TIDAL FORCE AND TORQUE ON THE CRUST OF NEUTRON STARE

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Signature of Author

For my brother.

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Abstract

The long term evolution of the relative rotation of the core super fluid in a neutron stare with respect to the rest of the star is determined through Legase modal.The core superfluid rotates at a different rate ,while spinning down at the same steady-state rate as the rest of the star, because of the assumed pinning between the superfluid vortices and the superconductor fluxoids.We find that the magnitude of this rotational lag changes with time and also depends on the distance from the rotation axis; the core superfluid supports an evolving pattern of differential rotation. We argue that the predicted change of the lag might occur as discrete events which could result in a sudden rise of the spin frequency of the crust of a neutron star, as is observed at glitches in radio pulsars. This new possibility for the triggering cause of glitches in radio pulsars is further supported by an estimate of the total predicted excess angular momentum reservoir of the core superfluid. The model seems also to offer resolutions for some other aspects of the observational data on glitches. The goal of this project is to show the tidal effect on the differential motion of the crust with respect to the core of the neutron star from that we find the redistribution of angular velocity and tidal locking time for the crust core interaction of neutron stare.

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Introduction

Glitches are observed in radio pulsars as sudden changes .Ω_c in the rotation frequency of the crust, with observed values of the jump in the range$10^{-9} \leq \Delta\Omega_c/\Omega_c \leq 10^{-6}$. In younger pulsars the jump in Ω_c is also accompanied by an increase.$\Delta\dot{\Omega}_c$ in the observed spin-down rate $\dot{\Omega}_c$ of the crust, which causes a recovery or relaxation back towards the pre-glitch behaviour of Ω_c over timescales of days to years (Radhakrishnan and Manchester 1969; Lyne 1995).It is generally understood that glitches should be caused by mechanisms related to the internal structure of the star. This is because no correlated variation in the electromagnetic signature (intensity, polarization, pulse profile, etc.) of a pulsar has been observed at the time of their glitches. The two generally accepted mechanisms for glitches thus invoke starquakes (Baym et al. 1969) and unpinning of the vortices of a superfluid component in the crust (Anderson Itoh 1975). In the latter mechanism, which is more relevant to the present discussion, a sudden release and rapid outward motion of a large number of otherwise pinned vortices act as the source of the excess angular momentum, which is transferred to the crust, hence causing the observed jump in Ω_c. Suggested mechanisms for the sudden release of a large number of initially pinned vortices include catastrophic unpinning due to an intrinsic instability, breaking down of the crustal lattice by magnetic stresses, and thermal instability resulting in an increase in the mutual friction between the vortices and the superfluid (Anderson and Itoh 1975; Ruderman 1976; Greenstein 1979; Jones 1991; Link and Epstein 1996; Ruderman, Zhu and Chen 1998). The core superfluid, on the other hand, is not commonly considered to play any major role in driving the glitches; there have been some earlier attempts in this regard which do not seem to have gained much support and acceptance (Packard 1972; Muslimov and Tsygan 1985; see, however, Sedrakian and Cordes 1997 for a recent suggestion based on a different model of pinning in the core from that invoked here). The coupled evolution of the neutron vortices and the proton fluxoids has nevertheless been discussed in various other respects, including its role in the post-glitch relaxation and also in driving glitches indirectly through crustal effects (Sauls 1989; Srinivasan et al. 1990; Jones 1991; Chau, Cheng and Ding 1992; Ruderman et al. 1998). Our aim here is to point at a so far unexplained property of the rotational evolution of the core superfluid that might serve to cause glitches, directly. This is suggested based on the calculated long-term evolution of the rotational lag between the superfluid core and its crest by tidal torque ; the latter being in corotation, on time-scales larger than a few thousand years, with the crust.So that after this time the pulsar will not show any glitch

The organization of the project is as follows ,In the first chapter we give definition and derivation of tidal force both Newtonanlly and relativestically .this leads to the discussion of tidal force on the gyroscope and its effect on the Shwarzchild spacetime

Beginning with chapter 2 we present the physics of neutron star that is how they formed ,rotate and their structure .In chapter 3 we study the effect of gravitomagnetism ,basic equation of gravitomagnatismand their physical meaning in terms of the Lense-Thirring effect .Finaly we discuss and made a conclusion on gravitomagnetic crest core coupling of the neutron star

Chapter 1

Tidal force

1.1 Newtonian tidal force

We begin with a discussion of gravitational effect of the earth on the drop of water . To sea this consider a spacecraft around the earth . The spacecraft is in free fall , the astronaut find themselves in zero gravitation environment. The reference frame attached to the spacecraft simulates an inertial reference frame; a test particle at rest relative to the spacecraft remain at rest , a test particle in motion remain in motion at a constant velocity. However, for an observer at rest relative to the fixed star, the spacecraft will be in accelerated motion.

We now ask ; are the gravitational effect of the earth completely eliminated by free fall ? Is there some local experiment that permit the astronaut to find out that they are falling in a gravitational field rather at rest in some region far away from any attracting mass ?

The answer is the astronaut can detect the gravitational field by tidal effects it produces. If the astronaut place a drop of liquid at the center of their spacecraft, they will find that this drop of liquid is not exactly spherical, but has two bulges (fig 1.1). One bulge point toward the earth, one away.

Sine, in the absence of external forces,surface tension would make the drop spherical, the deviation from a spherical indicate the existence of the gravitational field.The bulg

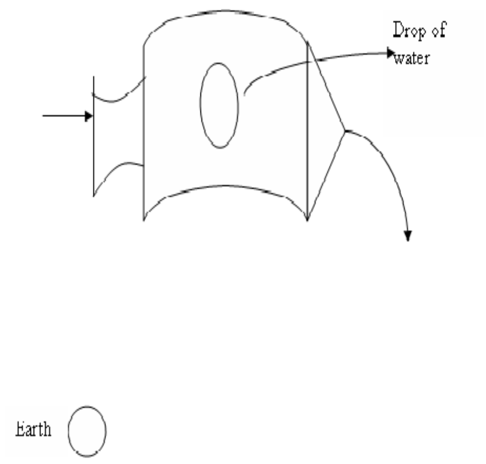

Figure 1.1: A space craft in orbit.the drop of water floating in this space craft is destroyed by the gravitational pull of the earth .

result from the inhomogeneity of the gravitational field:the end of the drop nearer the earth is pulled too much by gravitation, the other end is not pulled enough. The force that produce the bulges is called the tidal force[9]

We can calculate the tidal force as follows:Consider a reference point moving in free fall; take this point as the origin of a freely falling coordinate system with the Z-axis parallel to the radial line(fig1.2) A particle at position (0,0,z) in this reference frame

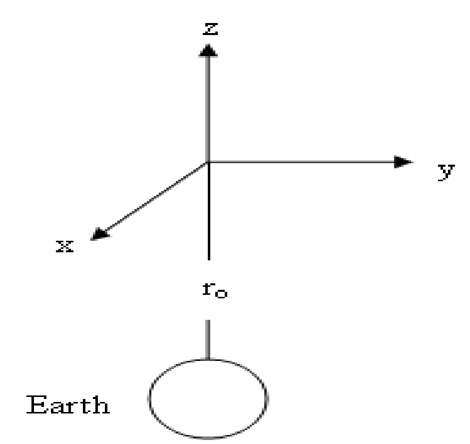

Figure 1.2: The x,y,z belongs to the reference frame in free fall, whose origin is instantaneously a distance r from the Earth.

experience a gravitational acceleration

$$-\frac{GM}{(r_0+z)^2} \tag{1.1.1}$$

Since the origin has acceleration $-\frac{GM}{r_0^2}$,the acceleration of the particle relative to the origin is, in the limit of small z,

$$-\frac{GM}{(r_0+z)^2}+\frac{GM}{r_0^2}=2z\frac{GM}{r_0^3}, \tag{1.1.2}$$

Hence, relative to our origin the particle move as though subjected to a force

$$f_z = 2z\frac{GMm}{r_0^3} \tag{1.1.3}$$

This is the tidal force. Note that it directly proportional to the distance of the particle from the origin and is repulsive. For a particle at point (0,y,0, the tidal force point in the -y direction and is given by

$$f_y = -y\frac{GMm}{r_0^3} \tag{1.1.4}$$

This is the tidal restoring force, towards the origin.For a particle at point (x,0,0), there is a similar restoring force

$$f_x = -x\frac{GMm}{r_0^3} \tag{1.1.5}$$

If the particle has simultaneous x,y and z displacements, then all three forces $(1.1.3), (1.1.4), and (1.1.$ are of course present simultaneously Note that these expression for the tidal force only apply if the displacement x,y,z are small compared to r_0

From these result it is obvious that a drop of liquid, consisting of many particles, will be stretched in radial direction and compressed in the transverse direction. In general, given an arbitrary gravitational field, the tidal force in a reference frame whose origin is in free fall can be expressed as follows

$$f^k = \sum_l x^l \frac{\partial F^k}{\partial x^l} = -\sum_l m \frac{\partial^2 \Phi}{\partial x^l \partial x^k} \tag{1.1.6}$$

Where F^k the gravitational force on the particle and Φ is the potential and where the derivatives are evaluated at the origin ($x^l = 0$) .Equation 1.1.6 is of course valid only near the origin (small x^l). We well call the quantity

$$R^k_{0l0} = -\frac{1}{mc^2} \frac{\partial F^k}{\partial x^l} = \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial x^k \partial x^l} \tag{1.1.7}$$

is the tidal tensor

The nine quantity R^k_{0l0} are components of the Riemann curvature tensor $R^\mu_{\nu\alpha\beta}$; the letter tensor describe the tidal field in the fore- dimensional space time of general relativity

The tidal force that acts on a particle placed at x^l (with $x^l \ll r_0$)is then.

$$f^k = -mc^2 \sum_l R^k_{0l0} x^l \tag{1.1.8}$$

which produce an acceleration

$$\frac{d^2 x^k}{dt^2} = -c^2 \sum_l R^k_{0l0} x^l \tag{1.1.9}$$

In the special case considered above, the tenserR^k_{0l0} has component;

$$R^k_{0l0} = \frac{GM}{r_0^3 c^2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2, \end{pmatrix} \tag{1.1.10}$$

