



NEUTRINO TRANSPORT IN STRONGLY  
MAGNETIZED PROTO-NEUTRON STARS AND THE  
ORIGIN OF FAST SPINNING PULSAR KICKS

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This is to certify that the thesis prepared by **TESFAYE TEKLEHAIMANOT**, entitled “**NEUTRINO TRANSPORT IN STRONGLY MAGNETIZED PROTO-NEUTRON STARS AND THE ORIGIN OF FAST SPINNING PULSAR KICKS**” and submitted in partial fulfillment of the requirements for the degree of **Master of Science in physics** compiles with the regulations of the University and meets the accepted standards with respect to originality.

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# Abstract

The Fermi LAT collaboration has recently reported the discovery of the pulsations of the  $\gamma$ -ray pulsar J1823-3021A with a luminosity which is the highest observed to date for any millisecond pulsar (MSP). This large luminosity implies a large spin down rate  $\dot{P}$  and therefore a large magnetic field which seems to be incompatible with the observed short rotation period  $P$ . In proto-neutron stars with strong magnetic fields, the cross section for  $\nu_e$  ( $\bar{\nu}_e$ ) absorption on neutrons (protons) depends on the local magnetic field strength resulting from the quantization of energy levels for the  $e^-$  ( $e^+$ ) produced in the final state. If the neutron star possesses an asymmetric magnetic field topology in the sense that the magnitude of magnetic field in the north pole is different from that in the south pole, then asymmetric neutrino emission may be generated. We calculate the absorption cross sections of  $\nu_e$  and  $\bar{\nu}_e$  in strong magnetic fields as a function of the neutrino energy. These cross sections exhibit oscillatory behaviors that occur because new Landau levels for the  $e^-$  ( $e^+$ ) become accessible as the neutrino energy increases. By evaluating the appropriately averaged neutrino opacities, we demonstrate that the change in the local neutrino flux caused by the modified opacities is rather small. To generate appreciable kick velocity ( $\sim 300 \text{ km s}^{-1}$ ) to the newly formed neutron star, the difference between the field strengths at the two opposite poles of the star must be at least  $10^{16} \text{ G}$ . We also consider the magnetic field effect on the spectral neutrino energy fluxes. The oscillatory features in the absorption opacities give rise to modulations in the emergent spectra of  $\nu_e$  and  $\bar{\nu}_e$ .

# Acknowledgements

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# Chapter 1

## Introduction

The pulsar J1823-3021A, located in the globular cluster NGC 6624, was discovered in 1994 (Biggs et al. 1990, 1994). It has been recognized shortly after the discovery of pulsars that neutron stars have velocities much in excess of those of any other normal stellar populations in our Galaxy (Gunn and Ostriker 1970 ; Minkowski 1970). However, it is only in the last few years that a significant body of evidence has come into place to support the view that Type II supernovae are nonspherical and that neutron stars receive large kick velocities at birth. Lyne and Lorimer (1994) analyzed pulsar velocities in light of new proper-motion measurements (Harrison, Lyne, and Anderson 1993) and up-to-date pulsar distance scale (Taylor and Cordes 1993), and they concluded that pulsars were born with a mean speed of  $\sim 450 \text{ km s}^{-1}$ , much larger than previously thought. More recent studies of pulsar velocities have adopted more sophisticated statistical methods and included better treatment of selection effects and uncertainties (Lorimer, Bailes, and Harrison 1997 ; Hansen and Phinney 1997 ; Cordes and Chernoff 1997).

They all yielded a result for the pulsar birth velocity in qualitative agreement (although not in quantitative details) with that of Lyne and Lorimer (1994). In particular, Cordes and Chernoff (1997) found that the three dimensional space velocities of their sample of 47 pulsars have a bimodal distribution, with characteristic speeds of  $180 \text{ km s}^{-1}$  and  $700 \text{ km s}^{-1}$  (corresponding to 80 % and 20 % of the population). They also estimated that pulsars with velocities greater than  $1000 \text{ km s}^{-1}$  may be under represented owing to selection effects in pulsar



surveys (The uncertainty in the high velocity end of the pulsar velocity distribution function has also been emphasized by Hansen and Phinney 1997). Concrete evidence for the existence of pulsars with velocities of  $\gtrsim 1000 \text{ km s}^{-1}$  has come from the observation of the Guitar Nebula pulsar (B2224+65), which produces a bow shock when plowing through the interstellar medium (Cordes, Romani, and Lundgren 1993). In addition, studies of pulsar supernova remnant associations have uncovered a number of pulsars having velocities much greater than  $1000 \text{ km s}^{-1}$  (Frail, Goss, and Whiteoak 1994), although in many cases the associations are not completely secure.

Compelling evidence for supernova asymmetry and pulsar kicks also comes from the detection of geodetic precession in the binary pulsar PSR 1913+16 (Cordes, Wasserman, and Blaskiewicz 1990) and the orbital plane precession in the PSR J0045-7319/B star binary (Lai, Bildsten, and Kaspi 1995 ; Kaspi et al. 1996) and its fast orbital decay (which indicates retrograde rotation of the B star with respect to the orbit ; see Lai 1996). These results demonstrate that binary break up (as originally suggested by Gott, Gunn, and Ostriker 1970 ; see Iben and Tutukov 1996) cannot be solely responsible for the observed pulsar velocities and that natal kicks are required. In addition, evolutionary studies of neutron star binary population imply the existence of pulsar kicks ( Deway and Cordes 1987 ; Fryer and Kalogera 1997 ; Fryer, Burrows, and Benz 1998 ; see also Brandt and Podsiadlowski 1995). Finally, there are a large number of direct observations of nearby supernovae (see, e.g., Cropper et al. 1988 ; Trammell, Hines, and Wheeler 1993 ; McCray 1993 ; Utrobin, Chugai, and Andronova 1995) and supernova remnants ( Morse, Winkler, and Kirshner 1995 ; Aschenbach, Egger, and Trumper 1995) in radio, optical, and X-ray bands, which supports the notion that supernova explosions are not spherically symmetric.

The origin of the pulsar velocities is still unknown. Two classes of mechanisms for the natal kicks have been suggested. The first class relies on convective instabilities in the collapsed stellar core and within the rebounding shock ( Burrows

and Fryxell 1992 ; Burrows, Hayes, and Fryxell 1995 ; Janka and Muller 1994, 1996 ; Herant et al. 1994). The asymmetries in the matter and temperature distributions associated with the instabilities naturally lead to asymmetric matter ejection and/or asymmetric neutrino emission. Numerical simulations indicate that the local, post collapse instabilities are not adequate to account for kick velocities higher than a few hundred  $\text{km s}^{-1}$  (Janka and Muller 1994). A variant of this class of models therefore relies on the global asymmetric perturbations seeded in the pre-supernova cores (Goldreich et al. 1996 ; see also Bazan and Arnett 1994). Clearly, the magnitude of kick velocity depends on the degree of initial asymmetry in the imploding core (Burrows and Hayes 1996). Because of various uncertainties in the presupernova stellar models, it is not clear at this point whether sufficiently large initial perturbations can be produced in the precollapse core (Lai and Goldreich 1998).

In this thesis we focus on the second class of models in which the large pulsar kick velocities arise from asymmetric neutrino emission induced by strong magnetic fields. Since 99% of the neutron star binding energy (a few times  $10^{53}$  ergs) is released in neutrinos, tapping the neutrino energy would appear to be an efficient means to kick the newly formed neutron star. Magnetic fields are naturally invoked to break the spherical symmetry in neutrino emission. But the actual mechanism is unclear. We first briefly comment on previous works on this and related subjects.

## 1.1 Previous Works on Asymmetric Neutrino Emission Induced by Magnetic Fields

A number of authors have noted that parity violation in weak interactions may lead to asymmetric neutrino emission from proto-neutron stars (Chugai 1984 ; Dorofeev et al. 1985 ; Vilenkin 1995 ; Horowitz and Piekarewicz 1997). However, their studies are largely unsatisfactory for a number of reasons : they either

failed to identify the most relevant neutrino emission/interaction processes or the relevant physical conditions in proto-neutron stars, or stopped at estimating the magnetic field effects on neutrino opacities. Chugai (1984) and Vilenkin (1995) (see also Bezchastnov and Haensel 1996) considered neutrino-electron scattering and concluded that the effect is extremely small (e.g., to obtain  $V_{kick} = 300 \text{ km s}^{-1}$  would require a magnetic field of at least  $10^{16} \text{ G}$ ). However, neutrino-electron scattering is less important than neutrino-nucleon scattering in determining the characteristics of neutrino transport in proto-neutron stars. Similarly, Dorofeev et al. (1985) considered neutrino emission by Urca processes in strong magnetic fields. But as shown by Lai and Qian (1998), in the bulk interior of the neutron star, the asymmetry associated with neutrino emission is cancelled by the asymmetry associated with neutrino absorption.

Concerning the parity violation effect in proto-neutron stars, Horowitz and Li (1997) recently suggested that the asymmetry in neutrino emission may be enhanced owing to multiple scatterings of neutrinos by nucleons that are slightly polarized by the magnetic field. They estimated that a field strength of few times  $10^{12} \text{ G}$  is adequate to account for kick velocities of a few hundred  $\text{km s}^{-1}$ . However, the paper by Horowitz and Li (1997) only discusses the idealized situations of scattering media and ignores various essential physics that is needed in a proper treatment of neutrino transport in proto-neutron stars. In particular, it does not consider the effect of neutrino absorption, which one might suspect to wash out the cumulative effect from multiple scatterings. Our own study of the parity violation effect (Lai and Qian 1998) was flawed in the treatment of scattering terms in the neutrino transport equation. As shown by Arras and Lai (1998), detailed balance requires that there be no cumulative effect from multiple scatterings in the bulk interior of the proto-neutron star where local thermodynamic equilibrium applies to a good approximation. Enhancement of neutrino emission asymmetry from multiple scatterings obtains only after neutrinos thermally decouple from proto-neutron star matter and therefore is insignificant.

Bisnovatyi-Kogan (1993) attributed pulsar kicks to asymmetric magnetic field distribution in proto-neutron stars. Using the fact that neutron decay rate can be modified the magnetic field, the inferred that neutrino emissions from opposite sides of the neutron star surface are different. However, neutron decay is not directly relevant for neutrino emission from a newly formed neutron star. Roulet (1997) (whose paper was posted while this paper was being written) considered the relevant neutrino absorption processes. But his study was restricted to calculating the neutrino cross sections as functions of neutrino energy (corresponding to § 3.1 of this paper) and therefore was insufficient for addressing the issue of asymmetric neutrino emission.

