



**ADDIS ABABA UNIVERSITY**  
**ADDIS ABABA INSTITUTE OF TECHNOLOGY**  
**SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING**

**EVALUATION OF GLOBAL GRAVITY FIELD MODELS**  
**USING GPS AND LEVELING DATA OVER ETHIOPIA**

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## DECLARATION

I, the undersigned, declare that the thesis titled “**Evaluation of Global Gravity Field Models using GPS and Leveling Data over Ethiopia**” carried out under the supervision of **Dr. Tulu Besha Bedada** is my own work and has not been presented for a degree in any other university and that all sources of material used for the thesis have been appropriately acknowledged.

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## LIST OF ACRONYMS

AUSPOS	Australian Online GPS Processing Service
APREF	Asia-Pacific Reference Frame
ASPRS	American Society for Photogrammetry and Remote Sensing
CI	Contour Interval
CHAMP	Challenging Mini-Satellite Payload
DOP	Dilution of Precision
EGM2008	Earth Gravity Model 2008
EIGEN-6C4	European Improved Gravity Model of the Earth by New Techniques
ERS	European Remote Sensing Satellite
FGCS	Federal Geodetic Control Subcommittee
GECO	Global Gravity Model by Combining GOCE and EGM2008 data
GGM	Global Geopotential Model
GNSS	Global Navigation Satellite System
GOCE	Gravity Field and Steady-State Ocean Circulation Explorer
GRACE	Gravity Recovery and Climate Experiment
GEOSAT	Geodetic Satellite
GPS	Global Positioning System
ICGEM	International Centre for Global Earth Models
IGS	International GNSS service
ITRF	International Terrestrial Reference Frame
LAGEOS	Laser Geodynamic Satellite
NGA	Geospatial Intelligence Agency
OPUS	Online Positioning User Service
PPS	Precise Positioning Services
RMSE	Root Mean Square Error
RINEX	Receiver Independent Exchange Format
SCOUT	Scripps Co-ordinate Update Tool
SPS	Standard Positioning Services
XGM2019e-2159	Experimental Gravity Model

## LIST OF SYMBOLS

$C$	Geopotential Number
$g$	Gravity of the Earth
$GM$	Geocentric Gravitational Constant
$R$	Radius of the Earth
$W_0$	Geoid Potential
$W$	Gravity Potential of the Earth
$U$	Normal gravity potential
$T$	Disturbing Potential
$\Phi$	Centrifugal Potential
$A$	Matrix of Coefficients for the Unknowns
$L$	Matrix of Observations
$X$	Matrix of Unknowns
$r$	Geocentric Radius
$\varphi$	Geodetic Latitude
$\lambda$	Geodetic Longitude
$\theta$	Geodetic Colatitude
$\gamma$	Normal Gravity
$\gamma_{45^\circ}$	Normal Gravity at $45^\circ$ Latitude
$\pi$	pi
$h$	Ellipsoidal Height
$H_A$	Orthometric Height of Station A
$H^{\text{din}}$	Dynamic Height
$H^{\text{N}}$	Normal Height
$H^{\text{N-O}}$	Normal - Orthometric Height
$H_C(A)$	Potential Height of Station A
$\Delta_g$	Gravity Anomaly
$\delta_g$	Gravity Disturbance
$g_P^{\text{FA}}$	Free-Air Reduction

$St(\psi_{PQ})$	Stokes Function
$\bar{C}_{n,m} \bar{S}_{n,m}$	Fully Normalized Dimensionless Spherical Harmonic Coefficients
$\bar{P}_{n,m}$	Fully Normalized Legendre's Function
$t_{nm}$	The error of Spherical Harmonic Coefficients
n,m	Degree and Order of Spherical Harmonic Coefficients
N	Geoid Undulation
$N_{GGM}$	Geoid Undulation Computed from GGMs Model
$N_{h-H}$	Geoid Undulation Computed from GPS and Leveling Data
$dl_{Model}$	Height Differences Computed from GGMs Model
$dl_{Spirit\ leveling}$	Height Differences Computed from Spirit Leveling
$\Delta N$	Geoid Undulation Difference Between $N_{GGM}$ and $N_{h-H}$
$\Delta H$	Height Differences Between $dl_{Model}$ and $dl_{Spirit\ leveling}$

## ABSTRACT

This study presents the evaluation of the most recently released global gravity field models. The evaluation method involves the comparison of the geoid undulations and relative differences in heights computed from the model with those computed from the GPS and leveling data. The assumption made here is that the value obtained from the GPS and leveling data consider as a standard to which the value obtained from the model. The results of the geoid undulation differences between the GPS and leveling data, and the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159 has a standard deviation of 0.5176m, 0.5024m, 0.5102m, and 0.5243m, with the root mean square error of 1.176m, 1.1871m, 1.1772 m and 2.3058 m respectively. Similarly, the results of the differences between spirits leveling based on relative differences in height and models based on relative differences in height has a standard deviation of 0.172m, 0.172m, 0.1718m, and 0.1722m, with the root mean square error of 0.2617m, 0.2617m, 0.2615m and 0.2619m respectively. The error of the geoid undulation generated by the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159 are reach up to 9.04 cm, 3.74 cm, 4.66 cm, and 3.58 cm respectively. The global gravity field models are strongly correlated with the GPS and leveling data. The variances of each model with respect to each other are significantly the same at 95% confidence interval. Based on the attained results, it is concluded that the global gravity field models can be used to prepare a topographical map of 4 meter and 1 meter contour interval, absolutely and relatively.

**Keywords:** Global Gravity Field Models, EGM2008, EIGEN-6C4, GECO, XGM2019e-2159, GPS, Spirit Leveling, Geoid Undulations and Relative Differences in Height.

## CHAPTER 1 INTRODUCTION

### 1.1 Background

The orthometric height is the vertical distance of a point above or below a given reference level called a datum. The most commonly used datum is the mean sea level. Mean sea level is a part of a special equipotential surface in the Earth's gravity field. In particular, orthometric heights are very important, practical, and spontaneously used for many engineering works and construction projects. It also provides data for determining the shape of the Earth's surface or used to record the information necessary to draw a topographic map. There are a lot of geodetic methods for determining orthometric heights such as spirit leveling, trigonometric leveling, and GPS and Leveling data (Ceylan et al., 2005). Currently, Ethiopia uses spirit leveling for the determination of the orthometric height and it uses the Blue Nile Geodetic benchmark point as the vertical datum. Blue Nile Geodetic benchmark points are permanent geodetic monuments, which were accomplished under the authority of agreement between the imperial Ethiopian government and the government of the United States of American Surveying associations during the Ethiopian Geodetic Control Project. The Ethiopian Geodetic Control Project in operation from 1957 to 1961, completed basic horizontal and vertical control networks, the area comprising the Ethiopian watershed of the Blue Nile River. The surveys establishing the networks were conducted with first-order methods and procedures by a combined organization of Ethiopian and United States personnel, financed by a joint imperial Ethiopian Government and the Government of the United States. This agreement established the purpose, authority, and scope for the conduct of the geodetic control surveys of the Blue Nile River Basin of Ethiopia (Ethiopia Blue Nile Geodetic Control Project, 1957 - 1961).

The global gravity field model is the modern method of determining the orthometric height system from the combination of the Global Positioning System technology. Three-dimensional coordinates can be obtained by GPS in the geodetic latitude( $\phi$ ), geodetic longitude( $\lambda$ ), and ellipsoidal heights( $h$ ) according to the selected reference ellipsoid. The ellipsoidal heights obtained by GPS observations are not directly used for practical applications because they have no physical meaning. So, the ellipsoidal height has to be transformed to orthometric height ( $H$ )

usually used in surveying and mapping applications. The orthometric height ( $H$ ), which is measured along the plumb lines of the Earth's gravity field and thus follows the curved paths, can be obtained by subtracting the geoid height ( $N$ ) from the GNSS ellipsoidal height ( $h$ ) which is useful in most geodetic activities, surveying, and mapping applications. In the same way, it's possible to determine the orthometric height system from the relative differences in height. A better approach to determining geoid height is the use of global gravity field models and GPS technology (Kilicoglu et al., 2008, Odera and Fukuda, 2017, Ssengendo et al., 2011, Uzodinma et al., 2014).

Global gravity field models are the representations of the gravity field of the Earth in terms of spherical harmonic coefficients. Based on the coefficient of each degree and order, various gravimetric quantities that depend on the Earth's gravitational potential can be derived. Global gravity field models can be classified into three groups: satellite-only, combined, and tailored gravity field models (Barthelmes, 2014, Erol and Erol, 2012). A large number of global gravity field models have been released and used in geoid modeling. For this model to be used for geodetic activities anywhere on the Earth there is a need to quantify its accuracy. The evaluation of the global gravity field models is based on comparisons with other external data. These external data sets should be independent of the estimation procedures that were used in the development of the model. For this evaluation, the external data mainly include GPS and leveling observations, sea surface heights from altimetry band tide gauge, sea surface topography, airborne and surface gravity data, and deflections of the vertical are to be used in appropriate evaluation procedures (Ceylan et al., 2005, Ssengendo et al., 2011).

Many studies evaluated the global gravity field model in Ethiopia using GPS and leveling data. Studies in Addis Ababa and the western part of Ethiopia compared EGM2008 with GPS and leveling observations, and the result indicated that the mismatches of orthometric heights and mismatches of geoids are ~2cm and ~2.6cm respectively (Birbiraw, 2013). Similar studies have been done on the validation of the EGM2008 based orthometric height differences with height differences determined from spirit leveling around Debre Birhan has a standard deviation of 2.41cm with a mean of -3.14cm (Worku, 2013). ( Bedada, 2010) compared leveling heights with the EGM2008 derived geopotential heights in Ethiopia and obtained

an improved standard deviation from 5.3 cm with EGM2008 to 4.6 cm and 3.9 cm when airborne gravity, and airborne gravity plus topographic models are included, respectively.

The main purpose of this thesis is to evaluate the accuracy of the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159 on the existing Blue Nile Geodetic benchmark points and on the newly established benchmark points over Ethiopia. The evaluation method involves the comparison of the geoid undulations and relative differences in heights computed from the global gravity field models with those computed from the GPS and leveling data. The assumption made here is that the value obtained from the GPS and leveling consider as a standard to which the value obtained from the model. The applicability and accuracy assessment of the global gravity field models for height conversion is not yet well practiced over Ethiopia. Therefore, it is very important to define and evaluate the accuracy of the global geopotential models.

## 1.2 Description of the Study Area

The study area is Ethiopia, located in the North-Eastern part of the African continent, it's lies between the Equator and Tropic of Cancer, between the 3<sup>0</sup> N and 15<sup>0</sup> N Latitude or 33<sup>0</sup> E and 48<sup>0</sup> E Longitude. This study uses both primary and secondary data. The Primary data were collected from actual field survey measurements using surveying instruments. These Primary data points include DES-42, DES-12, DES-320, DES-330, DES-36, and DES-27 (Fig 1.1).

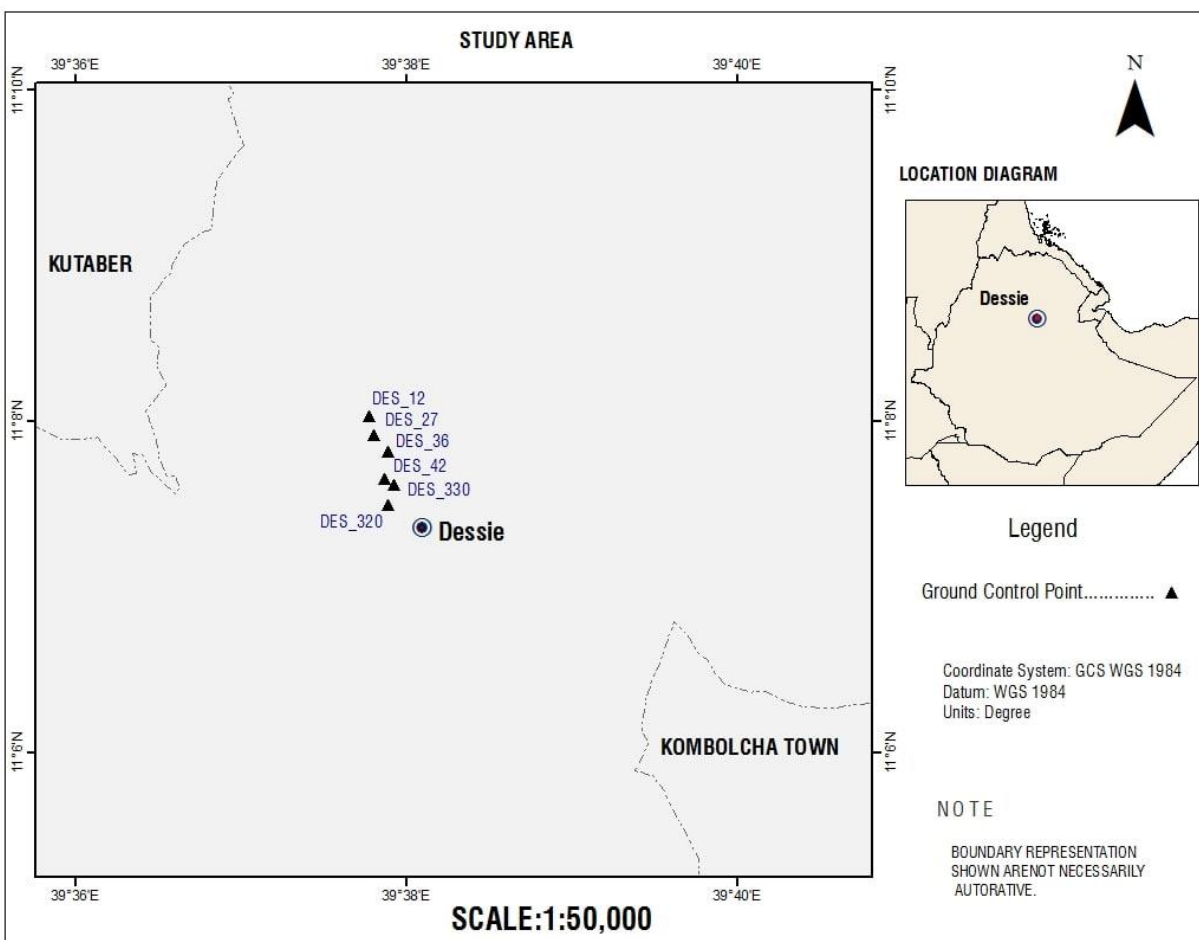


Figure 1. 1 Primary data points.

The secondary data points are those not collected under this study but collected previously and plotted in Fig (1.2). These secondary data points include ASOSA, GONDER , GEWANE , D/BERHAN , DUKEM , BALE ROBE , JIMMA , BEDELLE , GORE , TIE , TULU AMARA and WOLONKOMI (Ethiopia Blue Nile Geodetic Control Project, 1957 – 1961, Abubeker and Workaferahu, 2009). The remaining secondary data points include BMDZ1, BMJ1a, BMJ2a , BMJ3a , BMJ4a , BMJ5a , BMAaa and BMCaa (Bedada, 2010).

