NON-DIPOLAR COMPONENTS OF NEUTRON STAR MAGNETIC FIELDS

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By

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The undersigned hereby certify that they have read and recommended to the Faculty of Science School of Graduate Studies for acceptance a thesis entitled "**Non-dipolar components of Neutron star magnetic fields** " by **Mekuanint Kifle** in partial fulfillment of the requirements for the degree of **Master of Science in Physics**.

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my family.

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Abstract

Due to the existence of both observational and theoretical evidences for the presence of non-dipolar NS magnetic fields, it has become more and more clear that neutron stars posses magnetic field structures, which are much more complicated than the simple assumption of a dipolar neutron star (NS) magnetic field.

This thesis intends to show the presence of non-dipolar component of the NS magnetic fields as derived from a recent model by kebede[5]. We will derive both the dipolar and non dipolar components of the field separately. We approach the problem by applying Post-Newtonian approximation, to find the angle dependent charge distribution on the surface of the neutron star, which results in the non dipolar components of the NS magnetic field.

Table of Contents

Introduction

Neutron stars are built on a basic assumption that NS matter is electrically neutral composed of mainly heavy nuclei, neutrons, protons, and electrons. The spinning separated charges, which comes as a result of plasma diffusion are the sources for the magnetic fields of neutron stars.

Magnetic fields allow neutron stars to be distinguished from each other and classified into phenomenologically very different groups. Among isolated (or non-accreting) and binary NSs, we distinguish "classical" pulsars, millisecond pulsars, soft gamma-ray repeaters, anomalous X-ray pulsars, and inactive, thermal X-ray emitters. Binary systems associated with mass transfer onto a neutron star are divided into high-mass and low-mass X-ray binaries (according to the mass of the companion star), with substantially different properties.

It is widely accepted that the magnetic field structure of NSs may significantly differ from pure dipole. There are different observational hints for the existence of non-dipolar fields , among these the spectra from pulsars J0 218 + 4232 and J2144 - 3933 [1]. There are also other pulsars with non-dipolar fields like the vela pulsar, B1509 - 58.

The purpose of this thesis is to show NS magnetic fields inherently contain non-dipolar components even though these are of a lesser magnitude. The first chapter contains a review about the distinctive properties of the neutron stars relative to the other stars and some review about pulsars. In chapter two I try to mention some basic points about the NS magnetic fields and what is currently known about this field. The next two chapters (chap. 3 and 4)contain the derivation of the dipolar and the non dipolar components of the neutron star respectively based the model by kebede (2002). And finally discussions and conclusions included in Chapter 5.

Chapter 1

Neutron stars and pulsars

1.1 Neutron stars

Neutron stars are created as a result of the gravitational collapse of the core of massive stars, with a mass M $\geq 8M_{\odot}$, at the end of their life times. Their deaths are associated with type II supernova explosions[2]. A type II supernova occurs when the iron core of a super giant star collapses to the density of an atomic nucleus (a few hundred million tons per cubic centimeter). At such tremendously high densities, protons and electrons fuse together to form neutrons.[3] . Hence the name ”neutron stars”.

A neutron star is about 20 km in diameter and has the mass of about $1.4M_{\odot}$. This shows that neutron stars have very high density. One teaspoonful of NS material would weigh a billion tons! The neutron star which we currently rank as ”nearest to Earth” is a radio pulsar called J0108-1431. Because of its small size and high density, a neutron star possesses a surface gravitational field of which is about 2×10^{11} times that of Earth.[4].

The interior of the neutron star is mostly neutral matter with only traces of electrons and protons throughout the object. More than 95 percent of the content of

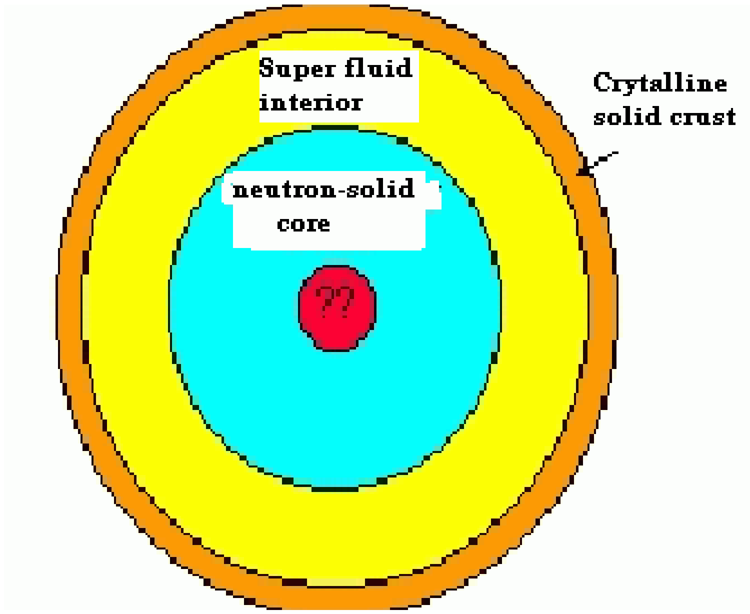


Figure 1.1: neutron star structure

NSs is neutrons with only a few percent of protons and an equal number of electrons .We are uncertain about the interior structure of a neutron star. We don't know a lot about how matter behaves at these amazingly high densities. One proposed model for the neutron star is shown in fig. 1.1 above.[3]. The crystalline solid crust consisting of nuclei, electrons and neutrons, the super fluid interior consists neutrons, protons and electrons, and possibly a neutron solid core with free electrons and protons.[5].

Neutron stars have both minimum and maximum mass limits. The maximum mass, which is of purely general relativistic origin, is unknown, but it lies in the range of 1.44 - 3 $M_{\odot}$, and the minimum stable neutron star mass is about $0.1 M_{\odot}$ [2]. The central region of the star collapses under gravity. It collapses so much that protons and electrons combine to form neutrons. Hence the name "neutron star".

Neutron stars may appear in supernova remnants, as isolated objects, or in binary systems. When a NS is in a binary system, astronomers are able to measure its mass. From a number of such binaries seen with radio or X-ray telescopes, NS masses has been found to be about 1.4 $M_{\odot}$, which is in agreement with what we put as a limit for the mass of NSs . For binary systems containing an unknown object, this information helps distinguish whether the object is a NS or a black hole, since black holes are more massive than NSs. NSs were predicted in the 1930s but not observed until 1967 as Pulsars[4].

1.2 pulsars

Pulsars are rapidly rotating highly magnetized NSs. As the name implies pulsars emit electromagnetic radiation in a broad frequency range from radio waves to $X - rays$ and $\gamma - rays$, and pulsars pulse because they rotate. The first neutron star was discovered by chance by Jocelyn Bell (Burnett) using a primitive antenna. With this antenna a faint but regular signal was detected[6]. Pulsars were first discovered as radio sources that blink on and off at a constant frequency. Pulsars are spinning neutron stars that have jets of particles moving almost at the speed of light streaming out above their magnetic poles. These jets produce very powerful beams of light. The magnetic and rotational axes of a pulsar are misaligned. Therefore, the beams of light from the jets sweep around as the pulsar rotates. We see pulsars turn on and off as the beam sweeps over the Earth. Neutron stars for which we see such pulses are called "pulsars", or sometimes "spin-powered pulsars," indicating that the source of energy is the rotation of the magnetized neutron star.

