RADIATIVE DECAY CONSTANT OF PION IN BETHE-SALPETER FRAMEWORK

By

Abebe Gucho

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

At

ADDIS ABABA UNIVERSITY

ADDIS ABABA, ETHIOPIA

JUNE, 2009

ADDIS ABABA UNIVERSITY
DEPARTMENT OF
PHYSICS

The undersigned hereby certify that they have read and recommend to the School of Graduate Studies for acceptance a thesis entitled "**RADIATIVE DECAY CONSTANT Of PION IN BETHE-SALPETER FRAMEWORK** " by **Abebe Gucho** in partial fulfillment of the requirements for the degree of **Master of Science in Physics**.

Dated: June, 2009

Advisor: ____________________
Dr. S. Bhatnagar

Examiners: ____________________
Dr. Fesseha Kassahun

____
Professor V. N. Mal'nev

ADDIS ABABA UNIVERSITY

Date: **June, 2009**

Author: **Abebe Gucho**

Title: **RADIATIVE DECAY CONSTANT Of PION IN BETHE-SALPETER FRAMEWORK**

Department: **Physics**

Degree: **M.Sc.** Convocation: **June** Year: **2009**

Signature of Author

TO:- MY FAMILY

Table of Contents

List of Figures

Abstract

In this work our purpose is to calculate the pion radiative decay constant using the formulation of Bethe-Salpeter equation under covariant Instantaneous Ansatz(CIA) after inclusion of both leading(LO) as well as next-to-leading(NLO) terms in the BS wave function of the pion. In this process we also calculate the pion Bethe-Salpeter Normalizer which is an extremely lengthy calculation.

Acknowledgements

First of all, I would like to thank Amara Regional Bureau for giving this chance next to God.
Secondly, I would like to thank my advisor Dr. Bhatnagar, for his many suggestions, constant support during this research, for many critical insights without which I could not have written this thesis, encouraging and providing the necessary materials for the accomplishment of this thesis gives me a great pleasure. I am also indebted to Abebe Fentie, Mekonnon Molla, Ykidem Megesha for interesting comments.
Lastly, I would like to thank Tsilat Adinew for best hospitality at Addis Ababa University.

Chapter 1

Introduction

Pseudoscalar meson is a meson with total spin 0 and odd parity with J^{pc} classification $O^{+\pm}$. Among all of the mesons known to exist, the pseudoscalars are perhaps the most well known in a sense. The masses of the pion, kaon, eta and eta prime particles are known with great precision. However, the decay properties of the pseudoscalar mesons, particularly of eta and eta prime, are somewhat contradictory to the mass hierarchy. While the eta prime meson is much more massive than the eta meson, the eta meson is thought to contain a larger component of strange and anti-strange quarks than the eta prime meson, which appears contradictory. The presence of an eta(1405) state also brings glueball mixing into the discussion. It is possible that the eta and eta prime mesons mix with the pseudoscalar glueball which should occur, in its pure state, somewhere above the scalar glueball in mass. This is one of a few ways in which the unexpectedly large eta prime mass of 957.78 MeV/c^2 can be explained, relative to its model-predicted mass around 250 to 300 MeV/c^2.

Pseudoscalar mesons are commonly seen in proton-proton scattering and proton-antiproton annihilation. The lightest Pseudoscalar meson was first proposed to exist

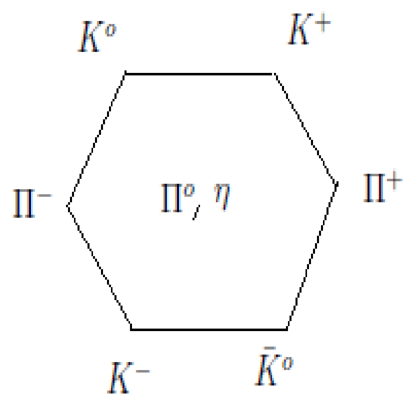

Figure 1.1: Pseudoscalar mesons consisting of up, down and strange quarks only form an octet

by Yukawa in the 1930s as the primary force carrying boson of the Yukawa Potential in nuclear interactions, and was later observed at nearly the same mass that he originally predicted for it. In the 1950s and 1960s, the pseudoscalar mesons began to proliferate, and were eventually organized into a multiplet according to Murray Gell-Mann's "Eightfold Way". Gell-Mann further predicted the existence of a ninth resonance in the pseudoscalar multiplet, which he originally called X. Indeed, this particle was later found and is now known as the eta prime meson. The structure of the pseudoscalar meson multiplet, and also the ground state baryon multiplets, led Gell-Mann (and Zweig, independently) to create the well known quark model.

QCD is the correct gauge theory of strong interactions. It is based on the gauge group $SU(3)$. In addition to the gauge fields, QCD involves fields of spin $\frac{1}{2}$ particles known as quarks. There are quarks of six types, or flavors. Quarks of each flavor come in three colours which furnish the defining representation 3 of the $SU(3)$ gauge group. The quantum field theories that have proved successful in describing the real world are all non-Abelian gauge theories, theories based on principles of gauge invariance more general than the simple $U(1)$ gauge invariance of quantum electrodynamics.

A great deal of attention has been dedicated to the electromagnetic decays of the neutral pseudoscalar mesons π^0 and η and calculations of such observables in the framework of different models. Since the great percentage of photons in the background of heavy-ion collisions is due to the decays of $\pi^0(\eta) \rightarrow 2\gamma$ these processes deserve special attention[1]. The two-photon decay of mesons can be used to identify the flavor of quark-antiquark states and provide an important tool for exploring the structure of these simplest bound states in QCD and for studying the non-perturbative(long distance) behavior of strong interactions[2]. Calculating hadron structure from QCD alone is very difficult, we can use specific models of hadron dynamics to gain some understanding of hadronic structures at low energies. A realistic description of pseudoscalar mesons at the quark–gluon level is an important element in advancing our understanding of hadron dynamics and reaction processes at scales where QCD degrees of freedom are relevant. The pseudoscalar mesons, Especially the pion have for a long time been a major focus of attempts to understand the internal structure of hadrons from nonperturbative QCD[3].

Now a number of recent studies have been carried out on pseudoscalar mesons at quark level of compositeness [2-5]. These studies have revealed that various mesons have many different covariant structures in their wave functions whose inclusion was also found necessary to obtain quantitatively accurate observables[3], and it was noticed that all Dirac covariants do not contribute equally and only some of the covariants are relevant for calculation of mass spectrum and decay constants. Towards this end, a power counting rule for incorporating various Dirac covariants in the structure of hadron-quark vertex function for a meson was developed where covariants are incorporated order by order in powers of the inverse of meson mass M, so as to systematically choose among different covariants from their complete set and write wave functions for different mesons . Recently, we have used this generalized hadron quark vertex function to calculate leptonic decay constants of pseudoscalar mesons such as π, K, D, D_s and B [6] as well as vector mesons [7] in the QCD oriented framework of Bethe-Salpeter Equation under covariant Instantaneous Ansatz(CIA) which is a Lorentz -invariant generalization of Instantaneous Approximation (IA) and which for the $q\bar{q}$ system ensures an exact interconnection between 3D and 4D forms of the BSE[7].

Furthermore, a number of recent studies in the framework of BSE [8,9] have been carried out on two photon decays of mesons. The decay into two photon is considered as an interesting experimental playground in the mesonic physics of the near future. Two photon couplings also provide a useful probe of the internal structure of mesons. The classic light $q\bar{q}$ states such as $\pi^o, \eta, \eta^{'}$ etc. have been observed to have $\gamma\gamma$ partial widths consistent with quark model predictions. However, mesons thought to be non-$q\bar{q}$ states have $\gamma\gamma$ widths far from $q\bar{q}$ quark model predictions. For this reason, it is clearly important to have accurate quark model predictions of $\gamma\gamma$ widths for all experimentally accessible $q\bar{q}$ mesons.

In this thesis, we are primarily interested in studying the application of our BSE

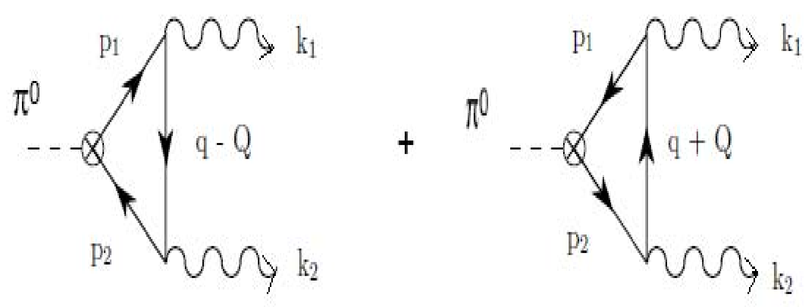

Figure 1.2: Triangular quark loops generating the decay $\pi^0 \to 2\gamma$

framework with the generalized vertex function described above to decays of a meson into two photons and calculating the Pion radiative decay constant F_P and Bethe-Salpeter normalizer for a pion which is a very lengthy calculation.

This thesis is organized as follows: Chapter 2 is devoted to a review of gauge theories QED and QCD. In chapter 3 we present the derivation of Bethe-Salpeter equation. In chapter 4 we calculate the radiative decay constants of pion after incorporation of NLO covariants in BS wave function. The calculation has also been done for pion BS normalizer in this section. Chapter 5 is devoted to summary and conclusion. We see the decay process in fig.1.2.

Chapter 2

Gauge Theory

2.1 Gauge Theories : QED and QCD

One of the most profound insights in theoretical physics is that interactions are derived from symmetry principles. The present belief is that all fundamental interactions originate by imposition of local gauge invariance on a free matter Lagrangian. This in turn is related with conservation of physical quantities(such as electric charge, color, etc.) The connection between symmetries and conservation laws is best discussed in the framework of Lagrangian field theory. In classical mechanics the particle can be obtained from Lagrange's equations[13], i.e

$$\frac{d}{dt}(\frac{\partial \pounds}{\partial \dot{q}_i}) - \frac{\partial \pounds}{\partial q_i} = 0, \tag{2.1.1}$$

where q_i are the generalized coordinates of the particles, t is the time variable. The Lagrangian is

$$L = T - V, \tag{2.1.2}$$

where T and V are the kinetic and potential energies of the system respectively. It is straightforward to extend the formalism from a discrete system, with coordinates $q(t)$, and generalized velocities $\dot{q}_i(t)$ to a classical field with generalized coordinates

$\phi(x)$ and generalized velocities $\partial_\mu \phi(x)$. The Lagrangian for a classical field is

$$\pounds = \pounds(\phi(x), \partial_\mu \phi(x)), \tag{2.1.3}$$

where the field ϕ is a function of the continuous space-time variable x_μ, and the corresponding Euler-Lagrange equation for $\phi(x)$ is

$$\frac{\partial \pounds}{\partial \phi} - \frac{\partial}{\partial x_\mu}\left(\frac{\partial \pounds}{\partial \frac{\partial \phi}{\partial x_\mu}}\right) = 0; \tag{2.1.4}$$

$$L = \int \pounds d^3x, \tag{2.1.5}$$

Here $\pounds$ is the Lagrangian density and L is the total Lagrangian for the system. We know that the invariance of Lagrangian under space translations, time transformations and rotations, leads to the conservation of linear momentum, energy and angular momentum respectively[13]. We are now interested in internal symmetry transformations on the Lagrangian. We first briefly discuss gauge transformations in connection with electromagnetism as well.

The transformations which A and ϕ may undergo while preserving E and B as well as the form of Maxwell‘s equations are called gauge transformations. The electric and magnetic fields are given in terms of scalar potential ϕ and vector potential $\vec{A}$ [12,13]as

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t} \tag{2.1.6}$$

and

$$\vec{B} = \vec{\nabla} \times \vec{A} \tag{2.1.7}$$

If we transform

$$\phi \rightarrow \phi' = \phi - \frac{\partial \Lambda(\vec{x}, t)}{\partial t} \tag{2.1.8}$$

and

$$A \rightarrow A' = A + \nabla\Lambda(\vec{x}, t), \tag{2.1.9}$$

Where $\Lambda(\vec{x}, t)$is an arbitrary function, we see that$\vec{E}$ and $\vec{B}$ and hence Maxwell's equations remain unchanged. These gauge transformations can be combined into a single equation by introducing the 4-vector potential [12,13]

$$A_\mu = (\vec{A}, i\phi). \tag{2.1.10}$$

The gauge transformations (2.1.6) and (2.1.7) can be combined into a single transformation equation[12,13],

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu\Lambda(x). \tag{2.1.11}$$

Maxwell's equations can also be written in a covariant form using the 4-current j_μ, where $j_\mu = (\vec{J}, ic\rho)$ [12,13] as

$$\partial_\mu F_{\mu\nu} = j_\nu, \tag{2.1.12}$$

where we have defined the field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{2.1.13}$$

which is invariant under the gauge transformation A_μ. That is

$$F'_{\mu\nu} = F_{\mu\nu}. \tag{2.1.14}$$

This will preserve the form of Maxwell's equations. Now the Lagrangian for electromagnetic field is given by

$$\pounds_{em} = -\frac{1}{4}F^2_{\mu\nu}. \tag{2.1.15}$$

Since the $F_{\mu\nu}$ is invariant under the gauge transformation on A_μ, $\pounds_{em}$ is also invariant under the gauge transformations(2.1.11).

There are two types of gauge transformations

1. Global Gauge Transformations, which are transformations that do not vary from point to point in space-time. and

2. Local Gauge Transformations, which are transformations that vary from point to point in space-time . Maxwell‘s equations are invariant under the local gauge transformations on $A_\mu(x)$. We now discuss quantum electrodynamics, the gauge theory of electromagnetic interactions.

