ADDIS ABABA UNIVERSITY

ADDIS ABABA INSTITUTE OF TECHNOLOGY

SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

PARAMETRIC STUDY ON STRUCTURAL FORCES AND MOMENTS OF ELLIPTICAL SHELL ROOF

A thesis submitted to the school of Civil and Environmental Engineering of Addis Ababa University in Partial Fulfillment of the Requirements for the Degree of Masters of Science in Civil Engineering

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BY **Gizachew Aderaw**

September 2019

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DECLARATION

I, the undersigned, declare that this thesis is my work and all sources of materials use for the thesis have been duly acknowledged.

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Date of submission: October 2019

This work is dedicated to

My Sister Yadamnesh Aderaw

And

My advisor Dr. Shifferaw Taye

Acknowledgment

First and foremost I would like to thank the Almighty God for his unending blessings. I would like to express my deepest gratitude and respect to my advisor Dr.Shifferaw Taye for his unreserved assistance, constructive, and timely comments of my work. I am grateful to express my deepest gratitude to my families, my sisters, Yadamnesh and Birtukan, my brother Biniyam and my friend for their support towards the completion of this study. I also put on record to express my deepest gratitude to wolkite University for their financial support towards the completion of this study. Last but not least, I wish to acknowledge all those who have made their contribution towards the success of this work.

ABSTRACT

Concrete shell roofs have been widely used in the construction field for reasons concerning aesthetic appearance or achieving an economical design of a building with large spans. This paper deals with the influence of aspect ratio and height to miner radius on the structural forces and the moments developed on the surface of elliptical shell roofs. The parameters considered in this study are major to miner radius ratio and height to miner radius ratio of elliptical shell roof. This thesis studies the parametric influences of aspect ratio and height to miner radius ratio on elliptical shell roofs. The influences of aspect ratio and height to miner radius should be considered in the analysis and design of the elliptical shell roof using both membrane and bending theory analysis. The structural forces and the moments developed in elliptical shell roof are predicted using mathematical method and the model proposed by ANSYS software.

As the height ratio increases, the in-plane forces linearly increase while the moment forces decrease in a quadratic fashion. Therefore, it is noted that the aspect and height ratios greatly affect the moment and in-plane forces, respectively.

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$N_\emptyset$	$normal\ internal\ stress\ in\ merdional\ direction\ per\ unit\ width$
N_θ	$normal\ internal\ stress\ in\ Circumferencial\ direction\ per\ unit\ width$
$P_n\ and\ P_\emptyset$	$external\ load\ components$
g	factored loads per unit area
$M_\emptyset$	$normal\ internal\ moment\ in\ merdional\ direction\ per\ unit\ width$
M_θ	$normal\ internal\ moment\ in\ Circumferencial\ direction\ per\ unit\ width$
a	major radius of ellipse
b	minor radius of ellipse
c	height of ellipse
t	thickness of ellipse
Ø	angle between the edge of ellipse to the center of ellipse
Θ	angle between minor and major axis

CHAPTER ONE

INTRODUCTION

Shell roofs with single and double curvature have been commonly used for different purposes. Different shapes can be made, namely, circular cylindrical shell, hyperbolic shell, elliptic shell and catenary shell etc.

Thin shell structures have found wide application in practice all over the world. These include, among others, roofs of various types, water tanks and reservoirs as well as liquid containers and storage facilities, pressure vessels, bodies of transportation structures including aircrafts, space crafts, motor vehicles and ships, egg shaped sewage digesters, cooling towers at thermal power stations. Elliptical shells are widely used in practice for stadium roof structures. They are mainly used as roof coverings from simple and small elliptical shells common in stadium structures to large-span ellipse for stadium. This thesis reported on the influence of height of elliptical shell, radius of the shell, and thickness of the shell on the membrane forces and the moments developed on the surface of elliptical shell roof. The ACI code defines a thin shell as a three-dimensional spatial structure made up of one or more curved slabs or folded plates whose thicknesses are small compared to their other dimensions. Thin shells are characterized by their three-dimensional load-carrying behavior, which is determined by the geometry of their forms, by the manner in which they are supported, and by the nature of the applied load. Concrete shell structures are able to span large distances with a minimal amount of material.

In this research it is tried to use thin concrete shell structures for the roofs of the ellipsoid shape. From among various types of roofs elliptical shell cover large spans and good esthetic view. The studies have been done on the influence of aspect ratio of elliptical shell on the membrane forces and moments developed on the surface of elliptical shell roofs. Elliptical shells can be analyzed by classical methods, namely membrane theory and bending theory. The elliptical shells can also be analyzed by numerical methods. The classical methods proposed by bending theory have been considered for the manual analysis. The numerical method proposed by ANSYS software has been considered for the computer analysis.

In this paper, a typical elliptical shell roof of standard dimension adopted in practice has been considered. The structural shell forces and the moments developed on the surface of elliptical shell roof are predicted using the model proposed by bending and membrane theory. The numerical analysis is carried out using ANSYS software. The results obtained from the classical method are compared with the corresponding results obtained from the numerical method. The comparison is done to check the degree of accuracy of the application of ANSYS software for further parametric study. In this paper, the influence of various parameters, namely, aspect ratio (a/b) and height to radius ratio (h/b) of a single elliptical shell roof on the structural shell forces and the moments developed on the surface of elliptical shell roofs have been reported. The parametric study is carried out using analytical method and ANSYS software.

1.1 Material

The material has always conditioned the engineer's imagination, maybe more in this kind of construction of elliptical shape roof structure.

From among the various construction material used for thin shell structures, the roof of the ellipsoid shape, in our case is constructed from reinforced concrete. In the world, some of the existing sport arena are made of reinforced concrete in order to increase the span ratio by considering the effect of buckling.

1.2 Statement of problem

It is difficult to know the design and construction of elliptical shell roof under aspect ratio on structural shell forces and moments on surface of elliptical shell roof.It is difficult to know the exact load analysis of elliptical shell roof on structural shell forces and moments on the surface of elliptical shell roof.

1.3 Objective

1.3.1 General objective

The objective of this thesis is to know the influence of aspect ratio of an elliptical shell roof on the structural shell forces and the moments developed on the surface of elliptical shell roofs.

1.3.2 Specific objective

The specific objective of this thesis is conducting the aspect ratio of elliptical shell roof on the structural forces and moments.

The objective of study are to:-

- Identify the influences of aspect ratio of elliptical shell roof on hoop forces.
- Identify the influences of aspect ratio of elliptical shell roof on meridian forces.
- Identify the influences of aspect ratio of elliptical shell roof on hoop moments.
- Identify the influences of aspect ratio of elliptical shell roof on meridian moments.

1.4 Scope of the Study

This thesis is limited to studies the influence of aspect ratio (a/b) of an elliptical shell roof on the structural shell forces and the moments developed on the surface of elliptical shell roofs. The studies have been reported on the influence of height of elliptical shell, radius of the shell, and thickness of the shell on the membrane forces and the moments developed on the surface of elliptical shell roofs. The structural shell forces and the moments developed on the surface of elliptical shell roof are predicted using the model proposed by bending theory and membrane theory. The analysis and design of elliptical shell will be also studied.

CHAPTER TWO

LETRATURE REVIEW

2.1 Introduction

The dome shape has been around in architecture and design for hundreds of years. Nearly every one of the famous architectural structures in the word have dome and vaulted shapes. The shape of the ellipse is based on spherical geometry and was taken from the shell form of some elements of nature. The elliptical shell behaviors follow the ideas of using curvature, a thin surface to establish strength and stability. The ellipse provides a structure with highly redundant strength and flexibility features. When an elliptical shell has been constructed as a roof structure, the main loading on the structure which consists of Self weight, snow loading, and wind loading should be considered. [6]

The shell roofs being discussed in this report is within the range of thin elastic shell theory. There are assumptions to develop this theory. The first assumption is that the shell must be thin. A shell is considered to be thin if $t/R \leq 1/20$, where t and R are respectively the thickness and the radius of the shell. Structurally, this assumption infers a small defection of shells, a negligible stress normal to the middle surface as reference surface of the shell, and a preservation of normal to the middle surface of the shell. [2]. The primary response of an elliptical to loading is development of membrane compressive stresses along the meridians, by analogy to the arch. The elliptical shell also develops compressive or tensile membrane stresses along lines of latitude. These are known as ‘hoop stresses’ and are tensile at the base and compressive higher up in the elliptical.[5] This thesis deals with the influence of aspect ratio on structural shell forces and moments of reinforced concrete elliptical shell roof structures.

2.2 Classification of shells

There are many different ways to classify shell structures. In order to better understanding the elliptical thin concrete shells, they were catalogued by three different criterions. The first criterion was the material, which the shell is made of. The second consideration is classification according to manner of generation, the third consideration is the shell thickness and the last criterion is classification according to Gaussian curvature to classify the shape of the shells.

2.2.1 Classification of shells according to material

The first criterion was the material, which the shell is made of like reinforced concrete, plywood or steel, because each one has different properties that can determine the shape of the building and therefore, these characteristics have to be considered in the design. In our case, the elliptical roof of the stadium is made of reinforced concrete.[15]

2.2.2 Classification based on thickness

There are two different classes of shells: thick shells and thin shells. A shell is called thin if the maximum values of the ratio t/R (where R is radius of curvature of the middle surface) can be neglected in compression with unity. For an engineering accuracy, a shell may be regarded as a thin if a condition is satisfied. $Max(t/R) \leq \frac{1}{20}$

For a large number of practical applications, the thickness of shell lies in range $\frac{1}{1000} \leq \frac{t}{R} \leq \frac{1}{20}$ i.e. in the range of thin shells. In our case the elliptical shell is thin. i.e. t/Rmin=1/100

2.2.3 Classification according to manner of Generation

Shells are generally classified into three categories based on their mode of formation as shell of revolution, cylindrical shells and shell of translation.

2.2.4 Classification according to Gaussian curvature

The geometry of the shell is completely described by the curved shape of the middle surface and the thickness of the shell at all points. The Gaussian curvature K is the product of the two principal curvatures, the maximum and the minimum, and it is used in order to classify the shells in three different groups, according to the K_g sign synclastic shells, anticlastic shells, both with double curvature, and developable shells with simple curvature. There is other one additional group, free-forms structures, which cannot be classified by the curvature. [17]

The domes of revolution are formed by a surface generated by a curve of any form revolving about a vertical line. This surface has double curvature. The simplest dome of revolution is a portion of a sphere. However, other curves are also satisfactory, such as the ellipse, the parabola, other conic sections, or random curves. [17]

2.3 Generation of surfaces

Surfaces described above can be created using geometric or non-geometric techniques. The former uses mathematical functions while the latter uses natural processes like form-finding.

2.3.1 Geometrical Surface generation

They include surfaces of revolution, surfaces of translation, ruled surfaces and free-form surfaces as shown in figure 2 Surface of revolutions are created by rotating a meridian curve about an axis of revolution.

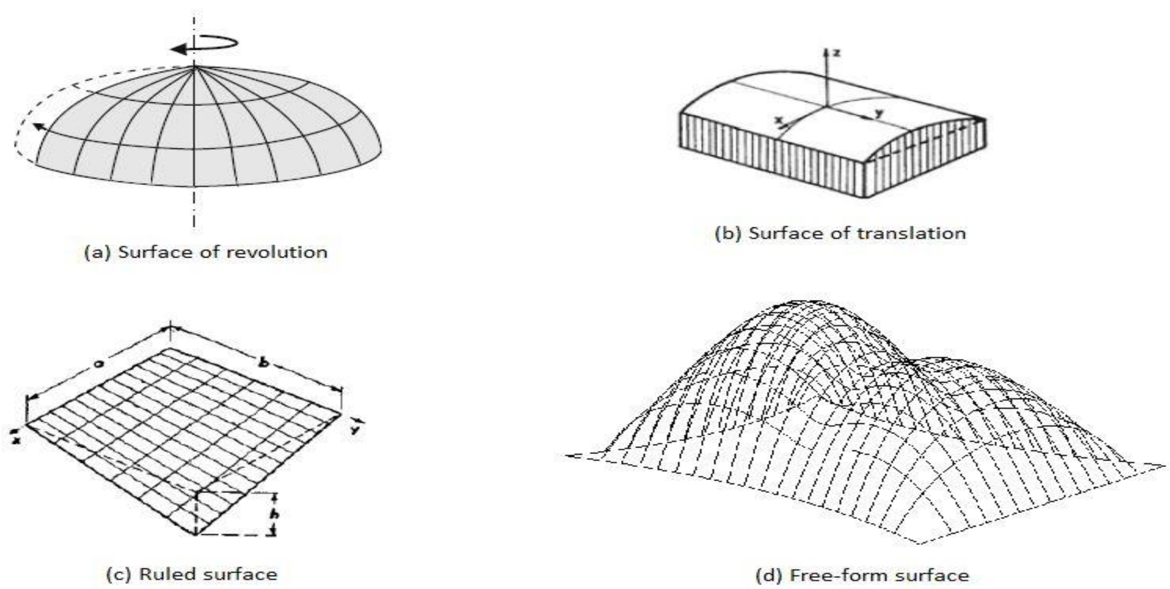


Figure 2.1 Examples of geometrically generated surface [16]

2.3.2 Non-Geometrical Surface generation

There are several physical and computational methods of form finding. They involve finding a shape which is at equilibrium with the forces that act upon it·

2.4 Elliptical Shells

This thesis try to analyses and model shells of revolution called elliptical shells. Shells of revolution are those formed by the rotation of a plane curve called the meridian, about a certain axis known as the axis of the shell. Elliptical shells constitute a fairly large proportion of axisymmetric shells in practice. They are mainly used as ellipsoid roof coverings from simple and small ellipse common in stadium structures and reservoir roofs to large-span, complete or mostly incomplete elliptical shell roof for auditoriums and sport facilities such as stadium, restaurant's and recreational area.

There are a lot of shapes that can be chosen for the stadium, restaurant and auditorium. It is ultimately going to influence the entire design process as shells are shape based structures. The type of geometry influences the way they transfer load and aesthetical view. The roof for this studies, having a large span, needs to carry its own self-weight and natural loads and for good aesthetical view.

In my thesis I try to use small to large elliptical shell roofs. The stress should be considered in the analysis and design of the elliptical roof using both membrane and bending theory analysis.

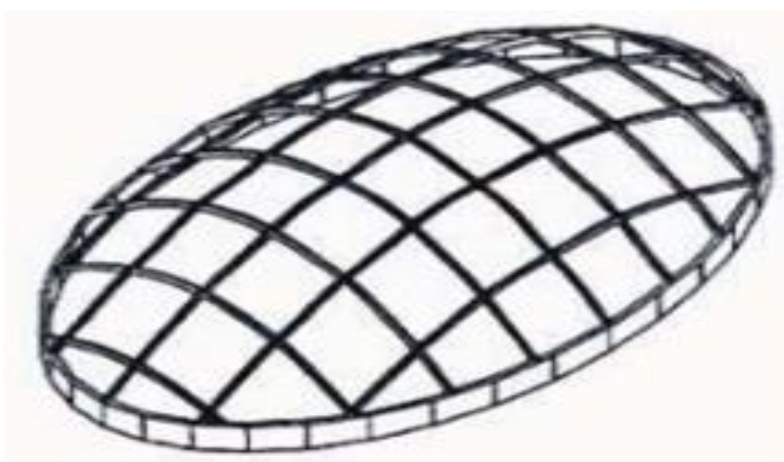


Figure 2.2 the scheme of disposition of ellipsoid arches of sport hall [18]

Shell structures are being widely used in many industries worldwide from monumental structures to storage structures for a multitude of different products. Shell structure is a classical and broad topic in reinforced concrete structures. A shell with one curved surface is known as a vault single curvature.

The shell roofs being discussed in this research is within the range of thin elastic shell theory. There are assumptions to develop this theory. The first assumption is that the shell must be thin. A shell is considered to be thin if $t/R \leq 1/20$, where t and R is respectively the thickness and the radius of the shell. Mathematically, this assumption implies that the ratio t/R can be neglected compared to unity in derivation of the thin elastic shell theory. Structurally, this assumption infers a small defection of shells, a negligible stress normal to the middle surface as reference surface of the shell, and a preservation of normal to the middle surface of the shell. It is noted that the designation of the middle surface as the reference surface assumes a homogeneous material of the shell. The second inference is not correct in the area of the shell subjected to concentrated loads. Therefore, this theory is applicable to uniform loads, which are dead load and live load in the report. The stresses in a shell element are ,as shown in figure ,only normal stresses $N\theta$, $N\varphi$, shear stress $N\theta\varphi$(membrane theory) or might include transverse shear stresses $Q\theta$, $Q\varphi$ and couples $M\theta$, $M\varphi$, $M\theta\varphi$ (bending theory),figure 2.3. [6]

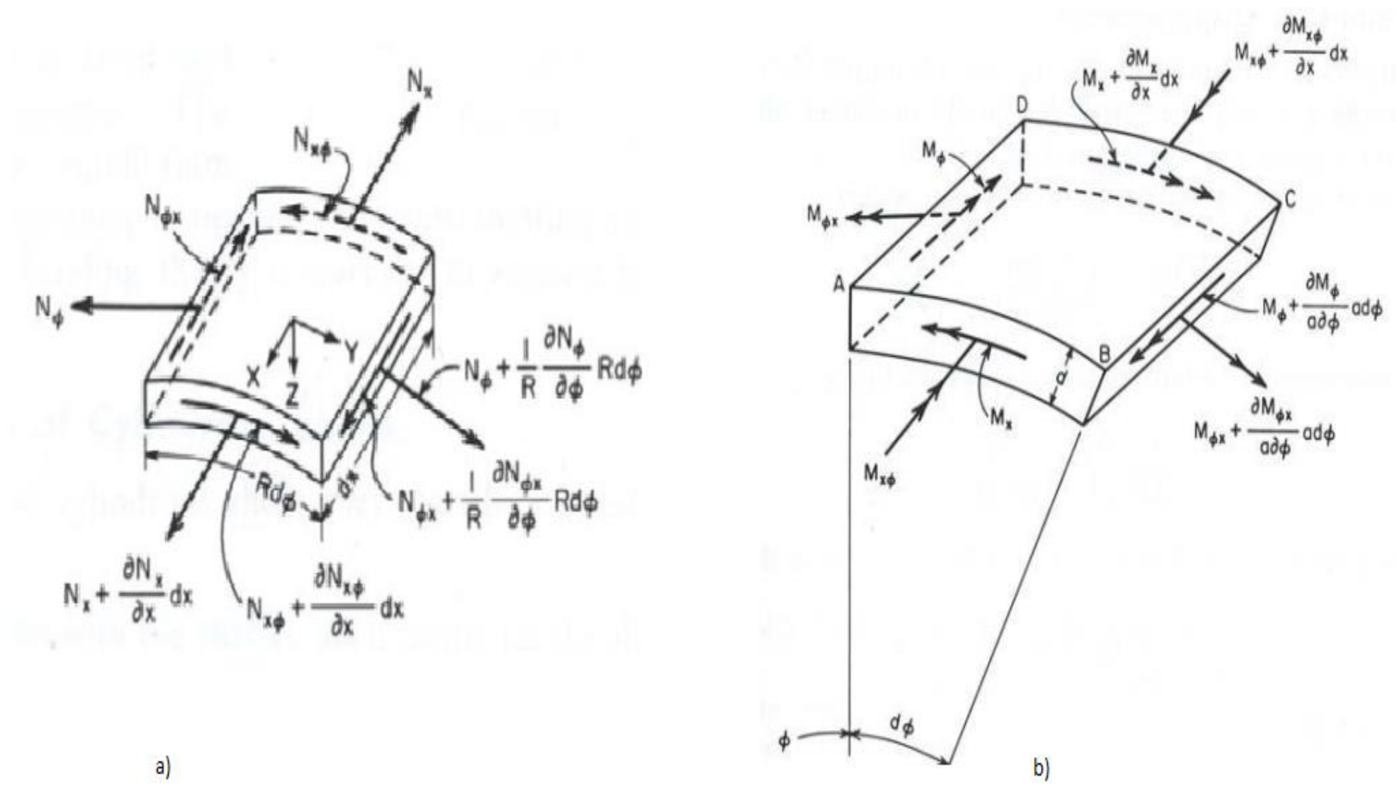


Figure 2.3 a) Stress in membrane theory and b) bending stress in bending theory [16]

