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Comparative Analysis of Blind Spectrum Sensing Techniques over Fading Channel

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Abstract

A spectrum is a scarce and precious natural resource and a matter of concern with the rapidly growing wireless communications. However, studies show that licensed spectrum is underutilized. Cognitive radio system allows unlicensed users also called secondary users to access licensed spectrum band also known as spectrum hole of primary user when they are not occupied. To do this the secondary users need to continuously monitor the licensed user's activity to find the unused band. Spectrum sensing is the key task for cognitive radio which prevents the unwanted interferences with authorized users to recognize available radio spectrum and enhance the usage of spectrum. Spectrum sensing is also a tough task because of shadowing, fading, and time-varying nature of wireless channels.

This thesis investigates the difference in performance of energy detection and eigenvalue based blind spectrum sensing techniques (without having prior knowledge) by implementing it over different environmental models and compares the Receiver Operating Characteristics (ROC) curves. Evaluation and analysis of performance is done by using Monte Carlo method with MATLAB software.

The implementation gives detail comparison between blind spectrum sensing technique performances over fading channel and multiple network nodes. In particular, the result shows the minimum required energy level for optimal detection (with acceptable interference) over fading channel and also how effectively cooperative spectrum sensing overcome the hidden primary user problem for both scheme and channels.

***Keywords:* Cognitive radio, energy detection, eigenvalue based detection, blind spectrum sensing, fading channels, spectrum hole.**

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Abbreviation

ADC - Analog-to-Digital Converter	IEEE – Institute of Electrical and Electronics Engineers
AP - Access Point	ISI - Intersymbol Interference
AWGN - Additive White Gaussian Noise	LPF - Low Pass Filter
BS - Base Station	MInT - Ministry of Innovation and Technology
BW - Bandwidth	MME - Maximum and Minimum Eigenvalue
CAF - Cyclic Autocorrelation Function	NP - Neyman Pearson
CCDF-ComplimentaryCumulative Distribution Function	OSA - Opportunistic Spectrum Access
CDF - Cumulative Distribution Function	PD - Probability of Detection
CR - Cognitive Radio	PDF - Probability Density Function
CSI - Channel-State Information	PFA - Probability of False Alarm
CSS (32) - Cooperative Spectrum Sensing	PM - Probability of Missed Detection
DSA - Dynamic Spectrum Access	PU - Primary User
ED - Energy Detection	PU TX1 - Primary Transmitter
EME - Energy with Minimum Eigenvalue	QOS - Quality-of-Service
ETA - Ethiopia Telecommunication Agency	RF - Radio Frequency
FC - Fusion Center	RMT - Random Matrix Theories
FCC - Federal CommunicationsCommission	ROC - Receiver Operating Characteristics
FIR - Finite Impulse Response	RV - Random Variable
FSA - Fixed Spectrum Allocation	SCF - Spectral Correlation Function

SDR - Software-Defined Radio

SU - Secondary User

SNR - Signal-to-Noise Ratio

WLAN - Wireless Local Area Network

SS - Spectrum sensing

WRAN - Wireless regional Area Network

Chapter 1: Introduction

1.1 Overview

From the beginning of wireless communications a use of available radio spectrum has always been a matter of concern because spectrum is a limited and valued natural resource. Then again a static spectrum allocation policy adopted by governments of many countries has caused underutilization of spectrum because a huge segment of licensed radio spectrum is not efficiently used [1]. Conventionally, licensed spectrum is allocated over relatively long time periods and is meant to be used only by licensees [2]. A government organization is responsible for allocating spectrum bands to operators. In Ethiopia, the ministry of innovation and technology (MInT) is responsible for this exercise; as federal communications commission (FCC) is for USA. This approach is termed as the fixed spectrum allocation (FSA) scheme and with this the radio spectrum is divided into bands allocated to distinct technology based services, e.g. mobile telephony, radio and TV broadcast services. The FSA supervision structure guarantees that the radio frequency spectrum is entirely licensed to an authorized party (primary user (PU)) without interference [3].

The current rapid growth of wireless communications has made the problem of spectrum utilization ever more challenging. On one side, the rising diversity (voice, short message, Web, and multimedia) and demand of high quality-of-service (QoS) applications have resulted in overcrowding of the allocated FSA spectrum bands leading to significantly reduced levels of user satisfaction. The challenge is particularly serious in communication-intensive situations such as in case of people gatherings or in a substantial emergency. On the other side, major licensed bands such as those allocated for television broadcasting have been found to be grossly underutilized which resulting in spectrum wastage. For example, the study report of FCC shows that the spectrum utilization in the 0–6 GHz band varies from 15% to 85%. In Ethiopia case, the national radio frequency spectrum allocation table from ETA shows that the spectrum in the 960M–3GHz band as highly underutilized [4,5]. This makes the benefit of cognitive radio significant to open licensed bands to unlicensed users which is known as dynamic spectrum access (DSA). Accordingly, the IEEE has formed a working group IEEE 802.22 to develop an air interface for opportunistic secondary access or dynamic spectrum access DSA via the cognitive radio technology [6].

Cognitive radio as a new concept was first proposed by Joseph Mitola III and Gerald Maguire [1] where presented as an extension of software defined radio enhancing flexibility of personal wireless services with radio domain model and computational intelligence. CR therefore is an exciting and new way of thinking about wireless communications. Indeed it is already being considered as one of the key candidate technologies for the fifth-generation (5G) wireless systems which aim to provide a higher data rate transmission, adequate capacity, cost efficiency and highly sophisticated services.

CR aims to improve spectrum utilization and efficiency of spectrum usage by opportunistically accessing the licensed spectrum without interfering the licensed users [7]. To avoid using the spectrum at the same time with the licensed users, CR has to determine the existence of primary user (PU) by sensing the wireless spectrum. It can communicate to its receiver if the spectrum is vacant. However, when PUs retransmits again, CR users should stop their transmission immediately to avoid creating interference to the PUs. Hence, spectrum sensing is vital for CR. The better CR knows about the PU's existence the better it can communicate and utilize the spectrum. There are several types of spectrum sensing techniques for CR such as energy detection, eigenvalue based detection, matched filter and Cyclostationary. This thesis focuses on studying the performance comparison of blind spectrum sensing techniques which are energy detection and eigenvalue based detection by applying them over fading channels.

1.2. Statement of the Problem

Spectrum sensing is a fundamental component of cognitive radio since continuous study of the environment is needed to determine the existence of licensed user. A failure in detection of a licensed user result either in causing harmful interferences to primary user or in missed opportunity for secondary user to utilize the spectrum hole.

Multipath fading or shadowing can cause hidden primary user problem which is one of the challenge of spectrum sensing for cognitive radio while scanning for primary user transmission [10]. Evaluating detection techniques by considering the channel effect will be necessary to avoid hidden primary user problem at the pre-deployment stage of cognitive radio.

The problem considered in this work is to determine an effective spectrum sensing technique that perform better in presence of fading channel without involving priory knowledge of the primary user signaling scheme.

1.3 Objectives

1.3.1. General Objective

- The main objective of this thesis work is to evaluate and compare the performance of the blind spectrum techniques over fading channel.

1.3.2. Specific Objectives

The specific objectives are

- To study and analyze the impact of fading channels on blind spectrum sensing techniques.
- To study the behavior of energy detection and eigenvalue based detection via Raleigh and Nakagami channel environments.
- To investigate the level of effectiveness of cooperative detection on the performance of the energy detection and eigenvalue based detection spectrum sensing technique over fading channels.

1.4. Literature Review

Different studies have been made in the area of spectrum sensing techniques of cognitive radio, among those the following are reviewed to frame the scope of the thesis and the existing problem.

In its simple form, the idea of blind spectrum sensing is to find primary transmitters operating at a given time by using local measurements and observations without prior knowledge from the transmitter [4]. With energy detection technique, the secondary user examines the signal strength generated from the primary user to exploit the free space (whitespace) within the channel and eigenvalue based detector determines the presence or absence of the primary signal based on the covariance matrix of the received signal.

In [10] Tevfik Y‘ucek and H’useyin Arslan outlined that one of the challenges of spectrum sensing for cognitive radio is the hidden primary user problem which can be caused by many factors

including severe multipath fading or shadowing observed by secondary users while scanning for primary users transmissions. Here, cognitive radio device causes unwanted interference to the primary user (receiver) as the primary transmitter's signal could not be detected because of the locations of devices.

The work in [11] stated the uncertainty of noise imposes fundamental limitations on the performance of the energy detector. Below a particular threshold SNR a reliable detection cannot be achieved by increasing the sensing duration. This SNR threshold for the detector is known as SNR wall. From experimental studies with the help of the primary user signal information (which needed prior information); the SNR wall was mitigated but it was not possible to eliminate it completely. Moreover, the energy detector cannot differentiate the primary user signal from the noise and other interference signals which may result to a high false alarm. The proposed approach in case of no information from primary user to set optimal detection with very low false alarm probability should be by evaluating the performance while considering the energy attenuating channel behaviors.

In the research paper [12], the author proposed new methods based on the eigenvalues of the covariance matrix of the received signal. It shows that the ratio of the maximum or average eigenvalue to the minimum eigenvalue can be used to detect the presence of the signal. Based on some latest random matrix theories (RMT) [12, 13] they quantify the distributions of these ratios and find the detection thresholds for the proposed detection algorithms. The probability of false alarm and probability of detection are also derived by using the RMT. The proposed methods overcome the noise uncertainty problem and can even perform better in case of highly correlated signals. Furthermore, the methods can be used for various signal detection applications without knowledge of the signal [14, 15].

In reference [8] the author shows comparison between AWGN and fading environments. The Gaussian model is popular in the parameter optimization problems e.g., optimizing the operating threshold or the power allocation so as to achieve the maximal throughput or minimal error rate. This model often gives a more convenient cost function which may result in a convex optimization problem. However, based on the Gaussian model the analysis of the average detection performance of blind detectors over different fading scenarios is not well practiced in the open literature.

In general, when we consider spectrum sensing techniques in cognitive radio without prior knowledge of primary user signal we can put most of the recent studies in two groups. Studies on one perspective focus on values such as less complicated, simple signal processing and less detection time which end up preferring energy detection over other schemes [4, 21, 27]. On the other perspective, study papers which deal with robustness to noise uncertainty and high correlation signals point to Eigenvalue based detection [12, 30, 31]. However, comparison in performance between energy detection and eigenvalue based detection over varying SNR was not done in fading channel environments. In this thesis work the performance comparisons between energy detection and eigenvalue based detection techniques in presence of Rayleigh and Nakagami-m fading channels is to be investigated. More so, the impact of employing cooperative secondary user nodes over fading channels is also considered. The designing of the detectors is based on consideration of fulfilling fundamental sensing requirements proposed in IEEE 802.22 WRAN (missed-detection and false alarm probabilities ≤ 0.1) [6].

1.5. Thesis Contributions

Spectrum scarcity is one of the challenging tasks in telecommunication industry due to technologies advancement as well as the increasing network size arising from wireless communication service demand from the public at large. Hence, special attention has to be given for cognitive radio network and its spectrum sensing performance. This study contributes on giving information which is useful in reducing interference caused by hidden primary user. It also helps to achieve the desired performance requirements before network deployment. It provides detail consideration of energy and eigenvalue based detection over fading environmental. Finally, the analyzing of cooperative network system in Rayleigh and Nakagami-m fading for blind spectrum sensing schemes are considered.

1.6. Limitation of the Study

The research:

- Is limited to simulation-based investigation which relies on theoretical analysis.
- Considers only the blind spectrum sensing techniques and two fading channel environments.

1.7. Organization of this Paper

The remaining sections of the paper are organized as follows.

Chapter 2 addresses the background studies of the cognitive radio system, introducing the network architecture and cycle of cognitive radio. Chapter 3 reviews spectrum sensing techniques: transmitter detection, cooperative detection and interference based detection and presents an overview of energy detection, eigenvalue based detection, matched-filtering and cyclostationary detection. Reasons are then given as to why the energy detector and eigenvalue based detector was chosen in this research. In Chapter 4, background theories on wireless communications and its characteristics are presented. Both large-scale and small-scale propagation are discussed along with common channel models. Chapter 5 describes performance analysis and detection threshold over Rayleigh and Nakagami-m fading channels. Chapter 6 provides the simulation results and discussion of single node and cooperative spectrum sensing of energy detection and eigenvalue based detection over Rayleigh and Nakagami-m fading channels. Finally, in Chapter 7, the conclusion and suggestions for future work were presented.

Chapter 2: Cognitive Radio Systems

2.1. Introduction

Cognitive radio is an intelligent wireless communication system that is aware of its surrounding environment (outside world) and uses the methodology of understanding by building to learn from the environment and adapt its internal states to statistical variations in the incoming RF stimuli by making corresponding changes in certain operating parameters in real-time with two primary objectives in mind [17]:

- Highly reliable communications whenever and wherever needed
- Efficient utilization of the radio spectrum

Spectrum utilization can be improved significantly by making it possible for a secondary user to access a spectrum hole unoccupied by the primary (licensed) user at the right location and time. Cognitive radio inclusive of Software-Defined Radio (SDR) has been proposed as the means to promote the efficient use of the spectrum by exploiting the existence of spectrum holes [1].

2.2. Cognitive Radio Networks

By dynamically switching between unoccupied spectrum gaps CRs take advantage of the locally unused spectrum. Cognitive radios possess the ability to observe their communication environment and adapt the parameters of their communication scheme to maximize the spectrum while minimizing interference to the primary users [18, 19]. To do so CRs must continuously sense the spectrum in use in order to detect reappearance of a primary user. This and other functions of CRs are contained in the basic cognitive network.

CR is defined as a radio that can change its transmission parameters based on interaction with the environment in which it operates. CR enables its users to

1. Determine which portion of the spectrum is available and detect the presence of licensed users when a user operates in a licensed band.
2. Select the best available channel.
3. Coordinate access to this channel with others.
4. Vacate the channel when PU is detected.

The figure below shows the concept of CR and Dynamic Spectrum Access. CR user dynamically and opportunistically accesses the spectrum hole or the white space while avoiding an access to the spectrum which is currently in use by the PUs [18].

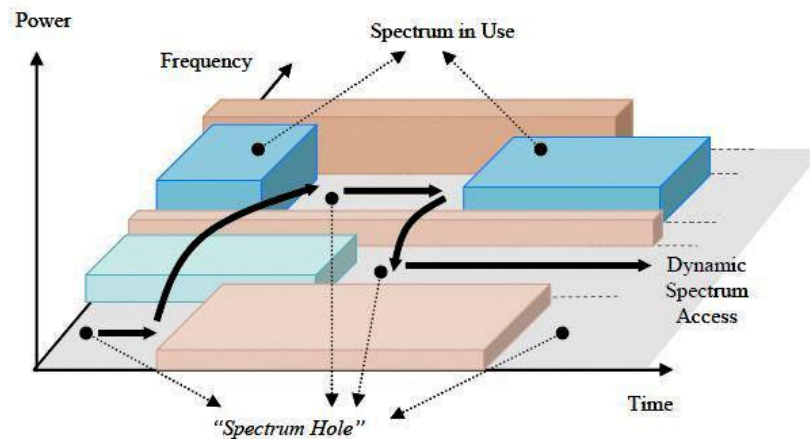


Figure 2. 1: Cognitive radio concepts [19]

2.3. Cognitive Radio Architectures

Cognitive Radio network architecture can be categorized into two groups; the primary network and the cognitive radio network [19].

The primary network is an existing infrastructure which has an exclusive right over a certain spectrum bands for example the cellular networks and TV broadcast networks. The components of the primary networks are

- Primary User (licensed user): a user which has a license to operate in a licensed band. The PU operation should not be affected by the operations of CR users.
- Primary Base-Station (licensed base-station): a fixed infrastructure network component with spectrum license.

The CR network does not have license to operate in a licensed band and its spectrum access is allowed opportunistically. The components of the cognitive radio networks are

- Cognitive Radio User (unlicensed user): a user who has no license over the spectrum. CR user can access the spectrum opportunistically only when PU is not present and CR user must vacate the channel immediately when the PU is detected.
- Cognitive Radio Base-Station (unlicensed base-station): a fixed infrastructure component with CR capabilities providing a single-hop connection to CR users. In cooperative spectrum sensing, the CR Base-Station also serves as a fusion center to gather the information from cooperative users and make the final spectrum sensing decision.
- Spectrum Broker (scheduling server): a central network entity that controls spectrum resource sharing among the CR users.