2.2 Quantum Electrodynamics:

It is well known that the best understood particle interaction, quantum electrodynamics, is based on the concept of local gauge invariance[14]. Lets consider free Dirac Lagrangian

$$\pounds = -\bar{\psi}(x)(\gamma_\mu \partial_\mu + m)\psi(x). \tag{2.2.1}$$

We transform the Dirac field

$$\psi(x) \to \psi^{'}(x) = \exp(i\alpha)\psi(x) \tag{2.2.2}$$

and

$$\bar{\psi}(x) \to \bar{\psi}^{'}(x) = \bar{\psi}(x)\exp(-i\alpha), \tag{2.2.3}$$

where α is a real constant and has the same value at all space-time points. These are global gauge transformation on ψ and $\bar{\psi}$. The Lagrangian $\pounds$ would transform as

$$\pounds^{'} = -\bar{\psi}^{'}(x)(\gamma_\mu \partial_\mu + m)\psi^{'}(x). \tag{2.2.4}$$

It can be easily checked that

$$\pounds^{'} = \pounds. \tag{2.2.5}$$

since α is constant and the derivatives of field transform as,

$$\partial_\mu \psi(x) \rightarrow \exp(i\alpha)\partial_\mu \psi(x). \tag{2.2.6}$$

Therefore, $\pounds$ is invariant under global gauge transformation on $\psi(x)$. From eqn(2.2.3), we see that the phase α can be chosen arbitrarily. Lets now try to generalize global gauge transformations to local gauge transformations, where $\alpha = \alpha(x)$ which varies from point to point in space-time[14]. Lets now consider the local gauge transformation on $\psi(x)$ i.e

$$\psi(x) \rightarrow \psi^{'}(x) = \exp(i\alpha(x))\psi(x), \tag{2.2.7}$$

where $\alpha(x)$ now depends on space and time.

$$\bar{\psi}(x) \rightarrow \bar{\psi}^{'}(x) = \bar{\psi}(x)\exp(-i\alpha(x)). \tag{2.2.8}$$

The set of phase transformations $\{U(x) = \exp(i\alpha)\}$, forms a unitary U(1) group, which is abelian since elements of this group can commute [13]i.e.

$$[U(\alpha_1), U(\alpha_2)] = 0. \tag{2.2.9}$$

It is observed that the Lagrangian $\pounds$ is not invariant under the above local gauge transformations. Since the term $\partial_\mu \psi(x)$ transform as,

$$\partial_\mu \psi(x) \rightarrow \exp(i\alpha(x))\partial_\mu \psi(x) + i\exp(i\alpha(x))\psi(x)\partial_\mu \alpha(x), \tag{2.2.10}$$

where the second term in eqn(2.2.10) breaks the invariance of $\pounds$. Lets modify $\pounds$ to incorporate a term $i\bar{\psi}(x)\gamma_\mu \psi(x)A_\mu(x)$ and also pullout " e " from $\alpha(x)$ so that $\alpha(x) = e\Lambda(x)$. The modified Lagrangian becomes

$$\pounds = -\bar{\psi}(x)(\gamma_\mu \partial_\mu + m)\psi(x) + ie\bar{\psi}(x)\gamma_\mu \psi(x)A_\mu(x). \tag{2.2.11}$$

This modified Lagrangian is invariant under the combined local gauge transformations on ψ and $A_\mu(x)$.

We transform

$$\psi(x) \to \exp(i\theta(x))\psi(x) = \exp(ie\Lambda(x))\psi(x), \tag{2.2.12}$$

$$\bar{\psi}(x) \to \bar{\psi}(x)\exp(-ie\Lambda(x)) \tag{2.2.13}$$

and

$$A_\mu(x) \to A_\mu^{'}(x) = A_\mu(x) + \partial_\mu \Lambda(x) \tag{2.2.14}$$

The Lagrangian also transforms as

$$\pounds^{'} = -\bar{\psi}^{'}(x)(\gamma_\mu \partial_\mu + m)\psi^{'}(x) + ie\bar{\psi}^{'}(x)\gamma_\mu \psi^{'}(x)A_\mu^{'}(x). \tag{2.2.15}$$

Comparison of eqn(2.2.11) and eqn(2.2.15) shows that the Lagrangian in eqn(2.2.11) is invariant under the combined local gauge transformations on $\psi(x)$, $\bar{\psi}(x)$ and $A_\mu(x)$fields. It was seen that initially free Dirac Lagrangian was not invariant under local gauge transformations. To make $\pounds$ locally invariant requires introduction of a carrier gauge field $A_\mu(x)$ which should couple to matter fields ψ and $\bar{\psi}$ through a term $ie\bar{\psi}(x)\gamma_\mu\psi(x)A_\mu(x)$ in the Lagrangian, where "e" plays the role of a coupling constant between matter fields and electromagnetic (A_μ)fields. A simple(minimal) prescription to make the Dirac Lagrangian locally invariant requires replacing ∂_μ by D_μ[12,13] that is,

$$D_\mu = \partial_\mu - ieA_\mu(x). \tag{2.2.16}$$

we can then write the full locally invariant QED Lagrangian as

$$\pounds_{QED} = -\bar{\psi}(D_\mu + m)\psi - \frac{1}{4}F_{\mu\nu}F_{\mu\nu}. \tag{2.2.17}$$

In summary, we see that by imposing the local phase invariance on the free Dirac Lagrangian, leads to the introduction of a carrier gauge field which couples to matter in a mathematically self consistent manner and hence describing the interactions between charged particles and e.m field. This is a simple example of how electromagnetic interactions are generated by a gauge principle.

2.3 Quantum chromodynamics (QCD):

QCD is a theory of strong interaction (color force) describing the interactions of the quarks and gluons making up hadrons. We now discuss the non-Abelian gauge theory such as QCD. We can infer the structure of quantum chromodynamics from local gauge invariance. QCD is based on the extension of the QED idea but the $U(1)$ gauge group is replaced by the $SU(3)$ group of phase transformations on the quark color fields[13].

Lets consider a free Dirac Lagrangian for a given quark flavor

$$\pounds = -\bar{\psi}_R(\gamma_\mu \partial_\mu + m)\psi_R - \bar{\psi}_B(\gamma_\mu \partial_\mu + m)\psi_B - \bar{\psi}_G(\gamma_\mu \partial_\mu + m)\psi_G. \tag{2.3.1}$$

We can write $\pounds$ as

$$\pounds = -\bar{\psi}(\gamma_\mu \partial_\mu + m)\psi, \tag{2.3.2}$$

where $\psi = \begin{pmatrix} \psi_R \\ \psi_B \\ \psi_G \end{pmatrix}$ and $\bar{\psi} = \begin{pmatrix} \bar{\psi}_R & \bar{\psi}_B & \bar{\psi}_G \end{pmatrix}$

are a triplet of color fields each of which is a 4×1 Dirac *spinor* and $\begin{pmatrix} m_R & 0 & 0 \\ 0 & m_B & 0 \\ 0 & 0 & m_G \end{pmatrix}$ is a mass matrix. Let ψ transform as

$$\psi \rightarrow \psi' = U\psi; U = \exp(i\alpha(x)), \tag{2.3.3}$$

while $\bar{\psi}$ should transform as,

$$\bar{\psi} \rightarrow \bar{\psi}^{'} = \bar{\psi}U^{\dagger}. \tag{2.3.4}$$

$$\bar{\psi}^{'}\psi^{'} = \bar{\psi}U^{\dagger}U\psi = \bar{\psi}\psi, \tag{2.3.5}$$

where $U^{\dagger}U = 1$ and $\mid U \mid= 1$[15]

$$U = \exp(i\alpha(x)). \tag{2.3.6}$$

$$H = \alpha(I) + \lambda^{a}\alpha^{a}(x). \tag{2.3.7}$$

Since ψ is a 3×1 matrix, $\alpha(x)$ should be a 3×3 matrix, which can always be expressed as a linear superposition of a unit matrix and eight Gell-Man matrices $\lambda_1, ..., \lambda_8$ which are 3×3 traceless matrices and are the generators of $SU(3)$ group[13]. Thus we can write $\alpha(x)$ as

$$\alpha(x) = \alpha_o I + \lambda^{a}\alpha^{a}(x), \tag{2.3.8}$$

where α_o and $\alpha^{a}(x)(a = 1, ..., 8)$ are arbitrary parameters. Pulling out a constant g from $\alpha(x)$ and expressing it as $\alpha(x) = g\theta(x)$, we can write

$$U = \exp(ig\theta_o). \exp(ig\lambda^{a}\theta^{a}(x)). \tag{2.3.9}$$

We can explore the consequence of requiring $\pounds$ to be invariant under $SU(3)$ local gauge transformations of the form

$$\psi(x) \rightarrow \psi^{'}(x) = U\psi(x) = \exp(ig\theta^{a}(x)\lambda^{a})\psi(x), \tag{2.3.10}$$

where U is 3×3 unitary matrix. A summation over the repeated suffix is implied. λ^{a} with a=1,...,8 are a set of linearly independent traceless 3×3 matrices, and θ^{a}

are arbitrary parameters. The group is non-*abelian* since the generators λ^a do not commute with each other[13,15]. In general,

$$[\lambda^a, \lambda^b] = if_{abc}\lambda^c, \tag{2.3.11}$$

where f_{abc} are the structure constants of the $SU(3)$ group[14]. To impose $SU(3)$ local gauge invariance on the free Dirac Lagrangian, we follow the same steps as in the previous section.

We consider transformations

$$\psi(x) \to \exp(ig\theta^a(x)\lambda^a)\psi(x). \tag{2.3.12}$$

Then $\partial_\mu\psi$ would transform as,

$$\partial_\mu\psi(x) \to \exp(ig\theta^a(x)\lambda^a)\partial_\mu\psi(x) + ig\lambda^a\psi(x)\partial_\mu\theta^a. \tag{2.3.13}$$

The term $\partial_\mu\theta^a$ breaks the invariance of $\pounds$[13], we then proceed exactly as for QED. That is, we introduce eight massless gauge fields A^a_μ, now we first obtain the general transformation property of non-abelian gauge fields. Consider the transformation

$$\partial_\mu \to D_\mu = \partial_\mu - ig\lambda^a A^a_\mu(x), \tag{2.3.14}$$

where D_μ is a covariant derivative introduced to make $\pounds$ locally invariant. We let $D_\mu\psi$ transform as,

$$D_\mu\psi \to (D_\mu\psi)^{'} = U(\alpha)D_\mu\psi. \tag{2.3.15}$$

Substituting eqn(2.3.14) into eqn(2.3.15) then we obtain

$$\partial_\mu U - ig\vec{\lambda}^{a'}\vec{A}^{a'}U = -igU(\vec{\lambda}^{a'}.\vec{A}^{a'}). \tag{2.3.16}$$

Multiplying this equation both sides by U^{-1} from the right, then we get

$$(\partial_\mu U)U^{-1} - ig\lambda^{a'}.\vec{A}^{a'} = -igU(\vec{\lambda}^{a'}.\vec{A}^{a'})U^{-1}. \tag{2.3.17}$$

We can rewrite this equation as,

$$\vec{\lambda}^{a'}.\vec{A}^{a'} = U(\vec{\lambda}^{a'}.\vec{A}^{a'})U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1}. \tag{2.3.18}$$

Lets denote $\lambda^{a'}.A^{a'} = A_\mu(x)$, where A_μ is a 3×3 matrix. Thus, the transformation property of non-abelian gauge fields A_μ is given by

$$A_\mu^a \to UA_\mu^a U^{-1} - i\frac{1}{g}(\partial_\mu U)U^{-1}. \tag{2.3.19}$$

Using this gauge fields the covariant derivative can be written as,

$$D_\mu = \partial_\mu - igA_\mu^a(x). \tag{2.3.20}$$

Then, we replace ∂_μ byD_μ in the Lagrangian and we obtain

$$\pounds = -\bar{\psi}(x)(\partial_\mu + m)\psi(x) - ig\bar{\psi}(x)\gamma_\mu\lambda^a\psi(x)A_\mu^a(x). \tag{2.3.21}$$

To obtain the gluon field strength tensor, we consider two covariant derivative D_μ and D_ν that is, we make use of the relation connecting $G_{\mu\nu}$ with the covariant derivatives[15]

$$G_{\mu\nu} = [D_\mu, D_\nu]. \tag{2.3.22}$$

We find the commutator of these two covariant derivatives is

$$[D_\mu, D_\nu] = -ig(\partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu])[15]. \tag{2.3.23}$$

Thus we get the gluon field strength tensor $G_{\mu\nu}$[15] as,

$$G_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]). \tag{2.3.24}$$

This is the general expression of the field strength tensor for both QED and QCD. We first see the invariant of $G_{\mu\nu}$ this field tensor should transform as,

$$G_{\mu\nu} \to G'_{\mu\nu} = (\partial_\mu A'_\nu - \partial_\nu A'_\mu - ig[A'_\mu, A'_\nu]). \tag{2.3.25}$$

Using the transformation of $A_\mu(x)$, we can write $G'_{\mu\nu}$ as,

$$G'_{\mu\nu} = [[\partial_\mu(UA_\nu U^{-1} - \frac{i}{g}(\partial_\nu U)U^{-1})] - [\partial_\nu(UA_\mu U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1})]$$
$$-ig[UA_\mu U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1}, UA_\nu U^{-1} - \frac{i}{g}(\partial_\nu U)U^{-1}]]. \quad (2.3.26)$$

We expand the above equation and using $\partial_\mu = U(\partial_\mu)U^{-1}$ we cancel a number of terms and we get,

$$G'_{\mu\nu} = U[(\partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu])]U^{-1}. \quad (2.3.27)$$

Thus, we see that the gauge field tensor in QCD transform as[14],

$$G_{\mu\nu} \rightarrow G'_{\mu\nu} = UGU^{-1}. \quad (2.3.28)$$

Using this gauge field strength we can find the free gauge field Lagrangian $\pounds'_{gaugefield}$ can be written as:

$$\pounds'_{gaugefield} = -\frac{1}{4}Tr[G'_{\mu\nu}G'_{\mu\nu}] = -\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} = \pounds gaugefield. \quad (2.3.29)$$