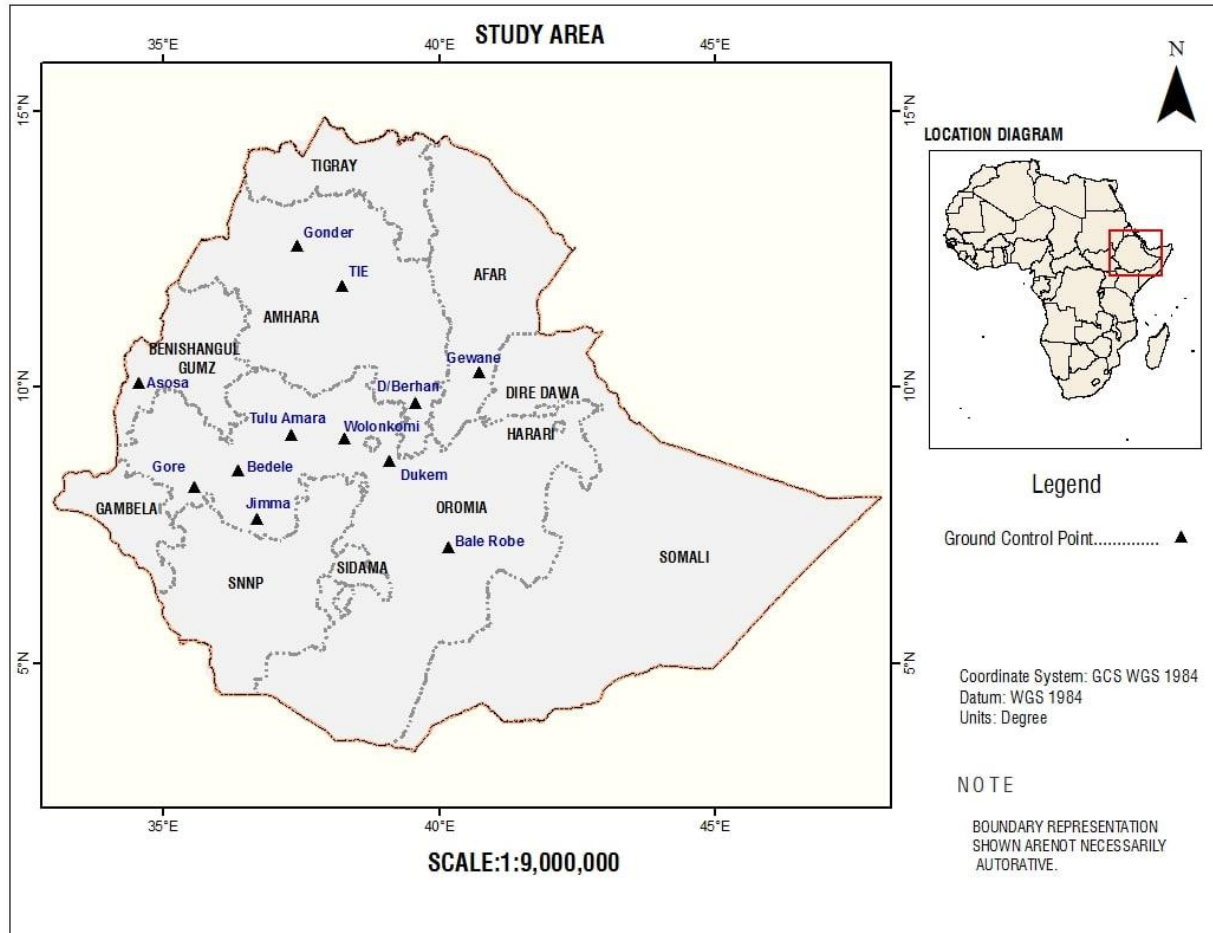


Figure 1. 2 Secondary data points.

### 1.3 Statement of the Problem

In many surveying, geophysics, and engineering applications orthometric heights are required. In particular, orthometric height is needed for the design of highways, railways, canals, for locating the gradient lines, and used to determine the catchments area, the volume of the reservoir. It is the vertical distance of a point above or below a given reference level, called a datum. The most commonly used datum is the mean sea level. The most commonly used method for the determination of the orthometric height over Ethiopia is spirit leveling and it is needed the physical connecting the particular point to a given benchmark that refers to mean sea level surface (Ceylan et al., 2005, Heiskanen and Moritz, 1967).

Currently, Ethiopia uses Blue Nile Geodetic benchmark point as the vertical datum. These Blue Nile Geodetic benchmark points are connected to tide gauge stations located at the port of Alexandria, Egypt. The Ethiopian geodetic control project completed approximately 120,000 square miles of west-central Ethiopia and it cannot cover the whole of Ethiopia (Ethiopia Blue Nile Geodetic Control Project, 1957 - 1961). Now, Blue Nile Geodetic benchmark points are mostly destroyed due to man-made factors and cannot provide the required service for determining the orthometric height (Ethiopian Geospatial Information Institute). The field procedures of spirit leveling are tedious, time-consuming, and labor-intensive over long distances and it's prone to systematic, random, and human errors. In some areas, it is impossible to apply the spirit leveling due to weather and terrain conditions. In general, it's uneconomical to apply and becomes less accurate as the leveling distance becomes longer. Because of those problems, it cannot provide the required service for determining the orthometric height (Abubeker and Workaferahu, 2009, Birbiraw, 2013, Fajemirokun, 1980, Featherstone et al., 1998).

Therefore, those problems initiated to know a modern method of determining the orthometric height system from global gravity field model and Global Positioning System technology. The application of the modern method of height system is not yet well practiced over Ethiopia. The orthometric heights obtained using this method can only be trusted if the global gravity field models have been assessed in the study area and proved to give high accuracy geoid undulations and relative differences in height.

## **1.4 Objectives**

### **1.4.1 General Objective**

The general objective of this study is to evaluate the accuracy of the global gravity field models by using GPS and leveling data over Ethiopia.

### **1.4.2 Specific Objectives**

- ✓ To evaluate the accuracy of the global gravity field models based on geoid undulation.
- ✓ To evaluate the accuracy of the global gravity field models based on the relative height difference.

## **1.5 Significance of the Research**

The significance of this research helps for many surveying, geophysics, and engineering applications to determine the modern methods of determining the orthometric height system from the combination of the global gravity field models and Global Positioning System technology using the GRAVSOFTE program. A GRAVSOFTE program is the python graphical interface that calculates the geoid undulations at a specified location on the Earth using global gravity field models and Global Positioning System technology. From Global Positioning System technology it's possible to obtain ellipsoidal heights with good accuracy. Using the combination of the geoid undulations and ellipsoidal heights or from the relative differences in height the surveyor, geophysics, and engineers can determine orthometric heights which are useful in most geodetic activities, surveying, and mapping applications.

## **1.6 Thesis Outline**

This thesis is organized in to five chapters. The first chapter is an introduction which includes background, description of the study area , statement of the problem, objectives and significance of the thesis are discussed.

Chapter two describes in detail the literature review on the gravity field of the Earth, anomalous quantities of the gravity field, and global gravity field models. This chapter discusses the height systems, types of height systems such as geopotential height, dynamic heights, normal heights,

ellipsoidal height, and orthometric heights. It also describes the methods of determining orthometric heights.

Chapter three explains the data acquisition and processing that provide necessary information about the data sources and GPS data processing. This chapter discusses the general frame work of data processing technique and methodology.

Chapter four describes the results and discussions. The results of geoid undulation, results of the relative height difference, and results of error of the geoid undulation generated by GGMs are given in this chapter.

Finally, Chapter five is deals about the conclusion and recommendations.

## CHAPTER 2 LITERATURE REVIEW

### 2.1 Gravity Field of the Earth

The gravity field of the Earth is the resultant of gravitational force and the centrifugal force of the Earth's rotation. As a vector, it has magnitude and direction. The magnitude  $g$  is called gravity and the direction of the gravity vector is the direction of the plumb line or the vertical. The gravity field is constant defines a closed surface, are called equipotential surface or level surfaces of the Earth's geopotential. The equipotential surface of the Earth which coincides with the mean sea level is called the geoid. The geoid is a representation of the Earth's figure and it is used as the reference surface for height measurement (Bjerhammar, 1975, Heiskanen and Moritz, 1967, Torge and Müller, 2012).

$$\begin{aligned}
 W(r, \varphi, \lambda) = & \frac{GM}{r} \sum_{n=n_{\min}}^{n_{\max}} \left(\frac{R}{r}\right)^n \sum_{m=0}^n (\bar{C}_{n,m} \cos m\lambda + \bar{S}_{n,m} \sin m\lambda) \bar{p}_{n,m}(\sin\varphi) \\
 & + \frac{1}{2} \omega^2 r^2 \cos^2 \varphi
 \end{aligned} \tag{2.1}$$

The normal field is an ellipsoidal approximation to the real gravity field. The gravity and the potential field that are consistent with such an ellipsoid are called normal gravity and normal potential. The normal gravity is the negative of the radial derivative of the normal potential. The normal potential is derived as the normal gravitational potential plus the centrifugal potential (Bjerhammar, 1975, Heiskanen and Moritz, 1967, Sanso, 1986).

$$\begin{aligned}
 U(r, \theta, \lambda) = & \frac{GM}{R} \sum_{n=0}^{\infty} \left(\frac{R}{r}\right)^{n+1} \sum_{m=0}^n (\bar{C}_{n,m} \cos m\lambda + \bar{S}_{n,m} \sin m\lambda) \bar{p}_{n,m}(\cos\theta) \\
 & + \frac{1}{2} \omega^2 r^2 \sin^2 \theta
 \end{aligned} \tag{2.2}$$

Where  $GM$  is the geocentric gravitational constant,  $r$  is the spherical radius,  $\varphi$  is the spherical latitude,  $\theta$  is the polar distance,  $\lambda$  is the spherical longitude,  $R$  is the equatorial radius of the

mean earth's ellipsoid,  $\bar{p}_{n,m}$  denotes the associated fully normalized Legendre functions of the first kind of degree  $n$  and order  $m$ ,  $\bar{C}_{n,m}$  and  $\bar{S}_{n,m}$  are fully normalized spherical harmonic coefficients of degree  $n$  and order  $m$  related to global geopotential model,  $n_{\max}$  and  $n_{\min}$  are the maximum and minimum degree of the spherical harmonic expansion, respectively,  $n$  and  $m$  are the spherical harmonic degree and order, respectively and  $\omega$  is the angular velocity of the Earth.

## 2.1.1 Anomalous Quantities of the Gravity Field

### 2.1.1.1 Disturbing Potential

The disturbing potential is the difference between the gravity potential and the normal potential. The disturbing potential field of the Earth ( $T$ ) satisfies the Laplace equation if there are no masses above the geoid (Heiskanen and Moritz, 1967, Torge and Müller, 2012).

$$T(r, \varphi, \lambda) = \frac{GM}{r} \sum_{n=n_{\min}}^{n_{\max}} \left(\frac{R}{r}\right)^n \sum_{m=0}^n (\Delta\bar{C}_{n,m} \cos m\lambda + \Delta\bar{S}_{n,m} \sin m\lambda) \bar{p}_{n,m}(\sin\varphi) \quad (2.3)$$

Where  $T$  is the disturbing potential,  $\Delta\bar{C}_{n,m}$  is the difference between the actual coefficients and those implied by the reference equipotential ellipsoid, and  $\Delta\bar{S}_{n,m}$  is equals to  $\bar{S}_{n,m}$ .

### 2.1.1.2 Gravity Disturbances

The difference between the measured gravity at a point,  $P$ , and the normal gravity at that same point,  $p$ , is called the gravity disturbance. As a vector, it has magnitude and direction. The difference in magnitude is the gravity disturbance and the difference in direction is the deflection of the vertical. The gravity disturbance ( $\delta_g$ ) can be expressed by the following equation (Caputo, 2016, Vermeer, 2018).

$$\delta_g = - \left( \frac{\partial W}{\partial H} \quad - \frac{\partial U}{\partial h} \right) \quad (2.4)$$

Where differentiation takes place along the plumb line for  $W$ , and along the ellipsoidal normal for  $U$ .

### 2.1.1.3 Gravity Anomalies

A gravity anomaly is a difference between the measured gravity at P and the normal gravity on the geoid Q. As a vector, it has magnitude and direction. The difference in magnitude is the gravity anomaly and the difference in direction is the deflection of the vertical. By using Stokes formula, the gravity anomaly ( $\Delta_g$ ) can be expressed by the following equation (Caputo, 2016, Vermeer, 2018).

$$\Delta_g = -\frac{\partial T}{\partial r} - \frac{2}{r}T \quad (2.5)$$

Where T is the disturbing potential, r is the spherical radius.

### 2.1.1.4 Geoid Undulation

The linear model is the linear observation model for the anomalous geopotential number, azimuth, zenith angle, latitude, longitude, and gravity. The actual application of the linear model is for the purpose of geoid computation. The so-called Stokes approach must be followed. By using Bruns's equation, the Stokes integral used for the computation of geoid undulation can be written as equation (2.6) and to reduce truncation errors must apply a remove-restore method (Denker, 2013, Fan, 2007, Vanicek and Krakiwsky, 2015).

$$N_P = \frac{R}{4\pi\gamma} \int_{\lambda=0}^{2\pi} \int_{\theta=0}^{\pi} St(\psi_{PQ}) \Delta_{gQ} \sin\theta \, d\theta \, d\lambda \quad (2.6)$$

Where: R is the radius of the Earth,  $\gamma$  is the normal gravity at the surface of the reference ellipsoid,  $St(\psi_{PQ})$  is the Stokes' function and  $\Delta_{gQ}$  is the gravity anomalies.

## 2.2 Global Gravity Field Models

Global gravity field models are the representations of the gravity field of the Earth in terms of spherical harmonic coefficients. It is a mathematical model which describes the gravity field of the Earth in three-dimensional space. Based on the coefficient of each degree and order, various gravimetric quantities that depend on the Earth's gravitational potential can be derived. The determination of the global gravity field model is one of the main tasks of geodesy (Barthelmes, 2014, Erol and Erol, 2012).

### 2.2.1 Types of Global Gravity Field Models

Global gravity field models can be classified into three groups: satellite-only, combined, and tailored gravity field models.

#### 2.2.1.1 *Satellite-Only GGMs*

The satellite-only GGMs are derived from the tracking and analysis of the orbits of satellite-based gravity observations (CHAMP and GRACE), or combined with other satellite missions such as Laser Geodynamic Satellite (LAGEOS), Geodetic Satellite (GEOSAT), and European Remote Sensing Satellite (ERS). Satellite only GGMs are the low-resolution models. The higher degree and order coefficients are heavily contaminated by noise and they exhibit a lack of power above degree and order 30. These models were limited in precision because of a weak signal and uncertainties in perturbations modeling (Sjöberg et al., 2015, Sulaiman et al., 2016).

#### 2.2.1.2 *Combined GGMs*

Combined GGMs are derived from the combination of satellite data, terrestrial gravity data (Land and Marine), satellite altimetry data, airborne gravimeter, topography, and bathymetry. This generally leads to an increase in the spectral resolution of the global gravity field models. The precision of the combined model depends on the bias in older satellite only GGMs, distribution, and quality of terrestrial data used (Krynski and Lyszkowicz, 2005, Mohamed, 2018).

#### 2.2.1.3 *Tailored GGMs*

The tailored GGMs are derived from an adjusted and extended of either satellite-only or combined models to higher degrees by using previously used or unused higher resolution gravity

data. This is usually obtained by adding corrections to the gravitational coefficients derived using the integral formula. Tailored GGMs should be applied only over the area where the tailoring was applied (Abdalla, 2009, Botai and Combrinck, 2012).

### **2.2.2 Experience of Gravity Field Models**

The evaluation of the accuracy of global gravity field models can be done by analyzing the differences between synthesized values and observation data. The accuracy of the EGM2008 model over the area of Poland is very accurate overall existing geopotential models (Łyszkowicz, 2009). Similar studies have been done to predict the local data in Kazakhstan and West Siberia and the result showed that only 66% of local data is compatible with global gravity field models (Karpik et al., 2016). (Foroughi et al., 2017) did the same test in Iran and show that the difference values reach up to ~2 m for geoid undulations. Similar studies have been evaluated EGM2008 in Turkey and the result of the evaluation indicated that the mean value and standard deviation of the differences are found to be -88.8 cm and 24.2 cm for GPS and leveling height anomalies (Kilicoglu et al., 2008). The evaluation of the global geopotential model over the area of Java Island in Indonesia shows that the mean difference value of 0.99 m and 0.701 m , absolutely and relatively (Heliani, 2016). Based on the absolute and relative methods undertaken the EGM2008 is superior to earlier EGM96 over the area of Greece (Gikas et al., 2013).

EGM2008 is superior to earlier global geopotential models over the northern part of Egypt and it provides their orthometric heights without any additional costs (Dawod et al., 2009). EGM2008 is consistently superior to other tested geopotential models in the Algerian region (Benahmed Daho, 2009). A similar study has been done on the validation of the EGM2008 model over the area of Nigeria and the result shows that the two sets of heights differ by a standard error of  $\pm 1.019$  m (Uzodinma et al., 2014).

A similar study has been done over Ethiopia. The agreement between the EGM2008 based orthometric height differences with height differences determined from spirit leveling around Debre Birhan has a standard deviation of 2.41cm with a mean of -3.14cm (Worku, 2013). The result obtained in Addis Ababa and the western part of Ethiopia indicated that the mismatches of orthometric heights and mismatches of geoids are ~2cm and ~2.6cm respectively (Birbiraw, 2013).

