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DESIGN AND OPTIMIZATION OF GEODETIC NETWORK: A CASE OF ETHIOPIA

BY

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DECLARATION

I certify that this research work entitled "*Design and Optimization of Geodetic Network: A Case of Ethiopia*" is my own work. The work has not been presented elsewhere for assessment. Where material has been used from other sources, it has been properly acknowledged and referred.

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As Master research advisor, I hereby certify that I have read and evaluated this MSc thesis prepared under my guidance.

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LIST ABBREVIATIONS

Abbreviations	Description
B	Coefficient matrix.
g	Constant terms in constraint equation.
n	No. of observations.
C	Constraints matrix.
N_L	Lower in Y-direction for each point
E_L	Lower in X-direction for each point
N_L	upper limit in Y-direction for each point
E_U	upper limit in X-direction for each point
ZOD	Zero Order Design.
FOD	First Order Design.
SOD	Second Order Design.
TOD	Third Order Design
COMD	Combined Design.

ABSTRACT

The optimization of a geodetic network is to enhance precision and efficiency in surveying practices. Precision involves in controlling the quality of a geodetic network. The research objective is to strategically position control points and minimize errors to improve the overall geodetic network. Finding the optimal design of geodetic network of Ethiopia is the main objective of this thesis by solving the zero order design and first order design problems by applying one of the classical methods that is the trial and error technique using a MATLAB programing language. Zero order design problem was applied to a case study network consists of 30 points and 70 designed distances with a priori deviation equal to 5mm, to determine the best points in the network to consider as control points. The results showed that P18 and P19 having the minimum ellipse of error and considered as control points. These points are therefore chosen as the control points since they have an area of ***0.094*** and ***0.101***, respectively, making them the best points. First order design problem was applied on a selected network to be analysed using the objective function, with selected range of movement of 100m to each point in each direction. This first order design problem optimization is done by the trial and error method. By taking P18 and P19 as control points the optimal design of the geodetic network with high precision is developed.

***Keywords*:** Control Points, First order design, Geodetic networks, Optimization, Precision, Zero order design.

CHAPTER ONE

1. Background

1.1 Introduction

Geodetic network is a collection of connected points on the surface of the Earth that have been carefully surveyed and measured. These networks are used to pinpoint the locations of things like landmarks, structures, and geographical characteristics on the surface of the Earth (Przewięźlikowska et al., 2021) . These networks are classified in to local (micro) which is generally designed for limited and specific purposes, on the other hand for multi-purposes design we use national control networks. To develop optimal design of a geodetic network it is important to follow the design, compute, measurement and adjustment process.

Geodetic network is designed for different purposes which pinpoint the locations of things like landmarks, structures, and geographical characteristics on the surface of the Earth. The objective or scalar function is maximized or minimized that describes the quality design criteria of a geodetic network is referred to us Optimization. Optimization problems are related to the adjustment of the relevant observations and are in fact formulated with the help of the same terms which appear in the adjustment (design matrix, weight matrix, covariance matrix of coordinates) (Grafarend, 1974), and these optimization problems are classified into different orders, zero-order design (ZOD) which is to define the optimum datum. First-order design (FOD), used to design of the optimum network configuration. Second-order design (SOD), targets to design the observation weights as a result the solution is able to accomplish set precisions. Third-order design (THOD), used for improving an existing network by adding extra points and/or observations. The fifth one is Combined Design (COMD), in this case first- and second-Order Design problems have to be optimally solved simultaneously (Alwan & Msaewe, 2012).

Optimization of a geodetic network needs different criteria's or constraints including variables which are taken as objective functions. The function that this research tries to improve is called objective or scalar function and it can be represented as A, D, E, S and N-optimality's that are Measures and Criteria for Precision. The covariance matrix of estimated coordinates serves as the foundation for a geodetic network's precision metrics and standards. A scalar or objective function of the components of the covariance matrix of the variations of the coordinate is one way to quantify precision. The goal is to satisfy the demand for a comprehensive depiction of a network's precision (Kuang, 1991).

Minimising the trace in the covariance matrix is known as A-optimality. Consequently, the parameter estimations' average variances are reduced. The goal of D-optimality is to reduce the covariance matrix determinant. Its statistical relevance lies in its ability to minimise the erroneous hyper ellipsoid's volume. E-optimality: given the parameter estimates, this seeks to minimise the biggest eigenvalue of the covariance matrix. S-Optimality: (Spectral Optimisation about the stiffness of the network). N-Optimality: Choosing how many and where to put measurement points in the network is the goal of the N-optimality setting (Grafarend, 1974)

Selecting the best optimality for the specific work that we are going to implement is based on the purpose of the geodetic network. Other than the objective function Criterion matrices-based methods and criterion matrix are the Measures and Criteria for Precision (Kuang, 1991).

Geodetic network optimization can be performed in different techniques this are classified in to Classical Optimization and Intelligent Optimization (Doma, 2013). Classically, a network can be optimized using the trial and error method or analytical methods such as Numerical and Differential methods. Under numerical practice linear programming and nonlinear programing are included (Rao, 2020)**.** Optimization problems may also be solved by intelligent or heuristic optimization techniques. Global and Local Optimization techniques are found in solving the problems and in Global Optimization there are three methods to perform optimization. These are genetic algorithms (GA), simulated annealing (SA) and particle swarm optimization algorithm (PSO) (Yetki et al., 2008).

For the optimization of first order design of a geodetic network the above optimization techniques are used. In the case of this thesis the trial and error method is applied, the design challenge can be solved by computer simulation, also known as "trial and error," which computes the design and cost criteria. If one of these requirements is not met, a new solution is proposed (often by making minor changes to the original postulate) and the requirements are recalculated. (Cross, 1985)

1.2 Statement of the problem

A geodetic network is a collection of interconnected, closely surveyed, and measured locations on Earth's surface. These networks are employed to determine the precise locations of objects on Earth's surface, such as buildings, landmarks, and geographical features (Przewięźlikowska et al., 2021). The process optimization, which involves maximizing or minimizing an objective function that specifies a geodetic network's quality design criteria, is required to modify these geodetic networks. In network analysis, control geodetic network design and optimization are crucial (Amiri-Simkooei, 2012).

Optimization of the geodetic network can be classified in to Classical Optimization and Intelligent Optimization. Trial and error method or analytical methods are classical optimization techniques (Rao, 2020) on the other hand intelligent or heuristic optimization is important to solve deformation monitoring network and it uses the optimization techniques such as Genetic (Evolutionary) algorithms (GAs), particle swarm optimization (PSO) and simulated anneal (SA) (Yetki et al., 2008).

In the contest of Ethiopia finding researches on the optimization of geodetic network design is difficult, so this paper may contribute additional information on the study of geodetic network optimization of Ethiopia. The research specifically focus on zero order design problem and first order design problem which is important to find the optimal geodetic datum and based on the findings configuration of the network will also be done.

Searching for an optimal coordinate reference frame or datum is performed by ZOD optimization problem that helps to make sure that the model is not influenced by fixed-coordinate inaccuracies, to become more efficient network, in terms of both cost and time, to be stable even when external environmental and other factors are present and can deliver a standardized reference system that multiple stakeholders can use, such as surveyors, cartographers, and geospatial analysts (Schmitt, 1982).

Collected GPS coordinates are the data used to perform the geodetic network optimization process and these data is included under the boundary of Ethiopia which is done in 2020. So by using this information ZOD and FOD optimization is performed using software called MATLAB. The technique for optimization used is Trial and error method which helps to find more precise design of the network.

1.3 Objective of the study

1.3.1 General Objective

The objective of this thesis work is to develop the optimal design of geodetic network with high precision in the case of Ethiopia. The outcomes will be optimized networks that fulfil all defined quality criteria.

1.3.2 Specific objective

- Evaluate the existing ZOD problem for a geodetic network of Ethiopia.

- Solve the FOD problem for a geodetic network of Ethiopia.

1.4 Research questions

Concern with the objectives of the study, in order to see whether the researcher has achieved the objectives or not, the following questions should be answered:

- What is the optimal design of a geodetic network in the context of Ethiopia?
- How does the research evaluate the existing Zero Order Design (ZOD) problem in the geodetic network of Ethiopia and what optimization strategies are employed?
- To what extent does the research address the First Order Design (FOD) problem for a geodetic network in Ethiopia?

1.5. Scope of the Study

This study was carried out in the case of Ethiopia to design and optimization of the geodetic network based on the existing data by using processing software called MATLAB and ArcGIS. The research covers Ethiopia's whole territory, that the zero order data is located. Taking into account the various topographical and geological characteristics found in each region.

1.6. Delimitation of the Study

The study recognises certain possible drawbacks, including limited data availability, financial limits, and the need for additional research in some specialised fields. Because it is very difficult to find an optimized geodetic network design by solving zero as well as first order design problems this thesis is conducted. Some equipment’s are also taken as a limitation for this research work.

1.6 Significance of the study

To take accurate and reliable measurement and positioning in a large number of applications, such as surveying, mapping, and navigation, optimization of geodetic networks plays a big role. From the different design orders available for the optimization process zero order and first order designs are very important in defining the optimum datum and geometric designing of a given area. And optimal placement and distribution of measurement points are selected. But in case of Ethiopia it is very difficult to find a research on optimization of geodetic network that solves ZOD and FOD design problems.

This study aims to fill this gap by giving a new optimization method that relays on zero and first order design optimization and making Shure that the result found is effective for the improvement of precision and reliability of geodetic networks. So the final selected coordinates will produce the optimum geodetic network design.

1.7 Thesis Structure

This thesis is organized into six chapters and a brief description of the chapters is presented below to get a summary of the general composition of the thesis.

Chapter one is the Introduction part of the thesis that gives a general overview of the proposed topic, background information, statements of the problem, objectives, research questions, scope, significances of the study, limitations are presented.

The second chapter is about the Literatures Reviewed that helps to conduct the research and this section includes a theoretical framework and background which include the concept behind the spectral built-up indices and evaluation based on a different approach.

Research Design and Methods are briefly explained in the third chapter of the thesis that includes the description of the study area, data and software packages, image processing techniques, classification, various indices computation methods, answer the questions raised in the statement of the problem

The results and its discussion is placed the fourth chapter includes the result of the present study by using different figures as well as tables and discussion focuses on the justification of the result based on the findings of the research output are presented.

The final chapter a clear conclusions and recommendations based on the process and result, this thesis concludes and recommends for further studies.

This study was concerned with the optimization of zero order design geodetic network which solves the geodetic datum problem and the first-order design used to find a result for the configuration problem for the network. In general the optimization of these design problems are made for the geodetic network of Ethiopia.

CHAPTER TWO

2. Literature Review

2.1. Theoretical Background

Geodetic network optimization is a critical procedure in geodetic surveying that plays an important role in the improvement of accuracy and reliability of geodetic measurements. Adjusting the geodetic network to lessen the effects of errors and uncertainties in the measurement data is the optimisation process in the context of zero order and first order design. First off, the accuracy of geodetic measurements is affected by a range of uncertainties and errors, including atmospheric conditions, instrument errors, and observational errors. To maximise the geodetic network, it is imperative to consider these sources of error and reduce their impact on the measurement data. (Dermanis, 1985).

One step in the optimisation process is the development of observation and functional equations that describe the link between the computed and observed values of the geodetic data. These equations need to be solved concurrently in order to yield the best approximation of the unknown parameters of the geodetic network. In mathematics, optimisation is the process of finding a goal function's maximum or minimum given a variety of constraints (either equalities or inequalities, or both). (Kuang, 1991).

For the optimization process to solve the system of equations and update the geodetic network, sophisticated mathematical algorithms and software tools must be used. MATLAB, ArcGIS, and GNSS processing software are the most often used software tools for geodetic network optimization. Every detailed explanation is given below which is very important for the geodetic network optimization.

2.1.1. Geodetic Network

A geodetic network is a collection of connected points on the surface of the Earth that have been carefully surveyed and measured. These networks are used to locate features on the surface of the Earth, including as buildings, landmarks, and geographical features. The term "geodetic control network" refers to a systematic grouping of easily recognised points that have been survey-marked in the field and whose locations have been established in the national spatial reference system in a way that makes sense for the particular type of control network and makes it possible to determine its accuracy. (Przewięźlikowska et al., 2021).

A network is just a collection of points, or more accurately, the relative locations of a finite number of points, according to the traditional conception of networks. Almost primarily the coordinates of the network points are the finite set of unknown parameters that are related to a finite collection of observations using a basic Euclidean model that is defined in terms of elementary mathematical relations from analytic geometry. (Dermanis, 1985).

The following table shows the classification and purpose of each geodetic network:

Table 1: Classification and purpose of each geodetic network

	Geodetic networks	Purpose
1	local or micro networks	Limited and specific purpose
2	National control or Reference networks	Multi-purposes

For the majority of engineering undertakings, including mining and building, geodetic networks are essential. They are also crucial for researching natural phenomena like crustal movements. The geodetic network can be used for monitoring, implementation, establishment, and maintenance depending on its stable and identifiable locations located on or near the Earth's surface and linked to a known coordinate reference system. The geodetic network should be constructed to meet the needs of each goal, which include cost, precision, and dependability, in order to accomplish these goals. (AbdAllah & Wang, 2022).

Many ground locations are connected to one another by making geodetic observations to build a geodetic network. Angle and distance measurements are the observations made in conventional terrestrial surveying. Nonetheless, geodetic networks have benefited from fresh observations in recent decades because to satellite and laser technologies. (Alizadeh Khameneh, 2017).

A geodetic network must be designed in advance in order to be established. The design process requires a number of a priori data in order to begin. This information can include, for example, the necessary precision of the network, the network's ability to identify errors, a priori knowledge of the area's geological status if the network is being used for monitoring, and so forth, depending on the goal of the geodetic network. An observation strategy for the network together with some suggestions for measurement performance is typically the result of a design process. The recommended list may include the correct datum, the quantity of control points required, the kind of observations (such as angle, distance, and satellite positioning observables), as well as the total number of observations.(Alizadeh Khameneh, 2017).

Geodetic control networks find widespread application in the study and observation of various geographical characteristics and events. Many human activities, such as surveying, civil and environmental engineering, GIS development, geodetic and geophysical measurements, and the collection of spatial data, place them at the top of the priority list. (Bielecka et al., 2020).

Creating an ideal network in terms of accuracy, dependability, and cost is the ultimate aim of network design. Geodetic networks offer a standard and consistent spatial reference system for the location of any geographical feature. Good quality for subordinate surveys in mapping, cadastre, engineering, and many other land administration-oriented applications is ensured by an even spatial distribution of geodetic control points. (Bielecka et al., 2020).

Although geodetic networks serve a variety of functions, one of its most significant uses is to track how the Earth or man-made structures are changing over time. (Kuang, 1991).

Generally speaking, an ideal geodetic network is one that is very precise, dependable, and economically developed. The network's reliability is determined by how resilient it is to potential large mistakes, while its accuracy is determined by the predicted inaccuracies of the net point coordinates. The zero-order design (ZOD), which is the initial stage of geodetic network design, determines the optimal datum for the network based on how it impacts network precision. There are various standards for the ZOD. (Eshagh & Alizadeh-Khameneh, 2015).

2.1.2. Optimization

Optimization is maximizing or minimizing an objective function that describes the quality design criteria of a geodetic network. The optimization and design of a control geodetic network play an important role in network analysis. (Amiri-Simkooei, 2012). Mathematically, optimization means determining the maximum or minimum of a target function under a number of constraints (equalities or inequalities or both). (Kuang, 1991)

The accuracy, robustness, and cost of a geodetic network can all be used to gauge its quality. Various objective functions that represent these standards may be employed during the optimisation process. In this article, only cost and precision criteria are taken into account. Criterion matrices are excellent instruments for establishing an objective function. They stand for the intended level of accuracy for the network's outcomes. The following objective function can be utilised in the optimisation process if a criterion matrix is used. (Yetki et al., 2008):

$$||C_X - C_S|| \rightarrow \min \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

The network quality is assessed using three broad criteria: economy, precision, and reliability. A network can be designed so that the predicted accuracy of the network's elements and the derived quantities can be achieved. It can be made as sensitive as possible, allowing for the identification of movements in deformation networks and outliers in measurements, as well as the marking of points and measurement performance that satisfies certain cost requirements.(Schmitt, 1982)

In geodesy, the term "optimisation" has just recently evolved to mean network design based on precise quantitative methodologies and considerations; it implies finding the best possible solution through planning. (Kuang, 1991)

In geodetic networks, optimisation is the process of identifying the optimal values of the unknown parameters by means of a mathematical model that links the unknown parameters to the observations and a set of observations. The goal of optimisation in a geodetic network is to lessen the difference between observed and model-predicted values. Optimisation is the process of choosing the optimal element from a range of input values in order to minimise or maximise a function. (Ghorbanian et al., 2019).

