One Dimensional Interacting Fermion System

By

Daya Koyira Koche

A thesis submitted in partial fulfilment of the requirements for the DEGREE OF MASTERS OF SCIENCE IN PHYSICS at the Graduate School of Addis Ababa University

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Department of Physics

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Abstract

Some properties of metals at a low - lying excitation is studied analytically based on the theory of Fermi liquid. The breakdown of Landau's theory of Fermi liquid in one dimension is analyzed. The one dimensional interacting system shows the power law behavior and logarithmic divergence in its correlation function. In contrast, the Fermi liquid shows the pole in its microscopic equation of the Green's function.

Contents

1 Introduction

This thesis is devoted to theoretical and qualitative analysis of the one dimensional interacting fermion system (singular Fermi liquid). The analysis starts with the usual Fermi liquid based on Landaus theory. The approach of the thesis is using the theory of linear response and adiabatic switching on of interaction so as to expand the energy near Fermi surface. Mathematically I used Taylor expansion as well as the Green function and bosonization method to suit for my work. The thesis has aimed at qualitative results of the problem hence rigorous mathematical derivation is excluded. This may be refereed from the indicated references.

The property of metals specially that associated with electron in metals is studied under free-electron model, tight band model, the Kroning-Penny model and others independent electron methods. Here our approach is to study various transport and thermodynamic properties of metals using the Landau theory of Fermi liquid which takes into account electro-electron interaction.

This theory first forwarded by Landau [1] aimed to describe the property of liquid helium-3. Latter on, others extended the theory to study the behavior of electrons in different materials. The Landau theory of Fermi liquid is applied in three dimensions. However, Luther [2], Haldane [3], Luttinger [4] and others extended his idea to one and two dimensions. The study of quantum dots, carbon nano tubes, quantum wires, etc. are the applications of the theory in one dimension.

The study of metals is always associated with determining the shape of the Fermi surface. Depending up on the shape of the Fermi surface various physical properties of a given metal can be determined. Landau's idea which is based up on the ideal fermions can be defined as: there is continuous and a one-to-one correspondence between the eigenstates (ground states and excited states) of the noninteracting and interacting systems.

Landau's theory gives a good account of the low lying single particle excitation of the system of interacting electrons. These single particle excitations are called

quasiparticles; they have a one-to-one correspondence with the single particle excitations of the free electron gas. A quasiparticle may be thought of as a single particle accompanied by a distortion cloud in the electron gas. One effect of the coulomb interaction between the electrons is to change the effective mass of the electrons; in the alkali metals the increase is of the order of 25 percent.

There are short comings to the Landau theory. It is not applicable to microscopic phenomena, those which involves distances of the order of inter particle spacing, or energies compared to that of a particle on the Fermi surface. Moreover, it is in a certain sense, too complete in that it provides far more information than any experiment will ever sample. It is therefore of interest to develop a direct description of experimental measurement on many particle systems. An exact formalism can be developed so long as the system responds linearly to the measuring apparatus [5].

The general theory of linear response is applicable to both microscopic and macroscopic phenomena. It is the base through which the phenomenological derivation of Landau's parameters are developed in this thesis. It establishes the connections between response and the correlation functions and the extent to which these may be related to the spectrum of elementary excitations. The theory provides a number of exact results of great importance.

The basic assumptions used by Landau in developing the theory are: the theory of linear response, adiabatic switching on of interaction, the system should be isotropic, the interaction force must be repulsive and also back-ward scattering is neglected.

Another important point in the development of the Landau theory is that the Fermi liquid must be normal. A normal Fermi liquid may be defined as a degenerate Fermi liquid in which the property of the system are not drastically modified by the particle interaction, no matter how strong they might be. In other words, the liquid retains the essential properties of the noninteracting system. It has a well defined Fermi surface, its specific heat varies linearly with temperature etc. Examples of normal Fermi liquids are He-3 above 4×10^{-3} degrees and conduction electrons in

metals which are not super conducting.

The thesis is divided in to four chapters. Chapter one is introduction and chapter two deals with neutral Fermi liquid. The neutral Fermi liquid corresponds to an interacting fermion system which is not charged. It is aimed to put a theoretical ground for a charged system which will be discussed in chapter three. In this chapter the basic assumptions of Landau theory, the quasiparticle concept, the Fermi-Dirac distribution function, equilibrium properties and transport equations will be discussed.

The third chapter deals with the real Fermi liquid i.e. the charged fermion system. The effect of interaction in the charged system is different from that of the neutral one. The electron-electron, the electron-phonon and the electrons with impurity interaction makes the system more complicated. In Landau's assumption the only interaction considered is the electron-electron one. The long range coulomb interaction becomes short ranged in order to satisfy the theory; this is due to screening effect.

The fourth chapter is the main body of the thesis. The previous two chapters help as a bridge to discuss about the interacting fermion system in one dimension. The quasi particle excitation system in 3D of the Fermi liquid concept never applies for interacting fermions in one dimension. Here the one particle Green function and bosonization technique for both Fermi and non Fermi liquid system is used as aground for further discussion. The one dimensional fermion system shows the power law behavior and logarithmic divergence in its correlation function. So this chapter is concerned to show where the Fermi liquid theory fails and how the one dimensional singular Fermi liquid behaves. This is the aim of the thesis. The bosonization technique, neutral and spin models, the Kondo problem, orthogonality catastrophe, X-ray edge singularity are discussed qualitatively and to some extent quantitatively. The last chapter is concerned with the summary and references.

2 Neutral Fermi liquids

The theory of Fermi liquid is based on the idea of a non interacting fermion system. The non interacting Fermion gas in equilibrium at a temperature T is described by the Fermi -Dirac distribution function

$$f(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/kT}+1}$$

where k is a Boltzman's constant. The constant μ, known as the chemical potential, is adjusted in such away as to give the correct total number of particles. At high temperature $f(\varepsilon)$ reduces to the usual Maxwell-Boltzman expression and the gas is classical. In the opposite limit, $T \to 0$, $f(\varepsilon)$ becomes a Fermi-Dirac step function, which jumps from 1 to 0 at the positive chemical potential μ_0: the gas is said to be fully degenerate.

In the degenerate region, the number of excited states available to the system is very much reduced by the exclusion principle, which acts to freeze the distribution; at a temperature T, only those particles whose energy is within kT of the Fermi energy are affected by a change in temperature. This reduction has striking physical consequences: the specific heat becomes proportional to T, instead of being constant; the spin susceptibility becomes temperature independent, instead of varying as $1/T$. For real fermion system, the particle interaction and the exclusion principle act simultaneously; we are thus led to study degenerate Fermi liquids, in which both effects are important. In some systems, the nature of the degenerate gas is drastically modified by the particle interactions. Such is the cause, for instance, in superconducting electron gas. Frequently, the interacting liquid retains many properties of the gas: it is then said to be normal. A normal Fermi liquid at $T = 0$ has a sharply defined Fermi surface S_F; its elementary excitation may be pictured as quasiparticles out side S_F and quasiholes inside S_F in close analogy with the single particle excitation of non interacting Fermi gas. Such a resemblance explains why so many properties of the liquid can be interpreted in terms of one-particle approximation. To consider another example, the one electron theory of solids pro-

vides a correct account of a large number of sophisticated phenomena in metals (de Haas-van Alpha effect, transport properties, etc) even though it ignores the particle interaction. Again, the explanation of this success is found in the concept of quasi-particle excitation. It is fact that such a single particle excitations are not complete; there exists many-body effects which arise as a consequence of particle interaction, and which are characteristics of the liquid state.

At present time we do not posses a theory that completely describes the properties of an interacting Fermi liquid at an arbitrary temperature. The problem may be formulated by means of sophisticated field theoretic techniques. However, the general solutions that have been obtained are of a somewhat formal character, and have not led, as yet, to explicit result which may be compared with experiment. Fortunately, one may obtain a number of simple results in the limit of low temperature ($T \ll T_F$), for phenomena occurring on a macroscopic scale. The relevant theory was constructed by Landau on a semi phenomenological basis. For this to be an acceptable hypothesis it is crucial that the interaction do not lead to any form of phase transition or symmetry -broken ground state. As we have pointed out in the introduction ^{3}He is the only degenerate Fermi liquid found in nature. With minor modifications the theory of Fermi liquids may also be applied to electrons in metals or semi metals (to nuclear matter). The extension of Landau's theory to charged system (such as electrons in metals) involves certain difficulties which arise from the long range character of coulomb interaction.

2.1 The quasi-particle concept

Consider a system of N noninteracting free fermions each of mass m enclosed in a volume Ω. The eigenstates of the total system are anti symmetrized combinations of N different single - particle states. Each single particle is characterized by two quantum numbers, its momentum $\mathbf{p}$, and its spin $\sigma = \pm 1/2$; its normalized wave

function in configuration is a simple plane wave:

$$\Psi_p(\mathbf{r}) = \frac{1}{\sqrt{\Omega}} \exp(i\mathbf{p}\cdot\mathbf{r}) \tag{2.1}$$

The total wave function is a slater determinant made up of N such plane waves. All the eigenstates of the system can be characterized by the distribution function $n_{p\sigma}$, which is equal to 1 if the state p, σ is occupied, to zero otherwise. (In what follows, we shall omit the spin index σ, and include it in p, unless specified otherwise).

A particle with momentum $\mathbf{p}$ posses a kinetic energy $p^2/2m$. In the absence of interaction, the energies of the particles are simply additive: the total energy of the system is given by

$$E = \sum_p n_p p^2/2m \tag{2.2}$$

The ground state is obtained by filling the N plane wave states of lowest energy. The Fermi momentum p_F being given by

$$N/\Omega = 1/3\pi^2(p_F/\hbar)^2 \tag{2.3}$$

All the plane wave states lying inside the Fermi surface S_F (here the sphere of radius p_F) are filled in the ground state; those lying out side S_F are empty.

Let us add a single particle to the system. The ground state of $(N+1)$ particle system is obtained if the additional particle added in the lowest available momentum state, one on the Fermi surface. The chemical potential μ, defined as

$$\mu = E_0(N+1) - E_0(N) = \frac{\partial E_0}{\partial N}$$

is thus given by

$$\mu = p_F^2/2m. \tag{2.4}$$

The chemical potential is equal to the energy of a particle on the Fermi surface. This result will be seem to apply equally well in the presence of particle interaction.

