

ADDIS ABABA UNIVERSITY  
FACULTY OF TECHNOLOGY  
SCHOOL OF GRADUATE STUDIES  
DEPARTMENT OF MECHANICAL ENGINEERING



The effect of the crack length and direction on the stresses and their distribution around the crack on cylindrical shell of a composite material.

ATHESIS SUBMITTED TO A SCHOOL OF GRADUATE STUDIES OF ADDIS ABABA UNIVERSITY IN PARTIAL FULFILLMENT FOR DEGREE OF MASTERS OF SCIENCE IN MECHANICAL DESIGN (MECHANICAL ENGINEERING)

BY  
Mulugeta Hagos

ADVISOR.

Prof . Idalberto Mendoz

July, 2010

## **DECLARATION**

I hereby declare that the work which is being presented in this thesis entitled” The effect of the crack length and direction on the stresses and their distribution around the crack on cylindrical shell of a composite material “is original work of my own, has not been presented for a degree of any other university and all resources of materials used for this thesis have been duly acknowledged.

---

Mulugeta Hagos  
(Candidate)

---

Date

This is to certify that the above declaration made by the candidate is correct to the best of my knowledge.

---

Prof. Idalberto Mendoza  
(Thesis Supervisor)

---

Date

# **Addis Ababa University Faculty of Technology**

## **Mechanical Engineering Department.**

The effect of the crack length and direction on the stresses and their distribution around the crack on cylindrical shell of a composite material.

By  
Mulugeta Hagos

### **Approved by Board of Examiners:**

Dr. Edessa Dribsa

Chairman Department of Graduate Committee

Prof. Idalberto Mendoza

Advisor

Dr. Daniel Tilahun

Internal Examiner

Dr-Ing Tamirat Tesfaye

External Examiner

---

---

---

---

## **Acknowledgment**

I would like to take this opportunity to express my sincere gratitude to Prof. Idalberto Mendoza, my advisor, for having patience to help me to work on this thesis and for his constant guidance and encouragement throughout the course of project. He is also greatly acknowledged for delivering his suggestion in streaming the project in the intended direction

## **Abstract**

Composite materials are artificially made materials used in various mechanical applications like shells and plates. Composite materials are nowadays commonly used in the aviation industry and marine application in the form of vessels or shells. In this work the stress and strain distributions around the surface cracked cylindrical shell structure have been presented by studying, the effect of the crack length and direction on stress, anisotropic elasticity relations and stress field equations of surface crack problems in composite materials, finally finite element method simulation is applied to investigate the stress and strain distribution around the surface crack in a composite material cylindrical shell structure.

Mathematical modeling and finite element simulation of the surface cracked cylindrical shell was performed and the problem was finally solved using ANSYS finite element method package.

In general, the maximum Von Misses stress at the crack tip generally increase as crack length increase and the internal axial crack orientation has shown the maximum critical von Misses stress distribution around the surface crack because of the nature of curved geometry of the shell structure and the far field tangentially applied stress tries to open the surface crack.

## Table of Contents

|                                                                                      |            |
|--------------------------------------------------------------------------------------|------------|
| <b>ACKNOWLEDGMENT .....</b>                                                          | <b>I</b>   |
| <b>ABSTRACT.....</b>                                                                 | <b>II</b>  |
| <b>TABLE OF CONTENTS .....</b>                                                       | <b>III</b> |
| <b>CHAPTER ONE THE PROBLEM STATEMENT .....</b>                                       | <b>1</b>   |
| 1.1 INTRODUCTION AND BACKGROUND OF THE PROBLEM .....                                 | 1          |
| 1.2 OBJECTIVES .....                                                                 | 2          |
| 1.3 SCOPE AND LIMITATION .....                                                       | 3          |
| <b>CHAPTER TWO LITERATURE SURVEY .....</b>                                           | <b>4</b>   |
| 2.1 LITERATURE REVIEW .....                                                          | 4          |
| <b>CHAPTER THREE INTRODUCTION TO COMPOSITE MATERIALS .....</b>                       | <b>12</b>  |
| 3.1 WHAT ARE COMPOSITE MATERIALS? .....                                              | 12         |
| 3.2 BACK GROUNDING HISTORY .....                                                     | 13         |
| 3.3 APPLICATION EXAMPLES.....                                                        | 14         |
| 3.3.1 Aircraft and Military aircraft application .....                               | 14         |
| 3.3.2 The sporting good industry.....                                                | 16         |
| 3.3.3 Marine application .....                                                       | 16         |
| 3.3.4 Consumer goods .....                                                           | 16         |
| 3.3.5 Construction and civil structures .....                                        | 17         |
| 3.3.6 Industrial applications.....                                                   | 17         |
| 3.4 CONSTITUENT AND MATERIAL .....                                                   | 17         |
| 3.4.1 Matrix.....                                                                    | 19         |
| 3.4.2 Fibers .....                                                                   | 20         |
| 3.5 CLASSIFICATION OF COMPOSITES .....                                               | 21         |
| 3.5.1 Based on matrix material .....                                                 | 22         |
| 3.5.2 Based on reinforcing material structure .....                                  | 23         |
| 3.6 DRAW BACKS OF COMPOSITE MATERIALS .....                                          | 27         |
| 3.7 ADVANTAGES.....                                                                  | 28         |
| 3.8 POTENTIAL ADVANTAGE OF COMPOSITE MATERIALS .....                                 | 29         |
| 3.9 FACTORS THAT CONTRIBUTE TO THE MECHANICAL PERFORMANCE OF COMPOSITE.....          | 30         |
| 3.10 HOOK'S LAW FOR TWO-DIMENSIONAL UNI-DIRECTIONAL LAMINA .....                     | 41         |
| 3.10.1 Hook's Law for a Two-Dimensional Angle Lamina.....                            | 44         |
| 3.10.2 Laminate Code.....                                                            | 47         |
| 3.10.3 Strain-Displacement Equations.....                                            | 48         |
| 3.10.4 Stress and Strain in a Laminates.....                                         | 50         |
| 3.10.5 Force and Moment Resultants Related to Mid-plane Strains and Curvatures ..... | 50         |
| <b>CHAPTER FOUR MODELING OF THE CYLINDRICAL SHELL STRUCTURE .....</b>                | <b>56</b>  |
| 4.1 FINITE ELEMENT METHOD THEORETICAL APPROACH .....                                 | 56         |
| 4.1.1 Mathematical Modeling.....                                                     | 56         |
| 4.1.2 Unified Near Crack Tip Stress and Displacement Field (Both Types) .....        | 59         |
| 4.1.3 Finite Element Modeling.....                                                   | 60         |

|                                                                    |           |
|--------------------------------------------------------------------|-----------|
| 4.1.4 Anisotropic Stress Functions .....                           | 65        |
| 4.1.5 Finite Element Method in Anisotropic Fracture Mechanics..... | 66        |
| 4.2 CONDITIONS AND METHODS.....                                    | 69        |
| 4.2.1 Cracked Composite Shell Modeling in ANSYS.....               | 69        |
| 4.2.2 Description of the Parameters of the Model.....              | 70        |
| 4.2.3 Loading and Boundary condition.....                          | 73        |
| <b>CHAPTER FIVE RESULT AND DISCUSSION .....</b>                    | <b>74</b> |
| 5.1 INTRODUCTION .....                                             | 74        |
| 5.2 DISCUSSING THE RESULTS .....                                   | 90        |
| <b>CHAPTER SIX CONCLUSION AND RECOMMENDATIONS .....</b>            | <b>94</b> |
| 6.1 CONCLUSION .....                                               | 94        |
| 6.2. RECOMMENDATION AND FUTURE DIRECTIONS.....                     | 95        |
| <b>REFERENCES .....</b>                                            | <b>96</b> |

## CHAPTER ONE

### THE PROBLEM STATEMENT

#### *1.1 Introduction and Background of the problem*

Fracture is a problem the society has faced for as long as there have been manmade structures. The problem may actually be worse today than in the previous centuries, because more can go wrong in our complex technological society.

Fortunately, advances in the field of fracture mechanics have helped to offset some of the potential dangers posed by increasing technological complexity. Our understanding of how materials fail and our ability to prevent such failure has increased considerably since World War II. Much remains to be learned, however, and existing knowledge of fracture mechanics is not always applied when appropriate especially for modern technological materials like composites

The combination of two or more materials can lead to a third material with highly desirable properties known as composite materials. The constituents of a composite material are usually combined on a macroscopic scale through physical rather than chemical means. Composite materials usually consist of a matrix and a reinforcing constituent. The matrix is often soft and ductile compared to the reinforcement. Various types of reinforcements are possible, including continuous fibers, chopped fibers, whiskers, flakes and particulates.

In many practical applications, using composites is more efficient. For example, in highly competitive airline markets; one is continuously looking for ways to lower overall mass of the aircraft without decreasing the stiffness and strength of its components like cylindrical shells or pressure vessels. This is possible by replacing conventional metal alloys with composite materials. Even if the composite material costs may be higher, the reduction in the number of parts in an assembly and the savings in the fuel costs make them more profitable

A shell is an initially curved plate defined by a middle surface, about which a constant or variable thickness is symmetrically situated. Typical ratio of a shell thickness to its radius is of the order 1:20. Shells are generally subjected to in plane loading and to the forces and couples normal to the surfaces, which are called here lateral loading. This nature of loading implies that the common mode of failure of shell structure is either in buckling or fracture (burst) of its walls. However, surface irregularities and micro cracks can be imparted to materials for various reasons. This could be while a joint is made during manufacturing,

assembly, etc. Hence, it is very important to analyze how the stress varies in the vicinity of crack in composite structures

A shell is in essence, a structure that can be derived from a plate by initially forming the middle surface as a singly or doubly curved surface. The way in which the shell supports the external loads is quite different from that of a flat plate. The stress resultant acting on the middle surface of the shell have both tangential and normal components, which carry a major part of the load, a fact that explains the economy of shell as load carrying structures and their well deserved popularity. The derivation of detailed governing equation for a curved shell problem presents many difficulties and, in fact, leads to many alternative formulations, each depending on the approximations introduced.

In the finite element treatment of shell problems, difficulties referred to above are eliminated, at the expense of introducing a further approximation. This approximation is of a physical, rather than mathematical nature. In this, it is assumed that the behavior of a continuously curved surface can be adequately represented by the behavior of a surface built up of small flat elements. Intuitively, as the size of the subdivision decreases it would seem that convergence must occur and indeed experiences indicates such a convergence.

Moreover, the nonlinearity imparted by material properties and geometric variations are also the contributing factors to the structural analysis of composite shells. The litigation in the analytical relation development for composite material in general is also another important factor that compels us to use the finite element method of stress analysis in the vicinity of a cracked composite structure.

Due to their high strength to weight and high stiffness to weight ratios, fiber reinforced composite laminated shell structures have been widely used in the aerospace, marine, automobile and other engineering industries. Thus, it is very important to study the behavior of composite material cylindrical shell structure for the distribution of stress and strains with an introduction of a surface crack

## ***1.2 Objectives***

The objective of this thesis is to analyze the effect of the crack length and direction (horizontal or vertical) on the stresses and their distribution around the crack on cylindrical shell of a composite material using finite element method (FEM). This will be done through modeling and simulating the cracked composite material cylindrical shell and applying the loads which actually exists in shells, tangential and axial loads with the following conditions will be justified.

### **Condition one**

A predefined dimension of the shell with tangential surface crack oriented on the interior and exterior surfaces of the cylindrical shell structure will be taken and the stress and their distributions around the crack will be analyzed and discussed in Model I loading condition subjected to far field axial stress

### **Condition two**

A predefined dimension of the shell with axial surface crack oriented on the interior and exterior surfaces of the cylindrical shell structure will be taken and the stress and their distribution around the surface crack will be analyzed and will be discussed in Model II loading conditions subjected to far field tangential stress

### ***1.3 Scope and limitation***

The aim of this thesis is to analyze surface cracked composite material cylindrical shell structure for the stress and strain and their distribution around the crack using finite element method package ANSYS software due to its availability. This will be done for the above objectives. The relevant equations governing the state of stress in a shell material will be derived using mechanics of materials and related with theoretical fracture mechanics for the analysis to be done showing an orthotropic material property on the software.

The scope of this thesis is limited to studying the properties of composite, analyze the effect of crack length and direction on stress, anisotropic, elastic relations, fracture mechanics approaches for composites, mathematical modeling of the surface cracked composite material cylindrical shell structures under in plane loading for stress intensity and fracture toughness, and finally applying finite element method to investigate the stress and strain distribution around the surface crack oriented on the composite cylindrical shell structure

## CHAPTER TWO

### LITERATURE SURVEY

#### *2.1 Literature Review*

As composite materials were introduced into the world, the initial impetus for using them was their high strength-to-weight and stiffness-to-weight ratios, along with very significant improvement in fatigue life and damage tolerance compared to most metallic materials. Furthermore, when composite materials are used to tailor a structure, they provide excellent opportunities for design flexibility and potential optimization of design criteria by controlling their directional natures to achieve certain performance goals or eliminate undesirable instabilities. Composite shell structures are one of those structures, finding applications in a variety of systems; including aircraft and submarine structures, space structures, automobiles, sport equipment, medical prosthetic devices and electronic circuit boards.

Exceptionally high stiffness and strength to weight ratios are the driving forces behind the success story of fiber reinforced composite structures in advanced lightweight constructions. For aerospace applications the triumphant progress is evident, culminating in the late announcement of a “First Composite Jetliner” [15]. But also high-speed ships, terrestrial vehicles and machines where high velocity or acceleration is requested utilize the advantage of this material more and more often. A set-up from layers with unidirectional reinforcement oriented in different directions, termed ‘multidirectional laminate’, has developed as a standard design. Homogenization of fiber and matrix behavior results in effective layer properties. In the assessment a homogeneous orthotropic layer is assumed as the smallest entity of the structure.

The structure with the cutouts of various shapes is adopted by the structural requirements or geometrical condition. Stress concentration of cylindrical shell with a cutout is important in many industrial structures such as aircrafts, vessels, submarines, nuclear reactors, etc. [7]. According to Lee *et al.*[7], the strain distribution around the un-stiffened and stiffened circular cutout of the isotropic stainless steel (ANSI type 304) and the GFRP laminated composite cylindrical shells subjected to axial compression have been studied and the obtained experimental results have been compared with numerical analysis. In their experiment, the strain was measured around the circular cutout by strain measuring

system under the axial uniform compression loading. The numerical analysis was performed by the commercial finite element code ANSYS and the strain concentration factor (SCF) was defined to investigate the strain concentration around the circular cutout. The SCF of the stainless steel cylindrical shell with a stiffened cutout was decreased 39% in experiment and 50 % in numerical analysis. The SCF of GFRP laminated composite cylindrical shell was decreased 22% in experiment and 31 % in numerical analysis.

Many composite components in aerospace structures are made of flat or curved panels with co-cured or adhesively bonded frames and stiffeners. Over the last decade a consistent stepwise approach has been developed which uses experiments to detect the failure mechanism, computational stress analysis to determine the location of first matrix cracking and computational fracture mechanics to investigate the potential for delamination growth [13]. According to Ronald Krueger [13] description, testing of thin skin stiffened panels designed for aircraft fuselage applications has shown that bond failure at the tip of the frame flange is an important and very likely failure mode. Debonding also occurs when a thin-gage composite fuselage panel is allowed to buckle in service. A methodology based on fracture mechanics has proven useful for characterizing the onset and growth of delamination in composites and has been used with limited success to investigate delamination onset and debonding in simple laboratory coupon type specimens. Future acceptance of a fracture mechanics methodology by industry and certification authorities however, requires the successful demonstration of the methodology on structural level.

A computer method to model crack growth in composite material shell finite elements undergoing large displacements and damage has been developed by Charoenphan *et al.* [19]. In the paper, they implemented a domain integral energy method for modeling crack growth in composite material shell structures using the finite element method. Volume integral expressions to evaluate the dynamic energy release rate in a through-thickness three-dimensional crack were derived. Using the domain integral, the energy release rate computation was implemented in the DYNA3D explicit non-linear dynamic finite element analysis program wherein crack propagation was modeled by releasing the constraints between initially constrained node pairs. The implementation enabled the program to either determine the energy resistance response for the material or predict the rate of crack propagation in shell structures. The numerical implementation was verified by simulating mode I and mode III slow crack growth problems in semi-infinite transversely isotropic

media, for which analytic solutions were available. Oscillations of energy following the release of nodal constraints as the crack propagates in discrete increments were suppressed using light mass proportional damping and a moving averaging scheme.

The behavior of an axis-symmetrical circumferential crack occurring in dissimilar materials represents such a non planar problem and has been investigated by Liu *et al.* [21]. In their study an axis-symmetrical fiber-matrix cylindrical model with a circumferential crack in the matrix of finite diameter was formulated within elastostatic scope. The problem was considered by means of integral transforms and a singular integral equation with a dominant generalized Cauchy kernel was obtained. Following the numerical solution technique developed by Erdogan, Gupta and Cook, the singular integral equation was reduced to a system of linear equations. By solving the linear equations, stress intensity factors associated with the crack length and the material properties were calculated and discussed. The solutions presented in their study were found to be general, including the solutions as special cases of their formulation for a homogeneous solid cylindrical bar and a thick-walled shell with an outer circumferential crack.

Jan Stegmann [23] has developed finite element based optimization techniques for laminated composite shell structures. The platform of implementation was the finite element based analysis and design tool MUST (MULTidisciplinary Synthesis Tool) and a number of features have been added and updated. This includes an updated implementation of finite elements for shell analysis, tools for investigation of nonlinear effects in multilayered topology optimization and a novel framework called discrete material optimization (DMO) for solving the material layout and orientation problem.

A necessary tool for optimization was robust finite elements and consequently, the finite element library in MUST was extended with a new three-node element and an updated four-node element. These were designated MITC3 and MITC4, respectively, since they used mixed interpolation of tensorial components to avoid problems with shear locking. The SHELLn family of standard isoparametric shell finite elements in MUST has also been updated for improved performance. All elements have laminate and geometrically nonlinear capabilities and tests showed that the performance and computational efficiency were very good.

In his work [23], geometrically nonlinear effects were investigated to determine if these should be taken into account when designing for maximum stiffness of laminated composite structures using structural optimization. Facilities for nonlinear topology optimization of multilayered shell structures was implemented using a Newton-Raphson scheme for the analysis, the adjoint variable method for sensitivity analysis and the MMA optimizer for solving the optimization problem. The SIMP method was used for layer-wise stiffness scaling to allow material to be added or removed in specific layers.

Existing methods for solving for optimal material orientation and maximum stiffness inherently suffer from problems with local optima, which inspired the development of discrete material optimization (DMO), which was a novel approach for simultaneous solution for material distribution and orientation. The DMO method uses an element level parameterization in a weighted sum formulation that allowed the optimizer to choose a single material from a set of pre-defined materials by pushing the weights to 0 and 1. As in his work [23], the success of the method was therefore dependent on the optimizer's ability to push the weights to 0 and 1 and several weighting schemes were implemented. Numerical examples of his work indicated that the method was indeed able to solve the combined material distribution and orientation problem. Furthermore, an industry related design problem of a wind turbine blade main spar was solved and the obtained results were very encouraging. The DMO method thus showed promising potential for application to problems of industrial relevance and no problems with local optima could be identified in the tested examples.

Anisotropy is an important characteristic of composite materials that must be taken into account both in elastic and inelastic analysis, including damage. Continuum damage mechanics (CDM) had important developments since the initial works of Kasyanov and Rabotnov, and constitutes now a practical tool to account for macroscopic damage in materials and structures [15]. Roberto and Guillermo [15], has presented an application of the anisotropic damage theory based in Murakami work. In his formulation, the fourth order damage tensor (that related Cauchy and effective stress tensors) was determined on the basis of the tensor (three-dimensional area density of damage) that, in turn, can be determined through experimental data. The analytical formulation has been set in incremental form and implemented into a finite element program (for plates and shells structure in composite material) taken in to account of geometrically non-linear effects. In

order to verify and validate the numerical model, comparisons with analytical and experimental results for simple situations has been presented.

Composite laminates are now widely used in both civil and military aircraft structures leading to weight saving. However, study of the behavior of such materials has shown that they are more damage-sensitive than metallic materials, specially to delaminations due to edge effects or low velocity impacts [17]. In his paper, Yves Ousset [17], described a numerical method to simulate delamination growth in layered composite plates within the framework of fracture mechanics. It was based on the search for the stationary points of the total energy of the plate (the sum of the mechanical energy and the fracture energy associated with the delamination growth). The resulting non-linear problem was solved by Newton's method. The expressions for the first and second derivatives of energy with respect to a crack front displacement were derived analytically and numerical examples were presented for DCB (double cantilever beam) specimens loaded in Mode I.

The characteristic properties of shell element with similar shapes are used to generate a so-called super element for the analysis of the crack problems for cylindrical pressure vessels. The formulation is processed by matrix condensation without the involvement of special treatment. This method can deal with various singularity problems and it also presents excellent results to crack problems for cylindrical shell. Specially, the knowledge of the kind of singular order is not necessary in super element generation; it is very economical in terms of computer memory and programming. This method also exhibits versatility to solve the problem of kinked crack at cylindrical shell [18].

Storakers *et al.* [25] have carried out a theoretical study for compressive loading of a thin circular embedded delamination located bellow a cylindrical surface. The delaminated member is subjected to nominally uniaxial and balanced bi axial loading with the main objective to analyze its influence on surface curvature specially as regards imminent crack growth. A finite element program earlier developed for delaminated plates has been generalized to apply also for shells and used to determine energy release rates and mode intensities along the delaminated front. A parameter study was made of the influence of curvature of the delaminated shell in particular as regards initiation and stability of crack growth.

Zheng-Ming Huang [26] in his paper focused on the ultimate flexural strength of a polymer based composite cylinder subjected to bending. According to the paper the outmost filament of the cylinder subjected to the maximum bending stress failed the first. The complexity, however, lied in the fact that the failure of this outmost filament generally did not correspond to the ultimate failure. Additional loads can still be applied to the cylinder and a progressive failure process will result. To deal with such a problem in his paper, the cylinder has been discretized into a number of lamina layers with different widths. The bridging micromechanics model [Huang, Z. M., composites part A, 2001] combined with the classical lamination theory has been applied to understand the progressive failure process generated in the cylinder. Only its constituent fiber and matrix properties under bending are necessary for this understanding and reasonably good accuracy has been achieved. However, the ultimate failure of the cylinder cannot be figured out only based on a stress failure criterion, as one cannot know a priori which ply failure corresponds to the ultimate failure. An additional critical deflection (curvature) condition must be employed also. By using both the stress and the deflection conditions, the estimated ultimate strength of the cylinder agreed well with an experimental measurement

Dynamic analysis of composite cylindrical shells has attracted much attention of researchers. A quick and accurate prediction of dynamic behavior of such cylindrical shells is of very much interest to designers and experimentalists alike. This normally requires a comprehensive development of a mathematical model. Due to the complexity of engineering characteristics of composite shell type structures, analytical solutions cannot be obtained in a straight forward manner. The differential quadrature method (DQM) is an efficient numerical technique which transforms governing equations of dynamic equilibrium to a matrix form by using weighted matrices. The DQM requires a small amount of computer capacity and provides accurate results. The DQM was successfully employed in various structural problems [14]. Haftchenari *et al.* [14] has presented a solution to the free vibration problem of laminated fibrous composite cylindrical shells. The governing equations of dynamic equilibrium were derived based on first order shear deformation theory. These were then solved by using the differential quadrature method by taking two types of boundary conditions (simply supported and clamped free ends) for a cylindrical shell. The results from the DQM solution were compared with some available theoretical as well as experimental results. It was observed that there was a very good

agreement between the results from this method and the corresponding ones found before by various researchers.

Khdier [16] has developed analytical solutions of the dynamic response of the classical, first-order and third-order theories of cross-ply laminated shallow shells for various boundary conditions. The solutions of his work are applicable to laminated shells with two opposite edges simply supported and the remaining ones can have arbitrary combinations of free, clamped and simply supported boundary conditions. A Levy type method in conjunction with generalized modal approach, were used to obtain these solutions. For thick shells, the classical shell theory predicts deflections and stresses significantly different from those of the third-order theory. The third-order theory and the first-order theory results were very close to each other for response and normal stress. However, the third-order theory did not require the use of shear correction factors as he concluded.

Hui-Shen Shen *et al.* [20] have presented post buckling analysis for an anisotropic laminated cylindrical shell of finite length subjected to combined loading of axial compression and torsion. The governing equations are based on classical shell theory with von Karman-Donnell-type of kinematics nonlinearity and including the extension-twist, extension-flexural and flexural-twist couplings. The nonlinear pre-buckling deformations and initial geometric imperfections of the shell were both taken into account. A singular perturbation technique was employed to determine interactive buckling loads and post buckling equilibrium paths. The numerical illustrations concern the post buckling response of perfect and imperfect, anisotropic laminated cylindrical shells for different values of load-proportional parameters. The results showed that the post buckling characteristics depend significantly upon the load-proportional parameter. The results revealed that in combined loading cases the post buckling equilibrium path was unstable and the shell structure was imperfection-sensitive.

Before 1992, substantial efforts have been devoted to developing techniques and standards for measuring fracture toughness and subcritical crack growth. These methods use specimens containing two-dimensional (2-D), through the thickness flaws because of the relative ease of fabrication and the availability of accepted analytical and numerical solutions. However, many defects observed in practice, and often responsible for failures or questions regarding structural integrity, are three-dimensional (3-D) surface flaws. The efficiency of data generated from standard specimens containing 2-D defects in predicting

crack growth behavior of 3-D flaws, including crack initiation, subcritical crack growth, and unstable fracture, is a major concern [30].

Raju and Newman [8], in their paper presented stress intensity factor influence coefficients for a wide range of semi-elliptical surface cracks on the inside or outside of an isotropic cylinder. The crack surfaces were subjected to four stress distributions: uniform, linear, quadratic, and cubic. These four solutions can be superimposed to obtain stress intensity factor solutions for other stress distributions, such as those caused by internal pressure and by thermal shock. The results for internal pressure were given for the ratio of crack depth to crack length ranged from 0.2 to 1; the ratio of crack depth to wall thickness ranged from 0.2 to 0.8; and the ratio of wall thickness to vessel radius was 0.1 or 0.25. The stress intensity factors were calculated by a three dimensional finite element method. The finite element models employ singularity elements along the crack front and linear strain element somewhere. The stress intensity factors were evaluated from a nodal-force method. The presented results were also compared to other analyses of surface cracks in cylinders. Similarly Nishioka and Atluri [9], Ezzat and Erdogan [10], and Newman and Raju [11] had focused on the stress intensity factor solutions for internal and external surface crack problems.

## Chapter Three

### Introduction to composite materials

#### *3.1 What are Composite Materials?*

As the term indicates, composite material reveals a material that is different from common heterogeneous materials. When two or more materials with very different properties are combined together they form a composite material. The different materials work together to produce a new material, which combines all of the properties of the previously separate materials. Within the composite it is still possible to easily tell the different materials apart. The purpose of composites is to allow the new material to have strengths from both materials, often times covering the original materials' weaknesses. Composites are different from alloys because alloys are combined in such a way that it is impossible to tell one particle, element, or substance from the other. Some common composite materials include concrete, fiberglass, mud bricks, and natural composites such as rock and wood. Currently composite materials refers to materials having strong fibers continuous or non-continuous surrounded by weaker matrix material. The matrix serves to distribute the fibers and also to transmit the load to the fiber. Composites can be easily found in nature. Wood is an example of a composite because cellulose fibers are held together by substance called lignin. These fibers can be found in cotton and thread, but it's the bonding power of lignin in wood that makes it much tougher.

The Composites are different from metals and are combinations of materials differing in composition or form. The constituents retain their identities in the composites, and do not dissolve or otherwise merge completely into each other, although they act together. Reinforced concrete is an excellent example of a composite structure in which the concrete and steel still retain their identities. The steel bars carry the tension loads, and the concrete carries the compression loads. In aircraft construction, the term composite structure refers to fabric resin combinations in which the fabric is embedded in the resin, but still retains its identity.

Advantages of composite materials are that, if well designed, they usually exhibit best qualities of their components or constituents and often some qualities that neither constituent possesses. Some of the properties that can be improved by forming a composite material are strength, stiffness, corrosion resistance, wear resistance, attractiveness, weight, fatigue life, temperature dependent behavior, thermal insulation, thermal conductivity and acoustical insulation. Naturally not all of these properties are improved at the same time nor are there usually any requirements to do so. In fact some of the properties are in conflict with one another. For example thermal insulation versus conductivity

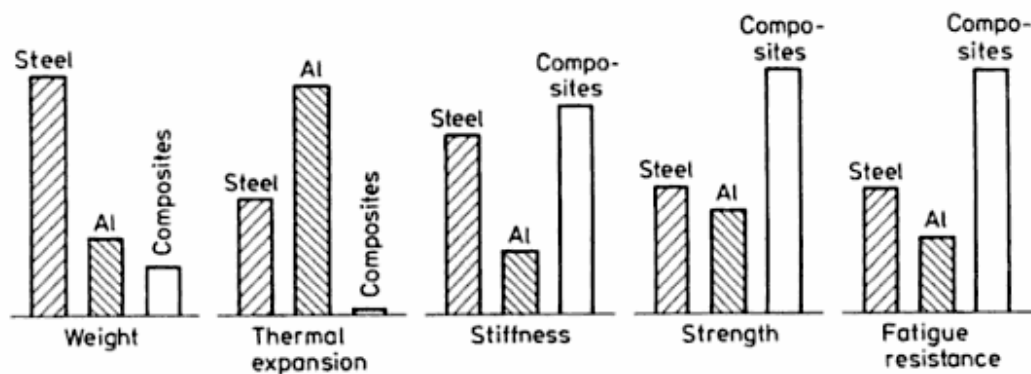


Fig 3.1 Comparison between monolithic and composite materials

### 3.2 Back grounding History

Composite materials have a long history of usage. Their precise beginnings are unknown, but all recorded history contains references to some of a composite materials. For example straw was used by the Israelites to strengthen mud bricks. Plywood was used by the ancient superior strength and resistance to thermal expansion as well as to swelling caused by the absorption of moisture. Medieval swords and armor were constructed with layers of different metals. More recently, fiber reinforced, resin matrix composite materials that have high strength to weight and stiffness to weight ratios have become important in weight sensitive applications such as aircraft and space vehicles.

