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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING



Effects of Skewed Supports on Horizontally Curved Box Girder Bridges

A Thesis in Structural Engineering

By

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A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of
Masters of Science

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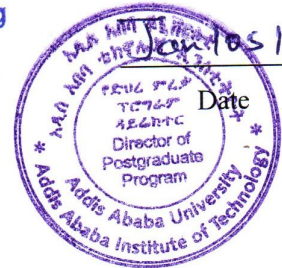
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Declaration

I the undersigned declare that the thesis entitled " **Effects of Skewed Support on Horizontally Curved Bridges**" Submitted to Addis Ababa Institute of Technology School of Civil and Environmental Engineering, is a record of original work and all sources of materials used for this thesis have been duly acknowledged. The results embodied in this document have not been submitted to any other University or Institute for the award of any type of work.

Ermias Gezahegn Taye



ABSTRACT

Reinforced concrete overpass bridges are commonly used for modern highway bridges and large urban interchanges. However, in urban areas, space constraints and alignment requirements often make it difficult to use conventional bridge designs. Bridges with in-plan curvature that rest on skewed supports fall into this category, presenting a complex geometrical arrangement. The ERA bridge design manual and AASHTO LRFD design manual have limitations in addressing the combined effects of curvature and support skewness on bridges. They specify an Angle of Curvature below which neglecting plan curvature in determining the preliminary effects of curved bridges is acceptable, as well as shear correction factors for skewed bridges. To assess the structural response of a bridge when these conditions occur concurrently, a three-dimensional Finite Element Analysis was conducted on a multi-cell cast in situ reinforced concrete box girder bridge. Two hundred bridges were modeled with varying parameters. The analysis showed that the bending moment results of skewed bridges are not significantly affected by the incorporation of in-plan curvature. However, there was a significant (15%) difference between the shear response of the approximation method stipulated in design codes and the outputs of this research. To address this issue, a shear correction and magnification factor is proposed for the outermost girders of skew-curved bridges. Additionally, it was found that the girder at the acute corner of the bridge experienced a downward support reaction force, known as an uplift force. This force is magnified when one-lane traffic loading is applied. Furthermore, it was observed that incorporating skewed supports reduces the live load deflection of curved bridges.

Keywords: Skewed support; Curved Bridges; Reinforced concrete

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Acronyms

AACRA: Addis Ababa City Roads Authority

AASHTO: American Association of State Highway and Transportation Officials

ACI: American Concrete Institute

ERA: Ethiopian Roads Administration

FEM: Finite Element Method

LRFD: Load and resistance factor design

NCHRP: National Cooperative Highway Research Program

Notations

T_{cr} : torsional cracking moment

A_{cp} : total area enclosed by the outside perimeter of the concrete cross-section

P_c : the length of the outside perimeter of the concrete section

f_{pc} : compressive stress in concrete after prestress

f_c' : cylindrical compressive strength of concrete

Φ : resistance factor specified in Article 5.5.4.2 of *AASHTO LRFD 2012*

V_u : equivalent factored shear force

T_u : factored torsional moment

A_0 : area enclosed by the shear flow path

d_s : effective depth of girder

f_{cm} : Crushing Concrete Strength

ϵ_{c1} : Strain at peak stress

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CHAPTER ONE

1. INTRODUCTION

1.1 Background

In urbanized or hilly areas, horizontal alignment issues are more critical. In resolving such issues, the construction of skewed and curved bridges might be mandatory. However, the behavior of such bridges is significantly different from that of straight bridges.

The *AASHTO LRFD Bridge Design Manual* has tried to deal with the above issues independently. It has provided a skew correction factor for straight bridges with skewed supports. It has also adopted some simple guidelines for the use of spine model idealization in bridge analysis. This provides bridge designers an alternative to modeling and analyzing every bridge as a 3D Finite Element Model. In summary, the LRFD Specification states that bridges with an Angle of Curvatures below [12 degrees for closed sections and 34 degrees for cast in situ multi-cell box girders] may be idealized as straight spine model bridges (AASHTO, 2012).

Designing girders and bearings of a skew-curved bridge can be a complex problem. This is because the reaction, shear, and moment demand due to gravity loads will vary across the girders of a skewed bridge deck. Furthermore, it is difficult to predict the response of a curved and skewed bridge with the analysis techniques commonly practiced in bridge design.

Hence, in this thesis, the effect of skewed supports on a horizontally curved bridge is studied in an attempt to know the response of the structure. In addition, the effect of variation in span length, skew angle, and Angle of Curvature on such bridges is studied.

The effect of skewed supports on horizontally curved bridges can be significant given the complex nature of the two parameters. Literature has documented some of the consequences that have appeared due to the curvature of the bridges and ignoring its effect at the design or construction stage.

A recently recorded problem was on a bridge at Wildcat Road in Shasta County, California. This Bridge was a single-span curved Bridge. It was found that Stirrups in the outside web were inadequate to resist the combined effects of torsion and flexural shear. Thus, the web was retrofitted with external prestressing tendons that will correct the problem. This repair was deemed to be preferable to adding extra mild reinforcement within a web overlay (*NCHRP REPORT 620-2008*).

1.2 Statement of the Problem

The increasing rate of urbanization and rapid infrastructure growth has induced the need for complex transportation systems. This has led to road and bridge geometries with unconventional and non-collinear configurations. This is especially true for overpass bridges being built in urban areas. One of these unconventional bridge arrangements is where the end abutments or bents are not perpendicular to the girders of the bridge found on a curved alignment.

The Governing manual has forwarded means to analyze the effects of such geometrical constraints if only they appear independently. i.e., It has provided a skew correction factor for skewed bridges (AASHTO, 2012). It has also forwarded a limitation or demarcation on the Angle of Curvature below which the curvature of the bridge can be ignored in determining the responses of a curved bridge (AASHTO, 2012).

Having the above points in mind the research tends to give an insight into the following questions:

- ↳ What will be the response of a bridge having a pan curvature resting on skewed supports?
- ↳ How will the variation in skew angle, angle of curvature, and span length affect the responses of skew-curved bridges?
- ↳ Will ignoring the curvatures of less than 34 degrees [*for cast in situ multicell girder Bridge*] be acceptable if the bridge has skewed support?

1.3 Objectives of the Study

1.3.1 General Objectives

- ↳ To study the structural response of reinforced concrete curved bridges that rest on skewed supports.

1.3.2 Specific Objectives

- ↳ To study whether ignoring curvatures of less than 34 degrees and adopting the approximation method is an acceptable procedure for analyzing skew-curved bridges.

↳ To study the effects of support skewness on the shear, longitudinal Bending moment, torsional moment, and deflection response of the longitudinal girders of a horizontally curved bridge.

1.4 Scope and Limitations

1.4.1 Scope of Study

A bridge is subjected to various load cases throughout its service life. This includes both static and dynamic loadings. However, for bridges with the above-mentioned geometries, this research work only investigates their structural response to gravity loading. Lateral loads that arise due to an earthquake or wind loads are not considered in this study.

1.4.2 Limitations of the Study

The following are the limitations of this research work.

↳ The study is conducted on a multi-cell cast in situ reinforced concrete box girder bridge. The Bridges are also simply supported [*single-span*] Bridges.

1.5 Methodology

The research work is conducted by implementing the following approaches. The first step is model generation using the finite element analysis program CSI Bridge 24.2. These models have various geometric properties. The span length ranges from 10 to 50 meters. The skew angle varies from 0^0 to 60^0 and Angle of Curvatures of 0^0 to 34^0 have been considered. The width of the bridges is considered as 8.90 meters (two-lane Bridges). Two hundred models were generated Using the parameters presented in Table 1. The Parameters of the study have also been explicated graphically in *Figure 12* of this study.

Table 1: Properties of the FEM Models

Aspect Ratio L/W	Span (L)	Overall Width of the bridge (W)	skew angle of bridge θ (deg)	The Angle of Curvature of bridge α (deg)
1.124 to 5.618	10 to 50m	8.9m	0	0
			15	5
			30	15
			45	25
			60	34

1.6 The Significance of the Study

As the construction industry is a sector that utilizes large capital, every means of uncertainty and gap shall be investigated and assessed. This is to have structures that are both safe and economically feasible. Thus, this paper has a positive impact on designers and interested academicians in forwarding insights on bridge structures with support skewness and in-plan curvature. The paper is also significant in providing a conclusion on whether adopting the directives stipulated in the governing manual is an acceptable procedure for analyzing skew-curved bridges.

1.7 Organization of the Thesis

Chapter 1 of this report explains the motivation for this study. The beginning of this chapter provides a general discussion of the problem being studied. It also contains the objectives along with the methodology, scope, limitations, and organization of the report.

Chapter 2 contains state of an art literature review on skewed bridges, curved bridges, and skew-curved bridges. It has also tried to assess various properties of these bridges.

Chapter 3 In this chapter, the paper deals with the finite element analysis of reinforced concrete box girder bridges. It also contains the model description of the bridge along with the assumptions made, material model properties, and verification models.

Chapter 4 Contains the analysis results of the generated model along with discussions. This chapter has also tried to answer the research questions. Results have been presented in plots and table formats.

Chapter 5 Conclusions and recommendations are given concerning the literature review and outputs of the model analysis.

CHAPTER TWO

2. LITERATURE REVIEW

2.1 Introduction

In urbanized or hilly areas, horizontal alignment issues are more critical. To resolve such issues and to have a smooth passage of congested traffic, the inception and construction of bridges with unconventional geometries are mandatory.

These types of bridges are not exclusive to urban areas and can also be found spanning rivers. The reason for altering their geometries in these cases is to conform to the natural channel of the river and ensure a smooth and unobstructed flow of water.

Unconventional bridge geometries include skewed and curved bridges. Skewed bridges have a support that is not perpendicular to the centerline of the bridge, despite being in a straight alignment. Curved bridges, on the other hand, are aligned horizontally and have supports that are typically perpendicular to the centerline of the bridge.

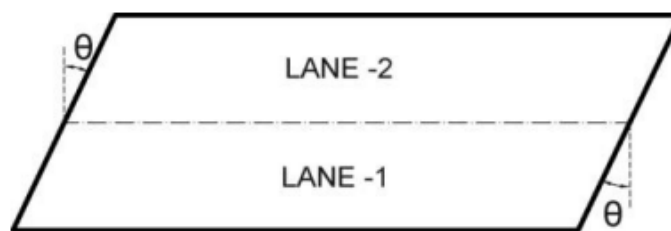


Figure 1: Typical arrangement of Skewed Bridges

A skew-curved geometry of a bridge presents itself when the above two conditions occur at the same time. i.e., curved bridges with skewed supports. These types of bridges are seldom encountered. They usually appear in urbanized areas where overpass bridges are being built. These bridges are a result of the difficulty of finding an acceptable or favorable road alignment in an already established town.

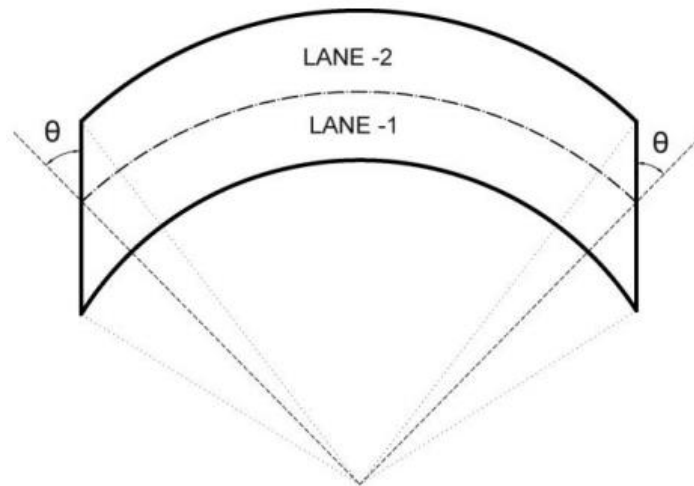


Figure 2: Typical arrangement of Skewed & Curved Bridges

The behavior of such bridges is significantly different from that of straight bridges as several parameters such as skew angle, the radius of curvature, and the cross-section of bridges are involved in the analysis. Given the above parameters; to understand the behavior of skew-curved bridges it is essential to understand the behavior of skewed and curved bridges separately.

2.2 Bridges with Skewed Support

Generally, the angle of skew in non-curved sections is known as the angle between a line perpendicular to the centerline of the bridge and the centerline of the support section (abutment or pier). For bridges curved in plan, the skew angle is defined as the angle between a centerline of support and a line along the radial direction of the bridge curvature.

The task of analyzing and designing bridges is very difficult even for conventional geometry. The designer shall account for the effects of a moving load and all the uncertainties that come along with it. Having this in mind the introduction of some peculiar arrangements and properties like that of skewed supports will further complicate the structural behaviors and their responses and shall be accounted for (*Chun,2010*).

The current practice in analyzing skewed bridges resides in utilizing an empirical approach to account for the effects of skewed supports. That is to say, the loadings will be imposed on the straight counterpart of the skewed bridges and the possible effects of skewed supports will be dealt with through the utilization of factors that can modify the results of straight bridges to determine the longitudinal reinforcement cutoff points and the increase in web reinforcement for shear in the obtuse corners of the bridge.

Finite element analysis was employed in a parametric study conducted on skewed bridges subjected to both concentric and eccentric loading. The influencing parameters were derived from the energy equilibrium condition. The study has concluded that three parameters influence the failure patterns of skewed bridges: (1) The geometry of the bridge such as skew angle, span length, bridge aspect ratio, and bridge continuity. (2) Loading conditions such as truck position and the number of loaded lanes. (3) The structural and material properties of the bridge components. These factors all determine the moment resistance of the bridge in both the longitudinal and transverse directions (*Helba and Kennedy,1995*).

A study was conducted using finite strip and finite element methods on three rectangular sections with four cells each. The sections had skew angles of 30° and 45°, and span lengths of 0.753m and 0.902m. The study compared various structural responses, including deflection, longitudinal bending stresses, and transverse bending moments, for the different skew cases. The deflections were noted at 11 locations and were found to be reduced by about 45% with the increase of skew angle from 30° to 45°. Moreover, it was concluded that results obtained via the finite strip method were as accurate as those obtained from the finite element method and experimental study (*Brown & Ghali,1975*).

To examine the impact of uneven supports, a study was conducted on a 1:2.82 scale model of a 61 m long two-lane, four-cell box-girder bridge in California. The bridge featured a 45° skew bent central support, and the experiment aimed to investigate its effects. Throughout the study, it was seen that the obtuse corner of the subject bridge attracts high reactions and non-negligible end moments and torsional moments. It reflected the idea that in skew bridges, loads tend to take the shortest path to the supports resulting in much higher reactions at the obtuse corner of the bridge (*Bouwkamp,1980*).

“Bridge engineers have recognized several peculiarities in skewed concrete box-girder bridge behavior. Shear cracks have developed in exterior girders near obtuse support corners. Transverse cracks have developed at the midspan of certain highly skewed, prestressed bridge decks. Tendencies toward uplift at acute support corners of long-span bridges have been noted. Although skewness produces analysis and design complications, it also affords design advantages. Since support reactions tend to concentrate at obtuse corners, reductions occur at acute corners. The thick-end diaphragms act to reduce longitudinal stresses in the superstructure. Hence, possibilities exist for material savings by refining structural analysis.” (*Tanmay Gupta & Manoj Kumar,2017*).

The obtuse corner reactions for any symmetrical loading condition will exceed acute corner reactions when a simple span, skewed box-girder superstructure presents itself. This is mainly because loads tend to take the shortest path to the support. Due to this, the reaction at the obtuse corner of a skewed bridge will be much greater than the one found at the acute corner.

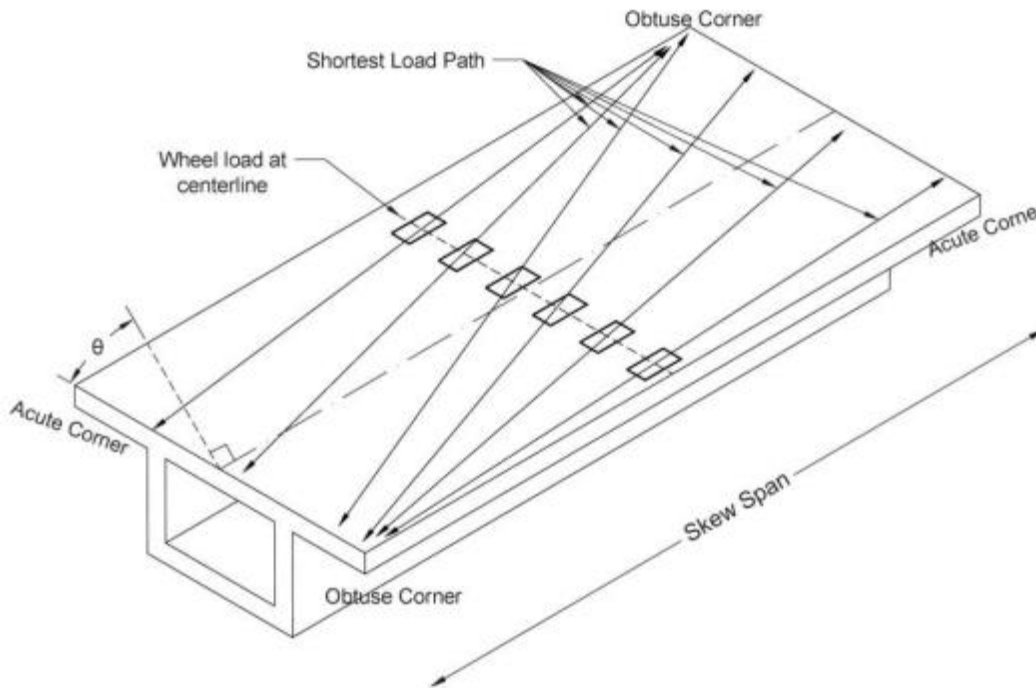


Figure 3: Wheel Load distribution path in skew box-girder bridges (Tanmay Gupta & Manoj Kumar)

In an attempt to study the effects of skewed supports on the responses of bridges, an experiment was done on a 1:8 scale model of 45° skewed prestressed concrete box girder bridges. The results from the experiment showed that the increment of skew angles will decrease the vertical moments and deformations of the bridge. On the other hand, structural responses such as torsional stresses, deformations, and vertical reactions at the obtuse corner have increased. It's also concluded that bridges that have a skew angle of more than 45° are not suitable for railway bridges. *It's also recommended that the use of multiple moving loads traveling in the opposite direction is advantageous in determining the possible uplift scenario that could happen in skewed bridges.* In addition, a parametric study has reflected that the increment of width to span ratio will contribute to the increment of torsional stresses while it has the opposite effect on vertical deformations of the bridge (He & XH, 2012).

The most adopted design practice in the United States of America (USA) and across the world is known as a "whole-width" design concept. This is a procedure in which the moment of inertia of all girders comprising the superstructure is lumped into one unit. This is sometimes

referred to as a spine model. The above procedure, however, leads to a conservative design. This is because the skew itself has the effect of reducing the maximum moment in the structure.

Mark R. Wallace, 1973 tried to demonstrate this effect with a skew deck that is under a concentrated load. Figure 4 shows the general arrangement of the load and geometries of the deck under consideration.

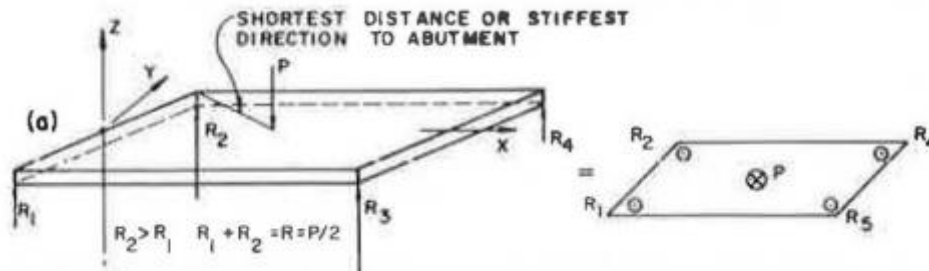


Figure 4:skewed deck under a concentrated load (Wallace)

The true reactions on the left may be described as a normal component (which sums to R) and a component that is skewed or unbalanced. The normal component affects longitudinal behavior exactly as a normal reaction affects a normal structure. The unbalanced component accounts for the portion of the reaction that is due to skew.

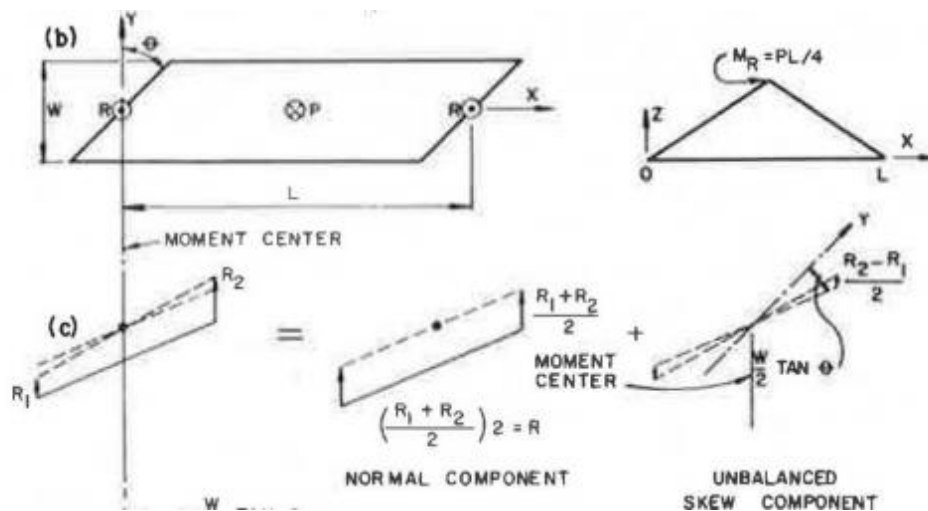


Figure 5:Normal and unbalanced reaction component of skewed Bridges (Wallace)

The unbalanced component of the reaction has the effect of applying a couple to the left support that causes a counterclockwise rotation about the moment center (y-axis in Figures), opposing the clockwise rotation caused by the load. Thus, a resisting couple results from the unbalanced longitudinal distribution of support reaction.

Its magnitude is calculated as shown in the figure below. “A clockwise couple of the same magnitude occurs at the other end of the span. The magnitude of the couple is a function of the reaction difference, structure width, and skew angle. When the preceding two effects are superimposed, a moment is obtained at midspan that is reduced from the moment of the simple span by the magnitude of the couple” (Mark R. Wallace, 1973).

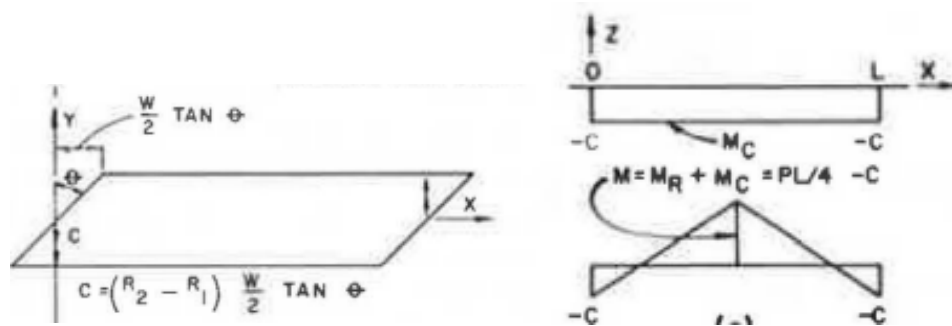


Figure 6: Reduced moment at Midspan of a skewed Bridge

After his studies, Wallace, 1973 has drawn the following important points about the behavior of skewed Box Girder Bridges. The points take into account the complexities of box girder behavior.

1. “The end diaphragm contributes measurably to the stiffness of a skewed structure. As its span lengthens, the support rotation increases. When this lengthening occurs, the end diaphragm provides a good mechanism for longitudinal as well as transverse distribution of reaction, carrying an increasingly greater load to the obtuse corner. Increases in skew angle accentuate this redistribution.”
2. “The unbalanced distribution of support reactions also produces torsional resisting moments at the supports, even under symmetrical loading. If moments are taken about the longitudinal centerline (x-axis) of the structure, a torsional couple is obtained that depends on the reaction unbalance and structure width.”