### 2.2.3 Global Gravity Field Models Considered in the Study

The global gravity field models considered in this study includes EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159.

**EGM2008:** - The global gravity field model developed by the US National Geospatial-Intelligence Agency (NGA) up to degree and order 2190. It was developed in 2008 based on altimetry data, ground data, the satellite model GOCO06s data, and GRACE-based satellite tracking data (Pavlis et al., 2012).

**EIGEN-6C4:-** A model released in 2014 that inferred from the combination of GRACE, LAGEOS, and GOCE data along with the altimetry and a global surface gravity anomaly grid data. The global gravity field model is up to degree and order 2190, developed by both the French CNES research center and Germany GFZ research center (Förste et al., 2014).

**GECO:** - A global gravity model utilizes the GOCE satellite-only global model and is used to improve the accuracy of the EGM2008 model in low and medium frequencies. It was developed, in 2015, up to degree and order 2190 (Gilardoni et al., 2016).

**XGM2019e-2159:** - The global gravity field model represented by spheroidal harmonics of up to degree and order 2190. It was developed in 2019 based on the satellite model GOCO06s data, altimetry data, ground data, and topography data (Zingerle et al., 2020).

**Table 2. 1 Main characteristics of global gravity field models used in this research**

No	Model	Time of releasing	Degree & order	Data	Reference
1	EGM2008	2008	2190	A, G, S(Grace)	(Pavlis et al., 2012)
2	EIGEN-6C4	2014	2190	A, G, S(Goce), S(Grace), S(Lageos)	(Förste et al., 2014)
3	GECO	2015	2190	EGM2008, S(Goce)	(Gilardoni et al., 2016)
4	XGM2019e-2159	2019	2190	A, G, S(GOCO06s), T	(Zingerle et al., 2020)
Where: S=Satellite, G = Ground, A =Altimetry Data and T = Topography Data.					

## 2.3 Height Systems

A height system is a one-dimensional coordinate system used to define the distance of a point from a given reference surface along a well-defined path. To determine the heights of Earth points a variety of surfaces can be taken as the reference. Of these, the most important reference surface is the geoid. The height of a point can be defined in many different ways; each way gives a different height coordinate for the same point. So, the definition and purpose of the term 'height' need great care. The largest influence for measuring the height of the point is the choice of the datum surface, but it's well known that the path that the distance (height) is measured over is also a significant contributing factor (Featherstone and Kuhn, 2010).

### 2.3.1 Types of Height Systems

Essentially, there are several types of height systems have been introduced. Different height systems were examined as follows. Such as geopotential height, dynamic heights, normal heights, ellipsoidal height, and orthometric heights (Fan, 2007).

#### 2.3.1.1 Geopotential Height

Geopotential height or geopotential number is defined as the potential difference between a surface point's potential and the geoid potential (*i. e.*,  $C = W_0 - W_A$ ). The geopotential height is equal in the same equipotential surface and measured in  $\text{kgal}\cdot\text{m}$ . So it has no distance dimension, it is the natural criterion for heights. Geopotential numbers govern fluid flow, thus forming the logical basis for physically meaningful and conceptually sensible heights. (Yilmaz, 2008).

#### 2.3.1.2 Dynamic Height

The dynamic height system is most closely related to the system of geopotential numbers and proposed by (Helmert, 1884). Dynamic height is the ratio of the geopotential number to the constant normal gravity value at  $45^\circ$  latitude. The constant normal gravity value at  $45^\circ$  latitude has been taken as the global value; the value is  $\gamma_{45^\circ} = 980.6294\text{gal}$  for international reference ellipsoids. The dynamic height system is not always preferable because it has no geometrical meaning, being a purely physical quantity (Jekeli, 2000, Hofmann-Wellenhof and Moritz, 2006).

### 2.3.1.3 Normal Height

In 1945 Molodensky introduced the concept of the normal height system to solve the problem of determining the mean gravity ( $g_A$ ) between benchmark A and the geoid along the plumb line, which supposed that the Earth's gravity field was normal, meaning the actual gravity potential equals normal gravity potential. In other words, we assume that  $W = U, g = \gamma, T = 0$ . The orthometric heights which correspond to this approximation are called normal heights. The theoretical replacement of the Earth's surface by the telluroid and the use of a reference ellipsoid with the associated gravity field. The telluroid is not an equipotential surface (Vaniček and Krakiwsky, 1986, Torge and Müller, 2012, Jekeli, 2000).

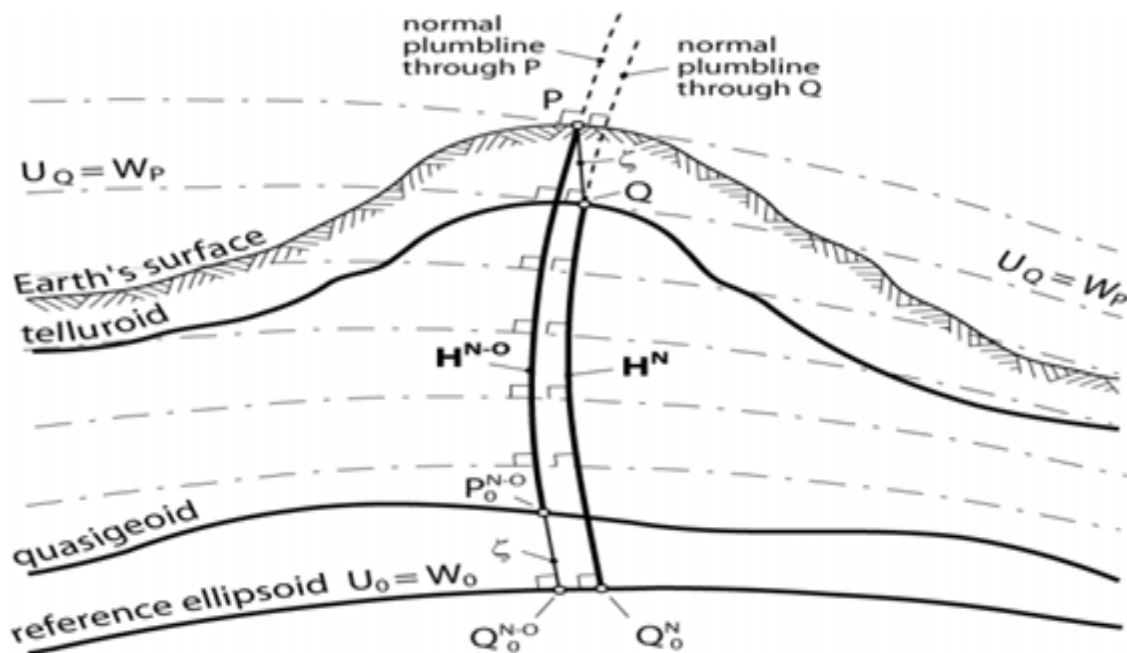


Figure 2. 1 The normal height  $H^N$ , The normal-orthometric height  $H^{N-O}$ , The quasigeoid height or height anomaly  $\zeta$ .

### 2.3.1.4 Ellipsoidal Height

The ellipsoid height is the linear distance measured along the ellipsoidal normal from the geometrical surface of a reference ellipsoid to the point of interest. The ellipsoidal height system is not directly related to gravity. Sometimes the reference ellipsoid is defined to generate its gravity field. Ellipsoidal heights can be derived from the geocentric cartesian coordinate system provided by GNSS observations. They are measured based on the mathematical shape of the

Earth and used for positioning purposes. They fail to determine what is downhill or uphill in the context of the energy flow of materials. Only geopotential heights can map the equipotential surface of the Earth (Badejo et al., 2016, Featherstone and Kuhn, 2006).

Ellipsoidal height is a function of the orientation, location, shape, and size of the reference ellipsoid used. Because of several ellipsoids in use, the same point can have different ellipsoidal heights on different ellipsoids, just as it can have different latitudes and longitudes. It is important to specify the reference ellipsoid used when dealing with ellipsoidal heights. The ellipsoid height is positive if the reference ellipsoid is below the topographic surface and negative if the ellipsoid is above the topographic surface. The orthometric height ( $H$ ), often referred to as mean sea level height, can be obtained by subtracting the geoid height ( $N$ ) from the GNSS ellipsoidal height ( $h$ ). The basic relationship between geoid, ellipsoidal, and orthometric heights is given by the following equation (Li and Götze, 2001, Meyer et al., 2006).

$$H = h_{GPS} - N_{geoid} \quad (2.7)$$

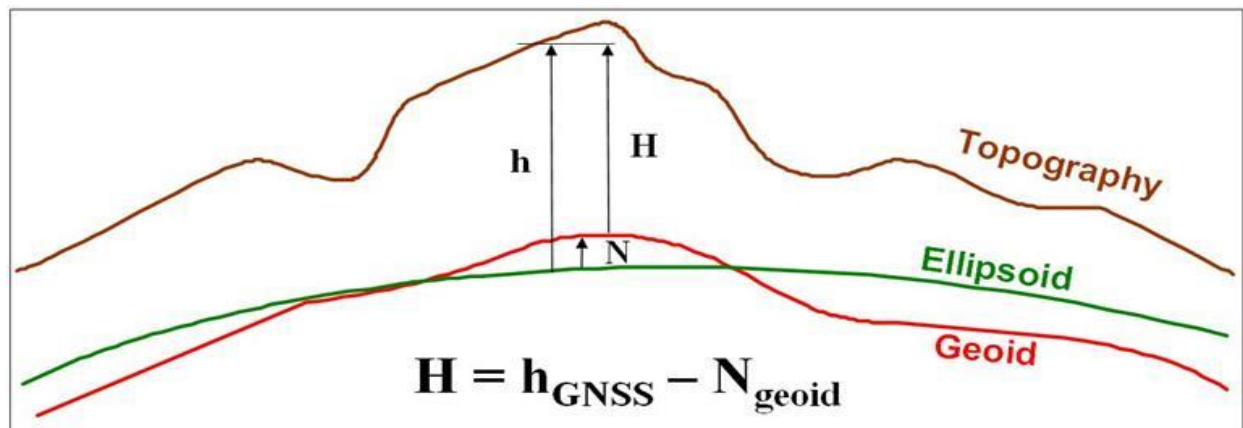


Figure 2. 2 The ellipsoidal height ( $h$ ), orthometric height ( $H$ ), and geoid undulation ( $N$ )

### 2.3.1.5 Orthometric Height

The orthometric height is the vertical distance of a point above or below a given reference level, called a datum. The most commonly used datum is the mean sea level. Mean sea level is a part of a special equipotential surface in the Earth's gravity field. In one sense, orthometric heights are purely geometric. However, the orthometric height is governed by the gravity field of the Earth and those that are naturally linked to the equipotential surfaces and measured along the plumb

lines of the Earth's gravity field and thus follow the curved paths. The vertical direction is parallel to the direction of gravity at any point. It is the direction of a freely suspended plumb-bob cord or string. A Plumb line is a line perpendicular to all equipotential surfaces of the Earth's gravity field. Therefore, orthometric heights are closely related to the gravity field of the Earth in addition to being a geometric quantity (Birbiraw, 2013, Featherstone and Kuhn, 2006, Meyer et al., 2006).

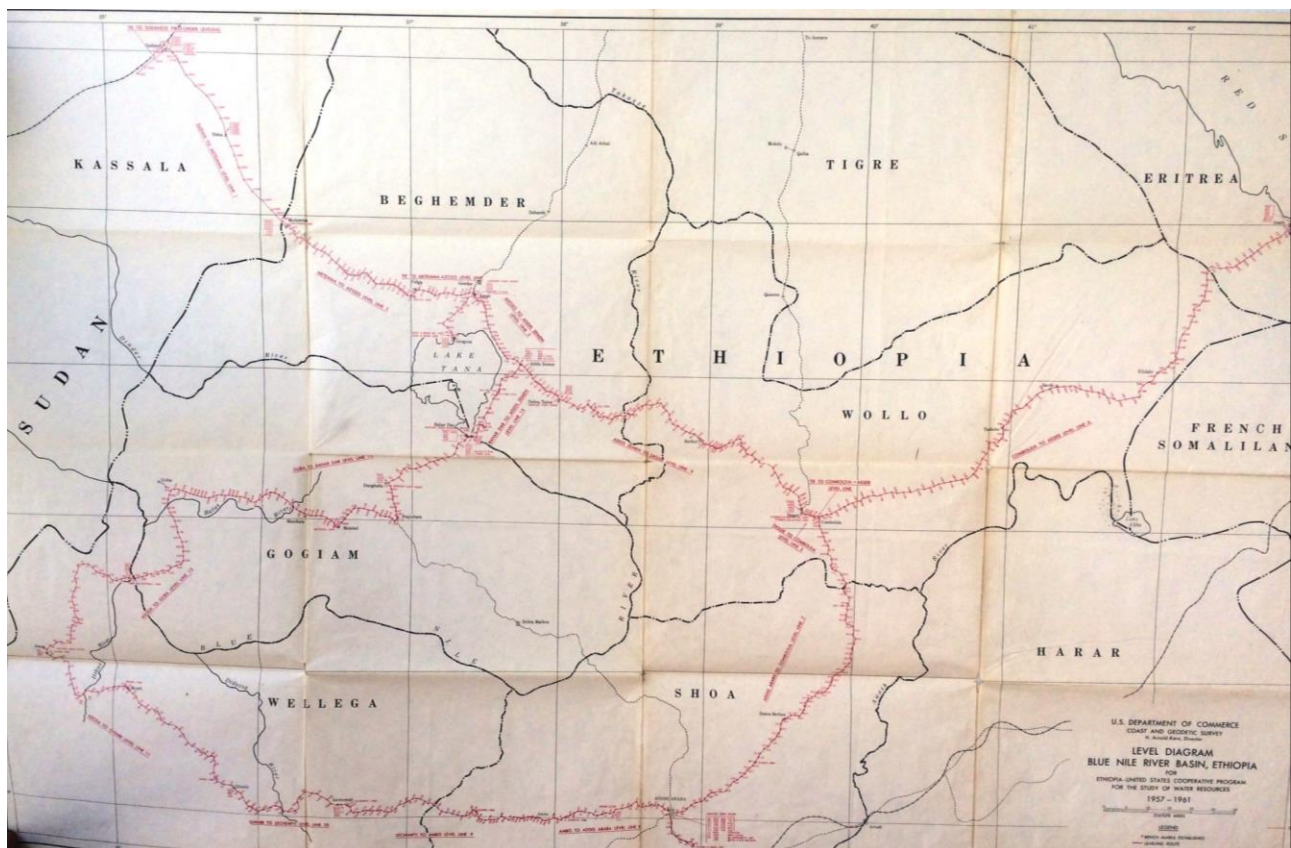
In particular, orthometric heights are very important, practical, and spontaneously used for many engineering works and construction projects. Orthometric height is needed for the design of highways, railways, canals, for locating the gradient lines, and used to determine the catchments area, the volume of the reservoir. It's used to determine the flow of water along with a drainage system, where water is expected to flow downhill from a higher to a lower height. However, it is the force of gravity that governs water flow, not height. It also provides data for determining the shape of the Earth's surface or used to record the information necessary to draw a topographic map (Fiedler, 1992, Yilmaz, 2008).

## **2.4 The Blue Nile Leveling Network**

The Ethiopian Geodetic Control Project, in operation from 1957 to 1961, completed the basic horizontal and vertical control networks throughout approximately 120,000 square miles of west-central part of Ethiopia, the area comprising the Ethiopian watershed of the Blue Nile River. The surveys establishing the networks were conducted with firsts-order methods and procedures by a combined organization of Ethiopian and United States personnel, financed by a joint Imperial Ethiopian Government and United States Government cooperative service. This agreement established the authority, purpose and scope for the conduct of the geodetic control survey on the Blue Nile river basin of Ethiopia.

