REVIEW OF MAGNETORESISTANCE AND HALL EFFECT IN METALS AND SEMICONDUCTORS

By

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Abstract

We discuss the theory of transport properties of charged particles in an electric and magnetic fields on aspects relevant to Hall effect and magnetoresistancce. We begin by reviewing magnetic properties of matter in relation to the external applied magnetic field. We show that charged particles moving in a magnetic field follow a curved path because of Lorentz force. When a current carrying conductor is placed in a magnetic field, the charge carriers begin following the curved path in the sample of a conductor until the field is balanced by the Hall field produced. This deflection of electrons from their line of path gives rise to the increase in path length of electrons in the conductor and in turn this reduces the effective current in the conductor. As a result at room temperature and rather low values of magnetic field, resistance of the material increases linearly with the magnetic field strength. And finally we tried to review that at high magnetic fields and low temperatures (about 4 K), the Hall resistance does not increase linearly with the field; instead, the plot showed a series of "stair steps". The explanation for this effect involves the circular paths in which electrons are forced to move by the field. As the field increases, the orbital radius decreases, permitting more orbits to bunch together on one side of the material. In this regard integer (IQHE) and fractional (FQHE) quantum Hall effects are discussed.

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Chapter 1

Introduction

It is the goal of this project to review on the Hall effects which are associated with the motion of charged particles in a magnetic field. These effects and the resulting change in resistance of a material (called magnetoresistance) when placed in a magnetic field demonstrate that the Lorentz force is responsible for such effects.

In 1879 Edwin Hall observed that when a magnetic field is applied at right angles to the direction of current flowing in a conductor, an electric field is created in a direction perpendicular to both the current and the magnetic field. We call this effect the Hall effect. The Hall effect describes the behavior of the free carriers in metal or in semiconductor when applying an electric as well as a magnetic field. Hall also observed that when an electric current passes through a sample placed in a magnetic field, a potential proportional to the current and to the magnetic field is created. This effect is the basis of many practical applications and devices such as magnetic field measurements, position and motion detectors. With the measurements he made, Hall was able to determine, for the first time, the sign of charge carriers in a conductor. Even today, Hall effect measurements continue to be a useful technique for characterizing the electrical transport properties of metals and semiconductors [1].

One may consider the Hall effect to result from the force on a charge moving in a magnetic field. Consider a positive charge moving at the velocity vector $\vec{v}$ through the magnetic field $\vec{B}$, the force would be in the direction of $\vec{v} \times \vec{B}$. If the current consisted of negative charges moving in the negative x-direction, the force would be in the direction of $-\vec{v} \times \vec{B}$. The potential difference induced in this effect is called the Hall voltage, and the sign of the Hall voltage allows one to determine the sign of carriers of current [2].

The Hall effect, the Quantum Hall effect and magnetoresistivity are phenomena that are important to develop an understanding of magnetoresistance (MR) in metals and semiconductors. The sources of insight are the effect of magnetic fields on resistivity, and the relationship that has to conduct electron orbital and angular momentum. Hall's original experiments measured the electrical potential at right angles to the current in a thin metal sheet was perpendicular to a magnetic field. The measured potential is generated in these experiments by cyclotron motions of the electrons in the magnetic field and the electric field derives the current. The electron cyclotron motion changes the path length for the current which results in the observed increase in resistivity with applied magnetic field. The electron orbital- angular momentum can also be influenced by the external magnetic field because of its connection with cyclotron motion. Orbital angular momentum is the quantum number that changes in the Quantum Hall effect. The influence of magnetic fields on electron orbital angular momentum and conducting electron path length are two major factors in MR.

The Hall effect is a widely used tool in semiconductors, and in combination with Landau levels in a two-dimensional electron gas gives the integral quantum Hall effect (IQHE), where the Hall conductance is an exact multiple of e^2/h [3]. QHE involves motion of electrons on a two-dimensional surface in the presence of a perpendicular magnetic field. The magnetic field has induced a great condensation of a continuous energy spectrum of a free particle in 2D into a discrete set of highly degenerate levels. These levels are called Landau levels, which are equally spaced by the cyclotron energy $\hbar\omega_c$, which itself proportional to the magnetic field strength. The gaps between the levels are void of electronic states.

For 3D electrons, no such gaps occur. The IQHE occurs where n number of electrons exactly fill an integral number of Landau levels resulting in an integral value of the filling factor i, where i is equal to an integer. There is an energy gap equal to $\hbar\omega_c$, between the filled states and the empty states. This makes the non-interacting electron system incompressible, because an infinitesimal decrease in the area A, which decreases in the electron concentration n that requires a finite energy to promote an electron across the energy gap in to the first un-occupied Landau level. This incompressibility is responsible for the IQHE.

To understand the minima in the diagonal resistivity ρ_{xx}, and the plateaus in the Hall resistivity ρ_{xy}, it is necessary to notice that each Landau level broadened by collisions with defects and phonon to have both extended sates and localized states. The extended states lie in the central broadened Landau level, and the localized states in the wings [4]. As the chemical potential sweeps through the Landau level, by varying either $\vec{B}$ or the particle number n, zeros of ρ_{xx} at $T = 0\ K$ and flat plateaus of ρ_{xy} occur when chemical potential lies within the localized states. When the filling factor $i = \nu$ is smaller than unity, the standard approach of placing n particles in the lowest energy single particle states is not applicable, because more degenerate states than the number of particles present in the lowest Landau level. For example, for the case of $\nu = 1/3$, it is not apparent how to construct antisymmetric product function for n electrons is $3n$ states to describe fractional quantum Hall effect (FQHE). In FQHE, no gap occurs in the absence of electron-electron interaction and it is not easy to understand why fractional quantum Hall states are incompressible. The incompressibility of the FQHE is completely the result of electron-electron interactions in a highly degenerate fractionally filled Landau level [5].

The Hall effect is basic to solid state physics and an important diagnostic tool for the characterization of materials particularly semiconductors. It provides a direct determination of both the sign of the charge carriers, e.g., n-types or p-types, and their densities in a given sample. The Hall effect may be used to determine the sign of the moving charges that form the electric current. It also has many important practical applications in Hall effect sensors in detecting and measuring magnetic fields. For example, electronic ignition systems for cars now days are based on Hall effect sensors, rather than mechanical breaker points used up to a few years ago [6,7].

The project is organized as follow: In Chapter 1: It deals with the introduction to MR and details of Hall effects are given. In Chapter 2: magnetic properties of materials are reviewed. In Chapter 3: general transport properties of charged particles through electric and magnetic fields in relation to the Hall effect and MR are discussed. General properties of two-dimensional electron gas (2DEG) in a magnetic field at low temperature are presented. In Chapter 4: results and discussion in semiconductor hetrostructure are given. In Chapter 5, conclusions are given.

Chapter 2

Magnetic Properties of Matter

There are a few basic concepts and ideas concerned with magnetic materials. The fundamental types of magnetic properties observed in an experiment in which magnetization $\vec{M}$, acquired in response to application of a magnetic field $\vec{H}$, is monitored. The fact that some atoms have atomic magnetic moments because of orbital and spin motion of electrons. Atomic magnetic moments are quantized. Because of unfilled $3d$ electron orbital, transition elements (solids) have the common crystals with atoms possessing a magnetic moment. Being a moving charge, electrons produce a small magnetic field having magnetic moment along the axis of rotation. The spin of electrons also produces magnetic moment along the spin axis [8].

2.1 Magnetization and susceptibility

Any system may be characterized by its response to external stimuli. For example, in electronics the proverbial "black box" is characterized by its output voltage when an input current is applied. This transfer impedance, as it is called, provides all the information necessary to understand the operation of the black box. If we know what is in the black box - for example, the detailed arrangement of resistors, diodes, etc, then we can predict through analysis, what the transfer impedance will be.

Similarly, a system of charges and currents, such as a crystal, may be characterized by a response function. In this case the output is the magnetization and the response function is the magnetic susceptibility. Determination of the susceptibility entails evaluations of the magnetization produced by an applied magnetic field. In general, this applied field may depend on space and time. The resulting magnetization also vary in space and time [7]. Most of the magnetic properties that we shall consider arises from electrons. It has been found experimentally that the electron possesses an intrinsic magnetic moment, or spin.

A magnetic field is produced by an electrical charge in motion; e.g., current flowing in a conductor, orbital movement and spin of electrons produce a magnetic field within a material. The magnetic field, $\vec{H}$, generated by a cylindrical coil called solenoid of N turns and length L is given by

$$H = \frac{NI}{L}. \tag{2.1}$$

The magnitude of the field strength within a substance subjected to a field $\vec{H}$ is defined as magnetic flux density, $\vec{B}$, given by

$$B = \mu H = \frac{\mu NI}{L}, \tag{2.2}$$

where μ is called permeability that measures the degree to which a material can be magnetized. In vacuum,

$$B = \mu_0 H, \tag{2.3}$$

where μ is the permeability of free space (vacuum) and is a universal constant whose value is $4\pi \times 10^{-7}\ H/m$. In a solenoid the magnetic field $\vec{H}$ of N turns per unit length L $(n = N/L)$ is equal to nI, where I is the current, so that

$$B = \mu_0 nI. \tag{2.4}$$

If there is an iron core of permeability, μ_r inside the coil,this does not change H. It remains nI. The magnitude of the magnetic flux density $\vec{B}$, however, is now $B = \mu H =$

$\mu_r\mu_0 H$. This is greater than $\mu_0 H$ and we rewrite [9,10]

$$\vec{B} = \mu_0(\vec{H} + \vec{M}), \tag{2.5}$$

where the quantity M is called the magnetization of the material. We can see that there are two components to B. These are $\mu_0 H = \mu_0 nI$, which is the externally imposed field and the component $\mu_0 M$, originating as a result of something that has happened within the material. So that the magnetization would be the excess of B over $\mu_0 H$. The ratio of the magnetization M (the result) to the magnetic field H (the cause), which is obviously the measure of how susceptible the material is to become magnetized is called the magnetic susceptibility χ_m of the material [11]. That is,

$$\chi_m = \frac{M}{H}. \tag{2.6}$$

The magnetic susceptibility can be dependent on the applied magnetic field $\vec{B}$.

Magnetism in a material arises due to alignment of magnetic moments. With the application of a magnetic field magnetic moments in a material tend to align and thus increase the magnitude of the field strength. This increase is given by the parameter called magnetization, $\vec{M}$, such that $\vec{B} = \mu_0(\vec{H} + \vec{M})$. Depending on the existence and alignment of magnetic moments with or without application of magnetic field, three types of magnetism can be defined.

2.1.1 Diamagnetism

Diamagnetism is a weak form of magnetism which arises only when an external magnetic field is applied. It arises due to change in the orbital motion of electrons on the application of magnetic field. There is no magnetic dipoles in the absence of externally applied magnetic field and when a magnetic field is applied, the dipole moment is aligned opposite (see Fig. 2.1) to the field direction [11,12]. The magnetic susceptibility $\chi_m = \mu_r - 1$ is negative so that $\vec{B}$ in such material is less than that of vacuum.