Finally, it is worthwhile to mention a speculative idea on pulsar kicks that relies on nonstandard neutrino physics. Kusenko and Segre (1996) suggested that asymmetric  $\nu_\tau$  emission could result from the Mikheyev-Smirnov-Wolfenstein flavor transformation between  $\nu_\tau$  and  $\nu_e$  inside a magnetized proto-neutron star because a magnetic field changes the resonance condition for the flavor transformation. With a  $\nu_\tau$  mass of  $\sim 100$  eV, they claimed that magnetic fields of  $3 \times 10^{14}$  G can give the pulsar a kick velocity of a few hundred  $\text{km s}^{-1}$ . However, their treatment of neutrino transport was over simplified : e.g., they ignored that neutrinos of different energies have different resonance surfaces. Furthermore, a simple geometric effect and realistic proto-neutron star conditions easily reduce their estimated kick velocity by an order of magnitude. Most likely, a magnetic field strength of  $\sim 10^{16}$  G is needed to produce the observed average pulsar kick velocity via their mechanism (see Qian 1997). Similarly strong magnetic fields are required in variants of the Kusenko and Segre mechanism, e.g., those considered by Akhmedov, Lanza, and Sciama (1997) (which relies on both the neutrino mass and the neutrino magnetic moment to facilitate the flavor transformation).

## 1.2 Plan of This Thesis

As should be clear from the brief review in § 1.1, in spite of quite a number of suggestions, there is at present no consensus on the magnitudes of the magnetic field induced asymmetry in neutrino emission from proto-neutron stars and the resulting kick velocities. In this paper we study the effect of asymmetric magnetic field topology on pulsar kicks. Since the energy levels of  $e^-$  and  $e^+$  in a magnetic field are quantized, the  $\nu_e$  and  $\bar{\nu}_e$  absorption opacities near the neutrinosphere and the neutrino-matter decoupling region depend on the local magnetic field strength. If the magnitude of magnetic field in the north pole is different from that in the south pole (the field does not need to be ordered), then asymmetric neutrino flux may be generated. Here we ignore the effects of magnetic fields on the proto-neutron star structure through the equation of state-these are secondary effects as far as neutrino transport is concerned.

The organization of this thesis is as follows. In section 2 we observed that Fast-Spinning Star J1823-3021A. In section 3.1 we calculate the  $\nu_e$  and  $\bar{\nu}_e$  absorption cross sections as functions of neutrino energy in strong magnetic fields. Section 3.2 summarizes the basic features of neutrino transport near the neutron star surface. In section 3.3 we derive and evaluate the “Rosseland mean “ neutrino opacities, which are directly related to the local neutrino flux. We then consider in section 3.4 the change in neutrino flux caused by the modified opacities, estimate the kick velocity and magnetic field resulting from an asymmetric magnetic field topology. In section 3.5 we briefly consider how the emergent neutrino spectra may be modified by the strong magnetic field. Finally, we present our conclusion.

Unless noted otherwise, we shall use units in which  $\hbar$ ,  $c$ , and the Boltzmann constant  $k$  are unity.

## Chapter 2

# Fast-Spinning Star

The spinning star, a millisecond pulsar called J1823-3021A, is located inside a packed conglomeration of stars called a globular cluster about 27,000 light years from Earth in the constellation Sagittarius. The pulsar emits incredibly intense high energy gamma rays, which the researchers detected and studied using NASA's Fermi Gamma Ray Space Telescope. Their analysis suggests the pulsar is just 25 million years old a baby as far as these stars go, for millisecond pulsars tend to be a billion years old or so, researchers said. The pulsar's extreme brightness and youth challenge current ideas about how super bright millisecond pulsars form and how widespread they may be, researchers said. "These anomalously energetic millisecond pulsars must be forming at a rate similar to the previously known, more normal millisecond pulsars at least in globular clusters, but possibly in the whole universe as well," said study lead author Paulo Freire of the Max Planck Institute for Radio Astronomy in Bonn, Germany. "In a sense, this pulsar would be the proverbial tip of a hidden new iceberg.

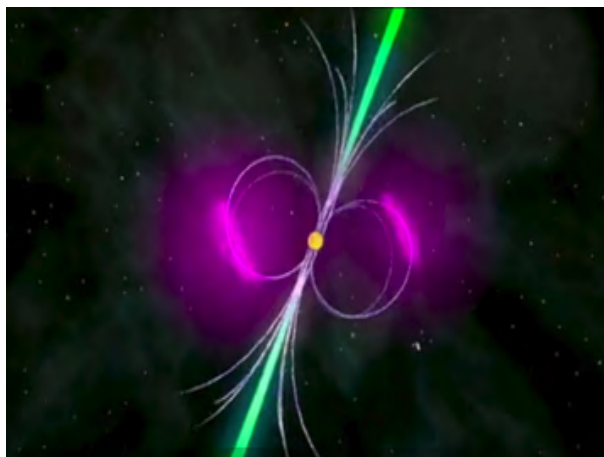


Figure 2.1: This still from a NASA animation depicts the super bright and super young pulsar J1823-3021A, which is the brightest and youngest pulsar yet discovered, and has a powerful magnetic field. The pulsar spins about 183.8 times a second and is located about 27,000 light years from Earth.

## 2.1 Exotic,fast-spinning star

Pulsars form when massive stars die in supernova explosions and their remnants collapse into compact objects made only of the particles called neutrons. When a mass as great as our sun's is packed into a space the size of a city, the conserved angular momentum causes the resulting neutron star to spin very rapidly and to emit a ray of high energy light that sweeps around like a lighthouse beam. This light appears to pulse because astronomers see the beam only when it's pointed at Earth. "Normal" pulsars rotate at a rate between 7 and 3,750 revolutions per minute, but millisecond pulsar can spin much faster up to 43,000 rotations per minute. These hyper-spinners are thought to be revved up by accretion of matter from a companion star. Indeed, roughly 80 percent of millisecond pulsars discovered to date are found in binary systems, researchers said. The new study could shed some more light on these exotic objects. The research is detailed online in the Nov. 3 issue of the journal *Science*. In a separate study, astronomers announced the discovery of nine previously unknown gamma-ray pulsars, also using the Fermi space telescope.

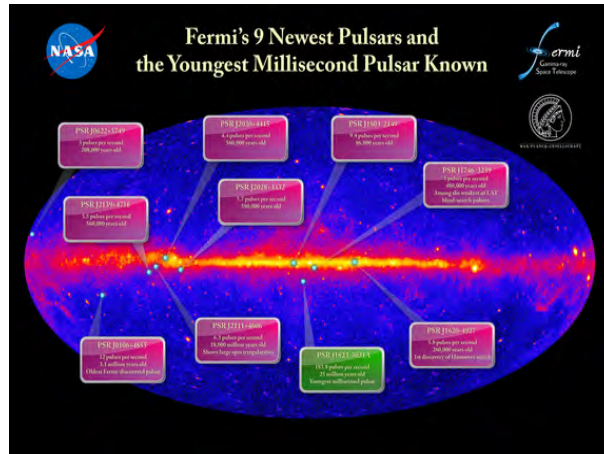


Figure 2.2: This plot shows the positions of nine new pulsars (magenta) discovered by Fermi and of an unusual millisecond pulsar (green) the youngest such object known found by the Fermi Gamma Ray Space Telescope. With this new batch of discoveries, Fermi has detected more than 100 pulsars in gamma rays.

## 2.2 Looking in gamma-ray light

For the J1823-3021A study, researchers trained the Fermi space telescope on the globular cluster NGC 6624. Globular clusters are good places to look for millisecond pulsars, because the dense packing of stars facilitates the formation of binary systems. Freire and his colleagues picked up a lot of gamma ray emission from the cluster so much that they initially thought the light was coming from 100 or so millisecond pulsars. But that wasn't the case."We now find that all detectable gamma ray emission comes from one single millisecond pulsar," Freire told Space.com in an email. That pulsar is J1823-3021A, which is spinning at about 11,100 revolutions per minute, or one complete turn every 5.44 milliseconds. The team didn't discover the pulsar; it has been known since the 1990s. But its incredible gamma ray brightness remained undetected until now.

J1823-3021A also appears to have a much stronger magnetic field than other millisecond pulsars. The exotic object's combination of characteristics is likely to have astronomers scratching their heads, Freire said. "It challenges the way we believe millisecond pulsars form," he said. "It was not thought that, for the spin period of this object (5.44 ms), they could be so energetic and have such a high magnetic field."



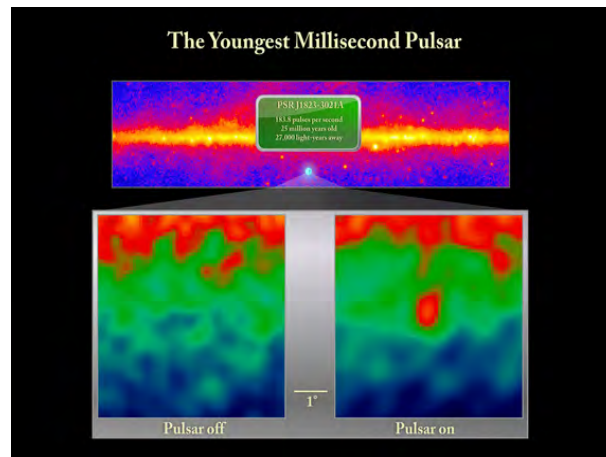


Figure 2.3: This image shows the on and off state of gamma rays from pulsar J1823-3021A as seen by Fermi's Large Area Telescope (LAT) .The object pulses 183.8 times a second and has a spin period of 5.44 milliseconds, which translates to 11,000 rpm.

Its amazing that all of the gamma rays we see from this cluster are coming from a single object. It must have formed recently based on how rapidly its emitting energy. Its a bit like finding a screaming baby in a quiet retirement home, said Paulo Freire, the studys lead author, at the Max Planck Institute for Radio Astronomy in Bonn, Germany.

## Chapter 3

# Effect of Asymmetric Magnetic Field Topology

A different kick mechanism relies on the asymmetric magnetic field distribution in proto-neutron stars (see Bisnovaty-Kogan 1993; however, he considered neutron decay, which is not directly relevant for neutrino emission from proto-neutron stars). Since the cross section for  $\nu_e$  ( $\bar{\nu}_e$ ) absorption on neutrons (protons) depends on the local magnetic field strength due to the quantization of energy levels for the  $e^-$  ( $e^+$ ) produced in the final state, the local neutrino fluxes emerged from different regions of the stellar surface are different. Calculations indicate that to generate a kick velocity of  $\sim 300 km s^{-1}$  using this mechanism alone would require that the difference in the field strengths at the two opposite poles of the star be at least  $10^{16}$  G (Lai and Qian 1998b). Note that unlike the kick due to parity violation, this mechanism does not require the magnetic field to be ordered, i.e., only the magnitude of the field matters.

### 3.1 $\nu_e$ And $\bar{\nu}_e$ Absorption Cross Sections In Strong Magnetic Fields

In this section we consider how magnetic fields modify the (total) neutrino absorption ( $\nu_e + n \rightarrow p + e^-$  and  $\bar{\nu}_e + p \rightarrow n + e^+$ ) cross sections for specific neutrino energies. We present our calculation of the  $\nu_e$  absorption cross section in detail. The  $\bar{\nu}_e$  absorption cross section can be obtained in a similar manner.