A pulsar can produce radiation by spinning its powerful magnetic field through

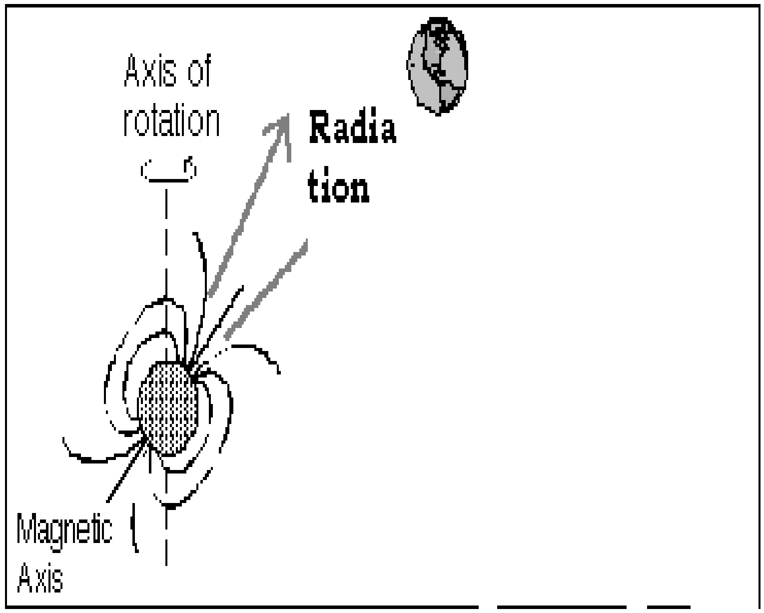


Figure 1.2: The earth receives the pulsed radiation

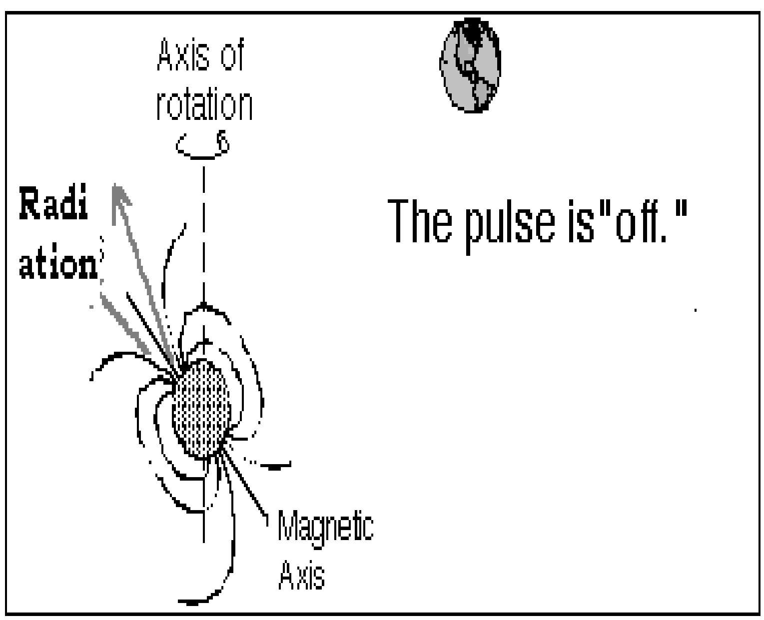


Figure 1.3: The earth does not detects the pulsed radiation

space. There are also 'accreting pulsars' which funnel matter from a companion star onto their magnetic polar caps as they rotate. A neutron star uses up a lot of its rotational energy moving its magnetic field around this way, and so it gradually slows down. When it slows down enough, it no longer radiates very much energy. In addition to this, the radiation reaction torques tend to align its magnetic moment with the spin axis in a few million years. At this conjecture the NS will cease pulsing and will not be considered as pulsars. This usually happens within a few million years.[4].

The rate of rotation of pulsars is so precise that some are known as the best timekeepers in the universe. However, every so often the rotation rate suddenly increases. It is thought that these glitches are related to the breakup of super fluid vortices pinned to the normal crust as a result of differential rotation.[6].

Most neutron stars in the Universe are old enough and tired enough that they are no longer pulsars. A recent paper estimates a thousand million old neutron stars in our Galaxy, even though the number of known pulsars is about a thousand. Every pulsar is a neutron star, but not every neutron star is a pulsar.

Chapter 2

Neutron star magnetic field

This chapter has the purpose of giving a general overview about what is currently known about neutron star magnetic fields, their origin and evolution.

We actually know surprisingly little about neutron star magnetic fields. In particular, most ”measurements” of neutron star magnetic fields are indirect inferences, which are put in doubt both by their inconsistency with other observational evidence and with plausible theoretical models for the physics of their surroundings.

2.1 Source of the field

Just recently it has been indicated that Plasma density gradients inherent to neutron star matter could lead to large scale plasma diffusion and subsequent charge separation with excess negative charge accumulating in the crust while at the same time, almost the same amount of excess positive charge is left behind at the solid core. Surface magnetic fields are then expected to result from the spinning of these separated charges.

In this case, the electron and proton currents resulting from the charge separation process are modelled by[5]

$$n_i = -D^0(\partial_i n_0 - \Gamma^0_{0i} n_0) + \mu n_0 g_{\beta i} F^{0\beta} \tag{2.1.1}$$

where; $n_\alpha \equiv (n_i, n_0)$ is the four particle current and D^0 represents the diffusion coefficient which we take as scalar.

2.2 Magnetic field evolution

It appears that only about five percent of neutron stars, the most strongly magnetized, undergo significant field decay but a recent finding by physicist at the University of Alacant in Alacant, Spain indicates that of about 30 neutron stars with fields in excess of about 10^{12} G show evidence for decay of their magnetic fields. Previously, such stars were assumed to have constant magnetic fields like stars with weaker fields.[7]

There are also Several evidences, which shows the possibility of an evolving magnetic field in neutron stars. I try to mention some of the evidences, which are discussed by different authors about the evolution of NS magnetic fields.

Kebede(2002) indicates that NS magnetic fields decay as a result of neutrino and photon emissions lead to the cooling of NS in general. According to this model all form of radiations resulting in the spin down of the pulsar are also responsible for field dissipation[5]

Even though, Ohmic decay is very slow and ineffective in producing the observed effects, more or less it could be counted as marginally responsible for field dissipation.

According to proponents of this model [8], all stars at all stages of their evolution have some magnetic field, due to electronic currents circulating in their interiors.