Excited states of the system are best specified with reference to the ground state. A given excited state is obtained by exciting a certain number of particles across

the Fermi surface. Such a procedure is equivalent to creating an equal number of particles out side S_F and of holes inside S_F. Particles and holes thus appear as elementary excitations whose configuration give rise to all excited states. The amount of excitation is characterized by the departure of the distribution function from its value in the ground state:

$$\delta n_p = n_p - n_p^0. \tag{2.5}$$

For noninteracting system, the excitation energy is simply

$$E - E_0 = \sum_p \frac{p^2}{2m} \delta n_p \tag{2.6}$$

At low temperature, particles and holes will only be excited near the Fermi surface; δn_p will typically be of order 1 in small region surrounding S_F and will otherwise be negligible. In an isolated system, the total number of particles is conserved: the number of particles is there fore be equal to that of excited holes. This restriction is some times inconvenient. It is then preferable to work with what is equivalent to the canonical ensemble of statistical mechanics, a system which is characterized by its chemical potential μ rather than by its number of particles N. Such a situation may be realized by imagining the system to be in contact with reservoir of particles. In such cases, the quantity of interest is the free energy, given by $F = E - \mu N$ at zero temperature. It follows from eq. (2.6) that the excitation free energy associated with the distribution δn_p is given by

$$F - F_0 = \sum_p (p^2/2m - \mu) \delta n_p \tag{2.7}$$

2.2 Definition of Quasi Particles and Quasi Holes - Basic Assumptions

On defining Fermi liquid, we are not going to use the Green's function and the mathematical apparatus of the many body perturbation theory, but we adopt an alternative approach. This consists in comparing the interacting real liquid with

the noninteracting ideal gas; we establish a-one-to-one correspondence between the eigenstates of the two systems. Such an approach will provide us with a qualitative understanding of the excitation spectrum of an interacting system.

Consider an eigenstate of the ideal system, characterized by a distribution function n_p. In order to establish a connection with real system, we imagine that the interaction between the particles is switched on slowly. Under such adiabatic conditions, the ideal eigenstates will progressively transform into certain eigenstates of the real interacting systems. We therefore assume that the real ground state may be adiabatically generated starting from some ideal eigesntate with a distribution n_p^0. This statement may be considered as the definition of a normal fermion system.

For reasons of symmetry, the distribution n_p^0 of an isotropic system is spherical. As a result, the spherical Fermi surface is not changed when the interaction between the particles is switched on: the real ground state is generated from the ideal ground state. Matters are otherwise when the Fermi surface for the noninteracting system is anisotropic. Under these circumstances, the Fermi surface will certainly be deformed when the interaction is switched on. In such cases, the real ground state may be shown to follow adiabatically from some excited state of the noninteracting system. This situation will not cause any major difficulty with the theory.

Let us now add a particle with momentum p to the ideal distribution n_p^0 and, again, turn on interaction between the particles adiabatically. We generate an excited state of the real liquid which likewise has momentum p, since momentum is conserved in particle collisions. As the interaction increased we may picture the particle as slowly perturbing the particles in its vicinity; if the change in interaction proceeds sufficiently slowly, the entire system of $N + 1$ particles will remain in equilibrium. Once the interaction is completely turned on, we find that our particle moves together with the surrounding particle distortion brought about by the interaction. In the language of field theory, we would say that is dressed with a self energy cloud. We shall consider the dressed particle as an independent entity, which we call a quasi particle. The above excited state corresponds to the real ground

state plus a quasi particle with momentum p.

Let S_F be the Fermi surface characterizing the un perturbed distribution n_p^0 from which the real ground state is built up. Because of the exclusion principle, quasi particle excitation can be generated only if their momentum p lies out side S_F. The quasi particle distribution in p space is thus sharply bounded by the Fermi surface S_F.

Using the same adiabatic switching procedure we can define a quasi hole, with momentum p lying inside the Fermi surface S_F; we may do likewise for higher configurations involving several excited quasi particle and quasiholes. The quasi particle and quasiholes thus appear as elementary excitation of the real system which, when combined, give rise to a large class of excited states. We have established our desired one- to- one correspondence between ideal and real eigenstates.

Any definition of elementary excitation is some what imprecise since damping occurs due to real collisions of quasiparticles and quasi holes. But the quasiparticle life time becomes sufficiently long in the immediate vicinity of the Fermi surface that the quasiparticle concept makes sense in that region. In pure systems at zero temperature, the life time varies as the inverse square of the energy separation from the Fermi surface S_F. Since the quasi article is better and better defined as one gets closer and closer to the Fermi surface, S_F remains sharply defined. It should be kept in mind that a quasi particle is only strictly defined if it is right on the Fermi surface. In order for our theory to make sense, we must careful to introduce only in the neighborhood of S_F.

Let us consider an eigenstate of the noninteracting system characterized by some distribution function n_p for the usual bare particles. By switching on the interaction adiabatically, we obtain an eigenstate of the real system, which may be labelled by the same function n_p. In the interacting system, n_p describes the distribution of quasi particles. The excitation of the system is measured by δn_p from the ground state distribution (2.5). At low temperature, one only sample low-lying excited states for which δn_p is restricted to the immediate vicinity of the Fermi surface.

Under such conditions quasi particle damping is negligible, and our over all picture becomes meaningful.

2.3 Energy of the Quasi Particle

For ideal system, there exists a simple linear relation between the energy of a given state and the corresponding distribution function, When particle interaction is taken into account, the relation between the state energy, E, and the quasi particle distribution function, n_p, becomes much more complicated. It may be expressed in a functional form, $E[n_p]$, which one can not in general specify explicitly. If, however, n_p sufficiently close to the ground state distribution n_p^0 we can carry out a Taylor expansion of this functional. On writing n_p in the form (2.5), and taking δn_p to be small, or to extend over small region in momentum space, we obtain

$$E[n_p] = E_0 + \sum_p \varepsilon_P \delta n_p + O(\delta n_p^2) \tag{2.8}$$

Where ε_p is the first functional derivative of E. since each summation over p carries a factor Ω, ε_p is of order $\Omega^0 = 1$.

If $\delta n_{\mathbf{p}}$ describes a state with one extra quasiparticle $\mathbf{p}$, the energy of the state is $(E_0 + \varepsilon_{\mathbf{p}})$: $\varepsilon_{\mathbf{p}}$ is the energy of the quasiparticle. According to Eq.(2.5), the energies of several excited quasiparticles are simply additive, with in correction of of order $(\delta n)^2$. We shall assume that ε_p is continuous when p crosses the Fermi surface. This statement is not obvious and should be considered as a characteristic of normal systems.

On the Fermi surface, ε_p must equal a constant ε_F: otherwise we could lower the ground state energy by transferring a particle from a state inside the Fermi surface to one of lower energy outside S_F. Since the ground state for $(N + 1)$ particles is obtained by adding a quasi particle on the Fermi surface, ε_F is simply the chemical potential, $\mu = \partial E_0/\partial N$, at zero temperature.

In practice, we need only values of $\varepsilon_{\mathbf{p}}$ in the vicinity of the Fermi surface, where

we can use a series expansion. The gradient of $\varepsilon_{\mathbf{p}}$,

$$\mathbf{v_p} = \nabla_{\mathbf{p}} \varepsilon_{\mathbf{p}} \tag{2.9}$$

plays the role of group velocity of the quasi particle: it is the velocity of the quasi particle wave packet. In the absence of a magnetic field, and for a system which is reflection invariant, ε_p and $\mathbf{v_p}$ do not depend on the spin σ. For anisotropic system, ε_p only depends on p, and may be denoted as ε_p. The velocity $\mathbf{v_p}$ is then parallel to $\mathbf{p}$, so that we can write

$$\mathbf{v}_{p_F} = \mathbf{p}_F / m^* \tag{2.10}$$

where m^* is called the effective mass of the quasi particle .

In an isotropic system, the velocity $|\mathbf{v_p}|$ varies over the Fermi surface; the notion of an effective mass is somewhat artificial. It is then convenient to introduce the density $\nu(\varepsilon)$ of quasi particle states having an energy $(\mu + \varepsilon)$:

$$\nu(\varepsilon) = \sum_p \delta(\varepsilon_p - \mu - \varepsilon). \tag{2.11}$$

At low temperatures all physical properties will depend on the density of states on the Fermi surface ν_0.

2.4 Interaction between Quasi Particles - Phenomenological Derivation

Coming back to the free energy concept, the excitation free energy as measured from the ground state value F_0 is given by

$$F - F_0 = E - E_0 - \mu(N - N_0). \tag{2.12}$$

Where N_0 is the number of particles in the ground state. In order to generalize the expansion (2.8) we need to calculate $(N - N_0)$. For that purpose, we note that by adding one particle to the ground state, we add exactly one bare particle to the system as a whole. This follows at once from adiabatic definition of quasi particles;

the state with one extra quasi particle is derived from an ideal state containing $(N+1)$ particles, and the total number of particles is conserved when interaction switched on adiabatically. The difference $(N - N_0)$ may thus be written as

$$N - N_0 = \sum_P \delta n_p \tag{2.13}$$

Using eqs.(2.8) and (2.13), we obtain, within correction of order $(\delta_n)^2$,

$$F - F_0 = \sum_P (\varepsilon_p - \mu)\delta n_p. \tag{2.14}$$

Most of the properties that we shall consider will involve a displacement of the Fermi surface by a small amount δ. The corresponding values of δn_p is equal to ± 1 in a thin sheet of width δ centered on the Fermi surface; it vanishes outside this sheet. Where δn_p is nonzero, $(\varepsilon_p - \mu)$ is of order δ. The difference $(F - F_0)$, given by eq.(2.14), is of order δ^2. The expansion (2.14), which looks like a first order expansion, is actually a second order one. Our approach is therefore consistent only if we push the Taylor expansion of the functional $(F - F_0)$ one step further, to include all terms of the second order in the displacement of the Fermi surface. We are thus led to write

$$F - F_0 = \sum_p (\varepsilon_p - \mu)\delta n_p + 1/2 \sum_{pp'} f_{pp'} \delta n_p \delta n_{p'} + O(\delta_n^3) \tag{2.15}$$

Equation (2.15) is the heart of the phenomenological theory of Fermi liquids proposed by Landau (1956). Its most important feature, new quadratic term, which describes the interaction between the quasi particles. The leading term of an expansion of $(F - F_0)$ in equation (2.15) in power of the relative number of excited quasi particles is given by

$$\alpha = \sum_p \frac{|\delta_{np}|}{N} = \frac{|N - N_0|}{N} \tag{2.16}$$

Landau's approximation is valid whenever α is small. The coefficient $f_{pp'}$ is the second variational derivative of E (or F) with respect to n_p. It is accordingly invariant under permutation of p and p'. Since each summation over a momentum index carries a factor $\Omega f_{pp'}$ is of order $\frac{1}{\Omega}$. This is easily understood if we realize

that $f_{pp'}$ is the interaction energy of the excited quasi particles p and p'. Each of the latter is spread out over the whole volume Ω, the probability that they interact with one another is of the order a^3/Ω, where a is the range of interaction.

We shall assume that $f_{pp'}$ is continuous when p or p' crosses the Fermi surface. Once again, this may be considered as characteristics of a normal system. In practice, we shall only need values of f on the Fermi surface, at points such that $\varepsilon_p = \varepsilon_{p'} = \mu$. Then $f_{pp'}$ depends only on the direction of p and p', and on the spins σ and σ'.