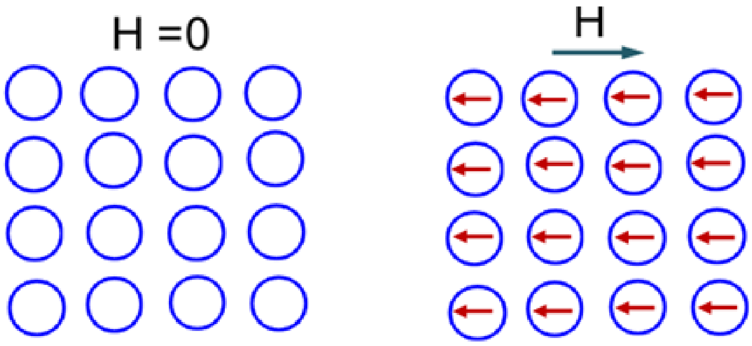

Figure 2.1: Diamagnetism in diamagnetic material.

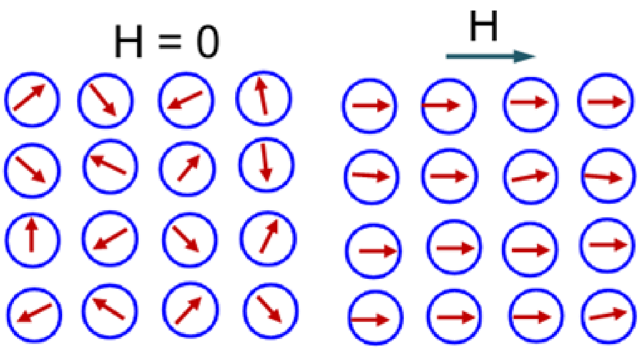

Figure 2.2: Paramagnetism.

2.1.2 Paramagnetism

In a paramagnetic material the cancelation of magnetic moments between electron pairs is incomplete and hence magnetic moments exist without any external magnetic fields. However,in the Fig.2.2 we can observe that the magnetic moments are randomly aligned and hence no net magnetization without any external field. when the magnetic field is applied all the dipole moments are aligned in the direction of the field [12]. Magnetic field intensity $\vec{B}$, in the paramagnetic material is slightly greater than that of vacuum.

2.1.3 Ferromagnetism

Certain materials posses permanent magnetic moments even in the absence of an external magnetic field. This is known as ferromagnetism.

Permanent magnetic moments in ferromagnetic materials arise due to un-canceled electron spins by virtue of their electron structure. The coupling interactions of electron spins of adjacent atoms cause alignment of moments with one another. The origin of this coupling is attributed to the electron structure by which the moments aligned in the same direction and results a net magnetic moments. Magnetic materials like $Fe(26 - ...4s^2 3d^6)$ have unpaired electron spins [13].

In addition to the tendency of these electrons to align themselves in the same direction as an applied magnetic field, they will also align themselves so that they are parallel to each other. This means that, even when the applied field is removed, the electrons in the material maintain a parallel orientation. Examples of ferromagnetic materials are nickel, iron, cobalt and their alloys.

Ferromagnetic materials exhibit small volume regions in which magnetic moments are aligned in the same directions. These regions are called domains. Adjacent domains are separated by domain boundaries. The magnitude of magnetization in the material is the vector sum of magnetization of all domains.

2.2 Charge transport in an electric field

The physics of transport in metals and semiconductors is treated foremost for charge transport. Charge and heat energy can be transported through the metal and the semiconductor in the presence of appropriate forces. Such a force can be an electric field or a temperature gradient. Both transport phenomena are coupled since electrons transport energy and charge simultaneously through the crystal.

2.2.1 Conductivity and mobility

Carriers move in the metal and semiconductor driven by a gradient in the Fermi energy. Conductivity in metals and semiconductors is determined by the carrier concentration (free electrons and holes) and scattering mechanisms (mobility). Many semiconductor properties, such as carrier concentration and the band gap, depend on the temperature.

Under the influence of an electric field the electrons accelerate according to

$$\vec{F} = m^* \frac{d\vec{v}}{dt} = \hbar \frac{d\vec{k}}{dt} = q\vec{E} = -e\vec{E}. \tag{2.7}$$

In the following q denotes a general charge while e is the elementary charge, and m^* is an isotropic effective mass. After a time δt the $\vec{k}$ vector of all conduction electrons (and the center of the Fermi surface) has been shifted by δk.

$$\frac{d\vec{k}}{dt} = \frac{-e\vec{E}}{\hbar}\delta t. \tag{2.8}$$

In the absence of scattering processes this goes on further (similar to an electron in vacuum). This regime is called ballistic transport.

In a real semiconductor, at finite temperatures impurities, phonon and defects will contribute to scattering. In the relaxation time approximation it is assumed that the probability for a scattering event, similar to friction, is proportional to the average carrier velocity. The average relaxation time τ is introduced via in an additional term $\dot{v} = -v/\tau$ that sums up all scattering events. Thus the maximum velocity that can be reached in a static electric field is given by (steady state velocity)

$$\vec{v} = -\frac{e\vec{E}}{m^*}\tau. \tag{2.9}$$

The current density per unit area is then linear in the field, i.e., fulfills Ohms law,

$$\vec{J} = nq\vec{v} = \frac{ne^2\vec{E}}{m^*}\tau = \sigma\vec{E}. \tag{2.10}$$

The conductivity σ in the relaxation-time approximation is given by

$$\sigma = \frac{1}{\rho} = \frac{ne^2}{m^*}\tau. \tag{2.11}$$

The specific resistivity is the inverse of the conductivity. Metals have a high conductivity. The mean free path $d = \tau v_F$ is

$$d = \frac{\sigma m^*}{ne^2} v_F, \tag{2.12}$$

where v_F is being the Fermi velocity in the Fermi energy ($E_F = \frac{1}{2} m^* v_F^2$).

In semiconductors, the carrier concentration depend on strongly on the temperature. At zero temperature the conductivity is zero. Also, the scattering processes and thus the relaxation time constant exhibit a temperature dependence. The conductivity spans a large range from insulating to almost metallic conduction.

2.2.2 Low and high-field transport

In the low-field regime the velocity is proportional to the electric field. The mobility is defined (scalar terms) as

$$\mu = \frac{v}{E}. \tag{2.13}$$

By definition, it is a negative number for electrons and positive for holes. However, the numerical value is usually given as a positive number for both carrier types. In an intrinsic semiconductor the mobility is determined by scattering with phonon. Further scattering is introduced by impurities, defects and alloy disorder. Using equation (2.11) the conductivity is

$$\sigma = qn\mu, \tag{2.14}$$

for each carrier type. And using equation (2.10) the mobility in the relaxation time approximation is

$$\mu = \frac{q\tau}{m^*}, \tag{2.15}$$

In the presence of both electrons and holes,

$$\sigma = \sigma_e + \sigma_h = -en\mu_n + ep\mu_p, \tag{2.16}$$

where μ_n and μ_p are the mobilities for electrons and holes, respectively. These are given by $\mu_n = -e\tau_n/m^*$ and $\mu_p = -e\tau_p/m^*$. As the unit for mobility, usually cm^2/Vs is used. At room temperature Cu has a mobility of 35 cm^2/Vs, semiconductors can have much higher values. In two dimensional electron gases, the mobility can reach several 10^7 cm^2/Vs at low temperature.

In the case of small electric fields the scattering events are elastic. The drift velocity is linearly proportional to the electric field. The average thermal energy is close to its thermal value $3kT/2$ and the carriers are close to their band edges. The scattering efficiency, however, is reduced at moderate fields. Then, the electron temperature becomes larger than the lattice temperature. With increasing electric field the carriers can gain more and more energy and will on average populate higher states.

In magnetic fields, electrons (holes) perform a cyclotron motion with frequency $\omega_c = eB/m^*$. The motion is perpendicular to the magnetic field on a line of constant energy in k-space. This line is the intersection of a plane perpendicular to the magnetic field and the respective iso-energy surface in k-space. The ballistic cyclotron motion can only occur between two scattering events. Thus, a significantly long path along the cyclotron trajectory and the connected magnetotransport properties are only possible when $\omega_c >> 1$, i.e. when the average scattering time τ is sufficiently large. This requires high mobility. And the magnetic field is sufficiently strong and the temperature is sufficiently low, i.e., $\omega_c >> KT$, such that thermal excitations do not scatter electrons between a different landau level [24].

Chapter 3

Galvanomagnetic effects

The kinetic phenomena that occur in a sample due to the simultaneous effect of an electric field $\vec{E}$ and a magnetic field $\vec{B}$ are called galvanomagnetic effects.

The galvanomagnetic effects are a well-studied subject in condensed matter physics. The subject is about the generation of electric field in a metal or in the semiconductors by an electric current. These effects are often used to probe the electronic properties of matter.

The galvanomagnetic effects include many well-known phenomena such as classical and quantum Hall effects, extraordinary MR, anisotropic MR and de Haas- Schubnikov oscillations as well as recently discovered spin Hall and inverse spin Hall effect.

Within the linear response region, the theories of galvanomagnetic effects are well-developed. For example, one can use the Kubo formula or Boltzmann theory to compute resistivity tensor of a given microscopic model in the presence or absence of a magnetic field. One can also use semiclassical dynamical equations for quasi-electrons or holes to obtain current[**14**].

3.1 The theory of magnetoresistance

The presence of magnetic field affects the resistivity of the sample. This effect of changing resistance of a material under the application of externally applied magnetic field is known as MR. It is caused by the trajectory that the moving charges follow according to the Lorentz force. By assuming that the modulus of the speed of the charge is kept to v_d, the additional distance traveled relative to the straight line is calculated. This deviation reduces the effective x-component of the current hence increases the resistance. This brings about the change of electrical resistivity of a material.

In ordinary metals at room temperature resistance vary by a few tenths percent but in semiconductors the MR is much larger and it depends on the impurity concentration, mobility of charge carriers and temperature. MR is quantitatively characterized by a scalar $\Delta\rho/\rho_0$, where $\Delta\rho$ is change of resistivity in the magnetic field and ρ_0 is resistivity without magnetic field. Let us consider the cause of an infinite semiconductor. The deviation of the carriers from the direction of electric field $\vec{E}$ is equivalent to the decrease of the mean free path of the carriers λ_0 in the direction of $\overrightarrow{E}$ [**15**].

$$\Delta\lambda \approx \lambda_0 \frac{\mu^2 B^2}{2} \tag{3.1}$$

where **B** is magnetic field intensity and μ is carriers' mobility. Decrease of λ_0 is equivalent to the decrease of the carriers' drift velocity, which in turn is proportional to the conductivity of the semiconductor,σ. Consequently,

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\Delta\rho}{\rho_0} \rightarrow \frac{\Delta\rho}{\rho_0} \sim \frac{\mu^2 B^2}{2}. \tag{3.2}$$

As a result the relative change in electric resistance of semiconductor is determined by

$$\frac{\Delta\rho}{\rho_0} = c\mu^2 B^2 \tag{3.3}$$

where c is a coefficient depending on the geometry of the semiconductor wafer.

