Addis Ababa University

Addis Ababa Institute of technology

School of Electrical and Computer Engineering

Railway Traffic Regulation Optimization: for case of AA-LRT

A Thesis Submitted to Addis Ababa University

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Declaration

I, the undersigned declare that some part of the work presented in this research work have been published in different articles stated in the reference section of this research paper.

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Abstract

The presented thesis is a study on train energy consumption calculation and optimal train driving strategies for minimum energy consumption and travel time of train by optimizing the train speed profile. Speed profile signifies time spent to complete the given journey and energy consumed for that period. It is desirable to develop optimal speed profiles for the operation of the trains on the AA-LRT network by considering both energy and time as the objectives and by taking into account the effect of every kind of system constraint. So that maximum possible energy savings can be made while at the same time improving network capacity.

This study is focus divided into four parts, the first parts discuss about the model for energy consumption calculation for train and second part discussed about optimization tool used for the study called Dynamic Programming (Background Approach) for obtaining optimal speed and control profiles leading to minimum energy consumption. The third part is about simulating the developed algorithm in MATLAB optimization toolbox and finally the fourth part is about traffic controller which control traffic pattern due to disturbances like train delays.

The main achievements of this thesis are: a) the development of a model that can be used to calculate energy consumption in trains based on the driving resistances. b) A preliminary algorithm that returns the optimal speed profile for minimum energy consumption by consideration the forces of rolling resistance, the acceleration force and the tractive force needed by the train. The result obtained from simulation helps the driver to attain the required driving performance by driving the train with the respective speed profile.

Keywords: Optimal Control, Train Energy Consumption, Dynamic Programming (Backward Approach), Speed Profile

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List of Symbols and Abbreviation

List Abbreviation

AACRA	Addis Ababa City Road Authority
ERC	Ethiopian Railways Corporation
EW	East to West
DP	Dynamic Programming
GA	Genetic Algorithm
HJB	Hamilton Jacobi Bellman
KW	Kilo Watt
Kphr	Kilo Meter per Hour
Mps	Meter per Second
QP	Quadratic Programming
TE	Tractive Effort
TPS	Train Performance Simulation

List of Symbols

A	Acceleration in Meter-Per-Second-Square
(A, B, C)	Rolling Resistance Coefficients
E	Energy Consumed (Joule)
Ftr	Tractive Effort (Newton)

Fμ	Frictional Force (Newton)
g	Gravitational Acceleration(M/S^2)
G	Gradient(%)
J	Joule
Mt	Mass of Vehicle
N	Newton
Pe	Total Power
R	Train Resistance
W	Watt
μ	Coefficient of Friction
τ	Torque
α	Acceleration (Mphps)

CHAPTER ONE

Introduction

1.1 Background

Railway transportation has played a major role in the economic development of the world in the last two centuries [17]. It represented a major improvement in land transport technology and obviously introduced important changes in movement of passengers and freight transportations.

The railway company has to achieve more regular and reliable train services, in order to meet their customer's expectation. One way to optimize their services is to improve quality in the train control process itself. Therefore, Railway Company has to reduce delays that propagate between trains and reduce the energy consumed by the trains.

Energy consumption is an essential element of economic development and this development is resulted from expansion of industrialization with huge amount of energy consumption for productivity. Industrialization is not only the advantage for development but has negative effect on environment and cost of energy consumed. With these and other core reasons, scientists tried to develop new technologies or enhance the performance of already existed technologies for better energy consumption with noticeable economic development in a given sector. For example, railway transportation system evolved from steam engine drive through diesel engine to purely electrical train [1].

In general, energy consumption scheme of any energy-consuming sector must be efficient enough so that wastage of power will reduced. Energy consumed by train operation can be reduced using mass reduction, aerodynamic and friction reduction by appropriate design of train and space utilization [13]. It is also possible to use optimal construction parameters of railway lines, modern design of railway stations, optimal traffic management and energy efficient dynamic train movement in order to achieve optimum train driving [19]. Controlling speed trajectory of the train is an efficient energy saving strategy and this can be achieved by generating efficient speed profile based on wayside signals, capacity of the train and track information (curve, gradient, speed restriction of the track) [17].

Train running between two stations has four running steps. The initial step of train driving is acceleration. In this step, the startup resistors are cutout one by one. When all the starting resistors are cutout the train, cruises (zero acceleration). When the train approaches the next station, it is possible to disconnect power from motors, which results Coasting. During coasting, the train decelerates. Further, the driver can apply break to stop the train faster.

In addition, the other element need to be optimized is the delay between trains. It is well known that LRT lines are unstable, because train delays are propagated to the following trains by the signalling systems when the minimum headway between successive trains is reached, and delays increase at each station due to the accumulation of passengers [7]. Traffic regulation strategies are then necessary to prevent the traffic degradation. Travel time is also the core section of railway transportation system to attract considerable number of passengers. Station to station speed trajectory determines not only energy consumed but also the total travel time. Therefore, generating appropriate speed trajectory of the train can result optimum journey time driving and/or energy consumption driving based on the interest of optimality. Train running along the track is based on the timetable, which defines journey time to complete the given journey. Timetable always adds slack time to counter balance unexpected time loss [6].

Regulation strategies try to recover train delays by certain time margins included both in the nominal dwell times at platforms and in the nominal runtimes. The train using its own time margins, being the control action local to each train, could compensate small delays or early departures. With larger delays, trains need several stations to compensate deviations, and a transient period is established to reach the nominal operation. The regulation strategy tries to minimize mainly headway time deviations during the transient.

In this thesis, optimizing train movement with respect to energy consumption and train journey time will be presented. Moreover, the model and design of corresponding suitable algorithm is used to optimize train movement problem for Addis Ababa Light Rail Transit System (LRT).

In general, having better journey time is in the expense of energy to overcome all driving resistance along the track. Practically, speed of the train cannot increase without bound because of so many constraints considered such as passenger comfort, speed limit of the track, mechanical design of the vehicle, maximum energy consumption level and speed limit of the vehicle itself. Therefore, energy consumption and journey time should be studied together in order to achieve optimum solution on both terms.

So in order to reduce time delay that propagate between trains, to reduce energy consumption , to give passengers reliable service and to maximize utilization of the resources etc., the operator should control in optimal way and the train should move in flexible manner.

The following sub-sections of document describes about the current condition of the targeted LRT system and about train movement in general.

1.1.1 Railway and Road Transport in Addis Ababa City

In the past, there was no active railway transportation service in Ethiopia. However according to Ethiopian Railway Corporation Report, the LRT projects is finished and start operation in 2008 E.C. In Addis Ababa, there is a fast population growth and small road network coverage. At the time of the City 125th year of founding anniversary in 2013 Addis Ababa City Road Authority (AACRA) reported, in the their official website, that that the city's road coverage was not more than 13.7%. By 2020, this coverage is expected to be 20% [27].

The LRT project on the other hand is believed to complement the existing road transport service and reduce the existing transportation problem in the city. The Electrified Addis Ababa Light Rail Transit is the first light railway line on the East Africa with two lines of total length of 34.25km.

1.2 Statement of the Problem

Every customer (passenger) in railway mode of transportation wants to get safe, punctual, stable and low cost transport from origin to destination. So the railway management process should consider different parameters in different angle to meet the good quality of rail transportation.

Ethiopian Railways Corporation (ERC) is in position of controlling Ethiopian railway transport sector and energy utilization of train and travel time of must be in optimal condition in order to attract considerable number of passengers. This thesis will focus on evaluating driving performance, which is the amount of energy consumed, and travel time of train by optimizing the train speed profile. Speed profile signifies time spent to complete the given journey and energy consumed for that period [18]. It is desirable to develop optimal speed profiles for the operation of the trains on the AA-LRT network by considering both energy and time as the objectives and by taking into account the effect of every kind of system constraint. So that maximum possible energy savings can be made while at the same time improving network capacity.

In order for any of optimization approaches to find an optimal solution in the control center, there must be some way of modeling the set of possible solutions to the train optimization problem.

1.3 Research Objective

1.3.1 General Objective

The main objective of this research work is to find a way to efficiently utilize the LRT infrastructure resource thereby reducing energy consumption, the waiting time of passengers and the effects of traffic delays on the operation of the LRT trains.

1.3.2 Specific Objectives

Specifically the aim of this research:

- Investigate various railway optimization techniques and analyze their role in railway industry.
- Understand and analyze the designed speed profile
- To formulate a multi-objective optimization problem by considering time and energy as the objectives.
- To model the operation of a train
- Describe the algorithm that can be suited to solve the formulated problem
- Develop MATLAB program for the analysis of the mode
- Search for optimal solution

1.4 Research Methodology

The problem described above is carried out by the following methodologies. In the first stage reference books, journal papers, conference reports related to optimization of train movement is studied. In parallel to this data are required as input for the research like design documents for Addis Ababa LRT system are collected.

After acquiring these data and knowledge of optimizing train movement the problem formulation (modeling) is developed, which used to calculate various parameters associated with the operation of the train. These parameters include energy consumption, travel time, acceleration and braking rate, tractive effort, train resistance, and stopping distance. Since Energy, consumption and riding time are considered as the objectives to be minimized. Then the model is converted into algorithm that meets realizable computational requirements. Finally, the simulation is done using MATLAB software. Figure 1.1 summarizes the methodology used in this thesis.

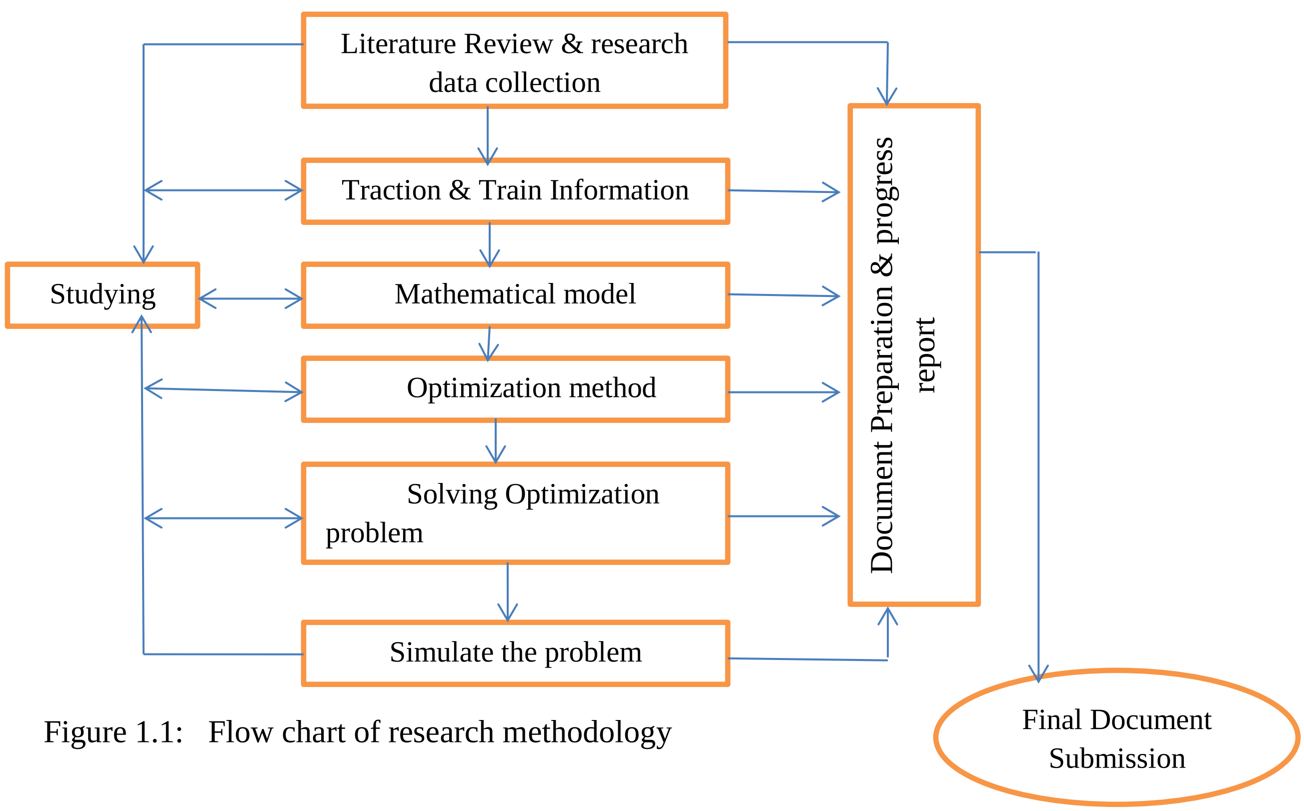


Figure 1.1: Flow chart of research methodology

1.5 Scope of the Thesis

This thesis develops means of generating speed profile for different traffic condition considering nearly all traction mechanics and track information for East to West line of Addis Ababa LRT. However, wind resistance and tunnel resistance is not considered. Tunnel resistance has an immense effect on train running resistance and wind resistance prevails for high-speed train. Signaling and curve speed limit are not also considered. The maximum time required to complete each sections of the track is taken to be the maximum headway between trains.

1.6 Thesis Organization

This thesis will organize into five chapters. The first chapter includes introduction, which provides clear information about the background of the thesis work, statement of the problem, research method and limitation of the thesis. Chapter two is about theoretical background and literature review. This section provides clear understanding of traction system, which is about types of traction system, tractive effort, tractive resistance and train optimization problem. Some information about railway development in Ethiopia and detail information about the newly constructing Addis Ababa light railway project are discussed. Finally, other researchers work on the area of optimum train driving will presented. Chapter three presents the mathematical formulation of the problem including traffic Optimization model ,Vehicle modeling for the energy consumption part and conclude by the algorithm implementation.

Chapter four presents Simulation and the results of MATLAB and discusses in detail various solutions. Chapter five concludes the study and suggests for future work.

CHAPTER TWO

Theoretical Background and Literature Review

2.1 Literature Review

This section reviews previously published research on train movement optimization. Although a number of researches have been carried out to optimize train movement problem in managing train service delays and reducing energy consumption, the importance of this idea is still new in Ethiopia.

Though there are many different approaches for the regulated train operation and minimization of train energy consumption, the most commonly used one is through proper management of the operation of trains [5]. This led to the development of optimal riding trajectories that could be used as references. It has been shown that the optimal trajectory consists of only four types of riding modes [14]. Therefore, the determination of optimal trajectory consists in the determination of the combination of riding modes together with the set of switching points.

The best strategy of optimizing traffic regulation, energy and journey time for train operation is generation of appropriate speed profile and forcing the system to follow it. Quadratic Programming, Dynamic programming, Mixed integer programming, Genetic programming, and analytical methods can be applied to generate appropriate speed profile depending on different operational scenarios. When we say operational scenario, it is to mean, operation of the train in either saving energy, minimizing delay, minimizing energy or jointly minimizing journey time and energy or others modes. Here, some paper works on the issue of regulated train operation and generating appropriate speed profile for optimum train operation are presented.

[7] Work on a predictive controller is proposed, which minimizes a quadratic cost function along a fixed-length optimization window. This work uses a non-linear event-based traffic model that includes bounded control actions and minimum interval constraints, and it is solved by non-linear programming techniques. The main drawback of this approach is that the optimization horizon has to be limited to five or six stations because of the real-time computation load.

Other predictive strategies try to minimize a cost function not based on the 'system' performance but on the passenger disturbance due to train delays. In reference [5], the cost function is independent of the timetable, and it is minimized using non-linear programming.

On the other hand, there are regulation strategies not based on optimization but on heuristic criteria. In reference [8], a heuristic predictive controller computes the complete trajectory of each train until delays are recovered. A heuristic feedback controller is proposed in reference [16], where the control law depends on the last measured delay of the train to be controlled, the delays of the train ahead, and the following train. This control tries to balance delays and headway deviations around the train to be controlled.

[22] Study on a linear state feedback approach is presented for the traffic regulation problem, where a very simple control law is obtained solving an optimization unconstrained quadratic problem. The main limitations of this solution for large delays are that control actions are not bounded and the minimum interval between consecutive trains due to the signaling system is not considered. Therefore, inequality constraints are not included. However, this model is useful to analyses the stability of the metro-type traffic regulation.

[14] Has done an extensive work on the analytical procedure for an electric train. He used nonlinear parametric programming to come up with the optimal driving strategy which defined the values of the set of driving modes, and the corresponding switching points. The paper defined the objective to be a weighted sum of both the energy consumption and ride time. Constraints included track profile, global speed limit, a resistance term which is linearly related to the speed, schedule ride time and station separation. The results of this paper put the optimal switching points in terms of system parameters and a computational method was required to calculate them.

