



**THE CONTRIBUTION OF DEGENERATE  
ELECTRON PRESSURE TO THE STABILITY OF  
THIN KEPLERIAN ACCRETION DISKS  
AROUND A NEUTRON STAR WITH  
AXISYMMETRIC MAGNETIC DIPOLE**

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# Abstract

The stability analysis of a geometrically thin, gas-pressure dominated accretion disk around a neutron star is presented. In purely radial perturbation case, thin disk is stable to thermal modes. The stability is analyzed at a small temperature, that is temperature approaching zero and at finite temperature. The contribution of both fully and partially degenerate electrons for the stability of the disk in its middle and outer regions are investigated. We find that the disk is stable in these regions, where the gas pressure is more dominant than radiation pressure.

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# Chapter 1

## Introduction

An accretion disk is a structure (often a circumstellar disk) formed by diffuse material in orbital motion around a central body. The central body is typically a younger star, a protostar, a white dwarf, a neutron star, or a black hole. The accretion disk is likely to be formed when the compact star is a member of a close binary system and matter transformed from a giant-type star on to its compact companion at high angular momentum. Following the pioneering works by Pringle and Rees (1972) [1], Shakura and Sunyaev(1973) [2], and Novikov and Thorne(1973) [3], a number of theoretical studies on the accretion disk has been made to account for observation of the X-ray sources. The importance of accretion is further manifested by the realization that probably a majority of all stars are members of binary systems which, at some stage of their evolution, undergo mass transfer. The study of interacting binary systems has revealed the importance of angular momentum in accretion. In many cases the transferred material can not land on the accreting star until it has rid itself of most of its angular momentum. This leads to the formation of accretion disks, which turn out to be efficient machines for extracting gravitational potential energy and converting it in to radiation.

There are two main reasons why many binaries transfer matter at some stage of their evolutionary life times [4]:

- (1) one of the stars in a binary separation shrinks, to the point where the gravitational pull of the companion can remove the outer layers of its envelope (Roche lobe overflow);

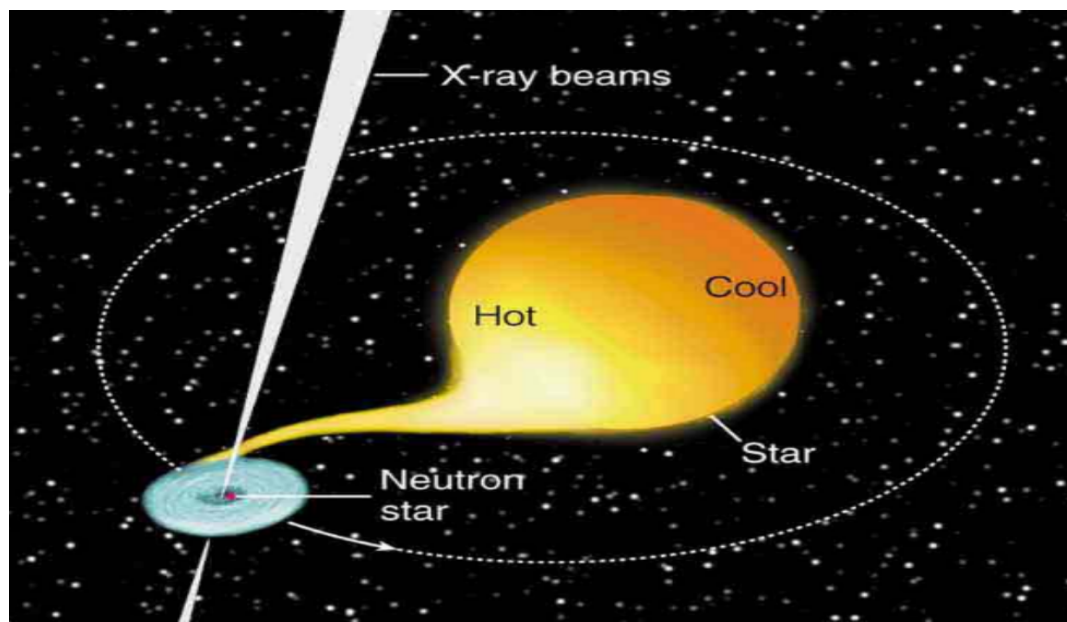


Figure 1.1: Accretion disk around a neutron star.

(2) one of the star may, at some evolutionary phase, eject much of its mass in the form of a stellar wind, some of this material will be captured gravitationally by the companion (stellar wind accretion).

The stability of geometrically thin accretion disk has been studied extensively and it has been found that the disk is thermally and viscously unstable if it is optically thick and radiation pressure dominated (Pringle, Rees and Pacholczyk, 1973; Lightman and Eardley, 1976) [5]. There is also a possible mode of pulsational over stability. In this case, one looks for instability in which oscillation on the orbital time scale grow in amplitude

because of the effects of viscosity(Lin and Paploizou,1996) [6]. Kato(1978) [7] considered the evolution of all three components of the fluid velocity. He found that the disk exits pulsation instability besides the viscous instability and thermal instability. Chan and Tamm(1995) [8] pointed out the galactic black hole candidates may be due to pulsational overstability. Some early analysis about the stability of gas pressure dominated disc have incorporated azimuthal perturbation (Ivov and Shaviv 1977, 1981; van Horn; Wesemal, and Wingert 1980) [9] . However, the radial perturbation were neglected in all those studies. McKee(1991) [10] has investigated the contribution of gas pressure to the stability of a standard disk. He found that the disk is stable when  $\beta \leq 0.6$  (where  $\beta$  is the ratio of radiation pressure to total pressure ). This implies that gas pressure dominated disk is more stable.

## 1.1 Electron Degeneracy Pressure

In order to estimate the electron pressure, we have to take account of the Pauli exclusion principle, which postulates that no two electrons can occupy the same quantum state, that is, have the same momentum and the same spin. Since an electron can have two spin states (up and down), this means that each element of phase space (location and momentum space) can be occupied by two electrons at most. The pressure generated by electrons that are forced in to higher momentum states as their density increases is called *electron degeneracy pressure* [11]. A state of complete degeneracy is obtained when all the available momentum states are occupied up to a maximum momentum value. Such an ideal situation can only be achieved at absolute zero temperature, but it constitutes a good approximation to states of high degeneracy and has the advantage of enabling a straight forward calculation of the pressure. Therefore, although the transition from a Maxwellian to a completely degenerate momentum distribution occurs gradually. The force provided by this pressure sets a limit on the extent to which matter can be squeezed together without it collapsing into a neutron star. When electrons are squeezed too close

together, the exclusion principle requires them to have different energy levels. To add another electron to a given volume requires raising an electron's energy level to make room, and this requirement for energy to compress the material appears as a pressure.

The density of electrons is described by Fermi-Dirac statistics since an electron has half-integral spin. For an electron with momentum  $p$  the density in the range  $(dp)$  can be described by [12]

$$n_e(p)dp = \frac{8\pi}{h^3} p^2 dp \left[ \exp\left(\alpha + \frac{E}{K_B T_c}\right) + 1 \right]^{-1} \quad (1.1.1)$$

From Pauli exclusion principle, two identical electrons can't occupy the same state, that is

$$P(p) = \left[ \exp\left(\alpha + \frac{E}{K_B T_c}\right) + 1 \right]^{-1} \quad (1.1.2)$$

which can't be greater than 1.

### 1.1.1 Non-relativistic Complete Degeneracy

We first consider a simple case which is  $T \rightarrow 0$ . When the density is high enough, all the electron states with energy less than a maximum energy are filled. Applying the Heisenberg and Pauli principle to a completely degenerate isotropic electron gas yields the momentum distribution; the number of electrons with momenta in the interval of  $(p, p + dp)$  per unit volume:

$$n_e(p) = \frac{2}{\Delta V} = \frac{2}{h^3} 4\pi p^2 dp \quad p \leq p_o \quad (1.1.3)$$

The maximal momentum  $p_o$  can be obtained by integration of  $n_e = \int_o^{p_o} n_e(p) dp$  and reversing the relation between  $n_e$  and  $p_o$

$$P_o = \left( \frac{3h^3 n_e}{8\pi} \right)^{1/3} \quad (1.1.4)$$

Using the pressure integral

$$P = \frac{1}{3} \int_o^\infty v p n(p) dp$$

where  $v$  is the particle velocity,  $p$  is it's momentum and  $n(p)dp$  is the number of particles per unit volume with momenta within the interval  $(p, p + dp)$  and substituting

equation(1.1.3), taking  $v = \frac{p}{m}$ , where  $m$  is the electron mass and carrying out the integral up to  $p_o$ , to obtain the electron degeneracy pressure as

$$P_o = \frac{8\pi}{15mh^3}p_o^5$$

$$P_e = \frac{h^2}{20mm_p^{\frac{5}{3}}}\left(\frac{3}{\pi}\right)^{2/3}\left(\frac{\rho_c}{\mu_e}\right)^{5/3} \quad (1.1.5)$$

where  $m$  is mass of electron,  $\mu_e$  is the ratio of electron number to proton number,  $h$  is Planks constant and  $\rho$  is a density. The equation can be simplified to give

$$P_e = 9.77 \times 10^6 \left(\frac{\rho_c}{\mu_e}\right)^{5/3} \quad (1.1.6)$$

We note that the degeneracy pressure is inversely proportional to the particle (electron) mass.

Even at absolute zero, the ideal degenerate Fermi gas pressure does not disappear since only one electron is able to have zero momentum according to the the Pauli exclusion principle, i.e.

$$P = \frac{2U}{3V} = \frac{2}{5}\frac{E}{V}\left[1 + \frac{5\pi^2}{12}\left(\frac{K_B T_c^2}{E}\right) + \dots\right] \quad (1.1.7)$$

Because of this zero point pressure, all the other electrons have some finite momentum. Although the argument presented for electrons could be equally applied to proton and neutrons, as they are nearly 2000 times more massive than electrons, quantum effects become important in their case under much more extreme conditions (much higher densities for a given temperature, and much more lower temperature for a given density), and are usually ignored. We also note that, in spite of the high densities characteristic of degenerate matter, the particles may still be considered free, since the particle energy of the order of  $P_o^2/2m$  is still higher than the Coulomb energy. If the electron density is increased further, the maximal momentum (Fermi momentum) in a completely degenerate electron gas grows larger.

### 1.1.2 Non-relativistic Partial Degeneracy

In the real situation, the temperature is finite, and therefore the system is not in a complete degenerate state. Now we consider partial degeneracy of non-relativistic electrons. That means  $v = \frac{p}{m}$  as in the non relativistic complete degenerate case. The energy of the electron particle can be evaluated as  $\frac{p^2}{2m}$ . Since the electrons are not in a complete degenerate state, there is no strict upper limit of energy level. We have to integrate the density as well as the pressure over all energy levels by extending the momentum upper limit to infinity [13].

$$n_e = \frac{8\pi}{h^3} \int_0^\infty \frac{p^2}{\exp(\alpha + \frac{p^2}{2mK_B T_c} + 1)} dp \quad (1.1.8)$$

where  $n_e$  is number density and  $\alpha$  is a parameter which depends on density and temperature. It approaches negative infinity in completely degenerate gas.

$$P_e = \frac{8\pi}{3h^3 m} \int_0^\infty \frac{p^4}{\exp(\alpha + \frac{p^2}{2mK_B T_c} + 1)} dp \quad (1.1.9)$$

where  $P_e$  is electron degenerate pressure in momentum space for a partially degenerate system.