2.5 Membrane theory

In membrane theory, the loads are considered to be carried only by in-plane direct stresses N*x*,N*φ*, and N*xφ* (Figure 2.3 and 2.4), which lay in the shell middle surface. This is quite reasonable because it has a small rigidity and is curved, not straight like slabs, which carry loads by bending moments. Membrane state of stress, also called moment less state of stress, is simple in analysis and can be used without considering the bending moment effect for certain geometries of shells. [7] It is noted that in order for moment less stress state, i.e. in-plane direct stresses, to exist, the shell must be closed elliptical shells or extend to infinity (case of parabolic or catenary shells) so that the stresses can be self-balanced (with the load, of course). However in the case of simply-supported shell roofs, which have free edges not satisfying the condition specified above. At the free edges, the bending moments develop, which requires bending theory. [7]

2.5.1. Equations of equilibrium

The shell problem in the membrane theory is straightforward in that it has three unknowns, two normal stresses N_{θ}, N_{φ}, and transverse shear stress $N_{\theta\varphi}$ and three equations of equilibrium are enough for solving the problem. [16]

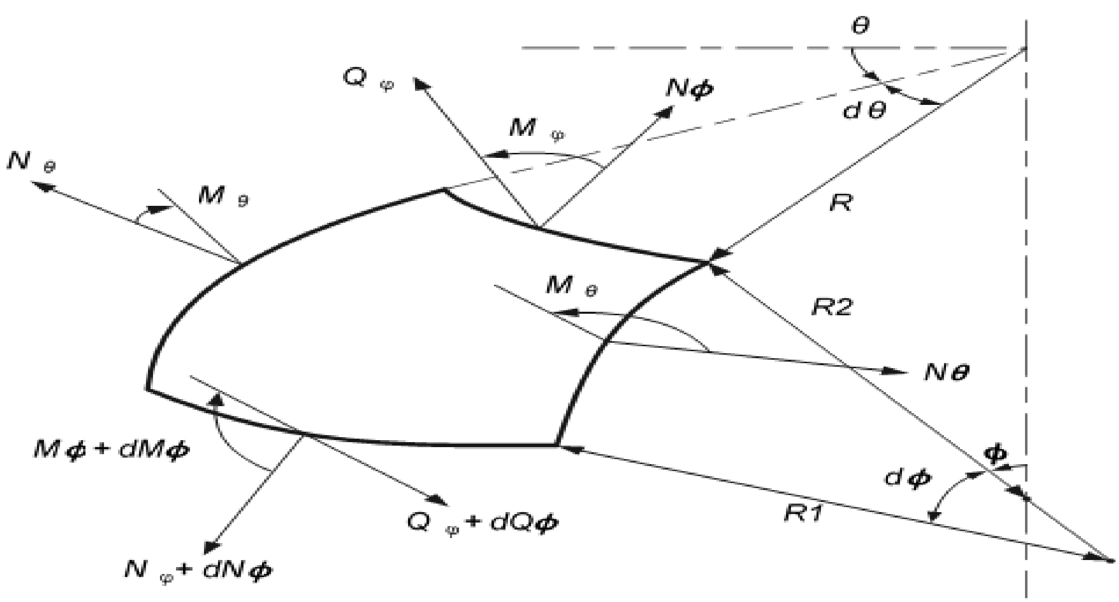


Figure 2.4. Free body diagram of a shell element [16]

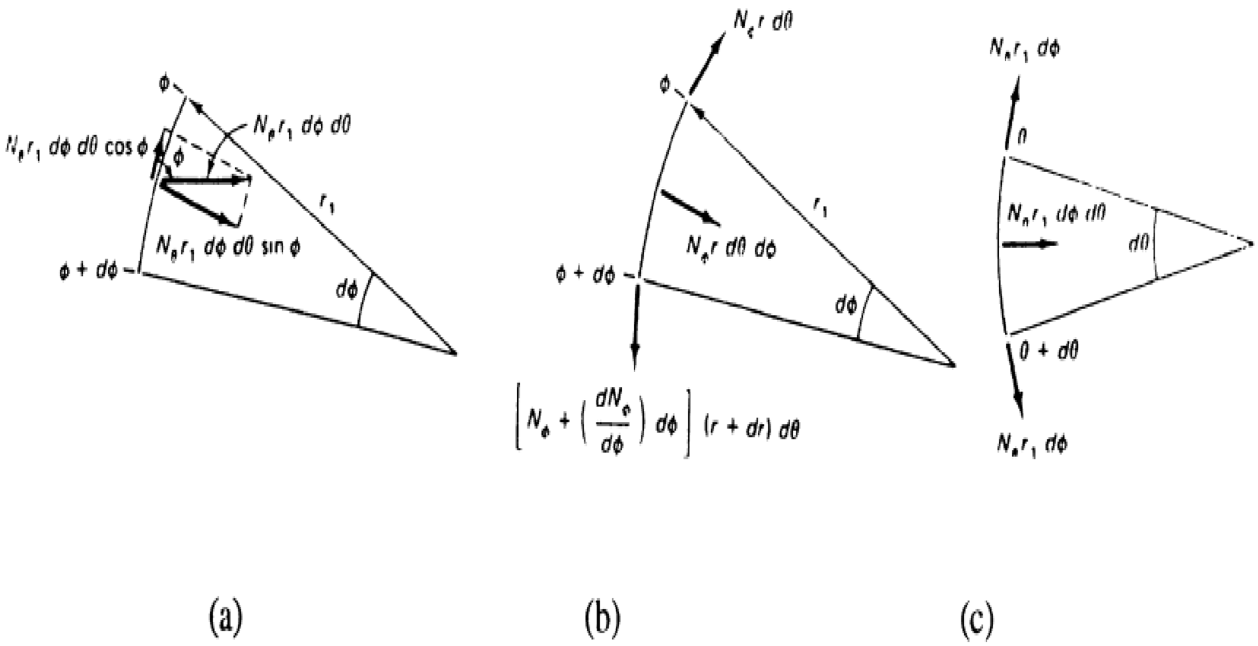


Figure 2.5 (a) Meridian and hoop sections through the shell of revolution, (b) meridian sections, (c) hoop section [16]

The equilibrium equation in the hoop direction from the figure above is

$$\frac{\partial N_{\theta\emptyset}}{\partial\theta} r_1 d\theta d\emptyset + \frac{\partial}{\partial\emptyset}(rN_{\emptyset}) d\theta d\emptyset - N_{\theta} r_1 d\theta d\emptyset cos\emptyset + P_{\emptyset} r r_1 d\theta d\emptyset = 0 \quad (2.1)$$

Dividing both sides by $d\theta d\emptyset$

$$\frac{\partial N_{\theta\emptyset}}{\partial\theta} r_1 + \frac{\partial}{\partial\emptyset}(rN_{\emptyset}) - N_{\theta} r_1 cos\emptyset + P_{\emptyset} r r_1 = 0 \quad (2.2)$$

Doing the same for the meridian direction we get

$$\frac{\partial N_{\theta}}{\partial\theta} r_1 + \frac{\partial}{\partial\emptyset}(rN_{\theta\emptyset}) + N_{\theta\emptyset} r_1 cos\emptyset + P_{\theta} r r_1 = 0 \quad (2.3)$$

The third equilibrium equation comes from projecting all the forces in the direction normal to the shell.

$$N_{\theta} r_1 sin\emptyset + rN_{\emptyset} - P_r r r_1 = 0 \quad (2.4)$$

Dividing by (rr_1) gives,

$$\frac{N_\theta}{r_2} + \frac{N_\emptyset}{r_1} = P_r \ldots\ldots (2.5)$$

By taking N_θ from (5) to (3) $N_\emptyset$ satisfies the linear differential equation

$$\frac{\partial}{\partial\emptyset}(rN_\emptyset) + N_\emptyset r cot\emptyset = r_1 r_2 (P_r cos\emptyset - P_\emptyset sin\emptyset) \ldots\ldots (2.6)$$

Solution:

$$N_\emptyset = \frac{1}{r_2 sin^2\emptyset}\left[\int_{\emptyset_0}^{\emptyset} r_1 r_2 \,(P_r cos\emptyset - P_\emptyset sin\emptyset) sin\emptyset d\emptyset + c\right] \ldots\ldots (2.7)$$

For our case, an elliptical cap r_1, r_2. Equation (5) and (7) become

$$r1 * N_\theta + r2 * N_\emptyset = r2 * r1 P_r \ldots\ldots (2.8)$$

$$N_\emptyset = \frac{r1}{sin^2\emptyset}\left[\int_{\emptyset_0}^{\emptyset} (P_r cos\emptyset - P_\emptyset sin\emptyset) sin\emptyset d\emptyset\right] \ldots\ldots (2.9)$$

For distributed load w per unit area of surface:

$$P_r = -g cos\emptyset \ldots\ldots (2.10)$$

$$P_\emptyset = g sin\emptyset \ldots\ldots (2.11)$$

2.6 Bending theory

The membrane theory cannot be solely used to solve the simply-supported shell roof problem. This is because the theory shows that the stresses Nφ and Nxφ exist everywhere in the shell whereas the free edges without edge beams dictate that the stresses must be null (free + no edge beams). Therefore, it is necessary to introduce fictive stresses, which are usually referred to as corrective line load, along edge to realize these particular boundary conditions (Fig. 3.1). The fictive stresses are the value from the membrane theory with the angle φ at the edge. However, this introduction will cause the shell to bend [1], which is not the scope of the membrane theory. Therefore, it is necessary to introduce the bending theory under the effect of the fictive stresses.

The result of the two theories, i.e. the membrane theory under actual loads, referred to as loaded membrane theory in this report, and the bending theory under fictive stresses only referred to as unloaded bending theory, is superimposed to get the final result of the shell roof problem. It should be noted immediately that the fictive stresses in the bending theory are lately treated as a boundary condition rather than loads. [1]

2.6.1 General bending theory

Instead of considering the loads to be carried by the three normal stresses Nθ, Nφ, and N$\theta\varphi$as in the membrane theory, the bending theory introduces the shear stresses Qθ ,Qφ , and three couple stresses Mθ , Mφ , Mθφ [1]

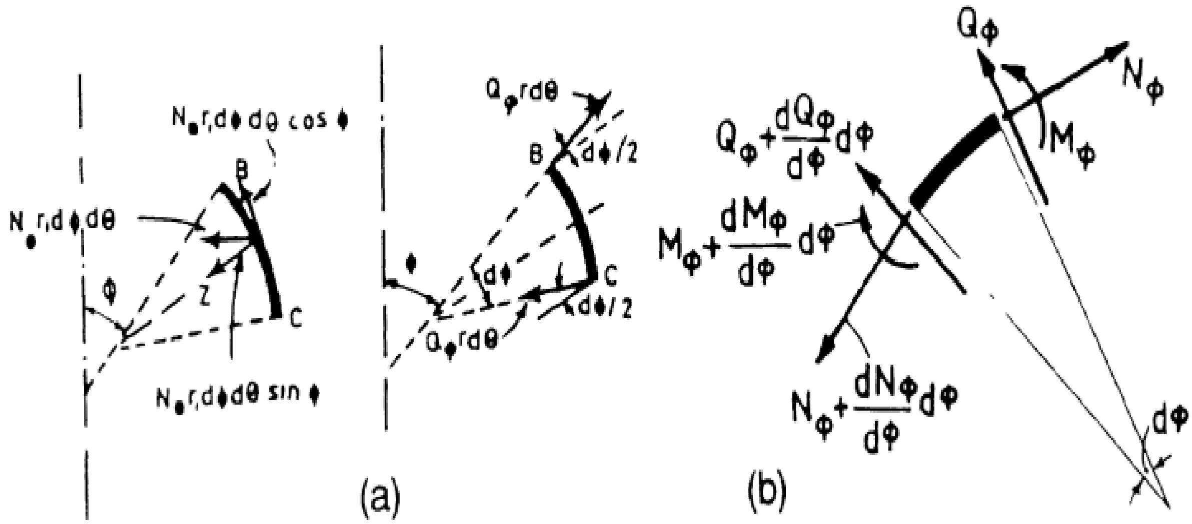


Figure 2.6 Meridian section through a shell element showing the internal forces and their projections, (a) membrane forces, (b) bending shear force [16]

2.6.2 Equations of equilibrium

The force equilibrium equation:[16]

$$\frac{\partial}{\partial\emptyset}(rN_{\emptyset}) - N_{\theta}r_1 cos\emptyset - rQ_{\emptyset} + P_{\emptyset}rr_1 = 0 \ldots\ldots (2.12)$$

$$rN_{\emptyset} + N_{\theta}r_1 sin\emptyset + \frac{\partial}{\partial\emptyset}(rQ_{\emptyset}) - P_r rr_1 = 0 \ldots\ldots (2.13)$$

The moment equation of equilibrium about θ axis is

$$\left[M_\emptyset + \frac{\partial(M_\emptyset)}{\partial\emptyset} d\emptyset\right]\left(r + \frac{\partial r}{\partial\emptyset} d\emptyset\right) d\theta - M_\emptyset r d\theta - r_1 M_\theta cos\theta d\emptyset d\theta - r_1 Q_\emptyset sin\emptyset r_2 d\emptyset d\theta = 0 \quad (2.14)$$

Simplified to:

$$\frac{\partial(M_\emptyset r)}{\partial\emptyset} - r_1 M_\theta cos\emptyset - Q_\emptyset r_1 r = 0 \quad (2.15)$$

Equations (12), (13), and (14) constitute three relations among six unknown force quantities. This means that the shell is statically indeterminate and three more relations are needed to find the internal forces. These additional relations are provided by the kinematic and constitutive equations.

2.7 Kinematic Relations

Kinematic relations relate the strains and the change of curvature with the components and gradients of the displacement vector. In our axisymmetric case, the displacement components are the displacement normal to the shell surface directed outward, w, and the displacement component along the meridian, v. These quantities are in general, functions of meridian parameter. Due to axial symmetry, the hoop displacement is identically zero. [1]

$$\varepsilon_\emptyset = \frac{dv}{r_1 d\emptyset} - \frac{w}{r_1} \quad (2.16)$$

$$\varepsilon_\theta = \frac{v}{r_2} cot\emptyset - \frac{w}{r_2} \quad (2.17)$$

The strain-displacement relation at any point o at a distance z from the middle surface, by replacing v, w,r1 and r2 with v_o,w_o,a+z and a+z

$$\varepsilon_\emptyset = \frac{dv_o}{(r_1+z)d\emptyset} + \frac{w_o}{r_1+z} \quad (2.18)$$

$$\varepsilon_\theta = \frac{v_o}{(r_2+z)} cot\emptyset + \frac{w_o}{(r_2+z)} \quad (2.19)$$

A simple kinematic relation can be established between v and w and $v_o$and w_o.

$$v_o = \left(\frac{r_1+z}{r_1}\right) v - \frac{z}{r_1}\frac{dw}{d\emptyset} \quad (2.20)$$

w_o=w……………………………………………………………………………….. (2.21)

Combining equations (18), (19), (20) and (21)

$$\varepsilon_{\emptyset} = \frac{dv}{r_1 d\emptyset} - \frac{z}{r_1(r_1+z)} + \frac{w}{r_1+z}$$ ………………………………………………………… (2.22)

$$\varepsilon_{\theta} = \frac{r_2+z}{r_2(r_2 r2+z)} v cot\emptyset - \frac{z}{r_2(r_2+z)} \frac{dw}{d\emptyset} cot\emptyset + \frac{w}{r_2+z}$$ ……………………………………….. (2.23)

2.7.1. Stress-strain relation

If $\sigma_{\emptyset}$ and σ_{θ} are the normal stresses, then using Hooke's law, the following stress-strain relationship can be written:

$$\sigma_{\emptyset} = \frac{E}{(1-v^2)}(\varepsilon_{\emptyset} + v\varepsilon_{\theta})$$ …………………………………………………………….…... (2.24)

$$\sigma_{\theta} = \frac{E}{(1-v^2)}(\varepsilon_{\theta} + v\varepsilon_{\emptyset})$$ ……………………………………………………………….... (2.25)

To find the stress over the thickness of the shell we integrate the stresses and replace the strains with equations (21) and (22) we find:

$$N_{\emptyset} = \frac{Et}{1-v^2}\left(\frac{1}{r_1}\left(\frac{dv}{d\emptyset} + w\right) + \frac{v}{r_1}(vcot\emptyset + w)\right)$$ …………………………………………..… (2.26)

$$N_{\emptyset} = \frac{Et}{1-v^2}\left(\frac{1}{r2}(vcot\emptyset + w) + \frac{v}{r2}\left(\frac{dv}{d\emptyset} + w\right)\right)$$ …………………….. (2.27)

$$M_{\emptyset} = -\frac{Et^3}{12(1-v^2)}\left(\frac{1}{r1}\left(\frac{d(\frac{v}{r1}+\frac{dw}{ad\emptyset})}{d\emptyset}\right) + \frac{v}{r1}(\frac{v}{r1} + \frac{dw}{r1d\emptyset})cot\emptyset\right)$$ ………………………………..……..(2.28)

$$M_{\theta} = \left(\left(\frac{v}{r_2} + \frac{dw}{r_2 d\emptyset}\right)\frac{cot\emptyset}{r_2} + \frac{v}{r_2}\left(\frac{d(\frac{v}{r2}+\frac{dw}{r2d\emptyset})}{d\emptyset}\right)\right)$$ ……………………………………………….... (2.29)

It is to be noted that in deriving these combined constitutive-kinematic relations, we have also used the kinematic relations for axisymmetric shell behavior.[16]

We may write these relations symbolically as:

$$N_{\emptyset} = D(\varepsilon_{\emptyset} + v\varepsilon_{\theta}) \quad (2.30)$$

$$N_{\theta} = D(\varepsilon_{\theta} + v\varepsilon_{\emptyset}) \quad (2.31)$$

$$M_{\theta} = -K(\mathrm{X}_{\theta} + v\mathrm{X}_{\emptyset}) \quad (2.32)$$

$$M_{\emptyset} = -K(\mathrm{X}_{\emptyset} + v\mathrm{X}_{\theta}) \quad (2.33)$$

Where

$$D = \frac{Et}{1-v^2} \quad (2.34)$$

$$K = -\frac{Et^3}{12(1-v^2)} \quad (2.35)$$

From the physical point of view, D signifies the membrane stiffness and K designates the bending stiffness of the shell.

