$\left\{-eQB\left(2ix\frac{\partial}{\partial x} + \sigma_3\right) + (eQB)^2 x^2\right\} \emptyset =$$
$$\{eQB(2xp_y) - eQB + (eQB)^2 x^2\} \begin{pmatrix} F_+(x) \\ 0 \end{pmatrix} e^{ip.X} \tag{2.2.29}$$

$$\Longrightarrow \text{RHS} = \{p_x^2 + p_z^2\} \begin{pmatrix} F_1(x) \\ 0 \end{pmatrix} e^{ip.X} - \begin{pmatrix} \frac{d^2 F_1(x)}{dx^2} \\ 0 \end{pmatrix} e^{ip.X}$$

$$+\{ eQB(2xp_y) - eQB + (eQB)^2 x^2\} \begin{pmatrix} F_1(x) \\ 0 \end{pmatrix} e^{ip.X} \tag{2.2. 30}$$

Equating LHS and RHS we get

$$\frac{d^2 F_1}{dx^2} - [eQBx + p_y]^2 F_1 + [E^2 - m^2 - p_z^2 + eQB]F_1 = 0 \tag{ 2.2.31}$$

Similarly, for $s = 2$,

$$\frac{d^2 F_2}{dx^2} - [eQBx + p_y]^2 F_2 + [E^2 - m^2 - p_z^2 + eQB]F_2 = 0 \tag{2.2. 32}$$

In general, for $s = 1 \; or \; 2$

$$\frac{d^2 F_s}{dx^2} - [eQBx + p_y]^2 F_s + [E^2 - m^2 - p_z^2 + eQB] F_s = 0 \tag{2.2.33}$$

Let us use the dimensionless variable

$$\varepsilon = \sqrt{e|Q|B}\left(x + \frac{p_y}{eQB}\right) \tag{2.2.34}$$

Where $\varepsilon = \varepsilon(x_1, p_2)$

Which transforms eq (2.2 33) to

$$\left(\frac{d^2}{d\varepsilon^2} - \varepsilon^2 + a_s\right) F_s = 0 \tag{2.2.35}$$

$$\text{Where, } a_s = \frac{E^2 - m^2 - p_z^2 + eQBs}{e|Q|B} \tag{2.2.36}$$

Eq. (2.2.35) have solution $a_s = 2\nu + 1$ for non-zero integer ν. Eq. (2.2.36) finally gives energy eigen values

$$E^2 = m^2 + p_z^2 + (2\nu + 1)\, e|Q|B - eQBs. \tag{2.2.37}$$

For particular a_s solution of f_s is

$$I_\nu(\varepsilon) = N_\nu e^{-\varepsilon^2/2} H_\nu(\varepsilon) \tag{2.2.38}$$

Where H_ν are Hermite polynomials of order ν and N_ν are normalization constants given by

$$N_\nu = \left(\frac{\sqrt{e|Q|B}}{\nu! 2^\nu \sqrt{\pi}}\right)^{1/2} \tag{2.2.39}$$

Completeness relation can be written as

$$\sum_\nu I_\nu(\varepsilon)\, I_\nu(\varepsilon_*) = \sqrt{e|Q|B}\,\delta(\varepsilon - \varepsilon_*) = \delta(x - x_*) \tag{2.2.40}$$

A charged particle in a magnetic field has a quantized *cyclotron* orbit with a discrete energy level called *Landau* level. The relativistic form of the Landau energy level is given as

$$E_n^2 = m^2 + p_3^2 + 2neB \tag{2.2.41}$$

Comparing (2.2.37) and (2.2.41) the solutions are two fold degenerate for :

$s = 1 \qquad \nu = n - 1 \qquad$ and

$$s = 2, \qquad \nu = n \tag{2.2.42}$$

But for n=0,

$(2\nu+1)e|Q|B = eQBs, Q = -|Q|$

$\Rightarrow \nu = -\frac{1}{2}(1+s)$

Sine ν and n are non negative integers, unlike the other n states, n=0 is a non-degenerate state. Here n represents the n-th landau level.

2.2.1 Positive energy solutions for spin up and down

Using eq. (2.2. 26) for the positive energy,

$$f_1^{(n)}(x) = \begin{pmatrix} \mathrm{I}_{n-1}(\varepsilon) \\ 0 \end{pmatrix}, \qquad f_2^{(n)}(x) = \begin{pmatrix} 0 \\ \mathrm{I}_n(\varepsilon) \end{pmatrix} \tag{2.2.43}$$

For n=0 , f_1 does not exist. We express this fact by

$$\mathrm{I}_{-1}(x) = 0 \tag{2.2.44}$$

Positive energy solution of the Dirac equation is written as

$$\psi = \mathrm{e}^{-\mathrm{ip.X}}\mathrm{U}_{\mathrm{s,n}}(x, \mathrm{P}) \tag{2.2.45}$$

Where $p.X = p^{\mu}X_{\mu}$. Comparing Eqs. (2.2.12), (2.2.22) and (2.2.45) we get

$$\mathrm{U}_{1,\mathrm{n}}(x_1, \mathrm{P}) = \begin{pmatrix} \mathrm{I}_{\mathrm{n}-1}(\varepsilon) \\ 0 \\ \chi_+ \end{pmatrix} \tag{2.2.46}$$

$$U_{2,n}(x_1, P) = \begin{pmatrix} 0 \\ I_n(\varepsilon) \\ \chi_- \end{pmatrix} \tag{2.2.47}$$

χ_1 and χ_2 can be solved using eqs (2.2.17) and (2.2.22). So that for $s = 2$,

$$(E+m)\begin{pmatrix} \chi_1^1 \\ \chi_1^2 \end{pmatrix} \mathrm{e}^{\mathrm{ip.X}} = \begin{pmatrix} -\mathrm{i}\nabla_3 & (-\mathrm{i}\nabla_1 - \mathrm{eQA}_1 - \nabla_2) \\ (-\mathrm{i}\nabla_1 - \mathrm{eQA}_1 + \nabla_2 & \mathrm{i}\nabla_3 \end{pmatrix} \mathrm{e}^{\mathrm{i}(\mathrm{p_y}x + \mathrm{p_z z})} \begin{pmatrix} \mathrm{I}_{\mathrm{n}-1}(\varepsilon) \\ 0 \end{pmatrix} \tag{2.2.48}$$

$$\Longrightarrow \qquad \chi_1^1 = \frac{p_z}{E_n+m} I_{n-1}(\varepsilon) \quad \text{and} \tag{2.2.49}$$

Using $I_n(\varepsilon) = \frac{1}{(2n)^{1/2}} [\varepsilon\, I_{n-1}(\varepsilon) \text{-} \frac{I_{n-1}(\varepsilon)}{\partial\varepsilon}]$ (2.2.50)

$$\chi_1^2 = -\frac{\sqrt{2neB}}{(E_n+m)} I_n(\varepsilon) \tag{2.2.51}$$

Thus,

$$U_{1,n}(x_1, P) = \begin{pmatrix} I_{n-1}(\varepsilon) \\ 0 \\ \frac{P_z}{E_n+m} I_{n-1}(\varepsilon) \\ -\frac{\sqrt{2neB}}{(E_n+m)} I_n(\varepsilon) \end{pmatrix} \tag{2.2.52}$$

Similarly, for $s = 2$,

$$U_{2,n}(x_1, P) = \begin{pmatrix} 0 \\ I_n(\varepsilon) \\ -\frac{\sqrt{2neB}}{(E_n+m)} I_{n-1}(\varepsilon) \\ -\frac{p_z}{E_n+m} I_n(\varepsilon) \end{pmatrix} \tag{2.2.53}$$

2.2.2 Negative energy solutions for spin up and down

To find negative energy solution where its energy eigen value is $E = -E_n$, we can start from Dirac equation as

$$\psi = e^{-i(-|E|)t} \binom{\emptyset}{\chi} \tag{2.2.54}$$

Following similar procedure done for positive energy, we can find the following negative energy spinors.

$$V_{1,n}(x_1,P) = \begin{pmatrix} \frac{p_z}{E_n+m} I_{n-1}(\tilde{\varepsilon}) \\ \frac{\sqrt{2neB}}{(E_n+m)} I_n(\tilde{\varepsilon}) \\ I_{n-1}(\tilde{\varepsilon}) \\ 0 \end{pmatrix} \quad \text{and} \quad V_{2,n}(x_1,P) = \begin{pmatrix} \frac{\sqrt{2neB}}{(E_n+m)} I_{n-1}(\tilde{\varepsilon}) \\ -\frac{p_z}{E_n+m} I_n(\tilde{\varepsilon}) \\ 0 \\ I_n(\tilde{\varepsilon}) \end{pmatrix}$$

(2.2.55)

2.3 Spin sum

2.3.1 Spin sum for positive energy solutions

For positive energy spin sum can be written as

$$P_U(x_1, y_1 \quad , P) = \sum_s U_{s,n}(x_1, P)\ \bar{\mathrm{U}}_{\mathrm{s,n}}(x_{1_*}P)$$

$$= \frac{1}{E_n+m} \sum_{i,j=n-1}^{n} I_i(\varepsilon) I_j(\varepsilon) T_{i,j} \qquad (2.3.1)$$

Where $T'_{i,j}s\ are\ 4 \times 4\ matrices$ and to find their values we must have matrix multiplication of the spinors.

Summing over all possible spin degree of freedom gives

$$P_U(x_1, y_1 \quad , P) = \sum_s U_{s,n}(x_1, P)\, \bar{\mathrm{U}}_{\mathrm{s,n}}\ (x_{1_*}P)$$

$$= U_{1,n}(x_1, P) + \bar{U}_{1,n}(x_{1_*}, P) + U_{2,n}(x_1, P)\bar{U}_{2,n}(x_{1_*}, P) \qquad (2.3.2)$$

But $\bar{U}_{1,n}(x_{1_*},\ P) = \mathrm{U}^{+}_{1,\mathrm{n}}(x_{1_*},\ P)\gamma^0$, (2.3.3)

where $+$ is adjoint and $\gamma^0 = \beta$

$$\Rightarrow \bar{U}_{1,n}(x_{1_*}, P) = \left(I_{n-1}(\varepsilon_*) \quad 0 \quad \frac{p_z}{E_n+m} I_{n-1}(\varepsilon_*\ ) \quad - \frac{\sqrt{2neB}}{(E_n+m)} I_n(\varepsilon_*\ )\right) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad (2.3.4)$$

$$\Rightarrow \bar{U}_{1,n}(x_{1_*}, P) = \left[\left(I_{n-1}(\varepsilon_*) \quad o \quad \frac{p_z}{E_n+m} I_{n-1}(\varepsilon_*\ ) \quad \frac{\sqrt{2neB}}{(E_n+m)} I_n(\varepsilon_*\ )\right]\right.$$

(2.3.5)

so that

$$U_{1,n}(x_1,P)\overline{U}_{1,n}(x_{1_*},P)=\begin{pmatrix} I_{n-1}(\varepsilon) \\ 0 \\ \frac{P_z}{E_n}I_{n-1}(\varepsilon) \\ -\frac{\sqrt{2neB}}{(E_n+m)}I_n(\varepsilon)\end{pmatrix}\left[\left(I_{n-1}(\varepsilon_*)\quad o\quad -\frac{p_z}{E_n+m}I_{n-1}(\varepsilon_*\ )\quad \frac{\sqrt{2neB}}{(E_n+m)}I_n(\varepsilon_*\ )\right]\right.$$

(2.3.6)

or

$U_{1,n}(x_1,P)\overline{U}_{1,n}(x_{1_*},P)=$

$$\begin{pmatrix} I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*\ ) & 0 & -\frac{p_z}{E_n+m}I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*\ ) & \frac{\sqrt{2neB}}{(E_n+m)}I_{n-1}(\varepsilon)I_n(\varepsilon_*) \\ 0 & 0 & 0 & 0 \\ \frac{p_z}{E_n+m}I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*\ ) & 0 & -\frac{(p_z)^2}{(E_n+m)^2}I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*\ ) & P_z\frac{\sqrt{2neB}}{(E_n+m)^2}I_{n-1}(\varepsilon)I_n(\varepsilon_*) \\ -\frac{\sqrt{2neB}}{(E_n+m)}I_n(\varepsilon)I_{n-1}(\varepsilon_*\ ) & 0 & P_z\frac{\sqrt{2neB}}{(E_n+m)^2}I_n(\varepsilon)I_{n-1}(\varepsilon_*\ ) & -\frac{\sqrt{2neB}}{(E_n+m)}I_n(\varepsilon)I_n(\varepsilon_*\ ) \end{pmatrix} \quad (2.3.7)$$

Similarly for $s=1$,

$$\overline{U}_{2,n}(x_{1_*},P)=\mathrm{U}^{+}_{2,\mathrm{n}}(x_{1_*},P)\gamma^0$$

(2.3.8)

$$\Longrightarrow\overline{U}_{2,n}(x_{1_*},P)=\left[0\quad I_n(\varepsilon_*)\quad \frac{\sqrt{2neB}}{(E_n+m)}I_{n-1}(\varepsilon_*\ )\quad \frac{p_z}{E_n+m}\ I_n(\varepsilon_*)\ \right] \quad (2.3.9)$$

Thus,

$$U_{2,n}(x_1,P)\overline{U}_{2,n}(x_{1_*},P)=\begin{pmatrix} 0 \\ I_n(\varepsilon) \\ -\frac{\sqrt{2neB}}{(E_n+m)}I_{n-1}(\varepsilon) \\ -\frac{p_z}{E_n+m}I_n(\varepsilon)\end{pmatrix}\left[0\quad I_n(\varepsilon_*)\quad \frac{\sqrt{2neB}}{(E_n+m)}I_{n-1}(\varepsilon_*\ )\quad \frac{p_z}{E_n+m}\ I_n(\varepsilon_*)\ \right]$$