Defining  $u = \frac{p^2}{2mK_B T_c}$  and separating the integrals in equations (1.1.8) and (1.1.9), we get

$$n_e = \frac{4\pi}{h^3} (2mK_B T_c)^{3/2} F_{1/2}(\alpha) \quad (1.1.10)$$

and,

$$P_e = \frac{8\pi}{3h^3 m} (2mK_B T_c)^{3/2} F_{3/2}(\alpha) \quad (1.1.11)$$

where  $F_{1/2}$  and  $F_{3/2}$  are given by

$$F_{1/2}(\alpha) = \int_0^\infty \frac{u^{1/2}}{\exp(\alpha + u) + 1} du \quad (1.1.12)$$

and,

$$F_{3/2}(\alpha) = \int_0^\infty \frac{u^{3/2}}{\exp(\alpha + u) + 1} du \quad (1.1.13)$$

respectively.

Examining  $F_{1/2}$  and  $F_{3/2}$  at different  $\alpha$ , we can measure the extent of degeneracy at

different radial location.

Combining  $P_e$  and  $n_e$  we find

$$P_e = n_e K_B T_c \left( \frac{2F_{3/2}(\alpha)}{3F_{1/2}(\alpha)} \right) \quad (1.1.14)$$

The quantity in the bracket defines the extent to which degenerate electron pressure differs from classical gas pressure.

Using the identity;

$$\int_0^{\infty} x^{z-1} \exp^{-x} dx = \Gamma(z)$$

the ratio in the equation (1.1.14) can be simplified to give

$$\frac{F_{3/2}(\alpha)}{F_{1/2}(\alpha)} = \frac{3}{2}$$

Therefore, the equation for electron degeneracy pressure becomes

$$P_e = n_e K_B T_c \quad (1.1.15)$$

The value of  $F_{1/2}(\alpha)$  is approximated using the series representation of

$$F_{1/2}(\alpha) = \frac{2}{3}(-\alpha)^{3/2} \left[ 1 + \frac{\pi^2}{8\alpha^2} + \frac{7\pi^4}{640\alpha^4} + \dots \right]$$

Approximating up to only the first term in the bracket  $F_{1/2}(\alpha)$  becomes

$$F_{1/2}(\alpha) = \frac{2}{3}(-\alpha)^{3/2} \quad (1.1.16)$$

Combining equations (1.1.10) and (1.1.16) we have

$$n_e = \frac{8\pi}{3h^3} (2mK_B T_c)^{3/2} |\alpha|^{3/2} \quad (1.1.17)$$

## 1.2 Continuity Equation (Conservation of Mass)

Any continuity equation can be expressed in an integral form (in terms of a flux integral), which applies to any finite region, or a differential form (in terms of a divergence operator)

which applies to a point. To express the condition that a flow must conserve mass, we consider a volume  $V$  containing a parcel of fluid given by the volume integral:

$$m = \int_V \rho dV \quad (1.2.1)$$

where  $m$  is the mass and  $\rho$  is the density of the fluid flow. The mass of the fluid flowing at velocity  $v$  in unit time through element  $ds$  is equal to the area of surface element, and it's direction is outward along the normal to the surface, which implies the mass flux is positive for fluid flow out of the volume. The total rate of flow out of the volume  $V$  is given by the surface integral  $\int_A \rho v \cdot ds$ , where  $A$  is the area of the surface bounding  $V$ . Then the rate of decrease of mass in  $V$  is given by [14]

$$\frac{\partial}{\partial t} \int_V \rho dV = - \int_A \rho v \cdot ds \quad (1.2.2)$$

The surface integral of the mass flux density  $\rho v$  can then be written as a volume integral of its divergence

$$\int_A \rho v \cdot ds = \int_V (\nabla \cdot \rho v) dV \quad (1.2.3)$$

This may be rewritten as

$$\int_V \left[ \frac{\partial \rho}{\partial t} + \nabla \cdot \rho v \right] dV = 0 \quad (1.2.4)$$

This equation holds true for any volume  $V$ , and hence the equation of continuity in differential form becomes:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho v = 0 \quad (1.2.5)$$

where  $v = (v_r, v_\Phi, v_z)$ , is the fluid velocity with radial, azimuthal and vertical components respectively.

Applying the divergence property of a vector for a thin cylindrically symmetric disk, using cylindrical coordinates,  $\nabla \cdot (\rho v) = 0$ , can be expressed as:

$$\nabla \cdot (\rho v) = \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r) + \frac{\partial (\rho v_\Phi)}{\partial \Phi} + \frac{\partial (\rho v_z)}{\partial z} \quad (1.2.6)$$

Due to cylindrical symmetry the fluid variables are independent of  $\Phi$ . Then it directly follows from the conservation of mass for steady state of axisymmetric disk.

$$\frac{\partial}{\partial r}(r\rho v_r) + \frac{\partial(r\rho v_z)}{\partial z} = 0 \quad (1.2.7)$$

Neglecting a vertical outflow from the disk, the gas will have a radial velocity component only. Then for steady flow, the continuity equation reduces to

$$\frac{1}{r} \frac{\partial}{\partial r}(r\rho v_r) = 0 \quad (1.2.8)$$

The surface density function is defined as:

$$\Sigma(r) = \int_{-H}^H \rho(r, z) dz = 2\rho H \quad (1.2.9)$$

where  $\rho$  is an average density and  $H$  is the half-thickness of the disk. Then equation (1.2.10) can be written as:

$$\frac{1}{r} \frac{\partial}{\partial r}(r\Sigma v_r) = 0 \quad (1.2.10)$$

This can be vertically integrated to yield the constant mass influx (accretion) condition:

$$\dot{M} = -2\pi r \int_{-H}^H \rho v_r dz = Constant. \quad (1.2.11)$$

Provided  $\rho v_r$  is negligible at  $Z=\pm H$ , a valid assumption except for the case of strong winds or magnetically driven outflows and  $\dot{M}$  is the constant mass inflow rate (accretion rate).

### 1.3 Momentum Equation(Euler Equation)

For real fluid the transfer of momentum occurs in part by the transport of fluid volumes having different velocities, which is expressed by the advective term in Euler's equation. From Newton's second law, conservation of momentum being written for an arbitrary control volume as:

$$\frac{dv}{dt} = \frac{-1}{\rho} \nabla p - \nabla \Phi_G \quad (1.3.1)$$

where  $v$  is the flow velocity,  $\rho$  is the fluid density, and  $P$  is the pressure.

If dissipative terms representing the action of viscous forces are included on the right hand side of equation (1.3.1), it becomes Navier-Stoke's equation. Using mathematical property ( $\frac{d}{dt} = \frac{\partial}{\partial t} + v \cdot \nabla$ ), the general form of the Navier-Stoke's equation for the fluid motion:

$$\rho \frac{\partial v}{\partial t} + \rho v \cdot \nabla v = -\rho \nabla \Phi_G - \nabla P + f \quad (1.3.2)$$

where  $\Phi_G = -GM[r^2 + z^2]^{-1/2}$  is the gravitational potential energy and  $f$  is the viscous force per unit volume responsible for momentum transport along velocity gradients by random (thermal and turbulent) motions.

Accretion disks are differentially rotating flows, radially varying azimuthal, so we can only consider kinematic viscosity, which depends on shearing motions and the viscous force per unit volume is given by

$$f = \nabla \cdot (2\rho\nu\xi) \quad (1.3.3)$$

where  $\nu$  is the coefficient of kinematic viscosity and  $\xi$  is the fluid shear tensor.

Using the continuity equation we can also rewrite the momentum equation as

$$\frac{\partial(\rho v)}{\partial t} + \nabla \cdot (\rho v v) = -\rho \nabla \Phi_G - \nabla P + \nabla \cdot (2\rho\nu\xi) \quad (1.3.4)$$

But it is possible to write the shear stress tensor as

$$\xi = \frac{1}{2} \left[ \nabla v - \frac{2}{3} (\nabla \cdot v) \right] \quad (1.3.5)$$

substituting this in to equation (1.3.4) we obtain

$$\frac{\partial(\rho v)}{\partial t} + \nabla \cdot (\rho v v) = -\rho \nabla \Phi_G - \nabla P + \nabla \cdot \left( \rho \nu \left( \nabla v - \frac{2}{3} (\nabla \cdot v) \right) \right) \quad (1.3.6)$$

From equation (1.3.6) the radial component of Navier-Stoke equation can be written as

$$\rho \left[ v_r \frac{\partial v_r}{\partial r} - \frac{v_\phi^2}{r} \right] = \frac{-\partial P}{\partial r} - \frac{\rho G M r}{(r^2 + z^2)^{3/2}} \quad (1.3.7)$$

Similarly, the vertical component of the momentum equation for the steady flow is

$$\rho \left[ v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} \right] = \frac{-\partial P}{\partial z} - \frac{\rho GM}{r^2} \quad (1.3.8)$$

Neglecting the vertical out flows, the equation reduces to the equation of hydrostatic equilibrium:

$$\frac{1}{\rho} \frac{\partial P}{\partial z} = -\frac{GMz}{r^3} \quad (1.3.9)$$

Using  $H$  as half thickness of the disk the pressure at the mid plane of the disk is

$$P = \int_0^H \rho \frac{GMz}{r^3} dz = \frac{1}{2}(\rho H) \frac{GM}{r^3} H$$

$$P = \frac{1}{2} H \Sigma \frac{GM}{r^3} \quad (1.3.10)$$

From equation (1.3.9) making a slight approximation:

$$\frac{1}{\rho} \left( \frac{\partial P}{\partial z} \right) \simeq \frac{1}{\rho} \frac{P}{z}$$

$$\frac{1}{\rho} \frac{P}{z} = \frac{GMz}{r^3}$$

Replacing  $z$  by the half-thickness  $H$  it becomes

$$\frac{1}{\rho} \frac{P}{H} = \frac{GMH}{r^3} \quad (1.3.11)$$

Using Keplerian velocity,  $v_k^2 = \frac{GM}{r}$  and  $C_s^2 = \frac{P}{\rho} \simeq \frac{dP}{d\rho}$ , we can write the hydrostatic equilibrium equation as

$$C_s^2 = v_k^2 \left( \frac{H}{r} \right)^2$$

$$\left( \frac{H}{r} \right) = \left( \frac{C_s}{v_k} \right) \quad (1.3.12)$$

where  $C_s$  is the speed of sound.