An optimisation problem is one that involves determining the best answer, or optimal solution, to a given objective function while accounting for a variety of constraints. The objective function, a mathematical function, quantifies the problem's aims, while the constraints are the conditions that the solution must satisfy. (Optimization, n.d.).

Optimization variables are those related to the optimization design problem under consideration (Berné & Baselga, 2004). To design an optimal geodetic network, proposed a four-step procedure to solve the design problem. These steps cover the methodology of the design procedure (Schmiti, 1985).

A geodetic network's configuration, or the point location and types of observations, as well as the distribution of observational effort and measurement precision, must be decided upon during the design phase. Using all relevant data, both absolute and relative, one must select an ideal datum during the adjustment stage. This is true for both newly designed networks and network extensions already in place. Typically, the various optimisation issues are categorised into several orders. (Schmiti, 1985).

Applied science relies on observation, modelling, and data analysis, and even with the best-intentioned efforts, optimisation issues will inevitably arise. Generally speaking, optimisation is a methodical attempt to provide answers to questions regarding what to look for, how to model, and how to analyse data in order to learn as much as possible about a particular aspect of the physical world, depending on the particular scientific field, within the constraints of predetermined actionable options. (Dermanis, 1985).

Geodetic network optimisation is ranked in various orders. The geodetic datum problem that is, the problem of converting geodetic data into absolute Cartesian coordinates is resolved by zero-order design. Within a geodetic net, finding the ideal configuration and observational weight is intimately tied to first- and second-order design. The problem of adding new data to an existing geodetic net, known as geodetic Bayesian estimation, is treated as a third-order design. (Grafarend, 1974) . The classification scheme used herein is the following:

a. Zero-order optimal design: the geodetic datum problem
b. First-order optimal design: the configuration problem
c. Second-order optimal design: the generalized weight problem
d. Third-order optimal design: the geodetic Bayesian approach

2.1.3. Zero-Order Optimal Design

There are various ways to solve the datum problem, particularly in "free networks." A free network is one whose internal structure is solely determined by relative measurements. Examples of such measurements include bearings, angles and distances or distance relations in two dimensions, solid angles and distances in three dimensions, and gravity and height differences in one dimension. Absolute measurements can eliminate the typical degrees of freedom in such networks, which are rotations, scales, and displacements. Absolute gravity data in a one-dimensional network and Doppler observations in two- and three-dimensional networks can be used to fix the datum (Schmitt, 1982).

Finding the best design parameters for a geodetic network to meet particular performance goals while satisfying a number of constraints is known as solving optimization design problems in geodetic networks. The location and orientation of survey markers, the frequency and length of measurements, and the kind and placement of survey instruments are frequently included in the design parameters for geodetic network optimization.

Zero-Order Design (ZOD) is the first step in the four-step design process. This stage looks for the network's optimal datum. A geodetic network's datum is the bare minimum of characteristics needed to link the network's configuration and observations to a recognised coordinate system (Alizadeh Khameneh, 2017).

It is necessary to relate any measurement to a coordinate system before we can begin to solve a geodetic problem. We are unable to determine the system's theoretical origin through geometric geodetic data. Mathematically speaking, there is no unique way to invert measurements into absolute coordinates; the only way to find a unique inversion is through extra postulates or optimal criteria (Grafarend, 1974).

The search for an ideal coordinate reference frame, or datum, is more accurately described as the zero order optimum design than it is as an optimisation challenge in the sense of optimising the network itself (Dermanis, 1985).

In a network with minimum restrictions datum, the optimal station placements, directions, and distances are found to be fixed in the network when the ZOD step is carried out. The maximum precision in network adjustment can be achieved by addressing these limits (Alizadeh Khameneh, 2017).

Generally speaking, there are two distinct features of ZOD that need to be differentiated. The statistical assessment of the datum solutions, namely their estimability and testability, is the secondary issue; the geometric interpretation of the results is the first. (Schmitt, 1982).

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2.1.4. First-Order Optimal Design

Determination of the optimum geometric design for a geodetic network is one of the classical design problems in geodesy, known as the first-order design problem (FOD) (Berné & Baselga, 2004). First-order design (FOD), which is to determine a configuration of instruments provided the stochastic model for observations (Jia & Lichti, 2017).

It may initially seem "out of time" to ask what geodetic net structure would be most effective given the observation spectrum and measurement accuracy. However, there are still a lot of unanswered concerns regarding the best way to construct a satellite orbit, a tracking station network, or networks used for navigation. Naturally, most geodetic net configurations are created based on an intuitive sense of what an ideal arrangement may be (Grafarend, 1974).

When the deterministic technique is used in the First Order Design issue, the formulation is exactly the same as in the case without signals. The risk function's correct formulation is the only issue (Dermanis, 1985).

The precision of the observations $\mathbf{P}_{\hat{x}}$ is the supplied element in the First Order Design (configuration issue), and the goal is to find an ideal configuration A that maximises the resulting coordinate accuracy. The optimality requirement is the minimization of a suitable risk function, which may also include the cost of the observations, which is dependent on the design matrix A, in addition to the coordinate precision $\mathbf{P}_{\hat{x}}$ (A. Dermanis, 1985).

For high-order triangulations, when topographical realities limit any major flexibility in the location of points and in the observation plan, the logic of FOD has long been questioned. Studies on the ideal configuration of geodetic standard problems, such as intersection and resection, are instances of classical FOD. The goals were isotropy (a circle-shaped error ellipse) and the trace and determinant of the cofactor matrix of the new location. (Schmitt, 1982).

2.1.5. Second-Order Optimal Design

Second-order design (SOD), the purpose of which is to optimize the stochastic model for observations (Jia & Lichti, 2017). The second order design (SOD) is one of the most important orders in designing a geodetic network. In this design order, the optimum weights of the observations are sought. SOD is traditionally carried out to obtain a network with high precision (Amiri-Simkooei, 2012).

In the Second Order Design (weight problem) the given elements are the configuration A and an idealized coordinates accuracy $\mathbf{P_x}$ or $\mathbf{\Sigma_x}$ (Criterion matrix). The optimal observation accuracy P or Σ is to be found in a way that the resulting coordinate accuracy $\mathbf{P_{\hat{x}}}$ or$\mathbf{\Sigma_{\hat{x}}}$ is as close as possible to its idealized counterpart $\mathbf{P_x}$ or $\mathbf{\Sigma_x}$ (A. Dermanis, 1985).

2.1.6. Third-Order Optimal Design

Last but not least, by including or excluding certain observations, a geodetic network that has already been designed can be made more precise, reliable, and sensitive. Third-Order Design (THOD) is the term for this step, which is the final order in the design process. The THOD can offer a fresh recommendation for the observation plan if a network's quality requirements alter before the next measurement campaign. To meet any new requirements, the network can be expanded by adding more nodes (network densification), expanding its observations, or altering the kinds of observations. (Alizadeh Khameneh, 2017), Third Order Design (THOD) is choosing a way to improve an existing network (Amiri-Simkooei, 2012).

A fifth design difficulty, known as the Combined Design (COMD), can be added to the ones mentioned above. (Vanicek and Krakiwsky, 1986) problem, where both the First- and Second- Order Design problems have to be optimally solved simultaneously with a preassigned covariance matrix of the parameters (Kuang, 1991).

It is impossible to optimise geodetic nets without first reviewing the optimisation procedures. Generally speaking, we must keep direct approaches and simulation apart. The majority of geodetic optimisation problems must be resolved using nonlinear programming approaches. (Grafarend, 1974).

The same arguments that justify the significance of the Third Order Design problem for networks without signals also seem to justify its lack of importance for geodetic networks with signals (Grafarend, 1974). The majority of classical networks, including national horizontal and vertical networks of all orders, have already been created and observed. Increasing points and/or observations is the only way to identify improvement in terms of optimisation. (Dermanis, 1985).

Third Order Design (improvement problem) the configuration **A** and the observational accuracy **P** are partly known and their modification is sought by additional network points and/or observations so that the resulting coordinate accuracy $\mathbf{P_{\hat{x}}}$ becomes satisfactory (Dermanis, 1985).

Generally speaking, geodetic networks can be used to learn more about the geometry of a certain region of the earth's surface, and the optimisation issues that arise from this are as follows:

- What to observe (first order design, the configuration problem)
- How to observe, i.e., with what accuracy to observe (second order design, the weight problem)
- How to best analyse data in their adjustment stage (zero order design, the datum problem)
- How to improve existing information with the help of additional data (third order design, the improvement problem).

It should be mentioned that, at least in the context of optimisation theory, the issue of optimum modelling has not received the attention it merits. In actuality, all design-optimization problems are articulated using the same terminology that exist in the adjustment (design matrix, weight matrix, covariance matrix of coordinates), and they are all related to the adjustment of the pertinent observations. (Dermanis, 1985).

The above descriptive classification can now be characterized by free or fixed parameters as shown below on the table.

Denoting the configuration or design matrix as A, and the weight matrix P as the inverse of the cofactor matrix Q_1 of the observations, then the cofactor matrix Q_x of the unknown parameters x, which are mainly coordinates, can be derived as a generalized inverse of the normal equations: (Schmiti, 1985)

$(A'PA)^{-} = Q_x$ ………………………………………………………………………………………. 2

The type of the inverse depends on the datum choice, for example $(\,)^{-1}$ for a constrained or $(\,)^{+}$ for a free network. The above descriptive classification can now be characterized by fixed and free parameters:

Table 2: Free and fixed parameters in different Design Orders

Design Order	Fixed Parameters	Free Parameters
Zero order	A , P	x , Q_x
First order	P , Q_x	A
Second order	A , Q_x	P
Third order	Q_x	A,P (Partly free)
COMD	Q_x	A,P

The characterization of Q_x as a fixed parameter implies that the object of the optimization problem is formulated in the elements of or as a function of Q_x.

The search for an optimal datum or coordinate system is the main result, so in this research paper Zero Order Optimal Design (ZOD) and First Order Optimal Design (ZOD) are the orders used for Optimization of geodetic networks among the different classifications.

2.2. Geodetic Network Designing and Optimization Exercise in Ethiopia

The official body conducting mapping, surveying, and remote sensing operations in Ethiopia is the Ethiopian Mapping Authority (EMA), which was founded in 1954. The majority of the regions with great potential for natural resources are included in the first, second, and third order benchmark points that make up the current national geodetic network, which covers an area of roughly 560000 km2 (Miskas & Molnar, 2009).

Conventional triangulation traversal was used to establish the majority of the network (EMA 2009). When compared to the accuracy achievable with modern technology, points made using this method are inaccurate and date back more than 50 years. In more recent times, GPS techniques were used to measure some sections. Due to inadequate documentation, details regarding the GPS techniques utilised to establish the new benchmarks were hard to come by (Miskas & Molnar, 2009).

As part of the worldwide GNSS service (IGS), the Institute of Geophysics, Space Science, and Astronomy (IGSSA) at Addis Ababa University's Arat Kilo Campus established Ethiopia's first Continuously Operating Reference Stations (CORS) in 2007. The University of Bahir Dar's Institute of Land Administration (ILA) constructed the TANA IGS station in 2015. The establishment of the CORS network for geodetic applications throughout the nation is within the purview of the Ethiopian Geospatial Information Institute (GII). (Ayele, 2022).

2.3. Optimization problems

Optimization problems are related to the adjustment of the relevant observations and are in fact formulated with the help of the same terms which appear in the adjustment (design matrix, weight matrix, covariance matrix of coordinates) (Grafarend, 1974).

A global optimization problem can be broken down into three distinct components*: the objective function, optimization variables, and solution approach.*

2.3.1. The objective function

The precision measures and criteria of a geodetic network are based on the covariance matrix of estimated coordinates. One measure of precision takes the form of scalar function of the elements of the covariance matrix of the coordinate's varieties. The purpose is to fill the need for an overall representation of the precision of a network (Kuang, 1991).

The objective function is the one we strive to enhance. An optimisation issue is to find the values of the variables that minimise or maximise the objective function. There are various definitions for optimisation depending on its type, or the type of minimization that is being done (Berné & Baselga, 2004).

A scalar function (objective function) may be one of the following (Grafarend, 1974):

A-optimality: here is the covariance matrix's trace being minimised. Consequently, the parameter estimations' average variances are reduced.

Objective function = Min. trace (Q_{XX}) ………………………………………………… 3

Constraint: $E_L \leq E_i \leq E_U$

$N_L \leq N_i \leq N_U$

D-optimality: technique aims to reduce the determinant of the covariance matrix. Its statistical relevance lies in its ability to minimise the erroneous hyper ellipsoid's volume. Objective function = Min. det. (Q_{XX}) ………………………………………………… 4

Constraint: $E_L \leq E_i \leq E_U$

$N_L \leq N_i \leq N_U$

Where:

E_i And N_i = coordinates of points,

i = number of point,

E_L And E_U = lower and upper limit in X-direction for each point, and

N_L And N_U = lower and upper limit in Y-direction for each point.

E-optimality: the biggest eigenvalue of the covariance matrix for the parameter estimations is intended to be minimised.

f = λ max → min ……………………………………………………………….. 4

S-Optimality: (Spectral Optimisation relates to network stiffness): this assumes that the optimisation problem is approached using eigenvalues and eigenvectors, i.e., the maximum flattening of the eigenvalue spectrum.

f = λ max - λ min → min …………………………………………………….... 5

N-Optimality: Finding the number and location of measurement sites in the network that will yield the most exact and accurate parameter estimates is the goal when using the N-optimality option. N-optimality attempts to minimise the determinant of the information matrix, which is a measure of the uncertainty or variance-covariance of the parameter values.

f = $\|C_{\underline{x}}\|$ --> min ………………………………………………………………… 6

With ‖ . ‖ denoting the norm of a matrix;

All of the aforementioned optimalities that are part of the objective function or scalar function are Measures and Criteria for Precision. On the other hand, Measures and Criteria for Precision also make use of Criterion matrix-based techniques, which offer a far more thorough control over precision. A criterion matrix is a synthetic variance-covariance matrix with an ideal structure, where "ideal" denotes that the matrix represents the best accuracy scenario inside the network that is designed. In the event where an optimal design approach introduces a criterion matrix in place of a scalar risk function, the optimisation problem's solution must as nearly as feasible approximate it. (Kuang, 1991).

The datum problem for the criterion matrix is the last step in determining the Measures and Criteria for Precision. Thus, in this approach, the criterion matrix ought to be established apart from any linear model that links the desired parameters to specific observations. That isn't feasible, though, in real point fields where it's necessary to provide a network datum for coordinates. The criteria matrix needs to be modified using the same datum parameters in order to be compared with a "real" covariance matrix that has a defined datum. A suitable S-transformation can be used to achieve this transformation. (Baarda, 1981).

S = I - H $(D^T H)^{-1} D^T$ …………………………………………………………………. 7

$C_{Trans.}$= S C S^T ……………………………………………………………………..... 8

Where C and $C_{Trans.}$ are the criterion matrices before and after transformation respectively; I is the identity matrix; H is a matrix satisfying A H = 0 with A the configuration matrix of the network and D characterizes the datum of the network with datum equation D_X^T = 0 and rank (D) = datum defects of the network.

Nonetheless, the inverse of the normal equation matrix of the deformation parameters can be fitted directly if a criterion matrix for datum-independent deformation parameters, such as relative block motions, strain parameters, and their derivative, is defined in the optimisation process. (Kuang, 1991).

One should remember that although deformation strictly means change of shape and dimension, detection of scale changes, rotations and displacements must also be included (Kuang, 1991).

2.3.2. Optimization variables

Optimization variables are those related to the optimization design problem under consideration (Berné & Baselga, 2004).

1. In the ZOD, the variables are the datum points, i.e. the coordinates that are to be fixed in the network.
2. In the FOD, the A-matrix of the variable as it represents the geometry of the network.
3. The SOD defines the P-matrix of observation weights as its variable.
4. For the TOD the variables are the A-matrix of observations and the P-matrix of their corresponding weights.

5. Combined Design (COMD) (Vanicek and Krakiwsky, 1986) problem, where both the First and Second- Order Design problems have to be optimally solved simultaneously with a preassigned covariance matrix of the parameters (Kuang, 1991).