[10] Developed a Train Performance Simulation model. The paper models the operation of a train using two different types of optimization problems. These are the shortest operation time and the proper operation time problems. The first case considers ride time between stations as the objective to be minimized while the later defines a term which is a function of both the ride time and energy consumption. In both cases it considers equations of motion and speed restrictions as constraints. Results show that the optimal trajectories that are achieved using this method are

accurate when compared to that of commercial software. Search techniques using heuristic algorithms seemed to deal with problem complexity and resulted in a more accurate solution although they were sometimes too slow when compared with gradient methods.

[21] The other research work aims at minimizing trip time under a certain energy constraint; the problem defined in this paper can be named as energy constraint operational problem. Since the paper is concerned about high-speed train, only aerodynamic resistance is applied to calculate appropriate speed profile. Trip time objective function is not subjected to jerk (comfort) and tractive effort constraints, which can influence passenger attraction and capacity of traction motor. Solving algorithm applied by this paper is analytical method. Energy, distance and speed are the constraints for this paper and works for one operational scenario.

[13] Propose a mechanism for optimization of speed profile in railway integrating energy saving. The paper developed an optimization model with three objective functions: reduction of travel time, reduction of delay and minimization of energy consumption. It tried to compare energy consumption, travel time and delay by solving the developed model considering one objective, two objectives and three objective functions. Evolutionary algorithm is the method used to solve the developed model. It is applied for continuously varying gradient. Comfort and jerk constraint, tractive effort constraint are not considered. It works for three operational scenarios.

[20] It aimed to determine the coasting point that will satisfy distance and time requirement, as well as minimizing energy consumption. Train Performance Simulation (TPS) and GA blocks are designed using Simulink. The objective in the paper was the minimization of energy consumption. Two trajectories are first calculated. These are the forward direction trajectory consisting of power driving and coasting, and a backward trajectory consisting of the braking curve. The coasting point is determined using genetic algorithm (GA). The algorithm defines the difference between actual driving time and the target driving time as the fitness function to be minimized. Several speed profiles, which could satisfy the distance and time requirements, are first generated. They are finally compared by their associated energy consumption value. This is achieved by repeatedly executing GA.

[28] Work focuses on energy minimization of railway system but not travel time. The paper used allocated travel time as one of the constraints for calculating the speed profile to obtain energy saving strategy. The mathematical model used by this paper is excellent, but for simulation, reason only rolling resistance is considered. The outcome of the simulation result allows saving about 24 % of energy compared to the applied speed profile during the time and works for one operational scenario.

[5] Used evolutionary algorithm to optimize the problem formulated using two objectives, energy and time. A special type of multi-objective optimization algorithm, Indicator Based Evolutionary Algorithm (IBEA) is used in this paper. The whole distance to ride between consecutive stations is partitioned into sub-sections. For each subsection are defined entrance speed (V0), exit speed (Vx) and intermediate speeds (V1 and V2). The evolutionary algorithm is responsible to calculate the three speeds (V1, V2 and Vx). Algorithms are developed for all types of riding modes, acceleration, cruising, coasting and braking. The particular mode to choose depends on the corresponding speed values. For instance, the entrance phase is between V0 and V1. If V0<V1, then an acceleration mode is executed. Similarly, the exit phase is between V2 and Vx. The intermediate phase is the profile section between V1 and V2.

[18] Used dynamic programming in the optimization of train speed profiles. They defined the optimization problem by using energy consumption as the objective to be minimized. A set of state equations that can act as constraints are then defines. Boundary conditions are transformed into a penalty function that can be included in the objective. State equations are time uniform discretized and linearized using first order Taylor series expansion and Trapezoidal rule for the approximation of integration. Finally, the overall problem equation is reduced into discrete sets such that dynamic programming (DP) can be applied. The final model contains an N-stage decision process, which is executed using digital computer. The paper then showed optimal speed profiles first for a plain track without speed limits, then for an actual track with varying gradient and speed restrictions. The results show that boundary conditions are satisfied within 0.6m and 0.1m/s. Computational error is also found very small to be applied for a practical scenario.

The problem proposed here is formulated using quadratic programming and Dynamic Programming.

2.2 Railway Traffic Regulation for LRT Type

A traffic regulation is control the movement or the activity that ensures high speed, train and all vehicle traffic flow and the safety of all concerned. LRT lines can be operated according to an offered timetable, typically during off-peak hours, or an offered headway, typically during peak hours. It is well known that these lines are unstable because train delays increase at each station because of the accumulation of passengers.

- Furthermore, delays can be transmitted to the following trains by the signalling system, when minimum headway between successive trains is reached
- Hence, traffic regulation is necessary to prevent the instability of the line.

Regulation strategies try to recover train delays by certain time margins included both in nominal dwell times at platforms and in nominal run times. Within small delays, each train could recover its own delay using the time margins. In a similar way, early departures could be compensated by extra run time (train coasting) or extra dwell time at platforms. Therefore, the regulation action is local to each train.

In LRT-type railways operated according to an offered and headway timetable, the main goal of regulation is a full timetable recovery, but the regularity of headways during the transient is usually considered to avoid the accumulation of passengers and seeks to compromise between timetable and headway deviations during the transient.

Different regulation strategies have been proposed on the basis of the minimization of a cost function containing the ‘system’ performance along an optimization horizon [9]. The performance criteria are delays referred to the timetable and headway deviations. The magnitude of control actions is also penalized in the cost function, so that nominal set point stands with null control actions.

In the following the railway traction and train kinematic model is described below which is related to optimization of energy consumption.

2.2.1 Railway Traction System

Traction means pulling or gripping of power. Traction system in general can be a system which doesn't use any electrical energy at any stage such as steam engine drive, internal combustion engine drive and a system which can use electrical energy at some stage such as diesel electric drive, battery electric drive and straight electric drive. Each of the drive system has its own advantage and disadvantage. The three kind of traction system used in railway transportation system are steam power, diesel power and electric power. These traction systems can be compared by looking at their efficiency and relationship between weight and output power. This time, electric traction system is the one widely applied in the railway industry. This is because of environmental friendliness, high starting torque, easy speed controllability and many other reasons [10] and [15].

The motion of a train between two consecutive stations is constrained by various parameters such as the alignment of the track, the status of the signals, the separation of the stations, the nature of the train and various requirements of the passengers. There are usually specified travelling time requirements by the passengers. At the same time, the operating company needs to conserve the operating energy. These two requirements of the operating company and of the passengers are usually antagonistic to one another that it is usually difficult to satisfy both of them at the same time. Various researches have been conducted in the past to come up with optimal riding modes that are meant to bring about the minimization of some kind of cost function [21].

2.3 Train Kinematics Model

The motion of a train can be modeled and represented by the various force components and the motion quantities that act on it at a particular time and location [24]. The force components that act on the train include weight of the train, Tractive Effort (TE), rolling resistance, air resistance, gradient resistance, curvature resistance, brake effort and adhesion.

2.3.1 Tractive Effort

To accelerate or decelerate the train, tractive effort must transfer between wheels and running surface of the rail through a friction force, called adhesion. Adhesion that determines tractive effort that can be applied before the wheels begin to slip is dependent on the weight per wheel.

In general, [16] defines tractive effort as follows:

It's the tractive force at the locomotive driving wheels (drivers) at the rail that starts and moves tonnage up various grades. The maximum tractive force that can be developed at the rail is equal to the weight on drivers multiplied by the adhesion (coefficient of friction) of the wheels on the rail.

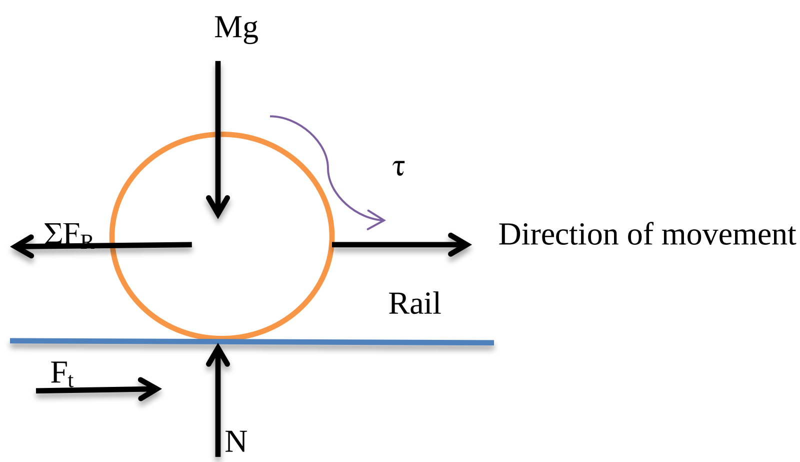


Figure 2.1: Force interactions between wheel and rail

At constant velocity, $\tau = r * F\mu$ (τ is torque and r is radius of the wheel) and tractive force is equivalent to frictional force $Ft = F\mu$. The friction (adhesion) limits the maximum tractive effort produced by traction motor.

$$Ft = \mu * Mt * g \qquad \text{......1}$$

Where: μ adhesive coefficient

Mt mass of the vehicle

g gravitational acceleration

Low value of μ results undesired wheel slip and wastage of energy. This situation also prevents increase of vehicle speed. Some researchers conducted wheel rail adhesion analysis and the following result were noticed: [19]

1. For a dry and clean wheel surface, the adhesion coefficient μ does not vary with vehicle speed;

2. For a track surface covered by oil, the adhesion coefficient μ can drop down to a very low level and this level is approximately maintained with increase in vehicle speed;

3. For water covering the wheel/rail contact surface, the adhesion coefficient μ reduces with increase in vehicle speed.

2.3.2 Train Movement (Tractive) Resistances

Train movement along the rail suffers from various resistance forces which opposes longitudinal traverse of the train. Those resistances are complex in nature and can be grouped in to four: [18],

a. Running (driving) resistance

b. Curve resistance

c. Gradient resistance

d. Tunnel resistance

All the resistance mentioned above will be explained in detail in mathematical model part.

2.4 Review of Optimization Techniques

Optimization is a key topic in engineering, economics and related fields. It is the process of trying to find the best possible solution to a single- or multi- parameter problem by an objective function or a performance index within a given time limitation [27]. An optimization problem has the form shown in Equation:

Minimize $f_o(x)$

Subject to $g_i(x) \leq b_i$, $i = 1,\ldots .m$

$h_j(x) = b_j$, $j=1,\ldots l$ ……………………………………..2

where vector x=(xi … xn) is the optimization variable to the problem; f_0 is the objective function; g_i is an inequality constraint function; h_j is an equality constraint function.

A vector x* is called optimal or a solution to the problem, if it has the smallest objective value among all vectors that satisfy the constraints, that is, for any z, there is $f_0(z) \geq f_0(x^*)$. The objective function can also be referred to as a cost function if it is to be minimized. If it is to be maximized, the function is known as fitness [28].

Optimization algorithms can be generally classified into two categories, namely exact algorithms and approximate algorithms, and the approximate algorithm is classified as Meta-heuristics and Ad-hoc heuristics [26].

The main advantage of using exact algorithms is that they are guaranteed to find optimal solutions and to prove optimality within instance dependent run time [24]. In this section, Dynamic Programming will be introduced, due to their applicability to a wide variety of problems.

2.4.1 Brute Force

Brute Force is a straightforward approach for solving a problem. It is usually directly based on the statement of a problem and the definitions of concepts involved. A brute force search enumerates all possibilities in the solution domain to find the optimum.

Selection Sort

Selection Sort is a typical brute force method. It is an in-place sorting algorithm, so it does not require additional storage space; therefore, it is suitable to be used where computer auxiliary memory is limited. The selection sort algorithm works as follows:

(1) The algorithm scans the entire list to find the smallest number and exchanges it with the first element of the list;

(2) The algorithm scans the entire list again, but starting with the second element, to find the smallest number among the remaining n-1 elements and exchanges it with the second element of the list;

(3) The algorithm repeats the scanning and exchanging among the remainder of the list (starting element is advancing each time Algorithm below After n-1 passes, the sorting is completed.

Exchange

A0 ≤ A1 ≤ ... ≤ Ai-1 | Ai ,…, Amin ,…, An-1

Elements have been sorted   Elements to be sorted

Algorithm of Selection Sort

```
// Input : An  array A[ …n-1]
// Output : An  array A[ …n-1] sorted in descending order
for i ←   to n-1 do
    min ← i
    for j ← i+1 to n-1 do
        if A[j] < A[min]    min ← j
    swap A[i] and A[min]
```

The complexity of the selection sort is O (n^2). Therefore, it is simple to implement but inefficient in solving problems with large lists.

2.4.2 Exhaustive Search

Many mathematical Modelling problems, especially combinatorial optimization problems, require finding an element in a solution domain that grows exponentially with an instance size. The element should maximize or minimize some desired characteristic. Exhaustive search is a brute force approach to such problems. It generates every element that satisfies the problems constraints in a solution domain then finds a desired element as the optimum result [28].

However, such a conventional brute force rapidly becomes impractical due to the processing time increases significantly when the problem is getting more complex.

2.4.3 Optimal Control Theory

Optimal control is a powerful tool that gives the ability to deal with complex control problems. It requires an advanced mathematical and dynamic programming background and is already famous for its adaptability and quality of results. Optimization techniques applied in railway system ranges from analytical solution through numerical method to evolutionary algorithm. For this thesis, optimization technique used is Dynamic Programming for the energy consumption optimization and discrete state space representation is used for delays of train disturbances.

2.4.4 Genetic Algorithm

Genetic algorithm is an optimization and search technique using the principle of genetics and natural selection. The number of initial population and how the population can be generated is the big challenge of this method. Too large initial population size will result unnecessary computational effort and too small initial population will narrow the search space.

2.4.5 Quadratic Programming

QP is a special type of mathematical optimization problem. It is the problem of optimizing (minimizing or maximizing) a quadratic function of several variables subject to bounds, linear equality constraints on these variables.

2.4.6 Dynamic Programming

Dynamic programming is a method for efficiently solving a broad range of search and discrete decision optimization problems. It uses a divide-and-conquer technique that partitions the problem into independent sub-problems and solves the sub-problems recursively. The optimal solution to the original problem can then be found from the optimal solutions to each sub-problem [22].

Mathematical Presentation of Dynamic Programming

A cost function can be formed in the sense that a sub-cost (g_k(Xn,Uk)) is incurred at each time k. The total cost can therefore be written as follows:

$$M = gn(Xn) + \sum_{k=0}^{N-1} gk(Xk, Uk) \quad \text{......}3$$

where:

- k is the discrete time;
- xk is the system state at time k. It also contains summaries of the past information which is relevant for future optimization;
- uk is the system decision at time k. It is associated with the state xk;
- N is the number of decisions;
- (gn(Xn)) is a terminal cost incurred at the end of the process

The aim of the dynamic programming is to select a set of decision U to minimize the cost function.

$U = \{U0, U1, U2 \dots Un - 1\}$……………………………………………………………………4

Based on the equation above, the following can be obtained:

$Mn(Xn) = gn(XN)$…………………………………………………………………………5

$$Mk(Xk) = \min uk \in \Delta k\{gk(Xk, Uk) + Mk + 1[fk(Xk, Uk)]\}$$

$$M0(Xk) = \min U0 \in \Delta 0\{g0(X0, U0) + Mj[f0(X0, Uo)]\}$$

where:

- Δk is the feasible decision space at time k.

It is important to note that M0 is obtained by the last step of the algorithm, which proceeds backward in time from N-1 to 0. In dynamic programming, all the decisions being made depends on the decision made for the previous stages, that is:

$Xk + 1 = fk(Xk, Uk)$……………………………………………………………………..6

Furthermore, let M* denotes the optimal cost, then:

$M^{*}=\min_{U0,..Un-1}$ [go(X0,U0)+g1(X1,U1)+..+g_{n-1}(Xn-1,Un-1)+gn(Xn)]………………………...7

Because k = 0,1,2,..… N-1 is conditional on xk and uk, equation above will become

$M^{*}=\min_{U0}$ [go(X0,U0)]+ $\min_{U1}$ [g1(X1,U1)+..+$\min_{Un-1}$[g_{n-1}(Xn-1,Un-1)+gn(Xn)..]…………..8

By appling equation 6 in 7 and introducing result into equation 9 the following result can be obtained:

$M^{*}=\min_{U0}$ [go(X0,U0)]+ $\min_{U1}$ [g1(X1,U1)+..+$\min_{Un-1}$[g_{n-1}(Xn-1,Un-1)+gn(Xn)..]……………9

= $\min_{U0}$ [X0,U0]+$\min_{U1}$[g1(X1,U1)+..Mn-1(Xn-1)]

= $\min U0 \in \Delta 0[g0(X0, U0) + M(X1)]$

The M* can therefore be solved as :

M^{*}= M0(X0) ………………………………………………………………………………..10

2.4.7 HJB Forwards and (Bellman's) Backwards Approach

Bellman's dynamic programming approach is quite general, but it is probably the easiest to understand in the case of purely discrete system. The general form of this equation is [18]:

$Xk + 1 = f(Xk, Uk) \quad , k = 0,1, \dots T - 1$……………………………………………………11

Where: $xk\epsilon X$ is a finite cardinality N, $uk\epsilon U$ is a finite set of cardinality M (T, N, M are positive integers).