From equation (1.3.12) we conclude that the Keplerian velocity is highly supersonic Using the general equation(1.3.6), the azimuthal component of Seiver-Stoke's equation is given by

$$\rho \left[ \frac{v_r}{r} \frac{\partial}{\partial r} (r v_\phi) \right] = \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^3 \rho \nu \frac{\partial}{\partial r} \left( \frac{v_\phi}{r} \right) \right] \quad (1.3.13)$$

Considering the radial component of momentum equation (1.3.7) the dominant terms are  $\frac{v_\phi^2}{r}$  and gravitational terms, so ignoring the other gradient terms yields

$$\begin{aligned}\frac{v_\phi^2}{r} &\simeq \frac{GM\rho}{r^2} \\ v_\phi^2 &\simeq \frac{GM}{r} \equiv v_k^2\end{aligned}\tag{1.3.14}$$

so radial moment is conserved when keplerian rotation prevails, i.e. Keplerian rotational motion provides support against gravity. This is the differential rotation: each infinitesimal disk annulus rotates with a different  $v_\phi(r)$ , which increases towards small  $r$ . This means there is a velocity shear between each annulus in the fluid flow. The Keplerian angular velocity is defined as

$$\Omega_k = \frac{v_k}{r} = \left(\frac{GM}{r^3}\right)^{1/2}\tag{1.3.15}$$

## 1.4 Conservation of Energy

An element of gas has two forms of energy: an amount  $\frac{1}{2}\rho v^2$  of kinetic energy per unit volume, and internal or thermal energy  $\rho\varepsilon$  per unit volume,  $\varepsilon$  the internal energy per unit mass, depends on the temperature  $T$  of the gas. According to the equipartition theorem of elementary kinetic theory, each degree of freedom of each gas particle is assigned a mean energy  $\frac{1}{2}KT$ .

For monatomic gas the only degrees of freedom are the three orthogonal directions of translational motion and [15]

$$\varepsilon = \frac{3KT}{2\mu m_p}\tag{1.4.1}$$

Molecular gases have additional internal degrees of freedom of vibration or rotation. In reality, cosmic gases are not quite monatomic and the effective number of degrees of freedom is not quite there; but in practice equation(1.4.1) is usually is a good approximation.

The energy equation for the gas is

$$\frac{\partial}{\partial t} \left( \frac{1}{2}\rho v^2 + \rho\varepsilon \right) + \nabla \cdot \left[ \left( \frac{1}{2}\rho v^2 + \rho\varepsilon + P \right) v \right] = f \cdot v - \nabla \cdot F_{rad} - \nabla \cdot q\tag{1.4.2}$$

The left hand side shows a family resemblance to the continuity equation(1.2.5), with the expected difference that the conserved quantity  $\rho$  is replaced by  $(\frac{1}{2}\rho v^2 + \rho\varepsilon)$ .

The last term in the square brackets represents the so called pressure work. The new quantities appear on the right hand side: first, radiative flux vector

$$F_{rad} = \int dv \int d\Omega n I_\nu(n, r) \quad (1.4.3)$$

where  $I_\nu$  is the specific intensity of radiation at the point  $r$  in the direction  $n$  and the integrals are over frequency  $\nu$  and solid angle  $\Omega$ . The term  $-\nabla \cdot F_{rad}$  gives the rate at which radiant energy is being lost by emission, or gained by absorption; by unit volume of the gas. In general, specific intensity governed by further equation, the conservation of energy equation for the radiation field.

The second new quantity in the energy equation(1.4.2) is the conductive flux of heat,  $q$ . This measures the rate at which random motions, chiefly those of electrons, transport thermal energy in the gas and thus act to smooth out temperature difference.

For a slow inflow of matter through an optically tick disk, the local viscous dissipation  $v \cdot f$  is balanced by the radiative losses  $\nabla \cdot F_{rad}$ . This is written as

$$\frac{9GM}{8r^3} \nu \Sigma = \frac{4\sigma T_c^4}{3\tau} \quad (1.4.4)$$

where  $T_c$  is the central temperature of the accretion disk,  $\sigma$  is the Stefan-Boltzmann constant and  $\tau$  is the optical depth of the disk.

The optical depth from the mid plane to the surface of the accretion disk,  $\tau_c$  is defined in terms of the Rossel and mean opacity  $K_R(\rho_c, T_c)$  by

$$\tau_c = \frac{\Sigma K_R}{2} = \rho_c h K_R \quad (1.4.5)$$

where  $\rho_c$  is the central density defined as  $\rho_c = \frac{\Sigma}{2H}$ .

## 1.5 Viscous Spreading

The shear flow between neighboring Kepler orbits in the disk causes friction due to viscosity. The frictional torque is equivalent to exchange of angular momentum between these

orbitals. But since the orbits are close to Keplerian, a change in angular momentum of a ring gas also means it must change its distance from the central mass. If the angular momentum transport (more precisely a non zero divergence of the angular momentum flux) there fore automatically implies redistribution of mass in the disk [16].

If at  $t=0$  this ring is released to evolve under the viscous torques, one find thats that it first spreads in to an asymmetric hump with a long tail to large distances. As  $t \rightarrow \infty$  the hump flattens in such a way that almost all the mass of the ring is accreted on to the center, while a vanishingly small fraction of the gas carries almost all the angular momentum to infinity. As a result of this asymmetric behavior essentially all the mass of a disk can accrete, even if there is no external torque to remove the angular momentum.

# Chapter 2

## Structure Equation

Reducing those equations discussed above to a single ordinary differential equations for the radial structure of the accretion disk and first assuming that the equation of state of an ideal gas:

$$P_g = \frac{\rho_c K_B T_c}{m_p \mu} \quad (2.0.1)$$

where  $K_B$  is the Boltzmann constant,  $\mu$  is the mean molecular weight, and  $m_p$  is the mass of proton. The equation of star pressure with the effect of electron degenerate pressure can be written using equations (1.1.6) and (2.0.1) as:

$$P = \frac{\rho_c K_B T_c}{m_p \mu} + 9.77 \times 10^6 \left( \frac{\rho_c}{\mu_e} \right)^{5/3} \quad (2.0.2)$$

where the first term stands for gas pressure and the second term stands for an ideal electron degenerate pressure on the right hand side of the equation.

### 2.0.1 Thickness of Accretion Disk

The thickness of an accretion disk is set by the balancing of pressure within the disk against the gravitational tidal force. As in the case of star, this structure depends on how the temperature of the gas varies with altitude above the plane of the disk. This in turn depends on the viscosity mechanisms that heat the disk and the radiative process and convection that transport the energy to the surface of the disk.

For an axisymmetric magnetized disk around a neutron star having a dipole moment

aligned with its rotation axis in steady state, the pressure can be expressed in terms of hydrostatic equilibrium as [16]:

$$P = P_g + P_e \quad (2.0.3)$$

where  $P$  is the total pressure.

$$\frac{GM}{2r^3} H \Sigma = \frac{\rho_c K_B T_c}{m_p \mu} + 9.77 \times 10^6 \left( \frac{\rho_c}{\mu_e} \right)^{5/3} \quad (2.0.4)$$

Using equation (1.2.10) this can be written componentwise as

$$\frac{H^2 \rho_c GM}{r^3} = \frac{\rho_c K_B T_c}{m_p \mu} + 9.77 \times 10^6 \left( \frac{\rho_c}{\mu_e} \right)^{5/3} \quad (2.0.5)$$

From equation (2.0.5) the half-thickness of the accretion disk can be calculated as:

$$H = \left( GM \right)^{-1/2} \left[ \frac{K_B T_c}{m_p \mu} + 9.77 \times 10^6 \left( \frac{\rho_c}{\mu_e} \right)^{5/3} \right]^{1/2} r^{3/2} \quad (2.0.6)$$

The shear stress (viscous stress) in a thin Keplerian disk is

$$f_{r\phi} = -\rho_c v r \Omega' = \frac{3}{2} \alpha \rho_c C_s H \Omega_k \quad (2.0.7)$$

Using  $\alpha = \frac{2}{3} \frac{f_{r\phi}}{P}$  and the differential motion between the disk rotating at Keplerian rate (velocity),  $\Omega_k(r) = \left( \frac{GM}{r^3} \right)^{1/2}$ , the viscous stress tensor can be given as:

$$f_{r\phi} = \frac{3}{4} \frac{\Sigma}{H} v \left( \frac{GM}{r^3} \right)^{1/2} = \alpha_{ss} P \quad (2.0.8)$$

## 2.0.2 Density of Accretion Disk

The density of an accretion disk in both middle and outer regions depend on the temperature and radius of the disk.

Substituting equation (2.0.2) and (2.0.6) into this equation (2.0.8) we have

$$\begin{aligned} \frac{3}{4} \Sigma v \left( \frac{GM}{r^3} \right)^{1/2} \left\{ \left( GM \right)^{-1/2} \left[ \frac{K_B T_c}{m_p \mu} + 9.77 \times 10^6 \left( \frac{\rho_c}{\mu_e} \right)^{5/3} \right]^{1/2} \right\}^{-1/2} r^{-3/4} \\ = \alpha_{ss} \left\{ \frac{\rho_c K_B T_c}{m_p \mu} + 9.77 \times 10^6 \left( \frac{\rho_c}{\mu_e} \right)^{5/3} \right\} \end{aligned} \quad (2.0.9)$$

This is simplified to give

$$\frac{K_B T_c}{m_p \mu} \rho_c^{4/3} + 9.77 \times 10^6 \frac{\rho_c^2}{\mu^{5/3}} - \left[ \frac{3 \Sigma v G M}{4 \alpha_{ss} r^3} \right]^{2/3} = 0 \quad (2.0.10)$$

$$\text{Let; } A = \frac{K_B T_c}{m_p \mu}, B = \frac{9.77 \times 10^6}{\mu^{5/3}}, \text{ and } D = \left[ \frac{3 \Sigma v G M}{4 r^3} \right]^{2/3} \quad (2.0.11)$$

Then, equation (2.0.10) is written as

$$A \rho^{4/3} + B \rho^2 - D = 0 \quad (2.0.12)$$

Again let:  $y = \rho^2, a = B^3, b = A^3 - 3B^2D, c = 3BD^2,$  and  $d = D^3$

This reduces the equation in to

$$ay^3 + by^2 + cy - d = 0 \quad (2.0.13)$$

This cubic polynomial function can be solved using Galios theory [17], in which we substitute  $y = x - \frac{b}{3}$ . Considering only the real part of the solution  $x$  is given by:

$$x = \left\{ \left( \frac{n}{2} \right) + \left[ \left( \frac{n}{2} \right)^2 + \left( \frac{m}{3} \right)^3 \right]^{1/3} \right\}^{1/3} - \left\{ \left( \frac{-n}{2} \right) + \left[ \left( \frac{n}{2} \right)^2 + \left( \frac{m}{3} \right)^3 \right]^{1/3} \right\}^{1/3} \quad (2.0.14)$$

$$\text{where; } m = c - \frac{b^2}{3}, \text{ and } n = -d + \frac{bc}{3} - \frac{2b^3}{27}$$

Substituting the values of  $b, c,$  and  $d$ , the values of  $m$  and  $n$  can be written as:

$$m = \left( \frac{2.93 \times 10^7}{\mu^{5/3}} \right) \left( \frac{3 \Sigma v G M}{4 r^3} \right)^{4/3} - \frac{1}{3} \left( \frac{K_B T_c}{m_p \mu} \right)^3 + \left( \frac{9.77 \times 10^6}{\mu^{5/3}} \right)^2 \left( \frac{3 \Sigma v G M}{4 r^3} \right)^{2/3} \quad (2.0.15)$$

and;

$$n = \left( \frac{3 \Sigma v G M}{4 r^3} \right)^2 + \left( \frac{9.77 \times 10^6}{\mu^{5/3}} \right) \left( \frac{K_B T_c}{m_p \mu} \right)^3 \left( \frac{3 \Sigma v G M}{4 r^3} \right)^{4/3} - \frac{27}{16} \left( \frac{9.77 \times 10^6}{\mu^{5/3}} \right) \left( \frac{\Sigma v G M}{r^3} \right)^2 - \frac{2}{27} \left\{ \left( \frac{K_B T_c}{m_p \mu} \right)^2 - 3 \left( \frac{9.77 \times 10^6}{\mu^{5/3}} \right)^2 \left( \frac{3 \Sigma v G M}{4 r^3} \right)^{2/3} \right\}^3 \quad (2.0.16)$$