2.3.3. Solution method

After defining the variables and the function by which they are related, the question remains how to achieve the solution. This paper focuses on the ZOD, so the variables are the datum points. As a result the to find the optimal solution it is very important to select which technique is used, to find that we must have a brief explanation for each Optimization Solution techniques.

It is impossible to optimise geodetic nets without first reviewing the optimisation procedures. Generally speaking, we must keep direct approaches and simulation apart. The majority of geodetic optimisation problems must be solved using nonlinear programming approaches. We are unable to provide a thorough analysis of this crucial subject for realistic optimisation processes in this short study. (Grafarend, 1974).

The first step in determining the best solution would be to formalise the problem. After the problem is established, we must determine the best solution, which is nothing more than making a choice from the viable region. Furthermore, the viable zone has an endless number of points, which makes it difficult to choose the best solution; a number of requirements must be met. Then, only we can announce the choice we are making that is best for ourselves. (Chakraborty & Kharagpur, 2014.).

For this reason, we must examine the sufficient and necessary requirements for the optimality of non-linear programming problems. Now that we have the required and sufficient criteria once more, we will search for two different types of non-linear programming issues in addition to mathematical programming issues. There are two types of optimisation problems: confined optimisation and unconstrained optimisation. (Chakraborty & Kharagpur, 2014.).

Unconstrained optimization problem: is something that only the objective function will be involved with. Furthermore, the system can be freely moved in the current scenario without any limitations. We are dealing with two different types of unconstrained optimisation problems in that scenario. There are two types of optimisation problems: one is the single variable optimisation problem and the other is the multivariable optimisation problem without any constraints. Because of this, it is unrestricted. (Chakraborty & Kharagpur, 2014.).

Constrained optimization problem: We'll handle two different types of situations. The multivariable function containing both the equality sign and equality in it is the first. And yet another with a multivariate function that is constrained by both inequality and equality. Since this context refers to a more broad form, we shall search for both necessary and sufficient conditions to find the optimal solution for both limited and unconstrained optimisation issues. (Chakraborty & Kharagpur, 2014.).

2.4. The Optimization Solution Techniques

When monitoring schemes are optimised, an objective function that reflects the standards used to determine the "quality of the scheme" is minimised or maximised. There are four broad standards by which this quality is judged: accuracy, dependability, tact, and economy (Kuang, 1991). The challenge of finding the maximum or minimum value of a function may often be solved using a variety of approaches (Doma, 2013).

A network can be optimised classically by trial and error or analytical techniques like linear or quadratic programming. In certain cases, generalised or iterative generalised inverses can be used to address optimisation problems. Intelligent optimisation methods like particle swarm optimisation, simulated annealing, and genetic algorithms can also be used to tackle optimisation problems (PSO) (Yetki et al., 2008).

Heuristic optimization this are the same as *intelligent optimization* which is important to solve deformation monitoring network and it uses the optimization techniques such as Evolutionary algorithms (EAs), particle swarm optimization (PSO) and simulated annealing.

The various Optimisation Solution approaches utilised for the geodetic network adjustment are depicted in the image below, along with a brief explanation of each. Since the focus of this work is on optimising zero order design (ZOD), it is crucial to choose the approach listed for ZOD (Jia & Lichti, 2017).

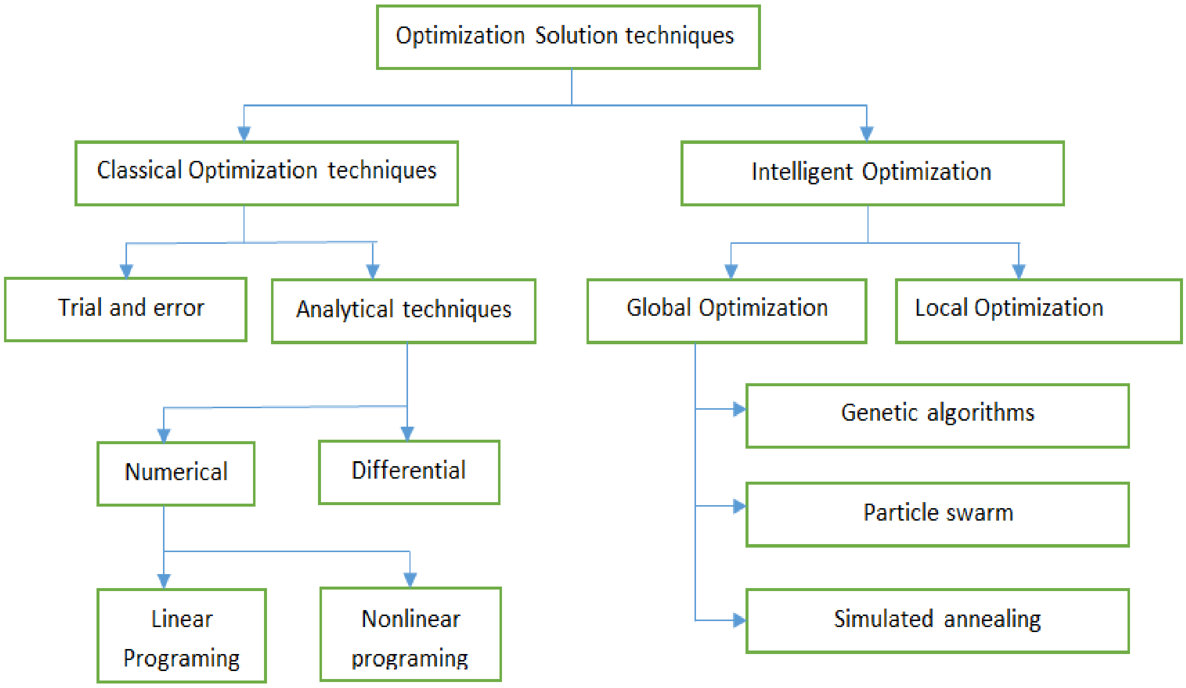

Figure 1: Optimization Solution Techniques

2.4.1. Classical Optimization techniques

The best solution for continuous and differentiable functions can be found using traditional optimisation techniques. These analytical approaches locate the optimum positions by applying differential calculus techniques. The practical uses of classical optimisation techniques are constrained due to the presence of objective functions that are not continuous or differentiable in certain real scenarios. Nonetheless, the majority of the numerical optimisation approaches that are covered in the following chapters are developed through an examination of the calculus methods of optimisation (Rao, 2020).

Three main types of problems can be handled by the classical optimization techniques (Rao,2020.) as listed below:

- single variable functions
- multivariable functions with no constraints,
- multivariable functions with both equality and inequality cons traints.

These Classical Optimization techniques can be grouped into two major types: trial and error techniques and analytical techniques and each of them are explain as follows;

Trial and error techniques

A method for adjusting geodetic networks that involves selecting initial values for unknown parameters at random and then adjusting the network to achieve the desired results is called trial and error optimisation. The beginning values of the unknown parameters are adjusted, and the modification is repeated, if the results are not satisfactory. This process is repeated until the intended results are obtained.

Trial and error optimisation is a technique used in geodetic network modification to increase a geodetic network's accuracy and improve estimates of unknown parameters (such station coordinates or instrument constants). In this method, the observed and computed values of the network quantities are brought as close to each other as possible by varying the initial values of the unknown parameters through trial and error.

The trial-and-error method involves making small changes to the estimated values of the unknown parameters while keeping an eye on how those changes impact the network quantities' computed values. If an adjustment improves the agreement between the computed and observed values, it is accepted and added to the estimations of the unknown parameters. Should the alterations.

The design challenge can be solved by computer simulation, also known as "trial and error," which computes the design and cost criteria. If one of these requirements is not met, a new solution is proposed (often by making minor changes to the original postulate) and the requirements are recalculated. The process is iterated until a network is found that is satisfactory, though probably not optimal. The stages below demonstrate how the optimisation process can be summed up as the trial and error technique. (Cross, 1985).

a) Specify precision and reliability criteria (e.g. ellipse of error).

b) Select an observation scheme (stations, observations and their precision (weights)).

c) Compute the covariance matrices of the desired least squares estimates and derive the values of the quantities specified as precision and reliability criteria.

d) If the computed criteria are close to those specified in (a), then go to the next stage; otherwise alter the observation scheme (by adding the observations or increasing the weight if they are not satisfied, or by removing the observations or decreasing the weights if they are too optimistic) and return to (c).

e) Compute the cost of the network and restart from (b) with completely different scheme (e.g. Trilateration instead of triangulation). Stop when it is believed that the optimum (minimum cost) network has been found.

Stated differently, the trial and error method involves computing the objective function using a proposed solution to the problem. If the suggested solution fails to meet the objective function, the answer is modified and the objective function is recalculated. Until the criterion is met, this process is repeated. The designer's experience determines the solution in the trial-and-error process. Though the solution may not always be found, in some circumstances it may be interesting due to its simplicity and lack of complex mathematical models (Alwan & Msaewe, 2012),

Recent research into the simulation method concentrates on the following(Cross, 1985) :

(i) Increasing the computational efficiency of the process, e.g. by using sequential least squares as in Baran(1982), Mepham(1983), and Tang(1990);

(ii) The establishment of general rules to help designers decide quickly on suitable networks to select in stage(ii) of the simulation process;

(iii) The use of interactive graphics;

(iv) The automation of the alternative process (stage (iv) above) so that the computer rather than the designer chooses which observations to add or remove.

The benefit of the simulation approach is that any set of choice criteria can be applied and contrasted to determine the necessary design. It is not necessary to transform these requirements into a robust mathematical formulation, which is essential when utilising solely analytical solutions involving discrete risk functions. The method's obvious drawbacks are that it might never find the ideal network and that it could require a significant amount of labour. (Kuang, 1991).

Analytical techniques

When a specific design challenge is resolved using a special set of mathematical methods, the approach is referred to as analytical design. Unlike simulation techniques, it results in a process that doesn't need human involvement (albeit it might not be straightforward; it might include iteration) (Cross, 1985).

certain algorithms are provided by analytical methods to address certain design issues. When such an algorithm is activated, it will automatically generate a network that meets the user quality requirements and is, mathematically speaking, optimal (Kuang, 1991).

The analytical approach gives some advantages rather than other existing methods for network optimization, as follows (Alwan & Msaewe, 2012) :

- Any type of geodetic observable can be considered.
- Any condition or constraint can be considered.
- All the criteria of precision, reliability and cost can be considered simultaneously in the optimal design.
- The optimization procedure can be performed in the sense of FOD and SOD separately or simultaneously.
- This methodology can be used for the optimal design of one, two, or three dimensional networks.

The main *disadvantage* of the analytical method is: the proper formulation of the mathematical model can be difficult (Alwan & Msaewe, 2012).

Numerical methods are used to solve equations and approximate mathematical functions. These methods usually include breaking down a problem into smaller, more manageable steps and using algorithms to do calculations at each level.

Differential method is employed, conversely, to solve an equation or assess a mathematical function. To find out more about the behaviour or features of the function or equation, these procedures typically entail taking its derivatives.

When it is difficult or impossible to find an analytical solution to a problem, numerical methods are generally more helpful. Additionally, we find two classes in this method: non-linear programming and linear programming, which will be discussed next.

Linear Programing

A field of mathematics known as "linear programming" is concerned with solving puzzles involving the optimisation of linear functions under sets of linear restrictions. Appendix A provides a broad description of the linear programming issue and an introduction to the simplex method, a way of solving it. (Cross, 1985).

Its focus is on optimising a linear function (minimization or maximisation) under a set of linear equality and/or inequality constraints or limitations. The "simplex method" of linear programming is used to solve linear programmes. It is widely accepted since it can represent significant and intricate management decision issues and may generate answers in a fair amount of time. We will discuss the simplex approach and its variations in later chapters of this book, with a focus on comprehending the methods. (Bazaraa, Jarvis & Sherali,2010).

Linear programs are constrained optimization models that satisfy three requirements. We can get more detailed information on, (Spivey, 1962)

1. The decision variables must be continuous; they can take on any value within some restricted range.
2. The objective function must be a linear function.
3. The left-hand sides of the constraints must be linear functions.

Thus, linear programs are written in the following form:

Maximize or Minimize $$Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

$Subject\ to$ $$a_{11}x_1 + a_{12}x_1 + \dots + a_{1n}x_n \begin{matrix} \le \\ = \\ \ge \end{matrix} b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \begin{matrix} \le \\ = \\ \ge \end{matrix} b_2$$

$$\begin{matrix} . \\ . \\ . \end{matrix}$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \begin{matrix} \le \\ = \\ \ge \end{matrix} b_m$$

Where the x_j values are decision variables and c_j, a_{ij}, and $b_i$values are constants, called **parameters** or **coefficients**, which are given or specified by the problem assumptions. Most linear programs require that all decision variables be nonnegative.

Non-Linear Programing

When the constraint or objective functions are non-linear, non-linear programming techniques are applied. (Cross, 1985). Thus, in maximization form, the general nonlinear program is stated as:

Maximize f$(x_1, x_2, \ldots\ldots, x_n,)$

Subject to: $g_1(x_1, x_2, \ldots\ldots, x_n,) \leq b_1,$

. .
. .
. .

$g_m(x_1, x_2, \ldots\ldots, x_n,) \leq b_m,$

where the constraint functions g1 through gm are listed one after the other. The previously discussed linear programme is a specific case.

The classical optimisation technique includes the trial-and-error, computer simulation, and analytical methods that were briefly discussed above. The various intelligent optimisation techniques that assist in optimising a geodetic network are then explained.

2.4.2. Intelligent Optimization techniques

Intelligent optimization techniques are a group of computational methods that uses heuristic algorithms to find the best answer for the optimal solution to a problem. These methods can be used to increase the precision and effectiveness of the geodetic network adjustment procedure.

In a manageable amount of time (a few hours), near-optimal solutions are found using heuristic techniques. They are employed to address large-scale issues that are impractical to resolve swiftly and optimally. A strong heuristic may yield the ideal solution or one that is quite similar to it, but it does not ensure convergence to the global optimal solution. This method is predicated on progressively enhancing an already-existing solution until the consumer is happy with the level of quality attained. The issue with this strategy is that, more often than not, a local optimum rather than a global optimum is reached. Using global optimisation techniques becomes essential to get the global optimum (Dare & Saleh, 2000).

Both local and global optimisation strategies are examples of intelligent optimisation strategies. Genetic algorithms and the Particle Swarm Optimisation (PSO) algorithm are two kinds of global optimisation techniques that are starting to be applied in geodetic science. (Doma, 2013). And simulated Annealing (SA) is one of the simplest and best-known intelligent optimization method (Delahaye et al., 2019).

Particle Swarm Optimization (PSO)

Originally designed to mimic the social behaviour of bird flocks, the PSO approach was later simplified to realise that the agents, here usually referred to as particles, were actually carrying out black-box optimisation. The population of particles in PSO is commonly referred to as a swarm. Particles are initially positioned at random points in the search space and move in randomly defined paths when using the PSO method. A particle's path is then gradually altered such that it begins to go in the direction of its best past locations as well as those of its peers. It searches the area around these positions in the hopes of finding even better ones (Doma, 2013).

Similar to a genetic method, particle swarm optimisation starts the system with a population of random solutions. However, it differs from a genetic algorithm in that instead of just assigning a random velocity to each possible answer, the potential solutions—referred to as particles—are subsequently "flown" across hyperspace (Eberhart & Kennedy, 1995).

Every particle is aware of its coordinates in hyperspace, which correspond to the best fit (or solution) it has found thus far. (That fitness's value is likewise stored.) We refer to this value as p-best. A second "best" value is monitored as well. The particle swarm optimizer's "global" version, known as g-best, maintains track of the total best value and its location that have been reached by every particle in the population thus far (Eberhart & Kennedy, 1995).

PSO is a population-based, stochastic optimisation technique used to address global optimisation issues. Particle swarm optimisation aims to change the velocity of each particle at each time increment in order to accelerate it towards its p-best and g-best. worldwide edition). A random term is used to weight the acceleration, and different random numbers are produced for acceleration towards the p-best and g-best (Eberhart & Kennedy, 1995).

The inputs for a PSO optimisation in that context are similar to those for a standard PSO optimisation, with a few additional variables specific to geodetic network modification. The inputs for a PSO as a whole, and specifically for geodetic network adjustment, are the objective function, swarm size, initial conditions, inertia weight, acceleration coefficients, and termination criteria.

