CONTRIBUTION OF MAGNETIC STRESS ENERGY TO SUPERNOVA BOUNCE

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To my family.

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Abstract

In this paper, we have analyzed the contribution of the magnetic stress energy to the supernova bounce. A calculation of Maxwell stress tensor is proposed when leads to the expression of the magnetic force density. By making the link between the the magnetic force density and the Maxwell stress tensor we derived the magnetic pressure, required to ensure core stability or support the star from gravitational collapse, we calculated the associated field stranght at the surface of the compact object (the neutron star- NS) to be $B \gtrsim 10^{18}$ G and showed that this field is exert a pressure $P_{mag} \sim 10^{35} \frac{g}{cms^2}$ which is able to cause supernova bounce of the infalling stellar material.

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Introduction

The contribution of magnetic stress energy to the supernova bounce is one of the most important aspects in the study of Astrophysics. Using magnetic stress energy tensor we can calculate the Maxwell stress tensor we use for the caculation of magnetic pressure, which is used to support the massive star from gravitational core collapse. If the magnetic pressure of the star is sufficiently stiff, matter which is falling to the center bounces back. Supernova explosions occur whenever massive stars ($\gtrsim 8M_\odot$) come to the end of their evolutionery life. In a core collapse and formation of a neutron star, gravitational energy release ($\sim 10^{53}$) erg, is carried away by neutrino (SN II, SN Ib,c). We know for sure from the observations of SN1987A that they are associated with the death of massive stars and that they emit a large number of neutrinos which corresponds to the typical gravitational binding energy of the neutron star over the diffusion time scale of neutrinos in the hot and dense supernova core. When the nuclear saturation density is reached at the center of the core, the core bounce occurs, producing a shock wave that starts to propagate outward in the core. In this paper, we focus on the ordinary supernova which will lead to neutron star formation. The core then gravitationally collapses to a neutron star leading to an explosion of an iron-core-collapse supernova (SN).

This thesis is organized as follows. In Chapter 1, I discuss about supernovae explosion evolution towards supernova, which includes nuclear burning, core-collapse supernovae, core bounce and shock formation and explosion mechanisms are discussed. In Chapter 2, I deliver derivation of the force volume density, electromagnetic Maxwell stress tensor, magnetic Maxwell stress energy and magnetic pressure. In chapter 3, I discuss about the structure of NS Finally in Chapter 4, discussion and conclusions are given.

Chapter 1

The Supernova Explosion

A supernova explosion is the end of the life of a massive star with mass, $M > 8M_{\odot}$ [$M_{\odot}$ is the solar mass]. The energy released during this exceeds by orders of magnitude and more the energy emitted our star (the sun) over its total life time reaching $\sim 10^{10}yr$. During a NS formation most of the energy released is in the form of scarcely observable neutrinos. The energy flash it self comes either from the thermal instability developed in the degenerate core, or from gravitational and partly nuclear energy release during the collapse. The magnetic field and rotation play an important role in converting gravitational energy in to the energy of an observable flash. A small number of stars (the most massive once) end their lives in gravitational collapse and black hole formation. The collapse in this case may be "silent" and not lead to supernova explosion [1].

Supernavae(SNe) eject over 90 percent of the dying star mass in to interstellar space with a kinetic energy of the order of 10^{51} erg [16]. The ejecta contain heavy elements that are important for the chemical evolution of galaxies, stars, planets, and life.

1.1 Pre-supernova Evolution of Massive Stars

Stars achieve progressively higher value of temperature and density in their center [11]. The evolution of star is mainly connected to the evolution of the core. Which is determined by temperature and density. In particular, the final stages depend on whether or not the core reaches a temperature high enough to ignite carbon burnings[3]. Bulk nuclear matter at supersaturation density is found only in gravitationally collapsed objects or in the early universe. We dscus below the major phases of star's core

1.1.1 Nuclear Burning

A star's core undergoes continued contraction with halts during a stage of nuclear energy generation. There is a succession of nuclear fuels which can be effective in producing such halts. These includes Hydrogen burning, Helium burning, Carbon burning, Neon burning, Oxygen burning and Silicon burning [11]. A series of nuclear burning stages transforms the star into an onion-like shell structure, until Silicon burning create a core of Iron. Each successive nuclear burning stage releases less energy than the previous stage, so the life time in each successive stage becomes progressively shorter. The pre-collapse period involves the staged fusion of light elements in the central core of the star and takes less than 10^7 years. The result of this slow burning phase is a concentric structure of shells of elements, the lightest outside (H, He) and the heaviest inside (C, 0, Si, Fe, etc) (see Fig.1.1). The final burning of Si to Fe takes only a few days. The central core eventually contains elements near iron in atomic number, i.e. near the maximum in the binding energy. Fusion is therefore no longer possible, and collapse becomes inevitable. The pre-collapse evolution provides the initial conditions leading to hydrodynamical collapse, and in particular the "iron" core mass is crucial [29].

Because iron is the most tightly bound nucleus (the "break-even point" between fusion and fission) the star is no longer able to produce energy in the core via further nuclear burning stages.

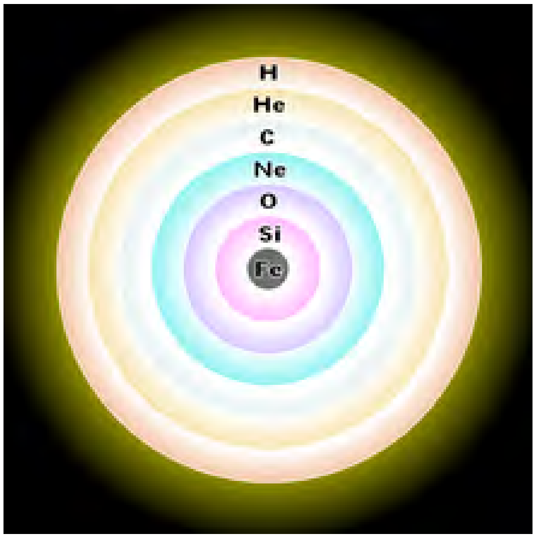

Figure 1.1: The onion-like layers of a massive, evolved star just prior to core collapse.

1.1.2 Core-Collapse Supernovae

During the last stage of a stellar evolution, nuclear burning takes place until an iron core is formed via H, He, C, O, Ne, and Si burnings. Since iron is the most stable element, further nuclear burning does not occur, and after a while the iron core begins to contract under the gravity of the star. The contraction increases the density and temperature, and when the density reaches upto the onset of electron capture, or when the temperature reaches upto the photodisintegration of the nuclei, the pressure decreases and gravitational collapse is set about. The collapse continues until the central density reaches the nuclear density, in which the pressure at the center of the core becomes sufficiently large to prevail the local gravity with the help of the nuclear force. Under such extreme conditions electron degeneracy cannot support the stellar core, and the free electrons are forced to join with protons to form neutrons[inverse beta decay]:

$$p + e^{-} \rightarrow n + \nu_e \tag{1.1.1}$$

The neutrinos, which escape directly from the core result in further energy loss and even faster collapse. The core collapses so rapidly that it effectively collapses out from under the stellar envelope. Collapse is halted only if nuclear matter under compression can stiffen sufficiently. Otherwise black holes result. The collapsing core divides into two regions: an inner core, homologously collapsing, subsonic core (infall velocities proportional to radius) and an outer supersonic shell.