Now let us consider a finite sample. In this case the charge carriers move along a straight line since the Hall field component influences the magnetic field and therefore MR should not exist. However, magnetic field influences are stronger than the Hall field, especially for a fast moving carriers. So the slow carriers deviate more under the influence of the Hall field. It is known that the velocity of electrons and holes are different so that the dispersion in the carriers' velocity leads to the increase of resistivity. The enhancement in the resistance or resistivity under the applied field is known as positive MR while the suppression in resistance or resistivity under applied field is termed as negative MR [**2**].

Negative MR is the term given to the large decrease in the electrical resistance when certain systems are exposed to a magnetic field. The negative MR is usually defined as a percentage ratio:

$$MR = -(\frac{\rho(0) - \rho(B)}{\rho_0}) \times 100\% = -\frac{\Delta\rho}{\rho_0} \times 100\% \tag{3.4}$$

Where $\rho(B)$ and ρ_0 are the resistivity in the presence and absence of magnetic field, B respectively. When the MR is dependent on the angle between the current direction and the orientation of the magnetization M, it is called anisotropic magnetoresistance (AMR). For symmetry reasons the MR is proportional to $\mathbf{B}$. Since MR arises from the complicated orbits of the electrons on the Fermi surface, the MR has large crystallographic anisotropy. Even in an isotropic conductor, there exists two possible configurations for MR. When the current is parallel to the applied magnetic field the longitudinal MR is measured (both the current density and the measured electric field are parallel to B). If the magnetic field is perpendicular to the current path then the transverse MR is being measured (both the applied current density and the measured electric field are in the plane perpendicular to B) [**15**].

The simplest example of MR is transverse MR associated with the Hall effect. when an electric conductor is subjected to an electric field $\mathbf{E}$ along x and simultaneously a magnetic field $\mathbf{B}$ along z, a new transverse field arises due to the Lorentz force of $\mathbf{B}$ on the electron moving along x. This field acts along y (because the Lorentz force is a cross product) giving rise to the Hall voltage and there is a MR associated with the transverse field. If the material itself is not isotropic, different crystallographic orientations will have distinct longitudinal and transverse MR. In low fields the magnitude of **M** does not change, only orientation. If M is aligned parallel or anti-parallel to the current, the resistance is different than if **M** were perpendicular to the current. This difference is the AMR. It is actually proportional to the magnetization rather than the magnetic field[**9**].

3.1.1 The Drude theory and the Hall effect

Before considering the effect of magnetic fields on conductors, we need some models to describe the flow of currents in response to electric fields. To do so we will use the Drude theory of conductors. The Drude model envisions a conductor as a gas of free current-carrying charges. The freely moving charges suffer randomizing collision events on an average of every τ seconds. The parameter τ is called the relaxation time, and is the only feature describing the collision events. Because of collision, particles in a conductor accelerates in the direction of the net force $\vec{F}$ consistent with its initial conditions dictated by the last collision.

From the basic assumptions of the Drude theory of metal, collisions are instantaneous events that abruptly alter the velocity of an electron. The dependence of resistance on the shape of the wire generally eliminates by introducing resistivity, a quantity characteristics only of the metal of which the wire is composed. The resistivity ρ is defined to be the proportionality constant between the electric field **E** at a point in the metal and the current density **J** that it induces ($\vec{E} = \rho\vec{J}$).

The current density J is a vector, parallel to the flow of charge, whose magnitude is the amount of charge per unit time crossing a unit area perpendicular to the flow of charge ($J = I/A$). The potential drop along the wire of length L will be $V = EL$. If n number of electrons per unit volume all move with velocity **v**, then the current density they give rise to will be parallel to $\vec{v}$ and is given by equation (2.10). In the equation, **v** is the average electronic velocity which is directed opposite to the field (as the electronic charge being negative).

The average velocity, called drift velocity, of an electron starting from zero during the average time $t = \tau$ as given in the equation (2.9) is:

$$\vec{v} = -\frac{e\tau}{m}\vec{E}. \tag{3.5}$$

And thus the current density per unit area can be rewritten as

$$\vec{J} = \frac{ne^2\tau}{m}\vec{E} \tag{3.6}$$

This result is usually stated in terms of the inverse of the resistivity, $\sigma = 1/\rho$

$$\rho = \frac{m}{ne^2\tau}. \tag{3.7}$$

To calculate the Hall coefficient and the magnetoresistance we first find the current densities J_x and J_y in the presence of an electric field with arbitrary components E_x and E_y, and in the presence of a magnetic field $\mathbf{B}$ along the z-axis. The net force acting on each electron is

$$\frac{d\vec{p}}{dt} = -e\left(\vec{E} + \frac{\vec{P}}{m} \times \vec{B}\right) - \frac{\vec{P}}{\tau}. \tag{3.8}$$

where $\mathbf{p}$ is the average momentum of each carrier. In the steady state, the current is independent of time and therefore, p_x and p_y will satisfy

$$0 = -eE_x - \omega_c p_y - \frac{p_x}{\tau}$$

and

$$0 = -eE_y + \omega_c p_x - \frac{p_y}{\tau} \tag{3.9}$$

where $\omega_c = eB/m$. We multiply these equations (3.9) by $-ne\tau/m$ and introduce the current density components through $\mathbf{J} = -ne\mathbf{v}$ to find

$$\sigma_0 E_x = \omega_c \tau J_y + J_x$$

and

$$\sigma_0 E_y = -\omega_c \tau J_x + J_y \tag{3.10}$$

where σ_0 is just the Drude model Dc conductivity in the absence of a magnetic field as given by $\sigma = (ne^2\tau)/m$. The transverse field E_y balances the Lorentz force, which is proportional to both the applied field $\vec{B}$ and to the current along the wire, (J_x). The Hall field is determined by the requirement that there will be no transverse current J_y. So that setting $J_y = 0$ in (3.10) we find that

$$E_y = -\frac{\omega_c\tau}{\sigma_0}J_x = -\left(\frac{B}{ne}\right)J_x. \tag{3.11}$$

One therefore defines a quantity known as the Hall coefficient by

$$R_H = -\frac{E_y}{J_xB}. \tag{3.12}$$

There are two quantities of interest. One is the ratio of the field along the wire E_x to the current density J_x

$$\rho(H) = \frac{E_x}{J_x}. \tag{3.13}$$

This is the magnetoresistance, which Hall found to be field independent. The other is the size of the transverse field E_y. Since it balances the Lorentz forces, one might expect to be proportional both to the applied field $\vec{B}$ and to the current along the wire J_x. When $J_y = 0$, equation (3.10) reduces to $J_x = \sigma_0 E_x$, and the expected result for the conductivity at zero magnetic field, $\rho_{xx} = E_x/J_x$ is field independent MR called longitudinal MR [**9**].

Note that for electrons, since the Hall field is in the negative y-direction, R_H should be negative. If on the other hand, the charge carriers were positive, the direction of their velocities would be reversed, and the Lorentz force would therefore be unchanged. As a result the Hall field would be opposite to the direction it has for negatively charged carriers. In the intrinsic semiconductor, two types of charge carriers, electrons and holes, in the electric field drift to wards each other and are deflected by the magnetic field in the same direction.

3.1.2 Electrical conductivity and Ohm's law

The vector equation of motion for a particle of charge q and mass m in a solid, undergoing an external force F_e and a friction $f = -kv$ is given by:

$$\frac{md\vec{v}}{dt} = F_e - \frac{m\vec{v}}{\tau} \tag{3.14}$$

The friction force describes the braking of the particle due to its interactions(collisions) with the ions of the crystal lattice and with the other charge carriers.

Conduction of charge carrier requires a conductor that is characterized by a large amount of free carriers which provide conduction current due an impressed electric field. Since the charge carrier is not in free space, it will not be accelerated under the influence of the electric field. Rather, it suffers constat collision with the atomic lattice and drifts from atom to another.

Here, we suppose that if the external forces go back to zero, the state returns to its equilibrium position exponentially with a relaxation time τ.

$$\frac{d\vec{v}}{\vec{v}} = -\frac{1}{\tau}dt \rightarrow \vec{v} = \vec{v_0}e^{t/\tau} \tag{3.15}$$

If the external force remain constant, the system goes to a stationary state, i.e. $dv/dt = 0$. Supposing that the external force is due to a homogeneous electric field $\vec{E}$, the new stationary velocity, or drift velocity $\vec{v}_d$ of the charge carriers becomes:

$$\vec{v}_d = \mu\vec{E} \tag{3.16}$$

For the solid material containing n charge carriers per unit volume, under the action of an electric field $\vec{E}$ and a permanent regime, the charge carriers move with an average drift velocity in the same direction as the electric field. we have seen that within a conductor the electric current density and the electric field are related by $\vec{J} = \sigma\vec{E}$. This can be written as

$$\vec{\nabla}.\vec{J} = \sigma\vec{\nabla}.\vec{E} = \frac{\sigma}{\epsilon}\rho \tag{3.17}$$

From the continuity equation ($\vec{\nabla}.\vec{J} + \partial\rho/\partial t = 0$)

$$\frac{\partial\rho}{\partial t} + \frac{\sigma}{\epsilon}\rho = 0 \tag{3.18}$$

So that the solution of the equation is

$$\rho(\vec{x}, t) = \rho(\vec{x}, 0)e^{-\sigma/\epsilon t} \tag{3.19}$$

For good conductors $\sigma/\epsilon = 10^{14}s^{-1}$, so that we can conclude that charges move almost instantly to the surface of the conductor. The ratio ϵ/σ is called the relaxation time,τ of the conducting medium. For perfect conductor $\sigma = \infty$, implies that the relaxation time is vanishing.

When electric field is applied, the force on the electron with charge $-e$ is $\vec{F} = -e\vec{E}$. In free space, the electron would accelerate at $\vec{a} = -e\vec{E}/m$. In a material, the electrons will suffer continual collisions. So that the electrons will move with different velocities between collisions. The average velocity, is called drift velocity, is given by equation (3.5) And the conduction current density is expressed using equation (3.6)

The equation $J = \sigma\vec{E}$ is known as Ohm's law. Ohm's law tells us how the current flows in response to an electric field. The proportional quantity σ is called the conductivity [9].