The following strategies are basic for the implementation of the Particle swarm optimization algorithm (Yetki et al., 2008),

1. Initialization

J = 0

a. Determine the objective function, optimization variables and constraints
b. Select PSO parameters such as inertia weight, constriction factor and social and cognitive parameters
c. Select neighbourhood topology
d. Randomly generate initial particle positions x_i^0 in D-dimensional search space
e. Set initial particle velocities to zero; $v_i^0 = 0$
f. Set j=1

2. Optimize
   (a) Evaluate objective function value f_i^j using particle positions x_i^j
   (b) If $f_i^j \leq f_i^{best}$ then $f_i^{best} = f_i^j$ and $p_i = x_i^j$ f_i^{best} is the particle's personal best cost.
   (c) If $f_i^j \leq f_{global}^{best}$ then $f_{global}^{best} = f_i^j$ and $p_g = x_i^j$ f_{global}^{best} is the best cost of whole swarm.
   (d) If stopping criterion is satisfied then go to 3
   (e) Update all particle velocities v_i^j by Eq. $\|C_X - C_S\| \rightarrow$ min
   (f) Update all particle positions x_i^j by Eq. $c_x = \sigma_0^2 (A^T P A)^{-1}$
   (g) Increase j
   (h) Go to 2(a)

   Stop

Genetic algorithms Optimization (GA)

A few significant points are brought up by the Genetic Algorithm. It is first and foremost a stochastic algorithm; in genetic algorithms, unpredictability plays a crucial role. Random processes are necessary for both reproduction and selection. The fact that genetic algorithms constantly take into account a population of solutions is a second crucial point. There are many benefits to remembering multiple solutions for every iteration. The algorithm can take advantage of variety by recombining various solutions to get superior ones. Parallelization of a population base algorithm is also quite feasible. It is also important to note that the algorithm's robustness is necessary for its success. The capacity to continuously perform effectively on a wide variety of problem types is referred to as robustness. GAs can be used to tackle any problem because they don't require any specific requirements on the problem before they can be implemented. With all those capabilities, GA is an extremely effective optimisation tool (Sivanandam & Deepa, 2008).

GAs are efficient search techniques covering a vast and expansive area. It has to do with achieving the best outcome imaginable, which is probably unachievable in one's lifetime. GAs differ significantly from earlier iterations of optimisation techniques. The design space of these algorithms needs to be modified to the genetic representation (Goldberg,1989).

GAs therefore handle a collection of encoded variables. One benefit of utilising encoded variables is that continuous functions can be encoded similarly to discrete functions. Random processing, or more precisely guided random processes, is the foundation of GAs. Consequently, a comparison analysis is conducted on random operators of the searching space. Basically, the following three crucial ideas need to be clarified in order to use GAs (Doma & Sedeek, 2014) :

a. Objective function: In order to solve the problem, mathematical relations and appropriate weights are used. The goal of every problem is to maximise or minimise a parameter or parameters.
b. Searching space: Solving an issue is about identifying the optimal solution out of several options. Searching space is the set of all possible states. Every outcome could be represented by a propriety-determining value.
c. Operators of GAs: After the objective function is met and the population has been encoded, the GA will begin to work. The three primary operators in simple GAs are typically reproduction, merging, and mutation operators. Deformation monitoring networks can be optimised by genetic algorithms.

The fundamental idea of GA is to generate a new population after the initial population. Darwinian evolutionary theory is used to produce a new population using the three genetic operators mentioned below. (Sivanandam & Deepa, 2008) :

- Selection: Given their goal functions, choose two chromosomes at random from a population;

- Crossover: To form two children chromosomes, elements of two parent chromosomes are crossed over in accordance with a specific rule;

- Mutation: With a mutation probability, components inside every given chromosome can undergo a mutation.

The process of creating a new population will then continue until the stop requirement is met and the chromosome with the lowest objective function is regarded as the optimal one.

Simulated Annealing Optimization (SA)

One optimisation technique that has been applied in several domains, including geodetic network correction, is called Simulated Annealing (SA). By simulating the annealing process in metallurgy, this metaheuristic optimisation technique finds the global minimum of a complex objective function. By minimising the residuals between the computed values and the observations, SA can be utilised in geodetic network adjustment to optimise the coordinate estimations of the points in a geodetic network.

This process involves bringing a solid to a low energy state after raising its temperature. The following two steps summarize it (Delahaye et al., 2019) :

- Bring the solid to a very high temperature until ”melting” of the structure;

- Cooling the solid according to a very particular temperature decreasing scheme in order to reach a solid state of minimum energy.

The dispersion of the particles is random in the liquid phase. It is demonstrated that if the initial temperature is high enough and the cooling period is long enough, the minimum-energy state is obtained. If not, the solid will be discovered in a metastable state with non-minimal energy; this process, known as hardening, entails a solid's abrupt cooling. (Delahaye et al., 2019).

One of the best methods for handling large-scale optimisation problems is simulated annealing, particularly when a global extremum needs to be identified amid numerous inferior local extrema (Berné & Baselga, 2004).

This method can be compared to thermodynamics in essence. A body's particles travel freely over a comparatively large range at high temperatures. The particles' mobility reduces with cooling; they can still move, but only inside constrained areas. When the cooling plan is slow enough, the particles can organise into lower energy states, leading to the eventual formation of a crystalline solid, which is the lowest energy state, as the temperature gets closer to zero. The metals industry is familiar with the annealing process, which involves gradually cooling a liquid until it solidifies into a crystalline form. The liquid can organise itself into a crystalline pattern (the global optimal state) if the cooling plan is slow enough. Should the temperature drop faster than necessary, the liquid will eventually reach an amorphous form with a somewhat higher energy (Berné & Baselga, 2004).

One of the key components of a simulated annealing algorithm would undoubtedly be the cooling plan, which includes the starting temperature, the rate of reduction, and the completion point of the plan (Berné & Baselga, 2004).

2.5 Summary of reviewed literature

This literature review gives a brief explanation and information about what geodetic networks and optimization of geodetic network mean. It is summarized as, geodetic network is a collection of connected points on the surface of the Earth that have been carefully surveyed and measured. On the other hand optimization refers to us maximizing or minimizing an objective function that describes the quality design criteria of a geodetic network. And optimization of geodetic networks designs are classified in to zero order design (ZOD), first order design (FOD), second order design (SOD), third order design (TOD) last not but list Combined Design (COMD), from these all designs our focus is on (ZOD).

The definition of optimisation issues pertaining to the adjustment of pertinent observations and which are actually stated with the aid of the same words appearing in the adjustment (design matrix, weight matrix, covariance matrix of coordinates) is necessary in order to obtain an optimal geodetic network (Grafarend, 1974) and they are classified in to into three distinct components*: the objective function, optimization variables, and solution approach.*

objective function can also be called scalar function are the precision measures and criteria of a geodetic network and there are different optimality's which are A-optimality, D-optimality E-optimality, S-optimality and N-optimality. Selection depends the purpose the geodetic network that is going to be optimized. In this paper precision is the main goal for ZOD optimization, so D-optimality and A-optimality are selected.

To get the optimal result there are different optimization techniques, which are classified in to two types. The first one is Classical Optimization technique under that trial and error as well as Analytical methods are included. The second one is Intelligent Optimization they are classified in to global and local optimization. Under global optimization there are three Heuristic techniques which are Genetic algorithms, Particle Swarm Optimization (PSO) and Simulated Annealing (SA).

The best optimization technique for geodetic network optimization depends on the specific problem and its constraints. If the objective function and constraints can be represented as linear equations, then linear optimization techniques such as the simplex method, interior-point method, or dual simplex method can be used. If the objective function and constraints are non-linear, then non-linear optimization techniques such as gradient descent, conjugate gradient, Newton's method, or quasi-Newton methods can be used.

For complex geodetic network optimization problems, meta-heuristic optimization techniques such as simulated annealing, genetic algorithms, or particle swarm optimization may be more appropriate. It is important to note that the choice of optimization technique will also depend on the computational resources available, the size of the problem, and the desired accuracy and convergence time. It may be necessary to try several different optimization techniques to determine the best approach for a specific problem. In this case the optimization solution technique that we will use is the trial and error for the ZOD and Analytical method to solve the FOD because of the size of the problem, the desired accuracy and convergence time.

CHAPTER THREE

3. Research methods, material and procedures

3.1 Description of the Study Area

Situated in the Horn of Africa, the Federal Democratic Republic of Ethiopia shares borders with Eritrea to the north, Djibouti to the northeast, Somalia to the east, Kenya to the south, South Sudan to the west, and Sudan to the northwest. This country has a very diverse topography, with several distinct regions distinguished by their topography and climate. The highlands, which comprise much of central and northern Ethiopia, are characterized by rugged mountains and plateaus, in contrast to the lowlands, which make up the majority of the east and southeast. The lowlands are also often flat and arid.

Geographically, the Federal Democratic Republic of Ethiopia is situated between 3° and 15° N latitude and 33° and 48° E longitude. With a land size of more than 1.1 million square kilometres, Ethiopia ranks 27th among all nations in the globe (426,372 square miles). Ethiopia's capital, Addis Ababa, is located approximately at latitude 9.01° N and longitude 38.76° E. Ethiopia's terrain is diverse, with plains, plateaus, and highlands. The highest mountain in the country is Rasdashen, which is located in the Simien Mountains and rises 4,550 metres (14,928 feet) above sea level. Ethiopia is the source of many rivers, one of which being the Blue Nile, which rises in the country's highlands and flows north to Egypt.

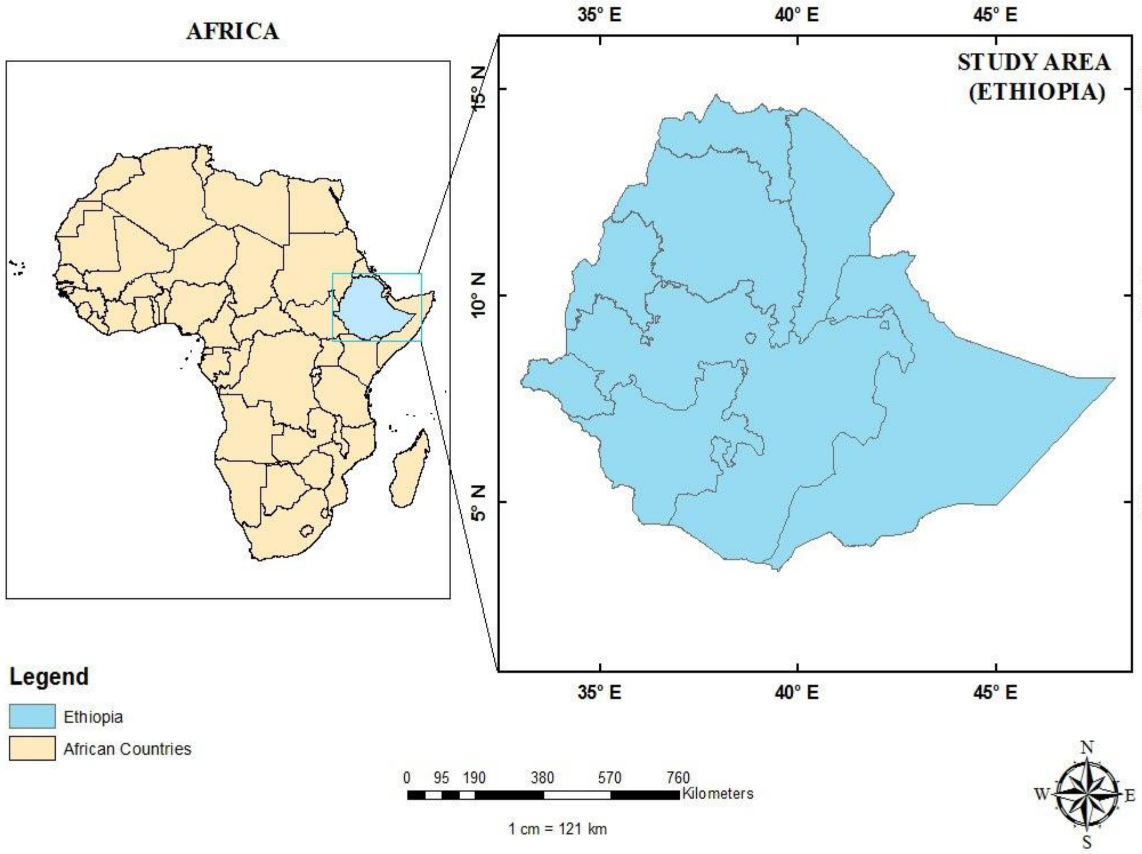

Figure 2: Study Area

3.2. Data

The exercise of obtaining information or data from multiple sources so that it can be examined and used to guide research or decision-making processes is known as data collection. Data collecting is a vital stage in establishing and enhancing the accuracy and dependability of the network in the context of design and optimization of geodetic network. Secondary data of coordinates are used for this research paper.

3.2.1. Data Collection

The process of acquiring authentic data directly from its source, usually through direct observation, surveying, or testing, is referred to as primary data collection. Primary data collection for geodetic network optimization may involve doing field surveys to set up new control points or increase the precision of current ones. As a result for this research paper a secondary data is use that is collected by modern technologies called global positioning system GPS which helps to acquire coordinates.

Data from previous surveys, including control point coordinates, benchmarks, and any existing network layout, are used to inform the current optimization process and avoid duplication of effort.

A data that have been already collected by other sources is used in this type of data collection. Such data comprise from government agencies, academic institutions, or commercial providers. Using pre-existing maps, satellite imagery, or other geospatial data to enhance or confirm primary data collecting efforts is referred to as secondary data collection in geodetic network optimization. In general from all the Data collected the next Table clarifies sources of data with their format.

Table 3: List of data used and their original sources

Data type	Source
Shapefiles	CSA
GPS data	Addis Ababa University

To solve ZOD and FOD problems a MATLAB language need to be prepared, to do so I used the initial coordinates of the network points. Latitudes and Longitudes are the coordinates for the point used as the initial data this is because the zones are different from one point to another so it may affect the result to use easting and northing. This initial coordinates are shown in the next table below.

Table 4: The initial Coordinates of the network points

P	PID	City	Easting	Northing	Zone	Lat	Long
1	ETTP	TEPI	767273.976,	795067.212	36	7.186454	35.420113
2	ETBR	BURE	730233.523,	895214.222	36	8.093397	35.089279
3	ETQR	QUARA	813875.526,	1351455.71	36	12.210077	35.884456
4	ETNJ	NEJO	773004.538,	1058494.558	36	9.566731	35.487051
5	ETMN	MANKUSH	748083.362,	1244989.092	36	11.253675	35.272325
6	ETAB	ATSEBI	578217.649,	1532892.696	37	13.864803	39.723861
7	ETNM	NEFAS MEWECHA	440667.290,	1294490.174	37	11.709517	38.455542
8	ETAM	AMBO	367823.000,	992808.000	37	8.979552	37.797603
9	ETHR	HUMERA	277009.443,	1558359.203	37	14.087294	36.934711
10	ETKM	KIBRE NENIGIST	496125.360,	649241.234	37	5.873682	38.964997
11	ETKC	KOMBOLCHA	576451.000,	1224247.44	37	11.073964	39.699979
12	ETRB	BALE ROBE	611445.472,	784349.34	37	7.094795	40.009158
13	ETNL	NEGELE	560854.757,	579811.639	37	5.24535	39.549162
14	ETZW	ZEWAY	466626.746,	875227.973	37	7.917848	38.697217
15	ETMS	M/SELAM	471447.643,	1185531.507	37	10.724517	38.738877
16	ETSR	SHIRE	422001.009,	1557101.428	37	14.083687	38.27748
R17	ETKS	KONSO	325008.902,	588727.817	37	5.324227	37.420828
18	ETCN	CHAGNI	224763.934,	1212253.494	37	10.955901	36.481666
19	ETHS	HAWASSA	459381.914,	778870.011	37	7.046181	38.632219
20	ETDK	DEBARK	378595.944,	1450714.344	37	13.120351	37.879986
21	ETFC	FICHE	469776.476,	1077884.142	37	9.750905	38.724435
22	ETGR	GINIR	686549.314,	786779.874	37	7.114796	40.689155
23	ETJK	JINKA	227770.617,	637360.584	37	5.760901	36.541952
24	ETMW	MAYCHEW	556108.413,	1411335.609	37	12.766189	39.516922
25	ETDM	D/MARKOS	360022.334,	1139636.772	37	10.307014	37.721661
26	ETMI	MEISSO	691011.833,	1018636.847	37	9.210936	40.738594
27	ETNT	NEKEMTE	230464.627,	1002616.731	37	9.062011	36.548058
28	ETML	MOYALE	503177.648,	390429.766	37	3.532295	39.028611
29	ETJJ	JIJIGA	256583.354,	1033983.652	38	9.347026	42.783764
30	ETHL	HARGELE	184517.41,	576228.902	38	5.206744	42.154328

The coordinate points that are listed above on the table have there own point number, point ID, PID, the City were they find, Easting as well as Northing with the converted Latitued and Longitued finaly the Zone. These coordinate points are located in Ethiopia as the figure below shows.