Since  $y = x - \frac{b}{3}$  and  $y = \rho_c^2$ , the central density is given by equation:

$$\begin{aligned}
\rho_c = & \left( \left( \frac{9}{32} \left( \frac{\Sigma v GM}{r^3} \right)^2 + \left( \frac{4.89 \times 10^6}{\mu^{5/3}} \right) \left( \frac{K_B T_c}{m_p \mu} \right)^3 \left( \frac{3 \Sigma v GM}{4 r^3} \right)^{4/3} - \left( \frac{8.24 \times 10^6}{\mu_e^{5/3}} \right) \right. \right. \\
& \times \left( \frac{\Sigma v GM}{r^3} \right)^2 - \left[ \frac{1}{27} \left( \frac{K_B T_c}{m_p \mu} \right)^2 - \left( \frac{2.93 \times 10^7}{\mu_e^{5/3}} \right)^2 \left( \frac{3 \Sigma v GM}{4 r^3} \right)^{2/3} \right]^3 \\
& + \left( \left\{ \frac{9}{32} \left( \frac{\Sigma v GM}{r^3} \right)^2 + \left( \frac{4.89 \times 10^6}{\mu^{5/3}} \right) \left( \frac{K_B T_c}{m_p \mu} \right)^3 \left( \frac{3 \Sigma v GM}{4 r^3} \right)^{4/3} \right. \right. \\
& - \left. \left. \left( \frac{8.24 \times 10^6}{\mu_e^{5/3}} \right) \left( \frac{\Sigma v GM}{r^3} \right)^2 - \frac{1}{27} \left[ \left( \frac{K_B T_c}{m_p \mu} \right)^2 - \left( \frac{2.93 \times 10^7}{\mu_e^{5/3}} \right)^2 \right] \right. \right. \\
& \times \left. \left. \left( \frac{3 \Sigma v GM}{4 r^3} \right)^{2/3} \right]^3 \right\}^2 + \left\{ \left( \frac{9.77 \times 10^6}{\mu_e^{5/3}} \right) \left( \frac{3 \Sigma v GM}{4 r^3} \right)^{4/3} - \frac{1}{9} \left( \frac{K_B T_c}{m_p \mu} \right)^3 \right. \\
& + \left. \frac{1}{3} \left( \frac{9.77 \times 10^6}{\mu_e^{5/3}} \right)^2 \left( \frac{3 \Sigma v GM}{4 r^3} \right)^{2/3} \right\}^3 \right)^{1/3} \left[ \frac{-9}{32} \left( \frac{\Sigma v GM}{r^3} \right)^2 \right. \\
& - \left. \left( \frac{4.89 \times 10^6}{\mu^{5/3}} \right) \left( \frac{K_B T_c}{m_p \mu} \right)^3 \left( \frac{3 \Sigma v GM}{4 r^3} \right)^{4/3} + \left( \frac{8.24 \times 10^6}{\mu_e^{5/3}} \right) \left( \frac{\Sigma v GM}{r^3} \right)^2 \right. \\
& + \left. \left[ \frac{1}{27} \left( \frac{K_B T_c}{m_p \mu} \right)^2 - \left( \frac{2.93 \times 10^7}{\mu_e^{5/3}} \right)^2 \left( \frac{3 \Sigma v GM}{4 r^3} \right)^{2/3} \right]^3 + \left( \left\{ \frac{9}{32} \left( \frac{\Sigma v GM}{r^3} \right)^2 \right. \right. \right. \\
& + \left. \left. \left( \frac{4.89 \times 10^6}{\mu^{5/3}} \right) \left( \frac{K_B T_c}{m_p \mu} \right)^3 \left( \frac{3 \Sigma v GM}{4 r^3} \right)^{4/3} - \left( \frac{8.24 \times 10^6}{\mu_e^{5/3}} \right) \left( \frac{\Sigma v GM}{r^3} \right)^2 \right. \right. \\
& + \left. \left. \frac{1}{27} \left[ \left( \frac{K_B T_c}{m_p \mu} \right)^2 - \left( \frac{2.93 \times 10^7}{\mu_e^{5/3}} \right)^2 \left( \frac{3 \Sigma v GM}{4 r^3} \right)^{2/3} \right]^3 \right\}^2 + \left\{ \left( \frac{9.77 \times 10^6}{\mu_e^{5/3}} \right) \right. \right. \\
& \times \left. \left. \left( \frac{3 \Sigma v GM}{4 r^3} \right)^{4/3} - \frac{1}{9} \left( \frac{K_B T_c}{m_p \mu} \right)^3 + \frac{1}{3} \left( \frac{9.77 \times 10^6}{\mu_e^{5/3}} \right)^2 \left( \frac{3 \Sigma v GM}{4 r^3} \right)^{2/3} \right\}^3 \right)^{1/3} \right]^{1/3} \\
& \left. \frac{1}{3} \left( \frac{K_B T_c}{m_p \mu} \right)^3 + \left( \frac{9.77 \times 10^6}{\mu_e^{5/3}} \right)^2 \left( \frac{3 \Sigma v GM}{4 r^3} \right)^{2/3} \right)^{1/2} \tag{2.0.17}
\end{aligned}$$

where;  $r$  is the radius of accretion disk,  $T_c$  is the central temperature and  $\rho_c = \rho_c(T_c, r)$ .

## 2.1 Central Temperature of the Disk

The central temperature in different regions of the disk should be high enough for the complete ionization of electron in the regions [18]. The value  $K_B T_C$  should be greater or equal to the critical value  $0.5 \text{ Mev}$  in both middle and outer regions of the accretion disk,

where the gas pressure is more dominant than radiation pressure, for complete ionization. Here,  $K_B$  is Boltzmann constant and  $T_c$  is the central temperature in both regions. It is possible to find the temperature of the accretion disk as a function of radius of the disk[19].

### 2.1.1 Temperature in the Middle Region of the Disk

The central temperature in the middle part of the accretion disk can be calculated using a formula:

$$T_c = \left(6.5 \times 10^7 K\right) \tilde{I}^{-1/5} \alpha_{ss}^{-1/5} \left(2\mu\right)^{1/5} \left(\frac{3r_g}{r_o}\right)^{9/10} \\ \times \left(\frac{M}{M_\odot}\right)^{-3/5} \left(\dot{M}_{14}\right)^{2/5} \left(\frac{r}{r_o}\right)^{-9/10} f^{2/5} \quad (2.1.1)$$

Moreover,

$$I = \frac{(2^N N!)^2}{(2N+1)!}, \quad \tilde{I} = \frac{3}{2} I(N+1), \quad r_g = \frac{2GM}{c^2}, \quad f = 1 - \left(\frac{r_o}{r}\right)^{1/2} \quad (2.1.2)$$

in which  $c$  is the speed of light,  $r_o = r_A$  is Alfvén's radius and  $N = 3$ , for electron scattering.

Constants	Values
$\alpha_{ss}$	0.01
$\mu$	0.62
$\frac{M}{M_\odot}$	1.4
$r_o = r_A$	$1 \times 10^6 m$
$\mu_e$	1
$M_\odot$	$1.99 \times 10^{30} kg$
$K_B$	$1.38 \times 10^{-23} m^2 kgs^{-2} k^{-1}$
$h$	$6.63 \times 10^{-34} m^2 kgs^{-1}$
$m_e$	$9.11 \times 10^{-31} kg$
$m_p = m_H$	$1.63 \times 10^{-27} kg$

Table 2.1: Some basic constants with their corresponding values used in this paper.

Substituting all the constants and writing the central temperature in terms of the radius of the accretion disk,  $r$  gives:

$$T_c = 2.22 \times 10^{9.3} r^{-11/10} \left[ r^{1/2} - 10^3 \right]^{2/5} K \quad (2.1.3)$$

## 2.1.2 Temperature in the Outer Region of the Disk

The central temperature in the outer part of the accretion disk can be calculated using a formula:

$$T_c = \left( 2.7 \times 10^7 K \right) \left( N + 1 \right)^{-1/20} \tilde{I}^{-1/5} \alpha_{ss}^{-1/5} \left( 2\mu \right)^{1/4} \left( \frac{3r_g}{r_o} \right)^{3/4} \\ \times \left( \frac{M}{M_\odot} \right)^{-1/2} \left( \dot{M}_{14} \right)^{3/10} \left( \frac{r}{r_o} \right)^{-3/10} f^{3/10} \quad (2.1.4)$$

Using the above constants given in Table 1.1 and other constants the central temperature in terms of radius,  $r$  is given as:

$$T_C = 5.57 \times 10^{8.45} r^{-9/10} \left[ r^{1/2} - 10^3 \right]^{3/10} K \quad (2.1.5)$$

The more simplified central density as a function of radius,  $r$  in the middle region of the accretion disk where gas-pressure is dominant is given by the equation [19]

$$\rho_c = \left( 1.4 \right) \frac{(N + 1)^{-1/2}}{\tilde{I}^{7/10}} \alpha^{-7/10} (2\mu)^{6/5} \left( \frac{3r_g}{r_o} \right)^{33/20} \\ \times \left( \frac{M}{\dot{M}} \right)^{-11/10} \left( \dot{M}_{14} \right)^{2/5} \left( \frac{r}{r_o} \right)^{-33/20} f^{2/5} \quad (2.1.6)$$

Using equation (2.1.2) to find the constants, the central density in the middle region becomes

$$\rho_c = 1.89 \times 10^{8.6} r^{-37/20} \left[ r^{1/2} - 10^3 \right]^{2/5} \text{ kg/m}^3 \quad (2.1.7)$$

Similar to equation (2.1.6) the equation of central density in the outer part of the accretion disk is given as:

$$\rho_c = \left( 5.3 \right) \frac{(N + 1)^{-17/40}}{\tilde{I}^{7/10}} \alpha^{-7/10} (2\mu)^{9/8} \left( \frac{3r_g}{r_o} \right)^{15/8} \\ \times \left( \frac{M}{\dot{M}} \right)^{-5/4} \left( \dot{M}_{14} \right)^{11/20} \left( \frac{r}{r_o} \right)^{-15/8} f^{11/20} \quad (2.1.8)$$

Substituting the constants in to the equation (2.1.8), it is simplified to be

$$\rho_c = 7.21 \times 10^{8.7} r^{-43/20} \left[ r^{1/2} - 10^3 \right]^{11/20} \text{ kg/m}^3 \quad (2.1.9)$$

## 2.2 Regions of Accretion disk

A disk around neutron star has three regions; inner, middle and outer regions. In this paper we do not consider the inner part of the disk, rather we study about stability of the disk in it's middle and outer regions. The disk around a neutron star differs from the disk around a black hole in that it should eventually release the gravitational binding energy of accreted matter (which is converted to internal energy of the disk and the rotational kinetic energy) more efficiently.

The energy equation of the disk is [19]

$$\Sigma \nu_r T \frac{ds}{dr} = Q^+ - Q^- \quad (2.2.1)$$

where  $s$  is the entropy per unit mass,  $r$  is the radius of a certain position in the disk, and  $Q^+$  and  $Q_-$  are energy input (heating) and energy loss (cooling) rate in the disk, respectively.