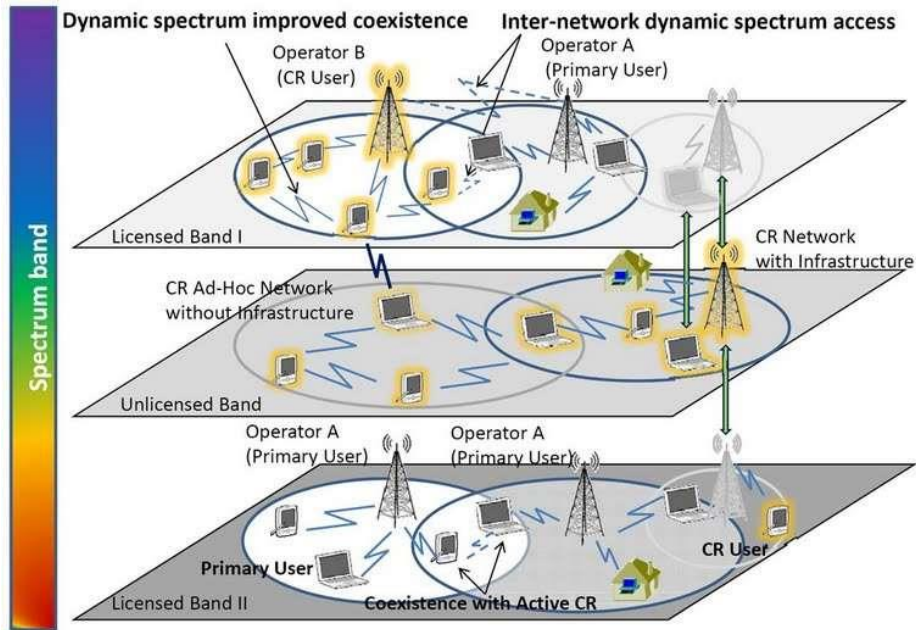


Figure 2. 2: Cognitive radio architectures [25]

2.4. Cognitive Radio Cycle

The cognition cycle by which a cognitive radio may interact with the environment is illustrated in figure 2.3 below. Stimuli enter the cognitive radio as interrupts dispatched to a cognition cycle for a response and Such a cognitive radio continually observes, orients itself, creates a plan, decides and then acts. In addition learning may be pursued in the background [2].The cognitive process starts with the passive sensing of RF stimuli and concludes with action.

Cognitive radios attempt to improve spectral utilization by performing three key online cognitive tasks [17]:

- a) Radio-scene analysis which encompasses the following:
 - Estimation of interference temperature of the radio environment to identify sources of interference.
 - Detection of spectrum holes to notify the absence of primary users or vacant RF spectrum.
- b) Channel identification which encompasses the following:
 - Estimation of channel-state information (CSI); which includes estimation of fading, shadowing and modeling signal propagation between receiver and transmitter.
 - Prediction of channel capacity for use by the transmitter.
- c) Transmit-power control to overcome signal fading and increase system performance and dynamic spectrum management to allocate frequency ranges based on the information obtained from the spectrum sensing part of the cognitive radio.

It is apparent that the cognitive module in the transmitter must work in a harmonious manner with the cognitive modules in the receiver. In order to maintain this harmony between the cognitive radio's transmitter and the receiver at all times a feedback channel connecting the receiver to the transmitter is needed. Through the feedback channel the receiver is enabled to convey information on the performance of the forward link to the transmitter.

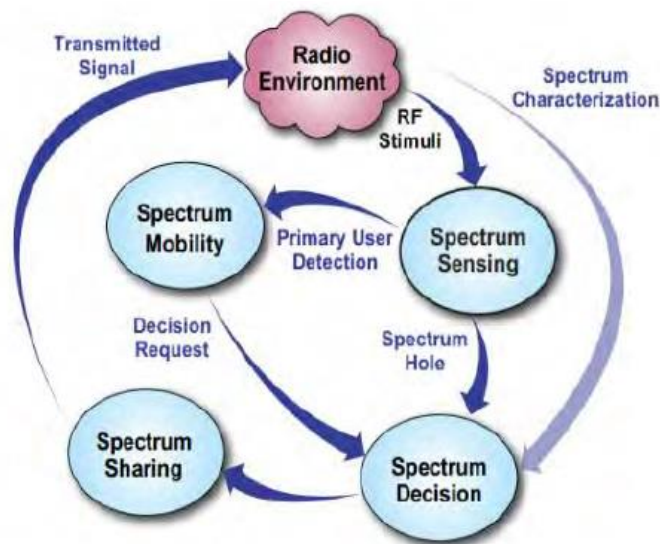


Figure 2. 3: Basic cognitive cycle [26]

CR function is basically including the following four tasks [17]:

- **Spectrum sensing** – CR continuously sense the environment and determine the unused spectrum. It is an active spectrum awareness process where cognitive radio monitors its radio environment and geographical surroundings to detect usage statistics of other primary and secondary users and determine possible spectrum space holes. Spectrum sensing can be done by one cognitive radio, by multiple cognitive radio terminals or by independent sensing network exchanging information in a cooperative way which improves overall accuracy.
- **Spectrum Management** – before using unallocated spectrum band CR checks data rates, transmission mode and capture the best available spectrum according to the user requirement. It involves making a decision based on spectrum sensing information to allow cognitive radio to select when to start its operation, operating frequency and its corresponding technical parameters. Cognitive radio primary objective is to transfer as much information as possible and to satisfy required quality of service without causing excessive interference to the primary users.
- **Spectrum Sharing** – defines fair spectrum scheduling methods by providing the fare and sophisticated scheduling mechanism among the coexisting users. Since there are a number of secondary users participating in usage of available spectrum holes, cognitive radio has to achieve a balance between its self-goal of transferring information.
- **Spectrum Mobility** – during the communication if CR users detect the primary user in its spectrum, CR must vacant the band to avoid interference with primary users. For faultless communication CR must switch to another unallocated spectrum band. It involves maintaining faultless connection during spectrum transition and handoff. That is, if a primary user starts to operate then cognitive radio has to stop its operation or to vacate currently used radio spectrum and change radio frequency. In order to avoid interference to primary licensed user this function has to be performed in real time. Therefore, cognitive radio has to constantly investigate possible alternative spectrum holes.

Chapter 3: Spectrum Sensing for Cognitive Radio

3.1. Introduction

Opportunistic spectrum access (OSA) is one that facilitates exploitation of local spectrum availability without affecting primary user [20]. The foundation on which the CR paradigm is built is the OSA. With this paradigm devices would be capable of sensing the environment over swaths of spectrum to find spectral holes and make use of frequency bands that are not occupied by primary users inducing no harm to the legacy system in the process. Basically, the secondary user identifies “gaps” in the spectrum known as a spectrum holes or white spaces and puts them to use. These white spaces originate from partial or no occupations by the licensed users (primary users (PU)) e.g. Digital TV broadcasters. SU also needs to vacate the channel immediately when the PU comes back. As a result, spectrum holes detection plays an important role in the CR system.

3.2 System Model

Analytically, when the decision on the availability of a primary user is to be made, it is reduced to an identification problem [24]. This is formalized as a hypothesis test as:

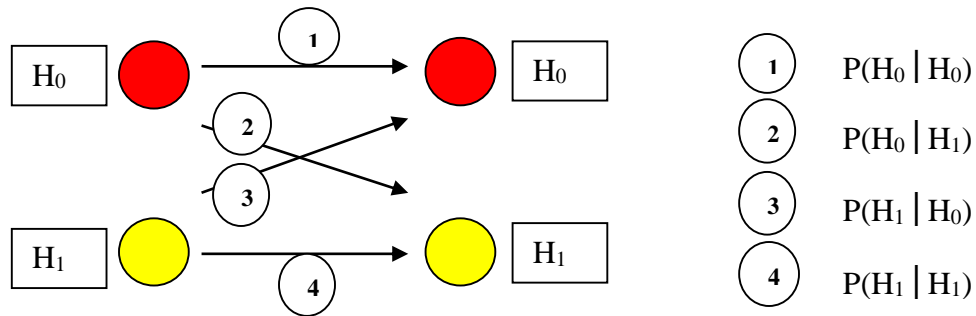


Figure 3. 1: Hypothesis test with possible outcomes and their corresponding probabilities [21].

- From Figure 3.1, four possible cases can be defined for the detected signal;
1. Declaring H_0 when H_0 is true ($H_0:H_0$)
 2. Declaring H_0 when H_1 is true ($H_0:H_1$)
 3. Declaring H_1 when H_0 is true ($H_1:H_0$)
 4. Declaring H_1 when H_1 is true ($H_1:H_1$)

Case 4 is known as a correct detection, whereas cases 2 and 3 are termed as false alarm and missed detection respectively. The goal of the signal detector is to achieve correct detection all the time. However, this cannot be accomplished absolutely in practice because of the statistical nature of the problem. Therefore, signal detectors are designed to function within minimum error levels. A prominent issue for spectrum sensing is missed detection; as it implies interfering with the primary system and also the false alarm rate has to be kept as low as possible to enable the system exploit possible transmission opportunities. The performance of the spectrum sensing technique is influenced by the probability of false alarm $PFA = P(H_1 | H_0)$ an important metric for spectrum sensing. As seen in [24, eq. (3.1)] a reliable method to differentiate a signal from noise is required.

$$x(k) = \begin{cases} n(k), & H_0 \\ h(k)s(k) + n(k), & H_1 \end{cases} \quad (3.1)$$

Where $x(k)$ represents the sample to be analyzed at instant k , $h(k)$ denotes the channel gain at each instant k , $s(k)$ represents the signal to be detected and $n(k)$ is the additive noise at the CR receiver, modeled as an independent and identically distributed zero-mean circularly symmetric complex Gaussian noise that is independent of $s(k)$ (of variance σ_w^2) that is $n(k) \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2 \mathbf{I})$ where \mathbf{I} denotes an identity matrix. H_0 is the null hypothesis; representing a sensed state with an absence of the licensed user signal. H_1 denotes the existence of a licensed user signal within the spectrum under consideration.

To evaluate the performance, a decision threshold (η) is set and compared to the test statistic (Y) generated. The detection rule is determined as:

$$Y > \eta \quad \text{primary user present}$$

$$Y < \eta \quad \text{primary user absent}$$

3.3 Spectrum Sensing Techniques

There are various ways to detect the spectrum holes: transmitter detection, cooperative detection and interference based detection. Transmitter detection methods consist of matched filter, eigenvalue based detection, cyclostationary and energy detection [11,21]. These techniques are further classified as [22] coherent, semi-coherent or non-coherent; that is either having complete partial or no prior knowledge of the transmitter respectively. Schemes that are cooperative include

centralized, distributed and relay-assisted sensing methods. Whereas transmitter and cooperative detection methods perceive spectrum to avoid interference to primary transmitters [23].

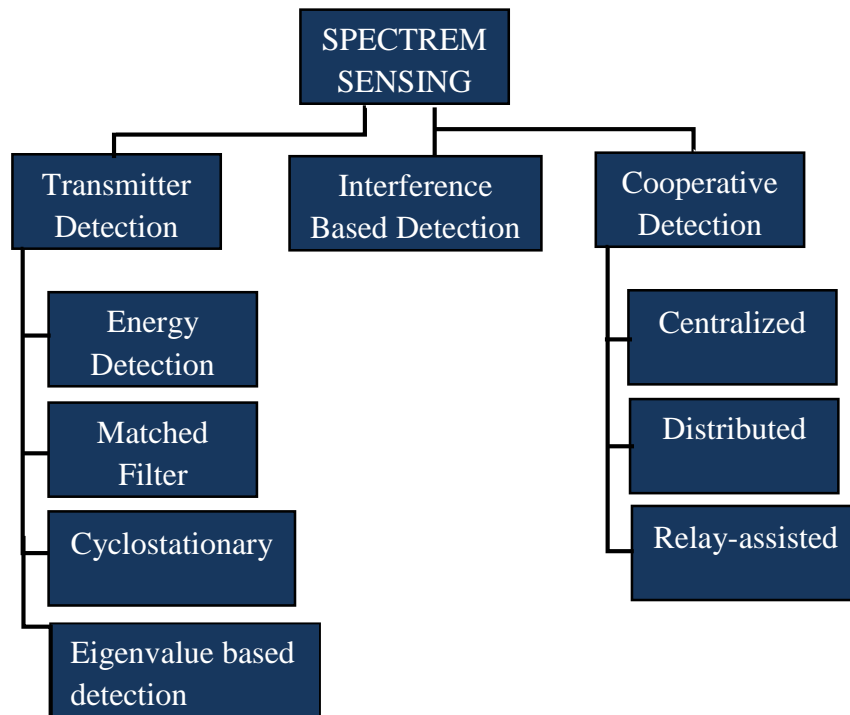


Figure 3. 2: Classifications of spectrum sensing techniques [11]

An efficient approach to identify spectral opportunities with low infrastructure requirement is to detect the primary receiver within operative range of a secondary user (SU). However, practically this is not feasible as the SU cannot locate a receiver since it is not intelligent enough. Hence, spectrum sensing methods rely on detecting the primary transmitter [11,21]. With this, a primary user transmitter is detected on the basis of the received signal at the secondary user end. The primary transmitter detection model represents analysis of the received signal at the secondary user. In its simple form, the idea is to find primary transmitters operating at a given time by using local measurements and observations. With these techniques the SU examines the signal strength generated from a PU to exploit the free space (whitespace) within the channel.

3.3.1. Blind Detection Techniques

Blind Signal Processing involves estimating the presence of a signal only based on the received signal at the receivers and minimize the error vector. This requires no knowledge of a PU signal.

3.3.1.1 Energy Detection

This is the most popular detection technique and has “the least computational and implementation complexity” [27]. Apriori knowledge of the primary user signal is not required for signal detection [21, 28]. Calculating the received signal’s energy gives the test statistic which is compared to a predetermined threshold. The threshold is determined from noise energy and its accuracy is a key to the performance of the energy detector. If the received signal’s energy at the cognitive radio is greater than the set threshold, the alternate hypothesis H_1 is validated and the primary user is concluded to be present. If the energy is lower, the null hypothesis H_0 is validated thus signifying the presence of a spectrum hole. The binary hypothesis is presented in Equation 3.1.

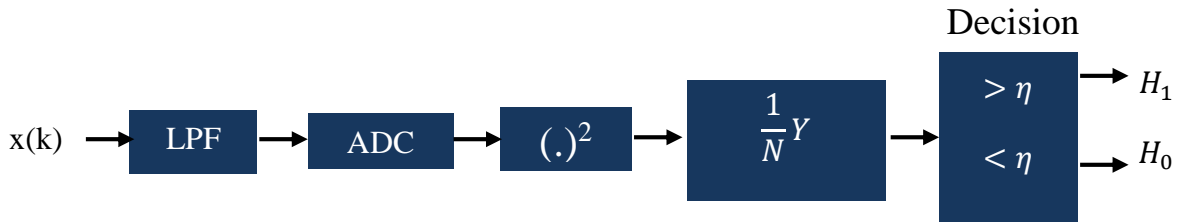


Figure 3. 3: General blocks of energy detection [11]

Figure 3.3 shows a simple energy detector which contains a Low Pass Filter (LPF), Analog-to-Digital Converter (ADC), the energy detector and the average N sample. The decision part will compare the threshold with Y and give its result.

The Neyman Pearson (NP) theorem states that the binary hypothesis test that maximizes the probability of detection (defined as the probability of deciding H_1 when H_1 is true) for a given probability of false alarm (the probability of deciding H_1 when H_0 is true) uses the likelihood ratio as the test statistic [21, 28, eq. (3.2)].

$$P(x) = \frac{p(x|H_0)}{p(x|H_1)} \leq \eta \quad (3.2)$$

Where, x denotes the observations and η the threshold.

The optimal Neyman-Pearson test is to compare the log-likelihood ratio to a threshold. That is [21, 28, eq. (3.3)].

$$P(x) = \log \left(\frac{p(x|H_1)}{p(x|H_0)} \right) \underset{H_0}{\overset{H_1}{\geq}} \eta \quad (3.3)$$

Clearly, the log-likelihood ratio depends on the distribution of the signal to be detected. In energy detection it is assumed that the signal to be detected does not have any known structure that could be used for detection. Thus it is assumed that the signal is also zero-mean complex Gaussian $s(k) \sim (\mathbf{0}, \gamma^2 \mathbf{I})$. Then $x|H_0 \sim (\mathbf{0}, 2\mathbf{I})$ and $x|H_1 \sim (\mathbf{0}, (\sigma_w^2 + \gamma^2))$. The log-likelihood ratio therefore becomes [21, 28, eq. (3.4)].

$$\log \left(\frac{p(x|H_1)}{p(x|H_0)} \right) = \log \left(\frac{\frac{1}{\pi^N (\sigma_w^2 + \gamma^2)^N} \exp\left(-\frac{\|x\|^2}{\sigma_w^2 + \gamma^2}\right)}{\frac{1}{\pi^N \sigma_w^{2N}} \exp\left(-\frac{\|x\|^2}{\sigma_w^2}\right)} \right) \quad (3.4)$$

By removing all the constants that are independent of the received vector $x(k)$ we obtain the optimal Neyman-Pearson test [21, 28, eq. (3.5)].

$$Y = P_c = \|x\|^2 = \frac{1}{N} \sum_{n=0}^{N-1} |x[k]|^2 \underset{H_0}{\overset{H_1}{\geq}} \eta_c \quad (3.5)$$

Where $|\cdot|$ denotes the Euclidean norm of a complex vector and N number of samples taken. In essence the energy detector measures the received energy during a finite time interval and compares it to a predetermined threshold.