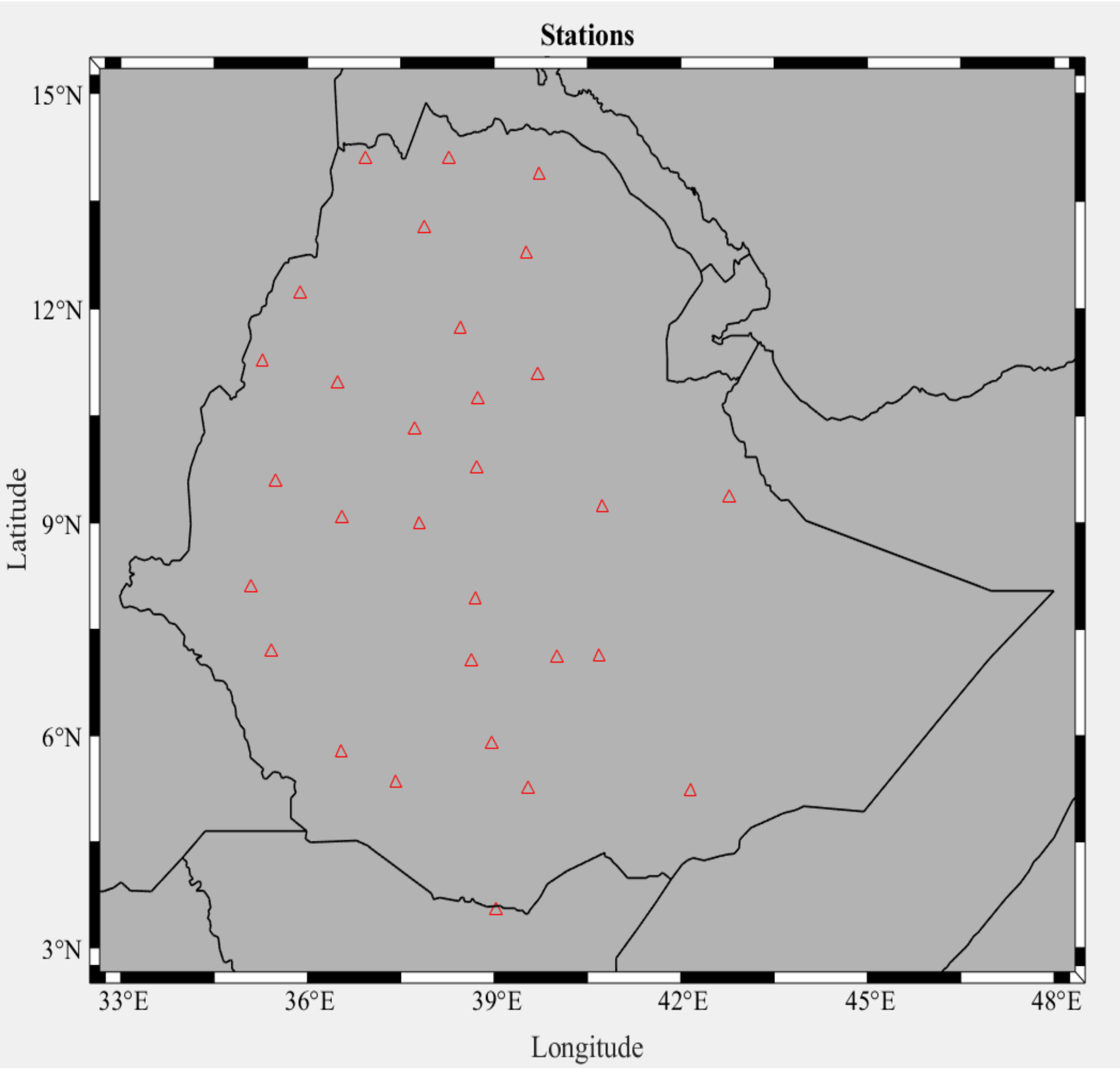

Figure 3: Observed Station points

3.2.2. Software used in the Analysis

In the optimization procedure of the zero order design of a geodetic network and adjustment process there are different software used. Esri created the geographic information system (GIS) program ArcGIS 10.2. Users may produce, modify, analyze, and display data in a variety of formats and projections with this robust application for working with geospatial data. Some of ArcGIS's most important attributes are mapping and visualization, geospatial analysis and data management. And all the above mentioned attributes are done by ArcGIS 10.2 software.

The computer language and numerical computing environment known as MATLAB (short for "matrix laboratory") is popular in many disciplines, including engineering, science, and economics. Users use MATLAB may create algorithms and applications, view data, and execute intricate computations. In general some of the key features of MATLAB include matrix operations, programming environment, libraries and toolboxes, visualization and integration with other tools. For zero order design and network adjustment process MATLAB Software in used. Data analysis and generation of Graphs, charts and indices are made by Microsoft Word and Microsoft Excel. In the table below all the software and there purposes are listed.

Table 5: Software’s used in the optimization process

Software	Purpose
ArcGIS 10.2	Image analysis, to generate different thematic maps and Accuracy assessment
MATLAB	For the geodetic network optimization that includes the production of the network and error ellipse plotting.
Microsoft Word and Microsoft Excel	Data analysis and generation of charts and indices. And some simple calculation

3.3. Methods

3.3.1. Zero Order Design problem

A geodetic network is an assemblage of interconnected, meticulously studied, and measured points on Earth's surface. The process of optimisation involves either maximising or minimising an objective function that characterises a geodetic network's quality design requirements. In network analysis, control geodetic network design and optimisation are crucial. ZOD is essential to building a reliable and accurate reference system. Every measurement and calculation that comes after it is based on the datum. (Amiri-Simkooei, 2012).

The main goal of this thesis is to find an optimized zero order design of a geodetic network and identifying the best network from the given data. And ZOD gives the solution of the datum problem, especially in 'free networks', is possible by different approaches. To get the aimed final result there are different steps in the method section as explained the next segment below.

A concise summary of the trial-and-error method is that the optimisation process begins with the specification of precision and reliability criteria (such as the ellipse of error). Next, an observation scheme consisting of stations, observations, and their precision (weights) is chosen. Finally, the covariance matrices of the desired least squares estimates are computed, and the values of the quantities specified as precision and reliability criteria are derived. Proceed to the next stage if the computed criteria closely match the first step's specifications; if not, modify the observation scheme (adding observations or raising weights if necessary, or removing observations or lowering weights if overly optimistic) and go back to (the treed step). When it's thought that the ideal (lowest cost) network has been identified, stop finally (Cross, 1985).

To begin the optimization process of the zero order design in the geodetic network adjustment approximate coordinates of points are taken as the data inputs. And by using this data the optimization procedure is started.

Formulate of Observation Equations

The observation equations are used to estimate the unknown parameters of the geodetic network based on the observed data. They provide the mathematical relationships between the observed data and the unknown parameters. Formulating observation equations for the optimisation of geodetic networks is a crucial step in the data processing and adjustment stage. Typically, the observation equations connect the seen data to the unknown parameters in the form of linear or nonlinear equations. These equations can be created based on the exact measurement techniques used in the geodetic network, such as GPS or levelling (Leick et al., 2015).

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The equation is often written using matrix symbols as

$$\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 10$$

Where **b** expresses a linear relationship between the residual observations (i.e., observed minus computed observations) and the unknown correction to the parameters **x**. The column matrix **v** contains all the noise terms, which are also unknown at this point. **A** represents the design matrix that relates the measured coordinates to the unknown coordinates. The above matrix equation the "linearized observation equations".

In cases such as optimization-based geodetic network design, observation equations can be used to define an optimisation problem where the objective is to minimise or maximise a criterion (e.g., A-optimality, least squares) by adjusting the unknown parameters. Formulating the observation equation is crucial to the process of network adjustment and optimisation. (Blewitt, 1997).

Generally speaking, observation equations offer the mathematical framework needed to comprehend and examine the connections between parameters and data. They are crucial for guaranteeing measurement quality, resolving geodetic issues, streamlining network architectures, and serving as a foundation for further research and decision-making (Blewitt, 1997).

Formulate of Normal Equations

Another crucial phase in the geodetic network optimisation process is the formulation of normal equations. Let's look at $\hat{X}$, the linearized observation equations' solution. The discrepancy between the actual observations and the newly estimated model for the observations is known as the "estimated residuals" (Blewitt, 1997).

The solution to the normal equations

$$\widehat{X} = (A^{T}A)^{-1}A^{T}b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 11$$

Formulation of constraints Matrix to solve deficient of rank defect

The least amount of constraints required to raise the system to full rank is equal to the rank deficiency. The rank deficiency is four for a horizontal network that only has angle observations. The rank deficiency is three for a horizontal network that has at least one distance observation. Since the existence of the required control points would define the datum, the rank inadequacies are known as datum defects. Of fact, there could be further reasons for rank deficiency; these are known as configuration faults and involve different issues entirely, like not having enough observations to identify a point. provides just enough constraint equations in one to meet the datum defect; these are referred to as minimal constraints (Alwan & Msaewe, 2012).

Solution of normal equation By Generalized Inverse (Helmert Method)

The Helmert Method is a mathematical technique used in geodetic network modification. It is used to adjust the coordinates of a network of points to match the coordinate system of a reference. The Helmert Method can be used to adjust in two or three dimensions, depending on the number of parameters used to specify the transformation. The Helmert Method is widely used for geodetic network modification because of its simplicity of use and consistent output for small to medium-sized networks. More advanced methods, such as least squares adjustment, may be more appropriate for vast networks. (Mataija et al., 2014).

Variance–Covariance matrix

A square matrix containing the variances and covariances related to many variables is called a variance-covariance matrix. The variances of the variables are contained in the diagonal components of the matrix, while the covariances between every conceivable pair of variables are contained in the off-diagonal elements. It is assumed that the "cofactor matrix," also known as the "covariance matrix," should be scaled by the variance of the input observation errors. It is customary to concentrate on the cofactor matrix, which, like A, is just a function of the satellite-receiver geometry at the times of the observations, because GPS observation mistakes are a strong function of the specific scenario (like, due to environmental conditions) (Blewitt, 1997).Variance–covariance matrix of coordinates of the analysed network is a basis for calculations of the relative positional errors.

The variance-covariance matrix is a common tool used by optimisation algorithms, including those used in geodetic network design like ZOD. The inverse of this matrix can be used by optimizers to give various parameters the right amount of weight, which will increase convergence and accuracy.

The variance-covariance matrix of the network's coordinates under investigation serves as the foundation for the computation of the relative positional errors. They make it possible to do the post analysis process, which yields the following figures that are commonly used to characterise positional accuracies (Alwan & Msaewe, 2012). The following is the algorithm for calculating the variance-covariance matrix:

Constraint: $\sigma_0^2 = \frac{v^t wv}{r}$ ………………………………………………………… 12

$\sum xx = \sigma_0^2 \, (B^t wB)^{-1}$ ………………………………………………………......................... 13

Where:

$\sum xx$ = variance-covariance matrix of adjusted unknowns,

σ_0^2 = variance of unit weight,

B= the coefficients matrix,

w = the weight matrix, and

r = redundancy.

Computation of ellipse of errors elements

The variance or standard deviation are precision metrics for one-dimensional instances, like an angle or distance. However, error ellipses can be created around a point to identify precision zones with different probability in two-dimensional scenarios, like the horizontal position of a point. The orientation of the ellipse relative to the E, N axes system, depends on the correlation between E and N. If they are uncorrelated, the ellipse axes will be parallel to E and N. If the two coordinates and of equal precision, or $\boldsymbol{\sigma_E} = \boldsymbol{\sigma_N}$, the ellipse becomes a circle. Considering the general case where the covariance matrix for the position of point p is provided as (Davis, et al. 1981) ,

$$\Sigma = \begin{bmatrix} \sigma_E^2 & \sigma_{EN} \\ \sigma_{EN} & \sigma_N^2 \end{bmatrix} \quad \ldots\ldots 14$$

The semi-major axis (often denoted as 'a') is half the length of the longest diameter of an ellipse and the semi-minor axis (often denoted as 'b') is half the length of the shortest diameter of an ellipse. The appropriate ellipse's semi major and semi minor axis are calculated as follows:

$$\sigma_{max}^2 = \frac{1}{2}\left(\sigma_N^2 + \sigma_E^2 + \sqrt{(\sigma_N^2 - \sigma_E^2)^2 + 4\sigma_{EN}^2}\right) \quad \ldots\ldots 15$$

$$\sigma_{min}^2 = \frac{1}{2}\left(\sigma_N^2 + \sigma_E^2 - \sqrt{(\sigma_N^2 - \sigma_E^2)^2 + 4\sigma_{EN}^2}\right) \quad \ldots\ldots 16$$

The ellipse's orientation is determined by computing (θ) between the E axis and the semi major axis from:

$$\tan 2\theta = \frac{2\sigma_{EN}}{\sigma_E^2 - \sigma_N^2} \quad \ldots\ldots 17$$

Where: θ = is laid off counter clockwise from the positive E-axis.

As Kepler's laws of orbital motion mentions, each orbit takes the approximate shape of an ellipse, with the Earth's center of mass at the focus of the ellipse. For a GPS orbit, the eccentricity of the ellipse is so small (0.02) that it is almost circular. The semi-major axis (largest radius) of the ellipse is approximately 26,600 km, or approximately 4 Earth radii. The "error ellipse" in the plane defined by the (z, y) coordinates (for example) can be computed using the elements σ_z^2 , σ_y^2 , and ρ_{zy} (Blewitt, 1997). And for the optimization of zero order design problem this step is basic, because it helps for plotting the points and the ellipse of errors. Then the final result will be presented as optimum datum is sought for the network. Plotting the points and the ellipse of errors is a graphical representation used in various fields to visualize and understand the distribution of errors or uncertainties associated with a set of data points. To solve ZOD, software written in the MATLAB language was created, as illustrated in the Fig. below. Which displays a flowchart outlining the phases of free network software, and which is capable of:

1. To find the free network adjustment, compute the general inverse, sometimes known as the "false inverse," using the Helmert Method.

2. Determine the components of the ellipse of error and compute the post-analysis for the adjustment, which includes the variance-covariance matrix.

3. Plot of the network's points together with each point's error ellipse.

3.3.2. First Order Design problem

ZOD is improved upon by FOD through network design optimisation. One of the traditional design challenges in geodesy is determining the best geometric design for a geodetic network; this is referred to as the first-order design problem (FOD). (Berné & Baselga, 2004). So from the input approximates of points this process is started and proceed to the next steps. Limits of the movement is another input which is used us search domain and formulating observation and normal equations are followed, which are explain on the previous section. To solve first order design problem in geodetic network adjustment the following phases are included and explained.

(FOD) in geodetic networks refers to a different measure related to the precision, accuracy, and reliability of the network configuration. FOD is used to evaluate the suitability of a network design for achieving specific geodetic objectives, considering factors like precision of position estimations, correlation between stations, and the geometry of the network.

The phrase "**input approximate coordinates of points**" refers to the process of giving approximation, approximate positional data for certain locations or data points inside a coordinate system. This step is the first of all the process because without providing the coordinates there will no further process in the optimization of geodetic network.

The next step is to "input limits of movement" and "search domain" describe the restrictions or boundaries that an object, system, or process must follow in order to move, operate, or seek for a certain task or aim. These restrictions and guidelines are crucial for controlling mobility and keeping it inside a predetermined and controllable area.

Formulate of observation equations and formulate of normal equations are briefly explained in the above section.

Rosenbrock Method

One kind of numerical optimisation technique is the Rosenbrock method. The method attributed to Rosenbrock employs a direct search methodology that leverages the Gram-Schmidt orthogonalization principle to provide acceleration in both the direction and step length. The process is thoroughly described in numerous references (Rosenbrock,1960).