The development of modern composite materials can be dated back to the early 1940s. It was in 1940 when fiberglass was first employed to reinforce plastics. The first successful application

was found in its use in nose radar domes, known as radomes, to protect aircraft antennas. Glass-polyester radomes were used to replace the plywood and canvas-urea domes. This type of fibrous composite demonstrated excellent load-bearing capacity, thermo stability and resistance to weathering. But, above all, it is their transparency to electromagnetic waves that made them most desirable for housing electronic equipment. Radomes made from fiberglass-reinforced plastics since been widely deployed on ships and ground installations, as well as aircraft. The first fiberglass boat was molded in 1942. In the same year, laminates of fiberglass with polyester resin were produced. Then, in 1946, the United States Government patented the first filament winding process

### ***3.3 Application examples***

#### **3.3.1 Aircraft and Military aircraft application**

The major reasons for the use of composite materials in spacecraft application include weight savings as well as dimensional stability. In low earth orbit (LEO), where temperature variation is from  $-100$  to  $+100^{\circ}\text{C}$ , it is important to maintain dimensional stability in support structures as well as in reflecting members. Carbon epoxy composite laminates can be designed to give a zero coefficient of thermal expansion. Typical space structures are tubular truss structures, face sheets for the payload bay door, antenna reflector, etc. In space shuttle composite material provide weight savings of 2688 lb per vehicle.



Fig 3.2 Boeing 777, the first American Commercial aircraft with composite empennage

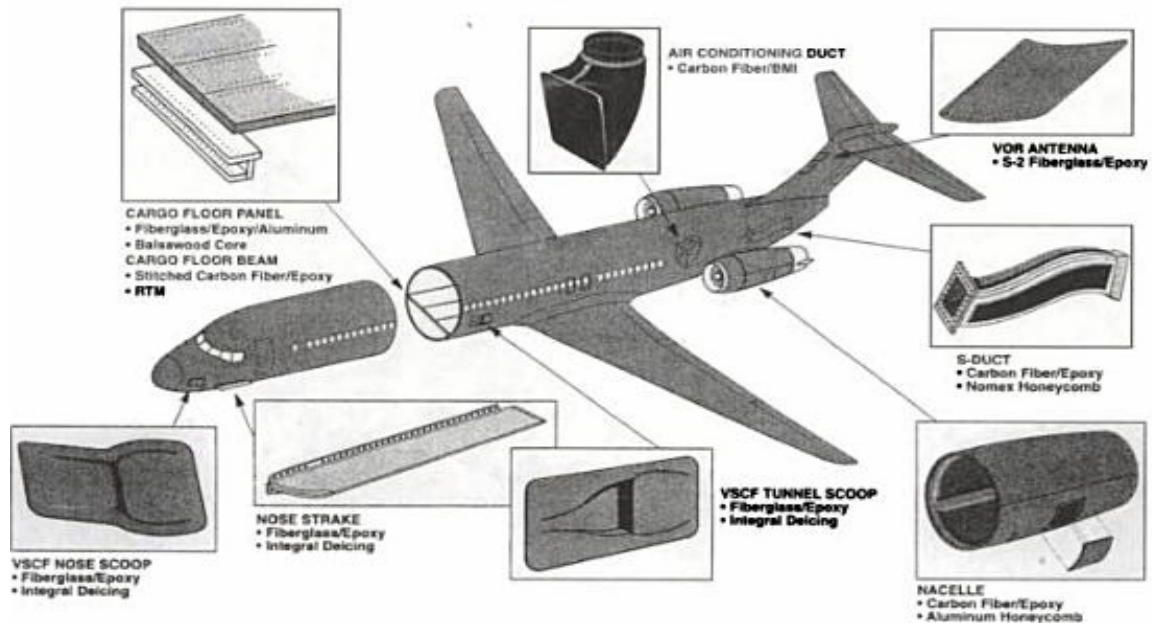


Fig 3.3 Typical composite structure used in commercial aircraft

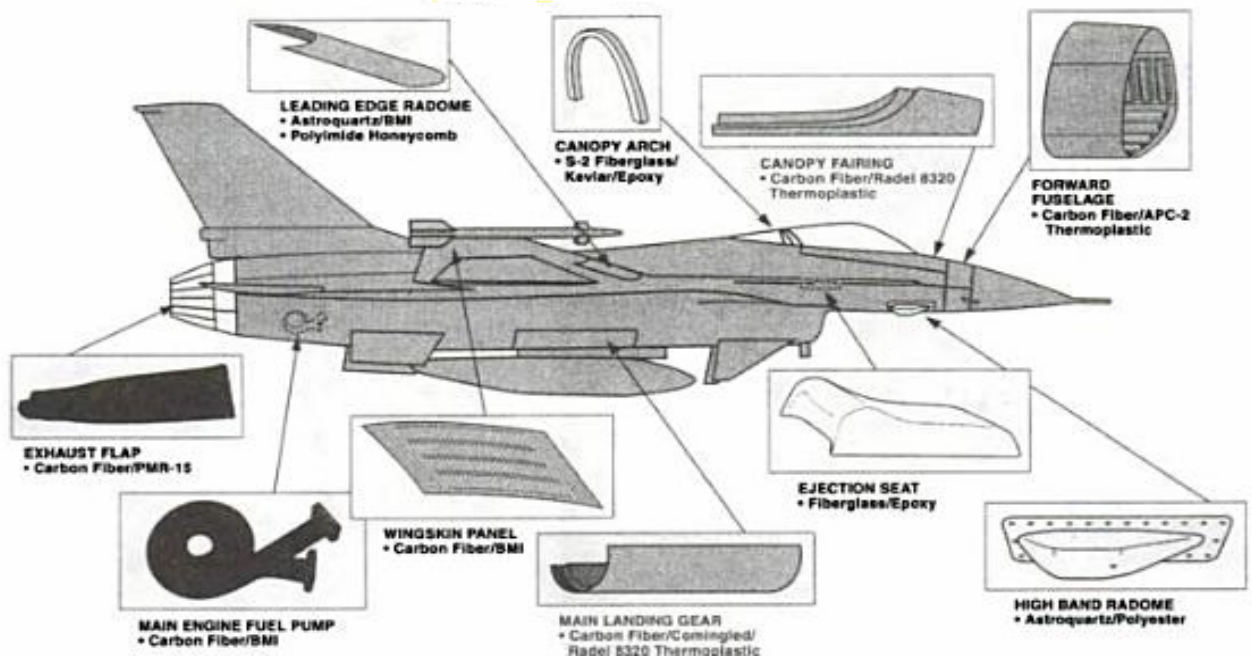


Fig 3.4 Typical composite structure used in military aircraft

### 3.3.2 The sporting good industry

Sports and recreation equipment suppliers are becoming major users of composite materials. The growth in structural composite usage has been greatest in high performance sporting goods and racing boats. Anyone who has visited a sporting goods store can see products such as golf shafts, tennis rackets, snow skis, fishing rods, etc made of composite materials. These products are light in weight and provide higher performance, which helps the user in easy handling and increased comfort

### 3.3.3 Marine application

Composite materials are used in a variety of marine application such as passenger ferries, power boats, buoys etc because of their corrosion resistance and lightweight, which gets translated in to fuel efficiency, higher cruising speed, and portability. The majorities of components are made of glass reinforced plastics with foam and honeycomb as core materials.



Fig 3.5 Composite material used for constructing ships

### 3.3.4 Consumer goods

Composite materials are used for a wide variety of consumer good applications such as sewing machines, doors, bathtubs, tables, chairs, computers, printers, etc. The majority of these components are short fiber composites made by molding technology such as compression molding, injection molding, RTM and SRIM

### **3.3.5 Construction and civil structures**

The driving force for the use of glass and carbon reinforced plastics for bridge application is reduced installations, handling, repair and life-cycle cost as well as improved corrosion and durability. It also saves a significant amount of time for repair and installations and thus minimizes the blockage of traffic

### **3.3.6 Industrial applications**

The use of composite materials in various industrial applications is growing. Composites are being used in making industrial rollers and shaft for the printing industry and industrial drive shafts for cooling-tower application. Filament winding shows good potential for the above applications. Injection molded, short fiber composite are also used for making robot arms and provide improved stiffness, damping, and response time

### **3.4 Constituent and Material**

Major constituents in fiber-reinforced composite materials are the reinforcing fibers and a matrix, which acts at a binder for the fibers. Other constituents that may also be found are coupling agents, coatings, and fillers. Coupling agents and coatings are applied on the fibers to improve their wetting with the matrix as well as to promote bonding across the fiber/matrix interface. Both in turn promote a better load transfer between the fibers and the matrix. Filler are used with some polymeric matrices primarily to reduce cost and improve their dimensional stability

Manufacturing of a composite structures starts with the incorporation of a large number of fibers into a thin layer of matrix to form a lamina (ply). The thickness of a lamina is usually in the range of 0.1-1 mm (0.004-0.04in). If continuous (long) fibers are used in making the lamina, they may be arranged either in a unidirectional orientation (i.e. all fibers in one direction) or in a bidirectional orientation (I.e. fibers in two directions, usually normal to each other). For a lamina containing unidirectional fibers, the composite materials have the highest strength and modulus in the longitudinal direction of the fibers. However, in the transverse direction, its strength and modulus are very low. For a lamina containing bidirectional fibers, the strength and modulus can be varied by employing different amount as well as different types of fibers in the longitudinal and transverse direction. For a balanced lamina, these properties are the same in both directions.

A lamina can be constructed using discontinuous (short) fibers in a matrix. The fibers can be arranged in a unidirectional orientation or in a random orientation. Discontinuous fiber-reinforced composites have lower strength and modulus than continuous fiber composites. However, with random orientation of fibers, it is possible to obtain nearly equal mechanical and physical properties in all direction in the plane of lamina

The thickness required to support a given load or maintain a given deflection in a fiber reinforced composite structure is obtained by stacking several lamias in a specified sequence to form a laminate. Various lamina in lamina in a laminate may contain fibers either all in one direction or in different direction. It is also possible to combine different or in different direction.

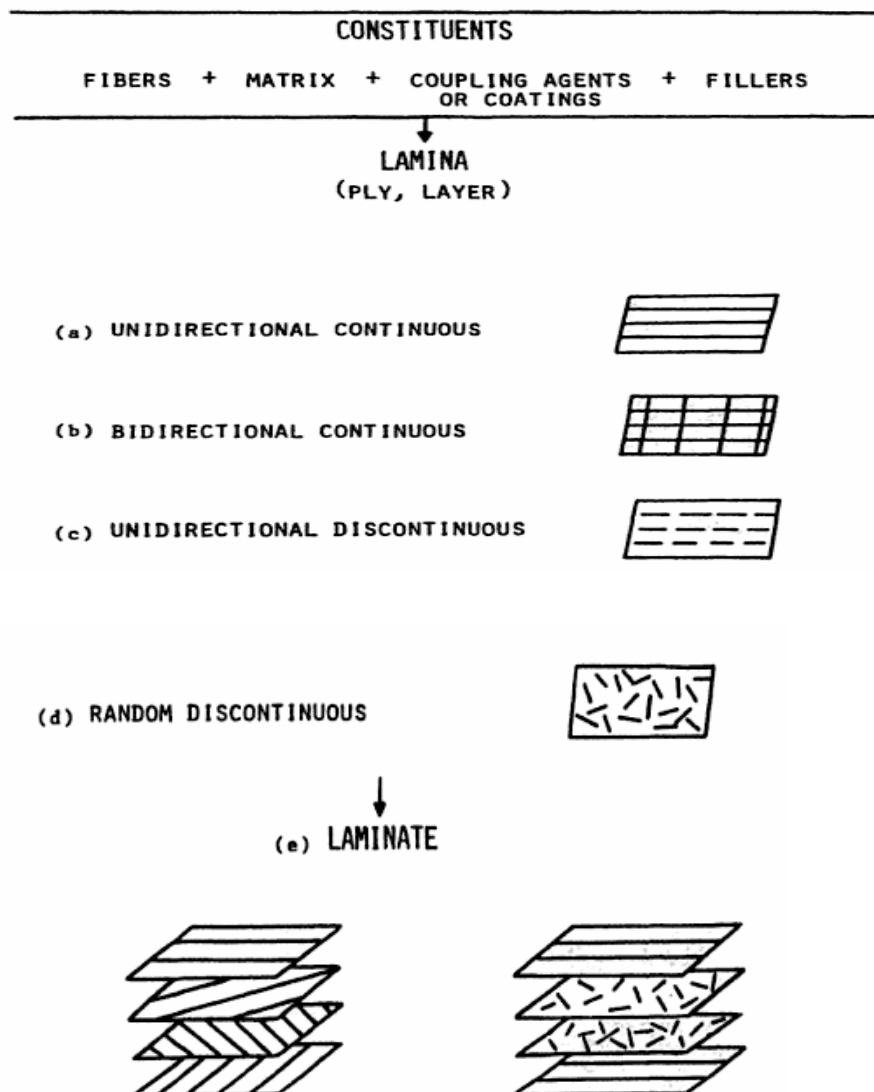


Fig 3.6 Basic building blocks in fiber-reinforced composite

### 3.4.1 Matrix

- Is the component that holds the filler together to form the bulk of the material of a composite and has several functions:
  - Binds the fibers together and transfers the load to the fibers. It provides rigidity and shape to the structure.
  - The matrix isolates the fibers so that individual fibers can act separately. This stops or slows the propagation of a crack
  - The matrix provides a good surface finish quality and aids in the production of net shape or near net shape parts.
  - The matrix provides protection to reinforcing fibers against chemical attack and mechanical damage( wear)
  - Depending on the matrix material selected, performance characteristics such as ductility, impact strength, etc. are also influenced. A ductile matrix will increase the toughness of the structure. For higher toughness requirements, thermoplastic
  - The failure mode is strongly affected by the type of matrix material used in the composite as well as its comparability with the fiber

The matrix plays minor role in the tensile load carrying capacity of composite structures. However, selection of a matrix has a major influence on the inter laminar shear as well as in –plane shear properties of the composite materials. The interlaminar shear strength is an important design consideration for structures under bending loads, where as the in-plane shear strength is important under torsional loads. The matrix provides lateral support against the possibility of fiber buckling under compression loading, thus influencing to some extent the compressive strength of the composite material. The interaction between fibers and matrix is also important in designing damaged-tolerant structures. Finally , the process ability and defects in a composite material depend strongly on the physical and thermal characteristic such as viscosity, melting point and curing temperature of the matrix.

## **Matrix materials**

### **Polymeric**

Thermoset polymers (resins)

Epoxies : Principally used in aerospace and aircraft application

Polyester,vinylesters: commonly used in automotive, marine ,chemical and electrical applications

Phenolics: used in bulk molding compounds

Polyimides, polybenzimidazoles(PBI),polyphenylquinoxaline(PPQ): for high temperature aerospace applications( temperature range:250-400c)

Thermoplastic polymers

Nylons ( such as nylon 6,nylon 6,6) , thermoplastic polyesters (such as PET, PBT), polycarbonate (PC),polyacetals: used with discontinuous fibers in injection-molded article

Polyamide-imide (PAI),polyether-ether ketone(PEEK),polysulfone(PSUL),polyphenylene sulfide(PPS),polyether(PEI): suitable for moderately high temperature applications with continuous fibers

### **Metallic**

Aluminum and its alloys , titanium alloys , magnesium alloys, copper-based alloys, nickel based super alloys, stainless steel: suitable for high temperature applications( temperature range:300-500c)

### **Ceramics**

Aluminum oxide ( $Al_2O_3$ ) , carbon , silicon carbide(SiC), Silicon nitride( $Si_3N_4$ ): suitable for high temperature applications

## **3.4.2 Fibers**

Fibers are the principal constituent in a fiber reinforced composite material. They occupy the largest volume fraction in a composite laminate and share the major portion of the load acting on

a composite structure. The fillers can be of any material such as carbon fiber, glass bead, Glass fiber, carbon fiber, extended chain polyethylene fiber, boron fiber ,sand, or ceramic. The main function of the fiber in the composite is:

- To carry the load. In a structural composite, 70 to 90% of the load is carried by fibers.
- To provide stiffness, strength, thermal stability, and other structural properties in the composite
- To provide electrical conductivity or insulation, depending on the type of fiber used.
- It reinforces the mechanical properties of the matrix

Proper selection of the type, amount, and orientation of fibers is very important since it influences the following characteristics of a composite laminate

- Specific gravity
- Tensile strength and modulus
- Compressive strength and modulus
- Fatigue strength as well as fatigue failure mechanism
- Electrical and thermal conductivity
- Cost

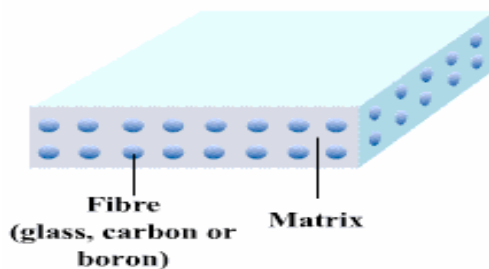


Fig 3.6 constituent of composite material

### ***3.5 Classification of Composites***

Composite material is a material composed of two or more distinct phases (matrix phase and dispersed phase) and having bulk properties significantly different from those of any of the constituents.

- Matrix phase

The primary phase, having a continuous character, is called matrix. Matrix is usually more ductile and less hard phase. It holds the dispersed phase and shares a load with it.

- Dispersed (reinforcing) phase

The second phase (or phases) is embedded in the matrix in a discontinuous form. This secondary phase is called dispersed phase. Dispersed phase is usually stronger than the matrix, therefore it is sometimes called reinforcing phase.

Many of common materials (metal alloys, doped Ceramics and Polymers mixed with additives) also have a small amount of dispersed phases in their structures, however they are not considered as composite materials since their properties are similar to those of their base constituents (physical properties of steel are similar to those of pure iron).

There are two classification systems of composite materials. One of them is based on the matrix material (metal, ceramic, and polymer) and the second is based on the material structure:

### **3.5.1 Based on matrix material**

#### **3.5.1.1 Metal Matrix *Composites* (MMC)**

Metal matrix composite (MMCs) as the name implies, have a metal matrix. Examples of matrices in such composites include aluminum, magnesium and titanium. Typical fibers include carbon and silicon carbide. Metals are mainly reinforced to increase or decrease their properties to suit the needs of the design. For example, the elastic stiffness and thermal and electrical conductivity of metals can be reduced by the addition of fibers such as silicon carbide

#### **3.5.1.2 Ceramic Matrix *Composites* (CMC)**

Ceramic matrix composite has a ceramic matrix such as alumina, calcium, aluminum silicate reinforced by fibers such as carbon and silicon carbide.

Advantages of CMCs

High strength, hardness, high service temperate, chemical inertness, and low density. However, ceramics by themselves have low fracture toughness. Under tensile or impact loading, they fail

catastrophically. Reinforced ceramics with fibers, such as silicon carbide or carbon, increases their fracture toughness because it causes gradual failure of the composite. This combination of a fiber ceramics matrix composite make more attractive for application where both high mechanical properties and extreme service temperatures are desired.

### **3.5.1.3 Polymer Matrix Composites (PMC)**

The most common advanced composite are polymer matrix composites (PMC). These polymers consist of a polymer reinforced by thin diameter fibers (graphite, aramids, boron, carbon). For example, graphite epoxy composites are approximately five times stronger than steel on a weight-for-weight basis. The reasons why they are the most common composite include their low cost, high strength, and simple manufacturing principles. The main drawbacks of PMCs include low operating temperatures, high coefficient of thermal and moisture expansion and low elastic properties in certain directions

### **3.5.1.4 Carbon Carbon composite**

Carbon carbon composites use carbon fibers in a carbon matrix. Carbon carbon composites are used in very high temperature environments of up to 6000 F (3315c) and are 20 times stronger and 30% lighter than graphite fibers.

Advantage of carbon carbon composite

Carbon by itself is brittle and flaw sensitive like ceramics. Reinforcement of a carbon matrix allows the composite to fail gradually and also gives advantages such as ability to withstand high temperatures, low creep at high temperature, low density, good tensile and comprehensive strengths, high fatigue resistance, high thermal conductivity and high coefficient of friction.

## **3.5.2 Based on reinforcing material structure**

### **3.5.2.1 Particulate composite**

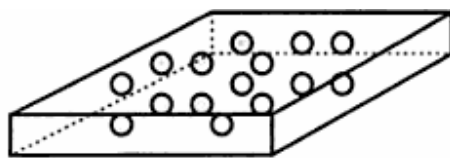
Consist of particles immersed in materials such as alloys and ceramics. They are usually isotropic since the particles are added randomly. Particulate composites have the advantage such as improved strength, increased operating temperature and oxidation resistance etc. Typical examples include use of aluminum particles in rubber, silicon carbide particles in aluminum, and gravel, sand and cement to make concrete

### 3.5.2.2 Flake composite

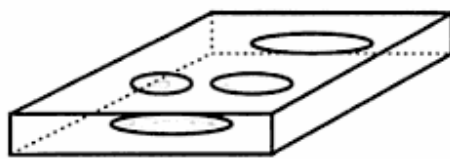
Consist of flat reinforcements of materials. Typical flake materials are glass, mica, aluminum and silver. Flake composite provide advantage such as high out of plane flexural modulus, higher strength and low cost. However, flakes cannot be oriented easily and only a limited number of materials are available for use

### 3.5.2.3 Fiber composite

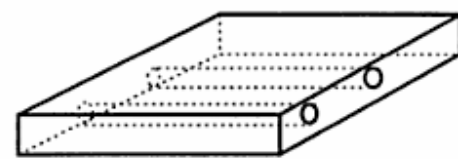
Consist of materials reinforced by short(discontinuous) or long (continuous) fiber. Fibers are generally anisotropic and examples include carbon and aramids. The fundamental unit of continuous fiber matrix composite is unidirectional or woven fiber laminas



Particulate Composites



Flake Composites



Fiber Composites

Fig 3.7 types of composite based on reinforcement shape

### Special Features of Composite

Composite have been routinely designed and manufactured for application in which high performance and light weight are needed. They offer several advantages over traditional engineering materials as discussed below

1. Composite materials provide capabilities for part integration. Several metallic components can be replaced by a single composite components
2. Composite structure provide in service monitoring or online process monitoring with the help of embedded sensors. This feature is used to monitor fatigue damage in aircraft structure
3. Composites offer good impact properties, the fig below shows impact properties of aluminum, steel, glass/epoxy, Kevlar/epoxy, and carbon/epoxy continuous fiber composites. Glass and Kevlar composites provide higher impact strength than steel and aluminum

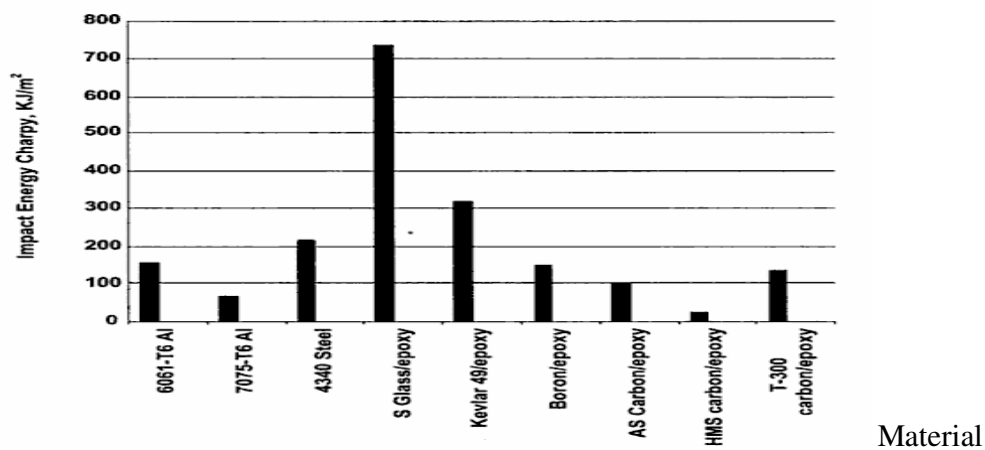


Fig 3.8 Impact properties of various engineering materials. Unidirectional composite materials with about 60% fiber volume fraction are used

4. Noise, vibration, and harshness (NVH) characteristics are better for composite materials than metals. Composite materials dampen vibrations an order of magnitude better than metals. These characteristics are used in a variety of applications, from the leading edge of an airplane to golf clubs.
5. By utilizing proper design and manufacturing techniques, cost effective composite parts can be manufacture. Composites offer design freedom by tailoring material properties to meet performance specifications, thus avoiding the over design of products. This is achieved by changing the fiber orientation, fiber type , and/or resin systems

6. Glass-reinforced and aramid-reinforced phenolic composites meet FAA and JAR requirements for low smoke and toxicity. This feature is required for aircraft interior panels. Stowbins and galley walls.
7. The cost of tooling required for composite processing is much lower than that of metals processing because of lower pressure and temperature requirements. This offer greater flexibility for design change in this competitive market where product lifetime is continuously reducing

### **Composite markets**

There are many reasons for the growth in composite applications, but the primary impetus is that the products fabricated by composites are stronger and lighter. To day it is difficult to find any industry that does not utilize the benefits of composite materials. The largest user of composite materials to day is transpiration industry, having consumed 1.3 billion pounds of composites in 2000. Composite materials have become the materials of choice for several industries

In the past three to four decades, there have been substantial changes in technology and its requirements. This changing environment created many new needs and opportunities, which are only possible with the advances in new material and their associated manufacturing technology

In the past decade, several advanced manufacturing technology and materials systems have been developed to meet the requirements of the various market segments. Several industries have capitalized on the benefits of composite materials. The vast expansion of composite usage can be attributed to the decrease in the cost of fibers as well as the development of automation techniques and high volume production methods. For example the price of carbon fiber decreased from \$150 lb in 1970 to about \$8 lb in 2000. This decrease in cost was due to the development of low cost-production methods and increased industrial use.

Broadly speaking, the composite market can de divided into the following industry categories: aerospace, automotive, construction, marine, corrosion resistant equipment,

consumer products , appliance/business equipments and others.

### **Barriers in composite market**

The primary barrier to the use of composite material is their high initial cost in some cases, as compared to traditional materials. Regardless of how effective the material will be over its life cycle, industry considers high upfront costs, particularly when the life-cycle cost is relatively uncertain. This cost barrier inhibits research into new materials

The recycling of composite materials presents a problem when penetrating a high volume market such as the automotive industry, where volume production is in the millions of parts per year. With the new government regulations and environmental awareness, the use of composites has become a concern and poses a big challenge for recycling

### ***3.6 Draw backs of composite materials***

- High cost of fabrication of composite is a critical issue, for example part made of graphite/epoxy composite may cost up to 10 to 15 times the material costs. Improvements in processing and manufacturing techniques will lower the cost. Already manufacturing techniques such as SMC( sheet molding compound) and SRIM( structural reinforcement injection molding ) are lowering the cost and production time in manufacturing automobile parts
- Mechanical characterization of a composite structure is more complex than that of a metal structure, unlike metals, composite materials are not isotropic, that is, their properties are not the same in all directions. So they require more material parameters. For example, a single layer of a graphite /epoxy composite requires nine stiffness and strength constants for conducting mechanical analysis. In the case of a monolithic material such as steel, one requires only four stiffness and strength constants. Such complexity makes structural analysis computationally and experimentally more complicated and intensive. In addition , evaluation and measurement technique of some composite properties such as compressive strengths are still being debated
- Repair of composite is not a simple process as compared to metals. Sometimes critical flaws and cracks in composite structure may go undetected
- Composites do not have a high combination of strength and fracture toughness vs. yield

strength for a 25 mm thick material. Metals show an excellent combination of strength and fracture toughness as compared to composites

- Composites do not necessarily give higher performance in all the properties used for material selection. The six primary material selection parameters are strength, toughness, formability, jointability, corrosion resistance and affordability
- Cannot be used for load-bearing or structural work
- It is very temperature sensitive and expands and contracts with temperature change
- If damaged or faded the material must be replaced

### **3.7 Advantages**

- lightweight (low density, high strength, super high strength-to-weight ratio)
- able to reduce part count and assembly by molding complex parts that would require sub assemblies if done in metal.
- Optimizable to further reduce the weight because properties can be tailored by varying the fiber type and their orientation.
- can be made transparent to radio frequency (antennas, radomes)
- lightweight (low density, high strength, super high strength-to-weight...ratio)
- Form ability. Can be shaped to any contour. This means aircraft will not need any rivets sticking out which would cause drag.
- Longer life. Composites have a longer life expectancy than steel or aluminum.
- Smooth, splinter free finish
- Pre-colored and never has to be painted, oiled or stained
- Easy to clean with a hose
- the corrosion resistance and weather ability has substantially reduced the maintenance cost and extends the product lifetime
- Composites exhibit a higher strength to weight ratio than steel or aluminum and can be engineered to provide a wide range of tensile, flexural and impact strength properties. For example, a composite's strength per unit density is roughly two times that of aluminum and four times that of steel.