If there is no applied magnetic field, the system is invariant under time reversal, which implies

$$f_{p\sigma,'p\sigma'} = f_{\_p-\sigma,-p'-\sigma'} \tag{2.17}$$

If furthermore the Fermi surface is invariant under reflection $p \longrightarrow -p$, eq.(2.17) becomes

$$f_{p\sigma,p'\sigma'} = f_{p-\sigma,p'-\sigma'} \tag{2.18}$$

In that case, $f_{p\sigma,p'\sigma'}$ depends only on the relative orientation of the spins σ and σ'; there are only two independent components, corresponding respectively to parallel spins and antiparallel spins. It is good to write this in the form

$$f_{pp'} = f^s_{pp'} + f^a_{pp'}; \qquad , f^{\uparrow\downarrow}_{pp'} = f^s_{pp'} - f^a_{pp\prime}, \tag{2.19}$$

where $f^s_{pp'}$ and $f^a_{pp'}$ are the spin symmetric and spin antisymmetric parts of the quasi particle interaction. If the system is isotropic, (2.19) may be further simplified. In that case, for p and $p\prime$ on the Fermi surface, $f^a_{pp'}$ and $f^s_{pp'}$ defends only on the angle ξ between the directions of p and p'. They may be expanded in a series of Legendere polynomials.

$$f^{s(a)}_{pp'} = \sum_{l=0}^{\infty} f^{s(a)}_l p_l(\cos\xi) \tag{2.20}$$

2.5 Equilibrium Properties

Now we are at a position to apply Landau theory to the study of a number of macroscopic properties, characteristic of the system at equilibrium. It will be evident that some of the properties are affected by interaction between quasi particles, while others are not.

2.5.1 Specific Heat

The specific heat is defined as

$$c_\nu = \left(\frac{\partial E}{\partial T}\right)_N . \tag{2.21}$$

Where the density of states $\nu(0)$ is defined by (2.11). For anisotropic system, $\nu(0)$ is given by

$$\nu(0) = \frac{\Omega m p_F}{\pi^2 \hbar^3} \tag{2.22}$$

from which we obtain the specific heat

$$C_\nu = \frac{m^* p_F}{3\hbar^3} k^2 T \tag{2.23}$$

A measurement of the slope of the linear specific heat therefore yields the effective mass of the quasi particles.

2.5.2 Compressibility and Sound Velocity

Let $E_0(\Omega)$ be the ground state energy of the system, regarded as a function of the volume Ω. The pressure P may be defined as

$$P = -\partial E_0 / \partial \Omega. \tag{2.24}$$

the compressibility κ is then given by

$$\frac{1}{\kappa} = -\frac{\Omega \partial P}{\partial \Omega} \tag{2.25}$$

For a large system, E_0 is an extensive quantity, proportional to the volume when the density $\rho = N/\Omega$ is kept constant. We may thus write

$$E_0 = \Omega f(\rho) \tag{2.26}$$

One finds directly from eqs.(2.24) and (2.26) that

$$\frac{1}{\kappa} = \rho^2 f\,''(\rho) \tag{2.27}$$

These quantities may be related to the chemical potential $\mu = \partial E_0/\partial N$. Indeed, it is straight forward to establish that

$$\frac{1}{\kappa} = N\rho \left(\frac{\partial \mu}{\partial N}\right)_\Omega \tag{2.28}$$

The compressibility is related to the velocity of sound, s, in the usual way:

$$s^2 = \frac{1}{\rho \kappa m} = \frac{N}{m}\left(\frac{\partial \mu}{\partial N}\right)_\Omega \tag{2.29}$$

2.5.3 Spin Susceptibility

The spin susceptibility χ_p is given by

$$\chi_p = \frac{M}{\Omega H} = \frac{m^* P_F \beta^2}{\pi^2 \hbar^3 (1 + F_0^a)} \tag{2.30}$$

It is clear from (2.30) that the spin susceptibility is modified by the exchange interaction F_0^a. Again eq.(2.30) is meaningful when $1 + F_0^a > 0$. Otherwise, the long wave length fluctuations of the magnetic moment become unstable: the system becomes ferromagnetic.

2.6 Transport Equation for Quasi Particles

2.6.1 Expansion of the Energy

Consider an in homogeneous state characterized by a distribution function

$$n_p(r,t) = n_p^0 + \delta n_p(r,t) \tag{2.31}$$

The departure δn_p from the ground state is supposed to be small. We assume δn_p to contain only long wave length fluctuations, and to be restricted to the vicinity of the Fermi surface, where quasi particles are well defined. The total energy is $E[n_p(r,t)]$ is a function of the distribution function and it can be expanded by a Taylor series for a small δn_p.

$$E = E_0 + \sum_p \int d^3r \varepsilon(p,r)\delta n_p(r) + \frac{1}{2}\sum_{pp'} \int\int d^3r d^3r' f(pr,p'r')\delta n_p(r)\delta n_{p'}(r) \quad (2.32)$$

For a translational invariant short range interaction force, $\varepsilon(p,r)$ is equal to ε_p and $f(pr,p'r')$ depend only on the difference$(r-r')$. Therefore the energy can be written as

$$E = E_0 + \int d^3r \delta E(r), \quad (2.33)$$

$$\delta E(r) = \sum_p \varepsilon_p \delta n_p(r) + \frac{1}{2}\sum_{pp'} f_{pp'}\delta n_p(r)\delta n_{p'}(r) \quad (2.34)$$

where the interaction energy $f_{pp'}$ is defined by

$$f_{pp'} = \int d^3r' f(p,r;p',r').$$

2.6.2 Transport Equations for Quasiparticles

In order to describe the transport properties, Landau considered the quasi particles as independent, described by a classical Hamiltonian $\tilde{\varepsilon_p}(r)$. Where the local excitation energy $\tilde{\varepsilon_p}(r)$is given by

$$\tilde{\varepsilon_p}(r) = \varepsilon_p + \sum_{p'} f_{pp'}\delta n_{p'}(r) \quad (2.35)$$

Consider a small volume element $d^3p\ d^3r$ in phase space according to the kinetic theory. The flow of the quasi particles through each side of the volume element is given by the linearized transport equation

$$\frac{\partial}{\partial t}\delta n_{\mathbf{p}} + \mathbf{v_p}\cdot\nabla_r \delta\tilde{n}_p + \frac{\partial n^0}{\partial \varepsilon_p}\mathbf{v_p}\cdot\mathbf{F_p} = I(\delta n_p) \quad (2.36)$$

Here again n_p is only defined in the vicinity of the Fermi surface. The concept of independent quasi particles assumed by Landau can not be true for all values of the momentum p: it is physically obvious that only excited quasi particles in the immediate vicinity of the Fermi surface can be considered as independent. The first term in eq.(2.36) the rate of change of quasiparticles. The second and the third terms describe the changes in the quasi particles due to diffusion and external force respectively. The right side term of the above equation is the collision term and it describes the collision between quasi particles. At a very low temperature and for excitations very close to the Fermi surface, we can neglect the collision term. Eq.(2.36) is very important in that it helps us to derive all the transport properties of a Fermi liquid and fluctuation spectrum of the system on a macroscopic scale.

2.6.3 The Current Density

The transport equation (2.36) in the absence of external force is written as

$$\frac{\partial \delta n_p}{\partial t} + v_p . \nabla_r \delta \tilde{n_p} = I(\delta n_p) \tag{2.37}$$

The charge density of a particle is related to its current density according to the conservation law;

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0 \tag{2.38}$$

This shows that the number of particles is everywhere conserved. The density fluctuation $\delta\rho$ may be written as

$$\delta\rho(r,t) = \sum_p \delta n_p(r,t) \tag{2.39}$$

If we sum over (2.37) and making the collision term zero, since collision conserve the number of particles and using (2.39), we get the expression

$$\frac{\partial \delta\rho}{\partial t} + \nabla_r \sum_p \delta\widetilde{n}_p \cdot \mathbf{v}_p = 0 \tag{2.40}$$

Comparing(2.40) with (2.39) we get

$$\mathbf{J} = \sum_P \delta\tilde{n}_p \mathbf{v}_p \tag{2.41}$$

where $\delta\bar{n}_p$, the departure from the local equilibrium, is given by

$$\delta\tilde{n}_p = \delta n_p - \frac{\partial n_0}{\partial \varepsilon_p} \sum_{p'} f_{pp'} \delta n_{p'}.$$

We thus write (2.41) as $J = \sum_p \delta n_p j_p$, where j_p is given by

$$j_p = v_p - \sum_{p'} f_{pp'} \frac{\partial n_0}{\partial \varepsilon_{p'}} v_{p'} \tag{2.42}$$

j_p thus appears as the current carried by the quasi particle p. According to eq.(2.42), the current j_p is different from the velocity v_p. This very important feature is characteristic of interacting system. When a quasi particle p is added to the system, it carries one extra particle at a group velocity v_p. Because of the particle interaction, the moving quasi particle tends to drag part of the medium along with it, thus producing an extra current: j_p will thus be different from v_p; the difference $(j_p - v_p)$ may be described as a drag current.

3 Charged Fermi Liquids

In chapter 2 we try to study about the behavior of neutral Fermi liquids. Here we try to see the interacting electrons properties beginning with the homogeneous system. In order to guarantee its stability, we assume that electrons are immersed in a uniform back ground of positive charge, of density equal to the average electron density. For the high electron densities and low temperatures the noninteracting electrons obey Fermi- Dirac statistics. The system may be regarded as a quantum plasma, in analogy with to the usual classical plasma, for which the noninteracting electrons are described by a Maxwell- Boltzman distribution. The quantum plasma is likewise the natural analog, for charged particles, of the neutral Fermi liquid; it may equally well be regarded as a charged Fermi liquid. Quantum plasmas are interesting in themselves. They also serve as a useful model for the behavior of electrons in the conduction bands of simple metals. For many purposes, the effect of the periodic array of ions in the latter system may be well approximated by the uniform background. The long range coulomb interaction between electrons falls off slowly with distance. This shows a charged Fermi liquid differs appreciably from its neutral counterpart.

3.1 Screening and Plasma Oscillation

When an electron moves, it tends to push other electrons out of its way, as a result of their mutual coulomb interaction. We may say that the electrons moves surrounded by a screening holes (the region in which one is not likely to find another electron). The screening holes corresponds to a distribution of positive charge, which is taken as a hole, tends to compensate the negative charge of the electron in the question. The electrostatic field of the electron is thus screened at large distance. As the electron moves it tends to carry along the screening hole. The central problem in developing a divergence free theory of electron systems is that of introducing the concept of dynamic screening in consistent fashion. Qualitatively speaking, each

electron in the system behaves like a moving test charge. It acts to polarize its surroundings. Another electron sees these electrons plus its accompanying time dependent polarization cloud; the effective interaction between the electrons is thus dynamically screened. By using such a concept of a dynamic screening, it is not difficult to reformulate the transport equations of the Landau theory of Fermi liquids, so that it can be applied to quantum plasmas with out difficulty.