If we substitute the constitutive-kinematic relations into the equilibrium equations we obtain three simultaneous differential equations for three unknown functions v, w and $Q_{\emptyset}$. Eliminating $Q_{\emptyset}$ the result would be two simultaneous differential equations for the two displacement components, v and w. introducing more convenient variables for the rotation of tangent to meridional curve and for the shear force:-

$$V^* = \frac{1}{r_2}\left(v + \frac{dw}{d\emptyset}\right) \quad (2.36)$$

$$U^* = r_2 Q_{\emptyset} \quad (2.37)$$

Re-writing the membrane forces in terms of new variables, U*and V* and differentiating to find the governing equations we get one of the two desired relations between U* and V*. To obtain another governing equation, we substitute for $M_{\emptyset}$ and M_{θ} from constitutive relations into equilibrium equations we reach at an expression of axisymmetric shells as two relations in terms of new variables U* and V* for a varying thickness dt/dΦ and higher order differential equations. [2]

2.8 Geckeler's Approximation

Geckler's approximation is an approximate bending analysis of axisymmetric shells. This analysis is valid for an elliptical shell of radius about the results can also be used, as an approximation, for other shell axisymmetric shells. In Geckeler's approximation only the functions themselves and their derivatives (gradients) of higher order are retained and the lower order derivatives are discarded. This approximation is based on the physical argument that the boundary effects are localized and thus the higher order gradients are of greatest importance. [2]

A unit moment and a unit shear are applied at the junction of roof and wall as shown below.

To find bending field of action use flexibility method. Let X_1 is horizontal force (Shear) and

$X_2$is moment.

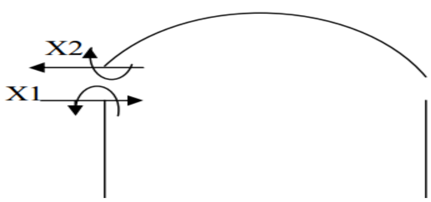


Figure 2.7 The flexibility Coefficient

With the Eq.(2.38) can be calculate meridian and hoop forces and moments.

$$Q\phi = e^{\lambda\phi}(C1\cos\lambda\phi + C2\sin\lambda\phi) + e^{-\lambda\phi}(C3\cos\lambda\phi + C4\sin\lambda\phi) \quad \text{.................. (2.38)}$$

where C_1, C_2, C_3 and C_4 are constants of integration to be evaluated from the boundary conditions of the shell.. Furthermore, let us assume that any adjacent edges of the shell are sufficiently far apart to allow decoupling of edge effects. This permits us to retain only two of the constants appearing in Eq. (2.38), and to rewrite the solution as

$$Q\phi = Ce^{-\lambda\psi}\sin(\lambda\psi + \beta) \quad \text{.. (2.39)}$$

where ψ is now the meridian angle measured from the normal at the edge of the shell to the normal at the point in question; C and β are new constants of integration. For the shell edge located in the equatorial plane, the relationship between ϕ and ψ is simply where the first equation refers to the outer edge of the elliptic shell, and the second refers to the inner edge.

Equilibrium considerations of the shell element yield

$$N\phi = -Q\phi \cot\phi \approx 0 \quad \ldots\ldots (2.40)$$

since cot $\phi \approx 0$ in the vicinity of the elliptical shell edges ($\phi = \pi/2$; $3\pi/2$). For the hoop stress resultant, we obtain, $N\phi \approx 0$

If cot ϕ *is not zero*

$$N\phi \approx C * Cot(\emptyset)e^{-\lambda\varphi}Sin(\lambda^{\varphi}) \ldots\ldots (2.41)$$

$$N\theta \approx C\lambda * \left(\frac{r2}{r1}\right) * e^{-\lambda\varphi}(cos(\lambda^{\varphi} + \beta) - Sin(\lambda^{\varphi} + \beta)) \ldots\ldots (2.42)$$

$$M\phi \approx \frac{2\lambda^3 D}{Etr1} * \left(\frac{r2}{r1}\right) * Ce^{-\lambda\varphi}(cos(\lambda^{\varphi} + \beta) - Sin(\lambda^{\varphi} + \beta)) \ldots\ldots (2.43)$$

$$M\theta \approx \boldsymbol{\mu}\mathrm{M}\emptyset \ldots\ldots (2.44)$$

The material properties E (Young's modulus of elasticity) and ν (Poisson's ratio) are assumed to be constant. The parameter t denotes shell thickness, while D is the flexural rigidity of the shell, given by

$$D = \frac{Et^3}{12(1-\mu^2)} \ldots\ldots (2.45)$$

For the elliptical shell, the principal radii of curvature r_1 and r_2 are functions of ϕ (as given by Eqs. (2.39) and (2.43), so the slenderness parameter λ is also a function of ϕ,[16]

Where the parameter:

$$\lambda = \sqrt[4]{3(1-\mu^2)(\frac{r1^2}{r2*t})^2} \ldots\ldots (2.46)$$

The edges of the elliptical shell subjected to uniformly distributed bending moments and horizontal shearing forces, where $\{M_e, H_e\}$ are the axisymmetric actions applied at the edge. In the formulation, we will therefore denote the applied edge bending moments and edge shearing forces simply by M_e and H_e, with ψ denoting the meridian angle from the edge.

When M_e only is applied at a given edge (in the absence of H_e), we want to choose the constants C and β such that the following boundary conditions are satisfied [16]:

$(M\phi)\psi=0 = Me$ *and* $(Q\phi)\psi=0 = 0$ ………………………………………………………… (2.47)

Applying these conditions to Eqs. (2.39) and (2.43), we obtain

$$\beta = 0,\ C = \frac{Etr1}{2\lambda^3 * D}\left(\frac{r1^2}{r2^2}\right) Me \quad \ldots\ldots (2.48)$$

Substituting these values of β and C into Eqs. (2.41) to (2.43), we obtain the interior stress resultants, bending moments and deformations due to the edge bending moment M_e as follows:

$$N\phi = -\frac{6r_1(1-\mu^2}{\lambda^3 t^2} * \left(\frac{r_1{}^2}{r_2{}^2}\right) * Cot(\emptyset)e^{-\lambda\varphi}Sin(\lambda^{\varphi})Me \quad \ldots\ldots (2.49)$$

$$N\theta = \frac{6r_1(1-\mu^2}{\lambda^2 t^2} * \left(\frac{r_1}{r_2}\right) * e^{-\lambda\varphi}(cos(\lambda^{\varphi}) - Sin(\lambda^{\varphi}))Me \quad \ldots\ldots (2.50)$$

$$M\phi = e^{-\lambda\varphi}(cos(\lambda^{\varphi}) + Sin(\lambda^{\varphi}))Me \quad \ldots\ldots (2.51)$$

$$M\theta = \mu M\theta \quad \ldots\ldots (2.52)$$

At the edge ($\psi = 0$), we have

When H_e only is applied at a given edge (in the absence of M_e), we want to choose the constants C and β such that the following boundary conditions are satisfied:

$(M_\phi)_{\psi=0} = 0$

$(Q_\phi)_{\psi=0} = H_e$

Applying the conditions to Eqs. (2.39) and (2.43), we obtain

$\beta = -\frac{\pi}{4}$ ()

$C = -\sqrt{2}H_e$

Substitution of these values of β and C into Eqs. (2.49) to(2.52) gives the interior stress resultants, bending moments and deformations due to the edge shear H_e as follows:

$$N\phi = -\sqrt{2}Cot(\emptyset)e^{-\lambda\varphi}Sin(\lambda\varphi)He \quad \ldots\ldots (2.53)$$

$N\theta = -2\lambda(\frac{r_2}{r_1})e^{-\lambda\varphi}cos(\lambda\varphi)He$ ………………………………………………………… (2.54)

$M\phi = -\lambda^3 t^2 \times \left(\frac{r_2^2}{3r_1^3}\right) \times e^{-\lambda\varphi}Sin(\lambda^{\varphi})He$ ……………………………………………….. (2.55)

Table 2.1 Geckler's approximation of NΦ, Nθ and MΦ [16]

		
$N_{\emptyset}$	$-\sqrt{2}Cot(\emptyset) \times e^{-\lambda\varphi} \times Sin(\lambda\varphi)H$	$-\frac{6r_1(1-\mu^2)}{\lambda^3 t^2} \times \left(\frac{r_1^2}{r_2^2}\right) \times Cot(\emptyset)e^{-\lambda\varphi} \times Sin(\lambda^{\varphi})M_{\emptyset}$
N_{θ}	$-2\lambda(\frac{r_2}{r_1}) \times e^{-\lambda\varphi} \times cos(\lambda\varphi)H$	$\frac{6r_1(1-\mu^2)}{\lambda^2 t^2} \times \left(\frac{r_1}{r_2}\right) \times e^{-\lambda\varphi}(cos(\lambda^{\varphi}) - Sin(\lambda^{\varphi}))M_{\emptyset}$
$M_{\emptyset}$	$-\lambda^3 t^2 \times \left(\frac{r_2^2}{3r_1^3}\right) \times e^{-\lambda\varphi} \times Sin(\lambda^{\varphi})H$	$e^{-\lambda\varphi} \times (cos(\lambda^{\varphi}) + Sin(\lambda^{\varphi}))$

This table is used by adding shear and moment effect for bending analysis to get the influences of forces and moments on elliptical shell roof. By using the above table we can get meridian and hoop forces and meridian and hoop moments of elliptical shell.

CHAPTER THREE

PARAMETRIC STUDY OF ELLIPTICAL SHELL ROOF

3.1 General

In this chapter, parametric studies of different cases are presented based on the theoretical background developed from previous chapters; to show the influence of aspect ratio on structural forces and moments of elliptical shell roof.

3.2 Modeling of Ellipsoid shell roof

Parametric studies are carried out for seven different cases, which are believed to verify the influence of aspect ratio on structural force and moments of elliptical shell roof. The parameters are major radius, miner radius and height of elliptical shell. In each parametric study, aspect ratio is:

Aspect =1 Aspect =1.5 Aspect =2

Aspect =2.5 Aspect =3 Aspect =3.5 and

Aspect =4 with height to minor radius ratio, h/b 0.25 up to 1 responses of structural forces and moments of ellipse.

The aspect ratio is selected due to know the influences of parameters

The material is selected on reinforced concrete

C-25

Steel, s-400

Passion ratio of concrete 0.25

Loading

Load cases including static and dynamic forces caused by dead load and wind in Euro code 3 prescribed combinations were analyzed.

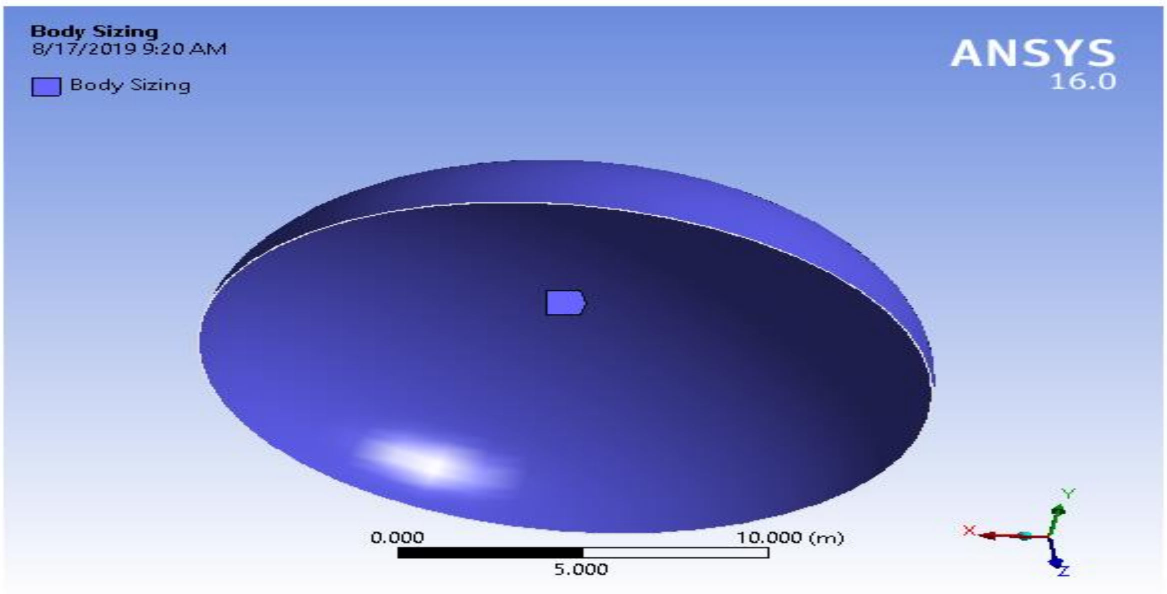


Figure 3.1 Model of ellipsoid shell roof

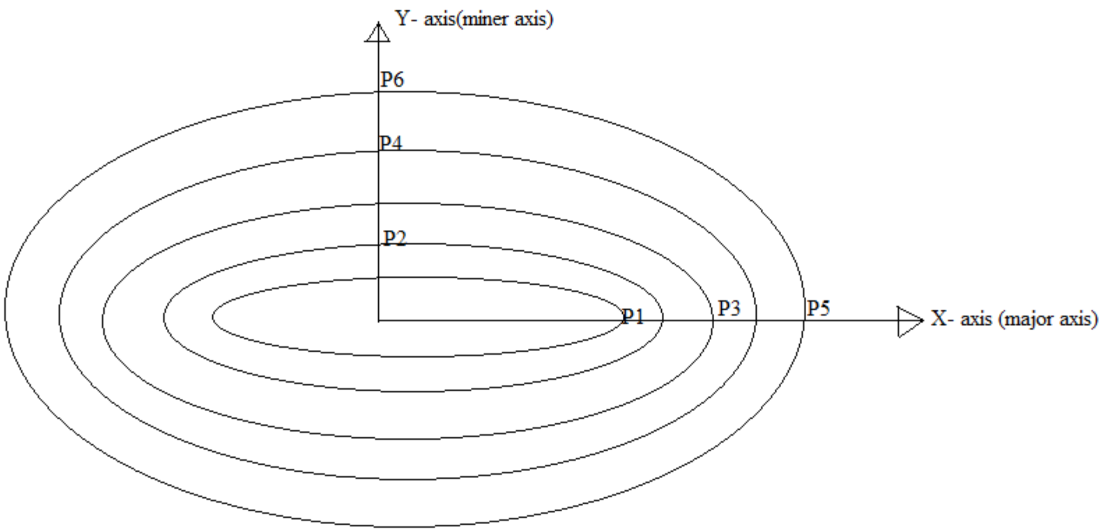


Figure 3.2 plane view of ellipsoid shell roof

3.4 Ellipse loading

The ellipse is subjected to wind load and self-weight.

Self-weight is calculated as:

g =γ_c × t ………………………………………………………………………………... (3.1)

where, $\gamma_c = 24\frac{kN}{m^3}$ (unit weight of concrete)

Wind load analysis of ellipse is the same as calculated as dome shape shell roof.

t =100mm (thickness of ellipse)

g= factored load

DL=1.35× $24\frac{kN}{m^3}$ ×0.1m =3.12 kN/m2

$g = 1.35Dl + 1.5Ll$--- (3.2)

$$g = 3.12 + 1.5 \times 0.25 = \frac{3.52kN}{m^2}$$

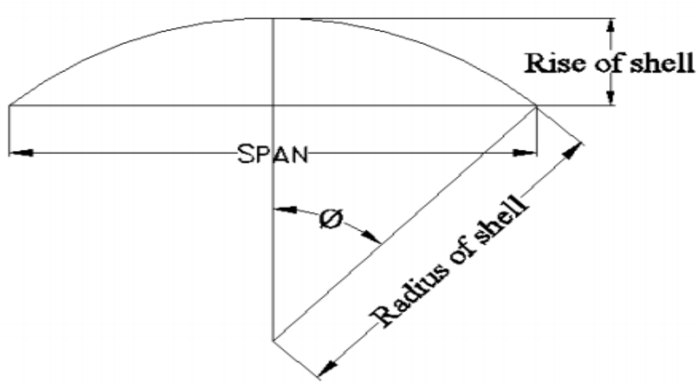


Figure 3.3 Elevation view of shell model

3.5 Design steps

The structural design involves the following activities:

1. Choice of a shell geometry which meets the overall architectural requirements. Our shell design models have a/b=1, a/b=1.5, a/b=2, a/b=2.5, a/b=3, a/b=3.5 and a/b=4 with height to minor radius h/b=1, h/b=0.75, h/b=0.5 and h/b=0.25.

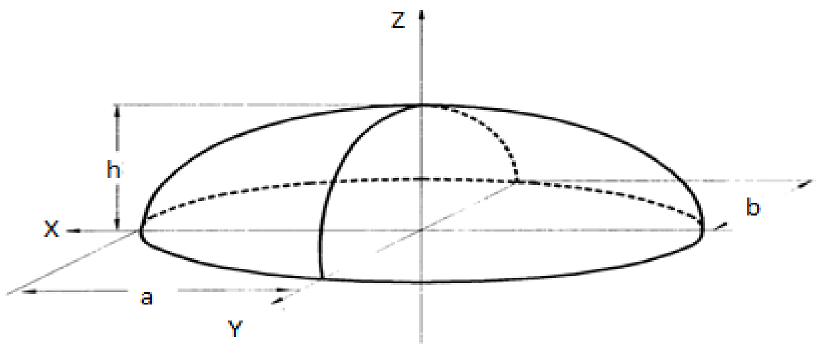


Figure 3.4 Elliptical shell

2. A choice of dimensions of the shell and its supporting members
3. A preliminary design and analysis of the shell. Membrane theory

3.6 Analysis

Analyses are carried out using ANSYS software and mathematical calculations which is discussed in section 2.5 to 2.8. A total of 28 model of elliptical shell roof structures are analyzed having a variable of major radius and height of elliptical shell roof. Typical 28 elliptical shell roof structures are selected and discussed below. Results for each case are presented below in tables and graphical format according to its category. The influence of aspect ratio and height to miner radius ratio both are analysis by using mathematical and software.

3.6.1 Membrane Analysis

All bending stresses in the shell are neglected in membrane analysis and the membrane stress resultants are calculated. All membrane stresses act in the plane of the shell. The membrane theory can sometimes provide a reasonable basis for design, but only if these conditions are satisfied [17]:

a) The displacements due to membrane stress resultants do not give rise to appreciable bending.
b) The loading is distributed smoothly over the surface of the shell.
c) The boundaries can supply the forces and permit the displacements required by the membrane stress resultants.

From equilibrium equation it is find the in finite element from the shell.

$N_\emptyset = \frac{1}{r_2 sin^2\emptyset}\left[\int_{\emptyset_0}^{\emptyset} r_1 r_2 (P_n Cos\emptyset - P_\emptyset Sin\emptyset)\, Sin\emptyset d\emptyset + C\right]$ …………………………………..….. (3.2)

$N_\theta = P_n r_2 - \frac{N_\emptyset r_2}{r_1}$ ……………………………………………………………….……. (3.3)

$N_\emptyset = normal\ internal\ stress\ in\ merdional\ direction\ per\ unit\ width.$

$N_\theta = normal\ internal\ stress\ in\ Circumferencial\ direction\ per\ unit\ width.$

$P_n \& P_\emptyset = external\ load\ components$

$\emptyset_o = 0$ because there is no opening at the top.

