COMPUTER PROGRAM FOR OPTIMAL DESIGN OF LOW-HEAD DIVERSION STRUCTURES

BY

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ABSTRACT

A computer program named HelaFlow is developed for optimal design of low-head diversion structures using visual basic.Net programming language. The program calculates the parameters of a diversion structure's components that are set in consideration of surface flow, subsurface flow, nature of foundation soil, structural stability and economy. It solves the uplifting pressure head distribution on the structure using finite difference technique, allowing for accurate design of structures built on anisotropic and/or shallow as well as isotropic and deep permeable media. It also optimizes the parameters of components for least cost design using cost of construction of components as input. The effects of anisotropy and variation in depth of permeable media were analyzed and found to affect the magnitude and distribution of uplifting pressure head on the apron of structures. Sample designs are presented to demonstrate the capability of the software program. This study equips the design engineer with a convenient decision tool for optimal design of low-head diversion structures.

Key words: diversion structure, subsurface flow, hydraulic jump, finite difference method, Anisotropy, optimization

LIST OF SYMBOLS

A_{ld}	=	downstream apron level
A_{lu}	=	Upstream apron level
b_d	=	downstream apron length
b_{dm}	=	minimum downstream apron length from hydraulic jump considerations
b_g	=	sloping apron length
b_T	=	total apron length
b_u	=	upstream apron length
C_d	=	discharge coefficient
$d_{d/s}$	=	depth of downstream pile
d_{dbl}	=	downstream pile bottom level
$d_{u/s}$	=	depth of upstream pile
d_{ubl}	=	Upstream pile bottom level
d_{dbl}	=	downstream pile bottom level
d_{ubl}	=	upstream pile bottom level
d_d	=	downstream pile depth
d_u	=	upstream pile depth
E_{f1} and E_{f2}	=	specific energy
f	=	lacey's silt factor
g	=	acceleration due to gravity
G_E	=	exit gradient
G_l	=	glacis level at a point
h	=	height of jump from Ds Apron level
H_b	=	head at weir proper portion
HFL	=	high flood level

HFLQ	=	high flood discharge
H_L	=	head loss
Hlp	=	horizontal location of jump
J_p	=	level of jump
K	=	coefficient of end contraction
L_u	=	width of under sluice
n_u	=	number of end contractions (twice the number of gated bays)
N_{ub}	=	number of bays at under sluice portion
N_{up}	=	number of piers at under sluice portion
N_{wb}	=	number of bays at weir proper portion
N_{wp}	=	number of piers at weir proper portion
O_{dal}	=	optimum downstream apron length
O_{dpd}	=	opitimum downstream pile depth
O_c	=	optimum cost of structure
O_{upd}	=	optimum upstream pile depth
Pl	=	pond level
plq	=	pond level discharge
Pp	=	ratio of discharge at under sluice to total discharge in percent
P_{pmax}	=	maximum ratio of under sluice discharge to total discharge allowed
P_{pmin}	=	minimum ratio of under sluice discharge to total discharge allowed
q	=	discharge intensity
Q_b	=	discharge capacity of weir proper portion
Q_t	=	total discharge
Qu	=	discharge capacity of under sluice portion
R	=	normal scour depth
S	=	gird spacing between nodes in finite difference calculation

SOR	=	successive over relaxation
TEL	=	total energy level
V_1,V_2	=	velocity before and after jump respectively
Wsl	=	water Surface level
X	=	ratio y_1 to Y_c
y	=	depth of water
Y	=	ratio y_2 to Y_c
y_1, y_2	=	initial and sequent depth of water respectively
Y_c	=	critical depth of water
Z	=	ratio of head loss H_L to critical depth Y_c
Ŭ	=	symbol alpha (ratio of total length of apron to depth of downstream pile)
ɸ	=	symbol delta(difference between nth iteration of a head at a point and the average head of its neighboring nodes in a finite difference calculation)
ŭ	=	infinitesimal difference used for approximating the derivative
ɗ	=	successive over relaxation parameter
ɚ	=	symbol lambda(a function of ratio of total length to depth of downstream pile)
ű	=	velocity potential function

DECLARATION

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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Chapter One

1 Introduction

Headwork structures are engineering facilities built across rivers or canals to store water and/or divert it from its original course. Among these, low-head diversion structures are extensively used in irrigation projects to divert water to a canal from either a canal or a natural river by raising the water level upstream. They are costly structures due to their usually massive structural volume. A typical of such structures includes a weir or barrage. The parameters of the components of these structures are interrelated and are set based on various considerations of surface and subsurface hydraulic and geologic conditions.

Various researchers have forwarded a wide range of surface and subsurface flow theories for the safe and economical design of low-head diversion structures. Among them are Bligh (1912) and Khosla (1954), who made significant contribution to the design of low head diversion structures by proposing subsurface flow theories and the corresponding design methods for structures founded on permeable medium.

Bligh's creep theory assumed the total head loss up to a point along the base of the structure to be proportional to the distance of the point from the upstream of the foundation. Although this theory has been found to be defective from actual field observations due to the inherent assumptions of creep length, it is still practiced for design of small diversion structures due to its simplicity in calculation.

Khosla evolved the method of independent variables. In this method the base of the structure is broken into simple and common profiles. The fundamental principle of the method is that, since the rigorous mathematical approach in the case of several cut-offs leads to rather intricate operations, an approximate result can be arrived at by splitting the complex foundation profile into several elementary forms. He established that the loss of head does not take place uniformly in proportion to the length of creep, but actually depends on the profile of the base of the weir. Khosla also established that the safety against undermining is not obtained by flat hydraulic gradient but should be kept below critical value. The ratio of uplift pressure head at any point along the base of a particular weir founded on permeable soil to the total head across the structure is constant and is independent of nature of subsoil as long as it is homogeneous.

The subsurface flow theories behind both Blighôs and Khoslaôs design methods depend on assumptions of isotropic and deep permeable media. Soil permeability is markedly anisotropic, with K_h, the coefficient of horizontal permeability, several times larger than K_v, the coefficient of vertical permeability (Novak *et al*, 2001). The average permeability of natural sandy soils in a horizontal direction may be from 2 to 10 times the value for the vertical direction (Vallentine, 1959). In such cases where the permeable media is anisotropic, graphical method of flow nets can be used to solve the subsurface flow for important structures (Garg, 2005). However, a graphical solution of the Laplace equation is a trial and error method and takes a great amount of time to solve a subsurface flow problem especially for geometrically complicated flow media such as those under low head diversion structure.

Moreover, in a rocky-country-side like those found in Ethiopia, low-head diversion structures are likely to be constructed on shallow permeable media. Using Blighôs and Khoslaôs mathematical methods in such circumstances, where the assumptions of isotropy and deep permeable media are not met, may be uneconomical or unsafe or even both uneconomical and unsafe..

Steady laminar subsurface flow can be described mathematically by the Laplace equation (Ragunath, 1983). The Laplace equation is an elliptic partial differential equation which, given the boundary conditions can be solved by using numerical methods such as finite difference method. Soil water movement has been investigated by approximating the partial differential equations for porous media flow with finite difference models (Hanks and Bowers, 1962; Reisenauer *et al.*, 1963; Taylor and Luthin, 1963, 1969; Freeze, 1971; Amerman, 1976a, 1976b).

Though the idea behind finite difference method is simple, enormous iterative calculation is needed to solve subsurface problems with it. Moreover, the complexity of permeable media profile under low-head diversion structures due to the existence of piles and varying apron thickness seems to have hindered the usage of finite difference method in designing of diversion structures.

Prediction of the surface flow of water above the structure is also an essential part of design of low-head diversion structures. This mainly involves finding the lowest level of jump formation on the sloping apron and solving the pre-hydraulic jump and post-hydraulic jump water surface profiles. For this, calculation methods and helper graphs are used. Still the procedure are lengthy making manual design of diversion structures cumbersome and preparation of alternative designs for finding the least cost design difficult.

Computers can be used for computations involving numerical methods. Moreover, routine design calculations and optimization for safe and least cost engineering structures in general and low-head diversion structures in particular can be done with due consideration of the actual site conditions and constraints by harnessing the power of computer machines once the right kind of computer program is developed. This study therefore, is taken up having the following objectives:

General Objective

To develop a computer program for optimal design of low-head diversion structures founded on all types of homogeneous permeable media including anisotropic and shallow permeable media.

Specific Objective

To develop a computer program for predicting the surface and subsurface flow of water at a low-head diversion structure

To analyze the effect of depth and anisotropy of permeable media on the uplifting pressure head distribution on the apron of head works

To develop and forward a tool for optimal design of low head diversion structures based on cost of construction of components of the structure.

Chapter Two

2 Literature Review

The design of diversion structures on permeable foundations has been thoroughly dealt with by different researchers. In this chapter, previous studies related to the work of this study are reviewed.

Until recently, Blighôs (1912) creep theory was being adopted for designing weirs with component parts on sand or alluvial soil. The theory assumed the total head loss up to any point along the base to be proportional to the distance of the point from the upstream of the foundation. Blighôs method does not discriminate between the horizontal and vertical creeps in estimating the exit hydraulic gradient. This theory has been found to be defective from actual field observations due to the inherent assumptions of creep length.

Pavlovsky (1922) first developed a general theory and a large number of individual solutions of the conformal transformation problem as applied to weir-foundation design. Apart from the purely mathematical analysis, his investigation comprised model-tank tests and ñelectric-analogyò method.

Lane (1935) based on his experiment on large number of dams, proposed a method in which the creep is weighted to allow for the variation in creep along vertical and horizontal directions. It is an improvement over the Blighôs creep theory but the method for determination of uplift pressure is criticized because it is an empirical method and not based on any mathematical approach.

The method of flow nets was first developed by Forcheimer and then formalized by Casagrand (1937). The method is a graphical solution of the Laplace equation for steady state flow. The flow nets are constructed by dividing the soil profile under the foundation into arbitrary number of equipotential (equal head) and flow lines. The solution is achieved by trial and error.

Khosla (1954) evolved the ñmethod of independent variablesò. In this method the base of the structure is broken into simple and common profiles. He established that the loss of head does not take place uniformly in proportion to the length of creep. But actually depends on the profile of the base of the weir. He also established that the safety against undermining is not obtained by flat hydraulic gradient but should be kept below critical value. The ratio of uplift pressure at any point along the base of a particular weir founded on permeable soil to the total head is constant and independent of nature of subsoil as long as it is homogeneous. The fundamental principle of the method is that an approximate result can be arrived at by splitting the complex foundation profile into several elementary forms.

Finite difference approximation was one of the earliest methods known to be used successfully for solutions of ground water problems (Richardson, 1911). Other approaches, such as finite element (Zienkiewicz, 1977) and boundary element (Liggett, 1977) have been introduced later. The finite difference method is simple and flexible that the non-linearityҫs arising from changes in parameter values, such as the change between confined and unconfined states can be include without difficulty (Amerman, 1977).

A steady-state model employed the SOR (Smith,1965; Forsythe and Wasow, 1960) technique to solve finite difference equations which simulated steady flow for either saturated or unsaturated conditions or for a combination of the two (water table condition).

Finite element method was suggested by Garg (2004) as an alternative to Khoslaҫs theory for subsurface flow prediction since it can also take into account soil non-homogeneity and anisotropy.

Ijam (2005) used conformal mapping technique to obtain an exact solution for seepage flow below a hydraulic structure founded on permeable soil of infinite depth for a flat floor with an inclined cut-off at the downstream end. The exit gradient decreases considerably along a distance beyond the floor end with an increase in cut-off inclination. He found that using an inclined cut-off increases the factor of safety in design against uplift and piping.

Gil *et al.* (2002) used spreadsheet program to solve Laplace equation using finite difference method with the appropriate boundary conditions. The calculation results were found to have excellent relations with experimental results.

Finite difference method based on boundary-fitted coordinate transformation was applied to analyze the steady seepage in a foundation pit, a lock foundation, and an embankment dam with a free surface (Jie *et al.*, 2003).

Garg (2002) developed a method of minimizing the cost of a barrage using an optimization technique by doing parametric analysis to gain insight on the effects of various parameters on the optimal barrage design.

A methodology of optimal designs of pileless, and single-pile sloping floor weirs that are structurally safe were proposed by Prabhata (1996).

The FLOWNS model was developed for generating flow nets for any saturated rectangular domain with any combination of constant head or constant flux boundary conditions. The program approximates with discrete values, the continuous distributions of potential and stream function using finite-difference approximations of Laplace's equation. The hydraulic conductivity distribution may be anisotropic and/or heterogeneous (Bramlett *et al.*, 2005)

Gabriel *et al*. (2004) studied analytical creep theory using two-dimensional finite difference computer model for the design of low head hydraulic structures. They found that seepage under hydraulic structure is a complex problem that can be properly solved using a numerical model. Comparison of numerical model results shows that actual distribution of potential along tin-creep length is non-linear against Bligh's Creep theory which suggests a linear distribution.

Turan (2004) developed a Windows-based program named WINDWEIR in Visual Basic.NET programming language for the optimum design of a diversion weir with sidewise intake. It determines the overall dimensions of each of the components of the diversion weir and the total cost of the whole structure. It also performs stability analysis.

For surface flow problems in a diversion structure, analysis of a hydraulic jump is required. Commonly in any hydraulic jump, eight variables are involved. These variables are related by six independent equations. If any two variables are known, the remaining six can be worked out by using these six equations mathematically. Since the mathematical solution is complicated, curves are used to avoid large-scale calculations by taking the q (discharge intensity) and H_L (head loss) as known variables (Garg, 2005).

Chapter Three

3 Model Development- Theoretical Consideration

3.1 Organization of the Computer Program

The computer program is equipped with a user interface that is organized in a way that follows the traditional way of manual design for low-head diversion structures with a sloping apron. That is, in a typical design procedure, first, the weir cross-section is designed to pass the desired discharges, and then the longitudinal sections are designed by considering the surface and subsurface flow conditions that depend on discharge intensity, head loss and geology of the site. The structure is also checked for structural stability against sliding, overturning and uplift. Finally, the material quantity needed for construction and the cost of material and construction are calculated. An option to search for the optimum parameter combinations that lead to least overall cost of the structure is also provided.

3.2 Water way and Afflux

The water way and afflux are correlated. If afflux is increased, water way is reduced and vice versa. Hence, a limit placed on maximum afflux shall limit the minimum waterway. It shall be seen that the cost of works as a whole is minimum for a certain water way and afflux. Attempts should therefore be made to attain the most economical combination of these two factors. This can be made by trial and error, generally limiting the maximum value of afflux (Garg, 2005).

In this computer program the decision on the maximum value of afflux is left for the designer (user of the program) since the maximum permissible value of afflux should be decided considering the results of backwater calculations and economic considerations of building upstream wing walls whose design and cost calculation is not covered in this study.

3.3 Dimensioning the Weir Cross Section

A low head diversion structure usually has two portions. One is the under sluice portion and the other is the weir proper portion with a raised crest. The dimensions of under sluice and weir proper portion is usually made such that the under sluice portion is capable of handling 15 to 20% of the high flood discharge. The user of the computer program is left with the choice for the minimum and maximum percentage of proportions of flow in under sluice to the total discharge at high flood flow.

