ADDIS ABABA UNIVERSITY
SCHOOL OF GRADUATE STUDIES

# INVESTGATION OF EFFECTS OF IMPERFECTION AND THEIR SIGNIFICANCE IN ANALYSIS RESULTS

Haimanot Tadesse

February 2007

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By

Haimanot Tadesse

A Thesis Submitted to School of Graduate Studies, Addis Ababa University
in Partial Fulfillment of the Requirements for the Degree
of MASTER OF SCIENCE in CIVIL ENGINEERING

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Approved by Board of Examiners

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# Description of Symbols

| Symbols | Descriptions |
|---|---|
| $W$ | Energy imparted to the system by the disturbing force |
| $T$ | Kinetic energy of the system |
| $V$ | Potential energy of the system |
| $N_{cr}$ | Critical load |
| $\delta$ | Displacement of one point of the system |
| $\xi$ | is the amplitude of the imperfection |
| R | resultant reaction between the block and the table |
| H | Horizontal component of resultant force R |
| V | Vertical component of the resultant force R |
| $N$ | Variable parameter |
| F | Horizontal Force |

$\theta_x$ angle of rotation of normal line with respect to x-axis

w weigth of block

**s** reaction between the ball and the spherical surface

$q_{ref}$ reference mean wind velocity pressure derived from reference wind velocity .

$C_{e(z)}$ exposure coefficient accounting for the terrain and height above ground z .

Z reference height appropriate to the relevant pressure coefficient ($z=z_e$) for external pressure and force coefficient, ($z=z_i$) for internal pressure coefficient)

$C_d$ dynamic coefficient accounting for both correlation and dynamic magnification

$W_e$ wind pressure acting on external surfaces

$W_i$ wind pressure acting on internal surfaces

$C_{pe}$ external pressure coefficient

$C_{pi}$ external pressure coefficient

$\phi_0$ basic value of imperfection

$\alpha_h$ reduction factor for height h applicable to columns

h height of the structure in meters

$\alpha_m$ reduction factor for the number of columns in a row

m number of columns in a row

$\delta_q$ in plane deflection of the bracing system due to *q* plus any external loads calculated from first order analysis.

$M_{Ed}$ maximum moment in the beam.

$H_{Ed}$ design horizontal action

$V_{Ed}$ design vertical action

h overall depth of the beam.

$n_c$ number of columns per plane

$n_s$ number of storys.

$n_r$ number of members to be restrained

# Abstract

Structural systems are prone to global as well as member imperfections in geometry. These imperfections are accounted for when considering structural stability and strength requirements of both sway and non-sway frames.

In the first part of this thesis, stability of imperfect systems is examined. With the intention of understanding the ideas necessary to tackle more practical aspects of instability, stability of equilibrium are first introduced by analysis of the behavior of some simple mechanical systems. Existence of imperfection cause the maximum load to be attained to fall below the critical value and the larger the imperfection, the larger is this reduction. The bifurcation point defines the form of instability of a structural system.

For simple mechanical systems, the behavior of the imperfect system is governed by the magnitude and the sign of the imperfection and also the critical load of the system without imperfection. In systems with more than one degree of freedom, it is the fundamental mode and load which governs the elastic portion of the behavior frames possessing imperfection. The pattern of imperfection of form and loading, considered as a whole determines the mode of buckling.

To evaluate strength requirements of imperfect structural systems, the imperfection effects are quantified in terms of an initial sway imperfection at the base of the column and then converted to equivalent horizontal forces. Analysis is carried out which includes these equivalent horizontal forces and the maximum bending moment in columns and beams for frames with varying number of bays and number of stories are examined. For a fixed number of stories, the effect on maximum bending moment in beams decreases with

increasing number of bays. The effect on columns is not as large as that on beams but is still significant for high rise buildings. For fixed number of bays the percentage increase rises with the story height.

# Introduction

## 1.1 Problem Background

In structural analysis procedures, design specifications and building codes prescribe the nature and magnitude of the loads to which the structure may be subjected. Once the loads are defined global analysis must be carried out to determine the internal resisting actions (forces & moments) which will be produced in the various member of the structural system, and also to ensure stability requirements.

The unloaded frame may be subjected to inevitable imperfections in geometry such as initial out-of-plumb, misalignment of the columns and girders and so on, due to erection procedures. These imperfections are introduced to frames, braces, individual systems of both sway and non-sway.

Building codes recommend the checking and allowing for possible imperfections in the geometry of the unloaded frame as an integral part of the structural analysis procedure.

In common practice global frame imperfections are not included in the analysis of framed systems. EBCS 2 clause 3.7.3 state that imperfection effects can be ignored when the effects of imperfection are smaller than the effects of design horizontal actions.[5] But it is a common practice to waive their effects in general. Other recent building codes such as the Euro code based British Standard BS EN 1992-1-1:2004 & BS EN 1993-1-:2005 recommend that imperfection effects be considered in the global analysis whenever $H_{Ed} < 0.15 V_{Ed}$.[7,8]

As part of the instability issue, imperfections should be given due attention and detailed study would provide useful information. Hence, the goal of this research is to investigate how significant its effect could be in analysis results and decide if the common practice of assuming them to be insignificant in general is valid.

## 1.2 Objectives

The general theme of this thesis work is to review the effects from the presence of imperfections, with specific objectives of

- Assessing factors affecting the imperfection values
- Evaluating the extent of the EBCS 2 & EBCS 3 provisions for frame imperfection and compare it with other international codes.
- Finding out the basic assumptions and basis of the code provision for the imperfection values.
- Evaluating significance of its effect on internal forces and moments.

## 1.3 Approaches and Methods used for the Study

To achieve the above objectives of the study, investigation is carried out by reviewing literatures on imperfection. Building code provisions for the problem specified are studied thoroughly. Using the procedures from the code structural analysis is carried out using the SAP2000 software for a total of 18 plane frames which were considered to represent a wide range of arrangements. Conclusions and recommendations are derived from the analysis results.

# 1

# Concepts of Stable and Unstable Elastic Equilibrium.

Stability theories are formulated in order to determine the conditions under which a structural system, which is in equilibrium, ceases to be stable. Instability is essentially a property of structures in their extremes of geometry; for example, long slender struts, thin flat plates or thin cylindrical shells. Normally, one deals with systems having one variable parameter N, which usually represents the external load but which might also be the temperature (thermal buckling) or other phenomena. For each value of N, there exists only one unbuckled configuration.

In classical buckling problems, the system is stable if N is small enough and becomes unstable when N is large. The value of N for which the structural system ceases to be stable is called the critical value $N_{cr}$. More generally the following should be determined:

- the equilibrium configurations of the structure under prescribed loadings.
- which amongst these configurations are stable.
- the critical value of the loadings and what behavioral consequences are implied at these load levels.

## 2.1 STABLE AND UNSTABLE EQUILIBRIUM STATES

In very general terms, stability may be defined as the ability of a physical system to return to equilibrium when slightly disturbed.

For a mechanical system, one can adopt the definition given by Dirichlet: "The equilibrium of a mechanical system is stable if, in displacing the points of the system from their equilibrium positions by an infinitesimal amount and giving each one a small initial velocity, the displacements of different points of the system remain, throughout the course of the motion, contained within small prescribed limits".

This definition shows clearly that stability is a quality of one solution - an equilibrium solution - of the system, and that the problem of ascertaining the stability of a solution is concerned with the "neighborhood" of this particular solution.

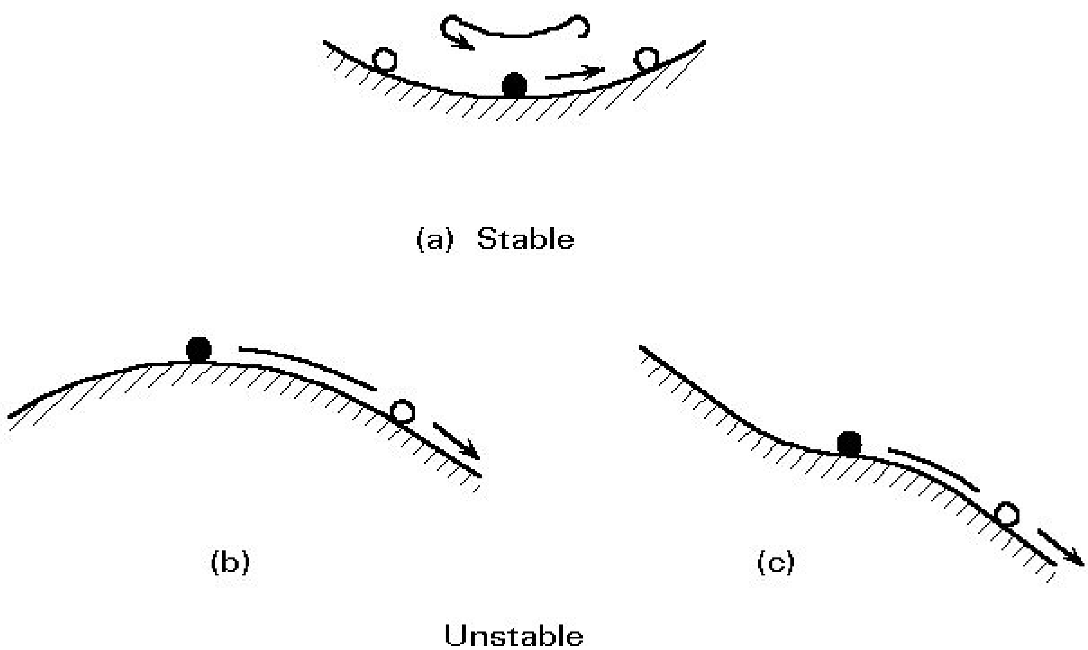
If one considers an elastic conservative system, which is initially in a state of equilibrium under the action of a set of forces, the system will depart from this equilibrium state only if acted upon by some transient disturbing force. If the energy imparted to the system by the disturbing force is W, then:

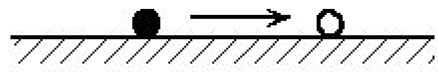
$$W = T + V \quad = \text{constant} \tag{2.1}$$

by means of the principle of conservation of energy.

In this relationship, T is the kinetic energy of the system and V is the potential energy. A small increase in T, is accompanied by an equally small decrease in V, or vice versa. If the system is initially in an equilibrium configuration of minimum potential energy, then the kinetic energy T during subsequent free motion decreases since V must increase. Hence the displacement from the initial state will remain small and the equilibrium state is a stable one.

For rigid bodies, the stability can be illustrated by the well-known example of a ball on a curved plane (Fig. 2.1). Resting on a concave surface (Fig. 2.1a) the equilibrium is stable; if one gives the ball a small initial velocity, it will begin to oscillate but will remain in the close neighborhood of its equilibrium state. On the other hand, if the system is not in a configuration of minimum V (potential energy), then an impulse leads to large deflections and velocities which develop very quickly, and the system is said to be unstable. This is the case where the ball rests on the crest of a convex surface (Fig. 2.1b) or at a horizontal point of inflection of the surface (Fig. 2.1c). If the ball rests on a horizontal plane, the equilibrium is called "neutral" (Fig. 2.1d).





(d) Neutral

Figure 2.1 The three states of equilibrium

## 2.2 MINIMUM POTENTIAL ENERGY

The intuitive example of the ball leads to the law of minimum potential energy of a system: "A conservative elastic system is in a state of stable equilibrium if and only if, the value of the potential energy is a relative minimum".

The word "relative minimum" is used because there may be other minima nearby at lower values of potential energy separated by small "hills" but the move from one minimum to another necessitates large disturbances (Figure 2). The existence of a relative minimum of the potential energy in the equilibrium configuration is, strictly speaking, only a sufficient condition for stability. However, this principle is, in practice, generally accepted as both a necessary and sufficient condition for stability.

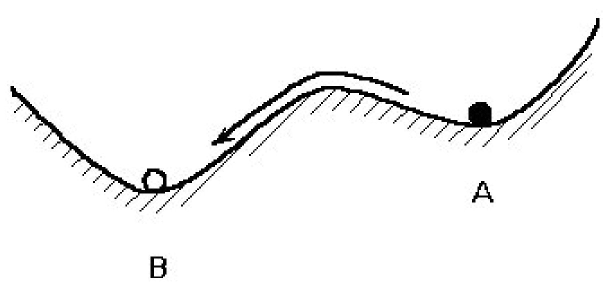



Fig.2.2 Relative character of the equilibrium

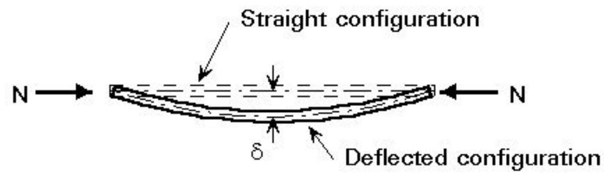
## 2.3 BIFURCATION BUCKLING

It has been shown that the stability concept is related to potential energy of the system. However, stability of a static elastic system, or structure, may also be explained by stiffness considerations. Referring to Figure 1a, one can see that the derivative of the potential energy with respect to displacement gives the stiffness (in the figure, the slope of the surface) of the system.

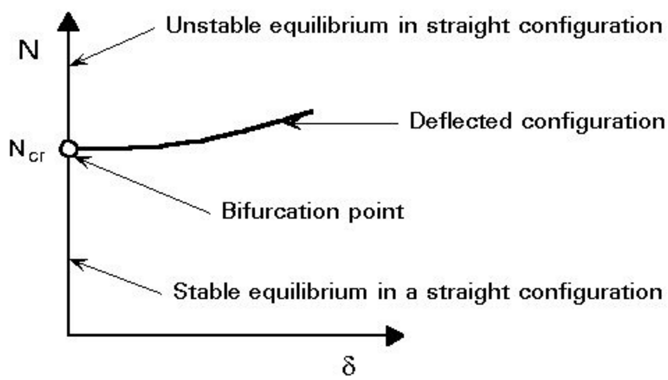
Thus, positive stiffness implies a stable state, whereas at a stability limit the stiffness vanishes. For a structure, the stiffness is given in matrix form, which if it has both a positive and definite condition, guarantees a stable state for the structure. The point at which the state of a system changes from stable equilibrium into neutral equilibrium is called "the stability limit".

