O N T H E L E N G T H O F D-M O D U L E S

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The undersigned hereby certifies that they have read this manuscript and recommends to the College of Natural science its acceptance. The title of the project is, **On The Length of D-Module** by Tewodros Argachew in partial fulfillment of the requirements for the degree of Master of Science.

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Abstract

This paper is concerned on the basic properties of weyl algebra, and length of D-modules over the weyl algebra.

I have only tied to present the length of A_3–modules and its composition series by considering polynomial rings with three variables.

Notations

- K denotes a field of characteristic zero.
- $\mathbb{C}$ denotes a field of complex numbers.
- $\mathbb{N}$ denotes the set of natural numbers.
- $\mathbb{N}_0$ denotes the set of natural numbers with 0.
- $\mathbb{Z}$ is the set of integers.
- K[x] denotes the ring of polynomials in one variable over a field.
- K[X] is the ring of polynomials in n variables over K.
- $\mathbb{C}$[X] is the ring of polynomials in n variables over $\mathbb{C}$.
- Differentiation (in the variable x) can be considered as a map

  ∂_x: $K[X] \to K[X]$.
- $End_K(K[X])$ denotes the set of endomorphisms from K[X] to K[X].
- ℓ (M) denotes length of an R-module M.

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INTRODUCTION

D-module is a module over a ring of differential operators. The major interest of such D-modules is as an approach to the theory of linear partial differential operator equations. The first case of algebraic D-modules are modules over the Weyl algebra A_n over a field of characteristic 0. The history of the Weyl Algebra begins with the birth of quantum mechanics, in the year 1925. What Heisenberg originally were introduced quantum theoretical analogies of the Fourier series.

This paper is concerned about the length of D-Modules, and the point of view that of modules over the Weyl algebra. I have only tied to present the length of A_3-modules and its composition series by considering polynomial rings in three variables. The work in this seminar report was partially supported by the idea of a S.C Coutinho, from his book a Primer of Algebraic D-Modules in preliminary part, so I have account for him.

The major objective of this paper is to provide basic properties of Weyl algebra and introduce the number of important examples of modules over the Weyl algebra, and mainly focused on the composition series and its length. This project report is divided in to two parts. The first part concerned with preliminaries; definition and basic properties of Weyl algebra. The second part of this project concerned with Simple A_n-module, composition series and length of A_3-Modules and collects a number of examples of modules over Weyl algebra. All examples we discuss here are the polynomial rings in three variables. It is also proved that every simple module is cyclic.