In a particular construction project, there may be preferred widths of bays due to availability of material such as barrage gates and construction equipment. A minimum of one bay is needed to pass water from upstream to downstream both in the under sluice and weir proper portions. Therefore, the program calculates the discharges as

$$Q_u = C_d (L_u - Kn_u H_u) H_u^{\frac{3}{2}} \quad \text{----(3.1)}$$

$$Q_b = C_d (L_b - Kn_b H_b) H_b^{\frac{3}{2}} \quad \text{---- (3.2)}$$

in which, L_u, L_b, n_u and n_b are given by

$$L_u = N_{ub} \times W_{ub} \quad \text{----(3.3)}$$

$$L_b = N_{wb} \times W_{wb} \quad \text{----(3.4)}$$

$$n_u = N_{up} \times 2 \quad \text{----(3.5)}$$

$$n_b = N_{wp} \times 2 \quad \text{----(3.6)}$$

Where,

W_{up} and W_{wp}= Widths of piers at under-sluice and weir-proper portions respectively

W_{ub} and W_{wb} = Widths of bays at under-sluice and weir-proper portions respectively

N_{ub} and N_{wb} = Number of bays at under sluice and weir proper portion respectively

N_{up} and N_{wp}= Number of piers at under sluice and weir proper portion respectively

Q_u and Q_b= discharge in under-sluice and weir proper portions respectively (m^3/s)

H_u and H_b= total head in meters including velocity head in under-sluice and weir proper portion respectively

n_u and n_b= number of end contractions (twice the number of gated bays) in under-sluice and weir proper portion respectively

L_u and L_b= clear water way length in meters in under-sluice and weir proper portion respectively

K= Coefficient of end contraction varying from 0.1 for thick blunt pier noses to 0.04 for thin pointed noises; generally taken as 0.1 in ordinary calculations.

C_d= coefficient of discharge

= 1.7 for broad crested weir and

= 1.84 for sharp crested weir

After calculating (Q_u and Q_b) using equations (3.1) and (3.2) the total combined discharge capacity of the two sections i.e. the under-sluice and the weir proper portions, is checked on whether it can pass the high flood discharge. And the ratio of the under-sluice discharge to the total discharge that is $Q_u/(Q_u + Q_b)$ is checked to be within the maximum and minimum limits set by the user.

If the above criteria are not met the program recalculates the discharges by increasing or decreasing the number of bays at the under sluice and weir proper portions iteratively until the criteria are met within permissible variation. In case of more than one bay, pier will have to be constructed so their effect will be included in the calculations as well. The width of a pier is usually dependent on the structural design. Because the program is not intended to design the structural aspects of the pier, the user must feed the value of the pier width manually. The output of the program for the weir cross-section dimensioning is the discharges capacities of the two portions and number of bays and number of piers at the under sluice and the weir proper portions. The algorithm for handling the weir section dimensioning in the program is presented in Figure (3-1)

3.4 Dimensioning the Longitudinal Section Profiles of the Diversion Structure

The longitudinal profile of the under sluice and weir proper portions of the structure are designed by considering the surface and subsurface flow of water and geology of the permeable media on which the structure is to be built. Designing a longitudinal profile of the structure includes setting the depths of the upstream and downstream piles, lengths and thickness at various points along the structure of the upstream, downstream and sloping aprons as well as setting the top levels of upstream and downstream aprons.

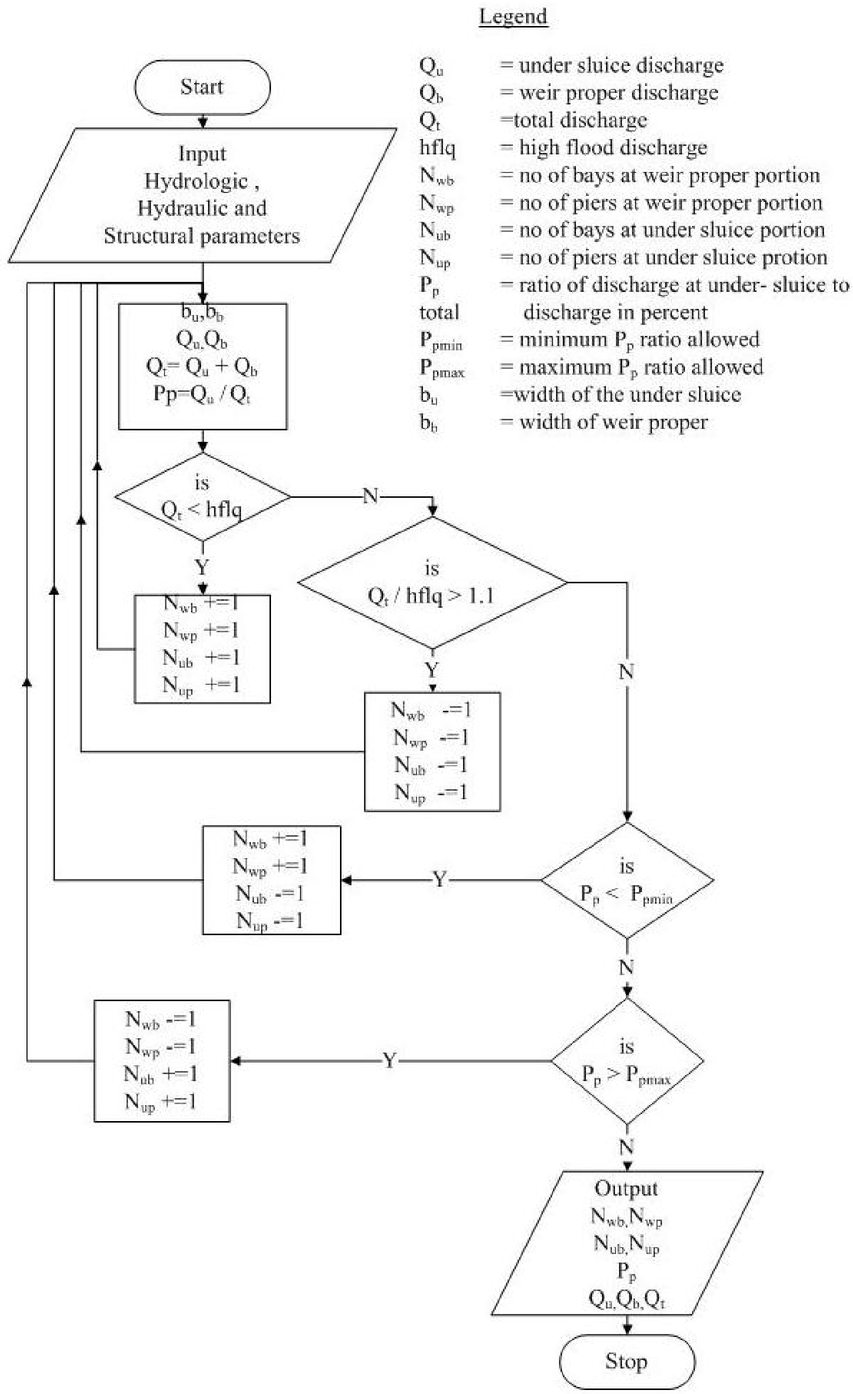


Figure 3-1 Algorithm for designing the weir cross section

The upstream and downstream piles are designed to guard against anticipated scouring action of surface water. Lacy (1939) found the scour depth (R) depends on soil property and discharge intensity as:

$$R = 1.35\left(\frac{q^2}{f}\right)^{\frac{1}{3}} \qquad \text{é . (3.7)}$$

Where,

R= Scour depth

q = discharge intensity

f = Lacyôs silt factor

The downstream pile depth is usually set at 1.5 times R below the high flood level (HFL) and the upstream pile depth is set at 1.25 times R below the high flood level.

$$d_{dbl} = HFL - 1.5 \times R \qquad \text{é . (3.8)}$$

$$d_{ubl} = HFL - 1.25 \times R \qquad \text{é . (3.9)}$$

$$d_d = A_{ld} - d_{dbl} \qquad \text{é . (3.10)}$$

$$d_u = A_{lu} - d_{ubl} \qquad \text{é . (3.11)}$$

Where,

A_{lu} =upstream apron level

A_{ld} =downstream apron level

d_{dbl} =downstream pile bottom level

d_{ubl} =upstream pile bottom level

d_d =downstream pile depth

d_u =upstream pile depth

The exit gradient is the hydraulic gradient of the seepage flow under the base of the weir floor. The rate of seepage increases with the increase in exit gradient. Such an increase would cause ôboilingô of surface soil, the soil being washed away by the percolating water. The flow concentrates into the resulting depression thus removing more soil and creating progressive scour backwards (to upstream). The piping phenomenon can be minimized by reducing the exit gradient. The total length of apron and the downstream pile depth are related in such a way that they act together to keep the exit gradient in safe limit so as to guard against undermining of the structure due to piping. Khosla gives the relation between these parameters:

$$G_E = \frac{H}{d_d \pi \sqrt{\lambda}}$$ é . (3.12)

Where,

G_E = the exit gradient

λ = a function of the relation between the total apron length and downstream pile depth

H = is the maximum head difference anticipated from high flood flow, pond level flow and static water cases

d_d = downstream pile depth

The value of λ for a permissible exit Gradient G_E and a downstream pile depth d_d is therefore calculated from eq. (3.12) as

$$\lambda = \left(\frac{H}{d_d \pi G_E} \right)^2$$ é . (3.13)

Where, λ is given by Khosla to be

$$\lambda = \frac{1 + \sqrt{1 + \alpha^2}}{2}$$ é . (3.14)

From eq. (3.14) the value of α is calculated as:

$$\alpha = \sqrt{(2\lambda - 1)^2 - 1}$$ é . (3.15)

The total apron length is calculated as

$$b_T = \alpha d_d$$ é . (3.16)

Where,

b_T =Total apron length

d_d =Depth of downstream pile

The downstream apron length is set to contain the full length of the hydraulic jump for all flow conditions. Viz. high flood flow and pond level flow with and without concentration of flow and retrogression of river with time considered.

The slope of glacis is set by considering of upstream apron level and downstream apron level and relation between the total apron length required from exit gradient considerations and the downstream apron length.

The thickness of aprons Viz. the upstream apron, sloping apron and the downstream apron are calculated taking the maximum unbalanced head between the subsurface uplift force and the surface flow for different case of flow, Viz. high anticipated flood flow; pond level flood flow and static water condition where the gates are closed with water at pond level upstream. Algorithm is presented in Figure 3-2 for standardizing the selection of the longitudinal section parameters.

The under-sluice and the weir-proper portions of the structure differ in their apron levels, discharge capacities, and widths. These differences in the longitudinal profiles will affect the difference in discharge intensity and the difference in Head loss upstream to downstream. The difference in these two variables will in-turn be reflected in the design of depths of pile, length of apron, thickness of aprons and ultimately on the variation on the design of the two sections.

3.4.1 Subsurface Flow Consideration

Water moves through porous materials in response to gradients in hydraulic head and in accordance with the law of continuity. Due to nature of the distribution of hydraulic head and of hydraulic soil properties, partial differential equations together with appropriate boundary and initial conditions are used to specify particular flow situations (Amerman, 1977).

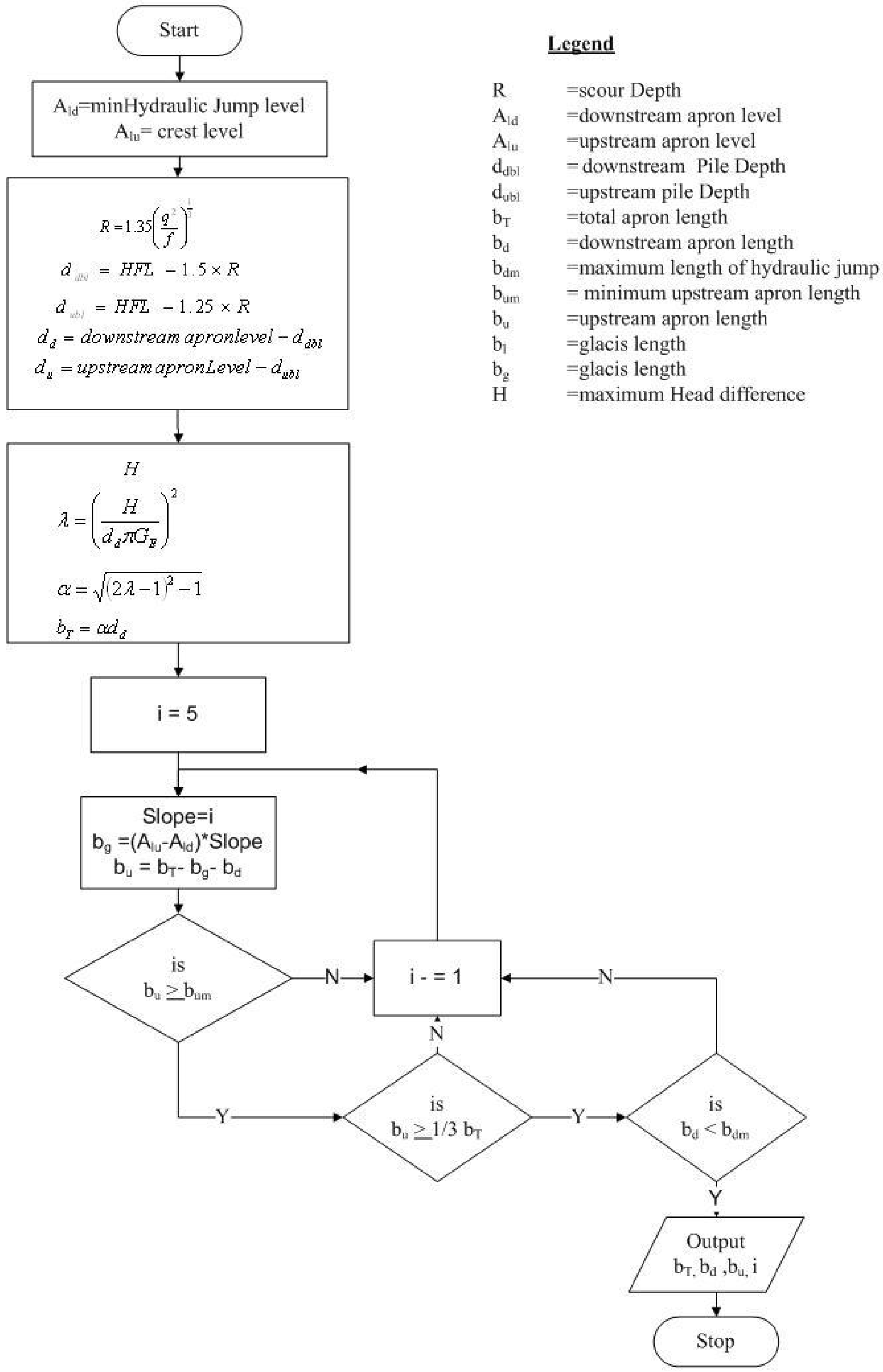


Figure 3-2 Algorithm for calculating the longitudinal profile section parameters

3.4.1.1 The Laplace Equation

The velocity of water flow in porous media was investigated by Darcy (1856) and found to be proportional to the head loss gradient. For anisotropic media, the permeability constant varies in different directions. In this study the method used to predict the subsurface flow of water in such circumstances, as well as in isotropic media, is to solve the Laplace differential equation that describes the subsurface water flow using numerical methods.
From conservation of mass, the continuity equation for steady incompressible fluid flow in a two-dimensional coordinate system is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{---- (3.17)}$$

Where,

u = velocity of flow in the x direction

v =velocity of flow in the yd direction

Introducing a velocity potential $\varphi = Kh$, defined as a scalar function of time and space, velocity components in the x, y directions are

$$u=-\frac{\partial \phi}{\partial x} \qquad v=-\frac{\partial \phi}{\partial y} \qquad \text{---- (3.18)}$$

Combining eq. (3.18) with the continuity equation for flow of incompressible fluid eq. (3.17), gives the Laplace equation.

$$\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}=0 \qquad \text{---- (3.19)}$$

For anisotropic permeable media with permeability coefficient K_x and K_y in the horizontal and vertical directions, respectively

$$K_x\frac{\partial^2 h}{\partial x^2}+K_y\frac{\partial^2 h}{\partial y^2}=0 \qquad \text{---- (3.20)}$$

The Laplace equation represents two sets of curves. One set of curves is called flow lines and the other is called equi-potential lines which represent points of equal head. The present study is concerned about the distribution of uplifting pressure head on the apron of a structure founded on permeable media.

3.4.1.2 Finite Difference Method

The finite difference method is simple and flexible that the non-linearity arising from changes in parameter values, such as the change between confined and unconfined states can be included without difficulty (Amerman, 1977). Therefore, it is selected for solving the Laplace equation that represents the subsurface flow of water under low-head diversion structures.