In the outer disk, the heat energy could be advected in ward, and we take  $Q_{adv}^- = \frac{1}{2} \Sigma T \nu_r \frac{ds}{dr}$  to be the quantity of the energy advection rate, where 1/2 is added because only the vertically integrated disk over a half-thickness  $H$ . Thus the energy-conservation equation equation (2.3.1) is written as

$$Q^+ = Q_{rad}^- + Q_{adv}^- + Q_\nu^- \quad (2.2.2)$$

where  $Q^+$  is viscous heat energy generation rate per unit surface area. according to the standard viscosity disk model, we have

$$Q^+ = \frac{3GM\dot{M}}{8\pi r^3} f \quad (2.2.3)$$

where  $f = 1 - (r_o/r)^{1/2}$ .

The quantity  $Q_{rad}^-$  in equation (2.3.2) is the photon cooling rate per unit surface area

of the disk. Since the disk is extremely dense, thus we can take  $Q_{rad}^- = 0$  as a good approximation. If the accretion rate is not very high, most of the energy generated in the disk is advected inward, and we call the disk as an advection-dominated disk. As the accretion rate increases, the density and temperature of the disk also increase, and neutron cooling becomes the dominant cooling mechanism in some regions of the disk. Thus, we say that this region becomes neutrino-dominated. When the accretion rate is sufficiently large, the disk may become entirely neutrino-dominated. In addition to the accretion rate, there are some other factors, such as the central mass of neutron star, the electron-nucleon ratio, that can influence the structure.

## Chapter 3

# Stability of the Accretion Disk

An equilibrium solution is physically relevant only if the disk is stable. When investigating the stability of an equilibrium condition we restrict ourselves to a linear analysis. One thus considers time-dependent perturbations and expands all equations which the system satisfies about the equilibrium condition, keeping only the linear terms. The system is stable provided the frequency of the corresponding normal modes all have negative imaginary parts. General theorems show that this result is unchanged by nonlinearities. On the other hand, if the imaginary part of the frequency of some normal mode is positive this mode grows exponentially, and one expects that the system is unstable.

For cold spherically symmetric configuration one considers as the first step only radial perturbations, which are also adiabatic. Since hydrodynamic time scales are usually much shorter than the characteristic times for energy transports, this is reasonable. In this case one obtains an eigenvalue problem (of the Sturm-Liouville type) for  $\omega^2$  since the equations for adiabatic perturbations are time reversal invariant (there is no dissipation). The equilibrium is stable provided  $\omega_o^2 > 0$  for the lowest mode frequency  $\omega_o$ . One can often find sufficient criteria for instability by making use of the Rayleigh-Ritz variational principle and simple trial functions sometimes the equilibrium solutions, whether or not it is stable against radial, adiabatic pulsations. There is a possibility that accretion disks around a neutron star or a black hole show non-linear oscillation caused by thermal instability,

similar to dwarf nova outbursts. For a disk with high accretion rate, the heat generated by viscosity close to the transonic part of the flow not only it locally radiates away, as standard Shakura-Sunyaev model, but also it is very efficiently transported by advection.

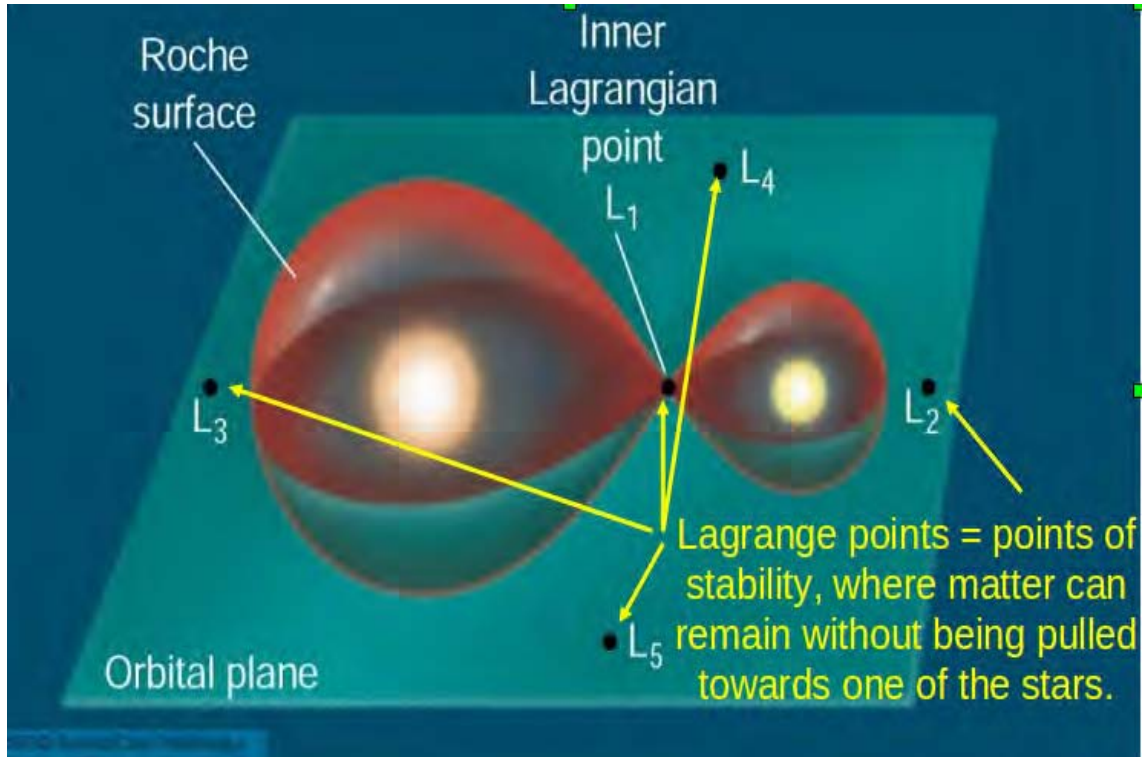


Figure 3.1: Lagrangian points of stability in binary stars.

The most development model for outbursts is the disk instability model of Osaki (1974) [20]. In this model the instability that gives rise to cyclic accretion is due to a temperature dependence of the resulting  $\alpha$ -parameter will be a function of the main dimensionless parameter of the disk, the aspect ratio  $H/r$ . If this is sufficiently rapidly increasing function, such that  $\alpha$  is large in hot disks and low in cool disks, an instability results by the following mechanism. Suppose we start the disk in stationary state at the mean accretion rate. If this state is perturbed by small temperature increase,  $\alpha$  goes up, and by the increased viscous stress the mass flux  $\dot{M}$  increase. This increase the disk temperature further, resulting in a runaway to a hot state. Since  $\dot{M}$  is larger than the average, the

disk empties partly, reducing the surface density and the central temperature. A cooling front then transforms the disk to a cool state with an accretion rate below the mean. The disk in this model switches back and forth between hot and cool states.

The contribution of electron degenerate pressure to the stability of accretion disk in different region of the disk is calculated from stability relation [19]

$$\beta = \frac{P_g}{P_g + P_e} \quad (3.0.1)$$

where;  $P_g$  is gas pressure and  $P_e$  is electron degenerate pressure.

For gas-pressure dominated regions the disk is stable if

$$\bar{\beta} < \frac{3}{5} \quad (3.0.2)$$

where  $\bar{\beta} = 1 - \beta$

### 3.0.1 Stability in the Middle Region

It is possible to find  $P_g$  from equation(2.0.1):

$$P_g = \left( \frac{K_B}{m_p \mu} \right) \rho_c T_c$$

where;  $\rho_c$  and  $T_c$  are the central density and temperature in the middle region.

$$P_g = 5.58 \times 10^{21.9} r^{-59/20} \left[ r^{1/2} - 10^3 \right]^{4/5} \quad (3.0.3)$$

Firstly, let us check the stability of the accretion disk due to a *complete* degeneracy electron pressure:

$P_e$  for complete degeneracy pressure can be calculated from equation (1.1.6)

$$P_e = 9.77 \times 10^6 \left( \frac{\rho_c}{\mu_e} \right)^{5/3}$$

$$P_e = 2.82 \times 10^{21.33} r^{-3.08} \left[ r^{1/2} - 10^3 \right]^{2/3} \quad (3.0.4)$$

The stability of accretion disk in the middle or outer region due to electron degeneracy and gas pressure is

$$\beta < 2/5$$

$$\frac{P_g}{P_g + P_e} < \frac{2}{5} \quad (3.0.5)$$

Substituting equations(3.0.3) and (3.0.4) in to equation (3.0.5) we can have

$$3.35 \times 10^{21.9} r^{-59/20} \left[ r^{1/2} - 10^3 \right]^{4/5} - 9.12 \times 10^{20.33} r^{-37/12} \left[ r^{1/2} - 10^3 \right]^{2/3} < 0 \quad (3.0.6)$$

Then the value of  $r$  for which the accretion disk is stable in the middle region is calculated to be [21]

$$1 \times 10^6 m < r < 5.33 \times 10^7 m$$

Similarly, we can check the stability of the accretion disk due to a *partial* electron degenerate pressure. Firstly, we can write the relation for electron and proton number density as

$$n_e = n_p = \frac{\rho_c}{m_p} = 1.13 \times 10^{18.4} r^{-37/20} \left[ r^{1/2} - 10^3 \right]^{2/5} \quad (3.0.7)$$

where  $m_p$  is mass of proton and  $\rho_c$  is the central density in the middle region.

Combining equations (1.1.17) and (3.0.7) we can have

$$|\alpha|^{3/2} = 9.4 \times 10^{-18.55} r^{-1/5} \left[ r^{1/2} - 10^3 \right]^{-1/5} \quad (3.0.8)$$

Combining equations (1.1.15), (1.1.17) and (3.0.8) the relation for partial degeneracy electron pressure is given by

$$P_e = 3.48 \times 10^{4.73} r^{-2.95} \left[ r^{1/2} - 10^3 \right]^{4/5} \quad (3.0.9)$$

When we consider the total pressure in the disk, we should include both the contribution from electron degeneracy pressure and also from ordinary particles, which are nuclear gas.

So, the total pressure is given as

$$P = 5.58 \times 10^{21.9} r^{-59/20} \left[ r^{1/2} - 10^3 \right]^{4/5} + 3.48 \times 10^{4.73} r^{-2.95} \left[ r^{1/2} - 10^3 \right]^{4/5} \quad (3.0.10)$$

where  $P$  is the sum of  $P_g$  and  $P_e$ .

The region of stability of the disk is checked by equation (3.0.5). That is

$$\frac{P_g}{P_g + P_e} < 2/5$$

$$3.35 \times 10^{21.9} r^{-59/20} \left[ r^{1/2} - 10^3 \right]^{4/5} - 1.39 \times 10^{4.73} r^{-3.05} r^{-2.95} \left[ r^{1/2} - 10^3 \right]^{4/5} < 0 \quad (3.0.11)$$

solving this for the radius of accretion disk, it is approximately equal to what we have found in the case of complete degenerate electron pressure

$$1 \times 10^6 m < r < 5.33 \times 10^7 m$$

### 3.0.2 Stability in the Outer Region

Again, we can use equation(2.0.1) to find the gas pressure in this region

$$P_g = \left( \frac{K_B}{m_p \mu} \right) \rho_c T_c$$

where  $\rho_c$  and  $T_c$  are the central density and temperature in the outer region.