The area covered by the Geodetic Control Project included the Ethiopian watershed of the Blue Nile River Basin. Also included in the project was the coverage of areas required to make the connection to Sudanese control to existing triangulation in Ethiopia. The triangulation extended from Khashum el Girba in Kassala Province of Sudan, eastward across the Ethiopian-Sudan border in to western Eritrea, thence southward along the border to the vicinity of

Metemma in Ethiopia, and thence in to the Blue Nile River Basin area. The leveling work starts from a control point in Sudan (Gedarif) comes from Alexandria, Egypt, and extended to the southeastward across the Ethiopia-Sudan border and into the river basin at Metema, Ethiopia. From Metemma the leveling survey continued to Azozo, Addis Zemen, Dessie, Addis Ababa, Nekemte, Asosa, Gubba, Metekel, Bahir Dar, with a loop tie-back into Addis Zemen. Spur lines were extended from Azozo to lake Tana at Gorgora, from Addis Ababa to Ducam, and from Kombolcha to Asseb on the Red sea coast of Ethiopia, where a connection was made to mean sea level of Red Sea (Ethiopia Blue Nile Geodetic Control Project, 1957 - 1961). A number of small loops were completed in the vicinity of Addis Ababa. The closure of the large loop was 0.40 meter. A level line was also run to the Red Sea with a closure of 0.18 meter. The maximum rate of correction for the long links of the adjustment is 0.26 mm/km (Blackwell, 1962).



**Figure 2. 3** Geodetic vertical control points.

## 2.5 Methods of Determining Orthometric Heights

There are several methods for determining orthometric heights. Based on the instruments used and applied measurements these methods are classified as spirit leveling, trigonometric leveling, and, GPS and leveling methods.

### 2.5.1.1 Spirit Leveling

Spirit leveling is the determination of the height difference in elevation between two points by using an instrument known as level with a leveling staff. The practical applications have shown that carrying out the instruments is very difficult on rough ground and is sensitive for both systematic and random errors. Preventative measures must be taken to eliminate or reduce these types of errors stemmed from instrumental, natural, and personal errors. These situations decrease the survey velocity, thus the cost of surveying rises. The effects of such errors can be reduced by applying appropriate measurement technique including by balancing the backsight (BS) and foresight (FS) distances, following backward forward forward backward (BFFB) or forward backward backward forward( FBBF) observation order, calibrating the instrument and surveying additional parameters such as pressure, temperature, and humidity at the time of survey moment (Baykal, 1989, Ceylan, 1988, Niemeier, 1986).

Nowadays, motorized spirit leveling has been done by establishing survey hardware on the land vehicle and successful results have been obtained. The advantages of motorized leveling are to improve 40-60% in production velocity, decrease in errors connected to time, observation rays are higher to decreases the refraction errors and more accuracy. The only disadvantage of motorized leveling is that the cost of instruments and vehicles is very high and level points must be on the edge of the road (Becker, 1986, Niemeier, 1986).

### 2.5.1.2 Trigonometric Leveling

Trigonometric leveling is an indirect procedure; the difference in elevation between two points can be determined by measuring the inclined or horizontal distance between them and the zenith angle or the altitude angle to one point from the other. According to the land, time, and observing the zenith angle, trigonometric leveling can be classified as unidirectional trigonometric leveling, Leap-Frog (jumped) trigonometric leveling, and reciprocal trigonometric leveling.

With the advent of total station instruments, trigonometric leveling has become an increasingly common method for rapid and convenient observation of elevation differences because slope distances and zenith angles are quickly and easily observed from a single setup, and then various studies have been made about this subject (Aksoy et al., 1993, Ceylan, 1993, Kasser, 2002, Kuntz and Schmitt, 1986, Rueger and Brunner, 1982).

Applications of the motorized trigonometric leveling have been done by establishing survey hardware on the land vehicle. The motorized trigonometric leveling is equal accuracy ( $\leq \pm 2\text{mm} / \text{km}$ ) and equal costs with the motorized spirit leveling when the motorized trigonometric leveling is done according to the following rules; the reciprocal and simultaneously vertical angle observations, the reciprocal distance measurements, the observation ranges are short (~250-300 m), using calibrated instruments, the survey is carried out by the experienced person. More than 27% of the production of velocity has been reached with motorized trigonometric leveling. Therefore, the studies have shown that trigonometric leveling may be an alternative method of spirit leveling (Chrzanowski, 1989, Chrzanowski et al., 1985, Whalen, 1985).

### ***2.5.1.3 GPS and Leveling***

The GPS and Leveling is the modern method of determining the orthometric height system from the combination of the global gravity field model technology. Three-dimensional coordinates can be obtained by GPS in the geocentric cartesian coordinate system( $x, y, z$ ). The Cartesian coordinate systems are transformed to geodetic latitude( $\varphi$ ), geodetic longitude( $\lambda$ ), and ellipsoidal heights( $h$ ) according to the selected reference ellipsoid. The ellipsoidal heights obtained by GPS observations are not directly used for practical applications. So, the ellipsoidal height has to be transformed to orthometric height (H) usually used in surveying and mapping applications. The orthometric height (H), which is measured along the plumb lines of the Earth's gravity field and thus follows the curved paths, can be obtained by subtracting the geoid height (N) from the GNSS ellipsoidal height (h) or the relative differences in height. However, a geoid height must be computed before computing orthometric heights. A better approach to determining geoid height is the use of global gravity field models and GPS technology (Abubeker and Workaferahu, 2009, El Shouny et al., 2018, Gikas et al., 2013, Sulaiman et al., 2016, Yılmaz et al., 2010).

## CHAPTER 3 DATA ACQUISITION AND PROCESSING

### 3.1 Data Source

This study uses both primary and secondary data. The Primary data were collected from actual field survey measurements using surveying instruments. These Primary data points include DES-42, DES-12, DES-320, DES-330, DES-36, and DES-27 (Fig 1.1). The other secondary data points are those not collected under this study but collected previously and plotted in Fig (1.2).

#### 3.1.1 Leveling Data

The leveling data collection comprises two types of data. These are primary data and secondary data. The primary leveling data observations are carried out at six selected leveling benchmark points using spirit level instruments in Dessie City. The reference benchmark used for computing the orthometric height is not a given benchmark that refers to the mean sea level surface. The accuracy of primary data collection is  $4 \text{ mm} * \sqrt{k}$  ( $k$  = length of the section in km) it is first-order, class I elevation accuracy standards (Ghilani et al., 2008). The primary leveling data observations are attached in Appendix (A). The secondary leveling data are previously existing Blue Nile Geodetic benchmark points (Ethiopia Blue Nile Geodetic Control Project, 1957 – 1961, Abubeker and Workaferahu, 2009). The remaining secondary leveling data with high accuracy are determined through a series of leveling heights connected to one particular reference station called BA202 (Bedada, 2010).

#### 3.1.2 Global Positioning System Data

The GPS data collection comprises two types of data. These are primary data and secondary data. The primary GPS data observations are collected at the six selected leveling benchmark points include DES-42, DES-12, DES-320, DES-330, DES-36, and DES-27 using sokkia atlas instruments belonging to Dessie City. The GPS measurement was undertaken to start 7<sup>th</sup> November to 9<sup>th</sup> November 2020 and the time of observation was made for 10 hours. The measurement was performed in three sessions. Session one includes two stations namely DES-42 and DES-12. The GPS measurement at these stations was undertaken on 7<sup>th</sup> November 2020. Session two includes two stations namely DES-320 and DES-330. The GPS measurement at

these stations was undertaken on 8<sup>th</sup> November 2020. Session three includes two stations namely DES-36 and DES-27. The GPS measurement at these stations was undertaken on 9<sup>th</sup> November 2020.

The secondary GPS data are previously collected coordinates (Latitude, Longitude, and Ellipsoidal height) at previously existing Blue Nile Geodetic benchmark points include ASOSA, GONDER , GEWANE , D/BERHAN , DUKEM , BALE ROBE , JIMMA , BEDELLE , GORE , TIE , TULU AMARA and WOLONKOMI. Unfortunately, no information is available about the accuracy of this secondary GPS data but the time of observation was made for a minimum of 24 hours and a maximum of 48 hours (Ethiopia Blue Nile Geodetic Control Project, 1957 – 1961, Abubeker and Workaferahu, 2009). The remaining secondary GPS data points include BMDZ1, BMJ1a, BMJ2a, BMJ3a , BMJ4a , BMJ5a , BMAaa and BMCaa are previously collected coordinates. Unfortunately, no information is available about the time of observation of this secondary GPS data but the accuracy is 2 - 3 cm (Bedada, 2010).

### **3.2 GPS Data Processing**

After the field observations, the raw data was downloaded from sokkia atlas instruments. The magnet tools are used to convert the raw data to RINEX (Receiver Independent Exchange Format). The RINEX data for each station was processed by using the online GPS processing service. Online GPS processing service presents two types of methods, which are the precise point positioning (PPP) method and the relative method. Three online GNSS processing services are based on relative methods such as Australian Online GPS Processing Service (AUSPOS), Online Positioning User Service (OPUS), and Scripps Co-ordinate Update Tool (SCOUT). For this study, AUSPOS was used because it's more reliable results than the other services. It is used for research around the world and to establish precise control surveys. (El-Mowafy, 2011) concluded that AUSPOS have a precision of a few millimeters to a couple of centimeters. The AUSPOS is designed for different applications such as very long baseline positioning, GNSS connections to IGS stations, DGPS reference station positioning, remote GNSS station positioning, and high accuracy positioning (Herbert and RAUFU, 2015, Jia et al., 2014, Mohamed et al., 2007).

AUSPOS is a free online GPS processing service using Bernese GNSS Software Version 5.2. The service uses the International Ground Station (IGS) realization of the ITRF 2014 reference frame to compute precise coordinates anywhere on Earth and the Geocentric Datum of Australia (GDA) within Australia. The Service is designed to process only dual-frequency GPS data. The users can submit the GPS RINEX data observed in ‘static’ mode, the antenna height and type, and the email address to the AUSPOS GPS processing service. An AUSPOS report will be emailed to the email address (Adam, 2017, Herbert and RAUFU, 2015).

This study uses a total of six International Ground Station (IGS) Such as MAL2 from Kenya, NKLG from Gabon, SOFI from Bulgaria, ANKR from Turkey, BHR4 from Bahrain, DRAG from Israel, and only one Asia-Pacific Reference Frame (APREF) station used in this study called ISNA from Iraq. The IGS and APREF stations were chosen based on the nearest to the local stations as the reference stations.

**Table 3. 1 Positional uncertainty of primary GPS data at 95 % C.L.**

Station	Longitude(East) (m)	Latitude(North) (m)	Ellipsoidal height(Up) (m)
DES-42	0.010	0.007	0.027
DES-12	0.017	0.010	0.052
DES-320	0.012	0.008	0.032
DES-330	0.013	0.009	0.038
DES-36	0.011	0.008	0.035
DES-27	0.010	0.008	0.029

**Table 3. 2 The Measured Coordinates Using GPS and Leveling**

Station Name	Latitude			Longitude			Ellipsoidal Height ( h ) in	Mean sea level height ( H ) in	Observa tion time in	Reference
	Deg	Min	Sec	Deg	Min	Sec	meter	meter	hour	
DES-42	11°	07'	38.60646"	39°	37'	52.04856"	2524.042	2529.7336	10hr	Primary Data
DES-12	11°	08'	01.43948"	39°	37'	46.42168"	2528.691	2534.0229		
DES-320	11°	07'	29.14314"	39°	37'	53.13036"	2524.756	2530.2664		
DES-330	11°	07'	36.30125"	39°	37'	55.72197"	2506.95	2512.7633		
DES-36	11°	07'	48.28313"	39°	37'	53.21807"	2515.041	2520.8345		
DES-27	11°	07'	54.28460"	39°	37'	48.23334"	2529.361	2534.7351		
ASOSA	10°	2'	58.3093"	34°	33'	7.62152"	1661.317	1665.675	48hr	(Ethiopian Geospatial Information Institute)
GONDER	12°	31'	20.21894"	37°	25'	9.13652"	2136.321	2137.529		
GEWANE	10°	14'	6.16067"	40°	42'	54.08716"	687.239	697.231		
D/BERHAN	9°	41'	31.88663"	39°	33'	7.2693"	2799.885	2805.841		
DUKEM	8°	48'	46.39374"	38°	53'	1.13449"	2044.497	2051.969		

BALE ROBE	7°	4'	27.23956"	40°	9'	54.14393"	2410.688	2424.729		
JIMMA	7°	34'	36.41934"	36°	40'	51.58268"	1868.659	1876.139		
BEDELLE	8°	27'	12.51338"	36°	20'	44.2757"	1975.519	1981.738		
GORE	8°	9'	17.71996"	35°	33'	1.63408"	2018.633	2024.864		
TIE	11°	48'	32.69247"	38°	14'	24.35521"	3112.919	3113.883	48hr	(Abubeker and Workaferahu, 2009)
TULU AMARA	9°	06'	11.95735"	37°	19'	16.90436"	3045.124	3049.974	24hr	
WOLONKOMI	9°	01'	53.90159"	38°	15'	53.30430"	2497.4	2503.431		
BMDZ1	8°	45'	6.768"	38°	57'	36.36"	1894.283	1901.928		(Bedada, 2010)
BMJ1a	8°	44'	53.808"	38°	58'	57"	1873.215	1881.194		
BMJ2a	8°	41'	57.228"	39°	1'	37.92"	1873.68	1881.712		
BMJ3a	8°	40'	29.568"	39°	2'	44.16"	1851.576	1859.76		
BMJ4a	8°	39'	21.204"	39°	3'	37.8"	1866.014	1874.243		
BMJ5a	8°	37'	24.24"	39°	5'	42"	1795.814	1804.252		
BMAaa	8°	46'	2.352"	38°	56'	12.84"	1896.823	1904.326		
BMCaa	8°	43'	6.096"	39°	0'	56.52"	1864.286	1872.323		

### 3.3 Absolute Methods

In the absolute method, the accuracy of the gravity field model is evaluated by computing and compare models based on geoid undulations with those obtained from the GPS and leveling based on geoid undulations.

#### 3.3.1 Computation of Geoid Undulation Using GGMs Model

A GRAVSOFTE program is the python graphical interface that calculates the geoid undulation using global gravity field models and GPS technology. By using Bruns's equation, the Stokes integral that is used for the computation of geoid undulation can be written as (El Shouny et al., 2018, Gikas et al., 2013, Sulaiman et al., 2016, Yilmaz et al., 2010).

$$N_{\text{Model}} = \frac{R}{4\pi\gamma} \int_{\lambda=0}^{2\pi} \int_{\theta=0}^{\pi} St(\psi_{PQ}) \Delta_{gQ} \sin\theta \, d\theta \, d\lambda \quad (3.1)$$

Where: R is the radius of the Earth,  $St(\psi_{PQ})$  is the Stokes' function and  $\Delta_{gQ}$  is the gravity anomalies.

According to the propagation law of random errors, the errors of the geoid undulation can be computed as (Eshagh, 2013).

$$eN_{\text{Model}} = \frac{GM}{R\gamma} \sqrt{\sum_{n=2}^{\infty} \left(\frac{R}{r}\right)^{2n+2} \sum_{m=-n}^n e^2 t_{nm} [\bar{Y}_{nm}(P)]^2} \quad (3.2)$$

Where  $t_{nm}$  is the error of spherical harmonic coefficients.

#### 3.3.2 Computation of Geoid Undulation Using GPS and Leveling Data

The geoid undulation ( N ) from GPS based ellipsoidal heights ( h ) relative to the reference ellipsoidal surface and the orthometric heights ( H ) related to mean sea level surface can be computed as (Abubeker and Workaferahu, 2009, El Shouny et al., 2018, Gikas et al., 2013, Sulaiman et al., 2016, Yilmaz et al., 2010).

$$N_{\text{Gps and levelling}} = h - H \quad (3.3)$$

### 3.4 Relative Methods

In the relative method, the accuracy of the gravity field model is evaluated by computing and compare model based on the relative difference in height with those obtained from the spirit-leveled based on the relative height difference (Featherstone, 2001, Gikas et al., 2013, Heliani, 2016, Kotsakis et al., 2009).

#### 3.4.1 Computation of Height Differences Using Spirit Leveling

Spirit leveling is the determination of the height difference in elevation between two or more points. The difference between the back sight ( BS ) and fore sight ( FS ) reading represents the elevation difference between the two points. There are two widely used methods for the determination of the height difference in elevation between two points such as the rise and fall method, the height of instrument method. The results of both methods are always the same but in this study, the following height of instrument methods was used (Roth and Field, 1991).

$$\text{Existing Elevation} + \text{BS} = \text{HI} \quad (3.4)$$

$$\text{New Elevation} = \text{HI} - \text{FS} \quad (3.5)$$

Where: BS is the back sight reading, FS is the fore sight reading and HI is the height of the instrument.