The finite difference techniques are based upon the approximations that permit replacing differential equations by finite difference equations. These finite difference approximations are algebraic in form, and the solutions are related to grid points. Thus, a finite difference solution basically involves the steps:

- Dividing the solution into grids of nodes
- Approximating the given differential equation by finite difference equivalence that relates the solutions to grid points
- Solving the difference equations subject to the prescribed boundary conditions and/or initial conditions

Desired accuracy is a function of individual preference and the parameters of given situations, but grids that are manifestly too coarse in either space or time are easily recognizable in the results. If Δ_x and Δ_y are too large, equipotentials exhibit irregular shapes. For small Δ_x and Δ_y, the finite difference calculation will take long computation time. Hence, compromise grid spacing should be selected. In this study, a grid spacing of 10cm is selected and found to give satisfactory results both in computation time and in stability of solution.

The first order differential of the velocity potential function is approximated on a fine grid as

$$\frac{\partial \phi}{\partial x} \approx \frac{\delta \phi}{\delta x} = K_x \frac{H_{i,j} - H_{i-1,j}}{\Delta_x} \quad \text{---- (3.21)}$$

$$\frac{\partial \phi}{\partial y} \approx \frac{\delta \phi}{\delta y} = K_y \frac{H_{i,j} - H_{i,j-1}}{\Delta_y} \quad \text{---- (3.22)}$$

The second order differential can be approximated on a fine grid as

$$\frac{\partial^2\phi}{\partial x^2} \approx \frac{\delta^2\phi}{\delta x^2} = \frac{K_x \frac{H_{i+1,j} - H_{i,j}}{\Delta_x} - K_x \frac{H_{i,j} - H_{i-1,j}}{\Delta_x}}{\Delta_x} \qquad \text{---- (3.23)}$$

Similarly,

$$\frac{\partial^2\phi}{\partial y^2} \approx \frac{\delta^2\phi}{\delta y^2} = \frac{K_y \frac{H_{i,j+1} - H_{i,j}}{\Delta_y} - K_y \frac{H_{i,j} - H_{i,j-1}}{\Delta_y}}{\Delta_y} \qquad \text{---- (3.24)}$$

Letting $\Delta_x = \Delta_y = s$ for equal mesh spacing in the x and y coordinate directions

$$\frac{\delta^2\phi}{\delta x^2} = K_x \frac{\frac{H_{i+1,j} - H_{i,j}}{s} - \frac{H_{i,j} - H_{i-1,j}}{s}}{s} \qquad \text{---- (3.25)}$$

$$\frac{\delta^2\phi}{\delta y^2} = K_y \frac{\frac{H_{i,j+1} - H_{i,j}}{s} - \frac{H_{i,j} - H_{i,j-1}}{s}}{s} \qquad \text{---- (3.26)}$$

Where,

ϕ = velocity potential at a point being considered

$\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}$ = First order differentials of potential function in the x and y coordinate directions respectively

$\frac{\partial^2\phi}{\partial x^2}, \frac{\partial^2\phi}{\partial y^2}$ = Second order differentials of potential function in the x and y coordinate direction respectively

K_x, K_y = permeability coefficients of the medium in x and y coordinate directions respectively

s= grid spacing

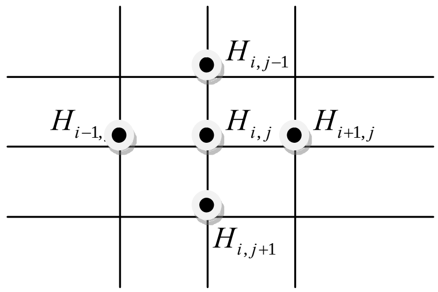


Figure 3-3 Grid for solving the Laplace equation using finite difference method

The Laplace equation (3.20) therefore becomes

$$K_x\left[\frac{\frac{H_{i+1,j}-H_{i,j}}{s}-\frac{H_{i,j}-H_{i-1,j}}{s}}{s}\right]+K_y\left[\frac{\frac{H_{i,j+1}-H_{i,j}}{s}-\frac{H_{i,j}-H_{i,j-1}}{s}}{s}\right]=0 \qquad \text{---- (3.27)}$$

Further, rearranging eq. (3.27), we get

$$\frac{H_{i,j+1}+H_{i,j-1}}{s}+\left(\frac{K_y}{K_x}\right)\left[\frac{H_{i+1,j}+H_{i-1,j}}{s}\right]=2\frac{H_{i,j}}{s}\left[1+\frac{K_y}{K_x}\right] \qquad \text{----(3.28)}$$

The head at a particular point is calculated using the head at neighboring points using the rearranged formula with the ratio of coefficients of the permeability of the porous medium in vertical to horizontal directions as

$$H_{i,j}=\left[\frac{\left(H_{i+1,j}+H_{i-1,j}\right)+\left(\frac{K_y}{K_x}\right)\left(H_{i,j+1}+H_{i,j-1}\right)}{2\left(1+\left(\frac{K_y}{K_x}\right)\right)}\right] \qquad \text{---- (3.29)}$$

Where,

$K_x, K_y =$	permeability of media in x and y direction respectively
$H_{i,j} =$	head at grid point i , j
$H_{i,j-1} =$	head at grid point i, j-1
$H_{i-1,j} =$	head at grid point i-1, j
$H_{i,j+1} =$	head at grid point i, j+1

$H_{i+1,j}=$ head at grid point i+1, j

For the ideal case of $\frac{K_y}{K_x}=1$ equation (3.28) becomes

$$H_{i,j}=\frac{H_{i,j+1}+H_{i,j-1}+H_{i+1,j}+H_{i-1,j}}{4} \qquad \text{---- (3.30)}$$

Thus,equation 3.30 shows that for isotropic condition the potential (head) at a grid-point is simply the mean potential of its neighbors.

To find the solution of equation (3.29), the values of the grid-points on the boundaries is fixed in accordance to the boundary conditions, Viz, Drichilet (constant head) and Neumann (no flow) boundary conditions of the site and the calculation are iterated until successive results agree within desired limits.

The boundary conditions that are considered to solve the Laplace equation in the two dimensional coordinate system are as shown in figure (3-4). The coordinates of their vertices are calculated from the dimensions of the components Viz. Length of aprons, thickness of piles, depth of piles and depth of permeable media. Then these boundary conditions are represented in computer program as

Constant head boundary conditions

Horizontal boundary

0,0	-	X_1,0	$H_{i,j} = H_{c1}$
X_6,V_7	-	X_7,V_7	$H_{i,j} = H_{c2}$

Vertical boundary

0,0	-	0,V_8	$H_{i,j} = H_{c1}$
X_7,V_7	-	X_7,V_8	$H_{i,j} = H_{c2}$

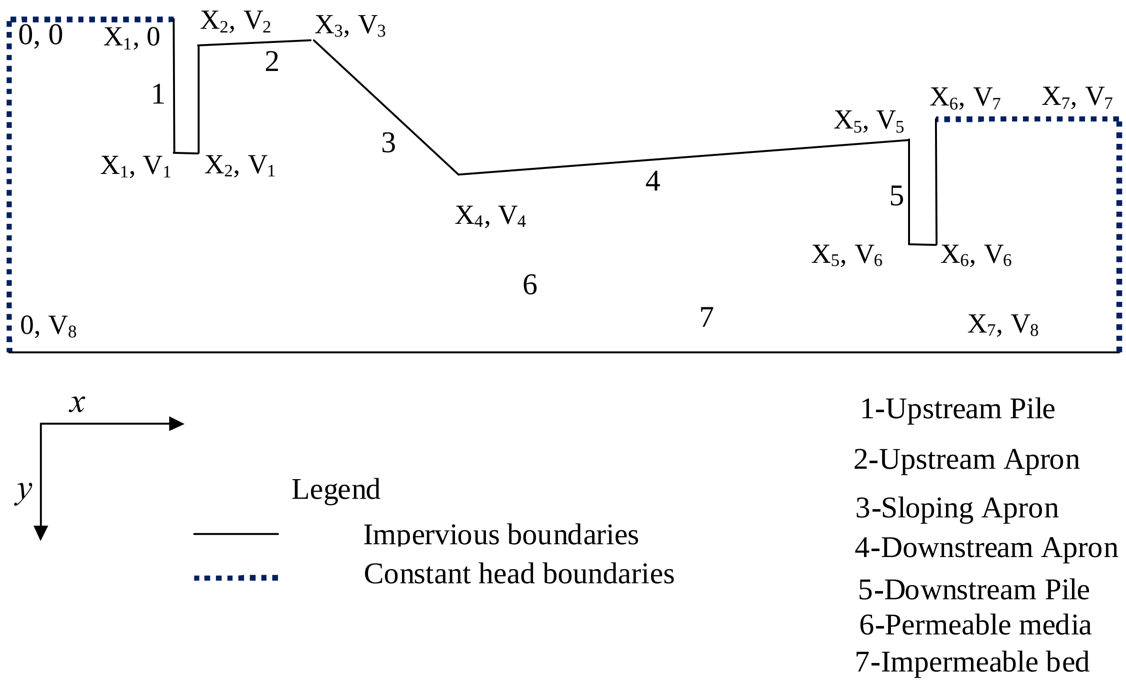


Figure 3-4 A typical longitudinal section of a low-head diversion structure with the structural components, the boundary conditions and coordinate representation of vertices for solving the Laplace equation with finite difference method

Where,

H_{c1} and H_{c2} = heads at upstream and downstream respectively

No-flux or impervious boundary conditions

For no-flux or impervious boundary conditions the velocity of flow across the boundary is zero. This can be represented by making $\Delta H=0$. For ΔH to be zero the head at an impervious boundary should be equal to the head at the adjacent node.

Horizontal boundary

X_1,V_1	-	X_2,V_1	$H_{i,j} = H_{i,j+1}$
X_2,V_2	-	X_3,V_3	$H_{i,j} = H_{i,j+1}$
X_5,V_6	-	X_6,V_6	$H_{i,j} = H_{i,j+1}$
$0,V_8$	-	X_7,V_8	$H_{i,j} = H_{i,j-1}$

Vertical boundary

$X_1,0$ - X_1,V_1		$H_{i,j} = H_{i-1,j}$
X_2,V_2 - X_2,V_1		$H_{i,j} = H_{i+1,j}$
X_5,V_5 - X_5,V_6		$H_{i,j} = H_{i-1,j}$
X_6,V_7 - X_6,V_6		$H_{i,j} = H_{i+1,j}$

Sloping boundary

X_3,V_3 - X_4,V_4		$H_{i,j} = H_{i,j+1}$
X_4,V_4 - X_5,V_5		$H_{i,j} = H_{i,j+1}$

For the gird points representing the inside of the permeable media, the head at a point is calculated using eq. (3.29) which is rewritten here as

$$H_{i,j} = \left[\frac{\left(H_{i+1,j} + H_{i-1,j}\right) + \left(\frac{K_y}{K_x}\right)\left(H_{i,j+1} + H_{i,j-1}\right)}{2\left(1 + \left(\frac{K_y}{K_x}\right)\right)} \right] \quad \text{---- (3.31)}$$

One of the most successful iterative methods for the solution of a system of algebraic equations is the SOR (successive over relaxation) method (Moin, 2001). In case of large quantities of data, it is useful to implement the SOR by introducing a so-called Over-Relaxation parameter θ . Over relaxation technique helps for rapid convergence to the final solution by amplifying each step towards the final solution when $0 < \theta < 1$.

In this method the process of finding the next solution is represented by

$$\phi_{n+1}(x, y) = \phi_n(x, y) + (\theta + 1)\Delta_n \quad \text{---- (3.32)}$$

Where, $0 < \theta < 1$ controls the degree of over relaxation and Δ_n is given by

$$\Delta_n = \frac{1}{4}(\phi(x+s, y) + \phi(x-s, y) + \phi(x, y+s) + \phi(x, y-s)) - \phi_n(x, y)$$

---- (3.33)

$\phi_n(x, y) =$ head at the given coordinate position in a cartesian coordinate system

n= represents the iteration number

$\theta =$ over relaxation parameter

= taken to be 0.5

$\Delta_n =$ the difference between the head at a point in the n[th] iteration and the average head of its neighbouring points

s= grid spaceing

The above representations are solved by iteration until subsequent solutions are sufficiently close. The algorithm for solving the Laplace equation by finite difference technique using the boundary conditions as input is given in figure 3-5.

3.4.2 Surface Flow Consideration

3.4.2.1 Hydraulic Jump

Hydraulic jump of water takes place when a super-critical flow changes into a sub-critical flow. This is generally accompanied by large-scale turbulence, dissipating most of the kinetic energy of super-critical flow. Such a phenomenon may occur in a canal below a regulating sluice, or at a place where a steep channel slope suddenly turns flat. In hydraulic jump, eight variables related by six independent equations are involved. These are,

$$y_1 y_2 (y_1 + y_2) = \frac{2q^2}{g} \quad \text{---- (3.34)}$$

$$H_L = \frac{(y_2 - y_1)^3}{4 y_1 y_2} \quad \text{---- (3.35)}$$

$$E_{f1} = y_1 + \frac{V_1^2}{2g} \quad \text{---- (3.36)}$$

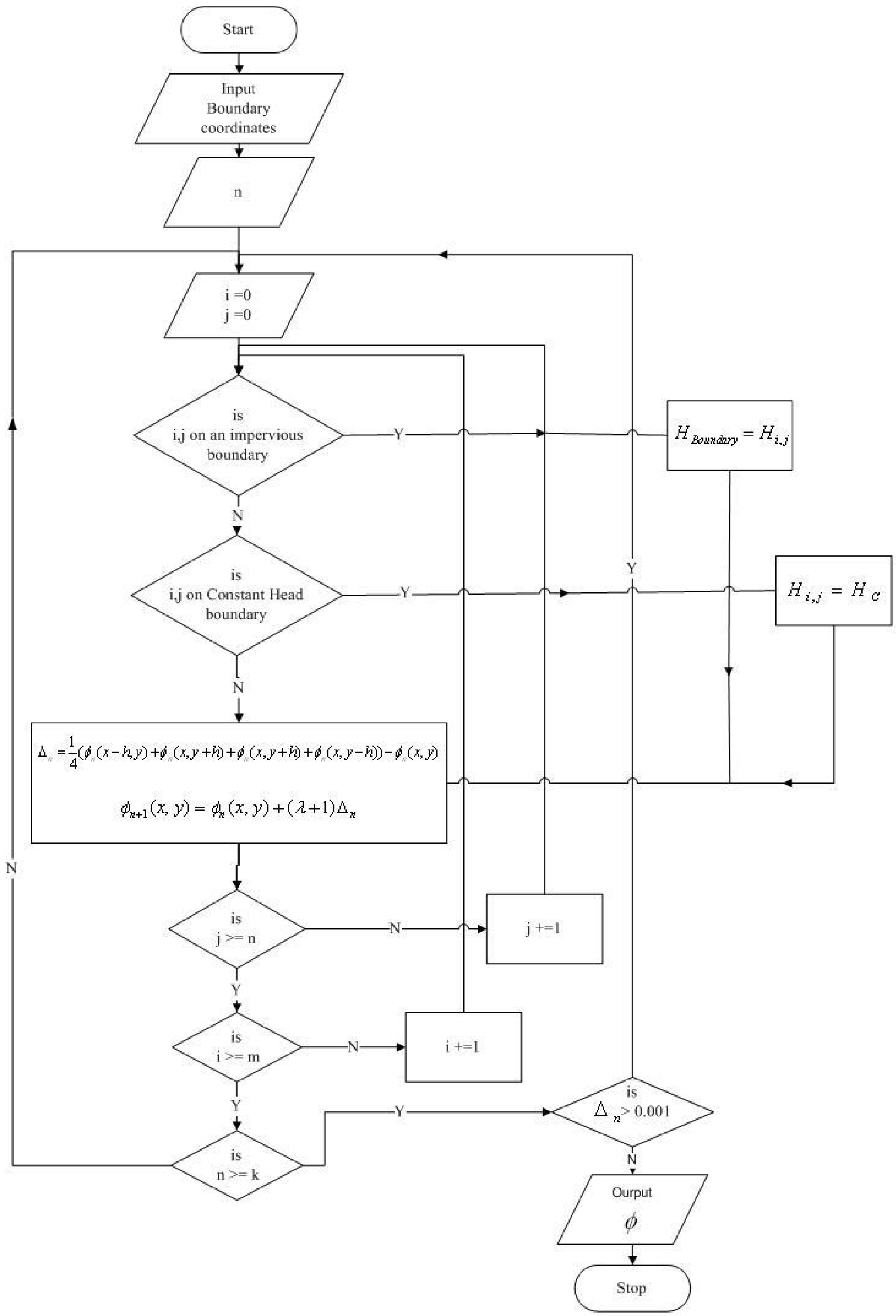


Figure 3-5 Algorithm for solving finite difference problem with over-relaxation technique

$$E_{f2} = y_2 + \frac{V_2^2}{2g}$$ ---- (3.37)

$$V_1 = \frac{q}{y_1}$$ ---- (3.38)

$$V_2 = \frac{q}{y_2}$$ ---- (3.39)

Where,

y_1 =depth of water before jump (initial depth)

y_2 = depth of water after jump (sequent depth)

v_1 =velocity before jump

v_2 =velocity after the jump

q =discharge intensity (discharge per width)

E_{f1}, E_{f2} = specific energies before and after the jump respectively

H_L = the head loss due to jump

g = acceleration due to gravity

The following mathematical equations are used to evaluate y_1 and y_2 using known values of q and H_L.