Experiments were done on a large-scale four-cell reinforced box girder bridge subjected to dead and working load. Various structural behaviors such as reactions, deflections, and moments are determined for skewed, right, and curved bridges. The results show that vertical reactions for skewed bridges vary significantly depending on the transverse positions of the load. That is to say, when the load is placed on the acute side of the bridge the adjacent end reactions are lower than the corresponding right and curved bridge cases. On the other hand,

when the load is placed on the obtuse side of the bridge the reaction of the adjacent support increases drastically than its right and curved counterpart bridges (*Wasti,1984*).

In addition, *Wasti,1984* reflected that the longitudinal bending moment is dependent on the transverse location of the load. When loads are placed on the acute side of the span higher deflections and moments are recorded. On the other hand, moments and deflections are decreased when loads are placed on the obtuse side of the span. It was deduced that for all load cases considered only the skew bridge has yielded end moments and torsional moments.

The Tsing Yi twin-cell box-girder continuous bridge has a 23m long first span and a 27m long second span. It has a carriageway width of 10.58m with a skew angle of 27° . The effect of skewness on the longitudinal bending moment was studied for this bridge. As a result of the finite element analysis, it was observed that the increment of the skew angle will decrease the maximum bending moment. The decrement of the maximum moment is more significant and swifter when skew angles are increased above 30° (*Ashebo,2007*).

In an experimental study on skew box-girder bridges to find out the effect of skewness on warping displacements and vertical reactions. In answering the research questions, it also reaffirmed that the obtuse corner of a skew bridge attracts higher vertical reactions than the acute corners and there also exists the possibility of bearing uplift at acute corners in unsymmetrically loaded skewed bridges (*Soliman,2000*).

In an experimental study on a 45° skewed reinforced concrete bridge model, it was observed that placing point loads on the obtuse side of the deck resulted in lower positive design moments while placing loads on the acute side produced the opposite effect. This response of the bridge was explained in terms of the attraction of negative moments at the end side due to the presence of skewness. It is also preferable that the use of a load distribution factor similar to the right bridge be avoided because the transverse moment is not uniformly distributed in skewed bridges (*Scordelis,1982*).

2.2.1 Diaphragms in Skewed Bridges

The provision of diaphragms in skewed bridges bears a significant advantage in resisting or enhancing the response of the bridge. This is particularly significant regarding the warping effects and hogging moments that may arise due to the geometry and loading condition of these bridges.

The construction of diaphragms poses a difficulty in bridges. This is highly magnified if the bridge has a non-linear geometry like skewed and curved. In general, skewed or orthogonal intermediate diaphragms can be provided for bridges that have a skew angle that is equal to or less than 20° . However, for skew angles greater than 20° the intermediate diaphragms shall be normal to the girders in a staggered formation (*Chen et al 2014*).

In an attempt to determine the best orientation of intermediate diaphragms an experimental test was performed on two-span continuous 45° skewed concrete box girders bridges with a total length of 80 m, width of 12 m, and depth of 2 m. Three diaphragm orientation cases were assessed. The results reflected that a bridge without any diaphragms attracted the highest bending moments and had the lowest torsional rigidity. On the other hand, the bridge with orthogonal diaphragms has attracted the highest obtuse corner support reactions and the lowest acute corner support reaction and provides high torsional rigidity. The alternative of providing no diaphragms has resulted in high torsional moments near the obtuse corner thus demanding an extra thickness of end slabs. This alternative can only be preferred for its ease of construction (*Dilger,1988*).

During a field test carried out on a five-decked bulb Tee Bridge, it was observed that the presence of intermediate diaphragms has a significant influence on the bending moment response of a bridge and also on the live load distribution of the bridge. It is also seen that the internal diaphragms have a lesser or close to no effect on the response of the shear bridge, rather this response of the bridge is highly affected by the location of the live load on the bridge (*Li & Ma,2010*).

A comprehensive research project was carried out using finite element models to analyze the impact of intermediate diaphragm arrangements on live load distribution factors of bending moment and shear force in bridges with a 40° angle. The study revealed that the orientation of concrete intermediate diaphragms parallel to the abutments did not significantly affect load distribution factors for both external and internal girders. However, it was observed that load factors for bending moment decreased by 19% when intermediate diaphragms were perpendicular to longitudinal girders, indicating that this arrangement has the most significant influence in multicell Box bridges. (*Iman Mohseni & Associates,2014*).

(*Iman Mohseni,2014*) has also indicated that in a system that contains a transversal diaphragm that is perpendicular to the girders the spacing between the internal diaphragms has no

significance on the load distribution of the bending moment. On the other hand, the number and spacing of intermediate diaphragms have a considerable effect on the shear distribution factors.

2.2.2 Live Load Distribution in Skewed Bridges

Multi-cell box-girder bridges are widely used for multi-lane highway bridges. During the application of vehicular live loads on the bridge decks, all the boxes experience longitudinal as well as transverse bending moments and shear forces. As an effect, the webs tend to twist along the major axis to maintain the comprehensive stability of deflection at the girder and slab interfaces. The participatory action of a box-shaped cell depends on the span length, spacing of cells, and stiffness of the slabs and webs. The calculation of this participatory effect of the cells in resisting the applied loads is known as the live load distribution factor which is accepted by many bridge design codes. The distribution of live load shear and the moment is crucial in the design of new bridges as well and it is an important parameter considered in the evaluation of the load-carrying capacity of existing bridges.

In a field test conducted on a 45° skewed prestressed concrete box girder Bridge, it was found that the maximum coefficient was recorded near the skew midspan of the bridge irrespective of the moving direction of the vehicle. In comparison with their counter right bridge the moment coefficient in skew bridges was found to be 13% less on the exterior girders and 19% less on the interior girders. Hence, it was concluded that girders of skew bridges that were designed as straight are less stressed than their right bridge counterparts (*Schaffer, 1967*).

Schaffer, 1967 also stated that deflections of skew bridges are relatively smaller than their right counterparts. However, it was observed that when girders have been directly loaded the deflection of skew bridges is higher than the deflections recorded at analogous locations of similar right bridges.

In an attempt to find the effect of skew angle on the load distributions of a bridge, an existing three-span Continuous skew box girder bridge was evaluated. Relative percentage changes in distribution factors were also studied in an analytical model. It's found that the distribution factor change rate remains slow for skew angles up to 30°. However, when higher values of skew angles are considered, this rate becomes much more rapid. In general, it is concluded that bridges having support skewness up to 30° load distribution are not much affected (*Ashebo, 2007*).

A dynamic live load test was done on a prestressed four-celled concrete box girder bridge with a skew angle of 12° . In parallel with the experiment, a finite element analysis was also employed to assess the parameters analytically. The calibrated finite element models were employed to determine the load distribution factors for the bridge. The results from the study indicated that the distribution factors taken from the AASHTO LRFD Specifications were 29% to 46% conservative for an interior girder compared to the results from the finite element analysis. The factors of the specifications are also found to be 2% to 9% un-conservative for the exterior girders (*Hodson,2011*).

Hodson,2011 also conducted static live-load tests on a cast-in-place 8° skew, prestressed concrete bridge, consisting of four box girders which were continuous over two equal spans each measuring 39.35 m & having 12.8 m width. It was seen that the presence of parapets severely affects distribution factors. The study has also reflected that the presence of a diaphragm has little effect on the values of the moment distribution factors. The moment distribution factor values with or without diaphragms have changed by about 1%.

A parametric study conducted by varying the span length and center-to-center spacing of the webs also reflected that the distribution factor for a moment decreased when the span length was increased. On the other hand, an increase in the center-to-center spacing of a bridge will also increase the distribution factors. In addition, it was also observed that the increase in the length of overhang lengths will raise the distribution factor for the exterior girder (*Hodson,2011*).

Hodson,2011 concluded that distribution factors tend to decrease for both interior and exterior girders when the skew angle reaches 15° and higher. The ratio of AASHTO to FEM distribution factors decreased for the interior girder while it increased for the exterior girder.

A study was conducted on multi-cell box girder skewed bridges by applying HS20-44 truck loading. It was found that the current AASHTO LRFD specifications overestimate the moment distribution factor for external girder by 40%. On the other hand, it has underestimated the moment distribution factors for the internal girder by about 25%. The study has also reflected that the live load distribution factor provided by the ASHTO LRFD Manual is highly conservative for exterior girders when the angle of skew varies from 0° to 60° . In contrast, it has underestimated the moment distribution factor for the interior girder when the skew angle reaches above 30° (*Mohseni and Khalim 2014*).

In an attempt to study the effect of skew angle, span length, and the number of lanes on a simply supported reinforced concrete slab bridge a finite element study was conducted on ninety-six models each representing various cases. Each model was subjected to an AASHTO HS20 vehicle load and the results were compared with the AASHTO standard specifications 2002. It was found that the Specifications had overestimated the maximum moment by 20% for a skew angle of 30°, 50% for a skew angle of 40°, and 100% for a skew angle of 50°. Henceforth, for bridges with a skew angle greater than 20°, It's recommended to utilize and conduct a three-dimensional finite element analysis instead of using the AASHTO provisions (*Menassa et al,2007*).

In a Finite Element study conducted on composite girders supporting concrete decks for medium-length bridges, 36 bridges with various span lengths, widths, and skew angles were analyzed. The study aimed to forward an expression for the live load distribution as a function of the parameters mentioned above. The skew angle for the assessed bridges has varied from 0 to 60°. The result of the study reflected that a large skew angle reduces the distribution factor for the moment and the AASHTO Specifications overestimate the moment for bridges having a large skew angle (*Bishara, 1993*).

In a study conducted on prestressed concrete girder bridges, it was determined that generally speaking the interior girders are more affected by the skew nature of the bridge than the exterior girders. It was also reflected that the skewness of a bridge has a minimum effect on bridges having a skew angle of less than 20°. The research has also reflected that for concrete bridges of medium span (23m to 54m) and skew angles less than 30°, the reduction in the distribution factor for interior girders was found to be less than 5%. This reduction was determined to be about 28% for bridges having a skew angle in the range of 30° to 60°. However, the skew effect differs for the exterior girders, the load distribution factor for a 20° skewed bridge was 10% higher than that of the straight bridge (*Barr et al. & Bishara et al. 2001*).

In a study conducted on continuous composite steel girder bridges, it was found that for skew angles between 0° and 30°, there is no real and significant relation between aspect ratio [*Length/width*] and skewness of the bridge that could affect the moment distribution factor. The study has also reaffirmed the fact that the skewness of a bridge has a greater effect on the design of the interior girder than that of the exterior girders. In addition, skewness has a small effect on the moment distribution factor for bridges bearing a skew angle of less than 30°. Empirical formulas for reactions and shear distribution factors at pier support for exterior and

interior girders for continuous composite skew bridges having equal span lengths have been forwarded by the paper (*Ebeido and Kennedy,1996*).

In a study conducted to assess the distribution factors of reactions at piers, it was found that these factors have higher values near the piers. As the skew angle increases the distribution factors for reaction increase faster than the shear distribution factor. This increase was very significant for the interior girder than the exterior girder. In a comparison made between the LRFD specification and the research findings, it was concluded that the LRFD's shear equations and the lever rule method could conservatively predict live load reactions for piers on exterior beams but undermine the live load reaction distribution for the interior girders (*Huo and Zhang,2008*).

2.3 Bridges at Curved Alignment

The bridge with a curved longitudinal layout has different structural behavior from the straight one due to the center of gravity's position being located outside the line connecting the extreme supports, generating an eccentricity about this axis.

The structural response of a curved box girder consists of a combination of longitudinal bending, transverse bending, torsion, and warping. Some of the force effects that are experienced by the curved bridge are peculiar to it. These force effects usually require an advanced analysis procedure to be figured out. The following are some of the force effects seen in curved bridges.

2.3.1 Force Effects in Curved Bridges

2.3.1.1 Longitudinal bending

The self-weight and other superimposed dead loads like the self-weight of the sidewalks and wearing surfaces create a bending moment and don't cause significant torsional moments and warping effects. These are resisted by the simple bending action of the rigid cross-section of the box girder. This longitudinal bending causes flexural stresses, which are considered membrane forces in the plates or slabs (*Alyaa Shatti Mohan,2017*).

2.3.1.2 Torsion

The traffic loads are usually placed asymmetrically along the cross-section of the bridge. This will in turn cause both longitudinal and transversal bending. The vertical reactions P_1 and P_2 at the web positions due to live loads can be resolved into a statically equivalent combination of

a set of symmetrical and asymmetrical loads as shown in the figure below. The symmetrical component of the load $(P_1+P_2)/2$ causes only longitudinal bending and the asymmetrical component $(P_1-P_2)/2$ creates the torsional and distortion effects. The torsional load component gives a resultant torque and it tends to rotate the section as a rigid body about the longitudinal axis of the box. The considerable strength of the box allows little twists (Alyaa Shatti Mohan,2017).

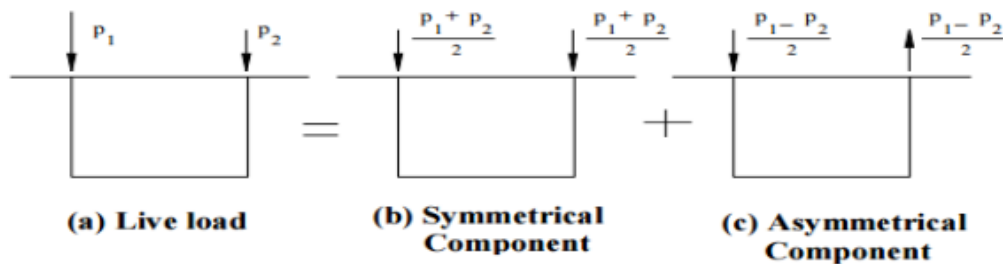


Figure 7:Resolution of asymmetrical live load (<https://theconstructor.org/structures/behaviour-of-box-girder-bridges/2194/>)

The distortional force can also occur when an eccentric loading is applied to a box girder. The distortional force occurs in a box girder when a torsional force, represented by a pair of coupled forces, is not completely balanced with the St. Venant shear flow.

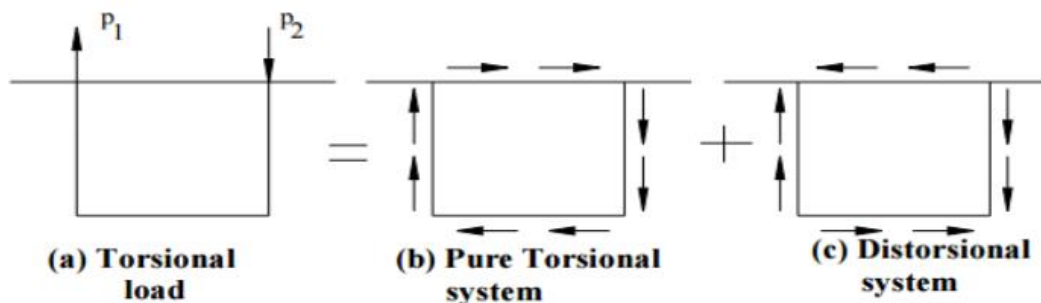


Figure 8:Resolution of Torsional load (<https://theconstructor.org/structures/behaviour-of-box-girder-bridges/2194/>)

Torsion in curved bridges is induced by vertical loads, including those that are symmetrical about the longitudinal axis of the bridge. The flexural tensile and compressive forces acting on a curved bridge element are statically equivalent to a radially directed couple of forces. The couple acts as a torque and must therefore be balanced by a closed shear flow in open sections, it is equilibrated primarily by differential bending of the webs referred to as warping either type of section must deform transversally in flexure to convert the torque into the torsional sectional forces. Although frame action is normally adequate to resist transverse bending in closed

sections, girders with open sections require closely spaced diaphragms which are expensive to build. Box sections are therefore preferred for curved girders in most cases (*Christian Menn, 1927*).

2.3.1.3 Centrifugal Load

The centrifugal force (*Horizontal Moving Load*) has two components, the radial force, and the overturning force. The radial component of the centrifugal force is assumed to be transmitted from the deck through the end cross frames or diaphragms to the bearings and the substructure. The overturning component of centrifugal force occurs because the radial force is applied at a distance above the top of the deck. The overturning component causes the exterior wheel line to apply more than half the weight of the truck and the interior wheel line to apply less than half the weight of the truck by the same amount. Thus, the outside of the bridge is more heavily loaded with a live load. The overturning force is computed by taking the sum of the moments about the inside wheel and setting the sum equal to zero. The result is that the outermost girder will receive a slightly higher load and the innermost girder will receive a slightly lower load.

The AASHTO LRFD manual has reflected the above point in the following statement in the commentary section “*Centrifugal force also causes an overturning effect on the wheel loads because the radial force is applied 1.80m above the top of the deck. Thus, centrifugal force tends to cause an increase in the vertical wheel loads toward the outside of the bridge, and the unloading of the wheel loads toward the inside of the bridge. Superelevation helps to balance the overturning effect due to the centrifugal force and this beneficial effect may be considered. The effects due to vehicle cases with centrifugal force effects included should be compared to the effects due to vehicle cases with no centrifugal force, and the worst case selected.*”

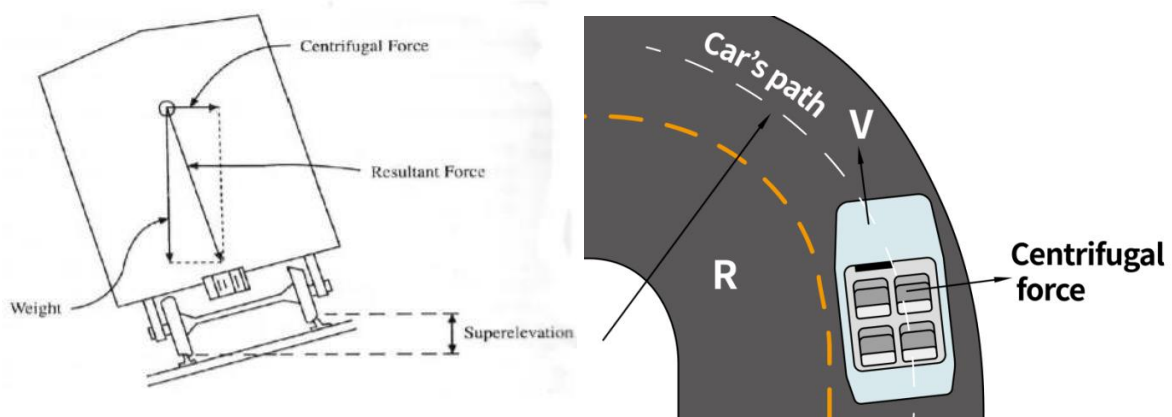


Figure 9: Centrifugal load in a curved bridge (Source - <https://www.midasbridge.com>)

The centrifugal forces shall be taken as the product of the axel weight of the design truck or tandem and the factor C taken as:

$$C=f*\frac{v^2}{g*R} \quad (\text{Equation-1})$$

where;

g = gravitational acceleration,

f = 4/3 for load combinations other than fatigue and 1.0 for fatigue

R = Radius of curvature of a traffic lane

V= highway design speed.

2.3.2 Live Load Distribution in Curved Bridges

An experimental study on three horizontally curved models that are designed and constructed according to AASHTO LRFD standard specifications was conducted. The bridges had a radius of curvature (12.5, 16.7, and 23.3) m and an angle of curvature (10^0 , 13.5^0 , 18^0). The experimental results showed that the AASHTO formula underestimates the Moment distribution factor [MDF] for the exterior girder by about (38%, 47%, and 53%) for curved models with θ (10^0 , 13.5^0 , 18^0) respectively. In contrast, the AASHTO formula overestimates the Moment distribution factor [MDF] with (44%,60,69%) for θ (10^0 , 13.5^0 , 18^0) respectively (*AbdulMutlib I. Said & Hashim Khalaf,2018*).

A Finite element study was conducted to study the live load distributions in curved bridges. The study generated various bridge models with a combination of span lengths and Angles of Curvatures. The result of the study reflected that for the same Angle of Curvature, the live load distribution factors (LLDFs) decrease with an increase in span lengths. In addition, the distribution factor for curved bridges with an Angle of Curvature of 5^0 and less does not vary significantly from the live load distribution factors obtained from the analysis for their counter-straight bridges (*Mohammed Ameen Zaki,2016*).

2.4 Curved Bridges with Skewed Supports

The increasing rate of urbanization usually requires bridge geometries that are very unconventional and unique. One of these unconventional bridge configurations is a curved bridge resting upon skewed support. generally, such bridges are referred to as skew-curved

bridges. In addition, there are no codal guidelines available to specify the impact of combined skew-curve upon bridge response. Moreover, only a handful of studies have been conducted in an attempt to assess the structural behavior of bridges with such geometry, some of which are summarized below.

In an attempt to study the support reactions of skew-curved bridges, over 800 single-span box girder bridges were modeled using three-dimensional finite element analysis software. The study reflected that Aspect ratios [*length/width*] play a major role in the response of skew-curved bridges. A significant increase in the reaction is also observed around the obtuse corner of the Curved Box girder bridges having a small Angle of Curvature. In contrast, there has been a significant decrease in the reaction at the inside corners of the bridge (*Lucas Richard Miner,2014*).

A finite element analysis was employed to better understand the deflection response of skewed curved bridges. The subject of the study was a single cell Box Girder with a span length of 24.4m simply supported bridge. The results of the analysis indicated that as the curvature angle (internal angle) increases the deflection of the bridge also increases. In addition, for constant curvature, the increase in skew angle will decrease the deflections of the girders. The study has also emphasized that for highly curved bridges presence of skewed support is beneficial to control its deflection (*Tanmay Gupta & Manoj Kumar,2018*).

(*Gupta, 2018*) has also extended the study to investigate the influence of skewness and curvature on the location of the critical section for the moment and the associated critical position of live load. Given it is very difficult to determine this point in a Cartesian coordinate system, a new skew-curve coordinate system was proposed by the paper to determine the critical positions more systematically.

2.5 Code Specifications and Current Practices

This section will try to assess the different codal provisions and recommendations given on how to deal with skew-curved bridges. Different Bridge design codes and manuals have been referred to get an insight into the guidelines provided for such bridges, some of the findings are discussed below:

2.5.1 AASHTO LRFD Bridge Design Specification

The American Association of State Highway and Transportation Officials has tried to handle the effects of such geometrical constraints if only they appear independently. i.e., a skew correction factor for straight skewed bridges is stipulated in *Section 4.6.2 of AASHTO LRFD*. It has also forwarded a limitation or demarcation on the Angle of Curvature at which the curvature of the bridge can be ignored. *AASHTO LRFD Section 4.6.1* also stated that bridges with Angle of Curvatures below [*12 degrees for closed sections and 34 degrees for cast in situ multi-cell box girders*] may be idealized as straight-spine model bridges. The manual has not given any clear-cut guidelines on how to deal with skew-curved bridges.

2.5.2 ERA Bridge Design Manual

The Ethiopian Roads Administration Bridge Design Manual 2013, has also adopted the provisions of the AASHTO LRFD specifications. In regards to the curvature of bridges, it has adopted the recommendations of Section 4.6.2 on AASHTO LRFD. The manual has also similar shear correction factors for skewed bridges as the AASHTO LRFD manual.

2.5.3 AACRA Bridge Design Manual

The Addis Ababa City Roads Authority Roads Administration Bridge Design Manual 2004, doesn't explicitly address how to deal with the skew curved bridges. However, section 5.4.3 of the manual, states that a skew angle of less than 20° may be ignored. It also suggests that a curved bridge may be treated as a straight bridge if the radius is greater than 90m.

2.6 Current Design Practice

In a typical bridge design situation, a curved skewed bridge will most likely be analyzed as a spine model. Current bridge design practice relies on the LRFD modification factors to approximate the skew or curved effect. Some of the major approximations that this introduces are:

- ↳ Bridge curves with an Angle of Curvature of less than 12 degrees for closed sections and 34 degrees for cast-in-place multi-cell Box Girder Bridges are ignored.
- ↳ Live load effects for exterior and interior girders are determined by approximating the results of the spine [*with no skewness and curvature*] models using distribution factors.
- ↳ No special considerations are given for the design of bearings and other substructure components for effects that may arise due to these geometries.