To begin with, it is useful to review various energy scales involved in the problem. Near the neutrinosphere and the neutrino-matter decoupling region, the matter density is typically  $10^{11}$ - $10^{12} \text{ g cm}^{-3}$ , and the temperature is about several MeV. The electrons are extremely relativistic and highly degenerate, with Fermi energy  $E_{F,e} \simeq 51.5 (Y_e \rho_{12})^{1/3} \text{ MeV}$ , where  $Y_e \sim 0.1$  is the electron fraction (the number electron per nucleon) and  $\rho_{12} = \rho / (10^{12} \text{ g cm}^{-3})$ . The neutrons and protons have Fermi energies  $E_{F,n} = 1.4[(1-Y_e)\rho_{12}]^{2/3} \text{ MeV}$ , and  $E_{F,p} = 1.4(Y_e \rho_{12})^{2/3}$  respectively, and thus they are essentially nondegenerate.

### 3.1.1 The Cross Sections

In strong magnetic fields the transverse motion of the electron is quantized into Landau levels. The electron energy dispersion relation is

$$E_e = [p_z^2 + m_e^2(1 + 2nb)]^{1/2}, \quad n = 0, 1, 2, \dots, \quad (3.1.1)$$

where  $p_z$  is the longitudinal momentum (along the magnetic field), and  $n$  is the Landau level index. The dimensionless field strength  $b$  is defined as

$$b \equiv \frac{B}{B_c}, \quad \text{with } B_c = \frac{m_e^2 c^3}{\hbar e} = 4.414 \times 10^{13} \text{ G}. \quad (3.1.2)$$

[ $B_c$  is the critical field strength defined by equating the cyclotron energy  $\hbar e B / (m_e c)$  to  $m_e c^2$ .] The quantization effect on the proton is extremely small because of the large proton mass and is neglected throughout this paper.

Since the cyclotron radius (characterizing the size of the Landau wavefunction)  $[\hbar c / (eB)]^{1/2} = [\hbar / (m_e c)] b^{1/2} = 386 b^{-1/2} \text{ fm}$  is much greater than the range of the weak interaction  $[\hbar / (m_w c)] \simeq 2.5 \times 10^{-3} \text{ fm}$ , where  $m_w \simeq 80 \text{ GeV}$  is the mass of W boson], one can use the V-A theory with contact interaction. Locally (at the point of interaction) the Landau wavefunction resembles a plane wave. Thus, we expect that the spin-averaged matrix element is the same as in the case without magnetic fields. This expectation has been borne out by detailed calculations (see, e.g., Fassio Canuto 1969 ; Matese and O'Connell 1969). Therefore, we only need

to consider the effect of magnetic field on the electron phase space.

Since the neutrino energy (a few to tens of MeV) of interest is much less than the nucleon mass, we first consider the case where the nucleon mass is taken to be infinity, an approximation adopted in the standard zero-field calculations (Tubbs and Schramm 1975). The cross section for  $\nu_e + n \rightarrow p + e^-$  can be written as a sum over the (spatial and spin) states of the electron (per unit volume)

$$\sigma_B^{(abs)}(E_\nu) = A \sum_e (1 - f_e) \delta(E_\nu + Q - E_e), \quad (3.1.3)$$

where we have defined

$$A \equiv \pi G_F^2 \cos^2 \theta_c (c_v^2 + 3c_A^2), \quad (3.1.4)$$

and the other symbol have their usual meanings:  $G_F = (293 \text{ GeV})^{-2}$  is the universal Fermi constant,  $\cos^2 \theta_c = 0.95$  refers to the Cobbibo angle,  $c_v = 1$  and  $c_A = 1.26$  are coupling constants,  $Q = m_n - m_p = 1.293 \text{ MeV}$  is the neutron-proton mass difference,  $E_\nu$  is the  $\nu_e$  energy, and  $(1 - f_e)$  is the pauli blocking factor. The electron occupation number  $f_e$  is

$$f_e = \frac{1}{\exp[(E_e - \mu_e)/T] + 1} \quad (3.1.5)$$

where  $E_e$  is the electron energy,  $\mu_e$  is the electron chemical potential, and T is the matter temperature. For zero-field, the sum becomes  $2 \int d^3p / (2\pi)^3$ , and we have

$$\sigma_{B=0}^{(abs)}(E_\nu) = \frac{A}{\pi^2} (E_\nu + Q) [(E_\nu + Q)^2 - m_e^2]^{1/2} (1 - f_e), \quad (3.1.6)$$

with  $E_e = E_\nu + Q$ . For  $B \neq 0$ , knowing that the degeneracy of the landau level (per unit area) is  $g_n eB / hc = g_n m_e^2 b / (2\pi)$ , with  $g_n = 1$  for  $n=0$  and  $g_n = 2$  for  $n>0$ , we have

$$\begin{aligned} \sigma_B^{(abs)}(E_\nu) &= A \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \sum_n g_n \frac{m_e^2 b}{2\pi} (1 - f_e) \delta(E_\nu + Q - E_e) \\ &= A \frac{m_e^2 b}{2\pi^2} \sum_{n=0}^{n_{max}} g_n \frac{E_e}{p_{z,n}} (1 - f_e), \end{aligned} \quad (3.1.7)$$

where  $E_e = E_\nu + Q$ , and  $p_{z,n} = [E_e^2 - m_e^2(1 + 2nb)]^{1/2}$ . The upper limit  $n_{max}$  for the sum is the maximum value of n for which  $p_{z,n}$  is meaningfully defined, i.e.,

$$n_{max} = \text{Int} \left[ \frac{(E_\nu + Q)^2 - m_e^2}{2m_e^2 b} \right] \quad (3.1.8)$$

where  $\text{Int}[x]$  stands for the integral part of  $x$ . Henceforth,  $n_{max}$  is similarly defined, and we shall not explicitly write out the lower and upper limits for the sum.

Figure 3.1, depicts  $\sigma_B^{(abs)}(E_\nu)$  for  $b=100$  and  $\sigma_B^{(abs)}(E_\nu)$  as functions of  $E_\nu$  for

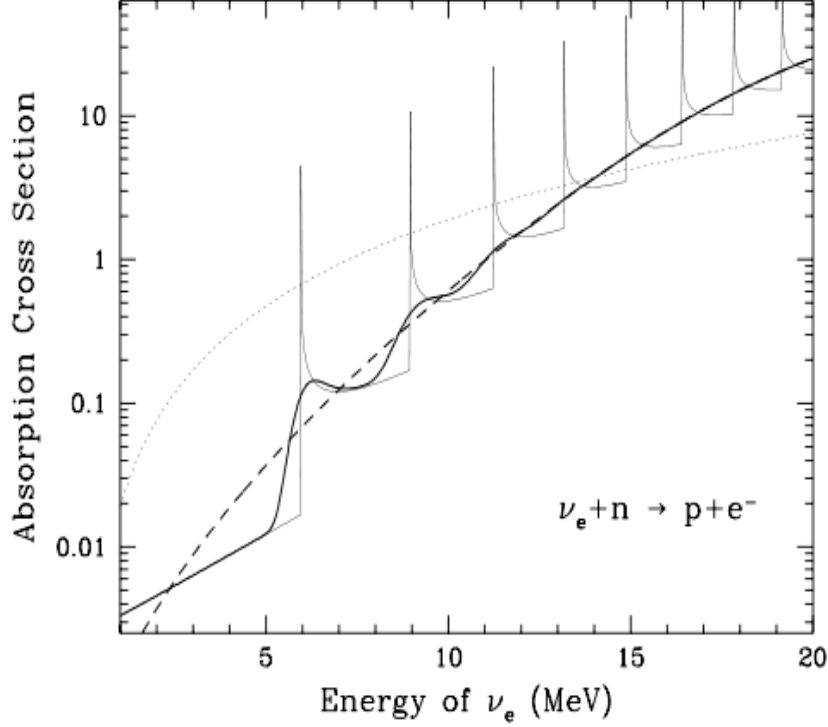


Figure 3.1: Absorption cross section (per nucleon) of  $\nu_e, (1 - Y_e) \sigma_B^{(abs)}(E_\nu)$ , in units of  $A \times \text{MeV}^2 = \pi G^2 \cos^2 \theta_c (c_V^2 + 3c_A^2) \text{MeV}^2 \simeq 0.91 \times 10^{-42} \text{cm}^2$ , as a function of the neutrino energy. The dashed line is the zero-field result (The effect of the nucleon thermal motion is included but is hard to discern). The heavy and the light solid lines are the results for  $b = 100$ , with and without the effect of the nucleon thermal motion, respectively. The dotted line is the elastic scattering cross section,  $\sigma_B^{(sc)}(E_\nu)$ , in the same units. All results are for  $T = 3 \text{ MeV}$ ,  $Y_e = 0.1$ , and  $\mu_e = 20 \text{ MeV}$  (corresponding to  $7.2 \times 10^{11} \text{ gcm}^{-3}$ ).

typical conditions ( $Y_e = 0.1$ ,  $\mu_e = 20 \text{ MeV}$ , and  $T = 3 \text{ MeV}$ ) in a proto-neutron star. Note that ideally, one should compare  $\sigma_B^{(abs)}(E_\nu)$  and  $\sigma_{B=0}^{(abs)}(E_\nu)$  at the same matter density. For  $\mu_e \simeq E_{F,e} \gg T$  (so that  $n_e^- \gg n_e^+$ ), the matter density is related to  $\mu_e$  via

$$\rho = \frac{m_n}{Y_e} \frac{1}{\pi^2} \int_0^\infty p^2 dp f_e \quad (\text{for } B = 0). \quad (3.1.9)$$

$$\rho = \frac{m_n}{Y_e} \frac{m_e^2 b}{2\pi^2} \int_0^\infty dp_z \sum_n g_n f_e \quad (\text{for } B \neq 0). \quad (3.1.10)$$

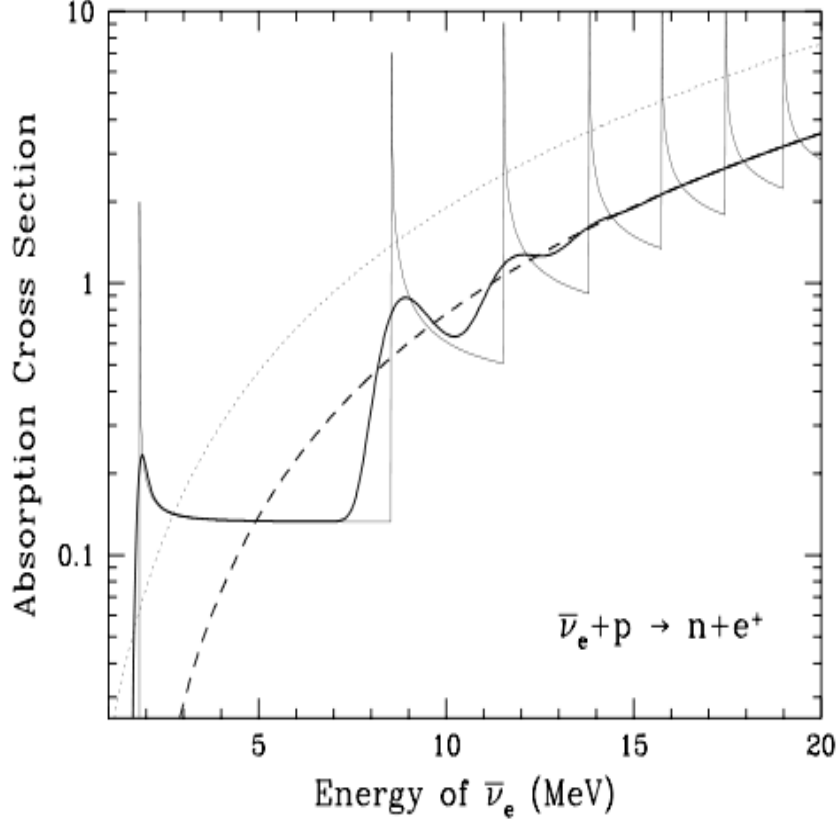


Figure 3.2: Absorption cross section (per nucleon) of  $\bar{\nu}_e$ ,  $Y_e \sigma_B^{(abs)}(E_\nu)$ . The physical parameters, units, and labels are the same as in Fig.3.1.