Compute y_c= critical depth= $\sqrt[3]{\frac{q^2}{g}}$ ---- (3.40)

Let now express $\frac{y_1}{y_c} = X$; $\frac{y_2}{y_c} = Y$; and $\frac{H_L}{y_c} = Z$ ---- (3.41)

Relations between Z and X; and Z and Y are worked out as (Garg, 2005)

$$Z = \frac{-X^6 + 20X^3 + 8 - (X^4 + 8X)^{3/2}}{16X^2}$$ ---- (3.42)

$$Z = \frac{-Y^6 - 20Y^3 - 8 - (Y^4 + 8Y)^{3/2}}{16Y^2} \quad \text{---- (3.43)}$$

For different values of X and Y, values of Z can be tabulated, and curves X-Z and Y-Z plotted, to read value of X and Y for known value of Z. (Garg, 2005)

Swamee (2004) gave an approximate solution to Y-Z curves:

$$Y = 1 + 0.93556\, Z^{0.368} \text{ For Z < 1} \quad \text{---- (3.44)}$$

$$Y = 1 + 0.93556\, Z^{0.24} \text{ For Z > 1} \quad \text{---- (3.45)}$$

With known values of Z from (3.41) and Y form (3.44) or (3.45), the value of X (y_1/y_c) is computed from the equation

$$Z = \frac{(Y - X)^3}{4XY} \quad \text{---- (3.46)}$$

Eq. (3.46) is an implicit equation for X and can be solved using Newton-Raphson numerical method after putting it in a solvable form.

$$F(X) = 4XYZ - (Y - X)^3 \quad \text{---- (3.47)}$$

When differentiated in X (3.47) it becomes

$$DF(X) = 4YZ + 3(Y - X)^2 \quad \text{---- (3.48)}$$

The program computes the value of X from (3.47) and (3.48) using the Newton-Raphson numerical method using the computer program algorithms shown in Figure 3-6.The values of E_{f1} and E_{f2} are calculated by using the values of X and Y as

$$\frac{E_{f1}}{y_c} = X + \frac{1}{2X^2} \quad \text{---- (3.49)}$$

$$\frac{E_{f2}}{y_c} = Y + \frac{1}{2Y^2} \quad \text{---- (3.50)}$$

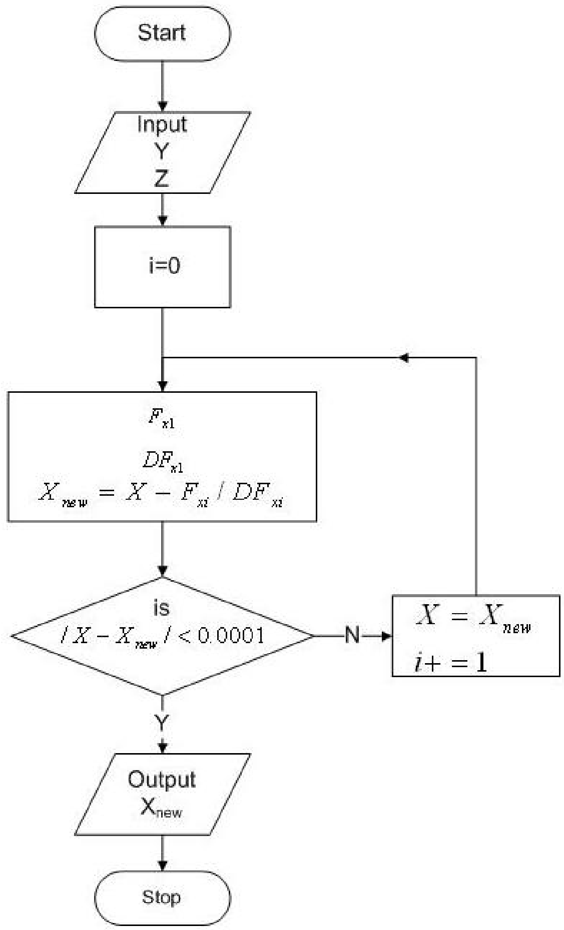


Figure 3-6 Algorithm for calculating the value of X (y_1/y_c)

3.4.2.2 Position of the Jump

The discharge per unit width when water is passing over the crest of a work with a certain head is

$$q = C_d H^{\frac{3}{2}} \qquad \text{---- (3.51)}$$

Where, H is the head measured from the total energy line to the crest and C_d is the coefficient of discharge.

Knowing H, the level of upstream TEL (Total Energy Level) is known. For the given discharge, the depth of water on the downstream side is known from gauge discharge curves of the channel. This fixes TEL on the downstream. The difference in the levels of upstream and downstream TEL gives the H_L (head loss).

The level at which the jump will form is obtained, and hence the position of jump is fixed by subtracting E_{f2} (specific energy after jump) from downstream TEL. The value of E_{f1} (specific energy before jump) is calculated as:

$$E_{f1} = H_L + E_{f2} \qquad \text{---- (3.52)}$$

The depths of water y_1 corresponding to specific energy E_{f1} is calculated by the numerical method called section method using the algorithm shown in Figure 3-7.

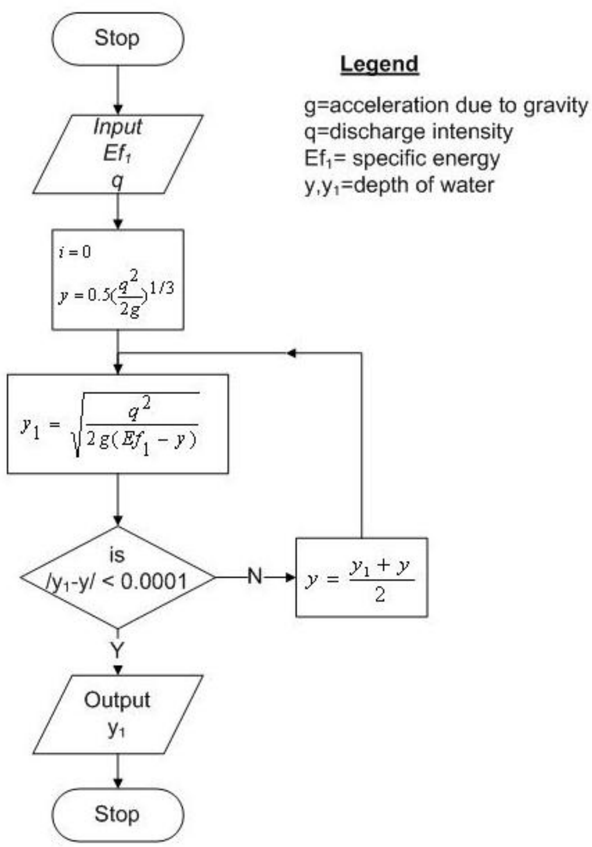


Figure 3-7Algorithm for calculating depth of water from a given specific energy

3.4.2.3 Pre-jump Water Surface Profile

For the different points on the glacis from upstream to the location of jump, the specific energy E_{f1} will vary. E_{f1} is the difference between TEL and glacis level at a point being considered. Since the glacis level is different at various points along the length of the structure, E_{f1} also varies and shall go on increasing (figure 3-8). For these different values of E_{f1}, corresponding values of y_1 are calculated using the algorithm given in figure 3-7.

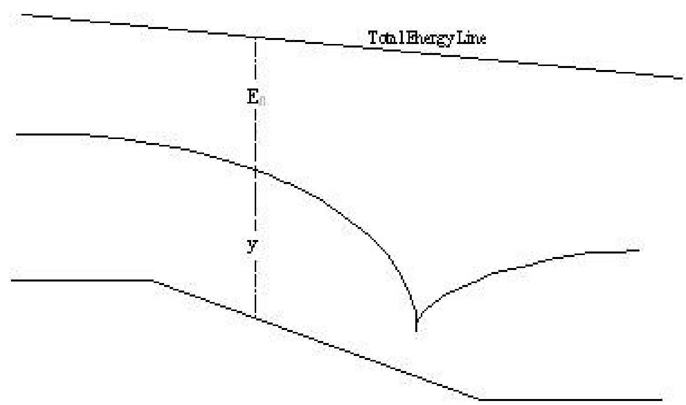


Figure 3-8 Pre-jump depth of water from total energy level

These values of y_1 go on reducing till the point of jump is reached. These values are plotted over the glacis, and hence, water surface profile before the jump is plotted. The pre-jump water surface profile differs for various flow conditions; this variation is handled by using the algorithm shown in figure 3-9.

3.4.2.4 Post-jump Water Surface Profile

To plot the water surface profile after the location of the jump, it is necessary to know the incoming Froude number, F_r.

$$F_r = \frac{q}{\sqrt{gy^2_1}} \qquad \text{---- (3.53)}$$

Knowing q and y_1, Fr is determined from equation (3.53). Graphs are available that relate (x/y_1) with (y/y_1) for different values of Fr. Taking different points beyond the location of the jump along the profile(figure 3-10), different values of x and hence that of x/y_1 is tabulated, corresponding to the value of x/y_1 for a fixed Fr. Different values of x and y are known, where (x, y) in any point in the direction of flow with respect to the point location of the jump as origin. Hence the water surface profile after the jump point is plotted. The method adopted to incorporate the post jump profile calculation in the computer program is to feed the data from available graph (Garg, 2005) and to develop a mechanism for calculating the depth of water using the Froude number for a specific case as input using the algorithm presented in Figure 3-11.

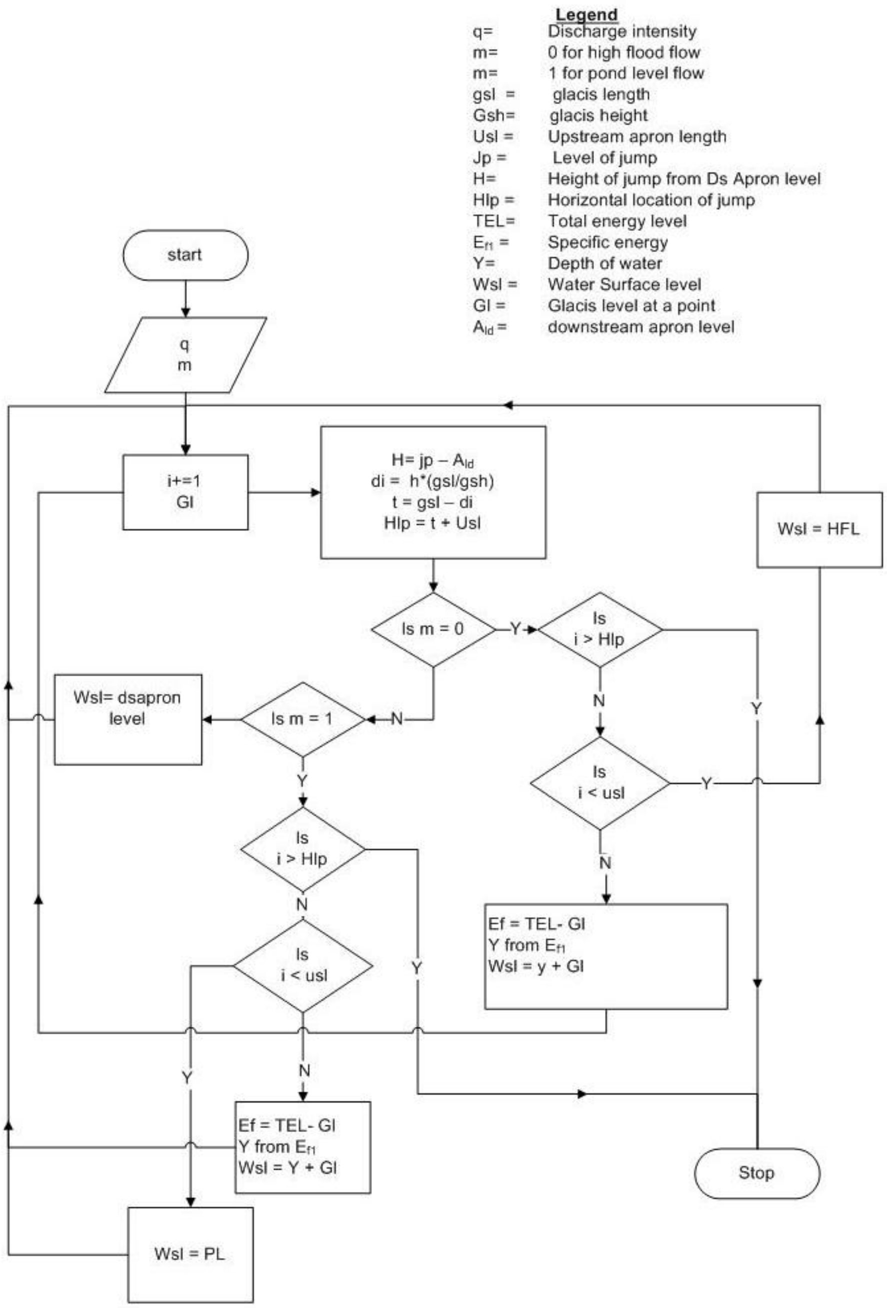


Figure 3-9 Algorithms for calculating the pre-jump water surface profile

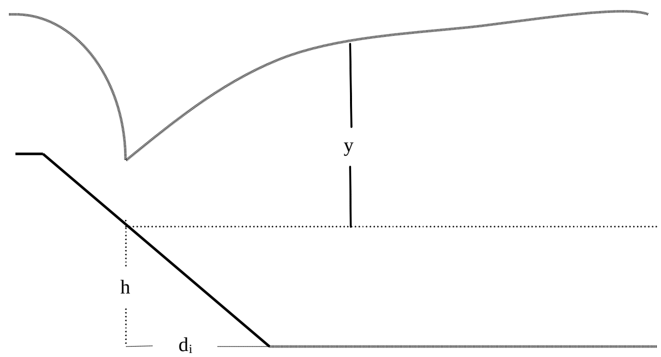


Figure 3-10 Post jump water surface profile

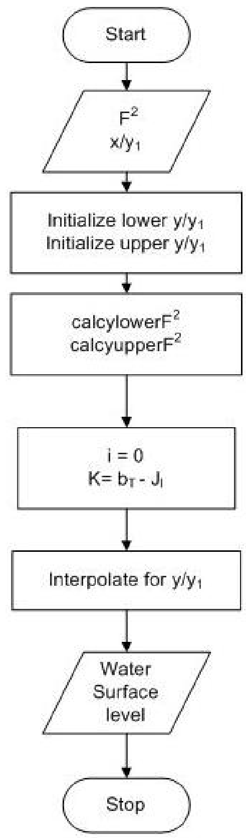


Figure 3-11 Algorithm for post-jump water surface profile calculation

3.4.3 Stability of Structure

After designing the components of the structure based on the surface and subsurface flow considerations, the computer program designs the structure for stability. First, the structure is designed for stability against uplift; this decides the thickness of aprons at various points along the longitudinal section of the structure based on the subsurface flow calculations. The forces and moments acting on the corresponding structure are then calculated and the structure is checked for its stability against overturning and sliding. If the structure is unsafe, the program makes alterations in the depths of aprons of structure. The forces that act on a diversion structure, specifically on the under sluice section, are shown in figure 3-12.

