ADDIS ABABA UNIVERSITY

MASTER'S THESIS

# Debt Collection Optimization:

## Commercial Bank of Ethiopia's case

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# CONTENTS

# LIST OF FIGURES

# LIST OF TABLES

# ABSTRACT

This thesis develops and analyzes a finite horizon Markov decision process model for commercial bank of Ethiopia's debt collection optimization problem. Two sectors namely domestic trade and services (DTS) and foreign trade (FT) have been studied primarily because of loan turnover rates and economic policy focus. A 2 years long data are used to determine cost of holding an outstanding loan in a certain state or delinquency status on a log scale where each state/status is scaled by the percentage of provision allocated to the states. Conclusions are made in terms of which loan segments banks need treat with more caution. Simple but important suggestions on ways of improving the operations of banks are made in the end.

DEFINITION

## Definition of the descriptors:

1. DTS: Domestic trade and services outstanding loans
2. FT: Foreign trade which is the sum of import and export outstanding loans
3. ST: loans' maturity date is 12 months or less
4. MT: loans' maturity date is between 12 months and 60 months
5. LT: loans' maturity date is above 60 months
6. PA: date past due of less than 30 days since last repayment
7. SM: date past due of 30 to 90 days since last repayment
8. SS: date past due of 90 to 180 days since last repayment
9. DO: date past due of 180 to 360 days since last repayment
10. LO: date past due of beyond 360 days since last repayment

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My good friends Ato Dagim and Ato Mesfin take their share of credit for their support and materials provision not to mention comments to make the work better.

Finally, I sincerely thank all the computational science department staff for their unfailing belief in the program the output of which solves the country's managerial decision making as well as scientific problems with seminal and practical research works.

# DECLARATIONS

**1. Candidate's Declaration**

I, Benyam Girma, hereby certify that the work presented in this thesis is, to the best of my knowledge and belief, original, except as acknowledged in the text, and that the material has not been submitted, either in whole or in part, for a degree at this or any other university.

I was admitted as a graduate student in 2010/11 and as a candidate for the degree of master of science in computational science; the higher study which this is a record was carried out in the Addis Ababa University.

I acknowledge that I have read and understood the University's rules, requirements, procedures and policy relating to my higher degree research award and to my thesis. I certify that I have complied with the rules, requirements, procedures and policy of the University.

| Benyam Girma | ________________ | ________________ |
|---|---|---|
| | Signature | Date |

**2. Supervisor's Declaration**

I hereby certify that the candidate has fulfilled the condition of the Resolution and Regulations appropriate for the degree of graduate study in Addis Ababa University and that the candidate is qualified to submit this thesis in application for that degree.

| | | | |
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| 1. | Tilahun Teklu (PhD) | ________________ | ________________ |
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# CHAPTER 1

# INTRODUCTION

## 1.1 Background

Financial institutions all over the world are the driving forces for development. The worldwide phenomenon of the credit crunch that started in early 2008 urged these institutions to think differently. They are now wary in their decisions especially in credit line provisioning.

Banks, like any other financial institutions, turn profit and survive primarily by lending money on interest. They assess the creditworthiness of the debtors using past credit history, current income, past and present behavior etc.

However, despite the complex behavior and circumstances modeling done, the past is never fully predictive of the future. That is why a systematic debt collection should be thought out and become an integral part of banks' system instead of relying solely on the predictions made prior to lending.

In the case of our country Ethiopia, the National Bank of Ethiopia has put forward guidelines with regards to debtors' status [1]. Then, Commercial banks would base their decisions to collect how much, when and how on Statuses' categorization, their need as well as resources at hand. Even though these banks have separate debt collection units, the process of debt collection is rarely consistent not to mention inefficient on many levels. One is the difficulty to make a decision among choices available for a debt collector at a given time. A case in point is what would happen if a weaker action is followed by a stronger one.

At times the decision of choice may be short sighted in a way that minimizes the likelihood of repayment in the long term. And that would absolutely defeat the purpose of having a system in the first place.

A question lenders are hard put to answer with confidence is: What should they do to change the status of a debtor for the better without violating certain regulations?

The best way to reduce default rates is through streamlining the debt collection process. Once a system is in place, banks may take actions without wasting time and other resources to recover what they are owed.

In all, the need for a consistent and efficient debt collection system is not only to make the lenders life easy; it is also about fairness towards the borrowers by avoiding unnecessary severe actions when following a procedural debt collection action.

One important implication of efficient debt collection is the rise of confidence of banks because of their ability to get their money back. Borrowers would therefore have a very easy time of getting loans, starting their own businesses and ultimately contributing to the economy.

## 1.2 Statement of the problem

Both the government and regulatory bodies like National Bank of Ethiopia measure the performance of banks by the level of loss provision they allocate every fiscal year. The Basel Accord international financial institutions employ has made it obligatory for the banks to periodically declare how much provision covers their bad assets.

Besides, economics makes it obvious that banks confidence to recollect a greater proportion of their assets would urge them to lend more than otherwise. Lenders would also be encouraged to repay due to the favorable conditions.

A common problem across the financial sector would be a deliberate underreporting of indices that measure underperformance. The loan collection units are not exceptions in this regard. Loan collection is largely inefficient and no system or individual takes responsibility for it.

As a result, one of the challenges banks seem to face is the lack of an objective measure of a loan's performance beyond the loan delinquency system outlined in the NBE directive mentioned above.

Therefore, this thesis attempts to show the significance of using debt collection modeling at a basic level which would help the loan collection units of banks to start thinking of putting in place a system that evaluates, prescribes a decision trajectory and makes assessment of its efficiency.

## 1.3 Scope

The case of the state owned bank Commercial bank of Ethiopia has been studied. This bank is by no stretch of imagination a representative of other banks. However, according to my observation the bank records data and attempts to streamline its processes the best it can. It also takes the biggest market share in the industry.

I study the outstanding balance evolution of two sectors, domestic trade and services (DTS) and Foreign trade (FT) for 8 evaluation periods/ quarters from March 2012 until December 2013.

The sectors are chosen based on the high loan turnover rate of DTS and the governments attention to FT especially export due to the sectors ability to bring in foreign currency thereby fueling the economy and reducing the trade deficit.

Further to this, in the ten years from 2003 to 2012 the share of disbursement of the two economic sectors is 28.7%. However, the share of Non-Performing loan (NPL) reaches 53% (34 from DTS and 19 from FT) .Thus, the justification to study the two sectors arises from the relatively higher proportion of loan turning bad requiring the redoubling of collection efforts for the two sectors.

## 1.4 Objectives

### 1.4.1 General

Show how a policy would be prescribed and the data input quality and form required to do so.

### 1.4.2 Specific

1. To do the policy prescription using the Markov decision process based on the outstanding loan amount of two sectors namely Domestic trade and Services and Foreign Trade

2. Demonstrate the probable level of efficiency attainable for different perceived value in the

difference between performing and non performing loans.

## 1.5 Methodology

### 1.5.1 Data Requirements

Data was collected from the bank's Management Information Systems department and its customer relations managers as well as loan officers. The major information required by the model are economic sector, delinquency status, tenure and outstanding amount. A 10 years data for the two sectors (DTS and FT) have been used to determine relationships between disbursement, collection and outstanding amounts. This particular period has seen interesting economic ups and downs thereby its properties could give a realistic dynamic picture of the whole debt provision as well as collection.

### 1.5.2 Mathematical Modeling

Loan provision involves an initial stage which assesses the individual's creditworthiness. This is mainly carried out by the loan appraisal unit with its own set of tools and criteria as to what factors have weight and when should the appraisal be made as a response to the status of the debtor. Then, the collection unit follows up and makes sure the loan is repaid at the set time before the tenure runs out.

However, there are a number of intermediate actions and stages that requires subtlety from the part of the bank. Debtors are averse to continuous insistence to repay on schedule because delaying repayment may benefit them. A typical case is a business owner wanting to use the capital accumulated to expand its business instead of keeping up on her repayment. Therefore, as long as they gauge the reaction of the bank correctly they would exploit the system well.

From the point of view of this thesis, modeling the whole loan life cycle requires a rigour beyond the scope envisioned for this research topic. Therefore, the thesis will be focusing on the aspect of the loan from the time it is issued until a certain future time to examine the evolution of repayment. Even with this basic aspect of the loan life cycle, a number of assumptions were made.

For instance taking legal action is not a linear and one directional process because the debtor would be given two or three rounds of rescheduling opportunities which may bring about the desired result or even make the whole collection effort meaningless by ending up writing off the debt. Of course write-offs are deliberately taken off the official record of banks for reasons mentioned in subsequent sections. Also, other assumptions are made to simplify the model within reasonable limits.

Markovian model assumes that future states depend only on the present state and not on the sequence of events that preceded it i.e Markov property is assumed.

**Definition 1.1** A state $S_t$ is Markov if and only if

$$T[S_{t+1}|S_t] = T[S_{t+1}|S_1, ..., S_t] \tag{1.1}$$

Equivalently this means once the state is known the history may be thrown away that is essentially what Eq(1.1) shows . However, higher order Markov chains are possible too.