This equation is referring to the forwards calculation. It can be supposed (to show the relation with the train motion problem) that xk is the speed of the train at step k taken from a pool X of possible speed values, and uk is the control input at step k taken from a pool U of possible control inputs (like accelerating, coasting etc.).

If the forward calculation method is used, the equation means that, at each step, the optimal speed of the next step will be obtain by minimizing a cost function, regarding the possible transitions from the current speed and control that can be applied.

To explain it better, a simple example (two dimensions-speed and time) will be presented briefly. Suppose that, at a step k, the speed of the train is xk, the pool of speed values X is 81 values (from 0 km/h to 80 km/h), the control input at a step k is uk and that the possible control inputs

U are accelerating, coasting and decelerating. The way that the forward HJB dynamic programming approach would find the optimal speed for the next step k+1 is the following:

In the start of the track (starting time, k=0) the speed x0=0. At this step, it is obvious that the only possible control input is accelerating (because current speed is zero). Given the maximum acceleration (speed difference) during one step is known (let's say that the train engine can accelerate at each step maximum 30 km/h, leveled straight track), the control u0 is accelerating and the possible transitions are 30 for the first step (each transition leads to a different speed value, from 1 km/h to 30 km/h). The HJB equation (above) would check all the transitions from the given point x0 to the next step k+1 including the non-possible transitions.

Then for each transition, an energy cost would be assign. The way that the non-possible transitions are avoided is assigning penalties for non-possible transitions, so that they will never be considered as optimal. A graphical representation of this simple problem is presented [22].

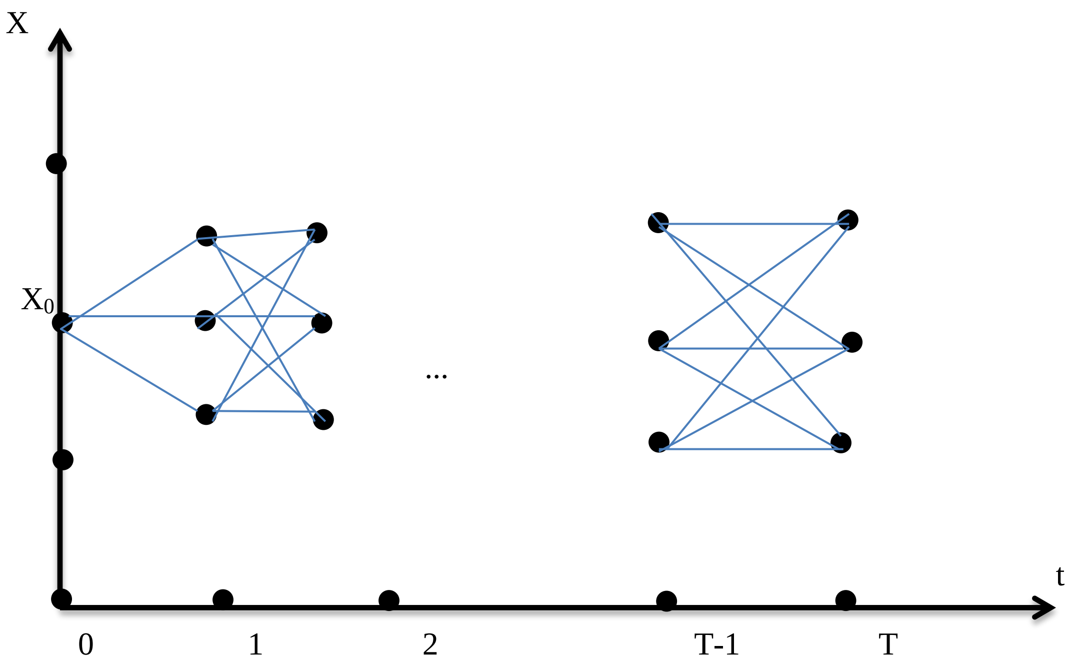


Figure 2.2: Discrete time: going forwards

In the example given above these transitions are 80 at each step.[22] explains that in this method, starting from x0, all the possible trajectories have to be enumerated, the cost for each must be calculated and then they have to be compared and select the optimal one for the whole problem.

The possible sequences that are possible in this problem are actually 81^T and to find the cost for each of these sequences T terms have to be added.

This means roughly $M^T T$ operations. This makes the calculation of the minimum-cost trajectory complicated and almost impossible to define due to the number of calculations needed.

Because of the difficulty of the forwards calculation, the best way to treat this problem is to start from a fixed final state and go backwards in steps (time). In order to use the backward calculation method, at each state a cost-to-go to the ending state must assigned.

At k=T, the terminal costs are known. The next step is to go to the previous step k=T-1. For each xT-1 all the possible transitions to k=T must be checked and the minimum one has to be chosen, to have the smallest "cost-to-go" (1-step transition cost plus terminal cost). This process must be repeated for all x's for k = T-2… 0. When this process is done, an optimal path is obtained from each x0 to some xT.

This method is providing the optimal solutions from each state xk to a fixed final xT that is the desired final state. The backward calculation implementation for this simplest case is presented graphically in the following Figure below.

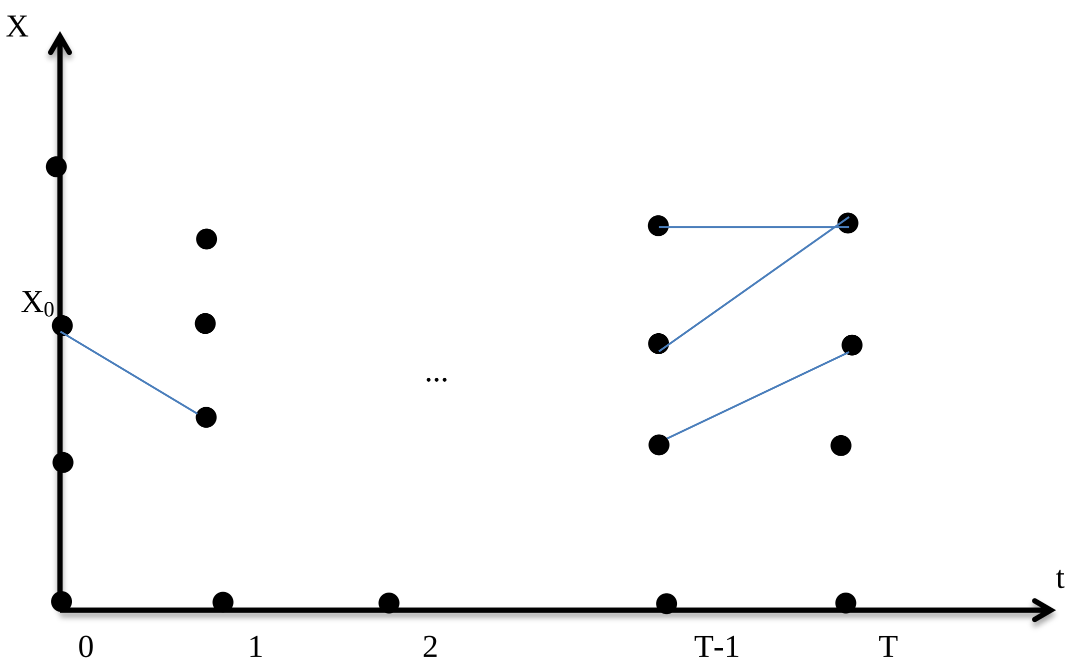


Figure 2.3: Discrete time: going backwards

The advantages of the backwards (dynamic programming) approaches are:

- The computational effort is clearly smaller than with the forwards approach. Furthermore, the backward approach gives an optimal policy for each initial condition (and $\forall k$). With the forward approach, to handle all initial conditions would require an enormous amount of calculations ($T*N*M^T$), and still it would not cover some states for k>0.
- Even more important is the fact that the backwards approach gives the optimal control policy in the form of feedback: given x, we know what to do (for any k). In the forwards approach, optimal policy does not depend on x but on the result of the entire forward pass calculation.

Railway systems are packed full of challenging combinatorial optimization problems. Yet, due to their complexity, the optimization process is usually carried out in a sequence, subdividing the full problem into several sub-problems, which are solved one after the other. For example, on a single-track section, optimizing train movement is realized by controlling the velocity curve of the train movement. Usually, the optimal objectives are energy consumption, enhance its comfort, make it run on schedule, etc.

2.5 Addis Ababa LRT System Description

The Addis Ababa Light Rail Transit (AA-LRT) project is composed of two routes called East-West Line which connects Hayat depot with the terminal point at Torhayloch (EW-22) and North-South line connects Kality depot with the terminal station (NS-27) at Minilik II Square near St George church. The trains are modern and they are 70% low floor trams. Primarily, ground lines are established, but in some sections, elevated or underground lines are employed. The project is a closed urban rail transit system. The LRT vehicles pass some intersections in the form of level crossing. The signal system is designed on the Principle that the LRT vehicles shall pass intersections first [1].

The planned lines are 75km in the overall length, and the lines in the E-W&N-S (Phase 1 Project to be executed currently is approximately 34.25km in length, including a common track section of 2.63km which extends from stadium to Lideta station.) Starting at the mileage of YCK5+046 and ending at the mileage YCK22+464, the main line of East-West line is nearly 17.410km long, while the main line of the North-South line is 16.561km long, with the starting mileage of YCK1+822 and the ending mileage of YCK18+381 [1]

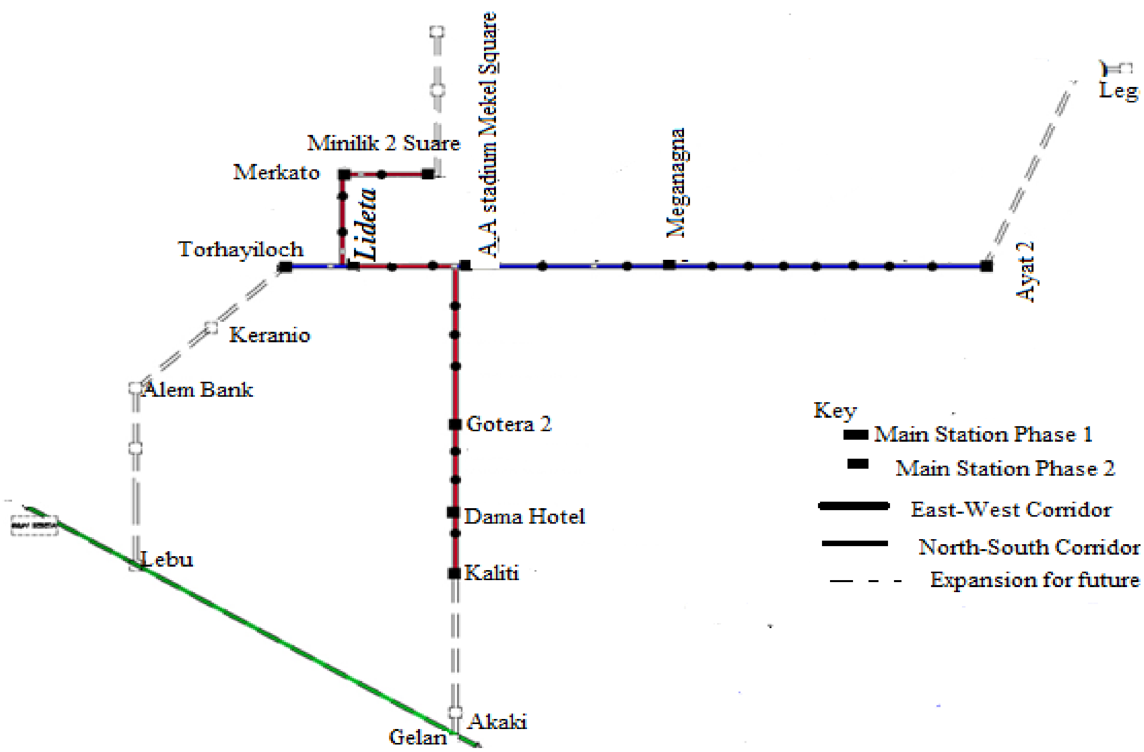


Figure 2.4: Addis LRT system topological drawing

Note: The above diagram contains only main stations & intermediate stations are not shown.

As we can see from the LRT topology, the project is divided into two phases. In the first phase, part of the route in East-West colored in blue and part of the route in the North-South colored in red will is completed whereas the remaining drawn in a broken line will be constructed in the second phase of the project. The first phase of the project Addis Ababa rail network coverage is 34.25km in two corridors that divides the city into four parts. The two corridors are called North-South line and East-West line. Their total length are 16.9km and 17.35km respectively and these routes share a common track of about 2.7km which starts from Addis Ababa Stadium ends at Lideta station as shown in the rough topology figure 2.4.

At the end of LRT 1st Project phase there will be a total of 39 stations of which five of them are located on the shared line (common track section). There are equal number of 17 stations on both EW line and NS line on the track sections other than the shared line [3].

For the EW line a parking lot is reserved near the starting point (Not build in current stage) and the Ayat depot is built near the ending point: for the North-South line, a parking lot is also reserved at the starting point (not build in current stage) and the Kality depot is built near the ending point. The parking lots reserved for the long term will not be constructed in short term [4].Refer table 2.1 for the summary of this topic.

Table 2.1: Addis Ababa LRT stations and depots summary table

Addis Ababa LRT stations and Depot		
Station type	Number of stations	Remark
Elevated stations	9	5 on the shared line, 1 on EW line, 3 on NS line
Underground stations	2	State here the location
Semi underground stations	1	State here its location and route
Ground station	27	13 on NS line and 14 on the EW line
Total Number of stations	39	Sate here the # of stations on each route
Depot	Location of the depot	Remark
Kality (main control center)	On South end of the NS	Main control center for the LRT system
Hayat	On East end of the EW	

2.5.1 AA LRT Stations

Addis Ababa LRT system has station capacity of two trains, i.e. one train one each track. A typical LRT station is shown in the figure 2.5.

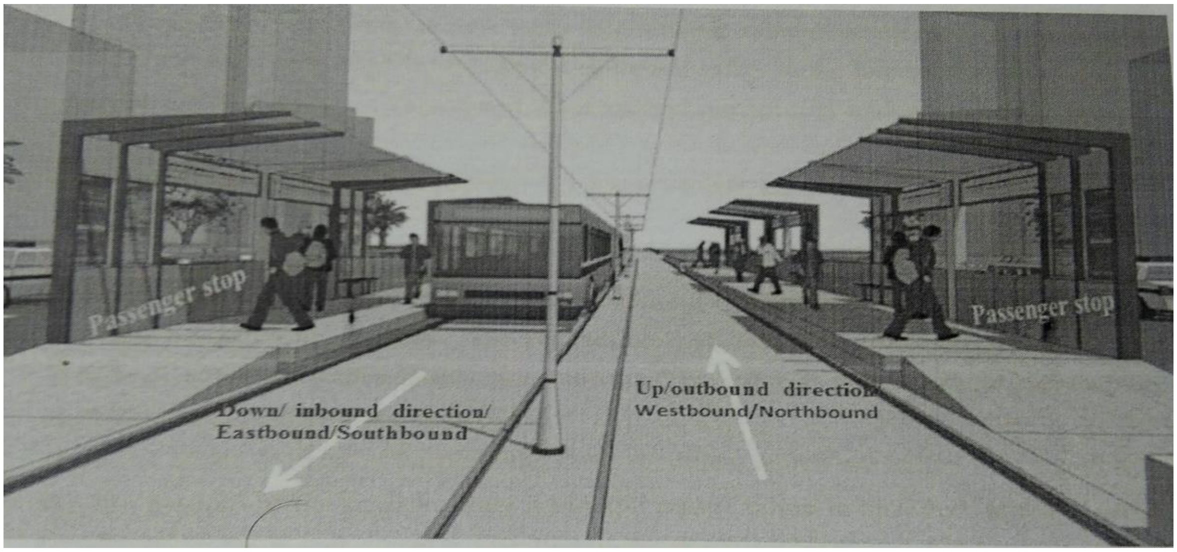


Figure 2.5: 3D view of typical AA-LRT station

2.5.2 Auxiliary Lines

The main type of auxiliary line is single crossover, which performs different functions in different stations. The four detailed functions are as follows:

1. **Reversing Station:** single crossover is placed behind reversing stations for the trains to reverse directions.
2. **Depot Junction:** the scheme of using two rails to connect depot is adopted. The two rails (double track) directly extend from the terminal to depot.
3. **Starting Station in the Shared-Rail Section**: single crossover is placed to ensure two trains travel within single rail.
4. A few of single crossovers are placed in intermediate station for the trains to temporally reserve directions.