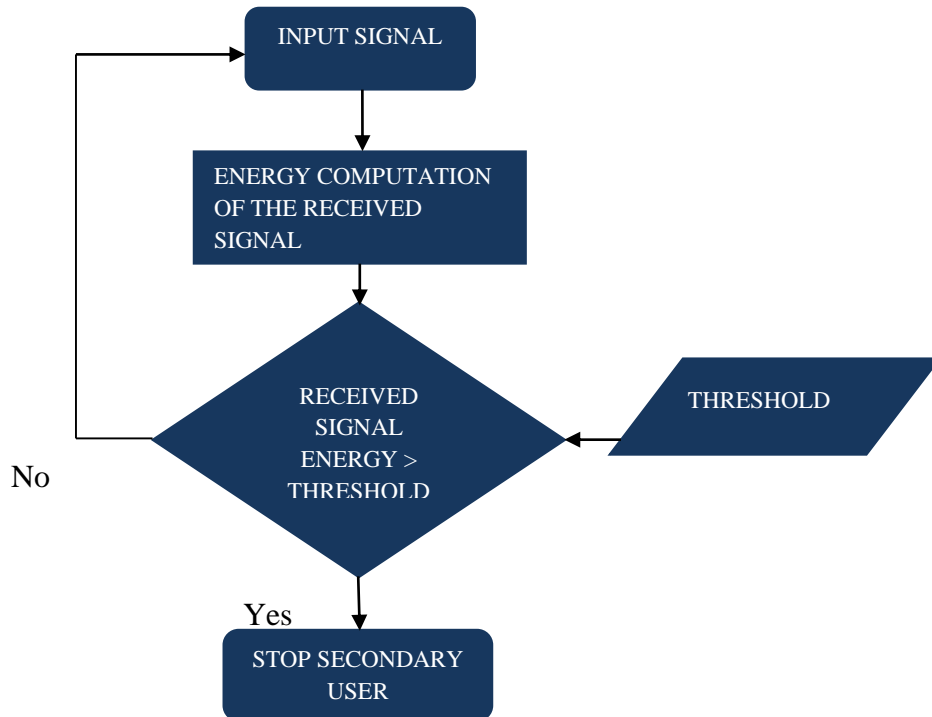


Figure 3. 4: Energy detection spectrum sensing algorithm flow chart [11]

3.3.1.2 Eigenvalues based detection

The key idea behind Eigenvalues based primary signal detection is that the primary signal received at the CR user is usually correlated. Such correlation can be utilized by the CR user to differentiate the primary signal from white noise [29]. Specifically, Eigenvalues based detector determines the presence or absence of the primary signal based on the covariance matrix of the received signal. Suppose that the received signal is given by [29, eq. (3.6)].

$$x(k) = hs(k) + n(k), \quad 0 \leq k \leq N - 1 \quad (3.6)$$

Then the sample covariance matrix of the received signal can be estimated as [29, eq. (3.7)].

$$\hat{R}_x = \frac{1}{N} \sum_{k=L-1}^{N-1} x[k]x^H[k] \quad (3.7)$$

Where

$$\mathbf{x}[k] = (x[k], x[k-1], \dots, x[k-L+1])^T \quad (3.8)$$

It can be shown [29] that \hat{R}_x converges in probability at [29, eq. (3.9)].

$$R_x = E\{x[k]x^H[k]\} = HR_s + \sigma_w^2 I_L \quad (3.9)$$

Where R_s is the $L \times L$ covariance matrix of the primary signal vector [29, eq. (3.10)].

$$s[k] = (s[k], s[k-1], \dots, s[k-L+1])^T \quad (3.10)$$

σ_w^2 is the noise power, and I_L is an $L \times L$ identity matrix.

Based on the sample covariance matrix of the received signal \hat{R}_x various test statistics have been developed for detection of the primary signal including the ratio of its maximum and minimum eigenvalue [29], the ratio of its diagonal and off-diagonal element [30], and its maximum eigenvalue [31]. As an example,

Let λ_{max} and λ_{min} denote the maximum and minimum eigenvalues of R_x respectively. Define $u = (\lambda_{max}, \lambda_{min})$ [30, eq. (3.11)]; then according to (3.9)

$$u = \begin{cases} (\sigma_w^2, \sigma_w^2), & H_0 \\ (\rho_{max} + \sigma_w^2, \rho_{min} + \sigma_w^2), & H_1 \end{cases} \quad (3.11)$$

Where ρ_{max} and ρ_{min} are the maximum and minimum eigenvalues of R_s , respectively. Because of the correlation among the sampled primary signals $\rho_{max} > \rho_{min}$. Therefore, if there is no primary signal [30, eq. (3.12)]

$$\frac{\lambda_{max}}{\lambda_{min}} = 1 \quad (3.12)$$

Otherwise

$$\frac{\lambda_{max}}{\lambda_{min}} > 1 \quad (3.13)$$

Based on the above empirical, the eigenvalue-based detection algorithm can be formulated as follows [29].

- 1) Estimate the covariance matrix of the received signal according to (3.7).
- 2) Calculate the maximum and minimum eigenvalues of the sample covariance matrix as $\hat{\lambda}_{max}$ and $\hat{\lambda}_{min}$ respectively, and compute the decision statistics [30, eq. (3.14)]

$$Y = \frac{\hat{\lambda}_{max}}{\hat{\lambda}_{min}} \quad (3.14)$$

- 3) Obtain the final decision

if $Y > \gamma$ primary user present

if $Y < \gamma$ primary user absent

In [29], the statistical characteristics of $\hat{\lambda}_{max}$ and $\hat{\lambda}_{min}$ under H_0 have been investigated. Based on that a closed-form expression of the decision threshold γ for a given false-alarm probability has been obtained. In general, it is tough to obtain a closed-form expression of the detection probability $P_D = P_r\{\hat{\lambda}_{max} > \gamma \hat{\lambda}_{min} | H_1\}$. However, it can be calculated using numerical methods (RMT).

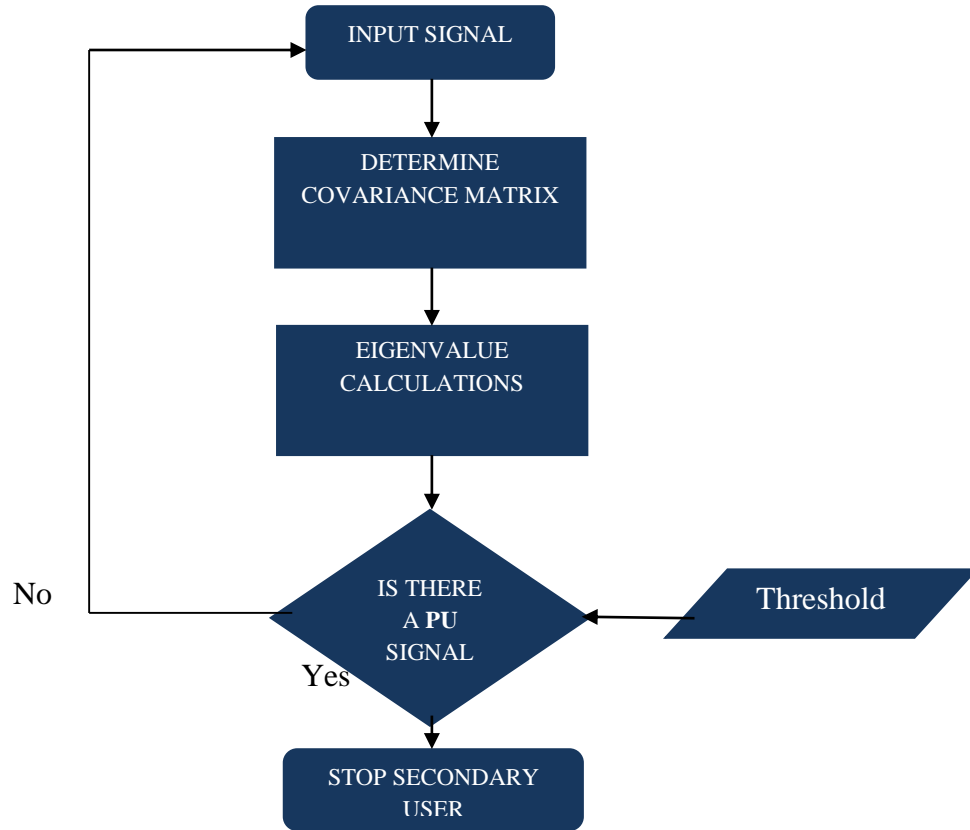


Figure 3. 5: Eigenvalue based spectrum sensing algorithm flow chart [11]

3.3.2. Other Detection Techniques

3.3.2.1. Matched Filtering

This is a coherent sensing technique and requires accurate knowledge of the transmitted signal from the PU to optimally detect the signal at the cognitive user. Such information includes the bandwidth, operating frequency, modulation and pulse shaping. The information is used for demodulating the received signal [32]. The apriori knowledge at the cognitive user increases the sensor's detection speed and accuracy. Matched filtering is therefore considered as the most efficient method for primary user detection as it takes the least time to determine the probability of false alarm [10]. Figure 3.4 shows the implementation of a matched filter using a Finite Impulse Response (FIR) of the known signal $S(k)$ to generate filter coefficients. The received signal $x(k)$ will then be matched to the response of the FIR filter as in [33, eq. (3.15)].

$$y[n] = \sum_{k=0}^n h[n - k]x[k] \quad (3.15)$$

Where, $y[n]$ is the output of the signal from FIR filter for the received signal and hence response $h[n] = S[N - 1 - n]$ for $n = 0, \dots, N - 1$.

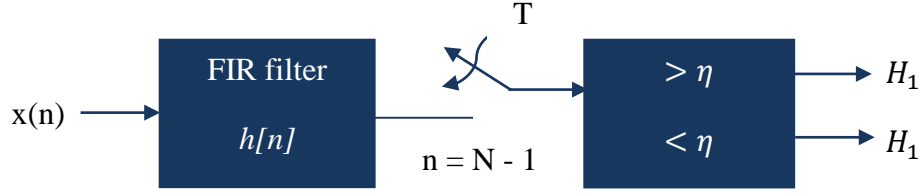


Figure 3. 6: Matched filter [11]

While matched filter detection needs less detection time due to a much better complexity compared to blind detection, matched filter detection requires a prior knowledge of every primary signal. If the information is not accurate, the detector performs poorly. Also a cognitive radio would need a dedicated receiver for every type of primary user [10].

3.3.2.2. Cyclostationary Feature Detection

Most man-made signals show periodic patterns related to symbol rate, chip rate, channel code or cyclic prefix that can be appropriately modeled as a cyclostationary random process [34]. Thus, radio communication signals can also be modeled as a Cyclostationary Processes as most modulated signal are associated with a signal wave that could give a cyclic property. Periodicity is introduced in signal transmission to insure receivers with good estimation of the signal sent. If we assume a signal $x(k)$ to be discrete zero-mean signal with autocorrelation function $R_x(n, k)$ cyclic with period T as in [35, eq. (3.16)].

$$R_x(n, k) = R_x(n + T, k + T) \quad (3.16)$$

The Cyclic Autocorrelation Function (CAF) is important parameter used to in feature detection. The Spectral Correlation Function (SCF) is also given by (3.17). That is frequency domain of (3.16).It can give an interpretation on the degree of correlation between frequency shifts of the signal as given by [35, eq. (3.17) & (3.18)]

$$S_x^\alpha(f) = \sum_{k=-\infty}^{\infty} R_x^\alpha(k) e^{i2\pi k f} \quad (3.17)$$

$$R_x^\alpha(k) = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N x(n) x^*(n-k) e^{-j2\pi \alpha n} e^{j\pi \alpha k} \quad (3.18)$$

Where, $R_x^\alpha(k)$ is the CAF of the discrete signal and α cyclic frequency. $S_x^\alpha(f)$ is the power spectrum of the signal for $\alpha = 0$. The detection method can be put as shown in Fig.3.5. the algorithms search for peak Cyclic Spectrum magnitude of the signal at any of the cyclic frequencies. If the peak is found signal exists otherwise signal does not exists. [12]

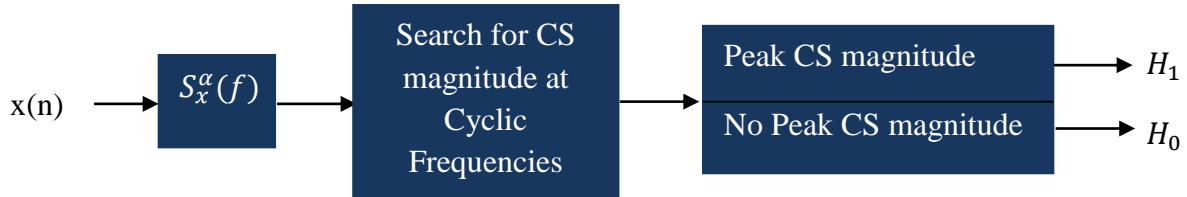


Figure 3. 7: Cyclostationary feature detection [11]

Phase information of signal is not problem for this method nevertheless the performance of feature detector will also depend on signal energy. Different modulation techniques incur different energy on the signal. Furthermore the method will require coherence of signal information like carrier frequency and is sensitive to sampling offset. This will increase the detection time as compared to energy detection. In that case using the spectrum holes is a run against time and spectrum occupancy need quicker response. More details could be obtained in [36].

3.4. Interference Based Detection

This theoretical method employs an interference temperature model; which is a measure of how well a radio operating within a particular modulation scheme and protocol can tolerate interference in its spectrum space [37]. This follows the fact that signal power received at a primary receiver reduces exponentially with distance; continuously till it reaches a level of the noise level [38]. Though a primary transmitter still operates at this point, the receiver handles this process as noise and not transmission. This makes it possible for a secondary user to utilize the channel, since no interference is introduced to the primary users' communication (as the primary receiver is not in receiving mode).

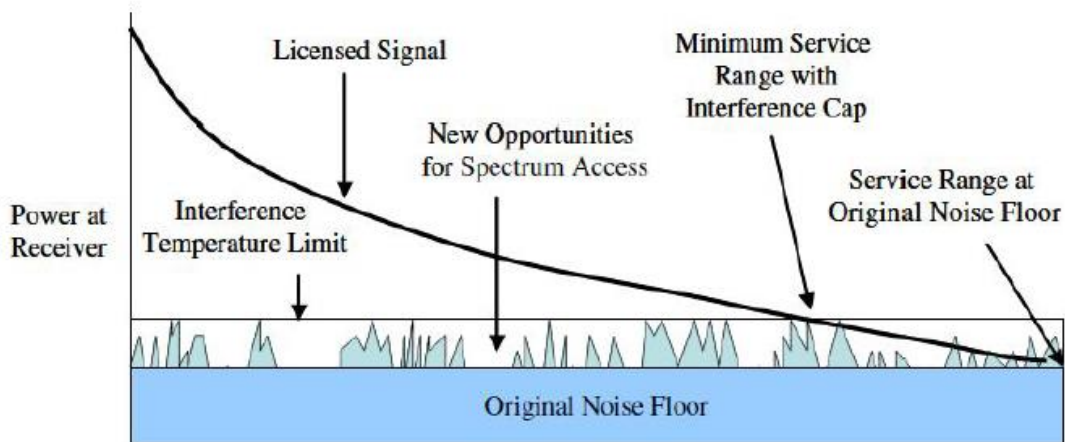


Figure 3. 8: Interference temperature model [19].

Above the maximum noise level an interference cap is introduced beneath this threshold; the primary receiver will treat this transmission as noise. An illustration of the interference temperature model is shown in Figure 3.8 above. The SU may exploit the channel if the detected primary signal level is below the interference temperature limit. More so, if the power of transmission of an SU stays below the interference gap, it may utilize any frequency parameter of its choice. With this approach it is hypothesized that the SUs will be allowed to transmit concurrently with the PUs under stringent interference avoidance constraints; wherein it is categorized as a spectrum underlay scheme. It is noteworthy nonetheless, that this method is far more challenging; since the prime problem faced with an implementation of this technique will be in determining specific receiver interference temperature levels for the various communication standards. Recently however, in [39] this approach to spectrum sensing was reportedly analyzed and declared to be non-implementable thus no further survey on this method will be conducted in this work.

3.5. Cooperative Detection

In cooperative detection multiple SUs collaborate in a centralized or decentralized manner to ascertain spectrum holes for opportunistic access. Each cooperating node within this context employs locally any of the sensing methods previously described while sharing the raw/refined sensing information with other node(s); dependent on a selected cooperation strategy [39] . This concept of collaboration is considered since effects of shadowing, multipath fading and receiver

uncertainty pose severe challenges to single user transmitter detection approach in SS [18]. A depiction of these phenomenons is depicted below and described in detail afterwards.

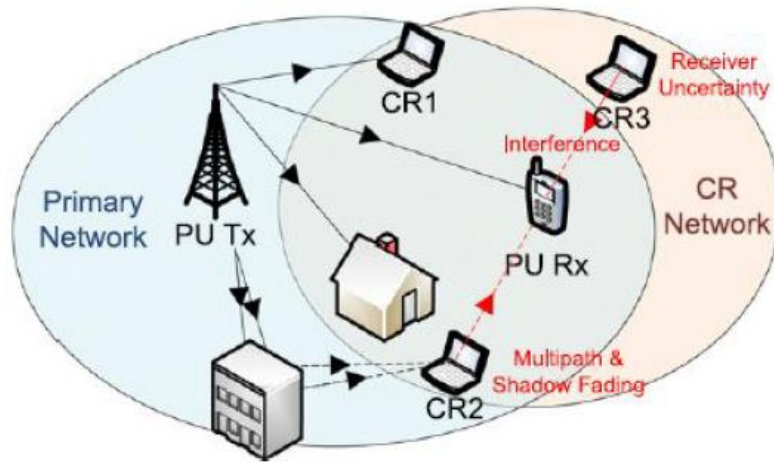


Figure 3. 9: Receiver uncertainty and multipath/shadow fading [18].

From the figure 3.7 above, CR1 and CR2 are within range of the primary transmitter (PU TX1) while CR3 is not. As a result of the obstruction from the house and due to multiple copies of the attenuated signal being sent CR2 suffers multipath and shadowing problems such that signals from the PU Tx may not be detected correctly. CR3 on the other hand is unaware of the transmission from PU Tx and the existence of primary receiver (PU Rx). Consequently, transmission from CR3 may interfere with reception at the PU Rx; this phenomenon is known as the receiver uncertainty problem.