To write the $L_{gaugefield}$ in terms of the structure constant f_{abc} we solve the commutator of A^a_μ and A^a_ν,

$$[A_\mu, A_\nu]^a = A^b_\mu A^c_\nu[\lambda^b, \lambda^c].$$
$$[A_\mu, A_\nu]^a = A^b_\mu A^c_\nu i f_{abc}\lambda^a. \quad (2.3.30)$$

The $\pounds_{gaugefield}$ becomes[15]

$$\pounds_{gaugefield} = -\frac{1}{4}[\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf_{abc}A^b_\mu A^c_\nu][\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf_{amn}A^m_\mu A^n_\nu].$$

$$\pounds_{gaugefield} = -\frac{1}{4}[(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2 + 2g f_{abc} A^b_\mu A^c_\nu (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) + g^2 f_{abc} f_{amn} A^b_\mu A^c_\nu A^m_\mu A^n_\nu]. \tag{2.3.31}$$

Finally, we may add to $\pounds$ a gauge invariant kinetic energy term for each of the A^a_μ fields. The final gauge invariant QCD Lagrangian is then

$$\pounds_{QCD} = -\bar{\psi}(\gamma_\mu \partial_\mu + m)\psi + ig\bar{\psi}\gamma_\mu \lambda^a \psi A^a_\mu - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}. \tag{2.3.32}$$

which can be written as,

$$\begin{aligned}\pounds_{QCD} = &-\bar{\psi}(\gamma_\mu \partial_\mu + m)\psi + ig\bar{\psi}\gamma_\mu \lambda^a \psi A^a_\mu - \frac{1}{4}[(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2 \\ &+ 2g f_{abc} A^b_\mu A^c_\nu (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) + g^2 f_{abc} f_{amn} A^b_\mu A^c_\nu A^m_\mu A^n_\nu].\end{aligned} \tag{2.3.33}$$

This is the Lagrangian for interacting colored quarks q and vector gluons A_μ, with coupling specified by g. Besides the fact that the matter field couples to the gauge field, the other complicating feature of the non-abelian case is that the gauge field couples to itself.

2.4 Feynman Rules For QCD

To see Feynman rules for QCD we take the Lagrangian of the gauge field L $_{gaugefield}$ from eqn(2.3.31) where we have three terms we see the Feynman rules for each terms. First we start with the first term that contains only the free gluon fields, that is given by

$$\pounds_{freegluon} = -\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu). \tag{2.4.1}$$

Using this Lagrangian we can obtain the free gluon propagator, to find the free gluon propagator we use the Euler-Lagrange equation, given by

$$\frac{\partial \pounds_{fg}}{\partial A_\mu^a} - \frac{\partial \pounds_{fg}}{\partial(\partial_\nu A_\mu^a)} = 0. \tag{2.4.2}$$

Substitute eqn (2.4.1) into eqn (2.4.2) we get

$$\Box^2 A_\nu^a \delta_{\mu\nu} - \partial_\mu \partial_\nu A_\nu^a = 0. \tag{2.4.3}$$

Imposing Lorentz gauge condition $\partial_\nu A_\nu^a = 0$ this equation becomes

$$\Box^2 A_\nu^a \delta_{\mu\nu} = 0. \tag{2.4.4}$$

Now we take the fourier transform of A_ν^a

$$A_\nu^a(x) = \frac{1}{(2\pi)^4} \int d^4k \exp(ikx) A_\nu^a(k). \tag{2.4.5}$$

Using the fourier transform of A_ν^a eqn (2.4.4) becomes

$$-k^2 A_\nu^a \delta_{\mu\nu} = 0. \tag{2.4.6}$$

Now taking the inverse of eqn (2.4.6) we can obtain the propagator for free gluon field[15], that is

$$D_{\mu\nu} = -\frac{\delta_{\mu\nu}}{k^2}. \tag{2.4.7}$$

Next, we see the second term in eqn(2.3.31). In this term we see that there is interaction between three gauge fields and we solve the vertex function for this 3-gauge fields. From eqn (2.3.31) the Lagrangian for the interaction of 3-gauge fields is given by

$$\pounds_{3-g} = -\frac{1}{2} g f_{abc} A_\mu^b A_\nu^c (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a). \tag{2.4.8}$$

Using all permutations of a,b,c we can rewrite $\pounds_{3-g}$ as,

$$\pounds_{3-g} = -gf_{abc}(\partial_\mu A^a_\nu)A^b_\mu A^c_\nu - gf_{acb}(\partial_\mu A^a_\nu)A^c_\mu A^b_\nu - gf_{bca}(\partial_\mu A^b_\nu)A^c_\mu A^a_\nu$$

$$-gf_{bac}(\partial_\mu A^b_\nu)A^a_\mu A^c_\nu - gf_{cab}(\partial_\mu A^c_\nu)A^b_\mu A^a_\nu - gf_{cab}(\partial_\mu A^c_\nu)A^a_\mu A^b_\nu. \tag{2.4.9}$$

Using the fourier transform of A^a_ν ,A^b_ν, A^c_ν from eqn (2.4.5) we again write $\pounds_{3-g}$ as,

$$\pounds_{3-g} = -igf_{abc}[(k_3 - k_1)A^b_\nu A^a_\nu A^c_\nu + (k_1 - k_2)A^c_\nu A^a_\nu A^b_\nu + (k_2 - k_3)A^a_\nu A^b_\nu A^c_\nu]. \tag{2.4.10}$$

The 3-gluon vertex function in covariant form is then given by[15]

$$-igf_{abc}[(k_\gamma - k_\alpha)_\beta \delta_{\alpha\gamma} + (k_\alpha - k_\beta)_\gamma \delta_{\alpha\beta} + (k_\beta - k_\gamma)_\alpha \delta_{\beta\gamma}]. \tag{2.4.11}$$

Similarly we can see the interaction between 4-gluon fields from the third term of eqn (2.3.31). Using this term we can find the 4- gluon vertex function. The Lagrangian for 4-gluon fields is given by

$$\pounds_{4-g} = g^2 f_{abc} f_{ade} A^b_\mu A^c_\nu A^d_\mu A^e_\nu. \tag{2.4.12}$$

we can rewrite $\pounds_{4-g}$

$$\pounds_{4-g} = g^2 f_{nab} f_{ncd} A^a_\mu A^b_\nu A^c_\mu A^d_\nu. \tag{2.4.13}$$

Using all permutations of a,b,c and d we again write L_{4-g}

$$\pounds_{4-g} = -\frac{1}{4}g^2[f_{nab}f_{ncd}((A^aA^c)(A^bA^d) - (A^aA^d)(A^bA^c))+$$

$$f_{nac}f_{nbd}((A^aA^b)(A^cA^d) - (A^aA^d)(A^bA^c)) + f_{nad}f_{nbc}((A^aA^b)(A^dA^c) - (A^dA^b)(A^aA^c)]. \tag{2.4.14}$$

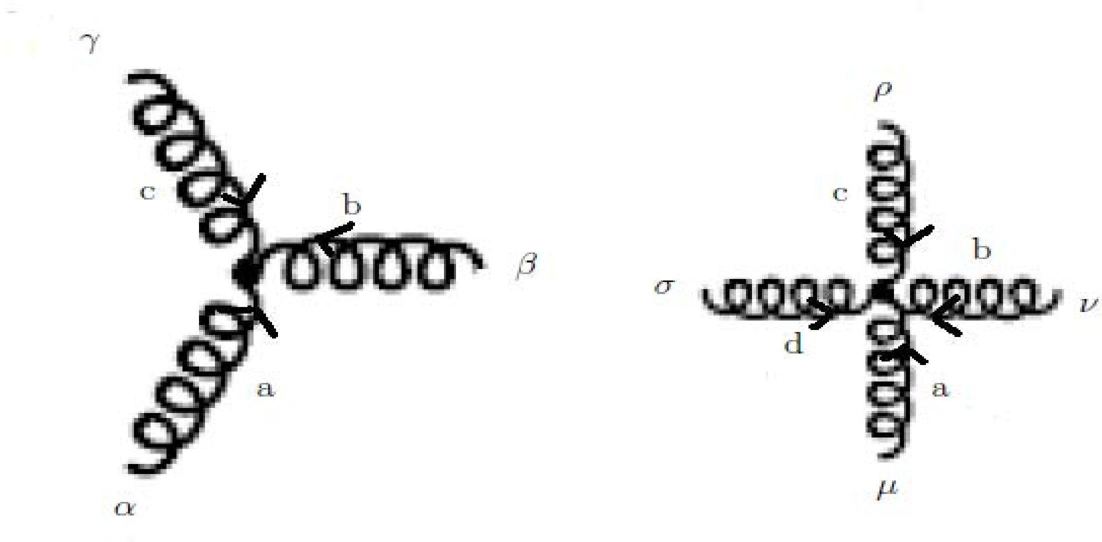

Figure 2.1: 3-gluon and 4-gluon vertex function

Finally the 4-gluon vertex function in covariant form is[15]

$$-\frac{1}{4}g^2[f_{nab}f_{ncd}(\delta_{\mu\rho}\delta_{\nu\sigma}-\delta_{\mu\sigma}\delta_{\nu\rho})+f_{nac}f_{nbd}(\delta_{\mu\nu}\delta_{\rho\sigma}-\delta_{\mu\sigma}\delta_{\rho\nu})+f_{nad}f_{nbc}(\delta_{\mu\nu}\delta_{\rho\sigma}-\delta_{\nu\sigma}\delta_{\rho\mu})]. \quad (2.4.15)$$

We can also see that the 3-g and 4-g vertex functions in fig.2.1.

Chapter 3

Bethe-Salepeter Equation

Bethe-Salpeter equation is a relativistic equation for bound state problems and the simplest n-body equation that follows from field theory, explicitly covariant and Intrinsically non-perturbative. The Bethe-Salpeter equation is a direct application of the Feynman rules, and a resummation of the finite set of diagrams by using integral equation.

Among the various approaches used in meson physics, the formalism of Bethe-Salpeter and Dyson-Schwinger equations (DSEs) plays a traditional and indispensable role. The Bethe-Salpeter equation (BSE) provides a field-theoretical starting point to describe hadrons as relativistic bound states of quarks and/or antiquarks. For instance, the DSE and BSE framework has been widely used in order to obtain nonperturbative information about the spectra and decays of the whole lightest pseudoscalar nonet, with an emphasis on the QCD pseudo-Goldstone boson the pion[16]. Moreover, the formalism satisfactorily provides a window to the next-scale meson sector, too, including vector, scalar[17] and excited mesons. Finally, electromagnetic form factors of mesons have been calculated with this approach for space-like momenta.

When dealing with bound states composed of light quarks, then it is unavoidable to use the full covariant BSE framework. Nonperturbative knowledge of the Greens function, which makes part of the BSE kernel, is required. Very often, the problem is solved in Euclidean space, where it is more tractable, as there are no Greens function singularities there. Different approaches have been developed to reduce the computational complexity of the full four dimensional (4D) BSE. The so-called instantaneous [18] and quasi-potential approximations can reduce the 4D BSE to a 3D equation in a Lorentz-covariant manner. In practice, such 3D equations are much more tractable, since their resolution is less involved, especially if one exploits the considerable freedom in performing the 3D reduction. Also note that, contrary to the BSE in the ladder approximation, these equations reduce to the Schrodinger equation of nonrelativistic Heavy-Meson Effective Theory and nonrelativistic QCD. However, the interaction kernels of the reduced equations often correspond to input based on economical phenomenological models, and the connection to the underlying theory (QCD) is less clear (if not abandoned from the onset). We now introduce the one-particle Dirac propagator in an external field.