Then Bounce occurs and a shock wave is generated at the boundary of the inner and the outer core. If the shock wave propagates through the core and the stellar envelope, blowing off matter, this results in a supernova explosion. On the other hand the inner core remains as a proto-NS which soon evolves into a NS after a cooling timescale of$\sim 5-10$s (see Fig. 1.3) [8]. The neutrinos generated are out gradually after the bounce with

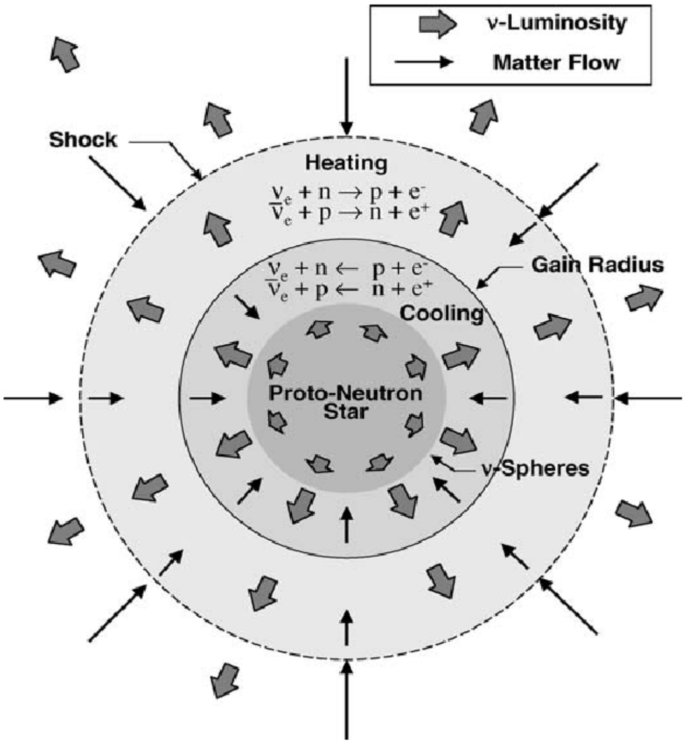

Figure 1.2: During the shock reheating phase, the stellar core is composed of a central radiating proto-neutron star whose surface is defined by the neutrinospheres (represented here by a single sphere) and a region above the neutrinosphere consisting of a net cooling region and a net heating region below the stalled shock, separated by the gain radius at which heating and cooling balance. Heating and cooling are mediated by electron neutrino and antineutrino absorption and emission. [8].

a timescale of $\sim$ 500 ms, and heat up matter behind the stalled shock wave via weak interaction. If the heat from the neutrinos is ample, the stagnated shock wave would revive and propagate through the core again. This is, however, denied by the recent spherically symmetric simulations which employ a realistic equation of state (EOS) and deal with sophisticated microphysics such as neutrino transport and/or electron capture, in which successful explosions have not been found [3].

The core resulting from the evolution of a star in the higher mass range collapses because it surpasses the critical Chandrasekhar limit. The core mass approaches $M_c \simeq 1.44 M_\odot$, the core radius $R_c \simeq 0.01 R_\odot$ where ($R_\odot = 6.9599 \times 10^{10} cm$) is a stellr radius. The interior matter density increases, speed up by the ongoing electron capture. When a stellar interior neutronizes, reducing the number of particles by a factor 2, but more importantly, eliminating the electric charges, the implosion generates a very densely packed core of neutrons of typically $R_N \simeq 10$ km. The collapse time (the free-fall time) is about 1 s. At the same time, shell material is pulled in and falls onto the core. Gravitational energy is released to the amount of

$$E_G = \frac{G M_c^2}{R_N} \simeq 3 \times 10^{53} erg \tag{1.1.2}$$

Note that the total observed energy of such a SN out-burst (kinetic energy, electromagnetic energy plus energy in neutrinos) is roughly 10^{53} erg. Above $\rho \simeq 4 \times 10^{11} g cm^{-3}$ neutrinos are captured [14].

1.1.3 Core Bounce and Shock Formation

When the core has collapsed down to a size of about 10 km, neutron degeneracy sets in cousing the core to stiffen and the infalling material from the envelope to rebound in a shock-wave outward from the core. This shock-wave drives the remaining material from the envelope outward, compressing it and heating it as it moves through. The net result is the formation of a neutron star in the stellar core and the total disruption of the remainder of the star with the liberation of about 10^{53}ergs of energy in neutrinos and 10^{51}ergs of kinetic and luminous energy. The luminous energy release is about the amount of energy that the sun will release in its 10 billion years lifetime [1]. Core rebound and shock formation occur closely in time, just about when the density at the edge of the homologous core reaches that of saturated nuclear matter. The shock's initial energy is a critical factor in determining its eventual fate [29].

The collapse is halted at an appropriate point, as the pressure of the degenerate neutrons takes hold, and the core bounces back. We can visualize the situation along the lines of the popular paradigm for nuclear matter introduced: as collapse progresses and the core density rises, the large, neutron-rich nuclei eventually begin to touch each other, and merge at just below nuclear density into essentially one gigantic stellar-mass nucleus, which overshoots the nuclear density by a factor of several orders before bouncing back. "The repulsive hard-core potential of the nucleus acts as a stiff spring, storing up energy in the compression phase, then rebounding as the compression phase ends". This rebounce is shown in in this (Fig.1.3) notice how the radial-velocity profile in the inner part of the core changes drastically at bounce, with infall changing into outflow. This inner core is essentially homologous. It is also roughly that part of the core which stays in sonic communication throughout. Just outside this inner core, a shock wave develops immediately as the rebounding core encounters matter that is continuing to fall in, since the impact between the two is supersonic. We can call this prompt hydrodynamic shock the bounce shock [3].

1.1.4 Explosion Mechanisms

It was this bounce shock that originally looked like a good candidate for the mechanism that produces the supernova explosion. The shock never reaches the outer layers of the star with enough energy to explode them, because of severe energy losses on the way. Energy losses to the shock are from two sources, dissociation of heavy nuclei into lighter components and production and escape of some neutrinos; both follow from the heat and entropy generated by the energetic shock [29]. The major way in which this loss occurs is through photodisintegration (a process that cools the shock on a hydrodynamical time-scale, immediately following core bounce, is the passage through it of neutrinos emitted from regions behind it, as the shock moves outward).