2.5.3 Addis Ababa LRT System Tramcar Operational Conditions

Safeguarding Measurement

1. The section with shared Rail: interlocking system is adopted in the section with shared rail in order to ensure the operation safety in the intersection point.
2. Other sections: drivers operation ensures the security and safety of the tramcars.

Basic Technical Conditions of AA- LRT Vehicles

Tramcar Type

1. After the execution of project I Addis Ababa LRT will use 6-axles double articulated 70% low-floor Light rail tramcar. The vehicles are 70% low floor articulated 6-axle modern trams, consisting of three modules, bi directional driving. Two tramcars will be able to operate with double heading. See fig showed below.
2. One unit of the tramcar is formed with three modules, which are called Mc.T and Mc. Under the Mc module of the tram car body a bi axial power bogie is installed, and a bi-axial driven bogie with independent wheel is installed under the T module.

Figure 2.6: One unit of AA LRT tramcar configuration

Marshaling Scheme

1. Initial Stage: one unit of the tramcar is arranged to be operated. The rated passengers are 286 persons in one tramcar. (with 64 seats and 6 standing persons/m^2)
2. Long-term: two units are coupled together to form one train. The rated passengers the train are 572 persons (with 128 seats and 6 standing persons/ m^2).

Vehicle type and train formation

Train formation:- Mc+Tp+Mc-

Mc module: motorcar with drivers cab

Tp module: trailor without drivers cab and with pantograph

+ : Articulation device

- : Hidden folding coupler

Main Dimensions of Vehicle

Length of car body: ~30m

Side doors of passenger compartment: four pairs per side

Clear opening of passenger compartment door (width x height):≥ 1300x1860mm

Seating Capacity of Vehicles

Table 2.2:   LRT vehicle loading capacity

Number of passengers (person)	Seated	Standing	Total
Seat (AW_1 →fully seated or standing:0 persons/m^2)	65	0	65
Seating capacity (AW_2→ standing:6 persons/m^2)	65	189	254
Overload capacity (AW_3 → standing:8 persons/m^2)	65	252	317

LRT Train Speed

Maximum operation speed: 70km/hr

Average travelling speed: ≥20km/hr (average dwelling time of 30 seconds at each station)

Operation speed during car wash: 3-4km/hr

Average Acceleration

Under rated load and rated voltage on straight and dry track. With half-worn wheels, average acceleration is as below:

Vehicle speed from 0 km/h to 40 km/hr $\geq 1 m/s^2$

Vehicle speed from 0km/hr to 70km/hr $\geq 0.5 m/s^2$

Average Braking Deceleration

Under rated load in straight and dry track, with half-worn wheels, average deceleration from maximum vehicle operation speed of 70km/hr until stop is below:

Maximum service brake deceleration: $\geq 1.1 m/s^2$

Emergency brake deceleration: $\geq 2.0 m/s^2$

2.6 Inputs for the Problem

- Mass of the train in tone
- Average acceleration in m/s^2
- Rolling resistance coefficients (a, b & c)
- Speed of the train
- Total time duration of the trip (for our case is t=9.00 seconds)
- Distance between two stations (0.85km).

Some of the most important information about the Light Rail Vehicles (LRV) was collected and is summarized in table 2.3 [2].

Table 2.3: AA-LRT vehicles data

Number of cars per train	3
Number of power bogies	2
Number of unpowered bogies	1
Number of axles per bogies	2
Number of electric motors per bogie	2
Power per electric Motor	130KW
Total loaded mass of train	63.02 ton
Mass per axle	10.05 ton
Maximum speed of train on level track	70kph
Maximum speed of train on a level crossing	50 kph
Maximum jerk	1 m/s^3
Minimum average acceleration	0.5 m/s^2 for 0<V<40 kph,1 m/s^2 for 0<V<=70 m/s
Minimum average deceleration from Maximum speed	1 m/s^2
Minimum average deceleration at emergency brake	2 m/s^2
Average travelling speed	≥20 kph
Average dwelling time	30 sec
Number of passengers per train	317 (8 persons/m^2), 254 (6 persons/m^2)

CHAPTER THREE

Mathematical Formulation of the Problem

3.1 Introduction

A way to represent and describe a problem clearly and easily has been a subject of intense research for many years. The result of this research effort is that many problems in the real world can be represented by constraints, and the satisfaction of these constraints gives solutions for the respectively represented problems.

Optimizing the velocity of train movement is one of the basic methods for reducing train energy consumption, enhancing its comfort, and ensuring that the train can arrive at station on time. Here a key step to determine the optimal velocity curve with the train can arrive at station on schedule. Theoretically, two types of model have been discussed, i.e., the mathematical model and the simulation analysis approach.

The problem to be solved is traffic regulation, and minimization of delay and energy consumption and these problems can be solved either in a separate way, multi-objective or proper operational mechanisms. Constraints that affect train operation are the key aspect of mathematical modeling and each of those constraints is explained briefly as follows:-

3.2 Traffic Optimization Model

In LRT loop lines with terminal stations, every station is composed of two platforms, one for each sense of circulation. Trains travel repeatedly from terminal to terminal station. Traffic is modeled as a set of N trains running through a sequence of M platforms, where each train has to stop in order to allow passengers to get on and off (Fig. 3.1).

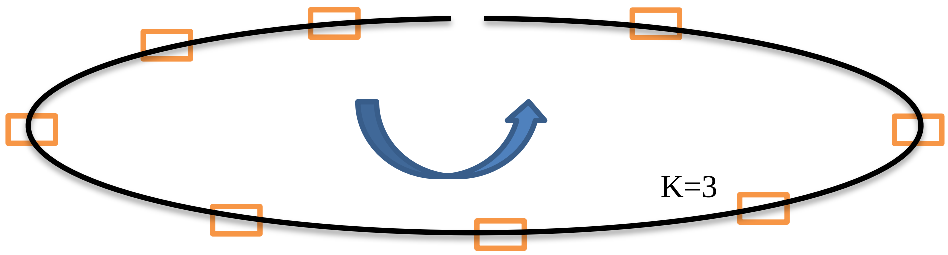


Figure 3.1: Loop line

3.2.1 LRT Line Modelling

Traffic Dynamical Model

The model is based on events related to arrivals of trains and passengers at stations. When an event takes place the system's state variables are updated in order to obtain the state of the system after the new event. Given a set of initial values for the states variables, the instant at which each train arrives at the next station is calculated assuming constant train speed over the line segments between stations:

The LRT system can be modelled by a system of equations in discreet time [8],

$tk+1^{i} = tk^{i} + rk^{i} + sk+1^{i}$……………………………………………………………12

where: $tk+1^{i}$is the total travel time for the train I to reach at station k+1

tk^{i} it is the departure time of train i of station k,

rk^{i} is the running time of train i of station k to station k+1,

$sk+1^{i}$ is the stopping time of train i of station k.

The LRT lines are unstable in presence of disturbances if no control action is applied for correcting the travel plan of the trains. Such kind of control action could be performed by manipulating stop times or in-between station running times. In other words, both actions are functionally equal. The running time is defined by,

$rk^{i} = Rk + Urk^{i}$……………………………………………………………………………13

Where: rk^i is the Running time of train

Rk it is the nominal running time which takes a train to reach station k+1 coming from station k ,

Urk^i is the control action on the train i in the route between the stations k and k+1.

State Space Model

Scheduling in ideal conditions (without disturbances [7]) is defined by equations $URk^i = 0$

and $t^i k+1 = T^i k+1$, where $T^i k+1$ is a previously established adequate timing. Disturbances in the model are captured by parameter $x^i k = T^i k - t^i k$ which is the difference between the scheduled timing per station and the real time.

If an ideal planning situation is considered as an equilibrium point the problem formulated as a discreet state space representation as follows:

$Xj + 1 = AXj + BUj + Wj$……………………………………………………………14a

$Y = Xj$…………………………………………………………………………………14b

Where: $Xj = \left[X1^{j-1} \ldots Xk^{j-k+1}\right]^T$

$Uj = \left[U0^{j} \ldots Uk - 1^{j-k+1}\right]^T$

$Wj = \left[W0^{j} \ldots Wk^{j-k+1}\right]^T$

$$A = \begin{bmatrix} -\frac{c1}{1-c1} & \cdots & 0 \\ \frac{1}{1-c2} & -\frac{c2}{1-c2} & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\frac{c}{1-ck} \end{bmatrix} \qquad B = \begin{bmatrix} -\frac{c1}{1-c1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\frac{c}{1-ck} \end{bmatrix}$$

The terms W^ik+1correspond to the disturbances, inherent to models and ck+1 is the delay rate capturing the effect of time intervals between departures of two successive trains.

Such dynamical system can be stabilized with the following sub-optimal ideal linear controller,

$Uj = \emptyset jXj$……………………………………………………………………………………15

Where: $\emptyset j = \begin{bmatrix} f & 0 & 0 & 0 & 0 & 0 \\ g & f & 0 & 0 & 0 & 0 \\ 0 & g & f & 0 & 0 & 0 \\ 0 & 0 & g & f & 0 & 0 \\ 0 & 0 & 0 & 0 & g & f \end{bmatrix}$

The f and g parameters is calculated by the following formulation,

$f = \frac{q+pCc}{(1-Cc)^2+pc+qc}$ ……………………………………………………………………………16a

$g = \frac{-p+p}{(1-Cc)^2+pc+qc}$……………………………………………………………………………16b

Where: $0 \leq p < \infty$; $0 \leq q < \infty$ and Cc is the maximum value of c_k and c_k is the delay rate value.

For nonlinear stability analysis there three important zones of stability in the system appear []:

a) First zone: it is a linear zone with guaranteed stability and it is defined by:

   $gx_k^i+f_{k+1}x_{k+1}^{i-1} \geq -k_k$

b) Second zone: it is a nonlinear zone with guaranteed stability and it is defined by:

   $1-r^2(c_{k+1}+f)^2-r^2(1+g)^2 > |r^2(-c_k+f_k)(1+g)|+|r^2(-c_{k+1}+f)(1+g)$

   Where $gx_k^i+fx_{k+1}^{i-1}<-k_k$

   $r^2=\max(1-c_{k+1})^{-2}=(1-c_c)^{-2}$

c) Third zone: it is a nonlinear zone where stability is not guaranteed and is formally defined by:

   $1-r^2(c_{k+1}+f)^2-r^2(1+g)^2 \leq |r^2(-c_k+f)(1+g)|+|r^2(-c_{k+1}+f)(1+g)$

   Where $gx_k^i+fx_{k+1}^{i-1}<-k_k$

Our previous formulation allows us to define a new set of stability indexes based on the distance from the origin (zero error) to the borders between the second (non-linear stable) and third (non-guaranteed stability) zone.

$$Vk+1^{i} = \frac{|r^2(-ck+f)(1+g)| + |r^2(-ck+1+f)(1+g)|}{1 - r^2(-ck+1+f)^2 - r^2(1+g)^2}$$

The dynamical system will have guaranteed stability if each of the indexes are in the interval:

$$0 \le V_{k+1}^{i} < 1$$

In case that any V^{i}_{k+1} is negative or equal or greater than one the system stability will not be guaranteed and the metro line could be unstable and need rescheduling.

3.3 Vehicle Modelling

3.3.1 The Physics of Vehicle Motion

In order to develop a train movement simulation, it is first necessary to consider the fundamental physics of train motion. The methods used to solve the dynamic movement equations are based on the equations of motion of the railway vehicle, subject to the constraints imposed on the vehicle by the route and driving style. This is the longitudinal equation of motion for the train and can’t be violated in any operational time.

$$Mtr\frac{d^2s}{dt^2} = Ftr - R - Fgrad \quad \text{.......} 17$$

Where:

- M_{tr} is the effective mass;
- F_{tr} is the traction force;
- F_{grad} is the force due to the gradient;
- R is the vehicle or train resistance.

3.3.2 The Force Due to the Gradient

The force due to the gradient shows the effect of the gradient profile and gravity acceleration. In an uphill situation, the train receives negative gravity component acceleration against the moving direction, while in a downhill situation the train receives positive gravity component acceleration, as shown in Figure 3.3 Such a force can be calculated as follows:

$Fgrad = Mtotal * g * sin\alpha$……………………………………………………………18

Where:

- M_{total} is the train mass plus passenger mass;
- α is the slope angle.

Gradient is expressed as the rise (in m) per a track distance of 100m and is called the percentage gradient G.

$Fgrad = m * g * \frac{G}{100} = 98Mtot * G$ ……………………………………………………………18a

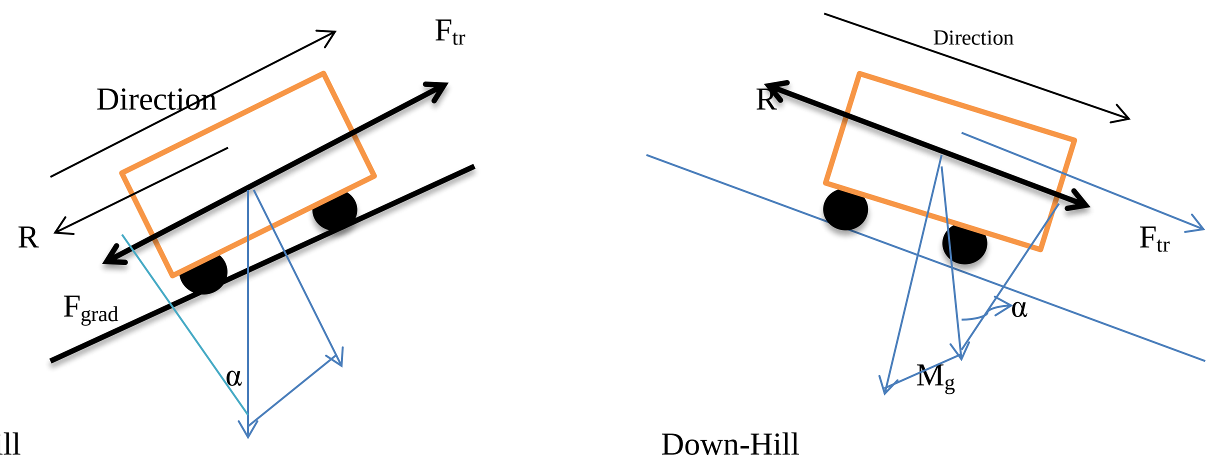
Where:

Fg: total gradient resistance (N)

m: total train static mass (tones)

g: acceleration of gravity (9.82 m/s)

G: gradient (%)



Up-Hill Down-Hill

Figure 3.3: Diagram of the force due to the gradient.

In a rail vehicle rolling along a straight level track, the force component perpendicular to the direction of gravity is zero.

3.3.3 Force due to Resistance

The formula that is used in this thesis for the calculation of the rolling resistance is:

$Fr = m * r = w * r$……………………………………………………………………19a

$Fr = a + b * v + c * v^2$…………………………………………………………………19b

Where:

Fr: total rolling resistance (N)

a, b, c: rolling resistance coefficients (depending on weight load)

a- mechanical resistance (constant), b-flange friction(internal friction) & c- for air resistance.

v: train speed (km/h)

The rolling force is the biggest resistance force during the train motion. The coefficients of the rolling resistance equations are different for different weight loads of the train. In this study, the train was supposed to be fully loaded so the rolling coefficients that were used were the ones associated with a train fully loaded.

3.3.4 Force due to Total Acceleration

Since it has rotating parts (wheels, axels, motor, etc) its effective accelerating mass (m_e) is more (8-15%) than its stationary mass. So force required for total acceleration:

$$Fa = Me * \alpha = 277.8 * Me * \alpha \quad \text{.......20}$$

Where :

F_a force required for total acceleration (N)

M_e is accelerating mass (tone)

α is acceleration (kmphps)

The acceleration force or acceleration resistance (FA) is coming from Newton's second law. In reality, and in the case under study, this force represents the force needed to achieve an acceleration value of 'a' for a train of mass 'm' when no resistances are applied to the train movement. This would be the only resistance in the ideal case of a straight-leveled track (no gradient and curve resistances) with no other parameters affecting the train motion.