(2.3.10)

or

$$U_{2,n}(x_1,P)\overline{U}_{2,n}\big(x_{1_*},P\big)$$
$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & I_n(\varepsilon)I_n(\varepsilon_*\ ) & \frac{\sqrt{2neB}}{(E_n+m)}I_n(\varepsilon)I_{n-1}(\varepsilon_*\ ) & \frac{p_z}{E_n+m}I_n(\varepsilon)I_n(\varepsilon_*\ ) \\ 0 & -\frac{\sqrt{2neB}}{(E_n+m)}I_{n-1}(\varepsilon)I_n(\varepsilon_*\ ) & \frac{-\sqrt{2neB}}{(E_n+m)^2}I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*\ ) & -P_z\frac{\sqrt{2neB}}{(E_n+m)^2}I_{n-1}(\varepsilon)I_n(\varepsilon_*) \\ 0 & -\frac{p_z}{E_n+m}I_n(\varepsilon)I_n(\varepsilon_*\ ) & -P_z\frac{\sqrt{2neB}}{(E_n+m)^2}I_n(\varepsilon)I_{n-1}(\varepsilon_*\ ) & -\frac{(p_z)^2}{(E_n+m)^2}I_n(\varepsilon)I_n(\varepsilon_*\ ) \end{pmatrix}$$

(2.3.11)

Thus,

$$P_U\big(x_1,y_1\ \ ,P\big) = U_{1,n}(x_1,P) + \overline{U}_{1,n}\big(x_{1_*},P\big) + U_{2,n}(x_1,P)\overline{U}_{2,n}(x_{1_*},P)$$

$$\begin{bmatrix} I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*\ ) & 0 & -\frac{p_z}{E_n+m}I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*\ ) & \frac{\sqrt{2neB}}{(E_n+m)}I_{n-1}(\varepsilon)I_n(\varepsilon_*\ ) \\ 0 & I_n(\varepsilon)I_n(\varepsilon_*\ ) & \frac{\sqrt{2neB}}{(E_n+m)}I_n(\varepsilon)I_{n-1}(\varepsilon_*\ ) & \frac{p_z}{E_n+m}I_n(\varepsilon)I_n(\varepsilon_*\ ) \\ \frac{p_z}{E_n+m}I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*\ ) & -\frac{\sqrt{2neB}}{(E_n+m)}I_{n-1}(\varepsilon)I_n(\varepsilon_*\ ) & -\frac{\left(2neB+p_z^2\right)}{(E_n+m)^2}I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*\ ) & 0 \\ -\frac{\sqrt{2neB}}{(E_n+m)}I_{n-1}(\varepsilon)I_n(\varepsilon_*\ ) & -\frac{p_z}{E_n+m}I_n(\varepsilon)I_n(\varepsilon_*\ ) & 0 & -\frac{\left(2neB+p_z^2\right)}{(E_n+m)^2}I_n(\varepsilon)I_n(\varepsilon_*\ ) \end{bmatrix}$$

(2.3.12)

2.3.2 Spin sum for negative energy solutions

Spin sum for negative spinors can be written as,

$$P_V\left(x_1, y_1 \quad , P\right) = \sum_s V_{s,n}(x_1, P)\bar{V}_{s,n}\left(x_{1_*}, P\right)$$

$$= \frac{1}{E_n+m}\sum_{i,j=n-1}^{n} I_i(\tilde{\varepsilon})I_j(\tilde{\varepsilon}_*)\tilde{T}_{i,j} \qquad (2.3.13)$$

Where $\tilde{T}_{i,j}$ s *are the* 4×4 *matrices.*

Summing over all possible spin degrees of freedom gives,

$$P_V\left(x_1, y_1 \quad , P\right) = \sum_s V_{s,n}(x_1, P)\bar{V}_{s,n}\left(x_{1_*}, P\right)$$

$$= V_{2,n}(x_1, P)\bar{V}_{1,n}\left(x_{1_*}, P\right) + V_{2,n}(x_1, P)\bar{V}_{2,n}\left(x_{1_*}, P\right) \qquad (2.3.14)$$

But $\quad \bar{V}_{1,n}\left(x_{1_*}, P\right) = \mathrm{V}_{1,\mathrm{n}}^{+}(x_{1_*}, P)\gamma^0 \qquad (2.3.15)$

$$\bar{V}_{1,n}\left(x_{1_*}, P\right) = \left[\frac{p_z}{E_n+m} I_{n-1}(\tilde{\varepsilon}_*) \quad \frac{\sqrt{2neB}}{(E_n+m)} I_n(\tilde{\varepsilon}_*) \quad - I_{n-1}(\tilde{\varepsilon}_*) \quad 0\right] \qquad (2.3.16)$$

Thus,

$$V_{1,n}(x_1, P)\bar{V}_{1,n}\left(x_{1_*}, P\right) = \begin{pmatrix} \frac{p_z}{E_n+m} I_{n-1}(\tilde{\varepsilon} \quad) \\ \\ \frac{\sqrt{2neB}}{(E_n+m)} I_n(\tilde{\varepsilon} \quad) \\ \\ I_{n-1}(\tilde{\varepsilon}) \\ \\ 0 \end{pmatrix} \left[\frac{p_z}{E_n+m} I_{n-1}(\tilde{\varepsilon}_*) \quad \frac{\sqrt{2neB}}{(E_n+m)} I_n(\tilde{\varepsilon}_*) \quad - I_{n-1}(\tilde{\varepsilon}_*)\right]$$

(2.3.17)

Similarly for $s = 2$,

$$\overline{V}_{2,n}\left(x_{1_*}, P\right) = V_2^{+}(x_{1_*}, P)\gamma^0 \tag{2.3.19}$$

$$\Longrightarrow \qquad \bar{V}_{2,n}\left(x_{1_*}, P\right) = \left[\frac{\sqrt{2neB}}{(E_n+m)} I_{n-1}(\tilde{\varepsilon}_*) \quad -\frac{p_z}{E_n+m} I_n(\tilde{\varepsilon}\ \ ) \quad 0 \quad -I_n(\tilde{\varepsilon}_*)\right] \tag{2.3.20}$$

So that

$$V_{2,n}(x_1, P)\bar{V}_{2,n}\left(x_{1_*}, P\right)$$
$$= \begin{pmatrix} \dfrac{\sqrt{2neB}}{(E_n+m)} I_{n-1}(\tilde{\varepsilon}) \\ -\dfrac{p_z}{E_n+m} I_n(\tilde{\varepsilon}_*) \\ 0 \\ I_n(\tilde{\varepsilon}) \end{pmatrix} \left[\frac{\sqrt{2neB}}{(E_n+m)} I_{n-1}(\tilde{\varepsilon}_*) \quad -\frac{p_z}{E_n+m} I_n(\tilde{\varepsilon}_*) \quad 0 \quad -I_n(\tilde{\varepsilon}_*)\right] \tag{2.3.21}$$

Thus,

$$V_{2,n}(x_1, P)\bar{V}_{2,n}\left(x_{1_*}, P\right) =$$

$$\begin{pmatrix} \dfrac{2neB}{(E_n+m)^2} I_{n-1}(\tilde{\varepsilon}) I_{n-1}(\tilde{\varepsilon}_*) & -P_z \dfrac{\sqrt{2neB}}{(E_n+m)^2} I_{n-1}(\tilde{\varepsilon}) I_n(\tilde{\varepsilon}_*) & 0 & -\dfrac{\sqrt{2neB}}{(E_n+m)} I_{n-1}(\tilde{\varepsilon}) I_n(\tilde{\varepsilon}_*) \\ -P_z \dfrac{\sqrt{2neB}}{(E_n+m)^2} I_n(\tilde{\varepsilon}\ \ ) I_{n-1}(\tilde{\varepsilon}_*) & \dfrac{(p_z)^2}{(E_n+m)^2} I_n(\tilde{\varepsilon}\ \ ) I_n(\tilde{\varepsilon}_*) & 0 & \dfrac{p_z}{E_n+m} I_n(\tilde{\varepsilon}\ \ ) I_n(\tilde{\varepsilon}_*) \\ 0 & 0 & 0 & 0 \\ \dfrac{\sqrt{2neB}}{(E_n+m)} I_n(\tilde{\varepsilon}\ \ ) I_{n-1}(\tilde{\varepsilon}_*) & -\dfrac{p_z}{E_n+m} I_n(\tilde{\varepsilon}\ \ ) I_n(\tilde{\varepsilon}_*) & 0 & -I_n(\tilde{\varepsilon}\ \ ) I_n(\tilde{\varepsilon}_*) \end{pmatrix} \tag{2.3.22}$$

There fore, spin sum for negative energy becomes,

$$P_V\left(x_1, y_1 \quad, P\right) = V_{1,n}(x_1, P)\bar{V}_{1,n}\left(x_{1_*}, P\right) + V_{2,n}(x_1, P)\bar{V}_{2,n}\left(x_{1_*}, P\right)$$

(2.3.23)

$$P_V\left(x_1, y_1 \quad, P\right)$$
$$= \begin{bmatrix} \frac{(2neB + p_z^2)}{(E_n + m)^2} I_{n-1}(\tilde{\varepsilon}) I_{n-1}(\tilde{\varepsilon}_*) & 0 & -\frac{p_z}{E_n + m} I_{n-1}(\tilde{\varepsilon}) I_{n-1}(\tilde{\varepsilon}_*) & -\frac{\sqrt{2neB}}{(E_n + m)} I_{n-1}(\tilde{\varepsilon}) I_n(\tilde{\varepsilon}_*) \\ 0 & \frac{(2neB + p_z^2)}{(E_n + m)^2} I_n(\tilde{\varepsilon} \quad) I_n(\tilde{\varepsilon}_*) & -\frac{\sqrt{2neB}}{(E_n + m)} I_n(\tilde{\varepsilon} \quad) I_{n-1}(\tilde{\varepsilon}_*) & \frac{p_z}{E_n + m} I_n(\tilde{\varepsilon} \quad) I_n(\tilde{\varepsilon}_*) \\ \frac{p_z}{E_n + m} I_{n-1}(\tilde{\varepsilon}) I_{n-1}(\tilde{\varepsilon}_*) & \frac{\sqrt{2neB}}{(E_n + m)} I_{n-1}(\tilde{\varepsilon}) I_n(\tilde{\varepsilon}_*) & -I_{n-1}(\tilde{\varepsilon}) I_{n-1}(\tilde{\varepsilon}_*) & 0 \\ \frac{\sqrt{2neB}}{(E_n + m)} I_n(\tilde{\varepsilon} \quad) I_{n-1}(\tilde{\varepsilon}_*) & -\frac{p_z}{E_n + m} I_n(\tilde{\varepsilon} \quad) I_n(\tilde{\varepsilon}_*) & 0 & -I_n(\tilde{\varepsilon} \quad) I_n(\tilde{\varepsilon}_*) \end{bmatrix}$$

(2.3.24)

Chapter 3 - Inverse Compton Scattering in strong magnetic field

3.1 Introduction

Inverse Compton scattering (ICS) involves the scattering of low energy photons to high energies by relativistic electrons so that the photons gain and the electrons lose energy. The process is called inverse because the electrons lose energy rather than the photons, the opposite of the standard Compton effect. ICS in a strong magnetic field can be used in studies of pulsar radiation and have important influence on theoretical models of pulsars. Relativistic electrons accelerated above the polar cap can Compton scatter off thermal radiation from the neutron star surface, producing high energy gamma rays.

3.2 Compton scattering in laboratory frame

Harold's study of the Compton scattering in strong magnetic fields is in the electron rest frame (ERF). His expression of cross section has been widely used in astrophysical calculations sine it is believed that ICS in strong magnetic fields near the surfaces of neutron stars is one of the possible mechanisms for X-ray and γ-ray emissions. However in actual calculations the cross section in the laboratory frame (LF) is required. To determine total cross section of ICS we first find Feynman propagator of an electron and then scattering amplitude.