Note that the first index k give the row and the second l the column

The tidal force given by equation (1.1.2),(1.1.3),and (1.1.4) satisfies the identity

$$\frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} = -\frac{GMm}{r_0^3} - \frac{-GMm}{r_0^3} + \frac{2GMm}{r_0^3} = 0 \tag{1.1.11}$$

That is the tidal force has zero divergence. This result hold in general for gravitational field in empty region of space We can prove this using 1.1.6

$$\begin{aligned}
\frac{\partial f^k}{\partial x^k} &= -\sum_{kl} \frac{\partial}{\partial x^k}\left(x^l m \frac{\partial^2 \Phi}{\partial x^k \partial x^l}\right) \\
&= -m \sum_{kl} \delta^l_k \frac{\partial^2 \Phi}{\partial x^k x^l} \\
&= -m \sum_{k} \frac{\partial^2 \Phi}{\partial x^k \partial x^k} = 0 \qquad (1.1.12)
\end{aligned}$$

The last equation hold true because in empty space, the gravitational potential satisfies the laplace equation. But in the presence of a mass density ρ,the divergence of the tidal force is given by

$$\frac{\partial f}{\partial x^k} = 4\pi m G \rho \qquad (1.1.13)$$

As we will sea later the tidal force given by general relativity (contained in equation of geodesic deviation) depend on the velocity of the particle with respect to the reference point . However, the general relativity tidal force agrees with above Newton value in the limit of weak gravitational field ($\frac{Gm}{rc^2}$) and low speed ($v \ll c$)

The detection of tidal field by means of deformation of the drops of water is not a practical method, because for a small drop, the surface tension will prevent the formation of tidal bulges. A somewhat more realistic method is the following; suppose the astronauts place a freely spinning rigid body in the spacecraft as shown in the figure 1.3 The tidal force will exert a torque on the rode and give it an angular acceleration.For the tidal field given by equation(1.1.3),(1.1.4),and (1.1.5), the torque about the x-axis is

$$\begin{aligned}
\tau_x &= \int \left[y(2z\frac{GM}{r_0^3}) - z(-y\frac{GM}{r_0^3}) \right] dm \\
&= \frac{3GM}{r_0^3} \int yz dm \\
&= \frac{3GM}{r_0^3}(-I_{yz}) \qquad (1.1.14)
\end{aligned}$$

Here $I_{yz} = I^{23}$ is the y-z component of the moment of inertia tensor defined by,

$$I^{kl} = \int (r^2 \delta^k_l - x^k x^l) dm \qquad (1.1.15)$$

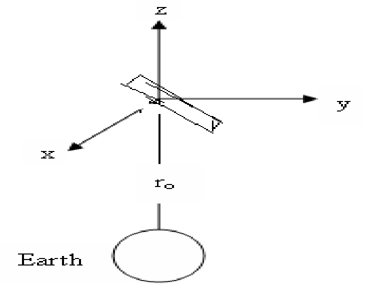

Figure 1.3: a rigid rode in free fall, instantaneously at rest at the origin of the coordinates

Where δ_l^k is the Kronecker delta. Expression similar to 1.1.14 can be found for the torques about the $y-$and$z-$ axes. Generally the torque exerted by a an arbitrary tidal field R^k_{0l0} on a body with an inertia tenser I^{ls} is given by

$$\tau^n = c^2 \sum_{kl} \epsilon^{nkl} R^k_{0l0} \Bigg(- I^{ls} + \frac{1}{3}\delta^l_s I^{rr} \Bigg), \tag{1.1.16}$$

where ϵ^{klm} is defined as follows

$$\epsilon^{123} = \epsilon^{231} = \epsilon^{321} = 1,$$

$$\epsilon^{321} = \epsilon^{213} = \epsilon^{132} = -1 \tag{1.1.17}$$

with all other components zero . If no other forces act equation 1.1.16 implies that the x-component of the spin of the rigid body changes at a rate

$$\frac{dS_x}{dt} = -\frac{3GM}{r_0^3} I_{yz} \tag{1.1.18}$$

Similar formula can be obtained for the rate of the change of the other component of the spin. This rate change of the spin can serve as a measure of the tidal field

1.2 Relativistic tidal force

The physical interpretation of the remann curvature tenser is it is tidal force field, that is it gives the relative acceleration between two particle in free fall. Before we can calculate this relative acceleration, we need to introduce the concept of differentiation along a curve to sea that: Consider a carve in the space time, possibly not necessarily, a geodesic. We can specify the displacement along the curve by giving the corresponding increment in length or proper time τ . Suppose that the curve is immersed in a vector field A^{μ} . If we went to know the rate of change of A^{μ} along the curve , we must calculate $\frac{dA^{\mu}}{d\tau}$. However ,this is not a good measure of how much A^{μ} is really changing, since part,or may be all, of the contribution of $\frac{dA^{\mu}}{d\tau}$ could be due to the curvilinear coordinates used to define the components A^{μ} . Also $\frac{dA^{\mu}}{d\tau}$ is not a vector. A better measure of the rate of change of A^{μ} along the curve is the vector quantity

$$\frac{DA^{\mu}}{D\tau} = A^{\mu}_{;\beta}\frac{dx^{\beta}}{d\tau} = \frac{dx^{\mu}}{d\tau} + \Gamma^{\mu}_{\alpha\beta}A^{\alpha}\frac{dx^{\beta}}{d\tau} \tag{1.2.1}$$

this is a vector, because it is the product of a tenser $A^{\mu}_{;\beta}$ and the vector $\frac{dx^{\beta}}{d\tau}$. The derivative $\frac{DA^{\mu}}{D\tau}$ is called the derivative along the curve [2]. In local geodesic coordinate($\Gamma^{\mu}_{\alpha\beta} = 0$),the derivative $\frac{DA^{\mu}}{D\tau}$ along a curve reduces to the ordinary derivative $\frac{dA^{\mu}}{d\tau}$. The expression (1.2.1)is valid for any curve .Let as now concentrate on geodesic and find the second derivative along a geodesic curve

$$\begin{aligned}
\frac{D^2A^{\mu}}{D\tau^2} \quad &= \frac{D}{D\tau}\left[\frac{DA^{\mu}}{D\tau}\right] \\
&= \frac{d}{d\tau}\left[\frac{DA^{\mu}}{D\tau}\right] + \Gamma^{\mu}_{\alpha\beta}\frac{DA^{\alpha}}{D\tau}\frac{dx^{\beta}}{d\tau} \\
&= \frac{d}{d\tau}\left[\frac{dA^{\mu}}{d\tau} + \Gamma^{\mu}_{\alpha\beta}A^{\alpha}\frac{dx^{\beta}}{d\tau}\right] + \Gamma^{\mu}_{\alpha\beta}\left[\frac{dA^{\mu}}{d\tau} + \Gamma^{\mu}_{\kappa\lambda}A^{\kappa}\frac{dx^{\lambda}}{d\tau}\right]\frac{dx^{\beta}}{d\tau} \\
&= \frac{d^2A^{\mu}}{d\tau^2} + \Gamma^{\mu}_{\alpha\beta,\nu}\frac{dx^{\nu}}{d\tau}A^{\alpha}\frac{dx^{\beta}}{d\tau} + 2\Gamma^{\mu}_{\alpha\beta}\frac{dA^{\alpha}}{d\tau}\frac{dx^{\beta}}{d\tau} \\
&-\Gamma^{\mu}_{\alpha\beta}A^{\alpha}\Gamma^{\beta}_{\kappa\lambda}\frac{dx^{\kappa}}{d\tau}\frac{dx^{\lambda}}{d\tau} + \Gamma^{\mu}_{\alpha\beta}\Gamma^{\alpha}_{\kappa\lambda}A^{\kappa}\frac{dx^{\lambda}}{d\tau}\frac{dx^{\beta}}{d\tau}
\end{aligned} \tag{1.2.2}$$

The next to last term comes from evaluating $\frac{d^2x^{\beta}}{d\tau^2}$ with the geodesic equation . We can now begin our calculation of the relative acceleration of two particles moving on a neighboring geodesic by writing down the equation of motion of each .

Suppose that for a given value of τ along each geodesic ,the coordinates of the particles are $x^\mu(\tau)$ and $x^\mu(\tau)+s^\mu(\tau)$ as shown in figure 1.4

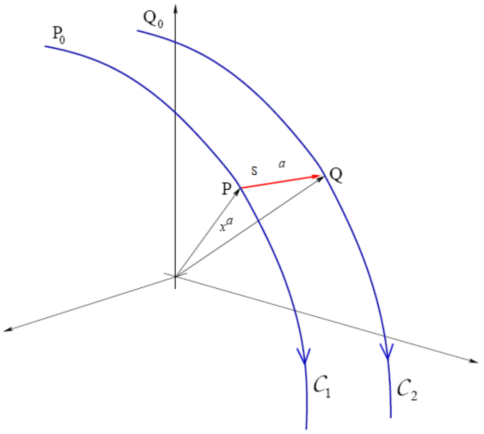

Figure 1.4: a rigid rode in free fall,instantaneously at rest at the origin of the coordinates

The displacement vector s^μ connects pair of points with the same value of τ on the two geodesics .Then the geodesic equations for the two particles are

$$\frac{d^2x^\mu}{d\tau^2}+\Gamma^\mu_{\alpha\beta}(x)\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}=0 \tag{1.2.3}$$

and

$$\frac{d^2(x^\mu+s^\mu)}{d\tau^2}+\Gamma^\mu_{\alpha\beta}(x+s)\left(\frac{dx^\alpha}{d\tau}+\frac{ds^\alpha}{d\tau}\right)\left(\frac{dx^\beta}{d\tau}+\frac{ds^\beta}{d\tau}\right)=0 \tag{1.2.4}$$

We will assume that s^μ and $\frac{ds^\mu}{d\tau}$ are infinitesimal; this means that the particles are near to each other and remain near for a fairly long time. If we approximate

$$\Gamma^\mu_{\alpha\beta}(x+s)=\Gamma^\mu_{\alpha\beta}(x)+\Gamma^\mu_{\alpha\beta,\delta}(s^\delta) \tag{1.2.5}$$