In practice, we have found that for a given  $\mu_e$ , the dependence of  $\rho$  on the field strength is rather weak for the range of parameters of interest in this paper. Thus, in all of our calculations here and below, we compare the cross sections at a given  $\mu_e$ . A prominent feature of  $\sigma_B^{(abs)}(E_\nu)$  shown in Figure 3.1. is the presence of “resonances,” which occur when  $p_{z,n} = 0$  (eq. [3.1.7]), i.e., at the neutrino energies for which the final state electron lies in a “stationary” Landau level (with zero longitudinal momentum). These resonances reflect the divergence of the density of states of electrons at these stationary Landau levels.

The cross section for  $\bar{\nu}_e + p \rightarrow n + e^+$  can be similarly calculated. One only needs to replace  $Q$  with  $-Q$  and  $\mu_e$  with  $-\mu_e$  in the expressions for  $\nu_e$  absorption to obtain the desired expressions for  $\bar{\nu}_e$  absorption. The numerical results for  $\bar{\nu}_e$  absorption cross section are shown for Figure 3.2. for the same physical parameters as used for Figure 3.1.

### 3.1.2 Thermal Averaging

The divergent cross sections at the "resonances" shown in Figures 3.1. and 3.2. are clearly unphysical. They are smoothed out by the thermal motion of the neutron. To the leading order, the width of the resonance is  $(kT/m_n)^{1/2} E_\nu$ , corresponding to the Doppler shift of the neutrino energy in the neutron's rest frame. The recoil energy, of order  $E_\nu^2/m_n$ , is much less effective in smoothing out the resonances and will be neglected. With this approximation, the smoothed cross section can be written as

$$\langle \sigma_B^{(abs)}(E_\nu) \rangle \simeq \frac{1}{(2\pi)^2} \int v_n^2 dv_n f_n \int d \cos \theta_n \sigma_B^{(abs)}(E'_\nu), \quad (3.1.11)$$

where  $E'_\nu = E_\nu(1 - v_n \cos \theta_n)$  and  $f_n = (2nm/T)^{3/2} \exp(-mv_n^2/2T)$  is the velocity distribution function for the neutron. If we replace the factor  $(1 - f_e)$  [which depends on  $(E'_\nu + Q)$ ] in  $\sigma_B^{(abs)}(E'_\nu)$  with the average value  $\langle 1 - f_e \rangle$ , which can be approximately taken as  $(1 - f_e)$  evaluated at  $E_e = E_\nu + Q$ , then the angular integral in equation (3.1.11) can be carried out analytically. For  $B = 0$ , we have

$$\begin{aligned} \langle \sigma_{B=0}^{(abs)}(E_\nu) \rangle &\simeq \frac{A}{(2\pi)^2} \langle 1 - f_e \rangle \int v_n dv_n f_n \frac{1}{3\pi^2 E_\nu} \\ &\times (E_e^2 - m_e^2)^{3/2} \Big|_{E_e = E_\nu(1-v_n)+Q}^{E_e} = E_\nu^{(1+v_n)+Q}, \end{aligned} \quad (3.1.12)$$

which is reduced to

$$\langle \sigma_{B=0}^{(abs)}(E_\nu) \rangle \simeq \frac{A}{\pi^2} (E_\nu + Q)^2 \langle 1 - f_e \rangle \times \left[ 1 + \left( \frac{E_\nu}{E_\nu + Q} \right)^2 \frac{T}{m_n} \right], \quad (3.1.13)$$

if we neglect the electron mass  $m_e$ . For  $B \neq 0$ , we find

$$\begin{aligned} \langle \sigma_B^{(abs)}(E_\nu) \rangle &\simeq \frac{A}{(2\pi)^2} \langle 1 - f_e \rangle \int v_n dv_n f_n \frac{m_e^2 b}{2\pi^2 E_\nu} \\ &\times \sum_n g_n [E_e^2 - m_e^2(1 + 2nb)]^{1/2} \Big|_{E_e = E_\nu(1-v_n)+Q}^{E_e} = E_\nu^{(1+v_n)+Q} \end{aligned} \quad (3.1.14)$$

We could have approximately included the neutron recoil effect by replacing  $Q$  and  $Q - (E_\nu + Q)^2/(2m_n)$  in equations (3.1.12)-(3.1.14). But we choose to leave out this effect since as mentioned previously, it is ineffective in smoothing out the

resonances in  $\sigma_B^{(abs)}(E_\nu)$ . We have numerically evaluated equations (3.1.12) and (3.1.14), and the results are depicted in Figure 3.1., together with the cross section are shown in Figure 3.2. As expected, we find that for  $B=0$ , the averaged cross section  $\langle\sigma_{B=0}^{(abs)}(E_\nu)\rangle$  is almost identical to  $\sigma_{B=0}^{(abs)}(E_\nu)$  (see eq. [3.1.13]). However, for  $B \neq 0$ , the thermal averaging is essential in obtaining the smoothed physical absorption cross sections.

### 3.1.3 Limiting Cases

The oscillatory behaviors of  $\langle\sigma_B^{(abs)}(E_\nu)\rangle$  for Figures 3.1. and 3.2. can be demonstrated analytically. Using the Poisson summation formula (Ziman 1979) :

$$\sum_{n=0}^{\infty} f(n + \frac{1}{2}) = \sum_{s=-\infty}^{\infty} (-1)^s \int_0^{\infty} f(x) e^{2\pi i s x} dx, \quad (3.1.15)$$

we can rewrite the sum in equation (3.1.14) as

$$\begin{aligned} m_e^2 b \sum_n g_n [E_e^2 - m_e^2 (1 + 2nb)]^{1/2} &= \frac{2}{3} (E_e^2 - m_e^2)^{3/2} \\ &+ (2m_e^2 b)^{3/2} \text{Re} \sum_{s=1}^{\infty} \frac{1-i}{4\pi i s^{3/2}} \exp \left[ 2\pi i s \left( \frac{E_e^2 - m_e^2}{2m_e^2 b} \right) \right]. \end{aligned} \quad (3.1.16)$$

The first term on the right-hand side of equation (3.1.16) corresponds to the zero-field result, and the second term (involving the sum of many oscillatory terms) corresponds to the correction owing to the quantization effect. The sum is clearly convergent, and thus for small  $B$  the averaged cross section  $\langle\sigma_B^{(abs)}(E_\nu)\rangle$  is reduced to the zero-field value  $\langle\sigma_{B=0}^{(abs)}(E_\nu)\rangle$

For sufficiently large  $B$  at a given  $E_\nu$ , or, equivalently, for sufficiently small  $E_\nu$  at a given  $B$ , only the ground state Landau level is filled by the electron, i.e.,  $n_{max} = 0$  (see eq [3.1.8]). In this limit,  $\langle\sigma_B^{(abs)}(E_\nu)\rangle > \langle\sigma_{B=0}^{(abs)}(E_\nu)\rangle/4$ , and only the first term in equation (3.1.7) needs to be retained. We then have

$$\langle\sigma_B^{(abs)}(E_\nu)\rangle \simeq A \frac{m_e^2 b}{2\pi^2} \frac{(E_\nu + Q)}{[(E_\nu + Q)^2 - m_e^2]^{1/2}} (1 - f_e) \quad (3.1.17)$$

for  $\nu_e$  absorption. A similar expression for  $\bar{\nu}_e$  absorption is obtained by replacing  $Q$  with  $-Q$  and  $\mu_e$  with  $-\mu_e$  in equation (3.1.17).



### 3.1.4 Discussion

The physical conditions that Figures 3.1 and 3.2 are based on ( $T = 3$  MeV,  $Y_e = 0.1$ , and  $\mu_e = 20$  MeV, corresponding to a matter density of  $\rho = 7.2 \times 10^{11} \text{gcm}^{-3}$ ) are typical of the neutrinosphere of the proto-neutron star. In Figure 3.3, we plot the  $\nu_e$  and  $\bar{\nu}_e$  absorption cross sections for a number of slightly different physical parameters, specified by  $b$ ,  $\mu_e$  and  $T$  (all with  $Y_e = 0.1$ ). The cross section for elastic neutral current scattering  $\nu + N \rightarrow \nu + N$ ,  $\sigma^{sc}(E_\nu) \simeq G_F^2 E_\nu^2 / \pi$  (Tubbs and Schramm 1975), is also shown for comparison. Absorption dominates the transport of  $\nu_e$  for high energies, with the ratio of absorption to scattering cross sections per nucleon  $(1 - Y_e)\sigma^{(abs)}/\sigma^{(sc)}$  approaching for  $E_\nu \gg \mu_e$ . For smaller