3.2.1 Feynman propagator of an electron

The Feynman propagator of an electron is defined as

$$S_F(x,y) = -i < 0|\, T\psi(x)\, \bar{\psi}(y)\, |0> \quad , \qquad (3.2.1)$$

for $t_x > t_y$ this becomes

$$= -< 0|\, \psi(x)\psi^{+}\gamma^{0}|0> \quad \text{where}$$

$$\psi(x) = \sum_{n=0}^{\infty}\sum_{s=1}^{2}\frac{1}{L}\sum_{p_2,p_3}[c_{s,n}(\boldsymbol{p},t)U_{s,n}(x_1,\boldsymbol{p}) + d_{s,n}^{+}(\boldsymbol{p},t)V_{s,n}(x_1,\boldsymbol{p})]\exp(ip_2x_2 + ip_3x_3) \qquad (3.2.2)$$

$$\psi(x)^{+} = \sum_{n=0}^{\infty}\sum_{s=1}^{2}\frac{1}{L}\sum_{p_2,p_3}[c_{s,n}^{+}(\boldsymbol{p},t)U_{s,n}^{+}(x_1,\boldsymbol{p}) + d_{s,n}(\boldsymbol{p},t)V_{s,n}^{+}(x_1,\boldsymbol{p})]\exp(ip_2x_2 - ip_3x_3) \qquad (3.2.3)$$

$$S_F(x,y) = -< 0 \left| \left\{ \sum_{n=0}^{\infty} \sum_{s=1}^{2} \frac{1}{L} \sum_{p_2,p_3} [c_{s,n}(\boldsymbol{p},t) U_{s,n}(x_1,\boldsymbol{p}) + d_{s,n}^{+}(\boldsymbol{p},t) V_{s,n}(x_1,\boldsymbol{p}) \right] \exp(ip_2x_2 + ip_3x_3) \right\} \{ \sum_{n=0}^{\infty} \sum_{s=1}^{2} \frac{1}{L} \sum_{p_2,p_3} [c_{s,n}^{+}(\boldsymbol{p},t) U_{s,n}^{+}(y_1,\boldsymbol{p}) + d_{s,n}(\boldsymbol{p},t) V_{s,n}^{+}(y_1,\boldsymbol{p})] \exp(ip_2x_2 - ip_3x_3)\} \gamma^0 |0 >$$

(3.2 .4)

$$\Rightarrow S_F(x,y) = -< 0 | \{ \sum_{n=0}^{\infty} \sum_{s=1}^{2} \frac{1}{L^2} \sum_{p_2,p_3} [c_{s,n}(\boldsymbol{p},t) U_{s,n}(x_1,\boldsymbol{p}) c_{s,n}^{+}(\boldsymbol{p},t) U_{s,n}^{+}(y_1,\boldsymbol{p})$$

$$+ c_{s,n}(\boldsymbol{p},t) U_{s,n}(x_1,\boldsymbol{p}) d_{s,n}(\boldsymbol{p},t) V_{s,n}^{+}(y_1,\boldsymbol{p})$$

$$+ d_{s,n}^{+}(\boldsymbol{p},t) V_{s,n}(x_1,\boldsymbol{p})$$

$$\times c_{s,n}^{+}(\boldsymbol{p},t) U_{s,n}^{+}(y_1,\boldsymbol{p})$$

$$+ d_{s,n}^{+}(\boldsymbol{p},t) V_{s,n}(x_1,\boldsymbol{p}) d_{s,n}(\boldsymbol{p},t) V_{s,n}^{+}(y_1,\boldsymbol{p})$$

$$\times \exp(ip_2(x_2-y_2) + ip_3(x_3-y_3))]\} \gamma^0 |0 >$$

(3.2 .5)

Using the commutation relations of annihilation and destruction operators,

$$\left\{ c_{s,n}(p_1,t), c_{r,m}^{+}(p_2,t) \right\} = \delta_{sr} \delta_{nm} \delta_{p_1 p_2}$$

(3.2 .6)

$$\{ d_{sn}(p_1,t), d_{rm}^{+}(p_2,t) = \delta_{sr} \delta_{nm} \delta_{p_1 p_2} , \qquad (3.\ 2.\ 7)$$

we have

$$S_F(x,y) = -\frac{i}{L^2} < 0 | \{ \sum_{n=0}^{\infty} \sum_{s=1}^{2} [c_{s,n}(p_2,t) U_{s,n}(x_1,p_2) c_{s,n}^{+}(p_2,t) U_{s,n}^{+}(y_1,p_2)$$

$$+ \quad c_{s,n}(p_3,t) U_{s,n}(x_1,p_3) c_{s,n}^{+}(p_3,t) U_{s,n}^{+}(y_1,p_3) \exp(ip_2(x_2-y_2)$$

$$+ \quad ip_3(x_3-y_3)\} \gamma^0 |0 >$$

$$= -\frac{i}{L^2} \sum_{n=0}^{\infty} \sum_{p_2 p_3} [U_{1,n}(x_1,p) U_{1,n}^{+}(y_1,p) + U_{2,n}(x_1,p) U_{2,n}^{+}(y_1,p] \exp(ip_2(x_2-y_2) + ip_3(x_3-y_3)\} \gamma^0$$

(3.2 .8)

Using equation (2.3.3) and (2.3.8),i.e,

$$U_{1,n}^{+}(x_1,p)\gamma^0 = \bar{U}_{1,n}(x_{1_*},P)\text{and}$$

$$U_{2,n}^{+}(x_1,p)\gamma^0 = \bar{U}_{2,n}(x_1 y_*,P)$$

$$\Longrightarrow S_F(x,y) = -\frac{i}{L^2}\sum_{n=0}^{\infty}\sum_{p_2 p_3}\ [\bar{U}_{1,n}(x_1 y_*,P) + \bar{U}_{2,n}(x_1 y_*,P)]$$

$$\exp(ip_2(x_2 - y_2) + ip_3(x_3 - y_3)$$

(3.2 .9)

From equation (2.3.12), we have

$$P_U(x_1,y_1,p) = \bar{U}_{1,n}(x_{1_*},P) + \bar{U}_{2,n}(x_{1_*},P)]$$

$$\Longrightarrow S_F(x,y) = -\frac{i}{L^2}\sum_{n=0}^{\infty}\sum_{p_2 p_3} P_U(x_1,y_1,P)\exp(ip_2(x_2 - y_2) + ip_3(x_3 - y_3) \quad (3.2\ .10\ )$$

Where

$P_U(x_1,y_1,P) =$

$$\begin{bmatrix} I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*\ ) & 0 & -\frac{p_z}{E_n+m} I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*\ ) & \frac{\sqrt{2neB}}{(E_n+m)} I_{n-1}(\varepsilon)I_n(\varepsilon_*\ ) \\ 0 & I_n(\varepsilon)I_n(\varepsilon_*\ ) & \frac{\sqrt{2neB}}{(E_n+m)} I_n(\varepsilon)I_{n-1}(\varepsilon_*\ ) & \frac{p_z}{E_n+m} I_n(\varepsilon)I_n(\varepsilon_*\ ) \\ \frac{p_z}{E_n+m} I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*\ ) & -\frac{\sqrt{2neB}}{(E_n+m)} I_{n-1}(\varepsilon)I_n(\varepsilon_*\ ) & -\frac{(2neB+p_z^2)}{(E_n+m)^2} I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*\ ) & 0 \\ -\frac{\sqrt{2neB}}{(E_n+m)} I_{n-1}(\varepsilon)I_n(\varepsilon_*\ ) & -\frac{p_z}{E_n+m} I_n(\varepsilon)I_n(\varepsilon_*\ ) & 0 & -\frac{(2neB+p_z^2)}{(E_n+m)^2} I_n(\varepsilon)I_n(\varepsilon_*\ ) \end{bmatrix}$$

(3.2 .11)

From landau level, $E_n^2 = m^2 + p_3^2 + 2neB$

$$\Longrightarrow p_3^2 + 2neB = E_n^2 - m^2 = (E_n - m)(E_n + m) \quad (3.2\ .\ 12)$$

If we use Eq. (3.2.12) in equation ((3.2.11) it gives

$$P_U(x_1, y_1, p) = \begin{bmatrix} I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*) & 0 & -\frac{p_z}{E_n+m} I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*) & \frac{\sqrt{2neB}}{(E_n+m)} I_{n-1}(\varepsilon)I_n(\varepsilon_*) \\ 0 & I_n(\varepsilon)I_n(\varepsilon_*) & \frac{\sqrt{2neB}}{(E_n+m)} I_n(\varepsilon)I_{n-1}(\varepsilon_*) & \frac{p_z}{E_n+m} I_n(\varepsilon)I_n(\varepsilon_*) \\ \frac{p_z}{E_n+m} I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*) & -\frac{\sqrt{2neB}}{(E_n+m)} I_{n-1}(\varepsilon)I_n(\varepsilon_*) & -\frac{(E_n-m)}{(E_n+m)} I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*) & 0 \\ -\frac{\sqrt{2neB}}{(E_n+m)} I_{n-1}(\varepsilon)I_n(\varepsilon_*) & -\frac{p_z}{E_n+m} I_n(\varepsilon)I_n(\varepsilon_*) & 0 & -\frac{(E_n-m)}{(E_n+m)} I_n(\varepsilon)I_n(\varepsilon_*) \end{bmatrix}$$

or

$$P_U(x_1, y_1, P) = \frac{1}{(E_n+m)} \begin{bmatrix} (E_n+m)I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*) & 0 & -\ p_3 I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*) & \sqrt{2neB} I_{n-1}(\varepsilon)I_n(\varepsilon_*) \\ 0 & (E_n+m)\, I_n(\varepsilon)I_n(\varepsilon_*) & \sqrt{2neB} I_n(\varepsilon)I_{n-1}(\varepsilon_*) & p_3\ I_n(\varepsilon)I_n(\varepsilon_*) \\ p_3 I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*) & -\sqrt{2neB} I_{n-1}(\varepsilon)I_n(\varepsilon_*) & -(E_n-m)I_{n-1}(\varepsilon)I_{n-1}(\varepsilon_*) & 0 \\ -\sqrt{2neB} I_{n-1}(\varepsilon)I_n(\varepsilon_*) & -\ p_3 I_n(\varepsilon)I_n(\varepsilon_*) & 0 & -(E_n-m)I_n(\varepsilon)I_n(\varepsilon_*) \end{bmatrix}$$

(3.2 . 13)

Let us denote the value in the bracket to be, $S_n(x_1, y_1, p)$. This gives

$$P_U(x_1, y_1, P) = \frac{1}{(E_n+m)} \quad S_n(x_1, y_1, P) \tag{3.2 . 14}$$

Using $\theta(t_x - t_y) = \lim_{\epsilon\to 0} \frac{-1}{2\pi i} \int_{-\infty}^{\infty} \frac{e^{-iw(t_x-t_y)}}{w+i\epsilon} dw$ (3.2 .15)

and Eqs 10 and 16 we get,

$$S_F(x, y) = \frac{1}{L^2} \sum_{p_2 p_3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sum_{n=0}^{\infty} \frac{S_n(x_1, y_1, P)}{(E_n+m)(w+i\epsilon)} \exp -\mathrm{i}\omega(t_x - t_y) + ip_2(x_2 - y_2)$$
$$+\ ip_3(x_3 - y_3)$$

(3.2 .16)

3.2.2 Scattering Amplitude

The initial and final states of an electron–photon scattering system can be represented by

$$|i,t_i\rangle = c_0^+(p_i,t_i)\ a_{\lambda_i}^+(k_i,t_i)|0\rangle$$

$$|f,\ t_f\rangle = c_0^+(p_f,t_f)\ a_{\lambda_f}^+(k_f,t_f)|0\rangle$$

Where $c_0^+ = c_{2,0}^+$ is the electron creation operator, a_λ^+ is the photon creation operator and $p_{i(f)}$ denotes the incident (outgoing) electron momentum. The scattering matrix is given by

$$S_{fi} = \lim_{t_i\to-\infty,t_f\to\infty} e^2\langle 0\left|Tc_0(p_f,t_f)a_{\lambda_f}(k_f,t_f)c_0^+(p_i,t_i)\ a_{\lambda_i}^+(k_i,t_i\right|0\rangle$$

This gives

$$S_{fi} = \lim_{t_i\to-\infty,t_f\to\infty} e^2\int d^4x_1d^4x_2\left\langle 0\left|Tc_0(p_f,t_f)\bar{\psi}\ (x_1)\right|0\right\rangle\gamma_\mu < 0|\ T\psi(x_1)\ \bar{\psi}(x_2)\ |0 > \gamma_\nu$$

$$\times\ [\langle 0|Ta_{\lambda_f}(k_f,t_f)A_\mu(x_1)\quad|0\rangle\langle 0|T\ a_{\lambda_i}^+(k_i,t_i)A_\nu(x_2)|0\rangle$$
$$+\ \langle 0|Ta_{\lambda_f}(k_f,t_f)A_\nu(x_2)|0\rangle\langle 0|T\ a_{\lambda_i}^+(k_i,t_i)A_\mu(x_1)|0\rangle]$$
$$\times\ \langle 0|T\psi(x_2))c_0^+(p_i,t_i)\ )|0\rangle$$

$$\psi^+(x) = \sum_{n=0}^{\infty}\sum_{s=1}^{2}\frac{1}{L}\sum_{p_2,p_3}[c_{s,n}^+(p,t)u_{s,n}^+(x_1,p)+d_{s,n}(p,t)v_{s,n}^+(x_1,p)]e^{-ip_2x_{2-ip_3}x_3} \tag{3.2 .17 }$$

$$\left\langle 0\left|Tc_0(p_f,t_f)\right|0\right\rangle = \langle 0|\ c_0(p_f,t_f)\ \bar{\psi}\ (r_1,t)\,|0\rangle\quad,\ t_f\to\infty$$

$$= \langle 0|\ c_0(p_f,t_f)\psi^+\ (r_1,t)\gamma^0\,|0\rangle$$

$$= \sum_{n=0}^{\infty}\sum_{s=1}^{2}\frac{1}{L}\sum_{p_2,p_3}\ \ [\langle 0|\ c_0(p_f,t_f)c_{s,n}^+(p,t)\ U_{s,n}^+(x_1,p)\gamma^0\,|0\rangle$$

$$+\langle 0|\ c_0(p_f,t_f)d_{s,n}(p,t)\ V_{s,n}^+(x_1,p)\gamma^0\,|0\rangle]e^{-i(p_2x_2+p_3x_3)}$$

$$= \sum_{n=0}^{\infty}\sum_{s=1}^{2}\frac{1}{L}\sum_{p_2,p_3}\ \ [\langle 0|\ c_0(p_ft_f)c_{s,n}^+(p\quad,t\quad) \tag{3.2 . 18}$$

$$U^{+}_{s,n}(x_1,p)\gamma^0\,|0\rangle e^{-i(p_2x_2+p_3x_3)}$$

$$=\frac{1}{L}\sum_{p_2,p_3}\ [\langle 0|\ c_0(p_f,t_f)c^{+}_{2,0}(p,t)\ U^{+}_{2,0}(x_1,p)\gamma^0\,|0\rangle e^{-i(p_2x_2+p_3x_3)}$$

$$=\frac{1}{\mathrm{L}}\sum_{p_2,p_3}\ \overline{U_0}(x_1,p_f)e^{-i(p_2x_2+p_3x_3)} \qquad (3.\ 2.\ 19)$$

The incident (out going) electron momentum can be expressed as $p_{i(f)}=(0, a_{i(f)}, p_{i(f)})$