### 3.8 Potential advantage of composite materials

| Potential advantages                              | Typical application or application areas                                                                  |
|---------------------------------------------------|-----------------------------------------------------------------------------------------------------------|
| Specific strength and stiffness                   | All applications, but particularly in aerospace, defense , sporting goods , industrial machinery, bridges |
| Fatigue properties                                | Aircraft , fan blades, pressure vessels, drive shafts                                                     |
| Damping properties                                | Drive shafts, sporting goods, musical instruments                                                         |
| Electrical insulation                             | Electrical housing , transformer spacers, cable trays ladder                                              |
| Corrosive resistance/low maintenance requirements | Industry, marine, off shore , bridges                                                                     |
| Thermal insulation                                | Refrigerated container, building panels, fire walls and doors                                             |
| Radar transparency                                | Aircraft radomes, patient supports for x-ray equipments                                                   |
| Non-magnetic                                      | Mine sweepers, patient supports for magnetic resonance scanners                                           |
| Biological inertness                              | Replacement joints , tendons                                                                              |
| Energy absorption                                 | Helmets, bulletproof partitions, armor , crash barriers                                                   |
| Fire and smoke properties                         | Fire walls and doors , aircraft interiors                                                                 |
| High tech image                                   | Sporting goods , automobile                                                                               |
| Geometrical complexity/parts integration          | All applications, but particularly in automobile and aircraft                                             |

|                                         |                                                                                                                     |
|-----------------------------------------|---------------------------------------------------------------------------------------------------------------------|
| Anisotropy                              | Aircraft, springs, pressure vessels, piping                                                                         |
| Shape flexibility                       | Used to achieve shapes that cannot ( at similar cost) be made with metals, for example in automobile body panels    |
| Thermal expansion/dimensional stability | Used to eliminate thermal expansion , for example in satellite antennae                                             |
| Tooling cost                            | All applications, but particularly in automobile and aircraft                                                       |
| Low scrap value                         | Used to eliminate theft, for example of reusable beer kegs and road signs( where steel/ aluminum designs are stolen |
| Manufacturing cost                      | All application , but particularly in automobiles and aircraft                                                      |

Table 3.1 Potential advantage of composite materials

### ***3.9 Factors that contribute to the mechanical performance of composite***

Fiber factors: Four fiber factors contribute to the mechanical performance of composites

**Length:** the fibers can be either long or short. Long continuous fibers are easy to orient and process, while short fibers cannot be controlled fully for proper orientation. Long fibers provide many benefits over short fibers. These include impact resistance, low shrinkage, improved surface finish, and dimensional stability. However, short fibers provide low cost, are easy to work with and have fast cycle time fabrication procedures. Short fibers have few flaws and therefore have higher strength

**Orientation:** fibers oriented in one direction give very high stiffness and strength in that direction. If the fibers are oriented in more than one direction, such as in a mat, there will be high stiffness and strength in the directions of the fiber orientation. However, for the same volume of fibers per unit volume of the composite, it cannot match the stiffness and strength of unidirectional composites

**Shape:** the most common shape of fibers is circular because handling and manufacturing them is

easy. Hexagon and square shaped fibers are possible but their advantages of strength and high packing factor do not outweigh the difficulty in handling and processing

**Material:** the material of the fiber directly influences the mechanical performance of a composite. Fibers are generally expected to have high elastic moduli and strengths. This expectation and cost have been key factors in graphite, armids and glass dominating the fiber market for composite

**Matrix factor:** Use of fiber by themselves is limited, with the exceptions of rope and cables. Therefore, fibers are as reinforcement to matrices. The matrix function includes binding the fibers together, protecting fibers from environment, shielding from damage due to handling, and distributing the load to fibers. Although matrices by themselves generally have low mechanical properties as compared to fibers, the matrix influences many mechanical properties of the composite. These properties include transverse modulus and strength, shear modulus and strength, compressive strength, inter-laminar shear strength, thermal expansion coefficient, thermal resistance and fatigue strength

### **Mechanical behavior of composite materials**

Composite materials are coming into prominence. Fiber-reinforced composite materials are uniquely useful because they can be constructed in such a manner that they have higher strength to density ratios, or stiffness to density ratios of any materials at moderate temperatures and the fibers can be oriented to provide minimum structural weight for a given structural geometry and given systems of loads.

Composite materials have many characteristics that are different from conventional engineering materials. Some characteristics are merely modifications of conventional behaviors; others are totally new and require new analytical and experimental procedures. Most common engineering materials are homogeneous and isotropic.

A homogeneous body has, uniform properties throughout, i.e. the properties are not a function of positions in the body. An isotropic body has material properties are not a function of orientation at a point in the body. In contrast, composite materials are often both in homogeneous and non isotropic (orthotropic or, more generally, anisotropic). An in homogeneous body has non-uniform

properties over the body, i.e, the properties are a function of positions in the body.

An orthotropic body has materials, properties that are different in three mutually perpendicular directions at a point in the body and, further have three mutually perpendicular planes of material symmetry. Thus, the properties are a function of orientation at a point in the body. An anisotropic body has material properties that are different in all directions at a point in the body; there are no planes of material property symmetry. Again, the properties are a function of orientation at a point in the body

Because of the inherent heterogeneous nature of composite materials, they are conveniently studied from two points of view: micromechanics and macro mechanics. Micromechanics – is the study of composite material behaviors wherein the interaction of the constituent materials are examined on a microscopic scale. This is particularly concerned with the composition of the composite in their early fabrication process. Macro mechanics- is the study composite material behavior wherein the material presumed homogenous and the effects of the constituent materials are detected, only as average apparent properties of the composite

### **Basic assumptions**

The basic assumptions made in material science approach model of fiber reinforced composite are

- The bond between fibers and matrix is perfect
- The fibers are continuous and parallel aligned in each. They are packed regularly; i.e. the space between fibers is uniform
- Fiber and matrix materials are linear elastic, they follow approximately Hooke's law and each elastic model is constant
- The composite is free of voids

### **Stress**

A mechanical structure takes external forces, which act upon a body as surface forces (for example, bending a stick) and body forces (for example, the weight of a standing vertical telephone pole on itself). These forces result in internal forces inside the body. Knowledge of the internal forces at all points in the body is essential because these forces need to be less than the strength of the material used in the structure. Stress, which is defined as intensity of the load per

unit area, determines this knowledge because the strength of a material are intrinsically known in terms of stress.

Imagine a body in equilibrium under various loads. If the body is cut at a cross section, forces will need to be applied on the cross-sectional area so that it maintains equilibrium as in the original body. At any cross-section, a force  $\Delta P$  is acting on an area of  $\Delta A$ . This force vector has a component normal to the surface,  $\Delta P_n$  and one parallel to the surface,  $\Delta P_s$ . The definition of stress then gives.

$$\sigma_n = \lim_{\Delta A \rightarrow 0} \frac{P_n}{\Delta A} \quad (a)$$

$$\tau_s = \lim_{\Delta A \rightarrow 0} \frac{P_s}{\Delta A} \quad (b)$$

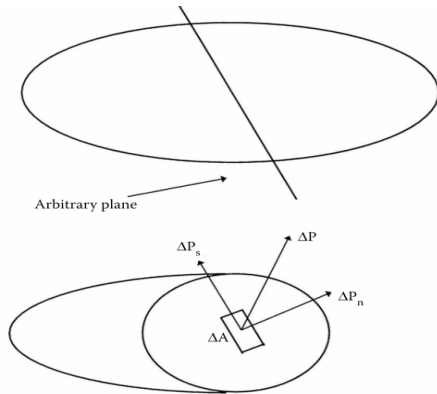


Fig.3.9 Force on an area

The components of the stress normal to the surface,  $\sigma_n$  is called the normal stress and the stress parallel to the surface,  $\tau_s$  is called the shear stress. If one takes a different cross-section through the same point, the stress remains unchanged but the two components of stress, normal stress,  $\sigma_n$  and shear stress,  $\tau_s$  will change. However, it has been proved that a complete definition of stress at a point only needs use of any three mutually orthogonal coordinate systems, such as a Cartesian coordinate system.

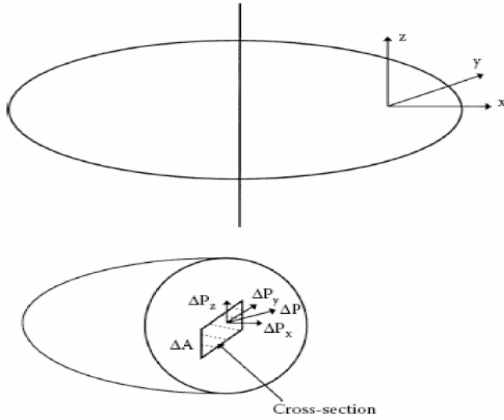


Fig 3.10 Forces on infinitesimal area on the y-z plane

$$\sigma_x = \lim_{\Delta A \rightarrow 0} \frac{\Delta P_x}{\Delta A}$$

$$\tau_{xy} = \lim_{\Delta A \rightarrow 0} \frac{\Delta P_y}{\Delta A}$$

$$\tau_{xz} = \lim_{\Delta A \rightarrow 0} \frac{\Delta P_z}{\Delta A}$$

Similarly, stress can be defined for cross-sections parallel to the xy and xz planes. For defining all these stresses, the stress at a point is defined generally by taking infinitesimal cuboids in a right hand coordinate system and finding the stresses on each of its faces. Nine different stresses act at a point in the body shown in the fig below.

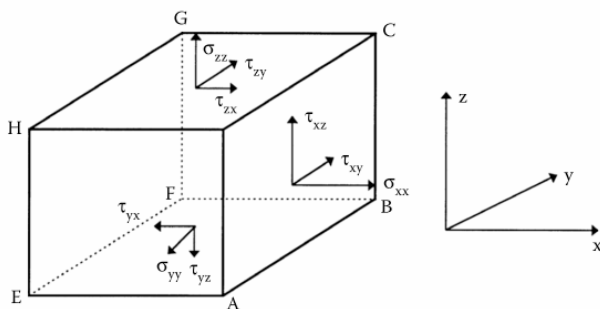


Fig 3.11 Stresses on a body

Strain

Let  $u = u(x, y, z)$  is the displacement along the x axis at a point  $(x, y, z)$

$v = v(x, y, z)$  is the displacement along the y axis at a point  $(x, y, z)$

$w = w(x, y, z)$  is the displacement along the z axis at a point  $(x, y, z)$

And the corresponding strain components of the above mentioned displacements are respectively  $\epsilon_x, \epsilon_y$  and  $\epsilon_z$ . Then we have the following relationships between the strain components and the corresponding displacements:

$$\epsilon_x = \frac{\partial u}{\partial x}, \epsilon_y = \frac{\partial v}{\partial y}, \text{ and } \epsilon_z = \frac{\partial w}{\partial z} \quad (d)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}, \text{ and } \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \quad (e)$$

Hooke's Law for Different Types of Materials

For three-dimensional elastic anisotropic the generalized Hook's law is expressed as

$$\sigma_{ij} = \sum_{k=1}^3 C_{ijkl} \sum_{l=1}^3 \epsilon_{kl} \quad (i, j = 1, 2, 3) \quad (3.1)$$

where  $\sigma_{ij}$  and  $\epsilon_{kl}$  are the stress and strain tensors, respectively, and  $C_{ijkl}$  are the elastic constants. Here the indices i, j, k and l can assume values of 1, 2 and 3. This implies that there may exist  $3^4 = 81$  independent elastic constants. However, it is known from the theory of elasticity, that both stress tensor  $\sigma_{ij}$  and strain tensor  $\epsilon_{kl}$  are symmetric. As  $\sigma_{ij} = \sigma_{ji}$ ,  $C_{ijkl} = C_{jilk}$

and as  $\epsilon_{kl} = \epsilon_{lk}$ ,  $C_{ijkl} = C_{ikjl}$

Thus,  $C_{ijkl} = C_{jikl} = C_{ijlk} = C_{jilk}$

The general equation (3.1) may be written as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \quad (3.2)$$

Where

$$\begin{aligned} C_{11} &= \frac{1 - \nu_{23}\nu_{23}}{E_2 E_3 \Delta}, \quad C_{12} = \frac{\nu_{21} + \nu_{31}\nu_{23}}{E_2 E_3 \Delta} = \frac{\nu_{12} + \nu_{32}\nu_{13}}{E_1 E_3 \Delta} \\ C_{13} &= \frac{\nu_{31} + \nu_{21}\nu_{32}}{E_2 E_3 \Delta} = \frac{\nu_{13} + \nu_{12}\nu_{23}}{E_1 E_2 \Delta} \\ C_{22} &= \frac{1 - \nu_{13}\nu_{31}}{E_1 E_3 \Delta}, \quad C_{23} = \frac{\nu_{32} + \nu_{12}\nu_{31}}{E_1 E_3 \Delta} = \frac{\nu_{23} + \nu_{21}\nu_{13}}{E_1 E_3 \Delta} \\ C_{33} &= \frac{1 - \nu_{12}\nu_{21}}{E_1 E_2 \Delta}, \quad C_{44} = C_{23} \quad C_{55} = C_{31} \quad C_{66} = C_{12} \\ \Delta &= \frac{1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13} - 2\nu_{21}\nu_{32}\nu_{13}}{E_1 E_2 E_3} \end{aligned}$$

Where [C] is stiffness matrix and ha 36 constants

If the strain energy density function, W, exists, where

$$W = \frac{1}{2} \sigma_{ij} \epsilon_{ij}$$

Such that

$$\frac{\partial w}{\partial \epsilon_{ij}} = C_{ijkl} \epsilon_{kl} = \sigma_{ij} \quad (3.3)$$

Then the independent components of elasticity tensor are reduced to 21 since eqn 3.3 requires

that  $C_{ijkl} = C_{klij}$ , or in short hand notation,  $C_{ij} = C_{ji}$  therefore eqn 3.2 can be written as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ & & C_{33} & C_{34} & C_{35} & C_{36} \\ & & & C_{44} & C_{45} & C_{46} \\ & & & & C_{55} & C_{56} \\ & & & & & C_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \quad (3.4)$$

*symmetric*

What happens if one changes the system of coordinates from an orthogonal system 1–2–3 to some other orthogonal system, 1'–2'–3'? Then, new stiffness and compliance constants will be required to relate stresses and strains in the new coordinate system 1'–2'–3'. However, the new stiffness and compliance matrices in the 1'–2'–3' system will be a function of the stiffness and compliance matrices in the 1–2–3 system and the angle between the axes of the 1'–2'–3' system and the 1–2–3 system.

Inverting Equation 3.2 the general strain–stress relationship for a three-dimensional body in a 1–2–3 orthogonal Cartesian coordinate system is

$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix}$$

Where  $[S_{ij}] = [C]^{-1}$

$$\begin{aligned}
 S_{11} &= \frac{1}{E_1}, \quad S_{12} = \frac{-\nu_{12}}{E_1}, \quad S_{13} = \frac{-\nu_{13}}{E_1} \\
 S_{22} &= \frac{1}{E_2}, \quad S_{23} = \frac{-\nu_{23}}{E_2}, \quad S_{33} = \frac{1}{E_3} \\
 S_{44} &= \frac{1}{G_{23}}, \quad S_{55} = \frac{1}{G_{13}}, \quad S_{66} = \frac{1}{G_{12}}
 \end{aligned}$$

### Monoclinic

There may exist several situations when the distribution and orientation of reinforcements may give rise to special cases of material property symmetry. When there is one plane of material property symmetry (say, the plane of symmetry is  $x_3 = 0$ , i.e., the rotation of 180 degree around the  $x_3$  axis yields an equivalent material), the elastic constant matrix  $[C_v]$  is modified as.

The stiffness matrix reduces to

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ & C_{22} & C_{23} & 0 & 0 & C_{26} \\ & & C_{33} & 0 & 0 & C_{36} \\ & & & C_{44} & C_{45} & 0 \\ & & & & C_{55} & 0 \\ & & \text{symmetric} & & & C_{66} \end{bmatrix}$$

The compliance matrix correspondingly reduces to

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ & S_{22} & S_{23} & 0 & 0 & S_{26} \\ & & S_{33} & 0 & 0 & S_{36} \\ & & & S_{44} & S_{45} & 0 \\ & & & & S_{55} & 0 \\ & & \text{symmetric} & & & C_{66} \end{bmatrix}$$

Therefore the elasticity equation becomes

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ & C_{22} & C_{23} & 0 & 0 & C_{26} \\ & & C_{33} & 0 & 0 & C_{36} \\ & & & C_{44} & C_{45} & 0 \\ & & & & C_{55} & 0 \\ & & & & & C_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \quad (3.5)$$

*symmetric*

### Orthotropic

Materials which have three mutually orthogonal planes of elasticity symmetry are called orthotropic (a short term for orthogonally anisotropic). For these orthotropic materials the elasticity tensor is written as follows where it is seen that for an orthotropic, three dimensional elastic body there are nine independent components. Hence nine independent elastic properties must be determined.

The stiffness matrix is

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ 0 & C_{22} & C_{23} & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

The compliance matrix correspondingly reduces to

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ 0 & S_{22} & S_{23} & 0 & 0 & 0 \\ 0 & 0 & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix}$$

Therefore the elasticity equation becomes

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ 0 & C_{22} & C_{23} & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \quad (3.6)$$

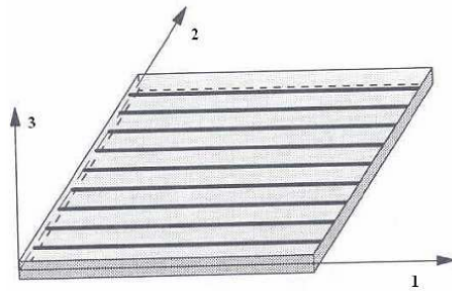


Figure 3.12 A unidirectional lamina with the material (local) coordinate system (1, 2, 3)

### Cylindrical Orthotropy

Consider cylindrical coordinates  $r, \theta, z$ , here the  $z$ -axis is assumed to coincide with the  $X_3$ -axis.

The stress and strain components are represented as

$$\begin{Bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \\ \tau_{r\theta} \\ \tau_{rz} \\ \tau_{\theta z} \end{Bmatrix} = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \epsilon_{rr} \\ \epsilon_{\theta\theta} \\ \epsilon_{zz} \\ \gamma_{r\theta} \\ \gamma_{rz} \\ \gamma_{\theta z} \end{Bmatrix} = \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{Bmatrix}$$

The stress-strain relations, in terms of compliances, become

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ & S_{22} & S_{23} & 0 & 0 & 0 \\ & & S_{33} & 0 & 0 & 0 \\ & \text{symmetric} & & S_{44} & 0 & 0 \\ & & & & S_{55} & 0 \\ & & & & & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix}$$

$$\{\epsilon_i\} = [S_p]\{\sigma_j\}$$

where

$$S_{11} = \frac{1}{E_{rr}}, \quad S_{22} = \frac{1}{E_{\theta\theta}}, \quad S_{33} = \frac{1}{E_{zz}},$$

$$S_{12} = S_{21} = -\frac{\nu_{\theta r}}{E_{\theta\theta}} = -\frac{\nu_{r\theta}}{E_{rr}},$$

$$S_{13} = S_{31} = -\frac{\nu_{rz}}{E_{zz}} = -\frac{\nu_{rz}}{E_{rr}},$$

$$S_{23} = S_{32} = -\frac{\nu_{z\theta}}{E_{zz}} = -\frac{\nu_{\theta z}}{E_{\theta\theta}},$$

$$S_{44} = \frac{1}{G_{\theta z}}, \quad S_{55} = \frac{1}{G_{rz}}, \quad S_{66} = \frac{1}{G_{r\theta}}$$

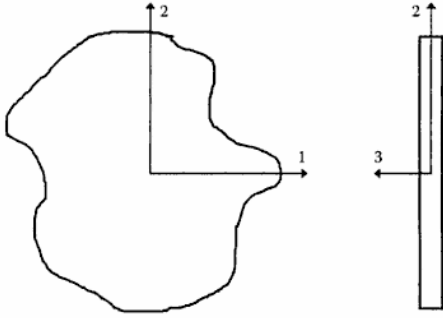
Unidirectional fiber composites can be regarded as orthotropic materials which possess three mutually orthogonal planes of symmetry. The directions perpendicular to these planes are called the material principal directions.

### 3.10 Hook's law for Two-Dimensional Uni-directional Lamina

Plane Stress Assumption

A thin plate is a prismatic member having a small thickness, and it is the case for a typical lamina. If a plate is thin and there are no out of plane loads, it can be considered to be under plane stress (Figure 3.13). If the upper and lower surface of the plate are free from external loads, then  $\sigma_3 = 0$ ,  $\tau_{23} = 0$  and  $\tau_{31} = 0$ . Because the plate is thin, these three stresses within the plate

are assumed to vary little from the magnitude of stresses at the top and the bottom surfaces. Thus, they can be assumed to be zero within the plate also. A lamina is thin and, if no out of plane loads are applied, one can assume that it is under plane stress. This assumption then reduces the three dimensional stress strain equations to two dimensional stress strain equations.



#### Reduction of Hook's law in Three Dimensions to Two Dimensions

A unidirectional lamina falls under the orthotropic material category. If the lamina is thin and does not carry any out-of-plane loads, one can assume plane stress conditions for the lamina. Therefore, taking eqn. (3.7) and inverse of eqn. (3.6) and assuming,  $\sigma_3 = 0$ ,  $\tau_{23} = 0$ , and

$\tau_{31} = 0$ , then

$$\epsilon_3 = S_{13}\sigma_1 + S_{23}\sigma_2,$$

$$\gamma_{23} = \gamma_{31} = 0$$

The normal strain  $\epsilon_3$  is not an independent strain because it is a function of the other two normal strains  $\epsilon_1$  and  $\epsilon_2$ . Therefore, the normal strain,  $\epsilon_3$  can be omitted from the stress-strain relationship eqn. (3.7) or its inverse. Also, the shearing strains,  $\gamma_{23}$  and  $\gamma_{31}$ , can be omitted because they are zero. Inverting Equation (3.6) for an orthotropic plane stress problem, it can then be written as

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \quad (3.8)$$

Where the  $S_{ij}$  are the elements of the compliance matrix. Note the four independent compliance elements in the matrix.

Inverting Equation (3.8) gives

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} \quad (3.9)$$

Where,  $Q_{ij}$  are the reduced stiffness coefficients which are related to the compliance coefficients as

$$Q_{11} = \frac{S_{11}}{S_{11}S_{22} - S_{12}^2} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}} \quad (3.10)$$

$$Q_{12} = Q_{21} = \frac{S_{12}}{S_{11}S_{22} - S_{12}^2} = -\frac{\nu_{21}E_{11}}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}} \quad (3.11)$$

$$Q_{66} = \frac{1}{S_{66}} = G_{12} \quad (3.13)$$

$$Q_{22} = \frac{S_{22}}{S_{11}S_{22} - S_{12}^2} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}} \quad (3.12)$$

Where,

$E_{11}$  = longitudinal young's modulus (in direction 1)

$E_{22}$  = transverse young's modulus (in direction 2)

$\nu_{12}$  = Major poison's ratio, where the general poison's ratio,  $\nu_{ij}$  is defined as the ratio of the negative of the normal strain in direction 'i' to the normal strain in direction 'j', when the only normal load is applied in direction 'i'

$G_{12}$  = in-plane shear modulus (in plane 1–2)

### 3.10.1 Hook's Law for a Two-Dimensional Angle Lamina

Generally, a laminate does not consist only of unidirectional lamina because of their low stiffness and strength properties in the transverse direction. Therefore, in most laminates some lamina is placed at an angle. It is thus necessary to develop the stress-strain relationship for an angle lamina.

The coordinate system used for showing an angle lamina is as given in figure 3.7. The axes in the 1-2 coordinate systems are called the local axes or the material axes. The direction 1 is parallel to the fibers and the direction 2 is perpendicular to the fibers. The axes in the x-y coordinate system are called the global axes or the off-axes. The angle between the two axes is denoted by an angle  $\theta$ . The global and local stresses in an angle lamina are related to each other through the angle of the lamina  $\theta$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = [T] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad (3.14)$$

Where [T] is called the transformation matrix and is defined as

$$[T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & (m^2 - n^2) \end{bmatrix} \quad (3.15)$$

where  $m = \cos\theta$ ,  $n = \sin\theta$

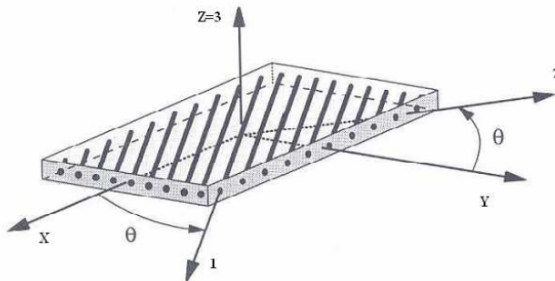


Figure 3.14 Local and global axes of an angle lamina [24]

Multiplying both sides of eqn by  $[T]^{-1}$  and remembering that  $[T]^{-1} [T] = [I]$  then

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \quad (3.16)$$

Substituting eqn. (3.9) into eqn. (3.16) results in

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T]^{-1} [Q] \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} \quad (3.17)$$

The global and local strains are also related through the transformation matrix

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} = [T] \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (3.18)$$

Finally substituting eqn. (3.18) into (3.17) results in the constitutive equations for a generally orthotropic lamina, that is to say a lamina composed of an orthotropic material in which the geometric and material axes are not aligned ( $\theta \neq 0, 90^\circ, 180^\circ, 270^\circ$ )

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T]^{-1} [Q] [T] \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = [\bar{Q}] \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (3.19)$$

Where  $[\bar{Q}]$  is the generally orthotropic lamina stiffness matrix, whose components are given by

$$[\overline{Q}] = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \quad (3.20)$$

where  $[\overline{Q}_{ij}]$  are called the elements of the transformed reduced stiffness matrix  $[\overline{Q}]$  and are given by:

$$\begin{aligned} \overline{Q}_{11} &= Q_{11} m^4 + 2(Q_{12} + 2Q_{66})m^2 n^2 + Q_{22} n^4 \\ \overline{Q}_{22} &= Q_{11} n^4 + 2(Q_{12} + 2Q_{66})m^2 n^2 + Q_{22} m^4 \\ \overline{Q}_{12} &= (Q_{11} - Q_{22} - 2Q_{66})m^2 n^2 + Q_{66}(m^4 + n^4) \\ \overline{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})nm^3 + (Q_{12} - Q_{22} + 2Q_{66})n^3 m \\ \overline{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})n^3 m + (Q_{12} - Q_{22} + 2Q_{66})nm^3 \end{aligned}$$

where ,

$m = \cos \theta$  and  $n = \sin \theta$

### Stress-Strain Relationship in a Laminate

A lamina (also called a ply or layer) is a single flat layer of unidirectional fibers or woven fibers arranged in a matrix. It is the building block of a laminate. A laminate is a stack of plies of composites. Each layer can be laid at various orientations and can be made up of different material systems. The stiffness, strengths, properties of a laminate will depend on:

- Elastic moduli
- The stacking position
- Thickness
- Angle of orientation
- Coefficients of thermal expansion
- Coefficients of moisture expansion of each individual lamina

### 3.10.2 Laminate Code

A laminate is made of a group of single layers bonded to each other. Each layer can be identified by its location in the laminate, its material, and its angle of orientation with a reference axis (Figure 4.1). Each lamina is represented by the angle of ply and separated from other plies by a slash sign.

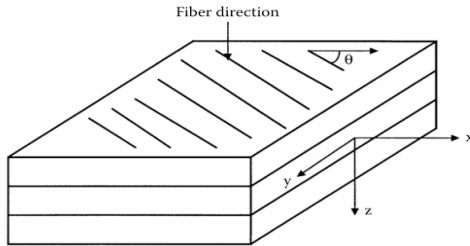


Fig 3.14 schematic of a laminate

Special notations are used for symmetric laminates, laminates with adjacent lamina of the same orientation or of opposite angles, and hybrid laminates. The following examples illustrates the laminate code

|     |
|-----|
| 0   |
| -45 |
| 90  |
| 60  |
| 30  |

[0/-45/90/60/30] denotes the code for the above laminate. It consist of five piles , each of which has a different angle to the reference x-axis. A slash separates each lamina. The code also implies that each ply is made of the same material and is of the same thickness.

|                |     |
|----------------|-----|
| Graphite/epoxy | 0   |
| Boron/epoxy    | 45  |
| Boron/epoxy    | -45 |
| Boron/epoxy    | -45 |
| Boron/epoxy    | 45  |
| Graphite/epoxy | 0   |

[0<sup>Gr</sup>/+45<sup>B</sup>], denotes the above laminate. It consists of six piles; the 00 piles are made of graphite/epoxy and the +45<sup>0</sup> angle piles are made of boron/epoxy. Note the symmetry of the laminate. Also, the + 45<sup>0</sup> notation indicates that the 00 ply should be followed by a +45<sup>0</sup> angle

ply and then by a  $-45^\circ$  angle ply. A notation of  $+45^\circ$  would indicate the  $-45^\circ$  angle ply is followed by a  $+45^\circ$  angle ply.

### 3.10.3 Strain-Displacement Equations

The following assumptions are made in the classical lamination theory to develop the relationships:

- Each lamina is orthotropic.
- Each lamina is homogeneous.
- A line straight and perpendicular to the middle surface remains straight and perpendicular to the middle surface during deformation ( $\gamma_{xz} = \gamma_{yz} = 0$ )
- The laminate is thin and is loaded only in its plane ( $\sigma_z = \tau_{yz} = \tau_{xz} = 0$ )
- Displacements are continuous and small throughout the laminate
- Each lamina is elastic.
- No slip occurs between the lamina interfaces.