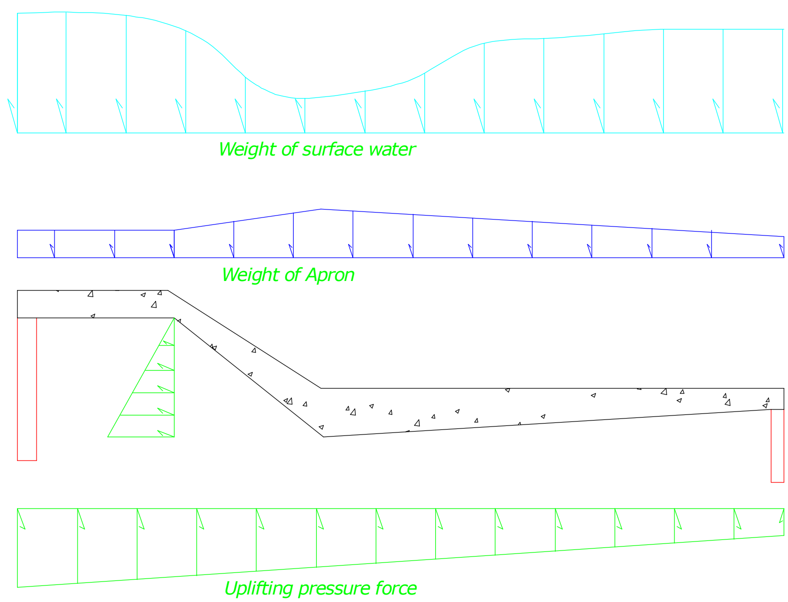


Figure 3-12Free hand drawing of forces acting on the under-sluice section of a typical diversion structure

3.4.3.1 Stability against Uplift

Uplift force exists on the structure because of the subsurface flow of water underneath it. This uplifting pressure head decreases from upstream to downstream. To insure the stability against uplift the necessary apron thickness at different points along a longitudinal section are provided. For this, first the maximum unbalanced head between the uplifting pressure head and depth of surface water above the apron is calculated at points along the structure from high flood flow, pond level flow and static water cases. The hydraulic grade line and the depth of surface water for the corresponding flow cases are calculated to find the maximum unbalanced head at a point from the three flow cases. Then the necessary thicknesses are calculated from the density of apron material.

$$t = \frac{h}{G-1} \qquad \text{---- (3.54)}$$

Where,

t= Thickness of apron at a point

h= The unbalanced head between the uplifting pressure head and surface water depth

G= Density of construction material for apron

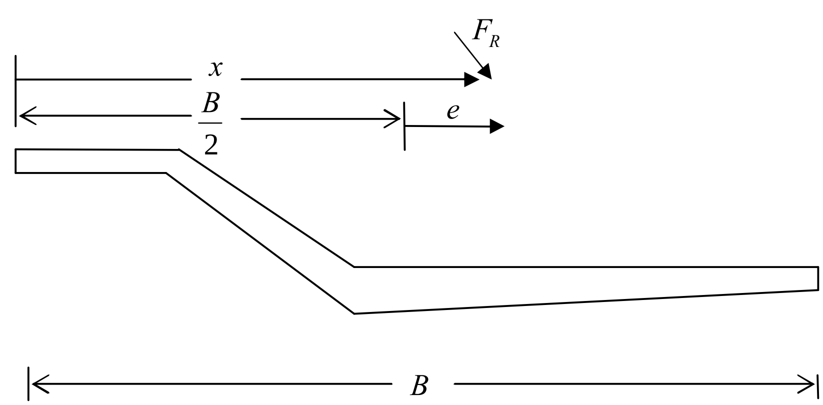
= 2.24 for concrete

3.4.3.2 Stability against Overturning

Stability against overturning is the next important factor to take into consideration. In big structures such as most low-head diversion structures, where the forces that act on them are distributed. It is necessary to keep the stabilizing moment more than the destabilizing moments.

Since unpredictable situation are likely to occur and cause the toppling moment to exceed the balancing one, a factor of safety of 1.5-2.0 is usually applied for safety against overturning (Baban, 1995)

In order to avoid lifting up the structureĝs heel and tension occurrence at the base, the resultant force must pass through the middle third of the structureĝs base



$$e < \frac{B}{6} \qquad \text{---- (3.55)}$$

$$e = /\frac{B}{2} - X / < \frac{B}{6} \qquad \text{---- (3.56)}$$

Where, $$x = \frac{\sum M}{\sum V_f} \qquad \text{---- (3.57)}$$

$e =$ eccentricity

$\sum M =$ summation of all moments about the structures toe

$\sum V_f =$ summation of vertical forces excluding the base reaction

$X =$ distance of the resultant of the forces from the toe

$B =$ width of the weir base

3.4.3.3 Stability against Shear and Sliding

The aprons of the structure and the weir body are considered for stability against shear and sliding. The structure may slide in the flow direction if there is not enough grip between the base and the foundation. To prevent this happening, the vertical forces are checked to be adequate, compared to the horizontal forces, to supply static friction that would keep the structure intact in its place. The US bureau of reclamation, as quoted by (Baban, 1995) suggests 0.35 for concrete structures on common soils. i.e.

$$\frac{\sum V}{\sum H} = 0.35 \qquad \text{---- (3.58)}$$

Where,

$\sum V =$ Sum of external vertical forces

$\sum H =$ Sum of external horizontal forces

3.4.4 Optimal Design (Least Cost Design)

Many design problems are usually associated with optimization. Optimization of an engineering problem does not normally seek a pure minimization of cost but a feasible solution that compromises cost and design. Optimization for least cost design of low-head diversion structures is a complex problem due to the existence of non-linear functions

involved Viz. relation between the exit gradient, the downstream pile depth and total apron length, the non-linear cost increase in construction of pile and apron with depth, etc. Therefore, it is difficult to make distinction between local optimal solutions and rigorous optimal solutions that may exist. Not to treat the former as actual solutions to the original problem, the optimization technique implemented in this computer program is to search for the least cost design from all possible design combinations of component parameters.
The optimization for least cost design of low-head diversion structures is done using

- The cost of construction of component materials as input
- The minimum parameter values for each component from consideration of surface and subsurface flow consideration and combinations of the component parameters that are needed to mitigate one or more destabilizing forces as constraints
- The total construction cost is the function to be minimized

The actual precision needed and/or is achievable for construction is also taken into consideration while searching for parameter values that give the least overall cost of the structure. Moreover, the possibility of the actual site condition to limit the parameters values of the structure such as, limit to the possible pile depth due to shallowness of the permeable bed is also taken into consideration.

The parameters that are manipulated to search for least cost design of structure are

- The downstream pile depth
- The upstream pile depth
- The downstream apron length

The minimum values of these parameters are calculated from scour and hydraulic jump considerations. These parameters can safely assume a value above their minimum. Due to the nature of diversion structures built on permeable media and their interaction with their environment, a change in one or more of these parameters is reflected in other parameters of the structure.

An increase in the downstream pile depth affects the total length required to limit the exit gradient. An increase in the upstream pile depth reduces the magnitude of the residual uplift pressure head and hence the thickness of aprons required to counter the uplifting force. The downstream apron length has a complex relation with the total apron length due to the involvement of slope of glacis, minimum upstream apron length and stability against overturning. The algorithm for searching for least cost design of low-head diversion structures is given in Figure 3-14.

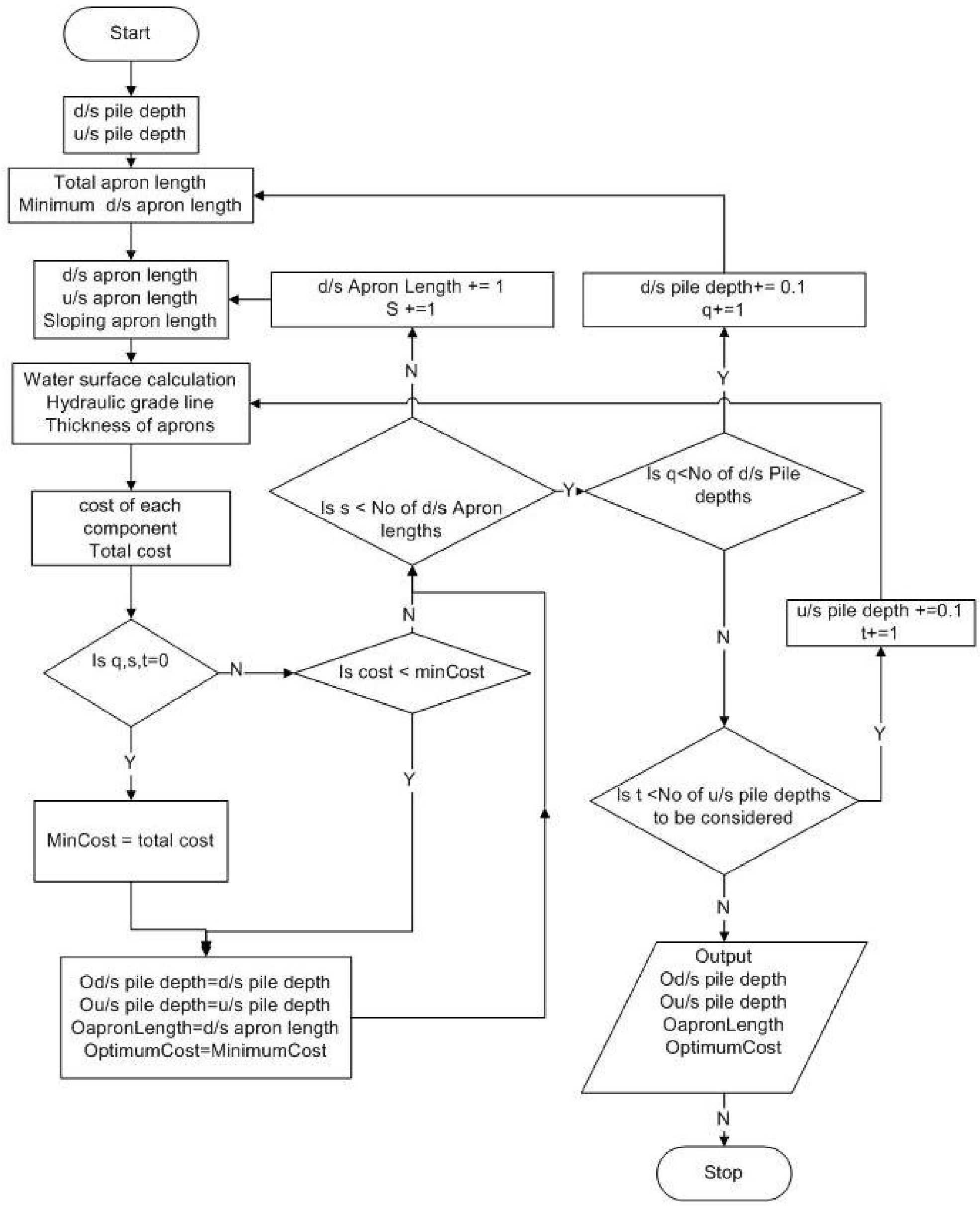


Figure 3-14 Algorithms for optimization

3.5 Model Testing and Validation

Testing and validation of the program was done by comparing the output results for sample problems with assumption of isotropic and deep permeable media, with that solved by hand calculation using Khoslaûs ñmethod of independent variablesò. And after validation of the program, further analysis was made to see

- Effect of anisotropy in the distribution of uplifting pressure head
- Effect of depth of the permeable foundation on the distribution of uplifting pressure head

Further, the capability of the program for designing and optimizing the parameters of the components of the structure based on cost consideration, and a scenario analysis on the effects of anisotropy and depth of permeable media on the material quantity required for construction are demonstrated.

3.5.1 Model Validation

Sample Problem 1

Here a simple low-head diversion structure is considered for the validation purpose. The magnitude and distribution of the uplifting pressure head is required to be calculated both with Khoslaûs method of independent variables and with finite difference method. The parameters of the structure considered are:

Table 1 Input used for sample problem 1

Parameters	Value
Total apron length	30m
Upstream pile depth	6m
Downstream pile depth	8m
Upstream head	100%
Downstream head	0%
Depth of permeable media	50m*

*A 50m depth of permeable media is considered as sufficient enough not to affect the shape of the flow net, satisfying Khoslaûs assumption of infinite depth.

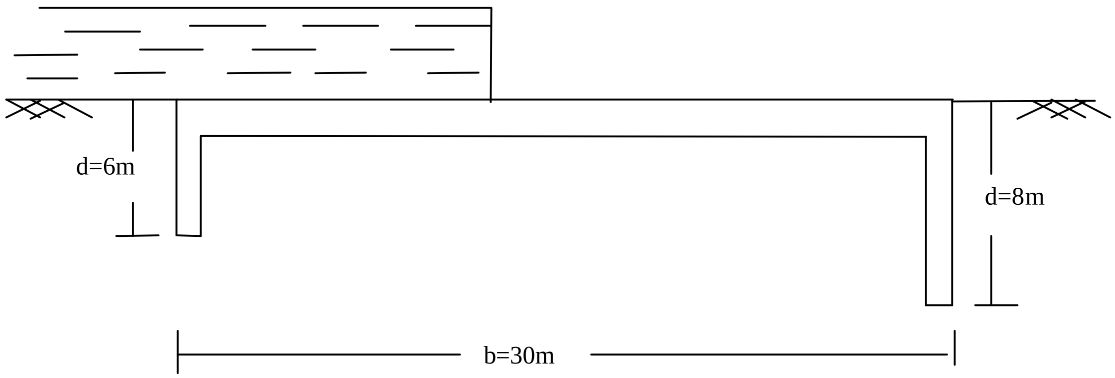


Figure 3-15 Drawing of structure for sample problem 1

Sample Problem 2

A sample problem from Garg(2005) is to be solved using the developed software program using Khoslaôs method of independent variables and finite difference method.

Table 2 Input river characteristics

Parameters	value
Reduced Level	257 m
Permissible Afflux	1 m
Silt Factor	1
Safe Exit Gradient	1/6
Retrogression of river	0.5 m
Discharge concentration	20 %

Table 3 Input Flood characteristic parameters

Parameters	value
High Flood Level	262.2 m
High Flood Discharge	8100 m^3/s
Pond Level	260.6 m
Pond Level Discharge	2690 m^3/s

Required is a complete hydraulic design of the under sluice section on the basis of hydraulic jump theory, Khoslaôs theory and finite difference method.

3.5.2 Model Analysis

A scenario analysis is made to see the effects of depth of permeable media and anisotropy on the uplifting pressure head distribution on the apron of the structure in sample problem 1 and on the design of structure for stability in sample problem 2.

For sample problem 1

Depths of permeable media that were considered for analysis are 10, 20, 30, 40 and 50m.

Analysis for the effect of anisotropy considers permeability ratios in the horizontal to vertical directions as 20, 10, 5, 2 and 1

For sample problem 2, a scenario analysis is made taking the value of horizontal to vertical ratio of permeability and depth as given in table-4.