During spirit leveling surveys two error checks should be conducted: the arithmetic check, and calculation of the allowable misclosure. The arithmetic check is conducted to check if the readings are written properly in the field book. For arithmetic check, the absolute value of the sum of the back sights minus the sum of the fore sights is equal to the difference between the absolute value of the sum of the rises and that of the sum of the falls is equal to the absolute value of the difference in elevation between the first and the last R.Ls. Expressed mathematically (Roth and Field, 1991).

$$\left| \sum B.S - \sum F.S \right| = \left| \sum Rise - \sum Fall \right| = |Last RL - First RL| \quad (3.6)$$

The second check is the allowable misclosure. The Federal Geodetic Control Subcommittee (FGCS) recommends the following equation to calculate the allowable misclosure.

$$C = m\sqrt{K} \quad (3.7)$$

Where: C is the allowable loop or section misclosure, in millimeters; m is a constant; and K is the total perimeter distance, in kilometers. In this study the accuracy of primary data collection is  $4 \text{ mm} * \sqrt{k}$  (k = length of the section in km) it is first-order, class I elevation accuracy standards (Ghilani et al., 2008).

### 3.4.1.1 Least Square Adjustment

Spirit leveling observations are subject to random errors. The observation equations of spirit leveling having equal or unit weights can be adjusted using the principle of least squares. This study uses the principle of weighted least squares. The fundamental principle of weighted least squares is stated that the sum of the weights times their respective squared residuals must be minimized. The equation form of weighted least squares adjustment is (Ghilani, 2017).

$$W_1V_1^2 + W_2V_2^2 + \dots + W_nV_n^2 = \sum WV^2 \rightarrow \text{minimum} \quad (3.8)$$

To apply the principle of least squares in spirit leveling adjustments, a prototype observation equation is first written for any elevation difference. The observation equation of each observed elevation difference between two stations I and J can be expressed as:

$$E_J - E_I = \Delta\text{Elev}_{IJ} + \Delta v\text{Elev}_{IJ} \quad (3.9)$$

Where:  $E_J$  and  $E_I$  are the unknown elevations of any two stations, J and I, respectively,  $\Delta\text{Elev}_{IJ}$  are the spirit leveling observation and its residual is  $\Delta v\text{Elev}_{IJ}$ .

The relative weights used to adjust the spirit leveling are inversely proportional to the lengths of the lines in kilometers (Ghilani, 2017, Ghilani et al., 2008).

$$W = \frac{1}{\text{Length}} \quad (3.10)$$

The least-squares solution of weighted normal equations is

$$X = (A^T W A)^{-1} A^T W L \quad (3.11)$$

Where: X is the matrix of unknowns, A is the matrix of coefficients for the unknowns, W is the weight matrix and L is the matrix of observations.

### 3.4.1.2 Computation of Normal Gravity

The potential and the gravity field that are consistent with reference ellipsoid are called normal potential and normal gravity. The normal field is an ellipsoidal approximation of the real gravity field. The equation used to derive the normal potential can be written as (Heikkinen, 1981).

$$\begin{aligned} U = & 62,636,860.8500 \\ & + [-9.780,326,77 \\ & - 0.051,630,75 \sin^2 \varphi - 0.000,227,61 \sin^4 \varphi - 0.000,001,23 \sin^6 \varphi] h \\ & + [0.015,438,99 \cdot 10^{-4} - 0.000,021,95 \cdot 10^{-4} \sin^2 \varphi - 0.000,000,10 \\ & \cdot 10^{-4} \sin^4 \varphi] h^2 \\ & - [-0.000,024,22 \cdot 10^{-8} + 0.000,000,07 \cdot 10^{-8} \sin^2 \varphi] h^3 \quad (3.12) \end{aligned}$$

Normal gravity defines as the negative of the partial derivative of the normal potential with respect to the height above the reference ellipsoid. The unit of potential is  $\text{m}^2/\text{s}^2$  and the unit of gravity is  $\text{m}/\text{s}^2$ . The equation used to derive the normal gravity can be written as (Heikkinen, 1981).

$$\begin{aligned} \gamma = -\frac{\partial U}{\partial h} = & 9.780,326,77 \\ & + 0.051,630,75 \sin^2 \varphi + 0.000,227,61 \sin^4 \varphi + 0.000,001,23 \sin^6 \varphi \\ & - [0.030,877,98 \cdot 10^{-4} - 0.000,043,90 \cdot 10^{-4} \sin^2 \varphi - 0.000,000,200 \\ & \cdot 10^{-4} \sin^4 \varphi] h \\ & - [-0.000,072,65 \cdot 10^{-8} + 0.000,000,21 \cdot 10^{-8} \sin^2 \varphi] h^2 \quad (3.13) \end{aligned}$$

Where:  $\varphi$  is geodetic latitude, h (in metres) is the ellipsoidal height above the reference ellipsoid.

### 3.4.1.3 Adjustment of Height Differences Using Normal Gravity

The height difference from spirit leveling is not unique but it depends on the route because equipotential surfaces are not parallel. Height difference cannot be uniquely determined from spirit leveling but it can be uniquely determined from geopotential. Precise leveling combines spirit leveling with gravity measurements to determine the geopotential. The determined geopotential can then be transferred into height difference. The potential heights of control benchmark stations can be computed as (Ghilani et al., 2008).

$$H_C(A) = H_A \left( g_A + \frac{0.0424}{1,000,000} H_A \right) \quad (3.14)$$

The height differences using spirit leveling ( $dl_{\text{spirit leveling}}$ ) between two benchmark stations can be computed as:

$$dl_{\text{spirit leveling}} = \frac{2[H_C(B) - H_C(A)]}{g_A + g_B} \quad (3.15)$$

Where  $H_C(A)$  and  $H_C(B)$  are the potential height of station A and station B respectively in units of kgal-meters,  $H_A$  is the orthometric height of station A and  $g_A$  and  $g_B$  are the modeled gravity value at station A and station B respectively in units of kgals. In this study uses the normal gravity instead of modeled gravity value.

### 3.4.2 Computation of Height Differences Using GGMs Model

The orthometric heights ( $H$ ) from GPS based ellipsoidal heights ( $h$ ) and the geoid undulation ( $N_{\text{Model}}$ ) can be computed as (Abubeker and Workaferahu, 2009, El Shouny et al., 2018, Gikas et al., 2013, Sulaiman et al., 2016, Yilmaz et al., 2010).

$$H = h - N_{\text{Model}} \quad (3.16)$$

The height differences using GGMs Model ( $dl_{\text{Model}}$ ) between two benchmark stations can be computed as:

$$dl_{\text{Model}} = H(B) - H(A) \quad (3.17)$$

Where:  $H(B)$  is the orthometric heights of benchmark station B and  $H(A)$  is the orthometric heights of benchmark station A.

### 3.5 Statistical Analysis

There are several statistical analyses used to evaluate the accuracy of the gravity field model. These include the minimum value, maximum value, mean, variance, standard deviation, and root mean square error (RMSE).

#### 3.5.1 Statistical Analysis of Absolute Methods

In computing the differences between models based on geoid undulations and, GPS and leveling based on geoid undulations, the assumption is the geoid undulation obtained from the GPS and leveling are standard to which the geoid undulation obtained from the model. The geoid undulation difference ( $\Delta N$ ) between model based on geoid undulations ( $N_{\text{Model}}$ ) and, GPS and leveling based on geoid undulations ( $N_{\text{Gps/leveling}}$ ) is computed by (Bluman, 2009, Peprah et al., 2017).

$$\Delta N = N_{\text{Gps/leveling}} - N_{\text{Model}} \quad (3.18)$$

The minimum and maximum value of geoid undulation difference ( $\Delta N$ ) is the smallest and largest value in the data set respectively.

The mean ( $\Delta N_{\text{mean}}$ ) of the geoid undulation difference is the sum of the geoid undulation difference divided by the total number of values.

$$\Delta N_{\text{mean}} = \frac{1}{n} \sum_{i=1}^n \Delta N_i \quad (3.19)$$

Where  $i=1, 2, 3, \dots, n$ , and  $n$  is the sample size.

The variance ( $S^2$ ) of geoid undulation difference can be computed by using the following equation.

$$S^2 = \frac{\sum_{i=1}^n (\Delta N_i - \Delta N_{mean})^2}{n - 1} \quad (3.20)$$

The standard deviation (  $S$  ) of the geoid undulation difference can be computed by using the following equation.

$$S = \sqrt{\frac{\sum_{i=1}^n (\Delta N_i - \Delta N_{mean})^2}{n - 1}} \quad (3.21)$$

Where  $n - 1$  is the degree of freedom. The standard deviation measures how much the values of the set of data deviate from the mean.

The root mean square error measures how much error there is between the model and, GPS and leveling data. The root mean square error (  $RMSE$  ) of the geoid undulation difference can be computed by using the following equation.

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (\Delta N_i)^2}{n}} \quad (3.22)$$

The American Society for Photogrammetry and Remote Sensing (ASPRS) recommends the root mean square error (  $RMSE$  ) of well-defined points must not exceed  $\pm (CI/3)$ , it is first-order, class I vertical accuracy standards. Where CI is the contour interval (Ghilani et al., 2008).

### 3.5.2 Statistical Analysis of Relative Methods

In computing the differences between model based on the relative difference in height and spirit leveling based on the relative height difference, the assumption made here is that the height differences obtained from the spirit leveling are standard to which the height differences obtained from the model. The height differences (  $\Delta H$  ) between model based on height differences (  $dl_{Model}$  ) and spirit leveling based on height differences (  $dl_{Spirit\ leveling}$  ) is computed by (Bluman, 2009, Peprah et al., 2017).

$$\Delta H = dl_{Spirit\ leveling} - dl_{Model} \quad (3.23)$$

The formulas used to calculate the minimum value, maximum value, mean, variance, standard deviation, and root mean square error (RMSE) are the same as the absolute methods.

### 3.6 Correlation Analysis

The correlation analysis is used to measure the strength of a linear relationship between two quantitative variables. The value of the correlation coefficients ranges from -1 to 1 and can be classified as positive and negative correlations. A positive correlation is a relationship between measurement and model outputs that moves in the same direction. Whilst negative correlation is a relationship between measurement and model outputs moves in the opposite direction. Correlation coefficients between two quantitative variables *X and Y* can be computed by using the following formula (Bluman, 2009).

$$r = \frac{n(\sum XY) - (\sum X)(\sum Y)}{\sqrt{[n(\sum X^2) - (\sum X)^2][n(\sum Y^2) - (\sum Y)^2]}} \quad (3.24)$$

Where: n is the number of data pairs.

### 3.7 Hypothesis Testing

A hypothesis testing is a mathematical formula used to decide to accept or reject the null hypothesis that is used to select the best global gravity field models includes EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159. The null hypothesis: the variance of model one is equal to the variance of model two. And the alternative hypothesis: the variance of model one is not equal to the variance of model two. For the comparison of the variance at 95% of probability level, the critical value must be computed. After computing the critical value the test value can be computed by using the following F test formula. If the test value is less than the critical value, the null hypothesis is accepted otherwise it is rejected (Bluman, 2009, Peprah et al., 2017).

$$F = \frac{S_1^2}{S_2^2} \quad (3.25)$$

Where:  $S_1$  is the variance of model one and  $S_2$  is the variance of model two.

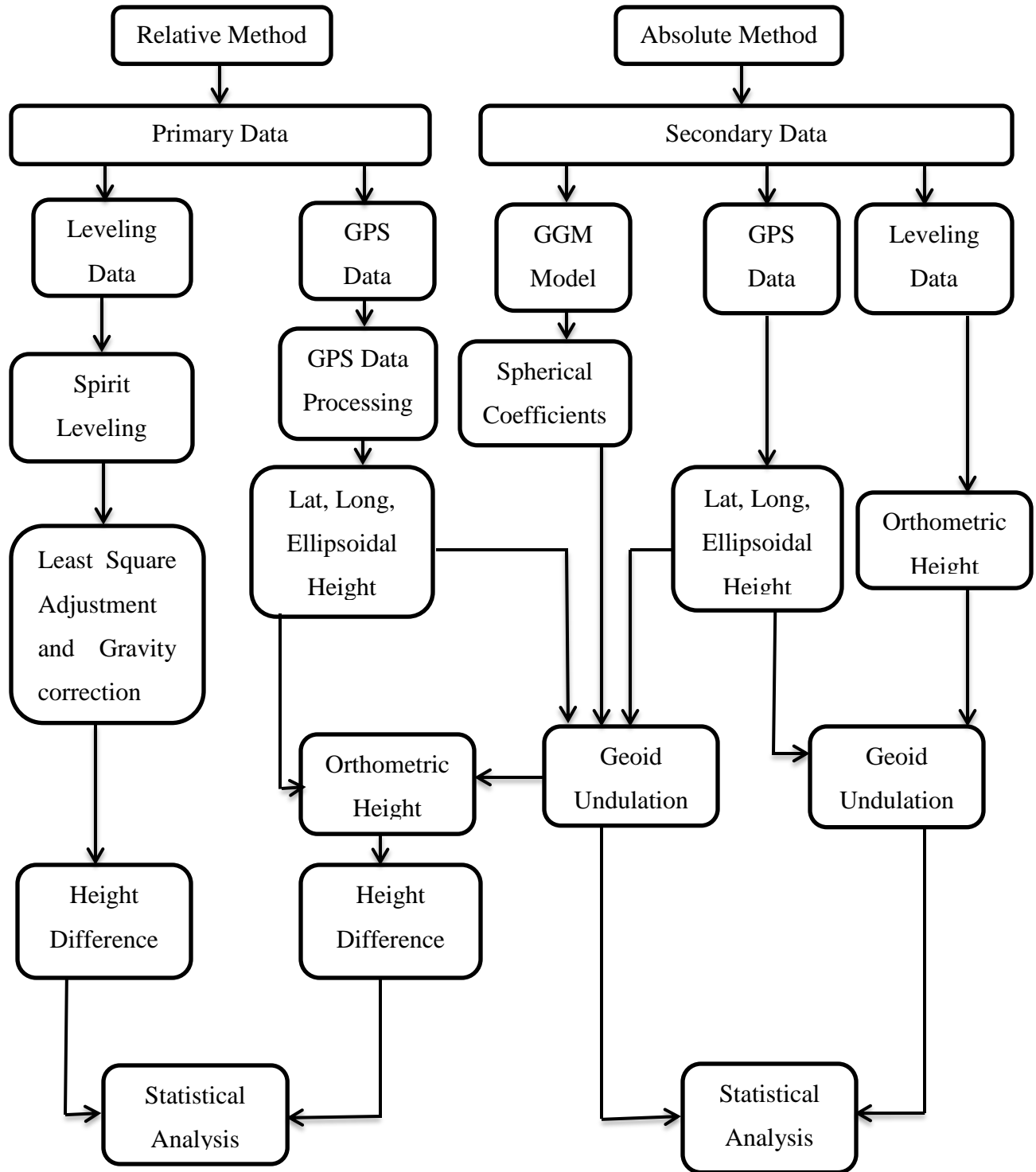


Figure 3. 1 General Frame Work of data processing Technique and Methodology

## CHAPTER 4 RESULTS AND DISCUSSION

### 4.1 Results of Geoid Undulation

#### 4.1.1 Computed Geoid Undulation Using GPS and Leveling Data

The geoid undulation ( $N$ ) of the 20 GCP points was computed from the GPS-based ellipsoidal heights ( $h$ ) and the orthometric heights ( $H$ ). The GPS - based ellipsoidal heights are relative to the reference ellipsoidal surface and the orthometric heights are related to mean sea level surface. Table (4.1) shows the computed geoid undulation from the ellipsoidal heights and the orthometric heights.