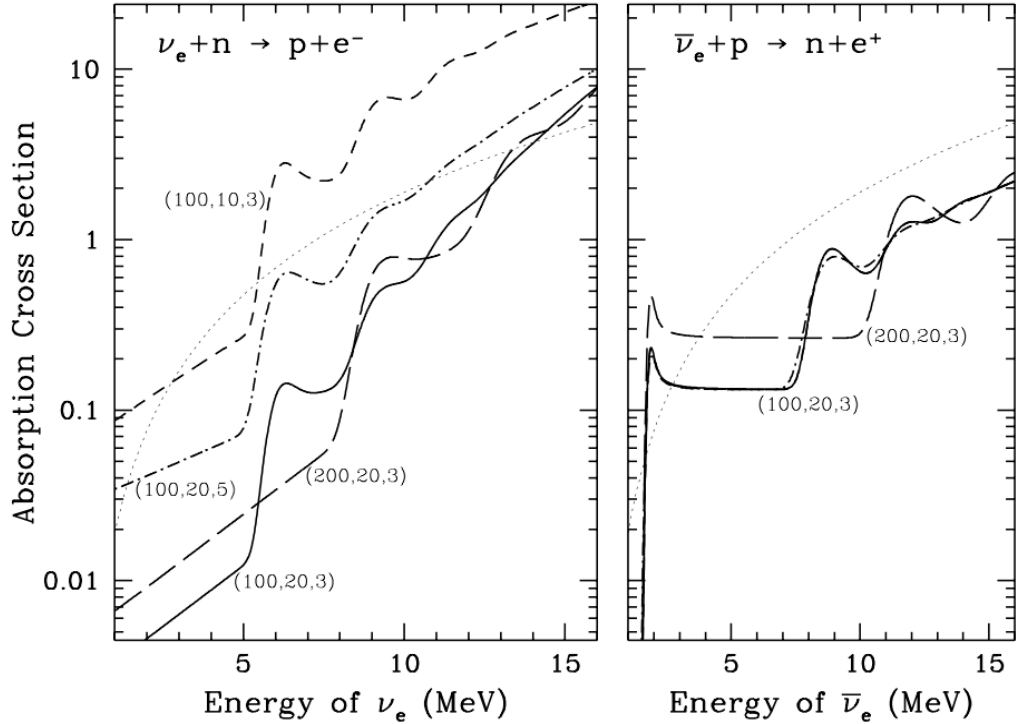


Figure 3.3: Absorption cross sections (per nucleon) of  $\nu_e$  (left) and  $\bar{\nu}_e$  (right), in units of  $\text{A} \times \text{MeV}^2 = \pi G_F^2 \cos^2 \theta_c (c_V^2 + 3c_A^2) \times \text{MeV}^2 \simeq 0.91 \times 10^{-42} \text{cm}^2$ , as functions of the neutrino energy for different physical parameters. All results assume  $Y_e = 0.1$ . The solid, short-dashed, long-dashed, and dot-dashed curves are for  $(b, \mu_e / \text{MeV}, T / \text{MeV}) = (100, 20, 3)$ ,  $(100, 10, 3)$ ,  $(200, 20, 3)$ , and  $(100, 20, 5)$ , respectively. [Note that for  $\bar{\nu}_e$ , the curve for  $(b, \mu_e / \text{MeV}, T / \text{MeV}) = (100, 10, 3)$  is almost identical to the curve for  $(b, \mu_e / \text{MeV}, T / \text{MeV}) = (100, 20, 3)$ , and both curves are close to the curve for  $(b, \mu_e / \text{MeV}, T / \text{MeV}) = (100, 20, 5)$ .] The light dotted lines are the elastic scattering cross sections in the same units.

energies ( $E_\nu \lesssim \mu_e$ ), absorption is severely suppressed by the Pauli blocking of the final electron states, and scattering becomes more important. In the case of  $\bar{\nu}_e$ , Pauli blocking is negligible, and scattering almost always dominates the transport, with the ratio of scattering to absorption cross section per nucleon  $\sigma^{(sc)}/[Y_e\sigma^{(abs)}] \simeq 2(0.1/Y_e)$  for  $E_\nu \gtrsim 10$  MeV.

As we can see from Figures 3.1-3.3, for a given set of physical parameters of the medium ( $T$ ,  $Y_e$ ,  $\mu_e$  or  $\rho$ ), the magnetic field effect on the absorption opacity is more prominent for lower energy neutrinos than for higher energy ones. This is because for a given field strength, as the neutrino energy increases, more and more Landau levels become available for the electron in the final state (see eq. [3.1.8]). Consequently, the electron phase space and hence the cross section should approach the zero-field values according to the correspondence principle. We found that typically when the number of Landau levels filled by the electron ( $n_{max}$ ) is greater than 2-3, the magnetic field effect on the absorption opacity becomes negligible. The "critical" neutrino energy is approximately given by

$$E_\nu^{crit} \simeq m_e(2bn_{max} + 1)^{1/2} \simeq 6\left(\frac{n_{max}}{3}\right)^{1/2} \left(\frac{B}{10^{15}G}\right)^{1/2} MeV. \quad (3.1.18)$$

for  $E_\nu \gtrsim E_\nu^{crit}$ , one can approximate  $\langle\sigma_B^{(abs)}(E_\nu)\rangle$  by the zero-field value  $\langle\sigma_{B=0}^{(abs)}(E_\nu)\rangle$  with good accuracy (within a few percent). Alternatively, we can define a threshold magnetic field,  $B_{th}$ , above which the  $\nu_e$  and  $\bar{\nu}_e$  absorption opacities are modified (by more than a few percent) by the magnetic field:

$$B_{th} \simeq \frac{\langle E_\nu \rangle^2}{2n_{max}m_e^2} B_c \simeq 3 \times 10^{15} \left(\frac{\langle E_\nu \rangle}{10 MeV}\right)^2 \left(\frac{3}{n_{max}}\right) G, \quad (3.1.19)$$

where  $\langle E_\nu \rangle \sim 10$  MeV is the average neutrino energy.

In the deeper layers of the neutron star, the typical neutrino energy is higher. Therefore, we expect the magnetic field effect on the absorption opacities to be smaller there. In addition, for  $\nu_e$ , absorption becomes unimportant as compared to scattering in the high-density region (although not necessarily in the deepest

interior, where  $\nu_e$  can be degenerate). Thus, in order to assess the importance of the magnetic field effect on the neutrino emission, we only need to focus on the surface region, the region between the neutrinosphere and the neutrino-matter decoupling layer. We note that because the absorption opacities in the magnetic field for different neutrino energies oscillate around the zero-field values, without a detailed calculation of the mean opacity (appropriately averaged over the neutrino energy spectrum) one cannot even answer the qualitative question such as whether the magnetic field increases or decreases the local neutrino energy flux.

In section 3.1-3.3, we study how the modified absorption cross sections affect the emergent  $\nu_e$  and  $\bar{\nu}_e$  energy fluxes and spectra from the proto-neutron star.

## 3.2 Neutrino Transport Near The Stellar Surface

As discussed in section 3.1.4, the quantization effect of magnetic fields on neutrino absorption is important in the region only near the neutron star surface (between the neutrino sphere and the neutrino-matter decoupling layer). In this section we consider neutrino transport in this relatively low-density region.

Because  $\nu_e$  and  $\bar{\nu}_e$  with typical energies of 10 MeV have comparable absorption and scattering opacities (see Fig. 3.3), the emergent  $\nu_e$  and  $\bar{\nu}_e$  energy fluxes and spectra are determined essentially in the same region where these two neutrino species just begin to thermally decouple from the proto-neutron star matter. The  $\nu_e$  and  $\bar{\nu}_e$  occupation numbers in this region are then approximately given by

$$f_{\nu_e}(E_\nu) = \frac{1}{\exp[(E_\nu/T) - \eta_\nu] + 1}, \quad (3.2.1)$$

and

$$f_{\bar{\nu}_e}(E_\nu) = \frac{1}{\exp[(E_\nu/T) + \eta_\nu] + 1}, \quad (3.2.2)$$

respectively, where  $\eta_\nu = \mu_\nu/T$ , and  $\mu_\nu$  is the  $\nu_e$  chemical potential. From these occupation numbers, we obtain

$$n_{\nu_e} - n_{\bar{\nu}_e} = \frac{\eta_\nu}{6} \left( 1 + \frac{\eta_\nu^2}{\pi^2} \right) T^3, \quad (3.2.3)$$

and

$$U_{\nu_e} + U_{\bar{\nu}_e} = \frac{7\pi^2}{120} \left( 1 + \frac{30\eta_\nu^2}{7\pi^2} \right) T^4, \quad (3.2.4)$$

where for example,  $n_{\nu_e}$  and  $U_{\nu_e}$  are the number and energy densities, respectively, for the  $\nu_e$ . We can estimate the magnitude of  $\eta_\nu$  as follows. The net deleptonization rate of the proto-neutron star is

$$-\dot{N}_e \sim \frac{Y_{e,i} M / m_N}{t_{diff}} \sim c(n_{\nu_e} - n_{\bar{\nu}_e}) \pi R^2, \quad (3.2.5)$$

where  $Y_{e,i}$  is the initial electron fraction of the proto-neutron star,  $M$  is the proto-neutron star mass,  $m_N$  is the nucleon mass,  $t_{diff}$  is the neutrino diffusion time-scale, and  $R$  is the proto-neutron star radius. The sum of the  $\nu_e$  and  $\bar{\nu}_e$  luminosities is

$$L_{\nu_e} + L_{\bar{\nu}_e} \sim \frac{1}{3} \frac{GM^2/R}{t_{diff}} \sim c(U_{\nu_e} + U_{\bar{\nu}_e}) \pi R^2 \quad (3.2.6)$$

From equations(3.2.3)-(3.2.6), we have

$$\eta_\nu \sim \frac{21\pi^2}{20} \frac{1 + (30/7\pi^2)\eta_\nu^2 + (15/7\pi^4)\eta_\nu^4}{1 + (\eta_\nu^2/\pi^2)}$$

$$\frac{Y_{e,i} T R}{GM m_N} \sim 0.1 \left( \frac{Y_{e,i}}{0.36} \right) \left( \frac{T}{5 \text{ MeV}} \right) \ll 1, \quad (3.2.7)$$

where the numerical value is obtained for  $M \simeq 1.4M_\odot$  and  $R \simeq 10$  km. Therefore, to a good approximation, the  $\nu_e$  and  $\bar{\nu}_e$  in the decoupling region have Fermi-Dirac energy distributions with zero chemical potentials.

Let  $I_E = I_E(r, \hat{\Omega})$  be the specific intensity of neutrinos of a given species, where  $r$  is the spatial position, and  $\hat{\Omega}$  specifies the direction of propagation. The general transport equation takes the form

$$\hat{\Omega} \cdot \nabla I_E = \rho \kappa_E^{(abs)*} I_E^{(FD)} + \rho \kappa_E^{(sc)} \frac{1}{4\pi} \int d\Omega' I_E'$$

$$- \rho [\kappa_E^{(abs)*} + \kappa_E^{(sc)}] I_E, \quad (3.2.8)$$

where  $I'_E = I_E(r, \hat{\Omega}')$ , and the subscript “E” indicates that the relevant physical quantities depend on the neutrino energy E. The opacities  $\kappa_E^{(abs)*}$  and  $\kappa_E^{(sc)}$  are related to the corresponding cross sections as, e.g.,  $\kappa_E^{(abs)*} = (1 - Y_e)\sigma_E^{(abs)*}/m_N$  and  $\kappa_E^{(sc)} = \sigma_E^{(sc)}/m_N$  for  $\nu_e$ . Note that we have included the effect of stimulated absorption for neutrinos (Imshennik and Nadezhin 1973) by introducing the corrected absorption opacity  $\kappa_E^{(abs)*} = \kappa_E^{(abs)}(1 + \exp(-E/T))$  (T is the matter temperature). Here after, we shall suppress the superscript “\*” in  $\kappa_E^{(abs)*}$ , and the notation  $\kappa_E^{(abs)}$  should be understood as having included the correction due to stimulated absorption. The quantity  $I_E^{(FD)}$  in equation (3.2.8) is the equilibrium neutrino intensity, given by the Fermi-Dirac distribution

$$I_E^{(FD)} = \frac{E^3}{c^2 h^3} \frac{1}{\exp(E/T) + 1}. \quad (3.2.9)$$

Note that the time derivative term  $(\partial I_E / \partial t)$  on the left-hand side of equation (3.2.8) has been neglected. This corresponds to an instantaneous redistribution of emission and absorption sources (i.e., an instantaneous redistribution of matter temperature). The timescale for the redistribution is of order the mean free path divided by c, much smaller than the neutrino cooling time of the star.