However, owing to spatial diversity it is unlikely that all SUs spread in space within a network will simultaneously experience receiver uncertainty or fading problems. Since secondary users that observe a strong signal from the PU Tx like CR1 in the figure can sense and communicate sensed result to other users. This collaborative paradigm should tackle flaws in observation at the other users considerably. By this technique of cooperation amongst users robustness is achieved without severe demands on individual radios enhancing effective primary detection [40].

For CSS SUs require two channels for local sensing to arrive at a decision. Initially SUs establish a link with the primary transmitter to carryout local sensing; this link between primary transmitter and the various cooperating SUs is known as the sensing channel. To share local spectrum sensing data with each other or the fusion center (FC) requires a control or reporting channel. So far, a

medium access protocol coordinates the shift between these two channels [39]. Considering the mode of collaboration between sensing nodes in a detection scheme CSS is broadly categorized as centralized, distributed and relay assisted based on how collaborating SUs convey sensing data within the network [18].

3.5.1. Centralized Cooperative Detection

In a centralized structure a central unit designated the fusion center (FC) or basestation (BS) determines eventual availability of spectrum holes after collating local SS information from cooperating SUs [40]. This opportunity is either broadcast to all SUs or the FC itself controls traffic by managing detected spectrum usage opportunity in an optimum fashion. The central node (FC) could be an access point (AP) in a wireless local area network (WLAN) or a base station (BS) in a cellular network; while in ad hoc networks any node - once identified - can act as a master to coordinate CSS. In operation the FC selects a control channel for the transmitter and tasks the various SUs to send their local sensing results via a reporting channel. Cooperating SUs would send collected data to the FC allowing it perform a data fusion to decide the presence of a primary signal or they could each send individual decisions and the FC conducts decision fusion to assume a decision. For the scenario where the SU sends complete local sensing data, the fusion process is termed soft combining. When the SU quantizes the local sensing information before sending to the FC this fusion process is termed quantized soft combining. For hard combining fusion an SU makes decision after sensing and sends one bit as its decision to the FC [40].

3.5.2. Distributed Cooperative Detection

In a distributed cooperation, SUs would not rely on an FC to make a cooperative decision rather it is conceived that the SUs communicate within nodes then converge to a joint (global) decision on the presence or absence of PU in an iterative manner [39]. This is accomplished in three steps defined by a distributed algorithm as follows. First, each cooperating user sends its local sensing data to other users in its neighborhoods (defined by the transmission range of the users). Next, cooperating users combine data with the received sensing information from other users to decide on the presence or absence of a PU based on the local criterion. The shared spectrum observations are usually in the form of soft sensing results or quantized (binary/hard) version of local decisions about spectrum hole availability. In a case where the spectrum hole is not identified, SUs send combined sensing information to other users in the next iteration. This process continues until the

scheme converges and a final unanimous decision on spectrum availability is achieved. In this manner each SU in a distributed scheme partially plays the role of an FC [39].

3.5.3. Relay- assisted Cooperative Detection

It is predicted that under realistic conditions the sensing and reporting channels in the schemes outlined previously may not function properly. For instance, a particular SU reporting channel may be weak while its sensing channel strong arising from shadowing or multipath consequences yet another SU may possess a strong reporting channel and a weak sensing channel [39, 40]. The relay-assisted detection paradigm provides a scheme where a SU serves as a relay forwarding sensed information. In [40], the centralized and distributed schema is considered one-hop cooperation, while the relay-assisted approach is thought of as a multi-hop cooperative scheme.

Chapter 4: Wireless Communication Channel

4.1. Introduction

Wireless communication operates through the media which is the air interface. The transmitted signal will travel along the transmission path from the transmitter to the receiver. This path can be obstructed not only by objects in the line-of-sight between them but also objects that cause changes in the electromagnetic field at the receiver [41]. As a result the received signal will degrade and become different to the transmitted signal. As the wireless channels are random, the wireless channel modeling is typically done in a statistical way [42].

Wireless channel modeling is classified upon the variation in the channel. The variation due to path Loss and shadowing occurring over relatively large distances is referred as the large-scale propagation. On the contrary, the variation due to rapid fluctuations and fading over a short distance or time is referred as the small-scale propagation [8,43].

4.2. Large-scale Propagation

Large-scale propagation can be classified into two models; path loss and shadowing. These models are used to predict the average received signal strength at large transmitter-receiver distance [42].

4.2.1. Path Loss

Path loss is used to model the attenuation of the signal which travels from the transmitter to the receiver. The further the signal travels the more attenuation it experiences. Hence, path loss increases exponentially with the increasing distance. The propagation model for path loss at the distance x is given as [42, eq. (4.1)].

$$\frac{P_r}{P_t} = \frac{L_0}{x^\eta} \quad (4.1)$$

Where η is path loss exponent which is different and dependent on the propagation environment for instance it is 2 for free-space propagation and 4 to 6 under shadowed urban cellular radio. P_r and P_t are respectively the received and transmitted power [42]. L_0 is the path loss at a reference distance d_0 which is given as [42, eq. (4.2) & (4.3)].

$$L_0 = \frac{(4\pi d_0)^2}{\lambda^2 G_t G_r} \quad (4.2)$$

$$L_0(dB) = 10 \log_{10} \left(\frac{(4\pi d_0)^2}{\lambda^2 G_t G_r} \right) \quad (4.3)$$

Where λ is the wavelength, G_t and G_r are the antenna gains for the transmitter and receiver respectively.

4.2.2. Shadowing

Shadowing is an effect of the variation in terrain and presence of obstacles. This affects the received power at different locations with a fixed distance x and is modeled as log-normal distribution. The effect of shadowing can be expressed as [44, eq. (4.4)].

$$L(dB) = L_0(dB) + 10\eta \log_{10} \left(\frac{x}{d_0} \right) + X_\sigma \quad (4.4)$$

Where the summation for first two terms (L_0 and the \log_{10} terms) are the path loss at distance d_0 and X_σ , which is a zero-mean Gaussian distributed random variable in log-scale, represents log-normal shadowing effect.

4.3. Small-scale Propagation

Small-scale propagation is to characterize rapid fluctuations over short distance or time. There are various factors which influence small-scale fading such as multipath propagation, speed of the mobile, speed of surrounding objects and the transmission bandwidth of the signal.

4.3.1. Multipath Fading

Multipath fading occurs when signal arrives at the receiver through different paths, as a result of reflection, diffraction or scattering. This creates many replicas of the transmitted signal as each path can arrive with different gains, phases and delays. Mathematically, the multipath channel with S number of multipath components can be modeled as [42, eq. (4.6)].

$$h(t, \tau) = \sum_{i=0}^{S-1} a_i(t, \tau) \exp[j\theta_i(t, \tau)] \delta(\tau - \tau_i(t)) \quad (4.6)$$

Where $h(t, \tau)$ is tap-filter representing multipath channel, $a_i(t, \tau)$, $\theta_i(t, \tau)$, $\tau_i(t)$ is gain, phase and path excess delay respectively. There are two time-related variables in this equation as t is time variation due to motion and τ is time variation due to multipath delay.

4.3.2. Doppler Shift

Doppler shift is the shift in the received signal frequency due to the relative motion of transmitter and receiver. This shift can influence small-scale fading and it depends on the velocity and direction of motion of the mobile. The Doppler shift f_d can be expressed as [42, eq. (4.7)]

$$f_d = \frac{v}{\lambda} \cos \psi \quad (4.7)$$

Where, v , λ , ψ is the speed of movement, the wavelength and the angle between the direction of motion and the wave's arrival path.

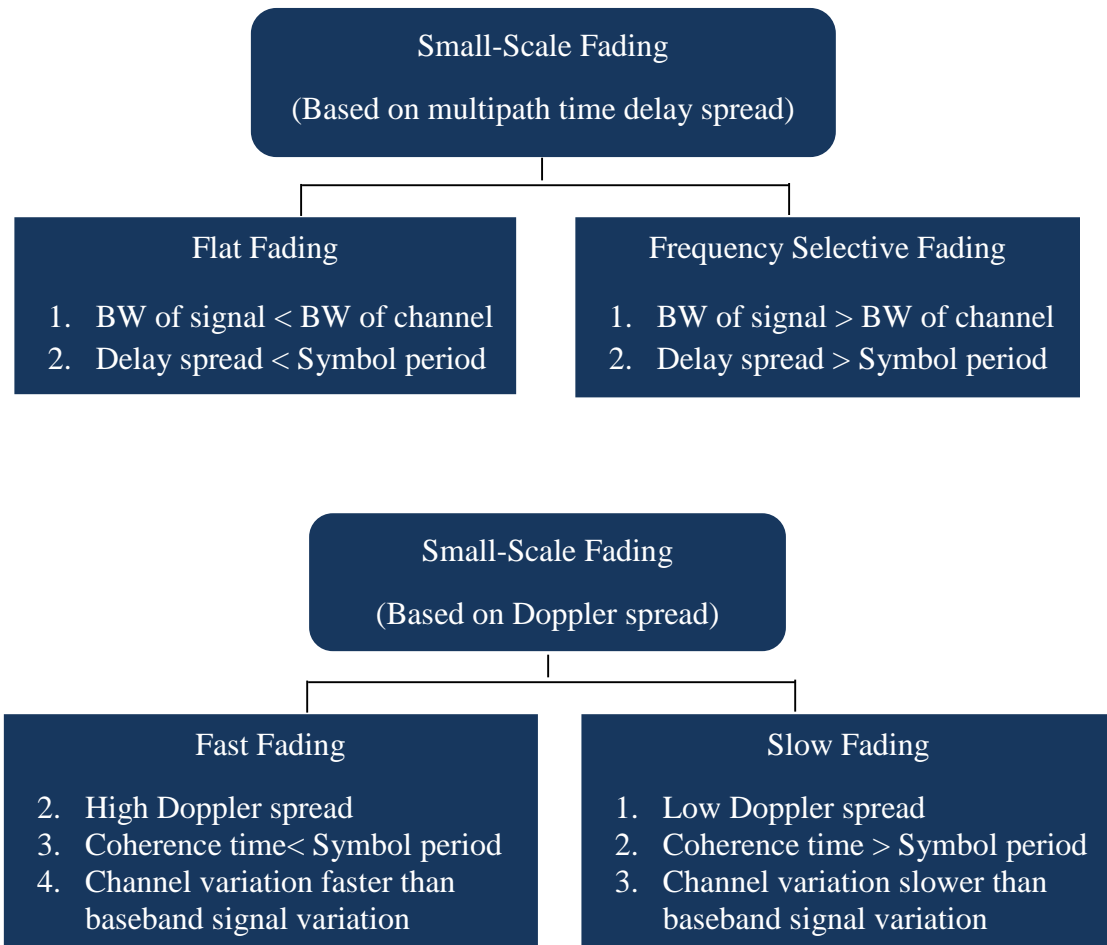


Figure 4. 1: Types of small-scale fading [42]

4.3.3. Types of Small-Scale fading

There are two types of small-scale fading. One is based on multipath time delay spread while the other is based on Doppler spread. However, these two types of fading are not mutually exclusive. Figure 4.1 summarizes small-scale fading based on multipath time delay spread and Doppler spread [42].

4.3.3.1. Fading Effects due to Multipath Time Delay Spread

Signals can experience either flat fading or frequency selective fading depending on the Multipath time delay spread.

➤ *Flat Fading*

Signal undergoes flat fading when coherence bandwidth, over which the mobile radio channel has a constant gain and linear phase response, is greater than the bandwidth of the signal, and the delay spread is smaller than the symbol period. Flat fading can be summarized as [42, eq. (4.8) & (4.9)].

$$B_s \ll B_c \quad (4.8)$$

and

$$T_s \gg \sigma T \quad (4.9)$$

Where B_s , B_c , T_s , σT is respectively the bandwidth of the signal, the coherence bandwidth, symbol period and the rms delay spread of the channel.

➤ *Frequency Selective Fading*

Signals undergo frequency selective fading when coherence bandwidth is smaller than the bandwidth of the signal and the delay spread is greater than the symbol period. In this case, the received signal is distorted because of multiple waveforms and channel induces intersymbol interference (ISI) to the signal. Frequency selective fading can be summarized as [42, eq. (4.10) & (4.11)].

$$B_s > B_c \quad (4.10)$$

and

$$T_s < \sigma T \quad (4.11)$$

4.3.3.2. Fading effects due to Doppler Spread

This type of fading is subject to the change of the channel due to motion. Channel can be classified as fast fading or slow fading based on the symbol duration and the change of channel.

➤ *Fast Fading*

In fast fading, the channel impulse response changes rapidly within the symbol duration. In other words, coherence time, which is time duration that the fading parameters remain fairly constant, is smaller than the symbol period. Fast fading is also called time selective fading. Fast fading can be summarized as [42, eq. (4.12) & (4.13)].

$$T_s > T_c \quad (4.12)$$

and

$$B_s > B_D \quad (4.13)$$

Where, T_c , B_D is the coherence time and the measure of spectral broadening caused by the time variation of the channel. B_D is defined as the range of frequencies where the received Doppler spectrum is non-zero.

➤ *Slow Fading*

In slow fading, the channel impulse response changes much slower than the transmitted baseband signal. In other words, coherence time is greater than the symbol period. The channel can be assumed to be static over one or several bandwidth intervals. Slow fading can be summarized as [42, eq. (4.14) & (4.15)].

$$T_s \ll T_c \quad (4.14)$$

and

$$B_s \gg B_D \quad (4.15)$$

4.4. Common Channel Models

In this section, four common channel models are presented; AWGN channel, Rayleigh, Rician and Nakagami-m fading channel.

4.4.1. AWGN Channel

The additive white Gaussian noise (AWGN) channel model is an ideal channel model as it merely adds the white Gaussian noise linearly into the signal. Being the non-fading channel model, the phenomenon of fading is not taken into account for this channel type, where the channel gain is always 1.

4.4.2. Rayleigh Fading Channel

Rayleigh fading channel model is normally used to describe the time-varying nature of the received envelope of a flat fading channel. This model is based on assumptions that there are infinite arrival paths at the same time at all angles, all paths have zero mean and similar variance, which means there is no dominant path and all path gains are statistically independent. By taking central limit theorem, I (In-phase) and Q (Quadrature) components of the Rayleigh fading channel are Gaussian distributed.

In Rayleigh fading channel, the instantaneous SNR follows the probability density function (PDF) given by [45, eq. (4.16)]

$$f_{\gamma}(\gamma) = \begin{cases} \frac{1}{\bar{\gamma}} \exp(-\frac{\gamma}{\bar{\gamma}}), & \gamma \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad (4.16)$$

Where γ and $\bar{\gamma}$ is respectively the instantaneous and average SNR [45].

4.4.3. Rician Fading Channel

Rician fading channel is used to model propagation paths between the transmitter and the receiver when there is a dominant line-of-sight path and many weaker multipath. Channel fading statistics in this case is Rician distributed as follows [46, eq. (4.17)]

$$f_{\gamma}(\gamma) = \begin{cases} \frac{\gamma}{\sigma^2} \exp(-\frac{(\gamma^2 + A^2)}{2\sigma^2}) I_0(\frac{A\gamma}{\sigma^2}), & A \geq 0, \gamma \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad (4.17)$$

Where, A denotes the peak amplitude of the dominant signal and $I_0(\cdot)$ is the modified Bessel function of the first kind and zero-order.

The Rician distribution is also described in terms of a parameter K , which is the ratio between the deterministic signal power and the variance of the multipath [46, eq. (4.18) & (4.19)].

$$K = \frac{A^2}{2\sigma^2} \quad (4.18)$$

$$K(\text{dB}) = 10 \log_{10} \frac{A^2}{2\sigma^2} \text{dB} \quad (4.19)$$

The Rayleigh distribution is a special case for Rician distribution where K approaches 0 or $-\infty$ dB.

4.4.4. Nakagami-m Fading Channel

This fading distribution is assumed in the analysis of many terrestrial wireless communication systems since it is flexible and embraces scattered, reflected and direct components of the original transmitted signal.

In Nakagami-m fading channel, the instantaneous SNR follows the PDF given by [47, eq. (4.20)]

$$f_{\gamma}(\gamma) = \frac{m^m \gamma^{m-1}}{\bar{\gamma}^m \Gamma(m)} \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right), \quad \gamma \geq 0 \quad (4.20)$$

Where, m is the Nakagami-m parameter which describes the severity of fading; $m < 1$ suggests severe fading, while $m > 1$ indicates less severe fading [47]. Rayleigh fading channel is a special case for Nakagami-m fading channel where m equals to 1. Also, in the limit where $m \rightarrow +\infty$, Nakagami-m fading channel converges to an AWGN channel [48].

4.5. Statistical Distributions and Random Variables

A random variable (RV) is a variable whose value depends on the randomness and it is statistically modeled by the distribution or the PDF it follows. In this section, commonly used statistical distributions in wireless communication are presented with its PDF and cumulative density function (CDF).