Substituting equations(2.1.5) and (2.1.9) in to this equation we have:

$$P_g = 5.34 \times 10^{22.15} r^{-61/20} \left[ r^{1/2} - 10^3 \right]^{17/20} \quad (3.0.12)$$

A *Complete* electron degenerate pressure is derived from equations(1.1.6) and (2.1.9):

$$P_e = 2.63 \times 10^{22.5} r^{-3.58} \left[ r^{1/2} - 10^3 \right]^{11/12} \quad (3.0.13)$$

The stability of accretion disk in the outer region due to electron degeneracy and gas pressure is obtained by substituting equation (3.0.12) and (3.0.13)in to equation (3.0.5) to calculate for the radius of the accretion disk in the outer region where the disk is stable.

That is

$$3.2 \times 10^{22.15} r^{-3.05} \left[ r^{1/2} - 10^3 \right]^{17/20} - 1.05 \times 10^{22.5} r^{-3.58} \left[ r^{1/2} - 10^3 \right]^{11/20} < 0 \quad (3.0.14)$$

Solving the inequality, the value of  $r$  for which the disk is stable in the outer region is [21]

$$5.33 \times 10^7 m \leq r \leq 1 \times 10^8 m$$

On the other hand, we can check the stability of the disk due to a *partial* degeneracy electron pressure in the outer region of the disk.

The electron number density is given as

$$n_e = n_p = \frac{\rho_c}{m_p} = 4.32 \times 10^{18.3} r^{-43/20} \left[ r^{1/2} - 10^3 \right]^{11/20} \quad (3.0.15)$$

where  $\rho_c$  is the central density in the outer region.

Combining equations (1.1.17) and (3.0.15) we can have

$$|\alpha|^{3/2} = 8.9 \times 10^{-17.38} r^{-16/20} \left[ r^{1/2} - 10^3 \right]^{-7/20} \quad (3.0.16)$$

Similar to that of the middle region we combine equations (1.1.15), (1.1.17) and (3.0.16) to get an equation for partial degeneracy electron pressure in the outer region .

$$P_e = 3.29 \times 10^{3.95} r^{-3.05} \left[ r^{1/2} - 10^3 \right]^{1/20} \quad (3.0.17)$$

So, the total pressure becomes

$$P = 5.34 \times 10^{22.15} r^{-61/20} \left[ r^{1/2} - 10^3 \right]^{17/20} + 3.29 \times 10^{3.95} r^{-3.05} \left[ r^{1/2} - 10^3 \right]^{1/20} \quad (3.0.18)$$

We use equation (3.0.5) to check the stability in this region

$$\frac{P_g}{P_g + P_e} < 2/5$$

$$3.2 \times 10^{22.15} r^{-61/20} \left[ r^{1/2} - 10^3 \right]^{0.85} - 1.32 \times 10^{-3.95} r^{-3.05} R^{-3.05} \left[ r^{1/2} - 10^3 \right]^{0.05} < 0 \quad (3.0.19)$$

Solving this for the radius of accretion disk, it is approximately equal to what we have found in the case of complete degenerate electron pressure

$$5.33 \times 10^7 m \leq r \leq 1 \times 10^8 m$$

# Chapter 4

## Analysis and Discussion

In this paper we have studied the contribution of degenerate electron pressure to the the stability of thin Keplerian accretion disks around neutron star with axisymmetric magnetic dipole, in which we ignored the situation when the particles are relativistic, so we took  $v = \frac{p}{m}$  as a non-relativistic degenerate gas. For our fiducial model We have taken a neutron star of  $M = 1.4M_{\odot}$ .

We included the complete non relativistic degeneracy electron pressure and partial non relativistic degeneracy electron pressure. Firstly, the central temperature and density of the disk is derived as a function of radius of accretion disk, which are the bases for the pressure derivation as a function of radius both in middle and outer regions of the disk. The degeneracy electron pressure is density dependent for complete degeneracy and temperature dependent for partial degeneracy.

### 4.1 Contribution of Degeneracy Electron Pressure in the Middle Region of Accretion Disk

This study indicates that the middle region of the disk is stable even though there is an additional pressure called degenerate electron pressure to a gas pressure. As it is discussed in chapter three, calculating the values of degenerate electron pressure in a complete and partial degenerate cases and adding to a gas pressure we have found that the disk is still

stable even after the degenerate pressure is added. This implies that the contribution of degenerate pressure is insignificant for the stability of the disk in the middle region.

For completely degenerate electron pressure contribution, which is on the basis that the central temperature approaches to zero, the disk is stable for the value of  $r$  in between  $1 \times 10^6 m$  and  $5.33 \times 10^7 m$ .

From fig. 4.1, we see that initially the pressure increases when the radius of the disk increases. Similarly in fig. 4.2 the pressure increases as the radius increases but in this case more rapidly. Similar phenomenon is observed in fig. 4.5 and 4.6. This is because of the additional term degeneracy electron pressure.

Although, the magnitude of the pressure increases as the radius of the disk increases, the stability of the disk holds in this region. we have seen a similar situation in the case of partial degeneracy pressure. That is the contribution of partial degeneracy electron pressure is insignificant for the instability of the accretion disk in the middle region, so the disk is stable.

## 4.2 Contribution of Degeneracy Electron Pressure in the Outer Region of Accretion Disk

The contribution of electron degeneracy pressure for the outer region of the accretion disk is also insignificant. We have derived the relation for the temperature and density in this region, which are less than the values obtained in the middle region. Then we have found that the pressure in complete and partial degenerate electron pressure cases which are the inputs for the stability calculation. Using the stability relation, the contribution of both complete and partial degeneracy electron pressure is calculated. For the complete degenerate electron pressure case, we have gotten the interval for which the disk is stable in the outer region in terms of the radius of the disk, that is  $5.33 \times 10^7 m \leq r \leq 1 \times 10^8 m$ . In similar manner for the partial degenerate electron pressure contribution, the radius in which the disk is stable is calculated to be the same as the complete degenerate case.

This has been illustrated by fig. 4.3 and 4.4. In both figures the pressure increases as the radius increases but not linearly. The increase in pressure in fig 4.4 with the existence of contribution of complete degeneracy electron pressure is greater pressure. Similar phenomenon is observed in fig. 4.7 and 4.8, for the contribution of partial degeneracy electron pressure. Even though, the pressures increase in the outer region with the contribution of electron pressure, it does not affect the stability of the disk.

Generally, the middle and outer regions, where the radiation pressure is dominated by the gas pressure, are stable with the presence of a complete and partial degenerate electron pressure.

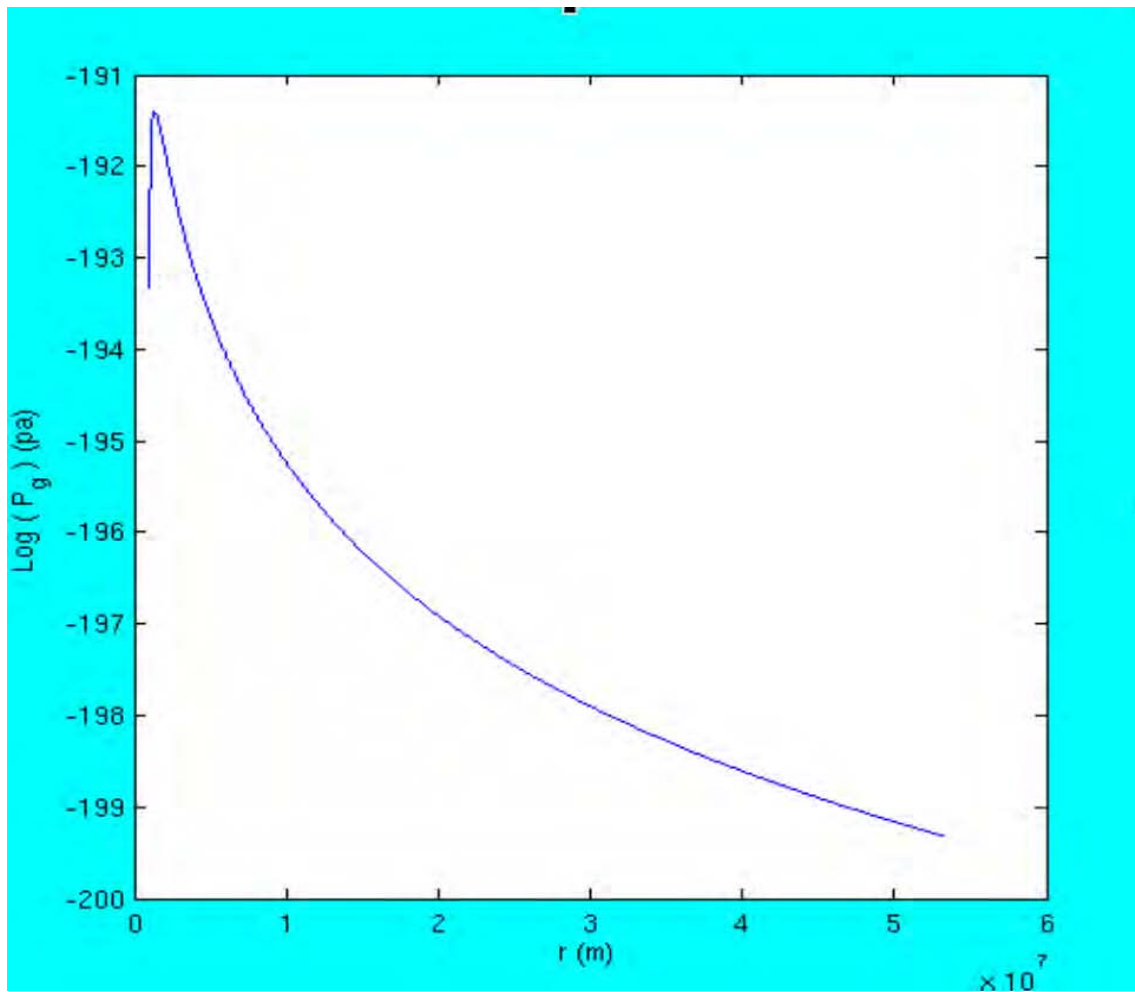


Figure 4.1: The relation between gas pressure and radius of the disk in the middle region.