,i.e, the incident (outgoing) electron momentum along the direction of the magnetic field (taken as the Z direction)is $p_{i(f)}$ *and the center of the Landau orbit is* $-\lambda^2 a_{i(f)}$

$$\Longrightarrow \langle 0|Tc_0(p_f,t_f)\bar{\psi}\,(r_1,t)|0\rangle=\frac{1}{L}\overline{U_0}(x_1,p_f)\ e^{(-iE_ft_f-ip_fz_1-ia_fy_1+iE_ft_1)} \qquad (3.\ 2.\ 20)$$

Similarly,

$$\langle 0|\ Ta_{\lambda_f}(k_f\ ,t_f\ )A_\mu(\overrightarrow{r_1},t)\quad |0\rangle\ =\frac{1}{\sqrt{\mathbb{V}}}\frac{1}{\sqrt{2\omega_f}}\epsilon_\mu^{(\lambda_f)}e^{(-iw_ft_f\ +i\omega_ft_1-i\overrightarrow{k_f}\cdot\overrightarrow{r_1})} \qquad (3.\ 2.\ 21)$$

but , $i\overrightarrow{k_f}\cdot\overrightarrow{r_1}=ik_{fx}\,x_1+ik_{fy}\,y_1+ik_{fz}\,z_1,$ (3.2 . 22)

Where, $k_{fz}=k_f\cos\theta_f$ (3. 2.23)

$$\langle 0|\ Ta_{\lambda_f}(k_f\ ,t_f\ )A_\mu\quad (\overrightarrow{r_1},t)\quad |0\rangle\ =\frac{1}{\sqrt{\mathbb{V}}}\frac{1}{\sqrt{2\omega_f}}\epsilon_\mu^{(\lambda_f)}e^{(-i\omega_ft_f\ +i\omega_ft_1-ik_{fx}x_1-ik_{fy}y_1-ik_f\cos\theta_fz_1\,)}$$

(3.2 .24)

$$\langle 0|T\psi\,(\overrightarrow{r_2}\ \ ,t)c_0^{+}(p_i,t_i)|0\rangle=\frac{1}{L}U_0(x_2,p_i)\ )\ e^{(-iE_it_i+ip_iz_2+ia_iy_2-iE_it_2)} \qquad (3.\ 2.25\ )$$

$$\langle 0|Ta^{+}_{\lambda_i}(k_i,t_i)\ A_\mu\ \ (\overrightarrow{r_1},t\,|0\rangle=\frac{1}{\sqrt{\mathbb{V}}}\frac{1}{\sqrt{2w_i}}\epsilon_\mu^{(\lambda_i)}e^{(i\omega_it_i-i\omega_it_2+i\overrightarrow{k_i}\cdot\overrightarrow{r_2})}$$

$$=\frac{1}{\sqrt{\mathbb{V}}}\frac{1}{\sqrt{2\omega i}}\epsilon_\mu^{(\lambda_i)}e^{(i\omega_it_i-i\omega_it_2+\,ik_{ix}x_2+ik_{iy}y_2+ik_i\cos\theta_iz_2\,)} \qquad (3.\ 2.\ 26)$$

Using Eqs. (3.2.16),20,24, 25, and 26 we get:

$$S_{fi} = \lim_{t_{i\to-\infty},t_{f\to\infty}} e^2 \int d^4x_1 d^4x_2 \frac{1}{L}\overline{U_0}(x_1,p_f)\, e^{-i(E_f t_f - E_f t_1) - i(p_f z_1 + a_f y_1)}$$

$$\times\ \gamma_\mu \langle 0|T\ \psi(x_1)\ \bar{\psi}(x_2)\ |0\rangle \gamma_\nu$$

$$\times \frac{1}{\sqrt{\mathbb{V}}}\frac{1}{\sqrt{2\omega_f}}\epsilon_\mu^{(\lambda_f)} e^{(-iw_f t_f\ + i\omega_f t_1 - ik_{fx}x_1 - ik_{fy}y_1 - ik_f \cos\theta_f z_1)}$$

$$\times \frac{1}{\sqrt{V\mathbb{V}}}\frac{1}{\sqrt{2\omega i}}\epsilon_\mu^{(\lambda_i)} e^{\left(i\omega_i t_i - i\omega_i t_2 + ik_{ix}x_2 + ik_{iy}y_2 + ik_i \cos\theta_i z_2\right)}$$

$$\times \frac{1}{\sqrt{\mathbb{V}}}\frac{1}{\sqrt{2wf}}\epsilon_\mu^{(\lambda_f)} e^{\left(-i\omega_f t_f - i\omega_f t_2) - ik_{fx}x_2 - ik_{fy}y_2 - ik_f \cos\theta_f z_2\right)}$$

$$\times (\frac{1}{\sqrt{V\mathbb{V}}}\frac{1}{\sqrt{2wi}}\epsilon_\mu^{(\lambda_i)} e^{(i\omega_i t_i\ - i\omega_i t_1) + ik_{ix}x_1 + ik_{iy}y_1 + ik_i \cos\theta_i z_1)}$$

$$\times \frac{1}{L} U_0(x_2,p_i)\ e^{i(E_i t_i - E_i t_2) + i(p_i z_2 + a_i y_2)} \qquad (3.2\ .27\ )$$

$$S_{fi} = \lim_{t_{i\to-\infty},t_{f\to\infty}} \frac{1}{L^2\mathbb{V}}\frac{1}{\sqrt{4\omega_i\omega_f}} e^2 \int d^4x_1 d^4x_2 \frac{1}{L}\overline{U_0}(x_1,p_f)\,\gamma_\mu$$

$$\times \frac{1}{L^2}\sum_{q_y,q_z} \int \frac{d_{q_0}}{2\pi}\sum_{n=0}^{\infty}\frac{\left(S_n\ (x_1,x_2,q)\right)}{q_0^2 - E_n^2(q_z) + i\epsilon}$$

$$\times\ e^{[iq_0(t_2-t_1) + iq_y(y_1-y_2) + iq_z(z_1-z_2)]}\gamma_\nu$$

$$\times\ \{[e^{[-i(E_f t_f - E_f t_1) - i(p_f z_1 + a_f y_1)]}\epsilon_\mu^{(\lambda_f)}$$

$$\times\ e^{[-i(\omega_f t_f\ - \omega_f t_1\ ) - ik_{fx}x_1 - k_{fy}y_1 - ik_f \cos\theta_f z_1]}$$

$$\times\ \epsilon_\nu^{(\lambda_i)} e^{[i\omega_i t_i - \omega_i t_2) + ik_{ix}x_2 + ik_{iy}y_2 + ik_i \cos\theta_i z_2]]}$$

$$+\ [e^{[-\ iE_f t_f - E_f t_1) - i(p_f z_1 + a_f y_1)]}\epsilon_\nu^{(\lambda_f)}$$

$$\times\ e^{[-i(\omega_f t_f\ - (\omega_f t_2) - ik_{fx}x_2 - ik_{fy}y_2 - ik_f \cos\theta_f z_2]}$$

$$\times\ \epsilon_\mu^{(\lambda_i)} e^{[i(\omega_i t_i - \omega_i t_1) + ik_{ix}x_1 + ik_{iy}y_1 + ik_i \cos\theta_i z_1]]\}}$$

$$\times\ U_0(x_2,p_i) e^{[i(E_i t_i - E_i t_2) + i(p_i z_2 + a_i y_2)]} \qquad (3.2\ .28\ )$$

$$S_{fi} = \frac{(2\pi)^5}{L^2\mathbb{V}} \frac{e^2}{2\sqrt{\omega_i\omega_f}} \sum_{n=0}^{\infty} \frac{1}{L^2} \sum_{q_2,q_3} \int d^4x_1 d^4x_2 \int d_{q_0}\, \overline{U_0}(x_1,p_f)$$

$$\times \left[\frac{\widehat{\epsilon_f}\left(S_n\ (x_1,x_2,q)\right)}{(E_i\ +w_i\ )^2 - E_n^2(q_z) + i\in}\right]$$

$$\times\ \widehat{\epsilon_i}\ \delta(\omega_f + E_f\ \text{-} q_0)\delta\ (q_0\text{-}\omega_i\text{-}E_i\ )$$

$$\times\, \delta(k_{iy} + a_i - q_y)\delta(q_y - k_{fy} - a_f)\delta(q_z - k_f \cos\theta_f - p_f)$$

$$\times\, \delta(k_i \cos\theta_i\ + p_i - q_z)\ exp(-ik_{fx}\ x_1 + ik_{ix}x_2)\ +\ \widehat{\epsilon_i}\ \ \frac{\left(S_n\ (x_1,x_2,q)\right)}{(E_i\ -w_i\ )^2 - E_n^2(q_z) + i\in}$$

$$\times\, \widehat{\epsilon_f}\, \delta(\omega_f\ - E_i\ + q_0)\delta(E_f\ - \omega_i\ \ - q_0)$$

$$\times\, \delta(k_{iy} - a_f\ + q_y)\delta(a_i - k_{fy} -\ q_y)\delta(q_z + k_i \cos\theta_i - p_f\ )$$

$$\times\, \delta(p_i - k_f\ \cos\theta_f\ - q_z)\ exp(k_{ix}\ x_1\ - ik_{fx}x_2)]U_0(x_2,p_i)$$

$$\times\ lim_{t_{i\to-\infty},t_{f\to\infty}}\ exp\ [-i(E_f + \omega_f)t_f\ +\ i(E_i\ -\ \omega_i)t_i] \qquad (3.2\ .\ 29)$$

Where,

$$\hat{e}_f = e_\mu^{(\lambda_f)}\gamma_\mu, \qquad \hat{e}_i = e_{\gamma_\mu}^{(\lambda_i)}. \qquad (3.2\ .30\ )$$

The phase factor at the end of Eq. (3.2.29) can be ignored , since the cross section ~ $\left|S_{fi}\right|^2$ The spinors $\overline{U}_{2,0}\left(x_1,p_f\right), U_0(x_2,p_i)$ can be expressed as:

$$\overline{U}_0\left(x_1,p_f\right) = \left|\frac{E_f+m}{2F_f}\right|^{1/2} I_0(x_1,a_f)\overline{U}_f(p_f) \qquad (3.2\ .31\ )$$

$$U_{2,0}^T(x_2,p_i) = \left|\frac{\mathrm{E_f+m}}{\mathrm{2F_f}}\right|^{1/2} I_0(x_2,a_i)u_i^T(p_i) \qquad (3.2\ .32\ )$$

Where

$$\overline{U}_f\left(p_f\right) = \begin{pmatrix}0 & 1 & 0 & \frac{p_f}{E_f+m}\end{pmatrix} \qquad (3.2\ .33\ )$$

$$U_i^T(p_i) = \begin{pmatrix}0 & 1 & 0 & \frac{-p_i}{E_i+m}\end{pmatrix} \qquad (3.2\ .34\ )$$

$$S_{fi} = \frac{(2\pi)^5}{L^2\mathbb{V}} \frac{e^2}{2\sqrt{\omega_i\omega_f}} \sum_{n=0}^{\infty} \frac{1}{L^2} \sum_{q_2,q_3} [\frac{(E_{f+}+m)(E_i+m)}{4E_iE_f}]^{1/2}$$

$$\times \int d^4x_1\, d^4x_2 \int dq_0 I_0(x_1, a_f) \overline{U}_f(p_f) I_0(x_2, a_i) U_i(p_i)$$

$$\times \{\hat{\epsilon}_f \frac{S_n(x_1,x_2,q)}{(E_i+\omega_i)^2 - E_n^2(q_z)+i\epsilon} \hat{\epsilon}_i \delta(\omega_f + E_f - q_0)\delta(q_0 - \omega_i - E_i)$$

$$\times \delta(k_{iy} + a_i - q_y)\delta(q_y - k_{fy} - a_f)\delta(q_z - k_f \cos\theta_f - p_f)$$

$$\times \delta(k_i \cos\theta_i + p_i - q_z) \exp(-ik_{fx}x_1 + ik_{ix}x_2)$$

$$+ \hat{\epsilon}_i \frac{S_n(x_1, x_2, q)}{(E_i + \omega_i)^2 - E_n^2(q_z) + i\epsilon + i\epsilon}$$

$$\times \hat{\epsilon}_f \delta(\omega_f - E_i + q_0)\delta(E_f - \omega_i - q_0)$$

$$\times \delta(k_{iy} - a_f + q_y)\delta(a_i - k_{fy} - q_y)\delta(q_z + k_i \cos\theta_i - p_f)$$

$$\times \delta(p_i + k_f \cos\theta_f - q_z) \exp(ik_{ix}x_1 - ik_{fx}x_2)\} \qquad (3.2\ .35)$$

Let $I_1 = \int dx_1^4 dx_2^4 e^{(-ik_{fx}x_1)} I_0(x_1, a_f)\, S_n(x_1, x_2, q) e^{(-ik_{ix}x_2)} I_0(x_1, a_i)$ (3.2 . 36)

and

$I_2 = \int dx_1^4 dx_2^4 \exp(-ik_{ix}x_1)\, I_0(x_1, a_f)\, S_n(x_1, x_2, q) e^{(-ik_{fx}x_2)} I_0(x_2, a_i)$ (3.2 . 37)

With the help of the integral:

$\int_{-\infty}^{\infty} d\varepsilon H_n(\varepsilon) e^{[-(\varepsilon-\alpha)^2]} = \sqrt{\pi}\, (2\alpha)^n$, (3.2 .38)

These two integrals give,

$$I_1 = \frac{\left(\lambda^3 k_f^+ k_i^-\right)^{(n-1)}}{2^n n!} A_n \exp[-\frac{\lambda^2}{4}(k_i^2(\sin\theta_i)^2 + k_f^2(\sin\theta_f)^2]$$

$$\times \exp[i\lambda^2\left(a_f k_{fx} + \frac{1}{2}k_{fx}k_{fy}\right) - i\lambda^2\left(a_i k_{ix} + \frac{1}{2}k_{ix}k_{iy}\right)] \qquad (3.2\ .39)$$