Table 4 Permeable media properties used for analysis of sample problem-2

Scenario	Ratios of horizontal to vertical permeability $(\frac{K_x}{K_y})$	Depth
1	1	25m
2	10	15m

3.5.3 Optimal Design (Least Cost Design)

The optimum design parameters are also calculated to exemplify the computer programôs capability. Optimization of the design of diversion structures is to be done using a secondary data of construction cost for components of the structure. The cost of construction for concrete structures is dependent on the depth at which it is to be constructed, a sample data is used based on average cost in Ethiopia in October 2009.

Table 5 Input cost of material and construction of aprons considered for optimizing sample problem-2

Depth interval	Cost of material and construction in birr.
d > 3m	1400
2 < d < 3m	1300
1 < d < 2m	1200
d < 1m	1100

(Teshome, 2009 Pers. Comm.)

Table 6 Input cost of material and construction of pile considered for optimizing design of sample problem-2

Depth interval	Cost of material and construction in birr.
d >10m	1600
7 < d < 10m	1500
5 < d < 7m	1400
3 < d < 5m	1300
d < 3m	1200

(Teshome, 2009 Pers. Comm.)

Chapter four

4 Results and Discussion

The results of calculations using the computer program are presented in this chapter. Hand calculation using Khoslaǿs method of independent variables is used in sample problem one for comparison purpose. The result of finite difference method calculations using varying depths of permeable media and horizontal to vertical permeability ratios is also presented. A comparison of design with method of independent variables of Khosla and that with finite difference method is also presented along with a scenario analysis for varying permeable media property both in depth and horizontal- to-vertical permeability ratios.

4.1 Design with Khosla’s Method of Independent Variables and Finite Difference Method

4.1.1 Sample Design One

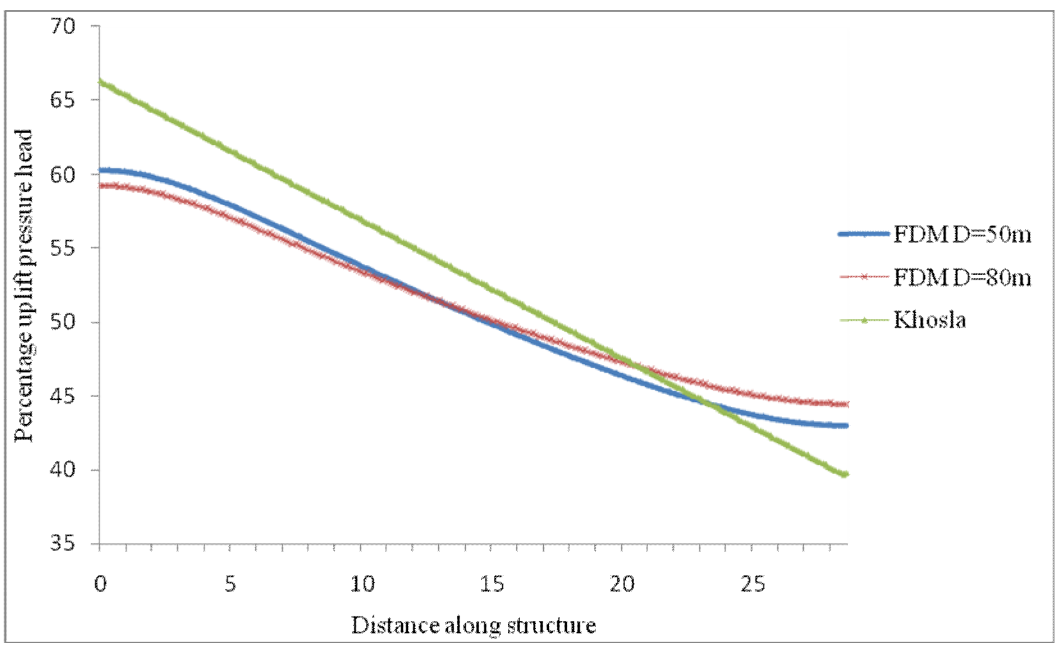


Figure 4-1 Results of uplift pressure head distribution calculated with Khosla's and finite difference method

The above figure shows the results of calculation for residual uplifting pressure heads on the apron of sample problem-1 from upstream pile to downstream pile in percentage of the total head difference. Khoslaǿs method of independent variables predicts higher value of uplift pressure at the upstream points, and lesser value of uplift pressure at downstream point compared to that found using the finite difference method. The design procedure with the method of independent variables adapts a linear distribution of uplifting pressure head. The finite difference calculation reveals the true nature of the uplifting pressure head distribution.

4.1.1.1 Effect of Depth of Permeable Media on Uplift Pressure Head Distribution

Various depths were considered to analyze the effect of depth of the permeable media on the magnitude and distribution of the residual uplift pressure head. The result is shown in figure 4-2.

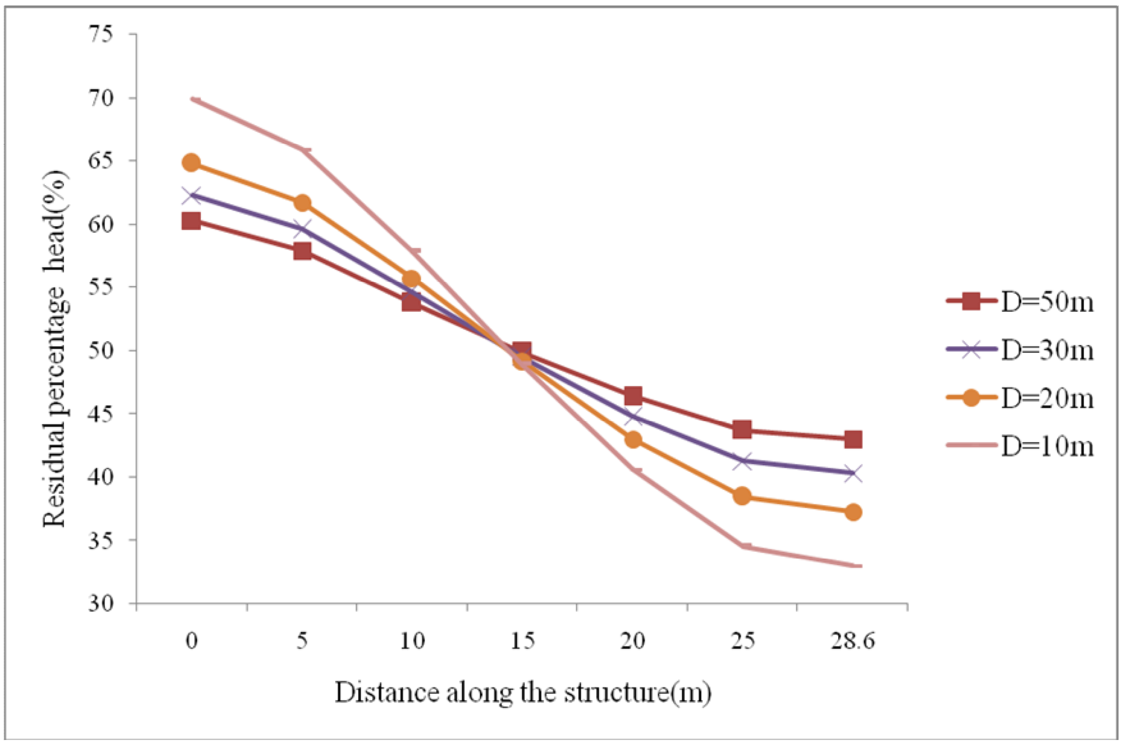


Figure 4-2 Results of uplift pressure head distribution for a structure founded on various depths of isotopic permeable media

The results shown in Figure 4-2 indicate that the percentage of residual uplift pressure head decreases for points at downstream of the structure as the depth of permeable media decreases. The percentage of residual uplift pressure head increases for points at upstream of the structure with decrease in depth of permeable media.

Solving the uplift-pressure-head distribution by finite difference method helps for a more accurate design when designing structures built on shallower permeable media. Though the uplift pressure head at upstream of structures is greater for shallow permeable media than that for a deep permeable media, there may be no need to increase the apron thickness depending on existence of surface water, which helps counter the uplift pressure with itös weight. .

4.1.1.2 Effect of Anisotropy on Uplift Pressure Head Distribution

The results of the analysis for the effect of anisotropy are as presented in figure (4-3). The analysis was done for a 30m deep foundation with various ratios of permeability in the horizontal to that in the vertical direction.

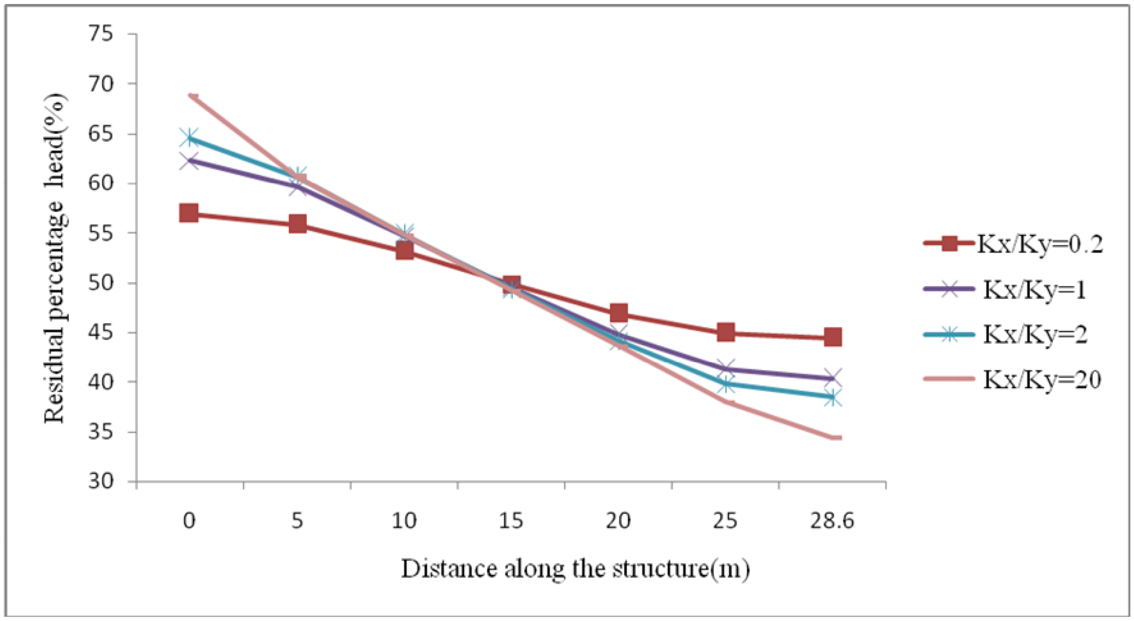


Figure 4-3 Results of uplift pressure head distribution solved using finite difference method for an anisotropic permeable media with depth of 30m and various ratios of horizontal to vertical permeability

The results indicate that the percentage of residual uplift pressure head decreases for points at downstream points of the structure as the ratio of horizontal to vertical permeability increase. This implies more economical design may be achieved by solving the subsurface flow under engineering structures founded on anisotropic soil of high horizontal to vertical permeability ratio, with the finite difference method.

In general, from the analysis on effects of depth of permeable media and anisotropy on residual uplift pressure, it was found that the variation in residual pressure head along the aprons of engineering structures built on permeable media is not linear from upstream to downstream. Moreover, design that is more economical and safe can be arrived at by using finite difference method for solving the subsurface flow under diversion structures. This is more pronounced for the case of diversion structure built on shallow anisotropic permeable media with high horizontal permeability to vertical permeability ratio.

4.1.2 Sample Design two

Weir cross section

Design output of weir section for under sluice portion

Number of bays	4
Number of piers	3
Under sluice Span	67.5 m

Design output of weir section for weir proper portion

Number of bays	30
Number of piers	29
Barrage Span	418 m

Total Span	488.5 m
Crest Level of Under sluice	257 m
Crest Level of Barrage	258.3 m

Table 7 Hydraulic Calculation Output for Under-Sluice Portion

No.	Parameter	Condition (1a)	Condition (1b)	Condition (2a)	Condition (2b)
1	Discharge q (m^3/s/m)	27.667	33.200	11.612	13.934
2	Upstream Water Level(m)	263.200	263.200	260.600	260.600
3	Downstream Water Level(m)	262.200	261.700	260.200	259.700
4	Upstream TEL(m)	263.422	264.252	260.822	261.065
5	Downstream TEL(m)	262.422	261.922	260.422	259.922
6	Head Loss(m)	1.000	2.330	0.400	1.143
7	Ef_2(m)	7.506	9.100	4.099	5.028
8	Lowest Level of jump formation(m)	254.916	252.822	256.323	254.894
9	Ef_1=Ef_2 + H_L (m)	8.506	11.430	4.499	6.171
10	y_1 corresponding to Ef_1(m)	2.562	2.509	1.518	1.447
11	y_2 corresponding to Ef_2(m)	6.615	8.280	3.555	4.550
12	Length of concrete floor required(m)	20.265	28.854	10.185	15.512
13	Froud no.	2.155	2.666	1.982	2.555

Condition (1a) = High flood flow without concentration and retrogression
Condition (1b)= High flood flow with concentration and retrogression
Condition (2a)= Pond level flow without concentration and retrogression
Condition (2b)= Pond level flow with concentration and retrogression

Table 8 Longitudinal Profile Calculation Output for Under-Sluice Bay Portion

Parameters	value
Scour Depth	10.69 m
RL of Bottom of downstream pile	245.66 m
Depth of downstream Pile	7.10 m
RL of bottom of upstream pile	249.80 m
Depth of upstream pile	7.20 m
Length of downstream apron	36.67 m
Length of upstream apron	5.70 m
Total length	55.00 m
Length of glacis	12.60 m
Slope of glacis	3.00 H:1V
Reduced level of downstream apron	252.80 m
Reduced level of upstream apron	257.00 m
Maximum static head	7.80 m
Head on Under-sluice	6.42 m
Q Under-sluice	1,504.57 m^3/s

4.1.3 Model Analysis Using Finite Difference Method for Sample Problem 2

Design drawings of the structure in sample problem 2 are shown in figure 4-4. The design drawings, viz. one designed with khoslaös independent variable method and the other with finite difference method are super-imposed to show the similarities and differences in apron thickness required according to the two methods.