At any point other than the mid-plane, the two displacements in the x-y plane will depend on the axial location of the point and the slope of the laminate mid-plane with the x and y-directions. For example, as shown in figure 3.15

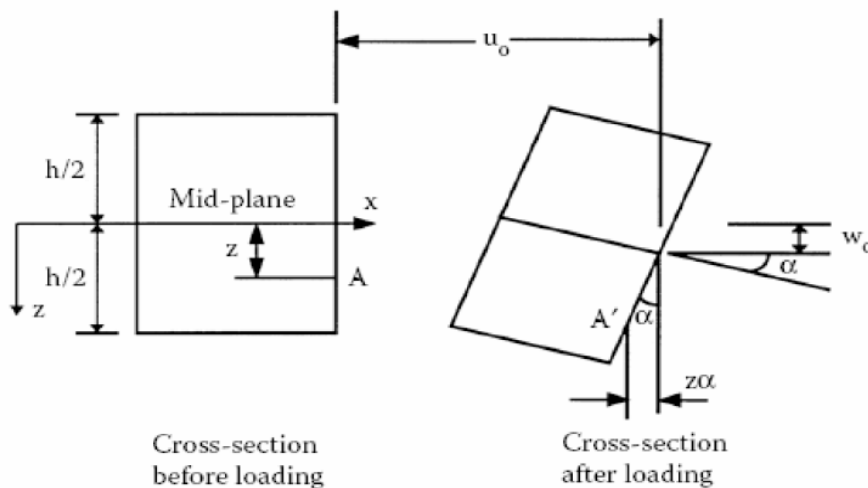


Fig 3.15 Relationship between mid-plane displacement and curvature

$$U = u_0 - z\alpha$$

$$\alpha = \frac{\partial u_0}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_0}{\partial x^2}$$

$$\epsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_0}{\partial x^2} \quad (3.21)$$

$$\epsilon_y = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} - z \frac{\partial^2 w_0}{\partial y^2} \quad (3.22)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z \frac{\partial^2 w_0}{\partial x \partial y} \quad (3.23)$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{13} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + Z \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ \gamma_{xy} \end{Bmatrix} \quad (3.26)$$

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} - z \begin{Bmatrix} \frac{\partial^2 w_0}{\partial x^2} \\ \frac{\partial^2 w_0}{\partial y^2} \\ \frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} \quad (3.24)$$

Mid plane strain                      Mid plane curvature

Laminate strain can be written as;

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} \quad (3.25)$$

Equation (3.25) shows linear relation between strains in the laminate to the curvatures of the laminate. Where,  $z$ = the distance from the mid-plane of the point in the laminate (figure 3.9).

### 3.10.4 Stress and Strain in a Laminates

Having formulated the constitutive relations for a lamina composed of a generally orthotropic material, eqns. (3.19), and (3.20), the next step is to derive the constitutive relations accruing from bonding several laminas together, each with any general orientation of material axes and composed of any material system. Substituting eqn. (3.25) in to (3.19) we obtain.

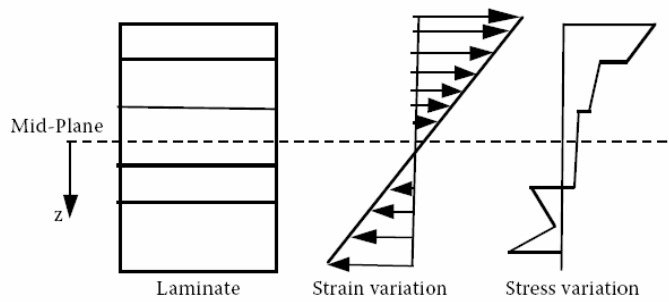


Fig 3.16 Stress and Strain variation through a laminate

### 3.10.5 Force and Moment Resultants Related to Mid-plane Strains and Curvatures

The mid-plane strains and shell curvatures in eqn. (3.25) are the unknowns for finding the lamina strains and stresses. However, eqn. (3.26) gives the stresses in each lamina in terms of these unknowns. The stresses in each lamina can be integrated through the laminate thickness to give resultant forces and moments (or applied forces and moments) as shown in figure 3.11. The forces and moments applied to a laminate will be known, so the mid-plane strains and shell curvatures can then be found. This relationship between the applied loads and the mid-plane strains and curvatures is developed in this section.

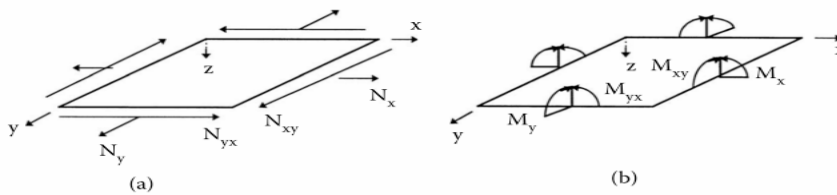


Figure 3.17 Resultant forces and moments on a laminate

Consider a laminate made of ‘n’ plies shown in figure 3.12, each ply has a thickness of  $t_k$ , then the total thickness of the laminate h is:

$$h = \sum_{k=1}^n t_k \quad (3.27)$$

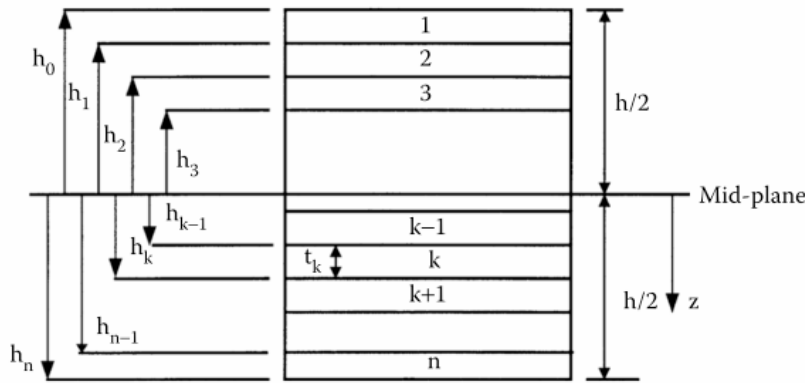


Figure 3.18 Coordinate system and layer numbering used for a typical laminate plate integrating

The global stresses in each lamina give the resultant forces per unit length in xy plane through the laminate thickness. Then, the location of the mid-plane is  $h/2$  from the top or the bottom surface of the laminate. The z-axis is taken positive upward from the mid-plane. The  $k^{th}$  layer is located between the points  $z = z_k$  and  $z = z_{k+1}$  in the thickness direction. The top surface and bottom surfaces are represented each by  $z = h/2$  and  $-h/2$  respectively.

$$N_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x dz \quad (3.28)$$

$$N_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y dz \quad (3.29)$$

$$N_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} dz \quad (3.30)$$

Where,  $N_x, N_y$  – normal forces per unit length

$N_{xy}$  – shear force per unit length

$\frac{h}{2}$  – half thickness of the laminate

Similarly, the resultant moment will be (x-y plane)

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x z dz \quad (3.31)$$

$$M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y z dz \quad (3.32)$$

$$M_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} z dz \quad (3.33)$$

Where,  $M_x, M_y$  – bending moments per unit length

$M_{xy}$  – twisting moment per unit length

$\frac{h}{2}$  – half thickness of the laminate

From eqns.(3.28) through (3.33) for a laminate can be written in a matrix form

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} dz \quad (3.34)$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} z dz \quad (3.35)$$

However, for a laminated plate the stresses that are integrated across the thickness of the plate or shells are the sum of the stresses across each lamina, or, the in-plane stress resultants can be written as using eqn. (3.26)

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} dz \quad (3.34)$$

$$= \sum_{k=1}^n \int_{h_{k-1k}}^{h_k} \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{66}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} dz + \sum_{k=1}^n \int_{h_{k-1k}}^{h_k} \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{66}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} z dz \quad (3.36)$$

Similarly

$$\begin{aligned}
 \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} z dz = \sum_{k=1}^n \int_{h_{k-1k}}^{h_k} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{66} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} dz + \\
 &\sum_{k=1}^n \int_{h_{k-1k}}^{h_k} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{66} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} z^2 dz
 \end{aligned} \tag{3.37}$$

However, since mid-plane strain  $[\varepsilon^0]$  and mid-plane curvature  $[k]$  are not functions of  $z$  and within each lamina  $[\overline{Q}]_K$  is not a function of  $z$ , eqns. (3.36) and (3.37) can be written as

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \left\{ \sum_{k=1}^n [\overline{Q}]_K \int_{h_{k-1}}^{h_k} dz \right\} [\varepsilon^0] + \left\{ \sum_{k=1}^n [\overline{Q}]_K \int_{h_{k-1}}^{h_k} dz \right\} k \tag{3.38}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \left\{ \sum_{k=1}^n [\overline{Q}]_K \int_{h_{k-1}}^{h_k} z dz \right\} [\varepsilon^0] + \left\{ \sum_{k=1}^n [\overline{Q}]_K \int_{h_{k-1}}^{h_k} z^2 dz \right\} k \tag{3.39}$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} \tag{3.40}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} \quad (3.41)$$

where

$$A_{ij} = \sum_{k=1}^n [\overline{Q}_{ij}]_k (h_k - h_{k-1}) \quad i, j = 1, 2, 6 \quad (3.42)$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^n [\overline{Q}_{ij}]_k (h_k^2 - h_{k-1}^2) \quad i, j = 1, 2, 6 \quad (3.43)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^n [\overline{Q}_{ij}]_k (h_k^3 - h_{k-1}^3) \quad i, j = 1, 2, 6 \quad (3.44)$$

The [A], [B] and [D] matrices are called the extensional, coupling and bending stiffness matrices, respectively. Combining eqns. (3.40) and (3.41) gives six simultaneous and six unknowns

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ k_x \\ k_y \\ k_{xy} \end{Bmatrix} \quad (3.45)$$

The extensional stiffness matrix [A] relates the resultant in-plane forces to the in-plane strains, and the bending stiffness matrix [D] relates the resultant bending moments to the plate curvatures. The coupling stiffness matrix [B] couples the force and moment terms to the mid-plane strains and mid-plane curvatures. Eqn. (3.45) will be used later in the modeling of the shell structure.

## CHAPTER FOUR

### MODELING OF THE CYLINDRICAL SHELL STRUCTURE

#### 4.1 Finite element method theoretical approach

##### 4.1.1 Mathematical Modeling

Mathematical modeling is the activity devoted to the study of the simulation of physical phenomena by computational processes. The goal of the mathematical simulation is to predict the behavior of some product within its environment. Mathematical modeling subsumes a number of activities, as illustrated by figure 4.1

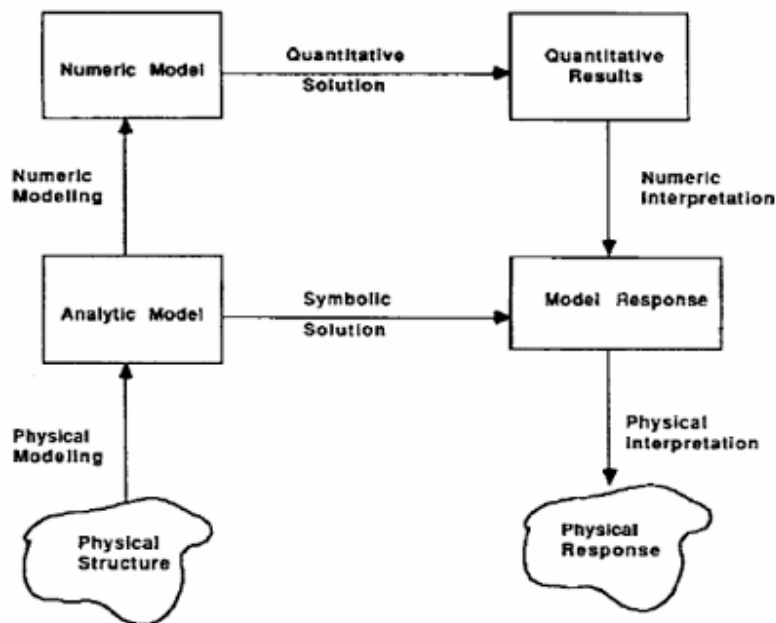


Fig 4.1 Mathematical Modeling Process [38]

The mathematical modeling activities presented include model generation, interpretation of numerical results, and development and control of numerical algorithms as a series of interrelated activities. All mechanical models are characterized mathematically by very complex expressions, many of which, until the advent of electronic computation, stood outside the reach of the scientific and engineering communities. Recently, the computer has made it possible to

solve many problems of mechanics, dramatically expanding the capabilities for mathematical modeling [33].

We will first consider here a first order shear deformation theory applicable to laminated cylindrical shells of shallow curvature, shown in figure 4.2. The thickness and in-plane dimensions of the shell are denoted by  $h$ ,  $a$  and  $b$  respectively. The displacements in the  $x$ ,  $\theta$ , and  $z$  are denoted by  $u$ ,  $v$  and  $w$  respectively. The radius,  $a$ , is assumed to be much larger than the thickness,  $h$ . In addition, transverse shear strains  $\gamma_{xz}$  and  $\gamma_{\theta z}$  no longer negligible, although the thickness strain)  $\epsilon_z$  is neglected.

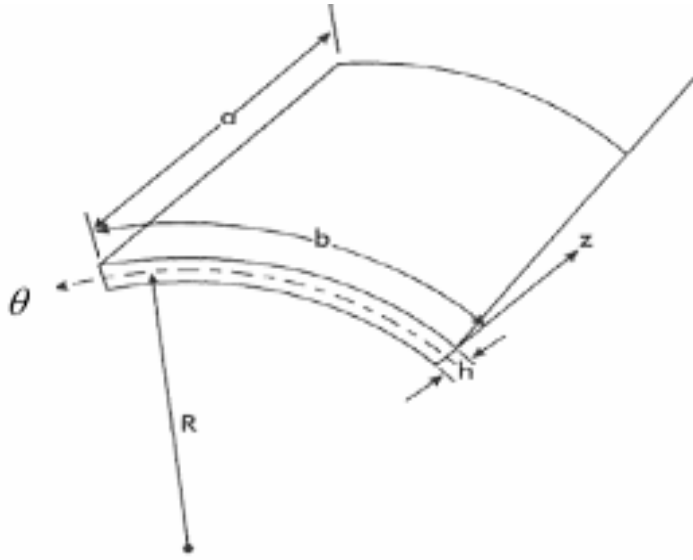


Fig 4.2 Nomenclature for Cylindrical Shell [33]

The classical theory of elasticity gives the following strain-displacement relations applicable to the geometry shown in figure 4.2 [33].

$$\begin{aligned}\epsilon_x &= \frac{\partial u}{\partial x}, \epsilon_\theta = \frac{1}{(1+Z/R)} \frac{\partial v}{\partial \theta} + \frac{w}{R}, \epsilon_z = \frac{\partial w}{\partial z} = 0 \\ \epsilon_{\theta z} &= \frac{1}{(1+Z/R)} \frac{\partial w}{\partial z} + \frac{\partial v}{\partial z} - \frac{v}{(1+z)R^2} \\ \gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \gamma_{x\theta} = \frac{\partial v}{\partial x} + \frac{1}{(1+Z/R)} \frac{\partial u}{\partial \theta}\end{aligned}\tag{4.1}$$

The first order theory assumes the following displacements

$$\begin{aligned}
 u &= u^0(x, \theta) + z\varphi_x(x, \theta) \\
 v &= v^0(x, \theta) + z\varphi_\theta(x, \theta) \\
 w &= w^0(x, \theta)
 \end{aligned} \tag{4.2}$$

Where  $u^0$ ,  $v^0$  and  $w^0$  are the axial, tangential, and transverse displacements of the mid-surface respectively, and  $\varphi_x$  and  $\varphi_\theta$  are the rotations of the cross-sections originally formed to  $x$  and  $\theta$  axes [33]. Since we assume that the shell is shallow,  $Z/R$  is small compared to unity. Eqns.(4.1) and (4.2) yield in matrix form:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{x\theta} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_\theta^0 \\ \gamma_{x\theta}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x \\ k_\theta \\ k_{x\theta} \end{Bmatrix} \tag{4.3}$$

Where  $[\varepsilon_i^0]$  and  $[k_i]$  are the mid surface strains and curvatures, respectively, given by [33]

$$\begin{aligned}
 \varepsilon_x^0 &= \frac{\partial u^0}{\partial x}, \varepsilon_\theta^0 = \frac{\partial v^0}{\partial \theta} + \frac{w}{R}, \gamma_{x\theta}^0 = \frac{\partial u^0}{\partial \theta} + \frac{\partial v^0}{\partial x} \\
 k_x &= \frac{\partial \varphi_x}{\partial x}, k_\theta = \frac{\partial \varphi_\theta}{\partial \theta} + \frac{w}{R}, k_{x\theta} = \frac{\partial \varphi_\theta}{\partial \theta} + \frac{\partial \varphi_x}{\partial x}
 \end{aligned} \tag{4.4}$$

In addition, the inter-laminar shear strains are

$$\gamma_{xz} = \varphi_x + \frac{\partial w}{\partial x}, \gamma_{\theta z} = \varphi_\theta + \frac{\partial w}{\partial \theta} - \frac{v}{R} \tag{4.5}$$

It is noted that these results are of exactly the same form as for flat, laminated plates of rectangular cross-section with the exception of the  $(W/R)$  term appearing in the mid-surface tangential strains  $\varepsilon_\theta^0$  and  $-V/R$  term appearing in the inter-laminar shear strains  $\gamma_{\theta z}$ . Since the analysis assumes that displacements  $u$ ,  $v$  and  $w$  are small compared to the plate thickness  $h$  and the radius of the plate  $R$  is much larger than  $h$ , the additional terms  $W/R$  and  $-V/R$  in eqns. (4.1) and (4.5) drop out. According to classical theory of shells, the membrane stress resultants  $N_\theta, N_x, N_{x\theta}$  and the stress couples  $M_\theta, M_x, M_{x\theta}$  for the shell are [33]

$$(N_\theta, N_x, N_{x\theta}) = \int_{-h/2}^{h/2} (\sigma_x^k, \sigma_\theta^k, \tau_{x\theta}^k) dz \tag{4.6a}$$

$$(M_\theta, M_x, M_{x\theta}) = \int_{-h/2}^{h/2} (\sigma_x^k, \sigma_\theta^k, \tau_{x\theta}^k) z dz \quad (4.6b)$$

where  $\sigma_x^k, \sigma_\theta^k$  and  $\tau_{x\theta}^k$  are the in-plane stresses in the  $k^{th}$  layer. Integration of the stresses yields the constitutive equations for extension and bending of cylindrical shell [33]

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon^0 \\ K \end{Bmatrix} \quad (4.7)$$

Where [A], [B] and [D] are the extensional, coupling and bending stiffness matrices, respectively, defined in reference [33]

$$[\epsilon^0] = [\epsilon_x^0 \quad \epsilon_\theta^0 \quad \gamma_{x\theta}^0] \text{ and } [k] = [k_x \quad k_\theta \quad k_{x\theta}] \text{ are as defined in eqn. (4.4)}$$

#### 4.1.2 Unified Near Crack Tip Stress and Displacement Field (Both Types)

This formulation is based on the work by Sih *et al.* [40] who used the stress function eqn. (4.8) for an infinite anisotropic plate with a central crack and solved the final displacement fields.

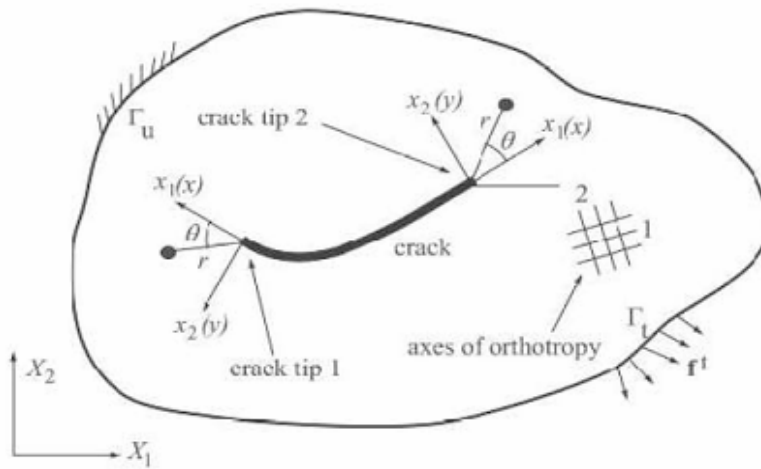


Fig 4.3 Local Coordinate at both crack tip [39]

Assume an anisotropic body containing a crack is subjected to arbitrary forces with general displacement and traction boundary conditions. Global Cartesian coordinates are  $(X_1, X_2)$  and local Cartesian coordinate  $(x_1=x_2, y_1=y_2)$  and local polar coordinate  $(r, \theta)$  are defined on the crack tip as illustrated in figure 4.3. Recalling the characteristic eqn. (4.11) of the governing fourth-order partial differential equation are always complex or purely imaginary ( $s_k = s_{kx} + i s_{ky}$ ,  $k=1,2$ ) and occur in conjugate pairs as  $s_1, \bar{s}_1$  and  $s_2, \bar{s}_2$

Sih et al. derived the two dimensional and stress fields in the vicinity of the crack tip by means of analytical functions and complex variable ( $s_k=s_{kx}+is_{ky}$ ,  $k=1,2$ ). The stress components for pure Model I are defined as

$$\begin{aligned}\sigma'_{11} &= \frac{K_1}{\sqrt{2\pi r}} \operatorname{Re} \left[ \frac{s_1 s_2}{s_1 - s_2} \left( \frac{s_2}{(\cos \theta + s_2 \sin \theta)^{1/2}} - \frac{s_1}{(\cos \theta + s_1 \sin \theta)^{1/2}} \right) \right] \\ \sigma'_{22} &= \frac{K_1}{\sqrt{2\pi r}} \operatorname{Re} \left[ \frac{s_1 s_2}{s_1 - s_2} \left( \frac{s_2}{(\cos \theta + s_2 \sin \theta)^{1/2}} - \frac{s_1}{(\cos \theta + s_1 \sin \theta)^{1/2}} \right) \right] \\ \sigma'_{12} &= \frac{K_1}{\sqrt{2\pi r}} \operatorname{Re} \left[ \frac{s_1 s_2}{s_1 - s_2} \left( \frac{s_2}{(\cos \theta + s_1 \sin \theta)^{1/2}} - \frac{s_1}{(\cos \theta + s_2 \sin \theta)^{1/2}} \right) \right]\end{aligned}\quad (4.16)$$

And the displacements are

$$\begin{aligned}u_1' &= K_1 \sqrt{\frac{2r}{\pi}} \operatorname{Re} \left\{ \frac{1}{s_1 - s_2} \left[ s_1 p_2 (\cos \theta + \sin \theta)^{1/2} - s_2 p_1 (\cos \theta + s_1 \sin \theta)^{1/2} \right] \right\} \\ u_2' &= K_1 \sqrt{\frac{2r}{\pi}} \operatorname{Re} \left\{ \frac{1}{s_1 - s_2} \left[ s_1 q_2 (\cos \theta + \sin \theta)^{1/2} - s_2 q_1 (\cos \theta + s_1 \sin \theta)^{1/2} \right] \right\} \\ u_3' &= 0\end{aligned}\quad (4.17)$$

#### 4.1.3 Finite Element Modeling

The finite element method (FEM) has undoubtedly become the most popular and powerful analytical tool for studying the behavior of a wide range of engineering and physical problems. Several general purpose finite element software have been developed, verified and calibrated over the years and are now available to almost anyone. One of the important applications of FEM is the analysis of crack propagation problems [39]. The ANSYS software is one of the finite element method packages available to analyze crack problems which have different layered elements for discretization and modeling composite materials. Among them SOLID46 (3-D layered structural solid shell, figure 4.4) is used for simulating shell structures. The element possesses the continuum solid element topology and features eight-node connectivity with three degrees of freedom at each node: translations in the nodal \$, %, and & directions. The element

allows to model up to 250 different material layers. Accuracy in modeling composite shells is governed by the first order shear deformation theory.

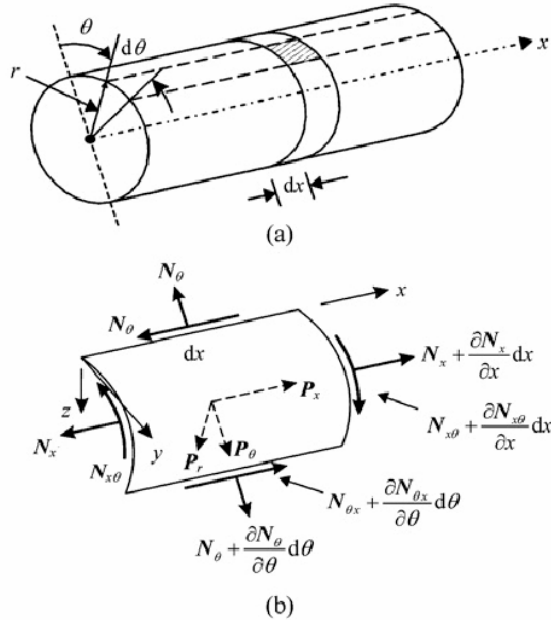


Figure 4.4 – Cylindrical shell and Membrane shell forces (a) Cylindrical shell (b) membrane shell forces [23].

Thus, the membrane theory cited in Sung et al.[32], an assumption of the element without flexural rigidity and carrying only the lateral loads by axial and transverse shear loads is applied for the analysis. Based on the assumptions of elastic material and small displacement, it is reasonable to use the potential energy theory to develop a cracked shell element. A free body diagram of membrane element including the forces shown in Figure 4.3b. The coordinate  $x$  and the angle  $\theta$  were used to describe the position of the shell element. The deformation gradients  $(u_i,1)$  and the stress  $\sigma_{ij}$  and the strain components  $\varepsilon_{ij}$  computed in the shell element are referred to element local coordinates. However, for the energy release rate determination that corresponds to a coordinate system oriented with respect to the crack tip co-ordinates. Thus for large deformation, the finite element quantities must be transformed into the crack tip coordinates to be able to

compute the energy release rate or stress intensity factor [29].

By the assumption of constant thickness shell, the nodal displacements  $u_i, v_i$ , and  $w_i$  shown in the parent element are related to the membrane strains  $\varepsilon_x, \varepsilon_y$  and  $\gamma_{x\theta}$  on the material coordinates as [23]

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \sum_{i=1}^8 N_i(\xi, \eta) \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \quad (4.8)$$

Where  $N_i(\xi, \eta)$  is the shape function associated with the parent element

Accordingly, the related displacements are [23]

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \sum_{i=1}^8 N_i(\xi, \eta) \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} \quad (4.9)$$

and the three types of strains related to the displacement vector  $\vec{u}$  are  $\vec{u} = \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix}$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{r\theta} \end{bmatrix} = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{1}{r} \frac{\partial N_i}{\partial \theta} & -\frac{N_i}{r} \\ \frac{1}{r} \frac{\partial N_i}{\partial \theta} & \frac{\partial N_i}{\partial x} & 0 \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} \quad (4.10)$$

Equation (4.10) may be denoted as  $\varepsilon = Bu$

The stress-strain (constitutive) relationship is  $\sigma = D\varepsilon$ , where

$$D = \begin{bmatrix} E_{xx} & E_{x\theta} & 0 \\ E_{x\theta} & E_{\theta\theta} & 0 \\ 0 & 0 & G_{x\theta} \end{bmatrix} \text{ is the material property matrix}$$

The stiffness K and force matrix Q are formulated as [23]:

$$K = t \iint B^T D B r dx d\theta \quad (4.11)$$

Eq (4.11) can be written in a normalized coordinate system as  $(\xi, \eta)$  as

$$K = t \int_{-1}^1 \int_{-1}^1 B^T DB \det|J| r d\xi d\eta \quad (4.12)$$

$$Q = \iint P^T q r dx d\theta = \int_{-1}^1 \int_{-1}^1 P^T q \det|J| r d\xi d\eta \quad (4.13)$$

Where. J is the Jacobian matrix and can be written as

$$J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial \theta}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial \theta}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \Lambda & \frac{\partial N_i}{\partial \xi} & \Lambda \\ K & \frac{\partial N_i}{\partial \eta} & \Lambda \end{bmatrix} \begin{bmatrix} \cdot & \cdot \\ x_i & \theta_i \\ \cdot & \cdot \end{bmatrix} \quad (4.14)$$

In Eqs. (4.11) and (4.12), matrices B and J are function of  $\xi$  and  $\eta$ , the symbols t and r represent the thickness and the radius of the cylindrical shell element respectively.

#### Stiffness for Similar Shell Elements

For two similar shell elements (1 and 2) with ratio  $\alpha$ , the relationship of the coordinates for the corresponding nodes can be represented by

$$\begin{cases} x_i^2 = \alpha x_i^1 \\ \theta_i^2 = \alpha \theta_i^1 \end{cases} \quad (4.15)$$

The matrix B in Eq.(4.12) can be decomposed as [23]

$$B = \bar{B} + \overline{\overline{B}} = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{1}{r} \frac{\partial N_i}{\partial \theta} & 0 \\ \frac{1}{r} \frac{\partial N_i}{\partial \theta} & \frac{\partial N_i}{\partial x} & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{N_i}{r} \\ 0 & 0 & 0 \end{bmatrix} \quad (4.16)$$

Therefore, from Equation (4.15) and (4.16), the matrices  $B_1$  and  $B_2$  for the elements 1 and 2 have the following relationship

$$\begin{cases} \overline{\overline{B}}_2 = \overline{\overline{B}}_1 / \alpha \\ B_2 = B_1 \end{cases} \quad (4.17)$$

And the relationships for the jacobian matrix are

$$\begin{cases} J_2 = \alpha J_1 \\ J_2^{-1} = J_1^{-1} / \alpha \\ \det|J_2| = \alpha^2 \det|J_1| \end{cases} \quad 63$$

By substituting Eq.(4.16) in Eq.(4.12) the stiffness matrix  $K$  becomes as

$$K = K^* + K^{**} + K^{***}$$

Where

$$K^* = t \int_{-1}^1 \int_{-1}^1 \overline{B}^T D \overline{B} \det|J| r d\xi d\eta$$

$$K^{**} = t \int_{-1}^1 \int_{-1}^1 \overline{B}^T D \overline{B} + \overline{B} \det|J| r d\xi d\eta$$

$$K^{***} = t \int_{-1}^1 \int_{-1}^1 \overline{B}^T D \overline{B} \det|J| r d\xi d\eta$$

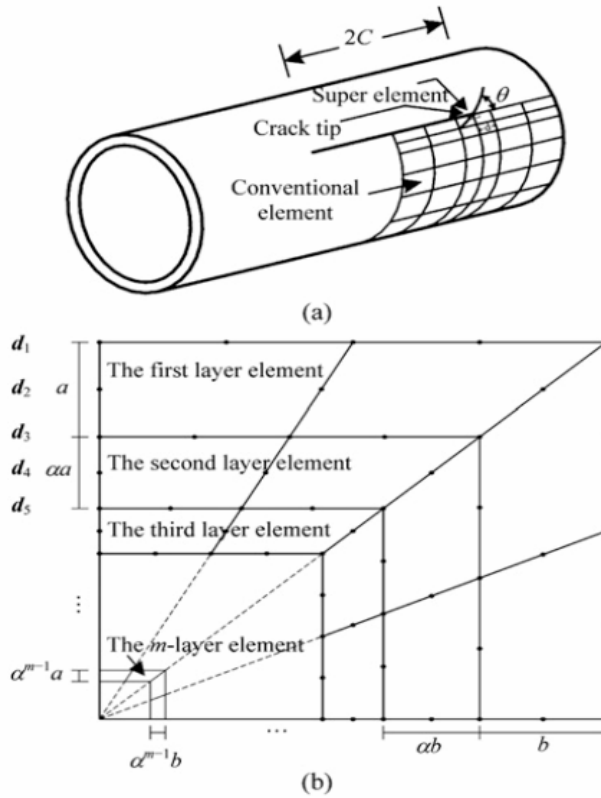


Figure 4.5 Mesh for the cylindrical shell around through-wall crack tip.(a) Cylindrical coordinate;(b) Plane projection.