CHAPTER THREE

3. FINITE ELEMENT MODELING AND ANALYSIS

In this chapter, the paper deals with the finite element analysis of a multi-cell reinforced concrete box girder bridge. The geometrical and material properties of the modeled bridges are also described in this chapter. The paper has tried to answer the basic research questions with the help of a Finite Element Analysis which is employed using a commercially available package *CSI Bridge V 24.2.0*

3.1 Model Description

As described in the previous sections the type of bridge selected for this study is a multi-cell box girder bridge. In addition to its Aesthetical values; the fact that it could accommodate longer spans and is preferable for overpass/flyover bridges, made Box Girder Bridge the viable choice as the subject of this analysis.

3.2 Geometrical Properties of Model

The following parameters describe the important geometric nature of the bridge.

Span Lengths

The span lengths of the modeled bridges are 10, 20, 30, 40, and 50m. The efficiency of box girders to support longer spans made it possible for having to assess larger spans for this study.

Width of Bridges

The model bridges will have a width that could accommodate two-lane traffic. According to Table 2-1 of the ERA geometric design manual, Width of One traffic lane is 3.65m.

Crush Barrier bottom width = 0.40m (New Jersey Shape (Type-2))

The exterior girders of the bridges will not be sloped. i.e., the models will have a straight exterior girder.

The geometric properties of the bridge models are presented in Table 2.

Table 2: Geometric Properties of Model Bridge

No.	Item	2-Lane Bridge (m)
1	Girder Depth	2.0
2	Web Width	0.35
3	Diaphragm Depth	1.45
4	Diaphragm Width	0.25
5	Girder Spacing	2.40
6	Total Width	8.90
7	Deck Thickness	0.20
8	Bottom Slab Thickness	0.20
9	Superelevation	8%

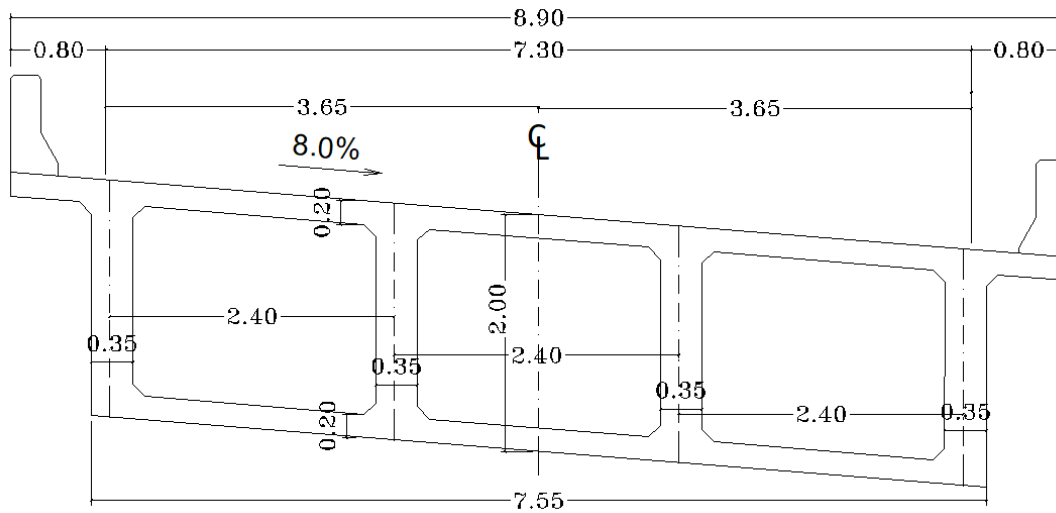


Figure 10: Cross Section of Two-lane bridge

3.3 Model Characteristics and Assumptions

Certain assumptions were made to simplify model development and to supplement the result interpretation of the parametric study.

- The diaphragm is oriented at the same angle as that of the skew angle of the abutment.
- A 5cm thickness of an asphalt layer is assumed as a wearing surface.
- Apart from the span length of the bridge, all other geometrical sections (*transversal*) are kept constant for all the models.
- The bridge is not located at a transition curve. Which implies a constant superelevation value.
- Three diaphragms are used for each model.

- The Bridge is assumed to have no longitudinal slope.
- Girder depth and web width are constant along the span of the bridge.

3.4 Material Properties

The following concrete properties will be assigned to the geometric sections defined for the bridge models.

Material Model for Concrete

Concrete Grade = C20/25

Weight per unit volume = 25.0kN/m³

Modulus of Elasticity = 31MPa

Coefficient of Thermal Expansion = 1.0*10⁻⁵

The property of the concrete material was supplemented for the program with the stress-strain relation definition per ES-EN 1992-11:2015.

$$\frac{\sigma}{f_{cm}} = \frac{k\eta - \eta^2}{1 + (k-2)\eta} \quad (\text{Equation-2})$$

where:

$$\eta = \frac{\epsilon_c}{\epsilon_{c1}}$$

ϵ_{c1} = is the strain at peak stress according to *ES-EN-1992*

$$k = 1.05 E_{cm} * \text{abs}(\epsilon_{c1}) / f_{cm} \quad (f_{cm} \text{ according to } ES-EN-1992)$$

The above expression is valid for $0 < \text{abs}(\epsilon_{c1}) < \text{abs}(\epsilon_{cu1})$ where ϵ_{cu1} is the nominal ultimate strain.

The stress-strain relation for the concrete model is presented in the table below:

- ✓ Crushing Concrete Strength f_{cm} (MPa) = 28
- ✓ Strain at peak stress ϵ_{c1} = 0.002
- ✓ Tensile strength of Concrete (95% fractile) (MPa) = 2.90

Table 3: Concrete Model Input for CSI Bridge

σ_c Stress (MPa)	Total Strain ξ_c (%)	Elastic Strain $\xi_{c, el}$ (%)	Inelastic Strain $\xi_{c, in}$ (%)
0	0	0	0
11.20	0.000411466	0.000373333	3.81328E-05
12.70	0.000478052	0.000423333	5.47189E-05
14.20	0.000548582	0.000473333	7.52485E-05
15.70	0.000623723	0.000523333	0.000100389
17.20	0.000704355	0.000573333	0.000131021
18.70	0.000791676	0.000623333	0.000168343
20.20	0.000887396	0.000673333	0.000214063
21.70	0.000994099	0.000723333	0.000270766
23.20	0.001116046	0.000773333	0.000342712
24.70	0.001261250	0.000823333	0.000437916
26.20	0.001448897	0.000873333	0.000575564
27.70	0.001771208	0.000923333	0.000847875
28.00	0.002000000	0.000933333	0.001066667
26.50	0.002531115	0.000883333	0.001647782
25.00	0.002759201	0.000833333	0.001925867
23.50	0.002937500	0.000783333	0.002154167
22.00	0.003090055	0.000733333	0.002356722
20.50	0.003226175	0.000683333	0.002542842
19.00	0.003350633	0.000633333	0.002717299
17.50	0.003466262	0.000583333	0.002882929
16.00	0.003574909	0.000533333	0.003041575
14.50	0.003677855	0.000483333	0.003194522
13.00	0.003776033	0.000433333	0.003342700
11.50	0.003870148	0.000383333	0.003486815
10.00	0.003960746	0.000333333	0.003627413
8.50	0.004048262	0.000283333	0.003764929
7.00	0.004133048	0.000233333	0.003899714

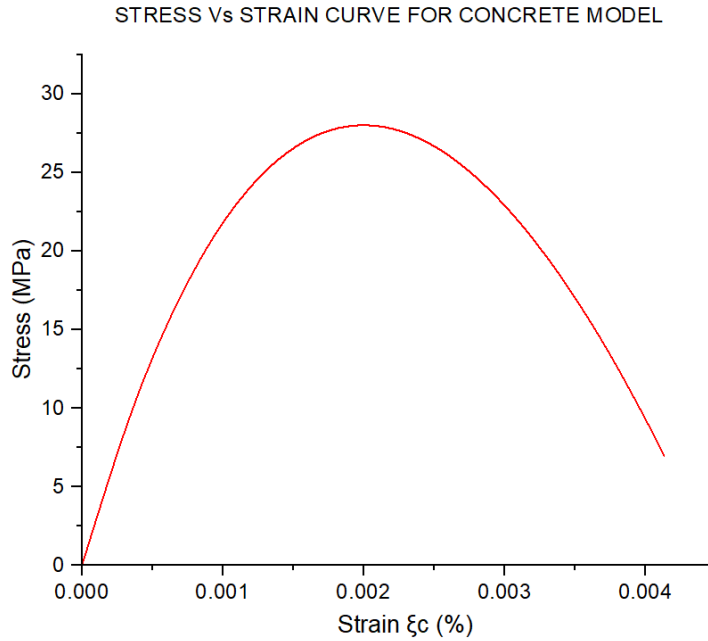


Figure 11: Stress-strain curve for concrete model

3.4 Description of Analysis Parameters

The analyzed models are of various geometric properties. The values of the parameters can be referred to in Table 1. Span of the bridge refers to the length measured at the centerline of the road. The width of the bridge includes the barriers. Figures 12 and 13 illustrate the parameters.

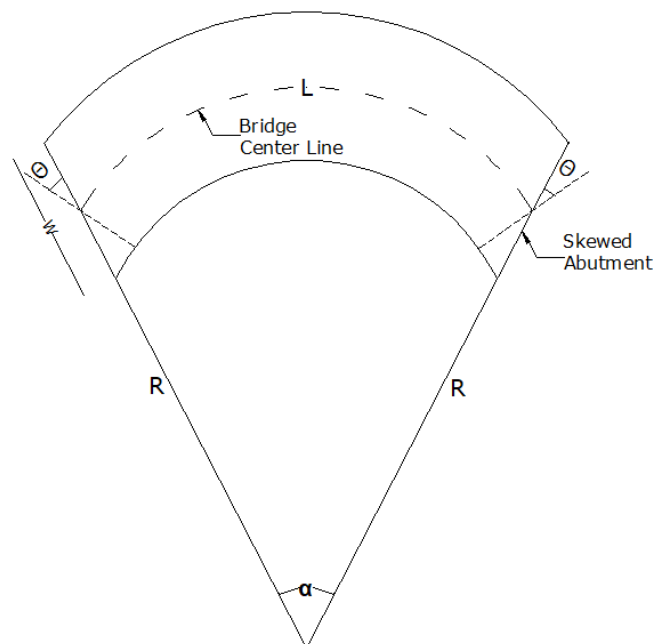


Figure 12: Schematic for Curved Bridge with skewed support

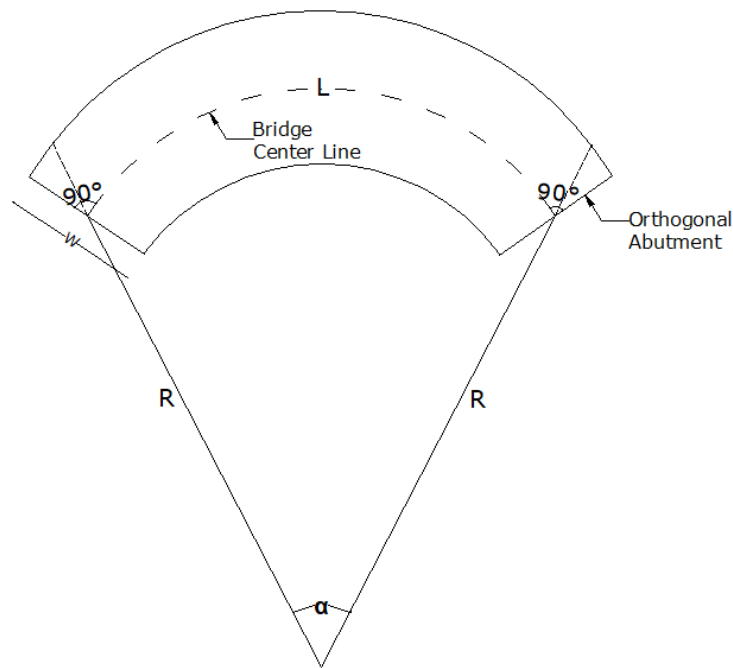


Figure 13: Schematic for Curved Bridge with orthogonal support

3.5 Element Discretization (Meshing)

In analyzing the bridge models, a three-dimensional linear elastic analysis method is employed. The model is comprised of a four noded shell element having six degrees of freedom (3 translations and 3 rotations) at each node. When employing a finite element analysis on curved bridges the element length shall not be coarser than 2% of the total span length (Lydzinski J. & Baber T. 2008). Having the aforementioned point as a benchmark the research made the element size to be one percent of the respective span length.

3.6 Boundary Conditions

The boundary condition for all models is similar. The bridge is simply supported single-celled bridge. The east support is idealized as pin support whereas the west support has been idealized as roller support. The program uses the following assignment for the boundary conditions.

Table 4: Boundary Conditions for Model

Support Properties		
DOF Direction	Release Type	
	Pin Support	Roller Support
Translation Vertical (U1)	Fixed	Fixed
Translation Normal to layout line (U2)	Fixed	Fixed
Translation Along Layout Line (U3)	Fixed	Free
Rotation About Vertical (R1)	Free	Free
Rotation about normal to layout line (R2)	Free	Free
Rotation About Layout Line (R3)	Free	Free

3.7 Loads Cases for Analysis

3.7.1 Permanent Loads

a) Dead load of structural components and non-structural attachments. This load is a dead load of the concrete superstructure and concrete crush barriers.

b) Dead load of wearing surfaces and utilities. Bituminous Wearing surfaces have been assumed on deck surfaces as a wearing surface (*unit weight 22.5 kN/m³*).

3.7.2 Vehicular Live Load

Design Truck Load

According to *ERA-BDM 2013*, the total design truck load of 325 kN is distributed between 3 axles as shown in Figure 14.

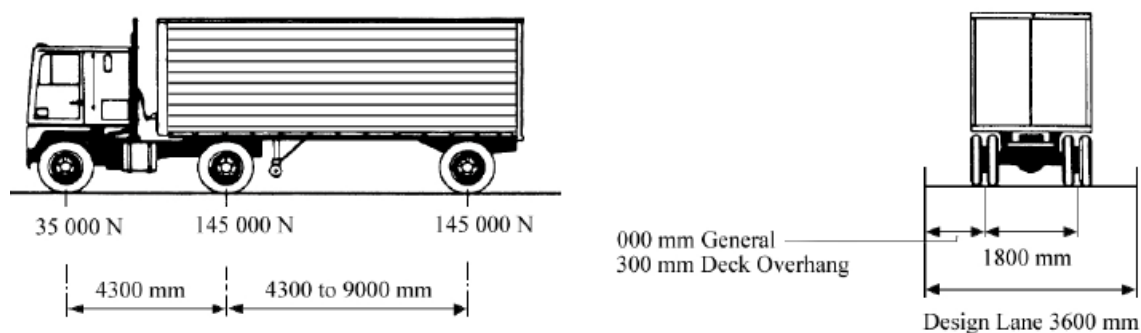


Figure 14: AASHTO HL-93 Truck Vehicle

The design load truck has a tire length single rectangle of 510mm and a length of 250mm.

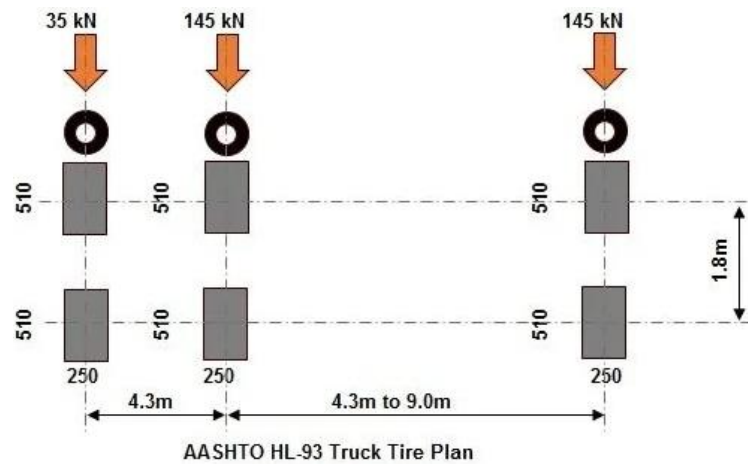


Figure 15:AASHTO HL-93 Truck Vehicle Tire Plan

3.7.3 Centrifugal Load

As per Article 3.6.3 of the AASHTO LRFD,2012, the centrifugal forces shall be taken as the product of the axel weight of the design truck or tandem and the factor C stated in Equation 1.

Centrifugal force is not required to be applied to the design lane load, as the spacing of vehicles at high speed is assumed to be large, resulting in a low density of vehicles following and/or preceding the design truck.

3.7.4 Vehicular Dynamic Load Allowance (IM)

The static effects of design truck or tandem, other than centrifugal and braking forces shall be increased by the percentage specified in the table below for dynamic load allowance. The governing manuals also forward the recommendation that the dynamic allowance shall not be applied to the lane loads (ERA BDM 2013 and AASHTO 2012).

Table 5:Dynamic load allowance factors

Component	IM
Deck joints – All limit states	75%
All other components	
• Fatigue and Fracture Limit State	15%
• All Other Limit States	33%

3.7.5 Design Speed

To capture the real response of the bridge under live load, it is vital to know the operating speed of the vehicle. The vehicular speed is also important in computing the centrifugal force. The analysis of this study assumes the bridge to be suited to urban areas. The design speed for roads in urban areas is 50km/hr. or 13.88m/s (*ERA Geometric Design Manual 2013*).

Table 6: Summary of Load Cases

No.	Load Case	Load Type
1	Self-weight of Bridge	Dead (DC)
2	Asphalt layer load	Dead (DW)
3	Crush Barrier	Dead (DC)
4	Truck Load	Vehicular Live (LL+IM)
5	Centrifugal Load	Centrifugal Load (CE)

3.8 Load Combination

The study aims to assess the response of a skew-curved bridge under dead and live load cases. The paper is not focused on designing these bridges or evaluating their capacities. Henceforth, all relevant load cases will have a load factor of unity. In other words, a service I load combination will be utilized. As per *Article 3.4* of the *ERA Bridge Design Manual, 2013*, this load combination is related to the normal operational use of the bridge with all the loads taken at their nominal values.

3.9 Model Validation

As with many other computer programs, *CSI Bridge V24.2.0* has specific engineering assumptions embedded in its code, which may or may not apply to each specific case. Accordingly, all output should be verified to the extent possible. Computer programs should be verified against the results of universally accepted closed-form solutions, other previously verified computer programs, or physical testing (*Robert G. Sargent, 2008*).

For program validation, the paper has used two independent verification data. The first one was having a closed-form solution for the subject bridge. The result of the program is compared with the solution acquired following the procedure provided by R.C. Hibbeler, 2006. In addition, the analysis results of the model bridge stipulated in Appendix GB of the *ERA Bridge Design Manual 2002* are also utilized.

Description of Model

The length of the bridge is 16.60m. The diaphragm has a depth of 1.0m and a width of 0.25m. It also has a width of 8.12m where 7.32m is dedicated to a carriageway. The total depth of the superstructure is 1.30m. where the top slab is 0.22m.

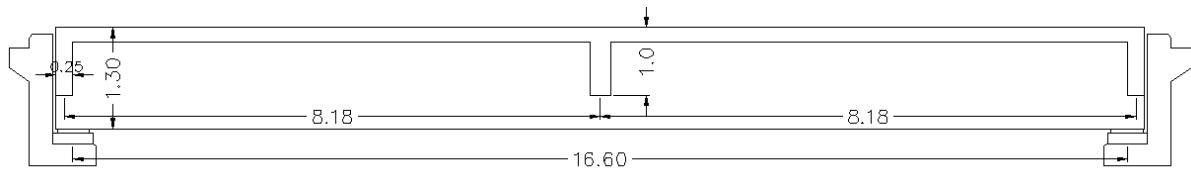


Figure 16: Elevation view for the proposed bridge (Dimensions are in meters)

The material properties of concrete stipulated in section 3.4 and table 3 of this document have been used.

Geometric Properties for the Model

The following figure illustrates the typical cross-section of the bridge under the study.

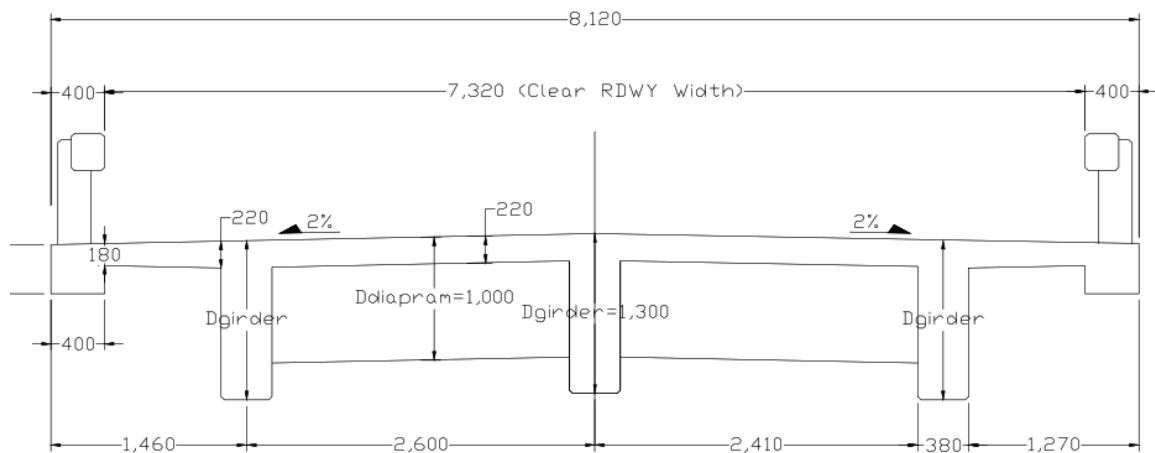


Figure 17: Typical cross-section of the model bridge ERA 2002

Boundary Conditions & Loading

The boundary condition for all the models was similar. The east and west abutments are modeled as simply supported. The east support is idealized as pin support whereas the west support has been idealized as roller support. The program uses the assignments stipulated in Table 4 for the boundary conditions.

A moving load was applied as per the AASHTO LRFD manual. HL- 93 truck load is applied on the two-lane bridge with a vehicular speed of 50km/hr.

The paper used the principle outlined by Hibbeler to obtain the closed-form solution for the bending moment of the model file. The principle is stated as:

“The absolute maximum moment in a simply supported beam occurs under one of the concentrated forces such that this force is positioned on the beam so that it and the resultant force of the system are equidistant from the beam centerline” (Hibbeler, 2006).

To obtain the structural responses of the bridge a moving load analysis of HL-93 is employed on CSI Bridge 24.2.0. The responses of the Program are verified by comparing its bending moment results with the statical results and the appendix tabulated below.