And keep only the first-order term in s^δ , the difference between equation 1.2.3 and 1.2.4 yields

$$\frac{d^2s^\mu}{d\tau^2}=-\Gamma^\mu_{\alpha\beta,\delta}s^\delta\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}-2\Gamma^\mu_{\alpha\beta}\frac{ds^\alpha}{d\tau}\frac{dx^\beta}{d\tau} \tag{1.2.6}$$

The second derivative of s^μ along the geodesic can then be calculated by substituting equation 1.2.5 in to 1.2.2 with the following result

$$\frac{D^2 s^\mu}{D\tau^2} = -R^\mu_{\alpha\delta\beta} s^\delta \frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} \tag{1.2.7}$$

Where $R^\mu_{\alpha\delta\beta}$ is the Rimen Christoffel curvature tenser

$$R^\alpha_{\beta\delta\gamma} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\beta\gamma,\delta} + \Gamma^\eta_{\beta\delta}\Gamma^\alpha_{\gamma\eta} - \Gamma^\eta_{\beta\gamma}\Gamma^\alpha_{\delta\eta} \tag{1.2.8}$$

Equation 1.2.7 is called the equation of geodesic this equation gives us the relativistic generalization of our Newtonian result for the tidal force

For a comparison of the relativistic and Newtonian equation for the tidal force, assume that the particle under consideration are moving slowly with

$$\frac{dx^\alpha}{d\tau} \simeq (1,0,0,0) \tag{1.2.9}$$

Further more assume that $s^0 = 0$; this simply means that the particle acceleration are compared at equal times. Then equation 1.2.7 reduce to

$$\frac{d^2 s^\kappa}{d\tau^2} = -mR^\kappa_{0\iota 0} s^\iota \tag{1.2.10}$$

The tidal force is therefore

$$f^\kappa = -mR^\kappa_{0\iota 0} s^\iota \tag{1.2.11}$$

where m is the mass of the particle and s^ι its displacement from the origin. Note that this equation is valid only if the displacement and the velocity are small($s^\iota \to 0, \frac{ds^\iota}{d\tau} \to 0$) To establish that this equation for the tidal force is in agrement with the Newtonian expression(1.1.8), We must check that the old equation (1.1.7)and a new(1.2.8) definition of $R^\kappa_{0\iota 0}$ considered in weak static gravitational fields In linear approximation, the two term quadratic in $\Gamma^\alpha_{\beta\mu}$ can be omitted from equation 1.2.8 Further more in this approximation

$$\Gamma^\alpha_{\beta\mu} = \frac{k}{2}\eta^{\alpha\delta}(h_{\delta\beta,\mu} + h_{\mu\delta,\beta} - h_{\beta\mu,\delta}) \tag{1.2.12}$$

This result in

$$R^\alpha_{\beta\mu\nu} \simeq -\frac{k}{2}\eta^{\alpha\delta}(h_{\mu\delta,\beta,\nu} - h_{\beta\mu,\delta,\nu} - h_{\nu\delta,\beta,\mu} + h_{\beta\nu,\delta,\mu}) \tag{1.2.13}$$

Therefore

$$R^{\kappa}_{0\iota o} \simeq -\frac{k}{2}(h_{\iota\kappa,0,0} - h_{0\iota,\kappa,0} - h_{0\kappa,0,\iota} + h_{00,\kappa,\iota}) \tag{1.2.14}$$

In the Newtonian limit all the terms containing time derivative can be omitted and using

$$\frac{1}{2}kh_{00} = \phi \tag{1.2.15}$$

We find

$$R^{\kappa}_{0\iota o} = \frac{\partial^2 \phi}{\partial x^{\kappa} \partial x^{\iota}} \tag{1.2.16}$$

Which is in agrement with equation 1.1.7 The tidal force equation 1.2.11 can be used for measuring the component of $R^{\kappa}_{0\iota o}$ of Riemann tensor, not that those measurement are performed locally What about the other component of the Riemann tenser ? These component can be obtained by measuring the component $R^{\kappa}_{0\iota o}$ in sufficiently large number of other reference frames that have some velocity relative to the original reference frame. Lorenth transformation between these frame relate the component $R^{\kappa}_{0\iota o}$ in one frame to the general component $R^{\alpha}_{\beta\mu\nu}$ in another and therefore can be used to find all the component. The reman tenser is therefore entirely determined by tidal force measurement. We will show that the tidal force satisfy

$$\frac{\partial f^{\kappa}}{\partial s^{\kappa}} = 0 \tag{1.2.17}$$

This means that the tidal force field can be represented graphically by filed line Of course, we already come across this condition in Newtonian theory but for the general proof we need to use the exact field equation. According to equation 1.2.11

$$\frac{\partial f^{\kappa}}{\partial s^{\kappa}} = -mR^{\kappa}_{0\iota o}\frac{\partial s^{\iota}}{\partial s^{\kappa}} = -mR^{\kappa}_{0\kappa o} \tag{1.2.18}$$

Since $R^{\alpha}_{\beta\mu\nu}$ is antisymmetric in μ and ν we have $R^{0}_{000} = 0$ is therefore equivalent to

$$\frac{\partial f^{\kappa}}{\partial s^{\kappa}} = -mR^{\mu}_{0\mu o} = mR_{oo} \tag{1.2.19}$$

Where $R_{00} = R^{\mu}_{0\mu o}$ is the component of the Rici tenser As we know Einstine field equation in vacco tell us that $R_{00} = 0$ and hence 1.2.17 is valid as long as we remain out side of the mass distribution that generate the gravitational field.

1.3 Geodetic Precession

One of the characteristics of curved space is that parallel transport of a vector alters its direction, which suggests that we can probably detect the curvature of the space-time near the Earth by actually examining parallel transport. But to perform such an experiment, we must first devise some physical procedure for the parallel transport of a vector. From non gravitational physics we know that if a gyroscope is suspended in frictionless gimbals the result is a parallel transport of its spin direction. However from this we can not immediately draw the conclusion that in gravitational physics the transport of such gyroscope will also result in parallel transport of the direction of its spin.

To find under what conditions transport of gyroscope result in Parallel transport, we start with equation of motion for the spin of a rigid body. According to Newton's theory a rigid body in a gravitational field is subject to a tidal torque given by equation 1.1.16 Which leads to a rate of change of spin given by

$$\frac{dS^n}{dt} = \epsilon^{nkl} R^k_{0l0} \bigg(- I^{ls} + \frac{1}{3}\delta^l_s I^{rr} \bigg), \tag{1.3.1}$$

Where n; k; l; s; r =1 ; 2 ; 3 . Here R^k_{0l0} is the Riemann tensor evaluated in the rest frame of the gyroscope, the presence of which signifies that this particular equation of motion does not obey the principle of minimal coupling, and that the gyroscope spin transport does not imitate parallel transport and the quantity ϵ^{klm} is defined by equation 1.1.17 For a spherical gyroscope we have that $I^{ls} \propto \delta^s_l$, then the tidal torque in the equation (1.1.16) becomes zero and the equation reads $\frac{dS^n}{dt} = 0$. I^s_l is the moment of inertia tensor defined in the equation 1.1.5

This Newtonian equation remains in tact when we are in curved space time, and in a reference frame that freely falls along a geodesic line. Thus the Newtonian time t must now interpreted as the proper time τ measured along the geodesic. In the freely falling reference frame the spin of the gyroscope remains constant in magnitude and direction, which means that it moves by parallel transport.

If now an extra non gravitational force acts on the gyroscope and as a result the gyroscope moves into a world line that is different from a geodesic, then we can not simply introduce local geodesic coordinates at every point on of this world line which makes the equation of motion for the spin $\frac{dS^n}{dt} \neq 0$. In flat space-time the precession of an accelerated gyroscope is called Thomas Precession. In a general coordinate system the spin vectorS^μ in parallel transport obeys the equation:

$$\frac{dS^\mu}{d\tau} = -\Gamma^\mu_{\nu\lambda} S^\nu \frac{dx^\lambda}{d\tau} \tag{1.3.2}$$

where $\Gamma^\mu_{\nu\lambda}$ are the Christoffel symbols second kind, and S^μ are the spin vector components (here$\mu; \nu; \lambda = 0; 1; 2; 3$).

From the above equation we can calculate how the spin of our gyroscope changes direction while the gyroscope moves along some free-fall trajectory. To be specific let us assume that the gyroscope in a circular orbit of radius r around the Earth. In real life somebody measures the change of the gyroscope spin relative to the fixed stars, which is also equivalent of finding this change with respect to a fixed coordinate system at infinity. We can use Cartesian coordinates since they are more convenient in calculating this change of spin direction than polar coordinates. The reason for this is that in Cartesian coordinates any change of the spin can be directly related to the curvature of the space-time, where in polar coordinates there is a contribution from both coordinate curvature and curvature of the space-time. We will not therefore use the exact Schwarzschild solution but only the approximate solution in isotropic rectangular coordinates x, y, z

$$ds^2 \simeq \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 + \frac{2GM}{r}\right)^{-1} (dx^2 + dy^2 + dz^2) \tag{1.3.3}$$

Further assume that our gyroscope is in orbit around the Earth and let the orbit be located in the x-y plane as shown in Figure 1.5 For convenience we will evaluate equation 1.3.2 at one point of the orbit say, the point $x = r, y = 0, z = 0$. In a circular orbit all points are equivalent and if we know the rate of the spin change at one point we can