The system of a ball on a curved plane (where the stability depends only on the shape of the surface) can be compared to a structure such as a compressed column. In this case, the column may be stable or unstable, depending on the magnitude of the axial load, which is the controlling parameter of the system (Figure 2.3a).

When the member is initially straight and the load is axial, the structure will be in stable equilibrium for small values of N; if a disturbing force produces deflections, the column will return to its straight position. When the load reaches a certain level, called "critical load", the stable equilibrium reaches a limit. At this load $N_{cr}$, there exists another equilibrium position in a slightly deflected configuration of the column; if, at this load, the member is deflected by some small disturbance, it will not return to the straight configuration.




(a) Simply supported column




(b) Axial force VS deflection relationship

Figure 2.3 Stability of a compressed column

If the load exceeds the critical value, the straight position is unstable and a slight disturbance leads to large displacements of the member and, finally, to the collapse of the column by buckling. The critical point, after which the deflections of the member become very large, is called the "bifurcation point" of the system (Figure 2.3b).

If the column is not initially perfectly straight, deflection starts from the beginning of the loading and there is no sudden buckling by bifurcation, but a continuous increase of the displacements (Figure 3.4). This phenomenon is called "divergence of the equilibrium" and there is no strict stability limit. If the material remains elastic, the stiffness of the column (given here by the slope of the N. δ curve) is always positive but a small disturbance will produce very large displacements.

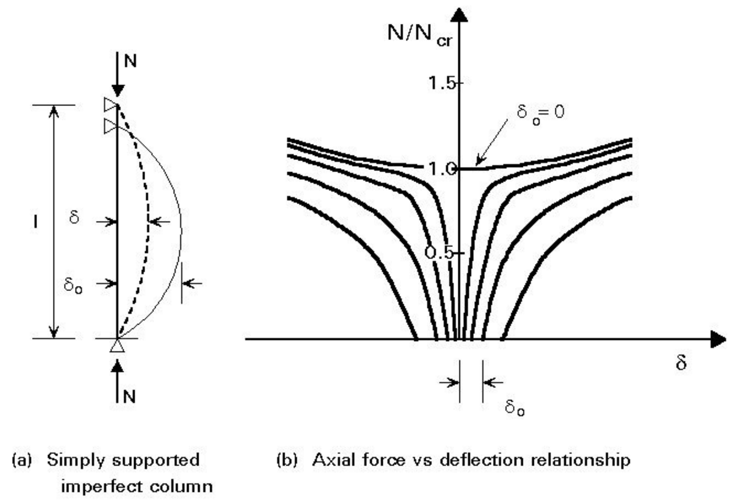
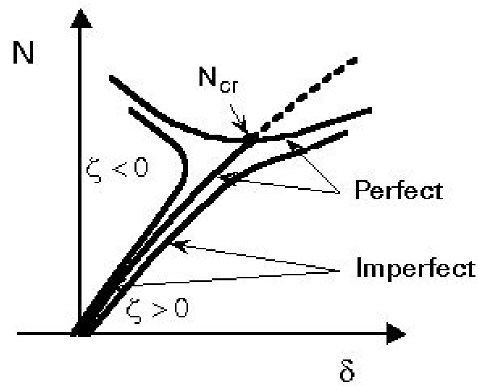


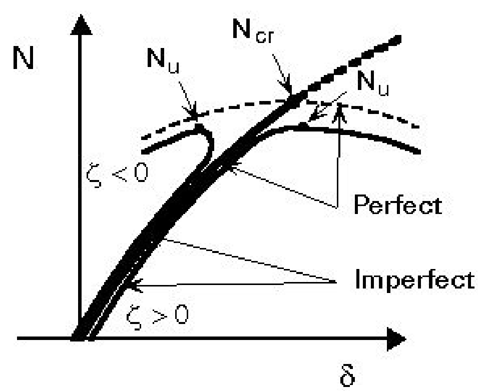

Figure 2.4 Stability of an imperfect compressed column

## 2.4 POST CRITICAL BEHAVIOUR OF PERFECT AND IMPERFECT SYSTEMS

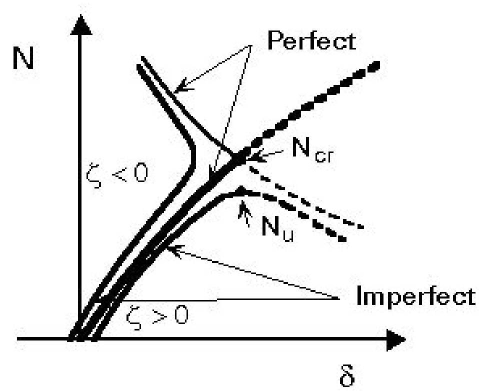
After the bifurcation point, three main situations can arise depending on the type of system under study (Fig. 2.5). In Fig. 2.5, N is the applied load, δ is a displacement of one point of the system and ξ is the amplitude of the imperfection. Heavy solid lines in Fig. 2.5 represent the equilibrium paths of the perfect system while light solid lines represent the equilibrium paths of imperfect systems, continuous lines representing stable equilibrium and dotted lines representing unstable equilibrium.




(a) Stable - symmetric point of bifurcation




(b) Unstable - symmetric point of bifurcation




(c) Asymmetric point of bifurcation

Figure 2.5 Post critical behaviors

- Figure 2.5a: stable-symmetric point of bifurcation.

Small positive and negative imperfections have similar effects and yield a stable and rising equilibrium path. The buckling is characterized by a rapid growth of the deflections when the critical load of the perfect system is approached. This case is of great practical importance because columns, beams and plates show this type of post critical behavior.

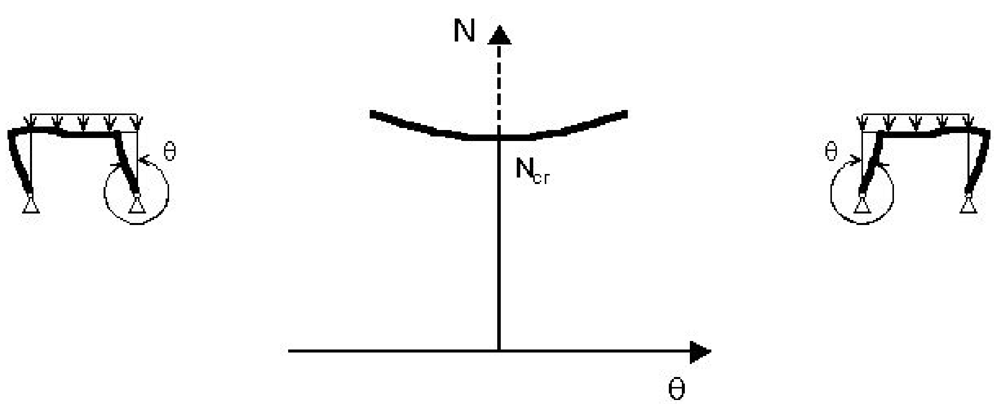
- Figure 2.5b: unstable-symmetric point of bifurcation.

The imperfections play an important role in modifying the behavior of the system. Small imperfections of both signs induce a reduced load with regard to the critical load. This is, for example, the case in some systems composed of hinged bars.

- Figure 2.5c: asymmetric point of bifurcation.

For small positive values of the imperfection, the system loses its stability at a limit point (ultimate load), largely reduced by comparison to the critical point. On the other hand, small negative imperfections lead to a rising stable path. Here, the system is mainly sensitive to initial positive imperfections. This is, for example, the case in some trusses. [2,3]

Figure 2.6 illustrates these three post-critical behaviors.



(a) Stable - symmetric point of bifurcation

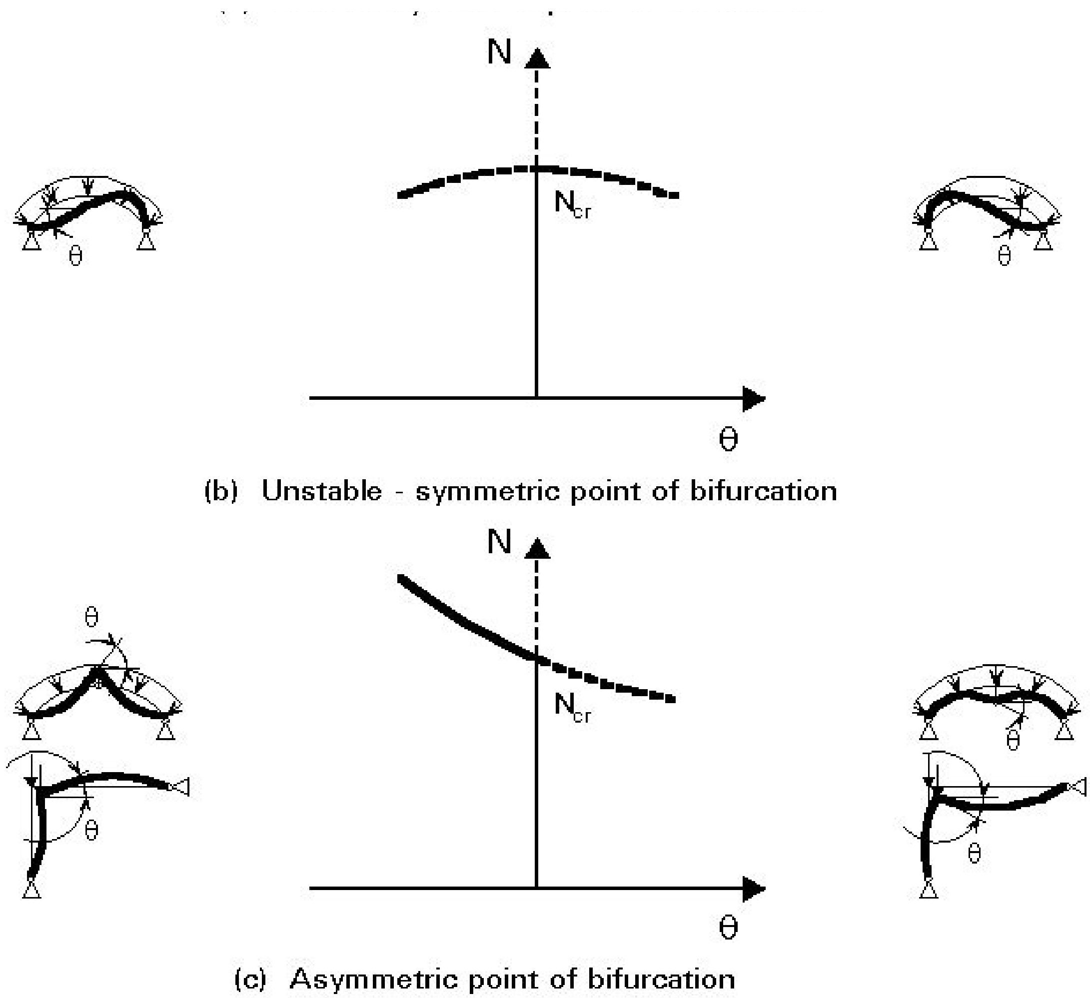



Figure 2.6 Examples of post-critical behaviors.

# 2

# Theoretical Investigation

## 3.1 Notions of instability

### 3.1.1 Systems with one degree of freedom of movement

In this topic the basic conceptions of stability of equilibrium are introduced by analysis of the behavior of some simple mechanical systems with the intention of understanding the ideas necessary to tackle more practical aspects of instability.

### a. The stability of a rigid block resting on a table

Fig. 3.1(a) shows a square block ABCD of side 2a, resting on a table. There is a hinge at A which permits rotation of the block in the plane of its cross-section, but prevents sliding across the table. The system has therefore one degree of freedom of movement:

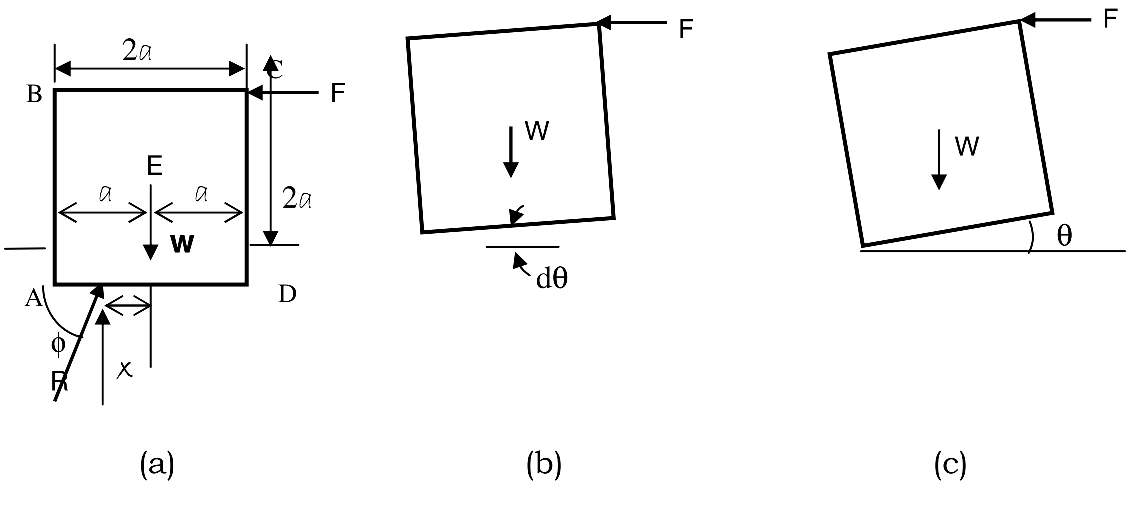



Figure 3.1 Equilibrium of block on table

The block can rotate about A, and this rotation shall be denoted by the angle $\theta$.The block is of weight W, and at C it is acted on by a horizontal force F. Consider the equilibrium of the block as the applied force F is gradually increased from zero. Let R be the resultant reaction between the block and the table; the horizontal and vertical components of R are H and V. Then for equilibrium of the block on the table, we have $\sum H=0$, hence

$H - F = 0$, where $H = R\cos\phi$ (3.1a)

$\sum V = 0$, hence

$V - W = 0$, where $H = R\sin\phi$ (3.1b)

$\sum M_E = 0$, hence

$$-Vx + Ha + Fa = 0 \tag{3.1c}$$

Equation (3.1) can be used to solve for x, the distance from the line of action of W at which the reaction R acts. It is

$$X = (Ha + Fa)/V = 2Fa/W \tag{3.2}$$

As F is increased from zero, there comes a stage when x=a, and R passes through the hinge A. If F is increased still further, x can no longer increase. Equilibrium demands that it must, and therefore the equilibrium condition (3.2) can longer be satisfied. The system does not remain in equilibrium, and the block tilts; it becomes unstable. The value of F at which this occurs is obtained by putting the limiting value x=a in equation (3.2), which gives

$$F = \frac{1}{2}W \tag{3.3}$$

Imagining the block to be just tilting i.e the geometry of the situation at the onset of instability, as shown in figure 3.1(b), where a rotation $d_\theta$ has occurred. The reaction between the block and the table must pass through the hinge A, the only point of contact. For equilibrium, taking moments about the hinge A, we have

$\sum M = 0$, hence 2aF-Wa=0,

Therefore $F = \frac{1}{2}W$

as before. This simple analysis of the system on the point of tilting has led quickly to the determination of the critical load corresponding to the geometrical mode of displacement pictured.