These currents may be expected to decay due to momentum transfer of an electron to a more massive particle through a coulomb or other collision within microscopic time scale τ_{coll}. However, any decrease in the current **I** implies a decrease of the magnetic flux $\Phi = cLI$, where c is the speed of light,, $L \sim R/c$ is the star's self-inductance, and R is its radius. According to Lenz's law, such a flux decline will induce an emf $\varepsilon = -cd\Phi/dt = -LdI/dt$ that tends to keep the current going as prescribed by Ohm's law, $\varepsilon = \mathbf{R}I$. The resistance **R** can be estimated in terms of a typical conductivity $\sigma = n_e e^2 \tau_{coll}/m_e$ (where n_e, -e, and m_e denote the electron concentration, charge, and mass) by $\mathbf{R} \sim c/(\sigma R)$.

Thus, the star is well described by an electric circuit with an inductance L and a resistance **R** connected in series, in which the current decays at such a rate that the induced emf is always as strong as required to maintain the instantaneous current against resistive decay. The exponential ("Ohmic") decay time is thus

$$\tau_{ohm} = \frac{L}{\mathbf{R}} \sim \sigma(\frac{R}{c})^2 = n_e r_e R^2 \tau_{coll} \qquad (2.2.1)$$

where $r_e = e^2/(m_e c^2)$ is the classical electron radius. Since the electron concentration is typically high, but specially since stellar radii (even in the very compact neutron stars) are large (e. g., compared to typical laboratory scales), in general $\tau_{ohm} \gg \tau_{coll}$ by many orders of magnitude. Even though, stellar magnetic fields can persist for very long times, there is some loss of magnetic field intensity.

Finally, the observational evidence, which shows the evolution of neutron star magnetic field is the evolutionary connection between young and old NSs. Generally speaking, young neutron stars appear to have strong magnetic fields $\sim 10^{11-15}$ G ("classical" radio pulsars, "magnetars", X-ray pulsars), whereas old neutron stars have weak fields $\sim< 10^9$ G (ms pulsars, low mass X-ray binaries) [8]. If these two groups have an evolutionary connection it can be considered as an indication for the evolution of neutron star magnetic field.

Chapter 3

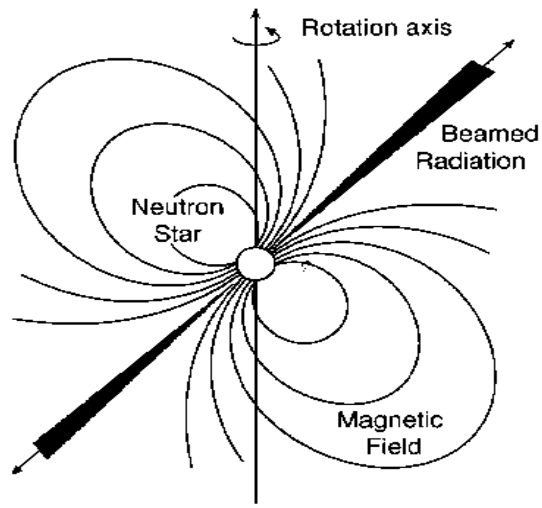
Dipolar field

Until recent years, it was believed that, the neutron star magnetic field is dipolar, and different works discuss about pulsars based on this kind of field.

Normally gravitational fields are so weak as compared to electro static driving forces, that is why different authors ignore this field in their discussions about NSs. In calculating the magnitude of the separated charges and the resulting dipole fields, Kebede (2002) ignored gravity. In the absence of gravity we expect to have dipolar surface fields for NSs as shown in fig.3 below.

In this chapter we are going to separately derive the dipolar field by ignoring gravity. A uniform spherical surface charge distribution is assumed. These are the result of the model equation taken at equilibrium and in the absence of gravity.

$$0 = D^0(\partial_i n_0 + \mu n_0 g_{\beta i} F^{0\beta})$$



field.png

Figure 3.1: Dipolar field

3.1 Derivation of the Vector potential

To derive the vector potential, for uniform spherical charge distribution with surface charge density σ, we have to first find the current density which is given by

$$\begin{aligned} \mathbf{J} &= \sigma\delta(r' - R)\mathbf{v} \\ &= \sigma\delta(r' - R)\vec{\omega} \times \mathbf{r} \end{aligned}$$

But

$$\vec{\omega} \times \mathbf{r} = \begin{pmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 0 & \omega \\ R\sin\theta'\cos\phi' & R\sin\theta'\sin\phi' & R\cos\theta' \end{pmatrix}$$

which implies,

$$\begin{aligned}
\vec{\omega} \times \mathbf{r} &= \hat{\mathbf{i}}(-\omega R \sin\theta' \sin\phi') + \hat{\mathbf{j}}(R \sin\theta' \cos\phi') \\
&= \omega R \sin\theta'(-\sin\phi'\hat{\mathbf{i}} + \cos\phi'\hat{\mathbf{j}}) \\
&= \omega R \sin\theta' \hat{\mathbf{e}}_{\phi'}
\end{aligned}$$

Then, the current density will be

$$\begin{aligned}
\mathbf{J} &= \sigma\delta(r' - R)\omega R \sin\theta' \hat{\mathbf{e}}_{\phi} \\
&= \sigma\delta(r' - R)\omega R \sin\theta'(-\sin\phi'\hat{\mathbf{i}} + \cos\phi'\hat{\mathbf{j}}) \qquad (3.1.1)
\end{aligned}$$

which clearly shows that $\mathbf{J}$ has only $\hat{\phi}'$ component as

$$J_{\phi} = \sigma\delta(r' - R)\omega R \sin\theta'$$

The vector potential is given by[9]

$$\mathbf{A}(x) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(x')}{|x - x'|} d^3x' \qquad (3.1.2)$$

Since the geometry is spherically symmetric we may choose the observation point in the xz plane ($\phi' = 0$). Since the azimuthal integration in equation (3.1.2) is symmetric in ϕ', the $\mathbf{i}^{th}$ component of the current does not contribute. This leaves only the $\mathbf{j}^{th}$ component, which is A_{ϕ}[9]. Thus

$$A_{\phi} = \frac{\mu_0}{4\pi} \int \frac{\sigma\delta(r' - R)\omega R \sin\theta'(\cos\phi')}{|x - x'|} d^3x' \qquad (3.1.3)$$

But we have[9]

$$\frac{1}{|x - x'|} = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{1}{2l+1} \frac{r_<^l}{r_>^{l+1}} Y_{l,m}^*(\theta', \phi') Y_{l,m}(\theta, \phi)$$

and

$$d^3x' = r'^2 dr' \sin\theta d\theta d\phi$$

Substituting these values in (3.1.3) we get

$$\begin{aligned} A_\phi &= \frac{\mu_0 \sigma 4\pi\omega R}{4\pi} \int \sum_{l,m} \frac{1}{2l+1} \frac{r_<^l}{r_>^{l+1}} Y_{l,m}^*(\theta',\phi') Y_{l,m}(\theta,\phi) \\ &* \; \sigma(r^{'} - R) \sin\theta^{'} \cos\phi^{'} r'^2 dr' sin\theta' d\theta' d\phi' \end{aligned} \quad (3.1.4)$$