Neutrinos are the key to supernova explosion: these are emitted copiously from the hot, collapsed core (and thereby cool the core, as we mentioned above, carrying away the gravitational binding energy released in the formation of the neutron star), and these neutrinos would stream through, deposit some fraction of their energy and momentum in the outer stellar layers ahead of the shock, and explode them away [3]. This figure shows a typical supernova.

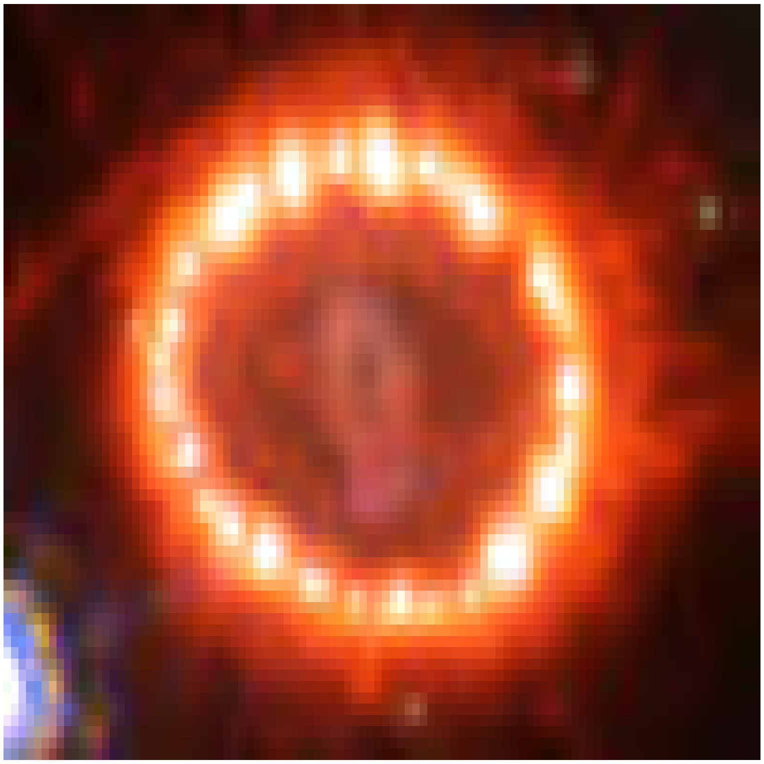

Figure 1.3: The Supernova explosion.

1.2 Observations about supernova

From the observational stand point supernovae are of two types; Type I supernova(SN I) and Type II supernova(SN II):

Type-I Supernovae: The (SN I) is with no hydrogen lines in their emission spectrum and numerous absorption lines of various heavy elements instead, and they are divided in to three groups SN Ia, SN Ib and SN Ic. In addition they are less massive stars for progenitors; many SN I are likely to occur in binaries. SN Ia never show a radio emission, so it is suggested that they are produced by nuclear explosions leading to the total disruption of the star while other types of SN results from hydrodynamics collapse, leading to formation of neutron star. In SN Ib,c the collapsing core is not surrounded by hydrogen foresumably orginated from less massive stars. Which have lost their hydrogen rich envelope in their preceding evolution may be in a binary. There are some special difference between the three types of SN I while they all do not show hydrogen lines, the main difference appears after ~ 250 days from the explosion. In SN Ia the strongest emission lines are represented by Fe II and Fe III ions.

Type II Supernova (SNe II): Type-II supernovae are sometimes called simply supernova, which are one of the most energetic explosive events that result from the explosion of a star after its main sequence life is over (when its nuclear fuel is exhausted). This occurs only if the star has mass greater than 5 $M_\odot$ so that when its core collapses, huge amount of gravitational energy is converted to heat. This in turn generates a powerful blast or shock wave which ejects the stars envelope into the interstellar space. It produces an extremely bright object made of plasma that remains visible over weeks or months. It outshines its entire host galaxy. The amount of energy release is terrific. Possibly, it would take 10 billion years for our Sun to deliver the same energy output that an ordinary Type-II supernova would produce. After the explosion, the remnant of the star could be a non-rotating neutron star, generally a pulsar, or could turn out to be a black hole [34].

With much hydrogen and nearly normal chemical composition, which is also established as a result of evolution of massive stars the collapsing core is surrounded by a large hydrogen envelope. The presence of hydrogen in the SN II spectra provides evidence that the explosion occurred befor the star has lost its hydrogen envelope [1]. The collapse-driven supernovae has been reported by the observation of the polarization. It is also well known that SN 1987A is globally asymmetric, which is directly confirmed by the images of the Hubble Space Telescope. The observed light curve also supports jet-like explosion. The asymmetry is likely to have originated from the core dynamics. Provided that the progenitors of the collapse-driven supernovae are rapid rotators on the main sequence and the fact that the recent theoretical studies suggest a fast rotating core prior to the collapse, it is important to incorporate rotations in simulations of core collapse [13].

Chapter 2

Magnetic Stress Energy tensor

2.1 The Magnetic Stress Energy Tensor in Magnetized Matter

The equilibrium thermodynamics of magnetized matter is important to understand the structure and evolution of compact astrophysical objects such as white dwarfs and neutron stars. Of particular relevance is the pressure, since it enters in an important way in the equations that determine the structure of the star (Newtonian hydrostatic equilibrium equation or its relativistic Oppenheimer-Volkov generalization) [30].

2.1.1 The Electromagnetic stress energy tensor

The stress-energy tensor for electromagnetic field, $T_{\mu\nu}^{(EM)}$ is given by:

$$T_{\mu\nu} = \frac{1}{4\pi}\left(g_{\alpha\mu}F^{\alpha\beta}F_{\beta\nu} - \frac{1}{4}g_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right) \tag{2.1.1}$$

where

$$g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{2.1.2}$$

which is the Minkowskian metric tensor. The electromagnetic tensor $F_{\mu\nu}$ can be related to avector potential $A_\nu = (-\phi, \mathbf{A})$,

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu} \tag{2.1.3}$$

where

$$F_{\mu\nu} = -F_{\nu\mu} \tag{2.1.4}$$

The antisymmetric tensor $F_{\mu\nu}$ and $F^{\mu\nu}$ are given explicitly in matrix form:

$$F_{\mu\nu} = \begin{bmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{bmatrix}, \tag{2.1.5}$$

and

$$F^{\mu\nu} = \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{bmatrix}, \tag{2.1.6}$$

respectively, where the electric field and magnetic field are given by:

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} \tag{2.1.7}$$

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{2.1.8}$$

Now let us consider The energy momentum tensor $F^{\mu\nu}$ for the electromagnetic field. The tensor $T_{\mu\nu}$ is a symmetric tensor [12].which may be writen explicitly in the form:

$$T_{\mu\nu} = \begin{bmatrix} \frac{1}{2}\left(\epsilon E^2 + \frac{B^2}{\mu_o}\right) & \frac{S_x}{c} & \frac{S_y}{c} & \frac{S_z}{c} \\ \frac{S_x}{c} & -\sigma_{xx} & -\sigma_{xy} & -\sigma_{xz} \\ \frac{S_y}{c} & -\sigma_{yx} & -\sigma_{yy} & -\sigma_{yz} \\ \frac{S_z}{c} & -\sigma_{zx} & -\sigma_{zy} & -\sigma_{zz} \end{bmatrix}, \tag{2.1.9}$$

where $\mathbf{S}$ is the Poynting vector given by:

$$\mathbf{S} = \frac{1}{4\pi}(\mathbf{E} \times \mathbf{B}). \tag{2.1.10}$$

Maxwell stress tensor is written as:

$$\sigma_{ij} = \epsilon_o E_i E_j + \frac{1}{\mu_o} B_i B_j - \frac{1}{2}\left(\epsilon_o E^2 + \frac{1}{\mu_o} B^2\right)\delta_{ij}, \tag{2.1.11}$$

where ϵ_o is the permittivity of free space, μ_o is the permeablity of a free space and δ_{ij} is the Kronecker delta. The indices i and j refer to the coordinates x, y and z, so the stress tensor has a total of nine compenents. Maxwell's stress tensor component T_{ij} is interpreted as the i component of stress directed parallel to the j component of a surface normal [7,20].

Note that

$$c^2 = \frac{1}{\epsilon_o \mu_o}, \tag{2.1.12}$$

where c is speed light. In free space in cgs-Gaussian units, we simply substitute ϵ_o with $\frac{1}{4\pi}$ and μ_o with 4π [5,4,6].

2.2 Maxwell Stress Tensor

In this work we address the problem by studying the electromagnetic force on a magnetized matter from a purely classical point of view and constructing thereby the corresponding magnetic stress tensor for the system. Our results imply that the magnetic stress energy tensor contribution to the magnetic pressure, defined as the negative of the diagonal components of the stress tensor.

2.2.1 Maxwell's Basic Equations

When currents are induced by the motion of a conducting fluid inside a magnetic field, a Lorentz (or $\mathbf{J} \times \mathbf{B}$) force will act on the fluid and modify its motion. The resulting theory is highly non-linear. In electromagnetic theory, we specify the spatial and temporal variation of either the electromagnetic field or its source, the electric charge density and current density. One choice is computable (at least in principle) from the other using Maxwells equations, augmented by suitable boundary conditions. It turns out that for the majority of applications, it is most instructive to deal with Maxwells equations electromagnetic field as primary:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_o} \tag{2.2.1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2.2.2}$$

which is Gauss's law for magnetism and which tells us that, we can represent the magnetic field, B by un ending lines of force.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2.2.3}$$

Maxwell-Faraday equation (Faraday's law of induction)

$$\nabla \times \mathbf{B} = \frac{\mu_o}{c}\mathbf{J} + \mu_o \epsilon_o \frac{\partial \mathbf{E}}{\partial t} \tag{2.2.4}$$

which is Ampre's law (with Maxwell's correction). The current density J, is now related to the electromagnetic field by a form of ohm's law appropriate to a moving media in the so called hydromagnetic approximation this is:

$$\mathbf{J} = \delta \left[\mathbf{E} + c\left(\mathbf{V} \times \frac{\mathbf{B}}{c}\right)\right], \tag{2.2.5}$$

where δ is the electrical conductivity, Essentially implies that the force due to the fluid velocity in the magnetic field moves charges in addition to exerting eletromotive force. Taking the curl of the equation (2.2.4), and using equation (2.2.3) We get:

$$\nabla \times (\nabla \times \mathbf{B}) = \delta \mu_o \nabla \times \left[\mathbf{E} + c\left(\mathbf{V} \times \frac{\mathbf{B}}{c}\right)\right] + \mu_o \epsilon_o \nabla \times \frac{\partial \mathbf{E}}{\partial t} \tag{2.2.6}$$

Normally $\frac{\partial \mathbf{E}}{\partial t}$ is neglected in astrophysics, so we shall drop it in our calculations here on. The result in this case will be:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \left[\frac{4\pi}{c^2 \mu_o}\right]\left(\frac{c^2}{4\pi\delta}\right)\nabla^2 \mathbf{B} \tag{2.2.7}$$

Thus the time rate of change of the magnetic field is governed by diffusion of the field $\frac{1}{\mu_o}\frac{\nabla^2 \mathbf{B}}{\delta}$, and convection $\nabla \times (\mathbf{V} \times \mathbf{B})$ of the field by the fluid. If we neglect the diffusion term we can approximate:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) \tag{2.2.8}$$

The Maxwell Stress Tensor given in equation (2.1.11) is a second rank tensor used in classical electromagnetism to represent the interaction between electric/magnetic forces and homogeneous magnetic field, it is easy to calculate the forces on any charge from the Lorentz force law. When the situation becomes more complicated, it is convenient to collect many of these terms in the Maxwell stress tensor and tensor arithmetic to find the answer to the problem at hand.

2.2.2 Magnetic force density on matter

Macroscopic matter interacts with a magnetic field through its electrical currents (due to the motion of free charges) and macroscopic magnetization (due to the alignment of microscopic magnetic dipoles, usually associated with quantized spins).

Consider the Lorentz force law which is written as

$$\mathbf{F} = q(\mathbf{E} + \mathbf{V} \times \mathbf{B}), \tag{2.2.9}$$

where $\mathbf{V}$ is the velocity of the charge.

The force d$\mathbf{F}$ on a tiny volume dV is found by substituting ρdV for the element charge q_i

$$d\mathbf{F} = (\rho\mathbf{E} + \rho\mathbf{V} \times \mathbf{B})dV, \tag{2.2.10}$$

where $\mathbf{J} = \rho\mathbf{V}$

$$d\mathbf{F} = (\rho\mathbf{E} + \ \mathbf{J} \times \mathbf{B})dV, \tag{2.2.11}$$

Now we define $\overrightarrow{f}$ as the force per unit volume exerted by the electromagnetic fields:

$$\overrightarrow{f} = \rho\mathbf{E} + \mathbf{J} \times \mathbf{B}, \tag{2.2.12}$$

This is the differential form of the Lorentz force law. The usual expresion for Lorentz force law $\mathbf{F} = q(\mathbf{E}\ + V \times \mathbf{B})$ can be obtained by integrating over volume. Using equation (2.2.1) and (2.2.4), we find from Eq(2.2.12)

$$\overrightarrow{f} = \epsilon_o(\nabla \cdot \mathbf{E})\mathbf{E} + \frac{1}{\mu_o}(\nabla \times \mathbf{B}) \times \mathbf{B} - \epsilon_o(\frac{\partial \mathbf{E}}{\partial t} \times \mathbf{B}), \tag{2.2.13}$$