If θ is the rotation of the block about the hinge A, as shown in Figure 3.1(c), a graph of F against θ can be plotted, depicting the behavior of the system. For values of F less than 1/2W, the system becomes unstable, and θ may take any value; see figure (3.2a), which depicts the result to which our investigation of small displacements of the block has led.

In the region OA, where $0 < F < \frac{W}{2}$

The block is in stable equilibrium, and θ is zero. At F=1/2W, an undefined rotation can occur, as indicated by the horizontal portion AB

The above ideas about the form of the graph of F against θ become modified when large displacements of the block are considered. Figure 3.1(c). In this case, taking moments about the hinge, we have

$$F(2a\cos\theta + 2a\sin\theta) - W(a\cos\theta - a\sin\theta) = 0 \ , \qquad (3.4a)$$

therefore

$$F = \frac{1}{2}W\frac{(\cos\theta - \sin\theta)}{(\cos\theta + \sin\theta)} \ , \qquad (3.4b)$$

Hence as θ increases, F falls below the value 1/2W; see figure 3.2(b).Thus the results of small displacements which gave a horizontal tangent at point A of figure 3.2(a) is in fact misleading. The whole graph is as shown in figure 3.2(b), where F rapidly reduces below 1/2W as θ increases.

It is evident however, that the simple analysis of small displacements which has led to figure 3.2(a) has shown up the essential features of the problem. As F is increased from zero, the system is stable up to up to F=1/2W.

At this value of F, the block is suddenly displaced. Suppose that the force F can follow the block, in such a way that it remains constant at the value of ½ W.

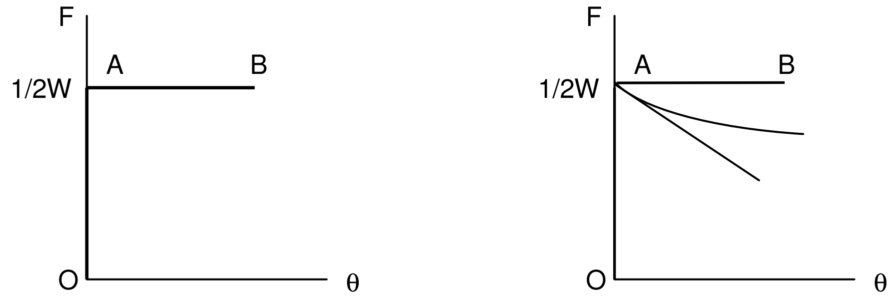



Figure 3.2 Equilibrium diagrams for block on table.
a) Approximate graph of F against θ for small displacements
b) Actual graph of F against θ for large displacements

Because the force required for equilibrium at any value of θ is less than 1/2W , the block does not reach an equilibrium position anywhere along its path; the excess of F=1/2W over the value of F required for equilibrium at any θ is used to accelerate the rotation of the block . Then for many practical purposes, the relation between F and θ may be taken to be that of figure 3.2(a)

With the horizontal portion AB prolonged, since it is the critical value of the F=1/2W to cause tilting that matters and the post-tilting behavior is not so important. [1]

## The stability of a slightly imperfect block

Suppose that the block has been accidentally manufactured with a small imperfection so that the face which rests on the table is very slightly rounded. This will somewhat modify its behavior form the previously dealt with. For even a small value of F the block will begin to tilt, so that instead of θ remaining zero up to the critical load F=1/2 W, θ assumes positive value whatever(non-zero) value is taken by F.

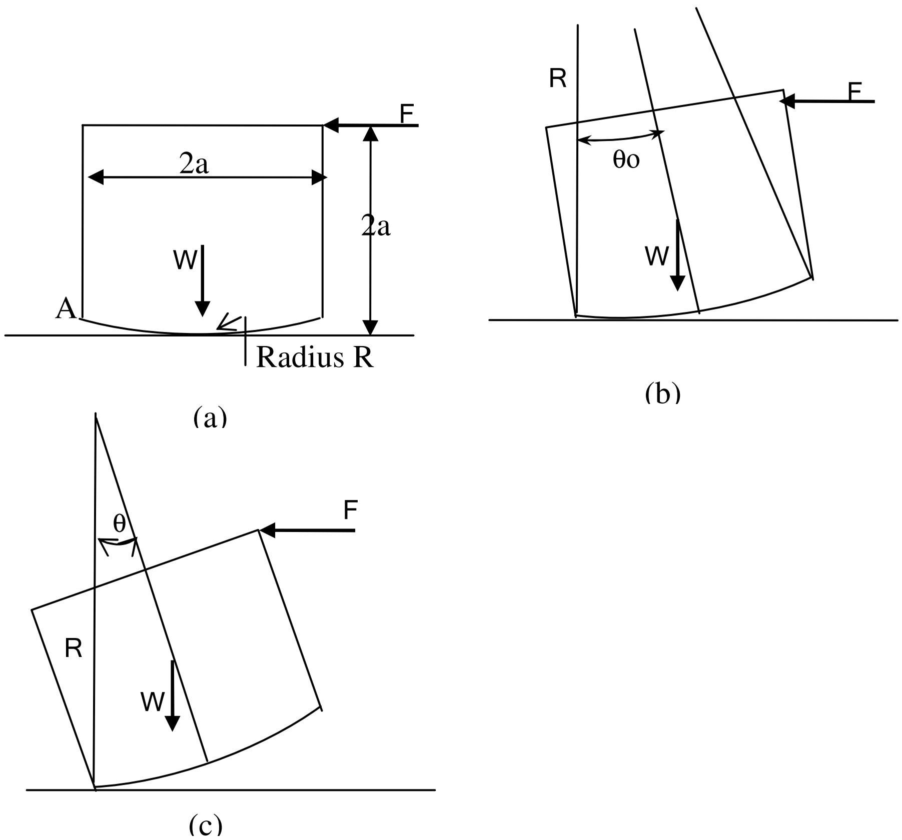



Fig 3.3 Imperfect block
(a) slight rounding of base of block
(c) large rounding of base

For the case of very slight rounding , we observe that at some value of F the block is just tilted on to its corner A, In this state , its angle of rotation from its original position is $\theta_o$ .Taking moments about A, provided $\theta_o$ is small , we have

$$2aF - aW = 0$$

Therefore:

F=1/2W as before.

$\theta_o = \frac{a}{R}$ is a measure of the imperfection of the block, and that provided $\theta_o$ is small , the value of F required to tilt the block on to its corner is independent of $\theta_o$,and equal to the critical load F=1/2W in the absence of imperfections.

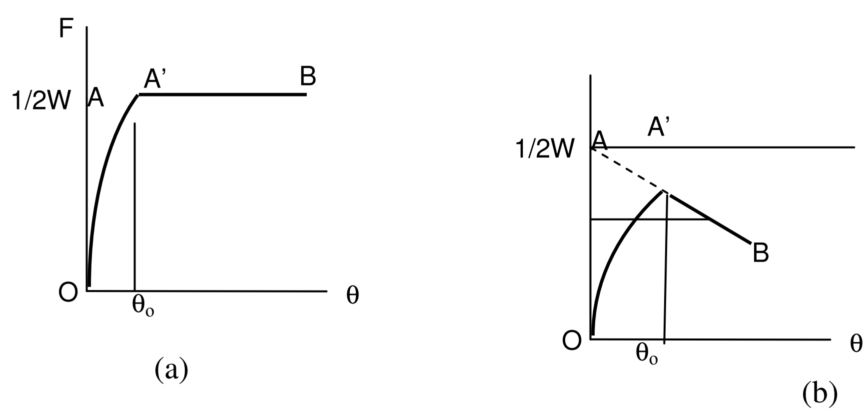



Figure . 3.4 Equilibrium diagram of imperfect block

Therefore the graph of F against θ is as shown in figure In the absence of imperfections, it is OAB (Fig. 3.2 a); the slight imperfections cause point A to move to A', giving the new curve OA'B (Fig. 3.4 a), but the maximum load attained is still equal to F=1/2W at which stage of the loading large rotations can occur without increase in F. For practical purpose, the system is stable in the region 0<F<1/2W, where changes in the force F cause well defined changes in the rotation θ. But at or above the load F=1/2W, the system is unstable as before.

Though the preceding analysis of the problem on the basis of small displacements is valid for slight imperfections, the errors involved become increasingly larger as $\theta_o$ increases. The result of the investigation of large displacements of the perfect system ($\theta$=0) have been plotted in Fig. 3.2 b. This graph can also be modified to take account of the imperfection. In the case where the angle of rotation $\theta$ is equal to $\theta_o$, as shown in Fig. 3.3 c, we have approximately

$$W(R-a)\sin\theta_O = F \quad , 2a(\sin\theta_O + \cos\theta_O) \qquad (3.5a)$$

Therefore, at tilting, where $\theta=\theta o$ and P reaches its maximum value,

$$F_{\max} = \frac{[W(R-a)\sin\theta_O]}{[\sin\theta_O + \cos\theta_O]} \qquad (3.5b)$$

For $\theta o$ very small and R very large, $F_{max}$ decreases below this value. The graph of F against $\theta$ is shown in Fig. 3.4b which may be compared with Fig. 3.2b Up to $\theta=\theta o$, the path OA' is followed. Beyond $\theta=\theta o$, the imperfections no longer have an effect. However, the existence of the imperfections has caused the maximum load attained to fall somewhat below the value 1/2W.the larger the imperfection, the larger is this reduction.

From Fig. 3.4b we note also that the system has two equilibrium positions for any value $F=F_1$ below the maximum load, one on the branch OA' and another on the branch A'B. The point 1 is a position of stable equilibrium against small displacements , though a large enough disturbance can cause the system to "flick" over to the position 2.This position of high instability , as the slightest disturbance towards increasing $\theta$ values brings the system to a position 3 where a smaller value of F is required for equilibrium. The excess of $F_1$ over F at the point 3 then rapidly causes increased rotation. [1]

## b. The stability of a loaded ball in a cup

Consider the system shown in Fig. 3.5a consisting of a spherical ball of radius r and weight W resting in a cup which is a portion of a sphere of radius R. The ball is acted on by a vertical force P applied vertically. Suppose the displacement of the ball is measured by the angle $\theta$ .Then it is evident that the ball can remain in equilibrium in the position θ=0, whatever the value of P. This is indicated in the equilibrium diagram of P against θ in fig. 3.6 by the line OB, coincident with the axis of P. Let us now investigate the stability of this equilibrium by applying a disturbance to the system. The ball has been displaced an amount which may be denoted by θ. The intention of the analysis is to determine whether it tends to return to its initial position, to be displaced further, or if it can remain there in equilibrium.

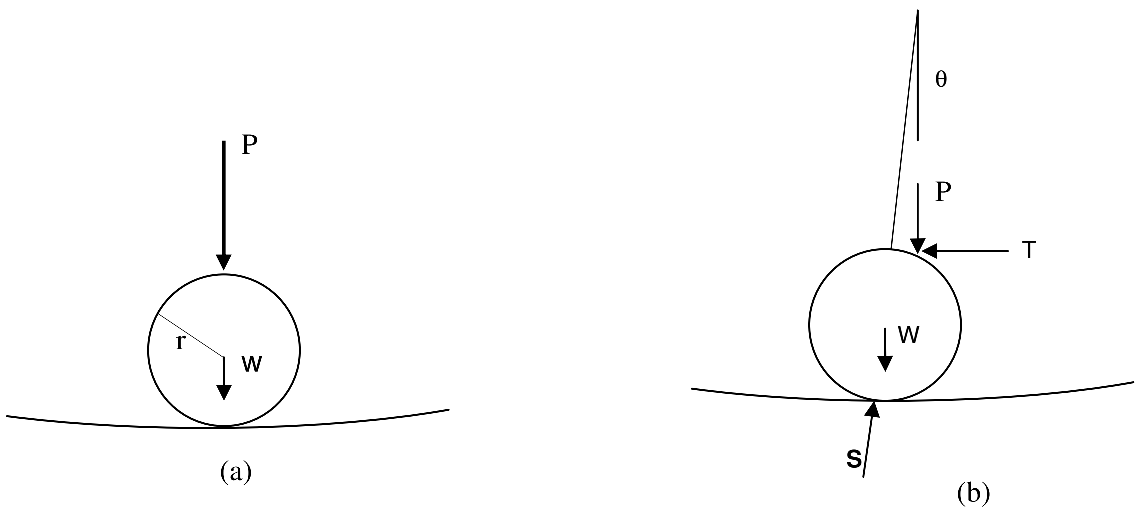



Fig. 3.5 Loaded ball in a cup

The displacement calls into action a horizontal force T at the point of the plunger. The reaction **S** between the ball and the spherical surface is at the angle θ if the cup is smooth.