But $sin\theta' \cos\theta' = -\frac{\sqrt{8\pi}}{3} Y_{1,1}(\theta',\phi')$, with $\phi' = 0$ [9]

Then A_ϕ will be

$$\begin{aligned} A_\phi &= \frac{\mu_0 \sigma 4\pi\omega R}{4\pi} \int \sum_{l,m} \frac{1}{2l+1} \frac{r_<^l}{r_>^{l+1}} Y_{l,m}^*(\theta',\phi') Y_{l,m}(\theta,\phi) \delta(r'-R) \\ &* \; Y_{11}(\theta',\phi')(-\frac{\sqrt{8\pi}}{3}) dr'^2 sind\theta' d\phi' \end{aligned} \quad (3.1.5)$$

From the normalization and orthogonality condition[9]

$$\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta Y_{l',m'}^*(\theta',\phi') Y_{l,m}(\theta,\phi) = \delta_{l'l}\delta_{m'm}$$

we have

$$\int Y_{lm}^*(\theta',\phi') Y_{11}(\theta',\phi') \sin\theta' d\theta' d\phi' = \delta_{l1}\delta_{m1} = \begin{cases} 1, & \text{l=1 and m=1;} \\ 0, & \text{l} \neq \text{1 or m} \neq \text{1} \end{cases} \quad (3.1.6)$$

The integration contributes only for l=1 and m=1

Also, from the relation

$$P_l^1(\cos\theta) = -(1-\cos^2\theta)^{\frac{1}{2}} \frac{dP_l(\cos\theta)}{d(\cos\theta)}$$

we have

$$P_1^1(\cos\theta) = -(\sin^2\theta)^{\frac{1}{2}} \frac{dP_1(\cos\theta)}{d(\cos\theta)}$$

$$= -\sin\theta \frac{d(\cos\theta)}{d(\cos\theta)}$$
$$= -\sin\theta$$

Therefore, the vector potential in (3.1.4) may be written as

$$A_\phi = \frac{\mu_0 Q\omega}{4\pi R}\int \sum_{l,m} \frac{1}{2l+1}\frac{r_<^l}{r_>^{l+1}} Y_{l,m}^*(\theta',\phi')Y_{l,m}(\theta,\phi) \tag{3.1.7}$$
$$\delta(r'-R)P_l'(cos\theta)cos\phi' r'^2 dr' d\phi' d(cos\theta')$$

Let the constant

$$\frac{4\pi}{\mu_o} = c$$

Finally the vector potential associated with the spinning crust containing the separated negative charge has only a ϕ component,which can be written as:

$$\begin{aligned} A_\phi &= \frac{-|Q|\omega}{RC}\int \sum_{l,m}(\frac{1}{2l+1})(\frac{r_<^l}{r_>^{l+1}})Y_{l,m}^*(\theta',\phi')Y_{l,m}(\theta,\phi) \\ &* \quad P_l^1(cos\theta')\delta(r'-R)cos\phi' r'^2 dr' d\phi' d(cos\theta') \end{aligned} \tag{3.1.8}$$

3.1.1 Vector potential inside the sphere(crust)

The vector potential inside the crust can be calculated from(3.1.8) by substituting $r_< = r$ and $r_> = R$ as:

$$\begin{aligned} A_{\phi,in} &= \frac{-|Q|\omega}{RC}\int \sum_{l,m}(\frac{1}{2l+1})(\frac{r^l}{R^{l+1}})Y_{l,m}^*(\theta',\phi')Y_{l,m}(\theta,\phi) \\ &* \quad P_l^1(cos\theta')\delta(r'-R)cos\phi' r'^2 dr' d\phi' d(cos\theta') \\ &= \frac{-|Q|\omega}{RC}\int \sum_{l,m}(\frac{1}{2l+1})(\frac{r^l}{R^{l+1}})Y_{l,m}^*(\theta',\phi')Y_{l,m}(\theta,\phi) \\ &* \quad \delta(r'-R)sin^2\theta' cos\phi' r'^2 dr' d\phi' d\theta' \end{aligned} \tag{3.1.9}$$

Integration over the delta function results

$$A_{\phi,in} = \frac{-|Q|\omega R^2}{RC} \int \sum_{l,m} \frac{r^l}{R^{l+1}} Y_{l,m}(\theta,\phi) Y^*_{l,m}(\theta',\phi') \sin^2\theta' \cos\phi' d\theta' d\phi'$$

But since $\sin\theta' \cos\phi' = -\sqrt{\frac{8\pi}{3}} Y_{11}(\theta',\phi')$ [9], then

$$A_{\phi,in} = \frac{-|Q|\omega R^2}{RC}(-\sqrt{\frac{8\pi}{3}}) \sum (\frac{1}{2l+1})(\frac{r^l}{R^{l+1}}) Y_{l,m}(\theta,\phi) \int Y^*_{l,m}(\theta',\phi') Y_{11}(\theta',\phi') \sin\theta' d\theta' d\phi'$$

By equation (3.1.6) the integration holds only for l=1, and m=1,

and$Y_{11}(\theta, phi) = \sqrt{\frac{3}{8\pi}} \sin\theta e^{i\phi}$, then we find

$$A_\phi, in = \frac{-|Q|\omega R^2}{RC}(-\sqrt{\frac{8\pi}{3}})(\frac{1}{2+1})(\frac{r}{R^2})(\sqrt{\frac{3}{8\pi}}) \sin\theta e^{i\phi}$$
$$= \frac{-|Q|\omega}{3C}(\frac{r}{R}) \sin\theta e^{i\phi}$$

Finally the vector potential inside the crust will be

$$A_{\phi,in} = \frac{-|Q|\omega}{3C}(\frac{r}{R}) \sin\theta \tag{3.1.10}$$

3.1.2 Vector potential outside the crust

The vector potential outside the crust can be derive from (3.1.8) by substituting $r_< = R$ and $r_> = r$ as

$$A_{\phi,out} = \frac{-|Q|\omega}{RC} \int \sum_{l,m} (\frac{1}{2l+1})(\frac{R^l}{r^{l+1}}) Y_{l,m}(\theta,\phi) Y^*_{l,m}(\theta',\phi') \delta(r'-R) \sin^2\theta' \cos\phi' dr' d\theta' d\phi'$$
$$= \frac{-|Q|\omega R^2}{RC}(-\sqrt{\frac{8\pi}{3}}) \sum (\frac{1}{2l+1})(\frac{R^l}{r^{l+1}}) Y_{l,m}(\theta,\phi) \int Y^*_{l,m}(\theta,\phi) \sin\theta' d\theta' d\phi$$