The best mathematical model we have at hand that captures the most important attributes of the problem of debt collection is the Markov Decision Process (MDP) as the system involves the sequential dependencies between states and the system modelled by the markovian process is controlled.

### 1.5.3 Classes of MDP

Markov decision processes may be classified according to the time horizon in which the decisions are made: finite and infinite-horizon MDPs. Finite-horizon and infinite-horizon MDPs have different analytical properties and solution algorithms. Because the optimal solution of a finite-horizon MDP with stationary rewards and transition probabilities converges to that of an equivalent infinite-horizon MDP as the planning horizon increases and infinite-horizon MDPs are easier to solve and to calibrate than finite-horizon MDPs, infinite-horizon models are typically preferred when the transition probabilities and reward functions are stationary.

However, in many situations, the stationary assumption is not reasonable, such as when the transition probability represents the probability of default that increases from one period to the next.

Markov decision processes can also be classified with respect to the timing of the decisions. In a discrete-time MDP, decisions can be made only at discrete-time intervals, whereas in a continuous-time MDP, the decisions can occur anytime. Continuous-time MDPs generalize discrete-time MDPs by allowing the decision maker to choose actions whenever the system state changes and/or by allowing the time spent in a particular state to follow an arbitrary probability distribution.

In MDPs, we assume that the state the system occupies at each decision epoch is completely observable. However, in some real-world problems, the actual system state is not entirely known by the decision maker, rendering the states only partially observable. Such MDPs are known as POMDPs, which have different mathematical properties than completely observable MDPs and are beyond the scope of this thesis.

Out of the above classes of MDP, Finite-horizon, discrete-time and fully observable MDP will be used. The reasons will become apparent in the methodology section (Chapter 3). Anchored in the above transition matrices (Eq. 1.1) the iterative non-linear equation of recursive Bellman equation is put forward as follows:

$$U^n(s) = R(s) + \gamma min_a \sum_{s'} T(s'|s,a)U^{n-1}(s') \tag{1.2}$$

Subject to

$$C(a^i_{t+1}) \leq \sum \sum_{t=1}^{t} C^i_t \tag{1.3}$$

$$B = B - C^i_t \tag{1.4}$$

Where U indicates the value or (dis)utility derived from the collection process which in this case is the debt owed and its value is different for each state. The Superscript n shows how many periods deep the iteration is in , in other words since iteration starts at the final period where there is no periods left n starts at 1 and $U^0$ is expected to be data. By virtue of the fact that the model we are dealing with here is a finite MDP, we would naturally know the final state of the system. However, in reality decision horizons are infinite so it should take an assumed arbitrary value based on the nature of the problem under study. Often times these values are vectors of 0's of size m where m is the number of states. Then the iterative scheme improves up on it at every iteration to reach the $\varepsilon$-optimal level indicated by a preset stopping criterion.

R(s) is the immediate reward for maximization problems or cost for minimization problems at state s at period n. Whenever the information is available and the reward is a result of the action taken, the value would be a function of the action taken.

The multiplier $\gamma$ just outside the summation is the discounting factor to account for time delay. The concept is the same as the one in project management or finance where the value of an asset today is worth than its value tomorrow.

What the sum states is the value of selecting an action a given a transition from s to all possible states denoted $s^{'}$ with their corresponding probability transitions. In other words it calculates the weighted average of leaving out of state s, due to an action a , and sinking into state $s^{'}$.

The action space needs to be exhaustively iterated through to reach a conclusion about particular state's value and the action that minimizes the cost of holding an asset where a proxy namely outstanding loan amount is employed.

One subtle information that could not be readily discerned from the above Bellman's equation is that the transition matrix should be uniquely determined for each possible action. This is the major weakness of the model from practical standpoint because the problem would be intractable for large action spaces. However, one our action space is small and two there are other modeling techniques widely popular in the Artificial Intelligence discipline that focused on learning algorithms that do not need an explicit calculation of the transition matrix. Another important assumption in this thesis is that all the states are reachable from all states. This assumption makes the transition ergodic with desirable properties in terms of finding optimum action, value pair for each state. Although, near-zero probabilities of transitions have been studied their application to the same problem we are attempting to solve is left for other researchers to explore.

The above model is non-linear due to the presence of the min operator; therefore matrix algebra would not handle it. That is why iterative techniques would be adopted. Conversion of the equation into its LP equivalent form is possible and recommended depending on the nature of the optimization problem.

The constraint equation is one of budgetary in nature; it is the financial constraint a particular loan collection unit would face where each time an action is taken it draws on what is left over from previously taken actions. B is the budget allocated at the beginning of each period for an asset portfolio and once we are at a juncture to take an action its cost should be considered. However , there are other constraints , one is legal permissibility. If for instance a debtor defaults on her loan and the optimal action from that point onwards is foreclosure, given sufficient budget to execute the action, it is not always possible to pursue with the optimal policy rule .One reason being, the current situation of the debtor who might be in dire circumstances that impaired her capacity to repay on schedule. Therefore, in more involved models several constraints most of which with intrinsic qualitative characters would be a part of the model.

### 1.5.4 Mathematical Motivation

Markov decision process has been proven to capture the system dynamics of this particular problem in its deterministic as well as stochastic nature (a case where past recovery actions have an impact on the efficacy of the current action). Generally speaking if the state and action spaces are finite, iterative algorithms converge since policies are finite and iterations are an improvement on previous iterations preventing infinite cycling.

As the number of states and actions get larger the Bellman equations become intractable to solve, this is what we call the curse of dimensionality. However, there are ways to represent the spaces in compact forms. State space reduction by function approximation, regression is mostly used, is one of the many ways available.

The rest of the thesis is structured as follows. Chapter 2 explores past researches in Markov decision process, its applications particularly in financial modeling. Chapter 3 outlines the mathematical modeling and analysis and the methodology used to solve the problem of debt collection optimization. A particular Case study of Commercial bank of Ethiopia is treated in Chapter 4. The following section or Chapter 5 contains a compact analysis of the results and the last and the $6^{th}$ Chapter concludes by summarizing the main points and putting forward possible related research areas that would make excellent extensions of this particular thesis.

# CHAPTER 2

## LITERATURE REVIEW

Markov decision process has been widely used to model business, health, communication, weather and other problems in diverse fields. They model phenomena for descriptive and predictive purposes.

However, the model's popularity has been limited due to the proprietary nature of the studies conducted using MDPs especially in business. Moreover, the large state space used to describe the system and immense data requirements to produce a valid analysis prevented the possible popularity the model might have enjoyed otherwise.

Both Puterman[2] and Onne Van der Weijde[3] treated road maintenance optimization via MDP modeling technique. The model is expected to prescribe a maintenance policy for a particular section of the road with a certain degree of deterioration. Onne Van der Weijde compared MDP with Expected annual cost method (EAC) where it was aptly demonstrated that MDP trumps EAC in Expected average cost and the time spent in the unacceptable state. A point worth mentioning though is that the alternative method would have an edge only when the state space is large. The superiority of MDP in this particular application led to its choice in the debt collection optimization case. However, policy prescription would take reliable data .

Another important area of application of MDPS is in medical decision making (MDM). Oghuzan et al.[4] conducted a typical case study of liver transplant for patients at various stages of health. Their results indicate as to when would a patient benefit most from the transplant and which patients should get the treatment and which ones should not, given the level of disease severity. MDP is thus a viable tool of decision making even in medicine where the stake is high where human lives are involved. As to its significance for this thesis, Oghuzan demonstrated that the computation time for solving the MDP model is significantly lower than standard Markov based models (i.e. total expected life years calculation).

What I have found to be an original thesis on Airline meal provisioning was done by Goto et al.[5]. The case of Canadian airway was studied. MDP was applied to determine how many meals should be cooked as well as loaded with the purpose of minimizing cost (fuel and meal processing) while making sure everyone on board gets at least one meal per flight. The model captured the important issue of uncertainty, typical for the Airline industry, in the number of passengers by giving provisions of a certain number of hours for last minute adjustments. The uncertainty dimension of the debt collection process in Goto's work is a parallel to the airline industry and thus MDP modeling of the debt collection process would output results that suggest an efficient way of loan recovery.

More and more papers and journals are being dedicated to the studying of the financial sector. We now have access to an impressive amount of data .Also analytical tools like data mining is being used to make sense of what the myriad data points could try to tell us so that they are utilized for better decision making.

Daniel Bookstaber[6] approached the age old finance problem of portfolio allocation by using MDPs. The paper explored cases of small state spaces, large state/action spaces and constrained MDPs to find the optimal policy of portfolio allocation. Finally, the paper concludes by comparing solution methods of value iteration, linear programming, Q-learning and SARSA (State Action Reward State Action). The widely used solution technique of value iteration will be applied to the debt collection process optimization to find optimal value or the minimum possible expected outstanding amount and optimal policy. Q-learning as an option will be given a cursory overview and comparison could be carried out in extensions of this thesis.