3.1.1 Plasma Oscillation

A plasma is a medium with equal concentration of positive and negative charges, of which at least one charge type is mobile. In a solid the negative charge of the conduction electron are balanced by an equal concentration of positive charges of the ion cores. A plasma oscillation in a metal is a collective longitudinal excitation of the conduction electrons. A plasmon is a quantum of plasma oscillation; we may excite a plasmon by passing an electron through a thin metallic film or by reflecting an electron or photon from a film [6,7]. The plasma oscillation which is the collective oscillation of electrons is a second characteristics of a homogeneous electron system. The existence of organized oscillations in the plasma is complementary to the existence of screening. When the electrons move to screen a charge disturbance in the plasma, they will in general, tend to over shoot the mark somewhat. They are consequently pulled towards that region, over shoot again, etc; in such away that an oscillation is setup about the state of charge neutrality. The restoring force responsible for oscillation is simply the average self consistent field of all the electrons, exactly the same mechanism as was operative for zero sound. Because of the long range coulomb force, the frequency of the oscillation is very high. For long wave lengths it is very nearly constant and equal to

$$\omega_p = (4\pi N e^2/m)^{1/2} \tag{3.1}$$

Plasma oscillations, like screening, represent a typical polarization phenomena in the plasma. Indeed both effects can be easily described with in the frame work of a

general treatment of polarization, or dielectric process, in an electron system.

3.2 Localized Quasiparticles Excitations

Here it is aimed to investigate the Landau transport equations in the absence of an external field and collision terms. We seek to determine the self consistent, spatially varying solutions of the resulting homogeneous equations. These solutions corresponds to the natural (resonant) frequencies for excitations in the Fermi liquid. The excitations are of two kinds: localized quasiparticles and collective modes. The corresponding solutions are very different in their structure

3.2.1 Qualitative Structure of an Inhomogeneous Excitation

Suppose that a single quasiparticle with momentum p_0 is added to the ground state. The response of the system differs according to whether the added quasiparticle is spread out uniformly throughout the system, or is localized. In the former case, the state of the system remains homogeneous; the interaction energy of the added quasi particle with the ground state distribution is uniform. There is no force tending to excite other quasi particles. On the other hand, if the added quasi particle is localized, the local energy $\tilde{\varepsilon_p}$ of other quasi particles varies in space. This in turn gives rise to a polarizing force, which tends to excite quasi particles other than p_0. In order to build a localized wave packet for the quasi particle p_0, we must find the equilibrium distribution of the other quasiparticles around it. Consider excitations which are periodic in space and time with wave vector q and frequency ω. They are characterized by the distribution function,

$$\delta n_p(r,t) = \delta n_p(q,w) \exp i(qr - \omega t) + c.c. \tag{3.2}$$

Assume that ω is much lager than the collision frequency, which means that we study the motion of quasi particles over times shorter compared to the collision time τ. Hence we can ignore collisions. The free flow of the excited quasi particles

is governed by the transport equation, $\frac{\partial}{\partial t}\delta n_{\mathbf{p}}(\mathbf{r},t) + \mathbf{v}_p \cdot \nabla_{\mathbf{r}}\delta\tilde{n}_{\mathbf{p}}(\mathbf{r},t) = 0$. This may be written in our plane wave geometry as

$$-\omega\delta n_p(q,\omega) + \mathbf{q}\cdot\mathbf{v}_p\delta\tilde{n}_p(q,\omega) = 0 \tag{3.3}$$

If we replace $\delta\tilde{n}_p$ by the expression then,

$$\delta\tilde{n}_p = \delta n_p - \frac{\partial n^0}{\partial\varepsilon_p}\sum_{p'} f_{pp'}\delta n_{p'},$$

then we have

$$-(qv_p - \omega)\delta n_p - qv_p(\frac{\partial n^0}{\partial\varepsilon_p})\sum_{p'} f_{pp'}\delta n_{p'} = 0. \tag{3.4}$$

This homogeneous equation will only have solutions for certain eigenvalues of the frequency ω; the solutions will provide the elementary excitations of the system which vary in space with a wave vector $\mathbf{q}$.

Consider an excitation of the form which would involve a single added quasi particle with momentum p_0. The corresponding distribution would be

$$\delta n_p = \delta_{p,p_0}$$

This state does not represent a real excitation of the system. The corresponding disturbance acts to polarize the medium; as a result, the bare quasi particle is dressed with an induced polarization cloud of other excited quasi particles. The equilibrium solution for this problem corresponds to some distribution function, δn_p, which is determined by (2.12) and takes the form

$$\delta n_p = \delta_{p,p_0} + \xi_p. \tag{3.5}$$

ξ_p, which describes the polarization cloud, is of order $1/N$. Nonetheless the total effect of polarization cloud may be appreciable, since the number of values of p is of order N. We shall call a solution such as (3.5) in which one component plays the leading role, the individual excitation of the system.

If $p = p_0$ in (3.3), since $f_{pp'}$is of order $1/N$, we find within correction of that order

$$\mathbf{q}\cdot\mathbf{v}_{p0} - \omega = o \tag{3.6}$$

Equation (3.6) expresses ω in terms of q. We see the plane waves moves along q with a velocity $v_{p_0}\cos\theta$, where θ is the angle between q and p_0. $v_(p_0)$ is there the group velocity of the quasi particle.

If $p \neq p_0$ in (3.6), on collecting terms of $1/N$, we find

$$(\mathbf{q}\cdot\mathbf{v}_p - \omega)\xi_p - \mathbf{q}\cdot\mathbf{v}_p(\frac{\partial n^0}{\partial\varepsilon_p})\sum_{p'} f_{pp'}\xi_{p'} = +\mathbf{q}\cdot\mathbf{v}_p(\frac{\partial n^0}{\partial\varepsilon_p})f_{pp'} \tag{3.7}$$

If $q = 0$, then $\xi_p = 0$: the excitation corresponds to a single bare quasi particle p_0. The corresponding state is homogeneous. If, however, q is finite, the interaction with the bare quasi particle $p_0$acts as a diverging force on the right- hand side of the transport equation (3.7), and the resulting value of ξ_p will be non vanishing.

We note that the distribution ξ_p has the same wave vector q and frequency $\omega = qv_{p,}$ as the bare quasi particle p_0: it propagates through the system at a velocity, v_{p_0}, not v_p. In other words, while the component p_0 undergoes a free oscillation, the motion of other component is forced by the bare quasi particle p_0. This distinction is important; it is a characteristic of an individual excitation. The total particle $\rho_{p_0}(q,\omega)$ and current J_{p_0} from (3.5) is given by

$$\rho_{p_0}(q,\omega) = \sum_p \delta n_p(q,\omega) = 1 + \sum_p \xi_p J_{p_0}(q,\omega) = \sum_p j_p\delta n_p(q,\omega) = j_{p_0} + \sum_p j_p\xi_p \tag{3.8}$$

The last terms on the right-hand side of these two equations corresponds to the charge and current of the polarization cloud carried by the dressed particle p_0. They represent a sizable correction to the first terms 1 and j_{p_0}, which corresponds to the bare quasi particle charge and current. In general we can conclude that a bare localized quasi particle p_0 does not represent an eigenstate of the system. It necessarily surrounds it self with a cloud of quasi particles, the distribution of which depends on the geometry of the wave packet. The cloud carries particle density and current: The dressed quasi particle thus has properties quite different from those of the bare quasi particles.

3.2.2 Electrons in Metals

The electron liquid concept which describes the particles motion in a uniform back ground of positive charge, subject only to the mutual coulomb interaction is an idealized physical system. One does not encounter such electron liquids in nature; the conduction electrons in a metal see a uniform ionic charge distribution, which is of course periodic when the metal is in a solid form.

The basic assumption of the Landau theory remain valid in the presence of periodic potential. The elementary excitations are still quasi particles, with energy ε_p and interaction $f_{pp'}$. Duet to anisotropy of the crystal lattice, ε_p depends on the direction of p, while $f_{pp'}$ depends on both p and p'. The Fermi surface S_F is defined by the relation

$$\varepsilon_p = \mu,$$

where μ is the chemical potential. In most metals S_F has a rather complicated geometrical shape. In some cases, for instance in alkali metals, the Fermi surface is nearly spherical. The system behavior is then approximately isotropic and the formulation of the theory is the same as the electron liquid. There is, however, an important difference: the current j_p carried by a quasi particle with momentum p is no longer equal to p/m, as was a case for translationally invariant system. We may instead write j_p in the form

$$j_p = p/m_c,$$

where m_c is a crystalline mass, which differs from the bare electron mass m because of the periodic potential acting on the electrons.

The real quasi particle effective mass, m^*, is defined in terms of the velocity, v_p, of a quasi particle on the Fermi surface,

$$v_p = \frac{p}{m^*}.$$

The quantity m^* determines the density of states at the Fermi surface, and thus the specific heat. The difference between $m*$ and m_c reflects that between v_p and

j_p, and is thus a consequence of quasi particle interaction. The relation between $v_p and j_p$ does not depend on translational invariance, hence for an isotropic metal, it is possible to write

$$j_p = v_p + \sum_{p'} f_{pp'}\delta(\varepsilon_{p'} - \mu) = v_p(1 + \frac{F_1^s}{3})$$

from which it is possible to relate m^* and m_c. Thus,

$$m^* = m_c(1 + F_1^s/3).$$

Here the bare mass m is replaced by the crystalline mass m_c for real metal system.

In general, the modified Landau - Silin theory provides a formal description of the influence of Coulomb interaction on electron motion in metals and, as such, is pleasing to the theoretical physicist. For experimentalists, the most important application of the Landau theory is the assistance it provides concerning the extent to which each phenomena of interest is affected by the interaction between electrons. This interaction is exhibited on two different levels: it modifies the energy and velocity of the quasi particles, and gives rise to an interaction between excited quasi particles. Phenomena as that are not affected by quasi particle interaction may be described in terms of one- electron picture.

3.3 Physical Quantities and Quasiparticle Interaction

A number of methods have been developed to the study of electrons on the Fermi surface of metals. The interpretation of the experimental results always relies on a one electron approximation. Consequently, only those effects which are not affected by the interaction $f_{pp'}$ can be used to obtain exact information concerning the Fermi surface. To the extent that the wave length and frequency are such that the Landau theory is applicable, it is possible to select physical effects on the basis of the Landau theory. The Landau theory thus provides a theoretical cheer on the validity of a given experimental determination of the Fermi surface. Some of the quantities which are not affected by quasi particle interactions are the transverse conductivity in the

anomalous region, the cyclotron resonance frequency, the de Haas-Van Alphen effect and the low frequency Hall constant in a high magnetic field. The anomalous skin effect can be used directly to determine the shape of Fermi surface however, in the simple case of an isotropic system, the resonance frequency involves the same mass m^* as that found from specific heat measurements [5];

$$\omega_c^* = \frac{eH}{m^* c}$$

Such experimental result can be described theoretically from Landau transport equations.