Again the integration holds for l=1 and m=1, then

$$A_{\phi,out} = \frac{-|Q|\omega R^2}{RC}(-\sqrt{\frac{8\pi}{3}})(\frac{1}{2+1})(\frac{R}{r^2})(-\sqrt{\frac{3}{8\pi}}) \sin\theta e^{i\phi}$$

$$= \frac{-|Q|\omega}{C}(\frac{R}{r})^2 \sin\theta e^{i\phi}$$

The vector potential outside the crust will be

$$A_{\phi,out} = \frac{-|Q|\omega}{C}(\frac{R}{r})^2 \sin\theta \tag{3.1.11}$$

3.2 Derivation of the magnetic field

In this section we will derive the dipolar component of the neutron star magnetic field. The magnetic field can be derived from vector potential as[9]

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{3.2.1}$$

which can be written as

$$\begin{aligned} \nabla \times \mathbf{A} &= \hat{\mathbf{e}}_r \frac{1}{r\sin\theta}[\frac{\partial}{\partial\theta}(\sin\theta A_\phi) - \frac{\partial A_\theta}{\partial\phi}] \\ &+ \hat{\mathbf{e}}_\theta[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{1}{r}\frac{\partial(rA_\phi)}{\partial r}] + \hat{\mathbf{e}}_\phi \frac{1}{r}[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}] \end{aligned} \tag{3.2.2}$$

But we have only $\hat{\phi}$ component of the vector potential, then the magnetic field will be

$$\begin{aligned} \mathbf{B} &= \nabla \times \mathbf{A} \\ &= \frac{1}{r\sin\theta}[\frac{\partial}{\partial\theta}(\sin\theta A_\phi)\hat{\mathbf{e}}_r + \frac{-1}{r}(\frac{\partial A_\phi}{\partial r})\hat{\mathbf{e}}_\theta] \end{aligned} \tag{3.2.3}$$

3.2.1 Magnetic field inside the crust

The magnetic field inside the crust can be calculated from (3.1.10) and (3.2.3) as

$$\begin{aligned}
\mathbf{B}_{in} &= \nabla \times \mathbf{A}_{in} \\
&= \frac{1}{r sin\theta}[\frac{\partial}{\partial\theta}(\sin\theta A_{\phi,in})]\hat{\mathbf{e}}_r + [\frac{1}{r}\frac{\partial A_{\phi,in}}{\partial r}]\hat{\mathbf{e}}_\theta \\
&= \frac{1}{r\sin\theta}[\frac{\partial}{\partial\theta}(-\sin\theta\frac{|Q|\,\omega}{3C}(\frac{r}{R})\sin\theta)]\hat{\mathbf{e}}_r \\
&+ [\frac{1}{r}\frac{\partial}{\partial r}(r\frac{|Q|\,\omega}{3C}(\frac{r}{R})\sin\theta)]\hat{\mathbf{e}}_\theta \\
&= \frac{1}{r\sin\theta}[-2\sin\theta\cos\theta\frac{|Q|\,\omega r}{3CR}]\hat{\mathbf{e}}_r \\
&+ \frac{1}{r}(2r\frac{|Q|\,\omega}{3CR}\sin\theta)\hat{\mathbf{e}}_\theta \\
&= [-2\cos\theta\frac{|Q|\,\omega}{3CR}]\hat{\mathbf{e}}_r + [\frac{2\,|Q|\,\omega}{3CR}\sin\theta]\hat{\mathbf{e}}_\theta
\end{aligned} \tag{3.2.4}$$

Then,

$$\mathbf{B}_{in} = \frac{-2\,|Q|\,\omega}{3CR}(\cos\theta\hat{\mathbf{e}}_r - \sin\theta\hat{\mathbf{e}}_\theta) \tag{3.2.5}$$

But we have,

$$\begin{aligned}
\hat{\mathbf{e}}_r &= \sin\theta\hat{\mathbf{j}} + \cos\theta\hat{\mathbf{k}} \\
\hat{\mathbf{e}}_\theta &= \cos\theta\hat{\mathbf{j}} - \sin\theta\hat{\mathbf{k}}
\end{aligned} \tag{3.2.6}$$

From(3.2.6)

$$\begin{aligned}
\cos\theta\hat{\mathbf{e}}_r - \sin\theta\hat{\mathbf{e}}_\theta &= \cos\theta(\sin\theta\hat{\mathbf{j}} + \cos\theta\hat{\mathbf{k}}) - \sin\theta(\cos\theta\hat{\mathbf{j}} - \sin\theta\hat{\mathbf{k}}) \\
&= \cos\theta\sin\theta\hat{\mathbf{j}} + \cos^2\theta\hat{\mathbf{k}} - \sin\theta\cos\theta\hat{\mathbf{j}} + \sin^2\theta\hat{\mathbf{k}} \\
&= \hat{\mathbf{k}}
\end{aligned}$$

Finally the magnetic field inside the crust will be

$$\mathbf{B}_{in} = \frac{-\,|Q|\,\omega}{3CR}\frac{2}{R}\hat{\mathbf{k}} \tag{3.2.7}$$

3.2.2 Magnetic field outside the crust

The magnetic field outside the crust can be calculated from (3.1.11) and (3.2.3) as

$$
\begin{aligned}
\mathbf{B}_{out} &= \nabla \times \mathbf{A}_{out} \\
&= \frac{1}{r sin\theta}[\frac{\partial}{\partial\theta}(\sin\theta A_{\phi,out})]\hat{\mathbf{e}}_r + [\frac{1}{r}\frac{\partial A_{\phi,out}}{\partial r}]\hat{e}_\theta \\
&= \frac{1}{r\sin\theta}[\frac{\partial}{\partial\theta}(-\sin\theta\frac{|Q|\,\omega}{3C}(\frac{R}{r})^2\sin\theta)]\hat{\mathbf{e}}_r \\
&+ [-\frac{1}{r}\frac{\partial}{\partial r}(-r\frac{|Q|\,\omega}{3C}(\frac{R}{r})^2\sin\theta)]\hat{\mathbf{e}}_\theta \\
&= \frac{1}{r\sin\theta}[-2\sin\theta\cos\theta\frac{|Q|\,\omega}{3C}(\frac{R}{r})^2]\hat{\mathbf{e}}_r \\
&+ [-\frac{1}{r}(\frac{|Q|\,\omega}{3C}(\frac{R}{r})^2\sin\theta)]\hat{\mathbf{e}}_\theta \\
&= [-2\cos\theta\frac{|Q|\,\omega}{3Cr}(\frac{R}{r})^2]\hat{\mathbf{e}}_r + [-\frac{|Q|\,\omega}{3Cr}(\frac{R}{r})^2\sin\theta]\hat{\mathbf{e}}_\theta
\end{aligned}
\tag{3.2.8}
$$

Then,

$$
\mathbf{B}_{out} = \frac{-\,|Q|\,\omega}{3C}(\frac{R^2}{r^3})(2\cos\theta\hat{\mathbf{e}}_r + \sin\theta\hat{\mathbf{e}}_\theta) \tag{3.2.9}
$$