3.1 One Particle Dirac Propagator In An External Field

We recall the Schrodinger's equation[19]

$$(i\frac{\partial}{\partial t} - H)\psi(x) = 0. \tag{3.1.1}$$

The propagator $K(\vec{x}, t; \vec{x}^{'}, t^{'})$ satisfies the equation(for $t > t^{'}$):

$$(i\frac{\partial}{\partial t} - H)K(\vec{x}, t; \vec{x}^{'}, t^{'}) = i\delta^3(\vec{x} - \vec{x}^{'})\delta(t - t^{'}). \tag{3.1.2}$$

If replacement is conducted between the Schrodinger hamiltonian and the Dirac hamiltonian since both these equations are linear in time derivatives, eqn(3.1.1) for a Dirac particle will be:

$$(i\frac{\partial}{\partial t} - \vec{\alpha}\vec{p} - \beta m)\psi(x) = 0. \tag{3.1.3}$$

This is the Dirac equation in the absence of A_μ field. In the presence of A_μ field, with using minimal coupling prescription[19]

$$p_\mu \rightarrow p_\mu - eA_\mu, \tag{3.1.4}$$

$$\vec{\nabla} \rightarrow \vec{\nabla} - ie\vec{A} \tag{3.1.5}$$

and

$$i\frac{\partial}{\partial t} \rightarrow i\frac{\partial}{\partial t} - e\phi. \tag{3.1.6}$$

Eqn(3.1.3) becomes

$$(i\frac{\partial}{\partial t} - e\phi + i\vec{\alpha}.\vec{\nabla} + e\vec{A}\vec{\alpha} - \beta m)\psi(x,t) = 0. \tag{3.1.7}$$

To make this equation covariant we multiply by $-\beta$ from the left on both sides of the above equation and obtain,

$$(\partial\!\!\!/_\mu - ieA\!\!\!/_\mu + m)\psi(x,t) = 0. \tag{3.1.8}$$

Lets consider $K(x,t;x^{'},t^{'})$ be the full Dirac propagator, then $K(x,t;x^{'},t^{'})$ would satisfy the equation[19],

$$(i\frac{\partial}{\partial t} - ieA\!\!\!/_\mu + m)K(x,t;x^{'},t^{'}) = -i\delta^4(x - x^{'}). \tag{3.1.9}$$

We can also write the above equation as,

$$(\partial\!\!\!/_\mu - ieA\!\!\!/_\mu + m)K(x,x^{'}) = -i\delta^4(x - x^{'}), \tag{3.1.10}$$

where $\gamma_\mu.\partial_\mu = \not{\partial}$. We can call equation (3.1.10) the full propagator equation. Whereas the propagator for free Dirac particle satisfies the equation,

$$(\not{\partial}_\mu + m)K_o(x, x^{'}) = -i\delta^4(x - x^{'}). \tag{3.1.11}$$

We can also write eqn(3.1.10) as

$$(\not{\partial}_\mu + m)K(x, x^{'}) = -i\delta^4(x - x^{'}) + ie\not{A}_\mu(x)K(x, x^{'}). \tag{3.1.12}$$

This can in turn be written as

$$(\not{\partial}_\mu+m)K(x, x^{'}) = -i\int d^4x^{''}\delta^4(x-x^{''})\delta^4(x^{''}-x^{'})+ie\int d^4x^{''}\not{A}_\mu(x^{''})K(x^{''}, x^{'})\delta^4(x^{''}-x). \tag{3.1.13}$$

But we have also $(\not{\partial}_\mu + m)K_o(x, x^{''}) = -i\delta^4(x - x^{''})$, then eqn(3.1.10) becomes

$$(\not{\partial}_\mu + m)K(x, x^{'}) = \int d^4x^{''}(\not{\partial}_\mu + m)K_o(x, x^{''})\delta^4(x^{''} - x^{'})$$

$$-e\int d^4x^{''}(\not{\partial}_\mu + m)K_o(x, x^{''})\not{A}_\mu(x^{''})K(x^{''}, x^{'}). \tag{3.1.14}$$

The complete Dirac propagator $K(x, x^{'})$ can thus be written as,

$$K(x, x^{'}) = K_o(x, x^{'}) - e\int d^4x^{''}K_o(x, x^{''})\not{A}_\mu(x^{''})K(x^{''}, x^{'}). \tag{3.1.15}$$

We can also write $K(x^{''}, x^{'})$ as

$$K(x^{''}, x^{'}) = K_o(x^{''}, x^{'}) - e\int d^4x^{'''}K_o(x^{''}, x^{'''})\not{A}_\mu(x^{'''})K(x^{'''}, x^{'}). \tag{3.1.16}$$

This can be written in a symbolic form by making substitutions;

$$x \Rightarrow 1, x^{'} \Rightarrow 2, x^{''} \Rightarrow 3, x^{'''} \Rightarrow 4. \tag{3.1.17}$$

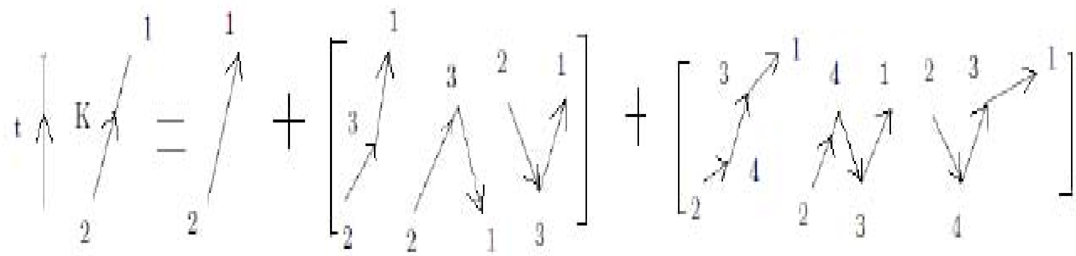

Figure 3.1: Single particle propagation

Therefore $K(1,2)$ is given by an infinite series;

$$K(1,2) = K_o(1,2) - e\int d^4 3 K_o(1,3) \not{A}_\mu(3) K_o(3,2)$$

$$-e\int d^4 3 \int d^4 4 K_o(1,3) \not{A}_\mu(3) K_o(3,4) \not{A}_\mu(4) K_o(4,2) + ... \tag{3.1.18}$$

In Dirac particle case, both the time directions ($\uparrow e^-$) and ($\downarrow e^+$) are allowed.

3.2 Two Particle Dirac Propagator

We now generalize to two-particle Dirac propagator with particles in mutual interaction. First define 2-particle free propagator for simultaneous propagation of two free particles. The free propagator for simultaneous propagation of two particles from $(3,4) \rightarrow (1,2)$ is

$$K_{oa}(1,3) K_{ob}(2,4) = K_o(12;34). \tag{3.2.1}$$

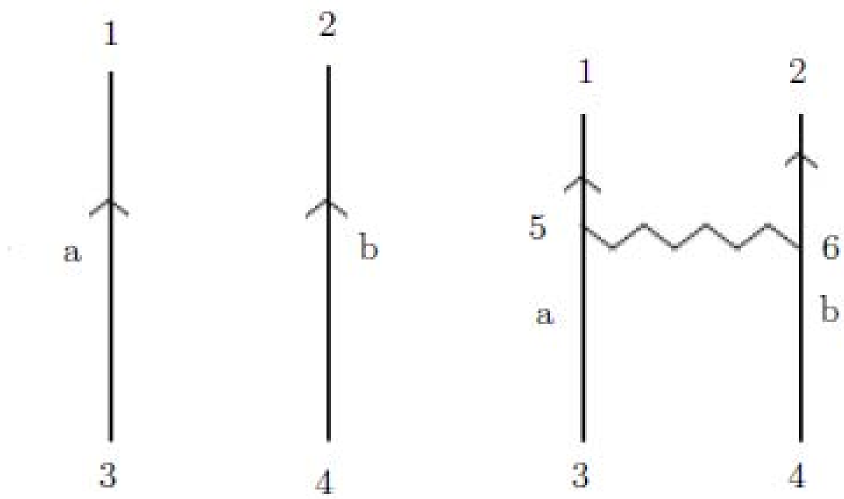

Figure 3.2: Propagation of two non interacting particles and two particles of the propagator interact through a single photon.

Lets consider the two particles a and b interact through the exchange of a single photon. The Coulomb interaction between these particles is

$$\frac{e^2}{r_{ab}}. \tag{3.2.2}$$

However, this Coulomb interaction is non-covariant. To write the mutual interaction between these two particles in a covariant form lets take the fourier transform of this equation in momentum space

$$\frac{e^2}{r_{ab}} \Rightarrow \frac{e^2}{(p_1 - p_2)^2}\gamma_4^a\gamma_4^b. \tag{3.2.3}$$

we can write this equation in covariant form

$$\frac{e^2}{r_{ab}} \Rightarrow -e^2\gamma^a_\mu(\frac{-1}{q^2})\gamma^b_\mu; q = p_1 - p_2,$$

where q is the momentum exchanged between the two electrons. Thus

$$\frac{e^2}{r_{ab}} \Rightarrow -e^2\gamma^a_\mu(D_F(x-x^{'}))\gamma^b_\mu = I(5,6), \tag{3.2.4}$$

where the RHS in the above equation is interaction potential and $D_F(x-x^{'})$ is the photon fourier transform of propagator on momentum space and is given as

$$D_F(x-x^{'}) = \frac{1}{(2\pi)^4}\int d^4q(\frac{-1}{q^2})\exp(iq(x-x^{'})). \tag{3.2.5}$$

To generalize a single particle propagator to a two particle propagator consider simultaneous propagation of two particles for two charged particles propagating from space-time points $(1,2)$ to $(3,4)$ and interacting through a photon exchange connecting them at space-time vertices 5 and 6, we can write the first order correction to the propagator

$$K_1(12;34) = \int d^45\int d^46K_o(12;56)I(5,6)K_o(56;34). \tag{3.2.6}$$

This is first order correction to two particle propagator due to simultaneous interaction of both Dirac particles through exchange of a single photon. Similarly we can write

$$K_2(12;34) = \int d^45\int d^46\int d^47\int d^48K_o(12;56)I(5,6)K_o(56;78)I(7,8)K_o(78;34). \tag{3.2.7}$$

which is second order correction to two particle propagator. If we iterate one quantum exchange between two particles, we can write the two particle propagator as an infinite series;

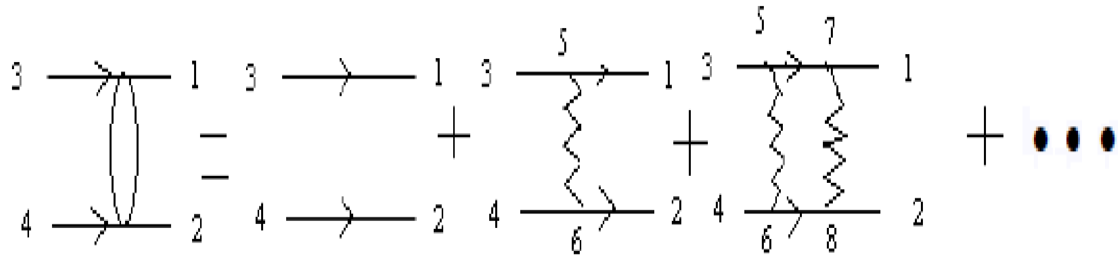

Figure 3.3: Two particle propagation

$$K(12;34) = K_o(12;34) + \int d^4 5 \int d^4 6 K_o(12;56) I(5,6) K_o(56;34)$$

$$+ \int d^4 5 \int d^4 6 \int d^4 7 \int d^4 8 K_o(12;56) I(5,6) K_o(56;78) I(7,8) K_o(78;34) + ... \quad (3.2.8)$$

which can in turn be written as

$$K(12;34) = K_o(12;34) + \int d^4 5 \int d^4 6 K_o(12;56) I(5,6) K(56;34), \quad (3.2.9)$$

where $K(56,34)$ is the full two-particle propagator for simultaneous propagation of two-particles from $(3,4) to (5,6)$, thus we arrive exactly to an integral equation written symbolically as

$$K = K_o + K_o I K. \quad (3.2.10)$$

which is complete Green's function for two particles as a sequential action of Moller interaction in ladder approximation.

To find the two particle wave function we can use the full two particle propagator. Consider the wave function $\psi(1,2)$ which can be written in terms of the initial wave function $\psi(3,4)$ as,

$$\psi(1,2) = \int d^3 3 \int d^3 4 \beta^a \beta^b K(12;34)\psi(3,4). \tag{3.2.11}$$

Substitute eqn(3.2.9) into eqn(3.2.11)and we obtain

$$\psi(1,2) = \int d^3 3 \int d^3 4 \beta^a \beta^b [K_o(12;34) + \int d^4 5 \int d^4 6 K_o(12;56) I(5,6) K_o(56;34)]\psi(3,4). \tag{3.2.12}$$

The complete two particle wave function is

$$\psi(1,2) = \psi_o(1,2) + \int d^4 5 \int d^4 6 K_o(12;56) I(5,6)\psi(5,6). \tag{3.2.13}$$

3.3 Derivation Of Bethe-Salpeter Equation

To obtain BSE we use eqn(3.2.13). Lets operate this equation both side by Dirac operator

$$(\partial\!\!\!/^a_\mu + m)(\partial\!\!\!/^b_\mu + m). \tag{3.3.1}$$

Then, we obtain

$$(\partial\!\!\!/^a_\mu + m)(\partial\!\!\!/^b_\mu + m)\psi(1,2) = (\partial\!\!\!/^a_\mu + m)(\partial\!\!\!/^b_\mu + m)\psi_o(1,2)$$
$$+(\partial\!\!\!/^a_\mu + m)(\partial\!\!\!/^b_\mu + m) \int d^4 5 \int d^4 6 K_o(12;56) I(5,6)\psi(5,6). \tag{3.3.2}$$

If two particles after interaction form a bound state, then

$$\psi_o(1,2) = 0. \tag{3.3.3}$$

Equation(3.3.2) is then reduced to

$$(\partial\!\!\!/^a_\mu + m)(\partial\!\!\!/^b_\mu + m)\psi(1,2) = (\partial\!\!\!/^a_\mu + m)(\partial\!\!\!/^b_\mu + m)\int d^4 5 \int d^4 6 K_o(12;56) I(5,6)\psi(5,6). \tag{3.3.4}$$

which gives

$$(\partial\!\!\!/^a_\mu + m)(\partial\!\!\!/^b_\mu + m)\psi(1,2) = \int d^4 5 \int d^4 6(-i\delta^4(1,5))(-i\delta^4(2,6)) I(5,6)\psi(5,6). \tag{3.3.5}$$

Thus, we get,

$$(\partial\!\!\!/^a_{1\mu} + m)(\partial\!\!\!/^b_{2\mu} + m)\psi(1,2) = -I(1,2)\psi(1,2). \tag{3.3.6}$$

This equation is two particle mutual interaction equation written down in covariant form. Where $I(1,2)$ is interaction potential. Lets now introduce two new co-ordinates,

$$\bar{X} = \frac{x_1 + x_2}{2} \qquad and \qquad x = x_1 - x_2. \tag{3.3.7}$$

Using equation (3.3.7) we can write $\partial_{1\mu}$ and $\partial_{2\mu}$ in the following way

$$\partial_{1\mu} = \frac{1}{2}\partial_{\bar{X}\mu} + \partial_{x\mu} \qquad and \qquad \partial_{2\mu} = \frac{1}{2}\partial_{\bar{X}\mu} - \partial_{x\mu}. \tag{3.3.8}$$