Where $k_i^{\pm} = k_{ix} \pm ik_{iy}$ and $k_f^{\pm} = k_{fx} \pm k_{fy}$ (3.2 .40) and A_n is defined by

$$A_n = \begin{pmatrix} 2n(q_0+m) & 0 & -2nq_z & -2nk_i^- \\ 0 & (q_0+m) & -2nk_i^+ & q_z\lambda^2 k_f^+ k_i^- \\ 2nq_z & 2nk_i^- & -2n(q_0-m) & 0 \\ 2nk_i^+ & -q_z\lambda^2 k_f^+ k_i^- & 0 & -(q_0-m)\lambda^2 k_f^+ k_i^- \end{pmatrix} \qquad (3.2\ .41\ )$$

Similarly,

$$I_2 = \int dx_1^4 dx_2^4 \exp(-ik_{ix}x_1)\, I_0(x_1, a_f)\, S_n(x_1, x_2, q) e^{(-ik_{fx}x_2)} I_0(x_2, a_i)$$

$$\Longrightarrow I_2 = \frac{\left(\lambda^3 k_f^+ k_i^-\right)^{(n-1)}}{2^n n!} \mathrm{B_n} \exp\left[-\frac{\lambda^2}{4}(k_i^2(\sin\theta_i)^2 + k_f^2(\sin\theta_f)^2)\right] \qquad (3.2\ .42\ )$$

Where B_n is given by

$$B_n = \begin{pmatrix} 2n(q_0+m) & 0 & -2nq_z & -2nk_f^- \\ 0 & (q_0+m)\lambda^2 k_f^- k_i^+ & -2nk_i^+ & q_z\lambda^2 k_f^- k_i^+ \\ 2nq_z & -2nk_f^- & -2n(q_0-m) & 0 \\ -2nk_i^+ & -q_z\lambda^2 k_f^- k_i^+ & 0 & -(q_0-m)\lambda^2 k_f^- k_i^+ \end{pmatrix} \qquad (3.2.43)$$

Therefore the value of scattering amplitude is,

$$S_{fi} = \frac{(2\pi)^5}{L^2\mathbb{V}} \frac{e^2}{2\sqrt{\omega_i\omega_f}} \sum_{n=0}^{\infty} \frac{1}{L^2} \sum_{q_2,q_3} [\frac{(E_i+m)(E_f+m)}{4E_iE_f}]^{1/2}$$

$$\times \frac{\left(\lambda^3 k_f^+ k_i^-\right)^{(n-1)}}{2^n n!} \exp\left[-\frac{\lambda^2}{4}(k_i^2(\sin\theta_i)^2 + k_f^2(\sin\theta_f)^2\right]$$

$$\times \{\frac{\bar{U}_f\left(p_f\right)\hat{\epsilon}_f A_{n\hat{\epsilon}_i} U_i\ \ (p_i)}{(E_i+w_i)^2 - E_n^2(q_z) + i\epsilon}$$

$$\times \ \exp\left[i\lambda^2\left(a_f k_{fx} + \frac{1}{2}k_{fx}k_{fy}\right) - i\lambda^2\left(a_i k_{ix} + \frac{1}{2}k_{ix}k_{iy}\right)\right]$$

$$\times \int dq_0\ \ \delta\left(\omega_f + E_f - q_0\right)\delta(q_0 - \omega_i - E_i)\delta(k_{iy} + a_i - q_y)$$

$$\times \ \ \delta\left(q_y + k_{fy} - a_f\right)\delta\left(k_f \cos\bar{\theta}_f - p_f\right)\delta(k_i\cos\theta_i - p_i - q_z)$$

$$\times \ \frac{\bar{U}_f(p_f)\hat{\epsilon}_i B_{n\hat{\epsilon}_f} U_i\ \ (p_i)}{(E_i - w_i)^2 - E_n^2(q_z) + i\epsilon}$$

$$\times \ \exp\left[i\lambda^2\left(a_f k_{ix} - \frac{1}{2}k_{ix}k_{iy}\right) + i\lambda^2\left(a_i k_{fx} + \frac{1}{2}k_{fx}k_{fy}\right)\right]$$

$$\times \int dq_0\ \ \delta\left(\omega_f - E_i - q_0\right)\delta\left(E_f - \omega_i - q_0\right)\delta(k_{iy} - a_f + q_y)$$

$$\times\, \delta\left(a_i - k_{fy} - q_y\right)\delta\left(p_i - k_f\cos\theta_f - q_z\right)\delta(q_z + k_i\cos\theta_i - p_f)\}$$

(3.2 .44)

$$\Longrightarrow S_{fi} = \frac{(2\pi)^5}{L^2\mathbb{V}}\frac{e^2}{2\sqrt{\omega_i\omega_f}}\sum_{n=0}^{\infty}\frac{1}{L^2}\sum\ \ [\frac{(E_i+m)(E_f+m)}{4E_iE_f}]^{1/2}$$

$$\times\ \frac{\left(\lambda^2 k_f^+ k_i^-\right)^{(n-1)}}{2^n n!}\exp\left[-\frac{\lambda^2}{4}(k_i^2(\sin\theta_i)^2 + k_f^2(\sin\theta_f)^2\right]$$

$$\times\ \frac{R_1}{(E_i-\omega_i)^2 - E_n^2(q_z) + i\epsilon}$$

$$\times\ \exp\left[i\lambda^2\left(a_f k_{fx} + \frac{1}{2}k_{fx}k_{fy}\right) - i\lambda^2\left(a_i k_{ix} + \frac{1}{2}k_{ix}k_{iy}\right)\right]$$

$$+\ \frac{R_2}{(E_i-\omega_i)^2 - E_n^2(q_z) + i\epsilon}$$

$$\times\ \exp\left[i\lambda^2\left(a_f k_{ix} - \frac{1}{2}k_{ix}k_{iy}\right) + i\lambda^2\left(a_i k_{fx} + \frac{1}{2}k_{fx}k_{fy}\right)\right]\}$$

$$\times\,\delta\left(\omega_f + E_f - w_i - E_i\right)\delta\left(k_{iy} + a_i - k_{fy} - a_f\right)$$

$$\times\,\delta(k_i\cos\theta_i + p_i - k_f\cos\theta_f - p_f),$$

(3.2 .45)

Where

$$R_1 = \overline{U}_f(p_f)\hat{\epsilon}_f A_n \hat{\epsilon}_i U_i\ \ (p_i)\ \text{ and } R_2 = \overline{U}_f(p_f)\hat{\epsilon}_i B_n \hat{\epsilon}_f U_i\ \ (p_i) \qquad (3.2.46\ )$$

$$R_1 = 2n\left[\frac{(q_0+m)p_i p_f}{(E_f+m)(E_i+m)} + (q_0 - m)\right]$$

$$+\ 2n\left[\frac{p_f}{(E_f+m)} + \frac{p_i}{(E_i+m)}\right](k_i^0 e_f^+ e_{iz} + k_f^+ e_{fz} e_i^- - q_z e_f^+ e_i^-)$$

$$+\ \ \lambda^2 k_f^+ k_i^-\left[\frac{(q_0+m)p_i p_f}{(E_f+m)(E_i+m)} + (q_0 - m) + \frac{q_z p_f}{E_f+m} + \frac{q_z p_i}{E_i+m}\right] e_{fz} e_{iz}$$

(3.2 .47)

$$R_2 = 2n[\frac{(q_0+m)p_i p_f}{(E_f+m)(E_i+m)} + (q_0 - m)]e_i^+ e_f^-$$

$$-2n\left[\frac{p_f}{(E_f+m)} + \frac{p_i}{(E_i+m)}\right]((k_i^+ e_f^- e_{iz} + k_f^- e_{fz} e_i^- - q_z e_-^+ e_f^-)$$

$$= \lambda^2 k_f^+ k_i^- [\frac{(q_0+m)p_i p_f}{(E_f+m)(E_i+m)} + (q_0 - m) + \frac{q_z p_f}{E_f+m} + \frac{q_z p_i}{E_i+m}]\, e_{fz} e_{iz}$$

(3.2 .48)

Using these values of R_1 and R_2 ,the scattering amplitude will be:

$$S_{fi} = \frac{(2\pi)^3}{L^2 \mathbb{V}} \frac{e^2}{2\sqrt{\omega_i \omega_f}} [\frac{(E_i+m)(E_f+m)}{4E_i E_f}]^{\frac{1}{2}} exp\, [-\frac{\lambda^2}{4}(\omega_i^2 (sin\, \theta_i)^2 + \omega_f^2 (sin\, \theta_f)^2$$

$$\times\ \exp\left[-i\lambda^2 a_i (k_{ix} - k_{fx}) - i\frac{\lambda^2}{2}(k_{ix}k_{iy} + k_{fx}k_{fy})\right] F^*$$

$$\times\ \delta(E_i + \omega_i - E_f - w_f)\delta(p_i + k_i \cos\theta_i - p_f - k_f \cos\theta_f)$$

$$\times\ \delta(a_i + k_{if} - a_f - k_{fy}) \qquad (3.2\ 49)$$

Where w_i and w_f are frequencies of the incident and scattered photons,E_i and E_f represent the energies of the incident and scattered electrons, respectively.θ_i and θ_f denotes the angle between the incoming (outgoing) photons and the magnetic field. X is expressed as

$$F^* = \sum_{n=0}^{\infty} \frac{1}{n!} [\left(\frac{\lambda^2 k_i^- k_f^+}{2}\right)^n \exp\,(i\lambda^2 k_{iy} k_{fx})(\frac{F_1^*}{(E_i + W_i)^2 - E_{i,n+1}^2} + \frac{F_2^*}{(E_i + W_i)^2 - E_{i,n}^2})$$

$$+\left(\frac{\lambda^2 k_i^- k_f^+}{2}\right)^n \exp\,(i\lambda^2 k_{ix} k_{fy})(\frac{F_1^{*\prime}}{(E_i - W_f)^2 - E_{i,n+1}^2} + \frac{F_2^{*\prime}}{(E_i - w_f)^2 - E_{f,n}^2})]$$

(3.2 .50)

Where $k_i^{\pm} = k_{ix} \pm ik_{iy}$, $k_f^{\pm} = k_{fx} \pm ik_{fy}$,

$$E_{i,n}^2 = m^2 + (p_i + w_i \cos\theta_i)^2\, 2neB \tag{3.2 .51 }$$

$$E_{f,n}^2 = m^2 + (p_i - w_f \cos\theta_f)^2\, 2neB \tag{3.2 .52 }$$

In which ,

$$F_1^* = \left[\frac{(E_i + \omega_i + m)p_i(p_i + \omega_i \cos\theta_i - \omega_f \cos\theta_f)}{(\mathrm{E_i} + \mathrm{m})(\mathrm{E_f} + \mathrm{m})} + (E_i + \omega_i - m)\right] e_i^- e_f^+$$

$$+ \left(\frac{p_i}{(E_i+m)} \frac{(p_i+\omega_i \cos\theta_i - \omega_f \cos\theta_f)}{(E_f+m)}\right)$$

$$\times \left[k_i^+ e_{fz} e_i^- + k_i^- e_{iz} e_f^+ - (p_i + w_i \cos)\, e_f^+ e_i^-\right] \tag{3.2 .53 }$$

$$\mathrm{F}_2^* = \left[\frac{(E_i+\omega_i+m)p_i(p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f)}{(E_f+m)(E_i+m)}\right.$$

$$+ \left(\frac{p_i}{(E_i+m)} \frac{(p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f)}{(E_f+\mathrm{m})}\right)$$

$$\times (p_i + \omega_i \cos\theta_i) + (E_i + \omega_i - m)\Big] e_{fz} e_{iz} \tag{3.2 .54 }$$

$$F_1^{*\prime} = \left[\frac{(E_i - \omega_f + m)p_i(p_i + \omega_i\cos\theta_i - \omega_f\cos\theta_f)}{(E_i + m)(E_f + m)} + (E_i - \omega_i - m)\right] e_i^+ e_f^-$$

$$- \left(\frac{p_i}{(E_i+m)} + \frac{(p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f)}{(\mathrm{E_f}+\mathrm{m})}\right)$$

$$\times \left[k_i^+ e_{iz} e_f^- + k_f^- e_{fz} e_i^+ + (p_i - w_f\cos\theta_f)\, e_i^+ e_f^-\right. \tag{3.2 .55 }$$

$$F_2^{*\prime} = \left[\frac{(E_i - \omega_f + m)p_i(p_i + \omega_i\cos\theta_i - \omega_f\cos\theta_f)}{(\mathrm{E_f} + \mathrm{m})(\mathrm{E_i} + \mathrm{m})} + (E_i - w_f - m)\right.$$

$$+ \left(\frac{p_i}{(E_i+m)} + \frac{(p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f)}{(\mathrm{E_f}+\mathrm{m})}\right)(p_i - \omega_f\cos\theta_f)\Big] e_{fz} e_{iz} \tag{3.2 .56 }$$

Where e_i and e_i are polarizations of the incident and scattered photons and

$$e_i^{\pm} = e_{ix} \pm i e_{iy} \tag{3.2 .57}$$

$$e_f^{\pm} = e_{fx} \pm i e_{fy} \tag{3.2 .58}$$

The conservation of energy and momentum along the z direction, i.e.

$$E_f = E_i + \omega_i - \omega_f \tag{3.2 .59}$$

And

$$(p_f + k_f \cos\theta_f = p_i + k_i \cos\theta_i) \tag{3. 2. 60}$$

leads to

$$w_f = \frac{1}{\sin^2\theta_f}\{E_i - p_i \cos\theta_f + w_i(1 - \cos\theta_i \cos\theta_f) - [(E_i - p_i \cos\theta_f)^2$$

$$+ 2\omega_i(E_i \cos\theta_f - p_i)(\cos\theta_f + \cos\theta_i) + \omega_i^2(\cos\theta_f - \cos\theta_i)^2]$$