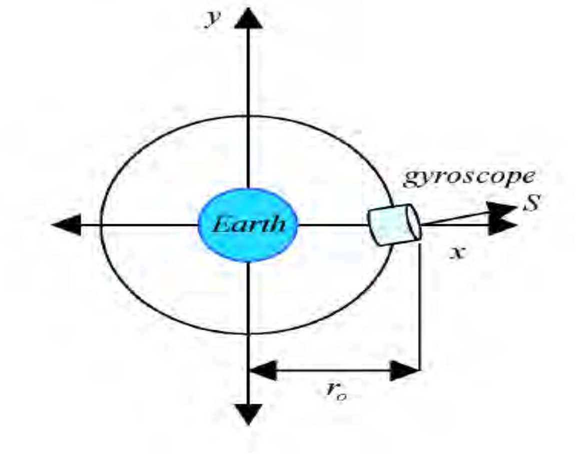

Figure 1.5: Gyroscope is in orbit around the Earth in x-y plane at one instant ,the one instance the gyroscope is at the point $x = r, y = 0, z = 0$

calculate the rate of change of the spin at any pion. For that let us write the line interval in the following way [2]:

$$ds^2 \simeq \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 + \frac{2GM}{r}\right)(dx^2 + dy^2 + dz^2) \tag{1.3.4}$$

which implies that:

$$g_{00} = \left(1 - \frac{2GM}{r}\right), \qquad g_{11} = g_{22} = g_{33} = \left(1 + \frac{2GM}{r}\right) \tag{1.3.5}$$

To evaluate the spatial components of the spin we will use equation (1.3.2), and the right hand symbols must be calculated. For that we need the four-velocity $v^\beta \approx (v_t; v_x; v_y; v_z) = (1; 0; v; 0)$. We also need the S^0 component of the spin, and for that we note that in the rest frame of the gyroscope $S'^0 = 0$ and$v'^\beta = (1; 0; 0; 0)$ and therefore $g'_{\mu\nu} S'^\nu v'^\nu = 0$, and also in our coordinate system we will also have that $g_{\mu\nu} S^\nu v^\nu = 0$, using the latter we have that:

$$S^0 = \frac{1}{g_{00}} \left(S^1 g_{11} \frac{dx^1}{d\tau} + S^2 g_{22} \frac{dx^2}{d\tau} + S^3 g_{33} \frac{dx^3}{d\tau} \right), \tag{1.3.6}$$

$$S^0 = \frac{1}{g_{00}} \left(S_x g_{11} \frac{dx}{d\tau} + S_y g_{22} \frac{dy}{d\tau} + S_z g_{33} \frac{dz}{d\tau} \right) \tag{1.3.7}$$

Substituting for the metric coefficients we obtain

$$S^0 = \frac{\left(1 + \frac{2GM}{r}\right)}{\left(1 - \frac{2GM}{r}\right)} vS_y \cong vS_y \tag{1.3.8}$$

Next letting $\mu = 1$ and summing over $\nu = 0; 1; 2; 3$ the component of the spin equation becomes:

$$\frac{dS^1}{d\tau} = -\Gamma^1_{0\lambda} S^0 v^\lambda - \Gamma^1_{1\lambda} S^1 v^\lambda - \Gamma^1_{2\lambda} S^2 v^\lambda - \Gamma^1_{3\lambda} S^3 v^\lambda \tag{1.3.9}$$

summing over $\lambda = 0;\ 1;\ 2;\ 3$ again we obtain:

$$\begin{aligned}\frac{dS^1}{d\tau} \quad &= -\Gamma^1_{00} S^0 v^0 - \Gamma^1_{01} S^0 v^1 - \Gamma^1_{02} S^0 v^2 - \Gamma^1_{03} S^0 v^3 \\ &-\Gamma^1_{10} S^1 v^0 - \Gamma^1_{11} S^1 v^1 - \Gamma^1_{12} S^1 v^2 - \Gamma^1_{13} S^1 v^3 \\ &-\Gamma^1_{20} S^2 v^0 - \Gamma^1_{21} S^2 v^1 - \Gamma^1_{22} S^2 v^2 - \Gamma^1_{23} S^2 v^3 \\ &-\Gamma^1_{30} S^3 v^0 - \Gamma^1_{31} S^3 v^1 - \Gamma^1_{32} S^3 v^2 - \Gamma^1_{33} S^3 v^3 \end{aligned} \tag{1.3.10}$$

Next we will calculate the Cristoffel symbols of the second kind for that we use:

$$\Gamma^\delta_{\mu\nu} = \frac{1}{2} g^{\delta\lambda} \left(\frac{\partial g_{\mu\delta}}{\partial x^\nu} + \frac{\partial g_{\nu\delta}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\delta} \right) \tag{1.3.11}$$

Since $\Gamma^\delta_{\mu\nu} = 0$ if $\mu \neq \nu \neq \delta$ equation (1.3.10) further simplifies to

$$\begin{aligned}\frac{dS^1}{d\tau} \quad &= -\Gamma^1_{00} S^0 v^0 - \Gamma^1_{01} S^0 v^1 - \Gamma^1_{02} S^0 v^2 - \Gamma^1_{03} S^0 v^3 \\ &-\Gamma^1_{10} S^1 v^0 - \Gamma^1_{11} S^1 v^1 - \Gamma^1_{12} S^1 v^2 - \Gamma^1_{13} S^1 v^3 \\ &-\Gamma^1_{20} S^2 v^0 - \Gamma^1_{21} S^2 v^1 - \Gamma^1_{22} S^2 v^2 - \Gamma^1_{23} S^2 v^3 \\ &-\Gamma^1_{30} S^3 v^0 - \Gamma^1_{31} S^3 v^1 - \Gamma^1_{32} S^3 v^2 - \Gamma^1_{33} S^3 v^3 \end{aligned} \tag{1.3.12}$$

The only non-zero Christoffel symbols calculated at$r = r_0$ are:

$$
\begin{aligned}
\Gamma^1_{10} \quad &= \Gamma^1_{01} = -\frac{GM}{r_0^2} \\
&\Gamma^1_{00} = -\frac{GM}{r_0^2} \\
&\Gamma^1_{11} = -\frac{GM}{r_0^2} \\
&\Gamma^1_{22} = -\frac{GM}{r_0^2} \\
&\Gamma^1_{21} = \Gamma^1_{12} = -\frac{GM}{r_0^2}\Gamma^1_{33} = -\frac{GM}{r_0^2} \\
&\Gamma^1_{31} = \Gamma^1_{13} = -\frac{GM}{r_0^2}
\end{aligned}
\tag{1.3.13}
$$

Thus equation (1.3.12) further becomes:

$$\frac{dS_x}{d\tau} = -\Gamma^1_{00}S^0 - \Gamma^1_{22}S^2v^2 \tag{1.3.14}$$

substituting we obtain:

$$\frac{dS_x}{d\tau} = -\frac{GM}{r_0^2}s_yv - \frac{GM}{r_0^2}s_yv \tag{1.3.15}$$

we can rewrite as follows:

$$\frac{dS_x}{d\tau} = -2\frac{GM}{r_0^2}s_yv \tag{1.3.16}$$

Similarly the equation for the S_y component of the spin becomes:

$$\frac{dS_y}{d\tau} = -\Gamma^2_{12}vS_x - \Gamma^2_{20}S_y - \Gamma^2_{22}S_yv - \Gamma^2_{32}S_yv \tag{1.3.17}$$

which becomes:

$$\frac{dS_y}{d\tau} = -\frac{GM}{r_0^2}s_xv \tag{1.3.18}$$

Finally the equation for the S_z component becomes

$$
\begin{aligned}
\frac{dS_z}{d\tau} \quad &= -\Gamma^3_{00}v^0S^0 - \Gamma^3_{01}v^0S^1 - \Gamma^3_{02}v^2S^0 - \Gamma^3_{03}v^3S^0 \\
&-\Gamma^3_{10}v^1S^0 - \Gamma^3_{11}v^1S^1 - \Gamma^3_{12}v^1S^2 - \Gamma^3_{13}v^1S^3 - \Gamma^3_{20}v^2S^0 \\
&-\Gamma^3_{21}v^2S^1 - \Gamma^3_{22}v^2S^2 - \Gamma^3_{23}v^2S^3 - \Gamma^3_{30}v^3S^0 - \Gamma^3_{31}v^3S^1 \\
&-\Gamma^3_{32}v^3S^2 - \Gamma^3_{33}v^3S^3
\end{aligned}
\tag{1.3.19}
$$

which finally becomes:

$$\frac{dS_z}{d\tau} = 0 \tag{1.3.20}$$

Equations (1.3.16), (1.3.18), (1.3.20) are valid at the chosen $x = r_0, y = z = 0$ point. These equations can also be written in a form that is valid at any point of the orbit, if we just recognize that all of them can be combined in the following single 3-D equation in the following way [5]:

$$\frac{dS}{d\tau} = -2\mathbf{v}.\mathbf{s}\nabla\Phi + v\mathbf{s}.\nabla\Phi \tag{1.3.21}$$

where $\Phi = -\frac{GM}{r_0}$is the Newtonian potential. Since both the velocity **v** and the gradient $\nabla\Phi$ vary with position around the orbit the behavior of **s** given by equation (1.3.21) is rather complicated with small periodic oscillation in both the magnitude and the direction of **s**. However we are not interested in such periodic wobbles but only in the long-term secular change in **s**. We can calculate this secular change by taking the average of equation (1.3.21) over an orbit. For this purpose we express **v** and $\nabla\Phi$ as a function of time

$$v = -\hat{i}v\sin\omega t + \hat{j}v\cos\omega t \tag{1.3.22}$$

$$\nabla\Phi = -\hat{i}\frac{GM}{r_0^2}\cos\omega t + \hat{j}\frac{GM}{r_0^2}\sin\omega t \tag{1.3.23}$$

If we insert these expressions in the right side of equation (1.3.21) and average over one period of the orbit, we find

$$\left\langle \frac{dS}{d\tau} \right\rangle = \frac{3}{2}\frac{GM}{r_0^2}v(-\hat{i}s_y + \hat{j}s_x) \tag{1.3.24}$$

We can rewrite this result more neatly as follows

$$\left\langle \frac{dS}{d\tau} \right\rangle = \frac{3}{2}\frac{GM}{r_0^3}(\mathbf{r}\times\mathbf{v})\times\mathbf{S} \tag{1.3.25}$$

From equation (1.3.25) we sea that on the average **S** presses about the axis of the orbit with an angular velocity

$$\mathbf{\Omega_G} = \frac{3}{2}\frac{GM}{r_0^3}\mathbf{r} \times \mathbf{v} \tag{1.3.26}$$

This is the geodetic precession or de sitter precession. For a gyroscope in orbit 650km above the Earth we can write an expression for the geodetic precession equal to:

$$\frac{3}{2}\frac{GM}{r_0^3}\sqrt{\frac{GM}{r_0}} \simeq 6.6 arcsecperyear \tag{1.3.27}$$

1.4 Tidal Forces in a Schwarzschild Space time

The Schwarzschild solution is one of the best-known exact solutions of the Einstein equations and was derived a few months after the theory was proposed[6]

Consider now a spherical coordinate system (t, r, θ, φ) Impose the constraints that the metric is spherically symmetric and static (i.e. none of the functions $g_{\mu\nu}$ depends on t,θ, φ) and that the spacetime is asymptotically flat (i.e. $g_{\mu\nu} = 1 for r \to \infty$) Under these conditions, the solution to the Einstein equations has line element

$$ds^2 = -dt^2\left(1 - \frac{2GM}{r}\right) - dr^2\left(1 + \frac{2GM}{r}\right)^2 + r^2(d\theta^2 + \sin\theta^2 d\varphi^2) \tag{1.4.1}$$

The spacetime described by (1.4.1) is that of a Schwarzschild black hole, where M is the black hole mass. Note that despite your intuition and the widely used concept of black hole mass, the Schwarzschild solution is a solution of the Einstein equations in vacuum,$i.e R_{\mu\nu} = 0$ Indeed it is the only spherically symmetric and asymptotically flat solution that the equations admit (this is the thesis of Birkhoff's theorem). Because of this, the spacetime exterior to a relativistic spherical (i.e. non rotating) star will be given by the line element (1.4.1).