By using (3.2.6) we get

$$
\begin{aligned}
2\cos\theta\hat{\mathbf{e}}_r + \sin\theta\hat{\mathbf{e}}_\theta &= 3\sin\theta\cos\theta\hat{\mathbf{j}} + (2\cos^2\theta - \sin^2\theta)\hat{\mathbf{k}} \\
&= 3\sin\theta\cos\theta\hat{\mathbf{j}} + 2\cos^2\theta\hat{\mathbf{k}} + \cos^2\theta\hat{\mathbf{k}} - \cos^2\theta\hat{\mathbf{k}} - \sin^2\theta\hat{\mathbf{k}} \\
&= 3\cos\theta(\sin\theta\hat{\mathbf{j}} + \cos\theta\hat{\mathbf{k}}) - (\sin^2\theta + \cos^2\theta)\hat{\mathbf{k}} \\
&= 3\cos\theta\hat{\mathbf{e}}_r - \hat{\mathbf{k}}
\end{aligned}
$$

Then, the magnetic field outside the sphere will be

$$
\mathbf{B}_{out} = \frac{-\,|Q|\,\omega}{3C}(\frac{R^2}{r^3})(3\cos\theta\hat{\mathbf{e}}_r - \hat{\mathbf{k}}) \tag{3.2.10}
$$

Which is dipolar

Chapter 4

Non dipolar field

In this chapter we will derive the non dipolar component of neutron star magnetic field resulting from the rotation of the NS, by using equation(2.1.1) and applying Post-Newtonian approximation.

The non dipolar field is derived by using an equation which is responsible for the non dipolar component, which can be written as

$$0 = D^0 \Gamma^0_{0i} n_0 + \mu n_0 g_{\beta i} F^{0\beta} \tag{4.0.1}$$

According to the definition of the affine connection[10]

$$\Gamma^\mu_{\mu\lambda} = \frac{1}{2} g^{\mu\rho} \frac{\partial g_{\rho\mu}}{\partial x^\lambda} \tag{4.0.2}$$

From this we have

$$\Gamma^0_{0i} = \frac{1}{2} g^{0\rho} \frac{\partial g_{\rho 0}}{\partial x^i} \tag{4.0.3}$$

which implies

$$\begin{aligned} \Gamma^0_{0i} &= \frac{1}{2} g^{00} \frac{\partial g_{00}}{\partial x^i} + \frac{1}{2} g^{0j} \frac{\partial g_{j0}}{\partial x^i} \\ &= \frac{1}{2} g^{00} \frac{\partial g_{00}}{\partial x^i} - \frac{1}{2} g_{0j} \frac{\partial g^{0j}}{\partial x^i} \end{aligned} \tag{4.0.4}$$

From (4.0.1) and (4.0.4) we can get

$$\frac{D^0}{2}(g^{00}\frac{\partial g_{00}}{\partial x^i} + g^{0j}\frac{\partial g_{j0}}{\partial x^i})n_0 + \mu n_0(g_{0i}F^{00} + g_{ki}F^{0k}) \quad = \quad 0 \qquad (4.0.5)$$

The electro magnetic field tensor is defined by

$$F^{\alpha\beta} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{pmatrix} \qquad (4.0.6)$$

Under Post-Newtonian approximation the metric tensors can be expanded as[10]

$$\begin{aligned} g_{00} &= -1 + g^2_{00} + ... \\ g_{ij} &= \delta_{ij} + g^2_{ij} + ... \\ g_{i0} &= g^3_{i0} + g^5_{i0} + ... \end{aligned} \qquad (4.0.7)$$

and

$$\begin{aligned} g^{00} &= -1 + g^{2oo} + ... \\ g^{ij} &= \delta_{ij} + g^{2ij} + ... \\ g_{i0} &= g^{3io} + g^{5io} + ... \end{aligned} \qquad (4.0.8)$$

By using equations (4.0.5) - (4.0.8) it is clear to write

$$\begin{aligned} \frac{D^0}{2}[(-1 + g^{2oo} + g^{4oo} + ...)\partial_i(-1 \quad &+ \quad g^{2oo} + ...) - (g^3_{0j} + g^5_{0j} \\ +...)\partial_i(g^{3oj} + ...)]n_o \quad &+ \quad \mu n_o[\delta_{ki} + g^2_{ki} + ...]F^{ok} = 0 \end{aligned} \qquad (4.0.9)$$

If we approximate the metric tensors to their first term we will get the following

$$\frac{D^0}{2}[(-1)\partial_i(-1) - g^3_{0j}\partial_i g^{30j}]n_o + \mu n_o[\delta_{ki}F^{ok}] = 0 \qquad (4.0.10)$$

which results

$$\frac{D^0}{2}[g^3_{0j}\partial_i g^{30j}]n_o + \mu n_o F^{oi} = 0 \tag{4.0.11}$$

But we have[10]

$$g^3_{0j} = g^{30j} = \zeta_j = \delta_{ij}\zeta_i \tag{4.0.12}$$

ζ_i is a vector potential which is defined by

$$\zeta_i = -4G\int \frac{T^{1io}(x',t)}{|x-x'|}d^3x' \tag{4.0.13}$$

where $T^{1io}(x',t)$ represents momentum density, then we can write

$$F^{oi} = \frac{D^0}{2\mu}\zeta_j\frac{\partial\zeta_j}{\partial x^i} \tag{4.0.14}$$

From the electromagnetic field tensor (4.0.6) we can see that F^{oi} represents the electric field E_i, then we can write E_i as (From here on, we change $D^0 \to D$)

$$E_i = \frac{3D}{2\mu}\zeta_i\frac{\partial\zeta_i}{\partial x^i} \tag{4.0.15}$$

4.1 Derivation of the charge density

From differential form of Gauss's law we have[9]

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_o} \tag{4.1.1}$$

where ρ and $\mathbf{E}$ represents the surface charge density and the electric field respectively. [4.0.15] and [4.1.1] results

$$\begin{aligned}\rho &= \epsilon_o\nabla\cdot\mathbf{E}\\ &= \epsilon_o\partial_i E_i\\ &= \epsilon_o\partial_i[\frac{3D}{2\mu}\zeta_i\frac{\partial\zeta_i}{\partial x^i}]\\ &= \frac{3\varepsilon_o D}{2\mu}[(\frac{\partial\zeta_i}{\partial x^i})^2 + \zeta_i\frac{\partial^2\zeta_i}{\partial x^{i^2}}]\end{aligned} \tag{4.1.2}$$

But

$$\partial^2 \zeta_i = \nabla^2 g^3_{i0} = 16\pi G T^{1io}[10] \tag{4.1.3}$$

and

$$\partial_i \zeta_i = \frac{-4\partial\phi}{\partial t} \tag{4.1.4}$$

Then by substituting (4.1.3) and (4.1.4) into (4.1.2), we find the charge density as

$$\rho = \frac{24\varepsilon_o D}{\mu}(\frac{\partial\phi}{\partial t})^2 + \frac{24\varepsilon_o D}{\mu}\pi G \zeta_i T^{1io} \tag{4.1.5}$$