Since MDPs are useful when the uncertainty in the system is a function of both the states and the actions, MDPs are too complicated a model as the buyers' action does not affect the asset movement. Hence, each stock follows its own Markov process independent of buyers' action. To solve the communication issue between stocks and avoid suboptimal policies, the paper suggests expansion of the number of state descriptors. In such cases, state space explosion sets in leaving us with a tradeoff of finding near optimal policy for each stock at the expense of intensive computational resources usage.

In the same vein as the previous study Chi So et al.[7] modeled the profitability of credit cards by MDP. An ingenious way of state space reduction was employed in this paper by introducing the behavioral score bands of credit card customers on top of their credit limit. The score bands serve as sufficient statistic of the risk of the account and already contain intelligence from a combination of characteristics. A combination of credit limits and behavioral scores were considered to generate a dynamic credit limit policy. Such examples of unexpected results like keeping the credit limit of customers with highest behavioral scores make for significant findings.

By taking an important lesson from this research, the possible state descriptors were reduced to six which are a combination of three tenure lengths and two economic sector categories. This is because although the CBE officials use behavioral score bands also known as 'risk grade' the notion is rudimentary at best. Even at the simplest level of complexity, risk grades are calculated before loans are given out only as pre-loan appraisal tools. These grades are hardly updated to reflect improvement or deterioration of repayment rates. Risk grades aren't updated unless the debtor defaults or in the case of multiple loan requests. This is the reason why the two economic sectors are considered separately as the common link of risk grade distribution is both inaccessible and unreliable. However, in the case of risk grade availability and continual updating , whether in defaulting cases or not, it is a strong tool to assess the probability of future debt status of a debtor with a better degree of accuracy. In other words, the risk grade statistic makes the outstanding-loan-status-based delinquency status classification more robust for the same reason Chi so et al. cited.

Another pragmatic assertion in the paper was that even though certain transitions to default are unlikely, making the Markov chain marginally non-ergodic hence non-robust, conservative estimates of higher probability as opposed to Maximum likelihood estimates are possible. This is because underestimating profitability is better than doing the opposite. Katja et al.[8] clearly showed how the probability of default for low default portfolios is estimated. In view of this important assumption, the transition probabilities to default whenever zero has been made to assume a small but non-zero probability hence conservative because expecting default is better than getting taken by surprise from the point of view of the banks .

A serious attempt to model and optimize the debt collections process was made by Naoki Abe et al[9]. The debt collection optimization research was the result of a number of optimization and data analytics researches' culmination. IBM and the New York State Department of Tax and Finance collaborated to develop software that makes use of CMDP (Constrained Markov Decision Process). The system went live in 2009 .It is claimed that so far the bureau saved in millions of dollars since the deployment. Despite the operational complexity and resources required to deploy similar systems at the CBE the purpose of this thesis may be regarded to have set out with the same objective of financial and operational efficiency gains.

The use of data modeling made the software perform better when more and better data accumulates over time. And this is achieved with little or no effort from the part of the modeler or user. It is also hoped that with proper system-wide modeling and integration the continual feeding of data in to the same could only elevate performance without the need for model calibration. This is an intrinsic property of learning based algorithms because more data is equated with better knowledge and hence better decision policy prescription.

Ethiopia's financial sector is entirely the jurisdiction of the government. As a result the market is too regulated with so few players even with the 14 plus private banks operating actively in the sector. In a recent development Kenyan banks have been allowed to open offices in the country.

Since 2012 major banks in the country have undergone a system overhaul one of which is the introduction of Core banking and migration from a paper data/records keeping to digitization. The new system helped in the interlinking of branch offices as well as creating a reliable intra banking networking. It is not the intention of this thesis to assess the impact of the new technology. However , despite streamlining services of primary sections of banks the technology has not been customized enough to adapt to the Ethiopian way of operation. System gaps are constantly being filled with time though. For example, according to a senior official of the bank, they no longer understand the new interest calculations even though to their knowledge there have not been any bank wide changes. This is the result of a lack of a team that comprises system analysts and bankers working towards the same goal.

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Commercial Bank of Ethiopia is at the forefront of implementing improved banking systems and operations- the recent churning out of new services such as mobile banking, Internet banking e.t.c could be cited as cases in point.

Notwithstanding, all the units involved with credit from loan application appraisal to recovery lack coordination to a level expected of the biggest state owned bank. For instance, risk grade revision would not be done unless the customer defaults or a new loan request is made. A second example and my central thesis question is what actions has the bank taken to recover the whole or part of the debt? None of the customer relations managers (CRMs) keep records of interventions from their part to measure the estimated impact of actions acted up on a group of customers or individuals.

I surmise that advancing loans with collateral may have made the bank's operation sluggish with a handful of loan takers that tend to be either risk lovers or those with the lowest estimated risk grade. Incidentally, sources indicate that about 12% of current loans advanced constitute unsecured loans that are not cushioned by collaterals. However, the figure could have been inflated for reasons beyond the comprehension of the writer of this thesis.

In light of numerous stories about foreclosures, especially of homeowners, one might wonder if such decisions are made by using a well thought out and consistent procedure.

Banks make loans and these loans can be either secured or unsecured. Both loan types are to be recoverable in as short time as possible. By doing so, banks can lend to more of their customers, have the smallest possible non-performing loans in their portfolio and the customers would develop a higher level of trust by these banks.

However, if debtors make irregular repayments and if these are few and far between, banks would be forced to follow a certain line of action to recover the debt. In the end, the relationships go sour making banks reluctant to giving future loans and harming the entire economy as a result.

Despite the lack of records made available to the public on debt and debt collection, it is evident that obtaining a loan in Ethiopia is pretty difficult. This is not to blame the banks as the information they can acquire on a particular applicant is so limited that making a sound assessment on the creditworthiness of an applicant would be practically impossible. These institutions then turn to asking for collaterals so large the candidate would end up giving up the application.

Customers on the other hand hide a lot of information to obtain the loan and if they succeed the repayment schedule can hardly be respected. The asymmetry in information makes the lender a victim of adverse selection and moral hazard [11].

Further to this, Joseph E. Stiglitz et al.[12] provide theoretical models with the implication that adverse selection and/or moral hazard can lead to credit rationing in a competitive market resulting in the dropping out of good borrowers because interest rates are simply too high for them.

Even though the problem of debt recovery is serious, banks have not been giving it equivalent attention. According to Adiel T.De et al[13] little research has been undertaken into the operations management of the collections process because banks may well want to 'bury their mistakes'. As can be referred in the Annex A with just eight entries, individual loan life cycle data are hard to obtain. The scarcity of such type of data was justified as being the bank's confidentiality and security protocol.

Because defaults are possible, banks determine how much capital they have to hold to cover against credit risk. [13] "The advent of the Basel Accord brought along analytic modeling of the collections process by drawing the attention of bankers and policy makers". CBE has been striving to keep the NPL rate below 5% of its portfolio at any reporting period to the NBE which they have allegedly declared to have succeeded in so doing.

Therefore, in this thesis debt collection modeling and optimization would be treated. The particular case of Commercial bank of Ethiopian would be studied and results discussed to recommend ways of bettering the system if there is one already or suggest one otherwise.

## 2.1 Solution Methods

MDPs are solved using value and policy iterations on the bellman optimality condition. Reinforcement learning (RL) versions of both model -based and non-model based RL are some of the options too. All the above methods converge to either an optimal or near-optimal policy. Sutton et al.'s [14] detailed treatise on reinforcement learning is the authority resource. One of the RL algorithms is Q learning and its convergence has been proven by Watkins and Dayan in 1992[15].This thesis will employ one of the iterative methods namely value iteration in order to determine optimal value and policy of the debt collection optimization problem.

CHAPTER 3

# METHODOLOGY

## 3.1 Framework

MDPs can be viewed as stochastic automata in which actions have uncertain effects, inducing stochastic transitions between states, and in which the precise state of the system is known only with a certain probability.

In addition, the expected value of a certain course of action is a function of the transitions it induces, allowing rewards to be associated with different aspects of the problem rather than with an all-or-nothing goal proposition.

This section primarily deals with the conversion of available data in to a form the MDP model can exploit.

n most applications of the Markov Chains ,the transition probability would be determined after which Stationarity and Markovity are tested using the log likelihood ratio test with asymptotical Chi-square distribution or alternatively other non-conventional method that has been efficiently employed in other research works.

Following this, value iteration algorithm would be used to determine the optimal policy prescription.

## 3.2 Data Analytics

Step 1.

Data at the Management and Information Systems (MIS) department is recorded in aggregate. That is to say collating customer-by- customer recordings have proved to be difficult for the analysts. As a result, the challenge was to estimate total 'net' OLA (outstanding loan amounts) (stocks) bar disbursements and collections collectively known as (flows) between the two observation periods say j and k. Otherwise, the asset quality at each period would be a running total of previous outstanding, disbursed and collection amounts.

$J < k$ where $j = 1, ...N$

$$OLA_k = OLA_j + \sum_{i=j}^{k}(disbursement_i - Collection_i) \tag{3.1}$$

Eq 3.1 models the stock of the outstanding loan amount between two consecutive periods by removing collected amounts and adding disbursed amounts.