If the relationship of the matrices  $K^*$ ,  $K^{**}$ , and  $K^{***}$  for elements 1 and 2 are derived from Equations (4.12), (4.14) and (4.17), then the following equation can be acquired to reflex the above mentioned relation as:

$$K_2 = K_1^* + \alpha K_1^{**} + \alpha^2 K_1^{***} \quad (4.19)$$

#### Supper Element Formulation

The element mesh around a singularity point is shown in figure 4.2. These elements are similar to each other. The inner-layer element to the adjacent outer-layer element is designated with the ratio  $\alpha$ . The stiffness of the outmost element may be expressed as

$$K_1 = K_1^* + K_1^{**} + K_1^{***}$$

$$= \begin{bmatrix} K_1^* & K_2^* & K_3^* \\ K_2^{*T} & K_4^* & K_5^* \\ K_3^{*T} & K_5^{*T} & K_6^* \end{bmatrix} + \begin{bmatrix} K_1^{**} & K_2^{**} & K_3^{**} \\ K_2^{**T} & K_4^{**} & K_5^{**} \\ K_3^{**T} & K_5^{**T} & K_6^{**} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K_6^{***} \end{bmatrix} \quad (4.20)$$

Then the stiffness matrix for the m-layer element, shown in figure 4.2(b) is

$$K_m = K_1^* + \alpha^{m-1} K_1^{**} + \alpha^{2(m-1)} K_1^{***}$$

This relation indicates that as the layer m-approaches the singularity point, the stiffness matrix for the element varies as per the above relationship derived.

#### 4.1.4 Anisotropic Stress Functions

The Airy stress function is limited to isotropic problems. For an extension to more complex problems, including anisotropic problems, the stress function written as  $\phi(x, y)$  can be written as

$$\phi(x, y) = 2 \operatorname{Re}[\phi_1(z_1) + \phi_2(z_2)] \quad (4.8)$$

Where  $\phi_1(z_1)$  and  $\phi_2(z_2)$  are arbitrary functions of  $z_1 = x + s_1 y$  and  $z_2 = x + s_2 y$ , respectively which are similar to eqn.(3.65). Combining the definition of stress components from the Airy stress function and satisfying the compatibility equation, the following relation is obtained for anisotropic solids in the absence of the body forces:

$$C_{22} \frac{\partial^4 \phi}{\partial x^4} - 2C_{26} \frac{\partial^4 \phi}{\partial x^3 \partial y} + (2C_{12} + C_{66}) \frac{\partial^4 \phi}{\partial x^2 \partial x^2} - 2C_{16} \frac{\partial^4 \phi}{\partial x \partial y^3} + C_{22} \frac{\partial^4 \phi}{\partial y^4} = 0 \quad (4.9)$$

This reduces to the following simplified equation for isotropic problems,

$$\frac{\partial^4 \phi}{\partial x^4} - 2 \frac{\partial^4 \phi}{\partial x^2 \partial x^2} + \frac{\partial^4 \phi}{\partial y^4} = \nabla^2 (\nabla^2 \phi) = 0 \quad (4.10)$$

The characteristic equation of the homogenous partial differential eqn.(4.10) is

$$C_{11}s^4 - 2C_{16}s^3 + (2C_{12} + C_{66})s^2 - 2C_{26}s + C_{22} = 0$$

$$C_{11}s^4 - 2C_{16}s^3 + (2C_{12} + C_{66})s^2 - 2C_{26}s + C_{22} = 0 \quad (4.11)$$

For an orthotropic material with axes of orthotropic (1, 2) coinciding with Cartesian(x,y) axes,  $C_{16}=C_{26}=0$ , the characteristic eqn.(4.11) is reduced to [39]:

$$s^4 + \left(\frac{E_1}{\mu} - 2\nu_1\right)s^2 + \frac{E_1}{E_2} = 0 \quad (4.12)$$

Finally, the stress components are defined from the second derivatives of the complex stress function  $\phi_i''$

$$\sigma_{xx} = 2 \operatorname{Re}[s_1^2 \phi_1''(z_1) + s_2^2 \phi_2''(z_2)]$$

$$\sigma_{yy} = 2 \operatorname{Re}[\phi_1''(z_1) + \phi_2''(z_2)] \quad (4.13)$$

$$\sigma_{xy} = 2 \operatorname{Re}[s_1 \phi_1''(z_1) + s_2 \phi_2''(z_2)]$$

And the displacements are obtained from the first derivatives of complex stress function  $\phi_i'$ :

$$u_x = 2 \operatorname{Re}[p_1 \phi_1'(z_1) + p_2 \phi_2'(z_2)] \quad (4.14a)$$

$$u_y = 2 \operatorname{Re}[q_1 \phi_1'(z_1) + q_2 \phi_2'(z_2)] \quad (4.14b)$$

where

$$p_i = C_{11}s_i^2 + C_{12} - C_{16}s_i, \quad q_i = C_{12}s_i + \frac{C_{12}}{s_i} - C_{26} \quad i = 1, 2 \quad (4.15)$$

#### 4.1.5 Finite Element Method in Anisotropic Fracture Mechanics

Saouma [5] extended the original isotropic maximum circumferential tensile stress theory to anisotropic solids. In this case, the fracture toughness is no longer uniquely defined. Instead, two values of  $K_{IC}^1$  and  $K_{IC}^2$  are required for characterizing the behavior of crack in a homogenous and transversely isotropic solid with elastic constants  $E_1$ ,  $E_2$  and  $\mu_{12}$  (Figure 4.5). For a crack arbitrarily oriented with respect to direction 1,  $K_{IC}^\theta$  would be a function of  $K_{IC}^1$  and  $K_{IC}^2$ :

$$K_{IC}^\theta = K_{IC}^1 \cos^2 \theta + K_{IC}^2 \sin^2 \theta \quad (4.28)$$

In order to avoid performing two separate fracture toughness tests, it is assumed that the ratio of the fracture toughness in both directions is equal to the ratio of the elastic modulus [5]:

$$K_{IC}^2 = K_{IC}^1 \frac{E_1}{E_2} \quad (4.29)$$

$$\sigma_\theta = \frac{K_I}{\sqrt{2\pi r}} \operatorname{Re} \left[ \frac{s_1 t_1 - s_2 t_2}{s_1 - s_2} \right] + \frac{K_{II}}{\sqrt{2\pi r}} \operatorname{Re} \left[ \frac{t_1 - t_2}{s_1 - s_1} \right] \quad (4.30)$$

where

$$t_1 = (s_2 \sin \theta + \cos \theta)^3$$

$$t_2 = (s_1 \sin \theta + \cos \theta)^3$$

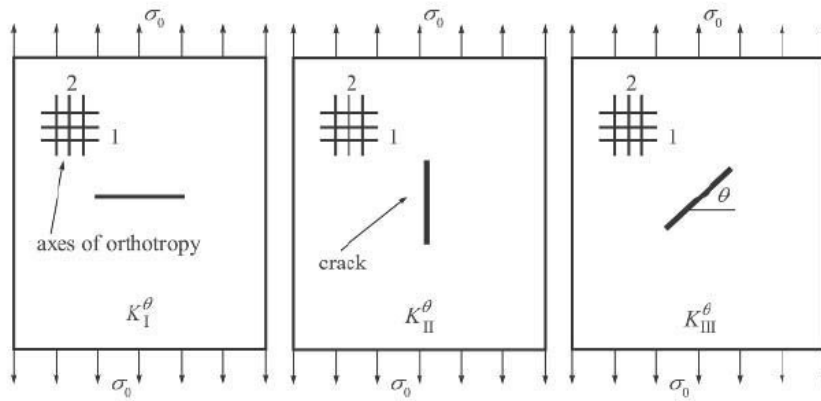


Figure 4.6 Fracture toughness for homogeneous anisotropic solid (Mohammadi, 200)

### Singular Quadratic Isoparametric Elements

The formulation of isoparametric element stiffness is well-documented. The geometry of an 8-noded plane isoparametric element is mapped into the normalized square space

$(\xi, \eta) (-1 \leq \xi \leq 1, -1 \leq \eta \leq 1)$  through the following transformations

$$x = \sum_{i=1}^8 N_i(\xi, \eta) x_i$$

$$y = \sum_{i=1}^8 N_i(\xi, \eta) y_i \quad (4.32)$$

$$N_i = (1 + \xi\xi_i)(1 + \eta\eta_i) - (1 - \xi^2)(1 + \eta\eta_i) - (1 - \eta^2)[(1 + \xi\xi_i)]\xi_i^2\eta_i^2/4 \\ + (1 - \xi^2)(1 + \eta\eta_i)(1 - \xi^2)\eta_i^2/2 + (1 - \eta^2)\xi_i^2/2 \quad (4.33)$$

Where  $N_i$  are the node shapes that correspond to the node  $i$ , whose coordinates are  $(x_i, y_i)$  in the  $x$ - $y$  systems and  $(\xi_i, \eta_i)$  in the transformed  $\xi, \eta$  systems.

$(\xi, \eta = +1)$  for corner points and zero for mid-side nodes. The displacements are interpolated by

$$u = \sum_{i=1}^8 N_i(\xi, \eta) u_i \\ v = \sum_{i=1}^8 N_i(\xi, \eta) v_i \quad (4.35)$$

Strain displacements relationship becomes

$$\{\epsilon\} = [B] \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \quad (4.35)$$

:

Where  $B$  is strain displacement matrix

$$B = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial z} \end{bmatrix} \quad (4.36)$$

$$\begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix} = [J^{-1}] \begin{bmatrix} \frac{\partial N_i}{\partial \xi_i} \\ \frac{\partial N_i}{\partial \eta_i} \end{bmatrix} \quad (4.37)$$

The stress is given by

$$\{\sigma\} = [D]\{\epsilon\} \quad (4.38)$$

$[D]$ — is the stress – strain matrix

The stiffness matrix then becomes the same form as presented above. To obtain a singular element to be used at the crack tip, the stress equation and the strain equation must be singular. This singularity is achieved by placing the mid-side nodes at the quarter point of the sides. For

simplicity, the strength of the singularity is found along the line  $\eta = -1$  as Reddy and Ramamurthy [25]. The shape functions along this line are,

$$\begin{aligned} N_1 &= -\frac{1}{2}(1 - \xi) \\ N_2 &= -\frac{1}{2}(1 + \xi) \\ N_5 &= -\frac{1}{2}(1 - \xi^2) \end{aligned} \tag{4.28}$$

The strain singularity along the line 1-2  $\eta = -1$  is  $\frac{1}{\sqrt{r}}$  which is calculated using equation 4.21 to 4.28 by substituting position of the node

## 4.2 Conditions and methods

### 4.2.1 Cracked Composite Shell Modeling in ANSYS

The composite shell is modeled using ANSYS, which is used for pre and post-processing of the model with FEA as the solver. The model is created in the preprocessing stage of the model generation and the results are viewed in the post-processing stage. The composite shell is created using SOLID46 (3-D layered structural solid shell) element. The properties of the lamina are given as inputs using ANSYS material library and the real constant command is used for defining layers of the composite material shell structure. The surface cracks in a shell are generated by cutting out an elliptical or circular cylindrical volume with negligible width on the outer or inner surfaces of the shell structure having circular or elliptical cross section and oriented in any direction on the surface of the cylinder which may be tangentially, axially or at an angle with respect to the cylinder axis as cracks in mechanical structure are very random in orientation and shape. It is in most cases propagating from one surface or side to the other. However, in order to simplify the analysis to the scope the FEA package such simple cracks are created to have circumferential or axial orientation.

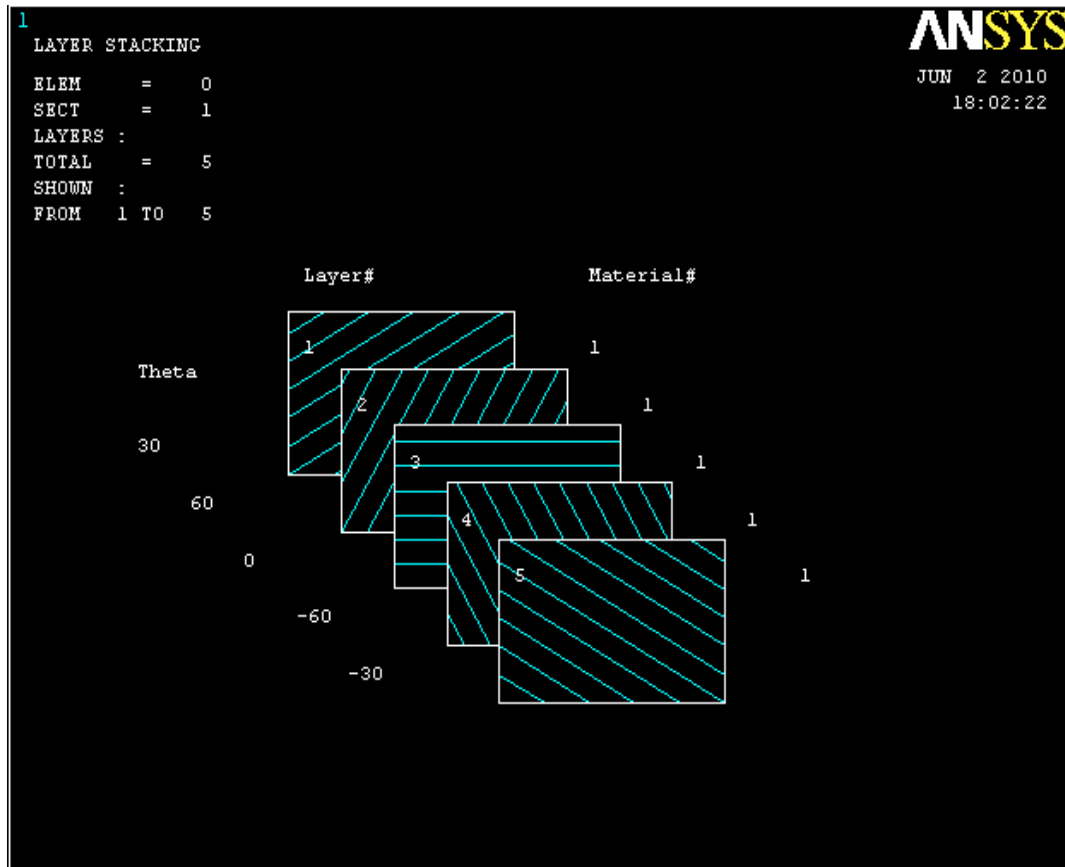


Figure 4.6 The staking sequence of the layers within the laminated shell structure

As a practical matter, we consider the material Hercules AS4/3501-6. This material is made of thin carbon fibers (approximately  $10\mu\text{m}$  or  $0.001\text{ cm}$  in diameter) which are 57 placed in an epoxy prepreg and pressed into thin sheets, with all the fibers running in one direction. The sheets are then stacked so that in each layer the fibers run perpendicularly to the fibers of the previous layer. The sheets are then heated and pressed together, so that the epoxy forms a continuous matrix that holds the fibers together. The resulting laminate is of the form  $[30^\circ/60^\circ/0^\circ/-60^\circ/-30^\circ]$  (figure 4.7). Moreover, each layer is quite thin (approximately  $500\mu\text{m}$ ), and a shell of this material, which has  $2.5\text{mm}$  thickness contains 5 layers.

#### 4.2.2 Description of the Parameters of the Model

Geometric parameters of the model

Outer diameter =  $80\text{mm}$

Layer thickness =  $0.5\text{mm}$

Inner diameter = 76mm

Length of the cylinder =150mm

Total thickness = 2.5mm

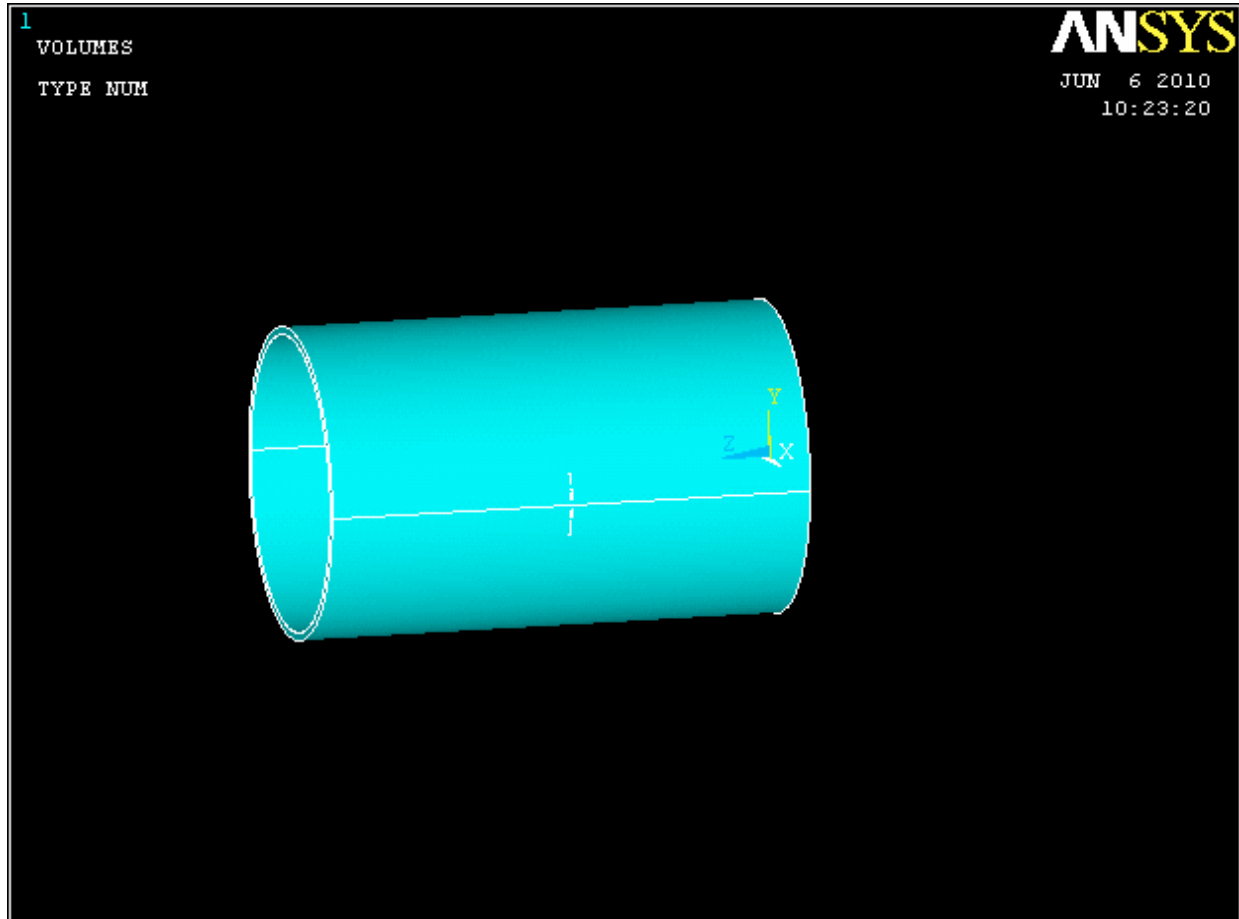


Figure 4.7 Model of composite shell having tangential elliptic surface crack.

### Meshing the Model

The model is meshed with automatic mesh generation command “Auto Mesh Generation”. As can be seen from figure 4.8, the model is meshed with more refined elements in the vicinity of the crack and coarsely meshed in areas away from the crack to economize the computation time.

Figure 4.7 Model of meshed composite shell having tangential elliptic surface crack for analysis

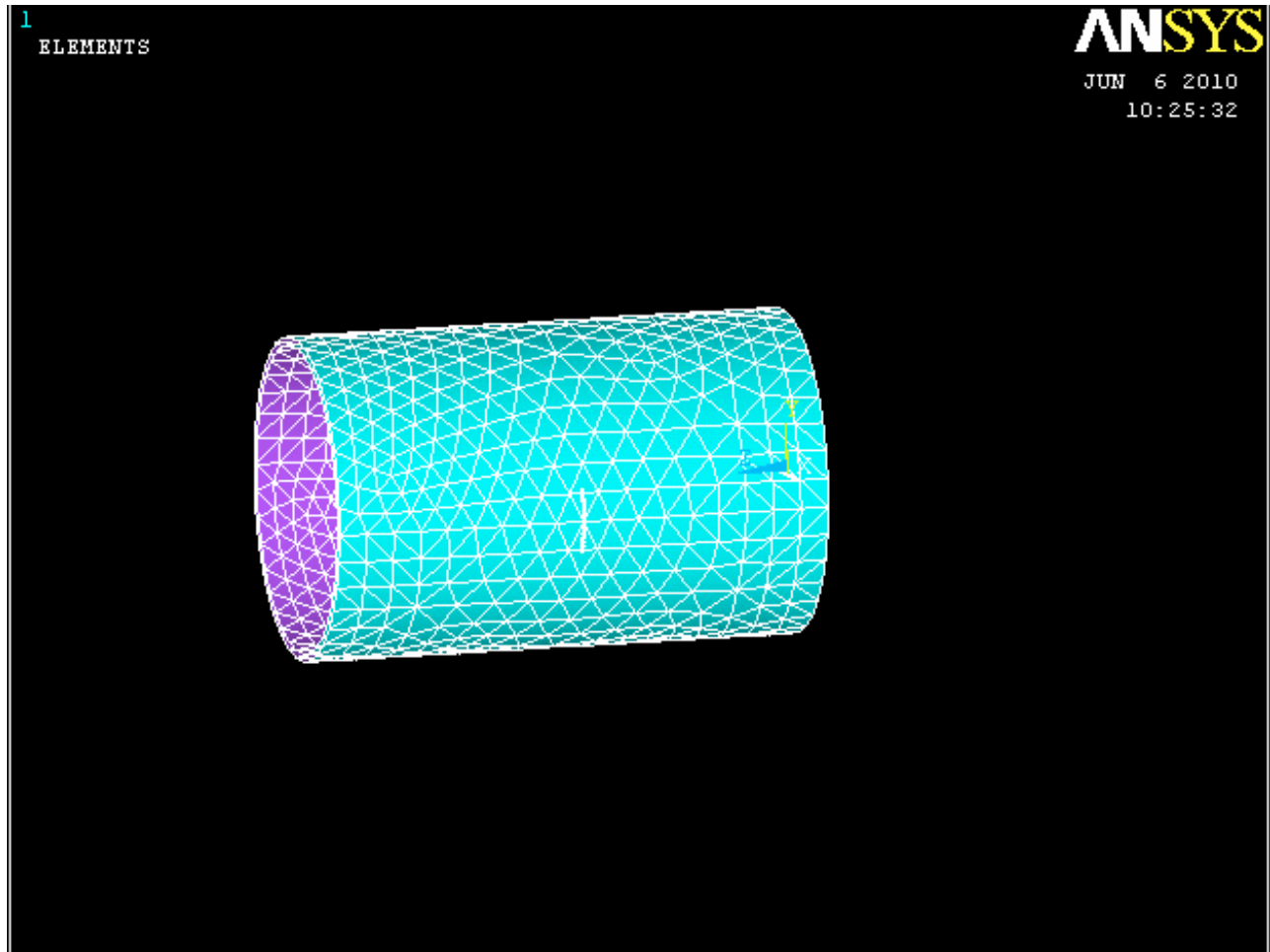


Figure 4.8 Model of meshed composite shell having surface crack for analysis

Material of the laminate

The material of the laminate is made of linear elastic orthotropic material with the following properties:

Material= Carbon/Epoxy with:

$\sigma_1 = 2000\text{Mpa}$ ,  $\sigma_2 = 52\text{Mpa}$  and  $\tau_{12} = 70\text{Mpa}$

Density =  $1550\text{kg/m}^3$

$E_x = 161\text{GPa}$   $E_y = 9\text{GPa}$   $E_z = 9\text{GPa}$

$\nu_{xy} = 0.26$   $\nu_{yz} = 0.01$   $\nu_{xz} = 0.26$   $G_{xy} = 6.1\text{GPa}$   $G_{yx} = 1\text{GPa}$   $G_{xz} = 6.1\text{GPa}$

Stacking sequence =  $[30^\circ/60^\circ/0^\circ/-60^\circ/-30^\circ]$

### 4.2.3 Loading and Boundary condition

The shell is constrained at one end with all its degrees of freedom fixed and subjected to a tensile load at the other end

#### Load

Pressure=9000Mpa

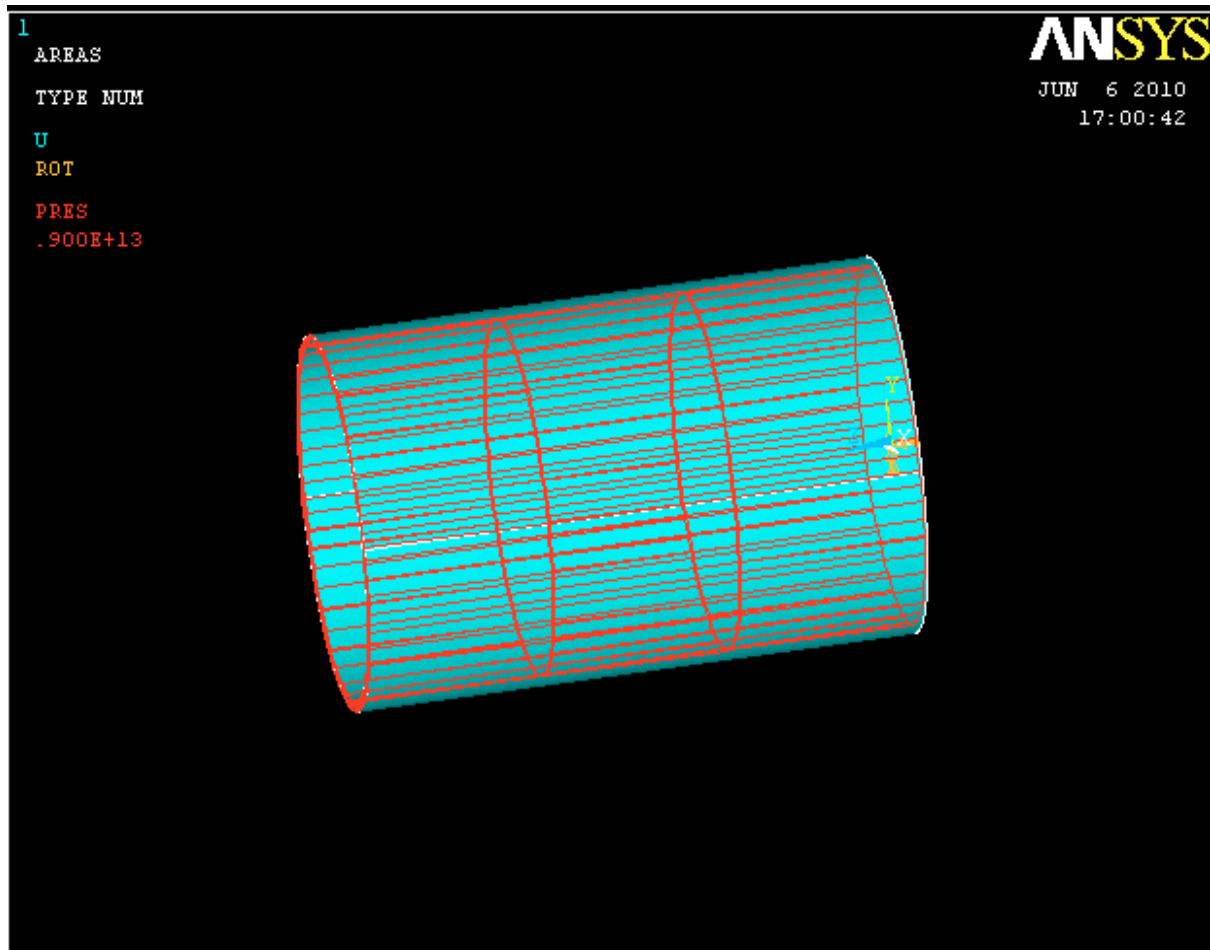


Figure 4.9 Model with boundary condition and load

## CHAPTER FIVE

### RESULT AND DISCUSSION

#### *5.1 Introduction*

Many have attempted to apply fracture mechanics to fibre-reinforced composites, and have met with mixed success. Conventional fracture mechanics methodology assumes a single dominant crack that grows in a self-similar fashion, i.e., the crack increases in size (either through stable or unstable growth), but its shape and orientation remain the same. Fracture of a fibre-reinforced composite, however, is often controlled by numerous microcracks distributed throughout the material, rather than a single macroscopic crack. There are situations where fracture mechanics is appropriate for composite, but it is important to recognize the limitations of theories that were intended for homogeneous materials.