(3.2 .61)

Let $F = F_1 + F_2$, which is given by

$$F_1 = \sum_{n=0}^{\infty} \frac{1}{n!}(\frac{\lambda^2 \omega_i \omega_f \sin\theta_i \sin\theta_f}{2} e^{-i(\emptyset_i - \emptyset_f)})^n \exp\,(i\lambda^2 \omega_i \omega_f \sin\theta_i \sin\theta_f \cos\emptyset_f \sin\emptyset_i)$$

$$\times [\frac{[A - B(p_i + \omega_i \cos\theta_i)]\; e_f^{+} e_i^{-} + B(k_f^{+} e_{fz} e_i^{-} + k_i^{-} e_{iz} e_f^{+}]}{(E_i + \omega_i)^2 - E_{i,n+1}^2}$$

$$+ \frac{[A + B(p_i + \omega_i \cos\theta_i)]\; e_{fz} e_{iz}}{(E_i + \omega_i)^2 - E_{i,n}^2}$$

(3.2 .62)

$$F_2 = \sum_{n=0}^{\infty} \frac{1}{n!} (\frac{\lambda^2 \omega_i \omega_f \sin\theta_i \sin\theta_f}{2} e^{-i(\emptyset_i - \emptyset_f)})^n \exp\,(i\lambda^2 \omega_i \omega_f \sin\theta_i \sin\theta_f \cos\emptyset_f \sin\emptyset_i)$$

$$\times\, [\frac{[A' - B(p_i - \omega_f \cos\theta_f)]\ \ e_f^- e_i^+ - B(k_f^- e_{fz} e_i^+ + k_i^+ e_{iz} e_f^-}{\left(E_i - \omega_f\right)^2 - E_{f,n+1}^2}]$$

$$+ [\frac{[A' + B(p_i - w_f \cos\theta_f)]\ \ e_{fz} e_{iz}}{\left(E_i - w_f\right)^2 - E_{f,n}^2}] \qquad (3.2\ .63\ )$$

Where the coefficients A, A′, B are defined by

$$A\ = (E_i + \omega_i - m) p_i p_f + (E_i + \omega_i - m)\left(E_f + m\right)(E_i + m) \qquad (3.2\ .64\ )$$

$$A' = (E_i + \omega_i - m) p_i p_i + (E_i + \omega_i - m)(E_i + m)(E_f + m) \qquad (3.2\ .\ 65)$$

$$B = p_i\left(E_f + m\right) + p_f(E_i + m) \qquad (3.2\ .66\ )$$

This gives the following,

$$|S_{fi}|^2 = \frac{(2\pi)^6}{L^4 \mathbb{V}^2} \frac{e^4}{4\omega_i \omega_f} [\frac{(E_i + m)\left(E_f + m\right)}{4E_i E_f}] \exp\,[\frac{-\lambda^2}{2} (\omega_i^2 \sin^2\theta_i + \omega_f^2 \sin^2\theta_f)]$$

$$\times\ exp[-i2\lambda^2 a_i\left(k_{ix} - k_{fx}\right) - i\lambda^2\left(k_{ix} k_{iy} + k_{fx} k_{fy}\right)] |F|^2 \frac{1}{(E_f + m)^2 + (E_i + m)^2}$$

$$\times\ \ \delta^2\left(E_i + \omega_i - E_f - \omega_f\right) \delta^2 (p_i + k_i \cos\theta_i - p_f - k_f \cos\theta_f)$$

$$\times\ \ \delta^2 (a_i - k_{iy} - a_f - k_{fy}) \qquad (3.2\ .67\ )$$

Let the final states of the scattered photon and electron be

$$\frac{\mathbb{V}d^3\omega_f}{(2\pi)^3} \text{ and } \frac{\mathbb{V}d^3p_f}{(2\pi)^3} \qquad (3.2.\ 68)$$

where $\frac{\mathbb{V}d^3\omega_f}{(2\pi)^3} = \frac{\mathbb{V}\omega_f{}^2 d\Omega_f}{(2\pi)^3}$

Using $e^4 = 16\pi^2 m^2 r_0^2$ (3.2. 69)

$$2\pi\delta^2\left(E_i + \omega_i - E_f - \omega_f\right) = T \qquad (3.2.70\ )$$

$$2\pi\delta^2\left(p_i + k_i\cos\theta_i - p_f - k_f\cos\theta_f\right) = L \qquad (3.2.71\ )$$

$$\delta(0) = 1 \qquad (3.2.\ 72)$$

We get,

$$\Longrightarrow \frac{|S_{fi}|^2}{T} = \frac{(2\pi)^6}{L^4\mathbb{V}^2}\frac{16\pi^2m^2r_0^2}{4\omega_i\omega_f}[\frac{1}{4E_iE_f}]\exp\left[-\frac{\lambda^2}{2}(\omega_i^2\sin^2\theta_i + \omega_f^2\sin^{-1}\theta_f)\right]$$

$$\times\ exp[-i2\lambda^2 a_i\left(k_{ix} - k_{fx}\right) - i\lambda^2\left(k_{ix}k_{iy} + k_{fx}k_{fy}\right)]|F|^2$$

$$\times\ \frac{1}{(E_f+m)(E_i+m)}\frac{1}{\mathrm{T}}\frac{\mathrm{T}}{2\pi}\frac{\mathrm{L}}{2\pi}\ \delta^2(a_i - k_{iy} - a_f - k_{fy}) \qquad (3.2\ .\ 73)$$

And finally dividing by the relative incident flux density,

$$\frac{(E_i - p_i\cos\theta_i)}{\mathbb{V}E_i} \qquad (3.2.74\ )$$

$$\frac{|S_{fi}|^2}{T}\frac{\mathbb{V}E_i}{(E_i - p_i\cos\theta_i)} = \frac{(2\pi)^6}{L^3\mathbb{V}^2}\frac{16\pi^2m^2r_0^2}{4\omega_i\omega_f}[\frac{1}{4E_iE_f}]\exp\left[-\frac{\lambda^2}{2}(\omega_i^2\sin^2\theta_i + \omega_f^2\sin^2\theta_f)\right]$$

$$\times\ exp[-i2\lambda^2 a_i\left(k_{ix} - k_{fx}\right) - i\lambda^2\left(k_{ix}k_{iy} + k_{fx}k_{fy}\right)]|F|^2\frac{\mathbb{V}E_i}{(E_i - p_i\cos\theta_i)}$$

$$\times\frac{1}{(E_f+m)(E_i+m)}\ \delta^2(a_i - k_{iy} - a_f - k_{fy}) \qquad (3.2.\ 75)$$

3.2.3 Total differential cross section of inverse Compton scattering corrected for spin sum

Summing over the final states of the scattered photon and electron, we obtained the rate of scattering probability. Then if we divide by the relative incident flux density we find

$$d\sigma = \frac{(2\pi)^6}{L^4\mathbb{V}^2}\frac{\pi^2 m^2 r_0^2}{16\omega_i\omega_f}[\frac{1}{E_i E_f}]\exp[-\frac{\lambda^2}{2}(\omega_i^2\sin^2\theta_i + \omega_f^2\sin^2\theta_f)]$$

$$\times\ exp[-i2\lambda^2 a_i(k_{ix}-k_{fx}) - i\lambda^2(k_{ix}k_{iy}+k_{fx}k_{fy})]$$

$$\times\ \frac{|F|^2}{(E_f+m)(E_i+m)}\frac{1}{(E_i-p_i\cos\theta_i)}\frac{\mathbb{V}\omega_f{}^2 d\Omega_f}{(2\pi)^3}\frac{\mathbb{V}d^3p_f}{(2\pi)^3}\delta^2(a_i-k_{iy}-a_f-k_{fy})$$

(3.2.76)

$$\frac{d\sigma}{d\Omega_f} = \frac{r_0^2}{4}\frac{\omega_f}{\omega_i}\frac{m^2}{(E_f+m)(E_i+m)(E_i-p_i\cos\theta_i)}$$

$$\times\ \exp[-\frac{\lambda^2}{2}(\omega_i^2(\sin\theta_i)^2+\omega_f^2(\sin\theta_f)^2]|F|^2$$

$$\times\ \{\frac{1}{E_f}exp[-i2\lambda^2 a_i(k_{ix}-k_{fx}) - i\lambda^2(k_{ix}k_{iy}+k_{fx}k_{fy})]$$

$$\times\ \delta^2(a_i-k_{iy}-a_f-k_{fy})d^3p_f \qquad (3.2.\ 77)$$

But $\{\frac{1}{E_f}exp[-i2\lambda^2 a_i(k_{ix}-k_{fx}) - i\lambda^2(k_{ix}k_{iy}+k_{fx}k_{fy})]$

$$\times\,\delta^2(a_i-k_{iy}-a_f-k_{fy})d^3p_f\} \qquad = \frac{1}{[E_i-(p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f)\cos\theta_f]}$$

(3.2.78)

Let $\quad k_i = \omega_i(\sin\theta_i\cos\emptyset_i, \sin\theta_i\sin\emptyset_i, \cos\theta_i)$ and (3.2.79)

$$k_f = \omega_f(\sin\theta_f\cos\emptyset_f, \sin\theta_f\sin\emptyset_f, \cos\emptyset_f) \qquad (3.2.\ 80)$$

Using $k_i^- k_f^+ = (k_{ix}-k_{fx})(k_{fx}+ik_{fy})$ (3.2. 81)

$$\Longrightarrow\left(\frac{\lambda^2}{2}k_i^- k_f^+\right)^n = [\frac{\lambda^2}{2}\omega_i\omega_f \sin\theta_i \sin\theta_f e^{-i(\emptyset_i-\emptyset_f)}]^n \tag{3.2.82 }$$

Therefore

$$\frac{d\sigma}{d\Omega_f} = \frac{r_0^2}{4}\frac{\omega_f}{w_i}\frac{m^2}{(E_f+m)(E_i+m)}$$
$$\times [\frac{\exp[-\frac{\lambda^2}{2}(\omega_i^2(\sin\theta_i)^2+\omega_f^2(\sin\theta_f)^2]|F|^2}{(E_i-p_i\cos\theta_i)[E_f-(p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f)\cos\theta_f]}] \tag{3.2 .83 }$$

We can also rearrange the other expressions as follows

$$E_i + m = m\left(\frac{E_i}{m}\right) + 1 = m(\gamma+1) \tag{3.2.84 }$$

$$E_f + m = E_i + \omega_i - \omega_f + m$$

$$= m(\frac{E_i}{m}+\frac{\omega_i}{m}-\frac{\omega_f}{m}+1)$$

$$= m\left(1+\gamma+\frac{\omega_i}{m}-\frac{\omega_f}{m}\right) \tag{3.2. 85}$$

From $\lambda^{-1} = \sqrt{eB}$, $\lambda^2 = \frac{1}{eB}$

$\Longrightarrow\lambda^2\omega_i^2 = \frac{\omega_i^2}{eB}$

But from $\mathrm{B_c} = \frac{\mathrm{m}^2}{\mathrm{e}}, e = \frac{m^2}{B_c}$

This gives $\lambda^2\omega_i^2 = \frac{\omega_i^2}{(m^2/B_c)B} = \frac{B_c}{B}\frac{\omega_i^2}{m^2}$ (3.2.86)

Similarly $\lambda^2\omega_f^2 = \frac{B_c}{B}\frac{\omega_f^2}{m^2}$ (3.2. 87)

$$(E_i - p_i \cos\theta_i) = E_i(1 - \frac{P_i}{E_i}\cos\theta_i = E_i(1 - \beta\cos\theta_i) = m\gamma(1 - \beta\cos\theta_i) \qquad (3.2.88\ )$$

$$[E_f - (p_i + w_i\cos\theta_i - w_f\cos\theta_f)\cos\theta_f]$$

$$= m[\gamma(1 - \beta\cos\theta_f) + \ \frac{\omega_i}{m} \quad (1 - \cos\theta_i\cos\theta_f) - \frac{\omega_f}{m}\sin^2\theta_f]\ (\text{App. B}) \qquad (3.2.89\ )$$

Let $\omega_0 = \frac{eB}{m}$ is the *cyclotron* frequency. summing over the polarizations of incident photons and over scattered photons, the total differential cross section is obtained:

$$\frac{d\sigma}{d\Omega_f} = \frac{r_0^2}{16}[\frac{\frac{\omega_f}{m}}{\gamma(1 - \beta\cos\theta_i)\frac{\omega_i}{m}(1 + \gamma)\left(1 + \gamma + \frac{\omega_i}{m} - \frac{\omega_f}{m}\right)}]$$

$$\times [\frac{\exp[-\frac{B_c}{2B}(\frac{\omega_i^2}{m^2}\ (\sin\theta_i)^2) + \frac{\omega_f^2}{m^2}(\sin\theta_f)^2]|F'|^2}{[\gamma(1 - \beta\cos\theta_f) + \frac{\omega_i}{m}(1 - \cos\theta_i\cos\theta_f - \frac{\omega_f}{m}\sin^2\theta_f)]}] \qquad (3.2\ .90\ )$$

In which $B_c = \frac{m^2}{e} \approx 4.414 \times 10^9 T \approx 4.414 \times 10^{13} G$, is the critical magnetic field .

$|F'|^2$ is given by

$$|F'|^2 = |F'|(1_i \to 1_f)|^2 + |F'|(1_i \to 2_f)|^2 + |F'|(2_i \to 1_f)|^2 + |F'|(2_i \to 2_f)|^2 \qquad (3.2\ .\ 91)$$

where

$\lambda_i \to \lambda_f, \lambda_{i(f)} = 1_{i(f)}, 2_{i(f)}$, represents the scattering of a photon from the polarization λ_i to λ_f .