**Table 4. 1 Computed geoid undulation from the ellipsoidal heights and the orthometric heights**

GCP Name	Ellipsoidal height ( $h$ ) in meter	Orthometric height ( $H$ ) in meter	Geoid undulation ( $N = h - H$ ) in meter
ASOSA	1661.317	1665.675	-4.358
GONDER	2136.321	2137.529	-1.208
GEWANE	687.239	697.231	-9.992
D/BERHAN	2799.885	2805.841	-5.956
DUKEM	2044.497	2051.969	-7.472
BALE ROBE	2410.688	2424.729	-14.041
JIMMA	1868.659	1876.139	-7.48
BEDELLE	1975.519	1981.738	-6.219
GORE	2018.633	2024.864	-6.231
TIE	3112.919	3113.883	-0.964
TULU AMARA	3045.124	3049.974	-4.85
WOLONKOMI	2497.4	2503.431	-6.031
BMDZ1	1894.283	1901.928	-7.645
BMJ1a	1873.215	1881.194	-7.979
BMJ2a	1873.68	1881.712	-8.032
BMJ3a	1851.576	1859.76	-8.184
BMJ4a	1866.014	1874.243	-8.229
BMJ5a	1795.814	1804.252	-8.438
BMAaa	1896.823	1904.326	-7.503
BMCaa	1864.286	1872.323	-8.037

**4.1.2 Computed Geoid Undulation Using GGMs Model**

The geoid undulation of the same 20 GCP points was computed using a global gravity field model. The global gravity field models used to compute the geoid undulation includes EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159. Table (4.2) shows the computed geoid undulation from the global gravity field models.

**Table 4. 2 Computed geoid undulation from the global gravity field models**

GCP Name	Geoid undulation in meter			
	EGM2008	EIGEN-6C4	GECO	XGM2019e-2159
ASOSA	-5.4251	-5.5898	-5.5373	-6.5113
GONDER	-2.0598	-2.1497	-2.1083	-3.2007
GEWANE	-11.505	-11.4704	-11.4657	-12.63
D/BERHAN	-6.9504	-7.0106	-7.0382	-8.2029
DUKEM	-8.5157	-8.5299	-8.4978	-9.6999
BALE ROBE	-13.1681	-13.2065	-13.1557	-14.273
JIMMA	-8.4824	-8.5009	-8.5757	-9.7268
BEDELLE	-7.4173	-7.4117	-7.4148	-8.6498
GORE	-7.2885	-7.3344	-7.3282	-8.519
TIE	-2.3953	-2.329	-2.4326	-3.4608
TULU AMARA	-6.1856	-6.2094	-6.2595	-7.3742
WOLONKOMI	-6.4434	-6.4942	-6.5061	-7.74
BMDZ1	-9.0061	-9.0105	-8.9664	-10.197
BMJ1a	-9.0654	-9.0692	-9.025	-10.2625
BMJ2a	-9.305	-9.3017	-9.2536	-10.5169
BMJ3a	-9.4423	-9.4357	-9.3867	-10.6498
BMJ4a	-9.5606	-9.5515	-9.5026	-10.7583
BMJ5a	-9.7714	-9.7583	-9.7114	-10.9413
BMAaa	-8.8901	-8.897	-8.8552	-10.0767
BMCaa	-9.2168	-9.2161	-9.1694	-10.4277

**4.1.3 Comparison of GPS and Leveling Data with GGMs Model**

In this section, the geoid undulation differences between the geoid undulation obtained from the GPS and leveling data, and the geoid undulation obtained from the global gravity field models at the same 20 GCP points were obtained. The statistics of the differences are also shown with respect to the minimum of the differences, maximum of the differences, mean of the differences, standard deviation of the differences, a variance of the differences, and root mean square error of the differences. Table (4.3) shows the geoid undulation differences obtained from GPS and

leveling data and global gravity field models. Table (4.4) shows the statistics of the geoid undulation differences obtained from GPS and leveling data, and global gravity field models.

**Table 4. 3 The geoid undulation differences**

GCP Name	$N_{GPS \text{ and leveling - } N_{EGM2008}}$ in meter	$N_{GPS \text{ and leveling - } N_{EIGEN-6C4}}$ in meter	$N_{GPS \text{ and leveling - } N_{GECO}}$ in meter	$N_{GPS \text{ and leveling - } N_{XGM2019e-2159}}$ in meter
ASOSA	1.0671	1.2318	1.1793	2.1533
GONDER	0.8518	0.9417	0.9003	1.9927
GEWANE	1.513	1.4784	1.4737	2.638
D/BERHAN	0.9944	1.0546	1.0822	2.2469
DUKEM	1.0437	1.0579	1.0258	2.2279
BALE ROBE	-0.8729	-0.8345	-0.8853	0.232
JIMMA	1.0024	1.0209	1.0957	2.2468
BEDELLE	1.1983	1.1927	1.1958	2.4308
GORE	1.0575	1.1034	1.0972	2.288
TIE	1.4313	1.365	1.4686	2.4968
TULU AMARA	1.3356	1.3594	1.4095	2.5242
WOLONKOMI	0.4124	0.4632	0.4751	1.709
BMDZ1	1.3611	1.3655	1.3214	2.552
BMJ1a	1.0864	1.0902	1.046	2.2835
BMJ2a	1.273	1.2697	1.2216	2.4849
BMJ3a	1.2583	1.2517	1.2027	2.4658
BMJ4a	1.3316	1.3225	1.2736	2.5293
BMJ5a	1.3334	1.3203	1.2734	2.5033
BMAaa	1.3871	1.394	1.3522	2.5737
BMCaa	1.1798	1.1791	1.1324	2.3907

**Table 4. 4 The statistics of the geoid undulation differences**

$N_{GPS \text{ and leveling - } N_{Models}}$	Minimum in meter	Maximum in meter	Mean in meter	SD in meter	Variance in meter	RMSE in meter
$N_{GPS \text{ and leveling - } N_{EGM2008}}$	-0.8729	1.513	1.0623	0.5176	0.2679	1.176
$N_{GPS \text{ and leveling - } N_{EIGEN-6C4}}$	-0.8345	1.4784	1.0814	0.5024	0.2524	1.1871
$N_{GPS \text{ and leveling - } N_{GECO}}$	-0.8853	1.4737	1.0671	0.5102	0.2603	1.1772
$N_{GPS \text{ and leveling - } N_{XGM2019e-2159}}$	0.232	2.638	2.2485	0.5243	0.2749	2.3058

#### 4.1.4 Correlation Analysis of Geoid Undulation

The linear correlation coefficients were computed to measure the strength of a linear relationship of the geoid undulation obtained from the GPS and leveling data, and global gravity field models. The correlation coefficients value of the geoid undulation obtained from the GPS and leveling data, and the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159 are 0.9842, 0.9855, 0.9855, and 0.9836 respectively. Its value tells that the global gravity field models are strongly correlated with the values of the geoid undulation obtained from the GPS and leveling data. The following four figures show that the correlation of the geoid undulation obtained from the GPS and leveling data, and global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159.

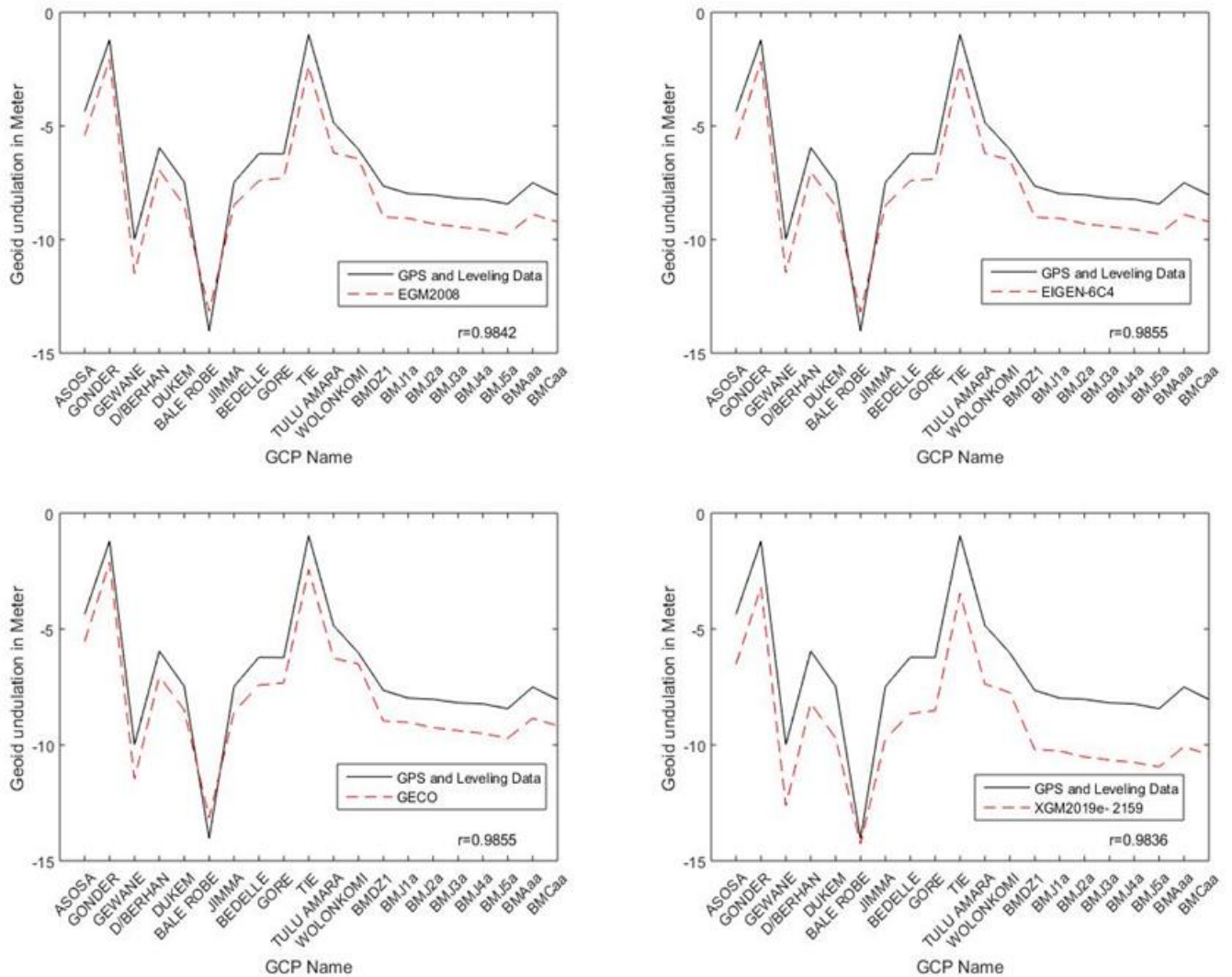


Figure 4. 1 The correlation between GPS and leveling data, and global gravity field models

#### 4.1.5 Hypothesis Testing of Geoid Undulation

These tests were performed to compare the variance of the geoid undulation differences obtained from the GPS and leveling data and GGMs that are used to select the best global gravity field model. For the comparison of the variance of the geoid undulation differences at 95% of probability level, the F test was used. Using the combinations of the 0.025 statistical table since the significance level equals 0.05 and the degree of freedom (n-1) equals 19, then the critical value is 2.532. This is a two-tailed test. The test value was computed between  $N_{\text{GPS and leveling}} - N_{\text{XGM2019e-2159}}$  and the other geoid undulation differences. Because  $N_{\text{GPS and leveling}} - N_{\text{XGM2019e-2159}}$  have larger variances than the other geoid undulation differences. The test value of the  $N_{\text{GPS and leveling}} - N_{\text{XGM2019e-2159}}$  with the  $N_{\text{GPS and leveling}} - N_{\text{EGM2008}}$ ,  $N_{\text{GPS and leveling}} - N_{\text{EIGEN-6C4}}$  and  $N_{\text{GPS and leveling}} - N_{\text{GECO}}$  are 1.0259, 1.089, and 1.0562 respectively. The test value is less than the critical value, because of this reason the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159 that are used to compute the geoid undulation are the same.

## 4.2 Results of Relative Difference in Height

### 4.2.1 Computed Height Differences Using Leveling Data

The height difference between the two benchmark stations was computed using spirit leveling data. Table (4.5) shows the computed spirit leveling height differences between two benchmark stations.

**Table 4. 5 Computed spirit leveling based height differences between the stations**

Line	From	To	Height Differences in meter
A	DES-320	DES-42	0.5328
B	DES-330	DES-320	17.5031
C	DES-42	DES-330	16.9703
D	DES-330	DES-36	8.0711
E	DES-36	DES-42	8.8991
F	DES-42	DES-27	5.0015
G	DES-27	DES-36	13.9007
H	DES-36	DES-12	13.1884
I	DES-12	DES-27	0.7122

### 4.2.2 Computed Height Differences Using GGMs Model

The height difference between two benchmark stations was computed using a global gravity field model. The global gravity field models used to compute the height difference includes EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159. Table (4.6) shows the global gravity field models based on the height difference between two benchmark stations.

**Table 4. 6 Computed global gravity field models based height differences between the stations**

Line	From	To	Height Differences in meter			
			EGM2008	EIGEN-6C4	GECO	XGM2019e-2159
A	DES-320	DES-42	0.7164	0.7166	0.7172	0.7161
B	DES-330	DES-320	17.8029	17.8031	17.8037	17.8026
C	DES-42	DES-330	17.0865	17.0865	17.0865	17.0865
D	DES-330	DES-36	8.0861	8.0859	8.0852	8.0866
E	DES-36	DES-42	9.0004	9.0006	9.0013	8.9999
F	DES-42	DES-27	5.3115	5.3113	5.3103	5.3122
G	DES-27	DES-36	14.3119	14.3119	14.3116	14.3121
H	DES-36	DES-12	13.638	13.638	13.6373	13.6386
I	DES-12	DES-27	0.6739	0.6739	0.6743	0.6735

### 4.2.3 Comparison of Leveling Data with GGMs Model

In this section, the differences between spirits leveling based on height difference and GGMs based on height difference of the same 9 lines were obtained. The statistics of the differences are also shown with respect to the minimum of the differences, maximum of the differences, mean of the differences, standard deviation of the differences, a variance of the differences, and root mean square error of the differences. Table (4.7) shows the height differences between leveling data and global gravity field models. Table (4.8) shows the statistics of the height differences between leveling data and global gravity field models.

**Table 4. 7 The differences between leveling data and global gravity field models**

Line	$dl_{\text{Spirit leveling}} - dl_{\text{EGM2008}}$ in meter	$dl_{\text{Spirit leveling}} - dl_{\text{EIGEN-6C4}}$ in meter	$dl_{\text{Spirit leveling}} - dl_{\text{GECO}}$ in meter	$dl_{\text{Spirit leveling}} - dl_{\text{XGM2019e-2159}}$ in meter
A	-0.1836	-0.1838	-0.1844	-0.1833
B	-0.2998	-0.3	-0.3006	-0.2995
C	-0.1162	-0.1162	-0.1162	-0.1162
D	-0.015	-0.0148	-0.0141	-0.0155
E	-0.1013	-0.1015	-0.1022	-0.1008
F	-0.31	-0.3098	-0.3088	-0.3107
G	-0.4112	-0.4112	-0.4109	-0.4114
H	-0.4496	-0.4496	-0.4489	-0.4502
I	0.0383	0.0383	0.0379	0.0387

**Table 4. 8 The statistics of differences between leveling data and global gravity field models**

$dl_{\text{Spirit leveling}} - dl_{\text{Models}}$	Minimum in meter	Maximum in meter	Mean in meter	SD in meter	Variance in meter	RMSE in meter
$dl_{\text{Spirit leveling}} - dl_{\text{EGM2008}}$	-0.4496	0.0383	-0.2054	0.172	0.0296	0.2617
$dl_{\text{Spirit leveling}} - dl_{\text{EIGEN-6C4}}$	-0.4496	0.0383	-0.2054	0.172	0.0296	0.2617
$dl_{\text{Spirit leveling}} - dl_{\text{GECO}}$	-0.4489	0.0379	-0.2054	0.1718	0.0295	0.2615
$dl_{\text{Spirit leveling}} - dl_{\text{XGM2019e-2159}}$	-0.4502	0.0387	-0.2054	0.1722	0.0297	0.2619

#### 4.2.4 Correlation Analysis of Relative Difference in Height

The linear correlation coefficients were computed to measure the strength of a linear relationship of the relative difference in height obtained from the leveling data and global gravity field models. The correlation coefficient value of the height difference obtained from the leveling data and the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159 is 0.9997. The correlation coefficient value of each model is the same. Its value tells that the global gravity field models are strongly correlated with the values of the relative difference in height obtained from the leveling data. The following four figures show that the correlation of the relative difference in height obtained from the leveling data and global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159.