Since the roof shell that we are designing is for its own weight and for variable loads,

$P_n \& P_\emptyset\ will\ be\ the\ component\ of\ its\ weight$

$P_n = -g\ Cos\emptyset$ Where ***g*** is factored weight per unit area

$P_\emptyset = +g\ Sin\emptyset$

Substituting P_n , $P_\emptyset$, $r_1\ and\ r_2$

$$N_\emptyset = \frac{r_1}{Sin^2\emptyset}\left[\int_0^{\emptyset} -g(Cos^2\emptyset + Sin^2\emptyset)\, Sin\emptyset d\emptyset + C\right]$$

$$= \frac{r_1}{Sin^2\emptyset}\left[\int_0^{\emptyset} -g\, Sin\emptyset d\emptyset + C\right]$$

$$= -\frac{r_1}{Sin^2\emptyset} g(Cos\emptyset - 1)$$

Substituting $Sin^2\emptyset = 1 - Cos^2\emptyset = (1 - Cos\emptyset)(1 + Cos\emptyset)$

$N_\emptyset = -\frac{r_1 g}{1 + Cos\emptyset}$ ………………………………………………………………………. (3.4)

$N_\theta = -gr_2 cos\phi - N_\emptyset * \frac{r_2}{r_1}$ ………………………………………………………………...… (3.5)

$N_\theta \ = -gr_2 \left[Cos\,\emptyset - \frac{1}{1+Cos\emptyset}\right]$ …………………………………………………….………..(3.6)

$r_1 = \frac{a^2 b^2 c^2}{\sin^2\phi * a^2 * b^2 + c^2(b^2 * \cos^2\theta + a^2 \sin^2\theta)} * \frac{1}{\sqrt{(\sin^2\phi(a^2\cos^2\theta + b^2\sin^2\theta) + c^2\cos^2\phi)}}$ [15]……......... (3.7)

$r_2 = \frac{a^2 b^2}{\sqrt{(\sin^2\phi(a^2\cos^2\theta + b^2\sin^2\theta) + c^2\cos^2\phi)}} * \frac{1}{(a^2\cos^2\theta + b^2\sin^2\theta)}$ [15] …………………………..… (3.8)

Case 1- aspect ratio a/b=1, with h/b=1

For the first model a/b=1 and h/b=1 in this case the ellipse is dome shape so it should be use dome shape equations. Take a=b=h=10m, so r1=r2=10m

Table 3.1 structural forces along major x- axis for aspect ratio a/b=1 with h/b=1

structural forces along major x- axis for aspect ratio a/b=1 with h/b=1 using eq.(3.4 and 3.6)													
angle	Ø	θ	SinØ	Sinθ	cosØ				cosθ	r1	r2	NØ	Nθ
0	0	0	0	0	1	a	B	c	1	10	10	-6.492	1.089
5	0.087	0	0.09	0	1	10	10	10	1	10	10	-6.479	1.044
10	0.175	0	0.17	0	0.98				1	10	10	-6.439	0.909
15	0.262	0	0.26	0	0.97				1	10	10	-6.375	0.688
20	0.349	0	0.34	0	0.94				1	10	10	-6.289	0.389
25	0.436	0	0.42	0	0.91				1	10	10	-6.186	0.019
30	0.524	0	0.5	0	0.87				1	10	10	-6.068	-0.41
33.69	0.588	0	0.55	0	0.83				1	10	10	-5.976	-0.76

Table 3.2 structural forces along major x- axis for aspect ratio a/b=1 with h/b=0.75

Structural forces along major x- axis for aspect ratio a/b=1 with h/b=0.75 using eq.(3.4 and 3.6)													
angle	Ø	θ	SinØ	Sinθ	cosØ				cosθ	r1	r2	NØ	Nθ
0	0	0	0	0	1	a	b	c	1	13.33	13.33	-8.656	1.4522
5	0.087	0	0.09	0	1	10	10	7.5	1	13.22	13.29	-8.562	1.3876
10	0.175	0	0.17	0	0.98				1	12.88	13.18	-8.292	1.1978
15	0.262	0	0.26	0	0.97				1	12.36	13.00	-7.876	0.8948
20	0.349	0	0.34	0	0.94				1	11.70	12.77	-7.359	0.4963
25	0.436	0	0.42	0	0.91				1	10.97	12.49	-6.786	0.0238
30	0.524	0	0.5	0	0.87				1	10.21	12.20	-6.198	-0.5003
33.69	0.588	0	0.55	0	0.83				1	9.66	11.98	-5.775	-0.9074

Table 3.3structural forces along major x- axis for aspect ratio a/b=1 with h/b=0.5

structural forces along major x- axis for aspect ratio a/b=1 with h/b=0.5 using eq.(3.4 and 3.6)													
angle	Ø	θ	SinØ	sinθ	cosØ				Cosθ	r1	r2	NØ	Nθ
0	0	0	0	0	1	a	b	c	1	20	20	-12.98	2.178
5	0.087	0	0.09	0	1	10	10	5	1	19.3	19.78	-12.53	2.064
10	0.175	0	0.17	0	0.98				1	17.6	19.15	-11.31	1.741
15	0.262	0	0.26	0	0.97				1	15.2	18.25	-9.687	1.256
20	0.349	0	0.34	0	0.94				1	12.7	17.21	-8.01	0.669
25	0.436	0	0.42	0	0.91				1	10.5	16.14	-6.50	0.031
30	0.524	0	0.5	0	0.87				1	8.64	15.12	-5.24	-0.620
33.69	0.588	0	0.55	0	0.83				1	7.5	14.42	-4.48	-1.093

Table 3.4 structural forces along major x- axis for aspect ratio a/b=1 with h/b=0.25

plane stress along major x- axis for aspect ratio a/b=1 with h/b=0.25 using eq.(3.4 and 3.6)													
angle	Ø	θ	SinØ	Sinθ	CosØ				Cosθ	r1	r2	NØ	Nθ
0	0	0	0	0	1	a	b	c	1	40	40	-25.969	4.357
5	0.09	0	0.09	0	1	10	10	2.5	1	34	37.9	-22.042	3.956
10	0.17	0	0.17	0	0.98				1	22.9	33.19	-14.716	3.017
15	0.26	0	0.26	0	0.97				1	14.1	28.25	-8.983	1.945
20	0.35	0	0.34	0	0.94				1	8.75	24.1	-5.502	0.937
25	0.44	0	0.42	0	0.91				1	5.67	20.85	-3.506	0.040
30	0.52	0	0.5	0	0.87				1	3.86	18.35	-2.345	-0.753
33.69	0.59	0	0.55	0	0.83				1	3.01	16.88	-1.796	-1.279

3.6.2 Bending Analysis

The more general and rigorous approach for solving shell problems is called bending theory. It considers the effects of the longitudinal moment and transverse shear (in the direction of the axis of the shell).

Both the membrane theory and the bending theory are very important and widely applicable to solving various kinds of thin shell structures. The edge-load solutions (bending solutions), together with the membrane solution, will yield the complete solution of a shell problem.

For solving shell problems, therefore, the general procedure will be to obtain membrane forces under a given loading, and then superimpose on these the bending theory solutions for edge loads. The edge loads are determined so that the membrane and edge load solutions together satisfy the boundary conditions; i.e., the edge loads are obtained by solving equations of compatibility at the boundaries.

The bending theory of general shells can be simplified to yield equations which can be solved numerically.

3.7 Parametric studies of cases and discussions

The parametric study is carried out by assuming a typical elliptical shell roof given in Fig.3.2. In Fig 3.2, a is the radius of major axis of elliptical shell roof, b is the radius of miner axis of the shell, h is the height of the shell, Φ is the semi central angle and θ is the angle between miner and major radius. The influence of various parameters, namely, major to miner radius ratio (a/b) and height to miner radius ratio (h/b) of an elliptical shell roof on the structural shell meridian forces ($N\Phi$) and hoop forces ($N\theta$) and meridian moments ($M\Phi$) and hoop moments ($M\theta$) developed on the surface of elliptical shell roofs have been studied. The parametric study is carried out using ANSYS software and numerical method. Analysis result and discussion of typical 28 structures of ellipse shell roof model that are categorized into seven parametric studies are presented hereafter.

Table 3.5 cases of parametric study

Cases	Parametric study
Case-1	Aspect ratio a/b=1 with h/b=1,h/b=0.75, h/b=0.5 and h/b=0.25
Case-2	Aspect ratio a/b=1.5 with h/b=1,h/b=0.75, h/b=0.5 and h/b=0.25
Case-3	Aspect ratio a/b=2 with h/b=1,h/b=0.75, h/b=0.5 and h/b=0.25
Case-4	Aspect ratio a/b=2.5 with h/b=1,h/b=0.75, h/b=0.5 and h/b=0.25
Case-5	Aspect ratio a/b=3 with h/b=1,h/b=0.75, h/b=0.5 and h/b=0.25
Case-6	Aspect ratio a/b=3.5 with h/b=1,h/b=0.75, h/b=0.5 and h/b=0.25
Case-7	Aspect ratio a/b=4 with h/b=1,h/b=0.75, h/b=0.5 and h/b=0.25

3.7.1 Case-1: parametric study of Aspect ratio a/b=1 with h/b=1

For the first model a/b=1 and h/b=1 in this case the ellipse is dome shape so it should be use dome shape equations. Take a=b=h=10m, so r1=r2=10m

Table 3.6 case-1: parametric study of Aspect ratio a/b=1 with h/b=1

Super impose Membrane and Bending analysis				
Angle Φ	NØ	Nθ	MØ	Mθ
0	-4.99125	1.08911	-1.419E-06	-3.55E-07
5	-6.71023	4.27624	0.69910	0.17477
10	-6.65972	-0.41834	0.59714	0.14928
15	-6.41163	0.89561	-0.18904	-0.04726
20	-6.27626	0.44231	-0.75861	-0.18965
25	-6.17821	-0.02458	-0.45893	-0.11473
30	-6.06778	-0.39779	0.36662	0.09165
33.69	-5.97654	-0.75462	0.75041	0.18760

This table is get from by using super imposing of membrane and bending theory equation.

In this case meridian forces at the edge is decrease from center of elliptical shell to the edge of shell but hoop forces are changes tensile forces from center of ellipse to increase compressive forces at the edge of ellipse.

Meridian moments at the edge and center of the ellipse are approximately to zero compressive force at the center and changes to compressive to tensile forces from p_2 to 6 at the edge.

Hoop moments at the center of the ellipse are approximately to zero compressive forces and increase tensile forces at the edge.

3.7.2 Case-2: parametric study of Aspect ratio a/b=1.5 with h/b=1

The 2nd model of elliptical shell for aspect ratio a/b=1.5 with height ratio h/b=1, take a=15m,b=10m and h=10m

Table 3.7 case-2: parametric study of Aspect ratio a/b=1.5 with h/b=1

Super impose membrane and bending analysis			
NØ	Nθ	MØ	Mθ
-13.6067	1.089114	-1.42E-06	-3.549E-07
-14.4855	0.040829	0.45441	0.11360
-13.6969	0.864339	-0.75846	-0.18961
-12.7134	0.673291	0.43539	0.10885
-11.5314	0.360808	0.56299	0.14075
-10.2870	0.016709	-0.26330	-0.06582
-9.0806	-0.35785	0.07418	0.01855
-7.9756	-0.74607	-0.46759	-0.11689
-7.0023	-1.13275	0.63252	0.15813
-6.1706	-1.50728	-0.17170	-0.04293

In this case meridian forces at the edge is decrease compressive force from center of elliptical shell to the edge of shell but hoop forces are changes tensile forces from center of ellipse to increase compressive forces at the edge of ellipse.

Meridian moments at the edge of the ellipse are approximately to zero compressive force for center and decreases compressive forces at the center.

Hoop moments at the edge and center of the ellipse are approximately to zero compressive force for edge and zero compressive at the center.

3.7.3 Case-3: parametric study of Aspect ratio a/b=2.0 with h/b=1

The 3rd model of elliptical shell for aspect ratio a/b=2 with height ratio h/b=1, take a=20m,b=10m and h=10m

Table 3.8 case-3: parametric study of Aspect ratio a/b=2 with h/b=1

Super impose membrane and bending analysis			
NØ	Nθ	MØ	Mθ
-25.2185	1.089114	-1.4E-06	-3.5E-07
-24.6632	0.948665	-0.67484	-0.16871
-21.3749	0.781196	0.085464	0.021366
-17.341	0.456247	-0.46474	-0.11619
-13.5659	0.080233	0.358401	0.0896
-10.4853	-0.31	0.527386	0.131846
-8.14358	-0.69239	0.622258	0.155564
-6.42261	-1.04789	-0.0395	-0.00988
-5.17654	-1.37381	-0.5986	-0.14965
-4.39181	-1.62394	-0.31505	-0.07876

In this case meridian forces at the edge is decrease compressive force from center of elliptical shell to the edge of shell but hoop forces are changes tensile forces from center of ellipse to increase compressive forces at the edge of ellipse.

Meridian moments at the edge of the ellipse are approximately to zero compressive force for center and decreases compressive forces at the center.

Hoop moments at the edge and center of the ellipse are approximately to zero compressive force for edge and zero compressive at the center.

3.7.4 Case-4: parametric study of Aspect ratio a/b=2.5 with h/b=1

The 4th model of elliptical shell for aspect ratio a/b=2.5 with height ratio h/b=1, take a=25m,b=10m and h=10m

Table 3.9 case-4: parametric study of Aspect ratio a/b=2.5 with h/b=1

Super impose membrane and bending analysis			
NØ	Nθ	MØ	Mθ
-39.9749	1.089114	-1.4E-06	-3.5E-07
-36.1079	0.962328	0.314061	0.078515
-26.7181	0.646338	0.723062	0.180765
-18.0885	0.247424	0.094267	0.023567
-12.0666	-0.15803	0.352494	0.088123
-8.24636	-0.53655	0.601239	0.15031
-5.86896	-0.87631	-0.03612	-0.00903
-4.37452	-1.1739	-0.54555	-0.13639
-3.41928	-1.4276	0.044452	0.011113
-3.11797	-1.52389	0.359504	0.089876

In this case meridian forces at the edge is decrease from center of elliptical shell to the edge of shell but hoop forces are changes tensile forces from center of ellipse to increase compressive forces at the edge of ellipse.

Meridian moments at the edge and center of the ellipse are approximately to zero tensile force for edge and zero compressive at the center.

Hoop moments at the edge and center of the ellipse are approximately to zero tensile force for edge and zero compressive at the center.

3.7.5 Case-5: parametric study of Aspect ratio a/b=3 with h/b=1

The 5th model of elliptical shell for aspect ratio a/b=3 with height ratio h/b=1, take a=30m,b=10m and h=10m

Table 10 case-5: parametric study of Aspect ratio a/b=3 with h/b=1

Super impose membrane and bending analysis			
NØ	Nθ	MØ	Mθ
-57.9293	1.089114	-1.4E-06	-3.5E-07
-49.1742	0.945741	-0.14768	-0.03692
-32.3244	0.609883	-0.66368	-0.16592
-19.5469	0.224844	-0.60411	-0.15103
-11.9848	-0.13962	-0.16077	-0.04019
-7.72672	-0.46495	0.559192	0.139798
-5.28498	-0.74936	-0.0334	-0.00835
-3.83338	-0.99478	-0.50221	-0.12555
-2.94037	-1.2022	0.040792	0.010198
-2.35136	-1.38136	0.460165	0.115041

In this case meridian forces at the edge is decrease from center of elliptical shell to the edge of shell but hoop forces are changes tensile forces from center of ellipse to increase compressive forces at the edge of ellipse.

Meridian moments at the edge and center of the ellipse are approximately to zero tensile force for edge and zero compressive at the center.

Hoop moments at the edge and center of the ellipse are approximately to zero tensile force for edge and zero compressive at the center.

3.7.6 Case-6: parametric study of Aspect ratio a/b=3.5 with h/b=1

The 6^{th} model of elliptical shell for aspect ratio a/b=3.5 with height ratio h/b=1, take a=35m,b=10m and h=10m

Table 11 case-6: parametric study of Aspect ratio a/b=3.5 with h/b=1

Super impose membrane and bending analysis			
NØ	Nθ	MØ	Mθ
-79.1005	1.089114	-1.4E-06	-3.5E-07
-60.8525	0.904424	-0.60703	-0.15176
-33.6287	0.519793	0.664325	0.166081
-17.7425	0.130333	-0.24282	-0.06071
-9.98622	-0.21004	0.068487	0.017122
-6.12614	-0.5002	0.34881	0.087202
-4.08117	-0.74619	-0.3253	-0.08132
-2.9295	-0.95343	-0.29763	-0.07441
-2.25036	-1.12296	0.368292	0.092073
-1.8708	-1.24496	0.248652	0.062163

In this case meridian forces at the edge is decrease from center of elliptical shell to the edge of shell but hoop forces are changes tensile forces from center of ellipse to increase compressive forces at the edge of ellipse.

Meridian moments at the edge and center of the ellipse are approximately to zero tensile force for edge and zero compressive at the center.

Hoop moments at the edge and center of the ellipse are approximately to zero tensile force for edge and zero compressive at the center.

3.7.7 Case-7: parametric study of Aspect ratio a/b=4 with h/b=1

The 7th model of elliptical shell for aspect ratio a/b=4 with height ratio h/b=1, take a=40m,b=10m and h=10m

Table 12 case-6: parametric study of Aspect ratio a/b=4 with h/b=1

Super impose of membrane and bending analysis along major axis			
NØ	Nθ	MØ	Mθ
-103.499	1.089114	-1.4E-06	-3.5E-07
-70.4795	0.856807	0.597962	0.149491
-32.5104	0.43377	-0.22349	-0.05587
-15.2893	0.052585	-0.32355	-0.08089
-8.09281	-0.2609	0.436379	0.109095
-4.81548	-0.51994	-0.14068	-0.03517
-3.16983	-0.73491	-0.43782	-0.10945
-2.2768	-0.91129	0.035516	0.008879
-1.76746	-1.05095	0.385538	0.096385
-1.5503	-1.12511	0.012005	0.003001

In this case meridian forces at the edge is decrease from center of elliptical shell to the edge of shell but hoop forces are changes tensile forces from center of ellipse to increase compressive forces at the edge of ellipse.

Meridian moments at the edge and center of the ellipse are approximately to zero tensile force for edge and zero compressive at the center.

Hoop moments at the edge and center of the ellipse are approximately to zero tensile force for edge and zero compressive at the center.

3.8 Influence of aspect ratio, a/b on structural forces

The influence of the ratio of height (h) of the shell to miner radius (b) of the shell, h/b of elliptical

shell roof is analyzed. Miner radius of the shell (b) is constant and is equal to 10m. The major radius of the shell (a) is varied from 10m to 40m. The semi central angle is also varied due to the variation of major radius from 33.69 degree to 69.4 degree. The height of the shell (h) is constant and equal to 10m. The thickness of the shell (t) is constant and is equal to 100mm. Hence the value of a/b ratios ranges from 1 to 4. The structural meridian and hoop forces (NΦ and Nθ) developed on the surface of the shell are predicted at various locations of the angles ranging from zero degrees to 69.4 degrees using numerical method. The predicted values of structural forces, NΦ and Nθ and the moments MΦ and Mθ are given in Fig.3 and Fig. 4 respectively.