So the total tractive force is expressed as

$$Ftr = Fa + Fr + Fgrad \quad \text{.......21a}$$

$$Ftr = \frac{mtrd^2s}{dt^2} + mr \pm Mtot * g\sin\alpha \quad \text{.......21b}$$

$$Ft = 277.8Me\alpha + mr \pm 98mG \quad \text{.......21c}$$

Curve resistance, aerodynamic resistance and the force due to gradient is not considered in this thesis because of:

When it comes to the curve resistance (resistance associated with the curves (turns) of the track), the ability to calculate it demands a very detailed knowledge of the tracks. This information is available for the line that is under study, but the process of adding these resistances in our calculations is time-consuming and needs to be further studied. This happens because, in order to use the dynamic programming model, the track had to be divided into equal distance intervals. This means that each of these intervals may include different curves, making the process of adding this type of resistances difficult.

Aerodynamic resistance is not considered in this thesis. Because this type of resistance is depending highly on the weather and is more or less random throughout the year. In fact, even if a detailed and accurate forecast for the wind direction and speed was available, still the calculation of the aerodynamic resistance is almost impossible. This happens because the train is changing direction through the line when taking turns, so the side where the wind is facing the train is changing. It can be said that, since the curve resistances are not taken under consideration, the train is not changing direction along the line, but even if this assumption is made, the aerodynamic resistance will not return realistic results. In order for this type of resistance to be reasonable and represent the real value, the direction of the train against the wind must be known in details. This is the reason why the aerodynamic resistance is not taken under consideration.

The gradient resistance is actually a major factor of the energy consumption formula. Just like the curve resistance, the gradient force needs detailed track data which will need to be further processed in order to be used, because of the method that was used. As mentioned above, for the formulation of the problem, the track had to be divided into equal intervals. Each interval might include different values of gradient, a fact that causes a problem when the gradient resistance is to be calculated.

In order for the gradient resistance to be included to the model that was developed, an approximation has to be used. One way to approximate the gradient resistance is to go across the line, starting from the first interval, consider the gradient changes inside the interval and calculate a unique approximated gradient resistance for each interval. The fact that for each position interval, a different coefficient would have to be used would make the formulation of the problem much more complex.

Therefore the mathematical expression for the total tractive force excluding the curve résistance, aerodynamic resistance and gradient resistance is as follow:

$Ft = 277.8me * \alpha + mr$……………………………………………………………………...23

The approach used to our problem is a mixture of equal-time and equal-distance dynamic programming model, with a focus on equal-time steps. This means that the total trip time between the stations under study, has to be divided into equal-time steps. Thus, whenever a calculation has to be done, it is done for time duration of one time step which will be referred to this text by the term tstep. This is a realistic approach since all of the transitions that are calculated in this thesis refer to one time step.

- **Initial and final condition**

Initial and final speed of the locomotive must be zero and can't be violated. At initial time the locomotive must reach to the initial running point and at final time it must reach to the destination point.

V(0,0)=V(S,T)=0; X(T)=S, X(0)=0

Our locomotive has rated power of 130KW with rated speed of 1800rpm and maximum speed of 4377rpm [3]. To find maximum power, first we have to find the maximum torque assuming that maximum torque is obtained during maximum power. Speed of the motor for maximum torque must be lower than synchronous speed.

The relationship between power and torque is shown below:

$Trated = \frac{Prated}{\omega rated},\ Tmax = \frac{Pmax}{\omega}$…………………………………………………………..24a

The margin of stability is expressed as the ratio of maximum torque to rated torque. This ratio is in the range of 2 to 3, but hard to obtain the ratio greater than 2.3 and it is good to have ratio of greater than or equal to 1.6.

$\frac{Tmax}{Trated} \geq 1.6 \rightarrow Tmax = 1.6Trated$ ……………………………………………………….…..24b

Substituting the expression

$1.6\frac{Prated}{\omega rated} = \frac{Pmax}{\omega} \rightarrow 116 = \frac{Pmax}{\omega}$……………………………………………………………..24c

Both values are unknown and the maximum torque happens relatively at the range of 20-30% slip.

The motor used by ERC train is six poles with rated operation frequency of 71Hz [2] and calculating operating speed of the train for maximum torque;

$\omega = (1 - s) * \omega sy \rightarrow \omega sy = 120 * \frac{f}{p} = 120 * \frac{71}{6} = \omega sy = 1420rpm$……………………….25

Synchronous speed is lower than rated speed, to mean the motor always works in generator mode and which is not right, so the rolling stock spec given at [2] has a problem and the pole is converted to 4 ($\omega sy = 2130$). Taking the maximum torque happened at 25%.

$$\omega = 0.75 * 2130 = 1597.5rpm$$

Substituting this to the original equation:

$Pmax = 116 * \omega = 116 * 1597.5 = 184.60KW$……………………………………………..26

Now, based on the manufacturer data sheet and the calculation performed above, we formulated the tractive effort limit of the motor used by ERC. At base speed, tractive force times by base speed must be greater than or equal to maximum power delivered by the four motors.

$$Ft * Vbase = 4 * 184.6KW$$

Knowing the base speed, this is 41 km/hr:

$$Ft = 64.83KN$$

Therefore:

Ft (v) ≤ 64.83KN   V≤ 41km/hr

Ft (v) ≤ -1.097v+109.777KN   V≥41km/hr

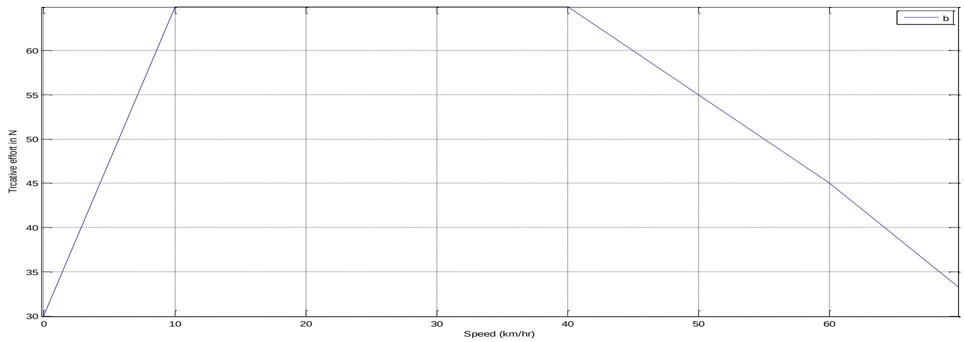


Figure 3.4: Tractive effort vs. speed curve updated to Ethiopian tramcar

3.3.5 Energy Consumption

There are two main ways to calculate the energy consumption. The first one is based on distance while the second one is based on time. The energy cost based on distance or time is obtained as follows:

Energy consumption calculation based on knowledge of driving resistance and distance

$$E = \int_{x1}^{x2} Ftot\, dx \quad \text{...27}$$

Where: Ftot is the sum of the driving resistances as mentioned above.

Energy consumption calculation based on power and time, the total power Pe needed can be calculated as

$$Pe = V * Ftot \text{...28}$$

Where

Pe: total power (W)

v: speed (m/s)

Ftot: sum of the driving resistances (see above) (N)

Based on that the energy consumption is obtained by integrating the total power needed for the total trip time. This gives:

$E = \int_0^t Pe\, dt$……………………………………………………………………………27a

Where :- E: total energy consumption (J)

Pe: total power (W)

t: elapsed time (s)

If Pe is constant, the energy consumption is:

$E = Pe * t$……………………………………………………………………………27b

In this thesis, the forces/resistances are calculated as follows:

a) Acceleration Force Fa (for one time step)

$Fa = m * \frac{V(t)-V(t-tstep)}{tstep}$ …………………………………………………………………..29

Where: $\frac{V(t)-V(t-tstep)}{tstep} = a$

is the equivalent constant acceleration that makes the train experience a difference in speed of (V(t)-V(t-tstep)) for one time step.

b) Rolling Resistance Fr (for one time step)

Since the speed is linear with time for each transition (time step), the rolling resistance for each time step can be calculated according to the average step speed

which is:

$$Vavg = V(t) + \frac{V(t-tstep)}{2}$$

So the rolling resistance for each time step will be:

$$Fr = a + b * Vavg + c * Vabg^2$$

So, in our case it will be:

$$Ft = Fa + Fr$$

From equations (above) and (eq. 14) taken from theory the total energy consumption of the train can be obtained from:

$$E = \int_0^T Pe * dt = \int_0^T Ft * v(t)dt$$

Where

T: total time duration of the trip (for our case is T=x seconds)

Pe: power consumption

Ftot: Total resistances the train has to overcome

E: total energy consumption

Given that the total trip time between the two successive stations under study is known the time horizon is discretized into equal time steps. Therefore, dt will be known and replaced by the term tstep. Now (above) becomes:

$$E = \sum_{ttstep}^{T} Ft * \frac{(v(t-tstep)+V(t))}{2} * tstep \qquad \text{30a}$$

Where

E: total energy consumption

T: total time duration of the trip

Ftot: sum of all driving resistances based on time

tstep: time step/interval (10 equal time intervals for a total trip time of 90 seconds gives a value of 9 seconds for each time step)

v(t-tstep): starting train speed of each time interval v(t): final train speed of each time interval

So, based on these formulas, the energy consumption equation becomes:

$$Estep = \sum_{t=tstep}^{T} (Fa + Fr) * \frac{V(t)+V(t-tstep)}{2} * tstep \ldots\ldots 30b$$

And for one time step (eq. 30b) becomes

$$Etstep = \left[m * V(t) - \frac{V(t-tstep)}{tstep} + a + b * Vavg + c * Vavg^2\right] * Vavg * tstep \ldots\ldots 30c$$

Distance/Position Calculation

In order for this model to be applied, the trip between two successive stations had to be divided into equal-time and equal-distance intervals. The model used for the energy consumption that is based on time and does not include distance, at every step (transition) that is made in the algorithm the distance covered had to be calculated. The distance covered is easy to get obtained by the ordinary equations of accelerated motion with constant acceleration.

The trip under study that means the distance between Kidus Yared to Magenagna covers a total distance of 0.85 km. The track is divided into 20 equal-distance steps, so every distance interval represents a distance of 42.5 m.

For a time step transition from state n-1 to state n from the equations of accelerated motion with constant acceleration the position covered can be calculated as follows:

$$\Delta Xn = Vn - 1 * \Delta tn + \frac{1}{2} * an * \Delta tn = Vn - 1 * \Delta tn + \frac{1}{2} * \frac{Vn - Vn - 1}{\Delta tn} * \Delta tn^2$$

$$\Delta Xn = Vn - 1 * \Delta tn + \frac{1}{2} * Vn * \Delta tn - \frac{1}{2} * Vn - 1 * \Delta tn$$

$$Xn - Xn - 1 = \frac{1}{2} * (Vn - 1 + Vn) * \Delta tn$$

$$Xn = Xn - 1 + \frac{1}{2} * (Vn - 1 + Vn) * \Delta tn \ldots\ldots 31a$$

And for 1 time step this becomes:

$$Xt = Xt - tstep + \frac{1}{2} * (Vt - tstep + Vt) * tstep \quad \text{.......31b}$$

Where:

Xt: final position of the train for one transition

Xt-tstep: Starting position of the train for one transition

vt: final train speed for one transition

vt-tstep: initial train speed for one transition

tstep: time interval for one transition

This formula gives the final position of a transition when the starting position, starting and final speed, and the time step are known.

3.4 Modes of Movement

As presented in figure 3.5, four movement modes are considered in a typical vehicle journey: motoring, cruising, coasting and braking. In the motoring mode, power is used to overcome resistance and the effects of gravity so that the vehicle can achieve the required acceleration rates. The train uses this mode to increase speed and move the vehicle from a low speed state to a higher speed state. In the cruising mode, power is used to keep the train speed constant and ideally. The acceleration should be zero. In the coasting mode, power is off. The speed is therefore, affected by resistance and the effects of gradient. The acceleration rate is usually negative unless the train is running on a steep downhill route. The selection of the coasting point plays an important part in the train control optimization for train journey time and energy saving. In the braking mode, the train applies the service brake or emergency brake to reduce the speed from a high level or to a standstill to a lower level in order to meet speed limits or stop at a station or signal.

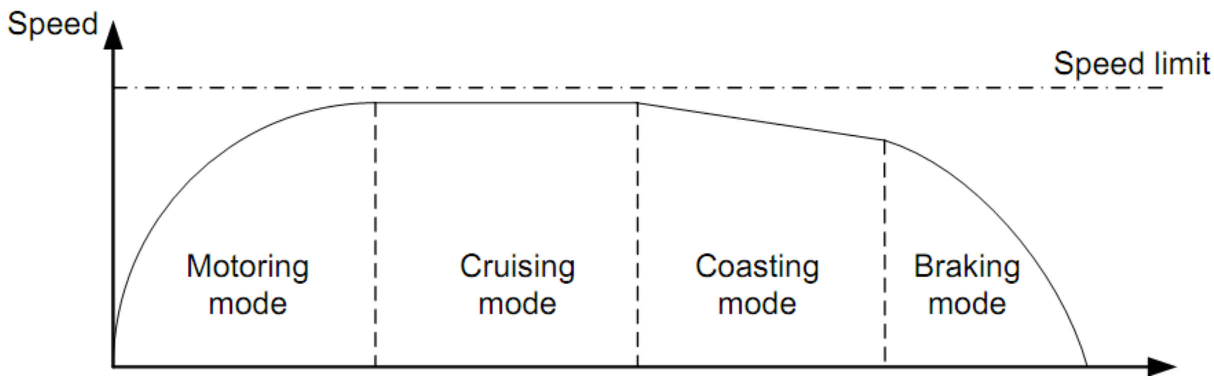


Figure 3.5: Train movement modes.

There are four train driving steps which are acceleration (high tractive effort and zero breaking effort), cruising (certain tractive power with zero acceleration and zero braking effort), coasting (energy saved due to this mode, both tractive and braking effort is zero) and finally braking (tractive effort is zero and braking effort probably with maximum braking effort).

Minimization of:

$$E = \int_{xst}^{xen} Ft * dt \ldots \ldots \ldots 32$$

$$Trun = \int_{xst}^{xen} \frac{1}{V} * dt \ldots \ldots \ldots 33$$

Subject to :

$$\frac{dx}{dt} = V \ldots \ldots \ldots 34$$

$$Mtr\frac{d^2s}{dt^2} = Ftr - R - Fgrad \ldots \ldots \ldots 35$$

3.5 Algorithm Implementation

The model built during this thesis based on the implementation of the backwards Hamilton-Jacobi-Bellman equation (dynamic programming approach).

As explained before in this report, the main idea behind our algorithm is to build a matrix that will hold the minimum cost-to-go (to the end of the line and time) for every combination of time elapsed, distance covered and current speed.

After this matrix is initialized and filled backwards, it becomes the basic tool by which the optimal sequence of control inputs and optimal speed values will be obtained. A time horizon of 90 seconds (which is the expected trip duration) is divided in 10 steps of 9.0 seconds each. This results in a total sum of 11 time steps (including t=0). Respectively, 20 distance steps for a trip of 0.85 km divided with 20 gives a distance step of 42.5 meters and a total of 21 distance steps (including x=0). The possible speed values are in total 71 (from 0 km/h to 70 km/h).

Precisely because the size of the matrices is that big, the (Dynamic programming) backwards calculation seems the only way for a program using this approach to run in reasonable time and give results. The way that the mode is decided and the energy cost is obtained is the following:

If the energy cost associated with each of these transitions is defined as e(vf) (calculated by equation 30. then this cost array is defined as:

$$C(vf) = J*(tf, xf, vf) + e(vf), \qquad vf = 0,1,2 \ldots, 70 \quad \text{...32}$$

Where: C (vf) is cost function and J* cost-to-go matrix

a) Coasting

The train mode by which a transition is achieved is coasting when the tractive effort needed for the transition is between 0 kN and 10 kN. In the ideal case, coasting would happen if the tractive effort needed for a transition is zero.

When a train is in coasting mode, the energy consumed is zero. This means that whenever a transition is achieved by coasting, the energy cost-to-go that is to be assigned to the associated element of the local array is simply the optimal energy cost-to-go of the ending point of this specific transition. So, when the mode of a transition is coasting the cost associated with the starting point is obtained by:

$$C(vf) = J*(tf, xf, vf)$$

Because in this case:

$$e(f) = 0$$

b) Acceleration

The mode of the train is acceleration when the tractive effort needed for the transition is between 10 kN and 403 kN (maximum tractive effort that the train can give). When the train is in acceleration mode, the energy cost-to-go assigned to the associated local array elements is the energy cost-to-go of the destination point plus the energy cost of the transition given by eq. 30.