3.1.3 Magnetoresistance in 3D and 2D systems

Consider a metal with a simple spherical Fermi surface and isotropic energy-independent effective mass. The magnetic field $\vec{B}$ will be parallel to z, and we shall use the symbols (effective mass, m^* scattering rate τ^{-1} and electronic charge, $-e$. We assume that the drift velocity $\vec{v}_d$ of each species of carrier can be treated using the relaxation time approximation:

$$m^*\left(\frac{d\vec{v}}{dt} + \frac{\vec{v}}{\tau}\right) = q\vec{E} + q\vec{v} \times \vec{B}. \tag{3.20}$$

So that the components of the equation of motion are found to be:

$$m^*\left(\frac{dv_x}{dt}+\frac{v_x}{\tau}\right)=qE_x+qv_yB$$

$$m^*\left(\frac{dv_y}{dt}+\frac{v_y}{\tau}\right)=qE_y-qv_xB$$

$$m^*\left(\frac{dv_z}{dt}+\frac{v_z}{\tau}\right)=qE_z. \tag{3.21}$$

We note this equation of motion is also valid for holes, given the convention positive effective mass and charge. In the equation $\vec{J}=\sigma\vec{E},\sigma$ is a tensor, $J\imath \vec{=} \sigma_{\imath\jmath}\vec{E}\jmath$. so that

$$\vec{J}=\sigma\vec{E}=\begin{pmatrix}J_x\\J_y\\J_z\end{pmatrix}=\begin{pmatrix}\sigma_{xx}&\sigma_{xy}&\sigma_{xz}\\\sigma_{yx}&\sigma_{yy}&\sigma_{yz}\\\sigma_{zx}&\sigma_{zy}&\sigma_{zz}\end{pmatrix}\begin{pmatrix}E_x\\E_y\\E_z\end{pmatrix} \tag{3.22}$$

From the conductivity tensor and using Ohm's law ($J=\sigma E$) current density is approximately defined by

$$\begin{aligned}J_x &= \sigma_{xx}E_x+\sigma_{xy}E_y+\sigma_{xz}E_z\equiv -nev_x,\\J_y &= \sigma_{yx}E_x+\sigma_{yy}E_y+\sigma_{yz}E_z\equiv -nev_y,\\J_z &= \sigma_{zx}E_x+\sigma_{zy}E_y+\sigma_{zz}E_z\equiv -nev_z.\end{aligned}$$

equation Thus from equation (3.21) and the relation that $\vec{J}=-ne\vec{v}$, we get

$$\bar{\sigma}=\begin{pmatrix}\sigma_{xx}&\sigma_{xy}&\sigma_{xz}\\\sigma_{yx}&\sigma_{yy}&\sigma_{yz}\\\sigma_{zx}&\sigma_{zy}&\sigma_{zz}\end{pmatrix}=\frac{\sigma_o}{1+\omega_c^2\tau^2}\begin{pmatrix}1&-\omega_c\tau&0\\\omega_c\tau&1&0\\0&0&1+\omega_c^2\tau^2\end{pmatrix} \tag{3.23}$$

where $\sigma_o=\frac{ne^2\tau}{m}$ and $\omega_c=eB/m$. The components of the conductivity tensors are

$$\sigma_{xx}=\sigma_{yy}=\sigma_0\frac{1}{1+\omega_c^2\tau^2}=\sigma_0\frac{1}{1+\mu^2B^2}$$

$$\sigma_{yx}=-\sigma_{xy}=\sigma_0\frac{\omega_c\tau}{1+\omega_c^2\tau^2}=\sigma_0\frac{\mu_B}{1+\mu^2B^2}$$

$$\sigma_{zz} = \sigma_0 = \frac{nq^2\tau}{m^*} = qn\mu \tag{3.24}$$

If only one type of carrier(charge q, density n) is considered, the Hall condition $J_y = 0$ leads to $E_y = \mu B E_x$ and $J_x = \sigma_0 E_x$. The Hall coefficient can be rewritten as

$$R_H = \frac{E_y}{J_x B} = \frac{\rho_{xy}}{B} = \frac{\mu}{\sigma_0} = \frac{1}{nq} \tag{3.25}$$

where the resistivity tensor ρ is the inverse of the conductivity tensor σ,

$$\rho = \sigma^{-1} = \begin{pmatrix} \rho_{xx} & \rho_{xy} & \rho_{xz} \\ \rho_{yx} & \rho_{yy} & \rho_{yz} \\ \rho_{zx} & \rho_{zy} & \rho_{zz} \end{pmatrix} = \frac{1}{\sigma_0} \begin{pmatrix} 1 & \omega_c\tau & 0 \\ -\omega_c\tau & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.26}$$

Thus we can see that the components of the resistivity tensor are

$$\rho_{xx} = \rho_{yy} = \rho_{zz} = \frac{1}{\sigma_0} = \rho_0 = \frac{m}{nq^2\tau} \tag{3.27}$$

$$\rho_{yx} = -\rho_{xy} = \frac{\omega_c\tau}{\sigma_0} = \omega_c\tau\rho_0 = \frac{B}{nq} \tag{3.28}$$

In the case of 2D system, all changes in $\vec{v}$ occur in the plane perpendicular to $\mathbf{B}$; therefore, it is sufficient to split equation (3.20) in to x and y components. For a steady state of a system [**16**], i.e.$(dv_x)/dt = (dv_y)/dt = 0$,solving equation (3.21) simultaneously, we have

$$\frac{m^* v_y}{\tau} = qE_y - \frac{qB\tau}{m^*}\Big(qE_x + qv_y B\Big). \tag{3.29}$$

Which can be rearranged to give

$$v_y\left(\frac{m^*}{\tau} + \frac{q^2B^2\tau}{m^*}\right) = qE_y - \frac{q^2B\tau}{m^*}E_x.$$

Dividing through by m^*/τ and making the identification $eB/m^* \equiv \omega_c$ gives

$$v_y\Big(1 + \omega_c^2\tau^2\Big) = \frac{q\tau}{m^*}\Big(E_y - \omega_c\tau E_x\Big)$$

and solving

$$v_x(1 + \omega_c^2\tau^2) = \frac{q\tau}{m^*}\Bigg(E_x + \omega_c\tau E_y\Bigg) \tag{3.30}$$

Hall effect experiments are usually quite deliberately carried out at law magnetic fields, such that $\omega_c\tau << 1$, implies that $\omega_c^2\tau^2$ can be neglected. Therefore equation (3.30) becomes

$$v_y = \frac{q\tau}{m^*}\Bigg(E_y - \omega_c\tau E_x\Bigg) = \frac{q\tau}{m^*}\Bigg(E_y - \frac{qB\tau}{m^*}E_x\Bigg).$$

and

$$v_x = \frac{q\tau}{m^*}\Bigg(E_x + \omega_c\tau E_y\Bigg) \tag{3.31}$$

For motion of electrons to be considered two things may be deduced from the equation. The motion of the electrons in the direction parallel to $\vec{B}$ is unaffected. Therefore, there will be no longitudinal MR: in this context, the longitudinal resistivity is measured in the direction parallel to the field $\vec{B}$ (i.e. both the applied current density and the measured electric field are parallel to B. There may well be transverse MR. Here transverse resistivity means that measured in the direction perpendicular to the field $\vec{B}$ i.e. both the applied current density and the measured electric field are in the plane perpendicular to $\vec{B}$. We are dealing with a steady state of the system, i.e.$(dv_x)/dt = (dv_y)/dt = 0$. Taking $\mathbf{B} = (0, 0, B)$ and $\mathbf{E} = (E_x, 0, 0)$ as defined above, we rewrite equation (3.21) in the form

$$v_x = -\frac{e\tau}{m^*}\Bigg(E_x + v_y B\Bigg)$$

and

$$v_y = \frac{e\tau}{m^*}v_x B \tag{3.32}$$

where $\vec{v}$ emphasis the fact that we are dealing with a drift velocity. Equation (3.22) shows that the magnetic field has made the conductivity anisotropic: it has become a tensor, rather than a scalar [**11**]. The slight novelty is that, in the presence of a magnetic field, σ is not a single number. It is a matrix that sometimes called the conductivity tensor.

The conductivity tensor shows that, in a magnetic field, the total current density J flows parallel to the applied field, E_x; instead, it now contains both x and y components.

The components of current density $J_x and J_y$ caused by electric field component E_x and magnetic field B_z. Equation (3.24) shows that as $B \to \infty, \sigma_{xx} \Rightarrow B^{-2} = 0$. We might therefore expect to see some MR. However, most experiments measure voltage drops in the x and y directions between pairs of contacts, rather than measuring the x and y components of the current density. In such a case the electric field will have components in both x and y directions. Therefore, we want the components

$$\rho_{xx} \equiv \frac{E_x}{J_x}$$

and

$$\rho_{yx} \equiv \frac{E_y}{J_X} \tag{3.33}$$

of the resistivity tensor, rather than the conductivity. The general conductivity tensor in 2D system is [**9**]

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \\ . & \end{pmatrix} \tag{3.34}$$

The derivations which start at Equations (3.32) and (3.33) can be repeated with $E = (0, E_y, 0)$ to yield $\sigma_{yx} = -\sigma_{xy}$ and $\sigma_{yy} = \sigma_{xx}$.

The structure of the matrix (tensor), with identical diagonal components, and equal but opposite off- diagonal components follows from rotational invariance. From the Drude model, we get the explicit expression for the conductivity

$$\sigma = \frac{\sigma_0}{1 + \omega_c^2 \tau^2} \begin{pmatrix} 1 & -\omega_c \tau \\ \omega_c \tau & 1 \end{pmatrix}. \tag{3.35}$$

This equation is called the static magnetoconductivity tensor in 2D system.

The off-diagonal terms in the matrix are responsible for the Hall effect. In equilibrium a current in the x-direction requires an electric field with a component in the y-direction. Putting an electric field in the x-direction gives rise to a current density J_x, but this current is deflected due to the magnetic field and bends towards the y-direction. In a finite material, this results in a building up of charge along the edge and an associated electric field E_y. This continuous until the electric field E_y cancels the bending of due to the magnetic field, and the electrons then travel only in the x-direction. It is this induced electric field which is responsible for the Hall voltage V_H. The resistivity is defined as the inverse of the conductivity. This remains true when both are matrices [**16**].

$$\rho = \sigma^{-1} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} = \frac{1}{\sigma_0} \begin{pmatrix} 1 & \omega_c\tau \\ -\omega_c\tau & 1 \end{pmatrix} \tag{3.36}$$

The components of interest are

$$\rho_{xx} = \rho_0$$

and

$$\rho_{yx} = -\rho_0\omega_c\tau = \frac{-B}{ne} \tag{3.37}$$

where $\rho_o = 1/\sigma_o$.

For a current I_x flowing in the x-direction and the associated electric field E_y in the y-direction, the Hall coefficient is redefined by

$$R_H = \frac{E_y}{J_x B} = \frac{\rho_{xy}}{B} = \frac{\omega_c\tau}{\sigma_0 B} = -\frac{1}{ne}. \tag{3.38}$$

We see that the Hall coefficient depends only on microscopic information about the material: the charge and the conducting particle. The Hall coefficient does not depend on the scattering time τ; it is insensitive to whatever friction process are at play in the material. The two resistivity should be $\rho_{xx} = m/(ne^2)\tau$ and $\rho_{xy} = B/ne$. Note that only ρ_{xx} depends on the scattering time τ, and $\rho_{xx} \to 0$ as scattering processes become less important and $\tau \to \infty$. Therefore we get no MR in the diagonal components of resistivity tensor and the familiar Hall effect for one carrier in the off-diagonal components [**16**].

Hall resistivity (ρ_{xy}) is just a straight line, and the diagonal resistivity is a constant when sweeping the magnetic field. The situation will be completely different when the cyclotron motion is quantized in a magnetic field. For example, ρ_{xx} will then oscillate with the magnetic field.