Table 3.13For meridian forces along major axis (x-axis) NΦ at p_5

For meridian forces along major axis (x-axis) NΦ at p_5							
height ratio, h/b	aspect ratio a/b						
	a/b=1	a/b=1.5	a/b=2	a/b=2.5	a/b=3	a/b=3.5	a/b=4
h/b=1	-5.976	-6.17	-4.392	-3.118	-2.351	-1.8708	-1.5503
h/b=0.75	-5.775	-4.3116	-2.68	-1.818	-1.346	-1.063	-0.877
h/b=0.5	-4.481	-2.286	-1.27	-0.83	-0.606	-0.476	-0.391
h/b=0.25	-1.7964	-0.643	-0.330	-0.2108	-0.1527	-0.1194	-0.0980

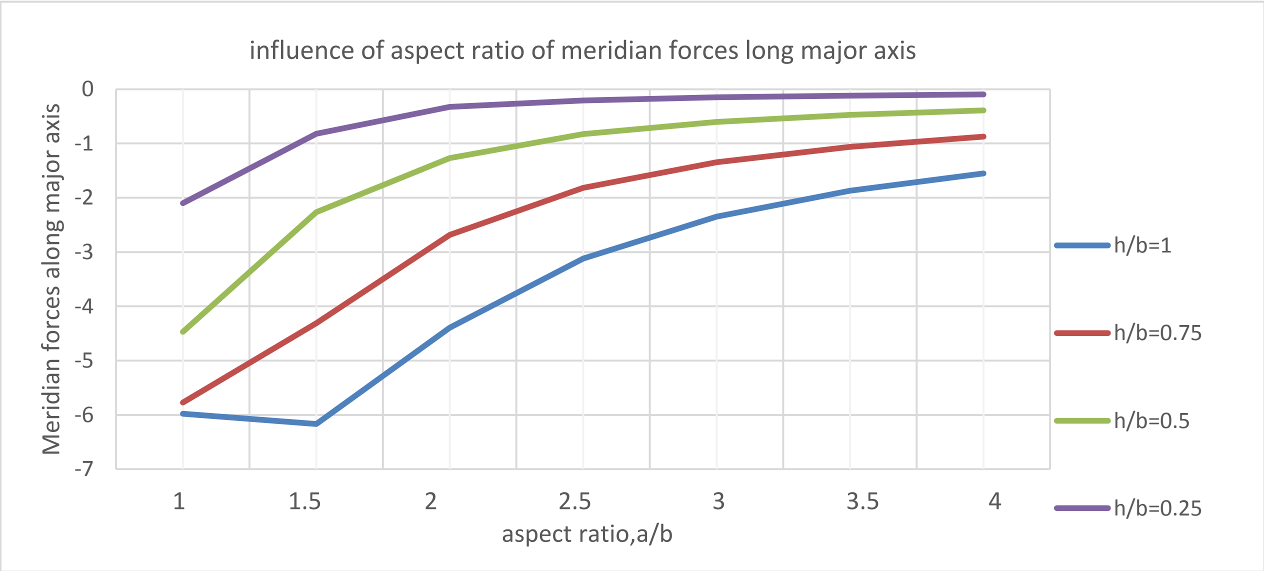


Figure 3.8 influence of aspect ratio of meridian forces along major axis

Table 3.14For meridian forces along miner axis (y-axis) NΦ at p_6

For meridian along miner axis (y-axis) NΦ at p_6							
height ratio, h/b	aspect ratio a/b						
	a/b=1	a/b=1.5	a/b=2	a/b=2.5	a/b=3	a/b=3.5	a/b=4
h/b=1	-5.97654	-9.71152	-12.4307	-14.2812	-15.5363	-16.4035	-17.0182
h/b=0.75	-5.77445	-8.3725	-9.93347	-10.8691	-11.4592	-11.8588	-12.1498
h/b=0.5	-4.46828	-5.60082	-6.19032	-6.55426	-6.7973	-6.95961	-7.06362
h/b=0.25	-2.09851	-2.20371	-2.12724	-2.00426	-1.87931	-1.7654	-1.6651

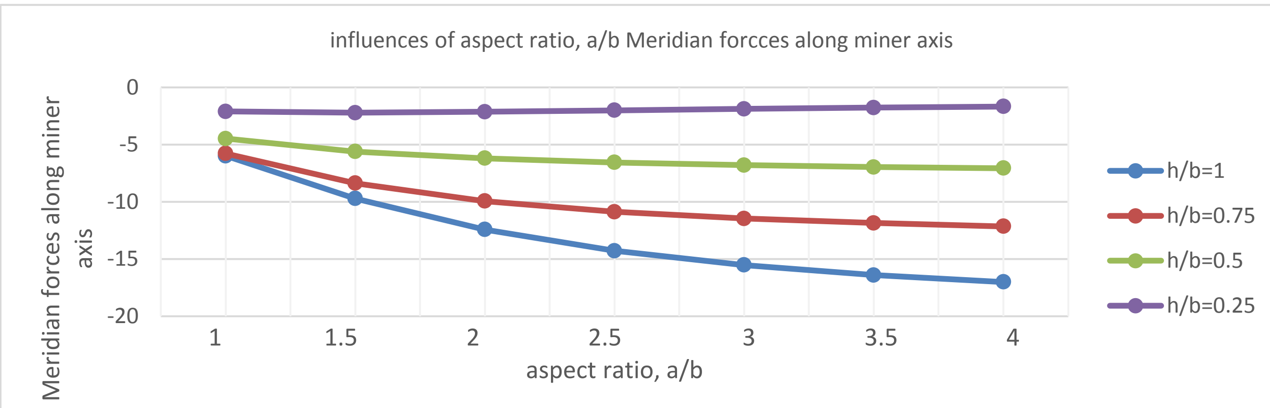


Figure 3.9 influence of aspect ratio of meridian forces along miner axis

Table 3.15For meridian forces along x and y-axis at p_5 and p_6

For meridian forces along miner and major axis (x and y-axis) NΦ at p_5 and p_6							
height ratio, h/b	aspect ratio a/b						
	a/b=1	a/b=1.5	a/b=2	a/b=2.5	a/b=3	a/b=3.5	a/b=4
h/b=1 along x-axis	-5.976	-6.17	-4.39182	-3.11797	-2.35136	-1.8708	-1.5503
h/b=1 along y-axis	-5.976	-9.711	-12.4301	-14.2809	-15.5376	-16.4083	-17.0275

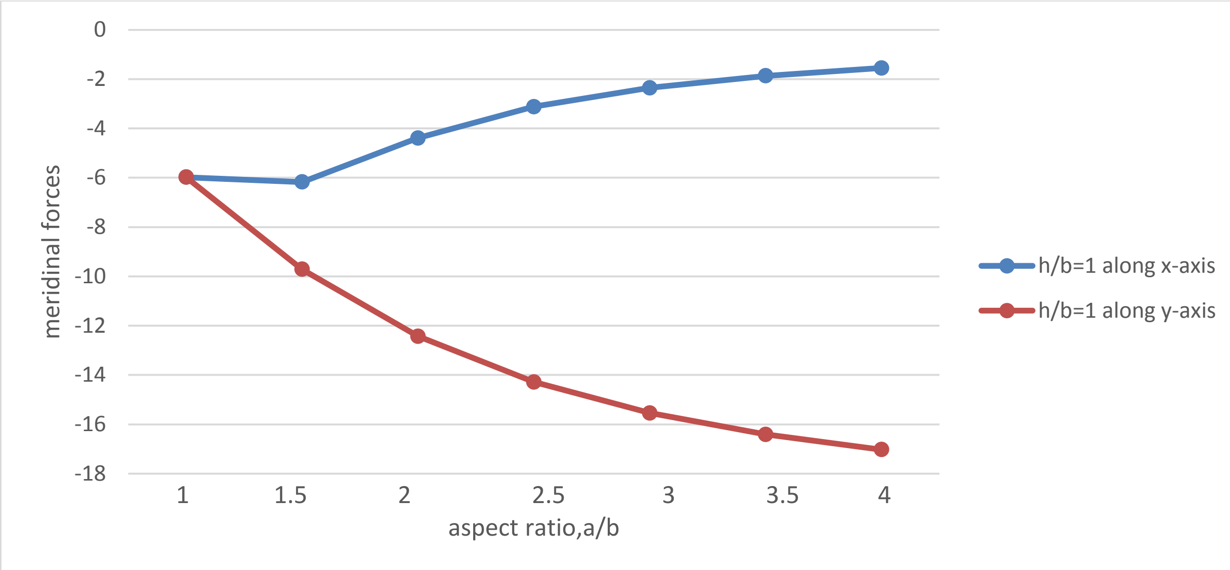


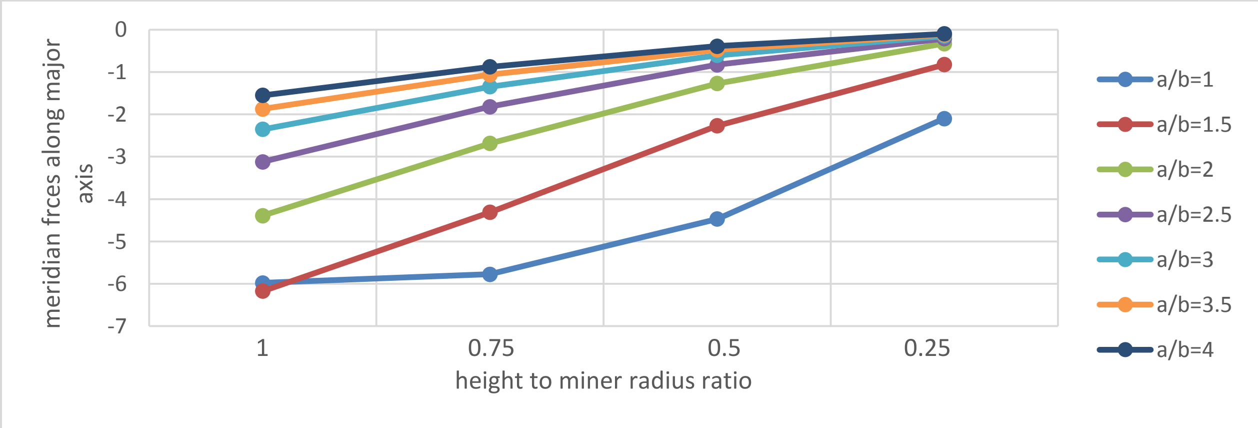
Figure 3.10 influence of aspect ratio of meridian forces along miner and major axis

3.8.1 Influence of a/b on meridian forces, NΦ

The influence of aspect ratio, a/b the meridian force NΦ developed at aspect ratio, a/b of ellipsoid shell ranging from 1 to 4 are given in Fig 3.9. From Fig 3.9, when the values of aspect ratio, a/b increases, the value of meridian forces NΦ is to be decrease along major axis of ellipsoid shell roof. Meridian forces (NΦ) is to be increase for all values of a/b ratio increases in miner axis of ellipsoid shell. In the meridian forces (NΦ) value is to be maximum at aspect ratio, a/b equal to 4 along miner axis. Meridian forces (NΦ) is to be increase for all values of a/b ratio increases in miner axis of ellipsoid shell. In the meridian forces (NΦ) value is to be maximum at aspect ratio, a/b is equal to 1 along major axis of ellipsoid shell roof. The meridian forces (NΦ) value is the same along miner and major axis of ellipsoid shell roof when the aspect ratio, a/b is equal to 1.

Table 3.16 For meridian forces on h/b along x-axis at p_5

For meridian forces on h/b along major axis (x-axis) $N\Phi \times g$ at p_5				
aspect ratio	height to miner radius ratio			
	h/b=1	h/b=0.75	h/b=0.5	h/b=0.25
a/b=1	-5.97654	-5.77445	-4.46828	-2.09851
a/b=1.5	-6.17062	-4.31134	-2.26837	-0.82349
a/b=2	-4.39181	-2.68465	-1.26995	-0.33001
a/b=2.5	-3.11797	-1.81844	-0.8298	-0.21081
a/b=3	-2.35136	-1.34646	-0.60619	-0.15273
a/b=3.5	-1.8708	-1.06261	-0.47558	-0.1194
a/b=4	-1.5503	-0.87705	-0.39141	-0.09809



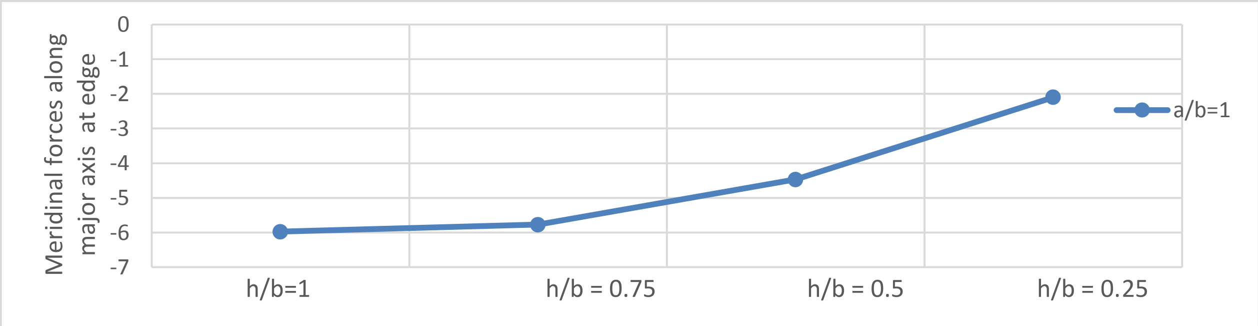


Figure 3.11 influence of height to miner radius ratio, h/b of meridian forces along major axis

Figure 3.12 influence of height to miner radius ratio for meridian forces along miner axis

3.8.2 Influence of h/b ratio on the structural forces and the moments

The influence of the ratio of height (h) of the shell to miner radius (b) of the shell, h/b of elliptical

shell roof is analyzed. The height of the shell (h) is varied from 2.5 m and 10 m. The miner radius of the elliptical shell (b) is made constant and is equal to 10 m. The major radius of the shell (a) is also constant and is equal to 10 m. The semi central angle is made constant and is equal to 33.69 degrees. The thickness of the shell (t) is constant and is equal to 100mm. Hence the value of h/b ratios ranges from 0.25 to 1. The structural shell forces ($N\Phi$ and $N\theta$) and the moments ($M\Phi$ and $M\theta$) developed on the surface of the shell are predicted at various locations of the angles ranging from zero degrees to 33.69 degrees using the model proposed by ANSYS software. The predicted values of structural forces $N\Phi$ and $N\theta$ and the moments $M\Phi$ and $M\theta$ are plotted and are given below in Fig.10 and Fig. 11 respectively.

3.8.3 Influence of height to miner radius, h/b on meridian forces, $N\Phi$

The influence of h/b ratios ranging from 0.25 to 1 on the structural force $N\Phi$ developed at the ratio h/b ellipsoid shell ranging from 1 to 0.25 are given in Fig 3.10 and 3.11. From Fig 3.10 and 3.11, it is found that as the values of h/b ratio increases, the value of $N\Phi$ is to be decrease along miner and major axis. When h/b ratio is equal to 0.25, the meridian forces, $N\Phi$ value is minimum at the edge of ellipsoid shell roof along miner and major axis. The value of meridian forces, $N\Phi$ is maximum when h/b is 1 at any aspect ratio along miner and major axis. When aspect ratio, a/b is equal to 1, the meridian forces, $N\Phi$ value is to be minimum rather than other aspect ratio on h/b ratio along miner axis of ellipsoid shell roof. When aspect ratio, a/b is equal to 4, the meridian forces, NΦ value is to be maximum rather than other aspect ratio on h/b ratio along miner axis of ellipsoid shell roof.

Table 3.17 for hoop forces along major axis at p_5

for structural hoop forces along major axis (x-axis) $N\theta \times g$ at p_5							
height ratio, h/b	aspect ratio a/b						
	a/b=1	a/b=1.5	a/b=2	a/b=2.5	a/b=3	a/b=3.5	a/b=4
h/b=1	-0.75761	-1.50728	-1.62389	-1.52391	-1.38136	-1.24496	-1.12511
h/b=0.75	-0.90739	-1.62028	-1.66954	-1.54239	-1.3896	-1.249	-1.12726
h/b=0.5	-1.09264	-1.71857	-1.70461	-1.55601	-1.39558	-1.25191	-1.1288
h/b=0.25	-1.7964	-1.78688	-1.72674	-1.56435	-1.3992	-1.25367	-1.12973

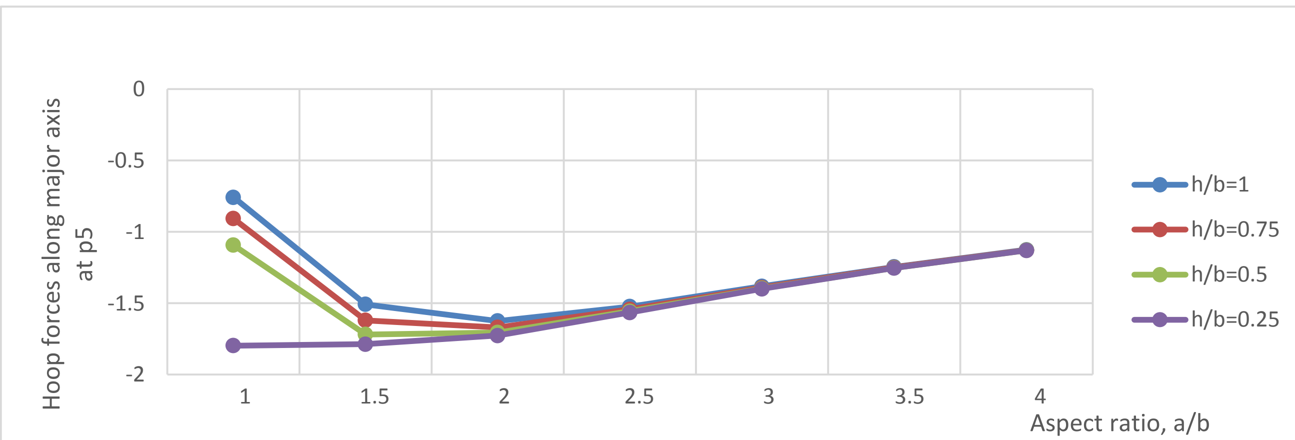


Figure 3.13 influence aspect ratio, a/b of hoop forces along major axis

Table 3.18 for hoop forces along miner axis (y-axis) $N\theta$ at p_6

for hoop forces along miner axis (y-axis) $N\theta \times g$ at p_6							
height ratio, h/b	aspect ratio a/b						
	a/b=1	a/b=1.5	a/b=2	a/b=2.5	a/b=3	a/b=3.5	a/b=4
h/b=1	-0.75761	-1.70463	-3.03045	-4.73507	-6.8185	-9.28074	-12.1218
h/b=0.75	-0.90739	-2.04163	-3.62956	-5.67119	-8.16652	-11.1155	-14.5183
h/b=0.5	-1.09264	-2.45845	-4.37058	-6.82902	-9.8338	-13.3849	-17.4823
h/b=0.25	-1.27884	-2.8774	-5.11537	-7.99277	-11.5096	-15.6658	-20.4615