For equilibrium in the displaced position, we have, provided θ is small

$$\sum V = 0 \quad \ldots\ldots\ldots\ldots \quad P + W - S\cos\theta = 0 \qquad (3.6a)$$

$$\sum H = 0 \quad \ldots\ldots\ldots\ldots \quad T - S\sin\theta = 0 \qquad (3.6b)$$

$$\sum M_C = 0 \quad \ldots\ldots\ldots\ldots \quad P(R - r)\sin\theta - Tr = 0 \qquad (3.6c)$$

These equations give, approximately,

P+W=S,

And $T = (P + W)\sin\theta$ ,

Therefore

$$P(R-r)\sin\theta = (P+W)r\sin\theta \tag{3.7a}$$

In this equation, the term sin θ cancels, and we obtain

$$P = \frac{Wr}{(R-2r)} \tag{3.7b}$$

Thus, assuming equilibrium in the displaced position, we have arrived at the condition that P must have a definite value, which we shall call $P_{critical}$. At this value of P, the ball is in equilibrium in the position θ=0 and also in neighboring positions.

The three equations of equilibrium have reduced to equation (3.7a), and it is now important to examine this equation carefully; in fact, for either

(I) ……sin θ=0

(II) $P = \frac{Wr}{(R-2r)} = P_{critical}$ (3.8)

q may take any (small) value, as its magnitude is not defined by eq. (3.7a)

(III) …..both (I) and (II) are true

These conditions are plotted in the diagram of Fig. 3.6. Condition (I) is given by the line OB, on which θ=0. Condition (II) is represented by C'AC, on which $P=P_{critical}$, and θ may take any small value .To prolong this line to the left of C' or to the right of C would involve an investigation of large displacements of the system. Condition (III) is represented by the point A at which the two characteristics intersect.

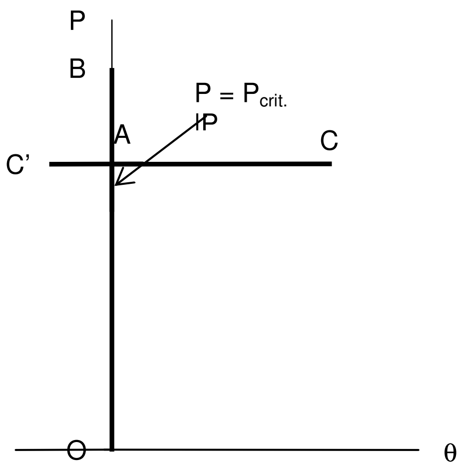



Fig. 3.6 Equilibrium diagram of the loaded ball in a cup

The equilibrium diagram for this system is therefore not a single line, but two intersecting lines. As P is increased from zero, the system apparently has a choice of path at the point A. It can continue up AB in the undisplaced position $\theta=0$, or at constant load $P=P_{critical}$ an undefined displacement can occur .The system is therefore in neutral equilibrium at Position A, with respect to small displacements. [1]

## Stability of the ball and cup system with an initial imperfection

Suppose the system is made up with an initial imperfection so that the force P is applied vertically but slightly to the left of the vertical axis of the cup, as shown in Fig.3.7a , where the eccentricity of P is denoted by $\delta_o$. In this case, the equations of statical equilibrium agre

$$\sum V=0 \text{ ............ } P+W-S=0 \tag{3.9a}$$

$$\sum H=0 \text{ ............ } T-S\theta=0 \tag{3.9b}$$

$$\sum Mc=0 \text{ .......... } P[(R-r)\theta+\delta_O]-Tr=0 \tag{3.9c}$$

Provided both $\delta_o$ and θ are very small. These equations give

$$P[(R-r)\theta+\delta_O]=(P+W)r\theta \tag{3.10a}$$

Therefore  $\theta = P\delta_o / [\ Wr - P(R-2r)\ ]$

$$\theta = \frac{P\delta_O}{(Wr)}\left(1 - \frac{P}{P_{critical}}\right) \tag{3.10b}$$

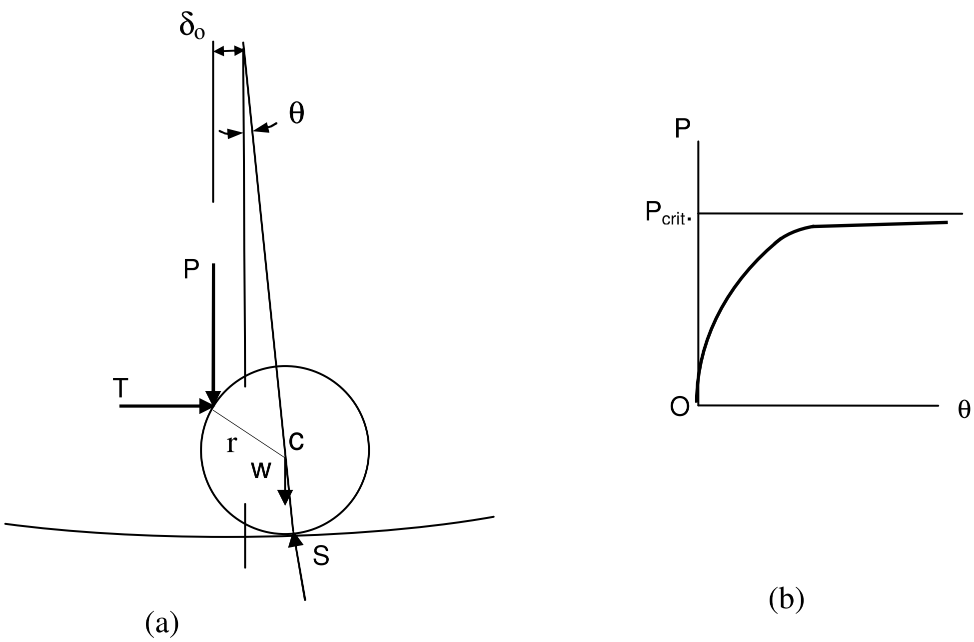



Fig. 3.7 Effect of imperfection on equilibrium of loaded

The graph of P against θ is shown in Fig. 3.7b. It may be compared with Fig. 3.6. The existence of the imperfection d causes the two straight lines OA and AC of Fig. 3.6 to become a curve which shows a definite θ value at any value of P less than $P_{critical}$, although P does become asymptotic to $P_{critical}$ as large displacement occur. These large values of θ will not in fact be determinable from eq.(3.10b), which has been derived on the assumptions of small displacements, but as in the previous problems, the small deflection analysis indicates the main aspects of the behavior of the system, though it can not quantitatively do with any degree of accuracy as P approaches its critical value.

The behavior of the imperfect system as described by eq.(3.10b) and Fig.3.7b is governed by

I. The magnitude of the imperfection
II. The sign of the imperfection
III. The critical load of the system without imperfections

## 3.1.2 Systems with more than one degree of freedom of movement

### Neutral elastic equilibrium

The stability of the equilibrium of systems which can move or deform in one manner has been discussed. However, many cases arise in which a structure or a system can deform in a number of ways, and this permits the possibility of instability occurring in a similar number of ways.

If a system can deform in any of n different patterns, then in general it is necessary to consider the stability of the equilibrium in each of these patterns, instability is likely to occur at some critical value of loading.

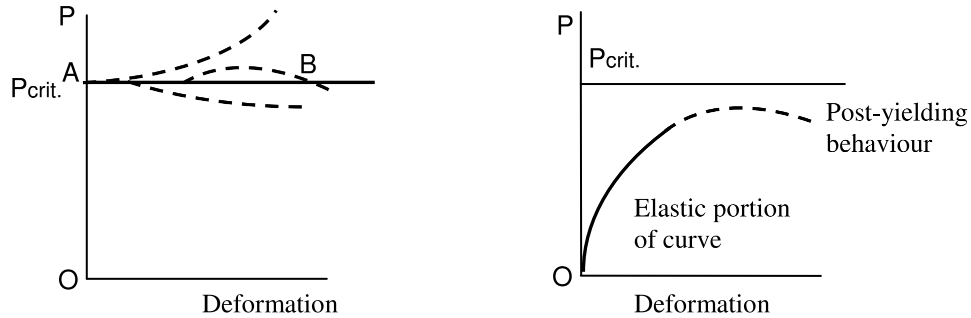
In general the number of ways in which a structure can deform is infinite or multiply –finite, and it usually becomes necessary to be able to find values for the modes of deformation which give low critical loads, though it should not be forgotten that a suitable pattern of imperfections and disturbances may well set off the structure in a mode of deformation for which the critical load to cause instability is not the lowest possessed by the system.

In framed structures, if the members of the frame are initially straight, if the center lines of the members framing into each joint intersect at coincident points (the nodes of the frame), if all loads or reactions act through these nodes, and if the frame remains linearly elastic, then there are certain critical loads for the frame at which a state of neutral equilibrium exists. The frame is in equilibrium in neighboring deflected position where the members remain straight and there is no rotation of the joints, and it is also in equilibrium in neighboring deflected positions where members bend somewhat and joints rotate.

Applying small deflection elastic beam theory to the frame members, that is assuming the distances between the node points of the frame are unaltered by the bending of the members, it was found that at the critical load the state of neutral equilibrium of the frame is unbounded as far as the effect of deformations is concerned. The equilibrium diagram shows a flat top AB as shown in Fig. 3.8(a)

If the effect of curvature of the members were taken into account, both with respect to its effect on the bending moment curvature relations for the members of the frame, then a departure from path AB would be expected. The actual curve might rise above AB or fall below it, or might do first one and then the other; various alternatives are indicated by the dotted lines in Fig. 3.8(a)

However, using small deflection theory there can be found critical loading conditions at which the fame is in neutral equilibrium with respect to small displacements of a certain kind, and associated with each critical loading there is a definite buckling mode; hence AB can be regarded as a tangent to the true equilibrium diagram, at zero deformations. The validity of the path AB, or of the associated critical loading $P_{crit.}$ as large deformations develop, depends on the departure of the dotted curves from the horizontal AB. For many frames quite large deformations can occur before the loading changes very much from $P_{crit}$ though for some frames the departure may be important.




(a)

(b)

Instability of Perfect frame, which remains elastic

Results of test on frame possessing initial imperfections of form and loading, and subjected to yielding.

Fig. 3.8 Perfect and imperfect frame compared

In addition to this, in many practical cases initial imperfections of the frame, either of its form or of its loading, alter the behavior or make it depart considerably from path OAB. Now the effect of initial imperfections is very difficult to assess. The relevant imperfections of form or of loading are very difficult to measure, particularly when it is realized that some imperfections affect behavior in one buckling mode but not in some other mode. Hence it is convenient to regard OAB as a limiting path which is approximated to only as imperfections are made very small. In many practical problems, because obtained from small deflection theory is as useful a measure of limiting behavior of the perfect frame as is the dotted curve obtained from large deflection theory.

Besides this no material has indefinite elasticity; after a certain amount of deformation, Yielding occurs, and elastic theory can no longer be applied. Hence OAB may be taken, in many practical cases and for practical purposes, as the limiting case for elastic behavior of the initially perfect frame, since elastic theory no longer holds in any case at large deformations except for very flexible structures made of high yielding point material.

Having established the notion of neutral equilibrium of an elastic frame with respect to displacements in a given mode, any given mode of buckling and the associated critical loading must always be considered together. Any particular mode and associated critical load have practical relevance only in so far as they govern the elastic portion of the behavior of an actual frame; this means that as the actual frame deforms, its deformation in the elastic range is in the same pattern as those of the buckling mode considered relevant, and the load is tending asymptotically to the corresponding load value $P_{crit.}$ as In Fig.3.8(b)

If a given initially perfect frame is taken and loaded according to some pattern,(eg. If the loads are increased proportionately, or according to some other rule), then there are a number of critical loads at which the frame is in neutral equilibrium with respect to deformations in certain modes. The lowest of these critical loadings is called the first critical load and the associated mode of deformation is called the first or fundamental buckling mode.

If an initially perfect elastic frame is loaded from zero load then it becomes unstable at the first critical load .If disturbed, it buckles in the fundamental mode. When this first critical load is exceeded, either because the disturbances to the frame have been smaller than some unavoidable restraints such as friction, or because the frame has been deliberately restrained against the occurrence of deformations in its fundamental mode, then the frame continues to be unstable with respect to first mode deformations; above the first critical load, continual restraints must be exerted against buckling in the fundamental mode. Hence the first critical load and the fundamental buckling mode govern the load-deformation behavior of an initially perfect frame in the absence of any particular restraint which prevents or reduces the occurrence of deformations in the first mode. [1]

## Frames possessing imperfections

Also, in many practical cases, it is the fundamental mode and load which governs the elastic portion of the behavior of a frame possessing imperfections. However this matter is entirely determined by the magnitude and sense of the imperfections throughout the whole structure. The pattern of imperfections of form and loading, considered as a whole, determines the mode of buckling. The imperfections can be considered as disturbances from the idealized conditions which set frame off in some mode. Often when frame is loaded, the pattern of imperfections throughout the structure is such as to set it off initially in a mode of deformation governed by a critical load higher than the first. However at larger loads the imperfections which tend to produce $1^{st}$ mode buckling have a greater effect.

The result is often that the mode of deformation then changes from a mode higher than the first, back to fundamental mode buckling. The curvature of portions of some members will be forced to reverse, while the curvature of other portions of the structure takes increasing values in the same sense. In such a case, the first part of the elastic load-deformation path of the structure is governed by a higher critical load and mode, while the second part of the behavior is governed by the first critical load.