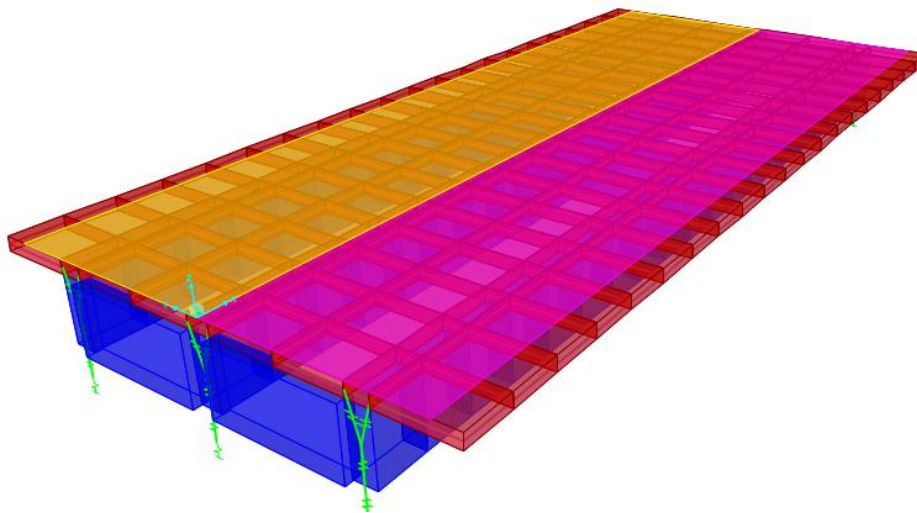


Figure 18: Finite Element Model for validation bridge

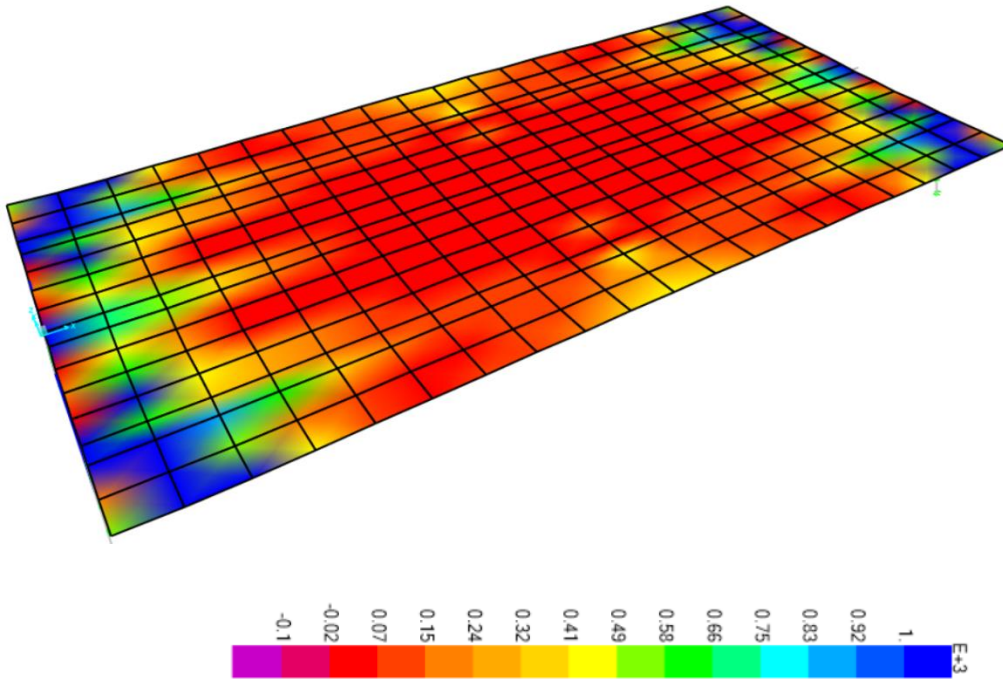


Figure 19: Bending stress on the Bridge due to truck Load

Table 7: Bending Moment Results from Different Approaches

X (m)	Hibbeler (Closed Form) kNm	CSI Bridge kNm	ERA Appendix kNm	Result check (Ratio)	
				CSI / Hibbeler	CSI / ERA
0.00	0.00	0.00	0.00	-	-
0.83		287.07	283.13		1.014
1.66		527.87	531.03		0.994
2.49		727.37	743.68		0.978
3.32		891.56	921.08		0.968
4.15		1022.67	1063.25		0.962
4.98		1122.64	1170.17		0.959
5.81		1201.70	1250.46		0.961
6.64		1256.72	1312.73		0.957
7.47		1281.92	1339.76		0.957
8.30		1278.73	1331.55		0.960
9.13	1303.07	1281.92	1339.76	0.984	0.957
9.96		1256.72	1312.73		0.957
10.79		1201.70	1250.46		0.961
11.62		1122.64	1170.17		0.959
12.45		1022.67	1063.25		0.962
13.28		891.56	921.08		0.968
14.11		727.37	743.68		0.978
14.94		527.87	531.03		0.994
15.77		287.07	283.13		1.014
16.60	0.00	0.00	0.00	-	-

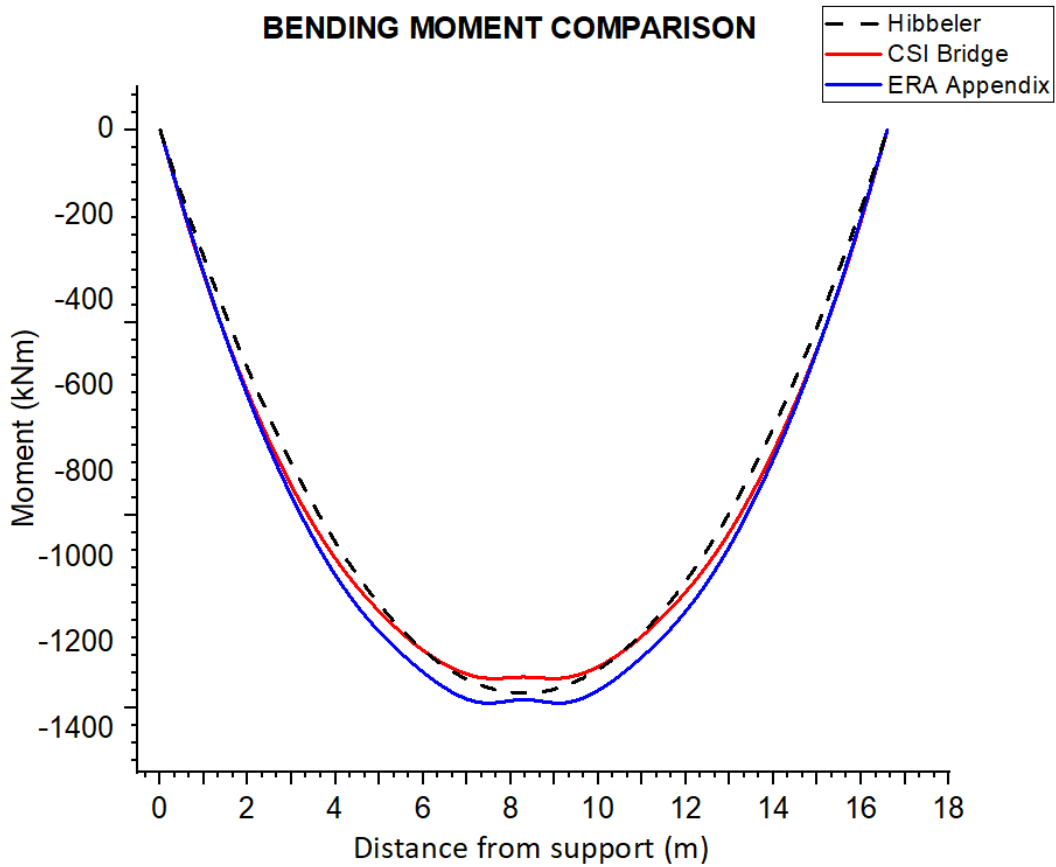


Figure 20: Comparison of Bending Moment results of bridge model against verification data

As shown in the above graph there is an acceptable conformity between the results obtained by CSI bridge and the verification data used by the paper.

CHAPTER FOUR

4. RESULTS AND DISCUSSION

In this chapter, the results of analysis performed on various models of bridges are presented. The analysis results are presented in a way that could meet the paper's objective. The defined vehicles (*AASHTO LRFD HL-93 truck Load*) are loaded in a predefined lane. The program offers two types of alternatives regarding the analysis of the bridge.

The Two types of vehicle live-load analysis that can be performed are influence-based enveloping analysis and step-by-step analysis with full correspondence. The CSI reference manual 2017 reflects that both analysis options may be applied to all bridges. However, the manual suggests that multi-step static analysis is the preferable option for special bridge types.

In a multi-step static analysis method any number of Vehicles can be simulated to run simultaneously on predefined Lanes, each with its own starting time, position, direction, and speed. Henceforth, the multi-step static analysis type is used to analyze the structure.

The paper has generated about two hundred finite element models to answer the research questions. Consequently, structural responses of the bridge models were recorded. The variable parameters of the analysis are the Angle of Curvature and a skew angle.

The design speed for the analysis has been specified in chapter three of the paper. However, the program also requires the direction of movement (*forward or backward*), starting time, the distance between consecutive vehicles, etc. Henceforth, the vehicles are programmed to start the movement at the same time in the same direction (*see Figure 21*). The two lanes defined for the bridge are loaded by the design vehicles having the appropriate speed.

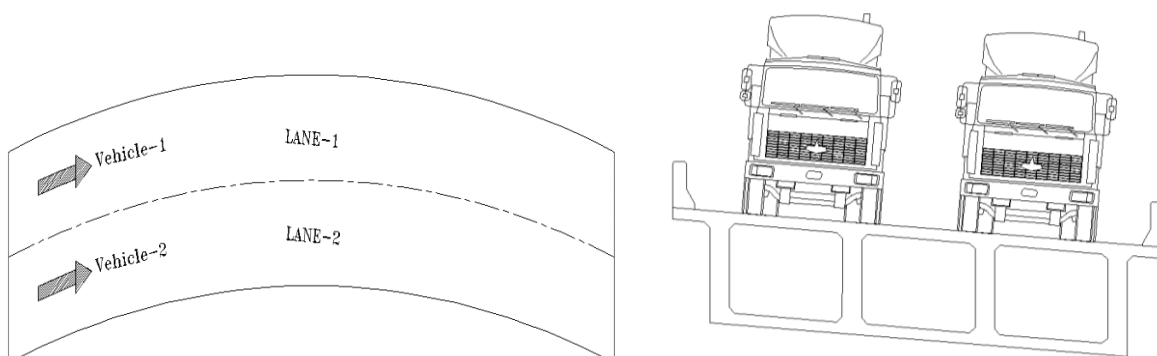


Figure 21: Vehicular arrangement for analysis of two-lane bridges

4.1 Visualization of Bridge Parametric Variation

To provide some visual clarity about the analysis parameters of the bridges, different models are presented below. The graphics are associated with the description of the model.

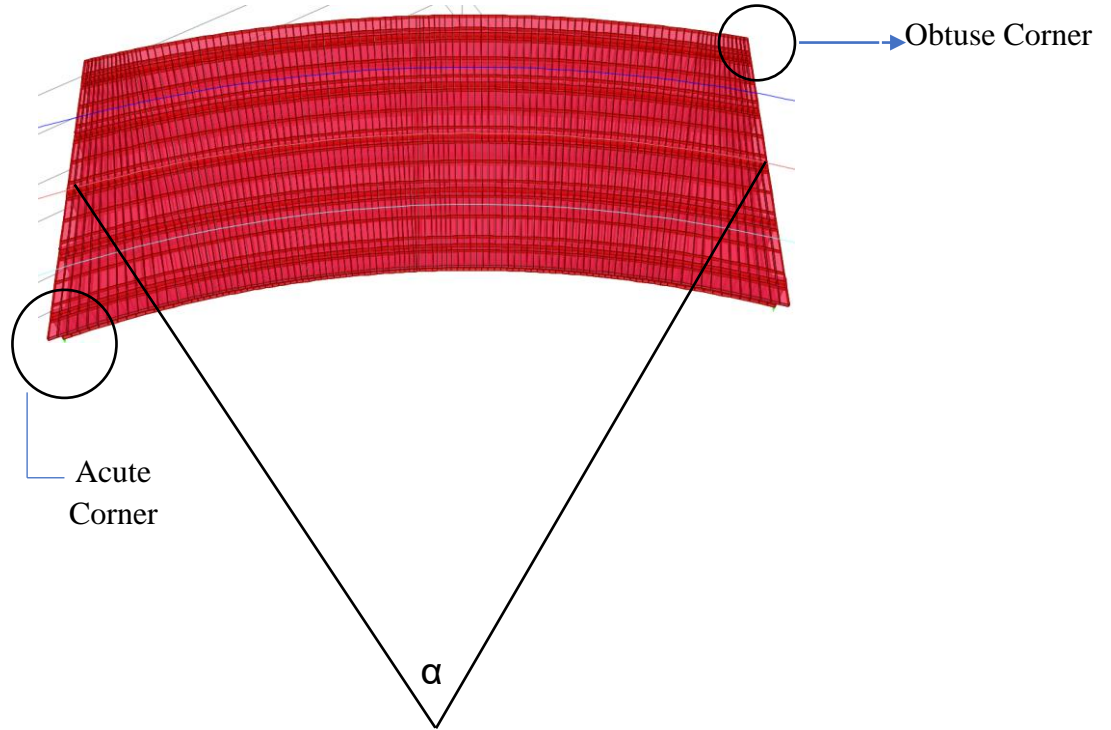


Figure 22: General Plan Layout for model Skewed-Curved Bridge

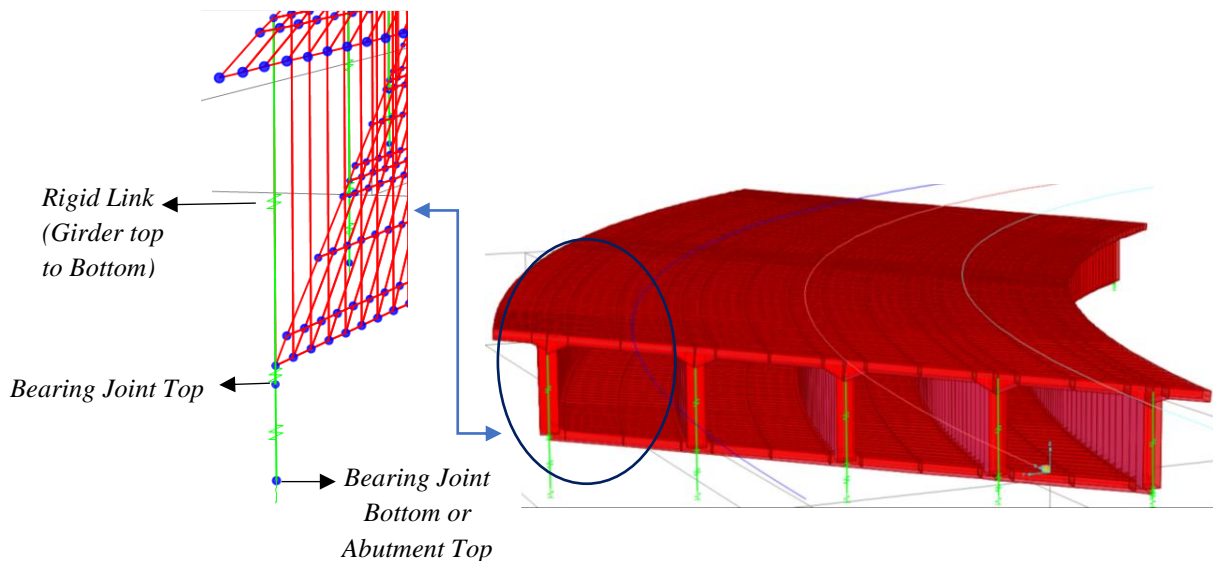


Figure 23: Cross section for a curved bridge

The bridges defined here have a predefined lane. All vehicles are restricted to stay within their predefined lanes.

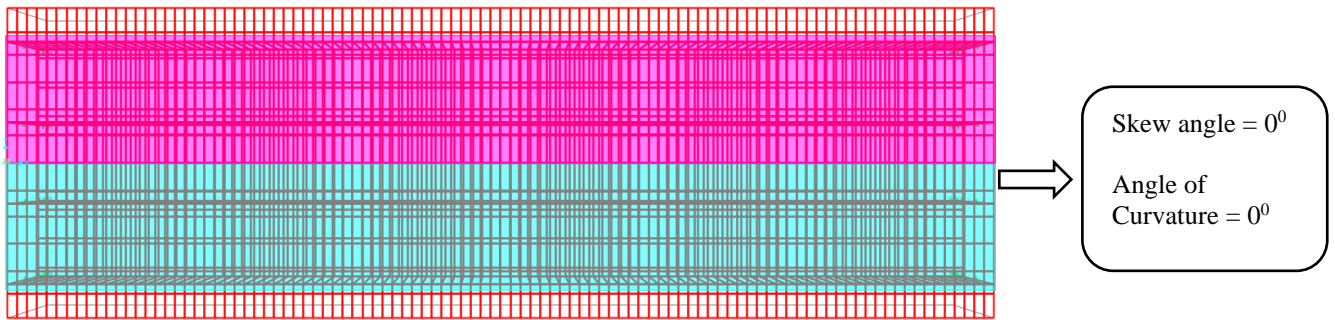


Figure 24: Plan Schematic for a straight bridge model

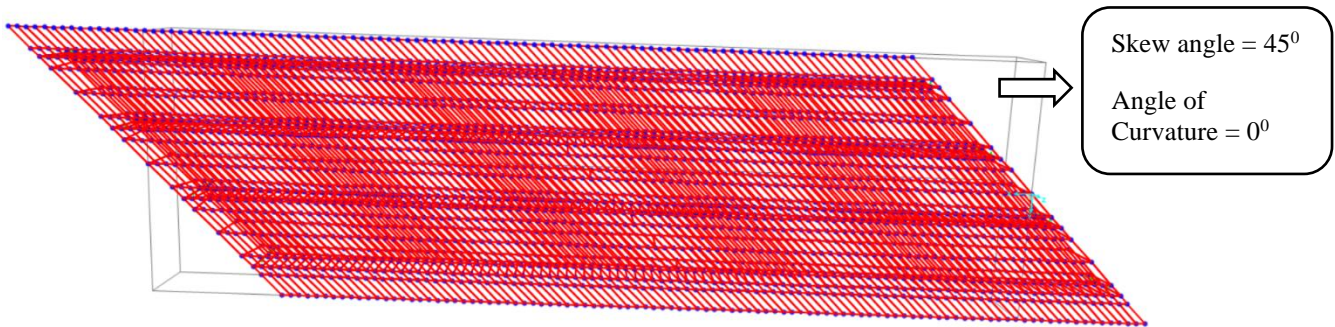


Figure 25: Plan Schematic for a skewed bridge model

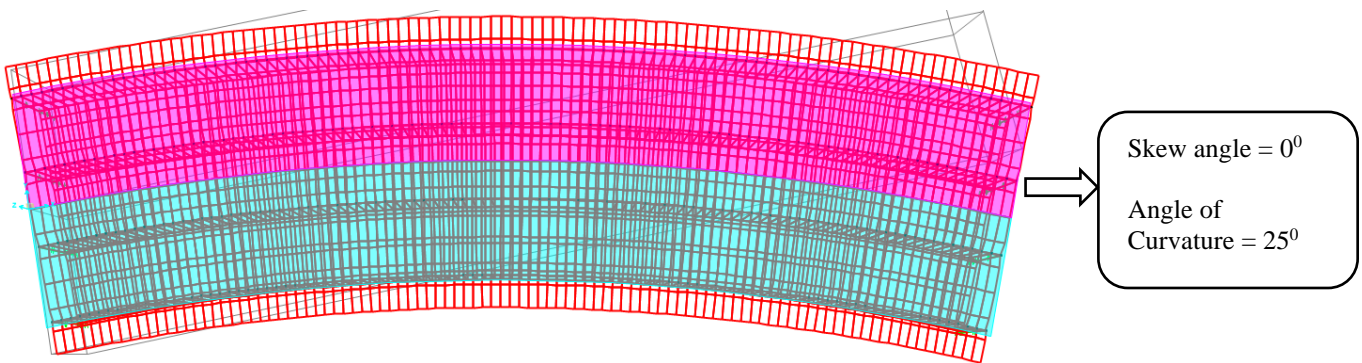


Figure 26: Plan Schematic for a curved bridge model

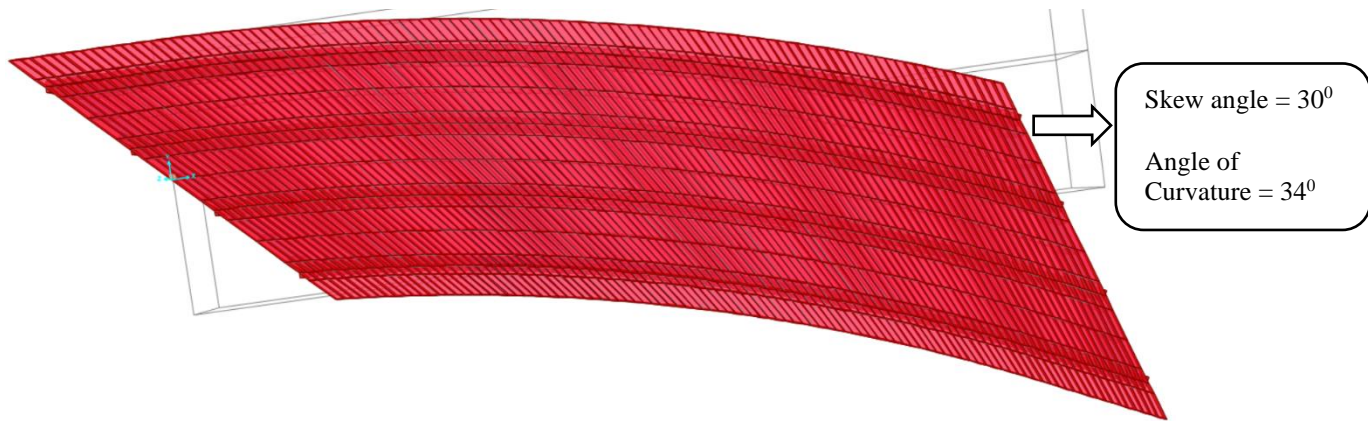


Figure 27: Plan Schematic for a curved skewed bridge model

4.2 Results of Analysis

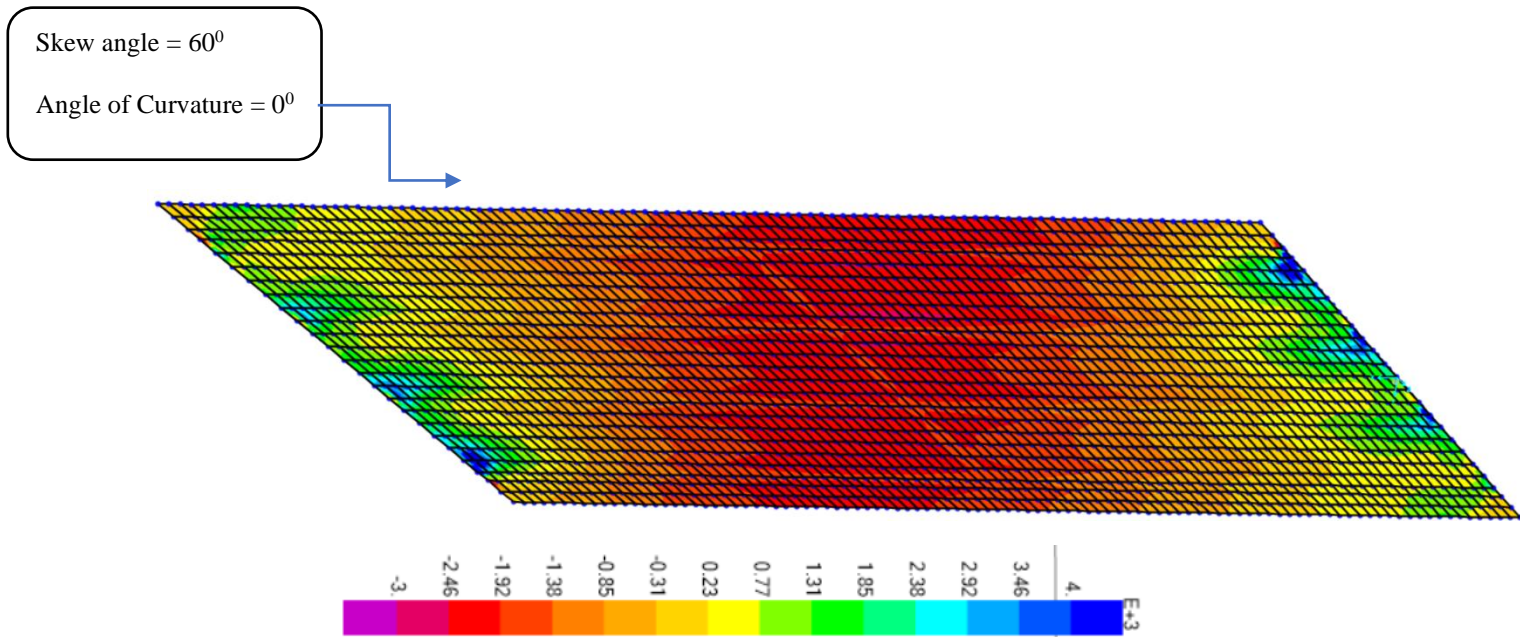
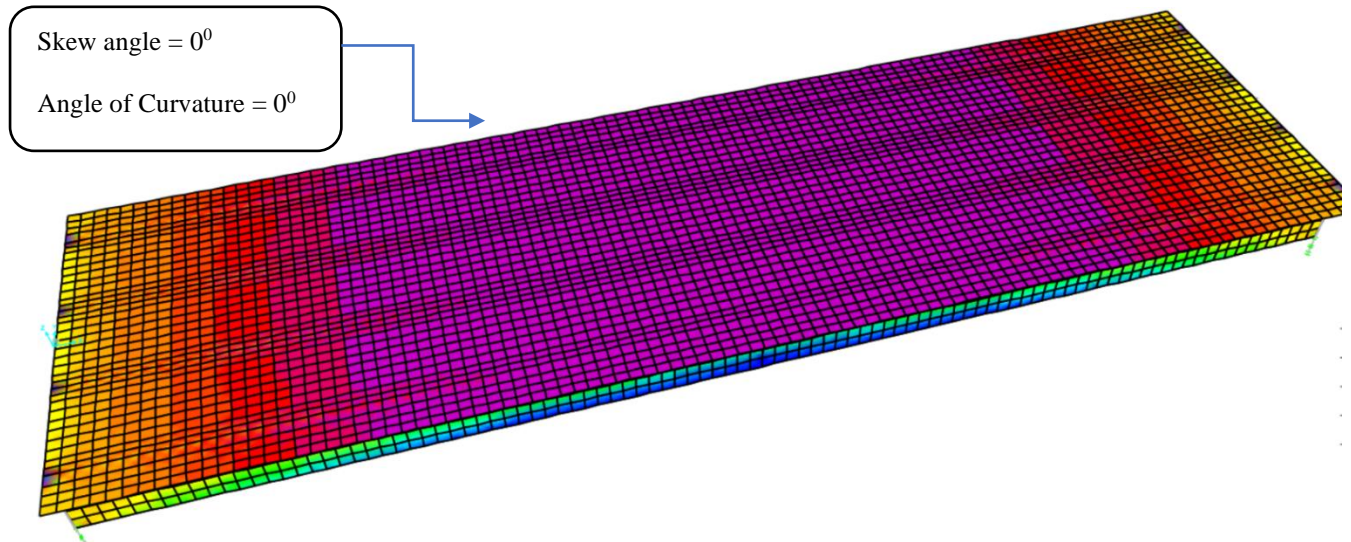
The paper has presented the analysis results obtained from generating models. To easily grasp the nature of the bridge response, the results of the models are presented using graphs and tables.