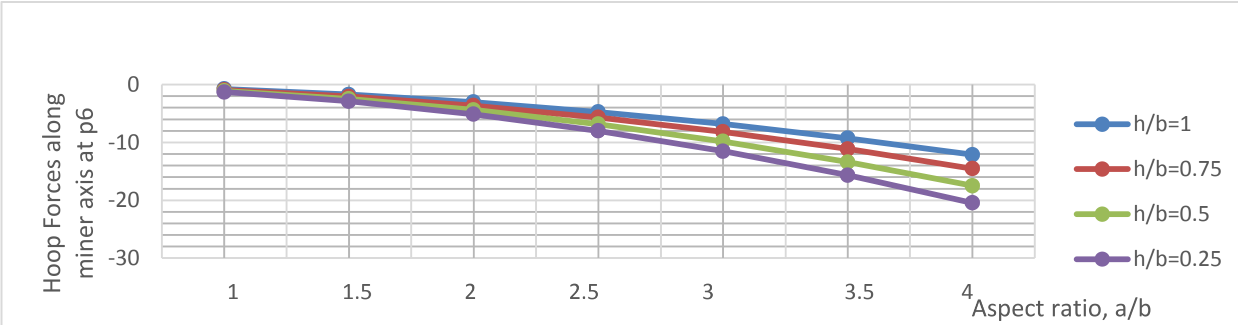


Figure 3.14 influence aspect ratio for hoop forces along miner axis

Table 3.19 For hoop forces along miner and major axis at p_5 and p_6

For structural hoop forces along miner and major axis (x and y-axis) $N\theta \times g$ at p_5 and p_6							
height ratio, h/b	aspect ratio a/b						
	a/b=1	a/b=1.5	a/b=2	a/b=2.5	a/b=3	a/b=3.5	a/b=4
h/b=1 along x-axis	-0.7576	-1.507	-1.624	-1.523905	-1.381	-1.245	-1.1251
h/b=1 along y-axis	-0.7576	-1.7046	-3.0304	-4.73507	-6.8185	-9.281	-12.122

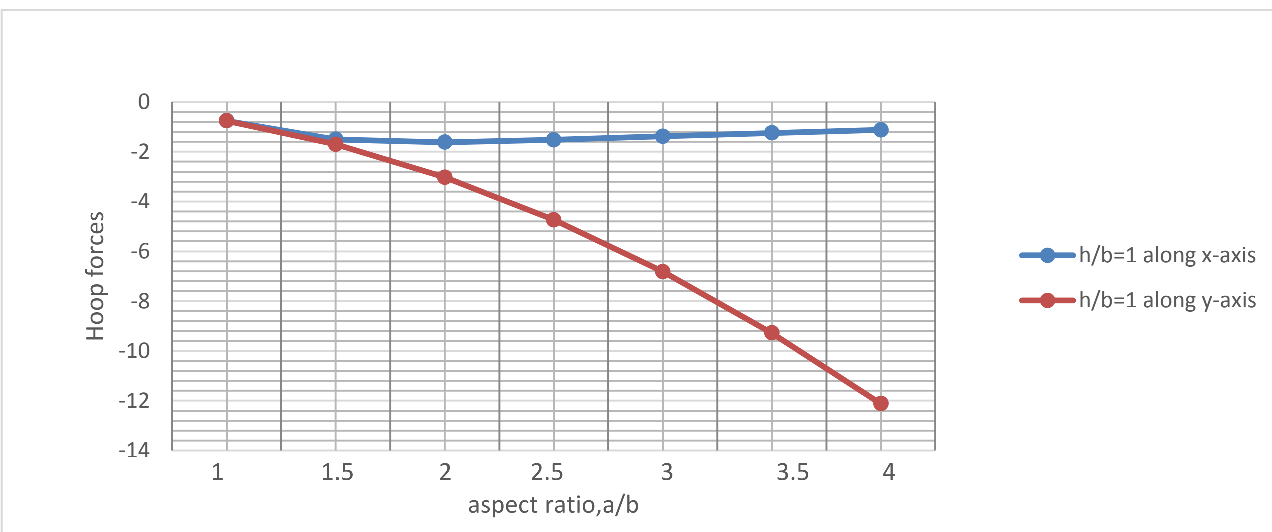


Figure 3.15 influence aspect ratio for hoop forces along miner and major axis

3.8.4 Influence of aspect ratio, a/b on hoop forces, $N\theta$

The influence of aspect ratio, a/b the hoop force $N\theta$ developed at aspect ratio, a/b of ellipsoid shell ranging from 1 to 4 are given in Fig 3.14. From Fig 3.14, when the values of aspect ratio, a/b increases, the hoop forces $N\theta$ is to be decrease along major axis of ellipsoid shell roof. Hoop forces, $N\theta$ is to be increase for all values of aspect ratio, a/b increases in miner axis of ellipsoid shell. In the hoop forces $N\theta$ value is to be maximum at aspect ratio, a/b equal to 4 along miner axis. Hoop forces $N\theta$ is to be increase for all values of aspect ratio, a/b increases in miner axis of ellipsoid shell. In the hoop forces $N\theta$ value is to be maximum at aspect ratio, a/b is equal to 2 along major axis of ellipsoid shell roof. The Hoop forces, $N\theta$ value is the same along miner and major axis of ellipsoid shell roof when the aspect ratio, a/b is equal to 1. when the values of a/b ratio increases, the value of hoop force ($N\theta$) is to be increase aspect ratio, a/b from ranging 1 to 2 of shell roof, but aspect ratio, a/b from ranging 2.5 to 4, the value of hoop force ($N\theta$) is to be decrease along major axis. Hoop forces ($N\theta$) is to be minimum at the aspect ratio, a/b equal to1 along miner radius of ellipsoid shell roof. The value of hoop force ($N\theta$) is maximum when h/b is 0.25 at any aspect ratio along miner and major axis. When aspect ratio, a/b is equal to 1, Hoop forces ($N\theta$) value is to be minimum rather than other aspect ratio on h/b ratio along miner axis of ellipsoid shell roof. When aspect ratio, a/b is equal to 4, the hoop forces ($N\theta$) value is to be maximum rather than other aspect ratio on h/b ratio along miner axis of ellipsoid shell roof.

Table 3.20 Hoop forces on h/b along major axis $N\theta$ at p_5

for structural hoop forces on h/b along major axis (x-axis) $N\theta \times g$ at p_5				
aspect ratio, a/b	height ratio, h/b			
	h/b=1	h/b=0.75	h/b=0.5	h/b=0.25
a/b=1	-0.75761	-0.907391	-1.092644	-1.79639985
a/b=1.5	-1.50728	-1.620281	-1.718568	-1.78688181
a/b=2	-1.62389	-1.66954	-1.70461	-1.72674381
a/b=2.5	-1.52391	-1.542387	-1.556007	-1.56435438
a/b=3	-1.38136	-1.389599	-1.395579	-1.39920348
a/b=3.5	-1.24496	-1.249001	-1.251913	-1.25367041
a/b=4	-1.12511	-1.127258	-1.128802	-1.12973147

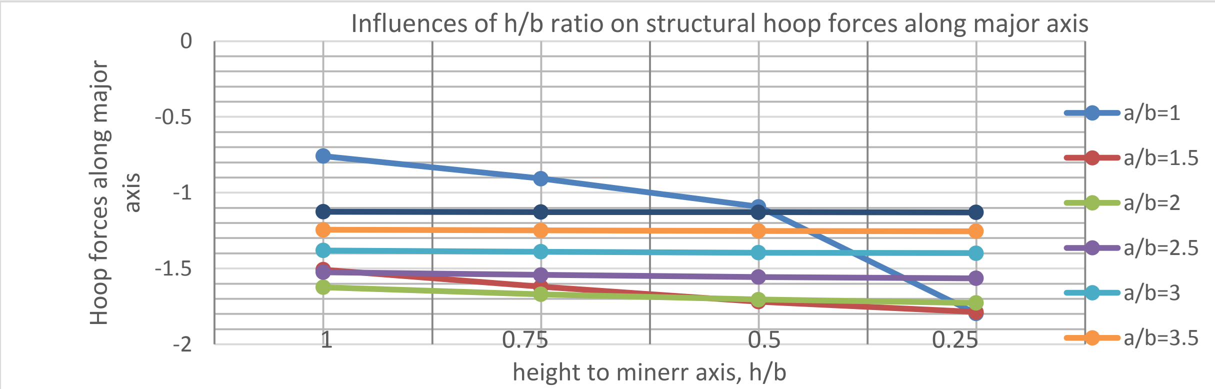


Figure 3.16 influence height to miner radius ratio for hoop forces along major axis

Table 3.21 Hoop forces on h/b along miner axis Nθ at p_6

Structural hoop forces on h/b along miner axis (y-axis) Nθ×g at p6				
aspect ratio a/b	height ratio, h/b			
	h/b=1	h/b=0.75	h/b=0.5	h/b=0.25
a/b=1	-0.75761	-0.90739	-1.09264	-1.278843743
a/b=1.5	-1.70463	-2.04163	-2.45845	-2.877398421
a/b=2	-3.03045	-3.62956	-4.37058	-5.115374971
a/b=2.5	-4.73507	-5.67119	-6.82902	-7.992773392
a/b=3	-6.8185	-8.16652	-9.8338	-11.50959369
a/b=3.5	-9.28074	-11.1155	-13.3849	-15.66583585
a/b=4	-12.1218	-14.5183	-17.4823	-20.46149988

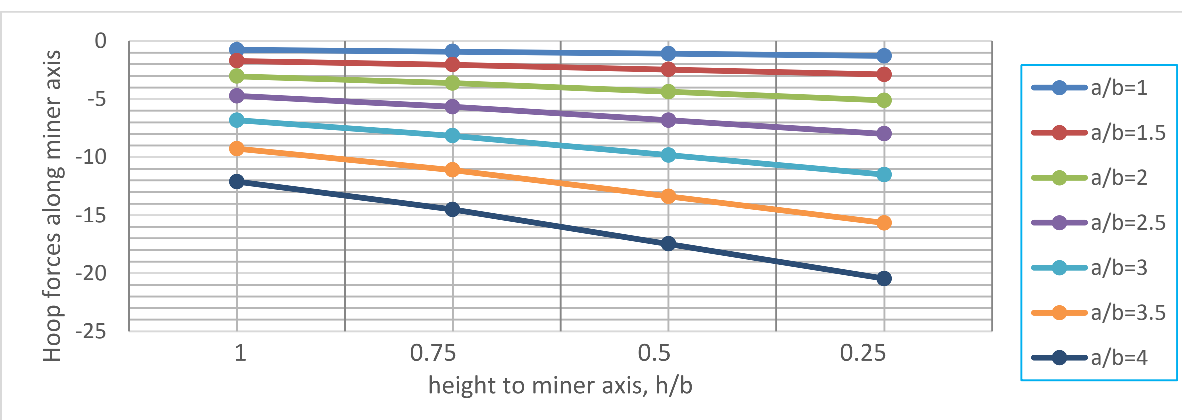


Figure 3.17 influence of height to miner radius ratio for hoop forces along miner axis

3.8.5 Influence of height to miner radius, h/b on hoop forces, Nθ

The influence of h/b ratios on the structural hoop force (Nθ) developed at the ratio h/b ellipsoid shell ranging from 1 to 0.25 are given in Fig 3.15 and 3.16. From Fig 3.15 and 3.16, the values of h/b ratio increases, the value of hoop force (Nθ) is to be decrease linearly along miner axis. When h/b ratio is equal to 0.25, the hoop force (Nθ) value is maximum at the edge of ellipsoid shell roof along miner and major axis. The value of hoop force (Nθ) is minimum when h/b is 1 at any aspect ratio along miner axis. When aspect ratio, a/b is equal to 1, the hoop force (Nθ) is to be minimum rather than other aspect ratio on h/b ratio along miner axis of ellipsoid shell roof. When aspect ratio, a/b is equal to 4, the hoop force (Nθ) is to be maximum rather than other aspect ratio on h/b ratio along miner axis of ellipsoid shell roof. When aspect ratio, a/b is equal to 1, the hoop force (Nθ) is to be maximum on h/b ratio equal to 0.25 along major axis of ellipsoid shell roof.

Table 3.22 Meridian moments along major axis at p5

Meridian moments along major axis (x-axis) $M\Phi \times g$ at p5							
height ratio, h/b	aspect ratio a/b						
	a/b=1	a/b=1.5	a/b=2	a/b=2.5	a/b=3	a/b=3.5	a/b=4
h/b=1	-1.90124	-1.63758	-1.1109	-0.31948	-0.3579	-0.12288	0.197277
h/b=0.75	-2.1889	-1.52174	-0.56304	0.392762	0.109672	0.390183	0.635862
h/b=0.5	-1.7842	-0.41066	0.405575	0.842747	0.707958	0.825462	0.907898
h/b=0.25	0.191988	0.8122	0.944138	0.98843	0.975533	0.986489	0.993355

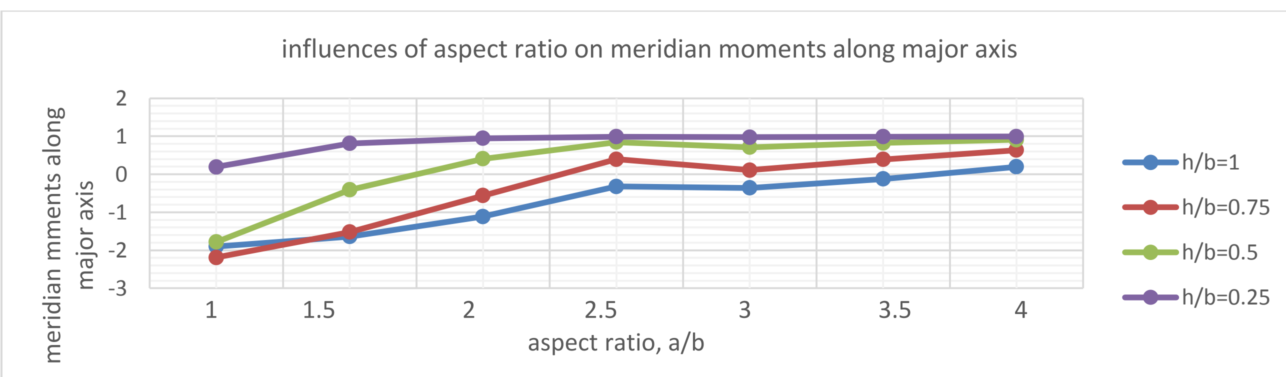


Figure 3.18 influence of aspect ratio for meridian moments along major axis

Table 3.23 Meridian moments along miner axis (y-axis) $M\Phi$ at p_6

Meridian moments along miner axis (y-axis) $M\Phi \times g$ at p_6							
height ratio, h/b	aspect ratio a/b						
	a/b=1	a/b=1.5	a/b=2	a/b=2.5	a/b=3	a/b=3.5	a/b=4
h/b=1	-1.90124	-1.40647	-0.96648	-0.61182	-0.33556	-0.1216	0.045234
h/b=0.75	-2.1889	-1.41986	-0.84411	-0.43599	-0.14614	0.063872	0.219785
h/b=0.5	-1.7842	-0.81132	-0.24854	0.090049	0.306114	0.451614	0.554115
h/b=0.25	0.191988	0.579426	0.742413	0.825176	0.872855	0.902859	0.923003

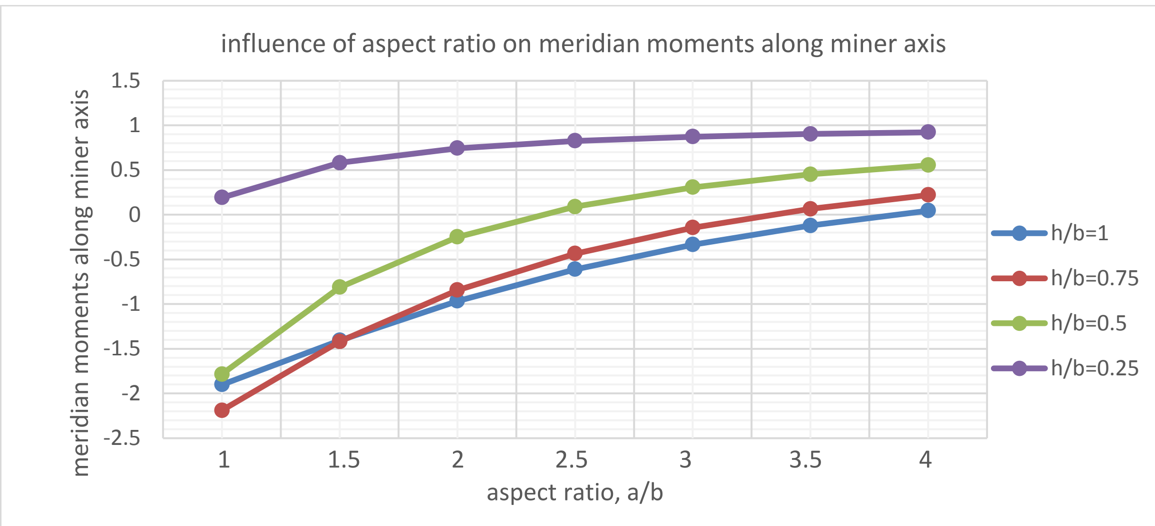


Figure 3.19 influence of aspect ratio for meridian moments along miner axis

3.8.6 Influence of aspect ratio, a/b on the moments ($M\Phi$ and $M\theta$)

The influence of aspect ratio, a/b on the structural moments meridian ($M\Phi$) and hoop moments ($M\theta$) are discussed. The predicted values of structural moments $M\Phi$ and $M\theta$ are plotted and are given in Fig.3.17 and Fig.3.17.

3.8.7 Influence of a/b on Meridian moments ($M\Phi$)

The influence of aspect ratio, a/b the meridian moment $M\Phi$ developed at aspect ratio, a/b of ellipsoid shell ranging from 1 to 4 are given in Fig 3.17 and Fig 3.18. From Fig 3.17 and Fig 3.18, when the values of aspect ratio, a/b increases, the meridian moment $M\Phi$ is decrease along major axis of ellipsoid shell roof. Meridian moment, $M\Phi$ is to be increase for all values of a/b ratio increases in miner axis of ellipsoid shell. In the meridian moment ($M\Phi$) value is to be maximum at aspect ratio, a/b equal to 4 along miner axis. Meridian moment ($M\Phi$) is to be increase for all values of a/b ratio increases in miner axis of ellipsoid shell. In the meridian moment ($M\Phi$) value is to be maximum at aspect ratio, a/b is equal to 1 along major axis of ellipsoid shell roof. The meridian moment ($M\Phi$) value is the same along miner and major axis of ellipsoid shell roof when the aspect ratio, a/b ii equal to 1.