The zeroth-order moment of the transport equation can be written as

$$\nabla \cdot F_E = \rho \kappa_E^{(abs)} c (U_E^{(FD)} - U_E), \quad (3.2.10)$$

where  $F_E \int I_E \hat{\Omega} d\Omega$  is the (spectral) neutrino energy flux,  $U_E = \int I_E d\Omega / c$  is the energy density, and  $U_E^{(FD)} = (4\pi/c) I_E^{(FD)}$  is the corresponding equilibrium energy density. With Eddington closure relation, the first moment of equation (3.2.8) becomes

$$F_E = -\frac{c}{3\rho\kappa_E^{(t)}} \nabla U_E, \quad (3.2.11)$$

where  $\kappa_E^{(t)} = \kappa_E^{(abs)} + \kappa_E^{(sc)}$ . Equations (3.2.10) and (3.2.11) completely specify the neutrino radiation field.

### 3.3 Rosseland Mean Opacities

Here we are concerned with the total neutrino energy flux emitted from a local surface region of the neutron star. The magnetic field strength in this local region is assumed to be constant. Since the scale heights for physical quantities such as the matter density and temperature are much smaller than the local radii, we adopt the plane-parallel geometry. Integrating equation (3.2.11) over the neutrino energy, we find that the total neutrino energy flux (of a specific species)  $F = \int F_E dE = F\hat{r}$  is given by

$$F = -\frac{c}{3\rho\kappa_R} \frac{dU}{dr} \quad (3.3.1)$$

where  $U = \int U_E dE$  is the total neutrino energy density, and  $\kappa_R$  is the “Rosseland mean opacity” defined as

$$\frac{1}{\kappa_R} = \frac{\int dE \left( \frac{1}{\kappa_E^{(t)}} \right) \left( \frac{dU_E}{dr} \right)}{\int dE \left( \frac{dU_E}{dr} \right)} \quad (3.3.2)$$

Since neutrino emission and absorption rates are extremely fast even in the regions near the proto-neutron star surface,

a radiative equilibrium approximately holds, i.e.,

$$\int \rho \kappa_E^{(abs)} (U_E^{(FD)} - U_E) dE \simeq 0. \quad (3.3.3)$$

From equation (3.2.10), we have  $dF/dr \simeq 0$ , or  $F$  is approximately constant.

If we introduce the Rosseland mean optical depth measured from the stellar surface via  $d\tau_R = \rho \kappa_R dr$ , with  $F$  constant, equation (3.3.1) can be integrated to yield

$$cU = 3F(\tau_R + \frac{2}{3}), \quad (3.3.4)$$

where we have used the boundary condition  $F = cU/2$  at the surface ( $\tau_R = 0$ ). Equation (3.3.4) resembles the standard Eddington profile. Note that outside the decoupling region the neutrino energy density  $U$  may not be equal to  $U^{(FD)} = (7\pi^4/240)T^4$  specified by the matter temperature  $T$ . Similarly,  $U_E$  may differ substantially from  $U_E^{(FD)}(T)$  (see § 3.5). However, we can still approximate  $U_E$

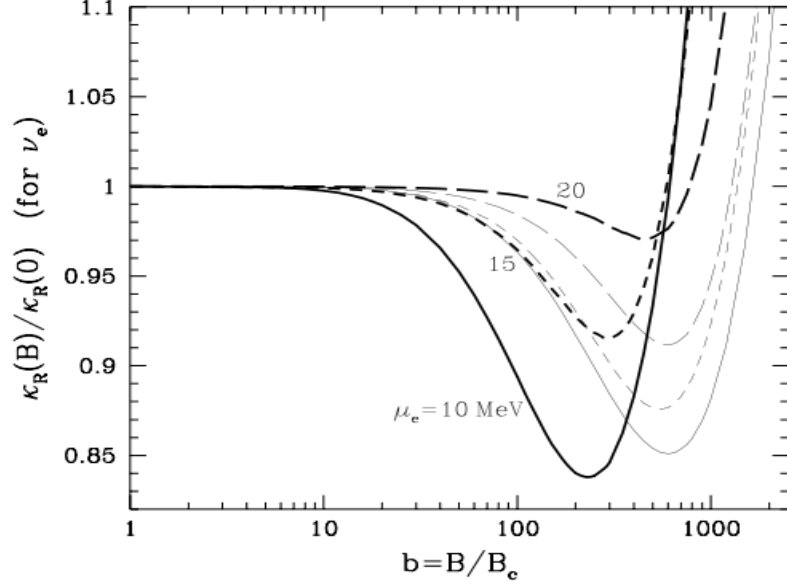


Figure 3.4: Ratio of the Rosseland mean opacity  $\kappa_R^{(B)}$  of  $\nu_e$  to its zero- field value  $\kappa_R^{(0)}$  as a function of the magnetic field strength  $b = B/B_c$  (with  $B_c = 4.414 \times 10^{13}$  G). All results assume  $Y_e = 0.1$ . The heavier curves correspond to  $T = T_\nu = 3$  MeV, and the lighter cures correspond to  $T = T_\nu = 5$  MeV. The solid, short-dashed, and long-dashed lines are for  $\mu_e = 10, 15$ , and  $20$  MeV, respectively.

by the Fermi-Dirac function at the neutrino temperature  $T_\nu$ , which is equal to the matter temperature at the decoupling layer. Equation (3.3.2) can then be rewritten as

$$\frac{1}{\kappa_R} = \frac{30}{7\pi^4 T_\nu^5} \int dE \frac{1}{\kappa_R^{(t)}} \frac{E^4 \exp(E/T_\nu)}{[\exp(E/T_\nu) + 1]^2}. \quad (3.3.5)$$

With the cross sections given in § 3.1, we can calculate the Rosseland mean opacity  $\kappa_R^{(B)}$  in magnetic fields using equation (3.3.5). Note that in equation (3.3.5)  $\kappa_R^{(t)}$  depends on the local matter temperature  $T$ . Only in the regions not far from the decoupling layer, do we have  $T_\nu = T$ .

Figure 3.4 shows the ratio of the Rosseland mean opacity  $\kappa_R^{(B)}$  of  $\nu_e$  to the zero-field value  $\kappa_R^{(0)}$  as a function of the magnetic field strength for typical conditions near the proto-neutron star surface. We choose these conditions to be  $T = T_\nu = 3, 5$  MeV, and  $\mu_e = 10, 15, 20$  MeV, all with  $Y_e = 0.1$ . Figure 3.5 shows similar results for  $\bar{\nu}_e$  with similar physical parameters ( $T = T_\nu = 3, 5, 7$  MeV ; Note that the results are insensitive to  $\mu_e$ ). We see that deviation of  $\kappa_R^{(B)}$  from the zero-field value by more than 5% generally requires field strength  $b = B/B_c \gtrsim 100$ .

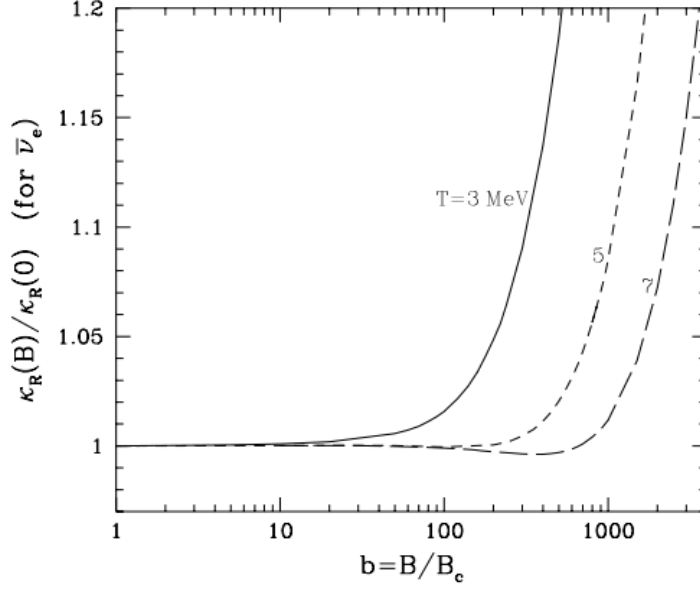


Figure 3.5: Ratio of the Rosseland mean opacity  $\kappa_R^{(B)}$  of  $\bar{\nu}_e$  to its zero- field value  $\kappa_R^{(0)}$  as a function of the magnetic field strength  $b = B/B_c$  (with  $B_c = 4.414 \times 10^{13}$  G). All results assume  $Y_e = 0.1$  and  $\mu_e = 20$  MeV (but the results are rather insensitive to the value of  $\mu_e$ ). The solid, short-dashed, and long-dashed lines are for  $T = T_\nu = 3, 5$ , and  $7$  MeV, respectively.

Note that the results mainly depend on  $T_\nu$  and are rather insensitive to  $T$ .

The behavior of the ratio  $\kappa_R^{(B)}/\kappa_R^{(0)}$  as a function of  $b$  is certainly not obvious a priori. Since the Rosseland mean integral is dominated by neutrinos with the smallest opacities, and since  $\kappa_R^{(abs)}$  scales linearly with  $B$  in the regime where only the ground Landau level is filled (see eq. [3.1.17]), we expect that for sufficiently large  $B$ , the Rosseland mean opacity increases linearly with increasing  $B$ . This is indeed what is indicated in Figures 3.4 and 3.5. For “intermediate” field strength, the  $\kappa^{(abs)}$  curve for  $\nu_e$  (see Fig. 3.1) has a significant portion that is below the zero-field value at low neutrino energies. For such intermediate fields, we would expect  $\kappa_R^{(B)}$  to be less than the zero-field value. This explains the nonmonotonic behavior of  $\kappa_R^{(B)}/\kappa_R^{(0)}$  as a function of  $B$  shown in Figure 3.4. For  $\bar{\nu}_e$ , the nonmonotonic behavior is much less pronounced (see the curve for  $T = 7$  MeV in Fig. 3.5) since the energy range in which  $\kappa_R^{(abs)} < \kappa_R^{(abs)} = 0$  is smaller.