Let us consider what happens to an extended body located outside the black hole horizon(as in fig 1.6) We will also assume that all the particles in the body move along geodesics and monitor how the separation between two nearby geodesics varies in time.

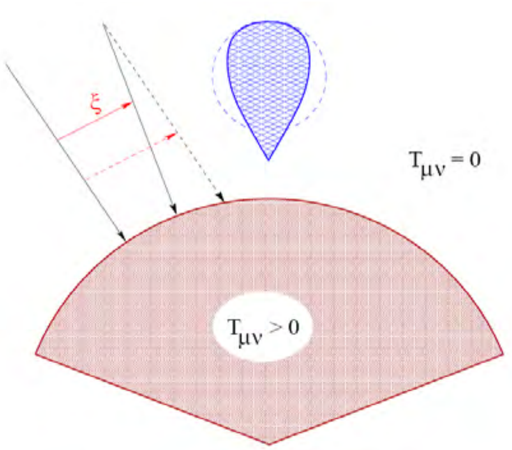

Figure 1.6: Schematic view of the geodesic deviation as well as of the deformation produced on a fluid body in the presence of a strong gravitational field. In the case considered here the source of the gravitational field is represented by a massive body (i.e.$T_{\mu\nu} > 0$) but a qualitative similar scheme would be true also in the case of a black hole.

Clearly, Jacobivector fields provide the connection between the behavior of nearby particles and curvature, via the equation of geodesic deviation (Jacobi equation)

$$\frac{D^2\eta^\mu}{D\tau^2} = -R^\mu_{\alpha\delta\beta}\eta^\delta v^\alpha v^\beta \tag{1.4.2}$$

where v^μ are the components of the tangent vector to geodesic and η^μ are the components of the connecting vector between two neighboring geodesics.

The equation of geodesic deviation gives the relative accelerations between free test particles falling in a gravitational field and is a cornerstone to the understanding of the physical effects of the gravitational field and the geometry of space-time. Geodesics deviations in vacuum space-times, namely, Schwarzschild are rigorously studied and The solution of the geodesic deviation equation (1.4.2) in the spacetime (1.4.1) leads to the following expressions for the spatial components of η

$$\frac{D^2\eta^r}{D\tau^2} = 2\frac{M}{r^3}\eta^r \tag{1.4.3}$$

This equation describe the tidal force in the radial direction .The deviation change is positive in this case it represent a radial tension or stretching effect

$$\frac{D^2\eta^\theta}{D\tau^2} = -\frac{M}{r^3}\eta^\theta \tag{1.4.4}$$

This equation describe a compression type pressure allied along the transverse direction

$$\frac{D^2\eta^\varphi}{D\tau^2} = -\frac{M}{r^3}\eta^\varphi \tag{1.4.5}$$

This equation also describe a compression type pressure this time aligned along the transverse direction. Note all the three component become extreme, as we gate close to $r = 0$ where the effect turns to infinity. Any astronaut falling in to the compact object will get elongated and simultaneously squashed in the middle (see fig 1.6)

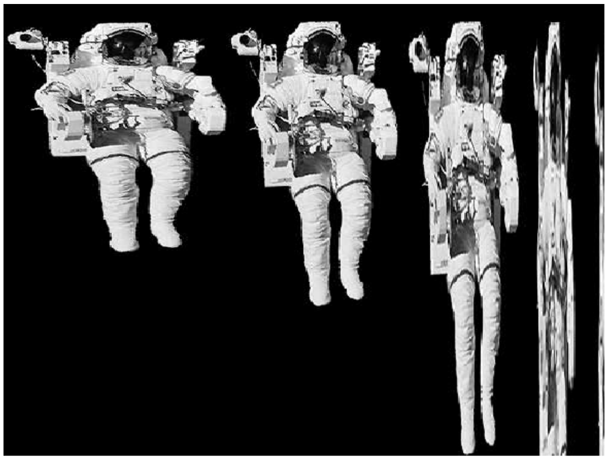

Figure 1.7: Astronaut stretched by the tidal field of a compact object

Chapter 2

Neutron star

A neutron star is a type of remnant that can result from the gravitational collapse of a massive star during a Type II, Type Ib or Type Ic supernova event. Such stars are composed almost entirely of neutrons, which are subatomic particles without electrical charge and roughly the same mass as protons. Neutron stars are very hot and are supported against further collapse because of the Pauli exclusion principle. This principle states that no two neutrons (or any other fermionic particle) can occupy the same place and quantum state simultaneously.

A typical neutron star has a mass between 1.35 and about 2.1 solar masses, with a corresponding radius of about 12 km if the Akmal-Pandharipande-Ravenhall (APR) Equation of state (EOS) is used.[8] In contrast, the Sun's radius is about 60,000 times that. Neutron stars have overall densities predicted by the APR EOS of 3.7×10^{17} to $5.9 \times 10^{17} kg/m3$(2.6×10^{14} to 4.1×10^{14} times the density of the sun), which compares with the approximate density of an atomic nucleus of $3 \times 10^{17} kg/m3$. The neutron star's density varies from below $1 \times 10^{9} kg/m3$ in the crust increasing with depth to above 6×10^{17} or $8 \times 10^{17} kg/m3$ deeper inside. This density is approximately equivalent to the mass of the entire human population compressed to the size of a sugar cube.

2.1 Formation of Neutron star

As the core of a massive star is compressed during a supernova, and collapses into a neutron star, it retains most of its angular momentum. Since it has only a tiny fraction of its parent's radius (and therefore its moment of inertia is sharply reduced), a neutron star is formed with very high rotation speed, and then gradually slows down. Neutron stars are known to have rotation periods between about 1.4 ms to 30 seconds. The neutron star's compactness also gives it very high surface gravity, up to $7 \times 10^{12} m/s^2$ with typical values of a few $\times 10^{12} m/s^2$ (that is more than10^{11}times of that of Earth). One measure of such immense gravity is the fact that neutron stars have an escape velocity of around 100,000 km/s, about $\frac{1}{3}$ of the speed of light. Matter falling onto the surface of a neutron star would be accelerated to tremendous speed by the star's gravity. The force of impact would likely destroy the object's component atoms, rendering all its matter identical, in most respects, to the rest of the star.

2.2 Structure of Neutron star

Current understanding of the structure of neutron stars is defined by existing mathematical models, but it might be possible to infer through studies of neutron-star oscillations. Similar to asteroseismology for ordinary stars, the inner structure might be derived by analyzing observed frequency spectra of stellar oscillations.

On the basis of current models, the matter at the surface of a neutron star is composed of ordinary atomic nuclei crushed into a solid lattice with a sea of electrons flowing through the gaps between them. It is possible that the nuclei at the surface are iron, due to iron's high binding energy per nucleon. It is also possible that heavy element cores, such as iron, simply drown beneath the surface, leaving only light nuclei like helium and hydrogen cores. If the surface temperature exceeds 10^6 kelvins (as in the case of a young pulsar), the surface should be fluid instead of the solid phase observed in cooler neutron stars (temperature $< 10^6$ kelvins).

The "atmosphere" of the star is roughly one meter thick, and its dynamic is fully controlled by the star's magnetic field. Below the atmosphere one encounters a solid "crust". This crust is extremely hard and very smooth (with maximum surface irregularities of 5 mm, because of the extreme gravitational field. Proceeding inward, one encounters nuclei with ever increasing numbers of neutrons; such nuclei would decay quickly on Earth, but are kept stable by tremendous pressures. Proceeding deeper, one comes to a point called neutron drip where free neutrons leak out of nuclei. In this region, there are nuclei, free electrons, and free neutrons. The nuclei become smaller and smaller until the core is reached, by definition the point where they disappear altogether. The exact nature of the superdense matter in the core is still not well understood. While this theoretical substance is referred to as neutronium in science fiction and popular literature, the term "neutronium" is rarely used in scientific publications, due to ambiguity over its meaning. The term neutron-degenerate matter is sometimes used, though not universally as the term incorporates assumptions about the nature of neutron star core material.

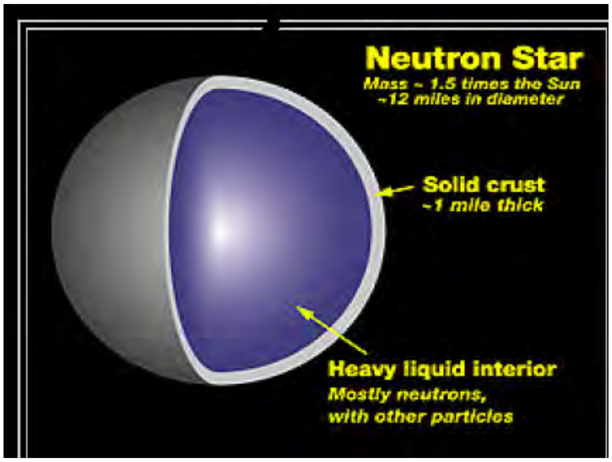

Figure 2.1: neutron star structure

Neutron star core material could be a superfluid mixture of neutrons with a few protons and electrons, or it could incorporate high-energy particles like pions and kaons in addition to neutrons, or it could be composed of strange matter incorporating quarks heavier than up and down quarks, or it could be quark matter not bound into hadrons. (A compact star composed entirely of strange matter would be called a strange star.) However, so far, observations have neither indicated nor ruled out such exotic states of matter.