However, where certain imperfections are large, or where all the imperfection of a structure are accidentally or deliberately such as to induce a higher mode of buckling, the whole of the elastic path may well be governed by a critical load higher than the first. The structure may never reach a stage where the deformations can change to the fundamental mode.

The instability of structures can not be studied merely by attempting to predict in isolation the behavior of compression members contained in it. The structure generally buckles as a whole and the deformations in the buckling mode usually affect the whole of the structure.

Major imperfections in a localized part of a structure may determine the final buckling mode but it is unusual for them to completely determine the deformation path by which that mode of buckling is reached. The whole pattern of imperfections has its effect here. [1]

## 3.2 Combinations of actions

Structural systems must be analyzed to ensure that the probability of reaching any of the limit states at which it would become unfit for its intended use.

The ultimate limit state considers the strength and stability requirements of the structure-essentially a collapse criterion. Individual types of characteristic loading are multiplied by the relevant factors to derive the design loads, and applied in the most unfavorable realistic combination.

The design values of the effects of actions($E_d$) should be determined, for each critical load case, by combining the values of actions which occur simultaneously, as follows

(a) Persistent and transient situations: Design values of the dominant variable actions and the combination design values of other actions.

(b) Accidental situations: Design values of permanent actions together with the frequent value of the dominant variable action and the quasi – permanent values of other variable actions and the design value of one accidental action.

(c) Seismic situations: Characteristic values of the permanent actions together with the quasi-permanent values of the other variable actions and the design value of the seismic action.

## Partial safety factors

In the relevant load cases, those permanent actions that increase the effect of the variable actions (i.e. produce unfavorable effects) shall be represented by their upper design values, those that decrease the effect of the variable actions (i.e produce favorable effects) by their lower design values.

Where the result of verification may be very sensitive to variations of the magnitude of a permanent action from place to place in the structure, the unfavorable and the favorable parts of this action shall be considered as individual actions. This applies in particular to the verification of static equilibrium.

## Simplified verification for building structures

The process for the persistent and transient situations may be simplified by considering the most unfavorable for the following combinations:

a. Design situations with only one variable action $Q_{k1}$

$$\sum_{j\geq 1} \gamma_{Gi} G_{kj} + \gamma_{Q1} Q_{k1} \qquad (3.11a)$$

b. Design situations with two or more variable action $Q_{k,i}$

$$\sum_{j\geq 1} \gamma_{Gi} G_{kj} + \gamma_{Q} Q_{k1} \qquad (3.11b)$$

In this case the effect of actions should also be verified for the dominant variable actions using Eq. (3.11a) [4]

## 3.3 Wind Actions

To examine the relative effect of equivalent horizontal imperfection load and other horizontal load the structures are analyzed for wind load also. Wind actions fluctuate with time .They act directly on the external surface of enclosed structures and, through porosity of the external surface, also act indirectly on the internal surfaces. They may also directly affect the internal surface of open structures. Pressures act on areas of the components. Additionally, when large areas of structures are swept by the wind, frictional forces acting tangentially to the surface, may be significant. To achieve the design aims account shall be taken of:

a. turbulent wind acting over part or all of the structure
b. fluctuating pressures induced by the wake behind the structure
c. fluctuating forces induced by the motion of the structure

The total response of structures and their elements may be considered as the superposition of a "background" component, which acts quasi-statically and resonant components due to excitation close to natural frequencies. For the majority of structures the resonant components are small and the wind load can be simplified by considering the background component only. Such structures can be calculated by a simplified method.

The wind action can be represented by a set of quasi-static pressures or forces whose effects are equivalent to the extreme effects of the wind.[4]

### 3.3.1 Modeling of wind actions

The wind action is represented either as a wind pressure or a wind force. The action on the structure caused by the wind pressure is assumed to act normal to the surface except where otherwise specified e.g. for tangential friction forces.

$q_{ref}$ reference mean wind velocity pressure derived from reference wind velocity. It is used as the characteristic value

$C_{e(z)}$ exposure coefficient accounting for the terrain and height above ground z .The coefficient also modifies the mean pressure to a peak pressure allowing for turbulence.

Z reference height appropriate to the relevant pressure coefficient ($z=z_e$) for external pressure and force coefficient, ($z=z_i$) for internal pressure coefficient)

$C_d$ dynamic coefficient accounting for both correlation and dynamic magnification

## Wind pressure on surfaces

### a. External pressure

The wind pressure acting on the external surfaces $W_e$ shall be obtained from:

$$W_e = q_{ref} c_e(z_e) c_{pe}$$

Where $C_{pe}$ is the external pressure coefficient

### b. Internal pressure

The wind pressure acting on the internal surfaces $W_i$ shall be obtained from:

$$W_i = q_{ref} c_e(z_i) c_{pi}$$

Where $C_{pi}$ is the external pressure coefficient [4]

# 3

# Comparison of Imperfection provisions of EBCS and EURO CODE (British Standard)

Structural members and frames carry residual stresses as well as geometrical imperfections, such as initial lean and curvature, due to erection and fabrication tolerances.

As part of the stability issue, the adverse effect of such imperfections , should be accounted for when considering frame stability, and the design of bracing and individual members of both sway and non-sway frames.

Frame imperfections are to be included in the global analysis, bracing systems are designed for the effects of an initial bow and imperfections of individual members are considered in the buckling analysis (alternately by a direct second order analysis of the member).

## 4.1 The European Code (British Standard) BSEN 1992-1-1:2004 & BSEN 1993-1-:2005

### 4.1.1 Frame imperfection

Considering Euro code [7,8], frame imperfections are introduced in the global analysis by means of an assumed slope of the columns. This slope is applied to produce an inclination in each horizontal direction separately and also, if applicable, torsion of the structure .This assumed shape of global imperfection may be derived from the elastic buckling mode of a structure in the plane of buckling considered.

Both in and out of plane buckling including torsion buckling with symmetric and asymmetric buckling shapes should be taken into account in the most unfavorable direction and form. The resulting forces of the analysis should be used in the design of members.

For frames sensitive to buckling in a sway mode the effect of imperfections should be allowed for in frame analysis by means of an equivalent imperfection in the form of an initial sway imperfection and individual bow imperfections of members. The imperfections may be determined from:

a) Global initial sway imperfections, see Figure: 4.1

This initial slope $\phi$ depends on the number of stories and the number of layout of columns. A simple and conservative option is to choose $\phi$ as 1/200.

$$\phi = \phi_0 \, \alpha_h \, \alpha_m \tag{4.1a}$$

where $\phi_0$ is the basic value: $\phi_0 = 1/200$

$\alpha_h$ is the reduction factor for height h applicable to columns:

$$\alpha_h = \frac{2}{\sqrt{h}} \text{ but } \frac{2}{3} \le \alpha_h \le 1.0$$

h is the height of the structure in meters

$\alpha_m$ is the reduction factor for the number of columns in a row:

$$\alpha_m = \sqrt{0.5\left(1 + \frac{1}{m}\right)} \tag{4.1b}$$

m is the number of columns in a row including only those columns which carry a vertical load $N_{Ed}$ not less than 50% of the average value of the column in the vertical plane considered

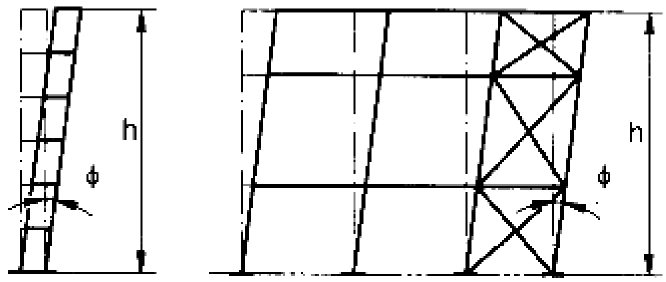



Figure 4.1 Global initial sway imperfection

Alternatively the initial slope may be substituted by an equivalent set of horizontal loads which are generally easier to use. For the determination of horizontal forces to floor diaphragms the configuration of imperfections as given in fig. 4.2 should be applied, where $\phi$ is a sway imperfection obtained from assuming a single storey with height h,

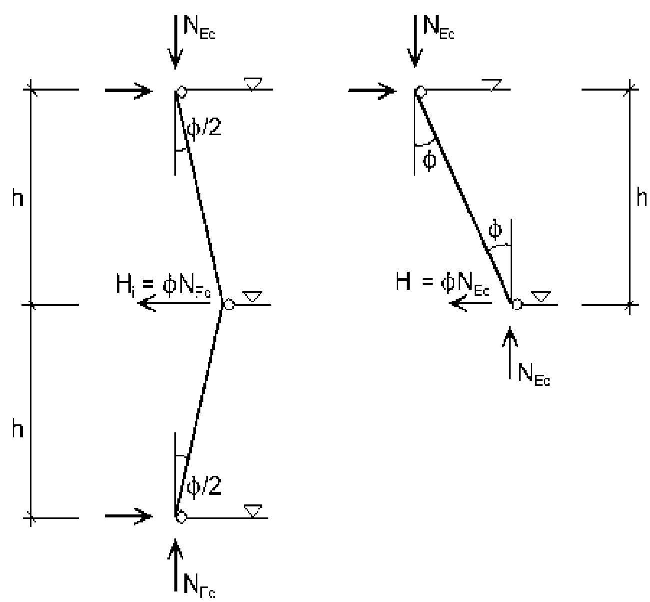



Figure 4.2 configuration of imperfection

Frame imperfections will produce internal forces and moments that will increase or decrease the effects of other loads. Both directions of the initial slope will have to be considered in combination with the other loading conditions unless some can be eliminated by inspection. A problem arises when the columns of the building have unequal lengths or when the bases are at different levels. In these cases, it is more convenient to use equivalent horizontal forces on the other hand.

Euro code specifies sway imperfections to be disregarded for building frames when $H_{Ed} > 0.15\ V_{Ed}$.

The possible torsional effect on a structure caused by anti-symmetric sways at the two opposite faces should also be considered. [7,8]

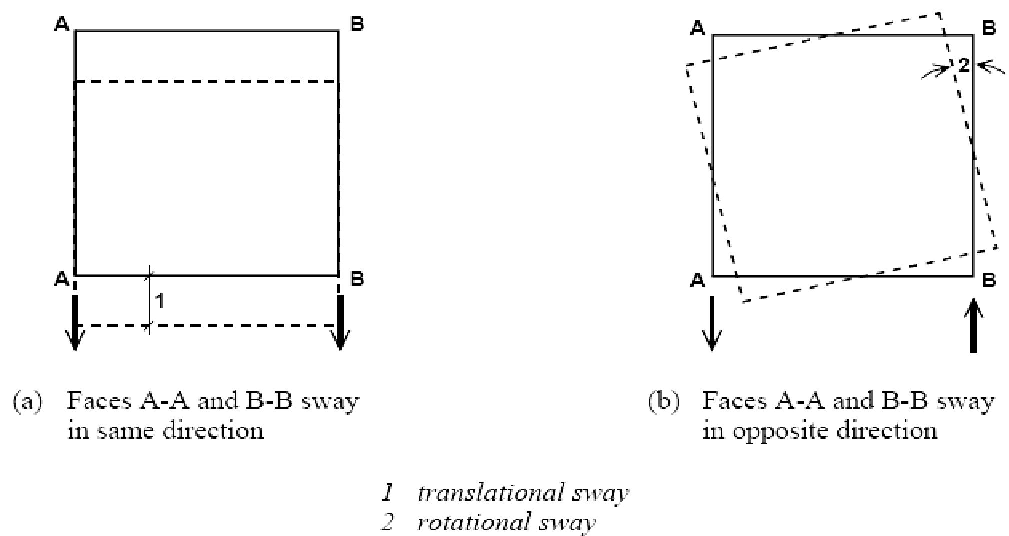



Figure 4.3 Translational & Rotational sway

### 4.1.2 Imperfections for analysis of bracing systems

In the analysis of bracing systems which are required to provide lateral stability within the length of beams or compression members the effects of imperfections is included by means of an equivalent geometric imperfection of the members to be restrained, in the form of an initial bow imperfection:

$$e_o = \alpha_m L / 500 \tag{4.2a}$$

Where $L$ is the span of the bracing system

And $$\alpha_m = \sqrt{0.5(1+\frac{1}{m})} \tag{4.2b}$$

In which m is the number of members to be restrained.

The effects of the initial bow imperfections of the members to be restrained by a bracing system may be replaced by equivalent stabilizing set of forces applied uniformly along the bracing.

$$q = \sum N_{Ed} 8 \frac{e_o + \delta_q}{L^2} \qquad (4.3)$$

Where $\delta_q$ is the in plane deflection of the bracing system due to $q$ plus any external loads calculated from first order analysis. It can be taken as 0 if second order theory is used.

We can also arrive at this value by analyzing a truss loaded at the top chord (eg. Through purlins) as shown in Fig.4.4a .Fig 4.4b shows bending moment diagram. Compressive forces in the top chord are shown in figure Fig 4.4c . The discontinuity in the diagrams is a result of components of other members at each joint. The initial bow imperfection is represented by small displacement **e** of the upper chord and fictious hinge at purlin location is assumed. Fig 4.4d shows such a mechanism.