The unconstrained minimization of the Rosenbrock function belongs to this class. It is obvious that the absolute minimum of this function is attained at $x = (1, 1)$. Therefore, it is quite surprising that this simple minimization problem serves as a benchmark example for numerical methods of unconstrained optimization. A glimpse at the graph of f shows the source of troubles – it has the form of a banana-shaped valley, where it is difficult to find the minimum iteratively. The situation becomes more difficult if more general constraints (e.g., inequalities) are given (Himmelblau, 1972).

$$f(x_1, x_2) = 100\,(x_2 - x_1^2)^2 + (1 - x_1)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 18$$

A general optimization problem is to select n decision variables $x_1, x_2, \ldots, x_n$ from a given feasible region in such a way as to optimize (minimize or maximize) a given objective function of the decision variables. The problem is called a nonlinear programming problem (NLP) if the objective function is nonlinear and/or thefeasible region is determined by nonlinear constraints. And nonlinear optimization problems are solved by Rosenbrock Method (Bazaraa and Shetty, 1979.)

A program is going to be prepared using MATLAB language to solve FOD with precision using Rosenbrock Method and it startes by Selecting points for the design variables. Then Search parallel to each of then directions sequentially, adopting the new point if the move is successful (objective function is less than or equal to the previous one), and keeping the prior position if the move is unsuccessful, continues until at success is followed by a failure in every direction then evaluate the new set of direction.

That lides to Take the best point obtained in the present stage, and repeat the same procedure of searching from step. When the objective function's starting and final values satisfy the convergence requirement, the method comes to an end. The flow chart below shows the methos to find FOD with high precision program.

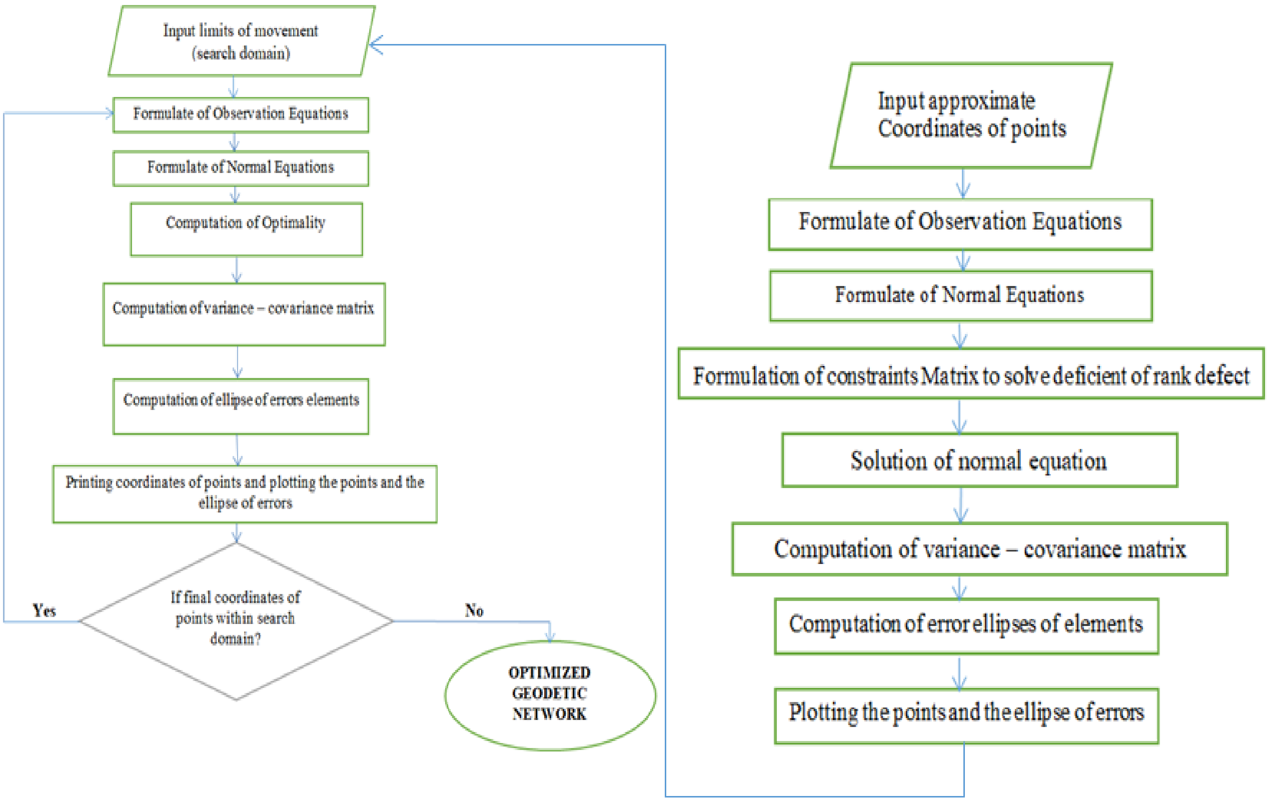

Figure 4: Flow chart of ZOD and FOD

CHAPTER FOUR

4. Results and Discussion

4.1. Zero Order Design Optimization

A geodetic network is a collection of closely surveyed and measured connected locations on the surface of the Earth, as was discussed in the preceding section. The purpose of these networks is to locate objects such as buildings, landmarks, and geographical features. (Przewięźlikowska et al., 2021). The precision and dependability of geodetic measurements need to be increased since these geodetic networks are crucial for many reasons. That's where optimisation comes into play. The process of optimisation involves either maximising or minimising an objective function that characterises a geodetic network's quality design requirements. An essential component of network analysis is the control geodetic network's design and optimisation (Amiri-Simkooei, 2012).

The initial step in selecting a few control points is to determine the best geodetic network architecture with high precision. The first step in doing such is to solve the Zero Order Design (ZOD) challenge. Different ways can be used to solve the datum problem, particularly in "free networks," by solving the ZOD problem. A network is said to be free if measurements of a relative kind only determine the network's internal structure.(Schmitt, 1982).

Using the ZOD, the best points that can be recognised as the network's control points were selected. In geodetic networks, the datum problem is crucial, and there are several approaches to creating a datum. The ZOD is the term used to define the optimal datum for a geodetic network. The modified coordinates of net points can be obtained in three different ways. It is possible to state that there are an endless number of solutions for the coordinates of the net points in a geodetic network if the datum is not defined; in other words, the points can have coordinates that are infinitely valued (Alwan & Msaewe, 2012).

Some limits should be incorporated into our adjustment procedure to limit the values and reduce our infinite number of solutions to a finite number. The network should be free from expansion and contraction, translation, and rotation by defining these constraints. These attributes can be included into the adjustment procedure through the use of least squares adjustment with minimum constraint, inner constraint, or even over constraint (Alwan & Msaewe, 2012).

Since there is no rank defect in our data it is not important to formulate the constraint matrix. If you calculate the rank and discover that it equals the number of rows (in our case, 30 coordinate points), which indicates that all of the points are linearly independent.

When given a list of latitude and longitude coordinates, we can arrange them into a matrix where each row corresponds to a point and the two columns correspond to the latitude and longitude, respectively.

Design the geodetic network

Plotting the geodetic network design by using the data given is the important part, to do so there are different methods of triangulating the points. In this case Delauny triangulation method is used because it helps us to find the shortest distance between the stations, the figure below shows the triangulated geodetic network design made from the points.

By using initial coordinates which are found in Ethiopia a geodetic network is prepared which helps to continue to the process of solving the datum problem in other words zero order design (ZOD) and by using the resulted found in the ZOD then the next step is the determination of the optimum geometric design for a geodetic network by the first order design problem (FOD). The network consists of 30 points and 70 designed distance with standard deviation equal to 5mm as shown in Figure below.

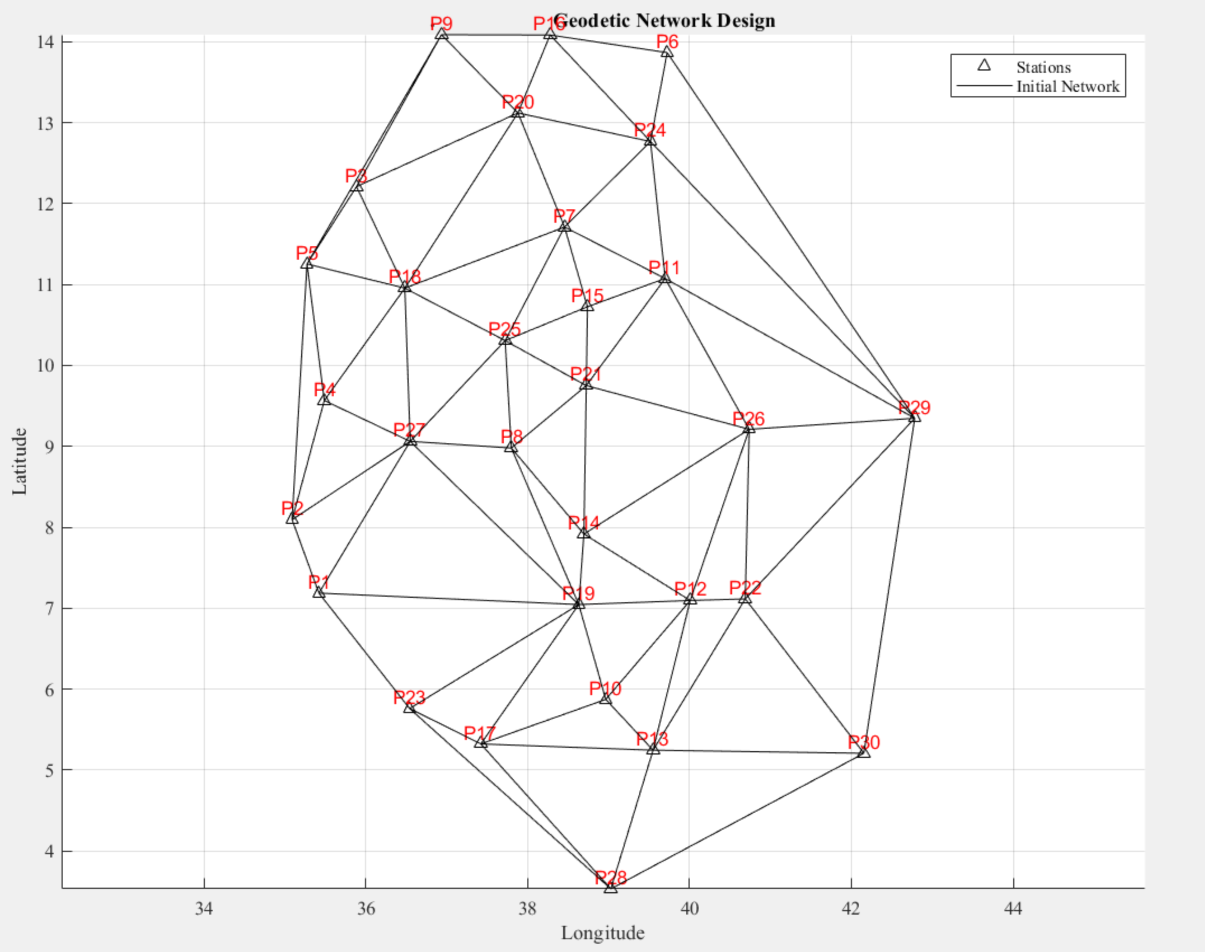

Figure 5: The Selected Geodetic Network as case study

4.1.1. The designed distances

The distances between survey stations or control points that are selected in compliance with the project's design standards are known as "designed distances." In order to comply with survey specifications, accuracy standards, and project objectives, these distances are usually specified and computed beforehand. A number of factors are considered while constructing distances, such as the survey's intended use, the required degree of precision, the topography or impediments in the survey region, the surveying tools and techniques that are available, and any applicable survey laws or rules (Wolf, n.d. 2006). For the process of geodetic network design the distance between one point to the other is important. So distances must be designed and these are derived from the network given above that uses a *Delaunay* triangulation (also known as a Delone triangulation) for a given set P of discrete points in a general position is a triangulation DT(P), such that no point in P is inside the circumcircle of any triangle in D jT(P). The next table lists the designed distances of the network.

Table 6: The Designed distances between the connected points

Station Occupied	Station Sighted	Distance, Km	Station Occupied	Station Sighted	Distance, Km
P1	P2	107.23	P10	P17	181.50
P1	P19	354.76	P10	P19	135.45
P1	P23	201.21	P11	P15	111.90
P1	P27	242.72	P11	P24	189.22
P2	P4	169.55	P11	P26	236.31
P2	P5	351.97	P11	P29	388.25
P2	P27	193.20	P12	P13	211.83
P3	P5	125.50	P12	P14	171.17
P3	P9	237.70	P12	P19	152.03
P3	P18	153.89	P12	P22	75.04
P3	P20	238.98	P12	P26	248.61
P4	P5	189.05	P13	P17	235.86
P4	P18	188.95	P13	P22	243.06
P4	P27	129.25	P13	P28	199.06
P5	P9	363.04	P14	P19	97.15
P5	P18	136.05	P14	P26	266.65
P6	P16	157.96	P15	P25	120.51
P6	P24	124.20	P15	P26	276.20
P6	P29	602.81	P16	P20	115.41
P7	P11	152.95	P16	P24	198.57
P7	P15	113.80	P17	P19	233.68
P7	P18	230.94	P17	P23	108.74
P7	P20	168.87	P17	P28	267.26
P7	P24	164.64	P18	P20	1284.69
P7	P25	175.32	P18	P25	153.52
P8	P14	154.03	P18	P27	210.72
P8	P15	219.72	P19	P23	271.61
P8	P19	233.84	P19	P27	320.77
P8	P25	147.84	P20	P24	181.71
P8	P26	323.88	P22	P26	233.12
P8	P27	137.53	P22	P29	338.74
P9	P16	144.82	P23	P28	370.57
P9	P20	148.31	P24	P29	521.15
P10	P12	178.20	P25	P27	188.98
P10	P13	95.19	P26	P29	224.95

4.1.2. Error Ellipses

An oval form known as an error ellipse is used to illustrate the statistical distribution of mistakes in the adjusted positions or measurements. It is obtained from the estimated parameters' covariance matrix. The error ellipse provides information on the degree of uncertainty along each axis as well as the correlation between the horizontal and vertical axes. The covariance matrix's standard deviations and covariances are used to build the error ellipse. The standard deviations along each of the ellipse's major and minor axes are represented, and the correlation between the parameters determines the ellipse's orientation. (Wolf, n.d. 2006).

Error ellipses have the benefit of allowing for a visual comparison of the relative precisions between any two stations, in addition to giving important information about the accuracy of an updated station position. Analysing the sizes, forms, and orientations of error ellipses allows for quick and insightful comparisons between different surveys. (Wolf, n.d. 2006).