Table 3.24 meridian moments on h/b along major axis at p_5

meridian moments on h/b along major axis (x-axis) $M\Phi \times g$ at p_5				
aspect ratio a/b	height ratio, h/b			
	h/b=1	h/b=0.75	h/b=0.5	h/b=0.25
a/b=1	-1.90124	-2.1889	-1.7842	0.191988
a/b=1.5	-1.63758	-1.52174	-0.41066	0.8122
a/b=2	-1.1109	-0.56304	0.405575	0.944138
a/b=2.5	-0.31948	0.392762	0.842747	0.98843
a/b=3	-0.3579	0.109672	0.707958	0.975533
a/b=3.5	-0.12288	0.390183	0.825462	0.986489
a/b=4	0.197277	0.635862	0.907898	0.993355

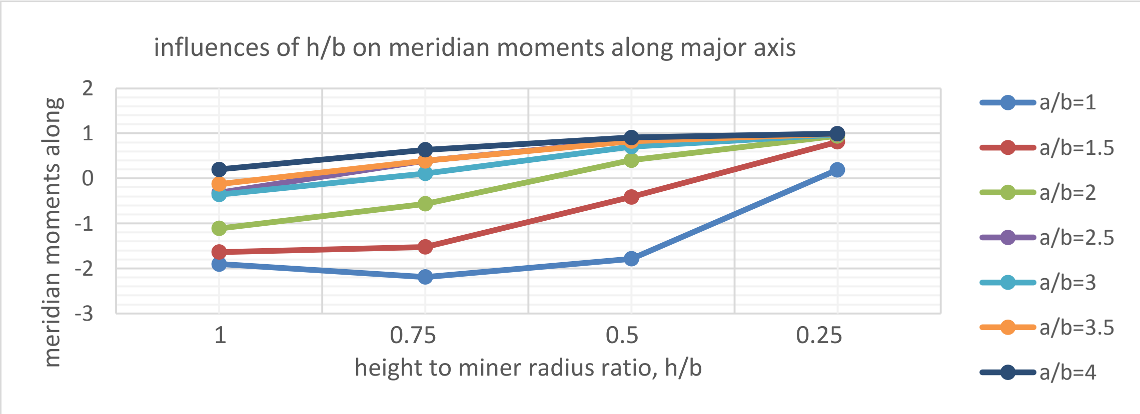


Figure 3.20 influence of height to miner ratio for meridian moments along major axis

Table 3.25 meridian moments on h/b along miner axis at p_6

Meridian moments along miner axis (y-axis) MΦ×g at p6							
height ratio, h/b	aspect ratio a/b						
	a/b=1	a/b=1.5	a/b=2	a/b=2.5	a/b=3	a/b=3.5	a/b=4
h/b=1	-1.90124	-1.40647	-0.96648	-0.61182	-0.33556	-0.1216	0.045234
h/b=0.75	-2.1889	-1.41986	-0.84411	-0.43599	-0.14614	0.063872	0.219785
h/b=0.5	-1.7842	-0.81132	-0.24854	0.090049	0.306114	0.451614	0.554115
h/b=0.25	0.191988	0.579426	0.742413	0.825176	0.872855	0.902859	0.923003

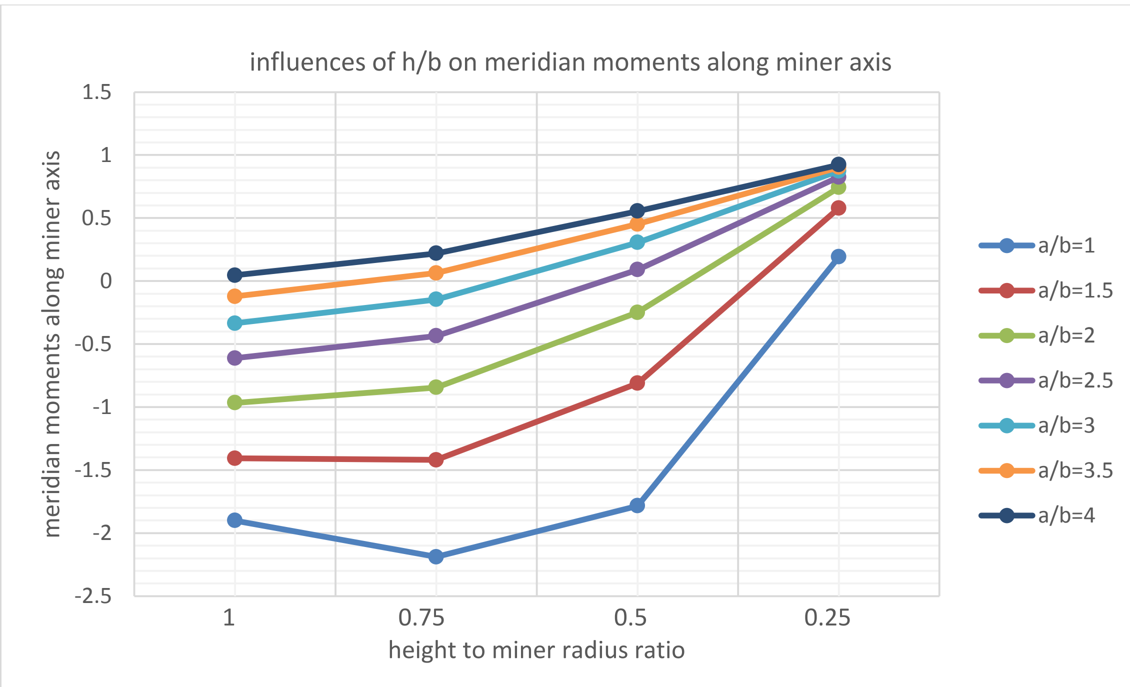


Figure 3.21 influence of height to miner ratio for meridian moments along miner axis

3.8.8 Influence of h/b on the moments MΦ and Mθ

The influence of h/b ratio on the structural moments MΦ and Mθ are discussed. The predicted values of structural moments MΦ and Mθ are plotted and are given in Fig.6.

3.8.9 Influence of h/b on meridian moments (MΦ)

The influence of h/b ratios ranging from 0.25 to 1 on the structural moments MΦ developed at the ratio h/b ellipsoid shell ranging from 1 to 0.25 are given in Fig 3.19 and 3.20. From Fig 3.19 and 3.20, it is found that as the values of h/b ratio increases, the value of meridian moments MΦ is to be decrease along miner and major axis. When h/b ratio is equal to 0.25, the meridian moments MΦ value is minimum at the edge of ellipsoid shell roof along miner and major axis. The value of meridian moments, MΦ is maximum when h/b is 1 at any aspect ratio along miner and major axis. When aspect ratio, a/b is equal to 1, the meridian moments, MΦ value is to be maximum rather than other aspect ratio on h/b ratio along miner axis of ellipsoid shell roof. When aspect ratio, a/b is equal to 4, the meridian moments MΦ value is minimum rather than other aspect ratio on h/b ratio along miner axis of ellipsoid shell roof.

Table 3.26 Hoop moments along major axis at p_5

Hoop moments along major axis (x-axis) $M\theta \times g$ at p5							
height ratio, h/b	aspect ratio a/b						
	a/b=1	a/b=1.5	a/b=2	a/b=2.5	a/b=3	a/b=3.5	a/b=4
h/b=1	-0.47531	-0.35162	-0.24162	-0.15295	-0.08389	-0.0304	0.011308
h/b=0.75	-0.54722	-0.35497	-0.21103	-0.109	-0.03653	0.015968	0.054946
h/b=0.5	-0.44605	-0.20283	-0.06213	0.022512	0.076529	0.112904	0.138529
h/b=0.25	0.047997	0.144857	0.185603	0.206294	0.218214	0.225715	0.230751

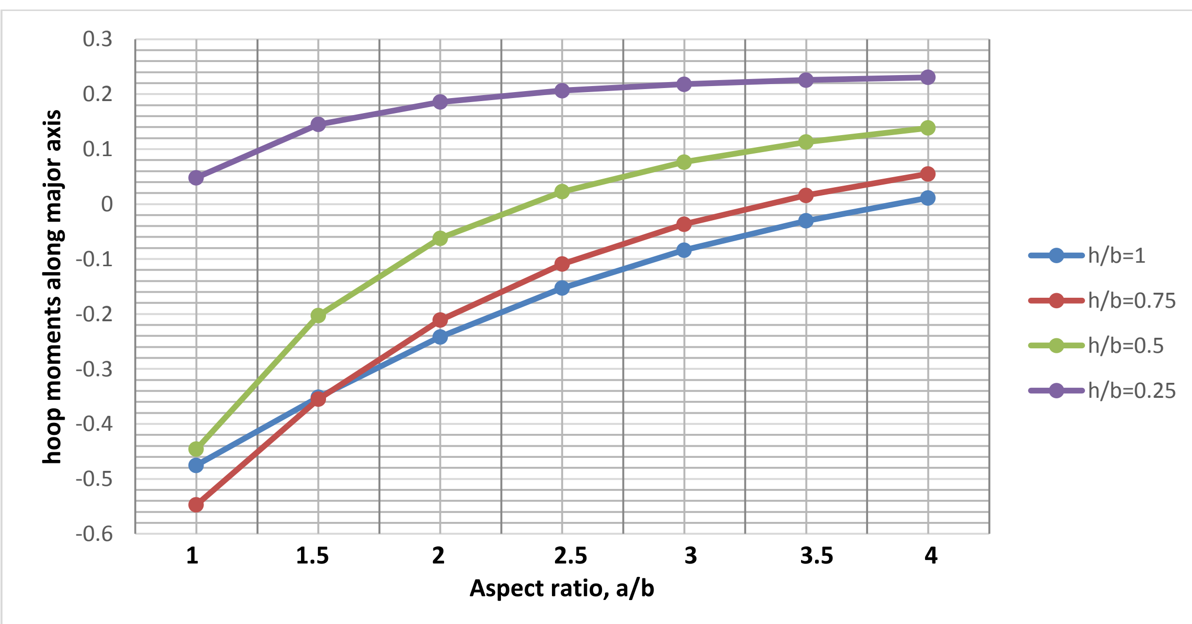


Figure 3.22 influence of aspect ratio for hoop moments along major axis

Table 3.27 Hoop moments along miner axis at p_6

Hoop moments along miner axis (y-axis) $M\theta \times g$ at p_6							
height ratio, h/b	aspect ratio a/b						
	a/b=1	a/b=1.5	a/b=2	a/b=2.5	a/b=3	a/b=3.5	a/b=4
h/b=1	-0.01876	-0.0956	-0.235	-0.324	-0.4045	-0.536	-0.58682
h/b=0.75	-0.0897	-0.123	-0.264	-0.4963	-0.6129	-0.7532	-0.864
h/b=0.5	-0.23194	-0.25436	-0.367	-0.5241	-0.695	-0.7892	-0.90557
h/b=0.25	-0.25	-0.35867	-0.41435	-0.44409	-0.46144	-0.47235	-0.47961

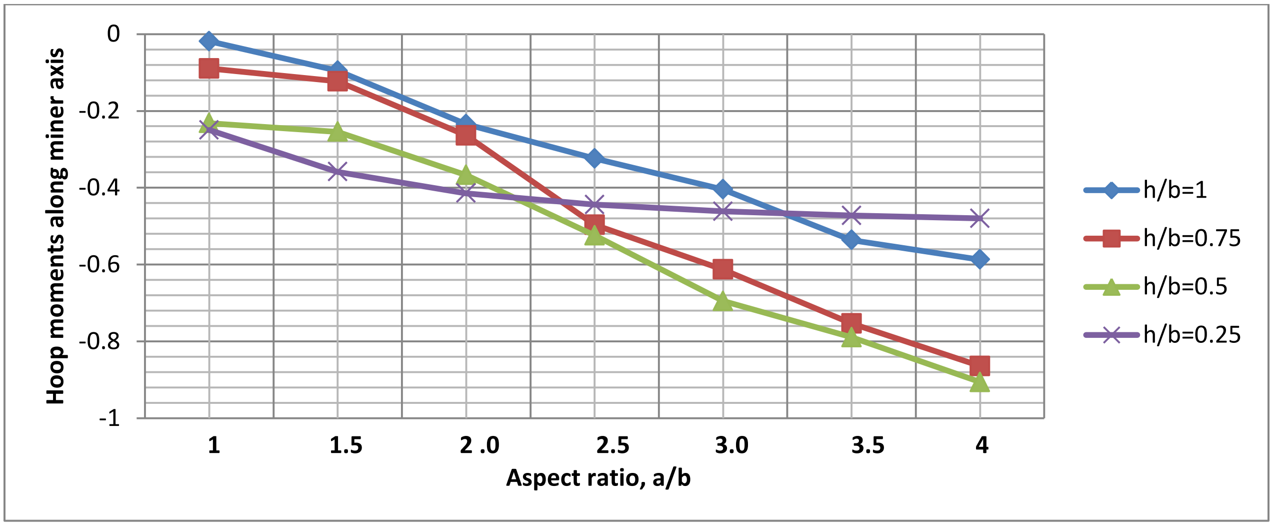


Figure 3.23 influence of aspect ratio for hoop moments along miner axis

3.8.10 Influence of aspect ratio, a/b on hoop moments ($M\theta$)

The influence of aspect ratio, a/b the hoop moments, $M\theta$ developed at aspect ratio, a/b of ellipsoid shell ranging from 1 to 4 are given in Fig 3.21 and 3.22. From Fig 3.21 and 3.22, when the values of aspect ratio, a/b increases, the hoop moments, $M\theta$ is decrease along major axis of ellipsoid shell roof. Hoop moments, $M\theta$ is increase for all values of aspect ratio, a/b increases in miner axis of ellipsoid shell. In the hoop moments, $M\theta$ value is maximum at aspect ratio, a/b equal to 4 along miner axis. In the hoop moments, $M\theta$ value is maximum at aspect ratio, a/b is equal to 1 on h/b equal to 0.75 along major axis of ellipsoid shell roof. The hoop moments, $M\theta$ value is not the same along miner and major axis of ellipsoid shell roof when the aspect ratio, a/b is equal to 1. The value of hoop moments, $M\theta$) is decrease along major axis. Hoop moments, $M\theta$ is minimum at the aspect ratio, a/b equal to1 along miner radius of ellipsoid shell roof. The value of hoop moments, Mθ is maximum when h/b is 0.25 at any aspect ratio along miner axis. When aspect ratio, a/b is equal to 1, hoop moments, Mθ value is minimum rather than other aspect ratio on h/b ratio along miner axis of ellipsoid shell roof. When aspect ratio, a/b is equal to 4, the hoop moments, Mθ value is to be maximum rather than other aspect ratio on h/b ratio along miner axis of ellipsoid shell roof.

Table 3.28 Hoop moments on h/b along major axis at p_5

Hoop moments on h/b along major axis (x-axis) Mθ×g at p_5				
aspect ratio a/b	height ratio, h/b			
	h/b=1	h/b=0.75	h/b=0.5	h/b=0.25
a/b=1	-0.47531	-0.54722	-0.44605	0.047997
a/b=1.5	-0.35162	-0.35497	-0.20283	0.144857
a/b=2	-0.24162	-0.21103	-0.06213	0.185603
a/b=2.5	-0.15295	-0.109	0.022512	0.206294
a/b=3	-0.08389	-0.03653	0.076529	0.218214
a/b=3.5	-0.0304	0.015968	0.112904	0.225715
a/b=4	0.011308	0.054946	0.138529	0.230751

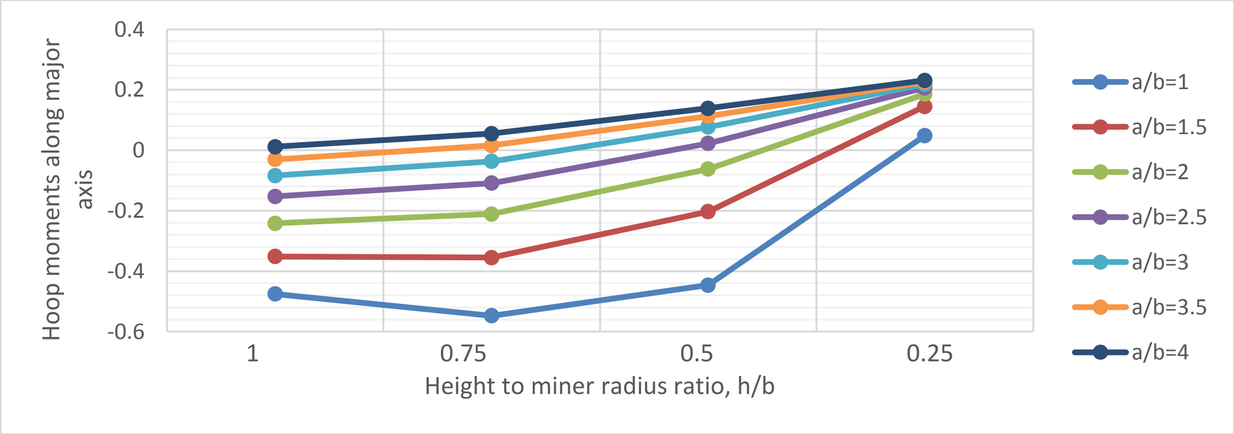


Figure 3.24 influence of aspect ratio for hoop moments along major axis

Table 3.29 Hoop moments on h/b along miner axis at p_6

Hoop moments on along miner axis (y-axis) $M\theta \times g$ at p_6				
height ratio, h/b	aspect ratio h/b			
	h/b=1	h/b=0.75	h/b=0.5	h/b=0.25
a/b=1	-0.01876	-0.0897	-0.23194	-0.25
a/b=1.5	-0.0956	-0.123	-0.25436	-0.35867
a/b=2	-0.235	-0.264	-0.367	-0.41435
a/b=2.5	-0.324	-0.4963	-0.5241	-0.44409
a/b=3	-0.4045	-0.6129	-0.695	-0.46144
a/b=3.5	-0.536	-0.7532	-0.7892	-0.47235
a/b=4	-0.58682	-0.864	-0.90557	-0.47961

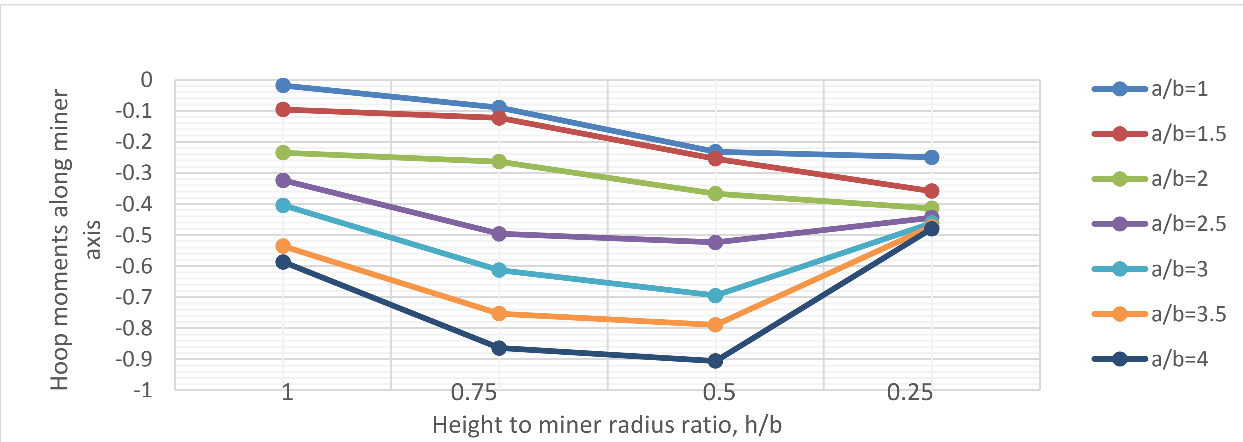


Figure 3.25 influence of aspect ratio for hoop moments along miner axis

3.8.11 Influence of h/b on hoop moment, Mθ

The influence of h/b ratios on the structural hoop moments, Mθ developed at the ratio h/b ellipsoid shell ranging from 1 to 0.25 are given in Fig 3.23 and 3.24. From Fig 3.23 and 3.24, the values of h/b ratio increases, the value of hoop moments, Mθ is decrease along miner and major axis. When h/b ratio is equal to 0.25, the hoop moments, Mθ value is maximum at the edge of ellipsoid shell roof along miner axis. The value of hoop moments, Mθ is minimum when h/b is 1 at aspect ratio 1 to 3 along miner axis. When aspect ratio, a/b is equal to 1, the hoop moments, Mθ is minimum rather than other aspect ratio on h/b ratio along miner axis of ellipsoid shell roof. When aspect ratio, a/b is equal to 4, the hoop moments, Mθ is maximum rather than other aspect ratio on h/b ratio along miner axis of ellipsoid shell roof. When aspect ratio, a/b is equal to

1, the hoop moments, Mθ is to be maximum on h/b ratio equal to 1 along major axis of ellipsoid shell roof.