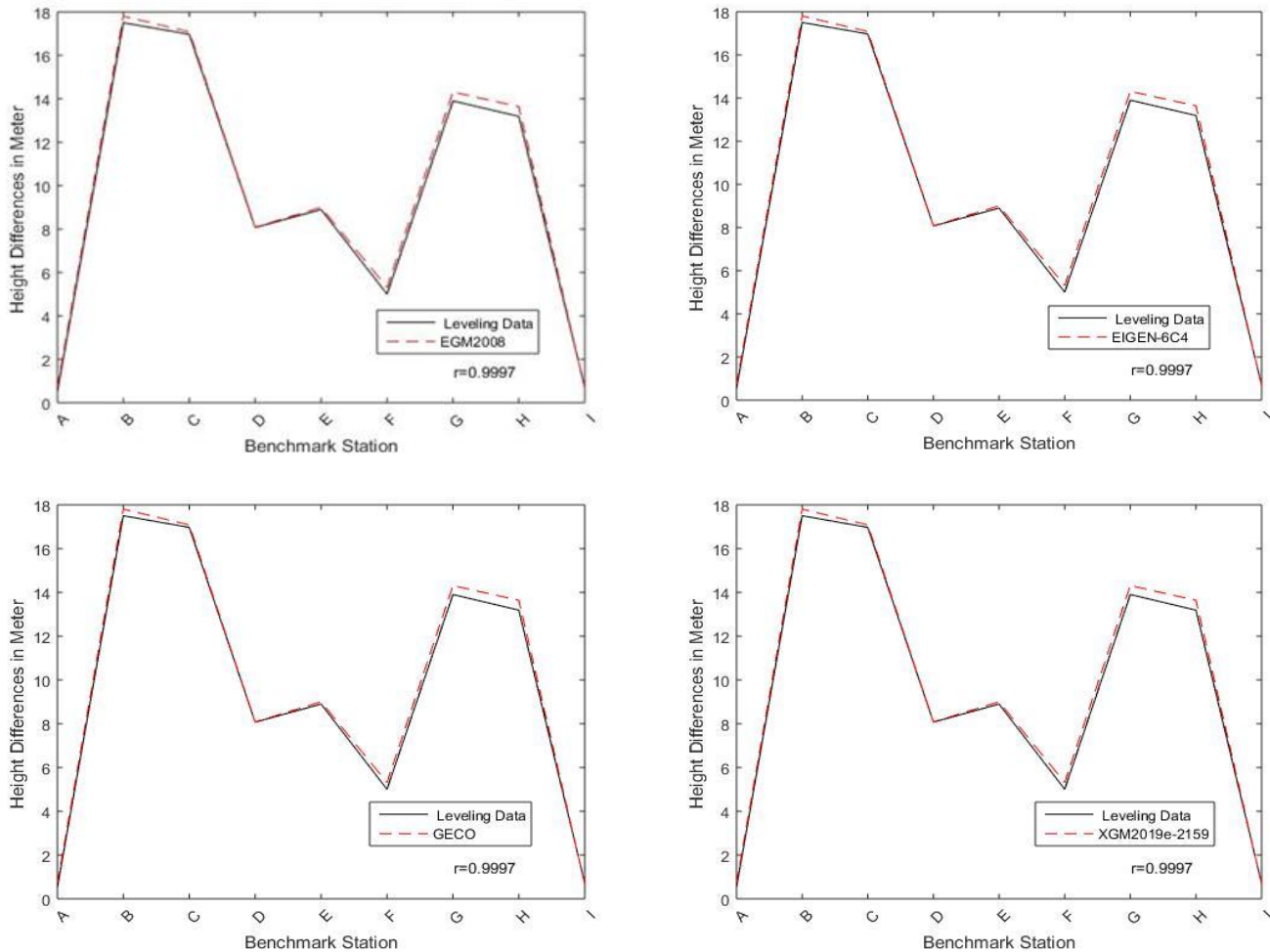


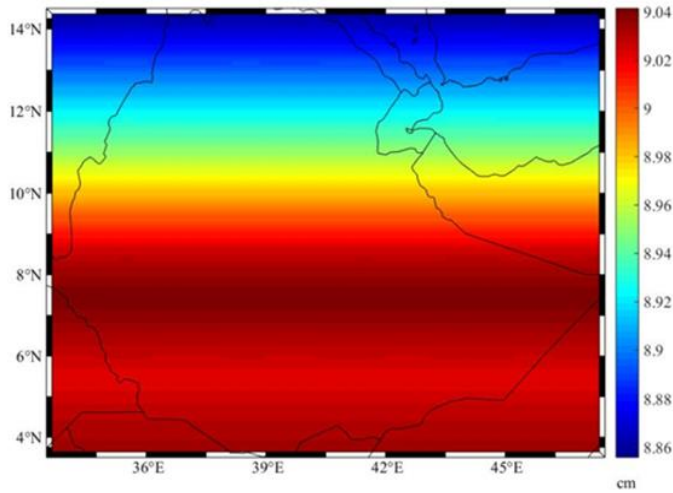
Figure 4. 2 The correlation between leveling data and global gravity field models

#### 4.2.5 Hypothesis Testing of Relative Difference in Height

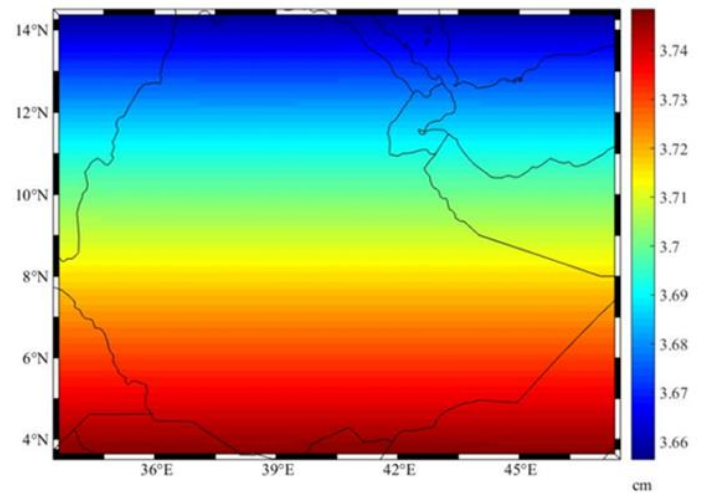
These tests were performed to compare the variance of the differences obtained from the spirits leveling based on height difference and GGMs based on height difference that is used to select the best global gravity field model. For the comparison of the variance of the differences at 95% of probability level, the F test was used. Using the combinations of the 0.025 statistical table since the significance level equals 0.05 and the degree of freedom (n-1) equals 8, then the critical value is 4.43. This is a two-tailed test. The test value was computed between  $dl_{\text{Spirit leveling}} - dl_{\text{XGM2019e-2159}}$  and the other height differences. Because  $dl_{\text{Spirit leveling}} - dl_{\text{XGM2019e-2159}}$  have larger variances than the other height differences. The test value of the  $dl_{\text{Spirit leveling}} - dl_{\text{XGM2019e-2159}}$  with the  $dl_{\text{Spirit leveling}} - dl_{\text{EGM2008}}$ ,  $dl_{\text{Spirit leveling}} - dl_{\text{EIGEN-6C4}}$ , and  $dl_{\text{Spirit leveling}} - dl_{\text{GECO}}$  are 1.0025, 1.0024, and 1.0052 respectively. The test value is less than the critical value, because of this reason the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159 that are used to compute the relative height difference are the same.

### 4.3 Results of Error of the Geoid Undulation Generated by GGMs

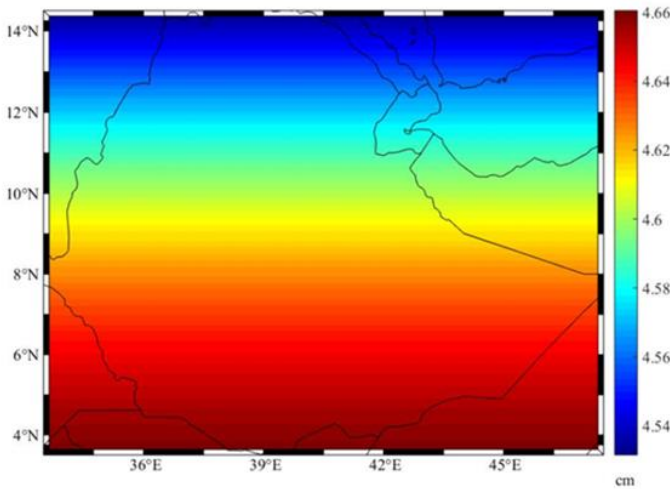
According to the propagation law of random errors, the errors of the geoid undulation were computed using a spherical harmonic coefficient. The error of spherical harmonic coefficients can propagate to the error of geoid undulation. The following four figures show that the errors of the geoid undulation computed by the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159.



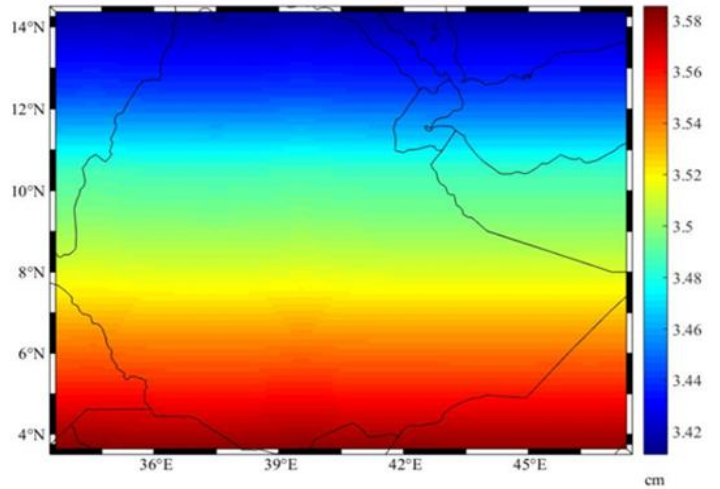
Error of geoid undulation computed by EGM2008



Error of geoid undulation computed by EIGEN-6C4



Error of geoid undulation computed by GECO



Error of geoid undulation computed by XGM2019e-2159

**Figure 4. 3 Errors of the geoid undulation computed by the global gravity field models**

#### 4.4 Discussion

This study has evaluated the most recently released global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159. The evaluation method involves the comparison of the geoid undulations and relative differences in heights computed from the global gravity field models with those computed from the GPS and leveling data. The results of the geoid undulation differences between the GPS and leveling data, and the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159 has a standard deviation of 0.5176m, 0.5024m, 0.5102m, and 0.5243m, with the root mean square error of 1.176m, 1.1871m, 1.1772 m and 2.3058 m respectively. Similarly, the results of the relative differences in height between spirits leveling and the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e\_2159 has a standard deviation of 0.172m, 0.172m, 0.1718m, and 0.1722m, with the root mean square error of 0.2617m, 0.2617m, 0.2615m and 0.2619m respectively. The error of the geoid undulation differences is much greater than the error of relative differences in height. Because the error of the geoid undulation differences contains the geoid error, random error and the datum shift between the geoid and MSL. There is no way to accurately measure the geoid so it was roughly approximated by MSL (Fraczek, 2003). The results of error of the geoid undulation generated by the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159 reach up to 9.04 cm, 3.74 cm, 4.66 cm, and 3.58 cm respectively.

The correlation coefficients value of the geoid undulation obtained from the GPS and leveling data, and the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159 are 0.9842, 0.9855, 0.9855, and 0.9836 respectively. Its value tells that the global gravity field models are strongly correlated with the values of the geoid undulation obtained from the GPS and leveling data. Similarly, the correlation coefficients value of the relative difference in height obtained from the leveling data, and the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159 is 0.9997. The correlation coefficient value of each model is the same. Its value tells that the global gravity field models are strongly correlated with the values of the relative difference in height obtained from the leveling data. The hypothetical tests were performed to compare the variance of the differences in geoid undulation and relative differences in height obtained from the GPS and leveling data, and global

gravity field models that are used to select the best model. Based on the attained results, the hypothetical test tells that the variances of each model with respect to each other are significantly the same at 95% confidence interval.

Many studies evaluated the global gravity field model in many countries using several methods. Studies in Ghana, Nigeria, and Uganda compared EGM2008 with GPS and leveling observations and the standard deviation showed that 3.47678 m, 0.419m, and 0.255m respectively (Abubeker and Workaferahu, 2009, Oluyori et al., 2018, Peprah et al., 2017). On the other hand, the standard deviation of the geoid undulation differences between the GPS and leveling data and the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159 in Egypt are  $\pm 0.23m$ ,  $\pm 0.30m$ ,  $\pm 0.30m$  and  $\pm 0.30 m$  respectively (El Shouny et al., 2018). (Foroughi et al., 2017) did the same test in Iran and the results showed that the difference values reach up to  $\sim 2$  m for geoid undulations. The evaluation of the global geopotential model over the area of Java Island in Indonesia showed that the mean difference value of 0.99 m and 0.701 m, absolutely and relatively (Heliani, 2016). Based on the absolute and relative methods undertaken the EGM2008 is superior to earlier EGM96 over the area of Greece (Gikas et al., 2013).

Similar studies have been done on the validation of the EGM2008 model over the area of Addis Ababa and the western part of Ethiopia, and the results showed that the mismatches of orthometric heights and mismatches of geoids are  $\sim 2$ cm and  $\sim 2.6$ cm respectively (Birbiraw, 2013). The relation between the EGM2008 based orthometric height differences with height differences determined from spirit leveling around Debre Birhan has a standard deviation of 2.41cm with a mean of -3.14cm (Worku, 2013). (Bedada, 2010) compared leveling heights with the EGM2008 derived geopotential heights in Ethiopia and obtained an improved standard deviation from 5.3 cm with EGM2008 to 4.6 cm and 3.9 cm when airborne gravity, and airborne gravity plus topographic models are included, respectively. In Fennoscandia, the computed propagated error of the geoid on the sphere reaches up to 7 cm but when it is computed on the ellipsoid the error reaches up to 3 m (Eshagh, 2013).

## CHAPTER 5 CONCLUSIONS AND RECOMMENDATIONS

### 5.1 Conclusions

A geoid model is important for converting the GPS-based ellipsoidal heights into orthometric heights which are useful in most geodetic activities, surveying, and mapping applications. However, it is well known that the accuracy of the determined local geoidal heights has an influence on converting GPS-based ellipsoidal heights to orthometric heights. So far, there is no available precise local geoid model that covers the entire territories of Ethiopia. A large number of global gravity field models have been released and used in geoid modeling. For this model to be used for geodetic activities anywhere on the Earth there is a need to quantify its accuracy. This study has evaluated the most recently released global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159. The evaluation method involves the comparison of the geoid undulations and relative differences in heights computed from the global gravity field models with those computed from the GPS and leveling data. The assumption made here is that the value obtained from the GPS and leveling consider as a standard to which the value obtained from the model.

The linear correlation coefficients were computed to measure the strength of a linear relationship of the geoid undulation and relative differences in height obtained from the GPS and leveling data, and global gravity field models. The global gravity field models are strongly correlated with the GPS and leveling data. The results of the geoid undulation differences between the GPS and leveling data, and the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159 has a standard deviation of 0.5176m, 0.5024m, 0.5102m, and 0.5243m, with the root mean square error of 1.176m, 1.1871m, 1.1772 m and 2.3058 m respectively. Similarly, the results of the relative differences in height between spirits leveling and the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159 has a standard deviation of 0.172m, 0.172m, 0.1718m, and 0.1722m, with the root mean square error of 0.2617m, 0.2617m, 0.2615m and 0.2619m respectively.

The results of error of the geoid undulation generated by the global gravity field models include EGM2008, EIGEN-6C4, GECO, and XGM2019e-2159 reach up to 9.04 cm, 3.74 cm, 4.66 cm, and 3.58 cm respectively. The hypothetical tests were performed to compare the variance of the differences in geoid undulation and relative differences in height obtained from the GPS and leveling data, and global gravity field models that are used to select the best model. The variances of each model with respect to each other are significantly the same at 95% confidence interval. Based on the attained results, it is concluded that the global gravity field models can be used to prepare a topographical map of 4 meter and 1 meter contour interval, absolutely and relatively.