2.3 Rotation of Neutron star

Neutron stars rotate extremely rapidly after their creation due to the conservation of angular momentum; like spinning ice skaters pulling in their arms, the slow rotation of the original star's core speeds up as it shrinks. A newborn neutron star can rotate several times a second; sometimes, the neutron star absorbs orbiting matter from a companion star, increasing the rotation to several hundred times per second, reshaping the neutron star into an oblate spheroid. Over time, neutron stars slow down because their rotating magnetic fields radiate energy; older neutron stars may take several seconds for each revolution The rate at which a neutron star slows its rotation is usually constant and very small: the observed rates of decline are between 10^{-10} and 10^{-21}seconds for each rotation. Therefore, for a typical slow down rate of 10^{-15}seconds per rotation, a neutron star now rotating in 1 second will rotate in 1.000003 seconds after a century, or 1.03 seconds after 1 million years. Sometimes a neutron star will spin up or undergo a glitch, a sudden small increase of its rotation speed. Glitches are thought to be the effect of a starquake - as the rotation of the star slows down, the shape becomes more spherical. Due to the stiffness of the 'neutron' crust, this happens as discrete events as the crust ruptures, similar to tectonic earthquakes. After the starquake, the star will have a smaller equatorial radius, and since angular momentum is conserved, rotational speed increases. Recent work, however, suggests that a starquake would not release sufficient energy for a neutron star glitch; it has been suggested that glitches may instead be caused by transitions of vortices in the superfluid core of the star from one metastable energy state to a lower one. Neutron stars have been observed to "pulse" radio and x-ray emissions believed caused by particle acceleration near the magnetic poles, which need not be aligned with the rotation axis of the star. Through mechanisms not yet entirely understood, these particles produce coherent beams of radio emission. External viewers see these beams as pulses of radiation whenever the magnetic pole sweeps past the line of sight. The pulses come at the same rate as the rotation of the neutron star, and thus, appear periodic. Neutron stars which emit such pulses are called pulsars. The most rapidly rotating neutron star currently known, PSR J1748-2446ad, rotates at 716 revolutions per second. A recent paper reported the detection of an X-ray burst oscillation (an indirect measure of spin) at 1122 Hz from the neutron star XTE J1739-285. However, at present this signal has only been seen once, and should be regarded as tentative until confirmed in another burst from this star.

2.4 Glitches induced by the core superfluid in a neutron star

It is quite remarkable that timing data for crab and Vela pulsars following their sudden spinups may provide evidence for superfluidity in neutron stars

At the time of the first Vela glitch, which was characterized by a sudden change in $\dot{\Omega}$,$\Delta\dot{\Omega}/\dot{\Omega} \sim 10^{-2}$ together with a smaller increase in Ω ,$\Delta\Omega/\Omega \sim 10^{-6}$, Baym et al.(1969) proposed a simple "two -component" neutron star model, to explain the spinup phenomenon. we present this model,which is largely phenomenological ,as prototype to show how in principle the observational data for glitches can be incorporated in a theoretical picture.

The model consist of a normal component, the crest and charged particles of moment of inertia I_c, weakly coupled to the superfluid neutrons, of moment of inertia I_n. The charged component is assumed to rotate at the observed pulsar frequency Ω(t),since all charged component are assumed to be strongly coupled to the magnetic field. The rotation of neutron superfliud is assumed to be quasi-"uniform", with average angular frequency

Ω_n(t).The coupling between the two components is described by a single parameter τ_c ,the relaxation time for friction dissipation.

In the model, it is imagined that the speed -up is triggered by star quake occurring in the crust. All that counts are the assumptions that (1) the crest spin up is rapidly communicated to charged particle in the interior by the strong magnetic field (time scale ∼100s), while (2) the response of the neutron superflid to the speedup of the crest -charged particle system is considerably slower (∼yr) due to a much weaker friction coupling between the normal and superflid components.

The glitch -free interaction between the two component after a star quake is governed by two linear equations:

$$I_c\dot{\Omega} = -\alpha - \frac{I_c(\Omega - \Omega_n)}{\tau_c} \qquad (2.4.1)$$

$$I_n\dot{\Omega}_n = \frac{I_c(\Omega - \Omega_n)}{\tau_c} \qquad (2.4.2)$$

Here α is the external braking torque on the crest due ,for example ,to magnetic dipole radiation-reaction forces. Taking α and τ_c to be constant over the time -scales of interest Eq(2.4.1) and(2.4.2) may be solved to give

$$\Omega = -\frac{\alpha}{I}t + \frac{I_n}{I}\Omega_1 e^{-t/\tau} + \Omega_2 \qquad (2.4.3)$$

$$\Omega_n = \Omega - \Omega_1 e^{-t/\tau} + \frac{\alpha\tau}{I_c} \qquad (2.4.4)$$

Where

$$I \equiv I_n + I_c \qquad \tau \equiv \tau_c\frac{I_n}{I} \qquad (2.4.5)$$

and Ω_1 and Ω_2 are arbitrary constants depending on the initial conditions.

Note the steady state solution ($\frac{t}{\tau} \to \infty$)is

$$\Omega_n - \Omega = \frac{\alpha\tau}{I_c} = \frac{\alpha}{I}\frac{\tau_c}{T}\Omega \qquad (2.4.6)$$

where

$$\frac{1}{T} = -\frac{\dot{\Omega}}{\Omega} = \frac{\alpha}{I\Omega} \tag{2.4.7}$$

is the characteristic age

Post glitch observations indicate $\tau \sim$ months for Vela and $\tau \sim$ weeks for the Crab. So, crudely assuming $I_n \sim I_c$, which is true for a massive $1.4M_0$ star built from stiff TI or TNI equation of state then Eq(2.4.5) and (2.4.6) predicted $\frac{\Omega_n - \Omega}{\Omega} \sim 10^{-5}$ for Vela and Crab.

Equation (2.4.3)has been used to fit the post-glitch timing data for the Crad and Vela pulsars .Assume that a glitch occurs instantaneously in the observed (crest) angular velocity at $t = 0$:$\Omega(t) \rightarrow \Omega(t) + \Delta\Omega_0$. Replacing$\Omega_1$ and $\Omega_2$in Eq (2.4.3) by the constants $\Delta\Omega_0$ and the "healing parameter" Q, so that the equation take the form

$$\Omega(t) = \Omega_0(t) + \Delta\Omega_0[Qe^{-\frac{t}{\tau}} + 1 - Q] \tag{2.4.8}$$

The healing parameter Q describes the degree to which the angular velocity relaxes back towards its extrapolated value: if $Q = 1$,$\Omega(t) \rightarrow \Omega(0)$ as t→ ∞. Here $\Omega_0(t) = \Omega(t) - \frac{\alpha t}{I}$ is the pulsar frequency in the absence of the glitch, where Ω_0 is a constant

The behavior indicated by Eq(2.4.8) and illustrated in Figure 2.1 gives a reasonable fete to all large glitches observed so far

Clearly the crucial test for this phenomenological two -component model is whether or note the post-glitch functions yield the same values of Q and τ for all the glitch of a single pulsar. Though there are obvious discrepancies. it is remarkable that to lowest order the values are reasonably constant. Analysis difficulties in timing date may be as much responsible for the apparent discrepancies as the over-simplification of the two component model. In general, however, the existence of at lest two distinct components, with relaxation timescales comparable to theoretical predictions for crest-superflid coupling, is supported by the data Consider the parameters Q and τ. Defining

$$\Delta\Omega(t) \equiv \Omega(t) - \Omega_0(t) \tag{2.4.9}$$

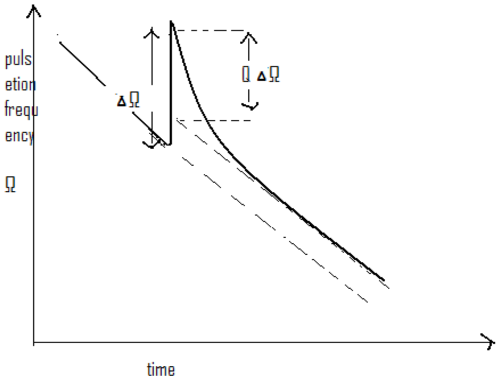

Figure 2.2: Time dependance of pulsar ,Ω,following a discontinuous speedup

We find from Eq(2.4.8) that

$$Q = \frac{\Delta\dot{\Omega}(t=0)}{\Omega_0} \tag{2.4.10}$$

and

$$\tau = \frac{\Delta\dot{\Omega}(t=0)}{\Delta\ddot{\Omega}(t=0)} \tag{2.4.11}$$

Thus Q and τ can in principle be determined directly from the post glitch data. Now Q can also be related to the moment of inerta of various components. For assume that the "star quake "responsible for the glitch result in changes $\Delta I_c, \Delta\Omega, \Delta I_n, \Delta\Omega_n$ and so on,but leaves τ_c and α roughly constant. Then on star quake time scale the angular moment of each component is separately conserved ,implying

$$\frac{\Delta I_c}{I_c} = -\frac{\Delta\Omega}{\Omega} \tag{2.4.12}$$

$$\frac{\Delta I_n}{I_n} = -\frac{\Delta\Omega_n}{\Omega_n} \tag{2.4.13}$$

Differentiating Eq(2.4.3) gives

$$\Delta\dot{\Omega} = \frac{\alpha\Delta I_c}{I_c^2} \tag{2.4.14}$$

But Eqs(2.4.12) and (2.4.7) gives

$$\frac{\alpha\Delta I_c}{I_c^2} = \frac{\alpha\Delta\Omega}{I_c\Omega} = -\frac{I}{I_c}\frac{\Delta\Omega}{T} \tag{2.4.15}$$