For a system that is at rest and spherically symmetric which rotates with angular frequency $\omega(r)$, the momentum density is given by[10]

$$T^{1io}(x', t) = T^{0oo}(r')[\vec{\omega}(r') \times \mathbf{x}']_i \tag{4.1.6}$$

From (4.0.13) and (4.1.6) the vector potential ζ, for the field out side the sphere, may be written as [10]

$$\zeta_i = \frac{2G}{r^3}(\mathbf{x} \times \mathbf{J}') \tag{4.1.7}$$

where

$$\mathbf{J}' = \frac{8\pi}{3}\int \omega(r')T^{0oo}(r')r'^4 dr'$$

From (4.1.6) and (4.1.7), then follows

$$\begin{aligned}
\zeta_i T^{1i0} &= \frac{2G}{r^3}(\mathbf{x} \times \mathbf{J}')(\omega(\vec{r'}) \times \mathbf{x}')T^{0oo} \\
&= \frac{2G}{r^3}T^{0oo}[-J' R \sin\theta(-\sin\phi\hat{\mathbf{i}} + \cos\phi\hat{\mathbf{j}})\omega R \sin\theta'(-\sin\phi'\hat{\mathbf{i}} + \cos\phi'\hat{\mathbf{j}}) \\
&= -\frac{2G}{r^3}T^{0oo}J'\omega R^2 \sin\theta\sin\theta'(\sin\phi\sin\phi' + \cos\phi\cos\phi') \\
&= -\frac{2G}{r^3}T^{0oo}J'\omega R^2 \sin\theta\sin\theta'\cos(\phi - \phi') \\
\zeta_i T^{i0} &\approx -\frac{2G}{r^3}T^{0oo}J'\omega R^2 \sin\theta\sin\theta' \qquad (4.1.8)
\end{aligned}$$

The scalar potential $\phi(x)$ can be expanded as[10]

$$\phi(x) = -\frac{Gm^0}{r} - \frac{G\vec{x}\cdot d^0}{r^3} + ... \tag{4.1.9}$$

where

$$\begin{aligned} m^0 &= \int T^{00} d^3x' \\ d^0 &= -\int x' T^{00} d^3x' \end{aligned}$$

Approximating $\phi(x)$ to the first term we get

$$\phi(x) = -G\frac{m^0}{r} \tag{4.1.10}$$

From conservation of mass we have[10]

$$\frac{\partial m^0}{\partial t} = 0$$

Then finally we can write

$$\frac{\partial \phi(x)}{\partial t} = 0 \tag{4.1.11}$$

From (4.1.5), (4.1.8) and (4.1.11) we can find the charge density as

$$\begin{aligned} \rho &= -(\frac{48\pi\epsilon_0 DG^2 T^{0oo}}{\mu})(\frac{J'\omega R^2 \sin\theta \sin\theta'}{r^3}) \\ &= -\alpha' J'\omega \frac{R^2}{r^3}\sin\theta\sin\theta' \end{aligned} \tag{4.1.12}$$

where

$$\alpha' = \frac{48\pi\epsilon_0 DG^2 T^{00}}{\mu}$$

4.2 Derivation of Vector potential

We have to first find the current density which is given by

$$\begin{aligned}\mathbf{J} &= \rho\delta(r'-R)\mathbf{v} \\ &= \rho\delta(r'-R)\vec{\omega}\times\mathbf{r}' \\ &= \rho\delta(r'-R)\omega R\sin\theta'\hat{\mathbf{e}}_\phi \qquad (4.2.1)\end{aligned}$$

Substituting (4.1.12) into (4.2.1) gives:

$$\begin{aligned}\mathbf{J} &= \frac{-\alpha' J'\omega R^2}{r^3}\sin\theta\sin\theta'\delta(r'-R)\omega R\sin\theta'\,\hat{\mathbf{e}}_\phi \\ &= \frac{-\alpha' J'\omega^2 R^3}{r^3}\sin\theta\sin^2\theta'\delta(r'-R)\hat{\mathbf{e}}_\phi \qquad (4.2.2)\end{aligned}$$

The vector potential outside the crust can be calculated by using (4.2.2) and (3.1.2) as:

$$\begin{aligned}A_{\phi,out} &= \frac{-\mu_0}{4\Pi}\int\frac{-\alpha' J'\omega^2 R^3}{r^3}\sin\theta\sin^2\theta'\delta(r'-R)\frac{d^3x'}{|x-x'|} \\ &= \frac{-\mu_0\alpha' J'\omega^2 R^3}{4\Pi r^3}\int\sin\theta\sin^2\theta'\delta(r'-R)4\Pi\sum_{l,m}\frac{1}{2l+1}\frac{r_<^l}{r_>^{l+1}} \\ &\times\ Y^*_{l,m}(\theta',\phi')Y_{l,m}(\theta,\phi)r'^2dr'\sin\theta' d\theta' d\phi' \qquad (4.2.3)\end{aligned}$$

Due to the reason that I stated before, in section (3.1), I can choose the observation point in the XZ plane($\phi = 0$). And after Integration over the delta function of (4.2.3) the vector potential will have the form

$$\begin{aligned}A_{\phi,out} &= \frac{-\mu_0\alpha' J'\omega^2 R^5}{4\Pi r^3}\sin\theta\sum_{l,m}\frac{4\Pi}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y_{l,m}(\theta,\phi) \\ &\times\int\sin^2\theta' Y^*_{l,m}(\theta',\phi')\sin\theta' d\theta' d\phi' \qquad (4.2.4)\end{aligned}$$

where $r_< = R$ and $r_> = r$.

We have $\sin^2\theta' = Y_{2,2}(\theta', \phi')4\sqrt{\frac{2\Pi}{15}}$, for$\phi' = 0$. Then the integration part in (4.2.4)will be:

$$\begin{aligned}\int \sin^2\theta' Y^*_{l,m}(\theta', \phi') \sin\theta' d\theta' d\phi' &= 4\sqrt{\frac{2\Pi}{15}} \int Y_{2,2}(\theta', \phi') Y^*_{l,m}(\theta', \phi') \sin\theta' d\theta' d\phi' \\ &= 4\sqrt{\frac{2\Pi}{15}} \delta_{l,2}\delta_{m,2} \qquad (4.2.5)\end{aligned}$$

The integration have non-vanishing value only for $l = 2$ and $m = 2$, then the vector potential outside the sphere will be:

$$A_{\phi,out} = \frac{-\mu_0\alpha' J'\omega^2 R^5}{4\Pi r^3} \sin\theta \frac{4\Pi}{5} \frac{R^2}{r^3} Y_{2,2}(\theta, \phi) 4\sqrt{\frac{2\Pi}{15}} \qquad (4.2.6)$$

But we have $Y_{2,2}(\theta, \phi) = \frac{1}{4}\sqrt{\frac{15}{2\Pi}} \sin^2\theta$, for $\phi = 0$