4.5.1. Gaussian Distribution

If a RV follows a Gaussian or Normal distribution, its PDF is defined by [49, eq. (4.21)]

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(y-\mu)^2/2\sigma^2}, \quad -\infty < y < \infty \quad (4.21)$$

Where, μ and σ^2 are the mean and variance [49]. The notation $y \sim N(\mu, \sigma^2)$ means that a random variable y is Gaussian-distributed, with mean μ and variance σ^2 . Its CDF is defined as [49, eq. (4.22)]

$$\begin{aligned} F(y) &= P(Y \leq y) \\ &= \int_{-\infty}^y f(y)dy \\ &= \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{x-\mu}{\sqrt{2}\sigma} \right) \right] \end{aligned} \quad (4.22)$$

4.5.2. Chi-squared Distribution

A Chi-Squared or a central Chi-Squared distribution is defined as a summation of squares of independent standard normal RVs, Y with zero mean and unit variance. A RV Z is [49, eq. (4.23)]

$$Z = \sum_{i=1}^k Y_i^2 \quad (4.23)$$

and it is distributed according to the Chi-Squared distribution with k degrees of freedom. It is also denoted as $z \sim X_k^2$. The PDF for the Chi-Squared distribution is [49, eq. (4.24)]

$$f(y) = \begin{cases} \frac{1}{2^{k/2} \Gamma(k/2)} y^{k/2-1} e^{-y/2}, & 0 \leq y < \infty \\ 0, & \text{otherwise} \end{cases} \quad (4.24)$$

Its CDF is defined as [49, eq. (4.25)]

$$\begin{aligned} F(y) &= P(Y \leq y) \\ &= \int_0^y f(y)dy \\ &= \frac{1}{\Gamma(k/2)} \gamma \left(\frac{k}{2}, \frac{y}{2} \right), \quad 0 \leq y < \infty \end{aligned} \quad (4.25)$$

and its CCDF is defined as [49, eq. (4.26)]

$$\begin{aligned} CCDF(y) &= P(Y > y) \\ &= 1 - F(y) \\ &= \frac{1}{\Gamma(k/2)} \Gamma \left(\frac{k}{2}, \frac{y}{2} \right), \quad 0 \leq y < \infty \end{aligned} \quad (4.26)$$

4.5.3. Non-central Chi-squared Distribution

A Non-Central Chi-Squared Distribution is defined as a summation of squares of independent Normal RVs, Y with mean μ and variance σ^2 . A RV Z is [49, eq. (4.27)]

$$Z = \sum_{i=1}^k \left(\frac{y_i}{\sigma_i}\right)^2 \quad (4.27)$$

and it is distributed according to the Non-central Chi-Squared distribution with k degrees of freedom and the non-centrality parameter ψ_{nc} , which is defined as [49, eq. (4.28)]

$$\psi_{nc} = \sum_{i=1}^k \left(\frac{\mu_i}{\sigma_i}\right)^2 \quad (4.28)$$

The PDF for Non-Central Chi-Squared distribution is [49, eq. (4.29)]

$$f(x) = \begin{cases} \frac{1}{2} e^{-(y+\psi_{nc})/2} \left(\frac{y}{\psi_{nc}}\right)^{k/4-1/2} I_{k/2-1}(\sqrt{\psi_{nc}y}) & , 0 \leq y < \infty \\ 0 & , \text{otherwise} \end{cases} \quad (4.29)$$

and its CDF is defined as [49, eq. (4.30)]

$$F(y) = 1 - Q_{\frac{k}{2}}(\sqrt{\psi_{nc}}, \sqrt{y}) \quad , 0 \leq y < \infty \quad (4.30)$$

Where, $Q_M(a, b)$ is Marcum-Q function.

4.5.4. Summation of Two Random Variables

Let X and Y be independent RVs with the PDF of $f_X(x)$ and $f_Y(y)$. The summation of these two independent RVs are $Z = X + Y$. The PDF of Z is given by [50, eq. (4.31)]

$$\begin{aligned} f_Z(z) &= f_X(x) * f_Y(y) \\ &= \int_{-\infty}^{\infty} f_X(x) \cdot f_Y(z-x) dx \end{aligned} \quad (4.31)$$

In other words, $f_Z(z)$ is the convolution of $f_X(x)$ and $f_Y(y)$ [50]. For instance, let Z be the summation of two independent Gaussian distributed X and Y with μ_x , σ_x^2 , μ_y and σ_y^2 . The PDF of X can be written according to (4.21) as [50, eq. (4.32)]

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-(x-\mu_x)^2/2\sigma_x^2} \quad (4.32)$$

and similarly the PDF of Y is [50, eq. (4.33)]

$$f_Y(y) = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_y^2}} e^{-(y-\mu_y)^2/2\sigma_y^2} \quad (4.33)$$

The PDF of Z, which is $Z = X + Y$, becomes [50, eq. (4.34)]

$$\begin{aligned} f_Z(z) &= f_X(x) * f_Y(y) \\ &= \int_{-\infty}^{\infty} f_X(x) \cdot f_Y(z-x) dx \\ &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sqrt{\sigma_x^2}} e^{-(x-\mu_x)^2/2\sigma_x^2} \cdot \frac{1}{\sqrt{2\pi}\sqrt{\sigma_y^2}} e^{-(z-x-\mu_y)^2/2\sigma_y^2} dx \end{aligned} \quad (4.34)$$

After some derivations, we have [50, eq. (4.35)]

$$f_Z(z) = \frac{1}{\sqrt{2\pi(\sigma_x^2 + \sigma_y^2)}} \exp\left[\frac{-(z - (\mu_x + \mu_y))^2}{2(\sigma_x^2 + \sigma_y^2)}\right] \quad (4.35)$$

This derivation shows that if X and Y are independent Gaussian distributed RVs such that [50, eq. (4.36) & (4.37)]

$$X \sim \mathcal{N}(\mu_x, \sigma_x^2) \quad (4.36)$$

$$Y \sim \mathcal{N}(\mu_y, \sigma_y^2) \quad (4.37)$$

then Z which is a summation of X and Y , $Z = X + Y$ is distributed as [50, eq. (4.38)]

$$Z \sim \mathcal{N}(\mu_x + \mu_y, \sigma_x^2 + \sigma_y^2) \quad (4.38)$$

4.5.4.1. Central Limit Theorem

The Central Limit Theorem is defined that a sum of n independent, identically distributed (IID) RVs can be approximated as a normal distribution when there is a large number of RVs and the contribution for each of them is small compared to the total.

For instance, let X_1, X_2, \dots, X_n be IID. RVs with mean μ and variance σ^2 and let $S_n = Y_1 + Y_2 + \dots + Y_n$, then

$$\lim_{n \rightarrow \infty} P\left(\frac{S_n - n\mu}{\sqrt{n}\sigma} \leq y\right) = F(y) \quad (4.39)$$

Where $F(\cdot)$ is the CDF for Gaussian distribution as in (4.21) [27, eq. (4.39)].

Chapter 5: Performance Analysis and Detection Threshold

5.1. Introduction

The accuracy of the spectral availability information is defined using sensing quality parameters. This feature make up the performance metrics. Sensing the performance of the energy detector is specified by the following general metrics:

- The probability of detection, (P_D)
- The probability of false alarm , (P_{FA})
- The probability of missed detection, (P_M)

In opportunistic spectrum sensing the probability of detection specifies that a detector makes a correct decision that a channel is occupied (H_1). The (P_D) is an indicator of the level of interference protection provided to the primary user. Hence, a large P_D denotes exact sensing; which translate to small chance of interference.

A false alarm event occurs when the detector assumes H_1 ; when the right decision is H_0 . The probability of this occurrence is specified as a probability of false alarm. When a false alarm event occurs the SU would not exploit the free spectrum thus missing a chance to utilize the free channel. P_{FA} should be kept as small as possible in order to prevent underutilization of transmission opportunities. The performance of the spectrum sensing technique is usually influenced by the probability of false alarm, since this is the most influential metric [51]. The probability of declaring the spectrum space vacant H_0 , when it is indeed occupied H_1 is referred to as the probability of missed detection (P_M). A high P_M implies an increase in the chance of interference between the PU and the SU. If the detection fails or miss detection occurs, the SU initiates a transmission which resulting in interference with the PU signal; contravening the opportunistic access concept. In essence, the spectrum sensing method should record a high probability of detection (low miss detection probability) and low probability of false alarm.

5.2. Performance Measurement

The receiver performance is quantified by depicting curves of probability of detection against SNR values (P_D versus SNR) and the receiver operating characteristics (ROC) curves. These curves serve as a tool to select and study the performance of a sensing scheme. P_D versus SNR graphs are

employed to show trade-offs between detection probability and SNR thus allowing the determination of an optimal scheme. Complementary ROC curves depict plots of probability of miss-detection ($P_M = 1 - P_D$) versus the probability of false-alarm (P_{FA}). These curves enable exploration of the relationship between sensitivity (probability of detection) and specificity (false alarm rate) [10]. To plot ROC curves, one parameter is varied while the other is fixed. This enables the study of various scenarios of interest.

5.3. Detection Performance over Fading Channels

Since signals take more than a path between a transmitter and receiver, they are generally modeled by fading distributions that account for uncertainties encountered in the channel. Among these are Rayleigh and Nakagami fading models. These channel models serve as tools for studying both multipath and path loss features of a typical environment where spectrum sensing is to be employed.

For Rayleigh fading, the signal is not received on a line-of-sight path; directly from the transmitting antenna [52]. This fading model considers urban multipath features, including effects of the ionosphere and troposphere. More so, it describes the statistical time varying nature of the received envelope of a flat fading signal or the envelope of an individual multipath component [42]. When this model is employed then attenuation of the signal is Rayleigh distributed making the SNR at every node exponentially distributed [53].

The Nakagami fading distribution is a convenient model for analyzing the performance of digital communication systems over generalized fading channels. This fading distribution is assumed in the analysis of many terrestrial wireless communication systems since it is flexible, embraces scattered, reflected and direct components of the original transmitted signal. For urban multipath environments the Nakagami-m fading model has been shown to be very suitable [53].

5.4. Probability of Detection and Threshold for Energy Detection

The noise $n(k)$ is considered a bandpass process consisting of two (2) components; the in-phase noise component $n_i(k)$ and quadrature phase component $n_q(k)$ whose sample function is written as [54, eq. (5.1)];

$$n(k) = n_i(k) \cos n_c t - n_q(k) \sin n_c k \quad (5.1)$$

Where n_c is the angular frequency. If $n(k)$ is restricted to bandwidth B_w , with power spectral density N_0 , then $n_i(k)$ and $n_q(k)$ are considered to be two low pass processes with bandwidth less than $B_w/2$. The power spectral density of each is equal to $2N_0$. When a sample function has bandwidth B and duration T then it is described approximately by a set of values $2B_wT$ or its degree of freedom is equal to $2B_wT$. Therefore, $n_i(k)$ and $n_q(k)$ each possess degrees of freedom $d=2B_wT$. Applying the approximation in [55, 56, eq. (5.2)], that;

$$\int_0^T n^2(k) dk = \frac{1}{2} \int_0^T [n_i^2(k) + n_q^2(k)] dk \quad (5.2)$$

Also, the integration $\int_{-\infty}^{\infty} n_i^2(k) dt$ over the interval 0 to T can be written as; [55, 56, eq. (5.3) & (5.4)]

$$\int_0^T n_i^2(k) dk = \frac{1}{B_w} \sum_{j=1}^{B_wT} c_{ij}^2 \quad (5.3)$$

Likewise,

$$\int_0^T n_q^2(k) dt = \frac{1}{B_w} \sum_{j=1}^{B_wT} c_{qj}^2 \quad (5.4)$$

$$\int_0^T n^2(k) dt = [\sum_{j=1}^{B_wT} d_{ij}^2 + \sum_{j=1}^{B_wT} d_{qj}^2] \cdot N_0 \quad (5.5)$$

In the same vein, considering the transmitted signal $S(k)$, as a band-pass process

We have that[55, 56, eq. (5.6) & (5.7)],:

$$\int_0^T s^2(k) dt = [\sum_{j=1}^{B_wT} b_{ij}^2 + \sum_{j=1}^{B_wT} b_{qj}^2] \cdot N_0 \quad (5.6)$$

Or,

$$\sum_{j=1}^{B_wT} (b_{ij}^2 + b_{qj}^2) = \frac{E_s}{N_0} \quad (5.7)$$

Where $E_s = \int_0^T s^2(k) dt$ is the energy of the signal the output of this filter is then squared and integrated over a time interval T to yield a measure of the energy of the received waveform. The output of the integrator denoted Y is the test statistic (testing the hypotheses H_0 and H_1) [55, 56, eq. (5.8)];

$$Y = \frac{1}{N_0} \int_0^T x^2(k) dk \quad (5.8)$$

Under Hypothesis H_0 (with the primary signal absent) the received signal is only noise, i.e. $x(k)=n(k)$. Applying (5.5), the test statistic Y is written as;

$$Y = \sum_{j=1}^{B_w T} (d_{ij}^2 + d_{qj}^2) \quad (5.9)$$

The test statistic under H_0 is said to be chi-square distributed with $2B_w T$ degrees of freedom, i.e. $Y \sim \chi_{2d}^2$ [55]. The chi-squared distribution is used to test for significant difference between the expected and observed result under the null hypothesis.

Under hypothesis H_1 the received signal is a sum of the signal and noise, i.e. $x(k)=s(k)+n(k)$. Therefore, using equations (5.2)-(5.6) we get;

$$\int_0^T x(k) dk = \left[\sum_{j=1}^{B_w T} (d_{ij} + b_{ij})^2 + \sum_{j=1}^{B_w T} (d_{qj} + b_{qj})^2 \right] \cdot N_0 \quad (5.10)$$

Applying the same approach as above i.e. using equation (5.8) and (5.10), the test statistic is written as;

$$Y = \left[\sum_{j=1}^{B_w T} (d_{ij} + b_{ij})^2 + \sum_{j=1}^{B_w T} (d_{qj} + b_{qj})^2 \right] \quad (5.11)$$

The test or decision statistic (output of the detector) under the case of H_1 is said to have a non-central chi-square distribution with $2B_w T$ degrees of freedom. Non-central chi-squared distribution offers a statistical test the chance to estimate departures from the null hypothesis. This presents an admissible hypothesis alternative to H_0 .

The non-centrality parameter ψ , is giving as $\frac{E_s}{N_0}$ [56]. Defining the SNR γ , in terms of the non-centrality parameter, gives [55, eq. (5.12)];

$$\gamma = \frac{E_s}{N} = \frac{E_s}{2N_0} = \frac{\psi}{2} \quad (5.12)$$

Which is $\psi = 2\gamma$.

Therefore, the decision statistic for the hypothesis H_1 (i.e. when the primary signal is present) is $Y \sim \chi_{2d}^2(\psi)$; also $Y \sim \chi_{2d}^2(2\gamma)$.

Following the notations so far the decision statistic for the energy of signal is [55, eq. (5.13)];

$$Y \sim \begin{cases} x_{2d}^2, & H_0 \\ x_{2d}^2(2\gamma), & H_1 \end{cases} \quad (5.13)$$

The probability density function (PDF) for a chi-squared distribution; for this case Y is (from [55, eq. 5.14]);

$$f_Y(y) = \begin{cases} \frac{1}{2^d \Gamma(d)} y^{d-1} e^{-\frac{y}{2}}, & H_0 \\ \frac{1}{2} \left(\frac{y}{\psi}\right)^{\frac{d-1}{2}} e^{-\frac{\psi+y}{2}} I_{d-1}(\sqrt{\psi y}), & H_1 \end{cases} \quad (5.14)$$

Where $\Gamma(\cdot)$ is the gamma function and $I_\nu(\cdot)$ is the ν^{th} -order modified Bessel function of the first kind.

The probability of detection is the probability that H_1 is selected when a signal is present. Probability of detection, P_D and false alarm P_{FA} for a given threshold (η), are represented respectively by [55, eq. (5.15) & (5.16)];

$$P_D = P(Y > \eta | H_1) \quad (5.15)$$

$$P_{FA} = P(Y > \eta | H_0) \quad (5.16)$$

Where, η is the decision threshold expressing the P_D and P_{FA} in terms of the probability density function yields [55, eq. (5.17)];

$$P_{FA} = \int_{\eta}^{\infty} f_Y(y) dy \quad (5.17)$$

Applying (5.14);

$$P_{FA} = \frac{1}{2^d \Gamma(d)} \int_{\eta}^{\infty} \left(\frac{y}{2}\right)^{d-1} e^{-\frac{y}{2}} dy \quad (5.18)$$

Also, substituting $\frac{y}{2} = k$, $\frac{dy}{2} = dk$ and changing the limits of (5.18);

$$P_{FA} = \frac{1}{\Gamma(d)} \int_{\frac{\eta}{2}}^{\infty} (k)^{d-1} e^{-k} dk \quad (5.19)$$

$$P_{FA} = \frac{\Gamma(d, \frac{\eta}{2})}{\Gamma(d)} \quad (5.20)$$

Where, $\Gamma(d, x)$ is the incomplete gamma function, defined by $\Gamma(d, x) = \int_x^\infty (k)^{d-1} e^{-(k)} dk$ [57]. Since the signal power is unknown, the false alarm probability P_{FA} is set to a constant; applying (5.20), the detection threshold Y can be determined. The value of η in real communication systems is influenced by the system requirements. Research efforts like [4] choose a threshold η in a way the P_{FA} is bounded by a target value. From (5.20), P_{FA} depends on two parameters: time-bandwidth product d and the threshold η . Thus the value of η is not related to SNR (γ).

Typically, P_{FA} is given a value between 10^{-1} - 10^{-2} . The IEEE 802.22 standard recommends $P_{FA} < 0.1$ for spectrum sensing [58]. Time-bandwidth product ($d=B_w T$) is between the range 1-25 [56]. P_{FA} easily computed using (5.20) for a given d .