After substituting eqn(3.3.8) into eqn(3.3.6) we obtain

$$(\frac{1}{2}\partial\!\!\!/_{\bar{X}\mu} + \partial\!\!\!/_{x\mu} + m_a)(\frac{1}{2}\partial\!\!\!/_{\bar{X}\mu} - \partial\!\!\!/_{x\mu} + m_b)\psi(\bar{X},x) = -I(x)\psi(\bar{X},x). \tag{3.3.9}$$

where $\psi(\bar{X},x) = \psi_1(\bar{X})\psi_2(x)$

Now we take the fourier transform of $\psi(\bar{X})$, $I(x)$ and $\psi(x)$ and the fourier transform of these functions are given by

$$\psi(\bar{X}) = \frac{1}{(2\pi)^4}\int d^4P \exp(iP\bar{X})\psi(P). \tag{3.3.10}$$

$$\psi(x) = \frac{1}{(2\pi)^4}\int d^4q \exp(iqx)\psi(q). \tag{3.3.11}$$

$$I(x) = \frac{1}{(2\pi)^4}\int d^4k \exp(ikx)I(k). \tag{3.3.12}$$

Inserting eqn(3.4.10), eqn (3.3.11) and eqn (3.3.12) into eqn (3.3.9) after simplification we get

$$\int d^4q(\frac{1}{2}i\not{P}_\mu + i\not{q} + m_a)(\frac{1}{2}i\not{P}_\mu - i\not{q} + m_b)\psi(P)\psi(q)\exp(iqx) =$$
$$-\frac{1}{(2\pi)^4}\int d^4q^{'}\int d^4k \exp(i[k+q^{'}])I(k)\psi(P)\psi(q^{'}). \tag{3.3.13}$$

Now to simplify eqn(3.3.13) we see the right hand side of this equation

Lets take $q = k + q^{'}$ that is $d^4k = d^4q$. This implies $q^{'}$ is constant. Then eqn (3.3.13) becomes

$$(\frac{1}{2}i\not{P}_\mu + i\not{q} + m_a)(\frac{1}{2}i\not{P}_\mu - i\not{q} + m_b)\psi(P)\psi(q) =$$
$$-\frac{1}{(2\pi)^4}\int d^4q^{'} I(q,q^{'})\psi(P)\psi(q). \tag{3.3.14}$$

For a quark and anti-quark forming a $q\bar{q}$ meson, the total momentum of the hadron is $P = p_1 + p_2$ and the relative momentum of quark and anti-quark is given by $q = \frac{1}{2}(p_1 - p_2)$, we can take the momenta of the constituents as, $p_1 = \frac{1}{2}P + q$ and $p_2 = \frac{1}{2}P - q$. And $\psi(P)\psi(q)$ also written as $\psi(P,q)$. Then we rewrite eqn (3.3.14) as

$$(m_1 + i\gamma^a p_1)(m_2 - i\gamma^b p_2)\psi(P,q) = \frac{1}{(2\pi)^4}\int d^4q^{'} K(q,q^{'})\psi(P,q). \tag{3.3.15}$$

Lets define

$$\psi(P,q) = (m_1 + i\gamma^a p_1)(m_2 - i\gamma^b p_2)\Phi(P,q), \tag{3.3.16}$$

where $\Phi(P,q)$ is the $4D$ wave function. Inserting eqn (3.3.16) into eqn (3.3.15) and after simplification we obtain

$$(m_1^2 + p_1^2)(m_2^2 + p_2^2)\Phi(P,q) = \frac{i}{(2\pi)^4}\int d^4q^{'}\bar{K}(q,q^{'})\Phi(P,q), \tag{3.3.17}$$

where $\bar{K}(q,q^{'}) = K(q,q^{'})(m_1 + i\gamma^a p_1)(m_2 - i\gamma^b p_2)$ is the qq interaction kernel. Now we also define the inverse propagators for two constituent quarks $\triangle_1 = (m_1^2 + p_1^2)$ and $\triangle_2 = m_2^2 + p_2^2)$. The full Bethe-Salpeter Equation for spinless quarks is given by

$$\triangle_1 \triangle_2 \Phi(P,q) = \frac{i}{(2\pi)^4} \int d^4 q^{'} \bar{K}(q,q^{'}) \Phi(P,q). \tag{3.3.18}$$

3.4 BSE under Covariant Instantaneous Ansatz (CIA)

We start with a 4D BSE for scalar $q\bar{q}$ system with an effective kernel K and 4D wave function $\Phi(P,q)$ as,

$$i(2\pi)^4 \triangle_1 \triangle_2 \Phi(P,q) = \int d^4 q^{'} \bar{K}(q,q^{'}) \Phi(P,q), \tag{3.4.1}$$

where $\triangle_{1,2}$ the inverse propagators of two scalar quarks,

$$\triangle_{1,2} = m_{1,2}^2 + p_{1,2}^2), \tag{3.4.2}$$

where $m_{1,2}$ are constituent masses of quarks. The 4-momentum of the quark and anti-quark, $p_{1,2}$ are related to the internal momentum q_μ and total momentum P of hadron of mass M as

$$p_{1,2\mu} = \hat{m_{1,2}} P_\mu \pm q_\mu, \tag{3.4.3}$$

where $\hat{m_{1,2}} = (\frac{1\pm(\frac{m_1^2-m_2^2}{M^2})}{2})$ are the Wightman-Garding(WG) definitions of masses of individual quarks. Using the CIA the kernel can be written as[6,7]

$$K(q,q^{'}) = K(\hat{q},\hat{q}^{'}), \tag{3.4.4}$$

where we split the internal momentum $q_\mu = (\hat{q}_\mu, iM\sigma)$ the transverse component of q_μ is[6,7]

$$\hat{q}_\mu = q_\mu - (\frac{q\cdot P}{P^2})P_\mu, \tag{3.4.5}$$

while its longitudinal component is[6,7]

$$M\sigma = M(\frac{q\cdot P}{P^2}). \tag{3.4.6}$$

The total 4-momentum P_μ of the hadron is orthogonal to $\hat{q}_\mu$ (i.e, $\hat{q}.P = 0$) irrespective of whether the individual quarks are on-shell or off-shell. The 4-dimensional volume element in momentum space can be expressed as

$$d^4q = d^3\hat{q}Md\sigma. \tag{3.4.7}$$

To obtain the hadron-quark vertex function $\Gamma(\hat{q})$ we first change the BSE to the $3D$ form, we define a $3D$ wave function $\phi(\hat{q}^{'})$ as

$$\phi(\hat{q}) = \int_{-\infty}^{\infty} Md\sigma\Phi(P,q). \tag{3.4.8}$$

Using the kernel and the $3D$ wave function BSE can also written as

$$i(2\pi)^4\triangle_1\triangle_2\Phi(P,q) = \int d^3\hat{q}^{'}K(\hat{q},\hat{q}^{'})\phi(\hat{q}^{'}). \tag{3.4.9}$$

Integrating LHS of eqn (3.4.1) $Md\sigma$ and noting that $d^4q = d^3\hat{q}Md\sigma$, we obtain the $3D$ BSE

$$(2\pi)^3\phi(\hat{q})D(\hat{q}) = \int d^3\hat{q}^{'}K(\hat{q},\hat{q}^{'})\phi(\hat{q}^{'}). \tag{3.4.10}$$

which was used earlier for determination of $q\bar{q}$ mass spectra. Here $D(\hat{q})$ is $3D$ Denominator function, its expression is given by

$$\frac{1}{D(\hat{q})} = \int \frac{M d\sigma}{2\pi i \triangle_1 \triangle_2}. \tag{3.4.11}$$

and whose value can be determined by carrying out pole integration over the complex σ-plane. Then by equating the left hand side equation of (3.4.9) and (3.4.10) we finally obtain the hadron-quark vertex function,

$$\Gamma(\hat{q}) = \triangle_1 \triangle_2 \Phi(P, q) = \frac{\phi(\hat{q}) D(\hat{q})}{2\pi i}. \tag{3.4.12}$$

In the process, we obtain an exact interconnection between $3D$ wave function $\phi(\hat{q})$ and $4D$ wave function $\Phi(P, q)$ where the 3D form serves for making contact with the mass spectrum of hadrons, whereas the 4D form provides the H$q\bar{q}$ vertex function $\Gamma(\hat{q})$ which is to be employed for calculating transition amplitude for various processes.

To generalize the above description to the case of fermionic quarks constituting a particular meson[6,7], first the scalar propagators $\triangle_i^{-1}$ in the above equation are replaced by the proper fermionic propagators S_F. Then for incorporation of relevant Dirac structures in the vertex function $\Gamma(\hat{q})$, we take guidance from some of the recent studies which have revealed that various mesons have many different covariant structures in their wave functions whose inclusion was also found necessary to obtain quantitatively accurate observables[3]. However, it was noticed that all covariants in a BS amplitude do not contribute equally to calculation of various meson observables[1] and only some covariants are considered to be relevant to calculation of decay constants. Towards this end a power counting rule [6,7] was developed whose motivation was to find a criterion so as to systematically choose among various Dirac covariant from their complete set to write wave functions for different mesons (P,V,S etc.). Various Dirac covariant in the structure of H$q\bar{q}$ vertex function for a particular meson (pseudoscalar, vector, scalar etc.), are incorporated order-by-order in powers of inverse of meson mass M.

As far as pseudoscalar meson is concerned, its $Hq\bar{q}$ vertex function which has a certain dimensionality of mass can be expressed as a linear combination of four Dirac covariants $\Gamma_i^p(i=0,...,3)$[3,5] each multiplying a Lorentz-scalar amplitude.

For adapting this decomposition to write the structure of vertex function $\Gamma(\hat{q})$, we re-express the $Hq\bar{q}$ vertex function by making this amplitudes dimensionless.Thus each term in the expansion of $\Gamma(\hat{q})$ is associated with a certain power of M. In detail, we can express Γ_p as a polynomial in various powers of $\frac{1}{M}$[6,7],

$$\Gamma_P = \Omega_P \frac{1}{2\pi i} N_P D(\hat{q})\phi(\hat{q}), \tag{3.4.13}$$

where

$$\Omega_P = \gamma_5 B_o - i\gamma_5(\gamma\cdot P)\frac{B_1}{M} - i\gamma_5(\gamma\cdot q)\frac{B_2}{M} - \gamma_5(\gamma\cdot P\gamma\cdot q - \gamma\cdot q\gamma\cdot P)\frac{B_3}{M^2}. \tag{3.4.14}$$

where $B_i(i=0,...,3)$ are four dimensionless constants to be determined. Now since we use constituent quark masses where the quark mass m is approximately half of the hadron mass M, we can use the Ansatz

$$q << P \sim M, \tag{3.4.15}$$

in the rest frame of the hadron. Then each of the four terms in eqn (3.4.14) receives suppression by different powers of $\frac{1}{M}$. Thus we can arrange these terms as an expansion in powers of $O(\frac{1}{M})$. We can then see in the expansion of Ω_P that the structures associated with the coefficients B_o, B_1 have magnitudes $O(\frac{1}{M^o})$ and are of leading order, while those with B_2, B_3 are $O(\frac{1}{M^1})$ and are next-to-leading order. This naive power counting rule[6,7] suggests that the maximum contribution to the calculation of any pseudoscalar meson observable should come from the Dirac structures of γ_5, $i\gamma_5(\gamma \cdot P)\frac{1}{M}$, $i\gamma_5(\gamma \cdot q)\frac{1}{M}$ and $i\gamma_5(\gamma \cdot P\gamma \cdot q - \gamma \cdot q\gamma \cdot P)\frac{1}{M^2})$ associated with the constant coefficients B_o, B_1, B_2 and B_3 respectively. Thus, we take eqn(3.4.13) the $Hq\bar{q}$ vertex function for a pion.

Chapter 4

Radiative Decay Of a Neutral Scalar Meson and Calculation of BS Normalizer N_P

4.1 Calculation Of Radiative Decay Constant For a Pion

The invariant amplitude for the decay of a neutral scalar meson into two photons given by the famous triangle diagram can be expressed by[19]

$$A(\pi^o \to 2\gamma) = \frac{e^2}{\sqrt{6}} \int d^4q Tr[\Psi(p,q) i\not{\epsilon}_1 S_F(q-Q) i\not{\epsilon}_2] + (1 \Leftrightarrow 2), \tag{4.1.1}$$

Where $(1 \Leftrightarrow 2)$ corresponds to the diagram with the two emitted photons exchanged and the Bethe-Salpeter wave function $\Psi(P,q)$ for a pion is expressed as

$$\Psi(P,q) = S_F(p_1)\Gamma(\hat{q})S_F(-p_2), \tag{4.1.2}$$

where $S_F(p_1)$ and $S_F(p_2)$ are the fermionic propagators for the two constituent quarks of the hadron, their expression is given by

$$S_F(p_1) = -i(\frac{m_1 - i\not{p}_1}{\triangle_1}). \tag{4.1.3}$$

$$S_F(p_2) = -i(\frac{m_2 - i\not{p}_2}{\triangle_2}). \tag{4.1.4}$$

In the following calculation, we take the structure of hadron-quark vertex function $\Gamma(\hat{q})$ as in eqn(3.4.13) containing all terms. Substituting the value of $\psi(P,q)$ and fermionic propagators S_F, the trace in equation (4.1.1) becomes

$$Tr = \frac{1}{\triangle_1\triangle_2\triangle_3} Tr[(m_1 - i\not{p}_1)[\gamma_5 B_o - i\gamma_5(\not{P})\frac{B_1}{M} - i\gamma_5(\not{q})\frac{B_2}{M} - \gamma_5(\not{P}\not{q} - \not{q}\not{P})\frac{B_3}{M^2}$$

$$(m_2 + i\not{p}_2)\not{\varepsilon}_1[m - (\not{q} - \not{Q})]\not{\varepsilon}_2]]. \tag{4.1.5}$$