3.9 Results and Discussion

There are 28 types of elliptical shell are used for modeling shell roofs in this thesis. It can be found in the comparison of the shell that the major and minor radius ratio with the height to minor radius ratio of elliptical shell roof of constant thickness. Aspect ratio 4 with h/b 0.25 is less meridian forces than the aspect ratio 1 with h/b 0.25 along major axis. For aspect ratio greater than 2.5, the in-plane forces, NΦ, adjacent to the vertex point on *Y*-axis tend to initially increase in a somewhat steep manner and then converge. On the other hand, as shown in Fig. 3.20 to 3.24 with an increase in aspect ratio, the `hoop' moment forces, M$\theta$, At points away from the vertex (i.e., points `P5' and `P6'), however, the hoop moments behave exactly opposite; with hoop moments along *X*-axis decreasing while the corresponding moments in *Y*-axis increasing with aspect ratio. For meridian moment forces, M$\Phi$ near the edges (i.e., points `P5'and `P6'), an interesting contrast is observed with meridian moments along *Y*-axis increasing much more rapidly than those in the *X*-axis when aspect ratio increases. Further, with an increase in the radius of *X*-axis, have a correspondingly significant effect on the total structural systems. This suggests that the behavior along the minor axis, (*Y*-axis) is the critical design factor and, therefore, such shells need to be reinforced at weak points for large aspect ratios. In general, the numerical results summarized in fig. 3.8 to 3.25 suggest that the stress resultant tends to gradually converge with increase in aspect ratio. Figure 3.8 to Figure 3.25 are helpful in a design process as the coefficient of stress resultants corresponding to any given aspect ratio a/b and height ratio of h/b can be directly read from these charts. It is known that the behavior of elliptical shells is largely influenced by aspect ratio, a/b and height to miner radius ratio, h/b. The stress resultants along *X*-axis are almost constant independent of the aspect ratio. In particular, the moment forces on *X*-axis for elliptical shells with a/b = 1.0, 1.25 and 1.50 are in very close agreement with those on *Y*-axis for a/b = 1.0. As the height ratio increases, the plane forces linearly increase while the moment forces decrease in a quadratic fashion. Therefore, it is noted that the aspect and height ratios greatly affect the moment and plane forces, respectively.

Generally height to minor radius, h/b 0.25 is the critical point for all aspect ratio of meridian and hoop forces and moments of elliptical shell roof. The height to minor radius ratio, h/b 0.25 is economical sections for all aspect ratio for meridian forces and moments of elliptical shell.

CHAPTER FOUR

CONCLUSION AND RECOMMENDATION

4.1 Conclusions

Based on the parametric study, the following conclusions are arrived at. On the structural forces and the moments developed on the surface of elliptical shell roofs, The parameters considered in this study are height to miner radius ratio and major radius to miner radius ratio of elliptical shell roof. The miner radius is made constant and is equal to 10m.The major radius of the shell is varied between 10m to 40m.The height of the shell is varied from 2.5 m to 10m.

We can conclude that both software and analytical methods agree with small divergent at the middle of the shell, this is because membrane theory satisfied. It difficult to conclude general the accuracy of the method, but particularly the software results resembles to realistic results at support and crown.

Design the shell using analytical is more uneconomical than software finite method analysis, even it is also safer with modification of bending effect. But analytical method is cumbersome in analysis of complex structures and could liable for large errors .But software could be more accurate for complex structures but modeling them will be difficult, besides data recognition also become more difficult for their local and global coordinate complexity.

Detailed parametric studies were carried out to bring out the effects of aspect ratio and height to miner radius ratio on the static behavior characteristic of elliptical shells. Based on the above theoretical developments and numerical results, the following concluding remarks are made:

- The major radius, the miner radius and the height of the elliptical shell roof greatly influences the structural forces and the moments developed on the surface of ellipsoid shell roofs.

- It is found that when the ratio of height of shell to the miner radius of the shell is equal to 0.25 to 1, the structural forces and the moments developed on the surface of ellipsoid shell roofs are significant for designing elliptical shell.
- When the aspect ratio, a/b of ellipsoid shell to the radius of the shell is equal to 4.0, the structural forces and the moments developed on the surface of ellipsoid shell roofs are significant for designing elliptical shell.
- The parametric study made in this paper is useful for the design engineers to arrive at economical sections of elliptical shell roofs made of concrete.
- As the height ratio increases, the plane forces linearly increase and the moment forces decrease in a quadratic fashion.
- Meridian moment is decrease linearly when aspect ratio increases along minor axis. Hoop force is increases linearly when aspect ratio increases along minor axis.
- Height to minor radius, h/b 0.25 is the critical point for all aspect ratio of meridian and hoop forces and moments of elliptical shell roof.
- The height to minor radius ratio, h/b 0.25 is used to design for all aspect ratio for meridian forces and moments of elliptical shell.

4.2 Recommendation

To be confident enough to compare the accuracy of the methods and the quality of the result, further detail studies shall be carried out. And detail verification works shall be done to further studies for different type of shell types for this study is limited to ellipse.

Besides, designers use software shall take care of modeling and data refining.

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APPENDIX

Table A1 bending analysis when aspect ratio is 1 with h/b 0.75 along major axis

									Nϕ		Nθ		Mϕ		Mθ	
λ	$^{\psi}$degree	ø radian	λ^{φ}degree	λ^{φ} radian	e-λ^{ψ}	cot(ø)	sin(λ^{φ})	cos(λ^{φ})	He	Me	He	Me	He	Me	He	Me
14.953	0	0	0	0	1	1.500984	0	1	0	0	-29.91	33.5	0	1	0	0.25
14.844	5	0.0872	74.219	1.294708	0.274	1.500984	0.9621	0.272594	-0.56	-0.894	-2.23	-6.34	-3.14	0.3383	-0.7851305	0.084572
14.526	10	0.1744	145.26	2.534048	0.079	1.500984	0.5709	-0.82105	-0.096	-0.157	1.937	-3.7	-0.52	-0.02	-0.1308738	-0.00496
14.034	15	0.2617	210.5	3.672138	0.025	1.500984	-0.506	-0.86253	0.0273	0.0461	0.648	-0.3	0.142	-0.035	0.0354188	-0.0087
13.411	20	0.3489	268.23	4.679046	0.009	1.500984	-0.999	-0.03334	0.0197	0.0348	0.009	0.3	0.096	-0.01	0.02398631	-0.0024
12.709	25	0.4361	317.74	5.542747	0.004	1.500984	-0.675	0.738173	0.0056	0.0105	-0.084	0.19	0.025	0.0002	0.00633109	6.22E-05
11.975	30	0.5233	359.26	6.267054	0.002	1.500984	-0.016	0.99987	6E-05	0.0001	-0.054	0.06	3E-04	0.0019	6.7503E-05	0.000467
11.436	33.69	0.5877	385.27	6.720837	0.001	1.500984	0.4238	0.905749	-0.001	-0.0022	-0.031	0.02	-0	0.0016	-0.0010562	0.000401

Table A2 bending analysis when aspect ratio is 1.5 with h/b 0.75 along major axis

									Nϕ		**Nθ**		**Mϕ**		**Mθ**	
λ	ψdegree	ø radian	λ^{φ}degree	λ^{φ} radian	e-λ^{ψ}	cot(ø)	sin(λ^{φ})	cos(λ^{φ})	He	Me	He	Me	He	Me	He	Me
33.645	0	0	0	0	1	1.0008	0	1	0	0	-29.907	33.54	0	1	0	0
32.711	5	0.0872	163.55	2.853119	0.06	1.0008	0.28449	-0.9587	-0.023	-0.02	1.644	-2.4	-0.43	-0	-0.107	-0
30.193	10	0.1744	301.93	5.267074	0.01	1.0008	-0.8501	0.52668	0.0062	0.005	-0.0795	0.238	0.102	-0	0.0255	-0
26.762	15	0.2617	401.43	7.002641	0	1.0008	0.65898	0.75216	-8E-04	-0	-0.0195	0.003	-0.01	0	-0.003	0
23.101	20	0.3489	462.02	8.059716	0	1.0008	0.97891	-0.2043	-4E-04	-0	0.0018	-0.01	-0	0	-0.001	0
19.679	25	0.4361	491.97	8.582173	0	1.0008	0.74638	-0.6655	-2E-04	-0	0.0034	-0.01	-0	0	-4E-04	0
16.716	30	0.5233	501.47	8.747941	0	1.0008	0.62633	-0.7796	-1E-04	-0	0.0032	-0.01	-0	-0	-3E-04	-0
14.262	35	0.6106	499.18	8.707854	0	1.0008	0.65707	-0.7538	-2E-04	-0	0.0031	-0.01	-0	-0	-2E-04	-0
12.281	40	0.6978	491.24	8.569325	0	1.0008	0.75487	-0.6559	-2E-04	-0	0.003	-0.01	-0	0	-2E-04	0
10.703	45	0.785	481.63	8.401748	0	1.0008	0.85369	-0.5208	-3E-04	-0	0.0028	-0.01	-0	0	-3E-04	0

Table A3 bending analysis when aspect ratio is 2 with h/b 0.75 along major axis

									Nϕ		Nθ		Mϕ		Mθ	
λ	ψdegree	ø radian	λφdegree	λφ radian	e-λψ	cot(ø)	sin(λφ)	cos(λφ)	He	Me	He	Me	He	Me	He	Me
59.81395	0	0	0	0	1	0.7505	0	1	0	0	-29.91	33.54	0	1	0	0.25
55.17132	6	0.1047	331.028	5.7746	0.003	0.7505	-0.487	0.873	0	7E-04	-0.08	0.142	0.065	0.0012	0.02	3E-04
44.6218	12	0.2093	535.462	9.3408	9E-05	0.7505	0.0838	-0.996	-0	-0	0.0025	-0.003	-2E-04	-8E-05	-0	-0
33.67128	18	0.314	606.083	10.573	3E-05	0.7505	-0.912	-0.41	0	2E-05	0.0003	4E-04	0.0005	-3E-05	0	-0
24.97699	24	0.4187	599.448	10.457	3E-05	0.7505	-0.858	-0.513	0	2E-05	0.0004	3E-04	0.0003	-4E-05	0	-0
18.76631	30	0.5233	562.989	9.821	5E-05	0.7505	-0.386	-0.923	0	3E-05	0.0012	-1E-03	0.0002	-7E-05	0	-0
14.475	36	0.628	521.1	9.0903	1E-04	0.7505	0.3283	-0.945	-0	-0	0.0024	-0.005	-2E-04	-7E-05	-0	-0
11.51517	42	0.7327	483.637	8.4368	2E-04	0.7505	0.8349	-0.55	-0	-0	0.0026	-0.01	-9E-04	6E-05	-0	2E-05
9.453364	48	0.8373	453.761	7.9156	4E-04	0.7505	0.9981	-0.062	-0	-0	0.0005	-0.013	-0.001	0.0003	-0	9E-05
8.181425	53.13	0.9268	434.679	7.5827	5E-04	0.7505	0.9634	0.268	-0	-0	-0.003	-0.012	-0.001	0.0006	-0	2E-04

Table A4 bending analysis when aspect ratio is 2.5 with h/b 0.75 along major axis

									Nϕ		Nθ		Mϕ		Mθ	
λ	$^{\psi}$degree	ø radian	λ^{φ}degree	λ^{φ} radian	e-λ^{ψ}	cot(ø)	sin(λ^{φ})	cos(λ^{φ})	He	Me	He	Me	He	Me	He	Me
93.4593	0	0	0	0	1	0.6016	0	1	0	0	-29.91	33.541	0	1	0	0.25
78.46378	7	0.122	549.2	9.5813	7E-05	0.6016	-0.156	-0.988	0	0	0.002	-0.002	0	-0	2E-04	-0
52.27257	14	0.244	731.8	12.766	3E-06	0.6016	0.198	0.9801	-0	-0	-7E-05	7E-05	-0	0	-0	0
33.02221	21	0.366	693.5	12.097	6E-06	0.6016	-0.452	0.8919	0	0	-1E-04	0.0003	0	0	1E-05	0
21.5957	28	0.488	604.7	10.548	3E-05	0.6016	-0.902	-0.433	0	0	0.0003	0.0004	0	-0	6E-05	-0
14.97812	35	0.611	524.2	9.145	0.0001	0.6016	0.276	-0.961	-0	-0	0.0021	-0.004	-0	-0	-0	-0
11.02809	42	0.733	463.2	8.0799	0.0003	0.6016	0.975	-0.224	-0	-0	0.0014	-0.012	-0	0	-0	0
8.575455	49	0.855	420.2	7.3301	0.0007	0.6016	0.866	0.5002	-0	-0	-0.006	-0.008	-0	0	-0	0
7.001717	56	0.977	392.1	6.8399	0.0011	0.6016	0.528	0.849	-0	-0	-0.016	0.0115	-0	0	-0	0
6.501322	59.04	1.03	383.8	6.6958	0.0012	0.6016	0.401	0.9161	-0	-0	-0.02	0.0214	-0	0	-0	0

Table A5 bending analysis when aspect ratio is 3 with h/b 0.75 along major axis

									Nϕ		Nθ		Mϕ		Mθ	
λ	$^{\psi}$degree	ø radian	λ^{φ}degree	λ^{φ} radian	e-λ^{ψ}	cot(ø)	sin(λ^{φ})	cos(λ^{φ})	He	Me	He	Me	He	Me	He	Me
134.5814	0	0	0	0	1	0.5	0	1	0	0	-30	33.541	0	1	0	0.3
104.6643	7	0.1221	732.7	12.78	0	0.5	0.213	0.9771	-0	-0	-0	7E-05	-0	0	-0	0
61.22044	14	0.2442	857.1	14.95	0	0.5	0.686	-0.727	-0	-0	0	-2E-05	-0	-0	-0	-0
35.16133	21	0.3663	738.4	12.88	0	0.5	0.309	0.951	-0	-0	-0	5E-05	-0	0	-0	0
21.6963	28	0.4884	607.5	10.6	0	0.5	-0.922	-0.388	2E-05	0	0	0.0004	0	-0	5E-05	-0
14.52857	35	0.6106	508.5	8.871	0	0.5	0.526	-0.85	-0	-0	0	-0.006	-0	-0	-0	-0
10.46507	42	0.7327	439.5	7.667	0	0.5	0.983	0.1855	-0	-0	-0	-0.013	-0	0	-0	0
8.022938	49	0.8548	393.1	6.858	0	0.5	0.544	0.8394	-0	-0	-0	0.0104	-0	0	-0	0
6.489009	56	0.9769	363.4	6.339	0	0.5	0.056	0.9984	-0	-0	-0	0.0558	-0	0	-0	0
5.452543	63.4	1.1065	345.9	6.033	0	0.5	-0.247	0.9689	4E-04	0	-0	0.0978	0	0	2E-04	0

Table A6 bending analysis when aspect ratio is 3.5 with h/b 0.75 along major axis

									Nϕ		Nθ		Mϕ		Mθ	
λ	ψ degree	ø radian	λ^{φ}degree	λ^{φ} radian	e-λ^{ψ}	cot(ø)	sin(λ^{φ})	cos(λ^{φ})	He	Me	He	Me	He	Me	He	Me
183.2	0	0	0	0	1	0.429	0	1	0	0	-29.9	33.5	0	1	0	0.25
125.4	7.5	0.131	940.63	16.41	0	0.429	-0.645	-0.764	3E-08	6E-09	2E-06	-0	0	-0	0	-0
61.58	15	0.262	923.75	16.11	0	0.429	-0.395	-0.919	2E-08	9E-09	2E-06	-0	0	-0	0	-0
31.95	22.5	0.393	718.96	12.54	0	0.429	-0.025	0.9997	5E-08	4E-08	-0	0	0	0	0	0
18.74	30	0.523	562.34	9.81	0	0.429	-0.375	-0.927	1E-05	2E-05	1E-03	-0	0	-0	0	-0
12.26	37.5	0.654	459.74	8.02	0	0.429	0.9863	-0.165	-2E-04	-0	9E-04	-0	-0	0	-0	0
8.755	45	0.785	393.99	6.873	0	0.429	0.5562	0.831	-3E-04	-0	-0.01	0.01	-0	0	-0	0
6.718	52.5	0.916	352.67	6.152	0	0.429	-0.131	0.9914	2E-04	6E-04	-0.03	0.08	0	0	0	0
5.474	60	1.047	328.43	5.729	0	0.429	-0.526	0.8505	0.001	0.004	-0.04	0.15	0	0	0	0
4.757	66.8	1.165	317.78	5.544	0	0.429	-0.674	0.7387	0.002	0.008	-0.04	0.19	0	0	0	0

Table A7 bending analysis when aspect ratio is 4 with h/b 0.75 along major axis

									Nϕ		Nθ		Mϕ		Mθ	
λ	ψdegree	ø radian	λ^{φ}degree	λ^{φ} radian	e-λ^{ψ}	cot(ø)	sin(λ^{φ})	cos(λ^{φ})	He	Me	He	Me	He	Me	He	Me
239.3	0	0	0	0	1	0.38	0	1	0	0	-29.9	33.54	0	1	0	0.25
140.4	8	0.14	1123	19.6	3E-09	0.38	0.68	0.73	-0	-0	-0	6E-09	-0	4E-09	-0	1E-09
58.52	16	0.28	936.3	16.3	8E-08	0.38	-0.585	-0.81	0	0	1E-06	-0	0	-1E-07	3E-07	-0
28.15	24	0.42	675.6	11.8	8E-06	0.38	-0.704	0.71	0	0	-0	4E-04	0	4E-08	1E-05	1E-08
16	32	0.56	511.9	8.93	1E-04	0.38	0.475	-0.88	-0	-0	0.002	-0.01	-0	-5E-05	-0	-0
10.35	40	0.7	413.8	7.22	7E-04	0.38	0.805	0.59	-0	-0	-0.01	-0.01	-0	0.001	-0	3E-04
7.386	48	0.84	354.5	6.18	0.002	0.38	-0.098	1	0	0	-0.03	0.076	0	0.002	7E-05	5E-04
5.706	56	0.98	319.5	5.57	0.004	0.38	-0.651	0.76	0	0.01	-0.04	0.18	0	4E-04	6E-04	1E-04
4.706	64	1.12	301.2	5.25	0.005	0.38	-0.857	0.52	0	0.01	-0.04	0.24	0	-0.002	9E-04	-0
4.267	69.44	1.21	296.3	5.17	0.006	0.38	-0.898	0.44	0	0.02	-0.03	0.255	0	-0.003	9E-04	-0