From (5.14), the probability of detection is obtained by the cumulative distribution function (CDF);

$$P_D = 1 - F_Y(y) \quad (5.21)$$

The CDF of Y is obtained (for an even number of degrees of freedom $-2d$ in this case) as;

$$F_Y(y) = 1 - Q_d(\sqrt{\psi}, \sqrt{y}) \quad (5.22)$$

This, from (5.22), the probability of detection, P_D for an AWGN channel is;

$$P_D = Q_d(\sqrt{\psi}, \sqrt{\eta}) \quad (5.23)$$

Equivalent to;

$$P_D = Q_d(\sqrt{2\gamma}, \sqrt{\eta}) \quad (5.24)$$

Where, $Q_d(.,.)$ is the generalized Marcum-Q function. Using eqns. (5.20) and (5.24); which are expressions for the P_{FA} and P_D respectively.

By averaging the conditional PD in the AWGN case as given in (5.24) over the SNR fading distribution we can have closed form expression for the PD in Rayleigh fading channels is expressed [55]. If the signal amplitude follows a Rayleigh distribution, the SNR follows an exponential PDF [55, eq. (5.25)];

$$f_R(\gamma) = \frac{1}{\gamma} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \quad \gamma \geq 0 \quad (5.25)$$

To obtain the Probability of Detection for Rayleigh channels, (5.24) is averaged over (5.25) i.e.

$$P_{D_{Ray}} = \int_D^\infty P_D f_R(\gamma) d\gamma \quad (5.26)$$

The probability of detection over Nakagami channel is determined by averaging the detection probability for a given SNR over the Nakagami distribution. If the signal amplitude follows a Nakagami distribution then PDF of SNR follows a gamma PDF given by [55, eq. (27)];

$$f_N(\gamma) = \frac{1}{\Gamma(m)} \left(\frac{m}{\bar{\gamma}}\right)^m \gamma^{m-1} \exp\left(-\frac{m}{\bar{\gamma}}\gamma\right), \quad \gamma \geq 0 \quad (5.27)$$

The average P_D in the case of Nakagami channels is obtained by averaging (5.27) over (5.24).

$$P_{D_{Nak}} = \int_D^\infty P_D(\gamma) f_N(\gamma) d\gamma \quad (5.28)$$

5.5. Probability of Detection and Threshold for Eigenvalue Based Detection

RMT has proved to give solution for two major problems in wireless communication. One is to provide mathematical tools to estimate and characterize wireless communications channels to efficiently use radio spectrum and power of transmitters and secondly it will give methods to improve channel capacity for fading, wideband, multiuser and multi-antenna channel features of wireless communications [13].

Recent year studies of RMT stretched to meet the demands of many areas of science. The development of RMT in relation to wireless communication found its place in detecting weak signals. The benefit obtained from RMT in signal detection is to improve existing channel and noise uncertainties. RMT takes a closer look at the Eigenvalues distribution large square matrices. It enables to characterize the existence of a signal by analyzing the distributions. The end results of RMTs are also used to set threshold values to draw decision for signal detection. Some important Random Matrices used in wireless communication include Gaussian, Wigner, Wishart and Haar Matrices [59, 13].

Assuming that, the frequency band of interest has got a central frequency f_c and bandwidth W . During a particular time interval the frequency band may be occupied by only one primary user

and several secondary users are randomly distributed in the cognitive radio network. Each secondary user is equipped with a single antenna. A non-cooperative spectrum sensing scheme is considered where the sensing work is completed by only one secondary user (only one transmitting source, only one receiver).

For signal detection two hypotheses: (1) hypothesis H_0 : there exists no signal (only noise); (2) hypothesis H_1 : there exists both the signal and additive white noise. The binary hypothesis test can be replaced by [59,13, eq. (5.29) & (5.30)]:

$$H_0: x(k) = n(k), \quad k = 0, 1, \dots \quad (5.29)$$

$$H_1: x(k) = \sum_{\tau=0}^N h(\tau)s(k - \tau) + n(k) \quad (5.30)$$

Where $x(k)$ denotes the discrete signal at the secondary receiver, $s(k)$ is the primary signal seen at the receiver, $h(k)$ is the channel response, N is the order of the channel and $n(k)$ are the noise samples.

Considering a sub-sample L of consecutive outputs and defining [59,13, eq. (5.31)-(5.35)]:

$$\hat{x}(k) = [x(k), x(k - 1), \dots, x(k - L + 1)]^T \quad (5.31)$$

$$\hat{n}(k) = [n(k), n(k - 1), \dots, n(k - L + 1)]^T \quad (5.32)$$

$$\hat{s}(k) = [s(k), s(k - 1), \dots, s(k - N_1 - L + 1)]^T \quad (5.33)$$

Yielding

$$\hat{x}(k) = H\hat{s}(k)\hat{n}(k) \quad (5.34)$$

Where H is $L \times (N+L)$ matrix, defined as

$$H = \begin{bmatrix} h(0) & \dots & h(N) & \dots & 0 \\ & \ddots & & \ddots & \\ 0 & \dots & h(0) & \dots & h(N) \end{bmatrix} \quad (5.35)$$

Considering the statistical properties of the transmitted signal and channel noise, let us assume that the noise is white and that the noise and the transmitted signal are correlated.

Let \mathbf{R} be the covariance matrix of the received signal, that is [59,13, eq. (5.36)],

$$R = \frac{1}{N_s} \sum_{k=M}^{M-1+N_s} \hat{x}(k) \hat{x}^H(k) \quad (5.36)$$

Where, N_s is the number of collected samples. If N_s is large based on the assumptions made earlier, we can verify that [59,13, eq. (5.37)]

$$R \approx \mathbb{E}[\hat{x}(k) \hat{x}^H(k)] = HR_s H^H + \sigma_w^2 I_L \quad (5.37)$$

Where, R_s is the statistical covariance matrix of the input signal; $R_s = \mathbb{E}[\hat{s}(k) \hat{s}^H(k)]$, σ_w^2 is the variance of the noise and I_L denotes an $L \times L$ identity matrix. Let $\hat{\lambda}_{\max}$ and $\hat{\lambda}_{\min}$ be the maximum and the minimum eigenvalues of R , and $\hat{\rho}_{\max}$ and $\hat{\rho}_{\min}$ be the maximum and minimum eigenvalues of $HR_s H^H$. Then [59,13, eq. (5.38)]

$$\hat{\lambda}_{\max} = \hat{\rho}_{\max} + \sigma_w^2 \quad \text{and} \quad \hat{\lambda}_{\min} = \hat{\rho}_{\min} + \sigma_w^2 \quad (5.38)$$

Obviously, $\hat{\rho}_{\max} = \hat{\rho}_{\min}$ if and only if $HR_s H^H = \delta I_L$ where, δ is a positive number. Again obviously, when the primary signal is absent, $\hat{\lambda}_{\max} = \hat{\lambda}_{\min} = \sigma_w^2$. Hence, if there is no signal $\hat{\lambda}_{\max}/\hat{\lambda}_{\min} = 1$; otherwise $\hat{\lambda}_{\max}/\hat{\lambda}_{\min} > 1$. The ration of $\hat{\lambda}_{\max}/\hat{\lambda}_{\min}$ can be used to detect the presence of the primary signal. However, $\hat{\lambda}_{\max}$ and $\hat{\lambda}_{\min}$ are the estimated eigenvalues. The real maximum eigenvalue $\hat{\lambda}_{\max}$ and minimum eigenvalue $\hat{\lambda}_{\min}$ of R_s will be obtained based on RMT.

5.5.1. Threshold Calculations

The threshold calculation for eigenvalue based detection algorithms are based on RMT. The detection algorithms are based on probability of false alarm as we have no information on the existence of signal or not. Hence, RMT theories will give up a relation between the false alarm probability and threshold. Obviously we can say that if we decrease the missed detection probability we have better chance to detect the signal i.e. probability of detection increases.

In the general model of the spectrum sensing algorithm a threshold must be determined to compare with a test statistic of the sensing metric in order to sense the presence of a primary user. Consequently, to find the threshold for the statistical test it is important to study the statistical distribution of the covariance matrix.

The eigenvalue distribution of \mathbf{R} is very complicated [60]. Moreover, there is little or no information about the signal. In fact, we don't even know whether the signal is present or not. This in turn makes the choice of the thresholds very difficult.

Therefore, in this subsection, we shall use random matrix theory to approximate the distribution of this random variable and derive the decision threshold based on the pre-defined probability of false alarm, P_{FA} .

When the primary signal is absent, \mathbf{R} turns to, the covariance matrix of the noise defined as

$$R_n = \frac{1}{N_s} \sum_{n=L}^{L-1+N_s} \hat{\mathbf{n}}(k) \hat{\mathbf{n}}^H(k) \quad (5.39)$$

\mathbf{R}_n is nearly a Wishart random matrix [60, eq. (5.39)].

The joint probability density function (PDF) of ordered eigenvalues of a Wishart random matrix has been known for many years [60]. However, since the expression for the PDF is very complicated and no closed form expression has been found for the marginal PDF of ordered eigenvalues.

Recently though, researchers have found the distribution of the largest eigenvalue [61] and smallest eigenvalue [62] for real and complex matrices as described in the following theorems:

Theorem 1: Assume real noise and defining the quantities [61, 62 eq. (5.40)-(5.43)].

$$Q = \frac{N_s}{\sigma_w^2} \quad (5.40)$$

$$\mu = (\sqrt{N_s - 1} + \sqrt{ML})^2 = (\sqrt{K} + \sqrt{P})^2 \quad (5.41)$$

And

$$v = (\sqrt{N_s - 1} + \sqrt{ML}) \left(\frac{1}{\sqrt{N_s - 1}} + \frac{1}{\sqrt{ML}} \right)^{\frac{1}{3}} = (\sqrt{K} + \sqrt{P}) \left(\frac{1}{\sqrt{K}} + \frac{1}{\sqrt{P}} \right)^{\frac{1}{3}} \quad (5.42)$$

Where

$$K = N_s - 1 \text{ and } P = ML \quad (5.43)$$

Where, N_s is the number of collected samples L is the length of the covariance matrix and M is the number of receivers that is one.

Now assume that $\lim_{K \rightarrow \infty} \frac{P}{K} = a$ ($0 < a < 1$). Then the quantity $\frac{\lambda_{\max}(Q) - \mu}{v}$ converges with probability one to the Tracy-Widom distribution of the first order [63].

Theorem 2: Assume that the noise is complex [63, eq. (5.44)-(5.51)]

$$Q = \frac{N_s}{\sigma_w^2} R_n \quad (5.44)$$

$$\mu' = (\sqrt{N_s - 1} + \sqrt{ML})^2 = (\sqrt{K} + \sqrt{P})^2 \quad (5.45)$$

$$v' = (\sqrt{N_s - 1} + \sqrt{ML}) \left(\frac{1}{\sqrt{N_s - 1}} + \frac{1}{\sqrt{ML}} \right)^{\frac{1}{3}} = (\sqrt{K} + \sqrt{P}) \left(\frac{1}{\sqrt{K}} + \frac{1}{\sqrt{P}} \right)^{\frac{1}{3}} \quad (5.46)$$

Assuming $\lim_{K \rightarrow \infty} \frac{P}{K} = a$ ($0 < a < 1$), then the quantity $\frac{\lambda_{\max}(Q) - \mu'}{v'}$ converges with probability one to the Tracy-widom distribution of the second order.

Assume that

$$\lim_{K \rightarrow \infty} \frac{P}{K} = a \quad (0 < a < 1) \quad (5.48)$$

Then

$$\lim_{K \rightarrow \infty} \lambda_{\min} = \sigma_w^2 (1 - \sqrt{a})^2 \quad (5.49)$$

Based on the theorems, we have the following results

$$\lambda_{\max} \approx \frac{\sigma_w^2}{K} (\sqrt{K} + \sqrt{P})^2 \quad (5.50)$$

$$\lambda_{\min} \approx \frac{\sigma_w^2}{K} (\sqrt{K} - \sqrt{P})^2 \quad (5.51)$$

The Tracy-Widom distributions were found by Tracy and Widom as the limiting law of the largest eigenvalue of certain random matrices [63].

Let F_1 and F_2 be the cumulative distribution function (CDF) of Tracy-Widom of the first and the second order respectively. There is no closed form expression for the distribution functions. It is therefore generally difficult to evaluate them. Fortunately, there are tables for the functions base on numerical computation [61].

Using the theories, from the derivation the decision threshold for the optimal detection method. Let γ represent the decision threshold, then the probability of false alarm of the optimal detection is [61, eq. (5.2)]

$$\begin{aligned}
P_{FA} &= P(\lambda_{max} > \gamma\lambda_{min}) \\
&= P\left(\frac{\sigma_w^2}{K} \lambda_{max}(Q) > \gamma\lambda_{min}\right) \\
&\approx P(\lambda_{max}(Q) > \gamma(\sqrt{K} - \sqrt{P})^2) \\
&= P\left(\frac{\lambda_{max}(Q) - \mu}{v} > \frac{\gamma(\sqrt{K} - \sqrt{P})^2 - \mu}{v}\right) \tag{5.52}
\end{aligned}$$

Now from the Energy with minimum eigenvalue (EME) method. When there is no signal, it can be verified that the average energy defined in (3.5).

$Y(N_s)$ is the average of MN_s statistically independent and identically distributed random variables. Since N_s is large, the central limit theorem tells us that $Y(N_s)$ can be approximated by the Gaussian distribution with mean σ_w^2 and variance $\frac{2\sigma_w^4}{MN_s}$. Hence the probability of false alarm is [61, eq. (5.53)- (5.56)]

$$\begin{aligned}
P_{fa} &= P(Y(N_s) > \gamma\lambda_{min}) \\
&\approx P\left(Y(N_s) > \gamma \frac{\sigma_w^2}{N_s} (\sqrt{N_s} - \sqrt{ML})^2\right) \\
&= P\left(\frac{Y(N_s) - \sigma_w^2}{\sqrt{\frac{2}{MN_s} \sigma_w^2}} > \frac{\gamma\sqrt{M}(\sqrt{N_s} - \sqrt{ML})^2 - \sqrt{MN_s}}{\sqrt{2N_s}}\right) \\
&\approx Q\left(\frac{\gamma\sqrt{M}(\sqrt{N_s} - \sqrt{ML})^2 - \sqrt{MN_s}}{\sqrt{2N_s}}\right) \tag{5.53}
\end{aligned}$$

Where

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^{+\infty} e^{-u^2/2} du \quad (5.54)$$

Hence, we should choose the threshold such that

$$\frac{\gamma\sqrt{M}(\sqrt{N_s}-\sqrt{ML})^2-\sqrt{MN_s}}{\sqrt{2N_s}} = Q^{-1}(P_{fa}) \quad (5.55)$$

That is,

$$\gamma = \frac{Q^{-1}(P_{fa})\sqrt{2N_s+\sqrt{MN_s}}}{\sqrt{M}(\sqrt{N_s}-\sqrt{ML})^2} = \left(\sqrt{\frac{2}{MN_s}} Q^{-1}(P_{fa}) + 1 \right) \frac{N_s}{(\sqrt{N_s}-\sqrt{ML})^2} \quad (5.56)$$

Similar to MME, here the threshold is not related to noise power. The threshold can be pre-computed based on N_s , L and P_{fa} .

5.5.2. Probability of Detection

When there is a signal the sample covariance matrix $R_x(N_s)$ is no longer a Wishart matrix and the distributions of its eigenvalues are unknown. To obtain a precisely closed form formula for the P_D we need to approximate [12].

Since N_s is usually very large, we have the approximation [12, eq. (5.57)-(5.59)]

$$R_x(N_s) \approx \mathbb{H}R_S\mathbb{H}^\dagger + R_n(N_s) \quad (5.57)$$

Note that $R_n(N_s)$ approximates to $\sigma_w^2 I_{ML}$. Hence, we have

$$\lambda_{max}(R_x(N_s)) \approx \rho_1 + \frac{\sigma_w^2}{\sqrt{N_s}} (\sqrt{N_s} + \sqrt{ML}) \quad (5.58)$$

$$\lambda_{min}(R_x(N_s)) \approx \rho_{ML} + \frac{\sigma_w^2}{\sqrt{N_s}} (\sqrt{N_s} + \sqrt{ML}) \quad (5.59)$$

Base on Appendix A and (5.59), an approximation for the P_d of EME as [12, eq. (5.60)]

$$\begin{aligned} P_d &= P\left(Y(N_s) > \gamma \lambda_{min}(R_x(N_s))\right) \\ &\approx P\left(\frac{T_r(R_n(N_s))}{ML} > \gamma \left(\rho_{ML} + \frac{\sigma_w^2}{\sqrt{N_s}} (\sqrt{N_s} - \sqrt{ML})\right) - \frac{T_r(\mathbb{H}R_S\mathbb{H}^\dagger)}{ML}\right) \end{aligned}$$

$$= Q \left(\frac{\gamma \left(\rho_{ML} + \frac{\sigma_w^2}{\sqrt{N_s}} (\sqrt{N_s} - \sqrt{ML}) \right) - \frac{\text{Tr}(\mathbb{H}R_S\mathbb{H}^\dagger)}{ML} \sigma_w^2}{\sqrt{\frac{2}{MN_s} \sigma_w^2}} \right) \quad (5.60)$$

From the formula, the P_d is related to the number of samples N_s and the average and minimum eigenvalues of the signal covariance matrix including channel effects.