We calculate the above quantity of Tr by evaluating the traces of γ-matrices by making use of the trace relations:

$$Tr(\gamma_5) = 0. \tag{4.1.6}$$

$$Tr(\gamma_5\not{a}\not{b}) = 0. \tag{4.1.7}$$

$$Tr(\gamma_5\not{a}\not{b}\not{c}) = 0. \tag{4.1.8}$$

$$Tr(\gamma_5\not{a}\not{b}\not{c}\not{d}) = 4\epsilon_{\mu\nu\rho\sigma}a_\mu b_\nu c_\rho d_\sigma. \tag{4.1.9}$$

$$Tr(\gamma_5\not{a}\not{b}\not{c}\not{d}\not{e}\not{f}) = 4(a.b)\epsilon_{\mu\nu\rho\sigma}c_\mu d_\nu e_\rho f_\sigma - 4(a.c)\epsilon_{\mu\nu\rho\sigma}b_\mu d_\nu e_\rho f_\sigma;$$

$$+4(a.d)\epsilon_{\mu\nu\rho\sigma}b_\mu c_\nu e_\rho f_\sigma + 4(d.e)\epsilon_{\mu\nu\rho\sigma}a_\mu b_\nu c_\rho f_\sigma;$$

$$-4(d.f)\epsilon_{\mu\nu\rho\sigma}a_\mu b_\nu c_\rho e_\sigma + 4(e.f)\epsilon_{\mu\nu\rho\sigma}a_\mu b_\nu c_\rho d_\sigma. \tag{4.1.10}$$

using these relations we obtain

$$2B_o m Tr(\gamma_5\not{P}\not{\varepsilon}_1\not{Q}\not{\varepsilon}_2) = 8B_o m\epsilon_{\mu\nu\rho\sigma}P_\mu\varepsilon_{1\nu}Q_\rho\varepsilon_{2\sigma}. \tag{4.1.11}$$

$$-2m^2\frac{B_1}{M}Tr(\gamma_5 \not{P}\not{\varepsilon}_1\not{Q}\not{\varepsilon}_2) = -8m^2\frac{B_1}{M}\epsilon_{\mu\nu\rho\sigma}P_\mu\varepsilon_{1\nu}Q_\rho\varepsilon_{2\sigma}. \tag{4.1.12}$$

$$2\frac{B_1}{M}Tr(\gamma_5 \not{p}_1\not{P}\not{p}_2\not{\varepsilon}_1\not{Q}\not{\varepsilon}_2) = 2\frac{B_1}{M}[2(p_1.P)+2(p_2.P)-4(p_1.p_2)]\epsilon_{\mu\nu\rho\sigma}P_\mu\varepsilon_{1\nu}Q_\rho\varepsilon_{2\sigma}. \tag{4.1.13}$$

$$-\frac{B_3}{M^2}Tr(\gamma_5 \not{P}\not{q}\not{p}_2\not{\varepsilon}_1\not{Q}\not{\varepsilon}_2) = -\frac{B_3}{M^2}[2(P.q)+4(p_2.q)-4(p_2.P)\sigma]\epsilon_{\mu\nu\rho\sigma}P_\mu\varepsilon_{1\nu}Q_\rho\varepsilon_{2\sigma}. \tag{4.1.14}$$

$$\frac{B_3}{M^2}Tr(\gamma_5 \not{q}\not{P}\not{p}_2\not{\varepsilon}_1\not{Q}\not{\varepsilon}_2) = \frac{B_3}{M^2}[2(P.q)-4(p_2.q)+4(p_2.P)\sigma]\epsilon_{\mu\nu\rho\sigma}P_\mu\varepsilon_{1\nu}Q_\rho\varepsilon_{2\sigma}. \tag{4.1.15}$$

$$-\frac{B_3}{M^2}Tr(\gamma_5 \not{p}_1\not{P}\not{q}\not{\varepsilon}_1\not{Q}\not{\varepsilon}_2) = -\frac{B_3}{M^2}[2(P.q)-4(p_1.q)+4(p_1.P)\sigma]\epsilon_{\mu\nu\rho\sigma}P_\mu\varepsilon_{1\nu}Q_\rho\varepsilon_{2\sigma}. \tag{4.1.16}$$

$$\frac{B_3}{M^2}Tr(\gamma_5 \not{p}_1\not{q}\not{P}\not{\varepsilon}_1\not{Q}\not{\varepsilon}_2) = \frac{B_3}{M^2}[2(P.q)+4(p_1.q)-4(p_1.P)\sigma]\epsilon_{\mu\nu\rho\sigma}P_\mu\varepsilon_{1\nu}Q_\rho\varepsilon_{2\sigma}. \tag{4.1.17}$$

Now the trace expression in terms of momentum of constituent quarks p_1, p_2 and hadron momentum P and internal momentum q, is given by

$$Tr = \frac{1}{\triangle_1\triangle_2\triangle_3}[8mB_o + \frac{B_1}{M}[-8m^2 + 4(p_1.P) + 4(p_2.P) - 8(p_1.p_2)];$$

$$+m\frac{B_3}{M^2}[-8(q.p_2) + 8(q.p_1) + 8(p_2.P)\sigma - 8(p_1.P)\sigma]]\epsilon_{\mu\nu\rho\sigma}P_\mu\varepsilon_{1\nu}Q_\rho\varepsilon_{2\sigma}. \tag{4.1.18}$$

After evaluating traces over γ-matrices over various terms and expressing the dot products of momentum of constituent quarks p_1 and p_2 with hadron momentum P and with internal momentum q by means of relations

$$p_1.P = \frac{1}{2}(\triangle_1 - \triangle_2 - M^2);$$

$$p_2.P = \frac{1}{2}(-\triangle_1 + \triangle_2 - M^2);$$

$$p_1.p_2 = m^2 - \frac{1}{2}(\triangle_1 + \triangle_2 + M^2); \tag{4.1.19}$$

$$p_1.q = -m^2 + \frac{1}{4}(3\triangle_1 + \triangle_2 + M^2);$$
$$p_2.q = m^2 - \frac{1}{4}(\triangle_1 + 3\triangle_2 + M^2).$$

we can express the complete trace Tr in eqn(4.1.1) as

$$Tr = \frac{1}{\triangle_1\triangle_2\triangle_3}[B_o(8m)+\frac{B_1}{M}[-16m^2+4(\triangle_1+\triangle_2)]+\frac{B_3}{M^2}[8m(\triangle_1+\triangle_2)+8m(\triangle_1-\triangle_2)\sigma]]\times$$
$$\epsilon_{\mu\nu\rho\sigma}P_\mu\varepsilon_{1\nu}Q_\rho\varepsilon_{2\sigma}, \tag{4.1.20}$$

where $(taking Q^2 << q^2)$, we can write,

$$\triangle_3 = m^2 + (q-Q)^2 \approx m^2 + q^2. \tag{4.1.21}$$

Here $\triangle_3$ is the inverse propagator of third quark in the quark triangle diagram, whereas $\triangle_{1,2}$ are inverse propagators of the two constituent quarks forming the meson. These inverse propagators of the three quarks can in turn be expressed in terms of the off-shell parameter σ as

$$\triangle_1 = w^2 - M^2(\frac{1}{2}+\sigma)^2; \tag{4.1.22}$$
$$\triangle_2 = w^2 - M^2(\frac{1}{2}-\sigma)^2; \tag{4.1.23}$$
$$\triangle_3 = w^2 - M^2\sigma^2. \tag{4.1.24}$$

Substituting the trace into eqn(4.1.1) the amplitude becomes

$$A(\pi^o \to 2\gamma) = \frac{e^2}{\sqrt{6}}\int d^4q \frac{1}{\triangle_1\triangle_2\triangle_3}[B_o(8m) + \frac{B_1}{M}[-16m^2 + 4(\triangle_1+\triangle_2)]+$$
$$\frac{B_3}{M^2}[8m(\triangle_1+\triangle_2) + 8m(\triangle_1-\triangle_2)\sigma]] \times \epsilon_{\mu\nu\rho\sigma}P_\mu\varepsilon_{1\nu}Q_\rho\varepsilon_{2\sigma}. \tag{4.1.25}$$

We can then express the invariant amplitude for two photon decay of pion as

$$A(\pi^o \to 2\gamma) = F_\pi\epsilon_{\mu\nu\rho\sigma}P_\mu\varepsilon_{1\nu}Q_\rho\varepsilon_{2\sigma}, \tag{4.1.26}$$

where $P = p_1 + p_2$ is the total momentum of the pion, k_1, k_2 are the momenta of the two emitted photons with polarizations ε_1, ε_2 respectively and $Q = k_1 - k_2$ is the momentum difference of the emitted photons. The radiative decay constant F_P can then be expressed as

$$F_P = \frac{e^2}{\sqrt{6}} N_P \int d^3\hat{q} D(\hat{q}) \phi(\hat{q}) I, \tag{4.1.27}$$

where

$$\begin{aligned} I = \int \frac{M d\sigma}{\triangle_1 \triangle_2 \triangle_3} [B_o(8m) + \frac{B_1}{M}[-16m^2 + 4(\triangle_1 + \triangle_2)] + \\ \frac{B_3}{M^2}[8m(\triangle_1 + \triangle_2) + 8m(\triangle_1 - \triangle_2)\sigma]]. \end{aligned} \tag{4.1.28}$$

Here the denominator function $D(\hat{q})$ obtained from carrying out pole integrations in σ-plane, can be expressed as

$$D(\hat{q}) = \frac{D_o(\hat{q})}{\frac{1}{2\omega_1} + \frac{1}{2\omega_2}}, \qquad D_o(\hat{q}) = (\omega_1 + \omega_2)^2 - M^2. \tag{4.1.29}$$

In eqn(4.1.27) carrying out pole integration over $d\sigma$ by the method of contour integration by noting the pole positions in the complex σ-plane:

$$\begin{aligned} \triangle_1 = 0 \quad &\Rightarrow \quad \sigma_1^{\pm} = \pm\frac{\omega}{M} - \frac{1}{2} \mp i\varepsilon; \\ \triangle_2 = 0 \quad &\Rightarrow \quad \sigma_2^{\pm} = \pm\frac{\omega}{M} + \frac{1}{2} \mp i\varepsilon; \\ \triangle_3 = 0 \quad &\Rightarrow \quad \sigma_3^{\pm} = \pm\frac{\omega}{M} \mp i\varepsilon; \qquad \omega^2 = m^2 + \hat{q}^2. \end{aligned} \tag{4.1.30}$$

we can express F_P as

$$\begin{aligned} F_P = \frac{e^2}{\sqrt{6}} N_P \int d^3\hat{q} D(\hat{q}) \phi(\hat{q}) [B_o(8m)s_1 + \frac{B_1}{M}[-16m^2 s_1 + 4(s_2 + s_3)] + \\ \frac{B_3}{M^2}[8m(s_2 + s_3) + 8m(s_5 - s_4)]], \end{aligned} \tag{4.1.31}$$

where

$$
\begin{aligned}
s_1 &= \int \frac{M d\sigma}{2\pi i \triangle_1 \triangle_2 \triangle_3};\\
s_2 &= \int \frac{M d\sigma}{2\pi i \triangle_1 \triangle_3};\\
s_3 &= \int \frac{M d\sigma}{2\pi i \triangle_2 \triangle_3};\\
s_4 &= \int \frac{\sigma M d\sigma}{2\pi i \triangle_2 \triangle_3};\\
s_5 &= \int \frac{\sigma M d\sigma}{2\pi i \triangle_1 \triangle_3};
\end{aligned}
\tag{4.1.32}
$$

The results on whether the contour is closed from above the real axis or from below the real axis are the same. We now show how these integrals are evaluated. We can shift the poles either above the real axis or below the real axis, as shown in figure 4.1.

Lets first choose the contour C consisting of the semicircle C_R with center at the origin and radius R in the lower half-plane, such that it includes all the poles of $f(\sigma)$ and the line segment of the real axis from $-R$ to $+R$. Integrating the function clockwise around the boundary of the semicircle contour C, we get by Cauchy residue theorem[21].

$$\int_{-R}^{+R} f(\sigma) d\sigma + \int_{C_R} f(\sigma) d\sigma = 2\pi i \Sigma Residue. \tag{4.1.33}$$

If R be large enough i.e $R \to \infty$, the second term of the right hand side of the above equation becomes zero i.e

$$Lim_{R\to\infty} \int_{C_R} f(\sigma) d\sigma = 0. \tag{4.1.34}$$

Thus, eqn (4.1.33) becomes

$$Lim_{R\to\infty} \int_{-R}^{+R} f(\sigma) d\sigma = 2\pi i \Sigma Residue. \tag{4.1.35}$$