3.2 The classical Hall effect in 3D system

A conducting material with a magnetic field **B** applied in the z-direction and an electric field **E** applied in the x-direction leading to current I flowing in the x-direction results a voltage drop V_H, across it in the y-direction.

In 1879, E.H. Hall discovered this phenomenon and called the Hall effect. When an electron with charge $-e$ moves in a magnetic field, a Lorentz force $-e\vec{v} \times \vec{B}$, is acted on it that is perpendicular to its velocity and the magnetic field. The trajectories of the electrons are bent so that they move to a side boundary assumed in the y-direction. Upon a sufficient accumulation of electrons on the side boundary, a static electric field builds up and balances the Lorentz force. Electrons then drift in their original intended direction.

The resulting electric field gives rise to a potential difference along the y-direction called the Hall voltage difference. For simplicity, we consider the gas of electrons moving with the same velocity **v** along the x-axis. The Lorentz force on each electron is balanced by an electric field $E_y = v_x B$ that gives a current density $J_x = nev_x$ where n is the electron density, [**1**,**5**]. Thus

$$J_x = ne\frac{E_y}{B}. \tag{3.39}$$

This result is valid when the electric and magnetic fields are weak, that is if we interpret ne, as the charge density of the current carriers. When a current density $J = I/A$ goes through a sample (metal or semiconductor) that is immersed in a magnetic field **B**, perpendicular to the sample, a Hall voltage $V_H \sim J_x.B.w$ is generated and can be measured along the sides of the sample. Introducing the proportionality constant, we can have

$$V_H = R_H J_x B_z w \tag{3.40}$$

Where R_H is a proportionality constant called the Hall constant and w is sample width [**17**].

3.2.1 Current density in the presence of an electromagnetic field

In order to understand some of the ideas involved in the theory of the Hall effect in real materials, it is instructive to construct a more careful model for electric currents under electric and magnetic fields from a classical point of view.

We imagine that the charge carriers move in a medium that offers some resistance. The resistance is due to scattering between the carriers and impurities in the material and between the carriers and vibrations of the material's atoms. Each charge carrier is accelerated by the applied fields but every so often it scatters and loses energy. If we assume that the average time between scattering events is τ, then we have, on average a retarding force acting on the carriers of [**9**]

$$\vec{F}_{retard} = -\frac{m\vec{v}}{\tau} \tag{3.41}$$

where m is the mass of the carrier. So under the influence of applied electric and magnetic fields equation (3.21) can be rewritten as

$$m\frac{d\vec{v}}{\tau} = q\big(\vec{E} + \vec{v} \times \vec{B}\big) - \frac{m\vec{v}}{\tau} \tag{3.42}$$

where the velocity $\vec{v}$, is taken to be an average over all of the carriers.

At steady state, the time derivative of $\vec{v}$, will vanish. Under the usual convention that $\vec{B}$ points along the z axis, we obtain the component equations for $\vec{v}$ by setting the left hand side of Equation (3.42) to zero and rearranging:

$$v_x = \frac{q\tau}{m}E_x + \frac{q\tau}{m}B_z v_y$$

$$v_y = \frac{q\tau}{m}E_y - \frac{q\tau}{m}B_z v_x$$

and

$$v_z = \frac{q\tau}{m}E_z. \tag{3.43}$$

From the equation that $J_x = nqv_x$ (and correspondingly for y and z components), by solving the above equations for v_x, v_y and v_z in terms of the components of $\mathbf{E}$ and B_z we get

$$J_x = \sigma\left(\frac{E_x + \omega_c\tau E_y}{1+\omega_c^2\tau^2}\right)$$

$$J_y = \sigma\left(\frac{E_y - \omega_c\tau E_x}{1+\omega_c^2\tau^2}\right)$$

and

$$J_z = \sigma E_z \tag{3.44}$$

where $\sigma = nq^2\tau/m$ and $\omega_c = qB_z/m$.

In the presence of a magnetic field, the current density $\vec{J}$ is generally not parallel to the electric field $\vec{E}$. However, for metals, even under the large magnetic field $\vec{B}$, the corresponding anisotropy is very small, in such a way that $\vec{J} \approx \sigma\vec{E}$. i.e Ohm's law remains valid. The consequences of anisotropy are mostly relevant in semiconductors and depend on the geometry of the system [**11**].

3.2.2 Drift of carriers in electronic and magnetic fields

The Hall effect is the production of a transverse voltage (a voltage change along the y-direction) due to transverse magnetic field $\vec{B}$ in the z-direction with a current flowing in the x-direction . It is useful for determining information on the sign and concentration of carriers. Because of the magnetic field in the z-direction there are forces in the y-direction which end up creating an electric field E_y in that direction.

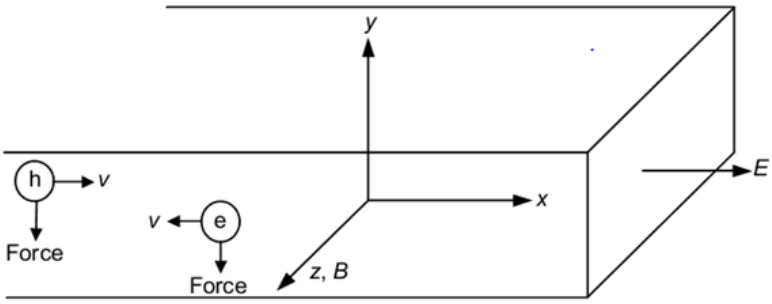

Figure 3.1: Geometry of a sample material for observing the deflecting magnetic force on an electron and a hole in relation to the Hall effect

In the Fig.(3.1) shown above, in the absence of a magnetic field, the holes flow in the positive x-direction. In a magnetic field, B_z, the holes experience an additional force $\vec{F}_B = q\vec{v}_x \times \vec{B}$, that pushes the holes in the negative y- direction. The holes thus collect at the bottom side of the surface and leave behind negatively charged acceptors at the top surface. These changes induce an electric field directed in the positive y- direction that creates an electric induced force opposite to the magnetic force. No current can flow in the y- direction because nothing is connected to the top and bottom surfaces. No current means that no net force in that direction. Therefore, the two opposite forces (B_z and induced E_y) must have equal magnitudes and $qE_y = qv_xB_z$ [**2**].

Under a magnetic field $\vec{B} = B_z\hat{e}_z$, the charge carriers are deflected towards the sides of the sample. On the other side a lack of charge carriers creates an effective charge of opposite sign [**18**]. This charge separation continues until the voltage generated in this way, called Hall voltage, counters the magnetic force. At the equilibrium state, there is no longer a drift velocity along the y- axis. Therefore, the Hall field E_H is defined by the condition $J_y = 0$.

$$E_H = E_y = \omega_c\tau\frac{J_x}{\sigma} = \left(\frac{1}{qn}\right)J_xB_z = R_HJ_xB_z \tag{3.45}$$

R_H is therefore an experimental measure of the algebraic quantity describing the mobile charge carrier density in a conductor and the sign of the carrier. Also, if σ is known, then a measure of R_H can be used to determine the mobility $\mu = \sigma R_H$, as long as there is only one type of carrier.

For the applied current I_x, the electrons will move in the negative x- direction. The force due to the magnetic field ,B_Z will push the electrons in the negative y- direction to the bottom surface leaving positively charged acceptors on the top side. In this case the electric field will point in the negative y- direction. Thus the polarity of the Hall voltage for p and n material is opposite and R_H is negative for n-type material [**9**].

The accumulation of charge at the edges continues until the electric field and the resulting electric force set up by the charge separation balances the magnetic force on the charge carriers ($F_{mag} = F_{electric}$). When equilibrium is reached, the electrons are no longer deflected downward. A voltmeter connected across the conductor can be used to measure the potential difference across the conductor, known as the Hall voltage V_H. Therefore, $qE_H = qv_dB$. and $E_H = v_dB$. If w is taken to be the width of the conductor, then the Hall voltage V_H measured by the voltmeter is:

$$V_H = E_H.w = v_dB.w. \tag{3.46}$$

The measured Hall voltage gives a value for the drift velocity of the charge carriers if w and B are known. The number of charge carriers per unit volume (charge density n), can also be determined by measuring the current in the conductor.

$$v_d = \frac{I}{nqA}$$

$$V_H = \frac{IBw}{nqA} \tag{3.47}$$

where $A = wt$ is the cross sectional area of the sample.Therefore

$$V_H = \frac{IB}{nqt}. \tag{3.48}$$

And again the Hall coefficient can be determined as

$$R_H = \frac{1}{nq} = \frac{V_H t}{IB}. \tag{3.49}$$

The sign and magnitude of R_H gives the sign of the charge carriers and their density [**15**]. Thus we can see, $R_H = -1/n|q|$ for electrons and $R_H = +1/n|q|$ for holes.

In most metals, the charge carriers are electrons and the charge density determined from the Hall effect measurements agrees with calculated values for metals which release a single valence electron and charge density is approximately equal to the number of valence electrons per unit volume. In metal the number of charge carrier is fixed and the increase of resistance in metal is because of the increase in number of thermally excited lattice vibrations(phonon) from which the charge carriers can scatter. In semiconductor, the increase in scattering is usually over whelmed by the exponential increase in the number of carriers as a result of thermal excitation across the energy gap.

3.2.3 The Hall effect in metals and semiconductors

The Hall effect is a conduction phenomenon which is different for different charge carriers. In most electrical applications, the convectional current is caused partly because it makes no difference whether you consider positive or negative charge to be moving. But the Hall voltage has a different polarity for positive and negative charge carriers, and it has been used to study the details of conduction in semiconductors and other materials which show a combination of negative and positive charge carriers.

For a simple metal where there is only one type of charge carrier (electrons) the Hall voltage, V_H can be derived by using the Lorentz force and seeing that in a steady state condition the charges are not moving in the y-axis direction because the magnetic force on each electron in the y-axis direction is canceled by a y-axis electrical force due to the build up of charges.

In metals the Hall voltage is given as

$$V_H = -\frac{I_x B_z}{net}$$

and the Hall coefficient is $R_H = -1/nq$. The Hall effect offered the first real proof that electric currents in metals are carried by moving electrons, not by protons. In semiconductors, the electric conductivity is often the result of two charge carriers of charges q_1 and q_2 of density n_1 and n_2 respectively. The total conductivity and the current density can thus be written:

$$\sigma_t = \sigma_1 + \sigma_2 = q_1 n_1 \mu_1 + q_2 n_2 \mu_2 \tag{3.50}$$

$$J_t = J_1 + J_2 = q_1 n_1 v_1 + q_2 n_2 v_2. \tag{3.51}$$

At room temperature the relaxation time τ is on the order of 10^{-14} up to $10^{-15}s$, so the term $\omega_c \tau$ is of the order of 10^{-3} and the second order terms $\omega_c^2 \tau^2$ are negligible. Shown that equation (3.44) can in this case be rewritten as

$$J_x = \sigma E_x + \big(\sigma_1 \mu_1 + \sigma_2 \mu_2\big) B_z E_y \tag{3.52}$$

$$J_y = \sigma E_y - \big(\sigma_1 \mu_1 + \sigma_2 \mu_2\big) B_z E_x \tag{3.53}$$

and that at the Hall condition $J_y = 0$ implies that

$$E_y = \big(\frac{\sigma_1 \mu_1 + \sigma_2 \mu_2}{\sigma}\big) B_z E_x \tag{3.54}$$

and

$$J_x = \left[1 + \big(\frac{\sigma_1 \mu_1 + \sigma_2 \mu_2}{\sigma}\big)^2 B_z^2\right] \sigma E_x \tag{3.55}$$

Since at low magnetic field, increasing temperature increases the conductivity. In such that $J_X \equiv \sigma E_x$.