To reach at the net OLA , I will regress both disbursement and collection on outstanding loan amounts separately. All the Gauss Markov assumptions for classical Linear Regression Model are assumed to be valid for this purpose.

$$\hat{disbursement} = \alpha_d + \beta_d * OLA \tag{3.2}$$

$$\hat{Collection} = \alpha_c + \beta_c * OLA \tag{3.3}$$

OLA in Eq 3.2 and 3.3 is the outstanding loan amount from 2003 to 2012-look at table 3.1- and thus different from the indexed OLA of Eq 3.1.The $\alpha$ and $\beta$ s are the regression coefficients.

Regression coefficients will be determined by ordinary least square estimate (OLS). This choice is based on the purpose of demonstration and not because it is particularly better than other estimation techniques. Moreover, the attempt is to bridge the gap of data unavailability and this application is not a standard way of getting data ready for the Markov model.

An important point to consider is that the initial asset quality vector or $OLA_1$ is assumed to be free from carried over assets. Moreover, regression is done on point-by-point basis assuming period cross transfers between states or statuses would have little impact on the nature of the regression line.

| Year | Disbursed | Collected | Outstanding |
|---|---|---|---|
| 2003 | 1130,477.00 | 701,491.00 | 3060,623.60 |
| 2004 | 1218,410.00 | 948,887.00 | 3496,566.60 |
| 2005 | 921,556.00 | 1075,457.00 | 3164,122.60 |
| 2006 | 1406,371.00 | 1831,684.00 | 2813,120.00 |
| 2007 | 2642,588.54 | 1554,983.18 | 3801,648.99 |
| 2008 | 1745,673.49 | 1980,298.50 | 3527,627.36 |
| 2009 | 1347,681.99 | 1749,104.64 | 3439,824.55 |
| 2010 | 1369,539.15 | 1634,615.46 | 3393,588.80 |
| 2011 | 3562,631.24 | 1872,401.04 | 5310,050.01 |
| 2012 | 1578,666.75 | 2106,064.39 | 4038,639.70 |

Table 3.1: Disbursed , Outstanding and Collected amounts for years 2003 - 2012

Step 2.

Once the net OLA at each period is determined the next step will be calculating the monthly OLA instead of the quarterly values. This is important because the decision epoch of the bank is mainly monthly whereas the data procured is quarterly.

To this end, I will use cubic Spline interpolation because of its relative high accuracy. [Oguzhan et al.[4] have effectively employed this interpolation method albeit in different context.

Timothy SAUER[16 ]– A cubic Spline S(x) through the data points $(x_1, y_1)$ is a set of cubic polynomials

$$S_x = y_1 + b_1(x - x_1) + c_1(x - x_1)^2 + d_1(x - x_1)^3 \quad \text{on} \quad [x_1, x_2] \tag{3.4}$$

$$\circ$$

$$\circ$$

$$\circ$$

$$S_x = y_{n-1} + b_{n-1}(x - x_1) + c_{n-1}(x - x_1)^2 + d_{n-1}(x - x_1)^3 \quad \text{on} \quad [x_{n-1}, x_n]$$

where $x = j^{th}$ period and $Y_j \in OLA^{\text{state i}}_{\text{period j}}$

The total Interpolated set of points is the Cartesian product of total number of periods and total number of states which are delinquency statuses.

Step 3:

Observe for any irregularities like non-positive values to apply normalization or standardization methods. Also, even though error terms may have accumulated both from the regression and interpolation the total amount of the asset quality in period 1 is expected to either decline or remain the same in the subsequent periods as the attempt has been to zoom-in on a certain loan portfolio segment and study its evolution over the eight quarters or equivalently twenty four months.

## 3.3 Discounting

Most Markov chains introduce discounting parameters in the system they are used to model. This parameter is employed to account for the time value of any sort of benefit or cost. It also makes sure the value of the function does not diverge. The discount factor ensures the maximum possible reward is collected sooner than later. Another way of looking at the discount factor is the present value of a reward at time t calculated while one is at a later period or t+1. This factor is more important for problems modeled over an infinite horizon than over finite horizon. The higher the discount factor the closer the value of a benefit/cost now to later. This factor usually denoted by the symbol $\gamma$ is the multiplicative inverse of 1+r where r is the interest rate applied for an economic sector.

## 3.4 Statistical Measurements

Both Stationarity and Markovity are tested by chi square goodness of fit Bartlett [17]. Almost all natural system are not order 1. However, to save computational time and resources, it is assumed the system under consideration to be so. In fact the tradeoff between robustness of the solution algorithm and a better or higher order Markov chain treatment makes the assumption valid and mild . This is the result of the state spaces explosion with higher order Markov chain i.e $|S|^m$. Similar assumptions are made to simplify computation without compromising results and interpretations. An example can be found in [7].

Maximum Likelihood Estimator [18]

$H_0$: transitions are independent or Markov chain is Zero order

$H_1$: transitions are dependent or MC is first order

$$\text{test stastic}: -2log_e\lambda = 2\sum_{i=j}^{n}\sum_{j=i}^{n} f_{ij} log \frac{logfifi..}{fi.f.i} \tag{3.5}$$

Asymptotically distributed as $\chi^2(n-1)^2$

Where f is the transition frequency matrix (contingency matrix) and is the likelihood ratio. However, higher order tests would have a similar but variable-wise different test statistic and different degrees of freedom for the Chi-square statistical distribution[19].

Had there been any and enough data on state-action-reward triples then the transition frequency table could have been constructed by :

$$f_{ij}^k = \frac{\text{customers switching from state i to j in period k}}{\text{Total customers in a state i in period k}} \tag{3.6}$$

## 3.5 Calculation of the Transition Probability

**Option 1**. Ordered Multinomial Logistic Regression Model

The transition probability matrix's elements can be calculated using ordered logistic regression (Ordered Logit). Since the regression is non-linear, ordinary least squares will not be useful to estimate parameters, instead the betas($\beta s$) are determined by the maximum likelihood estimator (MLE) method. This method has better accuracy than counting method but needs much more computation . Because of the intense computations involved, the numerical optimization choice of solution method would be Statistical software packages like STATA, R, and SPSS etc.

**Option 2:** A rather simpler but equally practical approach to calculating the transition probability of the Markov process is by estimating the contribution of a state now to any one of the chance nodes the system probably reached at the next period.

Lazlo et al[20] have efficiently used this approach to calculate the transition probability matrix of three competing plant species population.

I have adopted the same approach. The five statuses of the debtors can be considered as the competing population since whatever is left uncollected by the bank in the current period moves to one of these statuses in the next period. The analogy is thus viable.

$$Dev(i) = |X_{ik} - X_{ij}| \frac{X_{hk}}{X_{.k}} \tag{3.7}$$

Where period j precedes period k

i represents the population under consideration

$X_{ik}$ represents the number of species i in period k

$X_{ij}$ represents the number of species i in period j

$X_{hk}$ represents the number of species h in period k

$X_{.k}$ represents the number of all species in period k

Hypothesis

$H_0$ :Markov is order 0

$H_1$ :Markov is higher order than 0(order 1 & above)

$$M_k = M_j P \quad \text{and} \quad M_1 = X_1 \tag{3.8}$$

M is a Markov releve (statement)

Test statistic :

$$\sigma^2(D_X, D_M) = \sum_{j<k}^{c}(d_{Xjk} - d_{Mjk})^2 / \sum_{j<k}^{c}(d_{Xjk} + d_{Mjk})^2 \tag{3.9}$$

The square root of the above equation determines the discordance between Dx and DM . This is proportional to the closeness of the Markov model and the observed system evolution in distance configuration terms[20].

Where i and h are states in subsequent decision epochs. See appendix for data recorded without the effect of actions.

## 3.6 Defining the tuple < S, A, T, R, C>

### 3.6.1 State :

$s \in S : \bigcup s = \{pa, sm, ss, do, lo\}$

where

pa:pass , sm: special mention

ss: substandard , do: doubtful , lo: loss

The cost of holding an asset in the performing category is lower than the non- performing counterpart. Also, better states are worth more or cost less within the same category i.e performing or non-performing.

### 3.6.2 Action space (Randomized via a term q or efficiency of collection)

$a \in A : \bigcup a = \{RM, RE, LE\}$

where

RM: reminder (call, appointment, letter etc)

RE: rescheduling

LE: legal recourse (repossession, court order etc)

Therefore, based on expert opinion values be assigned to every state s.

$$q < S >= [q(a|s)] \tag{3.10}$$

Where q $< S >$ is the randomized action space on the vector S.

An important note to make here is that randomizing over actions does not result in a higher optimal value which is either a larger reward or a lower cost depending on the problem.