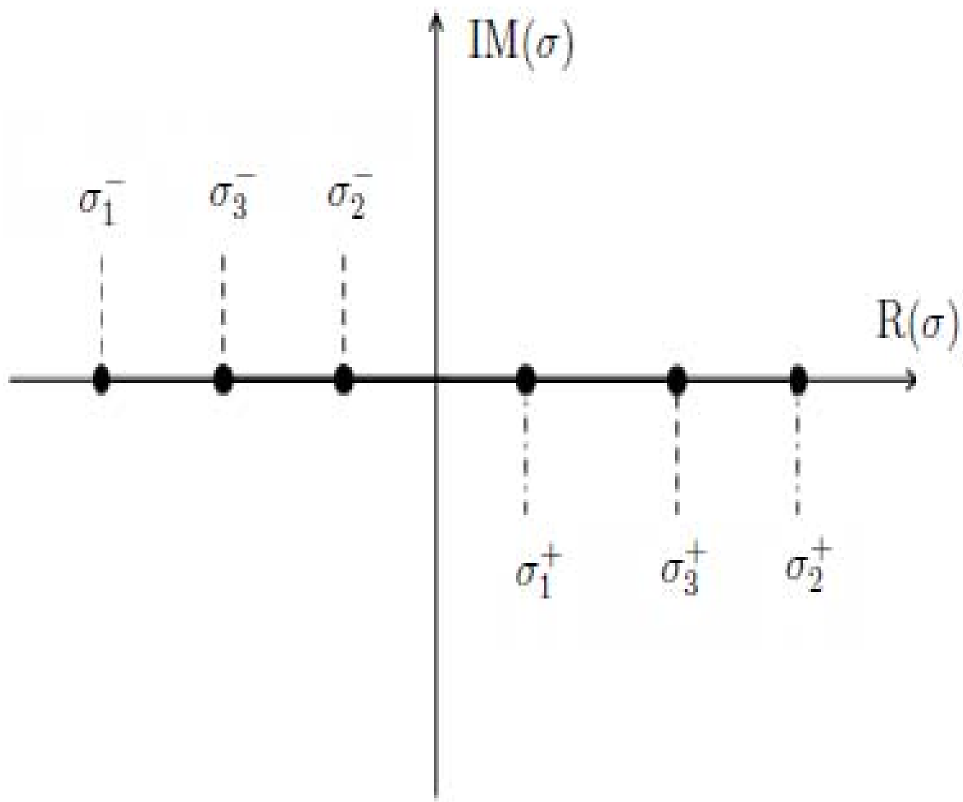

Figure 4.1: Contour integral

That is

$$\int_{-\infty}^{+\infty} f(\sigma)d\sigma = 2\pi i \Sigma Residue. \qquad (4.1.36)$$

For example we find the integral of s_1, now we calculate the residue for s_1 using the poles given in eqn(4.1.30), first for $\triangle_1$=0, i.e $\sigma_1^+ = +\frac{\omega}{M} - \frac{1}{2}$, we get,

$$R = Lim_{\sigma\to\sigma_1} f(\sigma)(\sigma - \sigma_1). \qquad (4.1.37)$$

$$R_1 = Lim(_{\sigma\to+\frac{\omega}{M}} - \frac{1}{2})\frac{M(\frac{\omega}{M} - \frac{1}{2} - \sigma)}{(\omega^2 - M^2(\frac{1}{2}+\sigma)^2)(\omega^2 - M^2(\frac{1}{2}-\sigma)^2)(\omega^2 - M^2(\sigma)^2)}. \qquad (4.1.38)$$

After simplification we obtain

$$R_1 = \frac{8\omega + 2M}{2\omega M^2(16\omega^2 - M^2)(2\omega - M)}. \qquad (4.1.39)$$

For $\triangle_2$=0, i.e $\sigma_2^+ = +\frac{\omega}{M} + \frac{1}{2}$, we get,

$$R_2 = Lim(_{\sigma\to+\frac{\omega}{M}} + \frac{1}{2})\frac{M(\frac{\omega}{M} + \frac{1}{2} - \sigma)}{(\omega^2 - M^2(\frac{1}{2}+\sigma)^2)(\omega^2 - M^2(\frac{1}{2}-\sigma)^2)(\omega^2 - M^2(\sigma)^2)}. \qquad (4.1.40)$$

$$R_2 = \frac{8\omega + 2M}{2\omega M^2(16\omega^2 - M^2)(2\omega + M)}. \qquad (4.1.41)$$

Similarly for $\triangle_3$=0, i.e $\sigma_3^+ = +\frac{\omega}{M}$, we get,

$$R_3 = Lim_{\sigma\to+\frac{\omega}{M}}\frac{M(\frac{\omega}{M} - \sigma)}{(\omega^2 - M^2(\frac{1}{2}+\sigma)^2)(\omega^2 - M^2(\frac{1}{2}-\sigma)^2)(\omega^2 - M^2(\sigma)^2)}. \qquad (4.1.42)$$

$$R_3 = -\frac{16}{2\omega M^2(16\omega^2 - M^2)}. \quad (4.1.43)$$

Now, adding these three residues R_1, R_2, R_3 and we obtain

$$\Sigma R = \frac{12}{M^4\omega - 20\omega^3 M^2 + 16\omega^5}. \quad (4.1.44)$$

Finally the integral for s_1 is

$$s_1 = \int \frac{M d\sigma}{2\pi i \triangle_1 \triangle_2 \triangle_3} = \frac{12}{M^4 w - 20M^2\omega^3 + 64\omega^5}. \quad (4.1.45)$$

Similarly we can solve the remaining integrals s_2, s_3, s_4, s_5 the results are given below

$$\begin{aligned}
s_2 &= \int \frac{M d\sigma}{2\pi i \triangle_1 \triangle_3} = \frac{4}{-M^2 w + 16\omega^3} \\
s_3 &= \int \frac{M d\sigma}{2\pi i \triangle_2 \triangle_3} = \frac{4}{-M^2 w + 16\omega^3} \\
s_4 &= \int \frac{\sigma M d\sigma}{2\pi i \triangle_2 \triangle_3} = \frac{1}{-M^2 w + 16\omega^3} \\
s_5 &= \int \frac{\sigma M d\sigma}{2\pi i \triangle_1 \triangle_3} = -\frac{1}{-M^2 w + 16\omega^3}
\end{aligned} \quad (4.1.46)$$

4.2 Calculation Of Bethe-Salpeter Normalizer N_P For a pion:

To calculate the normalization factor N_P, we use the current conservation condition[19],

$$2iP_\mu = (2\pi)^4 \int d^4q Tr[\bar{\Psi}(P,q)(\frac{\partial}{\partial P_\mu} S_F^{-1}(p_1))\Psi(P,q) S_F^{-1}(-p_2)] + (1 \leftrightarrow 2). \quad (4.2.1)$$

$$\bar{\Psi}(P,q) = \gamma_4 \Psi^\dagger(P,q)\gamma_4. \quad (4.2.2)$$

After substitution of the value of $\Psi(P,q)$, $\bar{\Psi}(P,q)$, $S_F^{-1}(p_1)$ and $S_F^{-1}(-p_2)$ the trace in the above equation becomes,

$$Tr = Tr[(m_2+ip_2)[\gamma_5 B_o + i(\gamma.P)\gamma_5\frac{B_1}{M} + i(\gamma.q)\gamma_5\frac{B_2}{M} + \gamma.q\gamma.P\gamma_5\frac{B_3}{M^2} - \gamma.P\gamma.q\gamma_5\frac{B_3}{M^2})]$$

$$(m_1-ip_1)\gamma_\mu(m_1-ip_1)[\gamma_5 B_o - i\gamma_5(\gamma.P)\frac{B_1}{M} - i\gamma_5(\gamma.q)\frac{B_2}{M} - \gamma_5\gamma.P\gamma.q\frac{B_3}{M^2} + \gamma_5\gamma.q\gamma.P\frac{B_3}{M^2}]]. \tag{4.2.3}$$

After expanding this equation we can solve the traces of γ's, there are 45 terms with traces over products of four γ matrices, 44 terms with traces over products of six γ matrices and 4 terms with eight γ matrices. For example we list some like,

$$Tr(\gamma_\mu \not{P}) = 4P_\mu. \tag{4.2.4}$$

$$Tr(\gamma_\mu \not{p}_1 \not{P} \not{q}) = 4[p_{1\mu}(P.q) - P_\mu(p_1.q) + q_\mu(p_1.P)]. \tag{4.2.5}$$

$$Tr(\not{p}_2 \not{q} \not{P} \gamma_\mu) = 4(p_2.q)P_\mu - 4(p_2.P)q + 4(q.P)p_{2\mu}. \tag{4.2.6}$$

$$Tr(\not{p}_2 \not{p} \not{q} \gamma_\mu) = 4(p_2.P)q_\mu - 4(p_2.q)P_\mu + 4(q.P)p_{2\mu}. \tag{4.2.7}$$

After solving the traces we express various terms in terms the dot product of momentum of constituent quarks, total momentum of hadron P and internal momentum q as

$$Tr = \frac{B_o^2}{M^2}[4m^2(p_2.P)+8m^2(p_1.P)-8(p_1.p_2)(p_1.P)+4p_1^2(p_2.P)]+$$

$$\frac{B_oB_1}{M^3}[-m^3P^2+16m(p_1.P)(p_1.P)-8mp_1^2P^2+8mP^2(p_1.p_2)]+$$

$$\frac{B_1^2}{M^4}[-4m^2P^2(p_2.P)-4m^2P^2(p_1.P)+4m^2P^2(p_1.q)+16(p_2.P)(p_1.P)^2$$
$$-4(p_2.P)p_1^2P^2-8(p_1.p_2)(p_1.P)P^2]+$$
$$\frac{B_oB_2}{M^3}[-8m^3(q.P)+16m(p_2.q)(p_1.P)+16(q.p_1)(p_1.P)-8m(q.P)p_1^2]+$$
$$\frac{B_1B_2}{M^4}[-8m^2(p_2.q)P^2+8m^2(q.P)(p_2.P)-8m^2(q.P)(p_2.P)-16m^2(q.P)(p_1.P)$$
$$+16(p_2.q)(p_1.P)^2+16(p_2.P)(p_1.q)(p_1.P)-16(p_1.p_2)(q.P)(p_1.P)-8(p_2.q)p_1^2P^2]+$$
$$\frac{B_2^2}{M^4}[-8m^2(p_2.q)(q.P)+4m^2q^2(p_2.P)-8m^2q^2(p_1.P)+16(p_1.q)(p_2.q)(p_1.P)$$
$$-8p_1^2(p_2.q)(q.P)-8q^2(p_1.p_2)(p_1.P)+4q^2p_1^2(p_2.P)]+$$
$$\frac{B_oB_3}{M^4}[16m^2(p_2.P)(q.P)-16m^2P^2(p_2.q)+32(p_2.q)(p_1.P)^2-32(p_2.P)(p_1.q)(p_1.P)$$
$$-16(p_2.q)p_1^2P^2+16(P.q)(p_2.P)p_1^2]+$$
$$\frac{B_1B_3}{M^5}[32m(p_2.P)(q.P)(p_1.P)-32m(p_2.q)(p_1.P)P^2-32m(p_1.P)^2(q.P)$$
$$+32m(p_1.q)(p_1.P)P^2]+$$
$$\frac{B_2B_3}{M^5}[-16m^3(q.P)^2+16m^3q^2P^2-32m(p_2.q)(q.P)(p_1.P)+32m(p_2.P)(p_1.P)q^2$$
$$+32m(q.P)(p_1.q)(p_1.P)_32mq^2(p_1.P)^2-16m(q.P)^2p_1^2+16mp_1^2P^2q^2]+$$
$$\frac{B_3^2}{M^6}[-16m^2(p_2.P)q^2P^2+16m^2(q.P)^2(p_2.P)-32m^2(q.P)^2(p_1.P)+32m^2(p_1.P)q^2P^2$$
$$+64(p_2.q)(p_1.q)(p_1.P)P^2-64(p_2.q)(P.q)(p_1.P)^2+16(p_1.p_2)(p_1.P)(q.P)^2$$
$$-32(p_1.p_2)(p_1.P)q^2P^2+64(p_2.P)(p_1.P)^2q^2-64(p_2.P)(q.P)(q.p_1)(p_1.P)$$
$$+16(p_2.P)(q.P)^2p_1^2-16(p_2.P)p_1^2q^2P^2] \tag{4.2.8}$$

where the momentum of constituent quarks can be expressed as in eqn(4.1.19). After substitution of the dot products of constituent quarks, hadron of momentum P and internal momentum q we obtain the complete trace Tr is

$$Tr = J_1 + J_2 + J_3 + ... + J_{10} \tag{4.2.9}$$

where

$$J_1 = \frac{BoBo}{M^4}\{2m^2M^4 - M^6 - 4m^2M^4\sigma - M^6\sigma - 8M^4\hat{q}^2 - 4M^4\hat{q}^2\sigma + 2M^6\sigma^2 + 4M^6\sigma^3\}. \tag{4.2.10}$$

$$J_2 = \frac{B0B1}{M^3}\{8m^3M^2 + 4mM^4 + 16mM^2\hat{q}^2 + 8mM^4\sigma\}. \tag{4.2.11}$$

$$J_3 = \frac{B1B1}{M^4}\{-4m^2M^4 - \frac{M^6}{2} - 4m^2M^2\hat{q}^2 + 2M^4\hat{q}^2 + (2m^2M^4 - M^6)\sigma + 12M^4\hat{q}^2\sigma + (4m^2M^4 + 2M^6)\sigma^2 + 4M^6\sigma^3\}. \tag{4.2.12}$$

$$J_4 = \frac{B0B2}{M^3}\{-4mM^2\hat{q}^2 + (8m^3M^2 + 2mM^4)\sigma - 8mM^2\hat{q}^2\sigma - 24mM^4\sigma^2 + 8mM^4\sigma^3\}. \tag{4.2.13}$$

$$J_5 = \frac{B1B2}{M^4}\{m^2M^4 + \frac{M^6}{4} + (-8m^2M^2 + 3M^4)\hat{q}^2 - 8M^2\hat{q}^4 - 12m^2M^4\sigma - 4M^4\hat{q}^2\sigma + (-12m^2M^4 - 2M^6)\sigma^2 - 4M^4\hat{q}^2\sigma^2 + 4M^6\sigma^3\}. \tag{4.2.14}$$

$$J_6 = \frac{B2B2}{M^4}\{(2m^2M^2 - \frac{3}{2}M^4)\hat{q}^2 + 2M^2\hat{q}^4 + (m^2M^4 + \frac{M^6}{4})\sigma + 4m^2M^2\hat{q}^2\sigma + 4M^2\hat{q}^4\sigma + (-6m^2M^4 + \frac{M^6}{2})\sigma^2 + (-8m^2M^4 - M^6)\sigma^3 - 4M^4\hat{q}^2\sigma^3\}. \tag{4.2.15}$$