This would be relatively straight forward to calculate if we only could get rid of those curls, which are more demanding. Fortunately we have the curl identity,

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = (\nabla \cdot \mathbf{B})\mathbf{B} - \mathbf{B} \times (\nabla \times \mathbf{B}), \tag{2.2.14}$$

The last term of this equation can be written as

$$\mathbf{B} \times (\nabla \times \mathbf{B}) = \frac{1}{2}\nabla B^2 - (\mathbf{B} \cdot \nabla)\mathbf{B}. \tag{2.2.15}$$

Since

$$\frac{\partial}{\partial t}(\mathbf{E} \times \mathbf{B}) = \frac{\partial \mathbf{E}}{\partial t} \times \ \mathbf{B} + E \times \frac{\partial \mathbf{B}}{\partial t}, \tag{2.2.16}$$

using equation (2.2.3), (2.2.14) and (2.2.16) we can write (2.2.13) as

$$\begin{aligned} \overrightarrow{f} = \quad & \epsilon_o \left[(\nabla \cdot \mathbf{E}) \ \mathbf{E} - \mathbf{E} \times (\nabla \times \mathbf{E}) \right] + \frac{1}{\mu_o} \left[(\nabla \cdot \mathbf{B})\mathbf{B} \ - \mathbf{B} \times (\nabla \times \mathbf{B}) \right] \\ & -\epsilon_o \frac{\partial}{\partial t}(\mathbf{E} \times \mathbf{B}), \end{aligned} \tag{2.2.17}$$

In similar manner the identity equation (2.2.14) again gives us:

$$\begin{aligned} \overrightarrow{f} = \quad & \epsilon_o \left[(\nabla \cdot \mathbf{E})\mathbf{E} + (\mathbf{E} \cdot \nabla)\mathbf{E} \right] + \frac{1}{\mu_o} \left[(\nabla \cdot \mathbf{B})\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{B} \right] \\ & -\frac{1}{2}\nabla \left(\epsilon_o E^2 + \frac{1}{\mu_o} B^2 \right) - \epsilon_o \frac{\partial (\mathbf{E} \times \mathbf{B})}{\partial t} \end{aligned} \tag{2.2.18}$$

The last term of this equation can be written as:

$$\epsilon_o \frac{\partial (\mathbf{E} \times \mathbf{B})}{\partial t} = \epsilon_o \mu_o \frac{\partial \mathbf{S}}{\partial t}, \tag{2.2.19}$$

where

$$\mathbf{S} = \frac{1}{\mu_o}(\mathbf{E} \times \mathbf{B}) \tag{2.2.20}$$

Then the force density can be writen as

$$\begin{aligned} \overrightarrow{f} = \quad & \epsilon_o \left[(\nabla \cdot \mathbf{E})\mathbf{E} + (\mathbf{E} \cdot \nabla)\mathbf{E} \right] + \frac{1}{\mu_o} \left[(\nabla \cdot \mathbf{B})\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{B} \right] \\ & -\frac{1}{2}\nabla \left(\epsilon_o E^2 + \frac{1}{\mu_o} B^2 \right) - \epsilon_o \mu_o \frac{\partial \mathbf{S}}{\partial t}. \end{aligned} \tag{2.2.21}$$

or as

$$\overrightarrow{f} = \epsilon_o \left[(\nabla \cdot \mathbf{E})\mathbf{E} + (\mathbf{E} \cdot \nabla)\mathbf{E}\right] + \frac{1}{\mu_o}\left[(\nabla \cdot \mathbf{B})\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{B}\right] - \frac{1}{2}\nabla\left(\epsilon_o E^2 + \frac{1}{\mu_o}B^2\right) - \frac{1}{c^2}\frac{\partial \mathbf{S}}{\partial t}. \tag{2.2.22}$$

This expression contains every aspect of electromagnetism and momentum and is relatively easy to compute. The last two terms of this expression are perhaps easier to rationalize. The second-to-last term is the gradient of the electromagnetic energy density. We will soon see that the last term is proportional the time-dependent change in momentum of the electromagnetic field from we derived the final expresion of the force density for the magnatice field above.

$$\overrightarrow{f} = \frac{1}{\mu_o}(\mathbf{B} \cdot \nabla)\mathbf{B} - \frac{1}{2}\nabla\left(\frac{1}{\mu_o}B^2\right). \tag{2.2.23}$$

We can also caculate the magnetic Maxwell stress tensor from equation (2.1.11), as

$$T_{ij} = \frac{1}{\mu_o}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right) \tag{2.2.24}$$

or

$$T_{ij} = \frac{1}{4\pi}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right) \tag{2.2.25}$$

The two terms in the magnetic Maxwell stress tensor can be identified as the “push” of an isotropic magnetic pressure of $\frac{B^2}{2\mu_o}$ that acts just like the gas pressure P , and the “pull” of a tension $\frac{B^2}{\mu_o}$ that acts parallel to the magnetic field. The combination of the tension and the isotropic pressure give a net tension $\frac{B^2}{2\mu_o}$ along the field and a net pressure $\frac{B^2}{2\mu_o}$ perpendicular to the field lines [21,10]. Equation (2.2.25) which is the correct, symmetric, well-known form in vacuum of the magnetic Maxwell Stress Tensor [10].

T_{ij} is sometimes written with double arrows: $\overleftrightarrow{T}$. We are used to the divergence of a vector field giving us a scalar. Here the divergence of a tensor field gives us a vector, the divergence of $\overleftrightarrow{T_{ij}}$ has as its jth componet [4]:

$$\begin{aligned}(\nabla \cdot \overleftrightarrow{T})_j = \quad & \epsilon_o \left[(\nabla \cdot \mathbf{E})E_j + (\mathbf{E} \cdot \nabla)E_j\right] + \frac{1}{\mu_o}\left[(\nabla \cdot \mathbf{B})B_j + (\mathbf{B} \cdot \nabla)B_j\right] \\ & -\frac{1}{2}\nabla_j \left(\epsilon_o E^2 + \frac{1}{\mu_o}B^2\right). \end{aligned} \tag{2.2.26}$$

Now considre the Maxwell Stress Tensor $\overleftrightarrow{T_{ij}}$ in equation (2.1.11), the divergence of $\overleftrightarrow{T_{ij}}$ is:

$$\begin{aligned}\nabla \cdot \overleftrightarrow{T} = \quad & \sum_{i=1}^{3}\sum_{j=1}^{3} \partial_i \overleftrightarrow{T}_{ij}\hat{x}_j \\ & = \sum_{i=1}^{3}\sum_{j=1}^{3} \partial_i \left[\epsilon_o \left(E_iE_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_o}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right)\right]\hat{x}_j \\ & = \epsilon_o \left[(\nabla \cdot \mathbf{E})\mathbf{E} + (\mathbf{E} \cdot \nabla)\mathbf{E}\right] + \frac{1}{\mu_o}\left[(\nabla \cdot \mathbf{B})\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{B}\right] \\ & -\frac{1}{2}\nabla \left(\epsilon_o E^2 + \frac{1}{\mu_o}B^2\right). \end{aligned} \tag{2.2.27}$$