### 3.6.3 Transition Probability:

I have opted for option 2, see section 3.5 . There are also cases where the model of the system is indeterminate. In this case we resort to Q learning, which is a type of reinforcement learning algorithm that attempts to determine state action pairs of all reachable states and finally pick the best trajectory that resulted in the most gain. To ensure ergodic i.e non-zero property, all state transitions with zero probabilities are adjusted to 1 part per billion (1 ppb) because the presumption behind this decision is that had they been given enough time or a longer horizon these zero- appearing transitions would have been non-zero . There are two major categories of state transitions an improvement and deterioration. Improvements happen when loan takers make repayments up to and beyond the cumulative outstanding amount, whereas deteriorations occur when repayments are missed beyond the allowed period. One scenario of big status deterioration jump would be a debtor declaring bankruptcy but such cases are few and far between.

### 3.6.4 Reward:

For the sake of simplicity this term will be kept intact or independent of a decision made to reach a state. However, the case at hand is minimization of cost of an asset on logarithmic scale of the outstanding loan amount drawn mainly from intuition which incidentally leads to tractable figures.

### 3.6.5 Cost of an Action:

Constrained Markov Decision Process (CMDP)

Similarly to unconstrained MDPs, there exists an optimal Markov policy under some mild assumptions, but this policy may need to be randomized. CMDP is completely described through its state and action pace, transition probabilities and cost criteria much like its unconstrained version. The set of randomized Markov policies

$$q(\cdot) \in p(A_s)$$

Therefore action a is selected with likelihood Note that the existence of an optimal policy requires that the action space is compact in order to ensure all optima are reached. Incidentally, A Markov policy is deterministic when the action distribution is degenerate; the set of deterministic policies $\pi$.

$$C(a^i_{t+1}) \leq B - \sum \sum_{t=1}^{t} C(a^{'i'}_t) \tag{3.11}$$

$$B = B - C(a^i_t) \tag{3.12}$$

Eq (3.11) and Eq (3.12) make sure whether the resources for a particular action are limiting or not.

Where B is the approximated total budget allocated for the loan collection department for one year or period and $a^i_t$ is action taken in state i and period t, $a^{'i'}_t$, is actions taken on states prior to period t.

The cost of an action for instance constitutes total number of man hours spent on system set up,meeting arrangement, making calls, interviewing, making correspondences etc.

Modeling the cost of an action is important because one cannot continue taking the same actions just for their superior efficacy or ease of implementation. Actions that are less costly, underutilized or suitable for a particular situation may need application from time to time.

However, these pieces of information are hard to come by. Personnel responsible for keeping records of such information are understandably unconvinced by the need of such practice unless they are obliged to do so. According to some of the customer relations managers and loan officers they may consider adopting new practices if it promises efficiency gains and delivers accordingly.

For the sake of completeness an algorithm for a minimization of finite horizon CMDP follows.

**Generalized algorithm for a minimization of finite horizon CMDP:**

1. Determine states, actions and transition probabilities
2. Set the horizon and initialize period or epoch
3. Determine the action space
4. Evaluate the value function for all permissible actions
5. Choose the action that results the least cost
6. Decide whether economic /budgetary , commercial or judicial constraints do not prohibit the application of the action.
7. When constraints work against the chosen action go back to step 3 making sure every element of the action space answers to step 6
8. Display optimal policy for each state and period and value at the end of the horizon for every state.
9. Go back to 2 until the horizon is reached

## 3.7 Solution methods

**On Bellman Optimality conditions**

The central equation here is the Bellman's equation and its optimality conditions. Adopting terminology from dynamic programming, I will refer to the function mapping states of the Markov chain to expected long-term cost as the cost-to-go function. The optimal cost-to-go function satisfies the Bellman equation.

The optimality principle works on the premise that all sub-policies of an optimal policy are optimal sub-policies.

### 3.7.1 Value Iteration

**Algorithm**

$State = s_1, ..., s_m$ , $Action = a_1, ..., a_p$, Reward= S→ ℝ , Decision rule: $A^S = \sigma^1, ..., \sigma^P$ where $P = p^m$ and Transition probability: $SXA \rightarrow P(S)$ .

1 Start at n=1

2 $U^0$ is the terminal value and known/given as data

3 Evaluate $U^n(S) = R(S) + \gamma min_a \sum_{s'} T(s^{'}|s, a)U^{n-1}(s^{'})$

4 And the policy that maximizes the above Bellman's optimality equation is $a_{s \in S}* \in argminU^*(s)$

5 End at $n = N$

$\gamma \in [0, 1]$

The max operator's output is a value but the argmin operator outputs a set.

Where U is the utility function, $\gamma$ the discount factor and n is the number of periods remaining or the depth of the decision epoch.

The bellman equation gives out the utility of a state. This also means that there are m non- linear (because of the min operator) equations to solve at every decision point where m is the number of states.

As the number of iterations goes to infinity,$U(s)_{i+1}$ converges to equilibrium $U(s)^*$. These are the solutions to the Bellman equations, optimal and unique. The vector of states that resulted in the optimal utility value would be the policy.

The vector of actions $-\pi^*(s)-$ that resulted in the optimal utility value would be the policy .

There are also other iterative techniques; one is policy iteration (PI). The linearity of the PI makes it an obvious and convenient choice. However, both solution methods have their own merit. In certain cases, we may need to exploit readily available software and routines; this can be achieved by converting the optimization problem into an Linear Programming (LP) solvable form.

### 3.7.2 Reinforcement Learning

**Model based:** The two place equivalent of value function U(s) in the preceding subsection is

$$Q^n(s,a) = \gamma \sum_{s^{'}} T(s^{'}|s,a)R(s^{'}) + min_{a'} Q^{n-1}(s',a') \tag{3.13}$$

Since T and R must be known this is a model based reinforcement learning algorithm.

**Model free:**

1. Given experience $< s, a, s^{'}, r >$
2. $a = argmin_{a'} f(Q(s,a^{'}, N(s,a^{'}))$
3. $Q(s,a) \leftarrow Q(s,a) + \alpha(error)$
   $Q(s,a) \leftarrow Q(s,a) + \alpha(sensed - predicted)$
4. $Q^n(s,a) \leftarrow Q(s,a)^{n-1} + \alpha(\gamma(R(s) + min_{a'}, Q(s^{'}, a^{'})^{n-1}) - Q(s,a)^{n-1})$

$\alpha$ is the learning rate

The learning rate shows the exploration-exploitation tradeoff. And the value is expected to decline with time by some function of decay $f(1/t)$.

The above RL algorithm is called temporal difference learning due to the estimation of the error term. Algorithms of both models are clearly outlined in (Sutton and Barto [14]).

A widely used RL model is the Q-learning which does not need a pre-determined transition probability matrix. Incidentally the Q function is the two place equivalent of the value function in the Bellman's optimality equation. The Q learning algorithm has also been proven to converge in a finite number of iterations .The convergence proof was presented by Watkins and Dayan[15].

However, reinforcement learning algorithm is not to be used for this particular thesis. Comparison of results of various algorithms is outside of the scope of this thesis and shall be left open for exploration.

CHAPTER 4

# THE CASE OF CBE'S DATA

## 4.1 Loan Life cycle Flow chart

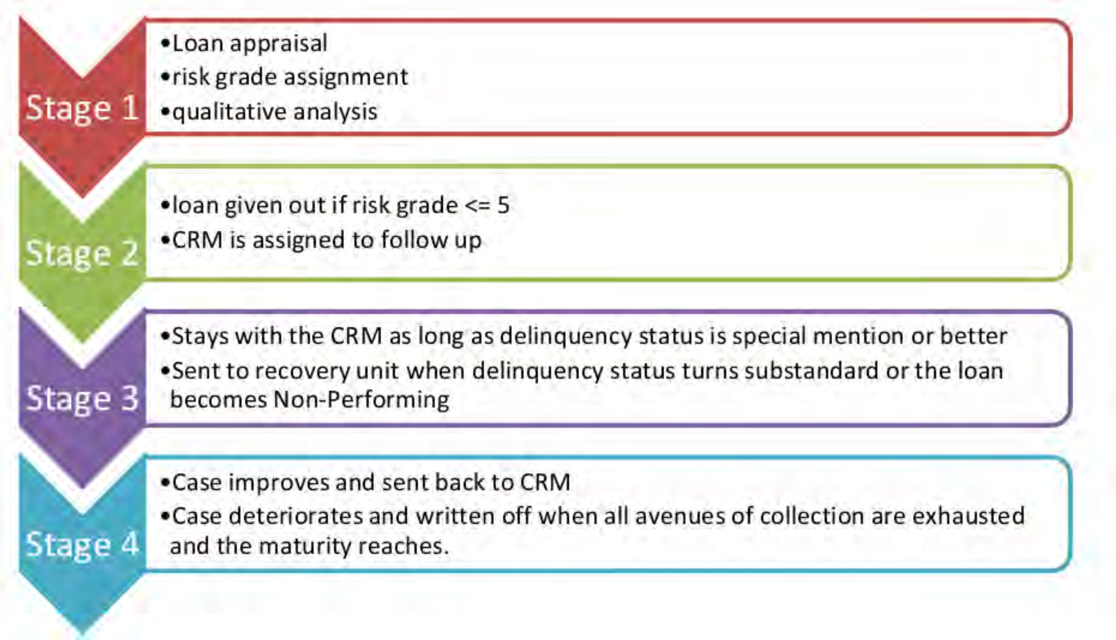


Figure 4.1: Stages of the loan life cycle

The bank assigns numbers from 1 to 5 to the equal categories of delinquency statuses. Operationally speaking the following is one way of going about calculating the delinquency status index as follows

Where $K_j$ is the number of missed repayment periods at the beginning of period j.

$$K_{cal} = floor\left(\frac{Ar_j}{Er_j}\right) \tag{4.1}$$

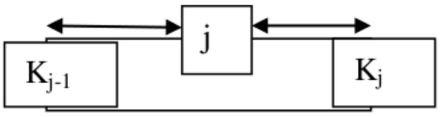

Figure 4.2: Depiction of a loan's segment's status in a single period

Where $Ar_j$ is actual repayment in period j and $Er_j$ is its expected repayment counterpart.

$$k = min(k_{j-1}, K_{cal}) \tag{4.2}$$

$$K_j = abs(k - K_{j-1}) \tag{4.3}$$

and $K_j \leq j$ which means the number of missed repayments cannot be larger than the current period since taking out the loan.