Unlike isotropic materials, one advantage of composite materials is that fracture seldom occurs catastrophically without warning, but tends to be progressive with subcritical damage widely dispersed through the material as shown in (figure 5.1). Tensile loading can produce matrix cracking, fiber bridging, fiber rupture, fiber pullout, and fiber matrix debonding. Ultimate tensile failure of a fiber-reinforced composite often involves several of these mechanisms. Out-of-plane stress can lead to delamination because the fibres do not contribute significantly to the strength in this direction

The finite element analysis done on ANSYS by simulating the composite cylindrical shell structure having surface crack is to determine the stress and their distribution around the crack tip. The loading conditions applied are all for the Mode I loading (opening mode) for which the cracks are oriented tangentially and axially on both surfaces of the cylindrical shell i.e., external and internal.

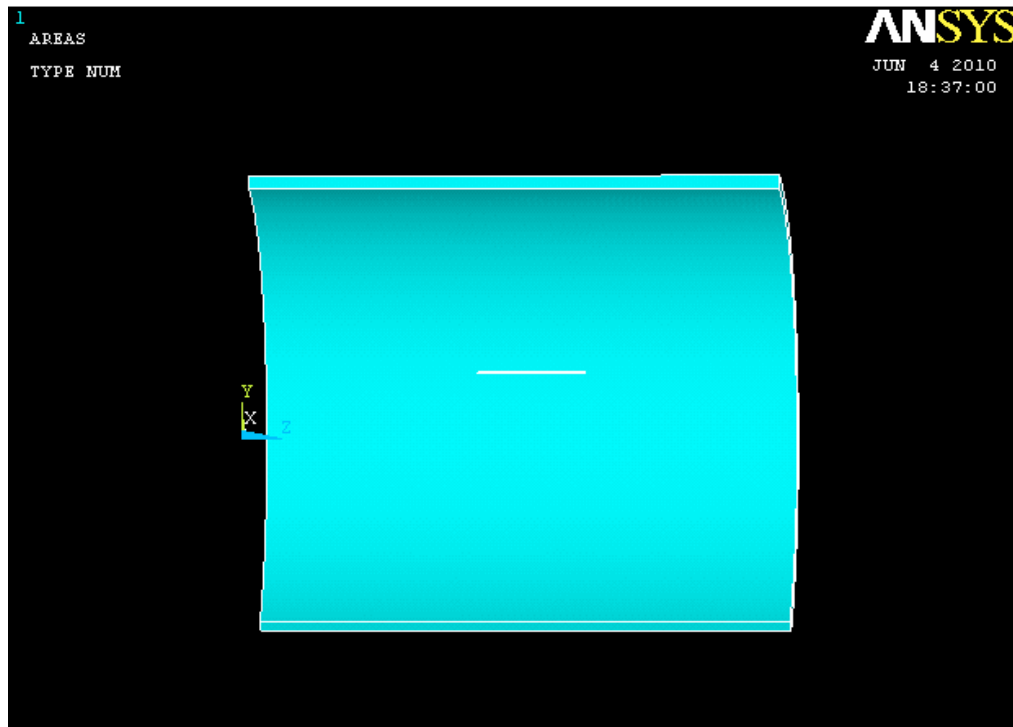


Fig 5.1 Model with internal axial crack

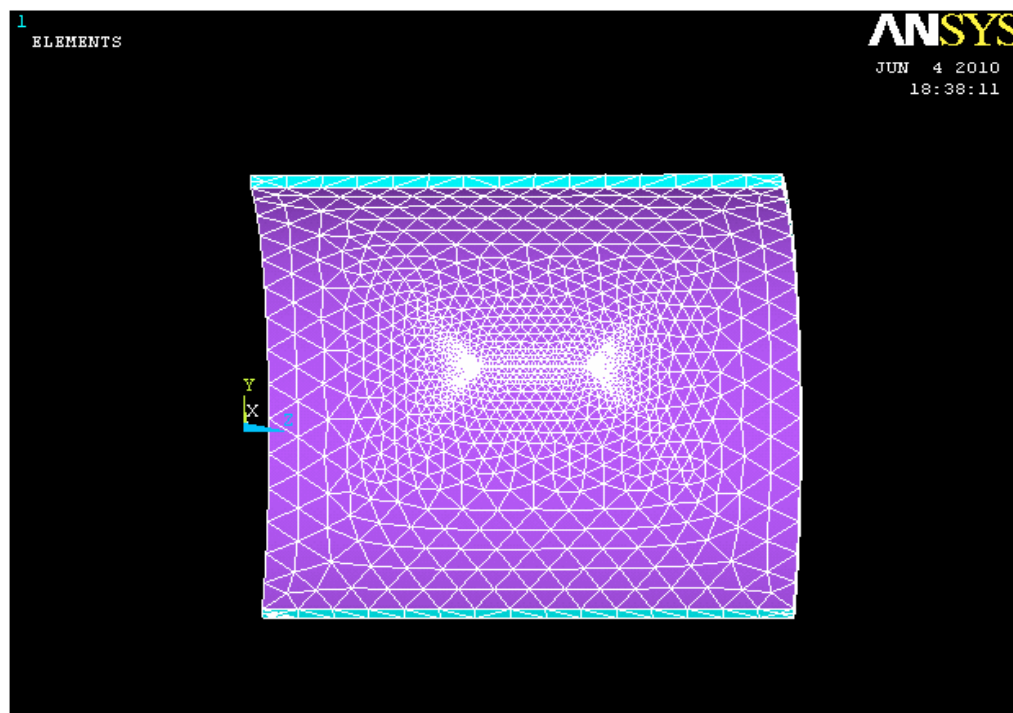


Fig 5.2 Meshed model with internal crack

When  $2a=30$  mm

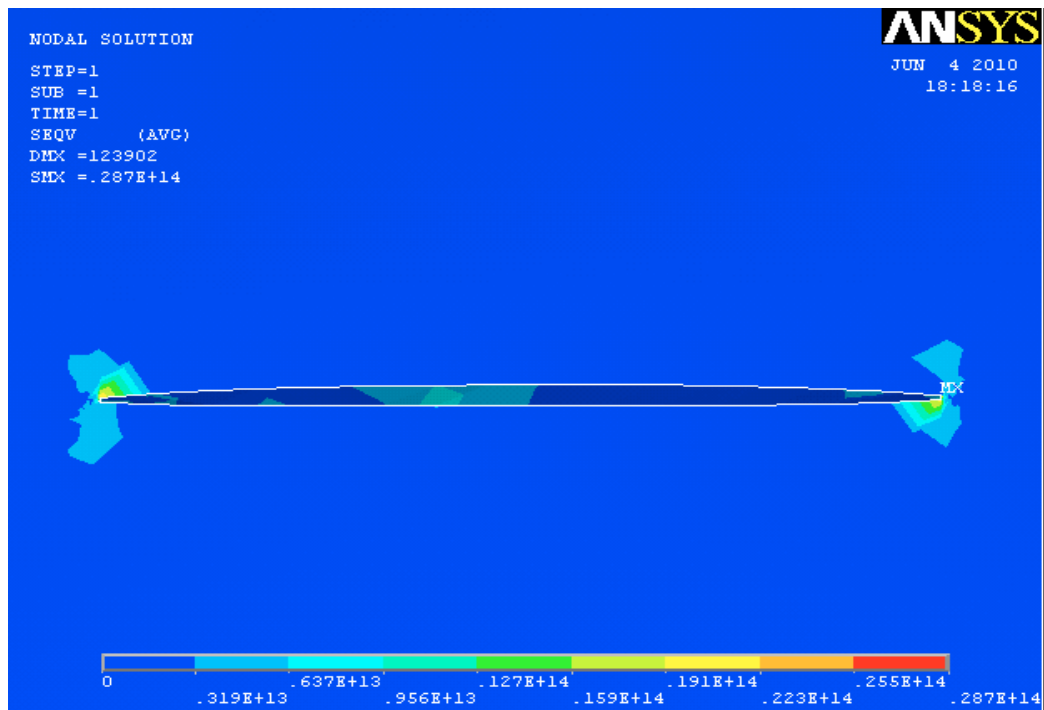


Fig 5.3 Von Misses stress around the crack

When  $2a=25$  mm

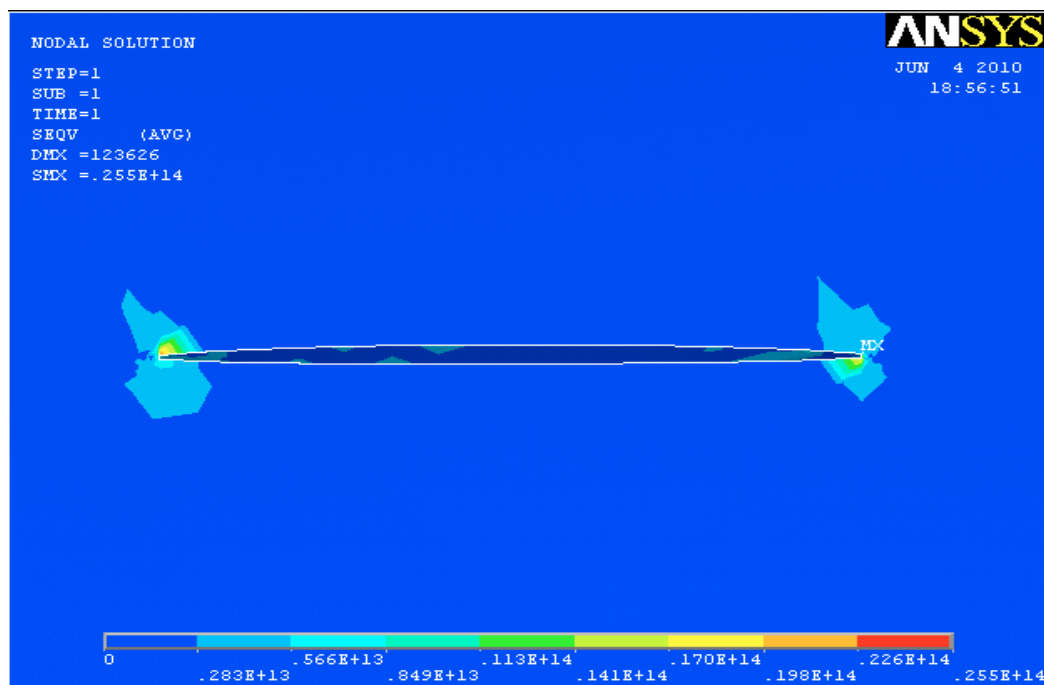


Fig 5.4 Von Misses stress around the crack

When  $2a=20\text{mm}$

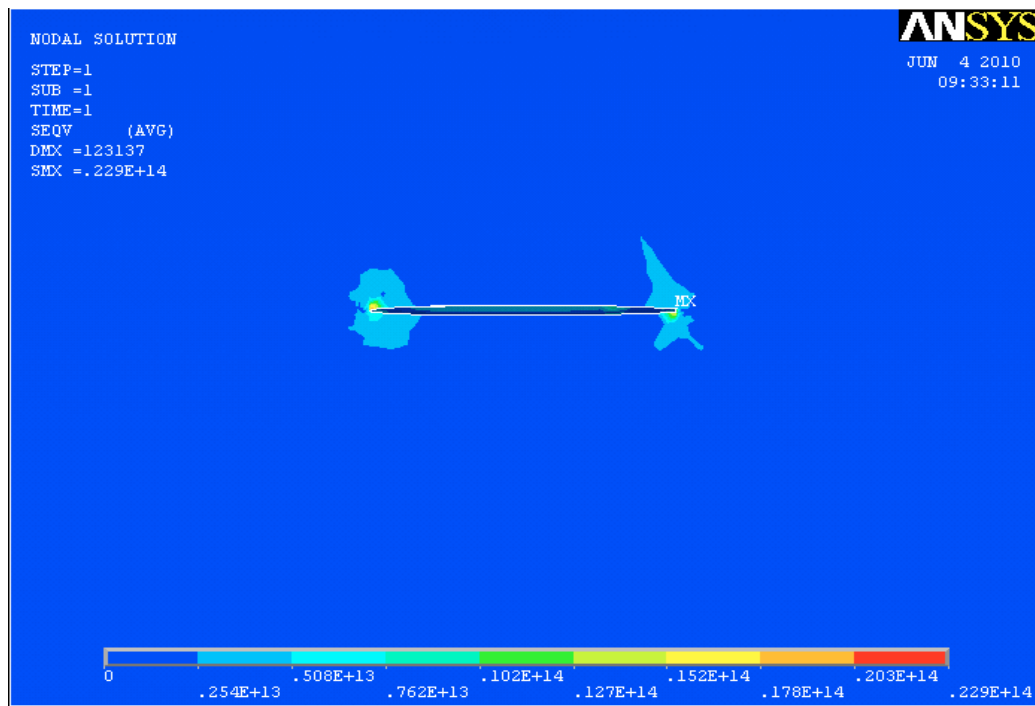


Fig 5.5 Von Mises stress around the crack

When  $2a=18\text{mm}$

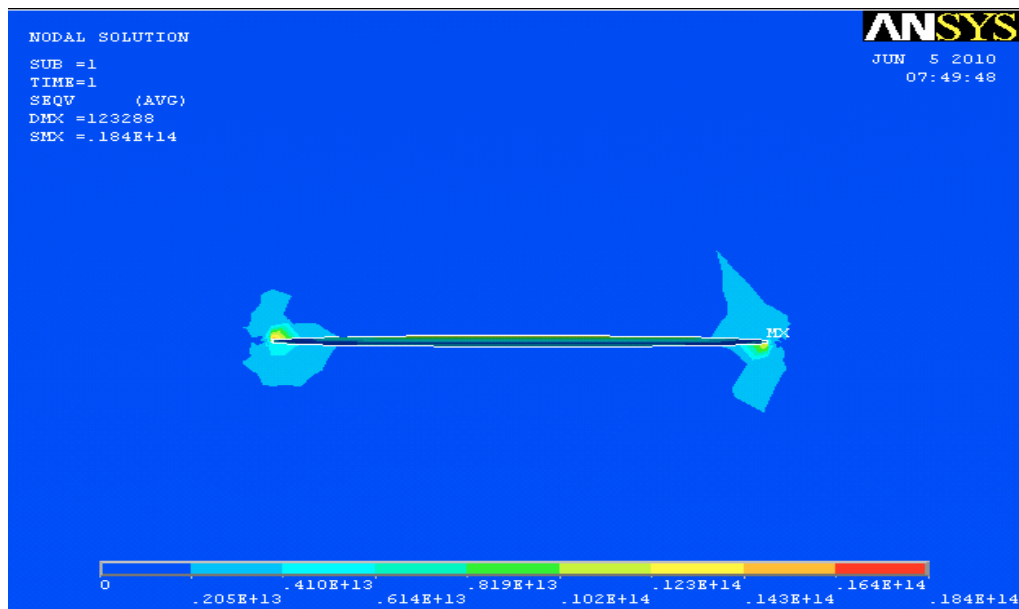


Fig 5.6 Von Mises stress around the crack

When  $2a=10\text{mm}$

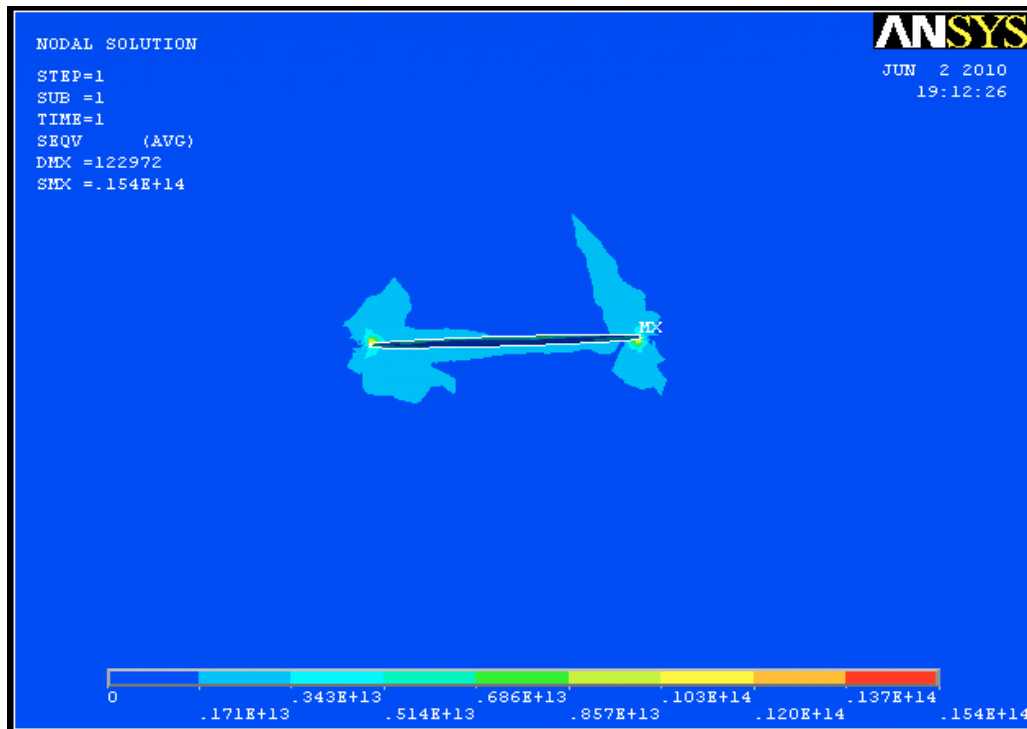


Fig 5.7 Von Misses stress around the crack

When  $2a=5\text{mm}$

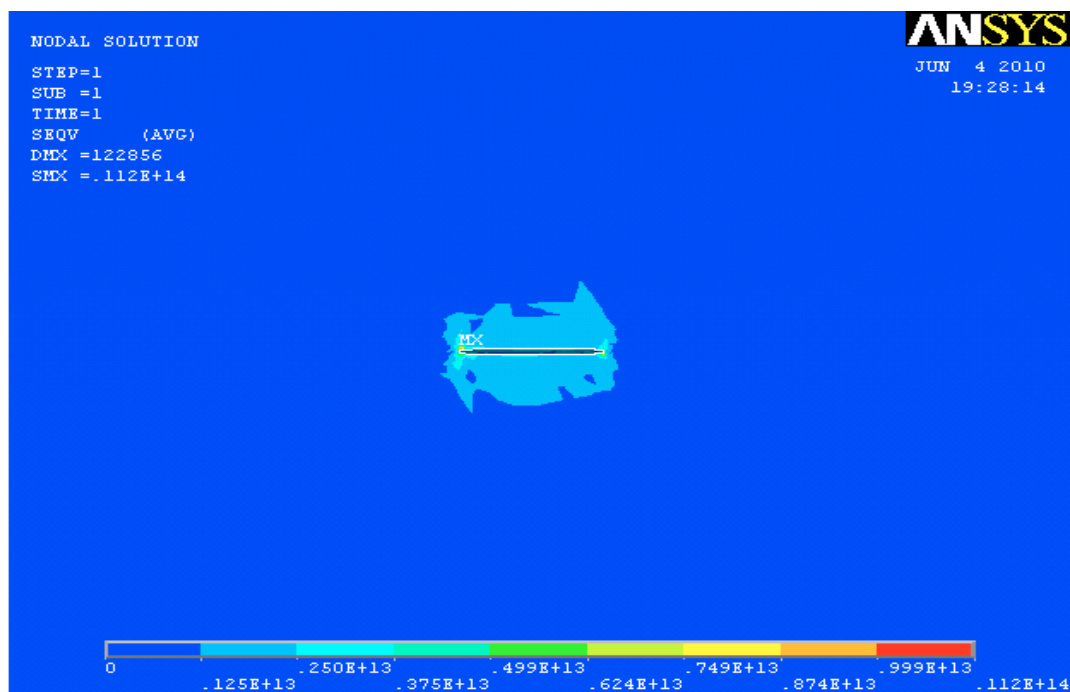


Fig 5.8 Magnified Maximum Stress intensity at the crack tip

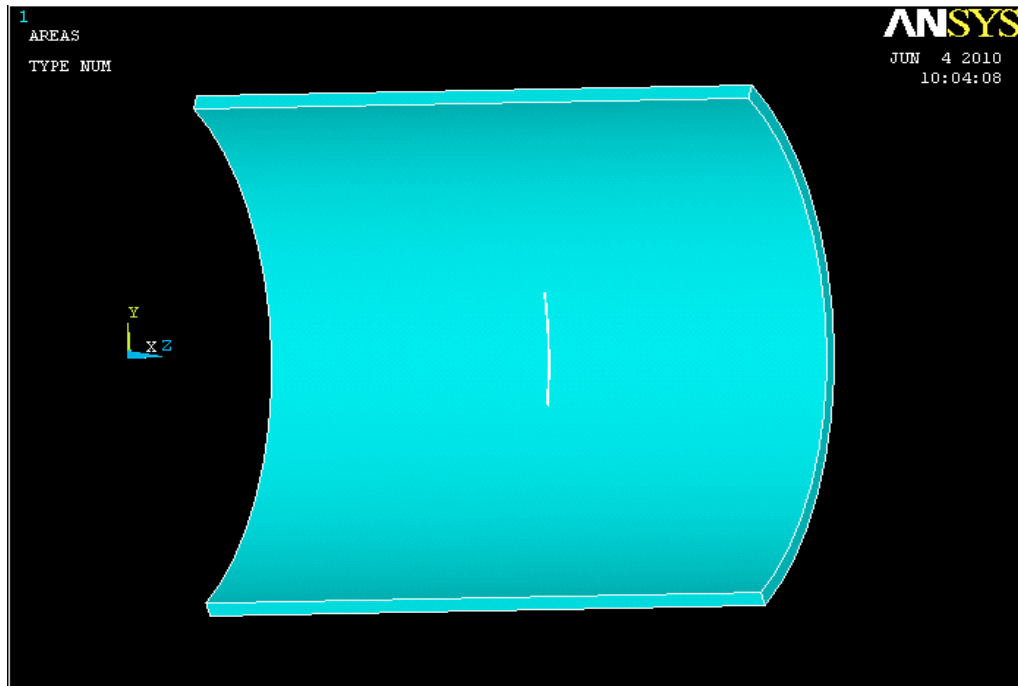


Fig 5.9 Model with internal tangential crack

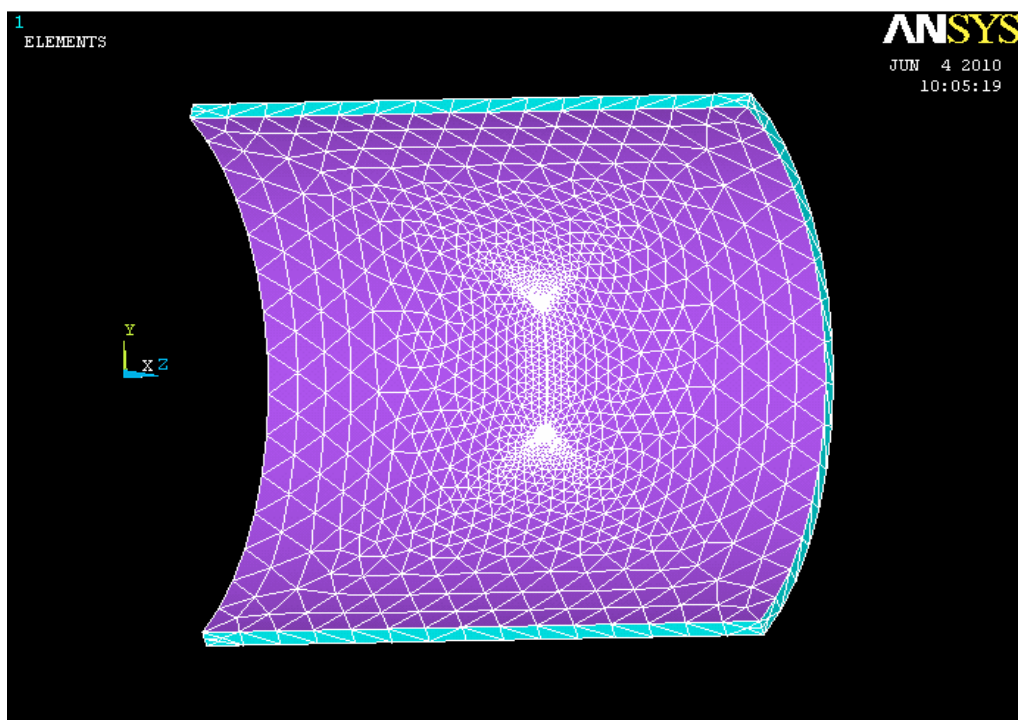


Fig 5.10 Meshed model with internal crack

When  $2a=30\text{mm}$

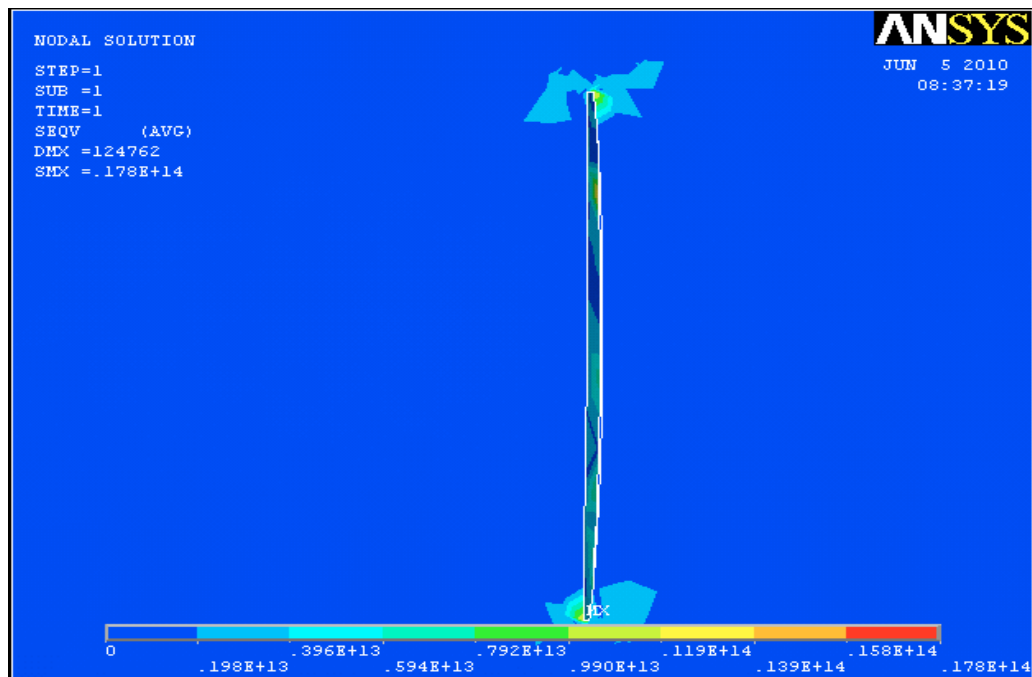


Fig 5.11 Von Misses Stress around the crack

When  $2a=25\text{mm}$

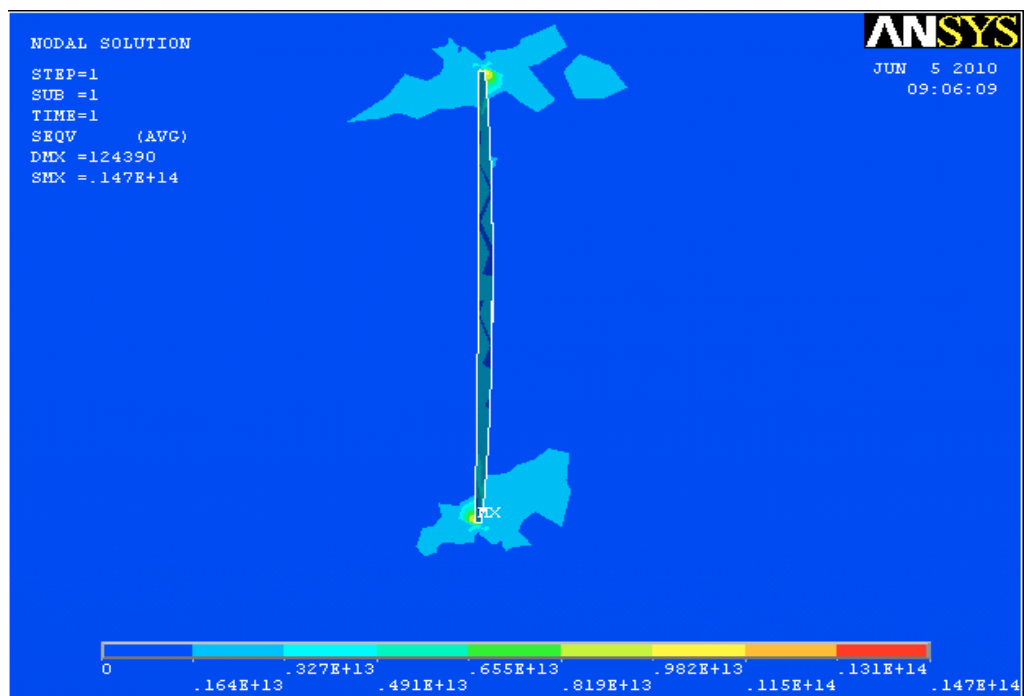


Fig 5.12 Von Misses Stress around the crack

When  $2a=20\text{mm}$

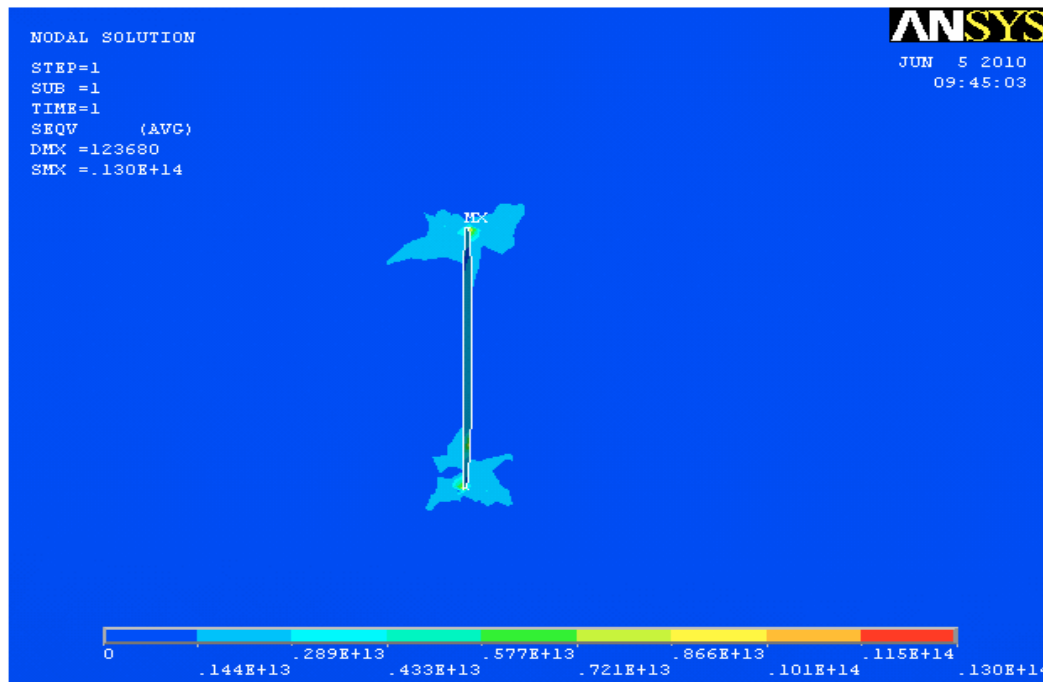


Fig 5.13 Von Misses Stress around the crack

When  $2a=18\text{mm}$

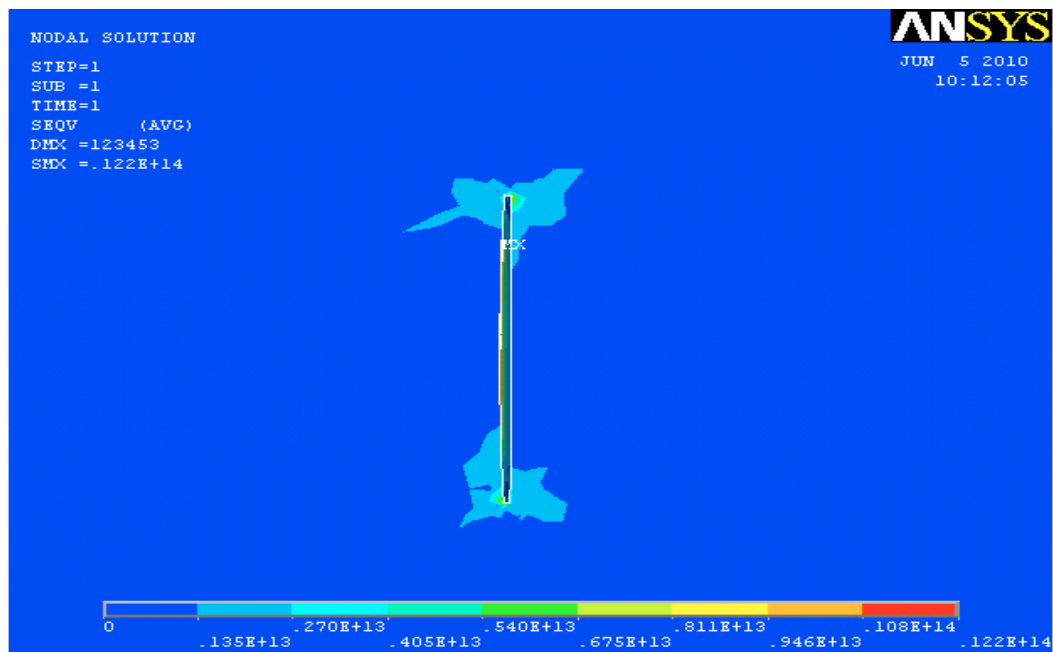


Fig 5.14 Von Misses Stress around the crack

When  $2a=10\text{mm}$

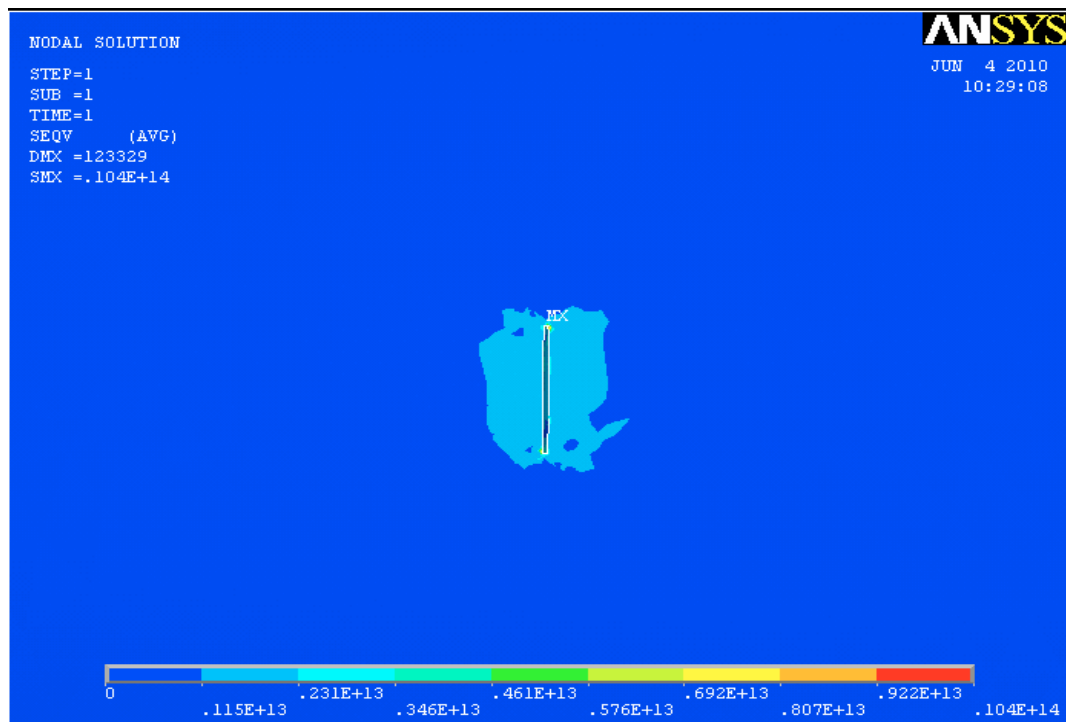


Fig 5.15 Von Misses Stress at the crack tip

When  $2a=5\text{mm}$

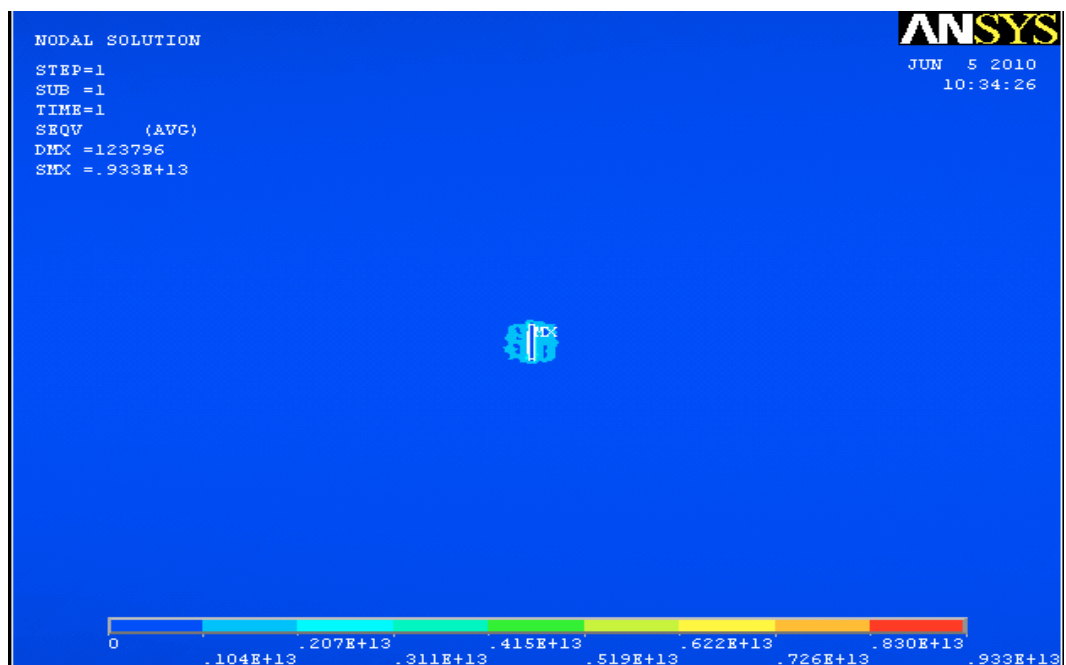


Fig 5.16 Von Misses Stress at the crack tip

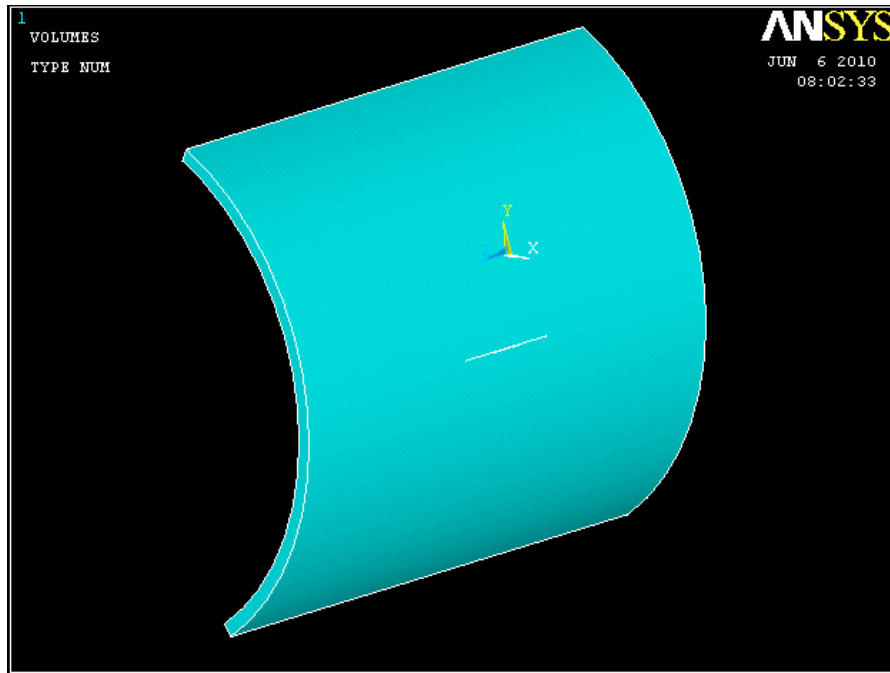


Fig 5.17 Model with external axial crack

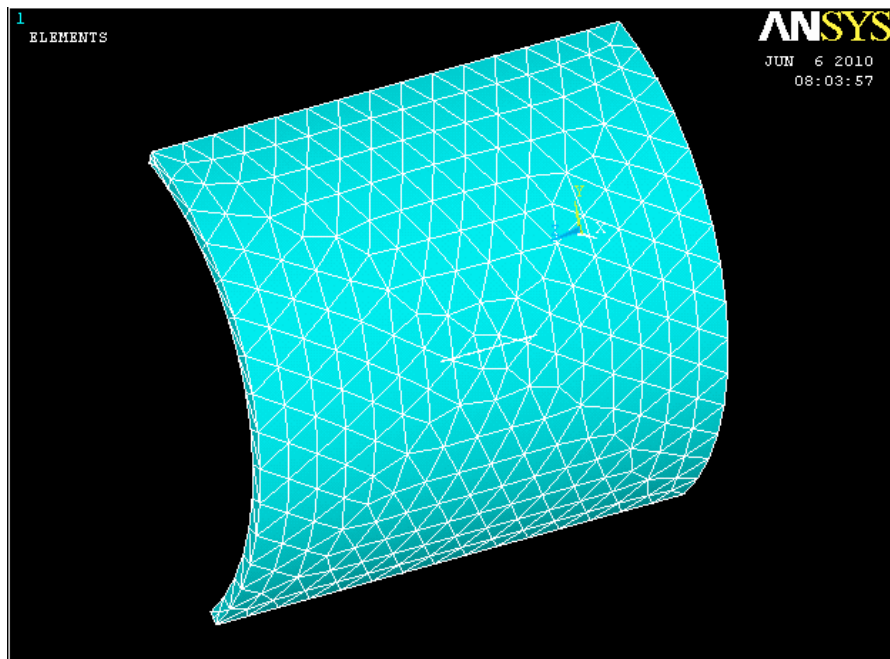


Fig 5.18 Meshed model with external axial crack

When  $2a=30\text{mm}$

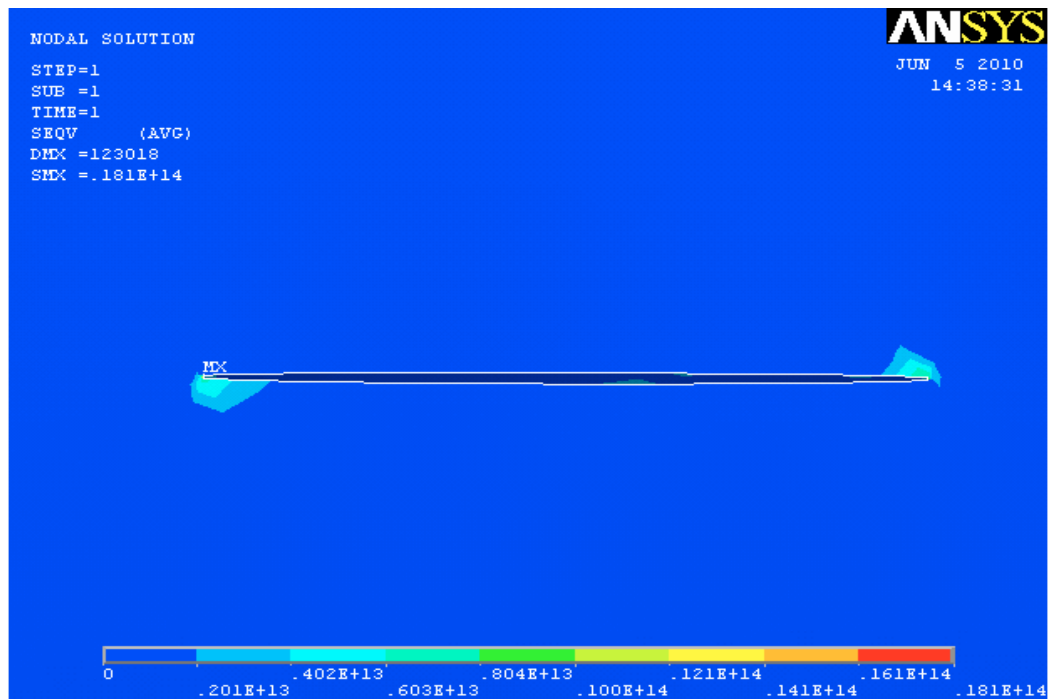


Fig 5.19 Von Misses Stress around the crack

When  $2a=25\text{mm}$

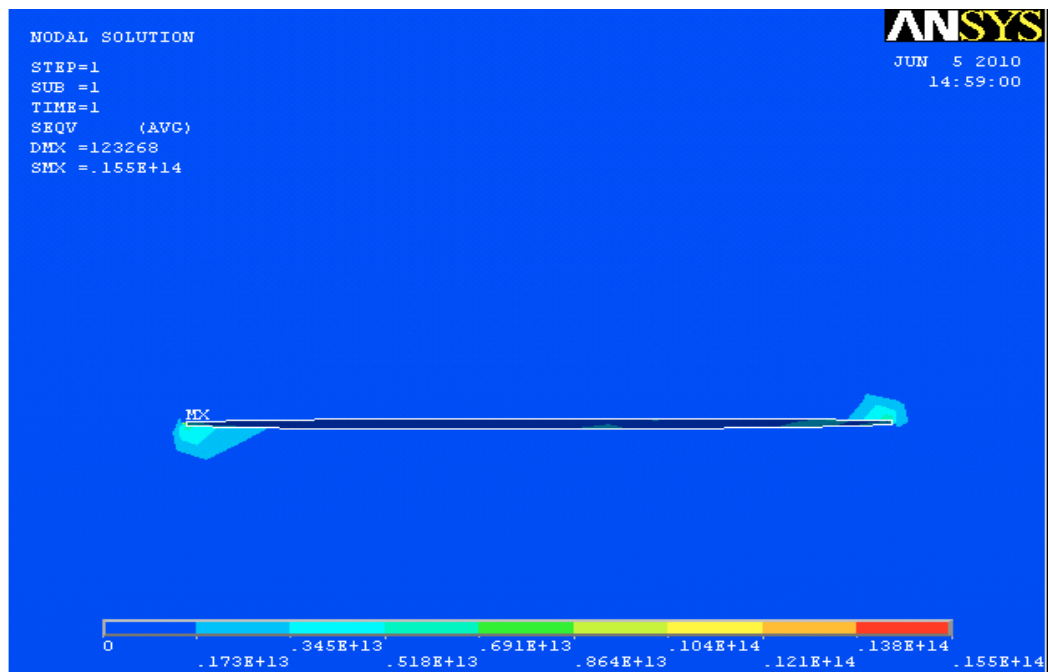


Fig 5.20 Von Misses stress around the crack

When  $2a=20\text{mm}$

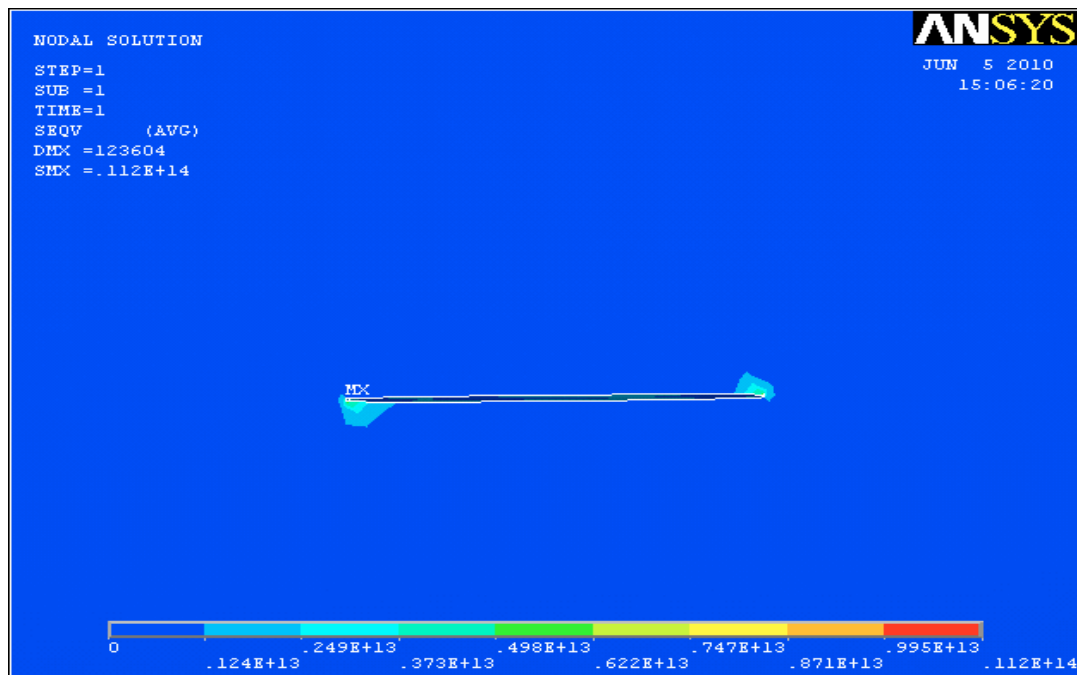


Fig 5.21 Von Misses stress around the crack

When  $2a=18\text{mm}$

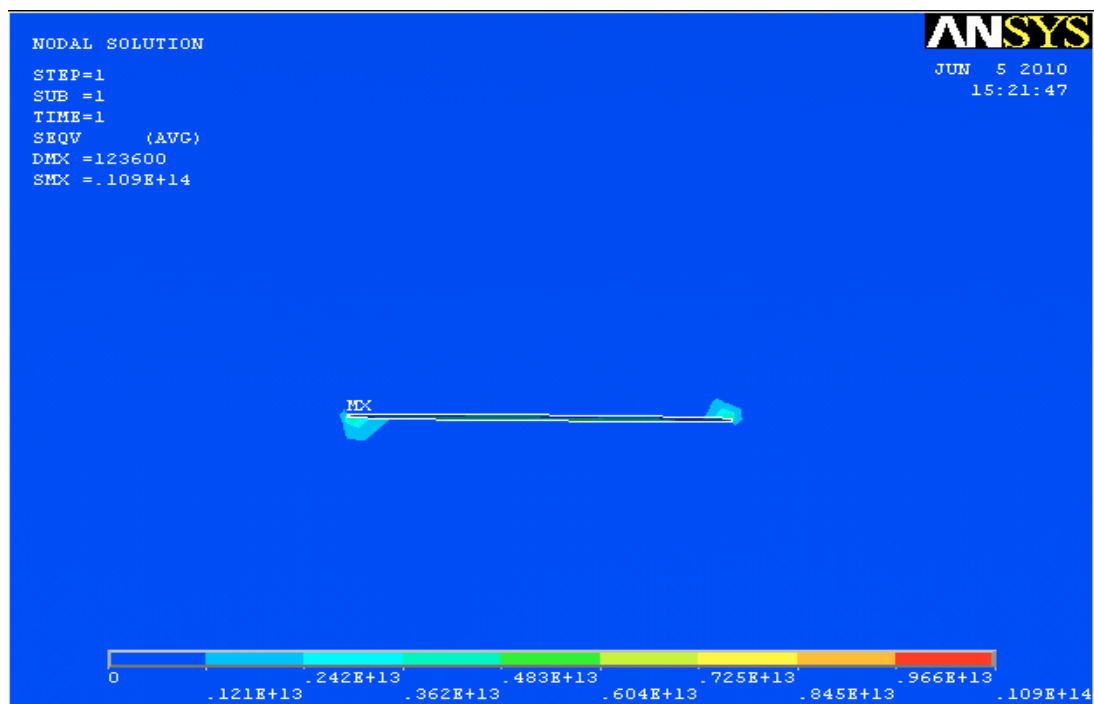


Fig 5.22 Von Misses stress around the crack

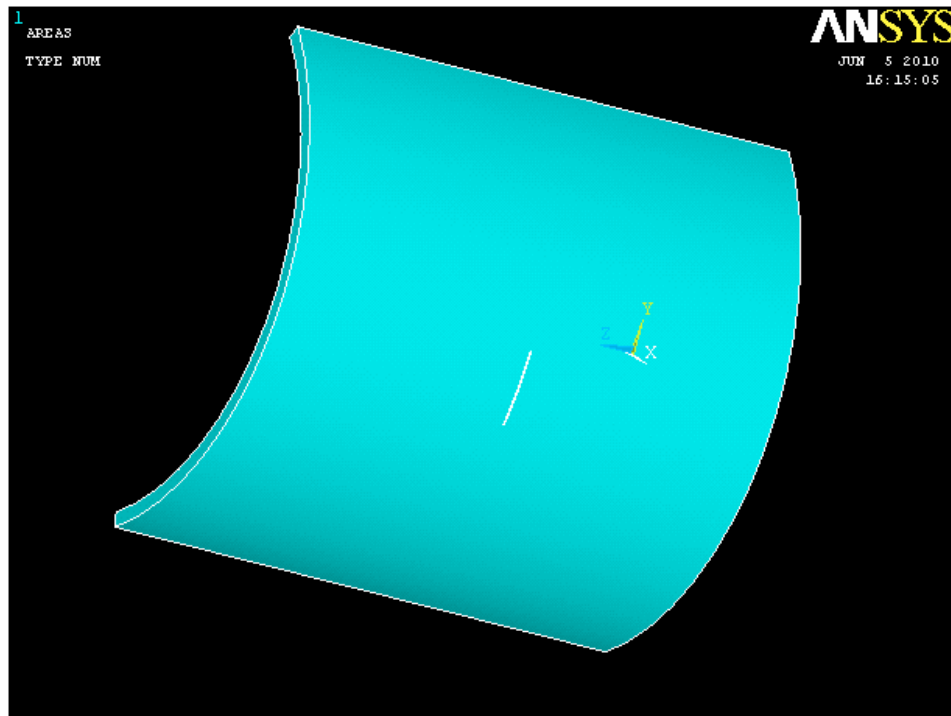


Fig 5.23 Model with external tangential crack

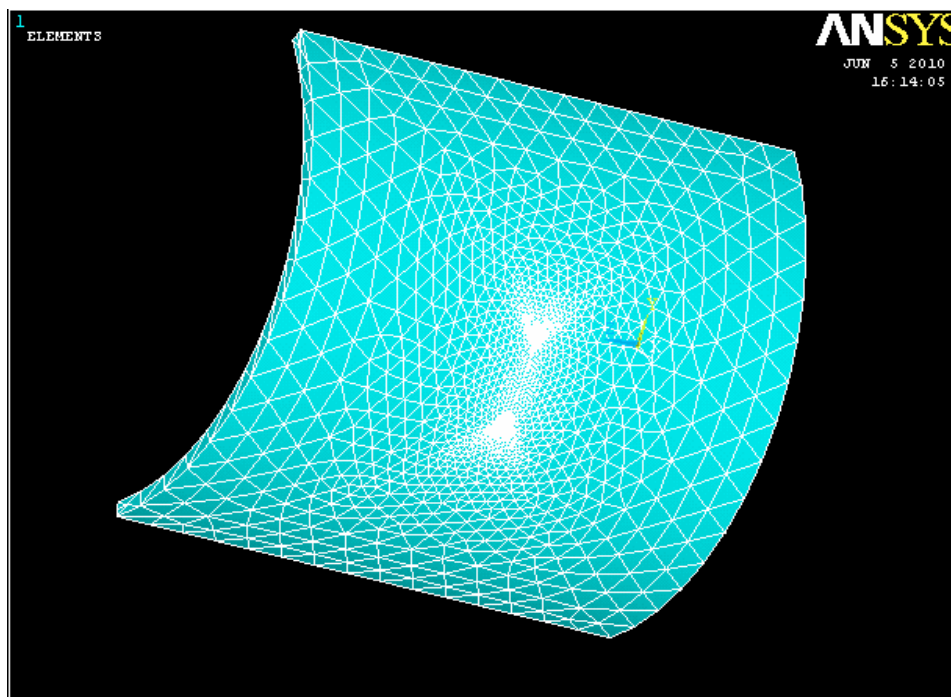


Fig 5.24 Meshed model with external tangential crack

When  $2a=30$

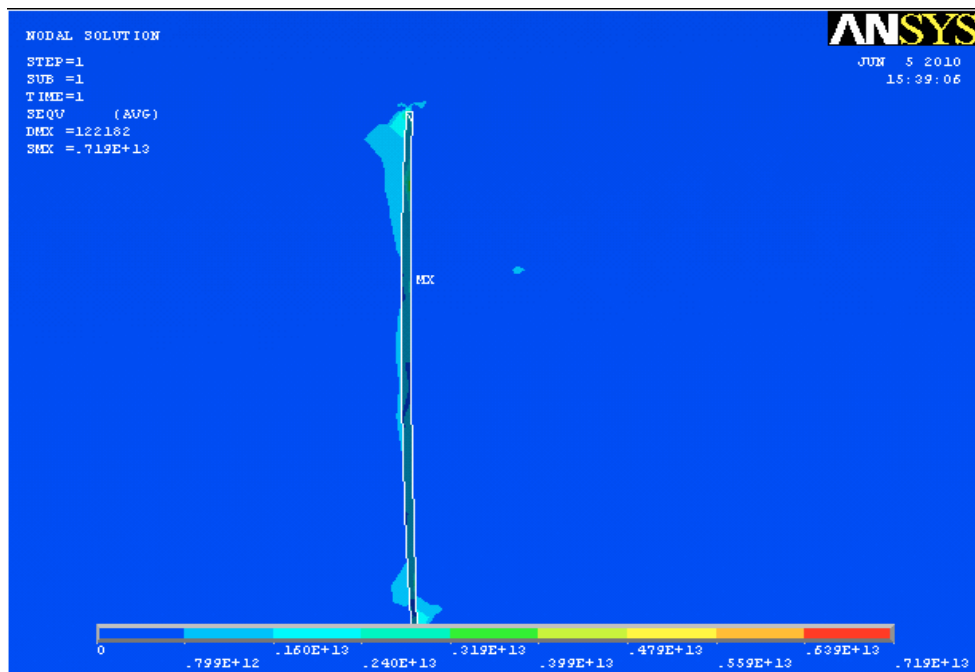


Fig 5.25 Von Misses stress around the crack

When  $2a=25\text{mm}$

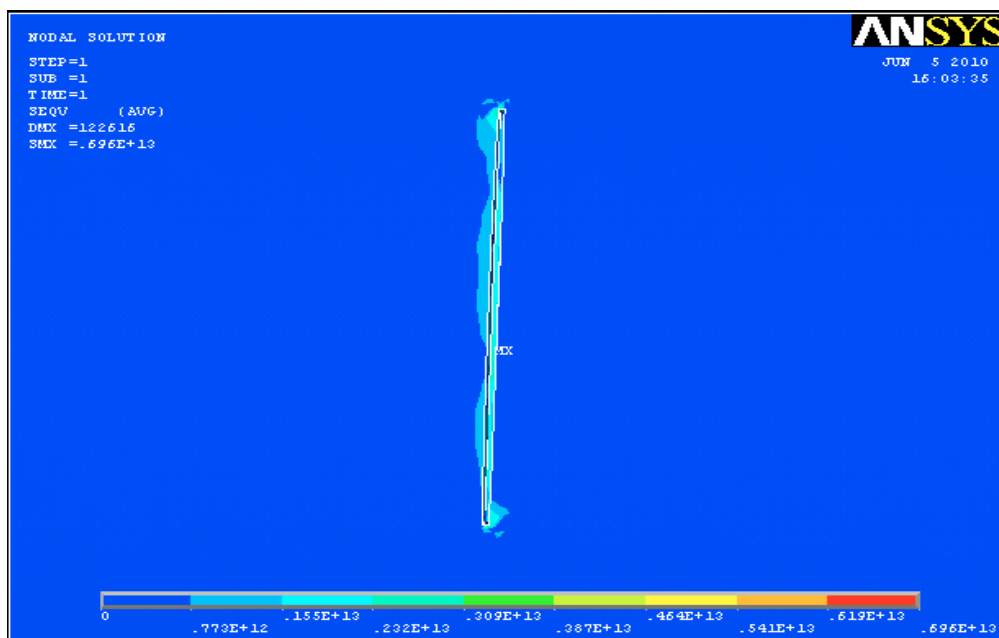


Fig 5.26 Von Misses stress around the crack

When  $2a=20\text{mm}$

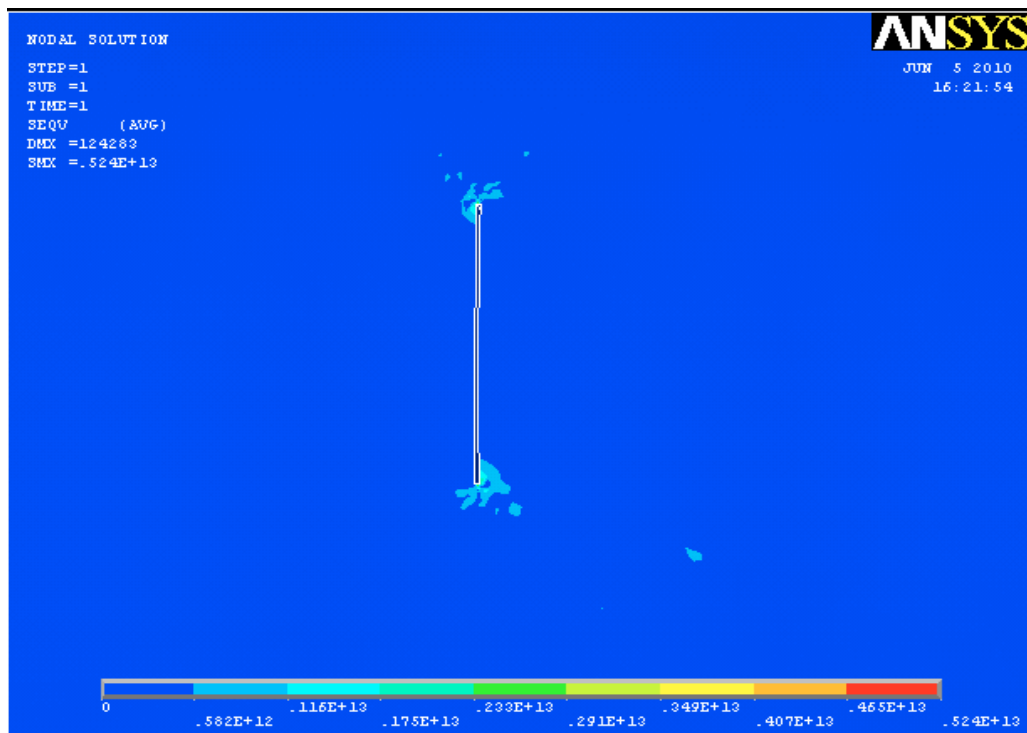


Fig 5.27 Von Mises Stress around the crack

When  $2a=18\text{mm}$

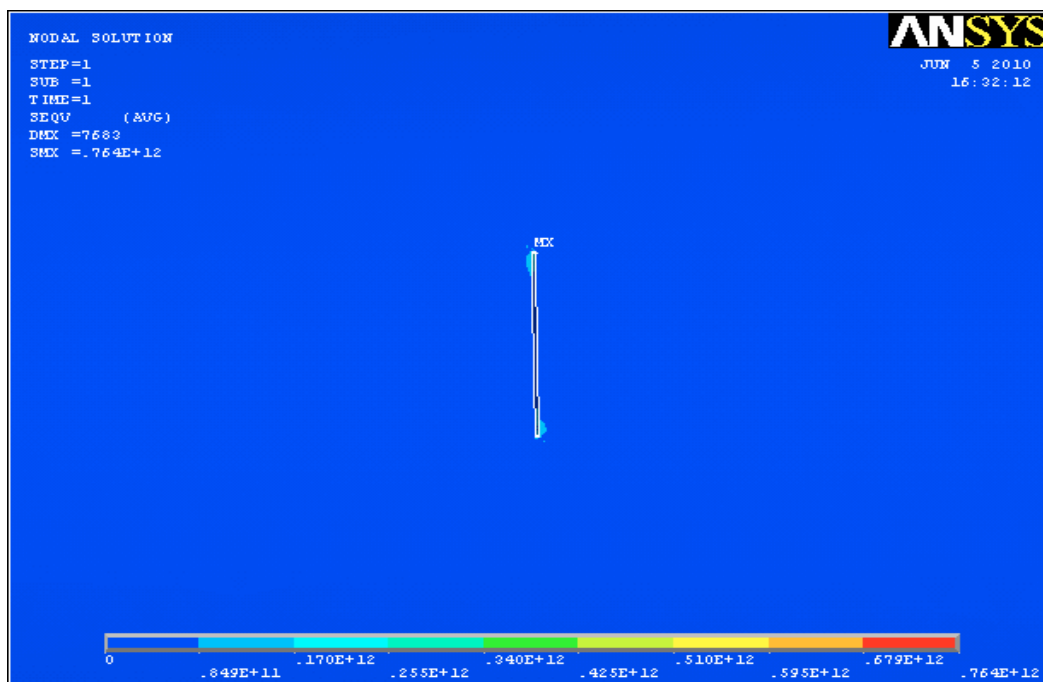


Fig 5.28 Von Mises Stress around the crack

When  $2a=10\text{mm}$

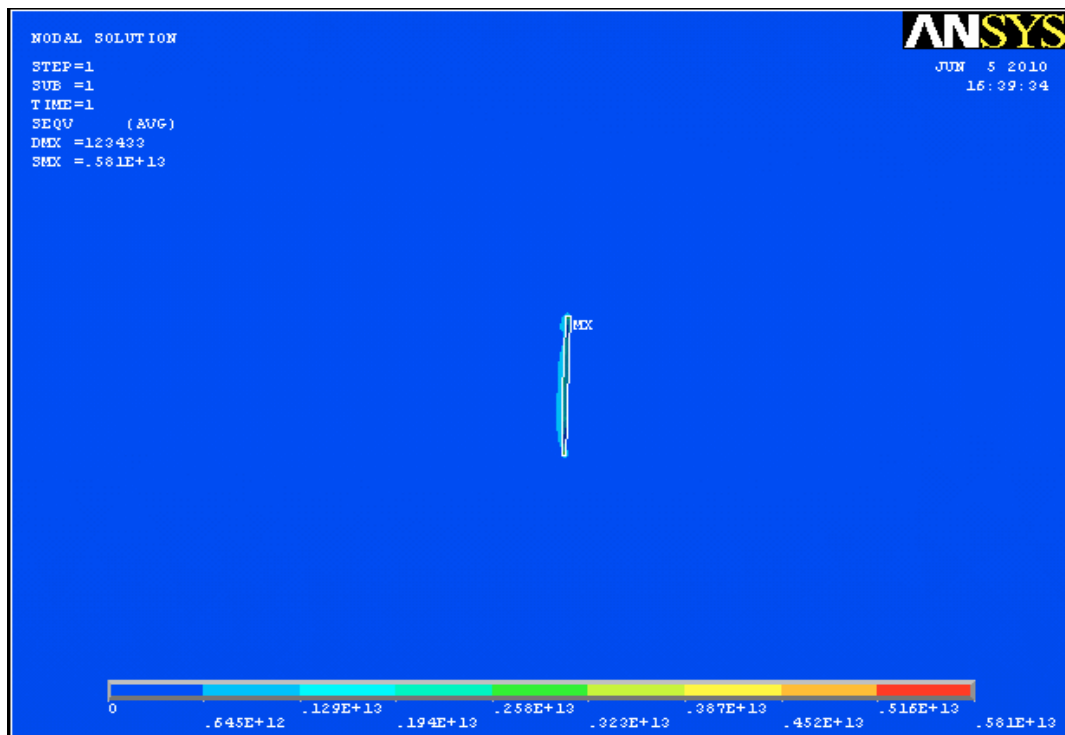


Fig 5.29 Von Mises stress around the crack

When  $2a=5\text{mm}$

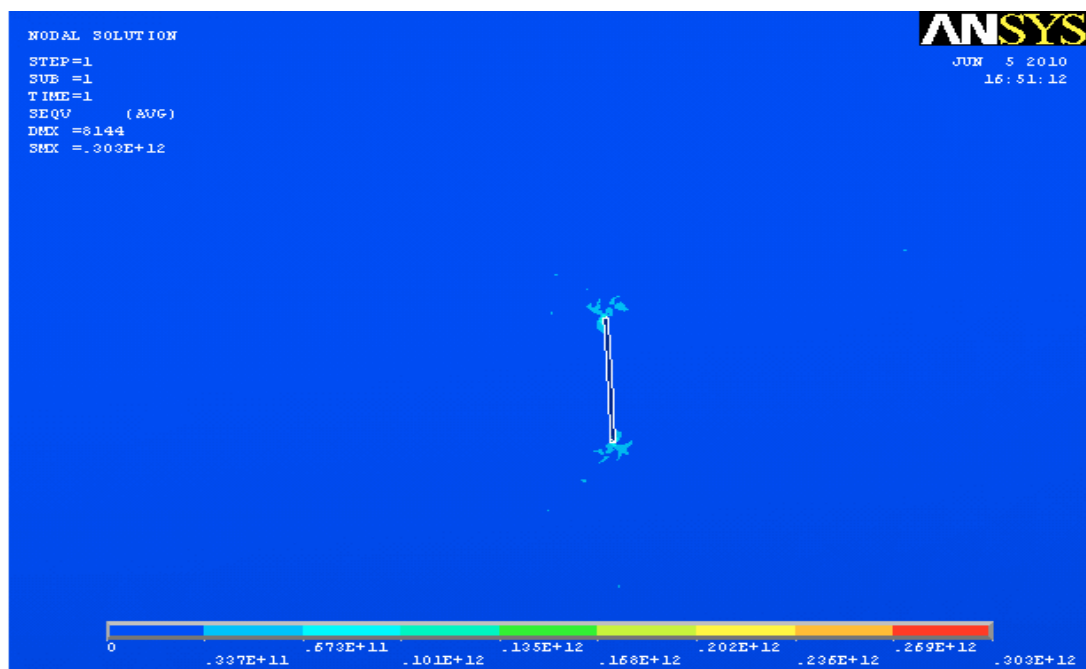


Fig 5.30 Von Mises stress around the crack

| Crack length | Internal                 |                               | External                 |                               |
|--------------|--------------------------|-------------------------------|--------------------------|-------------------------------|
|              | Axial<br>( $10^{12}$ )pa | Tangential<br>( $10^{12}$ )pa | Axial<br>( $10^{12}$ )pa | Tangential<br>( $10^{12}$ )pa |
| 30           | 28.7                     | 17.8                          | 18.1                     | 7.19                          |
| 25           | 25.5                     | 14.7                          | 15.5                     | 6.96                          |
| 20           | 22.9                     | 1.3                           | 11.2                     | 5.24                          |
| 18           | 18.4                     | 1.22                          | 10.8                     | 0.764                         |
| 10           | 15.4                     | 1                             | 7.44                     | 0.581                         |
| 5            | 11.2                     | 0.933                         | 3.52                     | 0.303                         |

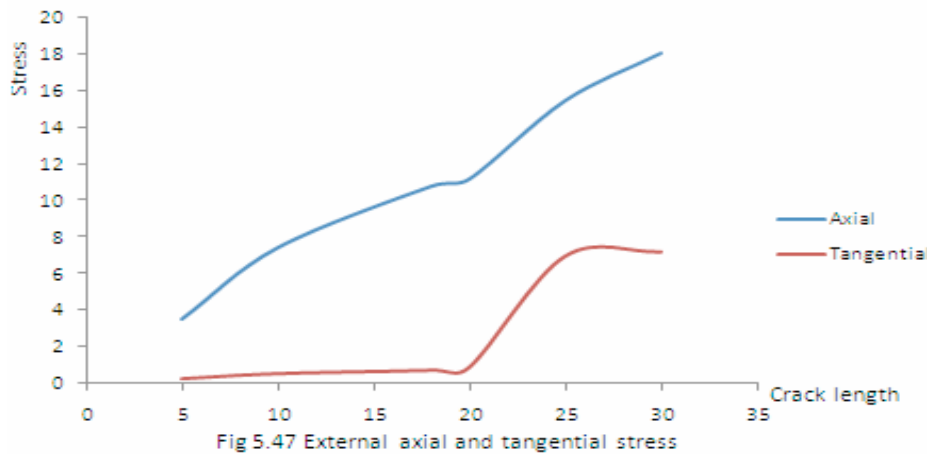
Fig 5.45 Summary of the results from the model

## 5.2 Discussing the results

As clearly seen in figures 5.3-5.45 the maximum stresses shown are distributed in the vicinity of the crack tip but are not uniform around the crack front rather localized somewhere on the crack face. Due to the reinforcement by the fibers in the matrix and differences in orientation of the layers in the laminate of the fiber reinforced composites, different locations in the shell structure have got different stress and strain intensity levels that shows spots of different stress distribution throughout the structure and because of this it is difficult to predict the positions as to where the maximum crack tip stresses will lie in the laminated shell structure in the absence of any experimental data on fiber fracture initiation and propagation during loading. The fiber failure is assumed to occur due to the maximum principal stress exceeding the ultimate tensile strength of the fiber in each lamina, but crack propagation is a cumulative effect of all the laminate in the structure that makes the prediction of maximum stress position more difficult. At the maximum penetration point of the crack tip, the opening action of the far field applied stress faces resistance from the fiber. If the stresses induced in the vicinity of the crack are larger than the fiber ultimate longitudinal strength, the fibers will break and the crack will propagate, if it is not the case, the maximum stress induced in the vicinity of the crack tip will find a way to be deflected towards the lower strength matrix material of the structure. The first failure corresponds to fiber breaking, whereas, the second failure corresponds to a tensile crack of the