Thus using equation (3.12), the Hall coefficient in semiconductor is defined as

$$R_H = \frac{\sigma_1\mu_1 + \sigma_2\mu_2}{\sigma^2} = \frac{\sigma_1\mu_1 + \sigma_2\mu_2}{\big(\sigma_1 + \sigma_2\big)^2} = \frac{q_1n_1\mu_1^2 + q_2n_2\mu_2^2}{\big(q_1n_1\mu_1 + q_2n_2\mu_2 big)^2}. \tag{3.56}$$

For two different charge carries; electrons and holes with $q_1 = -q$ and $q_2 = q$

$$R_H = \frac{n_p\mu_p^2 - n_e\mu_e^2}{q\big(n_p\mu_p + n_e\mu_e\big)^2}. \tag{3.57}$$

Note that if $n_p = 0$ so, $R_H = -1/\big(qn_e\big)$ and if $n_e = 0$, then $R_H = +1/\big(qn_p\big)$. Both the sign and the concentration of carriers are included in the Hall coefficient [**16**].

In metals the sign of R_H indicates whether the electric current is due to the motion of electrons ($R_H < 0$) or holes ($R_H > 0$). Holes are apparent positive charges, and could only be explained using wave mechanics. In semiconductors, the value and sign of R_H is strongly dependent on the dopant density (impurities). R_H can even be zero if $n_p\mu_p^2 = n_e\mu_e^2$.

3.2.4 Applications in Hall effect: Hall effect sensors

The Hall effect has been important for many reasons. For example, in semiconductors it can be used for determining the sign and concentration of the charge carriers. The fractional quantum Hall effect, in terms of basic physics ideas, may be the most important discovery in solid state physics in the last quarter of a century.

The Hall effect will be used to study some of the physics of charge transport in metal and semiconductor. It yields a direct measure of the majority carriers' sign, mobility and density and is the basis of many practical applications and devices such as magnetic field measurements, and position and motion detectors through Hall effect sensors.

A Hall effect sensor is a transducer that varies its output voltage in response to a magnetic field. Hall effect sensors are used for proximity switching, positioning, speed detection, and current sensing applications. Using groups of sensors, the relative position of the magnet can be deduced. Smart phones like iphone 3Gs are equipped with magnetic compasses. These compasses measure Earth's magnetic field using 3-axis magnetometer. These magnetometers are sensors based on Hall effect. Frequently, a Hall sensor is combined with threshold detection so that it acts as a switch.

Commonly seen industrial applications such as the pictured pneumatic cylinder, they are also used in consumer equipment; for example some computer printers use them to detect missing paper and open covers. When high reliability is required, they are used in keyboards. Hall sensors are commonly used to time the speed of wheels and shafts, such as for internal combustion engine ignition timing, tachometry and anti-lock braking systems. They are used in brushless DC electric motors to detect the position of the permanent magnet. In the pictured wheel with two equally spaced magnets the voltage from the sensors will peak twice for each revolution. This arrangement is commonly used to regulate the speed of disk drives.

The Hall sensor is used in some automotive fuel level indicators. The main principle of operation of such indicator is position sensing of a floating element. This can either be done by using a vertical float magnet or a rotating lever sensor. In a vertical float system a permanent magnet is mounted on the surface of a floating object. The current carrying conductor is fixed on the top of the tank lining up with the magnet. When the level of the fuel rises, an increasing magnetic field is applied on the current resulting in higher Hall voltage. As the fuel level decreases, the Hall voltage will also decrease. In a rotating lever sensor, a diametrically magnetized ring magnet rotates about a linear Hall sensor. The sensor only measures the perpendicular (vertical) component of the field. The strength of the field measured correlates directly to the angle of the lever and thus the level of the fuel tank [**6**]. And there are many more applications of Hall effects.

3.3 The quantum Hall effect

Quantum Hall effect (QHE) is observed in conductors whose thickness t is very low and comparable to the inter atomic distance. Such conductors are called two dimensional electron system (2DEG). If the temperatures are low enough and magnetic field is strong, a quantization of Hall resistance occurs differently to the classical Hall effect. In this case the Hall resistance gets discrete values of $h/(\mathrm{i}e^2)$, where i is an integer or a fraction number. If the energy for the electrons' motion in one direction is fixed and a strong magnetic field is perpendicular to the two dimensional plane, it will lead to fully quantized energy spectrum, which is necessary for the observation of quantum Hall effect. For sufficiently large value of induction$\vec{B}$, (when $\omega_c\tau >> 1$), the energy spectrum consists of separate equidistant Landau levels.

$$E_n = \big(n + \frac{1}{2}\big)\hbar\omega_c \tag{3.58}$$

where n = 1,2,3,...is the number of Landau levels completely filled by electrons [**17**].

The energy spectrum consists of a series of equally spaced levels and are called Landau levels. Each level is highly degenerate and has many states with the same energy. The number of states per level is proportional to the area of the 2D gas. Each Landau level has a total number of eB/h states per unit area. If an electric field E_y is applied in the y-direction an electron in the form of the wave function moves with a velocity of E_y/B in the x-direction.

When the lowest Landau level is full and the others are empty, the current density is

$$J_x = e\frac{eB}{h}\frac{E_y}{B} = \frac{e^2}{h}E_y \tag{3.59}$$

For which the Hall conductance is

$$\sigma_H = \frac{J_x}{E_y} = \frac{e^2}{h}. \tag{3.60}$$

When the lowest n Landau levels are full and all others are empty, the current density increases by a factor of n and

$$\sigma_H = \frac{ne^2}{h}$$

and the Hall resistivity is

$$\rho_H = \frac{1}{\sigma_H} = \frac{h}{ne^2}. \tag{3.61}$$

Quantum mechanics becomes important in quantum Hall effect (QHE) at low temperature and strong magnetic fields, where more interesting things can happen. It is useful to distinguish between two different QHEs which are associated to two related phenomena, IQHE and FQHE [**4**].

3.3.1 Energy quantization in a magnetic field: Origin of Landau level

The basic ingredient for the understanding of both the IQHE and the FQHE is Landau quantization, i.e. the kinetic energy-quantization of a free charged 2D particle in a perpendicular magnetic field. The spatial restriction of electron system in one dimension led to a quantization of the energy spectrum associated with dimension. Further quantization of the energy spectrum can be induced through the application of a large magnetic field. Physically, the magnetic field quantizes the angular momentum, where any motion perpendicular to the magnetic field can only occur within certain cyclotron paths having radii $R_c = v/\omega_c$.

If the magnetic field is applied such that it is perpendicularly oriented with respect to the 2-DEG, then the electron motion becomes restricted in all three spatial dimensions. However, in order to observe this fully quantized energy spectra, the charge carriers should not be scattered out of their cyclotron paths,i.e we must satisfy the condition $\omega_c\tau >> 1$. In addition to this, a second condition for observing the discrete energy spectra is that the energy spacing between the allowed energy levels must be greater than the thermal energy of the electrons (i,e the "sharpness" of the Fermi edge).

In a 2D particle system, the energy of a particle has contributions from the kinetic energy as well as the potential energy. However,by crude assumption that motion of charged particle in a crystalline environment in the same manner as a particle in free space, a free charged 2D particle there is no potential energy but only kinetic energy that we can write as

$$\mathbf{H} = \frac{\mathbf{p}^2}{2m} \tag{3.62}$$

In the case of a free particle for a non relativistic case, this is a very natural assumption. Let the electrons be confined to the $x - y$ plane and the magnetic field $\mathbf{B}$ be parallel to the z axis. The Hamiltonian for such a system is given by

$$\mathbf{H} = \frac{\left(\mathbf{p} + ie\mathbf{A}\right)^2}{2m} = \frac{\left[\left(p_x + eA_x\right) + \left(p_y + eA_y\right)\right]^2}{2m} \tag{3.63}$$

where $\mathbf{A}$ is a vector potential associated with the magnetic field $\mathbf{B}$ i.e. $\mathbf{B} = \nabla \times \mathbf{A}$. Under gauge choice, we are free to choose $\mathbf{A}$ under which $\mathbf{B}$ remains unchanged. If we choose $\mathbf{A} = -y\mathbf{B}i$ and substitute into equation (3.63), we get

$$H = \frac{p_y^2}{2m} + \frac{\left(p_x - eyB\right)^2}{2m}. \tag{3.64}$$

Since the coordinate x does not appear in $\mathbf{H}$ and since we know the commutators $[y, p_x] = [p_y, p_x] = 0$. It follows that p_x is a good quantum number and may be regarded as a constant parameter. We define $k = p_x/\hbar$, the eigen functions of the problem must now simultaneously satisfy:

$$p_x\psi(x, y) = \hbar k(x, y) \tag{3.65}$$

$$H\psi(x, y) = E\psi(x, y). \tag{3.66}$$

Since the operator p_x is given by $-i\hbar\partial/\partial_x$, the first equation yields the desired result:

$$\psi(x, y) = e^{ikx}\phi(y) \tag{3.67}$$

The quantum Hall effect has analogy to the one dimensional simple harmonic oscillator. We can see that the *schirödinger* equation $H\psi = E\psi$ reduces to the one dimensional problem. Equation (3.66) is the *schirödinger* equation and its solution determines the energy spectrum. we have

$$H\psi = \left(\frac{\left(p_x - eyB\right)^2 + p_y^2}{2m}\right)e^{ikx}\phi(y)$$

$$= e^{ikx}\left(\frac{\left(\hbar k - eyB\right)^2}{2m} + \frac{p_y^2}{2m}\right)\phi(y)$$

$$= e^{ikx}\left(\frac{e^2B^2}{2m}(y - \frac{\hbar k}{eB})^2 + \frac{p_y^2}{2m}\right)\phi(y)$$

$$= e^{ikx}\left(\frac{m\omega_c^2}{2}\left(y - y_0\right)^2 + \frac{p_y^2}{2m}\right)\phi(y) \tag{3.68}$$

where $y_0 = \hbar k/eB$. Here ω_c is known as the cyclotron frequency. The right hand side must equal $E\psi = Ee^{ikx}\phi(y)$. Thus the differential equation is obtained (after replacing p_y by $-i\hbar\partial/\partial y$):

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dy^2} + \frac{m\omega_c^2}{2}z^2\right)\phi(z) = E\phi(z) \tag{3.69}$$

where z is the dummy coordinate. Our problem looks very similar except we have a term $(y - y_0)^2$ instead of just y^2. But this simply means that the center of the oscillations is at $y = y_0$ rather than at y=0.