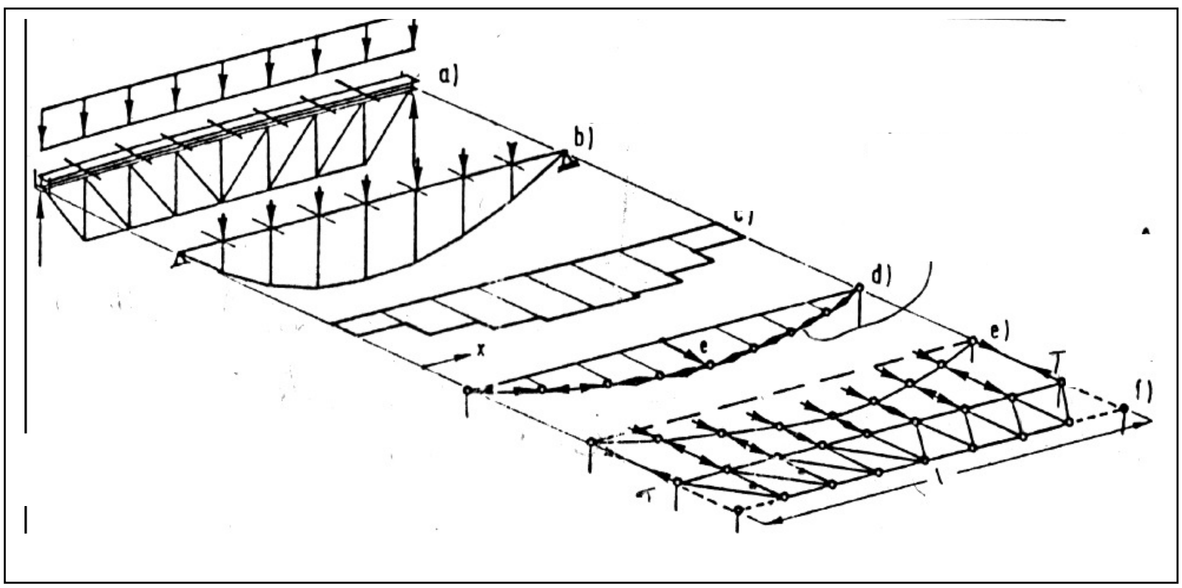



Figure: 4.4 Truss stabilized by a roof bracing

When the truss is stabilized by a roof bracing constructed between two such trusses and with the purlins as posts, because of the compression forces in the top chord, stabilizing forces are mobilized at the corner points of the mechanism. Fig. (4.4e & 4.4f)

These forces will be transferred to the cross bracing through the purlins. Compression forces result in the internal purlins and tension in the eve purlins. There exists an internal horizontal equilibrium. Assuming the form to be a sin function,

i.e. $V(x) = e \sin \frac{\pi x}{l} \Longrightarrow V''(x) = -e \frac{\pi^2}{l^2} \sin \frac{\pi x}{l}$ (4.4a)

Where l= span of truss

If V(x) is parabolic $\Longrightarrow V''(x) = -e \frac{\pi^2}{l^2}$ (4.4b)

Assuming a constant compressive force S, the stabilizing force will be continuous

$$-SV'' = Se \frac{8}{l^2} \quad (4.5)$$

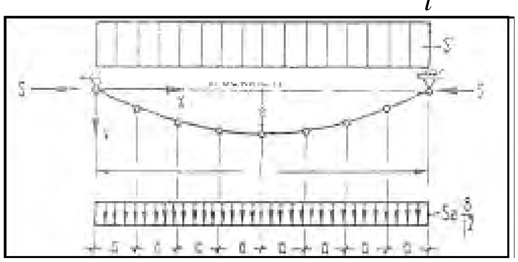


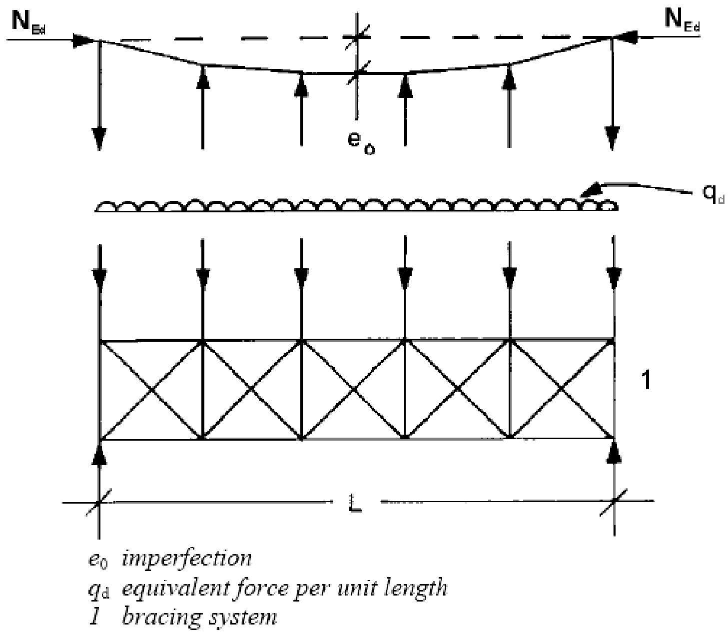
Figure: 4.5 continuous stabilizing force

Where the bracing system is required to stabilize the compression flange of a beam of constant height, the force $N_{Ed}$ in the fig.4.5 may be obtained from:

$$N_{Ed} = M_{Ed} / h \quad (4.6)$$

Where $M_{Ed}$ is the maximum moment in the beam.

and h is the overall depth of the beam.




The force $N_{Ed}$ is assumed uniform within the span L of the bracing system.
For non-uniform forces this is slightly conservative.

Figure 4.6 Imperfections in bracing systems

## 4.2 The Ethiopian building code standard (EBCS 3 -1995)

It requires appropriate allowances be incorporated to cover the effects of practical imperfections, including residual stresses and geometrical imperfections such as lack of verticality, lack-of-straightness, lack of fit and the unavoidable minor eccentricities present in practical connections.[6] Any possible unfavorable effects of these imperfections may be accounted for by suitable equivalent geometric imperfections, which reflect all types of such effects.

### Frame imperfection

Frame imperfections are accounted for in global frame analysis by means of equivalent geometric imperfections in the form of initial sway imperfection.

The coefficients in this value of the initial sway imperfection are, however, given in a different manner as given by:

$$\phi = k_c k_s \phi_o \qquad (4.7)$$

With $\phi_o = 1/200$

$k_c = (0.5+1/n_c)^{0.5}$ but $k_c \leq 1.0$

$k_s = (0.2+1/n_s)^{0.5}$ but $k_s \leq 1.0$

*where* $n_c$ *is the number of columns per plane*

$n_s$ *is the number of stories.*

While only columns which carry a vertical load $N_{sd}$ of less than 50% of the mean value of the vertical load per column in the plane considered, as in Euro code, EBCS 3 restricts those columns which do not extend through all the stories included in $n_s$ from being included in $n_c$.

$k_c$ has the same purpose as the reduction factor for height $\alpha_h$ in Euro code, however , in determining $k_c$ only those floor levels and roof levels which are connected to all the columns included in $n_c$ are included when determining $n_s$. In Euro code, the whole height of the building is used to compute the reduction factor for height $\alpha_h$.

When more than one combination of $n_c$ and $n_s$ satisfies these conditions, any such combination can safely be used.

Another deviation from Euro code is that Euro code allows sway imperfections to be disregarded only if $H_{Ed} > 0.15\ V_{Ed}$ and in EBCS sway imperfections are disregarded whenever their effect is smaller than the effect of design horizontal actions.

The initial sway imperfections apply in all horizontal directions, but need only be considered in one direction at a time. The possible torsional effects on the structure of anti-symmetric sways, on two opposite faces, shall also be considered.

Alternatively the initial sway imperfection may be replaced by a closed system of equivalent horizontal forces, see Figure 4.7.

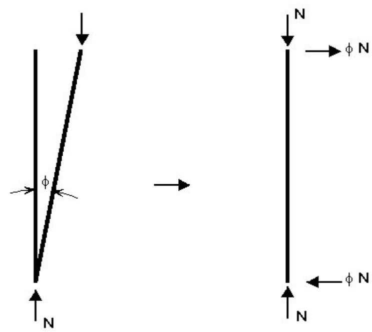



Figure 4.7 Equivalent horizontal forces

In beam-and-column building frames, these equivalent horizontal forces should be applied at each floor and roof level and should be proportionate to the vertical loads applied to the structure at that level, see Figure 4.8.

The horizontal reactions at each support should be determined using the initial sway imperfection and not the equivalent horizontal forces. In the absence of actual horizontal loads, the net horizontal reaction is zero. [6]

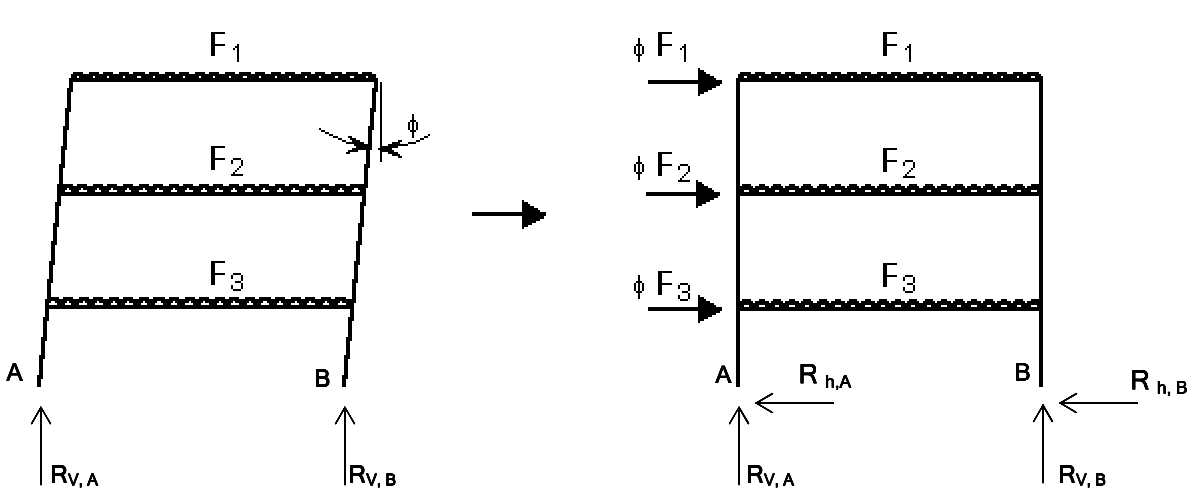


Figure 4.8 Equivalent horizontal forces in beam-and-column building frames

### 4.2.1 Imperfections for analysis of bracing systems

The specifications of EBCS 3 for Imperfections in bracing systems[6] are essentially the same as those of Euro code[8], but the coefficients are not, however, identical.

The effect of the imperfection is included in the analysis of bracing systems which are required to provide lateral stability within the length of beams or compression members, by means of an equivalent geometric imperfection of the members to be restrained in the form of an initial bow imperfection.

$$e_o = k_r \, L/500 \tag{4.8}$$

Where $L$ is the span of the bracing system

$K_r = (0.2+1/n_r)^{0.5}$ but $k_r \leq 1.0$

$n_r$ is the number of members to be restrained

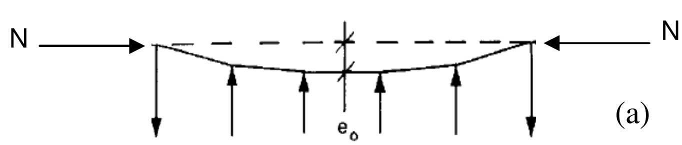
It is also possible to replace the initial bow imperfections of the members to be restrained by a bracing system by equivalent stabilizing forces shown in fig.4.7(a).

Where the bracing system is required to stabilize a beam, the force $N$ in fig should be obtained from:

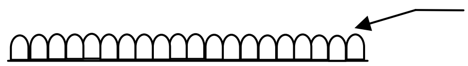
$N = M/h$

Where $M$ is the maximum moment in the beam

$h$ is the overall depth of the beam.

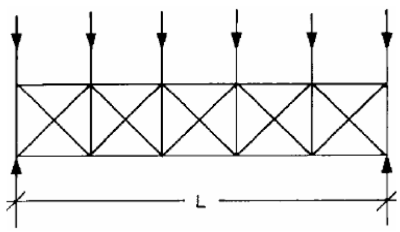



(a)



q = Equivalent stabilizing per unit length

(b)




Bracing system

(c)

Figure 4.9 Imperfection in bracing systems

The equivalent stabilizing force $q$ per unit length is calculated as follows:

a. For a single restrained member:

For $q = \dfrac{N}{50L} \quad : \quad \delta_q \leq \dfrac{L}{2500}$ (4.9a)

b. For multiple restrained members:

For $q = \dfrac{\sum N}{60L}(k_r + 0.2) \quad : \quad \delta_q \leq \dfrac{L}{2500}$ (4.9b)

For $q = \dfrac{\sum N}{60L}(k_r + a) \quad : \quad \delta_q > \dfrac{L}{2500}$ (4.9c)

Where $\delta_q$ is the in plane deflection of the bracing system due $q$ plus any external loads.

$a = 500\, \delta_q / L$ but $a \geq 0.2$

$k_r = (0.2 + 1/n_r)^{0.5}$ but $k_r \geq 1.0$

At points where beams or compression members are spliced, it shall also be verified that the bracing system is able to resist a local force applied to it by each beam or compression member which is spliced at that point, and to transmit this force to the adjacent points at which that beam or compression member is restrained.

When checking for this local force, any external loads acting on the bracing system shall also be included, but the forces arising from the imperfection given in (1) above may be omitted. [6]

# 4

# Analysis of plane frames for imperfection loads

To investigate the significances of effects of imperfections on analysis results, parametrical studies are made considering number of stories, number of columns and bay width as a parameter.

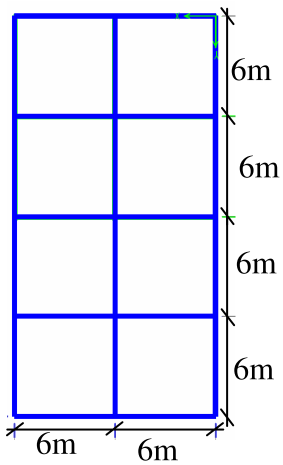
Because the EURO CODE(British Standard) means of allowance for imperfection are the same for both concrete and steel frames, the procedures of EBCS-3, which were described in chapter 4 of this paper are used for the concrete frames as well, although EBCS-2 ,1995 gives imperfections in concrete frames to be $\phi=1/200$.