So, in this case the cost associated with the starting point is obtained by:

$$C(vf) = J*(tf, xf, vf) + e(vf)$$

Where from equation 30:

$$e(vf) = (Fa + Fr) * \frac{vf + vs}{2} * tstep$$

Here:

Fa: Acceleration force (N)

Fr: Rolling resistance force (N)

vf: Final step speed value (m/s)

vs: Initial step speed value (m/s)

tstep: Time step (sec)

c) Constant Speed

The mode of the train is constant speed when the speed coordinate of the starting point is equal to the speed coordinate of the destination point (vs=vf).

When the train is in constant speed mode, the energy is calculated with the same way as in acceleration mode but, since the speed is constant, the acceleration force is zero so the energy comes only from the rolling resistance. Therefore, in the constant speed case, the cost associated with the starting point in the local array is:

$$C(vf) = J * (tf, xf, vf) + e(vf)$$

$$e(vf) = (Fa + Fr) * \frac{vf + vs}{2} * tstep,$$

$$where \quad vf = vs$$

$$e(vf) = Fr * vf * tstep$$

d) Braking

The train is in braking mode when the tractive effort needed for a transition is less than 0 kN. Most of today's trains have the ability under specific circumstances to regenerate energy and return it to the grid. Roughly, there are two different types of braking; regenerative braking and mechanical braking.

i) Regenerative Braking

The train can regenerate during braking as long as the speed is greater than 7 km/h. This means that, for the algorithm presented here there are two conditions in order for it to go into regenerative braking during one transition; first the tractive force must be between 0 kN and -380 kN and second, the starting and the ending speed of the transition must be greater than 6 km/h.

Returning energy to the grid is translated into negative energy consumption. The train can roughly return to the grid 70% of the energy that is associated with the braking transition. This means that during regenerative braking:

$$C(vf) = J * (tf, xf, vf) + 0.7 * e(vf)$$

When:

$$vs > \frac{6km}{h} \& vf > 6km/h$$

ii) Mechanical Braking

The train is in mechanical braking mode when the tractive effort is between -380 kN and 0 kN and the starting and final speed of the transition are less than 6 km/h.

During mechanical braking, the train is actually neither saving nor consuming energy. There are some minor losses but they are not taken under consideration. So, during mechanical braking:

$$C(vf) = J * (tf, xf, vf)$$

When $vs < \frac{6km}{h} \& vf < 6km/h$

iii) Partial Regenerative Braking

The mode partial regenerative braking refers to the braking cases when the starting speed of the transition is more than 6 km/h and the ending speed of the transition is less than 6 km/h. In this case the train can regenerate power until the speed goes down to 6 km/h. After that the train uses mechanical braking and does not return any power to the grid. Thus, in this case, the energy is calculated as if the final speed of the transition was 6 km/h and 70% of it is returned to the grid.

CHAPTER FOUR

Simulation Results and Discussion

In this section, the results from this thesis will be presented. Together with the presentation of the algorithm and the speed profile results for different cases that were obtained from the program in MATLAB, the main research questions will be answered respectively with the work done during this project.

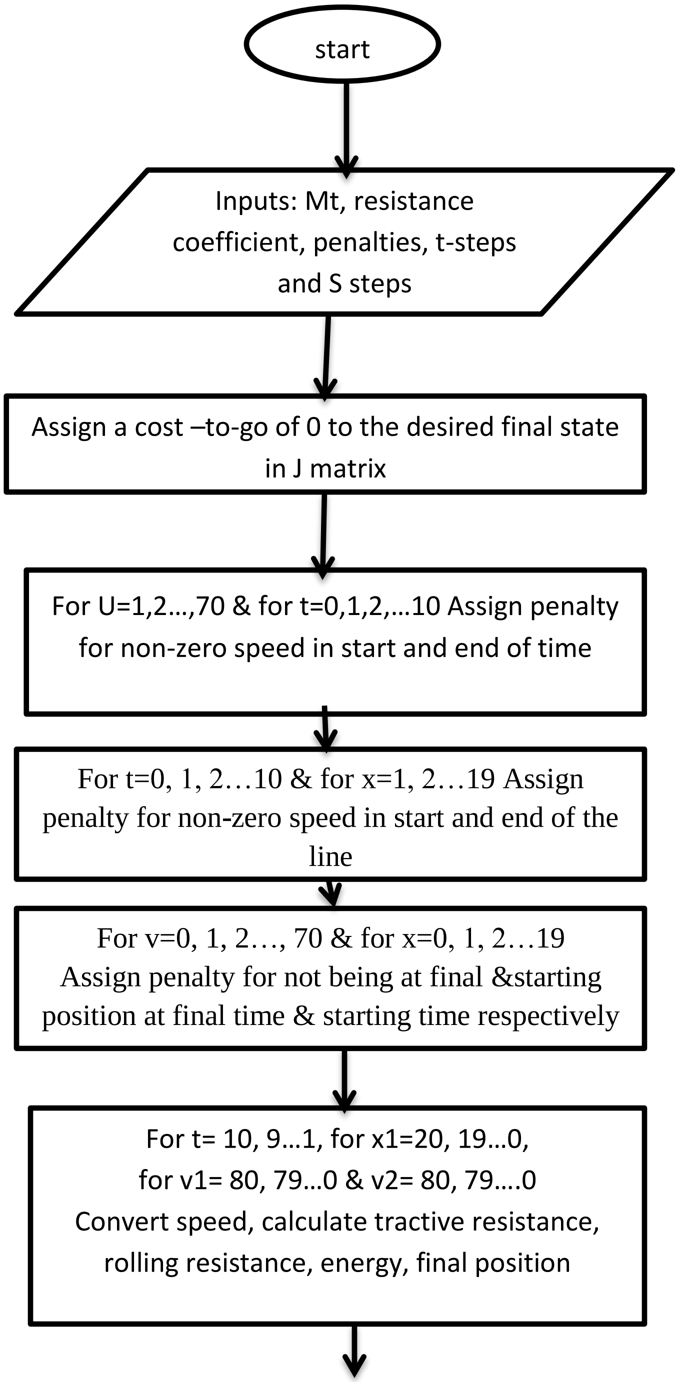
4.1 Algorithm Flow Chart and Simulation Parameter

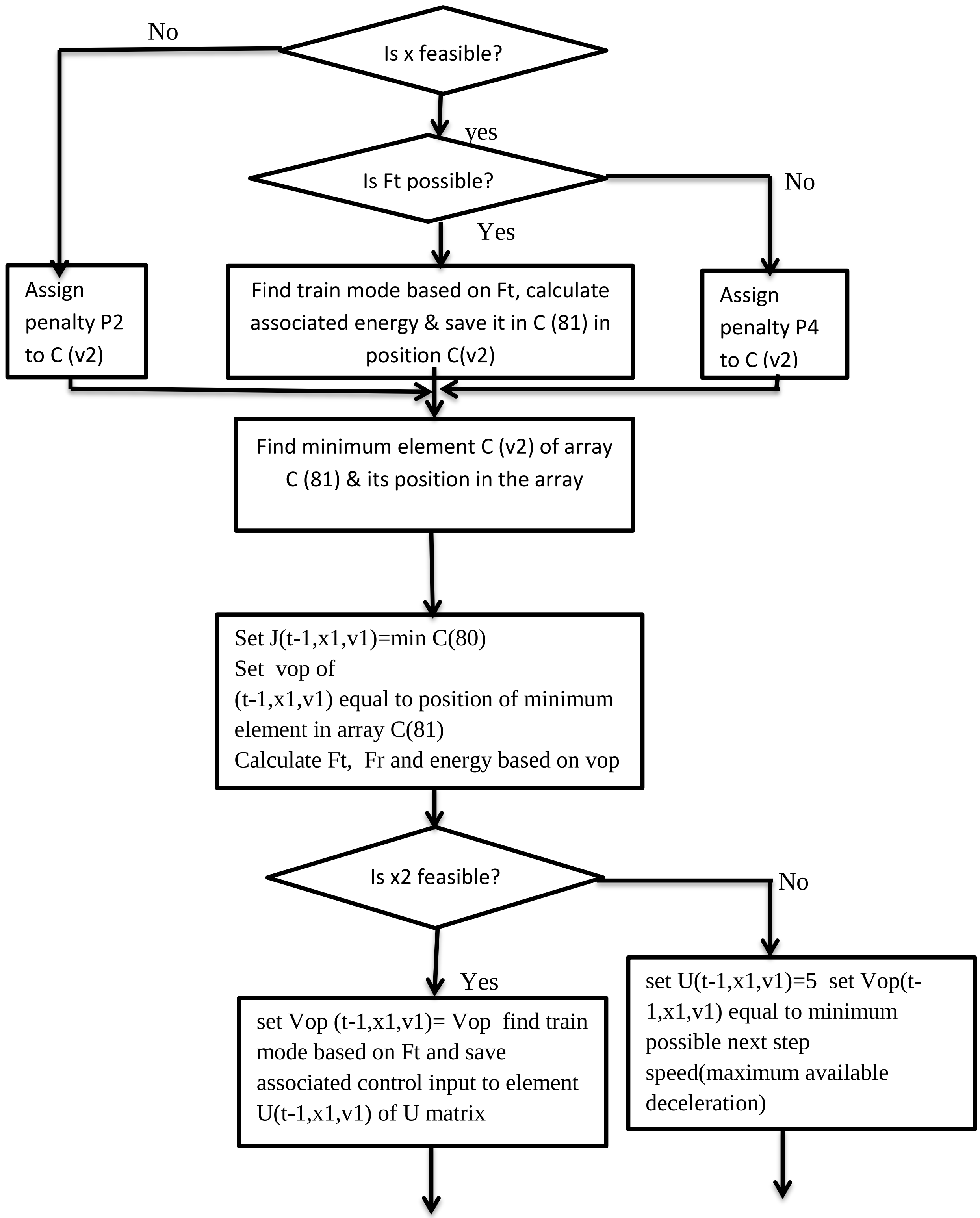
The following flow chart is presenting the logical steps that were followed in order to obtain optimal speed profiles regarding minimum energy consumption. Simulation parameter and algorithm flowchart is described below.

Table 4.1 Simulation Parameter

Input to algorithm	Simulation Output
M =630204kg/63.0204 tones	Speed Profile for different state
Rolling Resistances coefficient's	Energy Consumption for their respective state
T(time step)=9sec	
X(distance)=42.5m	
Total tractive effort calculated	

And the algorithm flow chart is as follow:


start
Inputs: Mt, resistance coefficient, penalties, t-steps and S steps
Assign a cost –to-go of 0 to the desired final state in J matrix
For U=1,2…,70 & for t=0,1,2,…10 Assign penalty for non-zero speed in start and end of time
For t=0, 1, 2…10 & for x=1, 2…19 Assign penalty for non-zero speed in start and end of the line
For v=0, 1, 2…, 70 & for x=0, 1, 2…19 Assign penalty for not being at final &starting position at final time & starting time respectively
For t= 10, 9…1, for x1=20, 19…0, for v1= 80, 79…0 & v2= 80, 79….0 Convert speed, calculate tractive resistance, rolling resistance, energy, final position


No
Is x feasible?
yes
Is Ft possible?
No
Yes
Assign penalty P2 to C (v2)
Find train mode based on Ft, calculate associated energy & save it in C (81) in position C(v2)
Assign penalty P4 to C (v2)
Find minimum element C (v2) of array C (81) & its position in the array
Set J(t-1,x1,v1)=min C(80)
Set vop of
(t-1,x1,v1) equal to position of minimum element in array C(81)
Calculate Ft, Fr and energy based on vop
Is x2 feasible?
No
Yes
set Vop (t-1,x1,v1)= Vop find train mode based on Ft and save associated control input to element U(t-1,x1,v1) of U matrix
set U(t-1,x1,v1)=5 set Vop(t-1,x1,v1) equal to minimum possible next step speed(maximum available deceleration)

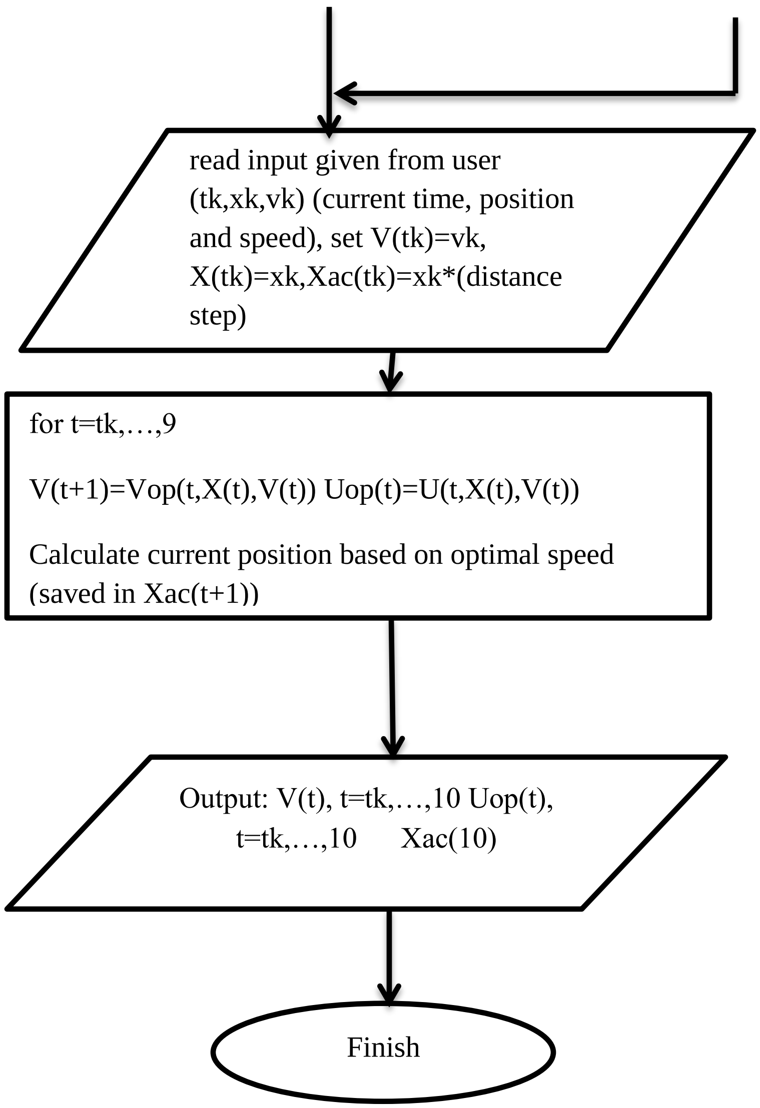


Figure 4.1: Algorithm Flow Chart

4.2 Numerical Results

In this part, the results that were obtained from the algorithm will be presented. The results that will be presented are for different input values regarding the current state of the train. For every current state here, the speed trajectory will be presented together with the graph of the profile (t,v(t)). Other measurements that will be presented for each input triplet is the energy needed or saved until the end of the line, and the approximated total distance covered since there is a deviation from the original target (0.85 km).

Speed and control trajectories

- **State (0,0,0)**

The input of the program is (0, 0, 0) when the train is in the start of the line, at starting time and with zero speed.

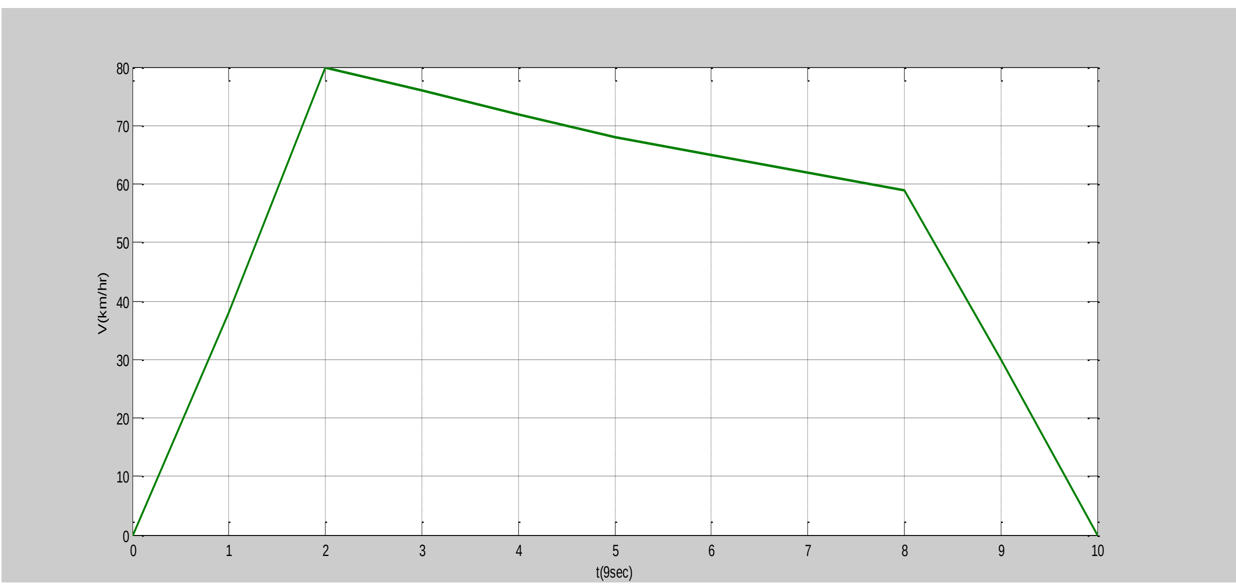


Figure 4.2: Speed Profile for State (0, 0, 0) (Whole Trip)

The speed profile, which is speed vs. time shown above present all types of train movement mode starting from acceleration to braking mode. And the state used above i.e. triplet input is the time, distance and speed respectively. The energy consumption in this case is 5.561×10^6 J. This value has to be compared with the other results and not to be taken as real energy consumption because of all of the approximations used.