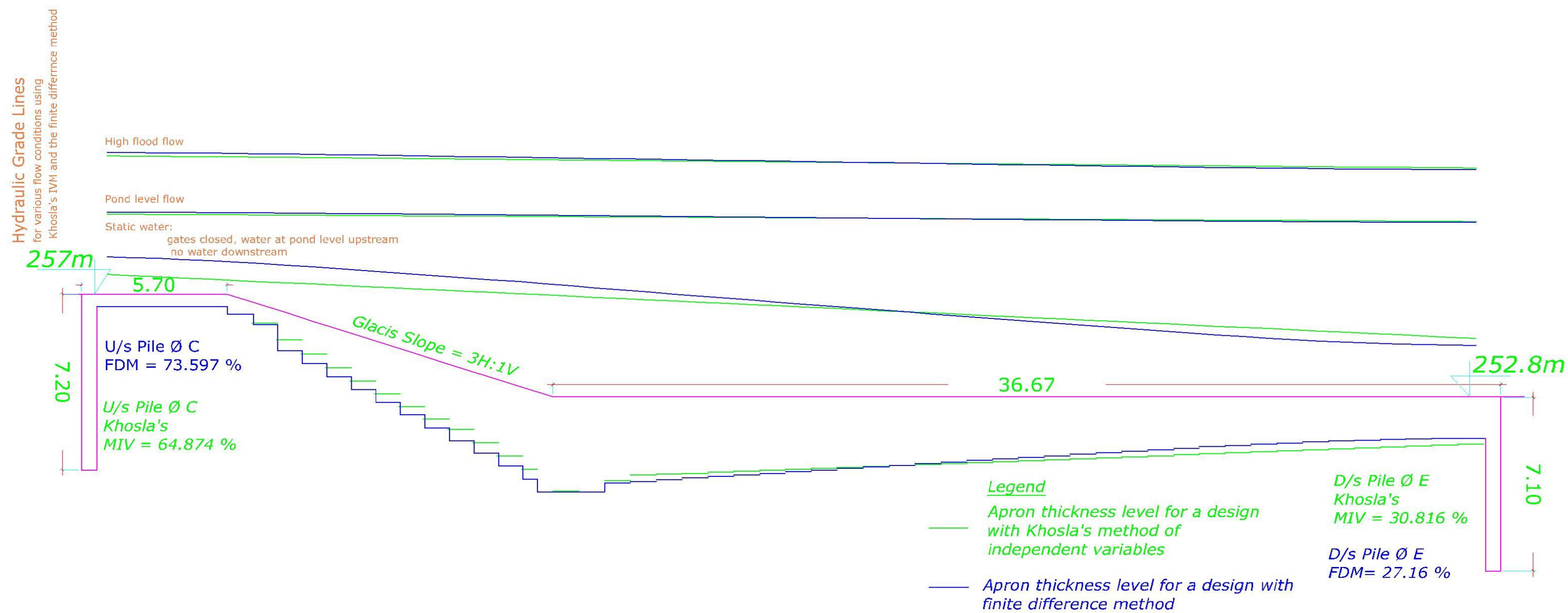


Figure 4-4 Super-imposed Design drawings of sample problem-2 for a structure built on an isotropic 20m deep permeable media designed using Khosla's IVM and finite difference methods

Table 9 Model analysis on the volume of construction material required for varying anisotropic and depth conditions of permeable foundation media using finite difference method

Component of structure	Depth (m)	Volume of concrete material required (m^3) according to the design with finite difference method*	
		Scenario 1[a]	Scenario 2[b]
U/s Apron	> 3	0.00	0.00
	2 to 3	0.00	0.00
	1 to 2	0.00	0.00
	0 to 1	2.50	2.50
Sloping Apron	> 3	0.63	0.21
	2 to 3	5.13	2.54
	1 to 2	9.96	5.48
	0 to 1	12.12	8.19
Downstream apron	> 3	3.67	4.70
	2 to 3	20.90	20.67
	1 to 2	35.90	29.0
	0 to 1	36.20	29.0

*finite difference method calculation requires the depth of permeable media and ratio of permeabilityês in the vertical to horizontal direction.

a - Scenario 1 horizontal to vertical permeability ratio of 10 and permeable media depth of 15m

b - Scenario 2 horizontal to vertical permeability ratio of 1 and permeable media depth of 25m

The result of the scenario analysis in Table-9 shows the variation in material quantity needed for different cases of permeable media property. This difference is accounted for in finite difference method calculation of subsurface flow problem, as opposed to Khoslaês method of independent variables which assumes isotropic and deep permeable media in all cases.

4.1.4 Optimized Least Cost Design with the Developed Program

Table 10 Optimum design parameters for sample problem 2

Parameter of component	Optimum Value (m)
U/s Pile Depth	9.2
D/s Pile Depth	8.5
D/s Apron Length	28.9
U/s Apron Length	6.1
Sloping Apron Length	8.4
Sloping Apron Slope	2H: 1V

The above table shows the optimum parameter values for least cost design of a diversion structure for a given cost as given in table 5 and table 6

Table 11 Volume of concrete required according to various design procedures for a structure founded on a 20m deep isotropic permeable media

Component of structure	Depth (m)	Volume with ordinary procedure (m^3)	Volume for optimized design			
			using Khosla's method (m^3)	Percentage difference	using Finite difference method (m^3)	Percentage difference
Upstream	> 3	0	0		0	
Apron	2 - 3	0	0		0	
	1 - 2	0	0		0	
	0 - 1	2.5	2.5		2.5	
Sloping	> 3	0.05	0		0.21	
Apron	2 - 3	3.13	1		2.54	
	1 - 2	8.12	5.21		5.48	
	0 - 1	11.97	8.14		8.19	
Downstream	> 3	2.97	3.46		4.7	
Apron	2 - 3	21.23	19.14		20.67	
	1 - 2	35.9	29		29	
	0 -1	36.2	29		29	
Total		122.07 m^3	97.45 m^3	-20.17%	102.29 m^3	-16.20%

Chapter five

5 Summary, Conclusion and Recommendations

5.1 Summary

A computer program is developed for design of low-head diversion structures. The program considers the surface and subsurface water flow phenomenon. Finite difference technique was successfully applied for predicting the uplifting head distribution under diversion structures founded on anisotropic and/or shallow permeable medium as well as isotropic and deep permeable media. The computer program solves variable equations involved in the occurrence of hydraulic jump while considering the surface flow dynamics. The program also optimizes the parameters of components of the structure to find the parameter combination that gives the design with least overall cost of structure taking the cost of construction of component structures as input. From a comparison of the computer program solution with that of Khoslaõs method of independent variables, it was found that the program would be a convenient tool

Moreover, it was found that the variation in residual uplift head along the aprons of structures built on permeable media is not linear from upstream to downstream as opposed to the assumptions in previous methods such as Blighõs and Khoslaõs method of independent variables. Anisotropy and depth of permeable media were found to affect the distribution of the residual uplift pressure head.

5.2 Conclusion

From this particular study it was found that the residual uplift pressure head doesnõt linearly vary from upstream to downstream, implying that accurate representation of the subsurface flow phenomenon is quite important for proper design of diversion structures in particular, and any hydraulic structures in general. Moreover, the residual uplift pressure was found to be affected by depth of permeable media and anisotropic soil condition. Therefore, effects of anisotropy and depth of permeable media should be considered while designing head work structures built on permeable structures.

The computer program calculates the various forces, design parameters and the cost of construction of the component structures of low head diversion structures efficiently. This allows for preparation of alternative designs for selecting cost effective designs. Moreover, the computer program searches for the optimum design with least overall cost. Great amount of design work-force-time and construction cost can be saved by using this computer program.

5.3 Recommendations

The different parameters of the components of a diversion structure are interrelated. Their optimum combination is dependent on the cost of construction of the components at a particular site. The practical situation in construction of water work structures is that the cost of material and construction vary; making an optimum design at one point in time (while in design stage) to be obsolete at other (during construction). However, with the development of such computer programs as this one, the optimum design can easily be prepared at any time (including at the construction stage), saving the client a great amount of expenses.

The finite difference method is a versatile technique for solution of subsurface flow. Its nature allows for study of subsurface flow problems even in complicatedly shaped permeable media, allowing for study and implementation of unusually shaped piles that may be economical. The author recommends more study in the practicality of using inverted-T and L shaped piles and inclined piles for diversion structures founded on permeable media.

This particular study focused on developing a computer program that would solve the surface and subsurface flow problems for diversion structure with a sloping apron and founded on porous media. Little or no effort is made to include the structural design of gates, piers and some other component structures that need special structural design considerations in the computer program. Moreover, the optimization procedure considers only a fraction of the parameters that can be manipulated for least cost design. I.e. Optimization can and should include consideration of permissible afflux, water- way width, wing walls and the top levels of aprons. The author recommends future line of study on optimization for least cost design of diversion structure to include consideration of these factors.

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*: original paper not seen

Ϙ: only abstract was seen

APPENDIX – I Hand calculation results and finite difference method calculation results

Sample Problem 1

Table 12 Validation of application of finite difference method using sample problem 1

parameters	Finite difference method for a 50m deep permeable media	Khoslaồs method of independent variables	Relative error
Percentage of residual head at Upstream pile ϕ_{C2} =	60.3%	66.383%	10%
Percentage of residual head at downstream piel ϕ_{E2} =	42.98%	39.762%	7.49%
Time Taken for Calculation=	2 min	-	

Table 13 Result of analysis for effect of anisotropy of permeable media on the magnitude of the uplift pressure head in percentage of the total head difference at upstream and downstream

$\frac{K_y}{K_x}$	Percentage of residual head at upstream	Percentage of residual head at downstream
0.05	68.82	34.31
0.1	67.91	35.21
0.2	66.82	36.35
0.5	64.52	38.46
1	62.3	40.32
2	59.9	42.21
5	56.88	44.44
10	54.87	45.97

Table 14 Result of analysis for effect of depth of permeable media on the magnitude of the uplift pressure head in percentage of the total head difference at upstream and downstream

Depth of permeable medium	Percentage of residual head at upstream of apron	Percentage of residual head at downstream apron
10	69.94	33
15	67.07	34.74
20	64.89	37.29
25	63.35	39.07
30	62.3	40.32
40	61.02	41.98
50	60.3	42.98

Hand calculation using Khoslaŝ method of independent variables for solution of sample problem 1.

Upstream pile

b= 30m

d=6m

$\alpha = 5$

$\lambda = 3.05$

$\phi_{E1} = 100\%$
$\phi_{C1} = 100 - \phi_{E} = 100 - 31.826 = 61.187\%$
$\phi_{D1} = 100 - \phi_{D} = 100 - 22.015 = 73.462\%$

Correction due to interference of downstream pile

Correction=4.1585 % (+ve)

Correction due to thickness of floor

Correction =1.038% (+ve)

Corrected ϕ_{C1}= <u>66.3835</u>

Downstream pile

b= 30m

d=8m

$\alpha = 3.75$

$\lambda = 2.441$

$\phi_{C2} = 0\%$
$\phi_{E21} = 44.218\%$
$\phi_{D2} = 29.90\%$

Correction due to interference of upstream pile

Correction=3.5611 % (-ve)

Correction due to thickness of floor

Correction=0.8949% (-ve)

Corrected ϕ_{E2} =<u>39.762%</u>

APPENDIX ï II User interface of the computer program

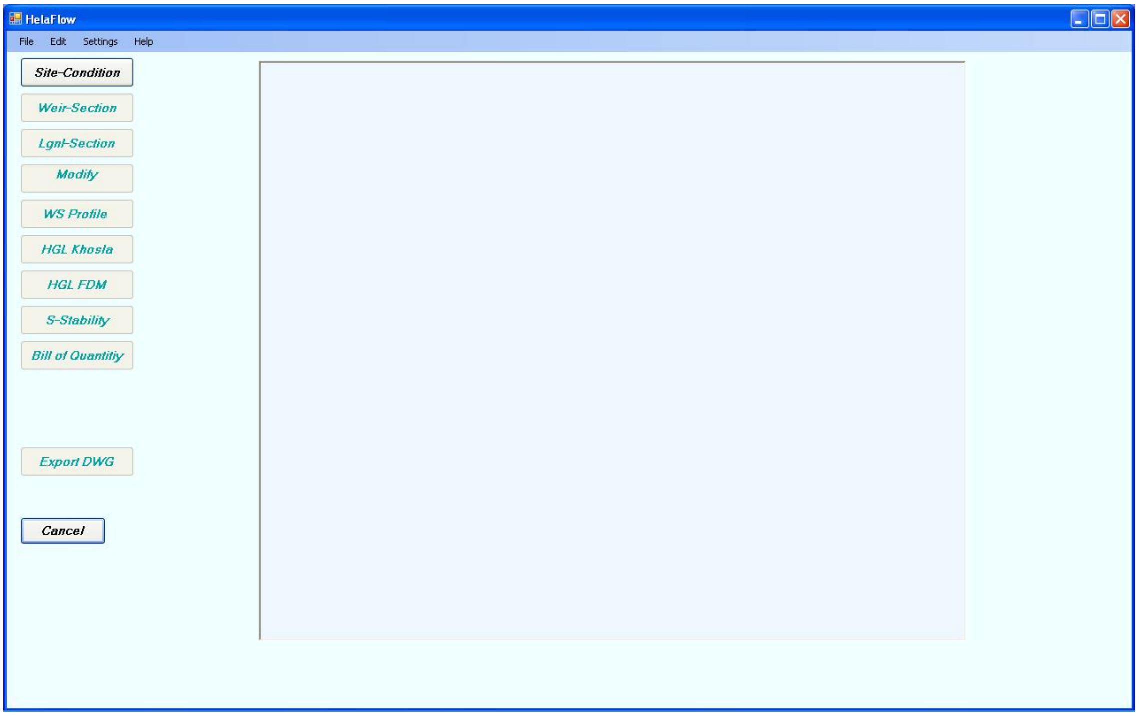

Figure 1- Main dialog box

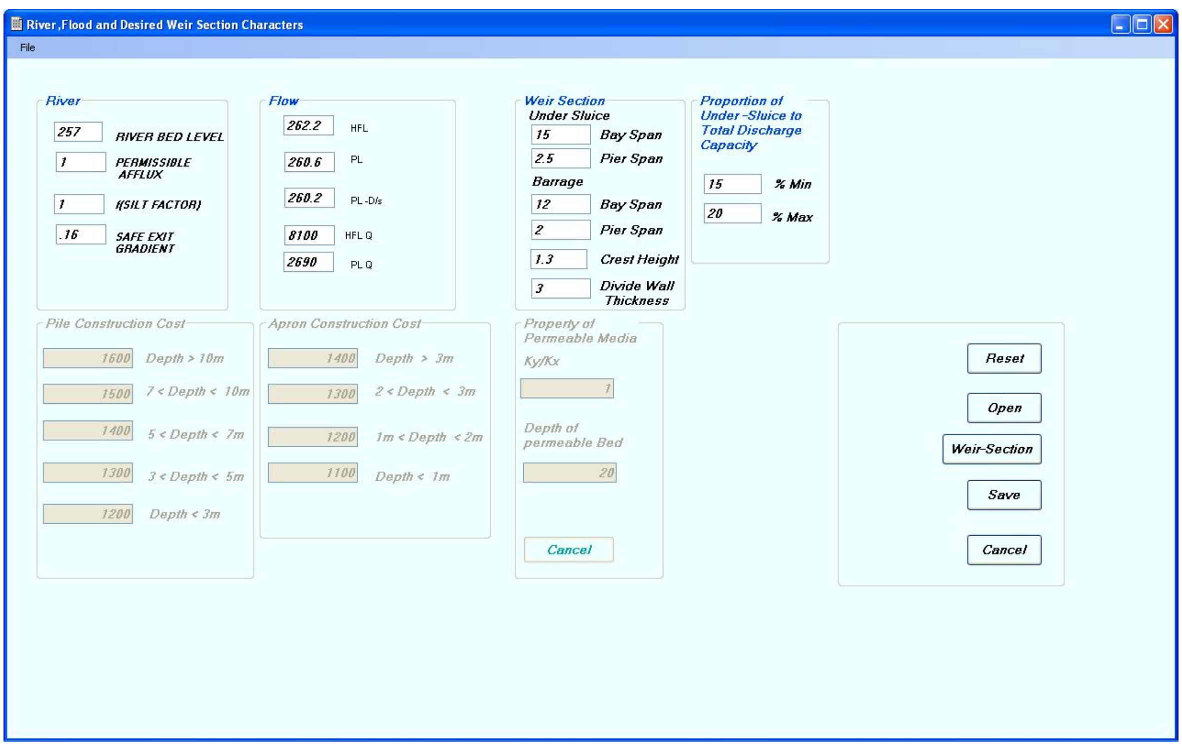

Figure 2 - Input dialog box for inserting the site characteristics Viz. River, Flood and preferred weir cross section settings