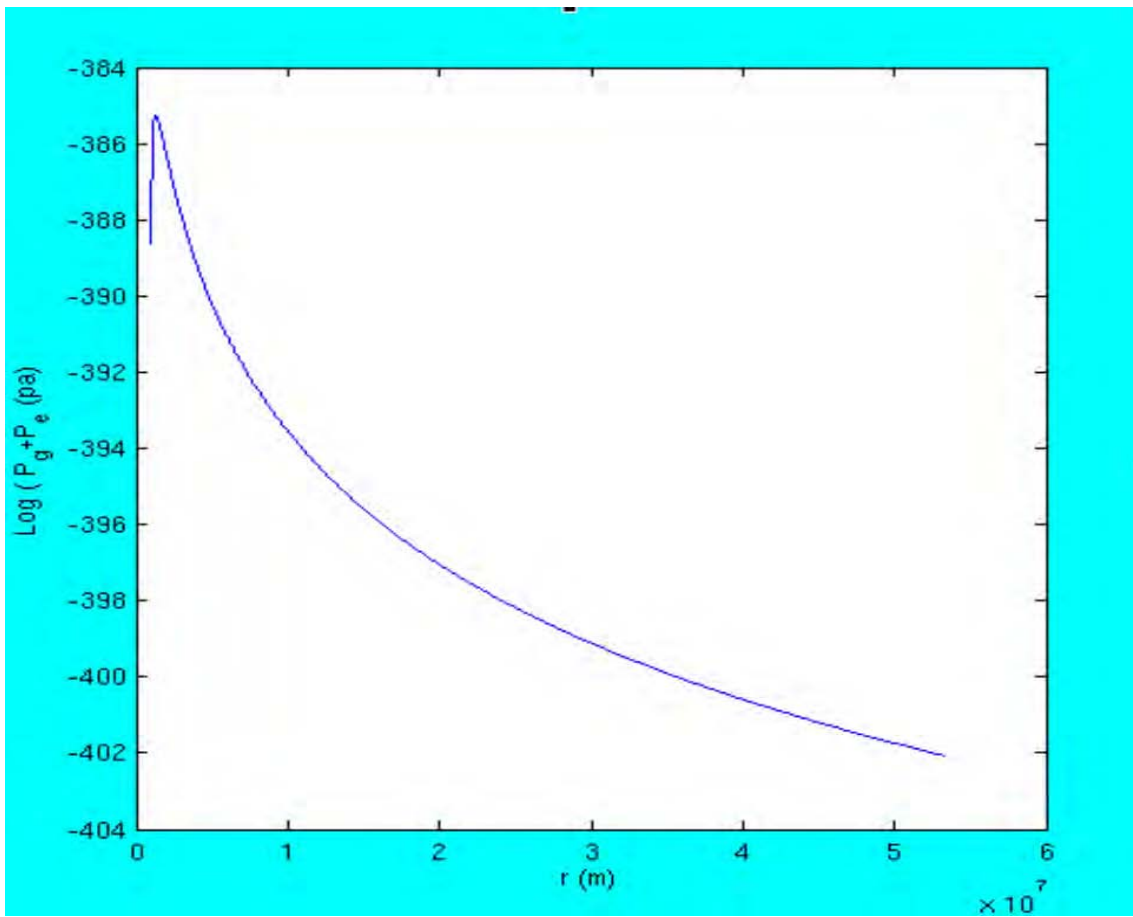


Figure 4.2: The relation between total pressure and radius of the disk for a complete degenerate electron pressure in the middle region.

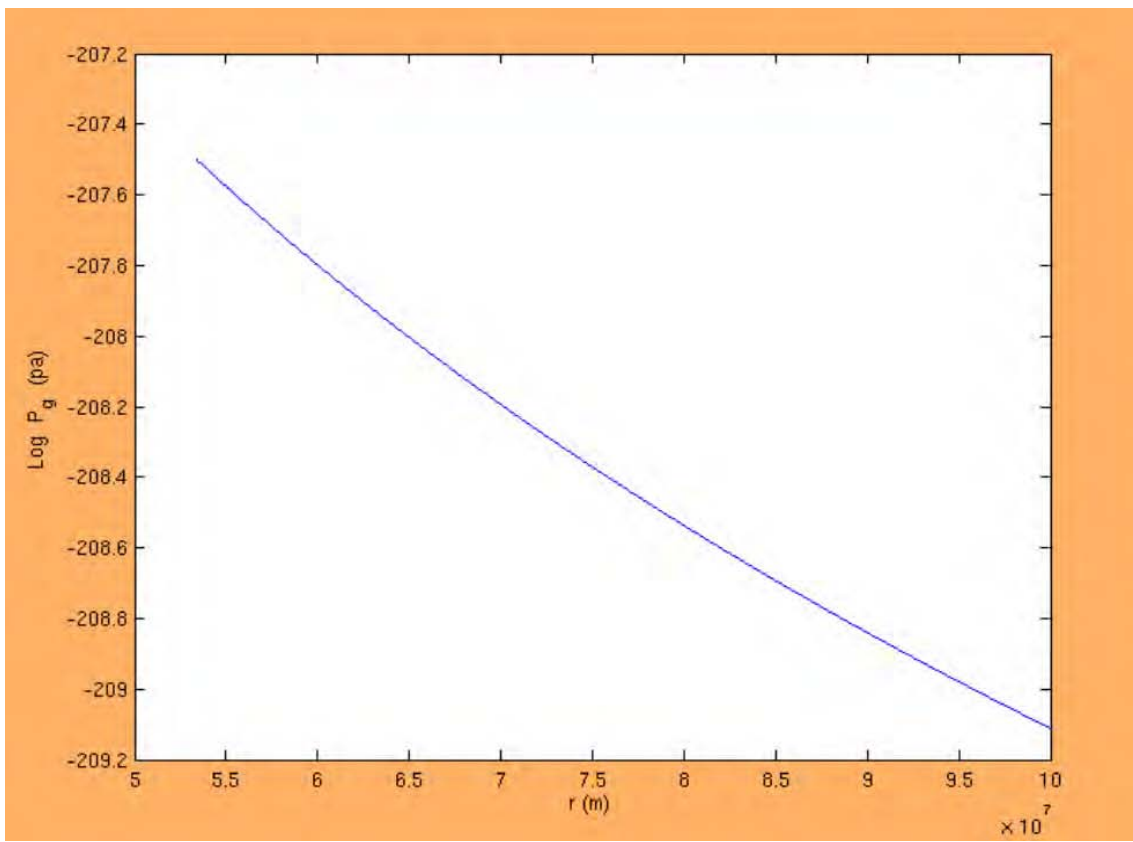


Figure 4.3: The relation between gas pressure and radius of the disk in the outer region.

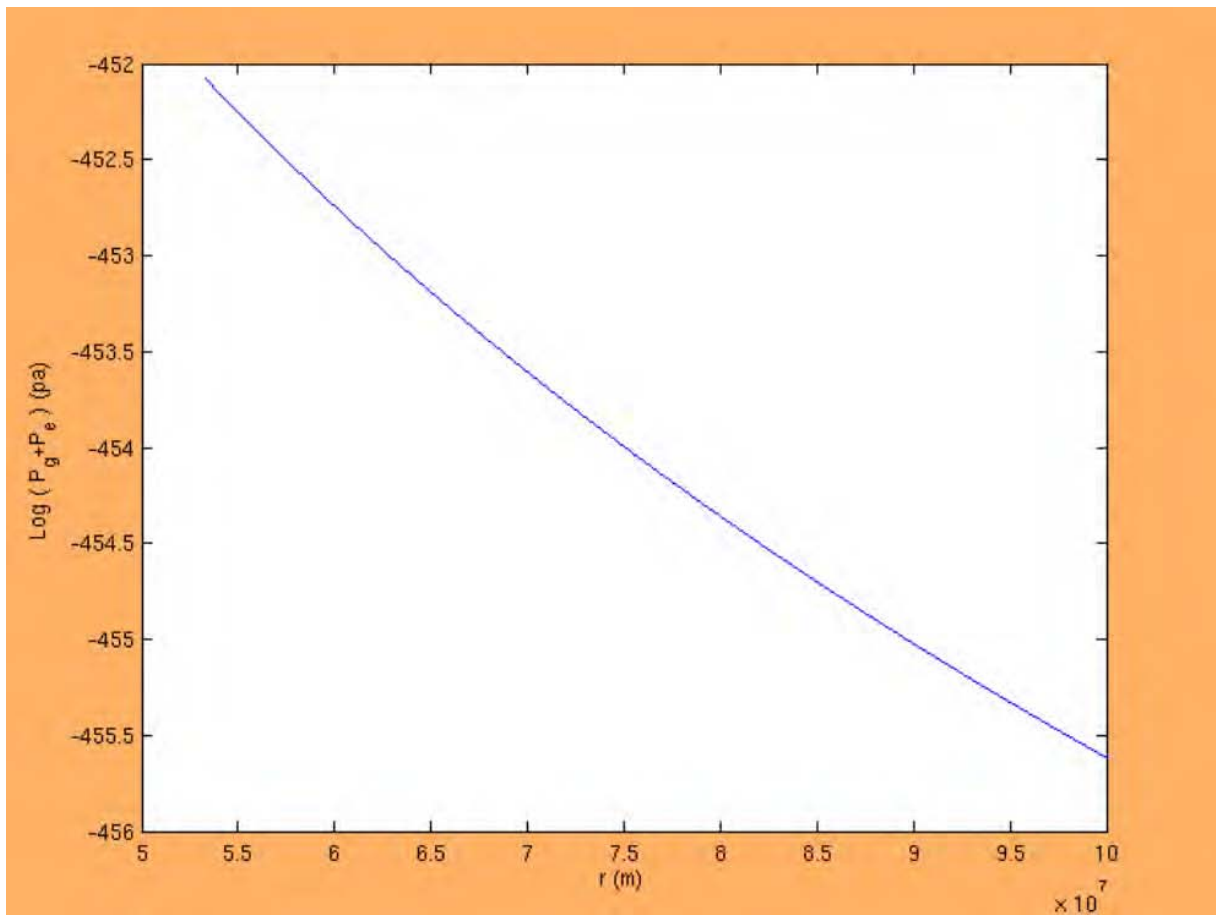


Figure 4.4: The relation between total pressure and radius of the disk for a complete degenerate electron pressure in the outer region.

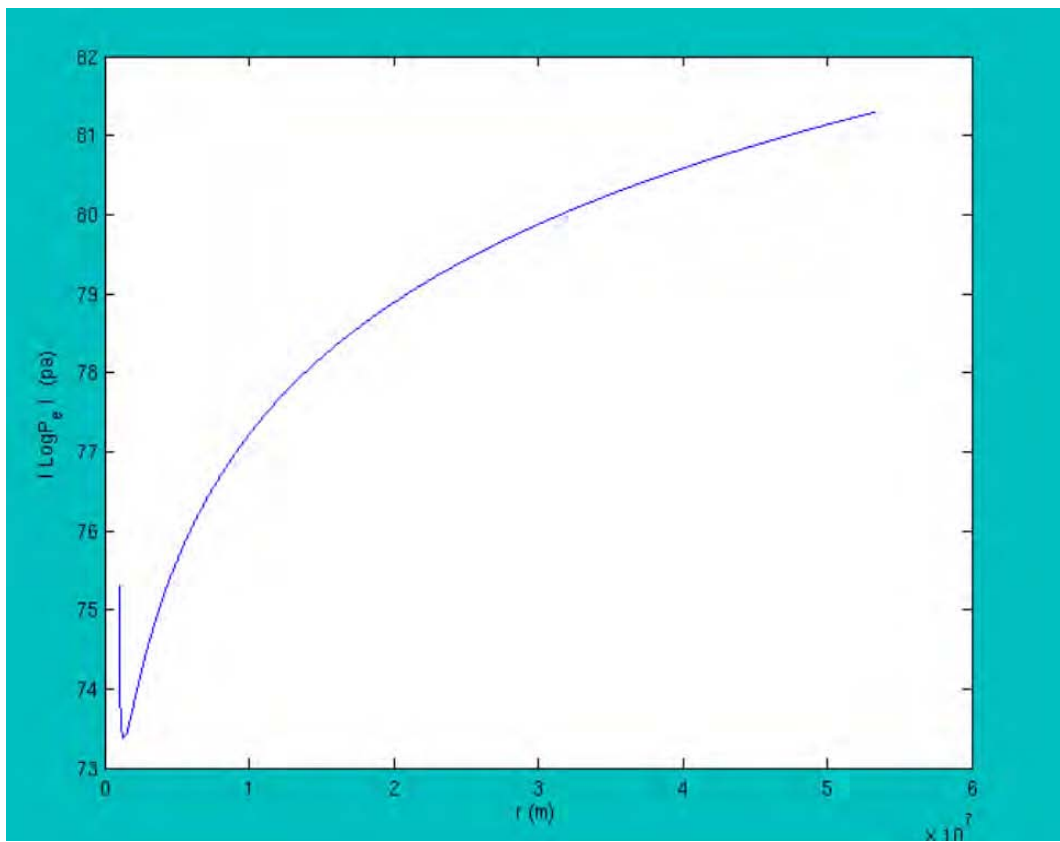


Figure 4.5: The relation between partial electron degeneracy pressure and radius of the disk in the middle region.