### 3.4 Differential Neutron Fluxes And neutron Star Kick From Asymmetric Magnetic Field Topology

Imagine a proto-neutron star with an asymmetric magnetic field topology, e.g., the north pole and the south pole have different magnitudes of magnetic field (say, by a factor of a few). What is the difference between the neutrino fluxes from these two regions resulting from the different absorption opacities of  $\nu_e$  and  $\bar{\nu}_e$ , and what is the resulting kick velocity of the neutron star? We recall that the fractional asymmetry  $\alpha$  in the total neutrino emission (of all species) required to generate a kick velocity  $V_{kick}$  is  $\alpha = 0.028[V_{kick}/(10^3 km s^{-1})][E_{tot}/(3 \times 10^{53} ergs)]$ , where  $E_{tot}$  is the total neutrino energy radiated from the neutron star (the neutron star mass has been set to  $1.4M_{\odot}$ ). In reality, rotation (if it is misaligned with the magnetic axis) tends to reduce the kick velocity. We also note that the asymmetric  $\nu_e$  and  $\bar{\nu}_e$  emission from the proto-neutron star would give rise to asymmetric neutrino heating behind the stalled shock in the delayed supernova mechanism. This asymmetric heating could lead to asymmetric supernova explosions.

To quantitatively understand how the magnetically modified opacities change the local neutrino flux, we will have to evaluate these opacities at different depths (corresponding to different physical conditions) below the neutron star surface, and to perform complete cooling calculations that take into account these opacities. However, Figures 3.4 and 3.5 clearly indicate that, even near the stellar surface, where one expects the modification to the opacities to be most prominent, and even with the most favorable conditions for the magnetic field to be effective in changing the opacities (i.e., low density and low temperature), a 5% change in the Rosseland mean opacity (for  $\nu_e$  or  $\bar{\nu}_e$ ) requires a magnetic field with  $b \gtrsim 100$ , or  $B \gtrsim 4 \times 10^{15}$  G.

In the following, we shall content ourselves with merely estimating the change in the local neutrino flux from the modified opacities.

We first consider the case in which the matter temperature throughout the neutron star is unaffected by the magnetic field. One might imagine this as a possibility if convection dominates energy transfer between different parts of the star. Under the neutrino-matter decoupling layer the specific neutrino energy density  $U_E$  is the same as the Fermi-Dirac function  $U_E^{(FD)}(T)$  specified by the matter temperature. The emergent neutrino flux is then inversely proportional to the Rosseland mean optical depth  $\tau_R$  from the decoupling layer to the stellar surface (see eq. [3.3.4]). Since  $\tau_R$  for  $\nu_e$  or  $\bar{\nu}_e$  at the decoupling layer is of order a few, the flux  $F$  is inversely proportional to  $\kappa_R$  evaluated near the decoupling region. If we use the curve for  $T = 3$  MeV and  $\mu_e = 10$  MeV in Figure 3.4 (which has the most pronounced magnetic correction), we find that the fractional change in the  $\nu_e$  flux is  $\delta F_{\nu_e}/F_{\nu_e} \sim 0.1$  at  $b = 100$  (If we had used the curve for  $\mu_e = 15$  or 120 MeV, which more closely represents the physical condition at the decoupling region, the modification  $\delta F_{\nu_e}/F_{\nu_e}$  would have been smaller by a factor of a few or more). Similarly, using the curve for  $T = 3$  MeV in Figure 3.5 to maximize the magnetic correction, we find  $\delta F_{\nu_e}/F_{\nu_e} \sim 0.02$  at  $b = 100$ . Since the fluxes of other types neutrinos are unchanged, and since different neutrino species more or less carry away the same amount of energy, the change in the total neutrino flux is only  $\delta F_{\nu_e}/F_{\nu_e} \sim 0.01$  at  $b = 100$ . If we now imagine that the north pole of the neutron star has  $b \sim 100$ , while the south pole has a field strength a factor of a few smaller, then, taking into account the geometric reduction ( $\sim \frac{1}{3}$ ), the flux asymmetry is about  $\alpha \sim 0.003$ , just enough to give a kick velocity of  $\sim 100 \text{ km s}^{-1}$  to the neutron star. With  $\kappa_R$  evaluated at more realistic conditions, we expect the resulting  $\alpha$  to be even smaller.

The assumption that the matter temperature is unaffected by the magnetic field (and therefore spherically symmetric) is unlikely to be valid. In reality, when a region of the star (say the north pole) emits more flux, it also cools faster. With reduced temperature, this region will tend to emit less flux. In the extreme case, one would find  $F \sim cU_o/\tau_R$ , with  $U_o$  the neutrino energy density in the stellar

core, and  $\tau_R$  the Rosseland mean optical depth from the core to the surface. Since in the bulk interior of the star, modification to  $\kappa_R$  because of the magnetic field is negligible, we would conclude that the change in the neutrino flux is much smaller than what is estimated in the last paragraph. To summarize, the modifications to the absorption opacities of  $\nu_e$  and  $\bar{\nu}_e$  due to quantized Landau levels give rise to rather small change in neutrino fluxes from the neutron star. To generate appreciable kick velocities ( $\sim 300 \text{ km s}^{-1}$ ) from this effect requires the difference of field strengths at the two opposite poles to be at least  $10^{16}$  G (and possibly much larger).

### 3.4.1 Asymmetric Emission of Neutrinos from a Proto-Neutron star

The inverse beta decay cross-section is direction sensitive through its dependence on the angle  $\theta$  is known, the angle between the neutrino 3-momentum and the magnetic field direction (see below). This anisotropic effect in the cross-section can have interesting astrophysical applications in explaining the high velocities of pulsars, of the order of  $450 \pm 90 \text{ km s}^{-1}$ . Typical pulsars have masses between  $1.0M_\odot$  and  $1.5M_\odot$ , i.e., about  $2 \times 10^{33} \text{ g}$ . The momentum associated with the proper motion of a pulsar would therefore be of order  $10^{41} \text{ g cm/s}$ . On the other hand the energy carried off by neutrinos in a supernova explosion is about  $3 \times 10^{53} \text{ erg}$ , which corresponds to a sum of magnitudes of neutrino momenta of  $10^{43} \text{ g cm/s}$ . Thus an asymmetry of order of 1% in the distribution of the outgoing neutrinos would explain the kick of the pulsars [This is our goal]. It has been suggested [28,29,30] that an asymmetry of this order in the distribution of outgoing neutrinos can be generated by the anisotropic cross-sections of the various neutrino related processes in presence of a constant magnetic field. A simple calculation given below makes this point clear.

The typical size of a neutron star is about  $R \approx 10 \text{ km}$ . Thus, when a neutrino, generated at the core reaches a density where its mean free path is about  $R$ , it

escapes from the star. Therefore the condition for the neutrino to escape can be written as

$$n_n \sigma = 1/R, \quad (3.4.1)$$

where  $n_n$  is the neutron number density. We already observed that  $\sigma$  is direction dependent. Therefore, the value of  $n_n$  on the "neutrino sphere" depends on the direction as well, and the surface is no longer a sphere. Different values of  $n_n$  will correspond to different temperatures. Thus, neutrinos will be emitted with different momenta in different directions. This can result in a kick to the star.

It is known that [31,32] the scattering cross-section for the inverse beta decay processes is given by

$$\begin{aligned} \sigma = \sum_{n'=0}^{n'_{max}} \sigma_{n'} = \frac{eBG_B^2}{2\pi} \sum_{n'=0}^{n'_{max}} \left[ g_{n'} \{ (G_V^2 + 3G_A^2) + 2G_A \delta(G_V + G_A) \cos\theta \} + \delta_{n'}, 0 \right. \\ \left. + \delta_{n'}, 0 \{ (G_V^2 - G_A^2) \cos\theta + 2G_A \delta(G_V - G_A) \} \right] \times \frac{k + E_\nu}{\sqrt{(k + E_\nu)^2 - m^2 - 2n'eB}}, \quad (3.4.2) \end{aligned}$$

$(k + E_\nu)^2 - m^2 - 2n'eB = 0$ , positive

$$n'_{max} = \frac{(k + E_\nu)^2 - m^2}{2eB} \quad (3.4.3)$$

where,  $k = \sqrt{\frac{E_n + m}{E_n}}$  and  $E_\nu$  = neutrino energy.

Rearrange equation (3.4.2),

when  $g_{n'}=1$  and  $n'=0$ ,

$$\sigma = \frac{eBG_B^2}{2\pi} [(G_V^2 + 3G_A^2) + (G_V^2 - G_A^2) \cos\theta] \quad (3.4.4)$$

and,

$$a = \frac{G_V^2 + 3G_A^2}{2\pi}, \quad \text{and} \quad b = \frac{G_V^2 - G_A^2}{2\pi}, \quad (3.4.5)$$

which can be put in the form of

$$\sigma = eBG_B^2(a + b \cos\theta). \quad (3.4.6)$$

This can be to estimate the magnitude of the kick, we abbreviate the electron-neutrino cross-section. If we now consider the directions  $\theta = 0$  and  $\theta = \pi$ , combined

with equation (3.4.1 and 3.4.5) the difference in neutron density on the corresponding points on the neutrino surface is given by

$$\Delta n_n = 1/eBG_B^2 R[1/(a-b) - 1/(a+b)] = 2b/(eBG_B^2 a^2 R), \quad (3.4.7)$$

neglecting corrections of order  $b/a$ .

The neutron gas in a typical proto-neutron star can be considered to be non-relativistic and degenerate. The number density of neutrons is thus given by

$$n_n = p_F^3/3\pi^2[1 + (\pi^2 m_n^2 T^2/p_F^4) + \dots], \quad (3.4.8)$$

where  $p_F$  is the Fermi momentum, and we have neglected higher order terms in the temperature. In equation (3.4.4) differentiate by temperature, this gives

$$dn_n/dT = m_n^2/3(T^2/p_F) \quad (3.4.9)$$

Substitute equation (3.4.6) into equation (3.4.8), So the temperature difference between the points on the neutrino surface in the  $\theta = 0$  and  $\theta = \pi$  directions is

$$\Delta T = 3/m_n^2(p_F/T^2)(2b/eBG_B^2 a^2 R). \quad (3.4.10)$$

The momentum asymmetry can now be written as

$$\Delta|q|/q = 1/6(4\Delta T), \quad (3.4.11)$$

where we have assumed a black body radiation luminosity ( $\propto T_4$ ) for the effective neutrino surface. The factor  $1/6$  comes in because the asymmetry pertains only to  $\nu_e$ , where as 6 types of neutrinos contribute to the energy emitted equation (3.4.9) and equation (3.4.10) are combined. This gives

$$\Delta|q|/q = 4/m_n^2(p_F/T^2)(b/eBG_B^2 a^2 R). \quad (3.4.12)$$

To find  $p_F$ , we use the leading term in the expansion for  $n_n$  given above and estimate  $n_n$  from the equation

$$n_n = \rho(1 - Y_e)/m_p, \quad (3.4.13)$$

where  $Y_e$  is the electron fraction and  $\rho$  is the mass density. Taking  $Y_e = 1/10$ , we obtain

$$p_F = 24\rho_{11}^{1/3} \text{ MeV}. \quad (3.4.14)$$

where  $\rho_{11}$  is the mass density in units of  $10^{11} \text{ gcm}^{-3}$ . Putting this back in the ratio  $|\Delta|q|/q$  just given above we obtain

$$|\Delta|q|/q = 27\rho_{11}^{1/3} B_{14}^{-1} T^{-2} \text{ MeV} (|b|/a^2), \quad (3.4.15)$$

where,  $B_{14} = B/(10^{14} \text{ Gauss})$  and  $T_{\text{MeV}} = T/(1 \text{ MeV})$ . For  $\Omega \gg m_e$  which is the relevant case and  $S = 0$  as a typical values of neutron polarization, we can find out the values of the constants  $a$  and  $b$ . The values of both of them will in general depend upon  $n'_{\text{max}}$  (the maximum allowed Landau number), but if we take only the contribution from the  $n' = 0$  level then

$$a = (G_V^2 + 3G_A^2)/2\pi = 9.2 \times 10^{-1},$$

and

$$b = (G_V^2 - G_A^2)/2\pi = -9.3 \times 10^{-2}$$

using  $G_V = 1$  and  $G_A = 1.26$

This gives

$$|\Delta|q|/q = 3\rho_{11}^{1/3} B_{14}^{-1} T^{-2} \text{ MeV}. \quad (3.4.16)$$

Obviously, with reasonable choices of  $\rho, B$  and  $T$ , it is possible to obtain a fractional momentum imbalance of the order of 1% which is necessary for explaining the pulsar kicks.