To simplify Eq. (3.2.90) further we have to find each values of Eq. (3.2. 91).

$$\frac{1}{(E_i + \omega_i)^2 - E_{i,n+1}^2} = \frac{1}{m^2} G_{i,n+1}\ [\text{App. C}] \qquad (3.2.92\ )$$

Where $E_{i,n+1}^2 = m^2 + (P_i + \omega_i \cos\theta_i)^2 + 2(n+1)eB$ (3.2.93)

Similarly,

$$\frac{1}{(E_i+\omega_f)^2 - E_{f,n+1}^2} = G_{f,n+1} \quad \text{and} \qquad (3.2.94\ )$$

$$\frac{\lambda^2 \omega_i \omega_f \sin\theta_i \sin\theta_f}{2} = \frac{B_c}{B} \Delta_i \Delta_f \sin\theta_i \sin\theta_f = \zeta \qquad (3.2.95\ )$$

$$e^{i(\emptyset_i - \emptyset_f)} = e^{-i\emptyset}$$

$$e^{[i\lambda^2 \omega_i \omega_f \sin\theta_i \sin\theta_f \sin\emptyset_f \cos\emptyset_i]} = e^{\frac{B_c}{B}\Delta_i \Delta_f \sin\theta_i \sin\theta_f \sin\emptyset} = e^{i2\zeta \sin\emptyset} = e^{i\zeta}$$

(3.2.96)

Where $\eta = 2\zeta \sin\emptyset$

To simplify calculations we can choose the coordinate system with $\emptyset_{\mathrm{i}} = 0$ so that $\emptyset_{\mathrm{f}} = \emptyset$.

So that

$$k_i = \omega_i(\sin\theta_i 0, \cos\theta_i) \qquad (3.2\ .97\ )$$

$$k_f = \omega_f(\sin\theta_f \cos\emptyset_f, \sin\theta_f \sin\emptyset, \cos\emptyset) \qquad (3.2\ .98\ )$$

Taking into account the fact that photons have two transversal polarizations, the polarizations of the incident and scattered photons can be chosen as

$$e_i^1 = (-\cos\theta_i, \quad 0, \quad \sin\theta_i) \qquad (3.2\ .99\ )$$

$$e_i^2 = (0, \quad -1, \quad 0) \qquad (3.2.100\ )$$

$$e_f^1 = (\cos\theta_f \cos\emptyset \quad , -\cos\theta_f \sin\emptyset \quad , \quad \sin\theta_f) \qquad (3.2.101\ )$$

$$e_f^{(2)} = (\sin\emptyset, \quad -\cos\emptyset, \quad 0) \qquad (3.2.102\ )$$

The above choice is not unique but is convenient for calculations.

For instance, to obtain $F'|(1_{\mathrm{i}} \rightarrow 1_{\mathrm{f}})|$, we can use

$$e_i^1 = (-\cos\theta_i, \quad 0, \quad \sin\theta_i) \qquad \text{and}$$

$$e_f^1 = (\cos\theta_f \cos\emptyset \quad , \qquad -\cos\theta_f \sin\emptyset \quad , \quad \sin\theta_f)$$

Since the polarization is from $1i\ to\ 1f$. So that

$$e_f^+ e_i^- = e_{fx}e_{ix} + e_{fy}e_{iy} + ie_{fy}e_{ix} - ie_{fx}e_{iy}$$

$$= \cos\theta_i \cos\theta_f \cos\emptyset + i\cos\theta_i \cos\theta_f \cos\emptyset \tag{3.2.103 }$$

$$K_f^- e_{fz}e_i^+ + K_i^+ e_{iz}e_f^- = \left(K_{fx}e_{ix} + K_{fy}e_{iy} + iK_{fx}e_{ix} - iK_{fy}e_{ix}\right)e_{fz}$$

$$+(K_{ix}e_{fx} + K_{iy}e_{fy} + iK_{iy}e_{fx} - iK_{ix}e_{fy})e_{iz} \tag{3.2.104 }$$

$$e_{fz}e_{iz} = \sin\theta_i \sin\theta_f \tag{3.2.105 }$$

The rest are written in [App. D]

Therefore $F'(1_i \to 1_f)$ can be written as [App. A].

$$F'(1_i \to 1_f) = [(A_- \cos\theta_f - B_1\cos\theta_i - B_2\cos\theta_f]\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n G_{i,n+1}e^{[i(n+1)\emptyset]}$$

$$-\ [(A'_- \quad \cos\theta_f + B_1\cos\theta_i - B_2\cos\theta_f]\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n G_{i,n+1}e^{-[i(n+1)\emptyset-\eta]}$$

$$+\ \sin\theta_i \sin\theta_f\ [A_+ \textstyle\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n G_{i,n}e^{in\emptyset}\text{-}A'_+ \sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n G_{f,n}e^{-i(n\emptyset-\eta)\emptyset]} \tag{3.2 .106 }$$

$$F'\left(1_i \to 2_f\right) = i(A_- \cos\theta_i - B_2)\textstyle\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n G_{i,n+1}e^{i(n+1)\emptyset}$$

$$+i(A'_- \cos\theta_i + B_2)\textstyle\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n G_{i,n+1}e^{-i(n+1)\emptyset+i\eta} \tag{3.2 . 107}$$

$$F'\left(2_i \to 1_f\right) = -i(A_- \cos\theta_f - B_1)\textstyle\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n G_{i,n+1}e^{i(n+1)\emptyset}$$

$$-\ i(A'_- \cos\theta_f + B_1)\textstyle\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n G_{f,n+1}e^{-i(n+1)\emptyset+i\eta} \tag{3.2 .108}$$

$$F'\left(2_i \to 2_f\right) = A_- \textstyle\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n G_{i,n+1}e^{i(n+1)\emptyset} - A'_- \sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n G_{f,n+1}e^{-i(n+1)\emptyset+i\eta} \tag{3.2.109}$$

In astrophysics we are more interested in the inverse Compton scattering of low energy photon gas with high-energy electrons in a strong magnetic field. When $\Delta_i \ll 1$ the summation of Equ. 106 – 109 converge and the n=0 terms dominate the contribution.

For further simplification, we integrate over the azimuthal angle $\emptyset$ $[App. E]$ and introduce the Bessel functions

$$J_n(-\xi) = \frac{1}{2\pi}\int_0^{2\pi} d\emptyset \cos[n\emptyset - \xi \sin(\emptyset)] \quad n=0,1,2 \; . \qquad (3.2\,.\,110)$$

Then summing up to n=0 we get

$$F'' = C_1 G_{i,1}^2 + C_2 G_{f,1}^2 + \left[\left(A_+ G_{i,0} - A'_+ G_{f,0}\right)^2 + 2\left(1 - J_0(\xi)\right) A_+ A'_+ G_{i,0} G_{f,0}\right] (\sin\theta_i \sin\theta_f)^2$$

$$+ 2[(A_- \cos\theta_i - B_2)(A'_- \cos\theta_i + B_2) + (A_- \cos\theta_f - B_1)$$

$$\times\ (A'_- \cos\theta_f + B_1) - D_1 D_2 - A_- A'_-]$$

$$\times\ J_2(-\xi) G_{i,1} G_{f,1} - 2[D_1 A'_+ G_{i,1} G_{f,0} + D_2 A_+ G_{i,0} G_{f,1}]$$

$$\times\ \sin\theta_i \sin\theta_f J_1(-\xi) \qquad (3.2.111)$$

For $\Delta_i \ll 1$ $J_1(-\xi)$ and $J_2(-\xi)$ are negligible and the above expression can be simplified to

$$F'' = C_1 G_{i,1}^2 + C_2 G_{f,1}^2 + [\left(A_+ G_{i,0} - A'_+ G_{f,0}\right)]^2 (\sin\theta_i \sin\theta_f)^2$$

$$+2(1 - j_0(\xi)) A_+ A'_+ G_{i,0} G_{f,0}](\sin\theta_i \sin\theta_f)^2 \qquad (3.2.112)$$

The differential cross section (3.2.90) can therefore be reduced to

$$d\sigma = \sigma \sin\theta_f \, d\theta_f \qquad (3.2\,.\,113)$$

which means that the total cross section is given as

$$\sigma = \frac{\pi r_0^2}{8} \frac{\omega_f/m}{\Delta_{ir}(1+\gamma)\left(1+\gamma+\omega_i/m - \omega_f/m\right)}$$

$$\times \frac{exp\,[-\frac{B_C}{2B} + \frac{\omega_f^2}{m^2}(sin\,\theta_f)^2]|F''|^2}{[\gamma(1-\beta\, cos\,\theta_f) + \omega_i/m(1 - cos\,\theta_i\, cos\,\theta_f) - \omega_f/m\, sin^2\theta_f)]} \qquad (3.2\,.114)$$

Chapter 4 - Result, Discussion and Conclusion

4.1 Results and Discussion

According to the results obtained in this thesis, if we have charged particles in a magnetic field then their cyclotron orbit will be quantized and the charged particles can occupy orbits with discrete energy levels, called Landau levels given by

$$E_n = m + p + 2neB$$

where n represents the nth Landau level. The solutions are twofold degenerate for S=1 and 2. But for n=0, it is a non-degenerate state and has only one solution for both the positive and negative energies.

In actual calculations when the strength of the magnetic field is high we require to work with n=0 solutions. For lower magnitude of the magnetic field the other Landau levels will start to contribute in the electron energy. In general, for a fixed energy of an electron and for very low magnetic field we will have as many possible landau levels. Charged particles on higher Landau levels have very short life time. Therefore it is reasonable to assume that electrons either before or after the magnetic scattering should be on the ground landau level.

The discussions of this thesis also show that the spin sum is written in compact form as shown below for positive energy solutions (positive spinors) and negative energy solutions (negative spinors) respectively. This spin-sum is highly important in astrophysical calculations of cross section

$$P_{U,n}(y, y_*, P) = \sum_s U_{s,n}(y, P)\bar{U}_{s,n}(y_* P)$$

$$P_{V,n}(y, y_*, P) = \sum_s V_{s,n}(y, P)\bar{V}_{s,n}(y_*, P)$$

for positive energy solutions (positive spinors) and negative energy solutions (negative spinors) respectively. This spin-sum is highly important in astrophysical calculations of cross section and the total cross section we have got is:

$$\sigma = \frac{\pi r_0^2}{8} \frac{\omega_f/m}{\Delta_{ir}(1+\gamma)(1+\gamma+\omega_i/m-\omega_f/m)}$$

$$\times \frac{exp\,[-\frac{B_c}{2B}+\frac{\omega_f^2}{m^2}(sin\,\theta_f)^2]|F\prime\prime|^2}{[\gamma(1-\beta\,cos\,\theta_f)+\omega_i/m(1-cos\,\theta_i\,cos\,\theta_f)-\omega_f/m\,sin^2\,\theta_f)]}$$

This expression also shows the strong dependence of total scattering cross section on the direction of photons and incident electron energy.

4.2 Conclusion

In this thesis we calculated total cross section of the inverse Compton scattering in the presence of uniform and strong magnetic field corrected for spin-sum. The equations show that the solutions of the Dirac equation are dependent on the landau levels and the energy of the electrons is seen to be degenerate except the lowest landau energy level (n=0), which dominates in the case of ICS.

Spin-sum is a corner stone in calculations of scattering cross-section involving charged fermions in the presence of strong magnetic field.

The magnitude of external field will be around B_c. Magnetic fields can enhance scattering cross sections from their vacuum values and on elementary particle interactions it can produce non trivial results which can explain some of the interesting observations in astrophysics.

Inverse Compton scattering is a possible mechanism for producing high energy photons by relativistic electrons. The expression found also shows that total cross section depends on the direction of photon and initial energy of the electron.

Finally, we can say that the main application of the properties of inverse Compton scattering in a strong magnetic field is that in a study of pulsar radiation it can have some important influence on the theoretical model of neutron stars.