matrix material. As the crack moves towards the fiber, the induced maximum tensile principal stresses on the fiber becomes high enough to cause failure of the fiber. At the same time the maximum principal stresses in the fiber is also tensile just away from and on both sides of the crack tip. However, these stresses are less than the failure strength of the fiber, therefore it can be concluded that a combination crushing and bending causes the fiber failure and these fiber failure subsequently lead to the formation of a crack advance in the shell structure

For the external tangential and axial surface crack produced by removing an elliptical volume having a depth of 2 mm contains layers of -30/-60/0/60/30 degrees of orientation with respect to the X-axis. On the outer layer, 30 degree ply, the force will cut the fibers at an angle of 60 degree with the Y-axis. Although a constant pressure is applied on the composite material, the stress varies due to the area, i.e.  $F=PA$ , the area for the axial crack is the crack length times the depth of the crack and for the tangential crack, the area is the crack tip length times the depth, since the crack length is greater than the crack tip length, there is more force on the axial direction than the tangential, subsequently the axial stress is higher than the tangential stress. The area is dependent on the crack length and as the crack length increases the force and the stress increases and this numerical analysis meets with the results found from ANSYS and is show below on Fig 5.47.



For the internal tangential and axial surface crack produced by removing an elliptical volume having a depth of 1 mm contains layers of -30/-60/0/60/30 degrees of orientation with respect to the X-axis. On the outer layer, -30 degree ply, the force will cut the fibers at an angle of 60 degree with the Y-axis, and with the same argument we can deduce that axial stress is greater than the tangential stress and the stress increases as the crack length increases.

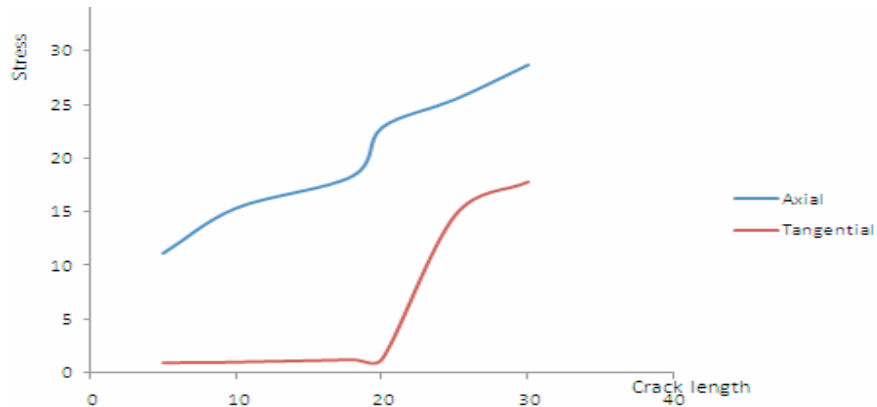


Fig 5.46 Internal axial and tangential stresses

Comparing the internal and external axial stresses, even though equal amount of force is applied the internal axial crack experiences higher stress than the external axial stress, this is due to the fact that the pressure applied inside the pressure vessel and has direct contact with the internal crack where as in the case of the external crack, the laminas before the external crack creates resistance against the pressure, hence fraction of the pressure is applied on the external crack and this in turn decreases the stress on the external crack.

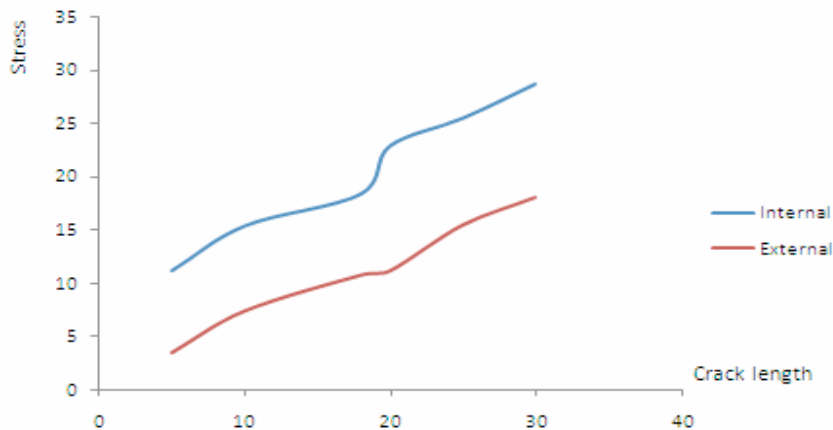
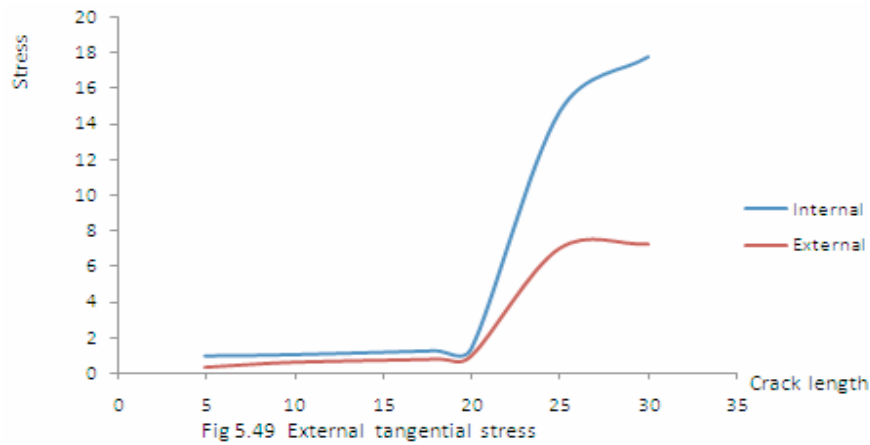
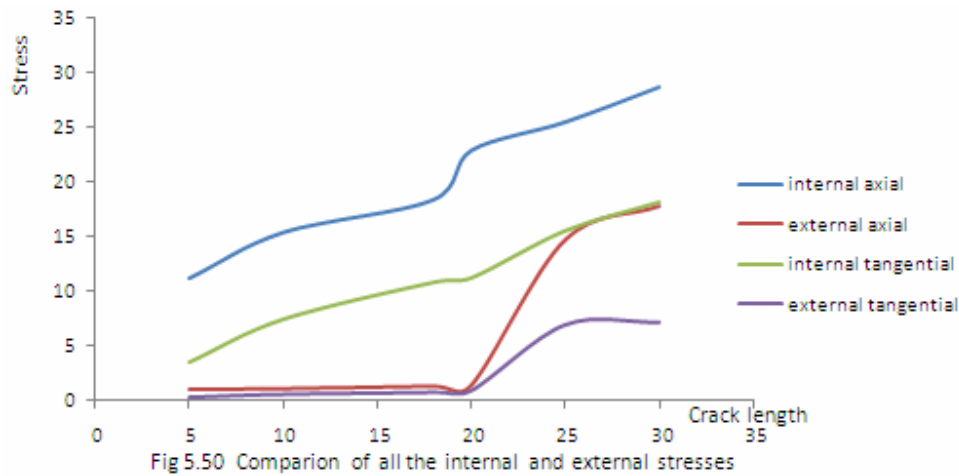


Fig 5.48 Axial stresses

Comparing the internal and external tangential stresses, with the same argument stated above, we can deduce that the internal tangential stress is greater than the external tangential stress and the stress increases as the crack length increases.



The internal axial crack orientation has shown the maximum critical von Mises stress distribution around the surface crack because of the nature of curved geometry of the shell structure and the far field tangentially applied stress tries to open the surface crack. When  $F_{(t)}$  equal to  $F_{yld}$ , first crack is initiated on the -30 degree layer on the crack tip and if the force is not controlled the crack propagates and goes forwards to the next lamina. Hence, the first condition for crack initiation involves the transverse bending stress in the fiber when there is an axial internal crack.



## CHAPTER SIX

### CONCLUSION AND RECOMMENDATIONS

#### *6.1 Conclusion*

Methods and corresponding computer codes for probabilistically assessing composite structure fracture have been discussed. The fracture in composite structures was simulated via a finite package (ANSYS) approach independent of stress intensity factors and fracture toughness parameters. The approach described herein is inclusive in that it integrates composite mechanics (for composite behaviour) with finite element analysis (for global structural response) and incorporates the assessment of stress analysis at the crack tip during composite structural fracture. Compared to Isotropic materials, Compared to traditional materials (like aluminum for which composites are substitutive), composites have higher resistance to fracture and have unpredicted stress distribution pattern as shown in figure 6.1 and 6.2. Furthermore, the crack tip contouring mode is different for both materials due to the difference in microstructural composition. However, the amount of deflection experienced is more for composites towards the cracked side. The sharp corner also here becomes the critical area for stress initiation

From the results obtained, the following conclusions may be drawn:

1. The Maximum Von Misses stress at the crack tip generally increase as crack length increase
2. The internal axial crack orientation has shown the maximum critical von Misses stress distribution around the surface crack because of the nature of curved geometry of the shell structure and the far field tangentially applied stress tries to open the surface crack.
3. The cumulative distribution for strain and stresses is most sensitive to the crack region and still composite material has shown much more resistance to crack than isotropic materials due to the fiber blocking of the crack advance