The divergence of the magnetic maxwell stress tensor (equation (2.2.24)) can also be expresed as:

$$\begin{aligned}\nabla \cdot \overleftrightarrow{T} = \quad & \sum_{i=1}^{3}\sum_{j=1}^{3} \partial_i \overleftrightarrow{T}_{ij}\hat{x}_j \\ & = \sum_{i=1}^{3}\sum_{j=1}^{3} \partial_i \left[\frac{1}{\mu_o}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right)\right]\hat{x}_j \\ & = \frac{1}{\mu_o}(\mathbf{B} \cdot \nabla)\mathbf{B} - \frac{1}{2}\nabla\left(\frac{1}{\mu_o}B^2\right). \end{aligned} \tag{2.2.28}$$

Comparing Eq (2.2.22) with Eq (2.2.27) shows that the divergence of the electromagnetic Maxwell-stress tensor is exactly all the terms in the force density expression except the term proportional to the time derivative of **S**. i.e

$$\overrightarrow{f} = \nabla \cdot \overleftrightarrow{T} - \frac{1}{c^2}\frac{\partial \mathbf{S}}{\partial t} \tag{2.2.29}$$

From equation (2.2.23) and (2.2.28) thus follows the force per unit volume (for the magnetic component) $\overrightarrow{f}$ can be written as:

$$\overrightarrow{f} = \nabla \cdot \overleftrightarrow{T} \tag{2.2.30}$$

$$f_i = \frac{\partial}{\partial x_j} T_{ij}. \tag{2.2.31}$$

If the sum of the momenta of all particles in the volume V is denoted by p_{meh}, equation (2.2.11) can be written in the form of Newton's second law as

$$\frac{dp_{mech}}{dt} = \int_V (\rho\mathbf{E} + \mathbf{J}\times\mathbf{B})dV \tag{2.2.32}$$

$$\begin{aligned}\frac{dp_{mech}}{dt} = {} & \epsilon_o \int_V \left[(\nabla\cdot\mathbf{E})\ \mathbf{E} - \mathbf{E}\times(\nabla\times\mathbf{E})\right] + \frac{1}{\mu_o}\left[(\nabla\cdot\mathbf{B})\mathbf{B}\ \ - \mathbf{B}\times(\nabla\times\mathbf{B})\right] dV \\ & -\epsilon_o \frac{\partial}{\partial t}\int_V (\mathbf{E}\times\mathbf{B})dV,\end{aligned} \tag{2.2.33}$$

where

$$p_{field} = \int_V \epsilon_o(\mathbf{E}\times\mathbf{B})dV \tag{2.2.34}$$

This states that the momentum associated with the electromagnetic field.

$$\begin{aligned}\frac{d}{dt}(p_{mech} + p_{field}) = {} & \epsilon_o \int_V \left[(\nabla\cdot\mathbf{E})\ \mathbf{E} - \mathbf{E}\times(\nabla\times\mathbf{E})\right] dV \\ & + \int_V \frac{1}{\mu_o}\left[(\nabla\cdot\mathbf{B})\mathbf{B}\ \ - \mathbf{B}\times(\nabla\times\mathbf{B})\right] dV\end{aligned} \tag{2.2.35}$$

Thus the right hand side is similar to the electromagnetic force density.

$$\frac{d}{dt}(p_{mech} + p_{field})_i = \int_V \frac{\partial}{\partial x_j} T_{ij} dV = \oint_{surface} T_{ij} n_j da, \tag{2.2.36}$$

Obviously, it represents the ith components of the momentum flow through the unit area. n_j represents the jth component of the normal vector perpendicular to the area, da.

$$dF_i = T_{ij} n_j da \tag{2.2.37}$$

Physically T_{ij} is the force per unit area (or stress acting on the surface), more precisely, T_{ij} is the force per unit area in the ith direction acting on the element of surface oriented in the jth direction.

Now consider the magnetic force on the magnetized star. Each charge is subjected to the Lorentz force $\frac{e}{c}(\mathbf{V}\times\mathbf{B})c$. The magnetic field also generates a force $\mathbf{F}$ which acts up on the medium carrying eletric current density $\mathbf{J}$. So the expression for the magnetic force density is given by,

$$f_{mag} = \mathbf{J} \times \frac{\mathbf{B}}{c} \tag{2.2.38}$$

where $\mathbf{J} = \left[\frac{c}{\mu_o}\right] \nabla \times \mathbf{B}$, then we can have:

$$f_{mag} = \frac{1}{\mu_o}(\nabla \times \mathbf{B}) \times \mathbf{B} \tag{2.2.39}$$

We can also write this as

$$f_{mag} = \frac{4\pi}{\mu_o}\left\{-\nabla\frac{B^2}{8\pi} + \frac{1}{4\pi}(\mathbf{B}.\nabla)\ \mathbf{B}\right\} \tag{2.2.40}$$

This has a form $f = -\nabla P$. In the direction of magnetic field there is a tension which is equivalent to a force per unit area $\frac{B^2}{2\mu_o}$ and in the other perpendicular directions there is a pressure, opposite in sign but of the same magnitude, $-\frac{B^2}{2\mu_o}$. For a magnetic field of $\sim 10^{18}$ G or higher the magnetic pressure could be of the same order of magnitude to the matter pressure [21]. Here we keep only the components perpendicular to the magnetic field. The fact that $f_{mag} = \mathbf{J} \times \mathbf{B}$ guarantees that the net force parallel to $\mathbf{B}$ must vanish, so we can throw away the component along $\mathbf{B}$ in each term. This transversality of f_{mag} means that the magnetic force does not inhibit nor promote motion of the fluid along the magnetic field. The magnetic force density has two parts: first, the negative of the gradient of the

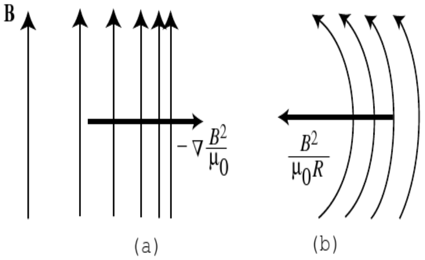

Figure 2.1: Contributions to the electromagnetic force density acting upon a conducting fluid in a non-uniform field. There is a magnetic pressure $\frac{-B^2}{2\mu_o}$ acting perpendicular to the magnetic field and a magnetic tensile stress $\frac{B^2}{\mu_o R}$ where R is the radius of curvature, acting in the direction of the curvature vector. There is no net force acting along the field.

magnetic pressure $\frac{B^2}{2\mu_o}$ orthogonal to **B** (Fig. 2.1a), and second, an orthogonal curvature force $\frac{(\mathbf{B}\cdot\nabla)\mathbf{B}}{\mu_o}$, which has magnitude $\frac{\mathbf{B}^2}{\mu_o R}$, where R is the radius of curvature of a field line (see in Figure 2.1). This curvature force acts toward the center of curvature (Fig. 2.1b) and is the magnetic-field-line analog of the force that acts on a curved object under tension [10].