The results of the analysis have reflected that the nature of the skew-curved bridge is not something to be overlooked. The paper also presents the effect of this geometry on the major force effects.

The force effects that are discussed are bending moment, shear force, torsion, and deflections. The deflection of the bridge is evaluated only for the vehicle load, i.e., it has excluded the dead load effects. All other effects are computed for the service I load combination.

4.2.1 Bending Moment Results

The following figures illustrate the bending stress in a way that can describe the general behavior of the variation in longitudinal bending stress for different skew and curve combinations.



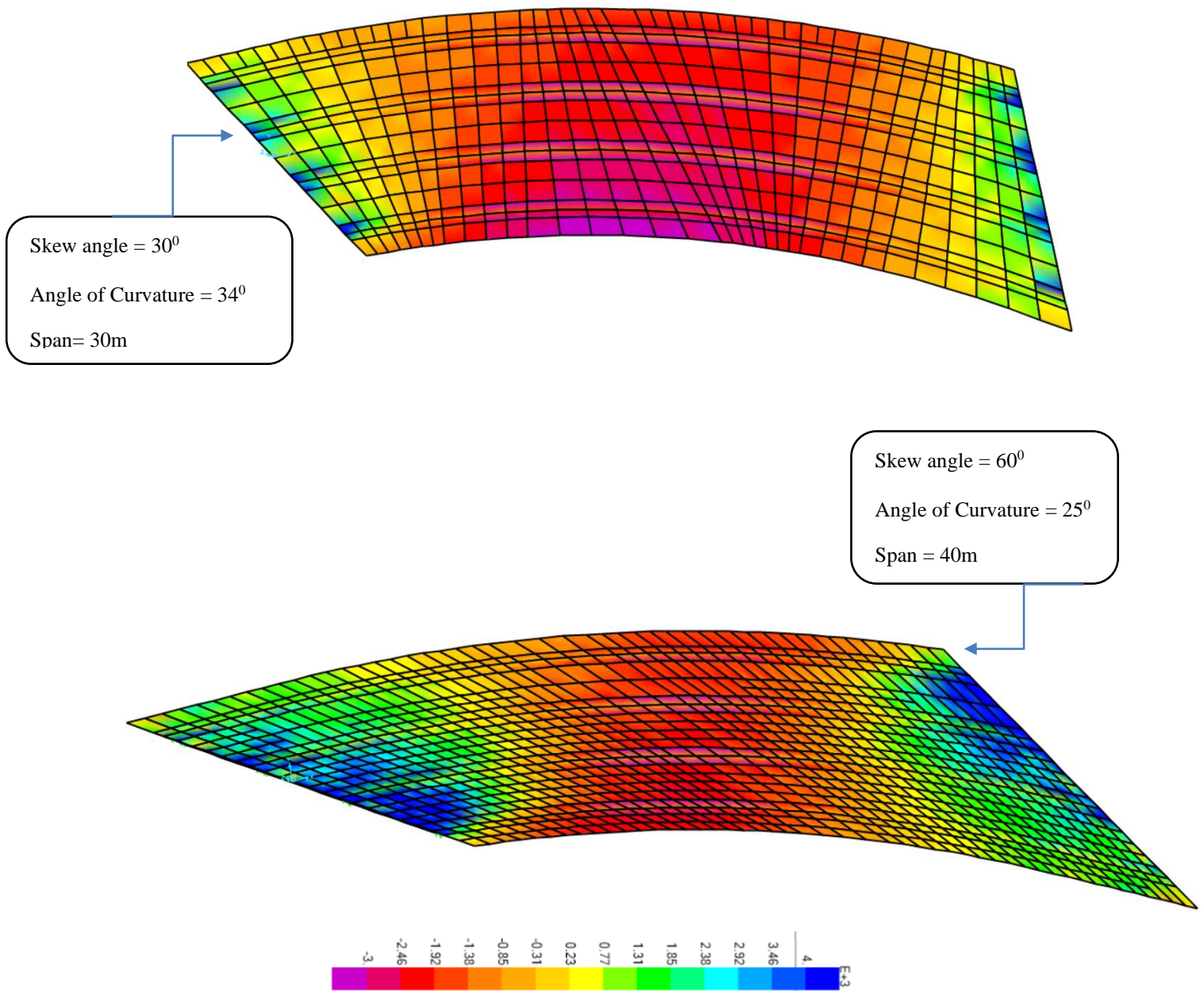


Figure 28: Effect of Curvature (α) on Variation of Dead Load Longitudinal Bending stresses (kN/m^2) in modeled bridges.

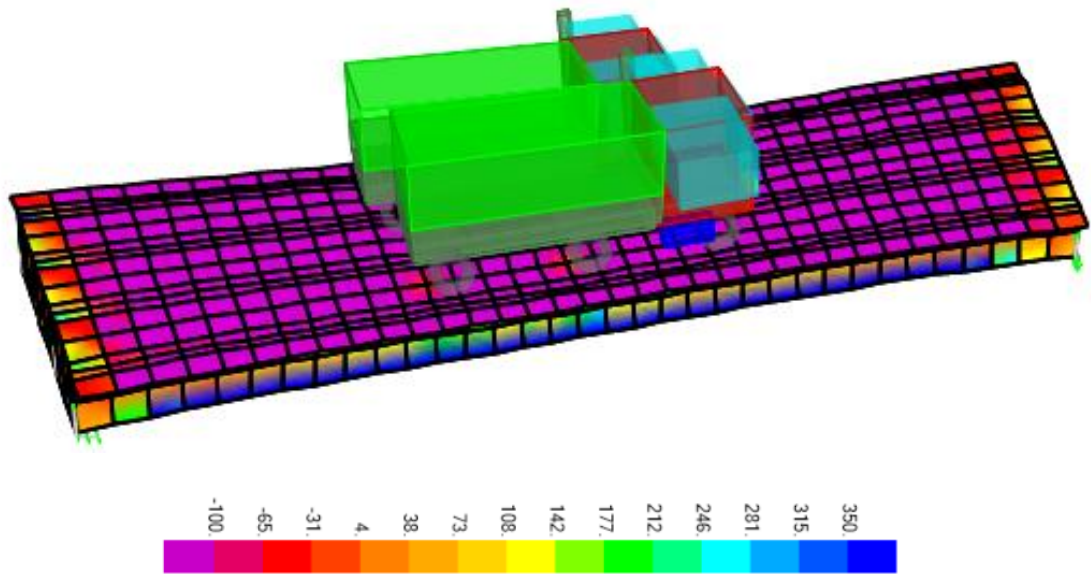


Figure 29: Maximum Longitudinal Bending stresses for Live Load Case (kN/m^2) for Bridge with 0° Skewed angle (θ) and 34° -degree Angle of Curvature (α) bridge.

The following graphs illustrate the longitudinal bending moment results. Graphs are plotted for the values obtained for the exterior and interior girders. The following figure illustrates the girders of the bridge mentioned above.

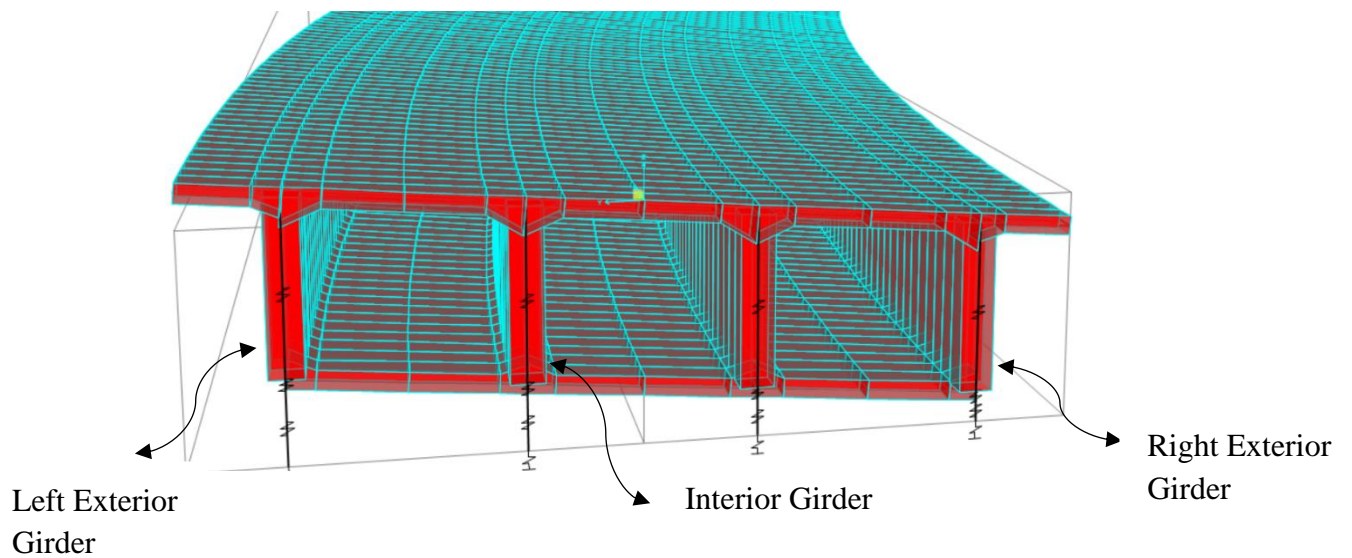
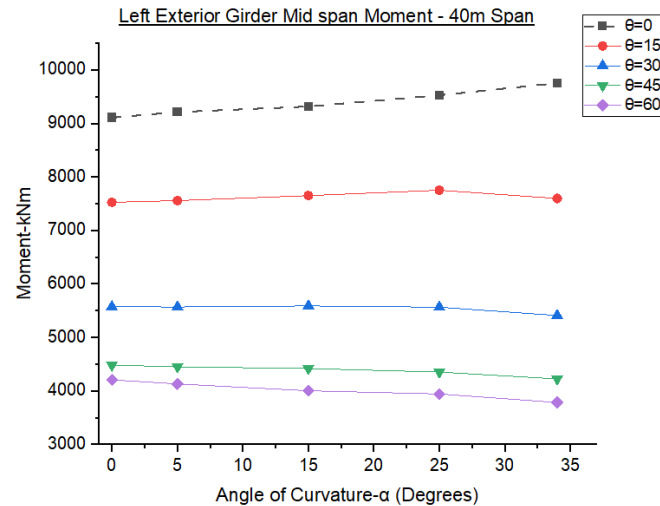
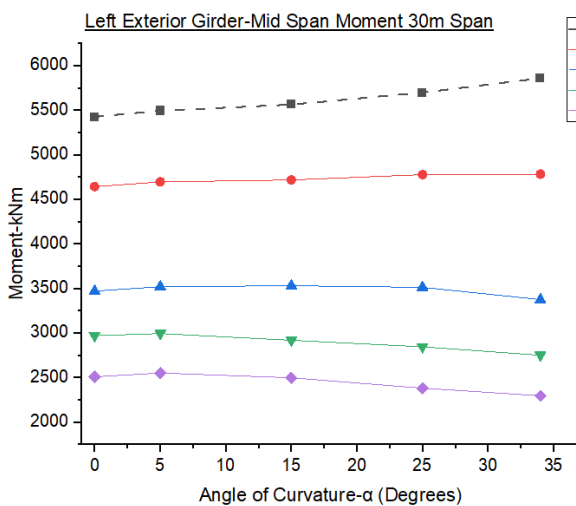
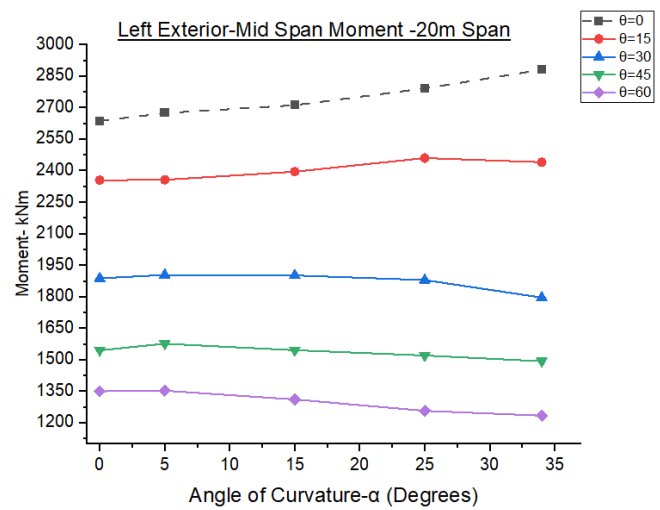
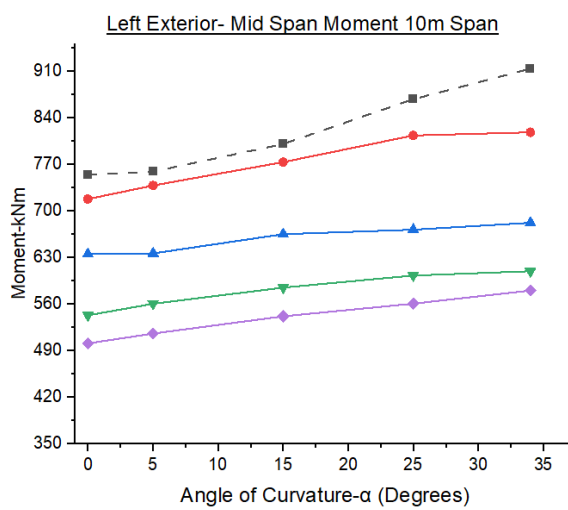


Figure 30: Girder notations for modeled bridges

The analysis reaffirmed a well-known fact for a skewed bridge. The longitudinal bending moment (M_3) at the mid-span has decreased with an increase in the skew angle of a bridge. This concept has been clearly explained by (Wallace, 1973). The Presence of skew angle will create a counteracting moment to the positive moment at the supports thus decreasing the span moment. This moment is a function of skew angle, width of the bridge and support reactions.

The results have also pointed out that for a constant skew angle, the bending moment increases with an average increment rate of 1% for all assessed span lengths of bridges. It can be said that the bending moment for skewed bridges is not significantly affected by the incorporation of curvature.



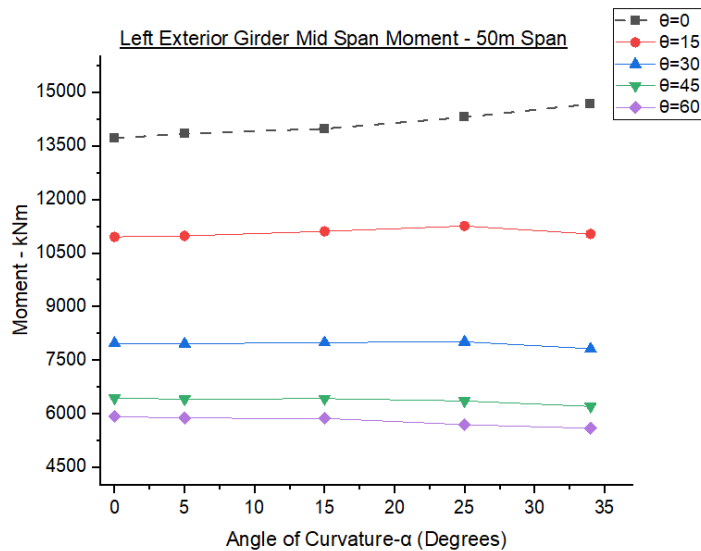


Figure 31: Maximum Longitudinal Bending Moment for Left Exterior Girder

The result also indicates that the increment rate described above has also been seen on all girders. However, comparing the values of the bending moment of each girder, the interior girders have experienced larger moments compared to the right and left exterior girders. This is because the interior girders are subjected to two lines of wheels.

The analysis has also assessed the validity of ignoring curvature effects on determining the bending moment of a bridge. The broken (*dashed*) line on the above graphs indicates the bending moment of a purely curved bridge. The graphs have reflected that the bending moment of a purely curved bridge increases with an increment of the Angle of Curvature. This increment is about 2% in every angle increment step of the analysis.

The bending moment for a non-skewed bridge with an Angle of Curvature of 34° has experienced a bending moment increment of 6% to 8% compared to the straight bridges for all assessed spans. The paper concludes that this is not a value to be ignored contrary to the governing manual suggestion (*AASHTO LRFD Section 4.6.1*).

4.2.2 Shear Force Results

The shear force behavior of the skewed-curved bridge is by far the most important force effect to be studied. This is because all the relevant works of literature forward the conclusion that; shear force on girders will increase as the support skewness of a bridge increases. The governing manuals have also affirmed this idea. It incorporated a skew correction factor to

magnify the shear force results acquired from a live load analysis using a spine model (AASHTO LRFD Section 4.6.2).

The research tried to assess the shear force effects on skewed curved bridges. The results from the three-dimensional finite element analysis have been plotted on the graphs below.

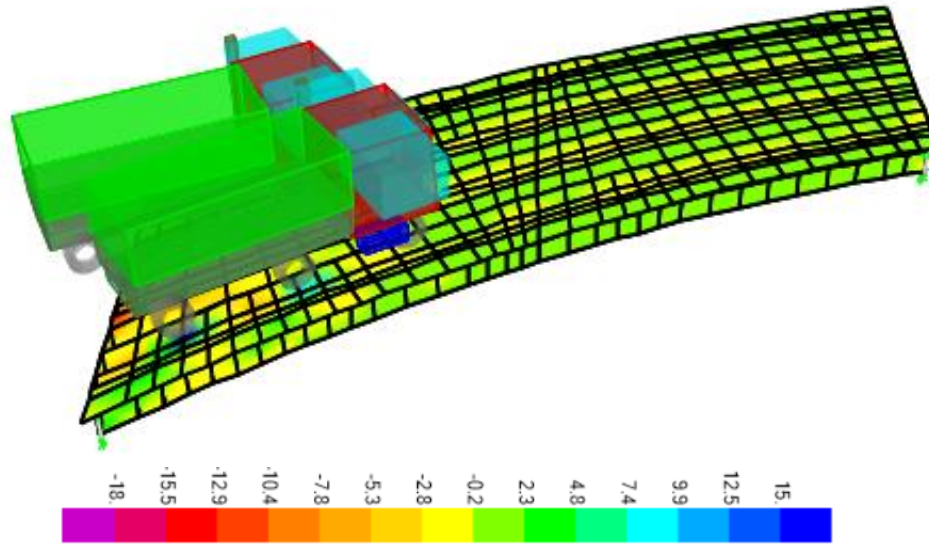
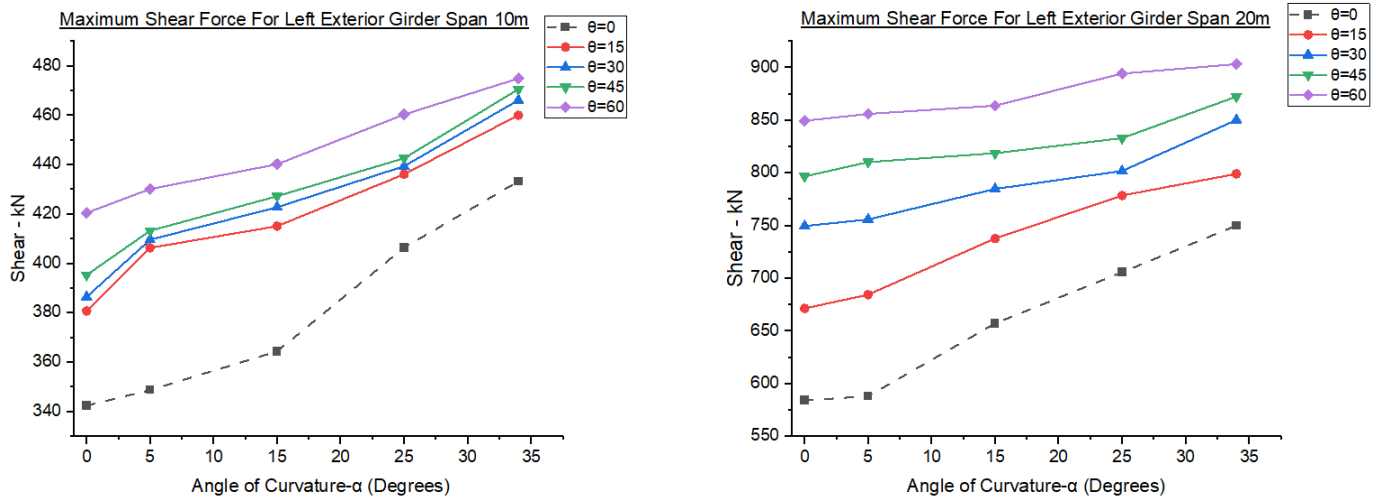


Figure 32: Maximum Shear Force (Service I Load Combination) for Bridge with skew angle (θ) of 45° and Angle of Curvature (α) of 25°