- **State (1,2,20)**

The train is ahead compared to the time that it is in in this case. In this case, the current time is 9 seconds, current position is 85 meters from start and current speed is 20 km/h.

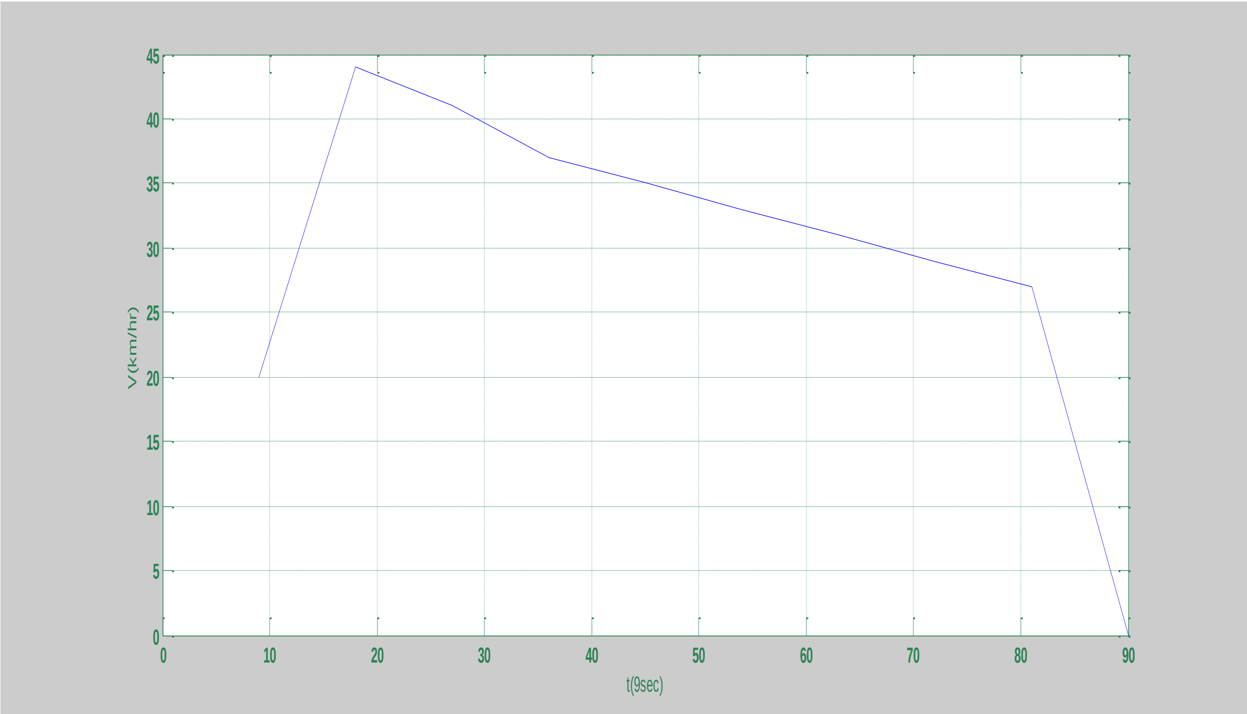


Figure 4.3: Speed Profile for State (1, 2, 20) (Whole Trip)

The speed and control trajectories in this case are:

Table 4.2 Speed and Control Trajectories for State (1, 2, 20)

t(sec)	9	18	27	36	45	54	63	72	81	90
V(km/hr)	20	44	41	37	35	33	31	29	27	0
U	1	1	3	3	3	3	3	4	4	

Note: u=1 is acceleration, u=2 is constant speed, u=3 is coasting, u=4 is regenerative and partial regenerative braking and u=5 is mechanical braking

The energy consumption in this case is $4.043x10^6$ J. As mentioned above in this case the train is at a position, which is further than it would normally go in 9 seconds. This means that the train does not have to accelerate to a higher speed than 44 km/h and then coast and break until the end of the line. The energy value has a big difference because a big part of the acceleration has already been done so a big amount of energy has already been consumed. Energy is now consumed only for the first 9 seconds. After that the train coasts with zero energy consumption until the final parts of regenerative and partial regenerative braking where the train saves energy.

- **State (3,2,30)**

This a state in which the train is behind in position compared to current time. In this case the train at current time 27 seconds is at a position 85 meters from starting position with a current speed of 30 km/h.

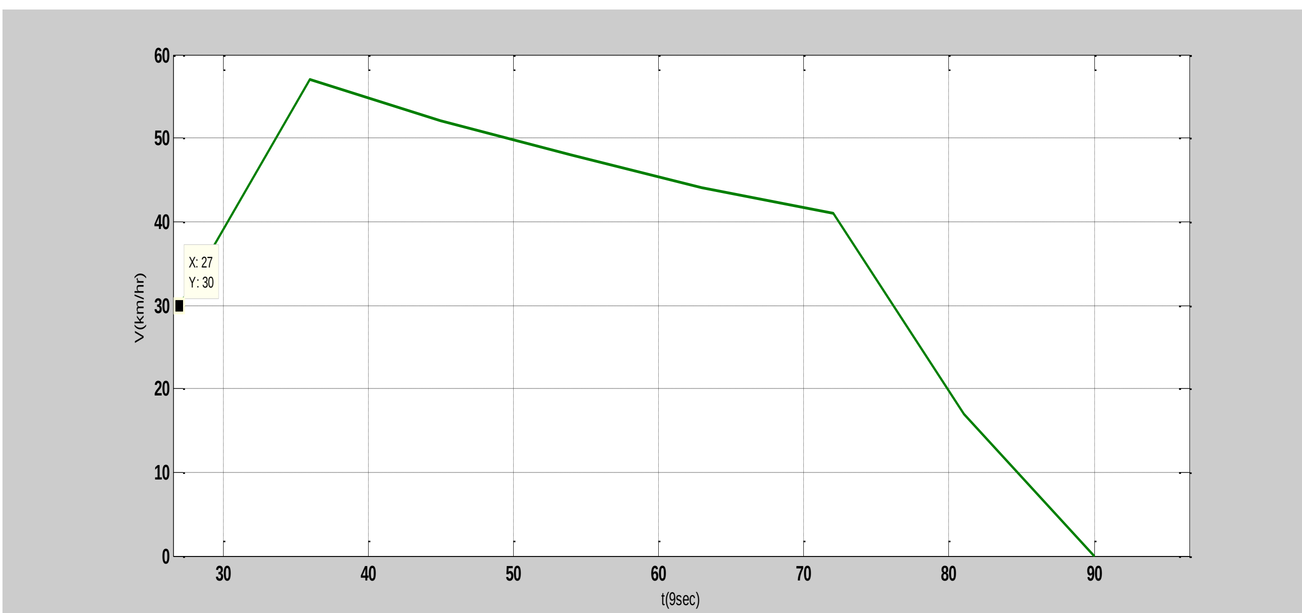


Figure 4.3: Speed Profile for State (3, 2, 30)

The speed and control trajectory in this case are:

Table 4.3 Speed and control trajectories for state (3,2,30)

t(sec)	27	36	45	54	63	72	81	90
V(km/hr.)	30	57	52	46	44	41	17	0
U	1	3	3	3	3	3	4	4

The energy consumption for the remaining trip in this case is 0.076*10^6 J. In this case where the train is behind it can be seen that it accelerates until maximum speed of 70 km in order to meet the timetables and at the same time coast and regenerate as much as possible.

- **State (4,5,40)**

This a state in which the train is behind in position compared to current time. In this case the train at current time 36 seconds is at a position 212.5 meters from starting position with a current speed of 40 km/h.

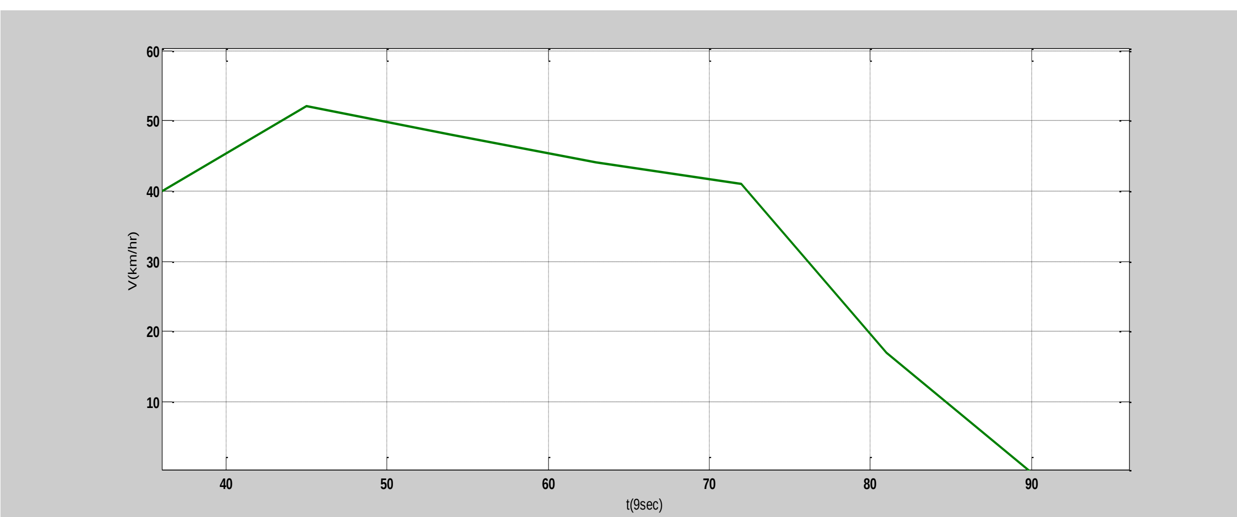


Figure 4.5: Speed profile for state (4, 5, 40)

The speed and control trajectory in this case are:

Table 4.4 Speed and control trajectories for state (4, 5, 40)

t(sec)	36	45	54	63	72	81	90
V(km/hr.)	40	52	48	44	41	17	0
U	3	3	3	3	3	4	4

In this case the train saves energy equal to $2.632*10^6$ J. This energy result is expected since the train has already consumed almost all of the energy needed to accelerate. The remaining acceleration is only for the first interval and is an acceleration of 20 km/h. After that, the train is coasting and braking with regenerative and partial regenerative braking, thus saving more energy than consuming for the rest of the trip remaining.

- **State (5,10,25)**

This a state in which the train is behind in position compared to current time. In this case the train at current time 45 seconds is at a position425 meters from starting position with a current speed of 25 km/h.

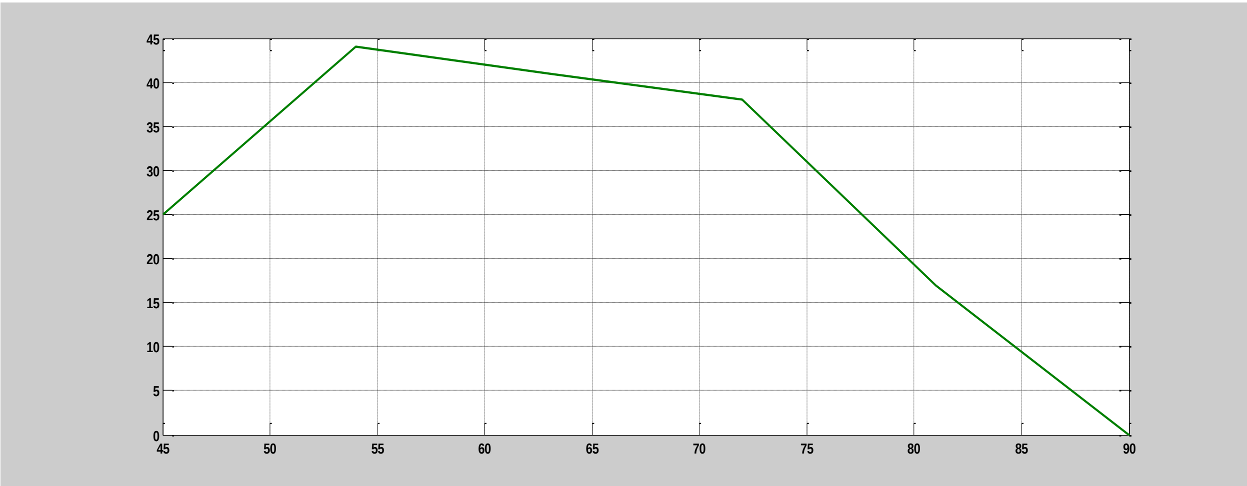


Figure 4.6: Speed profile for state (5, 10, 25)

The speed and control trajectory in this case are:

Table 4.5 Speed and control trajectories for state (5, 10, 25)

t(sec)	45	54	63	72	81	90
V(km/hr.)	25	44	41	38	17	0
U	3	3	3	3	4	4

- **State (6,10,30)**

This a state in which the train is behind in position compared to current time. In this case the train at current time 54 seconds is at a position425 meters from starting position with a current speed of 300 km/h.

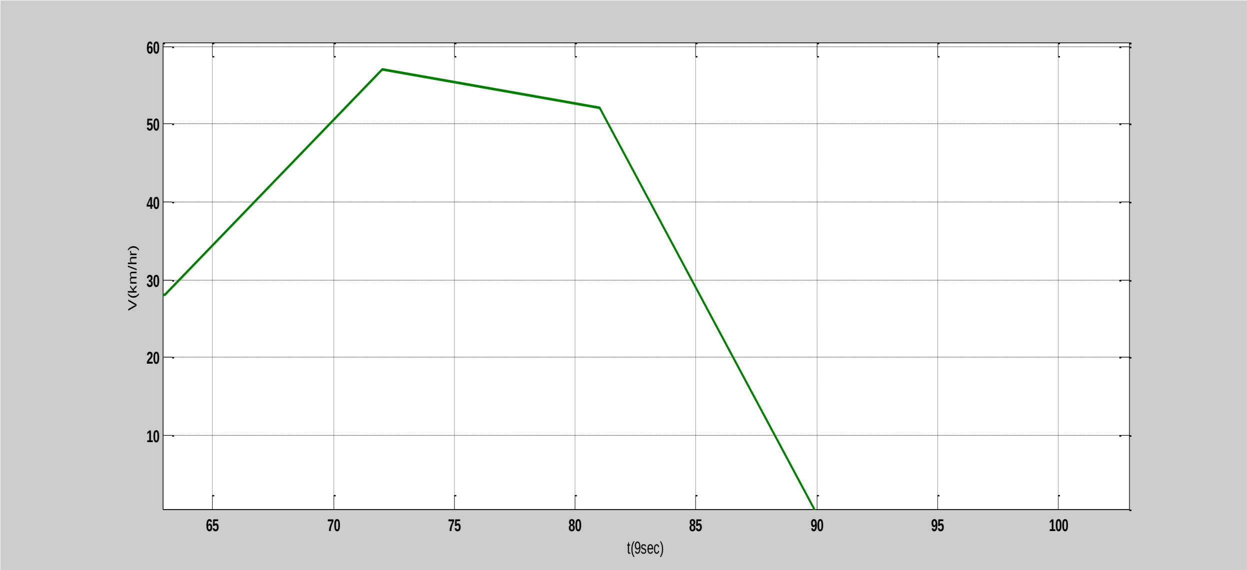


Figure 4.7: Speed profile for state (6, 10, 30)

In this case the train returns energy equal to $4.11*10^6$ J. In this case, the train only saves energy, since the remaining actions include only regenerative and partial regenerative braking.

The following is the summary of optimal speed profile for different state with its respective energy consumption.