5.6. Cooperative Spectrum Sensing over Various Fading Channels

In real communication environments the hidden terminal problem, deep fading and shadowing, would deteriorate the signal detection performance of cognitive users. To address this issue multiple cognitive radios can be coordinated to perform spectrum sensing. Several recent works have shown that cooperative spectrum sensing can greatly increase the probability of detection in fading channels [64, 65]. Let N denote the number of users sensing the PU. Each CR user makes its own decision regarding whether the primary user present or not and forwards the binary decision (1 or 0) to fusion center (FC) for data fusion. The PU is located far away from all CRs. All the CR users receive the primary signal with same local mean signal power i.e. all CRs form a cluster with distance between any two CRs negligible compared to the distance from the PU to a CR. For simplicity we have assumed that the noise, fading statistics and average SNR are the same for each CR user. We consider that the channels between CRs and FC are ideal channels (noiseless). Assuming independent decisions, the fusion problem where k out of N CR users are needed for decision can be described by binomial distribution based on Bernoulli trials where each trial represents the decision process of each CR user. With a hard decision counting rule, the fusion center implements an n -out-of- M rule that decides on the signal present hypothesis whenever at least k out of the N CR user decisions indicate . Assuming uncorrelated decisions, the probability of detection at the fusion center [66, eq. (5.61)] is given by

$$P_d = \sum_{l=k}^N \binom{N}{l} P_{d,i}^l (1 - P_{d,i})^{N-l} \quad (5.61)$$

Where, $P_{d,i}$ is the probability of detection for each individual CR user as defined by (5.26),(5.28) and (5.61).

5.6.1. Logical AND-rule

In this rule, if all of the local decisions sent to the decision maker are one, the final decision made by the decision maker is one. The fusion center's decision is calculated by logic AND of the received hard decision statistics. Cooperative detection performance with this fusion rule can be evaluated by setting $k=N$ in (5.61).

$$P_{d,AND} = P_{d,i}^N \quad (5.62)$$

5.6.2. Logical OR - rule

In this rule, if any one of the local decisions sent to the decision maker is a logical one, the final decision made by the decision maker is one. Cooperative detection performance with this fusion rule can be evaluated by setting $k=1$ in (5.61).

$$P_{d,OR} = 1 - (1 - P_{d,i})^N \quad (5.63)$$

5.6.3. Logical MAJORITY- rule

In this rule, if half or more of the local decisions sent to the decision maker are the final decision made by the decision maker is one. Cooperative detection performance with this fusion rule can be evaluated by setting $k \geq N/2$ in (5.61).

$$P_{d,Majority} = \sum_{l=\lfloor N/2 \rfloor}^N \binom{N}{l} P_{d,i}^l (1 - P_{d,i})^{N-l} \quad (5.64)$$

Chapter 6: Simulation Results

6.1. Introduction

In this chapter, we simulate based on the previous chapter concept and interpret the simulation results using the MATLAB2017a for both energy detection and eigenvalue based detection spectrum sensing techniques. On the performance comparison over the Rayleigh and Nakagami-m fading scenarios for a single and multiple (cooperative) nodes compared. On the other hand, simulations in this work are executed using Monte Carlo (MC) method which is a stochastic techniques based on the use of random numbers. The parameters used for simulation are given Table 6.1.

Parameters	Type /value	Remark
Channel	Rayleigh and Nakagami-m fading	In case of Nakagami-m (m=0.5)
Detectors	Energy detection and Eigenvalue based detection	ED and EME algorithms
Performance Metrics	P_D , P_{FA} and P_M	$P_{FA} \leq 0.1$ (based on IEEE 802.22)
Sample Size	1000000	
Number of Transmitters and Receiver	1,1	In case of CSS ([2,5,10]),1
Number of Monte Carlo	10000	
Temporal smoothing factor(L)	8	
Noise	White Gaussian	Assumption
SNR range	[-10,10]dB	$SNR = \frac{E(\ x(k)\ - \ \eta(k)\)^2}{E(\ \eta(k)\ ^2)}$
Cooperative network	Center fusion- AND Rule	
Number of nodes for CSS	[2,5,10]	

Table 6. 1: Simulation parameters

6.2. Detection Performance Comparison over Varying SNR

Fig6.1. shows the Probability of Detection Curves for ED and EME spectrum sensing with single CR users in the presence of Rayleigh and Nakagami $m=0.5$ fading channel. The results are taken for SNR vary form -10dB to 10 dB to imitate low SNR condition. The probability of false alarm is fixed to 0.1 which is the maximum allowed according to the IEEE standard. As shown in the figure6. 1 and illustrated in table 6.1 at a very low SNR over Rayleigh environment. ED cannot detect and in case of Nakagami channel ED has poor detection. However, EME achieve more than 80% detection in both fading channel cases. But the performance of ED increases rapidly while the SNR value increase. It can be seen that a transition from -4dB to 2dB leads to performance improvement of 65% (Rayleigh) and 26% (Nakagami). Perhaps the same improvement for EME scheme allows more than 95% detection. Due to severity difference of fading for Nakagami- $m=0.5$ and Rayleigh channel, the channel capacity loss of Nakagami- $m=0.5$ with respect to SNR also different from Rayleigh channel. For that reason the high-SNR power offset riches up to 3dB. However at low SNR region the Nakagami- m has independent fading for the case of ED. The two curve cross around at 1.75 dB which indicates that Nakagami- m concur Rayleigh fading channel characteristics at that point [67].

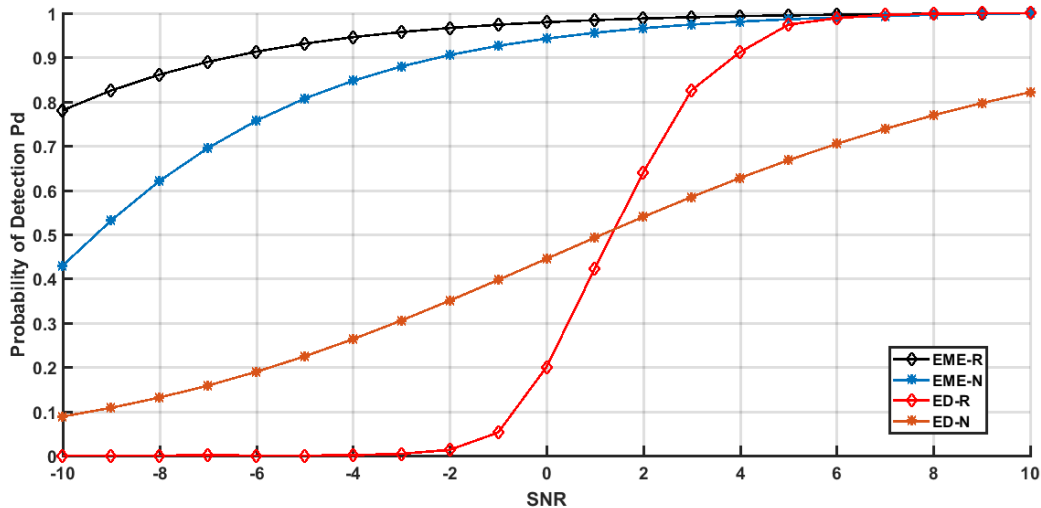


Figure 6. 1: Detection performance comparison over varying SNR over Rayleigh and Nakagami- m fading channel

The simulation result shows us how severe the effect of fading channel on ED and how EME perform better in comparison to ED. We also notice Nakagami fading has much effect on both ED and EME than Rayleigh flat fading.

SNR dB	Pd for ED (Rayleigh)	Pd for EME (Rayleigh)	Pd for ED (Nakagami)	Pd for EME (Nakagami)
(-10 , -4]	0% (no detection)	(80 – 95)%	(10 – 27)%	(43 – 85)%
(-4 , 2]	(0 – 65)%	(95 – 99)%	(28 – 54)%	(85 – 96)%
(2 , 6]	(65 – 98)%	(99 – 100)%	(55 – 70)%	(97 – 100)%
(6 , 10]	(98 – 100)%	100%	(70 – 82)%	100%

Table 6. 2: Detection performance comparison over varying SNR

6.3. Performance Variation between 0.1 and 0.05 Probability of False Alarm over Rayleigh and Nakagami-m Fading Channel

Obtained curves from figure 6.2 & 6.3 presents the performance of the detection probability Pd of ED and EME in terms of the signal noise relation in the receiver. Probability of detection for different false alarm probability of ($P_f = 0.1$ and $P_f = 0.05$) were calculated by using 10000 Monte Carlo sample over Rayleigh and Nakagami fading channels. The curves show us the tradeoff between probability of false alarm and probability of detection which is the increased probability of false alarm raise probability of detection in both fading channels. The simulation result of EME over Rayleigh shows more than 10% variation of detection performance between 0.1 and 0.05 probability of false alarm.

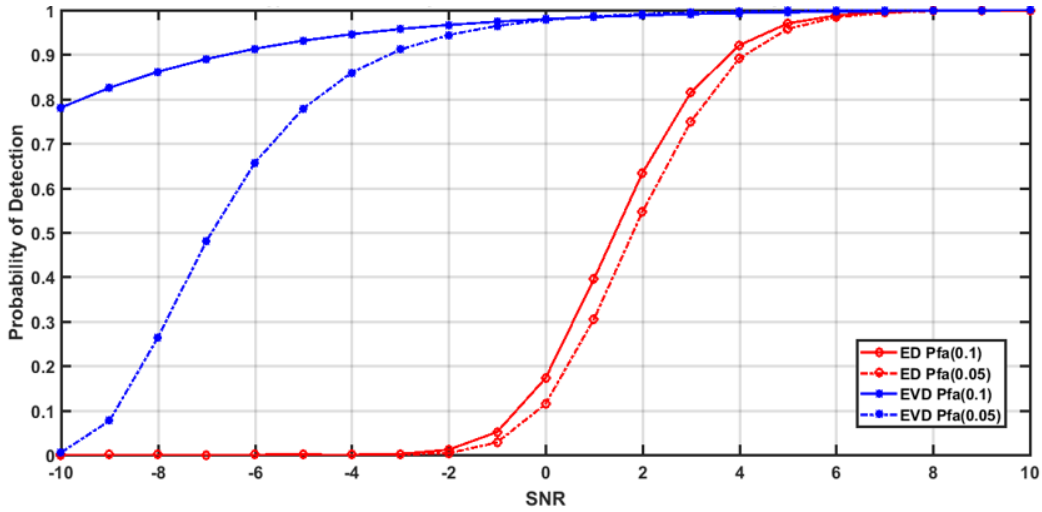


Figure 6. 2: Detection performances between 0.1 and 0.05 probability of false alarm over Rayleigh fading channel.

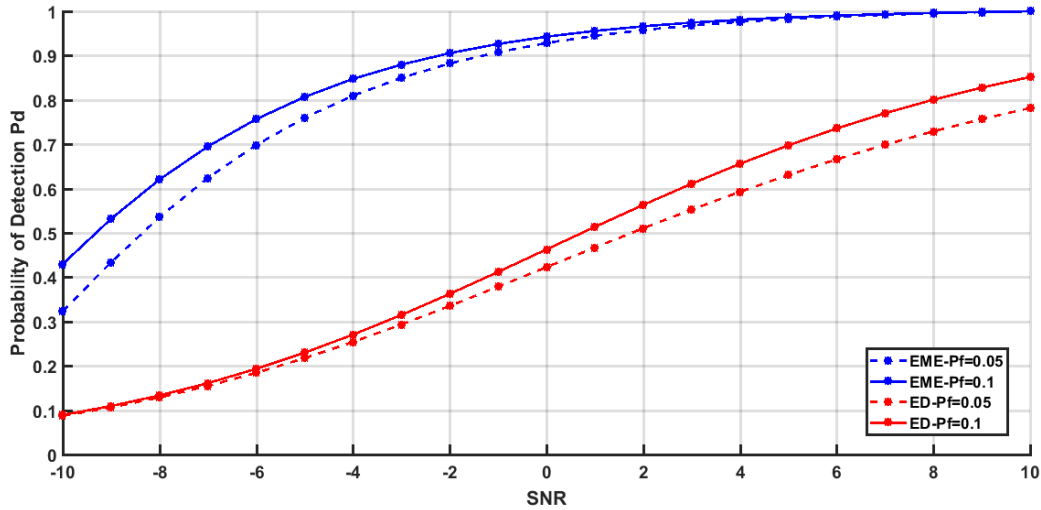


Figure 6. 3: Detection performances between 0.1 and 0.05 probability of false alarm over Nakagami-m fading channel.

6.4. Complementary ROC Curves for Energy Detection and Eigenvalue Based Detection over Rayleigh and Nakagami Fading Channel.

Fig 6.4 and 6.5 describes the detectors through complimentary ROC curves for different values of probability of false alarm and Number of Cognitive Radio. From the figure we can see curve of 2, 5 and 10 cognitive radios cooperative spectrum sensing in Rayleigh and Nakagami-m fading channel with AND rule fusion. The curves show that the increment of the Number of Cognitive

Radio improves the performance of ED and EME method in both Rayleigh and Nakagami- m channels.

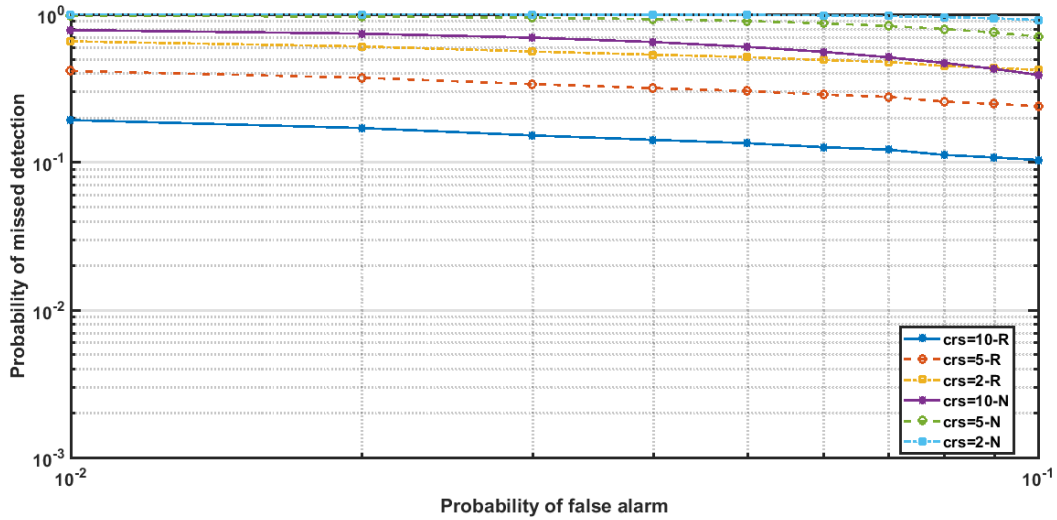


Figure 6. 4: Complementary ROC curves for energy detection over Rayleigh & Nakagami- m fading channel.

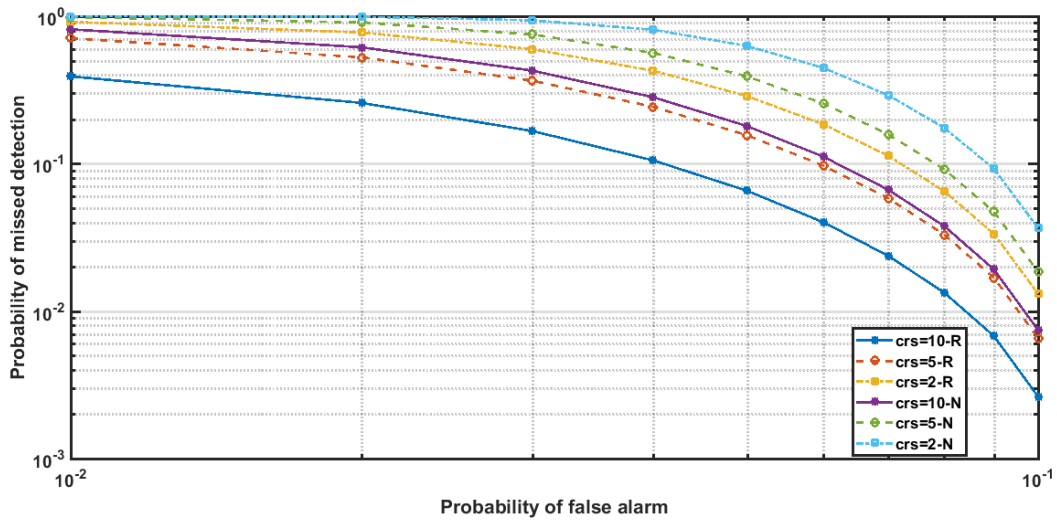


Figure 6. 5: Complementary ROC curves for eigenvalue based detection over Rayleigh & Nakagami- m fading channel.

6.5. Performance Comparisons of ED and EME Schemes Cooperative Spectrum Sensing

Fig 6.6 depict the performance comparisons of ED and EME schemes cooperative spectrum sensing for different number of CRs (5,10) and fading channel (Rayleigh and Nakagami). Performance of EME is much better for higher value of false alarm in relative to ED (for all number of CRs). From the figure we note almost equivalent performance between only 5 numbers of CRs of ED and 10 CRs of EME at PFA=0.01. The performance achieved by the method summarized in table 6.2, it's seen that performance of ED is better to overcome the effect of the fading channels by using less number of network nodes than EME for low PFA values.