Those quantities which are affected by quasi particle interactions are compressibility, spin susceptibility, static dielectric constant, general expression for conductivity $\sigma(0,\omega)$, etc. For instance, the general conductivity is given by

$$\sigma(0,\omega) = \frac{iNe^2}{m_c\omega} = \frac{iNe^2}{m^*\omega}(1 + \frac{F_1^S}{3}).$$

From the above equation it is possible to determine directly the crystalline mass m_c.

Now that it is important to present the various values of Landau parameters F_0^s, F_0^a, F_1^s, m_c, etc as determined experimentally for a number of metals. Unfortunately, this is not possible for a number of reasons. First, there is only a wide spread data available on the specific heat: it is not possible to measure the electronic compressibility, κ, directly while χ_p has thus far been determined directly only for lithium and sodium. Second, the value for m^* obtained in specific heat experiment includes the effect of electron phonon interaction, as well as electron- electron and crystalline field effects. Third, optical experiments, which permit a direct determination of m_c^* are not easily carried out. Therefore, the theory is of a little quantitative importance. Qualitatively, however, the predictions are important.

3.4 Breaking down of the Fermi Liquid

The familiar Fermi liquid picture in one dimension is based on the existence of long lived quasi particles / or quasi hole excitations as the energy of excitation

approaches to zero or equivalently as the energy approaches the Fermi energy. These quasi particles(quasi holes) can be traced back to bare electrons (holes) with a one -to- one correspondence, starting from the noninteracting situation followed by adiabatic turning on of the electron- electron interaction. Phenomenologically supposing the existence of such low -energy excitations, Landau was able to account for a rich variety of physical phenomena exhibited by conventional metals in the presence of non - negligible interactions. The liquid picture finds more rigorous justification in many body perturbation theory through in depth analysis of (i) the quasi particle life time, which varies as $|p - p_F|^{-2}$ when p approaches p_F for $D \geqslant 2$, or as $|E - E_F|^{-2}$ when E approaches the Fermi energy E_F, and (ii) the interaction operators γ and Γ (reduced) operators, leading to the well known ward identities reflecting underlying conservation laws. Example continuity equations reflecting the conservation of charge and enabling us to relate vertex function and interaction operators to the Landau parameters. The slow rate of decay for excitation near E_F is central to the success of the Fermi liquid picture.

The Fermi liquid picture breaks down in one dimension. This is a direct consequence of the unique phase structure in one dimension, notably the fact that the Fermi surface consists of two discrete points ($\pm p_F$) rather than a line or surface (or multiple lines or surfaces), and that for each branch of momentum energy dispersion - left or right - moving - the one dimensional wave vector uniquely determines the energy. Some of the most significant interrelated consequences include the following: (1)The reduced 1D phase space leads to quasi particle scattering rate($\mathrm{Im}[\Sigma(p,\omega)]$) $\propto |p - p_F|$ and $\propto \omega$, rather than the $|p - p_F|^2$ and ω^2 dependencies in two and three dimensions, and to a logarithmic divergence in $Re(\Sigma)$ at $E = E_F(\omega \to 0)$. Here Σ denotes the self - energy. Note that the real and imaginary parts of Σ are related by Kramers - Kroning relations.

(2) Logarithmic divergences appear in the two-particle interaction operator commencing in second order perturbation theory related to (1) above.

(3)A logarithmic divergence at E_F directly implies that the quasi particle re nor-

malization, or quasi particle weight vanishes. Hence the one- to one correspondence of the unperturbed k electron state to the elementary excitations of the interacting system is lost.

(4) Low energy electron - hole excitations arising from interactions can occur about the two discrete Fermi points $\pm p_F$ with a small momentum transfer $q \approx 2p_F$. This indicates that as the excitation energy approaches zero, there is a forbidden for electron - hole excitation at $0 < q < 2p_F$. Generalizations to multiple electrons-hole processes yield a series of forbidden regions, $2(n-1)p_F < q < 2np_F$, where n denotes the number of electron-hole pairs.

(5) Spin - charge separation occurs, i.e, the spin and charge bosonic elementary excitations propagate at different velocities. The presence of low - energy, un gapped bosonic charge and spin modes is a direct consequence of the finite spin and charge - density response at low q and ω.

4 One - Dimensional Interacting Fermion System

4.1 Microscopic bases of Landau's theory Bosonization Approach

At our current knowledge, it does not seem generally possible to derive Landau's theories starting from some microscopic Hamiltonian, apart possibly in perturbation theory for a small interaction. It is however possible to formulate the basic hypotheses in terms of microscopic quantities, in particular one - and two - particle Green function.

4.1.1 Quasi Particle - The Green function

As far as single particle properties are concerned it is sufficient to consider the one - particle Green function

$$G(k,\tau) = - < T_\tau a_k(\tau) a_k^+(0) >, \tag{4.1}$$

where τ is the usual (Matsubara)[] imaginary time. In this quantity, interaction effects appear via self energy corrections Σ in the fourier transformed function

$$G(k,\omega) = \frac{1}{i\omega - \varepsilon_k^{00} - \Sigma(k,\omega)} \tag{4.2}$$

Here $\varepsilon_k^{00}$is the bare particle energy, without any effective mass effect. Excitation energies of the system then are given by the poles of $G(k,\tau)$. In this terms, Landau's assumption about the existence of quasi particle is equivalent to assuming that $\Sigma(k,\omega)$ is sufficiently regular close to the Fermi surface as to allow an expansion for small parameters. Regularity in (k,ω) space implies that in real space the self - energy has no contributions that decay slowly in time and/ or space. Given that the self - energy can be calculated in terms of the effective interaction between particles; this is certainly a reasonable assumption when the particle - particle interaction is short range. For coulomb interactions, screening has to be invoked to make the effective interaction short ranged.

One can further notice that $\Sigma(k_F, 0)$ just renormalizes the chemical potential. Given that we want to work at fixed particle number we can absorb this term in the effective μ. Expanding then to first order around the Fermi surface, the green function takes the form

$$G(k, \omega) = \frac{z}{i\omega - \varepsilon_k^0} \tag{4.3}$$

where ε_k^0 has the form

$$\varepsilon_k^0 = \frac{k_F}{m^*}(|k| - k_F)$$

of the phenomenological approach, with

$$m^* = m\left(1 - \frac{\partial\Sigma}{\partial\omega}\right)\left(1 + \frac{m}{k_F}\frac{\partial\Sigma}{\partial k}\right)^{-1} \tag{4.4}$$

and the quasi particle renormalization factor is

$$z = \left(1 - \frac{\partial\Sigma}{\partial\omega}\right) \tag{4.5}$$

All derivatives are to be taken at the Fermi surface and at $\omega = 0$. One should note that a sum rule imposes that the frequency - integrated spectral density

$$A(k, \omega) = -\frac{1}{\pi}\,\mathrm{Im}\,G(k, i\omega \to \omega + i\delta) \tag{4.6}$$

equals unity. consequently, in order to full the sum rule, if $z < 1$ there has to be a contribution in addition to the quasi particle pole in eq.(4.3). This is the so called incoherent background from single and multiple particle - hole pair excitation which can extend to a rather high energies but becomes small close to the Fermi surface.

The form eq.(4.3) gives to a jump in the momentum distribution function at k_F of height z, instead of unity in the noninteracting case Fig. (4.1). In addition to the jump, the incoherent background gives rise to a contribution which is continuous through k_F. A finite quasi particle life time arises if the expansion of $\Sigma(k, \omega)$ is carried to second order. Then eq.(4.3) generalizes to

$$G(k, \omega) = \frac{z}{i\omega - \varepsilon_k^0 + i\,\mathrm{sin}(\omega)\tau(\omega)^{-1}}, \tag{4.7}$$

where $\tau(\omega)$ is typically given by

$$\tau^{-1} \propto m^{*3}\frac{(\pi T)^2 + \varepsilon_P^2}{1 + \exp(-\beta\varepsilon_P)}$$

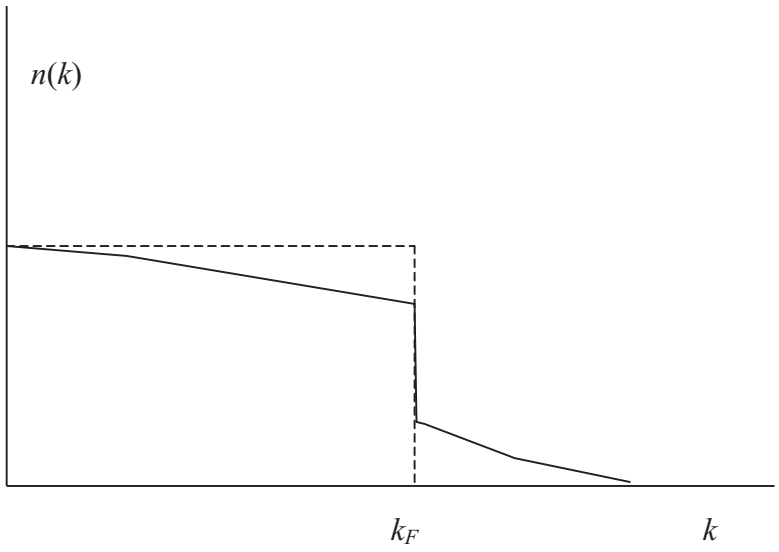

Figure 4.1: The momentum distribution function $n(k) =< a_k^+ a_k >$ in interacting (full line) and noninteracting (dashed line) cases

4.2 Bosonization and the Luttinger liquid

The Fermi liquid picture described previously is believed to be relevant for most three dimensional itinerant electron systems ranging from simple metals like sodium to heavy - electron materials. The best understood non -Fermi liquid properties is that of interacting fermion in one dimension. Here the results of bosonization approach as example for spin model is discussed.

4.2.1 Model with spin; the concept of Luttinger liquid

For spin 1/2 fermions, all the fermion operators acquire an additional spin index s. The kinetic energy of the fermion system is given by

$$\begin{aligned} H_0 &= v_F \sum_{k,s}((k-k_F)a^\dagger_{+,k,s}a_{+,k,s} + (-k-k_F)a^\dagger_{-,k,s}a_{-,k,s}) \\ &= \tfrac{2\pi v_F}{L}\sum_{q>0,\alpha=\pm,s}\rho_{\alpha,s}(q)\rho_{\alpha,s}(-q), \end{aligned} \tag{4.8}$$

where density operators for spin projections $s =\uparrow,\downarrow$ have been introduced:

$$\rho_\pm(q) = \sum_k a^\dagger_{\pm,k+q,s}a_{\pm,k,s} \tag{4.9}$$

There are now two types of interactions. First, the back ward scattering $(k_F, s; -k_F, t) \rightarrow (-k_F, s; k_F, t)$which for $s \neq t$ can not be re-written as an effective forward scattering (contrary to the spin less case). The corresponding Hamiltonian in the final form is

$$H = H_\rho + H_\sigma + \frac{2g_1}{(2\pi\alpha)^2} \int dx \cos\left(\sqrt{8}\phi_\sigma\right) \tag{4.10}$$

Here α is a short - distance cutoff, and for $\nu = \rho, \sigma$

$$H_\nu = \int dx \left(\frac{\pi u_\nu K_\nu}{2} \Pi_\nu^2 + \frac{u_\nu}{2\pi K_\nu} (\partial_x \phi_\nu)^2 \right), \tag{4.11}$$

with

$$u_\nu = \sqrt{(v_F + \frac{g_{4\nu}}{\pi})^2 - (\frac{g_\nu}{2\pi})^2}, \qquad K_\nu = \sqrt{\frac{2\pi v_F + 2g_{4,\nu} + g_\nu}{2\pi v_F + 2g_{4,\nu} - g_\nu}}, \tag{4.12}$$

and $g_\rho = g_1 - 2g_2$, $g_\sigma = g_1$, $g_{4,\rho} = g_4$, $g_{4,\sigma} = 0$.