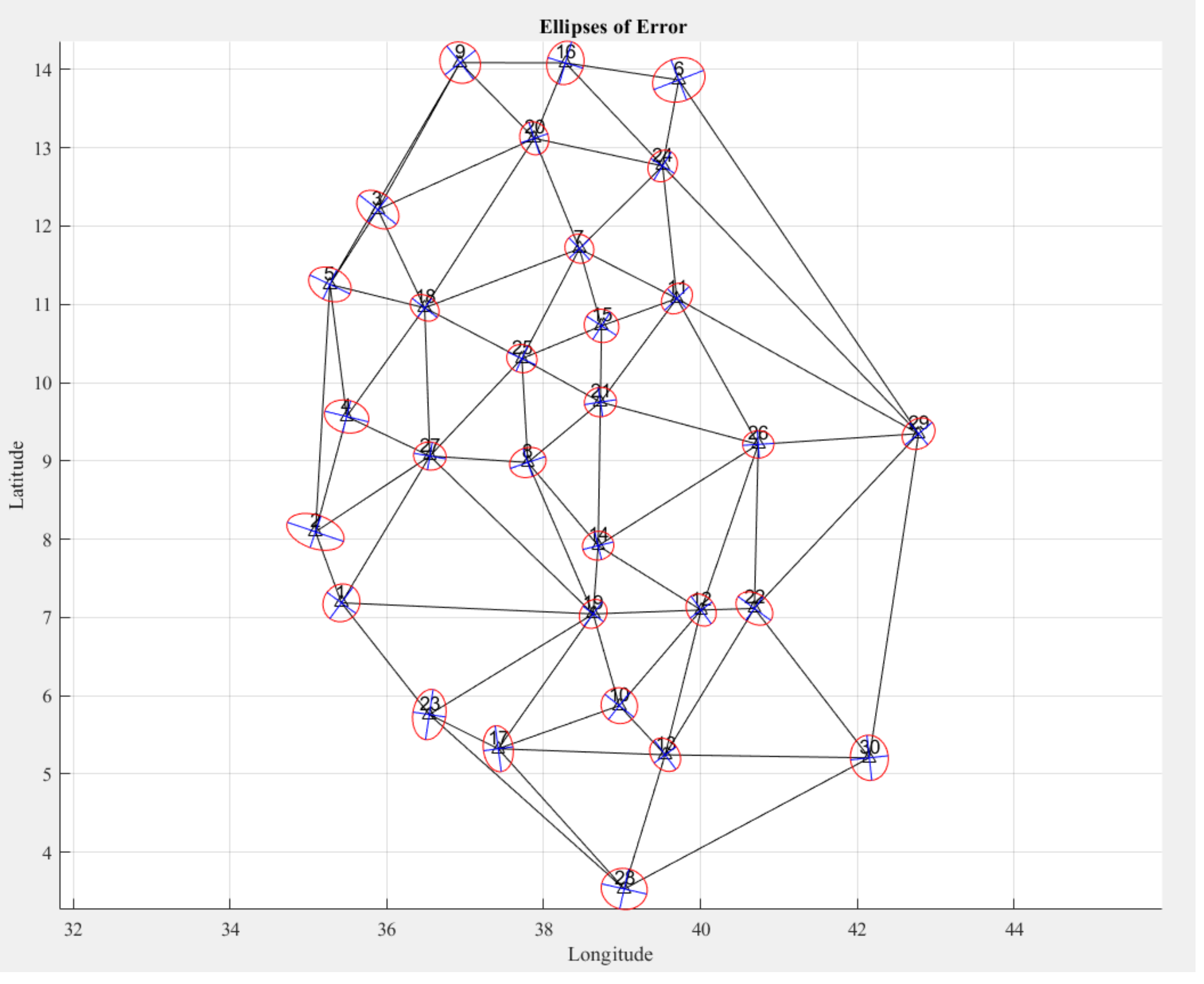

Figure 6: The Error ellipses for each point in the network

A program was prepared using MATLAB language to solve ZOD so to solve this problem ellipse of error is the main part because it helps to find the best datum point. The above figure shows the ellipses of error for each point in the network exaggerated by a factor of 400.

Each and every coordinate point has ellipse, and those ellipses contains semi-major, semi-minor axis and area, after the application of zero order design the values for calculated and presented in the next table below.

The area of the error ellipse is calculated by using this formula.

$$A = \pi * \mathrm{a} * \mathrm{b} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 19$$

Were A = area

a = Semi-major

b = Semi-minor

Table 7: The semi-major, semi-minor axis and area of ellipse of errors for each point in the network

Point	**Semi-major axis (m)**	**Semi-minor axis (m)**	**Area of ellipse of errors (m^2)**
P1	0.00062052	0.00057078	0.178
P2	0.00094549	0.00053144	0.253
P3	0.00075011	0.00051566	0.194
P4	0.00071663	0.00051604	0.186
P5	0.00070656	0.0005156	0.183
P6	0.00085402	0.00068613	0.295
P7	0.00048191	0.00046778	0.107
P8	0.00059141	0.00046797	0.139
P9	0.00068392	0.00062315	0.214
P10	0.00058931	0.00056145	0.166
P11	0.00054631	0.00043206	0.119
P12	0.00054913	0.00043748	0.121
P13	0.00057663	0.00043754	0.127
P14	0.00050198	0.00045661	0.115
P15	0.00056297	0.00051863	0.147
P16	0.00070220	0.00058403	0.206
P17	0.00073803	0.00047406	0.176
P18	0.00050184	0.0003731	**0.094**
P19	0.00048843	0.00040972	**0.101**
P20	0.00053581	0.00045336	0.122
P21	0.00051239	0.00046778	0.120
P22	0.00064367	0.00046202	0.149
P23	0.00080820	0.00052646	0.214
P24	0.00052659	0.00044504	0.117
P25	0.00048952	0.00044300	0.109
P26	0.00049839	0.00043939	0.110
P27	0.00052175	0.00044584	0.117
P28	0.00073682	0.00065199	0.241
P29	0.00056209	0.00045896	0.130
P30	0.00072350	0.00060268	0.219

As the area calculation shows the ellipse of error for the network points varies between **0.094m²** and **0.295 m²**. From all the pointes included in the network P18 and P19 have the smallest area of the error ellipse compared to the other points. As a result these points having the area of **0.094** and **0.101** respectively are considered as the best points and selected to be ***Control points***.

4.2. First Order Design Optimization

From the different classical design problems the determination of the optimum geometric design for a geodetic network is one of them in geodesy, known as the first-order design problem (FOD) (Berné & Baselga, 2004). First-order design (FOD), which is to determine a configuration of instruments provided the stochastic model for observations (Jia & Lichti, 2017).

Design matrix or (B-Matrix geometry of the network) are the observations variables in FOD problem. To ensure the intended accuracy and dependability of the network, it is essential to provide a range of movement or permitted error for the first-order design in the context of geodetic networks. A geodetic network normally consists of a number of survey points or control stations with known coordinates as well as newly established points with undetermined coordinates. The first-order design entails setting up the criteria for the new points in accordance with the network's needs and desired accuracy (Przewięźlikowska et al., 2021).

The choice of range of movement in a First Order Design (FOD) for a geodetic network depends on several factors, including the specific project requirements, accuracy goals, the environment in which the network is being established, and the intended use of the geodetic data. Here are some considerations for choosing an appropriate range of movement:

In First Order Design (FOD) the selection of range of movement for a geodetic network depends on several factors, including the specific project requirements, accuracy goals, the environment in which the network is being established, and the intended use of the geodetic data. Since our goal is to consider the accuracy requirements of the project. If the project demands very high accuracy, a smaller range of movement (e.g., 100 meters) may be more appropriate. But if the concern was the scale of the project and the geographic area it covers. For a large-scale project covering a significant geographical area, a larger range of movement (e.g., 1000 meters) might be suitable.

So the range of movement for the FOD project is 100 meters in every direction, that with in this range finding the most accurate coordinate points for each collected data except the datum points selected in the above zero order design problem optimization process. The upper and lower limits of movement for each point can provided as:

E_i^0-100m$\leq E_i \leq E_i^0 + 100\text{m}$ -

N_i^0-100m$\leq N_i \leq N_i^0 + 100\text{m}$

The angle of rotation in finding the best solution is by checking every point found in every 15 degree for all the 360^0.

The computation and presentation of the limits of movement for each coordinate points of the network in east and north directions is shown on Table 8 below. The lower and upper bounds, determined below, shall be compared to the final coordinates of network sites obtained according to the FOD.

If the final coordinates are within the allowed range of movement, the solution moves on to the next stage of FOD computation until it exceeds the allowed range of movement for all locations. The three cases listed above were subjected to FOD with high accuracy, high reliability, and high precision.

From the result points that are found in the process of ZOD optimization the coordinates of control points are selected as a datum points, these points are P7 and P24. By taking the datum points as fixed stations, FOD was applied to the network.

Table 8: The range of movement for the Coordinate points of the network

Point	Limits of E_i^0, m		Limits of N_i^0, m	
	Lower	Upper	Lower	Upper
1	767173.976	767373.976	794967.212	795167.212
2	730133.523	730333.523	895114.222	895314.222
3	813775.526	813975.526	1351355.71	1351555.71
4	772904.538	773104.538	1058394.558	1058594.558
5	747983.362	748183.362	1244889.092	1245089.092
6	578117.649	578317.649	1532792.696	1532992.696
7	440567.290	440767.290	1294390.174	1294590.174
8	367723.000	367923.000	992708.000	992908.000
9	276909.443	277109.443	1558259.203	1558459.203
10	496025.360	496225.360	649141.234	649341.234
11	576351.000	576551.000	1224147.44	1224347.44
12	611345.472	611545.472	784249.34	784449.34
13	560754.757	560954.757	579711.639	579911.639
14	466526.746	466726.746	875127.973	875327.973
15	471347.643	471547.643	1185431.507	1185631.507
16	421901.009	422101.009	1557001.428	1557201.428
17	324908.902	325108.902	588627.817	588827.817
18	224663.934	224863.934	1212153.494	1212353.494
19	459281.914	459481.914	778770.011	778970.011
20	378495.944	378695.944	1450614.344	1450814.344
21	469676.476	469876.476	1077784.142	1077984.142
22	686449.314	686649.314	786679.874	786879.874
23	227670.617	227870.617	637260.584	637460.584
24	556008.413	556208.413	1411235.609	1411435.609
25	359922.334	360122.334	1139536.772	1139736.772
26	690911.833	691111.833	1018536.847	1018736.847
27	230364.627	230564.627	1002516.731	1002716.731
28	503077.648	503277.648	390329.766	390529.766
29	256483.354	256683.354	1033883.652	1034083.652
30	184417.41	184617.41	576128.902	576328.902

"Computation of the elements of an ellipse of errors" entails figuring out its parameters using the measurements' covariance matrix. An illustration that sheds light on the precision and accuracy of a series of measurements is the ellipse of errors. The semi-major axis, semi-minor axis, orientation angle, and center coordinates are among the ellipse's components.

Then after the computation of the error ellipse using a MATLAB algorithm the final step is to let the code Print coordinates of points that are found by applying the FOD and plotting the points and the ellipse of errors. This figure shows the result of plotting the points and the ellipse of errors,

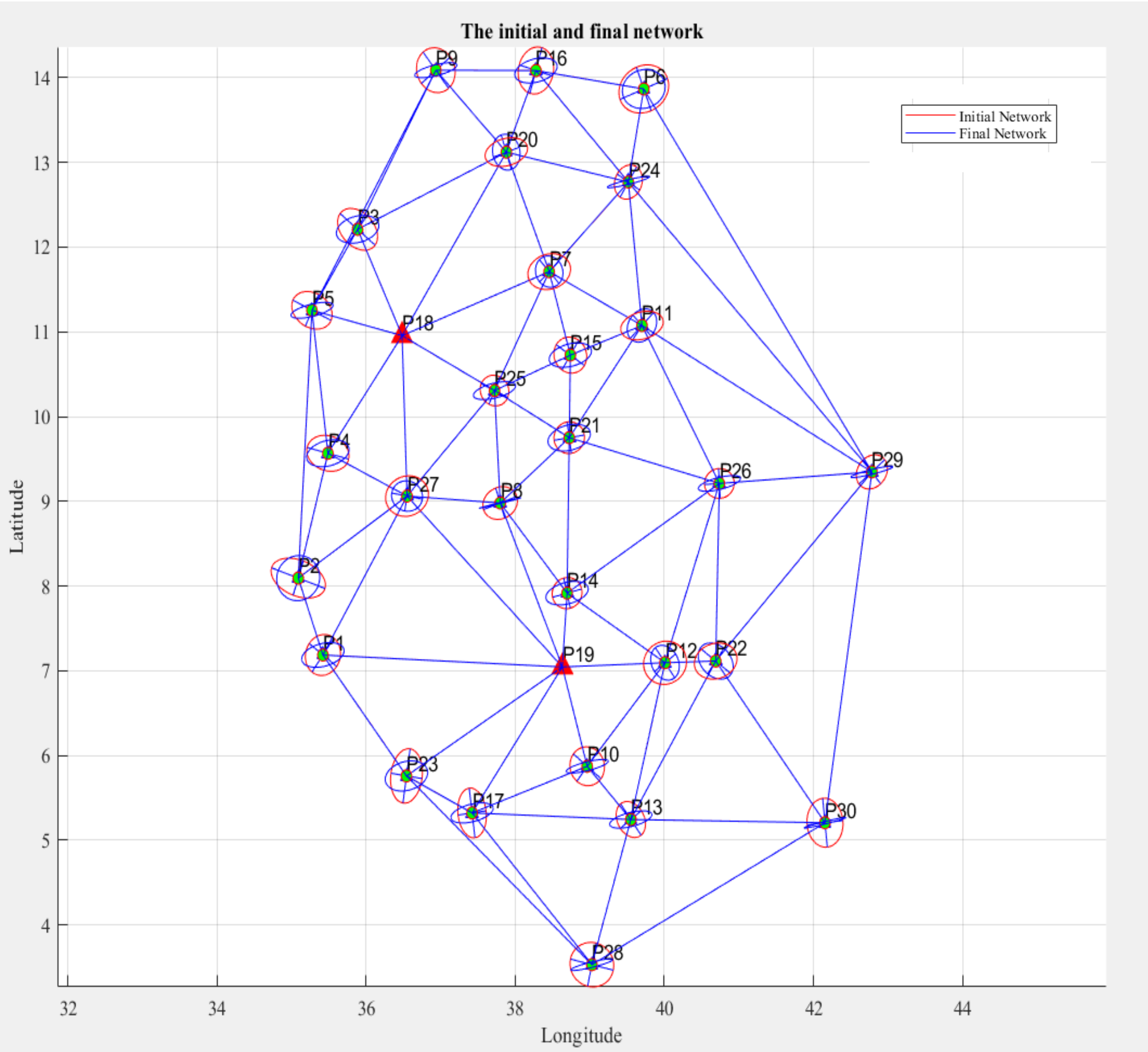

Figure 7: The initial and final network

As discribed in the method section every errore ellipses has there own semi-majore and semi-minor axises at the same time they got standared divation in the east as well as north direction and based on these calculated data it is posible to fined the valuse of the area for each ellipse of error. And the table below listes the initial and final coordinates along the latitued and longitued and the values of the areas for both the initial and final coordinate points.

Table 9: The results of FOD with high precision

Point	Latitude		Longitude		Area of ellipse of error, mm^2	
	Initial	**Final**	**Initial**	**Final**	**Initial**	**Final**
P1	7.186454	7.187039	35.420113	35.420027	0.178033	0.123049
P2	8.093397	8.093738	35.089279	35.089299	0.252569	0.240671
P3	12.210077	12.209557	35.884456	35.885346	0.194428	0.141284
P4	9.566731	9.567002	35.487051	35.486386	0.185889	0.136661
P5	11.253675	11.254174	35.272325	35.273271	0.183116	0.065654
P6	13.864803	13.865727	39.723861	39.723977	0.294538	0.206549
P7	11.709517	11.710397	38.455542	38.455774	0.188679	0.106884
P8	8.979552	8.979971	37.797603	37.797408	0.139113	0.004624
P9	14.087294	14.087363	36.934711	36.934970	0.214223	0.058805
P10	5.873682	5.873221	38.964997	38.964873	0.166312	0.047114
P11	11.073964	11.074466	39.699979	39.699744	0.146175	0.118647
P12	7.094795	7.094840	40.009158	40.009601	0.230277	0.120753
P13	5.24535	5.245262	39.549162	39.549801	0.126819	0.075641
P14	7.917848	7.916632	38.697217	38.695270	0.115214	0.116499
P15	10.724517	10.725332	38.738877	38.738592	0.146762	0.120855
P16	14.083687	14.084587	38.27748	38.277932	0.206142	0.125127
P17	5.324227	5.322814	37.420828	37.420890	0.094355	0.175864
P18	10.955901	10.955043	36.481666	36.482349	0.094116	0.094116
P19	7.046181	7.045263	38.632219	38.632106	0.100589	0.100589
P20	13.120351	13.121145	37.879986	37.880320	0.146717	0.122102
P21	9.750905	9.751435	38.724435	38.723958	0.120477	0.136091
P22	7.114796	7.114980	40.689155	40.688715	0.189996	0.149482
P23	5.760901	5.758983	36.541952	36.541209	0.213873	0.155270
P24	12.766189	12.767484	39.516922	39.517036	0.117799	0.024031
P25	10.307014	10.306861	37.721661	37.722110	0.109004	0.080100
P26	9.210936	9.210473	40.738594	40.737607	0.110077	0.063792
P27	9.062011	9.061729	36.548058	36.548515	0.116924	0.214798
P28	3.532295	3.531972	39.028611	39.027235	0.241475	0.028581
P29	9.347026	9.347244	42.783764	42.782757	0.129673	0.028581
P30	5.206744	5.206539	42.154328	42.154255	0.219175	0.013055

By comparing the areas of the error ellipses there was an improvement at the final result which is found after the application of FOD in comparison with the initial coordinates. With these improvements the network becomes more precise than the initial network. The final result of the optimizing geodetic network of Ethiopia is shown in the figure bellow.