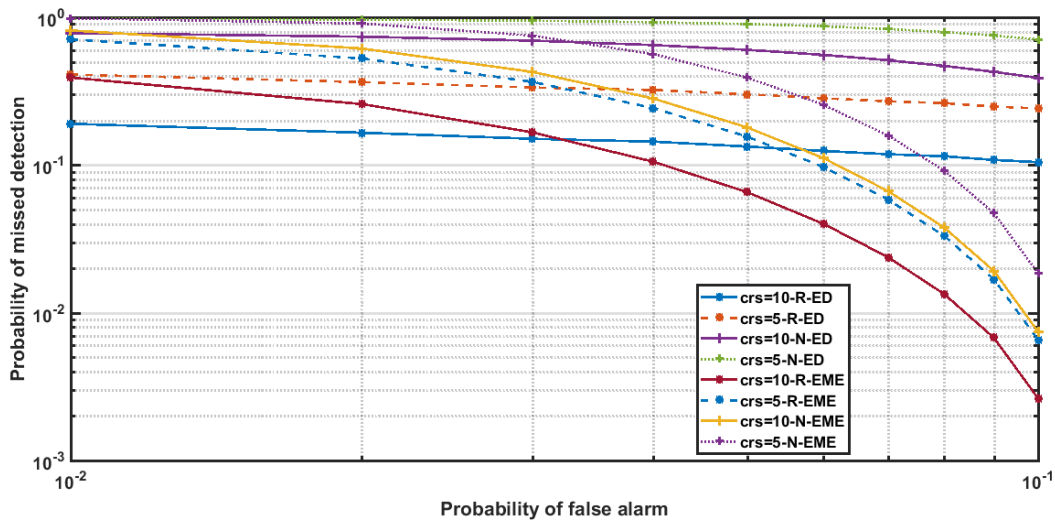


Figure 6. 6: Performance comparisons of ED and EME schemes cooperative spectrum sensing

PFA	PM of EME for 10CRs (Nakagami)	PM of EME for 10CRs (Rayleigh)	PM of ED for 10CRs (Nakagami)	PM of ED for 10CRs (Rayleigh)
0.01	0.84	0.4	0.78	0.19
0.02	0.7	0.27	0.72	0.18
0.03	0.45	0.18	0.68	0.17
0.04	0.27	0.11	0.64	0.16
0.05	0.19	0.065	0.6	0.15
0.06	0.11	0.04	0.55	0.14
0.07	0.065	0.025	0.5	0.13
0.08	0.04	0.014	0.48	0.12
0.09	0.02	0.007	0.42	0.11
0.1	0.008	0.0028	0.4	0.1

Table 6. 3: Performance comparisons of ED and EME schemes cooperative spectrum sensing

Chapter 7: Conclusion and Future Work

In this thesis, the performance comparison between the two energy detection and eigenvalue based blind spectrum detection schemes over fading channel are investigated. Blind spectrum sensing techniques are preferable in a sense that they can detect any type of signal and do not require any knowledge about the signal to be detected. The challenge is that cognitive radio must be able to detect very weak primary signal by resisting the fundamental limitations arise from the fading channel mainly during detections at low SNR.

The simulation results of single cognitive radio case shows that for lower SNR values range between -10dB to -2dB energy detection face sever challenge from the fading environment and cannot detect properly but eigenvalue based detection achieved satisfactory detection performance above 80% detection. For increased value of SNR of $-4\text{dB} < \text{SNR} < 4\text{dB}$ the detection probability of energy detection increased rapidly and achieves more than 65% detection in case of Rayleigh and 26% detection in case of Nakagami- $m=0.5$ environment perhaps the same improvement of SNR allows eigenvalue based detection to reach reliable detection zone having more than 95% detection in both fading environments. Eigenvalue based detection reaches its full detection performance at 2dB (Rayleigh) and 5dB (Nakagami). But we need more than 10dB to achieve full detection performance of ED under Nakagami and it reaches its peaks at 6dB (Rayleigh).

The cooperative spectrum sensing scenario for energy detection and eigenvalue based detection shown through ROC curves by varying values of probability of false alarm. From the results of different number of node, both detection technique's performance increased with increased number of nodes and it is possible to overcome the effect of the fading channels. For instance Energy detection at fixed PFA = 0.01 achieves PM= [0.22, 0.41, 0.65] in presence of Rayleigh and PM= [0.82, 0.92, 0.99] in presence of Nakagami- $m=0.5$ respectively for (10,5 and 2) nodes. Again, Eigenvalue based detection at fixed PFA = 0.01 achieves PM= [0.42, 0.74, 0.81] in presence of Rayleigh and PM= [0.91, 0.97, 0.99] in presence of Nakagami- $m=0.5$ respectively for (10,5 and 2) nodes.

In conclusion, Eigenvalue based detection achieves better resistance to sever fading environment in comparison to Energy Detection for single Cognitive Radio. However, it is important to note

that it costs more complicated signal processing and longer detection time to use Eigenvalue based detection.

Finally, for cooperative spectrum sensing we can conclude that Energy detection is much better than Eigenvalue based detection for lower value of Probability of false alarm. The simulation show that we can use 5 CRS of ED instead of 10 CRS of Eigenvalue Based Detection for fixed value of PFA=0.01 to achieve reliable detection performance. When we bring the cost of EME in to the picture, the difference becomes very significant.

Further Work

In order to find an optimal sensing technique future studies focuses on combining these two schemes in a cascade model and evaluation of the performance over practical medium by implementing a real-time platform using open source toolkit GNU radio.

References

- [1] J. Mitola III, J. Maguire, G. Q. “Cognitive Radio: Making Software Radios more Personal,” *in Proc. IEEE Personal Commun. Mag.*, vol. 6, no. 4, Aug. 1999, pp. 13-18.
- [2] E. Axell, G. Leus, E. G. Larsson and H. V. Poor. “Spectrum sensing for cognitive radio; state-of-the-art and recent advances,” *IEEE Signal Processing Magazine*, 29(3):101, 2012.
- [3] S. D. Tanuja and S. Dina. “Spectrum sensing algorithm for cognitive radio networks for dynamic spectrum access for IEEE 802.11 standard,” *International Journal of Research and Reviews in Wireless Sensor Networks (IJRRWSN)*, 2(1):77, 2012.
- [4] J. Ma, G. Ye Li and B. Hwang. “Signal Processing in Cognitive Radio,” *Proceedings of the IEEE* | Vol. 97, No. 5, May 2009.
- [5] ITU’s Survey on Radio Spectrum Management, [Online]. Available: http://www.itu.int/ITU-D/study_groups/SGP_2002-2006/JGRES09/ETHIOPIA.doc, 2007.
- [6] IEEE P802.22/D0.1 Working Group, “Draft Standard for Wireless Regional Area Networks,” [Online]. Available: <http://grouper.ieee.org/groups/802/22/>, May 2006.
- [7] J. Mitola. “Software radios: Survey, critical evaluation and future directions,” *IEEE Aerospace and Electronic Systems Magazine*, vol. 8, no. 4, 1993, pp. 25–36.
- [8] A. Goldsmith. *Wireless Communications*, Cambridge University Press, USA, 2005, pp. 644.
- [9] K. M. Noga and B. Pałczyńska. “Overview of Fading Channel Modeling,” *International journal of electronics and telecommunications*, vol. 56, no. 4, 2010, pp. 339–344.
- [10] T. Yucek and H. Arslan. “A survey of spectrum sensing algorithms for cognitive radio applications,” *in proc. IEEE communications surveys & tutorials*, vol. 11, no. 1, 2009.
- [11] S. Saini and C. Charan. “A Review on spectrum sensing techniques for cognitive radio networks,” *Krishi Sanskriti Publication*, Kurukshetra, India, Nov. 2014, pp. 152-156.
- [12] Y. Z. Ying-Chang. L. “Eigenvalue based Spectrum Sensing Algorithms for Cognitive Radio” *IEEE Institute for Infocomm Research*, Singapore. Nov. 23, 2009
- [13] A. M. Tulino and S. Verdú. “Random Matrix Theory and Wireless Communications,” Hanover, USA: now Publishers Inc., 2004.
- [14] I. M. Johnstone. “On the distribution of the largest eigenvalue in principle components analysis,” *The Annals of Statistics*, vol. 29, no. 2, 2001, pp. 295–327.

- [15] K. Johansson. "Shape fluctuations and random matrices," *Comm. Math. Phys.*, vol. 209, 2000, pp. 437–476.
- [16] Z. D. Bai. "Methodologies in spectral analysis of large dimensional random matrices, a review," *Statistica Sinica*, vol. 9, 1999, pp. 611–677.
- [17] S. Haykin. "Cognitive radio: brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, Feb. 2005, pp. 201-220.
- [18] I. F. Akyildiz, F. L. Brandon, and R. Balakrishnan. "Cooperative spectrum sensing in cognitive radio networks: A survey," *Physical Communication*, 4:40, 2010.
- [19] I. F. Akyildiz, W. Lee, M. Vuran, and S. Mohanty. "Next generation/dynamic spectrum access/cognitive radio wireless networks: a survey," *Computer Networks*, vol. 50, no. 13, 2006, pp. 2127–2159.
- [20] *IEEE. Standard definitions and concepts for dynamic spectrum access terminology relating to emerging wireless networks, system functionality and spectrum management*, IEEE standard 802.22,2008.
- [21] J. D. Gadze, O. A. Michael and A. N. Damilola. "A performance Atudy of Energy Detection Based Spectrum Sensing for Cognitive Radio Networks", *Int.J. Emerging Technology and Advanced Engineering*, April 2014.
- [22] A. A. Tabassam, M. U. Suleman, S. Kalsait, and S. Khan. "Building cognitive radios in matlab simulink; a step towards future wireless technology in wireless advanced (WiAd)," 2011, 2011, pp 15-20.
- [23] L. N. T. Perera and H. M. V. R. Herath. "Review of spectrum sensing in cognitive radio," *In Industrial and Information Systems (ICIIS), 2011 6th IEEE International Conference on*, 2011, pp 7-12.
- [24] L.S. Cardoso, M. Debbah, S. Lasaulce, K. Mari, and J. Palicot. "Spectrum sensing in cognitive radio networks. Cognitive Radio Networks: Architectures, Protocols and Standards," 2009.
- [25] Cognitive Radio Architecture, [Online]. Available: <http://spmarchitecture.com/cognitive-architecture/cognitive-radio-architecture-84277/>, 2017
- [26] I. F. Akyildiz, W.-Y. Lee, and K. R. Chowdhury. "CRAHNs: Cognitive radio ad hoc networks," *Ad Hoc Networks*, vol. 7, no. 5, 2009, pp. 810–836.

- [27] S. Maharjan, K. Po, and J. Takada. “Energy Detector Prototype for Cognitive Radio System,” Japan, 2016.
- [28] M. A. Matin. “Developments in Wireless Network Prototyping, Design, and Deployment: Future Generations,” 1st Editio. Hershey, PA: IGI Global, 2012.
- [29] Y. Zeng and Y. C. Liang. “Maximum-minimum eigenvalue detection for cognitive radio” *In Proc. IEEE PIMRC*, Athens, Greece, Sep. 2007.
- [30] Y. Zeng and Y. C. Liang. “Covariance based signal detections for cognitive radio” *In Proc. IEEE DYSPAN*, Dublin, Ireland, Apr. 2007, pp. 202–207.
- [31] Y. Zeng et al “Maximum eigenvalue detection: Theory and application,” *In Proc. IEEE ICC*, Beijing, China, May 2008, pp. 4160–4164.
- [32] F. M. Salem et al. “Matched-Filter-based Spectrum Sensing for Secure Cognitive Radio Network Communications,” *Int. J. Comput.* vol. 87, no. 18, 2014, pp. 41–46.
- [33] S. M. Kay. *Fundamentals of Statistical Signal Processing: Detection Theory Vol. 2*, Prentice Hall Signal Processing Series, 1993.
- [34] V. Dandawat'e and G. B. Giannakis. “Statistical tests for presence of cyclostationarity,” *IEEE Transactions on Signal Processing*, vol. 42, no. 9, 1994, pp. 2355–2369.
- [35] A. Al-Dulaimi et al. “Cyclostationary detection of undefined secondary users”, *IEEE NGMAST'09*, 2009, pp. 230-233.
- [36] A. Tkachenko et al. “Cyclostationary Feature Detector Experiments using Reconfigurable BEE2,” *2nd IEEE international Symposium on DySPAN'07*, 2007, pp. 216-219.
- [37] J. T. MacDonald and D. R. Ucci. “Interference temperature limits of IEEE 802.11 protocol radio channels,” *IEEE International Conference on*, 2007, pp 64-69.
- [38] L. N. T. Perera and H. M. V. R. Herath. “Review of spectrum sensing in cognitive radio,” *In Industrial and Information Systems (ICIIS), 6th IEEE International Conference on*, 2011, pages 7-12.
- [39] R. Umar and A.U.H. Sheikh. “A comparative study of spectrum awareness techniques for cognitive radio oriented wireless networks,” *Physical Communication*, pages 1{23, July 2012.
- [40] S. M. Mishra et al. “Sensing among cognitive radios,” *In Communications, 2006. ICC '06. IEEE International Conference on*, vol 4, 2006.

- [41] D. Tse and P. Viswanath. *Fundamentals of wireless communication*, Cambridge university press, 2005.
- [42] T. Rappaport, *Wireless communications: principles and practice*, Publishing House of Electronics Industry, 2004.
- [43] S. Haykin and M. Moher, *Modern Wireless communications*. Pearson Education, Inc. 2005.
- [44] V. Erceg et al. "An empirically based path loss model for wireless channels in suburban environments," *Selected Areas in Communications, IEEE Journal on*, vol. 17, no. 7, 1999, pp. 1205–211.
- [45] M. K. Simon and M.-S. Alouini. *Digital communication over fading channels*, Wiley-science, 2005, vol. 95.
- [46] N. C. Beaulieu and K. T. Hemachandra. "New simple solutions for the bivariate rician pdf and cdf," in *Wireless Communications and Networking Conference (WCNC)*, 2010, pp 1-4.
- [47] D. Horgan and C. C. Murphy. "Fast and accurate approximations for the analysis of energy detection in nakagami-m channels," *IEEE Communications Letters*, 2013, pp 83-86.
- [48] F. F. Digham et al. "On the energy detection of unknown signals over fading channels," *Communications, IEEE Transactions on*, 2007, pp21-24.
- [49] A. C. Rencher and W. F. Christensen. *Methods of multivariate analysis*, Wiley, vol. 709, 2012.
- [50] G. Schay. *Introduction to probability with statistical applications*, Birkh"auser, 2007.
- [51] L.S. Cardoso et al. "Spectrum sensing in cognitive radio networks" 2009.
- [52] M. S. Alouini and A. J. Goldsmith. "Capacity of rayleigh fading channels under different adaptive transmission and diversity combining techniques." *IEEE Transactions on Vehicular Technology*, July 1999.
- [53] D. Horgan and Murphy C. C. "Performance limits of cooperative energy detection in fading environments," *4th International Conference on Cognitive Radio and Advanced Spectrum Management (CogART)*, Barcelona, Spain, 26-29 October 2011.
- [54] W.C.V. Etten. *Introduction to Random signals and Noise*, John Wiley & Sons, 1st edition, 2005.
- [55] F. F. Digham, M. S. Alouini, and M. K. Simon. "On the energy detection of unknown signals over fading channels." *Communications, IEEE Transactions on*, 2007.

- [56] H. Urkowitz. "Energy detection of unknown deterministic signals." *Proceedings of the IEEE*, 1967.
- [57] I. S. Gradshteyn and I. M. Ryzhik. "Table of Integrals, Series, and Products," *Academic Press*, New York, 7th edition, 2007.
- [58] C. Stevenson et al. "IEEE 802.22: The first cognitive radio wireless regional area network standard," *Communications Magazine*, IEEE, 2009.
- [59] Mark Buchanan. *Enter the matrix: the deep law that shapes our reality*, newscientist, April 2010
- [60] Tulino, A. M. and S. Verdú. "Random Matrix Theory and Wireless Communications," *Now Publishers Inc.*, Hanover, MA, USA, 2004.
- [61] Johnstone, I.M. "On the distribution of the largest eigenvalue in principle components analysis," *The Annals of Statistics*, Vol. 29, 2001.
- [62] Bai, Z. D. "Methodologies in spectral analysis of large dimensional random matrices, a review," *Statistica Sinica*, Vol. 9, 1999.
- [63] C. A. Tracy and H. Widom, "On orthogonal and symplectic matrix ensembles," *Communications Mathematics Physics*, vol. 177, 1996, pp. 727-754.
- [64] S. H Lee and Y. H. Lee. "Hard Decision Combining-based Cooperative Spectrum Sensing in Cognitive Radio Systems," 2000.
- [65] G. Ganesan and Y. Li, "Cooperative spectrum sensing in cognitive radio networks," *In Proc. DySPAN*, Nov. 2005, pp. 137-143.
- [66] S. Kyperountas et al. "Performance analysis of cooperative spectrum sensing in Suzuki fading channels," *in Proc. of IEEE International Conference*, June 2008, pp. 428-432.
- [67] J. M. R. Jerez et al. "A new framework for the performance analysis of wireless communications under hoyt (Nakagami-q) fading"

Appendix

It is known that the summation of the eigenvalues of a matrix is the trace of the matrix. Let $\Delta(N_s)$ be the average of the eigenvalue of $R_x(N_s)$. Then

$$\begin{aligned}\Delta(N_s) &= \frac{1}{ML} \text{Tr}(R_x(N_s)) \\ &= \frac{1}{MLN_s} \sum_{n=L-1}^{L-2+N_s} \hat{X}^\dagger(n) \hat{X}(n) \\ \Delta(N_s) &\approx \frac{1}{MN_s} \sum_{i=1}^M \sum_{m=0}^{N_s-1} |x_i(m)|^2 = T(N_s) \\ T(N_s) &= \frac{\text{Tr}(R_x(N_s))}{ML} \\ &\approx \frac{\text{Tr}(\mathbb{H}R_s\mathbb{H}^\dagger)}{ML} + \frac{\text{Tr}(R_\eta(N_s))}{ML},\end{aligned}$$