Therefore, based on the above equations $K_j \in [0, tenure\_in\_Months]$

| Loan Type | Status | Days past due (DPD) | Loss Provision % | Number of missed repayments |
|---|---|---|---|---|
| Performing | Pass | <30 | 1 | 0 |
| | special mention | 30 < t < 90 | 3 | 1,2 |
| Non-performing | Substandard | 90 < t < 180 | 20 | 3,4,5 |
| | Doubtful | 180 < t < 360 | 50 | 6,7,8,9,10,11 |
| | Loss | >360 | 100 | ≥ 12 |

Table 4.1: loan type by days past due , loss provision , and number of missed repayments

## 4.2 Major Assumptions:

a Debtors comprising of an economic sector with the same tenure category behave similarly, this assumption is dictated by practicality rather than reality.

b Given that CBE has been able to keep percentage of NPL loans in check or under 5% since the introduction of the Basel Accord, the upper bound of this percentage will be the same figure.

c Tenure of a loan or expected maturity period affects the likelihood or rate of repayment because with time both personal and economic factors that impair capacity to repay alter.

d Cases of multiple loans to a single bank customer have not been treated because risk grade assigned to a particular loan to a customer remains valid for 6 months. The bank's recording system does not lend itself to easy access of this vital information.

e Loan collection gives precedence to the interest rate over the principal however this distinction is not considered

## 4.3 Components of the Markov Decision Process

### 4.3.1 States:

The state of a particular loan can be identified by the following 3 descriptors

1. Economic Sector $(DTS, FT)$
2. Tenure $(ST, MT, LT)$
3. Delinquency status $(PA, SM, SS, DO, LO)$

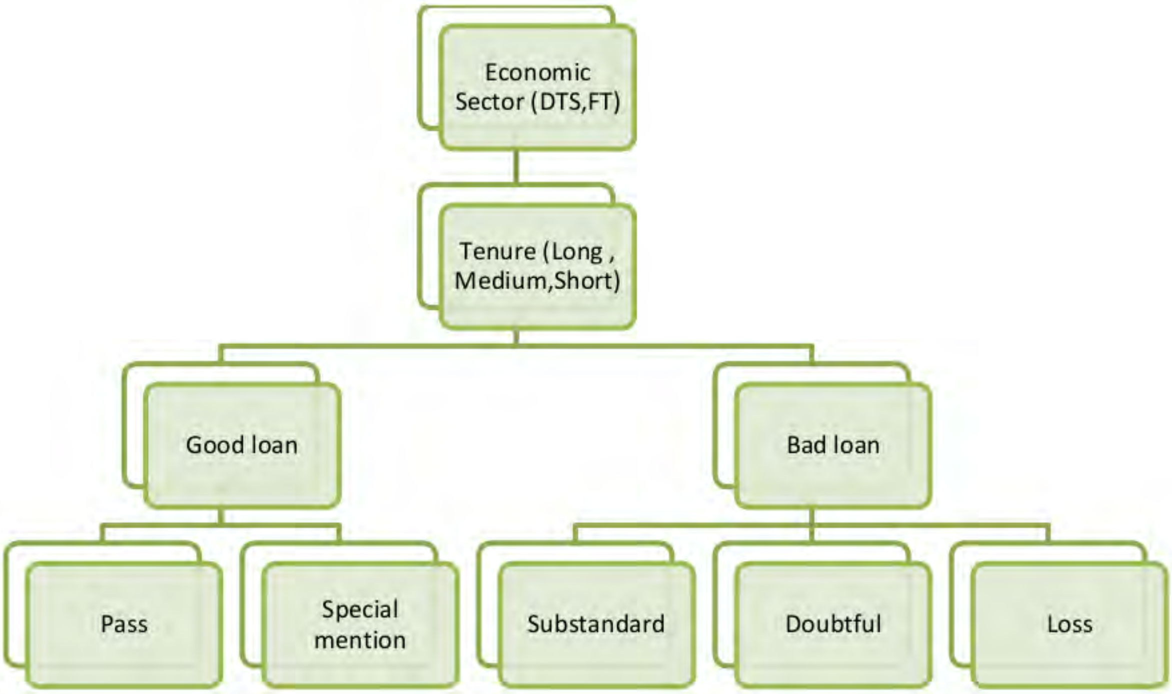


Figure 4.3: Loan segmentation

Number of status /tenure combinations or set is the Cartesian product of number of statuses and number of tenures.

*States Network*

Each column denotes the bank's debt asset portfolio at a particular time.

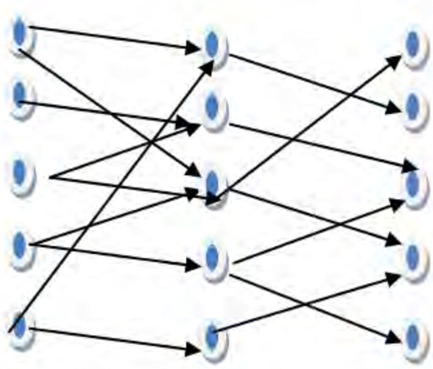

Figure 4.4: states transition network, all transitions are possible

### 4.3.2 Actions:

There are three possible actions that can be taken

1. Sending reminder
2. Reschedule payment
3. Legal action

These actions will be chosen randomly which means selecting a loan collection likelihood estimator

$q(\cdot)P(A_s)$

In which case action a is selected with likelihood $q(A)$

Maximizing Collection Amounts

$$q < S >= \begin{pmatrix} 0.9998 & 0.0001 & 0.0001 \\ 0.9998 & 0.0001 & 0.0001 \\ 0.5 & 0.4999 & 0.0001 \\ 0.3 & 0.6 & 0.1 \\ 0.0001 & 0.0001 & 0.9998 \end{pmatrix}$$

Minimizing Outstanding Amounts

$$q^* < S >= \frac{1 - q < S >}{2}$$

Thus

$$q^* < S >= \begin{pmatrix} 0.0001 & 0.5 & 0.5 \\ 0.0001 & 0.5 & 0.5 \\ 0.25 & 0.2501 & 0.5 \\ 0.35 & 0.2 & 0.45 \\ 0.5 & 0.5 & 0.0001 \end{pmatrix}$$

Rows represent state and columns actions.

### 4.3.3 Reward:

Outstanding loan amount of all the five states constitute the asset quality of the bank at the period of observation.

### 4.3.4 Transition probability: $m = n$

$$T(s^{'}|s,a) = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mn} \end{pmatrix} \tag{4.4}$$

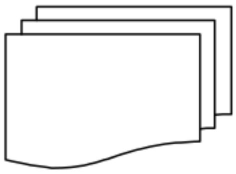

Figure 4.5: Transition probability depiction for three actions

### 4.3.5 Decision epochs:

The decision epochs are monthly from Mar $31^{st}$, 2012 to Dec $31^{st}$ 2013.

### 4.3.6 Cost of actions:

The cost of an action like reminder and rescheduling is assumed to be non-limiting. Also the third of the actions' i.e legal cost would be covered by the debtor from the sale of the collateral or other foreclosure means. What cost of actions does is determining the permissible set of actions for a given state.

### 4.3.7 Discount factor:

This value depends on the interest rate that varies for the two economic sectors.