The term involving ∇B^2 portrays a magnetic force due to pressure gradients perpendicular to the magnetic field.

$$P_{mag} = \frac{4\pi}{\mu_o}\left(\frac{B^2}{8\pi}\right) \tag{2.2.41}$$

The magnetic pressure at a raidal distance r is given by:

$$P_{mag} \cong \left(\frac{B^2}{2\mu_o}\right) \tag{2.2.42}$$

The infalling matter can be considered to have fallen from rest at infinity, so its infall velocity at a radius r is

$$v_{ff} = \sqrt{\frac{2Gm}{r}} \tag{2.2.43}$$

Stars more massive than $\sim 8M_\odot$ evolve to an onion-like configuration (Figure 1.1), with an iron core surrounded by successive layers of silicon, oxygen, carbon, helium, and finally hydrogen. In addition to iron nuclei, the core is composed of electrons, positrons, photons, and a small fraction of protons and neutrons. The pressure in the core, which supports it against the inward pull of gravity, is dominated at this stage by the electrons, and the balance between the electron pressure and gravity is only marginally stable. As a result of electron capture on the free protons and nuclei in the core and as a result of nuclear dissociation under the extreme densities and temperatures, electron and thermal pressure support are reduced, and the core becomes unstable and collapses [8]. The pressure in the inner core increases dramatically as the result of Fermi effects and the repulsive nature of the nucleon-nucleon interaction potential at short distances. As a result of this dramatic increase in pressure, the inner core becomes incompressible and rebounds. Here waves propagating radially outward reach the point at which the velocity of the infalling matter is supersonic. This sets up a density pressure, and velocity discontinuity in the flow i.e., a shock wave. This shock wave will ultimately propagate outward through the star, disrupting the star undergoing collapse. The shock wave is launched and energized by the rebounding inner core piston. If the shock wave were to propagate outward without stalling, we would have what has been called a prompt explosion. But this prompt-explosion is not likely to occur because the shock wave loses its energy due to the photodisintegration of nuclei and the thermal neutrino emission, and then the shock wave stagnates in route with in the outer core. In fact the thermal neutrinos take away most energy ($\sim 10^{53}$) erg which has been released by the gravitational energy. If one percent of this amount of energy were transported again to the stalled shock waves, it would lead to a successful explosion. This is an idea of the so-called delayed-explosion mechanism. During the collapse when the density reaches $\sim 10^{12} gcm^{-3}$neutrinos are trapped in the core [3]. The copious production of electron neutrinos occurs when the core electrons capture on the newly dissociation-liberated protons. These neutrinos are initially trapped but escape when the shock moves out beyond the electron neutrinosphere. This gives rise to the electron neutrino burst in a core collapse supernova, which is the first of three major phases of the three-flavor neutrino emission during these events. As a result of these two enervating mechanisms, the shock stalls in the iron core. At the time the shock stalls, the core configuration is composed of a central radiating object, the proto-neutron star (Figure 1.3), which will go on to form a neutron star or black hole. The proto-neutron star has a relatively cold inner part, composed of unshocked bulk nuclear matter, together with a hot mantle of nuclear matter that has been shocked but not expelled. The ultimate source of energy in a core collapse supernova is the $\sim 10^{53}$ erg of gravitational binding energy associated with the formation of the neutron star. This gravitational binding energy is released after core bounce over ~ 10 s in the form of a three-flavor neutrino pulse. The stalled supernova shock is thought to be revived, at least in part, by the charged-current absorption of electron neutrinos and antineutrinos that emerge from the proto-neutron star, a fraction of which are absorbed by protons and neutrons behind the shock. This is known as the delayed shock or neutrino-heating mechanism, originally proposed by Wilson and Bethe. Although the total energy emitted in neutrinos is two orders of magnitude greater than what is required for the generation of an $\sim 10^{51}$ erg explosion, deciphering the precise role of this neutrino heating in the supernova mechanism [8].

The pressure which this infalling gas exerts up on the magnetic field is known as the ram pressure, P_{ram} and is just the rate at which momentum is transported inwards at radius r per unit area [9]. that is

$$P_{ram} = \rho u^2 \tag{2.2.44}$$

This ram pressure is balanced by the magnetic pressure of the compact star at a radius r,pushing the matter out wards and thus generate a magnetohydrodynamical shock. A magnetohdrodynamical shock wave will then be generated when the core rebounds at around neuclar matter density [9,3], such that

$$P_{ram} = P_{mag} \tag{2.2.45}$$

$$\rho u^2 = \left(\frac{B^2}{2\mu_o}\right) \tag{2.2.46}$$

From this we can calculate the magnetic field, this will be

$$B = (2\rho u^2 \mu_o)\frac{1}{2} \tag{2.2.47}$$

A star whose mass of its Hydrogen exceeds $1.44 M_\odot$ (the chandrasekhar limit) reaches the end of its thermonuclear evolution and implodes to form NS whose radius which occured around the is of $\sim$ 10 km [18]. The collapse ends when the density becomes high enough ($\rho \simeq 3 \times 10^{14} gcm^{-3}$). Perhaps a shock wave emerges, somewhere half-way in the collapsed core [14]. The free fall velocity of the infalling matter prior to the bounce can be calculated as:

$$
\begin{aligned}
u &= \left(\frac{2GM}{r}\right)^{\frac{1}{2}} \\
&= \left(\frac{2(6.67\times10^{-8}\frac{dyncm^2}{g^2})(1.44M_\odot)}{10km}\right)^{\frac{1}{2}} \\
&= \left(\frac{2(6.67\times10^{-8}\frac{dyncm^2}{g^2})(1.44\times1.989\times10^{33}g)}{1\times10^{6}cm}\right)^{\frac{1}{2}} \\
&= \left((6.67\times10^{-14}\frac{dyncm}{g})(1.44\times1.989\times10^{33})\right)^{\frac{1}{2}} \\
&= \left(38.21\times10^{19}\frac{\frac{gcm}{s^2}cm}{g}\right)^{\frac{1}{2}} \\
&= \left(38.21\times10^{19}\frac{cm^2}{s^2}\right)^{\frac{1}{2}} \\
&= \left(6.18\times3.16227766\times10^{9}\frac{cm}{s}\right) \\
&= \left(19.55\times10^{9}\frac{cm}{s}\right). \qquad (2.2.48)
\end{aligned}
$$