Typically

$$\frac{I_c}{I}\frac{T}{\tau_C} \gg 1 \tag{2.4.16}$$

So the first term in Eq(2.4.14) can be ignored in comparison with the second

Recalling Eqs(2.4.5),(2.4.12) and (2.4.13) we get

$$Q = \frac{I_n}{I}(1 - \frac{\Delta I_n/I_n}{\Delta I_c/I_c}\frac{\Omega_n}{\Omega} \tag{2.4.17}$$

Now $\Omega_n - \Omega \ll \Omega$, so typically we expect

$$Q \approx \frac{I_n}{I} \tag{2.4.18}$$

Chapter 3

Gravitoelectromagnatism and Measurment of Lense-Thirring Effect

In ordinary circumstance such as terrestrial and include solar system, the gravitational effect can be adequately described to a very degree of accuracy with the Newtonian theory of gravity Thus it is assumed that the gravitational force depended on the mass of the interacting bodies and their mutual distance only For a strong gravitational object such as compact stares, it is necessary, to include genera relativistic effect[3]

However in general relativity where the space time curvature plays a fundamental role, the neither of these effect is drastiticaly different from the Newtonian theory. Fore instance the gravitational force, contrary to the Newtonian theory, depends on the velocity of the rotating gravitational source (Lense Thirring effect),on the spine of a relativistic test particle (Desitter precession), generation of gravitational waves etc Interestingly though these effect have no analogies in the Newtonian theory, these effects have equivalent in the classical electromagnetic theory (induce magnetism, Thomas precession ,electro magnetic wave)

This analogy has been found to be correct provided that the field of the interacting masses is not extremely intense (such as fore totally collapsed object Blake holes) and the gravitational source is rotating slowly (that is with in the special relativistic limit on the speed). with this approximation it has been shown that the gravitational field equations has equivalent in classical electrodynamics theory in the form of Maxwells equation and Lorentz force Lowe. Hence the approximation is called the gravitomagenetic approximation to the general theory relativity compact stars posses sufficient rotational velocity and enough weak field, hence many process related to these stars can be adequately studied with in the gravitationalelectromagnatic approximation

On Earth and inside the solar system gravitomagntic effects are extremely small, thus requiring extremely precise measurements so on the other hand the possibility to observe these effect in the strong gravitational field of compact stare have been also emphasized(Lemmerzah et.al,2001;Jentzen et.al,1996) especially for the fact that these object naturally posses very high rotational frequencies thus enhancing the typical general relativistic effects

3.1 The Gravitational Field of a Compact Stellar Sorce

The gravitational field surrounding a rotating mass differ from that of surrounding a none rotating mass. We can understand this by analogy with the case rotating ,uniformly charged sphere; such a sphere produces both electric and magnetic field whereas a none rotating spare produce only electric field

In the general theory of relativity gravitational force is referred to as a manifestation of the curvature of space time. thus any process occurring in vicinity of a gravitating source is effective by the geometry of the background space time. Basic to the geometrical structure of the theory is the general line element given by

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta \tag{3.1.1}$$

Where $g_{\alpha\beta}$ is the matrix tenser defined interims of products of partial derivatives of the coordinate transformation . Physically the component of the metrics tenser represent the gravitational field potential in fore dimensional space time .From these component the geometry of the space time is determined by the distribution of the mater content acting as the source of the gravitational field. This is similar to the electromagnetic field theory where the electromagnetic field is determined by the distribution of charge

Of a particular important for space time curvature is the Rimen Christoffel curvature tenser defined by equation 1.2.8. Where the chrestoffel symbol $\Gamma^{\alpha}_{\beta\gamma}$ is related to the metrics tenser by equation 1.3.11

The fundamental equation of general theory of relativity can now be expressed as

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = k T_{\alpha\beta} \tag{3.1.2}$$

Where $R_{\alpha\beta} = R^{\lambda}_{\alpha\lambda\beta}$ and $R = g^{\alpha\beta} R_{\alpha\beta}$ is the Ricci tenser and Ricci scalar obtained by contraction of the Rimen Christoffe tenser and k is a constant of proportionality equal to 8π in gravitational units

The solution to the gravitational field equations, namely the metrics component $g_{\alpha\beta}$, have been determined for various case of interest. However the solution obtained by Shwarze child for the case of a spherically symmetric space time and the Kerr fore an axially symmetric space time, have proved to be of particular interest for various astrophysical phenomenon such as for the gravitational field due to compact stellar object. For their particular interest to us in the following topic we outline these solutions here.

For a none -rotating, un-charged spherically symmetric star of mass M the space time is given by the Shwarze child metric as equation 1.4.1 But fore a rotating uncharged ,axially symmetric of mass M the metrics has a none vanishing off diagonal component $g_{\varphi t} = g_{t\varphi}$ reflecting the conserved angular momentum of the stare. Fore this case the space time is described by Kerr metrics.

$$g_{\alpha\beta} = \begin{pmatrix} -\left(1 - \frac{2mr}{\rho^2}\right) & 0 & 0 & -\frac{2Mar\sin^2\theta}{\rho^2} \\ 0 & \frac{\rho^2}{\Delta} & 0 & 0 \\ 0 & 0 & \rho^2 & 0 \\ -\frac{2Mar\sin\theta^2}{\rho^2} & 0 & 0 & (r^2 + a^2 + \frac{2Mra^2\sin\theta^2}{\rho^2})\sin^2\theta, \end{pmatrix} \tag{3.1.3}$$

Where

$$\rho^2 = r^2 + a^2\cos^2\theta \qquad\qquad \Delta = r^2 + a^2 - 2Mr \tag{3.1.4}$$

And the Kerr parameter $a = \frac{J}{M}$ denote the angular momentum of the rotating gravitational source per unit mass

3.2 Gravitomagnetism

Gravitomagnetism (sometimes Gravitoelectromagnetism, abbreviated GEM), refers to a set of formal analogies between Maxwell's field equations and an approximation, valid under certain conditions, to the Einstein field equations for general relativity.

According to general relativity, the gravitational field produced by a rotating object (or any rotating mass-energy) can, in a particular limiting case, be described by equations that have the same form as the magnetic field in classical electromagnetism. Starting from the basic equation of general relativity, the Einstein field equation, and assuming a weak gravitational field or reasonably flat spacetime, the gravitational analogs to Maxwell's equations for electromagnetism, called the "GEM equations", can be derived. GEM equations compared to Maxwell's equations in SI are: [6]

$$\nabla.E_g = -4\Pi G\rho \qquad\qquad \nabla.E = \frac{\rho_{em}}{\epsilon_0} \tag{3.2.1}$$

$$\nabla.B_g = 0 \qquad\qquad \nabla.B = 0 \tag{3.2.2}$$

$$\nabla\times E_g = -\frac{\partial B_g}{\partial t} \qquad\qquad \nabla\times E = -\frac{\partial B}{\partial t} \tag{3.2.3}$$

$$\nabla\times B_g = -\frac{4\Pi G}{c^2}J + \frac{1}{c^2}\frac{\partial E_g}{\partial t} \qquad\qquad \nabla\times B = \frac{1}{\epsilon_0 c^2}J + \frac{1}{c^2}\frac{\partial E}{\partial t} \tag{3.2.4}$$

where: E_g is the static gravitational field (conventional gravity, also called gravitoelectric for the sake of analogy); E is the electric field; B_g is the gravitomagnetic field; B is the magnetic field; ρ is mass density;ρ_{em} is charge density: J is mass current density $J = \rho V_\rho$, where V_ρ is the velocity of the mass flow generating the gravitomagnetic field); J_{em} is electric current density; G is the gravitational constant; ϵ_0 is the vacuum permittivity; c is the speed of propagation of gravity (equal to, by general relativity, the speed of light).

For a test particle whose mass m is "small" in a stationary system, the net (Lorentz) force acting on it due to a GEM field is described by the following GEM analog to the Lorentz force equation:

$$F_m = m(E_g + v \times 2B_g) \tag{3.2.5}$$

where: m is the mass of the test particle; v is the instantaneous velocity of the test particle. Acceleration of any test particle is simply:

$$a = E_g + v \times 2B_g \tag{3.2.6}$$

In some literature, all instances of B_g in the GEM equations are multiplied by $\frac{1}{2}$, a factor absent from Maxwell's equations. This factor vanishes if B_g in the GEM version of the Lorentz force equation is multiplied by 2, as shown above. The factors 2 and $\frac{1}{2}$ arise because gravitational field is caused by stress-energy tensor which is second rank tensor, as opposed to electromagnetic field which is caused by four-current which is first rank tensor. That difference becomes intuitively clear when non-invariance of relativistic mass is compared to electric charge invariance. This is often referred to as gravity being spin-2 field and electromagnetism being spin-1 field From comparison of GEM equations and Maxwell's equations it is obvious that $\frac{-1}{4\pi G}$ is the gravitational analog of vacuum permittivity ϵ_0. Adopting Planck units normalizes G, c and$\frac{-1}{4\pi\epsilon_0}$ to 1, thereby eliminating these constants from both sets of equations. The two sets of equations then become identical but for the minus sign preceding 4π in the GEM equations. These minus signs stem from an essential difference between gravity and electromagnetism: electrostatic charges of identical sign repel each other, while masses attract each other. Hence the

GEM equations are simply Maxwell's equations with mass (or mass density) substituting for charge (or charge density), and -G replacing the Coulomb force constant$\frac{-1}{4\pi\epsilon_0}$.