## 4.4 Numerical Evaluation

For the sake of demonstration one iteration of the Bellman equation using the empirical data would look like as follows. The segment considered is long tenure DTS.

| † Values less than 1 are rounded up to 1 to avoid negative values on the log scale

| S/P∗ | 1 | 2 | 3 | ... | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|
| PA | 2.22E+09 | 2.22E+09 | 2.21E+09 | | 1.93E+09 | 1.90E+09 | 1.87E+09 |
| SM | 54711 | 1.00E-09(1) † | 1.00E-09(1) | | 9.80E+05 | 2.47E+05 | 1.00E-09(1) |
| SS | 3.97E+06 | 2.59E+06 | 2.77E+06 | | 1.06E+06 | 1.00E-09(1) | 1.00E-09(1) |
| DO | 1.00E-09(1) | 1.00E-09 | 1.00E-09 | | 1.26E+07 | 1.37E+07 | 1.39E+07 |
| LO | 1.06E+05 | 3.00E+05 | 3.65E+05 | | 4.54E+06 | 3.93E+06 | 4.50E+06 |

Table 4.2: Outstanding loan amount over 24 Periods

S/P∗ States/Periods

$$U^1(S) = \Pr ovision\_\%(s)\left\{ OLA^1(S) + \gamma \min_{a_l} \sum_{s'} q(S,a)T(S' \mid S,a_l)U^0(S')\right\}$$

$$U^1(s) = \Pr ovision\_\%(s)\left\{ \log_{10}\begin{pmatrix} 1.90E+09 \\ 2.47E+05 \\ 1.00E-09 \\ 1.37E+07 \\ 3.93E+06 \end{pmatrix} + 0.913 * \min_{a_l} \begin{pmatrix} 0.99878 & 0.000144 & 0.000355 & 0.000468 & 0.000257 \\ 0.3445 & 0.65237 & 0.001546 & 0.001099 & 0.000484 \\ 0.22031 & 0.000174 & 0.77739 & 0.001595 & 0.000528 \\ 0.1386 & 0.000381 & 0.001002 & 0.8162 & 0.043814 \\ 0.14886 & 0.000163 & 0.001092 & 0.000622 & 0.84926 \end{pmatrix} \log_{10}\begin{pmatrix} 1.87E+09 \\ 1.00E-09 \\ 1.00E-09 \\ 1.39E+07 \\ 4.50E+06 \end{pmatrix} \right\}$$

$$U^1(s_1)=0.01\left[\log_{10}(1.90\text{E}+09)+0.913*\min\left\{\begin{pmatrix}0.0001\\0.5\\0.5\end{pmatrix}(0.99878\ \ 0.000144\ \ 0.000355\ \ 0.000468\ \ 0.000257)\log_{10}\begin{pmatrix}1.87\text{E}+09\\1.00\\1.00\\1.39\text{E}+07\\4.50\text{E}+06\end{pmatrix}\right\}\right]$$

$$U^1(s_2)=0.03\left[\log_{10}(1.00)+0.913*\min\left\{\begin{pmatrix}0.0001\\0.5\\0.5\end{pmatrix}(0.3445\ \ 0.65237\ \ 0.001546\ \ 0.001099\ \ 0.000484)\log_{10}\begin{pmatrix}1.87\text{E}+09\\1.00\\1.00\\1.39\text{E}+07\\4.50\text{E}+06\end{pmatrix}\right\}\right]$$

$$U^1(s_3)=0.2\left[\log_{10}(1.00)+0.913*\min\left\{\begin{pmatrix}0.25\\0.2501\\0.5\end{pmatrix}(\ 0.22031\ \ 0.000174\ \ 0.77739\ \ 0.001595\ \ 0.000528)\log_{10}\begin{pmatrix}1.87\text{E}+09\\1.00\\1.00\\1.39\text{E}+07\\4.50\text{E}+06\end{pmatrix}\right\}\right]$$

$$U^1(s_4)=0.5\left[\log_{10}(1.39\text{E}+07)+0.913*\min\left\{\begin{pmatrix}0.35\\0.2\\0.45\end{pmatrix}(0.1386\ \ 0.000381\ \ 0.001002\ \ 0.8162\ \ 0.043814)\log_{10}\begin{pmatrix}1.87\text{E}+09\\1.00\\1.00\\1.39\text{E}+07\\4.50\text{E}+06\end{pmatrix}\right\}\right]$$

$$U^1(s_5)=1.0\left[\log_{10}(1.39\text{E}+07)+0.913*\min\left\{\begin{pmatrix}0.5\\0.5\\0.0001\end{pmatrix}(\ 0.14886\ \ 0.000163\ \ 0.001092\ \ 0.000622\ \ 0.84926)\log_{10}\begin{pmatrix}1.87\text{E}+09\\1.00\\1.00\\1.39\text{E}+07\\4.50\text{E}+06\end{pmatrix}\right\}\right]$$

The above iteration evaluates $U^1(S)$ and continues until the horizon reaches.The optimal policy is the action that minimized U for each state and period.

## 4.5 Model's limitations

a) Should have incorporated risk grade

b) Should have considered a longer period data

c) Should have considered more economic sectors

Currently, collection is more relationship management than operations. But not sustainable when bigger competitors enter the financial sector State action pairs are too stochastic due to lack of records

# CHAPTER 5

# RESULTS ANALYSIS

## 5.1 Results Analysis

Transitions one to six are obtained by taking the average of 23 matrices of the respective loan segments. These matrices are taken to have been stabilized over the horizon.

The six matrices and Sigmas (closeness between Markov Chain and the observed system) corresponding to the entire loan segments are:

data.txt

```
LDTS Long tenure Domestic & Trade Services
Outstanding loan amounts estimate is good enough
Sigma 1 = 0.9866
Transition 1 =
0.99878       0.00014368    0.0003554    0.00046828    0.00025732
0.34450       0.65237000    0.0015462    0.00109880    0.00048367
0.22031       0.00017447    0.7773900    0.00159500    0.00052798
0.13860       0.00038142    0.0010023    0.81620000    0.04381400
0.14886       0.00016281    0.0010915    0.00062208    0.84926000
```

data.txt

```
MDTS Medium tenure Domestic & Trade Services
Outstanding loan amounts estimate is good enough
```

```
Sigma 2 = 0.8094

Transition 2 =

0.979320    0.0093077   0.0048773   0.0018121   0.0046857
0.173920    0.8051000   0.0076206   0.0034345   0.0099258
0.180160    0.0177450   0.7896300   0.0041834   0.0082871
0.207120    0.0182220   0.0050664   0.7584300   0.011163
0.046053    0.0059197   0.0044332   0.0016138   0.941980
```

data.txt

```
SDTS Short tenure Domestic & Trade Services

Outstanding loan amounts estimate is good enough

Sigma 3 = 0.9313

Transition 3 =

0.996790    0.00063778   0.00058914   0.00070196   0.0012792
0.204880    0.78998000   0.00100040   0.00108740   0.0030476
0.185710    0.00075329   0.81120000   0.00056366   0.0017741
0.160770    0.00115580   0.00075019   0.83528000   0.0020397
0.050147    0.00051031   0.00047677   0.00053578   0.9483300
```

data.txt

```
LIT Long tenure International Trade

Outstanding loan amounts estimate is off by[1.6786e+09]

Sigma 4 = 0.7696

Transition 4 =

0.898460    0.013838   0.00069868   0.042843   0.044156
0.382850    0.549920   0.00203180   0.032490   0.032708
0.088268    0.010898   0.83572000   0.032447   0.032666
0.085271    0.010459   0.00049789   0.882500   0.021271
0.085464    0.010504   0.00049989   0.021216   0.882320
```

data.txt

```
MIT Medium tenure International Trade

Outstanding loan amounts estimate is good enough

Sigma 5 = 0.9367

Transition 5 =

0.965030      0.016824      0.0077162      0.0103550      7.3107e-05

0.347700      0.607350      0.0182790      0.0238510      0.00281570

0.128120      0.013209      0.8483300      0.0101680      0.00016923

0.135880      0.013321      0.0076907      0.8429300      0.00017745

0.034648      0.014620      0.0072872      0.0097793      0.93366000
```

data.txt

```
SIT Short tenure International Trade

Outstanding loan amounts estimate is good enough

Sigma 6 = 0.9596

Transition 6 =

0.984910      0.00042747      0.0017891      0.0114230      0.0014525

0.286590      0.70074000      0.0021224      0.0084008      0.0021532

0.234940      0.00104150      0.7534300      0.0085811      0.0020099

0.151820      0.00088167      0.0042360      0.8417700      0.0012940

0.018671      0.00044739      0.0016717      0.0055082      0.9737000
```

An important point to raise concerning the above transition matrices is that the diagonal elements are relatively high. This can be attributed to a states' resistance to change whether it is an improvement or deterioration.

| Economic Sector | DTS | DTS | DTS | FT | FT | FT | |
|---|---|---|---|---|---|---|---|
| States /Tenure | Long | Medium | Short | Long | Medium | Short | Optimal Policy |
| PA | 0.093459 | 0.088999 | 0.094652 | 3.36E-007 | 0.085119 | 0.10179 | Reminder |
| SM | 1.02E-007 | 0.22996 | 0.19802 | 7.54E-007 | 3.78E-007 | 0.2521 | Reminder |
| SS | 1.3312 | 1.5025 | 0.003178 | 0.014795 | 1.0715 | 1.5766 | Reminder |
| DO | 0.041718 | 3.6166 | 2.8692 | 2.806 | 2.7967 | 3.0925 | Reschedule |
| LO | 5.4776 | 6.9205 | 7.379 | 5.1242 | 5.5382 | 7.8327 | Legal |
| THC∗ | 6.943977 | 142.35856 | 10.54405 | 7.944996 | 9.491519 | 12.85569 | |

Table 5.1: Optimal Value/Cost of holding an outstanding loan on a log scale and provision percentage scale of the states
THC∗ Total Holding Cost

Examination of the above table imparts the following indicators.

a Optimal policy for both sectors and all tenures is the same. A direct result of non- differentiation between neither sectors nor tenures when the bank takes an action. In other words, the efficiency collection of an action is the same. In reality however the actions produce different results depending on all the above factors.

b The total holding cost of domestic trade and services are spread farther apart than that of international trade's imparting the information that tenure length has more impact on DTS than FT.

c For all the loan segments of both DTS and FT: From 80%- 99% of the total holding cost comes from the loss and doubtful delinquency statuses indicating that once status deteriorates this bad it is likely to stay there. This could guide the banks to redirect resources particularly to these statuses to prevent it happening.

d Medium tenure DTS and short tenure IT shall be avoided at all costs because of their greater holding cost. For medium tenure DTS the reason could be loans in this segment have better chance of rescheduling or tenure extension which could be a disincentive to keep up with strict and regular repayment schedules.