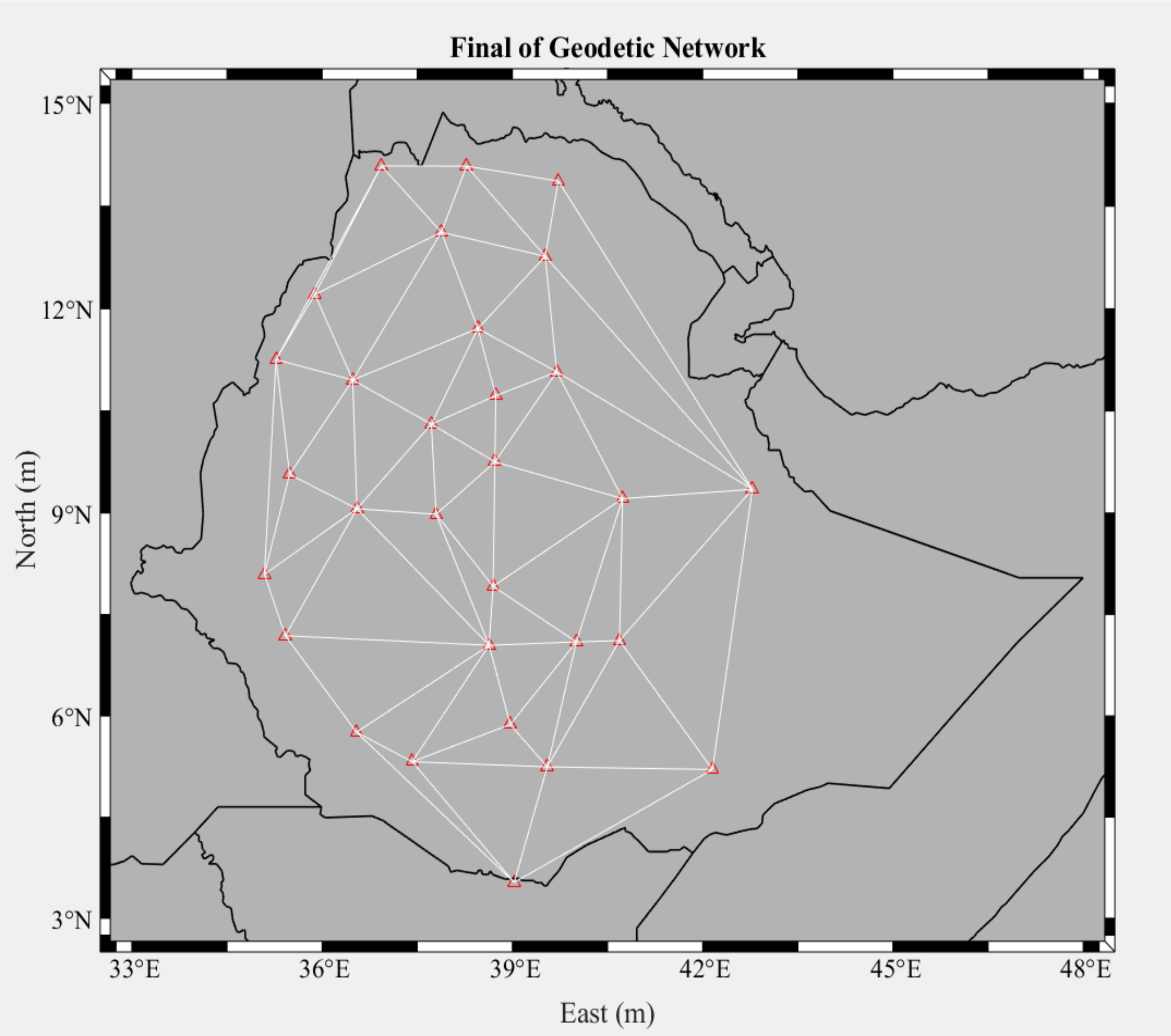

Figure 8: Optimized geodetic network of Ethiopia

4.2. Discussion

The aim of the discussion section is to clarify and examine the results of optimizing the geodetic network using the iterative Zero-Order Design (ZOD) and First-Order Design (FOD) procedures. The main goal of this study was to strategically place control points and reduce associated mistakes in order to improve the effectiveness and accuracy of the geodetic network.

Through ZOD, Initial investigation and rough control point placement were done. And since the main goal of applying ZOD in a geodetic network is to define the datum of a given study area and to select the most precise station coordinates of all the provided points, to do so error ellipse production has been done for each and every stations that are included in the network then from all the error ellipses plotted selecting the station that have the smallest area is done and use these points as a fixed coordinate and proceed to the next step.

On the other hand first order design (FOD) one of the classical design problems in geodesy which determines the optimum geometric design for a geodetic network. And this is done by applying different criteria's including the range of movement for every station coordinates. Then a new shifted station points are found except the ones that are taken as a fixed coordinates and by comparing the areas of the error ellipses for both the initial and final coordinates the optimum geodetic network is found.

It is very difficult to find a related research in this topic that optimize the zero order design and first order design problem in the case of Ethiopia. But some other studies used this process of optimization and produce a geodetic network. When employing the trial-and-error approach, the objective function and a suggested fix for the issue are computed together. The solution is adjusted and the objective function is recalculated if it turns out that the recommended solution is insufficient. This method is repeated until the requirement is satisfied (Alwan & Msaewe, 2012).

The outcomes of this stage of optimization provide important information on the best placement of control points and how that affects the geodetic network's accuracy and precision. The consequences of the optimized network, prospective revisions in the future, and the contribution of iterative design techniques to improving geodetic surveying methodologies are covered in this debate.

CHAPTER 5

5. Conclusions and Recommendations

5.1. Conclusions

In conclusion this study set out to optimize a geodetic network, aiming to improve its accuracy, efficiency, and overall precision of the network. Optimization is maximizing or minimizing an objective function that describes the quality design criteria of a geodetic network. The optimization and design of a control geodetic network play an important role in network analysis (Amiri-Simkooei, 2012). And the Optimization of geodetic networks is classified in different orders of problems that needed to be solved. It starts with Zero-order optimal design (ZOD) used to solve the geodetic datum problem the next one is First-order optimal design (FOD) which in the configuration geometric design problem solving, these two optimization design orders are conducted in this theses but there are two more orders that are Second-order optimal design (SOD) for the generation of weight problem solving and Third-order optimal design (THOD) for geodetic Bayesian approach.

There are different optimization techniques to solve the geodetic network design of the zero order design (ZOD) and first order design (FOD) problems. The classical and intelligent or heuristic optimization techniques are the two main parts. In the classical section trial and error method or analytical methods are found on the other hand Genetic or Evolutionary algorithms (EAs), particle swarm optimization (PSO) and simulated anneal are the methods the classes of Heuristic optimization techniques. From these techniques trial and error method is applied for solving both ZOD and FOD.

The solution of the datum problem, especially in 'free networks', can be done by different approaches. The zero order design (ZOD) solves this type of problem by applying the error ellipse for every floating stations of a given coordinates. From those error ellipses that have their own standard deviations in both east and north direction and semi-major with semi-minor axis's it is possible to calculate the areas of the ellipses. So to find the fixed stations of the network the error ellipses having the minimum areas of all are chosen. In this case area calculation shows the ellipse of error for the network points varies between $0.094116m^2$ and $0.294538m^2$. From all the pointes included in the network P18, P19 have the smallest ellipse of error compared to the other points. As a result these points are considered as the best points and selected to be fixed points.

Determination of the optimum geometric design for a geodetic network is one of the classical design problems in geodesy, known as the first-order design problem (FOD) (Berné & Baselga, 2004). Solving this problem in finding the optimum geodetic network of Ethiopia is the final stage and to perform that again trial and error method is applied. There are also different criteria's need including the limit of movement which is 100 meters in both east and north direction and angle rotating differences are applied. Then the final optimum geodetic networks are found by comparing the areas of the error ellipses between the initial and the modified final ellipses.

In summary, to attain the maximum level of accuracy, precision, and efficiency in geodetic surveying and mapping, a geodetic network is deliberately designed and configured with a network of control points and survey measurements. As a result solving the zero order design and first order design problems are very significant in finding the optimum fixed stations and designing optimum the geometric design of the network. This thesis tried to find the optimum geodetic network of Ethiopia by applying both zero and first order design problems and provides the most accurate and precise geodetic network.

5.2. Recommendations

It is clear from the results and experiences of this study that geodetic network optimization is a vital attempt with significant implications for surveying accuracy and effectiveness. The trial-and-error methodology used in this study gave insightful information about the optimization process. However, in order to increase efficiency and precision for larger, more complex geodetic networks, it is strongly advised to use cutting-edge optimization techniques.

In order to discover the best locations for control points and network architecture, analytical optimization approaches that make use of mathematical models and algorithms provide a methodical approach. In reducing errors and assuring the geodetic network's general robustness, they can be very effective.

Furthermore, effective tools for optimization are provided by evolutionary algorithms as the Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Simulated Annealing. These techniques are well suited for optimizing geodetic networks because they can handle challenging non-linear optimization issues. For instance, a genetic algorithm mimics the method of natural selection to find the best answer, making it extremely flexible and powerful.

In addition, Particle Swarm Optimization imitates social entity behavior, enabling effective solution space exploration. Simulated annealing, which was modelled after the metallurgical annealing procedure, can assist in overcoming local optima and achieving global optima in the solution space.

In general even if this thesis is conducted by using the trial and error method which is one of the classical optimization technique it is possible to use the Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Simulated Annealing that area included in intelligent techniques.

Reference

AbdAllah, A. A. G., & Wang, Z. (2022). Optimizing the geodetic networks based on the distances between the net points and the project border. *Scientific Reports*, *12*(1). https://doi.org/10.1038/s41598-021-04566-0

Alizadeh Khameneh, M. A. (2017). *Optimal Design in Geodetic GNSS-based Networks* (Issue December).

Alwan, H., & Msaewe, M. (2012). *Optimized Zero and First Order Design of Micro Geodetic Networks*. https://www.researchgate.net/publication/337317567

Amiri-Simkooei, A. (2012). A NEW METHOD FOR SECOND ORDER DESIGN OF GEODETIC NETWORKS: AIMING AT HIGH RELIABILITY. *Survey Review*, *37*(293), 552–560. https://doi.org/10.1179/003962604791482306

Ayele, N. A. (2022). *Optimum CORS Network Design for Geodetic Applications in Ethiopia*. 1–14.

Baarda, W. (1981). S-transformations and criterion matrices. *Netherlands Geodetic Commission*, *5*(1), 168.

Berné, J. L., & Baselga, S. (2004). First-order design of geodetic networks using the simulated annealing method. *Journal of Geodesy*, *78*(1–2), 47–54. https://doi.org/10.1007/s00190-003-0365-y

Bielecka, E., Pokonieczny, K., & Borkowska, S. (2020). Giscience theory based assessment of spatial disparity of geodetic control points location. *ISPRS International Journal of Geo-Information*, *9*(3). https://doi.org/10.3390/ijgi9030148

Blewitt, G. (1997). Basics of the GPS Technique : Observation Equations §. *Geodetic Applications of GPS*, 1–46.

BLUNDER DETECTION TECHNIQUES IN ADJUSTMENT. (2007). *13*(2), 1485–1497.

Chakraborty, D., & Kharagpur, T. (n.d.). *Classical Optimization Techniques : Single Variable Optimization*. *x*.

Cross, P. A. (1985). Numerical Methods in Network Design. *Optimization and Design of*

Geodetic Networks, *i*, 132–168. https://doi.org/10.1007/978-3-642-70659-2_7

Dare, P., & Saleh, H. (2000). GPS network design: Logistics solution using optimal and near-optimal methods. *Journal of Geodesy*, *74*(6), 467–478. https://doi.org/10.1007/s001900000104

Darwin, C., & Algorithms, G. (n.d.). *Genetic Algorithms 2.1*. 15–16.

Delahaye, D., Chaimatanan, S., & Mongeau, M. (2019). Simulated annealing: From basics to applications. *International Series in Operations Research and Management Science*, *272*, 1–35. https://doi.org/10.1007/978-3-319-91086-4_1

Doma, M. I., & Sedeek, A. A. (2014). *Comparison of PSO , GAs and Analytical Techniques in Second-Order Design of Deformation Monitoring Networks*. *8*(1), 21–30. https://doi.org/10.1515/jag-2013-0013

Eberhart, R., & Kennedy, J. (1995). New optimizer using particle swarm theory. *Proceedings of the International Symposium on Micro Machine and Human Science*, 39–43. https://doi.org/10.1109/mhs.1995.494215

Engineering Optimization Theory and Practice. (n.d.).

Eshagh, M., & Alizadeh-Khameneh, M. A. (2015). Two-Epoch optimal design of displacement monitoring networks. *Boletim de Ciencias Geodesicas*, *21*(3), 484–497. https://doi.org/10.1590/S1982-21702015000300027

Ghorbanian, A., Sahebi, M. R., & Mohammadzadeh, A. (2019). Comptes rendus - Geoscience Optimization approach to retrieve soil surface parameters from single-acquisition single-con fi guration SAR data. *Comptes Rendus - Geoscience*, *351*(4), 332–339. https://doi.org/10.1016/j.crte.2018.11.005

Goldberg_Genetic_Algorithms_in_Search.pdf. (n.d.).

Grafarend, E. W. (1974). Optimization of Geodetic Networks. *The Canadian Surveyor*, *28*(5), 716–723. https://doi.org/10.1139/tcs-1974-0120

Guanabara, E., Ltda, K., Guanabara, E., & Ltda, K. (n.d.). *No 主観的健康感を中心とした在宅高齢者における 健康関連指標に関する共分散構造分析Title*.

Himmelblau, D. M. (1972). Applied nonlinear optimization. *New York, US: McGraw Hill*.

Jia, F., & Lichti, D. (2017). A COMPARISON of SIMULATED ANNEALING, GENETIC ALGORITHM and PARTICLE SWARM OPTIMIZATION in OPTIMAL FIRST-ORDER DESIGN of INDOOR TLS NETWORKS. *ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, *4*(2W4), 75–82. https://doi.org/10.5194/isprs-annals-IV-2-W4-75-2017

Kuang, S. (1991). Optimization and Design of Deformation Monitoring Schemes. Ph.D. dissertation. *Fredericton, Canada: Department of Surveying Engineering.*, *157*, 1–196.

Leick, A., Rapoport, L., & Tatarnikov, D. (2015). *GPS Satellite Surveying.pdf*.

Mataija, M., Pogarčic, M., & Pogarčic, I. (2014). Helmert transformation of reference coordinating systems for geodesic purposes in local frames. *Procedia Engineering*, *69*, 168–176. https://doi.org/10.1016/j.proeng.2014.02.217

Miskas, A., & Molnar, A. (2009). *Establishing a Reference Network in Parts of Amhara Region , Ethiopia Using Geodetic GPS Equipment*. *3117*.

No Title. (n.d.).

Optimization, C. (n.d.). *Convex Optimization*.

Particle Swarm Optimization in Comparison with Classical Optimization for GPS Network Abstract : (2013). *3*(4), 250–257. https://doi.org/10.2478/jogs-2013-0030

Programming, N. (n.d.). *Nonlinear Programming 13*. 410–464.

Przewięźlikowska, A., Ślusarczyk, W., & Wójcik, K. (2021). *Methodology of Analysing Geodetic Network as Basis for Developing Mobile Application for Its Ongoing Update*. http://dx.doi.org/10.21203/rs.3.rs-959391/v1%0Ahttps://www.researchsquare.com/article/rs-959391/v1

Schmiti, G. (1985). *A . Review of Network Designs : Criteria , Risk Functions , Design Ordering*.

Schmitt, G. (1982). *OPTIMIZATION OF GEODETIC NETWORKS*. *20*(4), 877–884.

Spivey, W. A. (1962). Linear programming. *Science*, *135*(3497), 23–27. https://doi.org/10.1126/science.135.3497.23

The, I. (1985). *J . Optimization Problems in Geodetic Networks with Signals*. *1982*.

Wolf, P. R. (n.d.). *Adjustment Computation, SPATIAL DATA ANALYSIS*.

Yetki, M., Cevat, I., & Yigit, C. O. (2008). Optimal design of deformation monitoring networks using PSO algorithm. *13th FIG Symposium on Deformation Measurement and Analysis, 4th IAG Symposium on Geodesy and Geotechnical and Structural Engineering*, 12–15.