The frames considered are selected to represent response behaviors corresponding to low-rise, mid-rise, and high-rise reinforced concrete moment resisting frames.

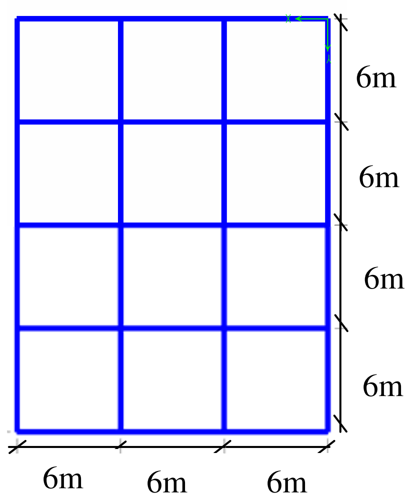
For convenience, the horizontal loads which are statically equivalent to the frame imperfections were applied on the plane frames and the relevant stress resultants for different sets of load combination are presented.

## 5.1 Description of the buildings

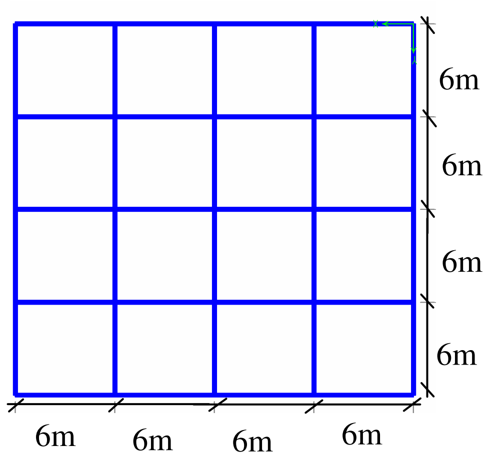
The frames considered are taken to be that of a building rectangular in plan with a width of 24 m. The number of bays in the longitudinal direction and the number of stories are taken to vary (Fig. 5a-5d shows the plan view). The bottom story height is 3m and the typical story height is 3.36m. The frame in the transverse direction consists of four bays of 6m width, two in each side of the longitudinal frame.



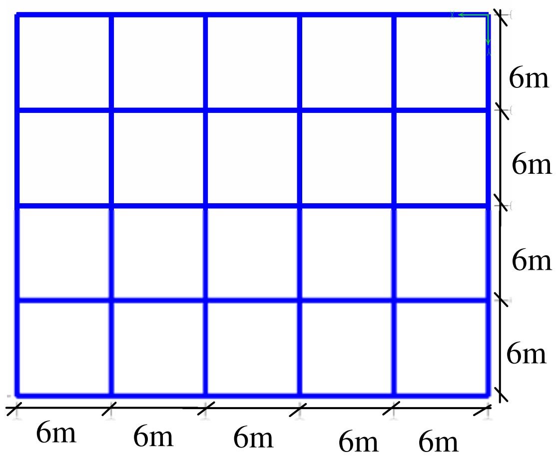

(a) Frame ID: CF6 2 bay




(b) Frame ID: CF6 3 bay




(c) Frame ID: CF6 4 bay




(d) Frame ID: CF6 5 bay

Figure: 5.1 plan view of buildings

The imposed live load on the floors is taken as 5 $KN/m^2$ to have the maximum effect. Analyzing the slab for this imposed live load and own weight, the resulting reactions are transferred to the supporting beams.

Axial forces in the columns which are contributions of the frame in the transverse direction are applied as external axial load to the frame in longitudinal direction. The equivalent horizontal imperfection loads are then calculated as a proportion of the total vertical load per story, that is, the factored dead load and live load plus the axial forces from the transverse direction. To have an insight on relative response behavior of structures to imperfection load and other lateral loads, analysis is carried out for wind load also.

Table:1 **Numerical comparison of imperfection values calculated using EBCS and EURO CODE for the selected frames.**

| Frame | height | No of columns | No of storeys | Reduction factor for Height | | Reduction factor for no of columns | | Imperfection Values | | % Decrease of EC from EBCS |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | EBCS (Ks) | EURO CODE($\alpha_h$) | EBCS(Kc) | EURO CODE($\alpha_m$) | EBCS($\phi$) | EURO CODE($\phi$) | |
| CF62-5 | 16.44 | 3 | 5 | 0.6325 | 0.4933 | 0.9129 | 0.8165 | 0.0029 | 0.0020 | 30.24 |
| CF62-10 | 33.24 | 3 | 10 | 0.5477 | 0.3469 | 0.9129 | 0.8165 | 0.0025 | 0.0014 | 43.35 |
| CF62-15 | 50.04 | 3 | 15 | 0.5164 | 0.2827 | 0.9129 | 0.8165 | 0.0024 | 0.0012 | 51.03 |
| CF62-20 | 66.84 | 3 | 20 | 0.5000 | 0.2446 | 0.9129 | 0.8165 | 0.0023 | 0.0010 | 56.24 |
| CF62-25 | 83.64 | 3 | 25 | 0.4899 | 0.2187 | 0.9129 | 0.8165 | 0.0022 | 0.0009 | 60.07 |
| CF62-30 | 100.44 | 3 | 30 | 0.4830 | 0.1996 | 0.9129 | 0.8165 | 0.0022 | 0.0008 | 63.05 |
| CF63-5 | 16.44 | 4 | 5 | 0.6325 | 0.4933 | 0.8660 | 0.7906 | 0.0027 | 0.0019 | 28.80 |
| CF63-10 | 33.24 | 4 | 10 | 0.5477 | 0.3469 | 0.8660 | 0.7906 | 0.0024 | 0.0014 | 42.18 |
| CF63-15 | 50.04 | 4 | 15 | 0.5164 | 0.2827 | 0.8660 | 0.7906 | 0.0022 | 0.0011 | 50.02 |
| CF63-20 | 66.84 | 4 | 20 | 0.5000 | 0.2446 | 0.8660 | 0.7906 | 0.0022 | 0.0010 | 55.34 |
| CF63-25 | 83.64 | 4 | 25 | 0.4899 | 0.2187 | 0.8660 | 0.7906 | 0.0021 | 0.0009 | 59.25 |
| CF63-30 | 100.44 | 4 | 30 | 0.4830 | 0.1996 | 0.8660 | 0.7906 | 0.0021 | 0.0008 | 62.29 |
| CF64-5 | 16.44 | 5 | 5 | 0.6325 | 0.4933 | 0.8367 | 0.7746 | 0.0026 | 0.0019 | 27.79 |
| CF64-10 | 33.24 | 5 | 10 | 0.5477 | 0.3469 | 0.8367 | 0.7746 | 0.0023 | 0.0013 | 41.36 |
| CF64-15 | 50.04 | 5 | 15 | 0.5164 | 0.2827 | 0.8367 | 0.7746 | 0.0022 | 0.0011 | 49.31 |
| CF64-20 | 66.84 | 5 | 20 | 0.5000 | 0.2446 | 0.8367 | 0.7746 | 0.0021 | 0.0009 | 54.70 |
| CF64-25 | 83.64 | 5 | 25 | 0.4899 | 0.2187 | 0.8367 | 0.7746 | 0.0020 | 0.0008 | 58.67 |
| CF64-30 | 100.44 | 5 | 30 | 0.4830 | 0.1996 | 0.8367 | 0.7746 | 0.0020 | 0.0008 | 61.75 |
| CF65-5 | 16.44 | 6 | 5 | 0.6325 | 0.4933 | 0.8165 | 0.7638 | 0.0026 | 0.0019 | 27.05 |
| CF65-10 | 33.24 | 6 | 10 | 0.5477 | 0.3469 | 0.8165 | 0.7638 | 0.0022 | 0.0013 | 40.76 |
| CF65-15 | 50.04 | 6 | 15 | 0.5164 | 0.2827 | 0.8165 | 0.7638 | 0.0021 | 0.0011 | 48.79 |
| CF65-20 | 66.84 | 6 | 20 | 0.5000 | 0.2446 | 0.8165 | 0.7638 | 0.0020 | 0.0009 | 54.23 |
| CF65-25 | 83.64 | 6 | 25 | 0.4899 | 0.2187 | 0.8165 | 0.7638 | 0.0020 | 0.0008 | 58.24 |
| CF65-30 | 100.44 | 6 | 30 | 0.4830 | 0.1996 | 0.8165 | 0.7638 | 0.0020 | 0.0008 | 61.36 |

## 5.2 Analysis result

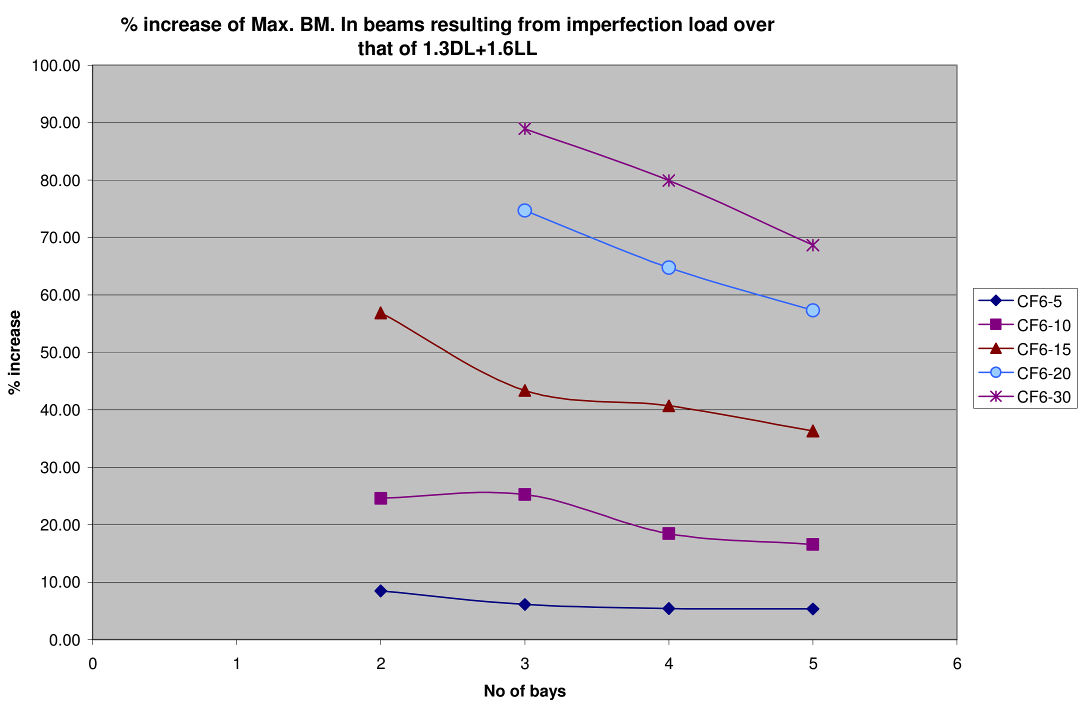
Of the different stress resultants, significant influence by imperfection loads is obtained for the negative bending moment in beams. Thus this result is displayed in Table: 2. And to investigate the relative change in the results due to inclusion of imperfection loads and other lateral load, i.e. wind load, the percentage increase of these loads combined according to the combination rules mentioned in chapter 3 of this paper, over that of 1.3DL+1.6LL is given in Table: 3.Maximum bending moments in columns are displayed in table 4.

Table:2

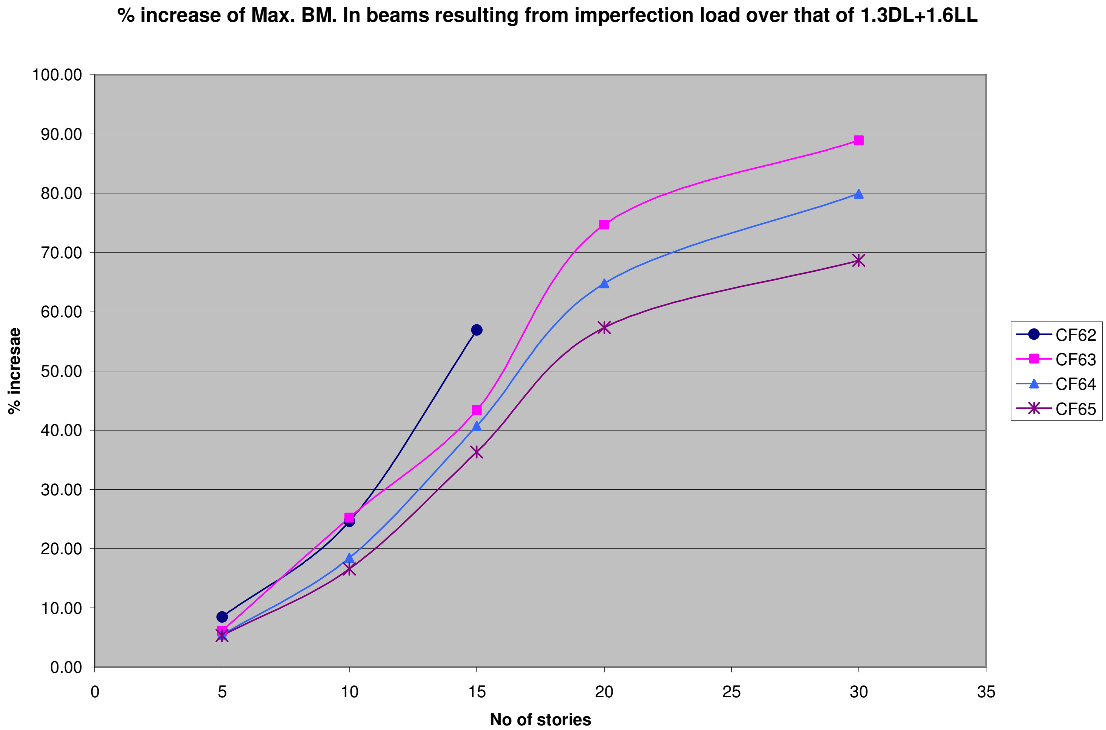
| Frame ID: | | Max. bending moments in beams | |
| --- | --- | --- | --- |
| | 1.3DL+1.6 LL | 1.3DL+1.6 LL+Impef. [9] | 1.3DL+1.35 LL+1.35WL |
| CF62-5 | 259.70 | 281.73 | 268.36 |
| CF62-10 | 287.03 | 357.74 | 330.26 |
| CF62-15 | 333.44 | 523.16 | 436.64 |
| CF63-5 | 258.78 | 274.64 | 258.92 |
| CF63-10 | 286.33 | 358.67 | 299.82 |
| CF63-15 | 340.38 | 488.07 | 374.42 |
| CF63-20 | 393.85 | 688.05 | 453.22 |
| CF63-30 | 722.85 | 1365.79 | 787.14 |
| CF64-5 | 260.25 | 274.39 | 254.26 |
| CF64-10 | 285.51 | 338.22 | 284.97 |
| CF64-15 | 339.61 | 477.90 | 348.77 |
| CF64-20 | 394.67 | 650.29 | 415.96 |
| CF64-30 | 738.53 | 1328.80 | 746.13 |
| CF65-5 | 261.28 | 275.23 | 251.32 |
| CF65-10 | 284.74 | 331.94 | 276.95 |
| CF65-15 | 338.60 | 461.65 | 335.47 |
| CF65-20 | 393.59 | 619.19 | 396.24 |
| CF65-30 | 739.99 | 1248.23 | 724.77 |