CHAPTER 6

# RECOMMENDATION AND FUTURE WORKS

## 6.1 Recommendation

The ideal scenario would have been getting data where the amount collected on a particular debtor or customer is shown as an immediate result of an action.

This information has not been recorded by the bank. Even though customers miss repayments and resume repayments with or without intervention, the action –reward relationship has a non zero probability. This means repayment via intervention can be captured by likelihood parameters. One way of doing this is comparing almost identical groups of customers where some are induced to repay and others are left alone.

However, all this is possible if either the bank keeps records to such detail or a researcher collects live data over a reasonably long period say 2 - 5 years to discern occurrences of seasonality.

From the bank's perspective, lending out loans is an investment. Thus, considering all economic sectors at a time gives a fuller picture of the bank's asset portfolio. But, to do so we need at least the risk grade distribution of individual loan takers that constitute the economic sectors. This is because, the higher the proportion of riskier debtors in an economic sector the smaller its contribution to the value of the bank's asset portfolio and vice versa.

In the final analysis, operations research applications are data hungry without which research significance is considerably diminished. The good news is that Ethiopian institutions are becoming progressive at a good speed by introducing business automation software like SAP and Microsoft ERP. This software enables OR researchers see gaps, study these gaps and try to fill those using better managerial decision making tools.

Comparison of algorithms for reinforcement learning and the one used here i.e.Value iteration could be done to evaluate efficiency and choose which algorithm to use and when. [21] treats the state aggregation of large state spaces, which is also an area of study in relation to debt collection optimization, the loan segments are bound to increase exponentially for every descriptor used to qualify a certain loan type.

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# APPENDIX A

| | Period | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Customer | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 1 | pa | sm | sm | sm | ss | ss | ss | ss | ss | do | do | do | do | do | do | do | do | lo | lo | lo | lo | lo | lo | lo |
| 2 | sm | sm | ss | ss | ss | ss | ss | do | do | do | do | do | sm | ss | ss | ss | sm | ss | sm | ss | ss | ss | ss | do |
| 3 | sm | sm | sm | sm | ss | ss | ss | do | do | do | do | do | do | lo | lo | lo | lo | lo | lo | lo | lo | lo | lo | lo |
| 4 | sm | sm | ss | ss | ss | do | do | do | do | do | do | lo | lo | lo | lo | lo | lo | lo | lo | lo | lo | lo | lo | lo |
| 5 | sm | sm | ss | ss | ss | ss | ss | do | do | do | do | do | do | do | do | do | ss | do | do | do | do | do | do | do |
| 6 | sm | sm | ss | ss | ss | do | do | do | do | do | do | do | do | do | do | do | lo | lo | lo | lo | lo | lo | lo | lo |
| 7 | pa | pa | pa | pa | pa | pa | sm | sm | sm | sm | pa | sm | sm | sm | ss | ss | ss | do | do | do | do | do | ss | sm |
| 8 | sm | sm | sm | ss | ss | ss | do | do | do | do | do | do | lo | lo | lo | lo | lo | lo | lo | lo | lo | lo | lo | lo |

Eight customers portfolio evolution

# APPENDIX B

**Loan life cycle**

**Customer Detail**

| | |
|---|---|
| *Customer ID* | |
| *Risk Grade* | |
| *Economic Sector* | |

**Loan Detail**

| | |
|---|---|
| *Tenure* | |
| *Interest Rate* | |
| *Date loan taken* | |
| *Amount loan taken* | |

| $I^{th}$ Period , Status: | **Expected Repayment** | **Actual Repayment** | **Action** |
|---|---|---|---|
| **Date** | | | |
| **Amount** | | | |

Sample data collector template for the debt collection unit of banks

# APPENDIX C

Directive No. SBB/32/2002 Amendment of provisions 3.10 “Non-performing” means loans or advances whose credit quality has deteriorated such that full collection of principal and/or interest in accordance with the contractual repayment terms of the loan or advance is in question. 3.10.1 For purposes of this Directive, loans or advances with pre-established repayment programs are non-performing when principal and/or interest is due and uncollected for 90 (ninety) days or more beyond the scheduled payment date or maturity.

# APPENDIX D SOURCE CODE

Transition Probabilities Matlab Code

```
function [  ] = kernel()
%This fucntion calculates the transition matrices of all the loan segments
addpath('splined');
addpath('OLA quarterly');

%x1=dlmread('splined1.m');
x1=dlmread('OLA quarterly1.m');

[m,n]=size(x1);
p1=zeros(m,m);

pav1=zeros(m,m);
% sum function call
%sumx1=sum_x(x1,m,n);
sumx1=zeros(1,n);
for k=1:n
    for i=1:m
    sumx1(k)=sumx1(k)+x1(i,k);
    end
end
for j=1:n-1
    k=j+1;
       for i=1:m
```

```
        diff=x1(i,k)-x1(i,j);
            for h=1:m
            if (diff < 0) && (i~=h)
                p1(i,h)=p1(i,h)+abs(diff)*(x1(h,k)/sumx1(k));
            elseif (diff < 0) && (i==h)
                p1(i,h) = max(x1(i,k),0.000000001);
            elseif (diff >=0) && (i~=h)
            p1(h,i)=p1(h,i)+abs(diff)*(x1(h,k)/sumx1(k));
            else
                p1(h,i) = max(x1(i,k),0.000000001);
            end
            end
        end
    row=sum(p1');
    row=row';
    for i=1:5
        p1(i,:)=p1(i,:)/row(i);
    end
    % p1=norma(p1,m);% normalization function call
     pav1=(pav1+p1);

end
  pav1=pav1/(n-1);
  dlmwrite('new11.m',pav1,'\t');
end
```

Bellman Equation Matlab Code

```
function [] = bellman()
m=5; % total number of states
n=24; % total number of periods
l=3;% total possible number of actions
scale=[0.01 0.03 0.2 0.5 1];
gamma1=0.913;% discount rate .this particular value applies only for DTS ...
    i,e splined1 to splined3
```

```
addpath('pavresults');
addpath('splined');
% Initialization of Expected values
EV1=zeros(1,3);
% optimal value
voptim1=zeros(m,1);
% optimum policy for every period except for the Nth period
poptim1=zeros(m,n-1);
% initialization and matrix generation for the  loan segment
v1=zeros(m,n);v1(:,2:n)=0;
% effciency/likelihood of collection for the 3 actions for the 5 states
a=[0.9998,0.0001,0.0001;0.9998,0.0001,0.0001;0.5,0.4999,0.0001;0.1,0.6,0.3
;0.0001,0.0001,0.9998];
q=0.5*(1-a);
% transition probability for the loan segments
T1=dlmread('T1.m');
%  splined values for the  loan segment
x1=dlmread('splined1.m');
% cost of an asset in log terms because 1000 is not 100 times costlier than
% 10
%for i=1:m
 %for j=1:n
%if x1(i,j) >1
%x1(i,j)=log10 (x1(i,j));
%else % because x can never be less than 0
       % x1(i,j)=1; % an amount this small would be neglected
        %x1(i,j)=log10 (x1(i,j));
%end
%end
%end
x1(x1<1)=1;% avoids negative cost when transformed into the log scale
x1=log10(x1);
% storing values until the final voptim, varies with each x
v1(:,1)=x1(:,n); % the last periods vector is known
for k=1:n-1  % total number of periods remaining
  j=k+1;
```

```
    for i=1:m % all states in period k

            for l=1:3 % all actions
    % 6 expected values for 6 loan segments

    EV1(l)=x1(i,n+1-j)+(gamma1)*q(i,l)* T1(i,:)*v1(:,k);
            end
   v1(i,j)=scale(i)*min(EV1);% provision percentage scaling
            poptim1(i,n+1-j)=find(EV1==min(EV1));% returning the argument ...
               or index or choice of action

  % writing optimal policy to a file for each state and period for the ...
      loan segments
dlmwrite('op1.m',poptim1);
end
end
% storing optimal values to the corresponding variables i.e each of the
% loan segments
voptim1=v1(:,n-1);
% writing optimal value to a file for each state for the loan segments
dlmwrite('ov1.m',voptim1);
end
```