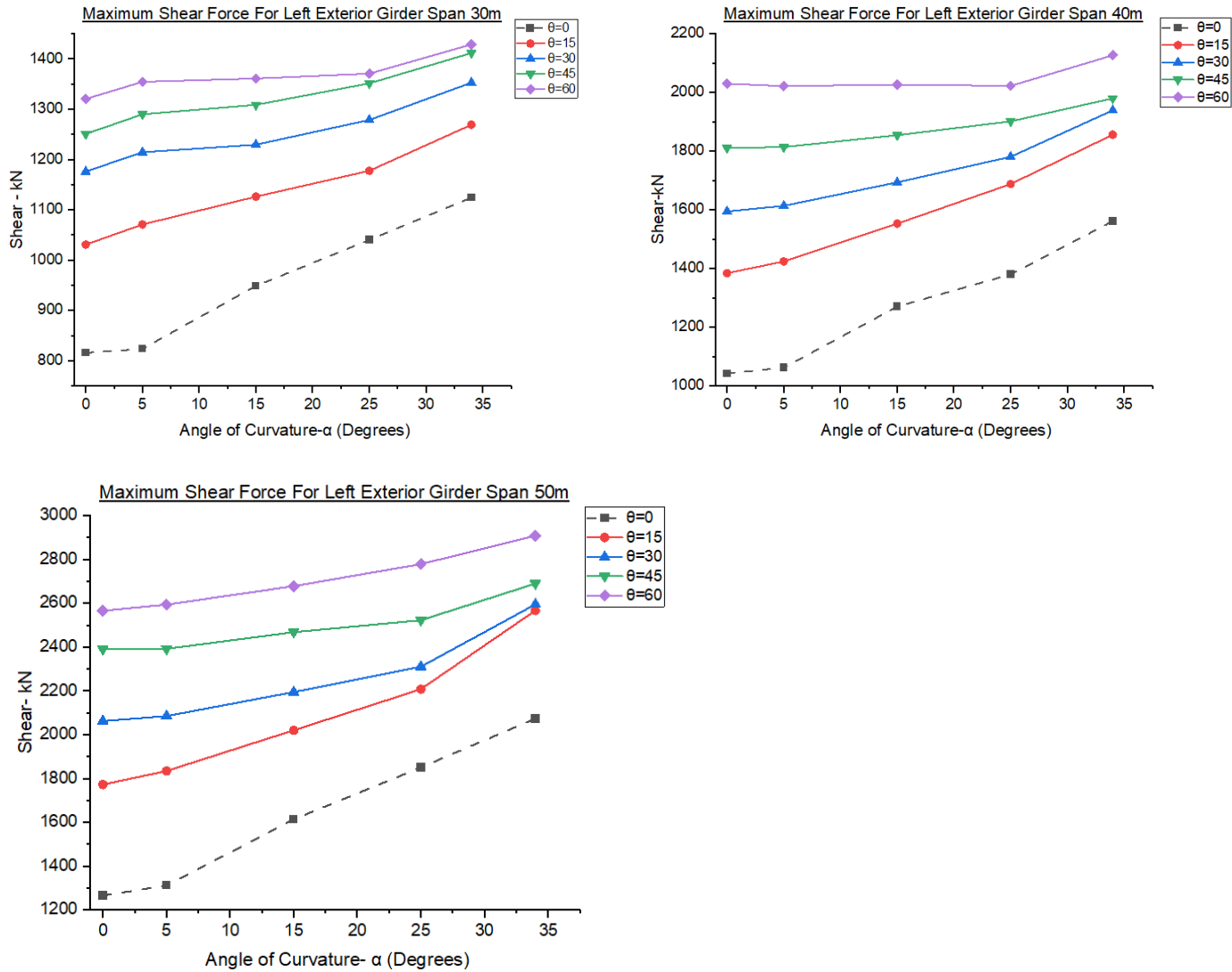


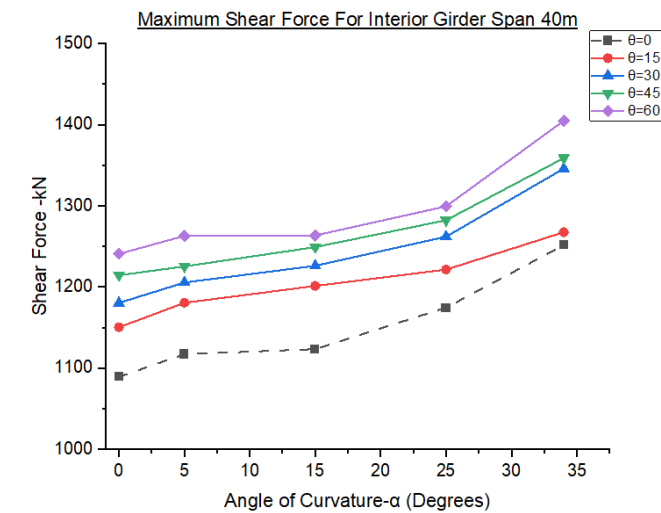
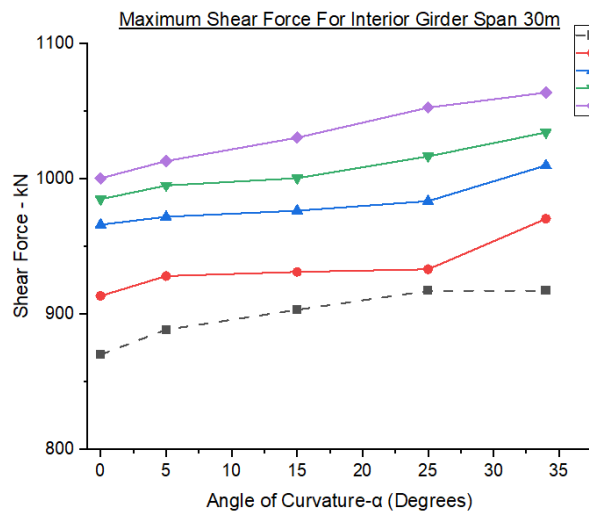
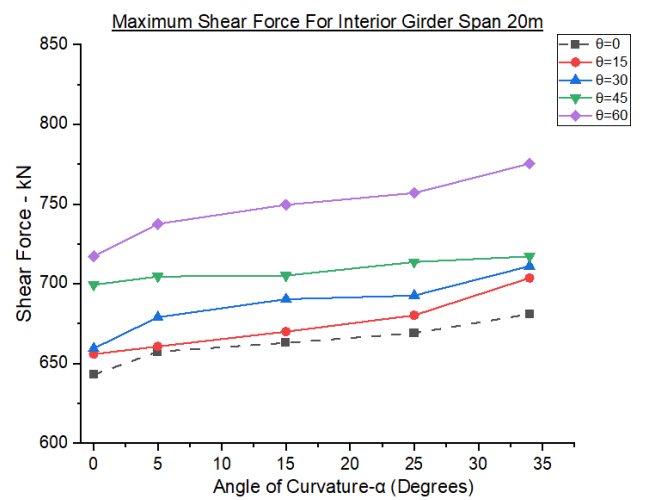
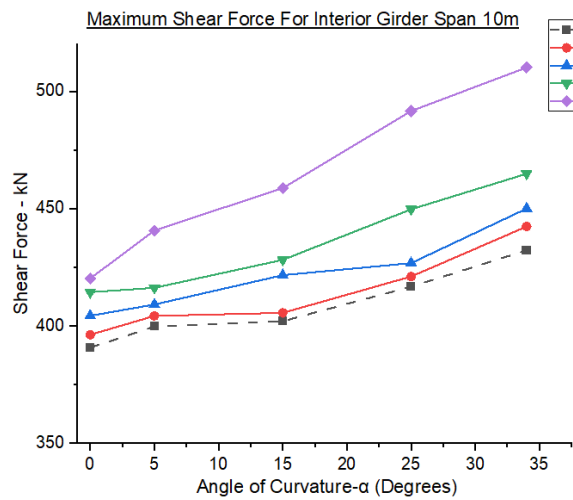
Figure 33: Maximum Shear force (V_2) for Left Exterior Girder at the support

The study reflected that for Left girders, the vertical shear force (V_2) of a non-skewed curved bridge (dashed line in the above graphs) increases as the Angle of Curvature of the bridge increases. The average increment rate is 6.3% for span lengths ranging from 10m to 30m. The average increment rate for spans 40m and 50m is 8.2% and 9.5% respectively. This can be attributed to the increase in shear spans due to curvature caused by Angle of Curvature increment.

The study has also reaffirmed the idea that for a given skewed bridge the shear force will increase with an increment in support skewness. This can be seen very clearly in the above graphs. The left exterior girder of a bridge with a support skewness of 60° experiences a higher shear force compared to the girder with a lower skew angle.

The incorporation of curvature on skewed bridges has increased the shear force on the left exterior girders of assessed bridges. This is due to the reaction concentration at the obtuse corners of the bridges. Upon the incorporation of curvature, the shear force for skewed bridges increases with an average rate of 3.2% for span lengths that range from 10m to 30m. This value is increased to 4.04% for a 40m span bridge and 5.02% for a 50m span bridge. An increment in centrifugal forces plays a major role in increasing the shear force of the exterior girders.

Interior Girders



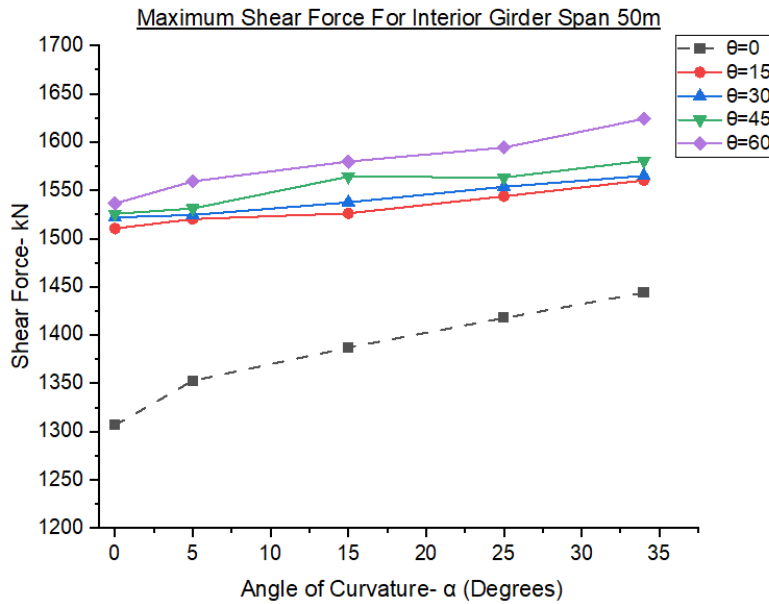


Figure 34: Maximum Shear force (V_2) for Interior Girder at the support

As it can be seen from the above plotted graphs the results solidify the fact that for a given skewed bridge the shear force will increase with an increment in support skewness. This is attributed to the concentration of reaction forces at the obtuse corners. Which leads to an increase in shear force. It can also be deduced that for a particular skewed bridge, the shear force for the interior girder has increased with an increment in the Angle of Curvature (*decrement of radius of curvature*). This will increase the centrifugal forces experienced by the girders. The results have also reflected that the increment seen with an increase in curvature is not significant.

The research has noted that the shear force on the right exterior Girder decreases as the Angle of Curvature increases for a given skewed bridge. This decrement is with an average percentage of 5%. This can be attributed to the decrease in shear span due to the increase in curvature.

4.2.3 Torsional Moment Results

It's a well-known fact that straight bridges (*non-skewed & and non-curved bridges*) experience insignificant torsional moments. However, this is not the case for skewed and curved bridges. The following graphs cement this idea by reflecting that considerable torsional moments will be experienced by skew-curved bridges.

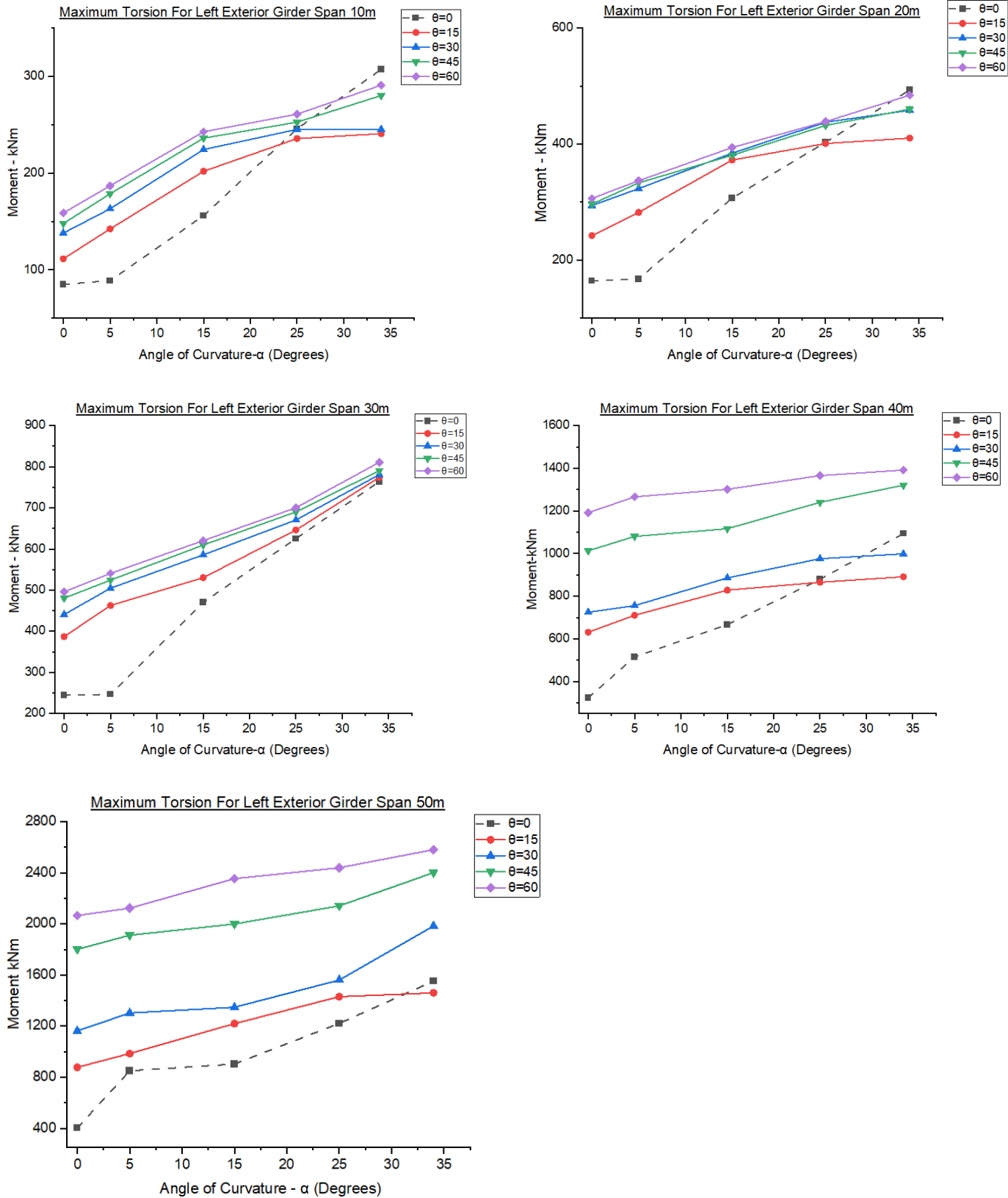


Figure 35: Maximum torsional moment for Left Exterior Girder

The above figures reflect two important points. One is that the incorporation of plan curvature on skewed bridges will increase the torsional moment for Exterior Girders. Second, for a constant Angle of Curvature, the torsional moment increased as the support skew angle increased. This rate of increment is seen to be not significant on bridges with smaller span lengths. However, for span lengths above 30m, a considerable increment in torsional moment has been noted as skew angles (θ) increase for a constant Angle of Curvature (α). For instance, for a span length of 40m with an Angle of Curvature of 15° the torsional moment has increased with an average rate of 15% as skew angle increased from 0° to 60° .

The results also indicated that the torsional moments experienced by purely curved bridges (*dashed line in the above graphs*) are very significant. For a skew angle (θ) of 0° Torsional moments increase with an increase in Angle of Curvature. In some instances, the torsional moment seen on a purely curved bridge with an Angle of Curvature of 34° has a higher torsional moment compared with skew-curved bridges. The reason for such results could not be discussed in this paper and more data sets shall be generated if it is desired to investigate that. The most relevant conclusion this paper could give regarding the above point is that torsional moments cannot be ignored even under an Angle of Curvature of 34° (*Which the code and trending practice recommends ignoring*)

The torsional effect of skewed-curved bridge geometry on the interior girder is also seen to be very significant. The two points mentioned above for the Left exterior girder have also been seen to hold for the interior girder as well. The torsional moment on the interior and exterior girder by a purely curved bridge (*dashed line in the above graphs*) is very significant.

4.2.4 Vertical Displacement [Deflection] Response

The paper tried to assess the vertical displacement of the generated models. This parameter is only studied for vehicular load actions. Generally, the analysis pointed out that the deflection of a bridge decreases with an increase in support skewness. It has also reflected that an increase in the Angle of Curvature [*curvature*] also increases the deflection of the bridge. The following figures represent the displacement values for the 40m and 50m span bridges.

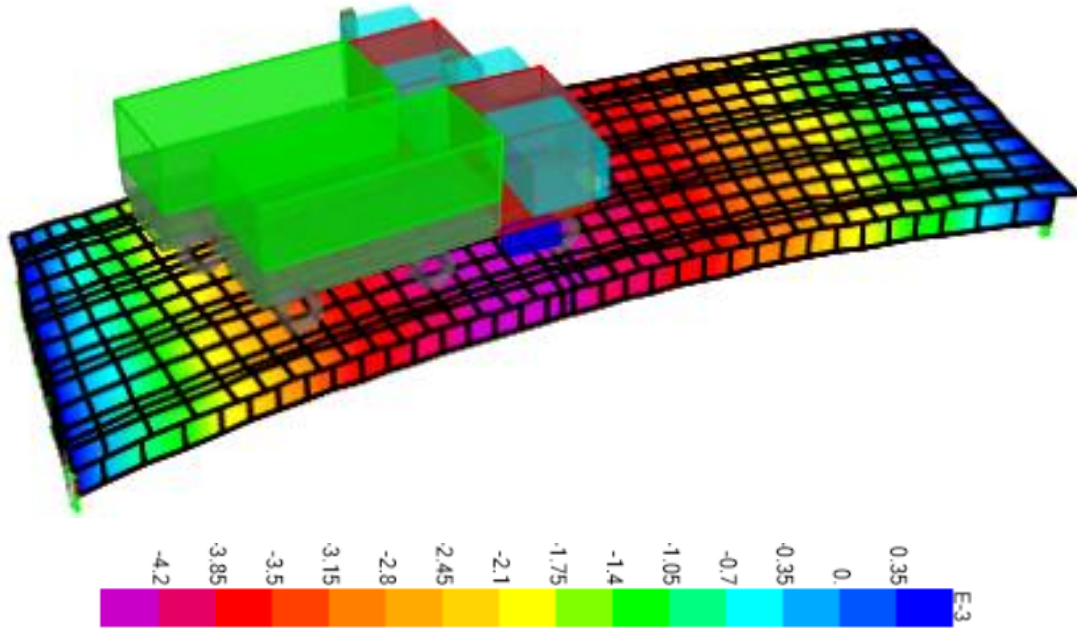


Figure 36: Uz for Live load case for Bridge with skew angle (θ) of 30° and Angle of Curvature (α) of 34°

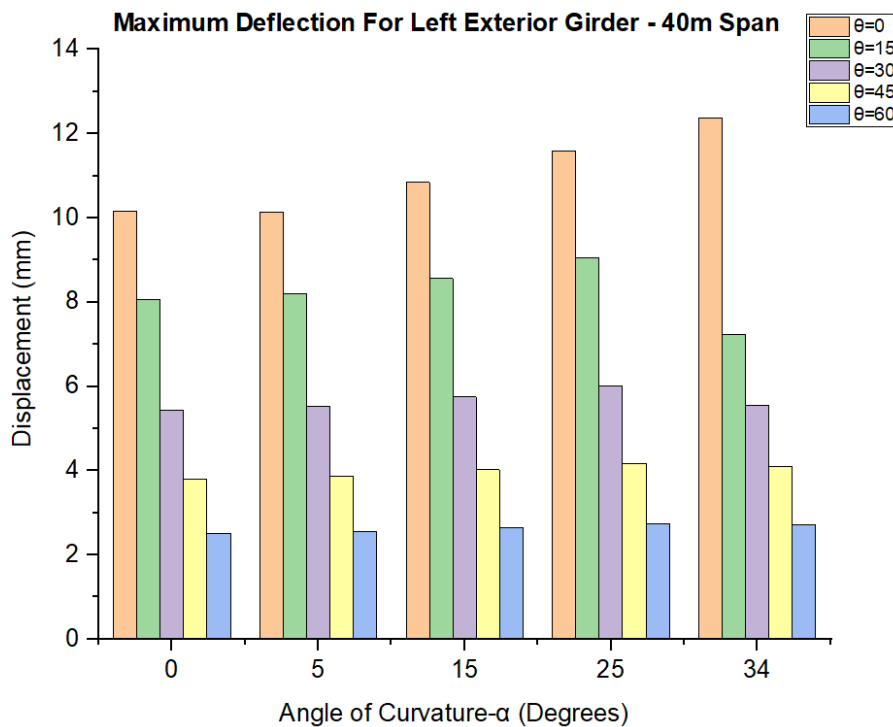


Figure 37: Maximum Vertical displacement for left exterior Girder of 40m span

The results for the left exterior girder reflected that deflections for the non-skewed bridge are significantly higher than skewed and skew-curved bridges. The study has also solidified the fact that the deflection for a bridge decreases as the skew angle increases.

The above graph for the left exterior girder implies that for a constant Angle of Curvature (α) the deflection due to live load decreases at a rate of 47% as the skew angle (θ) increases from 15° to 60° .

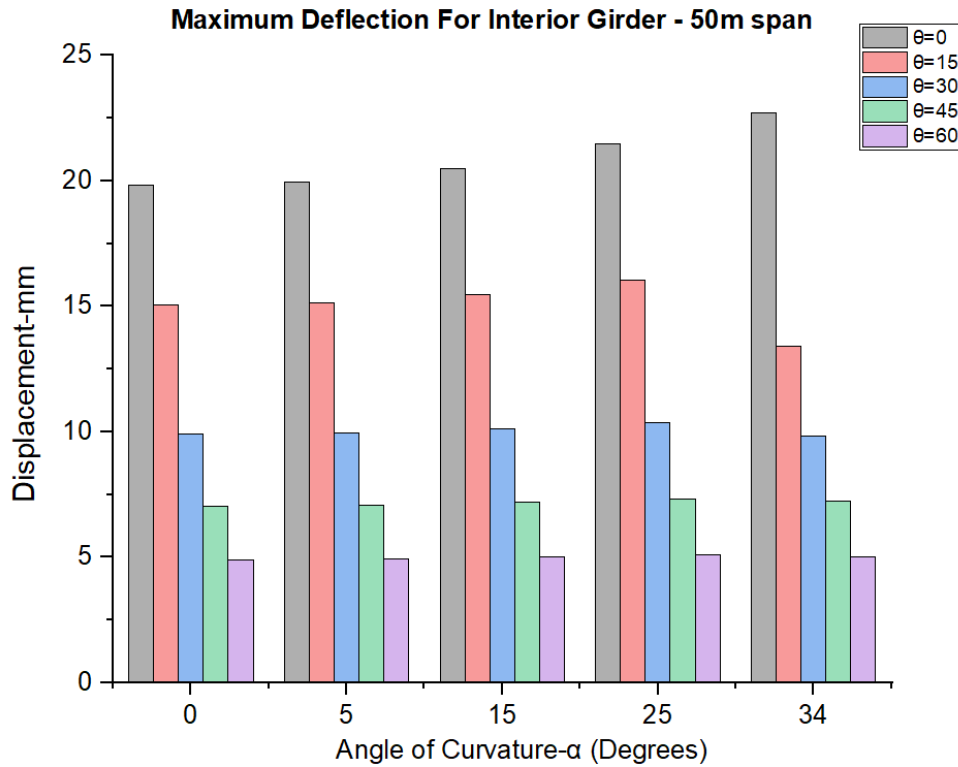


Figure 38: Maximum Vertical Displacement for Interior Girder of 50m span

The results for the right exterior girder also reflected that deflections for the curved bridges are significantly higher than skewed and skew-curved bridges. The study has also solidified the fact that deflection for a skew bridge decreases as the support skew angle increases. The above graph for the interior girder implies that for a constant Angle of Curvature (α) the deflection due to live load has decreased at a rate of 45% as the skew angle (θ) increases from 15° to 60° .

4.2.5 Bridge Reaction Response

The next task was to study the support reaction behaviors of skew-curved bridges. This helps in understanding the behavior of the bridges upon analyzing and designing substructures components. The peculiar nature of these bridges imposes an additional task of assessing every response of the bridge as much as possible.

To study the reaction forces of the bridge two load cases were selected. The first case assesses reaction forces under the effect of the self-weight of the bridge alone. The second case study is

the behavior of the support reaction forces under the service I load combination. However, contrary to previous analyses, only one of the lanes will be loaded.