For a noninteracting system one thus has $u_\nu = v_F$ (charge and spin velocities are equal) and $K_\nu = 1$. For $g_1 = 0$, (4.12) describes independent long - wave length oscillation of the charge and spin density, with linear dispersion relation $\omega_\nu = u_\nu |k|$, and the system is conducting. As in the spin less case, there are no single - particle or single - hole excited states. This model (no back scattering), usually called the Tomonaga - Lattinger model, is the one to which the bosonization method was originally applied [8, 9, 10].

For $g_1 \neq 0$ the cosine term has to be treated perturbatively. In particular, for repulsive interactions, $(g_1 > 0)$, g_1 is renormalized to zero in the long - wave length limit, and at the fixed point one has $K_\sigma^* = 1$. The three remaining parameters in (4.10)completely determine the long - distance low energy properties of the system.

4.3 Spin - Charge Separation

One of the more spectacular consequences of the Hamiltonian (4.10) is the complete separation of the dynamics of spin and charge degrees of freedom. For example, in general one has $(u_\sigma \neq u_\rho)$, i.e. the charge and spin oscillations propagate with different velocities. Only in noninteracting system or if some accidental degeneracy

occurs one does have $u_\sigma = u_\rho = v_F$. To make the meaning of this more transparent, let us create an extra particle in the ground state, at $t = 0$ and spatial coordinate x_0. The charge and spin densities then are easily found, using

$$\rho(x) = -\left(\frac{\sqrt{2}}{\pi}\partial_x\phi_\rho\right)$$

(note that $\rho(x)$ is the deviation of the density from its average value) and

$$\sigma_z = -\frac{\sqrt{2}}{\pi}\partial_x\phi_\sigma;$$

$$\begin{aligned}\Big\langle 0|\psi_+(x_0)\rho(x)\psi_+^\dagger(x_0)|0\Big\rangle &= \delta(x - x_0),\\ \Big\langle 0|\psi_+(x_0)\sigma_z(x)\psi_+^\dagger(x_0)|0\Big\rangle &= \delta(x - x_0).\end{aligned} \tag{4.13}$$

Now consider the time development of the charge and spin distributions. The time dependence of the charge and spin density operators is easily obtained from (4.10) (using the fixed point gauge $g_1 = 0$), and one obtains

$$\begin{aligned}\Big\langle 0|\psi_+(x_0)\rho(x)\psi_+^\dagger(x_0)|0\Big\rangle &= \delta(x - x_0 - u_\rho t)\\ \Big\langle 0|\psi_+(x_0)\sigma_z(x)\psi_+^\dagger(x_0)|0\Big\rangle &= \delta(x - x_0 - u_\sigma t)\end{aligned} \tag{4.14}$$

Because in general $u_\sigma \neq u_\rho$, after some time charge and spin will be localized at completely different points in space, i,e. charge and spin have separated completely.

Here a linear - momentum relation has been assumed for the electrons, and consequently the shape of the charge and spin distribution is time independent. If the energy momentum relation have some curvature (as is necessarily the case in lattice systems) the distribution will widen with time. However this widening is proportional to $\sqrt{t}$, and therefore, much smaller than distance between charge and spin. Thus, the quantitative picture of spin and charge separation is unchanged.

4.4 Physical Properties

The simple form of the Hamiltonian (4.10) at the fixed point $g_1^* = 0$ makes the calculation of physical properties rather straightforward. The specific heat now is

determined both by the charge and spin modes, and consequently the specific heat coefficient γ is given by

$$\frac{\gamma}{\gamma_0} = \frac{1}{2}\left(\frac{v_F}{u_\rho} + \frac{v_F}{u_\sigma}\right) \tag{4.15}$$

Here γ_0 is the specific heat coefficient of noninteracting electrons of Fermi velocity v_F.

The spin susceptibility and compressibility are equally easy to obtain. Note that in (4.10) the coefficient u_σ/k_σ determines the energy necessary to create a non zero spin polarization, and as in the spin less case, u_ρ/k_ρ fixes the energy needed to change the particle density. Given the fixed point value $k_\sigma^* = 1$, one finds

$$\frac{\chi}{\chi_0} = \frac{v_F}{u_\sigma}, \frac{\kappa}{\kappa_0} = v_F\frac{K_\rho}{u_\rho}, \tag{4.16}$$

where χ_0 and κ_0 are the susceptibility and compressibility of the noninteracting case. From equations (4.15)and (4.16) the Wilson ratio is

$$R_\omega = \frac{\chi\gamma_0}{\gamma\chi_0} = \frac{2u_\rho}{u_\rho + u_\sigma} \tag{4.17}$$

The quantity $\Pi_\rho(x)$is proportional to the current density. As before, the Hamiltonian commutes with the total current, one then has

$$\sigma(\omega) = 2K_\rho u_\rho \delta(\omega) + \sigma_{reg}, \tag{4.18}$$

i.e the product $K_\rho u_\rho$ determines the weight of the DC peak in the conductivity. If the total current commutes with the Hamiltonian σ_{reg} vanishes, however, more generally, this part of the conductivity varies as ω^3 at low frequencies [11].

The above properties , linear specific heat, finite spin susceptibility, and DC conductivity are those of an ordinary Fermi liquid, the coefficients u_ρ, u_σ and K_ρ determining renormalizations with respect to noninteracting quantities. Applying the single particle - Green function and Fourier transforming it, the one dimensional momentum distribution and density of states are respectively given by

$$n_k \approx n_{k_F} - \text{const sign}(k - k_F)|k - k_F|^\delta, \tag{4.19}$$

and

$$N(\omega) \approx |\omega|^{\delta} \tag{4.20}$$

In both cases $\delta = (K_\rho + 1/K_\rho - 2)/4$. Note that for any $K_\rho \neq 1$, i.e for any non vanishing interaction, the momentum distribution function and the density of states have power singularities at the Fermi level, with a vanishing particle density of states at E_F. This behavior is obviously quite different from a standard Fermi liquid which would have a finite density of states and step like singularity in n_k. The absence of a step at k_F in the momentum distribution function implies the absence of the quasi particle pole in the one particle Green function.

The coefficient K_ρ also determines the long- distance decay of all other correlation functions of the system: Hence the charge and spin density operators are

$$\begin{array}{l} O_{CDW}(x) = \sum_s \psi^+_{-,s}(x)\,\psi_{+,s}(x) = \lim_{\alpha\to 0} \frac{e^{2ik_F x}}{\pi\alpha} e^{-i\sqrt{2}\phi_\rho(x)} \cos\left[\sqrt{2}\phi_\sigma(x)\right] \\ O_{SDW}(x) = \sum_s \psi^+_{-,s}(x)\,\psi_{+,-s}(x) = \lim_{\alpha\to 0} \frac{e^{2ik_F x}}{\pi\alpha} e^{-i\sqrt{2}\phi_\rho(x)} \cos\left[\sqrt{2}\theta_\sigma(x)\right] \end{array} \tag{4.21}$$

Similarly the correlation function for singlet (SS) and triplet (TS) super conducting pairs are

$$\begin{array}{l} \left\langle O^\dagger_{ss}(x) O_{ss}(0)\right\rangle = C X^{-1-1/K_\rho} ln^{-3/2}(x) + ..., \\ \left\langle O^\dagger_{Ts_\alpha}(x) O_{Ts_\alpha}(0)\right\rangle = D X^{-1-1/K_\rho} ln^{1/2}(x) + ... \end{array} \tag{4.22}$$

The logarithmic corrections in these functions [12] have been studied in detail recently [13, 14, 15, 16] . The corresponding susceptibilities (i.e. the Fourier transform of the above correlations) behave at low temperature as

$$\begin{array}{ll} \chi_{CDW}(T) \approx T^{K_\rho - 1} |\ln(T)|^{-3/2}, & \chi_{SS}(T) \approx T^{1/K_\rho - 1} |\ln(T)|^{-3/2} \\ \chi_{SDW}(T) \approx T^{K_\rho - 1} |\ln(T)|^{1/2}, & \chi_{ST}(T) \approx T^{1/K_\rho - 1} |\ln(T)|^{1/2} \end{array} \tag{4.23}$$

i.e for $K_\rho < 1$ (spin or charge) density fluctuations at $2k_F$ are enhanced and diverge at low temperatures where as for $K_\rho > 1$ pairing fluctuations dominate. The phase diagram showing in which parameter space which type of correlation diverges for $T \to 0$ is shown in figure (4.2).

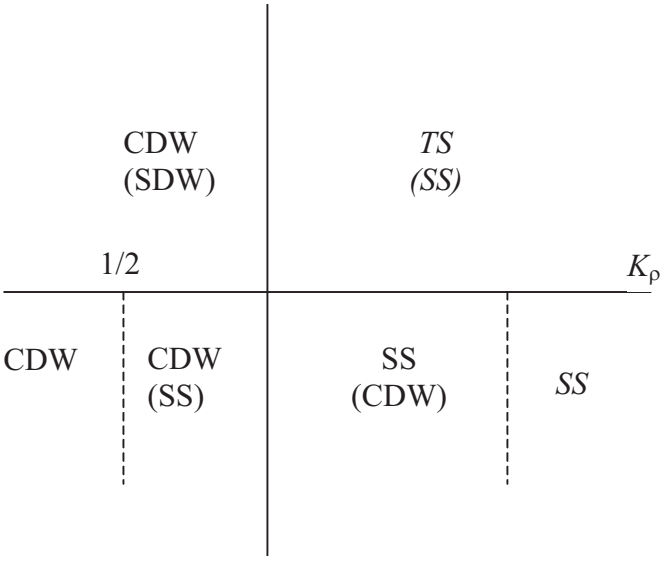

Figure 4.2: Phase diagram for interacting spin - 1/2 fermions [18]

These correlation functions with their power law variations actually determine experimentally accessible quantities: the $2k_F$ and $4k_F$ charge correlation leads to X-ray scattering intensities $I_{2K_F} \approx T^{K_\rho}$, $I_{4K_F} \approx T^{4K_\rho - 1}$, and similarly the NMR relaxation rate due to $2k_F$ spin fluctuation varies as $\frac{1}{T_1} \approx T^{K_\rho}$. The remarkable fact in all the above results is that there is only one coefficient, K_ρ, which determine all the asymptotic power laws.