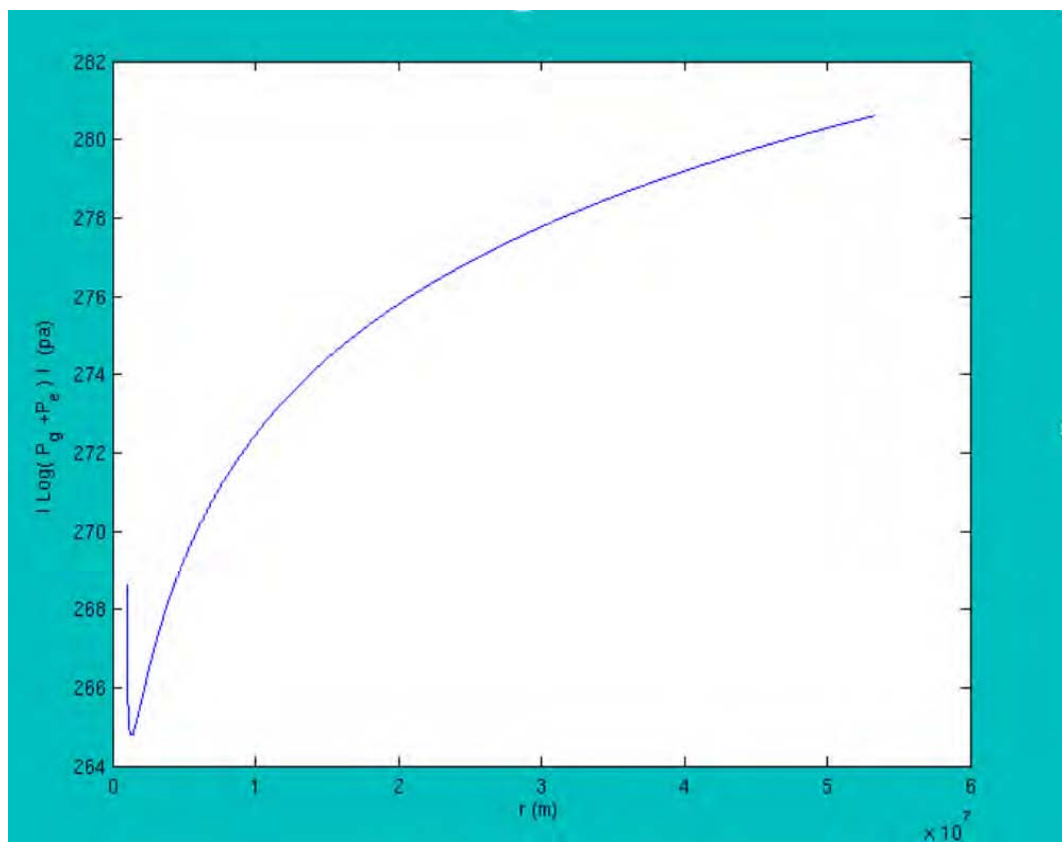


Figure 4.6: The relation between total pressure and radius of the disk for partial electron degeneracy pressure case in the middle region.

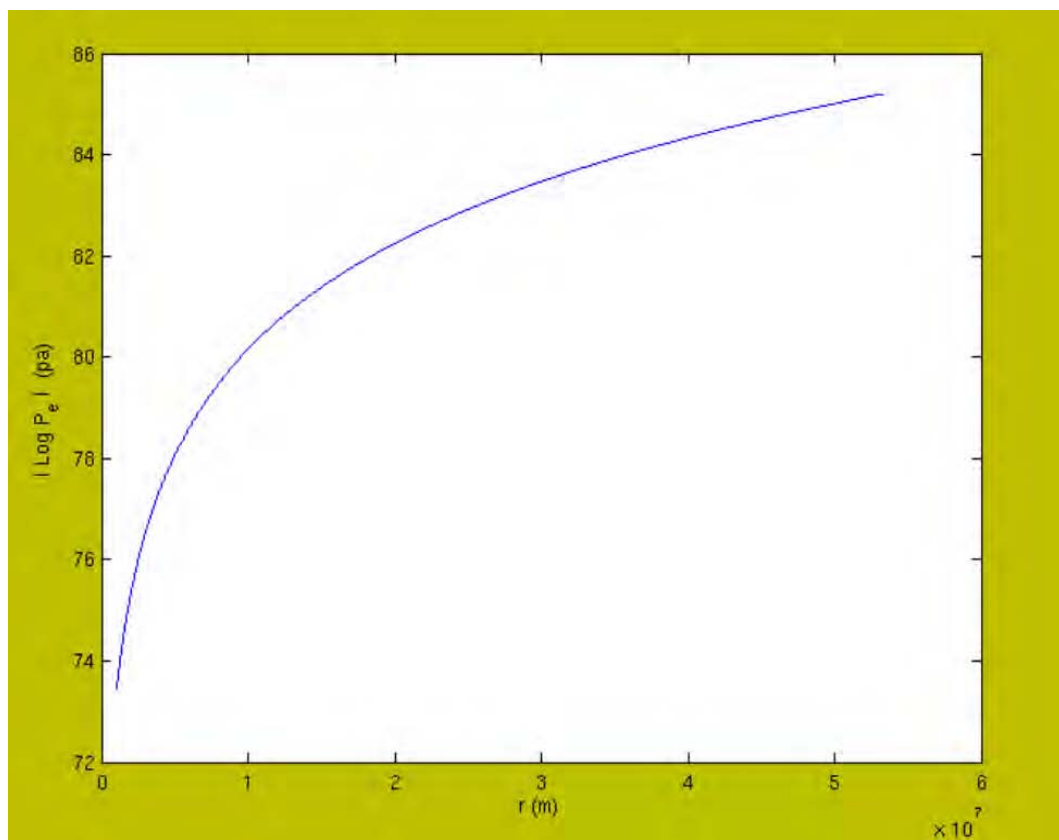


Figure 4.7: The relation between partial degenerate electron pressure and radius of the disk in the outer region.

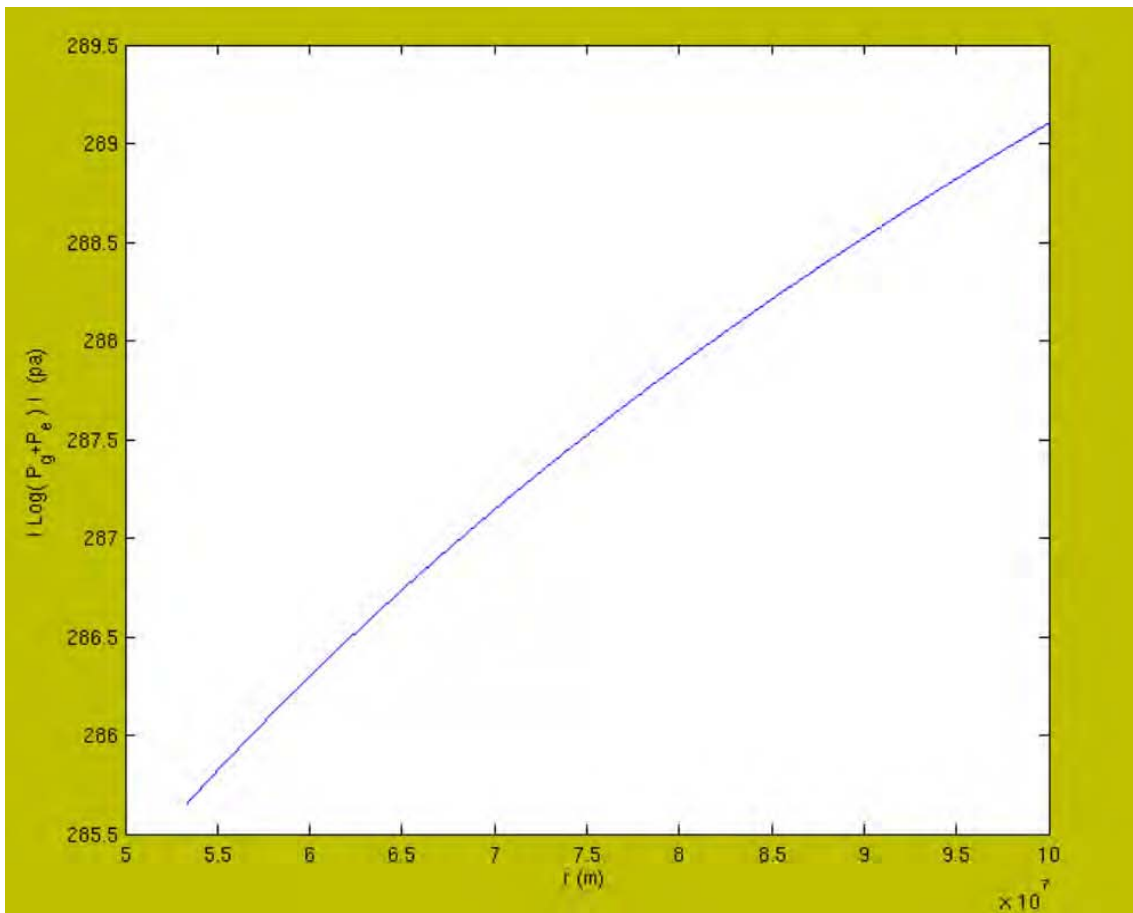


Figure 4.8: The relation between total pressure and radius of the disk for partial electron degeneracy pressure case in the outer region.

# Chapter 5

## Conclusion

In this study, our investigation is the stability in the case of gas-pressure dominated region of the accretion disk. According to the standard theory of accretion disk (Shakurra and Sunyaev 1973) [2], the middle and outer parts of a disk are dominated by the gas pressure. We have found that the temperature and density of the disk in the gas-pressure dominated region decreases as the radius of the disk increases. We have derived the relation for degeneracy electron pressure both in the case of complete and partial degeneracy. A complete degenerate electron pressure is calculated assuming that the temperature of the disk approaches to zero which means the temperature of th disk is less than Fermi temperature and a partial degenerate pressure is calculated at a finite temperature of the disk which is more practical. Comparing the gas pressure with the total pressure, we have observed that the total pressure rapidly increases when the radius of the disk increases. Though the pressure increase, the stability of the disk does not be altered in the middle and outer region of the accretion disk.

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**Declaration**

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

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**Place and time of submission: Addis Ababa University, June 2011**

This thesis has been submitted for examination with my approval as University advisor.

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