The following equation summarizes the results thus for: Common Structure of the Maxwell and GEM Equations given in Planck units.

$$\nabla . E = 4\iota\pi\rho \tag{3.2.7}$$

$$\nabla . B = 0 \tag{3.2.8}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{3.2.9}$$

$$\nabla \times B = \iota 4\pi J + \frac{\partial E}{\partial t} \tag{3.2.10}$$

where $\iota = 1$ for Maxwell and $\iota = -1$ for GEM

3.3 Gravitomagnetic fields of astronomical objects

Formula for gravitomagnetic field B_g near a rotating body can be derived from the GEM equations and is:

$$B_g = \frac{G}{2c^2} \frac{\mathbf{L} - 3(\mathbf{L}.\mathbf{n})\mathbf{n}}{r^3} \tag{3.3.1}$$

where $\mathbf{n}$ is the unite vector in the radial direction $\mathbf{n} = \frac{\mathbf{r}}{r}$ and $\mathbf{L}$ is the angular momentum of the body. At the equatorial plane,$\mathbf{r}$ and $\mathbf{L}$ are perpendicular, so their dot product vanishes, and this formula reduces to:

$$B_g = \frac{G}{2c^2} \frac{\mathbf{L}}{r^3} \tag{3.3.2}$$

Magnitude of angular momentum of a homogeneous ball-shaped body is:

$$L = I_{ball}\omega = \frac{2mr^2}{5}\frac{2\pi}{T} \tag{3.3.3}$$

Where: $I_{ball} = \frac{2mr^2}{5} = \frac{2mr^2}{5}$ is the moment of inertia of a ball-shaped body; ω is the angular velocity; m is the mass; r is the radius; T is the rotational period Therefore, magnitude of Earth's gravitomagnetic field at its equator is:

$$B_{gearth} = \frac{G}{5c^2}\frac{m2\pi}{rT} = \frac{2\pi Rg}{5c^2T} \tag{3.3.4}$$

where $g = \frac{GM}{r^2}$ is the earth's gravity

From this calculation it follows that Earth's equatorial gravitomagnetic field is about $B_{gearth} = 1.012 \times 10^{-14} Hz$, [13] or 3.1×10^{-7} in units of standard gravity (9.81 m/s2) divided by speed of light. Such a field is extremely weak and requires extremely sensitive measurements to be detected.

If the preceding formula is used with the second fastest-spinning pulsar , PSR J1748-2446ad (which rotates 716 times per second),assuming a radius of 16 km, and two solar masses, then the gravitomagnetic field is about 166 Hz. This would be easy to notice

3.4 Lense-Thirring Effect

The Lense-Thirring precession can be calculated by examining the equation of motion of a particle in the gravitational field(3.1.3). Hover the calculation is rather complicated ,and much simpler to obtain the answer by exploitation the analogy between the gravitational and electromagnatic equations discussed in section3.2

The Lense-Thirring precession is analogous to the precession of orbital angular momentum of a charged particle orbiting around another charged particle endowed with a magnetic dipole moment.[2] The time average magnetic moment **m** associated with the angular momentum of the orbiting particle then couples to the magnetic dipole moment **m′** of the central particle, via the the magnetic field, and this results in an effective torque on the orbital angular momentum. The magnetic field of the dipole $\mathbf{m}$ is given by the familiar formula

$$\mathbf{B} = \frac{\mathbf{3nn.m - m}}{r^3} \tag{3.4.1}$$

the torque exerted by the magnetic field on the dipole $\mathbf{m'}$ is

$$\tau = \mathbf{m'} \times \frac{\mathbf{3nn.m - m}}{r^3} \tag{3.4.2}$$

By analogy the torque exerted by the gravimagnetic field on the orbital angular momentum is

$$\tau = -G\mathbf{L} \times \frac{\mathbf{3nn.s - s}}{r^3} \tag{3.4.3}$$

The rate of change of the orbital angular momentum is then

$$\frac{d\mathbf{L}}{dt} = -G\mathbf{L} \times \frac{3 < \mathbf{nn.s} > -\mathbf{s}}{r^3} \tag{3.4.4}$$

From this, We can identify the precession angular velocity as

$$\boldsymbol{\Omega} = -G \times \frac{3 < \mathbf{nn.s} > -\mathbf{s}}{r^3} \tag{3.4.5}$$

If the polar orbit is of low altitude ($r = R_E$) , the magnitude of the Lense-Thirring precession amount to about 0.05 arcsec per year

Chapter 4

Result and Discussion

The gravitational redshift at a neutron star surface is ~ 0.3, and therefore relativistic effects, including the dragging of the inertial frames(Lense-Thirring Effect), are strong in and around neutron stars. In this section we analyze how frame dragging affects the relative precession of the crust and the core. Our post-Newtonian calculations rely on the usage of the gravitomagnetic field, B_g; [11]

To fined The Gravvitomaggnetic coupling torque on the crest of the neutron star by the core; Consider the gravitomagnetic force acting on a small region of the crust of mass dm. In the post-Newtonian approximation, it is given by

$$dF_{gm} = dm\mathbf{v} \times \mathbf{B_g} = dm(\mathbf{\Omega_c} \times \mathbf{r}) \times \mathbf{B_g} \tag{4.0.1}$$

where $\mathbf{v}$, $\mathbf{B_g}$, $\mathbf{\Omega_c}$, and $\mathbf{r}$ are the velocity of the small region, the gravitomagnetic field, the instantaneous angular velocity of the crust, and the radius vector of the region, respectively. The torque acting on this region of the crust is given by

$$\mathbf{d}\tau = \mathbf{r} \times \mathbf{dF_{gm}} = dm(\mathbf{r} \times (\mathbf{\Omega_c} \times \mathbf{r}) \times \mathbf{B_g}) \tag{4.0.2}$$

using the vector identity

$$(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = (\mathbf{C}.\mathbf{A})\mathbf{B} - (\mathbf{C}.\mathbf{B})\mathbf{A} \tag{4.0.3}$$

$$\mathbf{d}\tau = dm(\mathbf{B_g}.\mathbf{r})(\mathbf{\Omega_c} \times \mathbf{r}) \tag{4.0.4}$$

The field B_g is that of a dipole[12], and

$$\mathbf{B_g}.\mathbf{r} = -\frac{6}{r^3}\mathbf{J}.\mathbf{r} = -\frac{6I_0}{r^3}\mathbf{\Omega_0}.\mathbf{r} \tag{4.0.5}$$

where J, Ω_0, and I_0 are the angular momentum, the angular velocity, and the moment of inertia of the spherical core,

Now, substituting equation (4.0.5) into equation (4.0.4) and integrating over the crust, we arrive at the following form of the gravitomagnetic torque: respectively

$$\tau = -\int \rho dr^3 \left[(\frac{6I_0}{r^3})\mathbf{\Omega_0}.\mathbf{r} \right] (\mathbf{\Omega_c} \times \mathbf{r}) \tag{4.0.6}$$

$$\tau = \mathbf{\Omega_c} \times I_g \mathbf{\Omega_0} \tag{4.0.7}$$

where I_g is the linear operator (represented, generally, by a 3 × 3 matrix) defined as

$$I_g \mathbf{\Omega_0} = -\int \rho dr^3 \left(\frac{6I_0}{r^3} \right) (\mathbf{\Omega_0}.\mathbf{r})\mathbf{r} \tag{4.0.8}$$

We use Diracs bra and ket notation and express this operator as

$$I_g = -\int \rho dr^3 \left(\frac{6I_0}{r^3} \right) \mid \hat{r} >< \hat{r} \mid \tag{4.0.9}$$

Where we define the projection operator $\mid \hat{r} >< \hat{r} \mid$ by its action on an arbitrary vector **a**:

$$(\mid \hat{r} >< \hat{r} \mid)\mathbf{a} = (\mathbf{r}.\mathbf{a})\mathbf{r} \tag{4.0.10}$$

From equation (4.1.9) we see that I_g is a Hermitian operator: since the integrand $\propto \mid \hat{r} >< \hat{r} \mid$ in equation (4.1.9) is Hermitian, the integral must also be Hermitian. This means that the matrix representing I_g is symmetric.

If the crust is spherically symmetric, then $I_g = zI$, where z is a real number and I is a unit matrix. In this case, the torque acting on the crust is

$$\tau = z\mathbf{\Omega_c} \times \mathbf{\Omega_0} \qquad\qquad z \simeq I_c I_0 (\frac{2G}{c^2 R^3}) = 0.1 I_c \tag{4.0.11}$$

This torque term arises from the gravitational dragging, which gives place to a retardation of the angular velocity of the crest components of the star ,when it have different angular velocities from its companion core. In this case we neglect the gravitational radiation these the interaction is conservative.

But only the ϕ component of the torque is responsible for the retardation of the angular velocity of the crest, If the crest is rotating along the ϕ. For simplicity; if we decompose the torque using cylindrical coordinate, we will gate

$$\tau = z\Omega_{c\phi}\Omega_0\hat{\rho} - z\Omega_{c\rho}\Omega_0\hat{\phi} \tag{4.0.12}$$

So the ϕ component of the torque is

$$\tau_\phi = -z\Omega_c\Omega_0 \sin\theta \tag{4.0.13}$$

Where θ is the angle between $\mathbf{\Omega_c}$ and $\mathbf{\Omega_0}$,its value as shown on the observation is very small but not zero ($\sin\theta \simeq \theta$). Using the above expression we can determine the time when both the core and the crest have the same speed

$$\begin{aligned} t_{syc} &= \frac{I_c}{\tau_\phi}(\Omega_c - \Omega_0) \\ &\simeq \frac{I_c}{z\Omega_c\Omega_0\theta}(\Omega_c - \Omega_0) \end{aligned} \tag{4.0.14}$$

After this time both the crest and the core rotate with the same angular velocity.

Conclusion

When neutron stars are borne,their crest rotates at a different rate (faster in young pulsars)than the core;This differential motion cause the star to show a glitch event. But as it has been shown in the result of this project the tidal torque(equation4.1.11) will cause retardation of angular velocity of the crest and finally after some time the pulsar will made the corotation of the crest and the core. As a result of the elimination of differential velocity between the core and the crest the star will not show the glitch event after the time of syncroniyation. This is the resin whay young pulsar show the glitch while the elders not For example, if we consider the Crab pulsar, for which[1] $\Omega_c = 2001/s$,$\mathbf{\Omega_0 = 1001/s}$ and $\theta = 10^{-14}$ t_{syc} is$1.58 \times 10^3 yers$

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Declaration

This project is my original work, has not been presented for a degree in any other University and that all the sources of material used for the project have been dully acknowledged.

Name: Tesfaye Dagne

Signature:

Place and time of submission: Addis Ababa University, June 2010

This project has been submitted for examination with my approval as University advisor.
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