We here re-emphasize the two important properties of spin - 1/2 interacting fermions in one dimensions: (i) Correlation functions shows power- low decay, with interaction dependent powers determined by one coefficient, K_ρ; and (ii) spin charge separation: spin and charge degrees of freedom propagate with different velocities. Both this properties are typical of the Luttinger liquid and invalidate the Landau quasi partyicle concept in one dimension.

4.5 Singular Fermi liquids

4.5.1 Breakdown of Landau Theory

From Landau's phenomenological theory, one can only say that the theory breaks down when the physical properties - specific heat divided by temperature, compressibility, or the magnetic susceptibility - diverge or when the collective modes representing oscillations of the Fermi - surface in any harmonic and singlet or triplet spin combinations become unstable. The latter, called the Landau - Pomeranchuk singularities, are indeed one route to the breakdown of Landau theory and occur when the Landau parameters $f_l^{s,a}$ reach the critical value $-(2l+1)$. A phase transition to a state of lower symmetry is then indicated. The new phase can again be described in Landau theory by defining distribution functions consistent with the symmetry of the new ground state.

Landau theory breaks down when the quasiparticle amplitude $Z_{\mathbf{k}}$ given by

$$Z_{\mathbf{k}}^{1/2} = \left\langle \psi_{\mathbf{k}}^{N+1} \left| a_{\mathbf{k}}^{+} \right| \psi_{\mathbf{k}}^{N} \right\rangle$$

becomes zero. This requires that the single-particle self-energy be singular as a function of energy ε at $k \simeq k_F$. This in turn means that the Green's functions of singular Feremi liquid contain branch cuts rather than the poles unlike Landau Fermi - liquids. If a divergent number of low-energy particle - hole pairs is created upon addition of a bare particle, it means that the low - energy response functions (which all involve creating particle - hole pairs) of singular Feremi liquid are also divergent. Actually the single - particle self - energy can be written in terms of integrals over the complete particle - hole interaction vertex. The implication is that the interaction vertices are actually more divergent than the single-particle self-energy.

Another route to singular Fermi liquid is the case in which the interactions generate new quantum numbers which are not descriptive of the non-interacting problem. This happens most famously in the Quantum Hall problems and in one-dimensional

problems as well as problems of impurity scattering with special symmetries. In such cases the new quantum numbers characterize new low-energy topological excitations.

In the final analysis all breakdowns of Landau theory are due to degeneracies leading to singular low-energy fluctuations. If the characteristic energy of the fluctuations is lower than the temperature, a quasi-classical statistical mechanical problem results. We note the following routes to breakdown of Landau theory.

Landau-Pomeranchuk Singularities: Landau theory points to the possibility of its breakdown through the instability of the collective modes of the Fermi- surface. These collective modes can be characterized by the angular momentum l of oscillation of the Fermi-surface and whether the oscillation is symmetric "s" or antisymmetric "a" in spin.[17]

Critical regions of Large Q-Singularities: Landau theory concerns itself only with long wavelength response and correlations. A Fermi-liquid may have instabilities at a nonzero wave-vector, for example a charge-density wave (CDW) or spin-density wave (SDW) instability. An important point to note is that they arise perturbatively from repeated scattering between the quasiparticle parts of the Green's function while the scattering vertices are regular. The superconductive instability for any angular momentum is also an instability of this kind. [18,19]

Singular Fermi - liquid behavior is generally expected to occur only in the critical regime of such instabilities. [20] If the transition temperature T_c is finite then there is usually a stable low temperature phase in which unstable modes are condensed to an order parameter, translational symmetry is broken, and gaps arise in part or all of the Fermi - surface. For excitations on the surviving part of the Fermi - surface, Fermi - liquid theory is usually again valid. The fluctuations in the critical regime are classical, i.e. with characteristics frequency $\omega << k_B T_c/\hbar$. If the transition is tuned by some external parameter so that it occurs at zero temperature, one obtains a Quantum Critical Point (QCP). If the transition is approached at $T = 0$ as a function of the external parameter, the fluctuations are quantum-mechanical, while if it is approached as a function of temperature for the external parameter at

its critical value, the fluctuations have a characteristic energy proportional to the temperature.

Long - Range Interactions: Breakdown of Landau Fermi - liquid may come about through long - range interactions, either in the bare Hamiltonian through the irreducible interaction or through a generated effective interaction. The latter, of course, happens in the critical regime of phase transitions. Coulomb interactions will not do for the former because of screening of charge fluctuations. In other words, the longitudinal electromagnetic mode acquires mass in a metal. The latter is not true for current fluctuations or transverse electromagnetic modes which due to gauge invariance must remain massless. This will be discussed in the next section, where it is shown that no metal at low enough temperature is a Fermi - liquid. However, the cross - over temperature is too low to be of experimental interest.

An off - shoot of singular Fermi liquid through current fluctuations is the search for extra (induced) conservation laws for some quantities to keep their fluctuations massless. This line of investigation may be referred to generically as gauge theories. Extra conservation laws imply extra quantum numbers and associated orthogonality. The one-dimensional interacting electron problem and the Quantum Hall effect problems may be usefully thought of in these terms.

In the following subsections we discuss a particular simple form of Fermi - liquid formed by electrons interacting with a dilute concentration of magnetic impurity. Many of the concepts of Fermi - liquid theory are revisited in this problem. Variants of the problem provide an interesting array of soluble problems of singular Fermi-liquid behavior.

4.6 The Kondo Problem

The Kondo problem is one of the simplest and one of the most subtle examples of the effects of strong correlation effects in electronic systems. Experiments concern metals with a dilute concentration of magnetic impurities. In the Kondo model one

considers only a single impurity; the Hamiltonian then is

$$H = t \sum_{i,j} a^+_{i\alpha} a_{j\alpha} + J\mathbf{S} \cdot a^+_0 \sigma a_0$$

where $a^+_{i\alpha}$ ($a_{j\alpha}$) denote the creation (and annihilation) operators of a conduction electron at site i with projection a in the z - direction of spin s. The second term is the exchange interaction between a single magnetic impurity at the origin (with spin $S = 1/2$) and a conduction electron spin. When the exchange constant $J > 0$ the system is a Fermi - liquid. The ferromagnetic ($J < 0$) variant of this problem is one of the simplest examples of a singular Fermi - liquid.

There are two seemingly simple starting points for the problem: (i) $J = 0$: This turns out to describe the unstable high temperature fixed point. The term proportional to J is a marginal operator about the high temperature fixed point because as discovered by Kondo in a third order perturbation calculation, the effective interaction acquires a singularity. (ii) $t = 0$: The perturbative expansion about this point is well behaved. This turns out to describe the low temperature Fermi-liquid fixed point. One might be surprised by this, considering that typically the bare t/J is of order 10^3. But such is the power of singular renormalizations.

The interaction between conduction electrons and the localized electronic level is not a direct spin interaction. It originates from quantum-mechanical charge fluctuations that (through the Pauli principle) depend on the relative spin orientation. To see this explicitly it is more instructive to consider the Anderson model [21] in which

$$H = t \sum_{i,j} a^+_{i\alpha} a_{j\alpha} + \epsilon_d \sum_{\sigma} a^+_{0\sigma} a_{0\sigma} + U a^+_{0\uparrow} a_{0\downarrow} a^+_{0\downarrow} a_{0\uparrow} + \sum_{k,\sigma} \left(V_k a^+_{0\sigma} a_{0\sigma} + hc \right)$$

The last term in this Hamiltonian is the hybridization between the localized impurity state and the conduction electrons, in which spin is conserved. In the particle - hole symmetric case, $\epsilon_d = -U/2$ is the one - hole state on the impurity site in the Hartree - Fock approximation and the one - particle state has the energy $U/2$. Following a

perturbative treatment in the limit t/V , $U/V >> 1$ the Anderson model reduces to the Kondo Hamiltonian with an effective exchange constant $J_{eff} \sim (V^2/t)^2/U$.

The Anderson model has two simple limits. $V = 0$ (high temperature regime) describes a local moment with Curie susceptibility $\chi \sim \mu^2 B/T$. This limit is the correct point of departure for an investigation for the high temperature regime. As noted, one soon encounters the Kondo divergences. In the limit $U = 0$ (low temperature regime) the impurity forms a resonance of width $\Gamma \sim V^2/t$ at the chemical potential which in the particle - hole symmetric case is half - occupied. The ground state is a spin singlet. This limit is the correct starting point for an examination of the low temperature properties ($T << T_K$). A temperature independent contribution to the susceptibility and a linear contribution to the specific heat ($\sim N(0)T/\Gamma$) are contributed by the resonant state.

The passage from the high - temperature regime to the low - temperature regime was first done correctly by Wilson [22] through the invention of the Numerical renormalization Group by Anderson and Yuval [23, 24] by analytic methods). The analysis showed that under renormalization Group scaling transformations the ratio J/t increases monotonically continuous RG flows are observed from the high temperature extreme to the low temperature extreme and a smooth crossover between the two regimes occurs at the Kondo temperature

$$T_K \sim te^{-t/2J}$$

Because all flow is towards the strong-coupling fixed point, universal forms for the thermodynamic functions are found. For example, the specific heat C_v and the susceptibility χ are

$$C_v = T f_c\left(T/T_K\right), \qquad \chi = \mu^2 B f_\chi\left(T/T_K\right)$$

where the f's are universal scaling functions. An important theoretical result is that compared to a non-interacting resonant level at the chemical potential, the ratio of

the magnetic susceptibility to the specific heat changes,

$$R = \frac{\delta\chi/\chi}{\delta C_v/C_v}$$

for spin 1/2 impurities at $T << T_K$ is precisely 2 [25, 26]. In a noninteracting model, this ratio, nowadays called the Wilson ratio, is equal to 1, since both χ and C_v are proportional to the density of states $N(0)$. Thus the Wilson ratio is a measure of the importance of correlation effects. It is in fact the analogue of the Landau parameter f_0^a.

As we saw in earlier section, a Fermi - liquid description is appropriate as long as the spectrum retains a coherent single particle piece of finite weight $Z > 0$. If the evaluation of Z reduces to an overlap integral between two orthogonal wave functions then the system is a singular Fermi - liquid.

4.7 Orthogonality catastrophe

In the thermodynamic ($N \to \infty$) limit, an orthogonality catastrophe arises if the injection of an infinitely massive particle in more than one dimension produces an effective finite range scattering potential for the remaining N electrons. Such orthogonality is exact only in the thermodynamic limit: The single particle wave functions are not orthogonal. It is only the overlap between the ground state formed by their Slater determinants which vanishes as N tends to infinity.