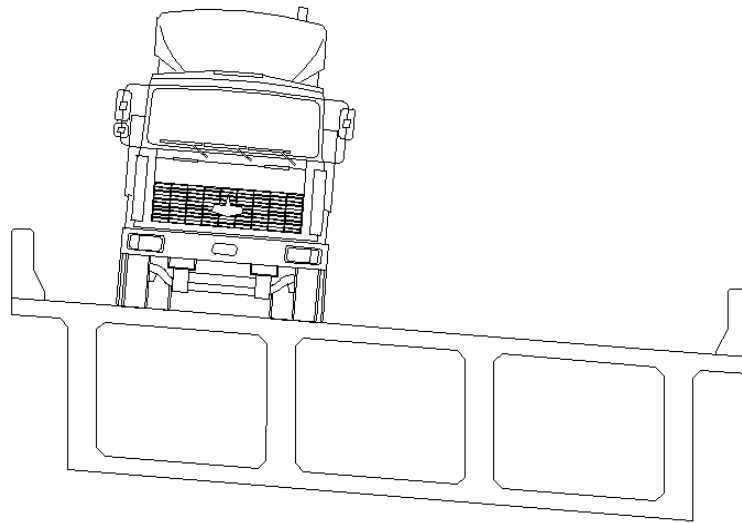


Figure 39: Vehicular Loading position (one lane) for assessing reactions

The vehicular load is not different from the previous analysis. The vehicle is programmed to stay only in its lane. There is no possibility of the vehicle accessing the second lane. The vehicle is placed at 0.6m from the edge of the design lane (*AASHTO LRFD Article 3.6.1.3*). This part of the analysis is done on models that have a sharper curve i.e., the generated models are a combination of skew angles (0° , 15° , 30° , 45° and 60°) and an Angle of Curvature of (25° and 34°). All span lengths 0 to 50m are assessed.

Figure 40 illustrates the support reaction annotations and positions. This helps in summarizing the findings easily and makes a clear distinction between each girder reaction. Reactions that align with the positive Z axis will have a positive value while negative values are given for reactions in the opposite direction to the positive Z axis.

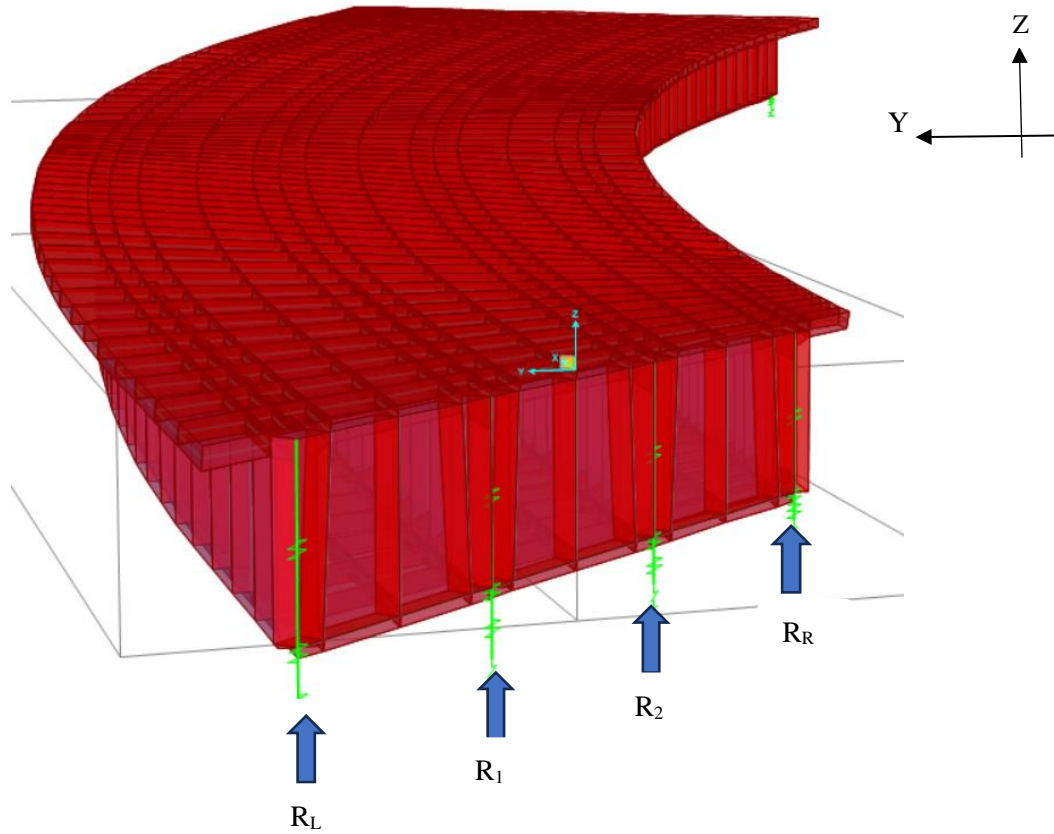


Figure 40: Support reaction Notation

Table 8: Support Reaction Forces with One Lane Loading for 10m Span Bridge

		SUPPORT REACTIONS @ Abutment X=0								
		Model Specification	Self-weight Load				Service I Load Combination			
			R_L	R_1	R_2	R_R	R_L	R_1	R_2	R_R
Span	Skew (θ)	Angle of Curvature (α)	kN	kN	kN	kN	(kN)	kN	kN	kN
10	0	0	199.50	237.62	237.62	199.50	366.79	380.66	279.52	242.73
		25	235.53	250.37	229.87	158.49	424.4	407.34	272.14	186.27
		34	246.52	254.95	228.77	143.71	451.43	418.45	271.37	168.15
	15	0	276.37	244.70	235.47	121.84	455.78	394.77	269.72	150.72
		25	140.14	256.20	250.66	238.73	263.59	414.30	308.14	294.60
		34	181.71	260.50	247.85	200.33	334.91	430.00	308.30	242.01
	30	0	314.89	253.20	240.50	83.73	508.17	383.61	268.36	109.77
		25	89.82	268.71	270.09	282.48	167.31	451.72	344.66	349.67
		34	140.87	274.37	269.32	235.06	256.84	466.72	344.89	284.43
	45	0	319.00	263.90	254.87	84.93	515.42	403.23	279.61	113.85
		25	88.34	291.59	296.24	285.88	151.00	478.24	383.98	341.96
		34	125.71	297.50	305.25	249.85	202.86	489.78	396.62	289.99
60	0	307.20	286.12	284.66	112.79	499.11	409.24	304.99	149.43	
	25	317.23	311.79	323.15	116.15	543.84	445.54	351.98	153.56	
	34	279.91	310.89	323.11	136.49	510.78	450.16	354.25	176.52	

Table 9: Support Reaction Forces with One Lane Loading for 20m Span Bridge

SUPPORT REACTIONS @ Abutment X=0										
Model Specification			Self-weight Load				Service I Load Combination			
Span	Skew (θ)	Angle of Curvature (α)	R _L	R _I	R ₂	R _R	R _L	R _I	R ₂	R _R
			kN	kN	kN	kN	kN	kN	kN	kN
20	0	0	389.70	425.90	425.90	389.70	591.51	630.33	502.40	475.32
		25	482.96	449.15	414.66	288.49	730.92	664.85	489.57	340.41
		34	513.52	459.06	413.00	250.89	777.94	680.04	488.36	296.53
	15	0	-93.96	400.82	449.83	879.21	-110.57	606.97	545.85	1115.32
		25	-75.20	438.58	477.03	809.40	-94.69	660.70	585.38	1028.47
		34	172.56	447.76	447.02	585.04	309.71	674.28	547.31	729.78
	30	0	-215.65	397.77	451.65	1017.24	-227.66	609.64	561.33	1292.18
		25	-243.04	443.65	499.82	970.53	-313.39	674.81	629.49	1238.11
		34	-1.44	455.97	481.38	947.20	77.88	693.45	607.14	941.97
	45	0	-133.55	414.34	454.60	947.05	-177.48	634.14	578.80	1192.28
		25	-162.38	457.09	514.79	919.04	-226.00	717.88	663.62	1111.54
		34	-26.68	473.17	522.59	781.21	-47.04	720.20	675.90	979.82
	60	0	-19.46	439.88	495.27	796.52	55.11	648.92	647.93	976.84
		25	-20.64	486.24	605.92	768.72	34.54	718.10	797.79	938.83
		34	-53.72	529.62	584.21	611.65	86.09	744.46	851.00	992.22

Table 10: Support Reaction Forces with One Lane Loading for 30m Span Bridge

SUPPORT REACTIONS @ Abutment X=0										
Model Specification			Self-weight Load				Service I Load Combination			
Span	Skew (θ)	Angle of Curvature (α)	R _L	R _I	R ₂	R _R	R _L	R _I	R ₂	R _R
			kN	kN	kN	kN	(kN)	kN	kN	kN
30	0	0	582.83	611.26	611.26	582.83	823.35	580.53	719.67	705.52
		25	784.04	655.89	584.98	369.68	1083.49	909.07	688.38	463.96
		34	854.16	672.4	579.04	290.71	1175.14	930.82	681.52	345.10
	15	0	-824.64	520.82	660.75	2037.5	-1073.77	593.61	904.34	2649.21
		25	-807.65	581.07	713.78	1928.79	-1019.24	831.84	868.69	2416.03
		34	-107.29	587.58	809.81	1325.27	-127.61	814.48	739.90	1652.71
	30	0	-917.38	481.07	621.36	2279.73	-1246.20	537.32	831.89	2967.90
		25	-1022.87	550.70	705.33	2218.02	-1308.63	809.27	878.32	2788.16
		34	-480.77	559.86	834.12	1741.13	-623.62	823.79	790.97	2191.48
	45	0	-615.52	464.66	571.23	2023.93	-787.20	707.42	727.36	2527.79
		25	-668.64	530.81	661.94	1982.24	-867.10	795.31	845.93	2483.44
		34	-417.37	552.54	655.26	1728.85	-553.82	824.12	839.83	2170.25
	60	0	-971.38	481.07	621.36	2279.73	-1228.90	720.01	768.70	2855.26
		25	-224.49	521.02	719.73	1612.70	-302.36	775.25	945.66	2002.29
		34	-132.39	541.85	749.22	1511.45	-189.29	802.57	985.48	1874.09

Table 11: Support Reaction Forces with One Lane Loading for 40m Span Bridge

SUPPORT REACTIONS @ Abutment X=0										
Model Specification			Self-weight Load				Service I Load Combination			
Span	Skew (θ)	Angle of Curvature (α)	R _L	R ₁	R ₂	R _R	R _L	R ₁	R ₂	R _R
			kN	kN	kN	kN	(kN)	kN	kN	kN
40	0	0	777.66	794.91	794.91	777.66	1051.08	1062.84	935.16	933.59
		25	1138.19	867.92	744.92	402.24	1502.82	1153.43	1014.38	615.35
		34	1221.69	942.70	723.29	267.44	1601.42	1251.39	849.89	320.24
	15	0	-2216.97	589.27	875.41	3904.15	-2216.97	589.27	875.41	3904.20
		25	-2214.45	674.01	966.15	3752.93	-2741.94	940.25	1166.84	4631.70
		34	-837.68	874.83	727.72	2612.77	-1036.80	942.45	877.55	3227.60
	30	0	-2235.58	477.72	763.69	4164.09	-2773.44	720.59	940.69	5137.70
		25	-2313.82	591.25	880.20	4059.23	-2885.98	858.94	1086.05	5025.10
		34	-1475.27	600.35	729.20	3367.94	-1857.39	871.26	905.22	4182.80
	45	0	-1393.65	452.22	562.19	3584.83	-1739.84	703.07	721.49	4423.10
		25	-1467.92	550.51	693.51	3499.28	-1844.41	822.72	886.06	4329.20
		34	-1091.08	536.02	678.61	3165.34	-1386.16	807.13	869.46	3928.40
	60	0	-509.86	409.30	562.69	2815.36	-643.02	649.28	752.45	3462.40
		25	-549.57	463.85	702.51	2774.21	-700.52	717.30	928.44	3419.80
		34	-430.93	502.01	722.09	2629.06	-558.90	764.02	955.03	3246.40

Table 12: Support Reaction Forces with One Lane Loading for 50m Span Bridge

SUPPORT REACTIONS @ Abutment X=0										
Model Specification			Self-weight Load				Service I Load Combination			
Span	Skew (θ)	Angle of Curvature (α)	R _L	R ₁	R ₂	R _R	R _L	R ₁	R ₂	R _R
			kN	kN	kN	kN	(kN)	kN	kN	kN
50	0	0	975.49	975.57	975.57	975.49	1276.32	1272.80	1146.70	1163.00
		25	1535.47	1075.65	897.56	403.14	1948.65	1393.16	1053.10	479.00
		34	1732.48	1112.23	876.92	192.26	2191.38	1437.20	1029.10	235.30
	15	0	-4484.62	519.61	1108.82	6766.03	-5461.82	757.87	1327.00	8235.20
		25	-4607.30	754.01	1299.62	6500.33	-5627.50	1035.48	1560.10	7926.46
		34	-2332.07	670.45	832.56	4769.61	-2860.11	937.86	1001.90	5831.30
	30	0	-4035.78	400.59	840.19	6723.78	-4936.35	634.77	1031.60	8200.34
		25	-4274.83	644.72	1038.71	6581.51	-5245.33	924.86	1273.50	8046.88
		34	-3046.07	539.43	775.03	5723.46	-3763.58	800.58	959.40	7021.53
	45	0	-2387.48	289.74	519.28	5534.03	-2932.08	527.63	678.80	6751.95
		25	-2551.93	487.75	670.21	5438.19	-3145.85	854.41	861.60	6650.90
		34	-2109.45	452.49	610.00	5102.89	-2612.85	700.52	799.20	6241.90
	60	0	-919.45	239.40	368.78	4350.07	-1134.08	518.31	534.50	5302.75
		25	-977.30	332.39	506.19	4294.20	-1213.15	570.51	704.90	5246.60
		34	-842.98	357.27	545.45	4118.69	-1054.41	599.91	751.00	5042.70

The results revealed that apart from the conventional straight bridge [*non-skewed with no curvature*] the support reactions of the girders are not uniformly distributed. This is true for both skewed and skew-curved bridges. This is mainly because loads tend to take the shortest path to the support. Due to this, the reaction at the obtuse corner (R_R) of a skewed & and skewed curved bridge will be much greater than the one found at the other supports (R_L).

The above table has also implied that the reactions at the acute corner (R_L) for bridges with severe support skewness and bridges having severe support skewness coupled with sharp planner curvature exhibit a reaction force that is directed downward. This is usually called an uplift force. The uplift force seen on the left girder is observed in all bridges except for the 10m span bridges.

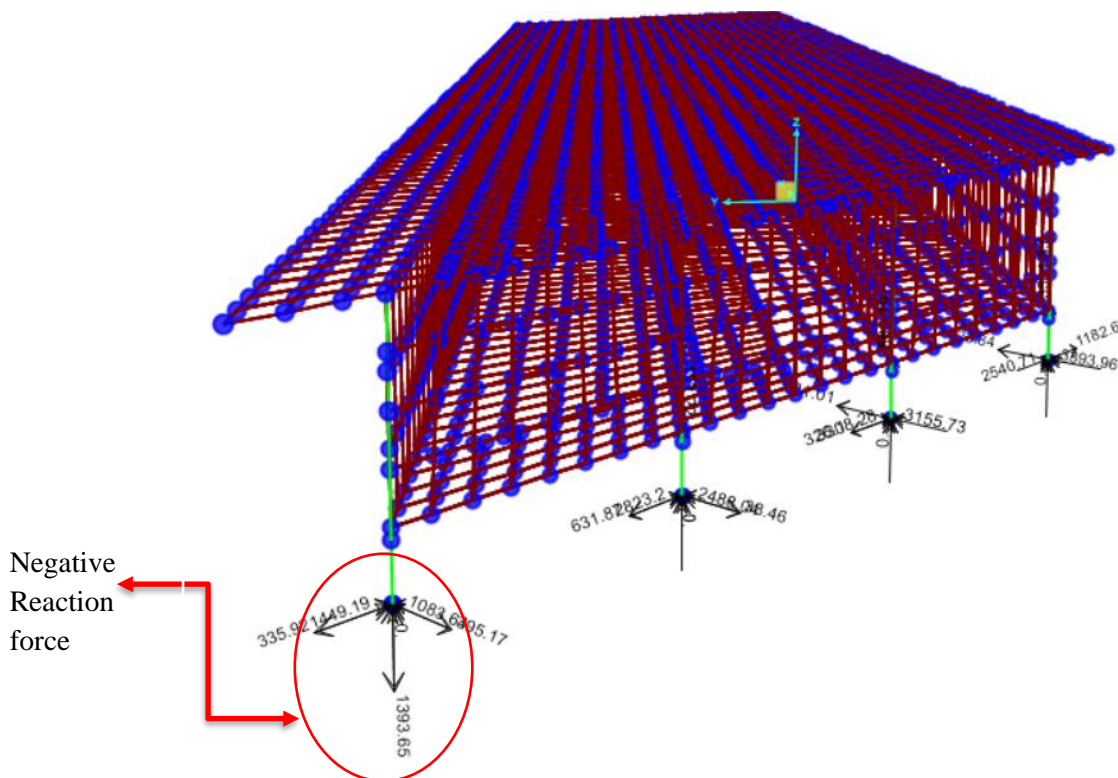


Figure 41: Support reaction force for $\theta=60^\circ$ and $\alpha=0^\circ$ under the effects of self-weight

The negative reaction force is witnessed even under the effect of the self-weight. The tables also show that the uplift force is exaggerated under the service load combination.

The results presented in the above tables indicate an uplift phenomenon is in good agreement with research findings forwarded by (Lucas Richard Miner, 2014). Thus, this issue shall be well addressed. To avoid such impact on bearings proper design of anchorages to the girders is also required.

The tabulated data presented above also indicate that for a skew curved bridge, the reaction of the girder found on the obtuse corner is about 29.83%, 39.32%, and 46.67% of the girder reaction found on the acute corner for 30m, 40m, and 50m span bridge respectively.

4.2.6 Combined Action of Shear and Torsion

On its bridge design specification issued in 2012, the American Association of State Highway and Transportation Officials (AASHTO) forwarded an empirical equation to determine the critical torsional moments of bridges. Article 5.8.2.1-3 of the aforementioned manual states that “If the factored torsional moment is less than one-quarter of the factored pure torsional cracking moment, it will cause only a very small reduction in shear capacity or flexural capacity and, hence, can be neglected.” For normal-weight concrete, torsional effects shall be investigated where: $T_u > 0.25 * \phi * T_{cr}$. In which,

$$T_{cr} = 0.12 \frac{A_{cp}^2}{P_c} \sqrt{1 + \frac{f_{pc}}{0.125 \sqrt{f_c}}} \quad (\text{Equation-3})$$

Where:

T_{cr} : torsional cracking moment

A_{cp} : total area enclosed by the outside perimeter of the concrete cross-section

P_c : the length of the outside perimeter of the concrete section

f_{pc} : compressive stress in concrete after prestress losses have occurred

f_c' : cylindrical compressive strength of concrete

Φ : 0.9; resistance factor specified in Article 5.5.4.2 of *AASHTO LRFD 2012*

The *AASHTO LRFD 2012* Bridge specification has also stated that the equivalent factored shear force V_u , for the box section can be determined as follows:

$$V_u + \frac{T_u ds}{2A_0} \quad (\text{Equation-4})$$

Where:

V_u : equivalent factored shear force

T_u : factored torsional moment

A_0 : area enclosed by the shear flow path

ds: effective depth of girder

In previous discussions, the paper has presented that for a skewed curved bridge geometry, the effect of shear and Torsion is gravely seen on the left exterior girder which is the outer girder. Henceforth, the combined shear and torsion effect is investigated for this specific girder. This will help in assessing the maximum possible shear effect that can be induced due to the combination of the two force effects.

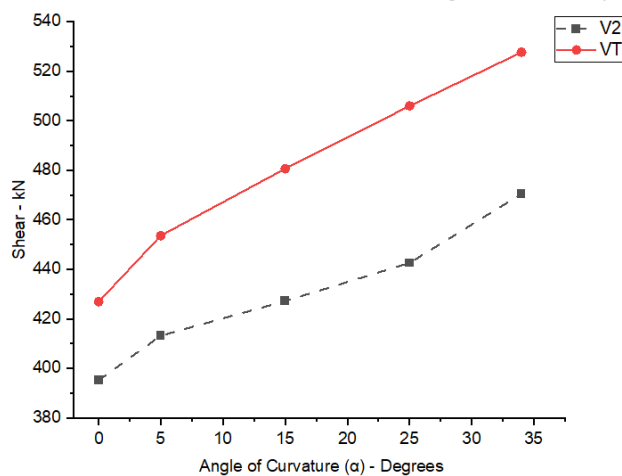
Considering the geometrical values of the bridge (Specified in Table 2) the critical torsional moment of the bridge is determined to be **4859.16 kNm**

$$0.25 * T_{cr} * \phi = 1093.33 \text{ kNm}$$

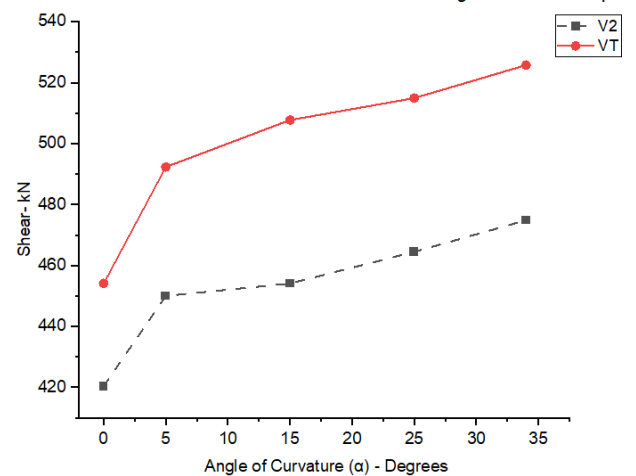
If the above demarcation value is to be considered the shear effect of torsion on skew-curved bridges shall not be investigated for bridges with spans 10,20 and 30m. This is because the maximum Torsion recorded on these bridges is less than the threshold value specified above. However, the assessment shows that for the aforementioned spans, the shear effect of torsion on the left exterior girder has increased the total shear force on average by 10.64%, 10.46%, and 10.34% for the 10m, 20m, and 30m span bridges respectively. This increment is seen in all combinations of the Angle of Curvature (α) and skew angle (θ) combination.

This force effect is not something that can be ignored. Thus, despite the guiding manual recommendation the torsional effect on shear shall not be overlooked when dealing with skew curved bridges. The following graphs will illustrate the combined shear and torsion effects for skew angle (θ) 45° and 60°

Shear Effect of Torsion on Left Exterior Girder - 45 Degree skew - 10m Span



Shear Effect of Torsion on Left Exterior Girder - 60 Degree skew-10m Span



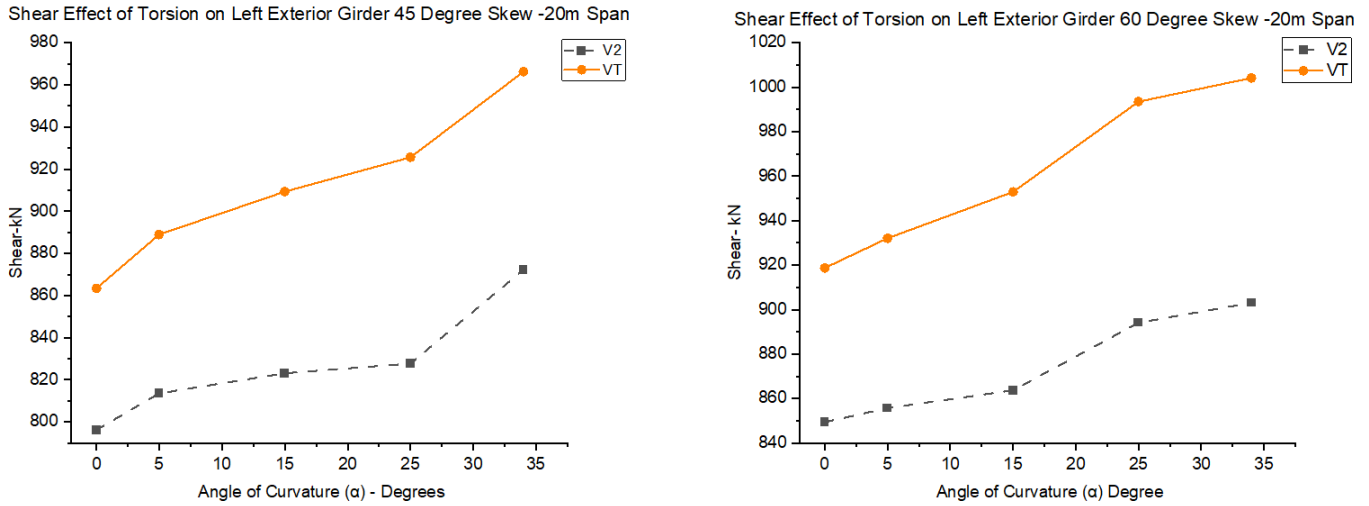


Figure 42: Shear effect of torsion on 10 and 20m span bridges

However, for bridge spans that are 40 and 50m, the torsional values have surpassed the critical torsional moments mentioned above. The following graphs will illustrate the shear effects of torsion on the left exterior girder for a 40m span bridge. Although all skew and Angle of Curvature combinations are assessed the following graphs will present the effects for skew angle (θ) of 45^0 and 60^0 where the maximum effects have been seen for service load combination.