As we can see by making the change of the variable $z = y - y_0$, for any given y_0, i.e. for every k our problem is exactly like that the corresponding 1D simple harmonic oscillator. For our knowledge of simple harmonic oscillators, it should be no surprise that

$$E_n = (n + 1/2)\hbar\omega_c$$

with n=0,1,2,... Because the smallest possible value of n is zero, the ground state of the harmonic oscillator has $E_0 = \frac{1}{2}\hbar\omega$.

This is a remarkable result. The allowed energy levels for a free 2D electron moving in a magnetic field are identical to a fictitious 1D simple harmonic oscillator. Note these levels are discrete, with the magnetic field the electron's energy is a continuous variable. Also note that the energy levels do not depend on the value of y_0 (or therefore k).

The momentum $\vec{k}$ in x direction creates no kinetic energy. Each value of n can have apparently, any value of $\vec{k}$ and thus the energy levels are highly degenerate. Thus the magnetic field has induced a great condensation of the continuous energy spectrum of a free particle in 2D into a discrete set of highly degenerate levels. These levels are known as Landau levels, are equally spaced by the cyclotron energy, $\hbar\omega_c$, which is itself proportional to the magnetic field strength.

Electrons are trapped in a thin layer made at the interface between semiconductors at sufficiently low temperature. Trapped electrons make a two-dimensional system. In a perpendicular magnetic field $\mathbf{B}$, the energy of an electron is quantized into Landau levels. The energy difference between two successive landau levels is $\hbar\omega_c$. The energy levels E_n are those of the harmonic oscillator as given in equation (3.58). They are called the Landau levels [**19**].

Classically, if we plot a graph between B and R_H, it should come out to be a straight line. But if we have very thin conductor and at a very low temperatures at around $T = 50mK$, it exhibits a series of plateaus as shown below.

In the Fig.(3.2), both the Hall resistivity and the longitudinal resistivity exhibit interesting behavior. Hall resistivity sits on a plateau for a range of magnetic field, before jumping suddenly to the next plateaus the longitudinal resistance becomes zero when quantum Hall effect reaches plateau.

One finds plateaus at values of $h/(ie^2)$ with e^2/h being called the quantum of conductance and i is an integer for Integral Quantum Hall effect and a fraction for the Fractional Quantum Hall effect.

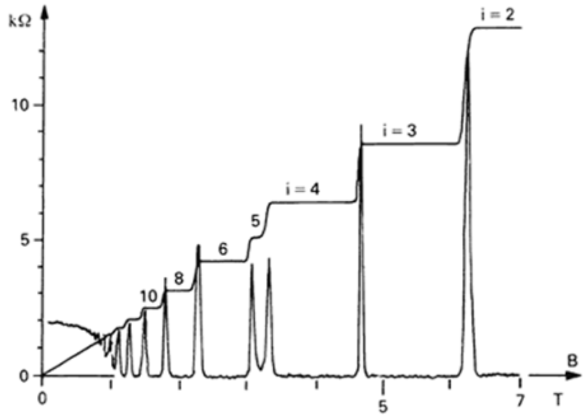

Figure 3.2: Relation between Hall resistance(R_{xy}) and magnetic field (B) in quantum Hall effect. Adapted from [**21**]

Generally, the quantum Hall effect requires two dimensions, low temperature, electrons and a large external magnetic fields. Two dimensions are necessary so that the gaps in between the Landau levels ($E_g = \hbar\omega_c$) are not obliterated by the continuous energy introduced by motion in the third dimension, Low temperatures are necessary so as not to wipe out the quantization of levels by thermal-broadening effects.

We imagine a conductor through which current I is flowing. If it is not superconductor, there is a voltage drop V along the direction of the current flow. But if we try to measure the voltage perpendicular to the current flow, no such voltage will be observed [**4**]. If a magnetic field $\vec{B}$ is applied perpendicular to the current flow, a voltage V_H will be produced perpendicular to the current flow and the corresponding resistance is given by

$$R_H = \frac{V_H}{I}. \tag{3.70}$$

The Hall resistance R_H, is dependent upon the material's temperature T, magnetic field B, electron density, and other physical properties. Under a very low temperature and a very high magnetic field the Hall resistivity ρ_H is a staircase function of the gate voltage V_g with extremely flat plateaus in the form of equation (3.61). This two dimensional actually means three dimensional but it is very thin of the order of around $100A^0$. The most common example of such system is MOSFET and semiconductor heterostructure. The important thing is that electrons can move freely in the 2D plane but not in the perpendicular direction [**21**].

3.3.2 The integer quantum Hall effect

The integral quantum Hall effect (at i = integer) was discovered by Klaus Von Klitzing in 1980, a century after the discovery of the Hall effect and for this he was awarded the Nobel prize in 1985. The integer quantum Hall effect (IQHE) involves filled or empty Landau levels. In IQHE as each Landau level is filled, there is a gap to the next Landau level. The gap is filled by localized non conducting states, and as the Fermi level moves through this gap no current is observed. The Landau level themselves are conducting.

For the IQHE the electron-electron interaction effects are really not important, but the disorder that causes the localized states in the gap is crucial. In 1980 experiments done by Von Klitzing and co- workers showed that for very high magnetic fields and low temperature, the Hall resistivity ρ_{xy} as a function of $\mathbf{B}$ has wide plateaus that correspond to integer value of i and the diagonal resistivity ρ_{xx} vanishes. From $F_y = -ev_xB$, for the current and the fact that the motion in the y-direction can be associated with an effective electric field E_y, we obtain

$$E_y = \frac{F_y}{-e} = \frac{J_xB}{-ne} \Rightarrow \frac{J_x}{E_y} = \frac{-en}{B} = \sigma_H = \rho_{xy} \tag{3.71}$$

Fig.(3.3) shows that in a classical electron gas the Hall resistivity R_{xy} is proportional to the magnetic field B as indicated by a thin line labeled classical theory. However, the Hall resistivity R_{xy} shows a stair case in an actual sample, with the plateau crossing the line at $i = 1, 2, 3...$ ($R_{xx} = \rho_{xx}$ and $R_{xy} = \rho_{xy}$).

The integral quantum Hall effect manifests itself as a series of plateaus in Hall resistance,R_H of materials containing 2D electron system. R_H is precisely (at i= 1) given by $R_H = h/(ie^2) = 25812.81/\Omega$.

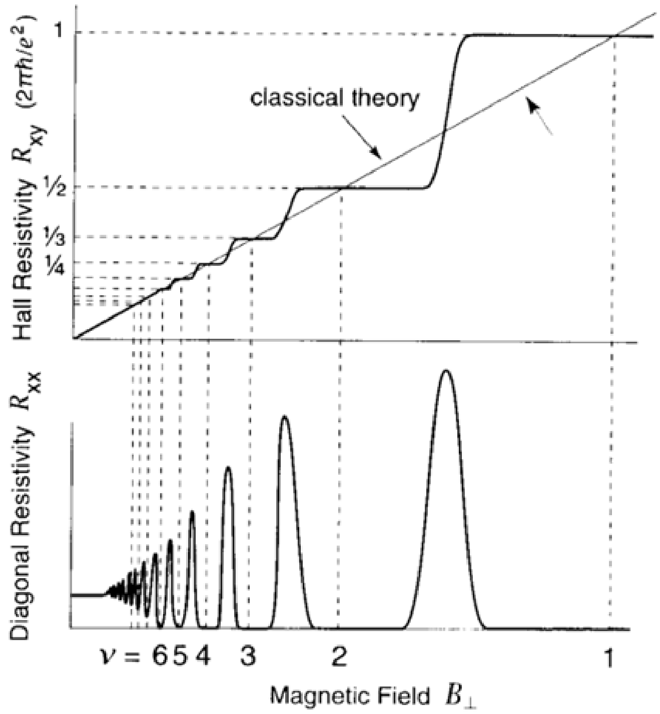

Figure 3.3: The integer quantum Hall effect is illustrated schematically. Adapted from [**21**]

3.3.3 Fractional quantum Hall effect

The fractional quantum Hall effect (at $i = \nu = n/m$ with integer n and odd integer m) was discovered by Tsui, Stormer, and Gossard in 1982. In a very high field, around filling factor $\nu = 1/3$, the Hall resistance develops a plateau at a value of $3h/e^2$ and the longitudinal resistance tend to zero as temperature is lowered.

A fractional form of the quantum Hall effect (FQHE) has been discovered, where the conductance steps are in rational fractions of the basic e^2/h unit. It is generally believed that this peculiar effect is due to interactions among the charge carriers requiring a quantum theory going beyond the effect of confinement. Gaps for the FQHE which involves partially filled Landau levels are introduced by electron- electron interaction.

The FQHE occurs for partially filled Landau levels and electron- electron interactions are crucial. Potential fluctuations cause localized states and plateau formation.

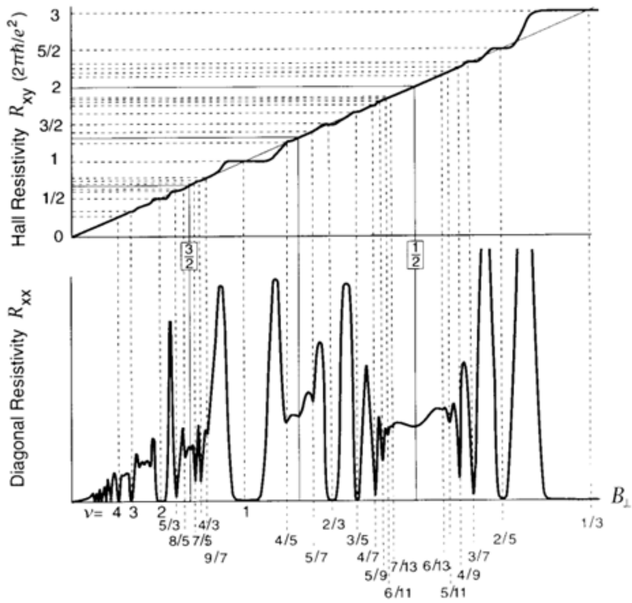

Figure 3.4: The fractional quantum Hall effect illustrated.

In the Fig (3.4) above, it is easier to identify FQHEs by searching for dips in the diagonal resistivity rather than plateaus in the Hall resistance. All FQHEs in this figure are on the principal sequences $v = n/2n \pm 1$ or its electron-hole conjugate sequences $v = 2 - n/2n \pm 1$

Many fractional quantum Hall states are observed in pure samples. It is easier to identify them by searching dips in the diagonal resistance ρ_{xx} rather than plateau in the Hall resistance R_{xy} [**22,23**].

Chapter 4

Results and discussions

The integer Quantum Hall effect was originally discovered in a Si MOSFET (this stands for "metal-oxide-semiconductor field- effect transistor"). It is a metal-insulator-semiconductor sandwich, with electrons trapped in the "inversion band" of width $30A^0$ between the insulator and the semiconductor. Meanwhile the fractional quantum Hall effect was discovered in a GaAs/AlGaAs heterostructure. A lot of the subsequent work was done on this system, and it usually goes by the name GaAs. In both these systems, the density of electrons is around $n \sim 10^{11} - 10^{12}\ cm^{-2}$ [23].