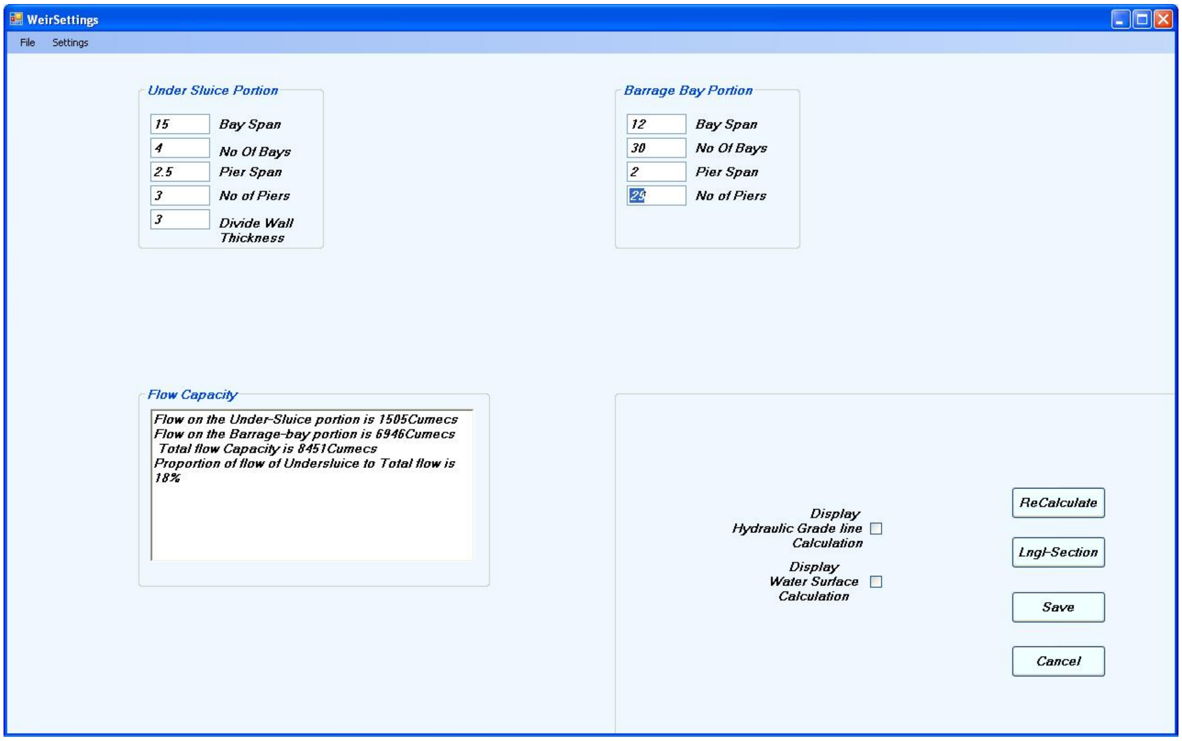

Figure 3- Outputs for weir cross section dimensioning

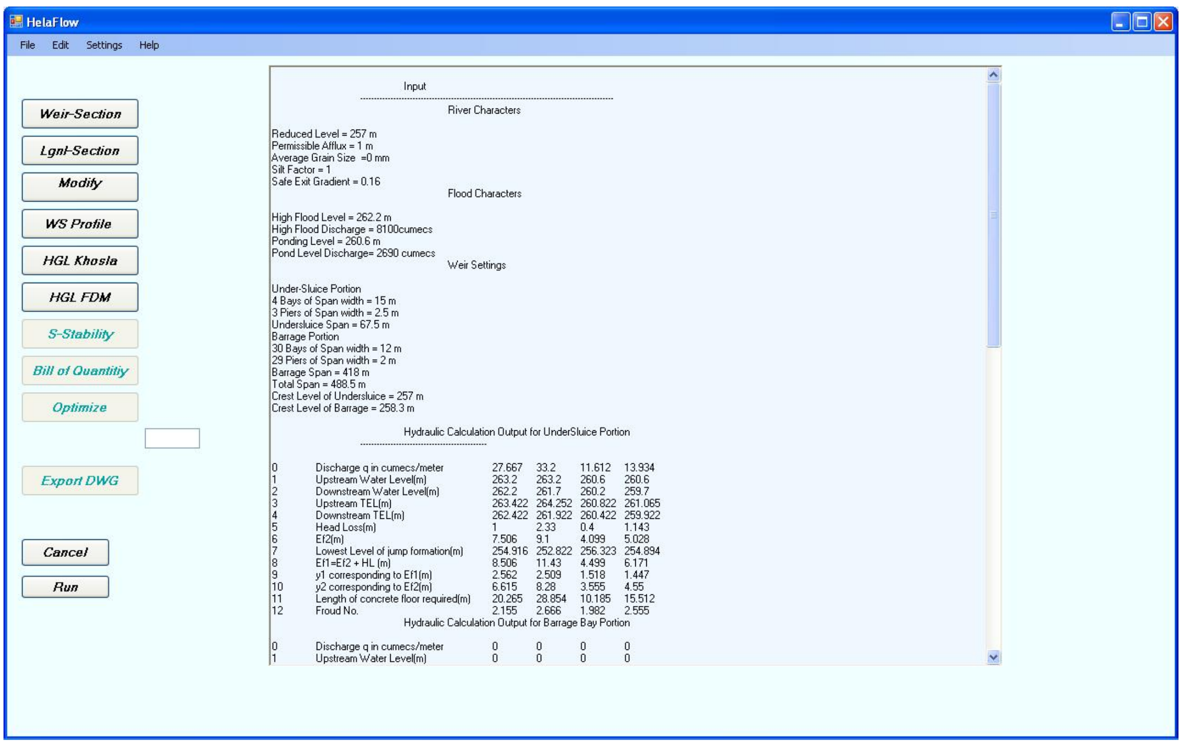

Figure 4- Longitudinal cross section dimensioning output

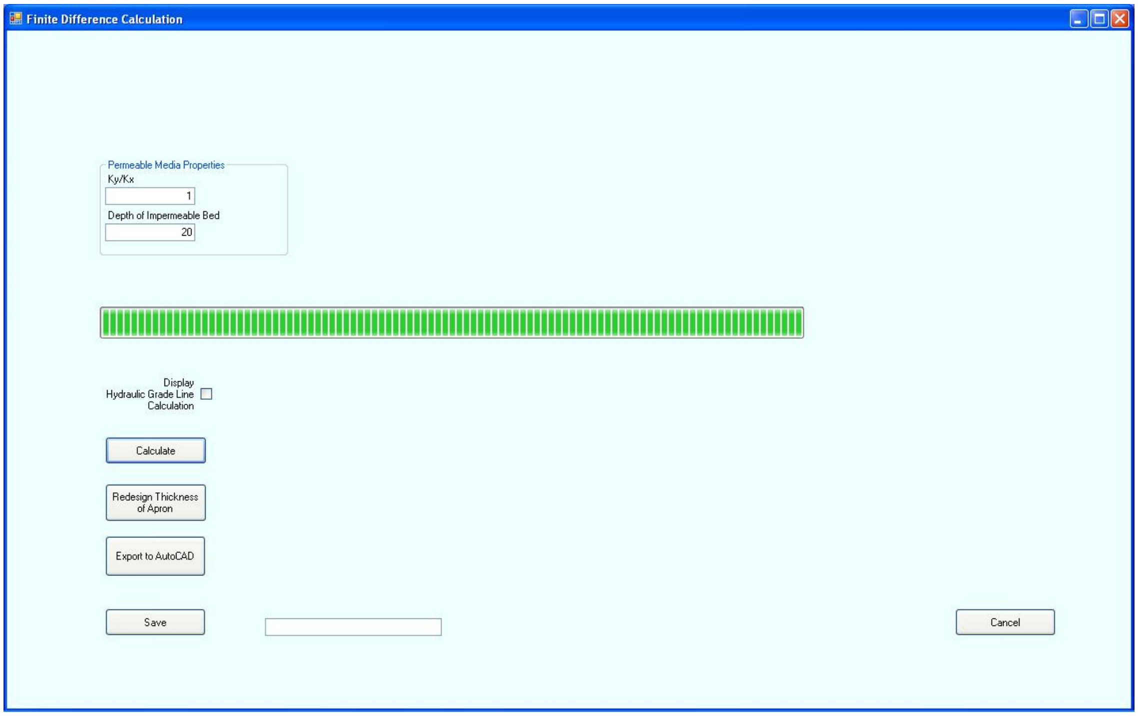

Figure 5- Finite difference calculation dialog

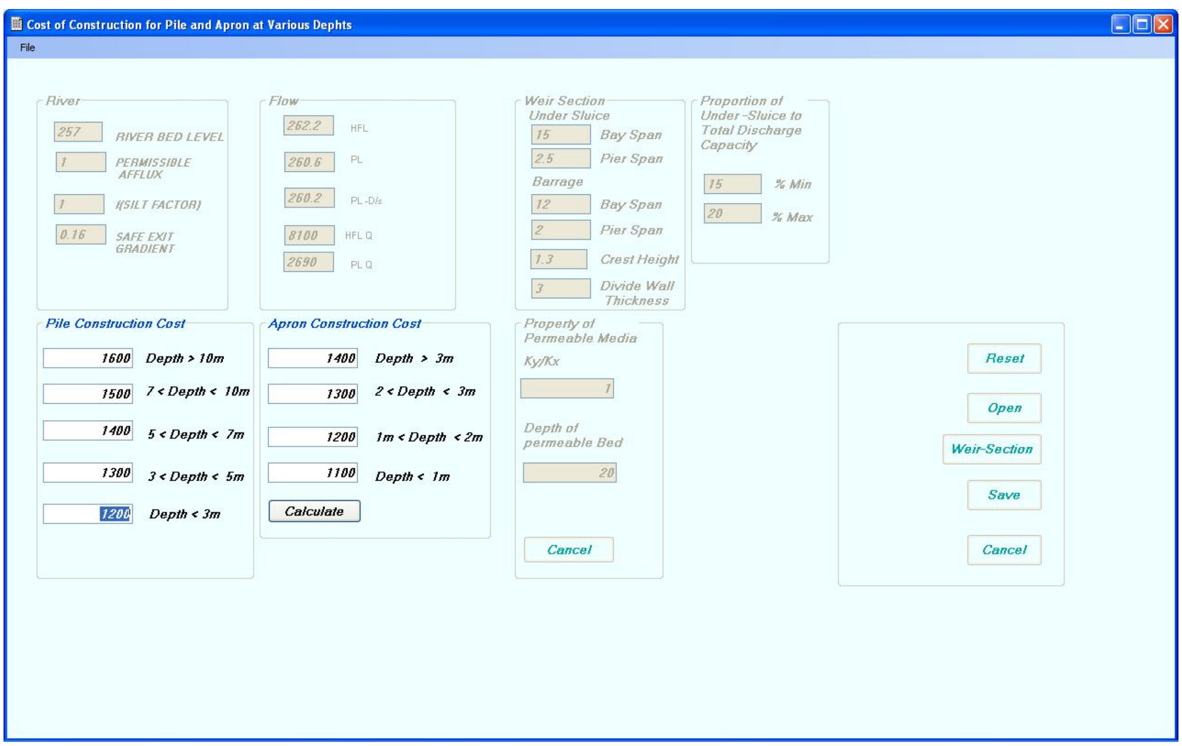

Figure 6- Input dialog box for cost of construction of component structures Viz. piles and aprons at various depths from ground level

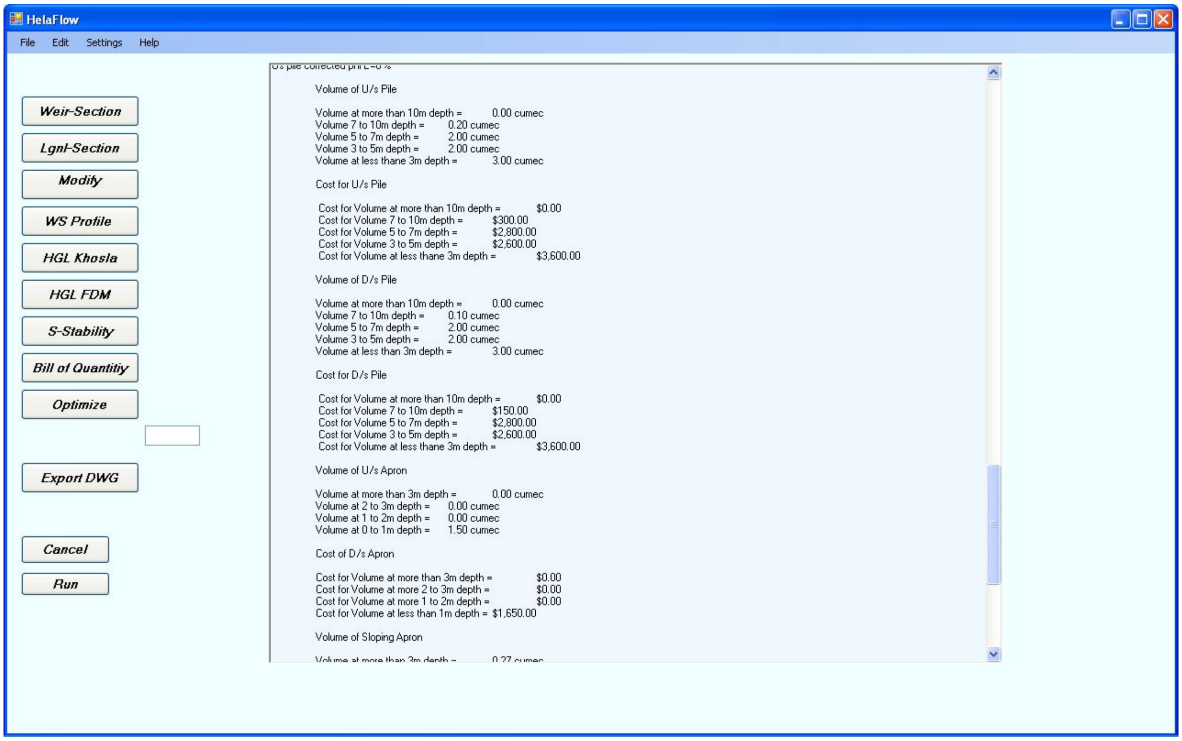

Figure 7- Cost calculation output for various components

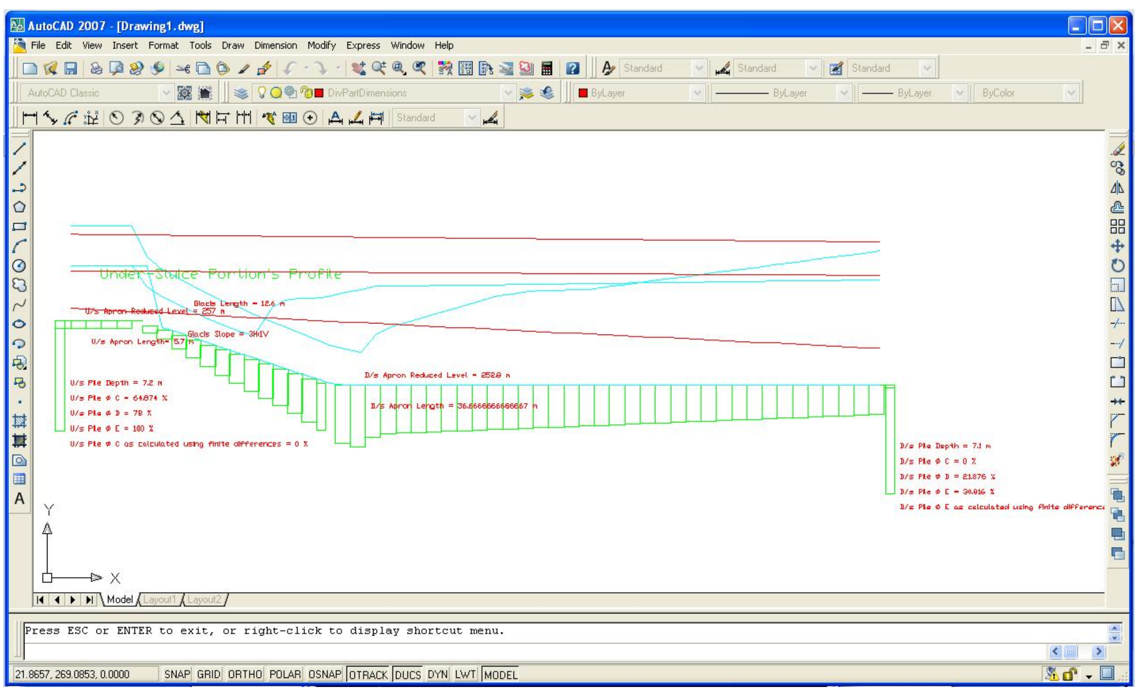

Figure 8- Sample output to AutoCAD by the computer program