Table 4.6 Summary of Speed Profile

State (1,2,20)											E(J)=5.561x10^6
State (1,2,20)											E(J)
t(sec)	9	18	27	36	45	54	63	72	81	90	
V(km/hr)	20	44	41	37	35	33	31	29	27	0	
U	1	1	3	3	3	3	3	4	4		4.04x10^6
State (2,2,20)											
t(sec)	18	27	36	45	54	63	72	81	90		
V(km/hr.)	20	52	48	54	41	27	26	25	0		
U	1	1	3	3	3	3	3	4	4		
State (3,2,30)											
t(sec)	27	36	45	54	63	72	81	90			
V(km/hr.)	30	57	52	46	44	41	17	0			
U	1	3	3	3	3	3	4	4			0.07x10^6
State (4,5,40)											
t(sec)	36	45	54	63	72	81	90				
V(km/hr.)	40	52	48	44	41	17	0				
U	3	3	3	3	3	4	4				2.6x10^6
State (5,10,25)											
t(sec)	45	54	63	72	81	90					
V(km/hr.)	25	44	41	38	17	0					
U	3	3	3	3	4	4					
State (6,10,30)											4.11x10^6

The above table summarizes all results for different speed profile and their respective energy consumption, where state (t,x,v) time step ,distance and train speed and U is control action. For the traffic model part the computation is as follow:

Starting from platform 6 and the fiftieth train, we introduced three delay time disturbances of 50, 200, 250 seconds respectively using suboptimal values p=1, q=1, cc=0.026, and Kk = 8.4 to compute the parameters of the discreet space state model.

In the first case, the LRT line is disturbed with 50 seconds delay. The system shows light deviations in three trains and stability indexes V^{i}_{k+1} reaching values between 0.3 and 0.50.

This means that the LRT line system is inside the stability zone with a possible increment of 0.50 from the upper non-guaranteed stability threshold and 0.30 from the lower non-guaranteed stability threshold. A 200 seconds delay produces values for the stability indexes between 0.1 and 0.60. The Summarized result are presented in the table 4.8

Table 4.7: Summarized result.

Disturbances	Stability Index		Train affected	Stability Zone	Recovery time
	Lower bound	Upper bound			
50	0.3	0.5	3	Stable	400
100	0.22	0.52	4	Stable	600
200	0.1	0.6	5	Stable	800
250	0	0.57	6	Critical Point/stability is not guaranteed	1000

5.3 Numerical Results Discussion

In this part, the numerical results obtained by the program will be discussed. The different results that will be discussed are the energy results and the speed profile results.

Energy Consumption Results

Given the approximations used in this model, the energy results cannot be fully accurate. The fact that not all the resistances are taken under consideration may have resulted in a significant deviation from the real energy consumption values giving lower consumption than the one this specific trip would require while the constant acceleration assumption might have caused also a deviation from reality. This deviation though does not mean that the results of this program are not useful. The energy consumption results are reasonable. This is a fact because the key to reach the optimal speed profiles is the relation between the energy consumption values. Based on that and on the fact that the same approximations are playing the same role in all the energy consumption results, the comparison between the different possible transitions can be considered as fully reasonable and the value of the speed profiles is not fully affected by the approximations and assumptions used.

Total trip results for different total time values

This is a type of a sensitivity analysis. The question that is to be answered here is what the speed profile would look like if the total trip time would be 200 or 300 seconds instead of 93 seconds. The time steps will still be 10 so, the speed profiles given here are based on time and the associated time steps are 20 seconds and 30 seconds respectively.

tstep=20 sec

When the total trip time is 200 seconds the time step is equal to 20 seconds. This means that the speed values that the algorithm is returning are the speed vales that the train should have every 20 seconds. The associated graph is:

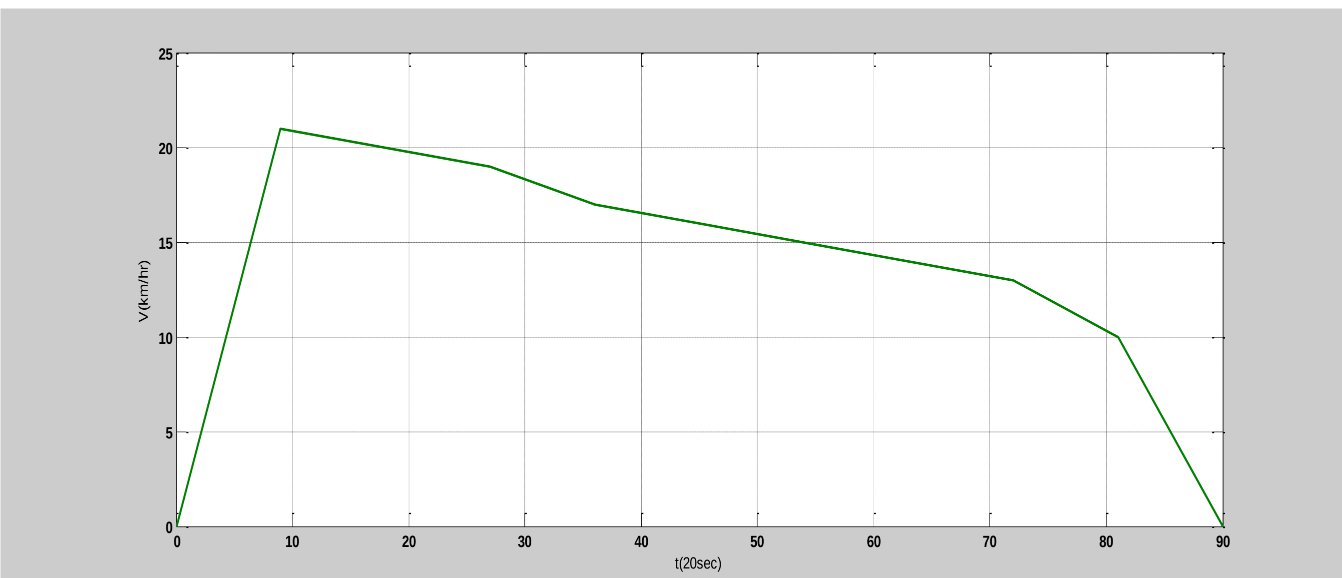


Figure 4.8: Speed Profile for Initial State (0,0,0) and Total Trip Time 200 sec

In this case, the train does not need to accelerate to high speed values, exactly because the total trip time is more than double now (200 sec) and the distance that has to be covered is the same (0.85km).

Tstep =25 sec

In the case that the total trip time is 250 seconds which means that the time step is 25 seconds and the total distance is still 0.85km the optimal speed profile based on time is given by the program as:

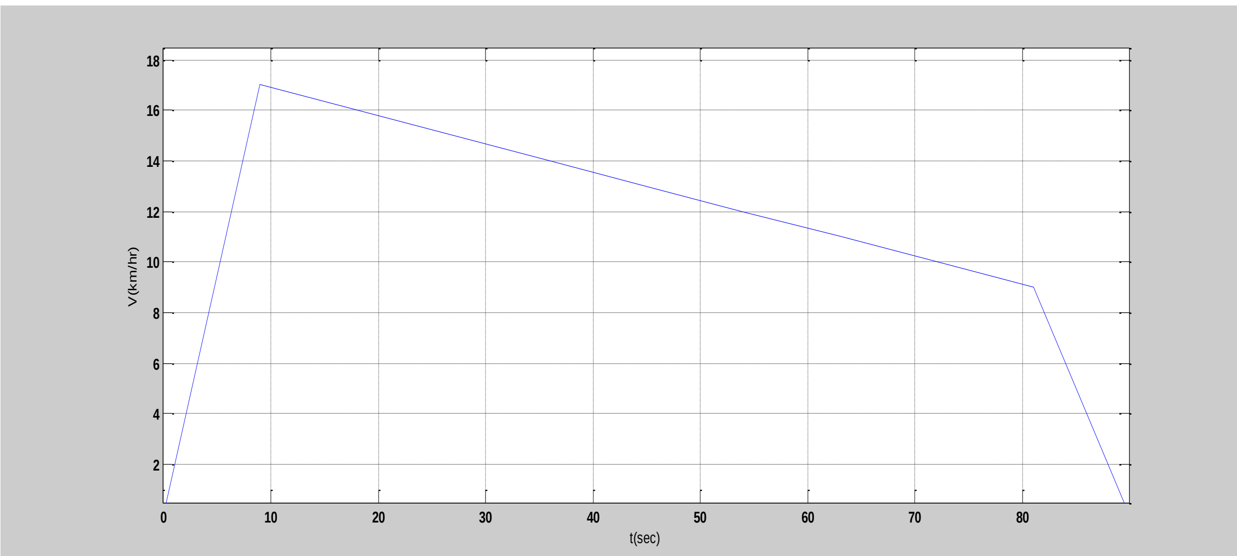


Figure 4.9: Speed Profile for Initial State (0,0,0) and Total Trip Time 250 Seconds

In this case, the train accelerates to even smaller speed values since there is much more time available for the same distance. From these two graphs it can be said that the algorithm is adoptable regarding changes in total trip time. The results seem to be logical here as well, since it is expected when there is much more time available for the same line the train to move slower.

CHAPTER FIVE

Conclusions and Recommendations

5.1 Conclusion

The main objective of this thesis was to find out which method should be followed in order to calculate the speed profile for AA-LRT that leads to minimum energy consumption for a given trip, to build algorithm based on the Bellman Backward approach. Another target of this thesis was a model that can be used in order to calculate the energy consumption for a given trip of a train.

In addition, the railway line in general and LRT line in particular show an inherent time delay disturbance from the previously planned time schedule, which lead the whole network disturbed. A discrete space state stability analysis is presented for traffic in LRT lines. This approach has allowed us to define a stability index and three zones of stability: a linear zone, a nonlinear zone with guaranteed stability and a nonlinear zone where stability is not guaranteed.

The main achievements of this thesis are: a) the development of a model that can be used to calculate energy consumption in trains based on the driving resistances. b) a preliminary algorithm that returns the optimal speed profile for minimum energy consumption by consideration the forces of rolling resistance, the acceleration force and the tractive force needed by the train.

5.2 Recommendations

Due to lack of real information, the models developed takes approximation and some datasheet from international railway standards and railway manufacturing companies. Thus, by considering the actual information we can improve the result obtained in this thesis.

In order for the program to be able to be used online in real-time as an advisory system for train drivers, accuracy is required in both energy consumption calculation and the calculation of the speed profiles.

- Energy consumption can be modeled based on the both distance covered and time taken.
- Use the least possible number of approximations and assumptions in energy calculation and algorithm implementation. It is also essential to take consideration all the different losses that might occur during train motion such as electrical losses or losses that occur while the power is transferred from motor to train wheels.
- Automatic train control systems use various algorithms to determine optimal train speed profiles by collecting instantaneous train status information. The algorithms developed in this paper can be further developed for the application to real time train control systems.
- By computing the stability index in real time it would be theoretically possible to predict well in advance the need for global rescheduling for the traffic part.

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Appendixes

```
function out = optimal_speed(ts,xs,vs)
m=63020;          %static mass for AW4 load 63.02 tonnes or 63020 kg
a=8547;           %rolling resistance coefficient(N)
b=64.2;           %rolling resistance coefficient(N/(km/h))
c=2.2452;         %rolling resistance coefficient(N/(km/h)^2)
P1=10000000000;     %penalty 1 for having non zero speed at final time and position
P2=20000000000;
P3=40000000000;
P4=80000000000;

tstep=9sec;           % time step is 9 sec. %first stop it is 1.51km and 1:33 sec trip time. thus trip
time is 90 sec divided by 10 steps we have 9 sec every step

xstep=42.5;     %respectively, 850 meters divided by 20 give a distance step of 42.5 meters
J = ones([11 21 71]); %initialization of matrices

Vop = ones([11 21 71]);
J(11,21,1)=0;

for v=2:71
    for t=1:11
         J(t,21,v) = P1;
          J(t,1,v) = P1;
             end
end
for t=1:11
    for x=2:20
        J(t,x,1)=P3;
            end
```

```
end
for v=1:71
   for x=1:20
      J(11,x,v)=P2;
   end
end
for v=1:71
   for x=2:21
      J(1,x,v)=P2;
         end
end
%MAIN
for t=11:-1:2
   for x1=20:-1:1
      for v1=71:-1:1
          for v2=71:-1:1
   v1c= (v1-1)*10/36;
   v2c= (v2-1)*10/36; % converting speed, In the cost matrix dimension v = 1 represents 0 km/h and v=81 stands for 80km/h so and subtract 1 and multiply by 10/36 to convert it into m/s for our calculations

  fA = m*(v2c-v1c)/tstep;   %calculation of acceleration force, use converted speed to m/s
   fRR  =  aRR+b*((v1+v2-2)/2)+c*(((v1+v2-2)/2)^2);  %we  use  average  transition  speed  to calculate rolling resistance. FRR is calculated using the speed in km/h
   fT = (fA+fRR); %total tractive effort needed for the transition
   xpr2 = ((v1c+v2c)*tstep)/2;
   xp2 = xpr2/xstep;
  x2 = x1 + round(xp2);
 if x2<=21
    if fT<=403000 && fT>=-380000
        if fT>10000         % in acceleration mode
```

```
        C(v2)= J(t,x2,v2) + fT*((v1c+v2c)*tstep)/2;
         elseif v1==v2    % constant speed mode
            C(v2) = J(t,x2,v2) + fT*((v1c+v2c)*tstep)/2;
             elseif fT>=0 && fT<=10000
        C(v2) = J(t,x2,v2);
        elseif fT<0  % in decceleration mode

           if v1>7 && v2>7         % in regen. braking,6km/h is represented from v=7 in the
matrix we put >7 instead of 6
              C(v2) = J(t,x2,v2) + 0.7*fT*((v1c+v2c)*tstep)/2;
              elseif v1<=7  %we are in mech brake
                 C(v2) = J(t,x2,v2);
                 elseif v1>7 && v2<=7
                    fA=m*(6*10/36-v1c)/tstep;
%so we calculate all the resistances with a final speed of 6 km/h
 fRR=aRR+b*((v1-1+6)/2)+c*(((v1-1+6)/2)^2);
                    fT=(fA+fRR);
                 C(v2) = J(t,x2,v2) + 0.7*fT*((v1c+v2c)*tstep)/2;
                      end
                 end
               else
                 C(v2)= P4;
                   end
            else
               C(v2) = P2;
                   end
       end
         [A,I]=min(C);
          J(t-1,x1,v1)=A;
           vop=(I-1)*10/36;
            v1co=(v1-1)*10/36;
```

```
        fA=m*(vop-v1co)/tstep;
         fRR=aRR+b*(v1+I-2)/2+c*(((v1+I-2)/2)^2);

     fT=(fA+fRR);

     xp2 = x1 + ((v1co+vop)*tstep)/(2*xstep);
      x2 = round(xp2);

     if x2<=21
         Vop(t-1,x1,v1) = I;
         if fT>10000
  U(t-1,x1,v1)= 1;
elseif v1co==vop
   U(t-1,x1,v1) = 2;
    elseif fT>=-10000 && fT<=10000
           U(t-1,x1,v1) = 3;
            elseif fT<-10000
     if v1>7 && v2>7
        U(t-1,x1,v1) = 4;
         elseif v1<=7 && v2<=7
           U(t-1,x1,v1) = 5;
            elseif v1>7 && v2<=7
             U(t-1,x1,v1) = 4;
                end
             end
     else
         U(t-1,x1,v1) = 5;
         Vop(t-1,x1,v1) = round(v1-(380000/m)*tstep*36/10);
           end
     if Vop(t-1,x1,v1)<=0
         Vop(t-1,x1,v1)=1;
```

```
            end
        end
    end
end

%GATHERING OPTIMAL SOLUTION
tk = ts + 1;   %inserting the inputs taken from the keyboard, this is current time
xk = xs + 1; %inserting the inputs taken from the keyboard, this is current position
vk = vs + 1;   %inserting the inputs taken from the keyboard, this is current speed
V(tk) = vk;
X(tk) = xk;
Xac(tk)=(xk-1)*xstep;
for t=tk:10
    V(t+1) = Vop(t,X(t),V(t));
      xp(t+1) =((V(t)+V(t+1)-2)*(10/36)*tstep)/(2*xstep);
       X(t+1) = X(t) + round(xp(t+1));
        Uop(t) = U(t,X(t),V(t));
         Xac(t+1) = Xac(t)+ xp(t+1)*xstep;                 %calculating total distance covered
end

       %out = Uop(tk:10);
out = V(tk:11)-1;
%out = J(tk,xk,vk);
End
```