## 5.2 Recommendations

To apply the global gravity field models for all over the country, validation has to be done for the remaining parts of Ethiopia because the GPS and Blue Nile Geodetic benchmark points that are used for this thesis work do not cover the whole Ethiopia. It is highly recommended to distribute and collocate the Blue Nile geodetic benchmark points with a precise differential GPS observation that is required to accurately validate the accuracy of the global gravity field models.

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## APPENDICES

### Appendix A

#### Leveling Field book-1

Observer Name: Muhamed Melesse    Instrument: spirit level    Project: Thesis    Date: 16/11/2021

Station	BS			Mean = $\frac{U + M + L}{3}$	FS			Mean = $\frac{U + M + L}{3}$	HI	RL
	U	M	L		U	M	L			
DES-320	1.022	0.945	0.867	0.9447					2531.2111	2530.2664
	0.283	0.165	0.046	0.1647	2.683	2.563	2.442	2.5627	2528.8131	2528.6484
	0.521	0.333	0.146	0.3333	2.83	2.675	2.521	2.6753	2526.4711	2526.1377
	2.581	2.445	2.311	2.4457	1.466	1.257	1.047	1.2567	2527.6601	2525.2144
	2.556	2.445	2.333	2.4447	0.391	0.221	0.049	0.2203	2529.8844	2527.4397
DES- 42	0.441	0.321	0.201	0.321	0.283	0.153	0.023	0.153	2530.0524	2529.7314
	0.768	0.664	0.559	0.6637	2.698	2.578	2.458	2.578	2528.1381	2527.4744
	0.564	0.526	0.488	0.526	2.37	2.25	2.13	2.25	2526.4141	2525.8881
	0.298	0.266	0.234	0.266	2.999	2.968	2.937	2.968	2523.7121	2523.4461
	0.415	0.384	0.354	0.3843	3.11	3.049	2.989	3.0493	2521.0471	2520.6627
	0.564	0.496	0.428	0.496	3.625	3.52	3.415	3.52	2518.0231	2517.5271
	0.67	0.628	0.586	0.628	3.17	3.09	3.01	3.09	2515.5611	2514.9331
DES-330	2.749	2.695	2.641	2.695	2.86	2.81	2.76	2.81	2515.4461	2512.7511
	2.9	2.84	2.78	2.84	0.549	0.512	0.475	0.512	2517.7741	2514.9341
	2.924	2.85	2.775	2.8497	0.322	0.244	0.166	0.244	2520.3797	2517.5301
	3.37	3.298	3.226	3.298	0.229	0.166	0.103	0.166	2523.5117	2520.2137
	3.249	3.21	3.172	3.2103	0.131	0.087	0.043	0.087	2526.6351	2523.4247
	0.648	0.477	0.307	0.4773	0.734	0.7	0.666	0.7	2526.4124	2525.9351
	2.5	2.349	2.199	2.3493	1.02	0.81	0.6	0.81	2527.9517	2525.6024

	2.359	2.258	2.157	2.258	0.301	0.184	0.067	0.184	2530.0257	2527.7677
	1.884	1.834	1.784	1.834	0.51	0.42	0.329	0.4197	2531.4401	2529.6061
DES-320					1.217	1.177	1.137	1.177		2530.2631

**Note:** The given elevation of benchmark DES-320 is 2530.2664 and it's taken from EGM2008. The number was rounded to four decimal places. The allowable error is 3.5869 mm but the closure error is 3.3333 mm.

**Leveling Field book-2**

Observer Name: Muhamed Melesse    Instrument: spirit level    Project: Thesis    Date: 17/11/2021

Station	BS			Mean = $\frac{U + M + L}{3}$	FS			Mean = $\frac{U + M + L}{3}$	HI	RL
	U	M	L		U	M	L			
DES-42	0.441	0.321	0.201	0.321					2530.0524	2529.7314
	0.768	0.664	0.559	0.6637	2.698	2.578	2.458	2.578	2528.1381	2527.4744
	0.564	0.526	0.488	0.526	2.37	2.25	2.13	2.25	2526.4141	2525.8881
	0.298	0.266	0.234	0.266	2.999	2.968	2.937	2.968	2523.7121	2523.4461
	0.415	0.384	0.354	0.3843	3.11	3.049	2.989	3.0493	2521.0471	2520.6627
	0.564	0.496	0.428	0.496	3.625	3.52	3.415	3.52	2518.0231	2517.5271
	0.67	0.628	0.586	0.628	3.17	3.09	3.01	3.09	2515.5611	2514.9331
DES-330	1.765	1.665	1.565	1.665	2.86	2.81	2.76	2.81	2514.4161	2512.7511
	2.11	1.825	1.54	1.825	1.046	0.888	0.73	0.888	2515.3531	2513.5281
	1.31	1.01	0.71	1.01	0.55	0.315	0.08	0.315	2516.0481	2515.0381
	2.925	2.625	2.325	2.625	0.705	0.385	0.065	0.385	2518.2881	2515.6631
	1.855	1.779	1.704	1.7793	1.52	1.29	1.06	1.29	2518.7774	2516.9981
	2.965	2.905	2.845	2.905	0.706	0.666	0.626	0.666	2521.0164	2518.1114
DES-36	0.232	0.188	0.144	0.188	0.232	0.188	0.144	0.188	2521.0164	2520.8284
	0.706	0.666	0.626	0.666	2.965	2.905	2.845	2.905	2518.7774	2518.1114
	1.42	1.17	0.92	1.17	1.855	1.8	1.745	1.8	2518.1474	2516.9774
	0.82	0.47	0.12	0.47	2.78	2.505	2.23	2.505	2516.1124	2515.6424

	0.79	0.56	0.33	0.56	1.34	1.06	0.78	1.06	2515.6124	2515.0524
	3.685	3.522	3.359	3.522	1.537	1.357	1.177	1.357	2517.7774	2514.2554
	3.686	3.618	3.55	3.618	0.184	0.144	0.104	0.144	2521.2514	2517.6334
	4.101	4.01	3.919	4.01	0.242	0.202	0.162	0.202	2525.0594	2521.0494
	2.436	2.403	2.369	2.4027	0.261	0.221	0.181	0.221	2527.2411	2524.8384
	2.775	2.73	2.685	2.73	0.384	0.354	0.324	0.354	2529.6171	2526.8871
	1.709	1.686	1.664	1.6863	0.835	0.81	0.785	0.81	2530.4934	2528.8071
DES-42					0.798	0.758	0.718	0.758		2529.7354

**Note:** The given elevation of benchmark DES-42 is 2529.7314 and it's taken from leveling field book-1. The allowable error is 4.1865 mm but the closure error is 4.0000 mm.

### Leveling Field book-3

Observer Name: Muhamed Melesse    Instrument: spirit level    Project: Thesis    Date: 18/11/2021

Station	BS			Mean = $\frac{U+M+L}{3}$	FS			Mean = $\frac{U+M+L}{3}$	HI	RL
	U	M	L		U	M	L			
DES-36	0.232	0.188	0.144	0.188					2521.0164	2520.8284
	0.706	0.666	0.626	0.666	2.965	2.905	2.845	2.905	2518.7774	2518.1114
	1.42	1.17	0.92	1.17	1.855	1.8	1.745	1.8	2518.1474	2516.9774
	0.82	0.47	0.12	0.47	2.78	2.505	2.23	2.505	2516.1124	2515.6424
	0.79	0.56	0.33	0.56	1.34	1.06	0.78	1.06	2515.6124	2515.0524
	3.685	3.522	3.359	3.522	1.537	1.357	1.177	1.357	2517.7774	2514.2554
	3.686	3.618	3.55	3.618	0.184	0.144	0.104	0.144	2521.2514	2517.6334
	4.101	4.01	3.919	4.01	0.242	0.202	0.162	0.202	2525.0594	2521.0494
	2.436	2.403	2.369	2.4027	0.261	0.221	0.181	0.221	2527.2411	2524.8384
	2.775	2.73	2.685	2.73	0.384	0.354	0.324	0.354	2529.6171	2526.8871
	1.709	1.686	1.664	1.6863	0.835	0.81	0.785	0.81	2530.4934	2528.8071
DES-42	1.75	1.655	1.56	1.655	0.798	0.758	0.718	0.758	2531.3904	2529.7354
	2.3	2.22	2.14	2.22	1.5	1.42	1.34	1.42	2532.1904	2529.9704

	1.79	1.63	1.47	1.63	1.146	1.086	1.026	1.086	2532.7344	2531.1044
	1.23	1.095	0.96	1.095	1.475	1.28	1.085	1.28	2532.5494	2531.4544
	1.486	0.881	0.274	0.8803	1.918	1.77	1.622	1.77	2531.6597	2530.7794
	1.928	1.784	1.64	1.784	0.62	0.485	0.35	0.485	2532.9587	2531.1747
	1.906	1.746	1.586	1.746	0.45	0.27	0.09	0.27	2534.4347	2532.6887
	1.418	1.328	1.238	1.328	1.838	1.688	1.538	1.688	2534.0747	2532.7467
	1.314	1.24	1.166	1.24	1.805	1.655	1.51	1.6567	2533.6581	2532.4181
	1.785	1.725	1.665	1.725	1.195	1.045	0.895	1.045	2534.3381	2532.6131
	2.036	1.996	1.956	1.996	1.348	1.268	1.188	1.268	2535.0661	2533.0701
DES-27	0.367	0.31	0.253	0.31	0.374	0.326	0.278	0.326	2535.0501	2534.7401
	0.75	0.58	0.41	0.58	1.717	1.633	1.55	1.6333	2533.9967	2533.4167
	1.145	1.06	0.975	1.06	1.58	1.28	0.98	1.28	2533.7767	2532.7167
	0.099	0.059	0.019	0.059	2.639	2.606	2.573	2.606	2531.2297	2531.1707
	0.63	0.6	0.57	0.6	2.586	2.542	2.498	2.542	2529.2877	2528.6877
	0.479	0.439	0.399	0.439	2.801	2.729	2.658	2.7293	2526.9974	2526.5584
	0.256	0.194	0.131	0.1937	2.868	2.812	2.757	2.8123	2524.3787	2524.1851
	0.26	0.205	0.15	0.205	2.634	2.584	2.534	2.584	2521.9997	2521.7947
	0.384	0.336	0.288	0.336	2.508	2.454	2.4	2.454	2519.8817	2519.5457
	1.64	1.565	1.49	1.565	2.43	2.37	2.31	2.37	2519.0767	2517.5117
	1.133	1.033	0.933	1.033	1.08	0.94	0.8	0.94	2519.1697	2518.1367
	1.855	1.8	1.745	1.8	2.298	2.188	2.078	2.188	2518.7817	2516.9817
	2.965	2.905	2.845	2.905	0.706	0.666	0.626	0.666	2521.0207	2518.1157
DES-36					0.232	0.188	0.144	0.188		2520.8327

**Note:** The given elevation of benchmark DES-36 is 2520.8284 and it's taken from leveling field book-2. The allowable error is 4.6737mm but the closure error is 4.3333mm.

**Leveling Field book-4**

Observer Name: Muhamed Melesse Instrument: spirit level Project: Thesis Date: 19/11/2021

Station	BS			Mean = $\frac{U + M + L}{3}$	FS			Mean = $\frac{U + M + L}{3}$	HI	RL
	U	M	L		U	M	L			
DES-27	0.367	0.31	0.253	0.31					2535.0501	2534.7401
	0.75	0.58	0.41	0.58	1.717	1.633	1.55	1.6333	2533.9967	2533.4167
	1.145	1.06	0.975	1.06	1.58	1.28	0.98	1.28	2533.7767	2532.7167
	0.099	0.059	0.019	0.059	2.639	2.606	2.573	2.606	2531.2297	2531.1707
	0.63	0.6	0.57	0.6	2.586	2.542	2.498	2.542	2529.2877	2528.6877
	0.479	0.439	0.399	0.439	2.801	2.729	2.658	2.7293	2526.9974	2526.5584
	0.256	0.194	0.131	0.1937	2.868	2.812	2.757	2.8123	2524.3787	2524.1851
	0.26	0.205	0.15	0.205	2.634	2.584	2.534	2.584	2521.9997	2521.7947
	0.384	0.336	0.288	0.336	2.508	2.454	2.4	2.454	2519.8817	2519.5457
	1.64	1.565	1.49	1.565	2.43	2.37	2.31	2.37	2519.0767	2517.5117
	1.133	1.033	0.933	1.033	1.08	0.94	0.8	0.94	2519.1697	2518.1367
	1.855	1.8	1.745	1.8	2.298	2.188	2.078	2.188	2518.7817	2516.9817
	2.965	2.905	2.845	2.905	0.706	0.666	0.626	0.666	2521.0207	2518.1157
DES-36	0.232	0.188	0.144	0.188	0.232	0.188	0.144	0.188	2521.0207	2520.8327
	0.706	0.666	0.626	0.666	2.965	2.905	2.845	2.905	2518.7817	2518.1157
	2.298	2.188	2.078	2.188	1.855	1.8	1.745	1.8	2519.1697	2516.9817
	1.08	0.94	0.8	0.94	1.133	1.033	0.933	1.033	2519.0767	2518.1367
	2.43	2.355	2.279	2.3547	1.64	1.565	1.49	1.565	2519.8664	2517.5117
	2.508	2.454	2.4	2.454	0.384	0.336	0.288	0.336	2521.9844	2519.5304
	2.634	2.584	2.534	2.584	0.26	0.205	0.15	0.205	2524.3634	2521.7794
	2.868	2.812	2.757	2.8123	0.256	0.216	0.176	0.216	2526.9597	2524.1474
	2.801	2.729	2.658	2.7293	0.479	0.439	0.399	0.439	2529.2501	2526.5207
	2.586	2.542	2.498	2.542	0.63	0.6	0.57	0.6	2531.1921	2528.6501
	2.639	2.606	2.573	2.606	0.099	0.059	0.019	0.059	2533.7391	2531.1331
	1.58	1.28	0.98	1.28	1.145	1.06	0.975	1.06	2533.9591	2532.6791
	2.185	2.07	1.955	2.07	0.75	0.58	0.41	0.58	2535.4491	2533.3791

	1.945	1.775	1.605	1.775	0.6	0.495	0.39	0.495	2536.7291	2534.9541
	0.382	0.262	0.142	0.262	1.75	1.65	1.55	1.65	2535.3411	2535.0791
	1.348	1.228	1.108	1.228	2.232	2.102	1.972	2.102	2534.4671	2533.2391
	1.816	1.742	1.668	1.742	1.4	1.28	1.16	1.28	2534.9291	2533.1871
	1.71	1.646	1.581	1.6457	1.238	1.164	1.09	1.164	2535.4107	2533.7651
DES-12	1.406	1.38	1.354	1.38	1.406	1.38	1.354	1.38	2535.4107	2534.0307
	1.238	1.164	1.09	1.164	1.71	1.646	1.581	1.6457	2534.9291	2533.7651
	1.4	1.269	1.139	1.2693	1.816	1.742	1.668	1.742	2534.4564	2533.1871
	2.232	2.102	1.972	2.102	1.348	1.228	1.108	1.228	2535.3304	2533.2284
	1.676	1.576	1.476	1.576	0.382	0.262	0.142	0.262	2536.6444	2535.0684
	1.34	1.26	1.18	1.26	1.901	1.731	1.561	1.731	2536.1734	2534.9134
	1.385	1.33	1.275	1.33	1.863	1.793	1.723	1.793	2535.7104	2534.3804
DES-27					1.026	0.966	0.906	0.966		2534.7444

**Note:** The given elevation of benchmark DES-27 is 2534.7401 and it's taken from leveling field book-3. The allowable error is 4.4623mm but the closure error is 4.3333mm.

**Computed height differences between two benchmark stations**

Number of Lines	From	To	Height Differences in meter		
			Without adjustment	with least square adjustment	with gravity correction
1	DES-320	DES-42	0.535	0.5335	0.5328
2	DES-330	DES-320	17.512	17.5136	17.5031
3	DES-42	DES-330	16.9803	16.9801	16.9703
4	DES-330	DES-36	8.0773	8.0755	8.0711
5	DES-36	DES-42	8.907	8.9046	8.8991
6	DES-42	DES-27	5.0047	5.0043	5.0015
7	DES-27	DES-36	13.9073	13.9089	13.9007
8	DES-36	DES-12	13.198	13.196	13.1884
9	DES-12	DES-27	0.7137	0.7128	0.7122