More quantitatively, if the injection of the additional particle produces an s-wave phase shift δ_0 for the single particle wave functions,

$$\psi(kr) = \frac{\sin(kr)}{kr} \to \frac{\sin(kr + \delta_0)}{kr}$$

then an explicit computation of the Slater determinants reveals that their overlap diminishes as

$$\langle \psi_N \, | \psi'_N \rangle \sim N^{-\delta_0^2/\pi^2}$$

Here $| \psi_N \rangle$ is the determinant Fermi sea wave function for N particles and $| \psi'_N \rangle$ is the

wavefunction of the system after undergoing a phase shift by the local perturbation produced by the injected electron.

Generally, such an orthogonality ($Z = 0$) arises also if two N particle states of a system possess different quantum numbers and almost the same energy. These new quantum numbers might be associated with novel topological excitations. This is indeed the case in the Quantum Hall Liquid where new quantum numbers are associated with fractional charge excitations. The singular Fermi liquid properties of the interacting one-dimensional fermions may also be looked on as due to orthogonality. Often orthogonality has the effect of making a quantum many-body problem approach the behavior of a classical problem.

4.8 X-ray Edge singularities

The term X-ray edge singularity is used for the line shape for absorption in metals by creating a hole in an atomic core-level and a particle in the conduction band above the chemical potential. In the non-interacting particle description of this process, the absorption starts at the threshold frequency ω_D. In this case, a Fermi edge reflecting the density of unoccupied states in the conduction band is expected to be visible the spectrum.

However, when a hole is generated in the lower level, the potential that the conduction electrons see is different. The relevant Hamiltonian is now

$$H = \varepsilon_d \left(d^+ d - 1/2\right) + \sum_k \varepsilon_k a_k^+ a_k + \frac{1}{L} \sum_{k,k'} V\left(k, k'\right) \left(a_k^+ a_{k'} - 1/2\right) \left(d^+ d - 1/2\right),$$

where the operators (d^+, d) annihilate or create holes in the core level, which is taken to be dispersionless. The first two terms in the Hamiltonian represent the unperturbed energies of the core hole and the free electrons. The last term depicts the screened Coulomb interaction between the conduction electrons and the hole in the core level.

As a consequence of the interactions, the line shape is quite different. There are two kinds of effects, (a) excitonic - the particle and the hole attract, leading to a

shift of the edge and a sharpening of the edge singularity - and (b) an orthogonality effect of the type just discussed above, which smoothens the edge irrespective of the sign of the interaction. This changes the absorption spectrum in the presence of interactions. The form of the singularity is

$$A(\omega) \sim (\omega - \omega_D)^{-2\delta_0/\pi + \delta_0^2/\pi^2}$$

The exponent δ_0^2/π^2 is a consequence of the orthogonality catastrophe overlap integral; the exponent $(-2\delta_0/\pi)$ is due to the excitonic particle hole interactions. If the hole has finite mass we have a problem with recoil which is not exactly solvable, but we know the essential features of the solution. The recoil removes the singularity in two and three dimensions and the absorption edge acquires a characteristic width of the order of the dispersion of the hole band. If the hole moves only in one dimension, the singularity is not removed.

4.9 A Spinless Model with Finite Range Interactions

A model, which is a generalization of the Ferromagnetic Kondo problem and in which the low - energy physics is dominated by the orthogonality catastrophe, is given by the following Hamiltonian:

$$H = \sum_{k,l} \varepsilon_k \gamma_{kl}^+ \gamma_{kl} + \frac{t}{\sqrt{L}} \sum_{k,k'} \left(\gamma_{k0}^+ d + hc \right) + \frac{1}{L} \sum_{k,k'} V_l \left(\gamma_{kl}^+ \gamma_{k'l} - 1/2 \right) \left(d^+ d - 1/2 \right).$$

The operators (γ^+, γ) are the annihilation and creation operators of spinless conduction electrons with kinetic energy ε_k. The local chemical potential has been set to zero ($\varepsilon_d = 0$) and the Hamiltonian is particle-hole symmetric. The new index l is an orbital angular momentum index (or a channel index). Hybridization conserves point-group symmetry, so the localized orbital hybridizes with only one channel ($l = 0$). By contrast, the impurity couples to all channels via the interaction V_l. As we are summing over all moments (k, k') this interaction is local.

This problem may be mapped onto the anisotropic Kondo model. Indeed the

transformation

$$d^+ \to S^+, \qquad d^+ d - 1/2 \to S_z,$$
$$t \to \tfrac{J_{\perp 0}}{\sqrt{2}\pi a}, \quad 2V_l \to \sqrt{2} J_{zl} - 2\pi v_F \left(\sqrt{2} - 1\right) \delta_{l0}$$

produces

$$H = \sum_{k,l,\sigma} \varepsilon_k a^+_{kl\sigma} a_{kl\sigma} + \frac{1}{2} \sum_{k,k'} J_{\perp 0} \left(S^+ s^-_l + hc \right) + \sum_l J_{zl} S_{zl} s_{zl}.$$

Here a is short distance cutoff. In the resulting (anisotropic multi-channel) Kondo Hamiltonian the spin operators $\mathbf{S}$ and $\mathbf{s}_l$ portray charge excitations of the local orbital and conduction band; the spin index in the resulting Kondo Hamiltonian should now be regarded as a charge label. Physically, this map- ping is quite natural. The impurity may or may not have an electron, this is similar in character to having spin up or spin down. Similarly, the kinetic hybridization term transforms into a spin flip interaction term of the form $(S^+ d^- + hc)$. As V_l couples to the occupancy of the impurity site, we might anticipate J_z to scale with V_l. The additional correction $\left(-2\pi v_F \left(\sqrt{2} - 1\right) \delta_{l0}\right)$ originates from the subtle transformation taking the original fermionic system into an effective spin model.

This problem has been solved by renormalization group methods. But simple arguments based on the x-ray edge singularity, orthogonality and recoil give the correct qualitative physics. When $t = 0$, the problem is that of the x-ray edge Hamiltonian (with $\varepsilon_d = 0$). When t is finite, the charge at the impurity orbital fluctuates (the impurity site alternately empties and fills). This generates, in turn, a fluctuating potential. The x-ray absorption spectrum is the Fourier transform of the particle-hole pair correlator

$$\Delta(\omega) \sim \left\langle \gamma^+(t)\, d(t)\, d^+(0)\, \gamma(0) \right\rangle_\omega .$$

This quantity should display the x-ray edge characteristics for large frequencies $(\omega > \Delta_{eff})$ where the effect of recoil is unimportant:

$$\Delta(\omega) \approx \Delta_0 (\omega/W)^\gamma ; \quad \gamma = -\frac{2\delta_0}{\pi} + \sum_l \frac{\delta_l^2}{\pi^2}.$$

The threshold frequency Δ_{eff} is determined by the recoil energy. W is the bandwidth. The bare hybridization width Δ_0 is $\sim t^2/W$. The exponent in the singularity contains an exitonic shift $(-\frac{2\delta_0}{\pi})$ as well as an orthogonality contribution $(\sum_l \delta_l^2/\pi^2)$. The recoil is cut off by Δ_{eff}. For $\omega < \Delta_{eff}$ the electron gas becomes insensitive to the change in the potential. As the x-ray edge singularity is cut off at $\omega = O(\Delta_{eff})$, self-consistency implies that $\Delta_{eff} = \Delta(\omega = \Delta_{eff})$. This leads to the identification

$$\Delta_{eff} = W(\Delta_0/W)^{\frac{1}{1-\gamma}}$$

so that for

$$\gamma < 1, \Delta_{eff} \to 0 as W \to \infty.$$

For $\gamma < 1$, a singular Fermi-liquid emerges in which the hybridization of the localized d-orbital with the electron gas scales to zero at zero frequency. The actual value of γ determines the singular properties at low energy or temperature. In the single channel problem such a scenario occurs if the potential V_0 is sufficiently attractive. On mapping to the spin problem we find that this region corresponds to the singular Fermi-liquid Ferromagnetic Kondo problem. The scaling of the hybridization to zero corresponds, in the spin-model, to $J_\pm = 0$.

5 Summary and References

5.1 summary

The basic assumption of Landau's theory is the existence of low - energy quasiparticles with a very long life time and their description in terms of a rather simple energy functionals. From this, a number of important qualitative results for low temperature thermodynamic properties were obtained.

Actual quantitative predictions are obtained when one extends the theory to non equilibrium properties using Boltzman transport equations. A new phenomena actually observed in $He-3$ is the existence of collective excitations, called zero sound. This approach also allows the calculation of the quasiparticle life time and its divergence as the Fermi energy is approached as well as the treatment of a number of transport phenomena.

As already mentioned, the ideal system for the application of Landau's theory is $He-3$, which has both short range interaction and is isotropic. The application to electrons in metals is more problematic. First, the interactions are long-ranged (coulombic). This can however be accommodated by properly including screening effects. More difficulties, at least at quantitative level, arise because metals are naturally anisotropic. This problem is not of fundamental nature; even when the Fermi surface is highly anisotropic, an expansion near the Fermi surface can still be written down thus interaction parameters can be defined. However, a simple Legendre expansion is not in general possible and the description of the quasi particle interaction in terms of a few parameters becomes impossible. An exception case, with a very nearly spherical Fermi surface, are the alkali metals, a determination of landau parameters can indeed be attempted.

It should be noticed that the difficulties with the Landau description of metals are not conceptual nature, and in particular do not invalidate the quasi particle concept but are rather limitations on the usefulness of the theory for quantitative purposes. Landau's theory can be interpreted in terms of microscopic quantities like

the greens functions(the quasi particle pole) and interaction vertices. It should however be emphasized that the arguments do provide a microscopic interpretation of landau's picture, rather than providing its correctness. Similar remarks apply to the calculated diverging quasi particle life time: this at best show the Landau's picture is internally consistent. Considerable progress towards a deeper formal understanding of Fermi liquid theory has been made in recent years.

The landau theory of Fermi liquid does'nt apply for interacting fermion systems in one and two dimensions. The one and two- particle Green's functions shows a power low behavior and the spin charge separation happens hens they travel with different velocities.

In this work we have also developed the basic bosonization formalism for one dimensional interacting fermions and some elementary and direct application of some physical properties, in particular the correlation functions. We have seen that the properties of the one dimensional interacting system the Luttinger liquid are fundamentally different from two or three - dimensional Fermi liquids. In particular the elementary excitations are not quasi particles but rather collective oscillations of the charge and spin density; propagating coherently, but in general at different velocities.

This give rise to the interesting phenomena of spin charge separation. Finally, and again contrary to the Fermi liquid case, most correlation function show non - universal power laws, with interaction- dependent exponents. However all there exponents depend only on one parameter, K_ρ, the spin analogue of which K_σ, being fixed to unity by spin rotation invariance ($K_\sigma \neq 1$ is possible if spin rotation invariance is broken). Beyond K_ρ, the only parameters that intervene in the low energy physics of luttinger liquid are the velocities of the spin and charge modes, $u_{\rho,\sigma}$. In the spinless case only two parameters, K and u are involved.

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Declaration

I, the undersigned, declare that this thesis is my original work and has not been presented for a degree in any other university and that all sources of material used for the thesis have been duly acknowledged.

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