Table:3 percentage increase of results of the combined lateral loads over that of 1.3DL+1.6LL(in beams)

| Frame Id. | **1.3D+1.6L+Impf.** [9] | **(1.3D+1.35L+1.35WL)** |
|---|---|---|
| CF62-5 | 8.48 | 3.33 |
| CF62-10 | 24.64 | 15.06 |
| CF62-15 | 56.90 | 30.95 |
| CF63-5 | 6.13 | 0.05 |
| CF63-10 | 25.26 | 4.71 |
| CF63-15 | 43.39 | 10.00 |
| CF63-20 | 74.70 | 15.07 |
| CF63-30 | 88.95 | 8.89 |
| CF64-5 | 5.43 | -2.30 |
| CF64-10 | 18.46 | -0.19 |
| CF64-15 | 40.72 | 2.70 |
| CF64-20 | 64.77 | 5.39 |
| CF64-30 | 79.92 | 1.03 |
| CF65-5 | 5.34 | -3.81 |
| CF65-10 | 16.58 | -2.74 |
| CF65-15 | 36.34 | -0.92 |
| CF65-20 | 57.32 | 0.67 |
| CF65-30 | 68.68 | -2.06 |




(a)




(b)

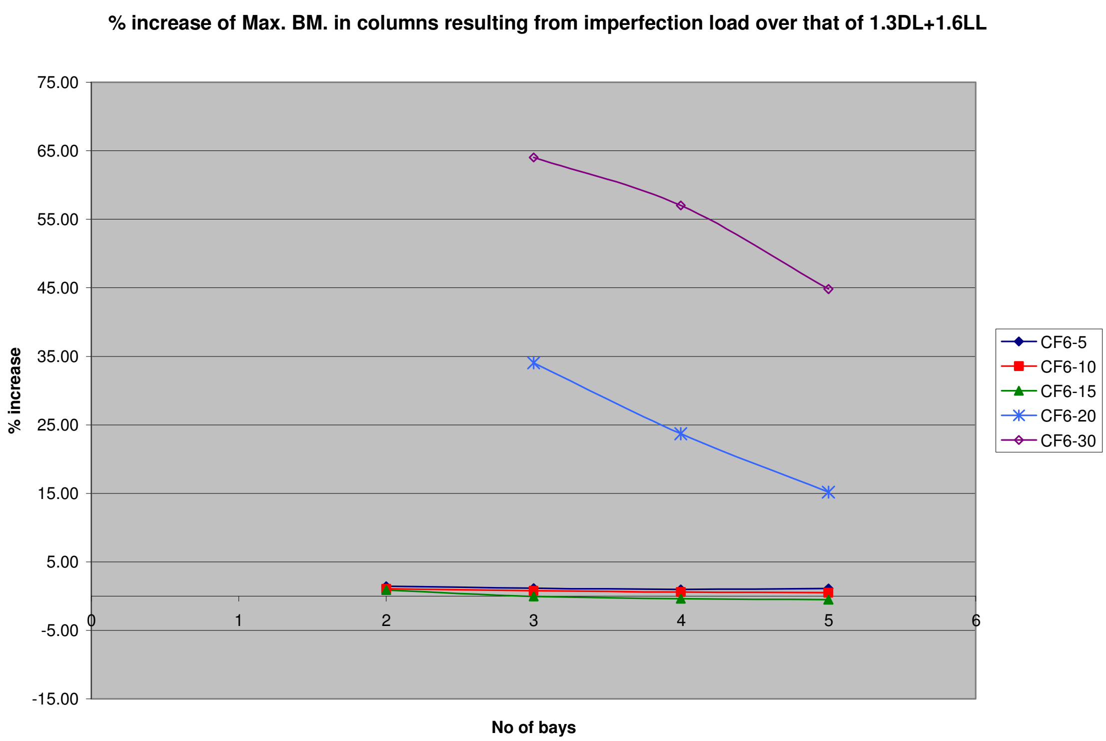
Figure 5.2 Percentage increase of bending moment in beams.

Table:4

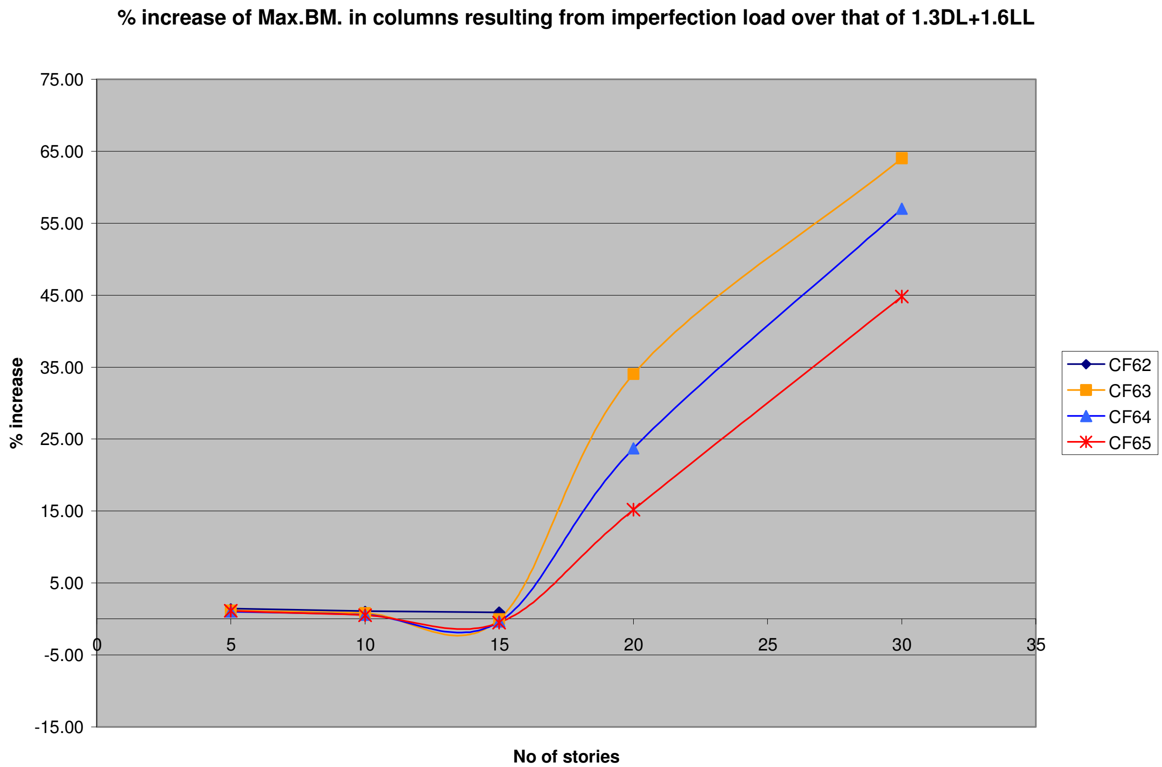
| Frame ID: | | Max. bending moments in columns | |
|---|---|---|---|
| | 1.3DL+1.6 LL | 1.3DL+1.6 LL+Impef. [9] | 1.3DL+1.35 LL+1.35WL |
| CF62-5 | 207.62 | 210.58 | 199.22 |
| CF62-10 | 240.80 | 243.41 | 232.19 |
| CF62-15 | 279.43 | 281.93 | 275.51 |
| CF63-5 | 206.85 | 209.23 | 196.36 |
| CF63-10 | 238.86 | 240.80 | 226.92 |
| CF63-15 | 281.53 | 281.47 | 266.00 |
| CF63-20 | 323.74 | 433.91 | 303.77 |
| CF63-30 | 589.71 | 967.41 | 560.65 |
| CF64-5 | 205.42 | 207.45 | 194.01 |
| CF64-10 | 237.09 | 238.48 | 223.86 |
| CF64-15 | 279.38 | 278.34 | 262.15 |
| CF64-20 | 322.19 | 398.56 | 299.78 |
| CF64-30 | 596.23 | 936.22 | 564.01 |
| CF65-5 | 204.27 | 206.55 | 192.35 |
| CF65-10 | 235.59 | 236.79 | 221.70 |
| CF65-15 | 277.37 | 275.95 | 259.44 |
| CF65-20 | 319.72 | 368.29 | 296.56 |
| CF65-30 | 593.35 | 859.26 | 560.74 |

Table:5 percentage increase of bending moments of the combinec lateral loads over that of 1.3DL+ 1.6LL(in columns)

| Frame ID: | 1.3D+1.6L+Impf. [9] | (1.3D+1.35L+1.35WL) |
|---|---|---|
| CF62-5 | 1.43 | -4.05 |
| CF62-10 | 1.08 | -3.58 |
| CF62-15 | 0.89 | -1.40 |
| CF63-5 | 1.15 | -5.07 |
| CF63-10 | 0.81 | -5.00 |
| CF63-15 | -0.02 | -5.52 |
| CF63-20 | 34.03 | -6.17 |
| CF63-30 | 64.05 | -4.93 |
| CF64-5 | 0.99 | -5.55 |
| CF64-10 | 0.59 | -5.58 |
| CF64-15 | -0.37 | -6.17 |
| CF64-20 | 23.70 | -6.96 |
| CF64-30 | 57.02 | -5.40 |
| CF65-5 | 1.12 | -5.84 |
| CF65-10 | 0.51 | -5.90 |
| CF65-15 | -0.51 | -6.46 |
| CF65-20 | 15.19 | -7.24 |
| CF65-30 | 44.82 | -5.50 |




(a)




(b)

Figure: 5.3 Percentage increase of bending moment in columns.

# 6

# Disscussion and Conclusions

To evaluate strength requirements of imperfect structural systems, the imperfection effects are quantified in terms of an initial away imperfection at the base of the column and then converted to equivalent horizontal forces. Analysis is carried out which includes these equivalent horizontal forces and the maximum bending moment in columns and beams for frames with varying number of bays and number of stories are examined.

As presented in tables and graphs, for the 5 story Building and the 10 story building with 4 bays and 5 bays, the percentage increase in maximum bending moment in beams is below 30%.For bending moment in columns in the 20 & 30 story buildings show high percentage increase for all number of bays.

In general for a fixed number of stories, the effect on maximum negative bending moment in beams decreases with increasing number of bays. The decrease is pronounced for the high rise buildings. The effect on columns is not as large as that in beams but still significant for the 20 & 30 story buildings considered.

For a fixed number of bays the percentage increase rises with the story height both in beams and columns, the effect being still higher on beams.

The percentage increase over that of 1.3DL+1.6LL was found to be very high for the high rise buildings with smaller number of bays. This could be the result of the high aspect ratio. From the frames considered those with aspect ratio greater than 2 give an increase above 50%.

## 6.1 Conclusions

The main goal of the research was to investigate the significance of imperfection effects in analysis results and decide if the common practice of assuming them insignificant in general is valid. Additional objective include evaluating the extent of the EBCS 2 & EBCS 3 provisions for frame imperfection and compare it with other international codes.

From the results discussed we can conclude that the imperfection effects should not be neglected, especially for buildings located on areas where other lateral loads are small. Special attention should also be given to buildings with high aspect ratio as they have been found to be more influenced.

According to EBCS imperfection effects can be ignored when their effects are smaller than the effects of design horizontal actions. This underestimates their effects when compared to Euro Code allowance, in which case they are neglected only when $H_{Ed} > 0.15V_{Ed}$. Therefore with regard to this provision EBCS needs to be revised.

## 6.2 Future Works

In this study in-plane imperfections have been dealt with. Future works can include accounting for imperfections in a 3D analysis as well as identification of other parameters influencing imperfection and magnitude of their influence. Torsional effects on a structure caused by anti-symmetric sways can also be included in further studies.

Part of the objective of this thesis work was to find out the background for the code provisions of the imperfection values. Due to lack of reference this objective could not be met; however, it can be included in future works dealing with imperfections.

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9. Access steel European codes Made easy

# CANDIDATE'S DECLARATION

I hereby declare that the work presented in this thesis which is entitled "**Investigation of Effects of Imperfection and Their Significance in Analysis Results**" is my original work and has not been presented for a degree in any other University and that all sources of material used for the thesis have been duly acknowledged.

____________________ ____________________

Haimanot Tadesse Date

(Candidate)

This is to certify that the above declaration made by the candidate is correct to the best of my knowledge.

____________________ ____________________

Dr.- Ing Adil Zekeria Date

(Thesis Advisor)