The mobility in MOSFETs, which is typically on the order of $\mu \sim 10^6\ cm^2/Vs$, is limited by the quality of the oxide semiconductor interface (surface roughness). This technical difficulty is circumvented in the semiconductor heterostructures - most popular are GaAs/AlGaAs heterostructures, where high quality interfaces with almost atomic precisions may be achieved, with mobilities on the order of $\mu \sim 10^6\ cm^2/Vs$. These mobilities were necessary to observe the FQHE, which was indeed first observed in a GaAs/AlGaAs sample.

In high magnetic fields, at low temperatures and for high mobility, 2D electron gases exhibit a deviation from the classical behavior, Recall that the classical Hall effect (i.e., considering the Lorentz force, classical Drude theory), the generation of a field E_y perpendicular to a current flow J_x was described with the conductivity tensor σ (for $x,\ y$ plane only) [24].

$$\sigma = \frac{\sigma_0}{1+\omega_c^2\tau^2}\begin{pmatrix} 1 & -\omega_c\tau \\ \omega_c\tau & 1 \end{pmatrix}. \tag{4.1}$$

$B\ kG$	20	40	60	80
$\rho_{xy}\ (\Omega m)$	3125	6250	9375	12500
$\rho_{xx}\ (\Omega m)$	0	0	0	0

Table 4.1: Data collected for Hall resistivity and longitudinal resistivity of GaAs/AlGaAs Heterostructure for a given values of magnetic field at $n = 4 \times 10^{11} cm^{-2}$ and $e = 1.6 \times 10^{-19}$ using equation (4.5)

$$\sigma_{xx} = \sigma_{yy} = \frac{\sigma_0}{1 + \left(\tau\omega_c\right)^2} \rightarrow 0, \tag{4.2}$$

$$\sigma_{xy} = -\sigma_{yx} = \frac{\sigma_0 \tau\omega_c}{1 + (\tau\omega_c)^2} \rightarrow \frac{ne}{B}, \tag{4.3}$$

where σ_0 is the zero-field conductivity $\sigma_0 = ne^2\tau/m^*$. The arrows denote the limit for $\omega_c\tau$, i.e., large fields. The resistivity tensor $\rho = \sigma^{-1}$ is given by Equation (3.35). Such that in the high magnetic field limit if $\omega_c >> 1$, we have

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \rightarrow 0, \tag{4.4}$$

$$\rho_{xy} = \frac{-\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} \rightarrow -\frac{B}{ne}, \tag{4.5}$$

Experiments yield strong deviations from the linear behavior of the transverse resistivity $\sigma_{xy} = E_y/J_x = R_H B$ with the Hall coefficient $R_H = -1/ne$ with increasing magnetic field is observed at low temperatures for samples with high carrier mobility, i.e. $\omega_c\tau >> 1$. The Hall resistivity exhibits extended Hall plateaus with resistivity values that are given by $\sigma_{xy} = h/ie^2$. Hall resistivity σ_{xy} and longitudinal resistivity σ_{xx} for a modulation-doped GaAs/AlGaAs-heterostructure are drown below using the following parameters.

$$n = 4 \times 10^{11}\ cm^{-2},$$

$$\mu = 8.6 \times 10^4\ cm^{-2}/Vs,$$

at 50 mK as a function of magnetic field $B = 10\ kG = 1T$. The number i refers to the quantum number [24]. The result is analyzed based on Equations (4.4) and (4.5). From the Figure (4.1) shown below, we can observe that both the Hall resistivity (ρ_{xy}) and longitudinal resistivity (ρ_{xx}) exhibit interesting behavior. After some characteristic magnetic field (10 kG) the longitudinal resistivity oscillates as a function of the magnetic field.

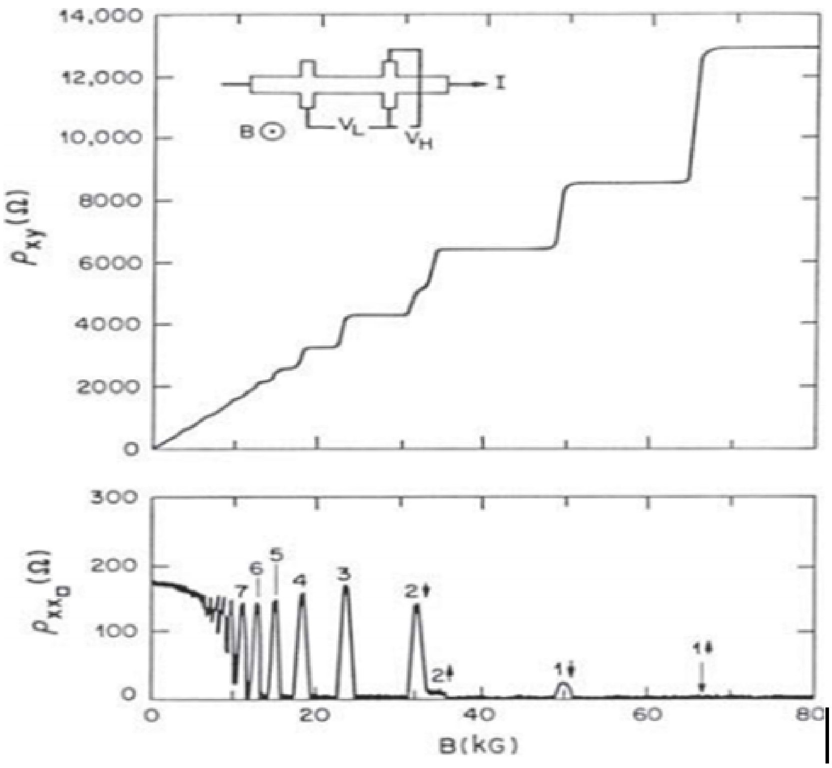

Figure 4.1: Measured Hall resistivity and longitudinal resistivity of GaAs/AlGaAs heterostructure, $n = 4 \times 10^{11} cm^{-2}$, $\mu = 8.6 \times 10^4 cm^2/vs$ as a function of magnetic field $(10KG = 1T)$. The numbers refer to the quantum number and spin polarization of the Landau level involved. the inset shows schematically the Hall bar geometry, $V_L(V_H)$ denotes the longitudinal (Hall) voltage drop. Reprinted from [**24**],third edition.

Perhaps the most striking features in the data is that the Hall resistivity sits on a plateau for a range of magnetic fields, before jumping suddenly to the next plateau. On these plateaus the resistivity takes the value of $\rho_{xy} = h/e^2\nu, \in z$.

When the Landau level is filled, the surprising is that the plateau exists, with the quantization persisting over a range of magnetic fields. The longitudinal resistivity ρ_{xx} also exhibits a surprise when ρ_{xy} sits on a plateau, the longitudinal resistivity vanishes $\rho_{xx} = 0$. It spikes only when ρ_{xy} jumps to the next plateau. Usually we would think of a system with $\rho_{xx} = 0$ as a perfect conductor. But there is something a little counter-intuitive about vanishing conductivity in the presence of a magnetic field. From equations (4.4) and (4.5), if $\rho_{xy} = 0$, then we get the familiar relation between conductivity and resistivity $\sigma_{xx} = 1/\rho_{xx}$. But if $\rho_{xy} \neq 0$, then we have the more interesting relation above.

In particular we have $\rho_{xx} = 0 \Rightarrow \sigma_{xx}$ if $\rho_{xy} \neq 0$ while we would usually call a system with $\rho_{xx} = 0$ a perfect conductor and we would usually call a system with $\sigma_{xx} = 0$ a perfect insulator. This particular surprise has more to do with the words to use to describe the phenomena than understanding physics. This behavior occurs in the Drude model in the limit $\tau \to \infty$, where there is no scattering. In this situation, the current is flowing perpendicular to the applied electric field, so $\vec{E} \cdot \vec{J} = 0$. But $\vec{E} \cdot \vec{J}$ has the interpretation as the work done in accelerating charges the fact that this vanishes means that we have a steady current flowing without doing any work and, correspondingly without any dissipation.

The fact that $\sigma_{xx} = 0$ is telling us that no current is flowing in the longitudinal direction (like an insulator) while the fact that $\rho_{xx} = 0$ is telling us that there is no dissipation of energy (like a perfect conductor).

In case of FQHE, as the disorder is decreased, more and more plateaus emerge but not all fractions appear. It seems plausible that in the limit of a perfectly clean sample, we would get an infinite number of plateaus which brings us back to the classical picture of a straight line for ρ_{xx} [**24**]

Chapter 5

Conclusions

This project has given more emphasis on Hall effect (classical and quantum Hall effect) in relation to the physics of charge transport in metals and semiconductors and the magnetoresistance effect which is a change in the resistance of a material upon the application of a magnetic field. The Hall effect is the production of a transverse electric field that is produced by the presence of a magnetic field perpendicular to the flow of charge carriers. For a simple metal and semiconductors the Hall effect combined with other data can reveal the sign and density of the charge carriers. The sign of the charge carriers can be determined from the polarity of the Hall voltage. If there is more than one type of charge carrier, the Hall effect is the weighted average of the Hall effect of the individual carriers and becomes non linear with the magnetic field. At room temperature the charge carrier density of semiconductor is much smaller than that of metals and thus the magnitude of the Hall voltage is much larger for a given current I, magnetic field $\vec{B}$ and film thickness t. For the normal 3D metal at room temperature and rather low values of magnetic field, the Hall resistance is proportional to the magnetic field strength and the graph should be straight line and a diagonal resistivity is a constant. In the transport of charge carriers in the field, the motion of charges in the direction parallel to the magnetic field is unaffected, so that there will be no longitudinal MR. However there will be transverse MR in the direction perpendicular to the field, $\vec{B}$, i.e. both the applied current density and the measured electric field are in the plane perpendicular to the magnetic field. This MR is associated with the transverse field, E_y. MR in metal arises from the increased path length of electrons traveling in metals as a consequence of the Hall effect. When the cyclotron motion of electron is quantized in a magnetic field the diagonal resistivity (ρ_{xx}) will then oscillate with the magnetic field and the Hall resistivity (ρ_{xy}) is quantized and sits in a plateau. In 2D structure the Hall resistance reveals a number of distinct steps called plateau as magnetic field strength increases. At the same time, in the applied magnetic field range where the Hall resistance shows the plateaus, the MR (i.e. the resistance measured along the direction of current flow) drops to negligible values. The integral quantum Hall effect can be explained solely by the filling of the Landau levels where as FQHE is explained by partially filled Landau level. In FQHE the disorder is decreased and more and more plateaus emerge but not all fractions appear. It seems plausible that in the limit of a perfectly clean sample, we would get an in finite number of plateaus which brings us back to the classical picture of a straight line for ρ_{xx}.

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DECLARATION

I hereby declare that this master of science project has given review on the title mentioned and that all sources of material used for the project have been dully acknowledged.

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December, 2017

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