Appendices

Appendix A

Let us first define reduced quantities,

$\Delta_i = \frac{\omega_i}{m}$, $\Delta_f = \frac{\omega_i}{m}$, $\Delta_0 = \frac{\omega_0}{m}$ and

Reduced Doppler frequencies by

$\Delta_{ir} = \gamma(1 - \beta\, cos\, \theta_i)\Delta_i$ *and* $\Delta_{ir} = \gamma(1 - \beta\, cos\, \theta_i)\Delta_i$

$$F'(1_\mathrm{i} \to 1_\mathrm{f}) = [(A_- \cos\theta_f - B_1 \cos\theta_i - B_2 \cos\theta_f] \sum_{n=0}^{\infty} \frac{1}{n!} \zeta^n G_{i,n+1} e^{[i(n+1)\emptyset]}$$

$$-[(A'_- \quad \cos\theta_f + B_1 \cos\theta_i - B_2 \cos\theta_f] \sum_{n=0}^{\infty} \frac{1}{n!} \zeta^n G_{i,n+1} e^{-[i(n+1)\emptyset - \eta]}$$

$$+ \sin\theta_i \sin\theta_f \, [A_+ \textstyle\sum_{n=0}^{\infty} \frac{1}{n!} \zeta^n G_{i,n} e^{in\emptyset} \text{-} A'_+ \sum_{n=0}^{\infty} \frac{1}{n!} \zeta^n G_{f,n} e^{-i(n\emptyset - \eta)\emptyset]}$$

$$F'(1_i \to 2_f) = i(A_- \cos\theta_i - B_2) \sum_{n=0}^{\infty} \frac{1}{n!} \zeta^n G_{i,n+1} e^{i(n+1)\emptyset}$$

$$+i(A'_- \cos\theta_i + B_2) \textstyle\sum_{n=0}^{\infty} \frac{1}{n!} \zeta^n G_{i,n+1} e^{-i(n+1)\emptyset + i\eta}$$

$$F'(2_i \to 1_f) = -i(A_- \cos\theta_f - B_1) \textstyle\sum_{n=0}^{\infty} \frac{1}{n!} \zeta^n G_{i,n+1} e^{i(n+1)\emptyset}$$

$$-i(A'_- \cos\theta_f + B_1) \textstyle\sum_{n=0}^{\infty} \frac{1}{n!} \zeta^n G_{f,n+1} e^{-i(n+1)\emptyset + i\eta} (3..)$$

$$F'(2_i \to 2_f) = A_- \textstyle\sum_{n=0}^{\infty} \frac{1}{n!} \zeta^n G_{i,n+1} e^{i(n+1)\emptyset} - A'_- \sum_{n=0}^{\infty} \frac{1}{n!} \zeta^n G_{f,n+1} e^{-i(n+1)\emptyset + i\eta}$$

Where

$$A_{\pm} = a \pm b(\beta\gamma + \Delta_i \cos\theta_i)$$

$$A'_{\pm} = a' \pm b(\beta\gamma - \Delta_f \cos\theta_f)$$

$$B_1 = b\Delta_f \sin^2\theta_f$$

$$B_2 = b\Delta_i \sin^2\theta_i$$

In which

$$a = \beta\gamma(1 + \gamma + \Delta_i)(\beta\gamma + \Delta_i \cos\theta_i - \Delta_f \cos\theta_f) + (\gamma - 1 + \Delta_i)(1 + \gamma)(1 + \gamma + \Delta_i - \Delta_f)$$

$$a' = \beta\gamma(1 + \gamma - \Delta_f)(\beta\gamma + \Delta_i \cos\theta_i - \Delta_f \cos\theta_f) + (\gamma - 1 - \Delta_i)(1 + \gamma)(1 + \gamma + \Delta_i - \Delta_f)$$

$$b = \beta\gamma(1 + \gamma + \Delta_i - \Delta_f) + (\beta\gamma + \Delta_i \cos\theta_i - \Delta_f \cos\theta_f)\,(1 + \gamma)$$

$$\eta = \zeta \sin\emptyset$$

$$\zeta = \frac{B_c}{B}\Delta_i\Delta_f \sin\theta_i \sin\theta_f$$

$$G_{i,n} = \frac{1}{2(\Delta_{ir} - n\Delta_0) + \Delta_i^2 \sin^2\theta_i}$$

$$G_{f,n} = \frac{1}{2(\Delta_{fr} + n\Delta_0) - \Delta_f^2 \sin^2\theta_f}$$

Appendix B

$[E_f - (p_i + w_i \cos\theta_i - w_f \cos\theta_f)\cos\theta_f] = E_f - P_i \cos\theta_f + \omega_i \cos\theta_i \, cos\,\theta_f + \omega_f \, cos\,\theta_f^2$

But $\mathrm{E_f} = E_i - \omega_i - \omega_f$ and $cos\,\theta_f^2 = 1 - \sin^2\theta_f$

$$= E_i - \omega_i - \omega_f - P_i \cos\theta_f + \omega_i \cos\theta_i \, cos\,\theta_f + \omega_f \, cos\,\theta_f^2$$

$$= E_i + \omega_i(1 - \cos\theta_i \, cos\,\theta_f) - P_i \cos\theta_f + \omega_f(\, cos\,\theta_f^2 - 1)$$

$$= E_i - P_i \cos\theta_f + \omega_i(1 - \cos\theta_i \cos\theta_f) - \omega_f \sin^2\theta_f$$

$$= E_i(1 - \beta\cos\theta_f) + m\Delta_i(1 - \cos\theta_i \cos\theta_f) - m\Delta_f \sin^2\theta_f$$

$$= m[\gamma(1 - \beta\cos\theta_f) + \ \Delta_i(1 - \cos\theta_i \cos\theta_f) - \Delta_f \sin^2\theta_f]$$

Appendix C

$$\frac{1}{(E_i+\omega_i)^2-E_{i,n+1}^2}=\frac{1}{E_i^2+\omega_i^2+2E_i\omega_i-m^2-P_i^2-\omega_i^2\cos^2\theta_i-2P_i\omega_i\cos\theta_i-2(n+1)eB}$$

Using $E_{i,n+1}^2=m^2+(P_i+\omega_i\cos\theta_i)^2+2(n+1)eB$

$$=\frac{1}{E_i^2+2E_i\omega_i-m^2-P_i^2-2P_i\omega_i\cos\theta_i-2(n+1)eB+\omega_i^2\sin^2\theta_i}$$

$$=\frac{1}{2E_i\omega_i-P_i^2-2P_i\omega_i\cos\theta_i-2neB+\omega_i^2\sin^2\theta_i}$$

$$=\frac{1}{2m\gamma m\Delta_i-2m\gamma\beta m\Delta_i\cos\theta_i-2nm^2\Delta_0+m^2\Delta_i^2\sin^2\theta_i}$$

$$=\frac{1}{m^2[2(\Delta_{ir}-n\Delta_0)+\Delta_i^2\sin^2\theta_i}$$

$$=\frac{1}{m^2}G_{i,n+1}$$

Similarly,

$\frac{1}{(E_i+\omega_f)^2-E_{f,n+1}^2}=G_{f,n+1}$ and

$$\frac{\lambda^2\omega_i\omega_f\sin\theta_i\sin\theta_f}{2}=\frac{B_c}{B}\Delta_i\Delta_f\sin\theta_i\sin\theta_f$$

$$e^{i(\emptyset_i-\emptyset_f)}=e^{-i\emptyset}$$

$$e^{[i\lambda^2\omega_i\omega_f\sin\theta_i\sin\theta_f\sin\emptyset_f\cos\emptyset_i]}=e^{\frac{B_c}{B}\Delta_i\Delta_f\sin\theta_i\sin\theta_f\sin\emptyset}=e^{i2\zeta\sin\emptyset}=e^{i\zeta}$$

Where $\eta=2\zeta\sin\emptyset$

Appendix D

Using

$$e_i^1 = (-\cos\theta_i, \;\; 0, \;\; \sin\theta_i)$$

$$e_f^1 = (-\cos\theta_f \cos\emptyset \;\; , \; -\cos\theta_f \sin\emptyset \;\; , \; \sin\theta_f) \text{ to find } |\;\; \mathrm{Y}(1\mathrm{i}\rightarrow 1f)|$$

$$e_f^- e_i^+ = e_{fx}e_{ix} + e_{fy}e_{iy} - ie_{fy}e_{ix} + ie_{fx}e_{iy} = \cos\theta_i \cos\theta_f \cos\emptyset - i\cos\theta_i \cos\theta_f \cos\emptyset$$

$$K_f^+ e_{fz} e_i^- + K_i^- e_{iz} e_f^+ = \left(K_{fx}e_{ix} + K_{fy}e_{iy} + iK_{fy}e_{ix} - iK_{fx}e_{iy}\right)e_{fz}$$

$$+(K_{ix}e_{fx} + K_{iy}e_{fy} + iK_{ix}e_{fy} - iK_{iy}e_{fx})e_{iz}$$

$$= (-\omega_f \sin\theta_f \cos\emptyset \cos\theta_i - i\omega_f \sin\theta_f \sin\emptyset \cos\theta_i) \sin\theta_f$$

$$+(-\omega_i \sin\theta_i \cos\emptyset \cos\theta_f - i\omega_f \sin\theta_i \sin\emptyset \cos\theta_f) \sin\theta_i$$

Similar procedure is used to find values for $|\;\; \mathrm{Y}(1\mathrm{i}\rightarrow 2f)| \;\; | \;\; \mathrm{Y}(2\mathrm{i}\rightarrow 1f)| \;\; | \;\; \mathrm{Y}(2\mathrm{i}\rightarrow 2f)|$

Appendix E

$$|F''|^2 = C_1 G_{i,1}^2 \int e^{-ti\theta} d\emptyset + C_2 G_{f,1}^2 \int e^{-2i(\emptyset-\eta)} + \sin^2\theta_i \sin^2\theta_f [A_+^2 G_{i,0}^2 + A_+'^2 G_{f,0}^2 \int e^{2i\eta} d\emptyset$$

$$-2A_+ A_+' G_{i,0} G_{f,0} \int e^{-ti\theta} d\emptyset] + 2[(A_- \cos\theta_i - B_2)(A_-' \cos\theta_i + B_2) + (A_- \cos\theta_f - B_1)$$

$$\times (A_-' \cos\theta_f + B_1) - D_1 D_2 - A_- A_-'] G_{i,1} G_{f,1} \int e^{i\emptyset} e^{-i(\emptyset-\eta)} d\emptyset - 2(D_1 A_+' G_{i,1} G_{f,0} \int e^{i(\emptyset+\eta)} d\emptyset$$

$$+ D_2 A_+ G_{\mathrm{i},0} G_{\mathrm{f},1} \int \mathrm{e}^{-\mathrm{i}(\emptyset-\eta)} \sin\theta_{\mathrm{i}} \sin\theta_{\mathrm{f}} + 2(D_1 A_+ G_{\mathrm{i},1} G_{\mathrm{i},0} \int \mathrm{e}^{\mathrm{i}\emptyset} \mathrm{d}\emptyset + D_2 A_+' G_{\mathrm{f},1} G_{\mathrm{f},0} \int \mathrm{e}^{-\mathrm{i}(\emptyset-\eta)}$$

$$\int e^{-ti\theta} d\emptyset = \int e^{-ti\theta} d\emptyset = \int e^{-ti\theta} = 2\pi$$

$$\int e^{-ti\theta} = 2\pi J_0(\varepsilon) = 2\pi \frac{1}{2\pi} \int_0^{2\pi} d\emptyset \cos\eta$$

$$= 2\pi \frac{1}{2\pi} \int_0^{2\pi} d\emptyset \cos(-\zeta \sin\pi) \int e^{i\emptyset} e^{-i(\emptyset-\eta)} d\emptyset$$

$$= 2\pi J_2(-\varepsilon)$$

For $\Delta_{\mathrm{i}} \ll 1, G_{i,1} G_{i,0}$ *and* $G_{f,1} G_{f,o}$ terms can be neglected

Let Q= $[(A_- \cos\theta_i - B_2)(A_-' \cos\theta_i + B_2) + (A_- \cos\theta_f - B_1)$

$$\times (A_-' \cos\theta_f + B_1) - D_1 D_2 - A_- A_-'] G_{i,1} G_{f,1}$$

$$\Longrightarrow |F|^2 = C_1 G_{i,1}^2 + C_2 G_{f,1}^2 + \left[\left(A_+ G_{i,0} - A_+' G_{f,0}\right)^2 + 2\left(1 - J_0(\zeta)\right) A_+ A_+' G_{i,0} G_{f,0}\right] (\sin\theta_i \sin\theta_f)^2$$

$$+2(Q) J_2(\zeta) G_{i,1} G_{f,1} - 2[D_1 A_+' G_{i,1} G_{f,0} + D_2 A_+ G_{i,0} G_{f,1}] \sin\theta_i \sin\theta_f J_1(-\zeta)$$

Then summing up to n=0 we get

$$F'' = C_1 G_{i,1}^2 + C_2 G_{f,1}^2 + \left[\left(A_+ G_{i,0} - A'_+ G_{f,0}\right)^2 + 2\left(1 - J_0(\xi\ \ )\right) A_+ A'_+ G_{i,0} G_{f,0}\right] (\sin\theta_i \sin\theta_f)^2$$

$$+2[(A_- \cos\theta_i - B_2)(A'_- \cos\theta_i + B_2) + (A_- \cos\theta_f - B_1)$$

$$\times (A'_- \cos\theta_f + B_1) - D_1 D_2 - A_- A'_-]$$

$$\times J_2(-\xi\ ) G_{i,1} G_{f,1} - 2[D_1 A'_+ G_{i,1} G_{f,0} + D_2 A_+ G_{i,0} G_{f,1}] \times \sin\theta_i \sin\theta_f J_1(-\xi\ )$$

For $\Delta_i \ll 1$ $J_1(-\xi)$and$J_2(-\xi)$ are negligible and the expression F''can be simplified to

$$F'' = C_1 G_{i,1}^2 + C_2 G_{f,1}^2 + [\left(A_+ G_{i,0} - A'_+ G_{f,0}\right)]^2 (\sin\theta_i \sin\theta_f)^2$$

$$+ 2(1 - j_0(\xi)) A_+ A'_+ G_{i,0} G_{f,0}](\sin\theta_i \sin\theta_f)^2$$

In which

$$D_1 = (A_- \cos\theta_f - B_1)\cos\theta_i - B_2 \cos\theta_f$$

$$D_2 = (A'_- \cos\theta_f + B_1)\cos\theta_i + B_2 \cos\theta_f$$

And

$$C_1 = D_1^2 + (A_- \cos\theta_i - B_2)^2 + (A_- \cos\theta_f - B_1)^2 + A_-^2$$

$$C_2 = D_2^2 + (A'_- \cos\theta_i + B_2)^2 + (A'_- \cos\theta_f + B_1)^2 + A'^2_-$$

For $\Delta_i \ll 1$ $J_1(-\xi)$and$J_2(-\xi)$ are negligible and the expression ____can be simplified to

$$F'' = C_1 G_{i,1}^2 + C_2 G_{f,1}^2 + [\left(A_+ G_{i,0} - A'_+ G_{f,0}\right)]^2 (\sin\theta_i \sin\theta_f)^2$$

$$+2(1 - j_0(\xi)) A_+ A'_+ G_{i,0} G_{f,0}](\sin\theta_i \sin\theta_f)^2$$

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DECLARATION

I hereby declare that this thesis is my original work and has not been presented for a degree in any other university. All sources of material used for the thesis have been duly acknowledged.

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