The momentum associated with the proper motion of a pulsar would there be of order  $10^{41} \text{ gcms}^{-1}$  and the sum of magnitudes of neutrino momenta of  $10^{43} \text{ gcms}^{-1}$ .

The ratio of  $|\Delta|q|/q = 10^{41} \text{ gcms}^{-1} / 10^{43} \text{ gcms}^{-1} = 1\%$

when,  $\rho_{11} = 7.2 \times 10^{11} \text{ gcm}^{-3}$

$$T = 3 \text{ MeV} = 3 \times 10^6 \text{ eV}$$

$$B_{14} = B/10^{14} \text{ G}$$

So, this values substitute in equation (3.4.15), we get

$$B = \frac{3(7.2 \times 10^{11})(10^{14})(10^2)}{9 \times 10^{12}} \text{ G}$$

$$\therefore B = 2.4 \times 10^{15} \text{ G}.$$

This is our estimated magnetic field.

### 3.5 Magnetic Field Effect On $\nu_e$ And $\bar{\nu}_e$ Energy Spectra

We now consider how the oscillatory behaviors (see Figs. 3.1-3.3) of the absorption opacities of  $\nu_e$  and  $\bar{\nu}_e$  may manifest themselves in the radiated neutrino energy spectra. For an absorption dominated medium, neutrinos with energy  $E$  decouple from the matter and start free-streaming at a column depth (in  $\text{g cm}^{-2}$ )  $y = \int \rho dr \sim 1/\kappa_R^{(abs)}$ . Thus, the neutrino energy spectrum is given by  $F_E \sim cU_E^{(FD)}$  with  $U_E^{(FD)}$  evaluated at the depth  $y \sim 1/\kappa_E^{(abs)}$ . In the presence of significant scattering, decoupling occurs at depth  $y \sim 1/\kappa_E^{eff}$  with the effective opacity  $\kappa_E^{eff} \sim (\kappa_E^{(abs)}\kappa_E^{(t)})^{1/2}$ , where as the neutrinosphere from which neutrinos start free-streaming is located at  $y \sim 1/\kappa_E^{(t)}$ . Near the decoupling region we have  $dU_E/dr \sim \rho\kappa_E^{eff}U_E^{(FD)}$ , and thus from equation (3.2.11) the neutrino spectral energy flux is given by

$$F_E \sim \frac{\kappa_E^{eff}}{\kappa_E^{(t)}} cU_E^{(FD)} \left( y \sim \frac{1}{\kappa_E^{eff}} \right). \quad (3.5.1)$$

The above qualitative argument can be made more rigorous by directly solving equations (3.2.10) and (3.2.11). Consider a plane parallel atmosphere and define the (energy dependent) optical depth via  $d\tau_E = -\rho\kappa_E^{(t)} dr = \kappa_E^{(t)} dy$ . Equations (3.2.10) and (3.2.11) can be combined to give

$$\frac{d^2}{d\tau_E^2} U_E = \beta_E^2 [U_E - U_E^{(FD)}], \quad \text{with } \beta_E^2 = 3 \frac{\kappa_E^{(abs)}}{\kappa_E^{(t)}}. \quad (3.5.2)$$

with a constant  $\beta_E$ , the above equation can be solved exactly. For the boundary condition  $F_E = (c/3)(dU_E/d\tau_E) = cU_E/2$  at  $\tau_E = 0$ , the neutrino energy flux at

the surface is given by

$$F_E = \frac{\beta_E/3}{1 + 2\beta_E/3} \int_0^\infty d\tau_E \beta_E c U_E^{(FD)}(\tau_E) \exp(-\beta_E \tau_E) \\ \simeq \frac{\beta_E/3}{1 + 2\beta_E/3} c U^{(FD)}(\tau_E \simeq \beta_E^{-1}). \quad (3.5.3)$$

Figure 3.6 shows the spectral energy fluxes of  $\nu_e$  calculated using equation (3.5.3) for different field strengths ( $b = 100$  and  $300$ ). The matter temperature profile is obtained from equation (3.3.3) assuming  $U \propto T^4$ . For a total energy flux of  $F = (c/4)U(T_{eff})$  (where  $T_{eff}$  is the effective temperature), we have  $T = T_{eff}[(\frac{3}{4})\tau_R + \frac{1}{2}]^{1/4}$ . We choose  $T_{eff} = 3$  MeV in these examples. For simplicity, we have neglected the variations of the opacities as a function of  $\tau_E$ . Thus, the temperature at decoupling ( $\tau_E \simeq \beta_E^{-1}$ ; see eq.[3.5.3]) is given by  $T \simeq T_{eff}[(\frac{3}{4})\kappa_R/\kappa_E^{(eff)} + \frac{1}{2}]^{1/4}$ , where  $\kappa_E^{(eff)} = \beta_E \kappa_E^{(t)} = (3\kappa_E^{(abs)} \kappa_E^{(t)})^{1/2}$ , and we take the relevant opacities to have the corresponding values for  $Y_e = 0.1$ ,  $\mu_e = 20$  MeV. and  $T=3$  MeV. This simplification should not introduce qualitative changes in the spec-

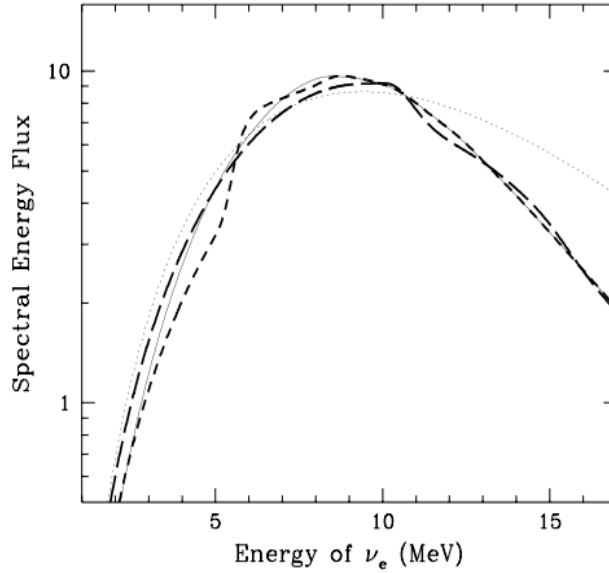


Figure 3.6: Illustrative examples of the  $\nu_e$  spectral energy fluxes (in arbitrary units) modified by the magnetic fields. The light solid, short-dashed, and long-dashed lines are for  $b = 0, 100$ , and  $300$ , respectively. The light dotted line is the Fermi-Dirac spectrum with  $T = T_{eff} = 3$  MeV.

trum. As seen from Figure 3.6, the spectrum is pinched at the high energies because of the  $E^2$ -dependence of the opacities. The magnetic field introduces



modulations in the spectrum. For lower magnetic field strengths, the modulations occur at the lower energy band. As the magnetic field gets stronger, the modulations shift to the higher energy part of the spectrum.

The distortions and modulations of the  $\nu_e$  and  $\bar{\nu}_e$  energy spectra by strong magnetic fields may offer an interesting possibility to limit the magnetic field strength in proto- neutron stars through the detection of neutrinos from a Galactic supernova. We note, however, that the  $\nu_e$  spectra shown in Figure 3.6 are based on very approximate calculations. They serve only as illustrative examples of the magnetic field effect on the neutrino spectra. More detailed calculations are needed to obtain realistic spectra that include the magnetic field effect.

# Chapter 4

## Conclusion

In this thesis, We have discussed first detection of  $\gamma$ -ray pulsations from an individual globular cluster MSP (PSR J1823-3021A). Whose  $\gamma$ -ray luminosity is among the highest observed for any MSP. The pulsar dominates the total emission of the cluster. The issue of whether asymmetric magnetic field topology in a proto-neutron star can induce asymmetric neutrino emission from the star through the modifications of the neutrino absorption opacities by the magnetic field. Another related (but distinct) kick mechanism relies on the fact that  $\nu_e$  and  $\bar{\nu}_e$  absorption opacities near the neutrinosphere depend on the local magnetic field strength (because of the quantization of  $e^-$  and  $e^+$  energy levels). If the magnitude of the magnetic field in the north pole is different from that in the south pole, then asymmetric neutrino flux can be generated. We show in Lai and Qian (1998b) that this kick mechanism requires a much larger field strength. These modifications arise from the quantized Landau levels of electrons and positrons produced in strong magnetic fields. By calculating the appropriate mean neutrino opacities in the magnetic field, we demonstrate that this mechanism is rather inefficient in generating a kick to the neutron star : To obtain appreciable kick velocities ( $\sim 300 km s^{-1}$ ), the difference between the field strengths at the two opposite poles of the star must be at least  $10^{16}$  G. The inverse beta-decay process, even for unpolarized Neutrons ( $S=0$ ), the cross-section depends on the direction of the neutrino momentum (anisotropy in the cross-section). The anisotropy in the cross-section comes only from the zeroth Landau level contribution of the

electron. However, for the higher levels, there is a cancellation between the two possible states in a Landau level which washes out all angular dependence in these levels, provided the neutrons are unpolarized. The asymmetry in cross-section will therefore come only from the zeroth Landau level state of the electrons and its amount will depend on the relative contribution of this state to the total cross-section. If the magnetic field is so high that only the zeroth Landau level state can be obtained for the electron, the asymmetry will be large, about 18%. For smaller and smaller magnetic fields, the asymmetry decreases with new Landau levels contributing. For polarized neutrons, however, there is an asymmetry even in the field-free case.

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### Declaration

I here by declare that this thesis is my original work and has not been presented for a degree in any other university. All sources of material used for the thesis have been duly acknowledged.

Name: Tesfaye Teklehaimanot

Signature: .....

This thesis has been submitted for the examination with my approval as university advisor.

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June, 2013