Then we get the vector potential as:

$$\begin{aligned}A_{\phi,out} &= \frac{-\mu_0\alpha' J'\omega^2 R^7}{5r^6} \sin\theta \frac{1}{4}\sqrt{\frac{15}{2\Pi}} \sin^2\theta 4\sqrt{\frac{2\Pi}{15}} \\ &= \frac{-\mu_0\alpha' J'\omega^2 R^7}{5r^6} \sin^3\theta \qquad (4.2.7)\end{aligned}$$

Let

$$\alpha = \frac{\mu_0\alpha'}{5}$$

Finally the the vector potential outside the sphere can be written as

$$A_{\phi,out} = \frac{\alpha J'\omega^2 R^7}{r^6} \sin^3\theta \qquad (4.2.8)$$

4.3 Derivation of the magnetic field

The magnetic field interms of the vector potential can be written as :

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad (4.3.1)$$

Since we have only ϕ component of the vector potential, the magnetic field will be:

$$\begin{aligned}
\mathbf{B}_{out} &= \hat{\mathbf{e}}_r \frac{1}{r\sin\theta}[\frac{\partial}{\partial\theta}(\sin\theta A_\phi)] - \hat{\mathbf{e}}_\theta \frac{1}{r}[\frac{\partial}{\partial r}(rA_\phi)] \\
&= \hat{\mathbf{e}}_r \frac{1}{r\sin\theta}[\frac{\partial}{\partial\theta}(-\sin\theta\frac{\alpha\omega^2 R^7}{r^6}\sin^3\theta)] \\
&- \hat{\mathbf{e}}_\theta \frac{1}{r}[\frac{\partial}{\partial r}(-r\frac{\alpha\omega^2 R^7}{r^6})] \\
&= \hat{\mathbf{e}}_r \frac{1}{r\sin\theta}[-4\sin^3\theta\cos\theta\frac{\alpha\omega^2 R^7}{r^6}] \\
&- \hat{\mathbf{e}}_\theta \frac{1}{r}[5\frac{\alpha\omega^2 R^7}{r^6}\sin^3\theta] \\
&= \hat{\mathbf{e}}_r[-4\sin^2\theta\cos\theta\frac{\alpha\omega^2 R^7}{r^7}] \\
&- \hat{\mathbf{e}}_\theta[5\sin^3\theta\frac{\alpha\omega^2 R^7}{r^7}] \\
&= -\alpha\frac{\omega^2 R^7}{r^7}[4\sin^2\theta\cos\theta\hat{\mathbf{e}}_r + 5\sin^3\theta\hat{\mathbf{e}}_\theta] \qquad (4.3.2)
\end{aligned}$$

Substitute (3.2.6) into (4.3.2),then we can find $\mathbf{B}_{out}$ as:

$$\begin{aligned}
\mathbf{B}_{out} &= -\alpha\frac{\omega^2 R^7}{r^7}[(4\sin^3\theta\cos\theta\hat{\mathbf{j}} + 4\sin^2\theta\cos^2\theta\hat{\mathbf{k}}) \\
&+ (5\sin^3\theta\cos\theta\hat{\mathbf{j}} - 5\sin^4\theta\hat{\mathbf{k}})] \\
&= -\alpha\frac{\omega^2 R^7}{r^7}[9\sin^3\theta\cos\theta\hat{\mathbf{j}} + (4\sin^2\theta - 4\sin^4\theta - 5\sin^4\theta)\hat{\mathbf{k}}] \qquad (4.3.3)
\end{aligned}$$

Finally, the magnetic field outside the crust can be written as:

$$\mathbf{B}_{out} = -\alpha\frac{\omega^2 R^7}{r^7}[9\sin^3\theta\cos\theta\hat{\mathbf{j}} + (4\sin^2\theta - 9\sin^4\theta)\hat{\mathbf{k}}] \qquad (4.3.4)$$

Chapter 5

Discussion and conclusion

The spinning separated charges, which comes as a result of plasma diffusion process is a source for NS magnetic field. The accumulated negative charges on the crust, which result this field, does not have uniform density.

If we have a total charge density ρ_{total}, this may be represented by

$$\rho_{total} = \rho_o + \rho(\theta) \tag{5.0.1}$$

where ρ_0 is constant resulting in the dipolar component of the NS magnetic field, where as ρ is zenith angle dependent, which generates the non-dipolar part. The dipolar component is symmetric around the NS magnetic axis.

To see the difference between the dipolar and the non-dipolar component of the NS magnetic field, let us first discuss how the dipolar component varies with angle θ. At a point of the rotational axis , that means when $\theta = 0^0$, equation(3.2.10), shows that the magnetic field $\mathbf{B} \propto c\hat{k}$, at $\theta = 90^0$, the result shows that the magnetic field $\mathbf{B} \propto \frac{-c}{2}\hat{k}$, and at $\theta = 180^0$ the magnetic field $\mathbf{B} \propto c\hat{k}$. The other point is that the magnetic field for the dipolar case is inversely proportional to r^3 i.e $B \propto \frac{1}{r^3}$.

Equation (2.1.1) at equilibrium can be written in the following form

$$\mu n_0 g_{\beta i} F^{0\beta} = D^0 \partial_i n_0 - D^0 \Gamma^0_{0i} n_0 \tag{5.0.2}$$

From (5.0.2), the first term responsible for the electric field, from which we derive ρ_o, where as the additional term, which contains Γ^0_{0i} is responsible for the electric field from which we derive the charge density $\rho(\theta)$, and this charge is the source for the non-dipolar component of the NS magnetic field.

In contrast to, what we discussed for the dipolar field of a NS, in the non-dipolar case, when the angle $\theta = 0$, the magnetic field becomes zero, again when $\theta = 180^0$, the field is zero. But ,when we vary the angle between $\theta = 0$ and $\theta = 180^0$, the variation of the dipolar and the non dipolar components of NS is different . In addition to this, in non-dipolar case, the magnetic field is inversely proportional to r^7, $\mathbf{B} \propto \frac{1}{r^7}$, this shows that the non-dipolar components of the NS magnetic field decreases enormously with the distance r as compared to the dipolar component. These shows that the neutron star magnetic field has completely different structure from what we observe by approximating the neutron star magnetic field as dipolar. Specially near the surface of the NS the non-dipolar component has greater contribution on the magnetic structure of the neutron star.

For the past years different authors ignore the non-dipolar field and they approximated the NS magnetic field as dipolar. But as I mentioned before, there are different evidences that shows the presence of this field, which is different from the dipolar component, our result also shows this.

Finally, modelling of the NS magnetic field as dipolar, is facing problems in explaining different phenomenon like in case of pulsars. To have more accurate explanation and interpretation about the NS we have to include the non-dipolar components of the NS magnetic field, which have great influence on the overall structure of the NS magnetic field.

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Declaration

I hereby declare that this thesis is my original work and has not been presented for a degree in any other university. All sources of material used for the thesis have been duly acknowledged.

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