$$J_7 = \frac{B0B3}{M^4}\{2m^2M^4 + \frac{M^6}{2} + (-16m^2M^2 - 10M^4)\hat{q}^2 - 16M^2\hat{q}^4 + 2M^6\sigma - 16M^4\hat{q}^2\sigma - 8m^2M^4\sigma^2 + 8M^4\hat{q}^2\sigma^2 - 8M^6\sigma^3\}. \tag{4.2.16}$$

$$J_8 = \frac{B1B3}{M^5}\{-2mM^6 + 32mM^4\hat{q}^2 - 4mM^6\sigma + 2mM^4\hat{q}^2\sigma + 8mM^6\sigma^2 + 16mM^6\sigma^3\}. \tag{4.2.17}$$

$$J_9 = \frac{B2B3}{M^5}\{(-16m^3M^2 + 4mM^4)\hat{q}^2 - 16mM^2\hat{q}^4 - 2mM^6\sigma + 16mM^4\hat{q}^2\sigma - 4mM^6\sigma^2 + 16mM^4\hat{q}^2\sigma^2 + 8mM^6\sigma^3\}. \tag{4.2.18}$$

$$J_{10}=\frac{B_3^2}{M^6}\{(8m^2M^4-2M^6)\hat{q}^2-24M^4\hat{q}^4+8m^2M^6\sigma+(48m^2M^4-4M^6)\hat{q}^2\sigma-16M^4\hat{q}^4\sigma+$$
$$(16m^2M^6-2M^8)\sigma^2-4M^8\sigma^3\}. \qquad (4.2.19)$$

and carrying out the $d\sigma$ integral in equation (4.2.1) (where the invariant four-dimensional volume element $d^4q = d^3\hat{q}Md\sigma$) by noting the pole positions in the complex σ-plane, we can express the BS normalizer as

$$N_P^{-2} = -(2\pi)^2 i\int d^3\hat{q}D^2(\hat{q})\phi^2(\hat{q}).J(J_1+J_2+J_3+,...,+J_{10}), \qquad (4.2.20)$$

where

$$J_1=\frac{BoBo}{M^4}\{(2m^2M^4-M^6)I_1+(-4m^2M^4-M^6)I_2-8M^4\hat{q}^2I_1-4M^4\hat{q}^2I_2+2M^6I_3+4M^6I_4\}.$$

$$J_2=\frac{B0B1}{M^3}\{(8m^3M^2+4mM^4)I_1+16mM^2\hat{q}^2I_1+8mM^4I_2\}.$$

$$J_3=\frac{B1B1}{M^4}\{(-4m^2M^4-\frac{M^6}{2})I_1+(-4m^2M^2+2M^4)\hat{q}^2I_1+(2m^2M^4-M^6)I_2+$$
$$12M^4\hat{q}^2I_2+(4m^2M^4+2M^6)I_3+4M^6I_4\}.$$

$$J_4=\frac{B0B2}{M^3}\{-4mM^2\hat{q}^2I_1+(8m^3M^2+2mM^4)I_2-8mM^2\hat{q}^2I_2-24mM^4I_3+8mM^4I_4\}.$$

$$J_5=\frac{B1B2}{M^4}\{(m^2M^4+\frac{M^6}{4})I_1+(-8m^2M^2+3M^4)\hat{q}^2I_1-8M^2\hat{q}^4I_1-12m^2M^4I_2$$
$$-4M^4\hat{q}^2I_2+(-12m^2M^4-2M^6)I_3-4M^4q^2I_3+4M^6I_4\}.$$

$$J_6=\frac{B2B2}{M^4}\{(2m^2M^2-\frac{3}{2}M^4)\hat{q}^2I_1+2M^2\hat{q}^4I_1+(m^2M^4+\frac{M^6}{4})I_2+4m^2M^2\hat{q}^2I_2+4M^2\hat{q}^4I_2$$
$$+(-6m^2M^4+\frac{M^6}{2})I_3+(-8m^2M^4-M^6)I_4-4M^4\hat{q}^2I_4\}.$$

$$J_7 = \frac{B0B3}{M^4}\{2m^2M^4+\frac{M^6}{2}I_1+(-16m^2M^2-10M^4)\hat{q}^2I_1-16M^2\hat{q}^4I_1+2M^6I_2-16M^4q^2I_2$$
$$-8m^2M^4I_3+8M^4\hat{q}^2I_3-8M^6I_4\}.$$

$$J_8 = \frac{B1B3}{M^5}\{-2mM^6I_1+32mM^4q^2I_1-4mM^6I_2+2mM^4q^2I_2+8mM^6I_3+16mM^6I_4\}.$$

$$J_9 = \frac{B2B3}{M^5}\{(-16m^3M^2+4mM^4)\hat{q}^2I_1-16mM^2\hat{q}^4I_1-2mM^6I_2+16mM^4\hat{q}^2I_2-4mM^6I_3+$$
$$16mM^4\hat{q}^2I_3+8mM^6I_4\sigma^3\}.$$

$$J_{10} = \frac{B_3^2}{M^6}\{(8m^2M^4-2M^6)\hat{q}^2I_1-24M^4\hat{q}^4I_1+8m^2M^6I_2+(48m^2M^4-4M^6)\hat{q}^2I_2-16M^4q^4I_2+$$
$$(16m^2M^6-2M^8)I_3-4M^8I_4\}.$$

and the integrals $I_1, I_2, I_3 and I_4$ over $d\sigma$ are:

$$I_1 = \int_{-\infty}^{+\infty}\frac{Md\sigma}{\triangle_1^2\triangle_2} = 2\pi i[\frac{-M^2+12\omega^2}{4\omega^3(M^2-4\omega^2)^2}];$$

$$I_2 = \int_{-\infty}^{+\infty}\frac{Md\sigma}{\triangle_1^2\triangle_2}\sigma = 2\pi i[\frac{-M^2+20\omega^2}{8\omega^3(M^2-4\omega^2)^2}];$$

$$I_3 = \int_{-\infty}^{+\infty}\frac{Md\sigma}{\triangle_1^2\triangle_2}\sigma^2 = 2\pi i[\frac{M^2+2\omega^2}{16\omega^3M^4}];$$

$$I_4 = \int_{-\infty}^{+\infty}\frac{Md\sigma}{\triangle_1^2\triangle_2}\sigma^3 = 2\pi i[\frac{1}{16\omega^3M^2}].$$

We have thus evaluated the general expressions for F_P and N_P in the framework of BSE under CIA, with Dirac structure introduced in the H$q\bar{q}$ vertex function we see that so far the results are independent of any model for $\phi(\hat{q})$. However, for calculating the numerical values of F_P, one needs to know the constant coefficients B_o, B_1, B_2 and B_3, which are associated with the Dirac structures $\gamma_5 B_o$, $i\gamma_5(\gamma.P)\frac{1}{M}$, $i\gamma_5(\gamma.q)\frac{1}{M}$

and $i\gamma_5(\gamma.P\gamma.q-\gamma.q\gamma.P)\frac{1}{M^2}$ respectively. Now, to find the radiative decay constant F_P, first we obtain the BS normalizer substituting all the values of hadron of mass M=140MeV, constituent quark mass m=300MeV and $\omega_0 = 250MeV$ into equation (4.2.8) of the BS normalizer N_P and integrate with respect to $\hat{q}$.

Then, now we parameterize B_o, B_1, B_2 and B_3 which are free parameters, by calibrating to the decay constants of P-mesons (π, K, D, D_s, B) as well as the pion radiative decay constant. For LO covariants alone it was found that the best fit of calculated values of decay constants of π, K, D, D_s, B mesons[10,11] to their experimental values are obtained for the range of parameter values $\frac{B_1}{B_o} = 0.14 - 0.16$. It was seen that at $\frac{B_1}{B_0} = 0.148$[10,11], we get the meson decay constant, $f_P = 130.7MeV^{-1}$[10,11] which is the experimental value of this quantity. It was earlier seen that the numerical values of this decay constant in BSE under CIA improve dramatically when Dirac structure $\gamma_5(\gamma.P)\frac{1}{M}$ and $i\gamma_5(\gamma.P)\frac{1}{M}$ is introduced in the vertex function in comparison to the values calculated with only γ_5 [10,11].

Using the value of $\frac{B_1}{B_0} = 0.148$, the BS normalizer N_P was obtained as,

$$N_P = 0.3198[20]$$

The pion radiative decay constant F_P using the BS normalizer calculated only LO covariants is,

$$F_P = 0.0256GeV^{-1}[20]$$

which is close to the corresponding experimental value of this quantity at 0.02424 GeV^{-1} obtained from the experimental value of decay width $\Gamma_{\pi^o\rightarrow 2\gamma} = 8.02eV$. Using the relation

$$\Gamma_{\pi^o\rightarrow 2\gamma} = \frac{F_P^2 M_P^3}{64\pi}.$$

In this thesis we calculate the pion radiative decay constant F_π using NLO covariants along with LO covariants. We have presented in chapter 4, the precise analitical results of calculation of F_P and N_P using with LO and NLO covariants along with the BS normalizer calculation for pion. As far as the numerical results of F_P are concerned, we need to parametrize F_P and N_P expressions eqn(4.1.31) and eqn(4.2.20) we expect the numerical results of F_P so improve considerably when we introduce both LO and NLO covariants in contrast to the earlier calculation where only LO covariants were used. We further expect the NLO contribution to be much less than the Lo contribution.

Chapter 5

conclusion

In QED the basic interaction vertex corresponding to the interaction Lagrangain $\pounds_{int} = ie\bar{\psi}(x)\gamma_\mu\psi(x)A_\mu(x)$ point like where e plays the role of a coupling constant between matter and electromagnetic field. However, in QCD though the quark-gluon vertex arising from the interaction Lagrangian $\pounds_{int} = ig\bar{\psi}(x)\gamma_\mu\lambda^a\psi(x)A^a_\mu(x)$ is again point like with g playing the role of a coupling constant between matter and gluon fields, however, the hadron quark vertex is an extended one due to nonperturbative (long distance) effects. This nonperturbative hadron-quark vertex function can not be derived from QCD entirely even now.

In this thesis we model this extended hadron-quark vertex using the QCD motivated model based on Bethe-Salpeter Equation framework. Towards this end we first derived the BSE in chapter 3. we then calculated the nonperturbative hadron-quark vertex function in BSE under CIA. For incorporating the various Dirac covariants in its structures. We proposed a power counting rule to know the relative importance of various covariants. We applied this hadron-quark vertex function to calculate the pion-photon radiative decay constant along with the Bethe-Salpeter normalizer which was a very extensive calculation.

In these calculations we incorporate both the leading order (LO) as well as the next-to-leading order(NLO) covariants. In an earlier calculation performed using Lo covariants alone the theoretical results extremely close to experimental value i.e, F_P= 0.0256 GeV^{-1}($F_{expt.V}$ = 0.02424 GeV^{-1}). With the incorporation of NLO covariants also in the present calculation we hope that the theoretical results on F_P will improve further.

Bibliography

[1] P. Costa, Phys. Rev. C **70** (2004) 048202.

[2] M. A. Ivanov, Yu. A. Kalinovisky and C. D. Roberts, Phys. Rev. D **60** (1999) 034018.

[3] P. Maris and P. C. Tandy, Phys. Rev. C **60** (1999) 055214.

[4] R. Alkofer and L. V. Smekel, Phys. Rev. **353** (2001) 281.

[5] R. Alkofer, P. Watson and H. Weigel, Phys. Rev. D **65** (2002) 094026.

[6] S. Bhatnagar, J. Mahacha, ICTP preprint IC/2008/08 (2008)

[7] S.Bhatnagar and S.-Y. Li, J.Phys. G **32** (2006) 949.

[8] C.R. Munz, hep-ph/9601206/ (1996).

[9] D. Kekez and D. Klabucar, hep-ph/9605219 (1996).

[10] S.Bhatnagar and S.-Y. Li, hep-ph/0612084/(2006).

[11] S.Bhatnagar and S.-Y. Li, Int. J. Mod. Phys. E (2009) In press.

[12] I. J. R.AITCHISON and A. J. G. HEY(1984). *Gauge Theories in Particle Physics*. Adam Hilger Ltd, Bristol.

[13] Halzen, Francis (1984). *Quarks and Leptons*. Jhon Wiley and Sons, Inc. Canada.

[14] R. N. Mohapatra, C. H. Lai (1981) *Gauge Theories Of Fundamental Interactions* Farrer Road Singapore 9128.

[15] Lewis H. Ryder.(1985) *Quantum Field Theory.* 2^{nd} edn.

[16] A. Hoell, A. Krassnigg and C. D. Roberts, Phys. Rev. C **70**, 042203 (2004); P. Watson, W. Cassing and P. C. Tandy, Few Body Syst. **35**, 129 (2004) and references therein.

[17] T. Umekawa, K. Naito, M. Oka and M. Takizawa, Phys. Rev. C **70**, 055205 (2004).

[18] J. E. Villate, D. S. Liu, J. E. Ribeiro and P. J. de A. Bicudo, Phys. Rev. D **47**, 1145 (1993).

[19] J.J. Sakurai (2005). *Advanced Quantum Mechanics.* 8^{th} edn. Pearson Education, Inc. Singapore.

[20] S. Bhatnagar, Int. J. Mod. Phys. E Vol. **17**, No. 3 (2008) 519-530.

[21] ARFKEN and WEBER, *Mathematical Methods For Physicists*, 5^{th} edn. United state of America (2001).

[22] Michael E. Peskin, Daniel V. Schroeder(1997). *An Introduction to Quantum Field Theory.*

Declaration

I, the undersigned, declare that this thesis is my original work and has not presented for a degree in any other university. All sources of material used for the thesis have been duly acknowledged.

Name: Abebe Gucho

Signature

Place and date of submission: physics department, Addis Ababa University, June, 2009

This thesis has been submitted for examination with my approval as university advisor

Name: Dr. S. Bhatnagar

Signature