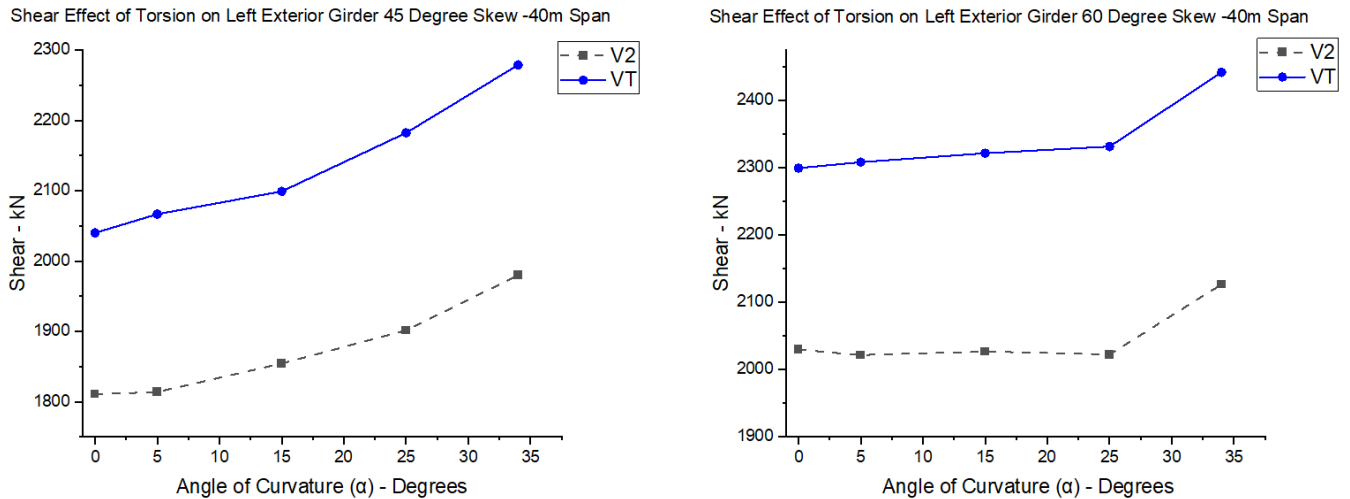


Figure 43: Shear effect of torsion on 40m span bridges

It can be seen that the effect of the torsional moment on the shear force for the exterior girder is very significant. The shear effect of torsion on the left exterior girder has increased the total shear force on average by 12.61% for 40m bridges. This increment is also seen in all combinations of the Angle of Curvature (α) and skew angle (θ) combination.

The graphs also indicate that for a given span length and support skewness of a bridge the combined effect of shear and torsion increases with the incorporation of curvature.

The results for the above assessment indicate that the torsional contribution by the curved nature of the bridge is significant and could not be overlooked in studying the overall responses of the skew curved bridges.

Efficiency of Skew correction Factor [Cf]

It is mentioned that the only method one can acquire from the manuals for handling skew-curved bridges is the adoption of the skew correction factor. This accounts for the shear force increment on the obtuse corners of such bridges. As it is stated on numerous occasions throughout the study, the effect of curvature on the responses of such bridges is ignored for an Angle of Curvature that is less than 34⁰.

It shall be noted that this correction factor is only forwarded for adjusting the live load distribution factor while a spine model is used. That is to say, a designer assumes the bridge is a spine model and determines the total live load effects. Next, distribution factors will be acquired by using various empirical equations forwarded in section 4.6.2.2.3b-1 of the *AASHTO LRFD Manual*. The skew correction factor comes into play here; in magnifying the amount of shear force that is taken by the exterior girder to account for the obtuse corner effect.

The skew correction factor for the exterior girder can be determined by Equation 5 specified in Table 4.6.2.2.3c-1 of *AASHTO LRFD,2004*. The skew correction factor is applied to the live load force effects determined for a spine model (straight bridge) with ($\alpha=0$ & $\theta =0$). The shear magnification factor for the obtuse girder of the bridge is:

$$C_f = 1.0 + \left(0.25 + \frac{L}{70d}\right) \tan \theta \quad (\text{Equation-5})$$

Where:

θ : skew angle of support

d: depth of girder (mm)

L: Length of a bridge (mm)

The results reflected that the skew correction factor in union with the distribution factors (*approximation method*) method failed to predict combined shear force results of bridges with

a span 20m above. The following selected graphs attempt to present the comparative results for the outer exterior girder shear force.

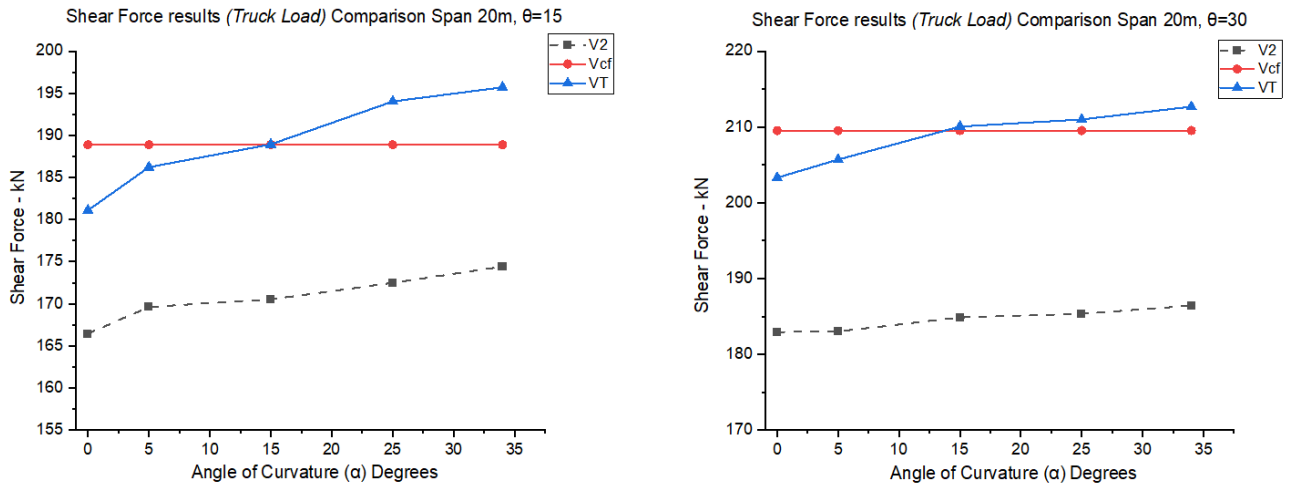


Figure 44: Combined shear and torsion action for left exterior Girder 20m span Bridge

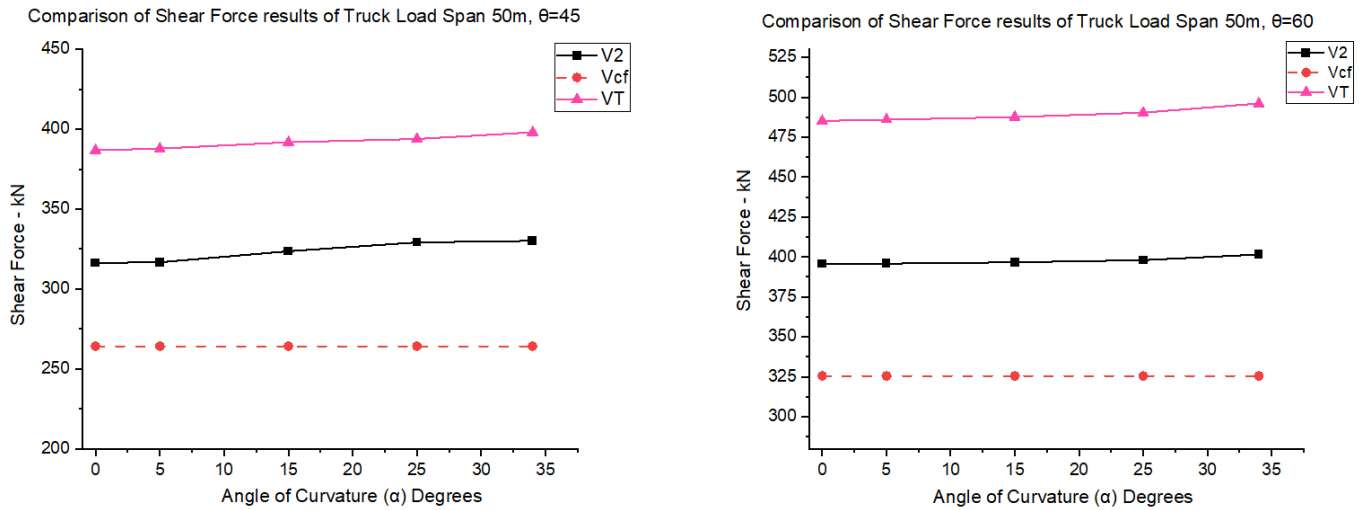


Figure 45: Combined shear and torsion action for left exterior Girder 50m span Bridge

The results and above graphs clearly show that the skew correction factor shall not be relied on in determining the shear responses of skew-curved bridges with spans 20m and greater.

Given the above issue the paper has assessed; it will be a more comprehensive approach to forward a means to address it as well. Therefore, with the data sets already generated a linear regression was performed.

A relation was derived for the shear force effect on outermost girders for truckload cases. The shear effect was determined to be the most critical issue for a skew-curved bridge. The correction factor tries to cover the bridge parameters of span lengths (L), skew angle (θ), and an Angle of Curvature (α). This relation has a coefficient of determination (R^2) of 0.9006.

$$C_v = 0.592 + 0.017 L + 0.43 \tan \theta + 0.211 \tan \alpha \quad (\text{Equation-6})$$

Where:

C_v : Correction factor for vehicular (*Truck Load*) shear Force

θ : Skew angle (*in Degree*), $0^\circ \leq \theta \leq 60^\circ$

α : Angle of Curvature (*in Degree*), $\alpha \leq 34^\circ$

L: Span of Bridge (*in meters*), $0 < L \leq 50m$

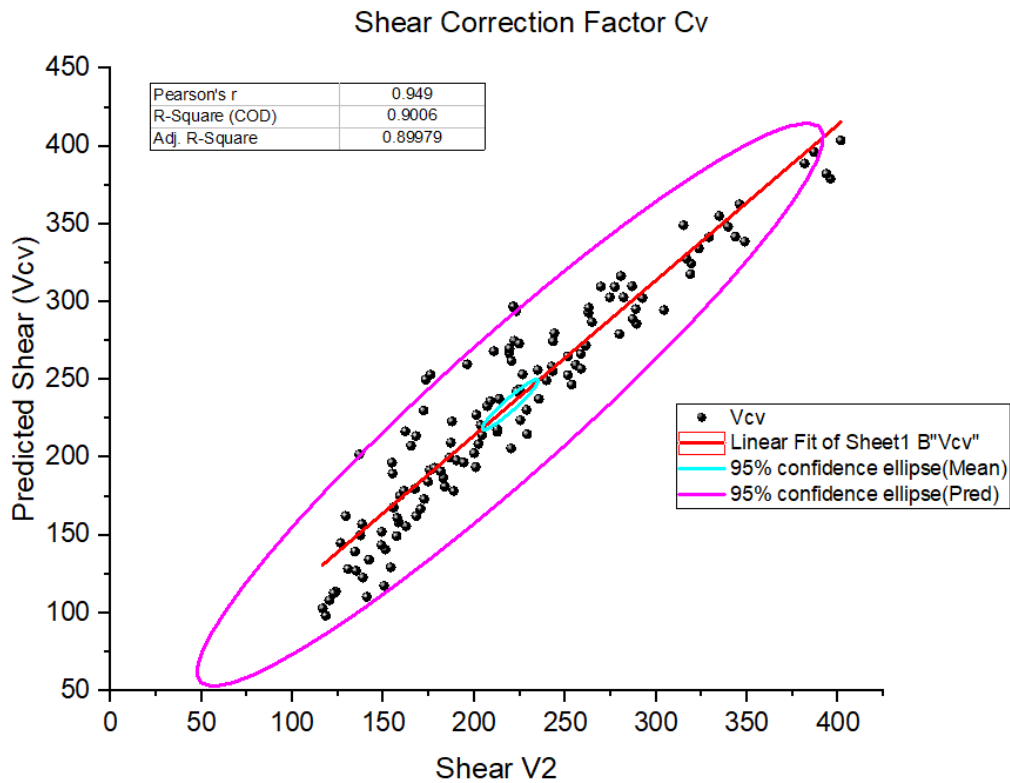


Figure 46: Shear correction factor for outer girders of skew curved bridges

The above correction factor can only correct the transversal shear force (V_2) of a straight bridge. In other word it will predict the shear force values of a straight bridge to the required skew curved bridge with a similar span length. However, as stated in the findings the torsional effect on such bridges is significant. Thus, the paper has tried to find the relevant magnification factor to account for the shear effect of torsion.

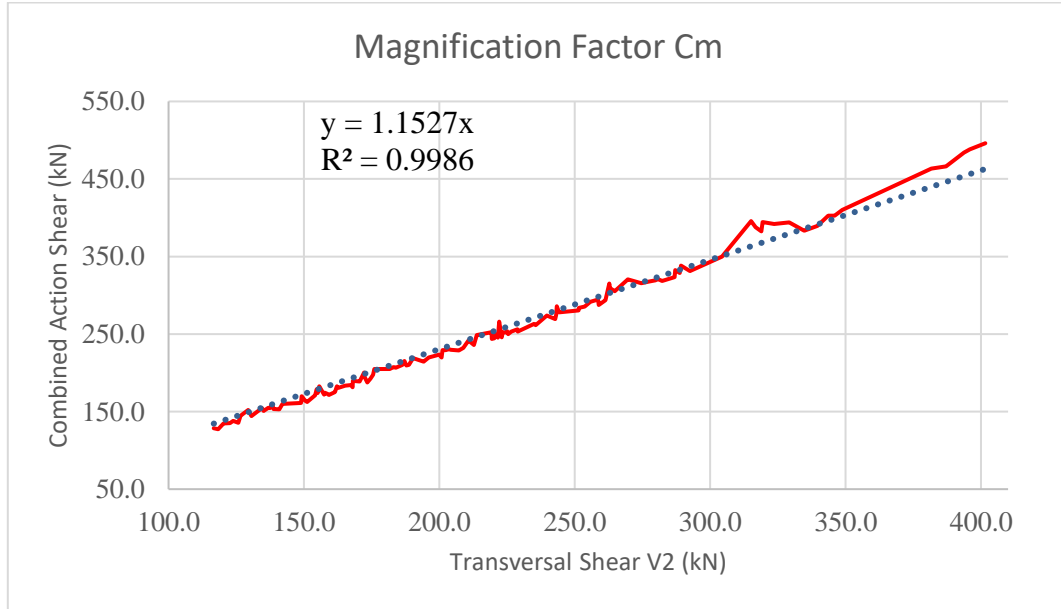


Figure 47: Magnification factor to predict total shear force

As can be seen from the above graph, the total shear force on the outer girder V_T is about 15% of the shear force not accounting for the torsional effects (V_2). Hence it is acceptable to have a magnification factor of C_m 1.15.

This factor shall be used in conjunction with the correction factor specified in equation 6 to have the total vehicular shear load on skew curve bridges. In this regard, the designer will be able to predict the total shear force on the outer girders of skew-curved bridges.

Hence, the total vehicular (Truck Load) design shear force on the outer girder of skew-curved bridges will be given by:

$$V_T = C_m C_v V_2 \quad (\text{Equation-7})$$

Where:

V_2 : Shear force on the outer girder of straight bridges

C_v : Shear correction factor (Equation-6)

C_m : Shear magnification factor = 1.15

By utilizing Equation-7, designers can effortlessly calculate the shear force on the outer girder caused by the HL-93 truck load for a required skew curved bridge, using only the shear force of a straight bridge with a similar span length. This equation is applicable across all specified parameter ranges.

Table 13: Summary of research findings compared with the code recommendations

Bridge Type	AASHTO/ERA	This Study	Remarks
Skew	Skew correction factor for shear $cf=1+0.25+L/70d) \tan \theta$	-	-
Curve	Ignores curvature less than 34°	-	The effects of in-plan curvature less than 34° on the shear response of the outermost, is not something to be ignored.
Skew Curved	No directives provided	Correction factors (C_v) & and magnification factor (C_m) are provided for the shear responses of the outermost exterior girder under Truck Load. $C_v = 0.592+0.017 L+ 0.43 \tan \theta + 0.211 \tan \alpha$ $C_m = 1.15$	<u>Range of Applicability</u> $0^{\circ} \leq \theta \leq 60^{\circ}$ $\alpha \leq 34^{\circ}$ $0 \leq L \leq 50m$ Design speed $\leq 50km/hr.$ Girder spacing $\leq 2.4m$

CHAPTER FIVE

5. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The results of the research work have led to the following noteworthy conclusions.

- The current design trends, which overlook the impact of curvature and rely on skew correction factors, are inadequate for accurately capturing the live load shear effects of skew-curved bridges with spans exceeding 20m.
- The support reactions of skew-curved bridges are not proportional to all girders. The girder located at the obtuse corner of the bridge has experienced the highest reaction force compared to the other three girders.
- The Left exterior girder (*acute corner*) has experienced a negative (*downward*) reaction force even under the effect of bridge self-weight. This phenomenon is magnified when the bridges are loaded with one-lane loading. This peculiar response of skew-curved bridges causes an uplift at the acute corner of the bridge.
- The incorporation of in-plan curvature on skewed bridges increases the torsional moment for exterior girders.
- Torsional moments for a bridge with a constant Angle of Curvature, increased as the support skew angle increased. The rate of increment is not significant on bridges with smaller span lengths. However, for span lengths above 30m, a considerable increment in torsional moment has been noted as skew angles increase for a constant Angle of Curvature.
- The effects of torsion on the shear force response of the bridge are very substantial and cannot be ignored. A 15% increment in shear force due to the torsional effect has been observed under truck live loading.
- The shear correction and magnification factor provided by the paper can be used to predict the shear force experienced by the outer girders of skew-curved bridges under live loading.

5.2 Recommendations

This study can be further expanded to address issues that are not addressed here. The following are the recommendations based on the findings of the paper.

- ↪ Professionals need to understand that the structural response of skewed curved bridges cannot be accurately determined using the approximation method provided in design manuals, particularly for the shear force impact on the outer girders. Therefore, a thorough investigation of the structural performance of skewed-curved bridges should be conducted using a *three-dimensional finite element analysis*.
- ↪ The responses of skew-curved bridges shall be investigated for a continuous bridge.
- ↪ The effects of such geometries on designing bearings and other substructure components shall be investigated extensively.
- ↪ The responses of these bridges under seismic & and wind loads shall be studied.

References

AASHTO (2012). American Association of State Highway and Transportation Officials LRFD Bridge design specifications.

AbdulMutlib I. Said and Hashim Khalaf Lateef., (2018). Experimental Study for Horizontally Curved Box Girder Bridges with Special Reference to the Live Load Moment Distribution Factor. Journal of the Association of Arab Universities for Basic and Applied Sciences.

Alyaa Shatti, (2017). the structural Behavior of Horizontally curved prestressed concrete Box girder bridges” University of Salford, United Kingdom.

Ashebo, DB, Chan, TH, Yu, L, (2007). Evaluation of Dynamic Loads on a Skew Box Girder Continuous Bridge Part I: Field Test and Modal Analysis. *Engineering Structures*.

Bishara, A.G. and Liu, M.C. and El-Ali, N.D., (1993). Wheel Load Distribution on Simply Supported Skew I-Beam Composite Bridges, Journal of Structural Engineering.

Bouwkamp, JG, Scordelis, AC, Wasti, ST, (1980). Failure Study of a Skew Box Girder Bridge Model, *IABSE Congress Report*.

Brown, TG, Ghali, A, (1975). Semi-Analytic Solution of Skew Box Girder Bridges, *Proc. of Institution of Civil Engineers*.

Chen, (2014) Bridge Engineering Handbook, Second Edition: Superstructure Design.

Christian Menn, (1927). Prestressed Concrete Bridges, Swiss Federal Institute of Technology, Zurich, Switzerland.

Chun, B, Skewed Bridge Behaviors (2010). Experimental, Analytical, and Numerical Analysis, Ph.D. Dissertation in Wayne State University.

Dilger, WH, Ghoneim, GA, Tadros, GS, (1988). Diaphragms in Skew Box Girder Bridges, Canadian Journal of Civil Engineering.

Ebeido, T. and Kennedy, J.B. (1996). Girder moments in continuous skew composite bridges", Journal of Bridge Engineering,

ERA (2013). Ethiopian Roads Administration Bridge design manual.

He, XH, Sheng, XW, Scanlon, A, Linzell, DG, Yu, XD. (2012). Skewed Concrete Box Girder Bridge Static and Dynamic Testing and Analysis, Engineering Structures.

Helba, A. and Kennedy, J.B., (1995). Skew composite bridges - analyses for an ultimate load, Canadian Journal of Civil Engineering.

Hodson, DJ, Barr, PJ, Halling, MW, (2011). Live-Load Analysis of Posttensioned Box-Girder Bridges, Journal of Bridge Engineering ASCE.

Huo, X.S., Zhang, Q. (2008). Effect of Skewness on the Distribution of Live Load Reaction at Piers of Continuous Skewed Bridges, Journal of Bridge Engineering.

Iman Mohseni, Abdul Khalim Abdul Rashid, and Junsuk Kang, (2014). Effect of Intermediate Diaphragm on Lateral Load Distribution Factor of Multicell Box-girder Bridges, KSCE Journal of Civil Engineering.

Li, L. and Ma, Z. J. (2010). Effect of intermediate diaphragms on decked bulb-tee bridge system for accelerated construction. J. Bridge Engineering.

Lydzinski J, Baber T. (2008). Finite element analysis of the Wolf Creek multi-span curved girder bridge. Charlottesville, Virginia: Virginia Transportation Research Council.

Menassa, C., Mabsout, M., Tarhini, K. and Frederick, G., (2007). Influence of Skew Angle on Reinforced Concrete Slab Bridges, Journal of Bridge Engineering.

Miner LR. (2014). Effect of abutment skew and horizontally curved alignment on bridge reaction forces Doctoral dissertation, California State University, Sacramento.

Mohammed Ameen Zaki (2016). Live load distribution factors for horizontally curved concrete box girder bridges. University of Massachusetts Amherst.

M. R. Wallace (1973). Studies of Skewed Concrete Box-Girder Bridges, California Department of Transportation, Research and Development Supplemental Report.

Robert G. Sargent (2008). Verification and validation of simulation Models, Syracuse University.

R.C Hibbeler (2006). Structural Analysis, Pearson Hall Inc.

Schaffer, T., (1967). Structural Response of A 45° Skew Prestressed Concrete Box-Girder Highway Bridge Subjected to Vehicular Loading Brookville Bridge, Fritz Laboratory Reports.

Scordelis, AC, Bouwkamp, JG, Wasti, ST, Seible, F, (1982). Ultimate Strength of Skew RC Box Girder Bridge, Journal of the Structural Division ASCE.

Soliman, MI, Abdel, FAS, Abdel, WOH, El-Behairy, HS, (2000). Warping Displacement of Skew Reinforced Concrete Box Girder Bridges.

Tanmay Gupta & Manoj Kumar (2017). Structural Response of Concrete Skew Box-Girder Bridges. International Journal of Bridge Engineering.

Tanmay Gupta, Manoj Kumar, (2018). Structural Response of Skew-Curved Concrete Box-Girder Bridges under Eccentric Vehicular Loading.

Wasti, ST, Scordelis, AC (1984). Comparative Structural Behavior of Straight, Curved, and Skew Reinforced Concrete Box Girder Bridge Models.