4. For the axial pressure in Model I loading condition the crack oriented internally in the tangential position of the cylindrical shell structure have also shown a maximum critical von Misses stress distribution as compared to the outer tangential surface crack orientation.

5. Composite materials have shown complex and unpredicted fracture behavior characterization as seen clearly from the stresses and strains distributions as compared to conventional isotropic materials which usually shows specific crack tip plastic zones.

In general, this study shows the possible location of failure in the fiber, the effect of the crack length, damage initiation and evolution in matrix material and interfacial debonding between the fibre and matrix. However, in comparison with the traditional materials, composite shell has shown very strong resistance to crack initiation and propagation. Hence, they are very suitable for structures that need very high curiosity of catastrophic failure.

### ***6.2. Recommendation and Future Directions***

The use of composites, advanced composites, in modern structural building is becoming predominantly important to assure the reliability of such structures which involve high risk and cost of fabrication. In this paper the stress analysis of composite shell structures with crack is presented by considering a cylindrical shell structure. However, in addition to material non linearity, there could be geometrical nonlinearity, which makes the analysis more difficult. Thus, one of the feature continuations of this work could be extending it to geometrically non linear shell structures.

In the current paper the stress analysis is carried out using the finite element method by developing the solid model in ANSYS environment. However, the possibility of developing the stress analysis in the vicinity of the crack by the use of mathematical model of the problem is still what could be conducted ahead. Furthermore, the possibility to have analytical solutions to be compared with the finite element solution is also what should be done in the future.

In this work the crack is modeled as a very small trip cut out of the surface of the shell and the tip has significant variation from the naturally occurring crack. Hence, the researcher proposes the development of a better model in a more versatile software packages which could simulate the reality to be conducted as future work. To further check the reality of the cracked surface waviness, the use very capable software is prominent.

## REFERENCES

1. Kaw, A., *Mechanics of Composite Materials*, CRC Press, 1997.
2. Marcel Dekker *Practical Fracture Mechanics in Design*, A series of text books and Reference books in the Ohio state university of Mechanical Engineering, Columbus, Ohio, 2005.
3. Perez, *Fracture Mechanics*, Kluwer Academic Publisher, New York, 2004.
4. Vasiliev, V. and Morozov, E., “*Mechanics and Analysis of Composite Materials*”, Elsevier Science, 2001.
5. Saouma, V. E., “Lecture Notes in Fracture Mechanics”, University of Colorado, Boulder, 2000.
6. Xiao H.T., Yue Z.Q., Tham L.G., Lee C.F., “Analysis of Elliptical Cracks Perpendicular to the Interface of Two Joined Transversely Isotropic Solids”, *International Journal of Fracture*, 133, 329–354, 2005.
7. Shang Xin-chun, “An Exact Analysis for Free Vibration of a Composite Shell Structure Hermetic Capsule”, *Applied Mathematics and Mechanics*, 22(9), 1035-1045, 2001.
8. Mahdi, E., Hamouda, A. M.S., Sahari, B. B., Khalid, Y.A., “Effect of Material and Geometry on Crushing Behaviour of Laminated Conical Composite Shells”, *Applied Composite Materials*, 9, 265-290, 2002.
9. Sadd H.M. *Elasticity: Theory, applications, and Numerics*, USA, Elsevier Inc., 2005.
10. Mohammadi S., *Extended Finite Element Method for Fracture Analysis of Structures*, Blackwell, UK, 2008.
11. Pinskunov V.G., Rasskazov A.O., “Evolution of the Theory of Laminated Plates and Shells”, *International Applied Mechanics*, 38, 135-165, 2002.
12. Zenkour, A. M., “Stress Analysis of Axis Symmetric Shear Deformable Cross-Ply Laminated Circular Cylindrical Shells”, *Journal of Engineering Mathematics*, 40: 315-332, 2001.
13. Abrosimov, N. A., “Numerical Modeling of Non-Linear Deformation and Buckling of Composite Plate-Shell Structures Under Pulsed Loading”, *Mechanics of Composite Materials*, 35 (6): 495-506, 1999.
14. Lee, Y. S, Lee, J. J, Kang, Y. K, Song, S. Y., “A study on the Structural Analysis of the Steel and GFRP Laminated Composite Cylindrical Shell with a Stiffened Cutout”, *Key Engineering Materials*, 326-328: 1837-1840, 2006. 87

15. Gerasimov, V. P., Gudramovich, V. S., Larionov, I. F., Mulyar, Y. M., Savvin, Y. I., “Ductile Fracture of composite Shells in Axial Compression”, *Institute of Mechanics*, 11: 58-71, 1979.
16. Ukadgaonker, V.G. and Kakhandki, V., “Stress Analysis for an Orthotropic Plate with an Irregular Shaped Hole for Different In-plane Loading Conditions Part-1”, *Composite structures*, 70: 255-274, 2005.
17. Folias, E.S., “The Stress Field in a [0/90] Laminated Composite Plate Containing a Finite Line Crack”, *International Journal of Fracture*, 114: 203- 224, 2002.
18. Sekhar, A. S. and Srinivas, B. N., “Dynamics of Cracked Composite Shafts”, *Journal of Reinforced Plastics and Composites*, 22 (7), 637-653, 2003.
19. Theocaris, P. and Petrou, L., “Stress Distributions and Intensities at Corners of Equilateral Triangular Holes”, *International Journal of Fracture*, 31: 271–289, 1986.
20. Daoust J. and Hoa S., “An Analytical Solution for Anisotropic Plates Containing Triangular Holes”, *Composite Structures*; 19:107–130, 1991.
21. Rybicki E. and David S. “Effect of Stacking Sequence and Lay-up Angle on Free Edge Stresses Around a Hole in a Laminated Plate Under Tension”, *Journal of Composite Materials*, 12:300–313, 1978.
22. Ozdil, F., Carlsson, L. A., Liv, X., ”Characterization of Mode II Delamination Growth in Glass/Epoxy Composite Cylinders”, *Journal of Composite Materials*, 34, 274-298, 2000.
23. Sung, W., Go, C., Shih, M.,” Finite Element Modeling for Analysis of Cracked Cylindrical Pipes”, *Journal of Zhejiang University of Science A*, 8,1373-1379, 2007.
24. Yang J. and Fu Y. “Delamination Growth of Laminated Composite Cylindrical Shells”, *Theoretical and Applied Mechanics*, 45:192-203, 2006.