We used

$$1dyn = g\frac{cm}{s^2}, \qquad (2.2.49)$$

$$M_\odot = 1.989\times10^{33}g, \qquad (2.2.50)$$

$$G = 6.67\times10^{-8}\frac{dyncm^2}{g^2}, \qquad (2.2.51)$$

where G is the Universal gravitational constant. We can also calculate this amount of free fall velocity from the kinetic enegy which is produced during the core cllapse,$\sim 10^{51}erg$. tha is

$$KE = \frac{1}{2}mv^2 =\sim 10^{51}erg \qquad (2.2.52)$$

From this we get the free fall velocity $\sim 10^9\frac{cm}{s}$. The ram pressure corresponding to this free fall velocity then becomes:

$$\begin{aligned}P_{ram} &= \rho u^2\\ &= \rho \times 38.21 \times 10^{19}\frac{cm^2}{s^2}\\ &= 3 \times 10^{14}\frac{g}{cm^3} \times 38.21 \times 10^{19}\frac{cm^2}{s^2}\\ &= 114.63 \times 10^{33}\frac{g}{cms^2}. \qquad (2.2.53)\end{aligned}$$

We have already indicated that, when the ram pressure and the magnetic pressure are equall the supernova bounce is likely to occur.

$$P_{mag} = 1.1463 \times 10^{35}\frac{g}{cms^2}. \qquad (2.2.54)$$

$$\begin{aligned}B &= (2\rho u^2\mu_o)^{\frac{1}{2}}\\ &= (2(\times 114.63) \times 10^{33}\frac{g}{cms^2}\mu_o)^{\frac{1}{2}}\\ &= (229.26 \times 10^{33}\frac{g}{cms^2}(4\pi \times 10^{-7}\frac{H}{m}))^{\frac{1}{2}}\\ &= (229.26 \times 10^{33}\frac{g}{cms^2}(4\pi \times 10^{-2}\frac{gcm}{s^2A^2}))^{\frac{1}{2}},\\ &= (2879.51 \times 10^{31}\frac{g}{cms^2}(\frac{gcm}{s^2A^2}))^{\frac{1}{2}}\\ &= (287.951 \times 10^{32}\frac{g^2}{s^4A^2})^{\frac{1}{2}}\\ &= (287.951 \times 10^{32}\frac{g^2s^2}{s^4c^2})^{\frac{1}{2}}\\ &= 2.87951 \times 10^{17}gs^{-1}c^{-1}\\ &= 2.87951 \times 10^{18}G. \qquad (2.2.55)\end{aligned}$$

We used

$$1G = 10^{-4}kgc^{-1}s^{-1}, \qquad (2.2.56)$$

$$\mu_o = 4\pi \times 10^{-7}\frac{H}{m}, \qquad (2.2.57)$$

$$H = \frac{Wb}{A} = \frac{m^2kg}{s^2A^2} = \frac{10^7gcm^2}{s^2A^2}. \qquad (2.2.58)$$

When the core density gets close to $10^{14}gcm^{-3}$, which is the density of a neutron star, the material becomes incompressible and the collapse of the central regions stops. As the compression phase ends abruptly, the core rebounds and the (supersonic) impact with the external layers that are continuing to contract produces a shockwave. In a perfectly elastic collision, the energy of the infalling outer portion of the core would bounce back after reflection to its position before the collapse. If one compares this energy ($\sim 10^{53}$ erg) with the binding energy of the outer envelope ($\sim 10^{51} erg$) it seems possible that the rebounding core could be responsible for the expulsion of the whole stellar envelope.

Chapter 3

Stellar Structure of Stars

3.1 Basic Equations of Stellar Structure

The standard theory of stellar evolution is based on the following assumptions.

- Stars are spherically symmetric systems made of matter plus radiation. The effects of rotation and magnetic fields are negligible.
- The evolution of the physical and chemical quantities describing a star is slow, i.e. the temporal evolution of the stellar structure can be described by a sequence of models in hydrostatic equilibrium.

The assumption of hydrostatic equilibrium implies that the pressure has to increase toward the center. In order to increase the pressure, the equation of state dictates that density and temperature have to increase too.

3.1.1 Matter in the Interior

The matter in the interior of the star is described in terms of the energy-momentum tensor, $T^{\mu\nu}$ that assumes the form of a perfect fluid

$$T_{\mu\nu} = (\rho c^2 + P)U_\mu U_\nu + P g_{\mu\nu} \tag{3.1.1}$$

Where $g_{\mu\nu}$ are the covariant components of the metric tensor, u_μ is the local fluid four-velocity vector $u^\mu = dx^\mu/d\tau$, where $d\tau = \frac{ds}{c}$. It satisfies the normalization $u^\mu u_\mu = -1$, ρ is the total mass-energy density, and P the corresponding pressure. Assuming that the star is static and spherically symmetric, the three-velocity component of the vector field vanishes. We consider now case of spherically symmetric compact star in hydrostatic equilibrium, Due to the high symmetries of these objects, all non-diagonal elements in the metric vanish, and, due to the static requirements for the gravitational fields, the metric elements are mere functions of the position of a spherically symmetric shell. Static and spherically symmetric non-rotating stars of a space-time has the following form

$$ds^2 = \exp(2\Phi(r))c^2dt^2 - \exp(2\lambda(r))dr^2 - r^2(d\theta + \sin^2\theta d\phi^2) \tag{3.1.2}$$

The coordinates θ and ϕ are polar and azimuthal of spherical coordinates, and the two functions $\Phi(r)$ and $\lambda(r)$ are uniquely given by the mass-energy distribution $\rho(r)$ in the star. According to the assumption of spherical symmetry all parameters describing the star depend only on one quantity, i.e. the distance r from the center with thickness dr. By denoting with ρ the value of the matter density at a generic point r within the star, the mass contained within a sphere of radius r centered on the center of the star, is given by

$$M(r) = 4\pi \int_0^r \rho(r) r^2 dr \tag{3.1.3}$$

which is the continuity of mass equation. For an observer at rest the velocity $U = (U^i)$ vanishes where i= 1, 2, 3, Generally based on to the metric due to a rotating compact star, from the normalization we have

$$-1 = g_{00}(U^0)^2, \tag{3.1.4}$$

where by definition

$$U^0 = \frac{dx^0}{ds}, \tag{3.1.5}$$

This can be written as:

$$U_0 = g_{o\alpha}U^\alpha = -\sqrt{-g_{00}}. \tag{3.1.6}$$

Where $U_1 = U_2 = U_3 = 0$, for spherically symetric star the source of the gravitational filed and the metric has the form

$$g_{\mu\nu} = \begin{pmatrix} -e^{2\Phi} & 0 & 0 & 0 \\ 0 & e^{2\lambda} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin(\theta) \end{pmatrix}, \tag{3.1.7}$$

where $\Phi(r)$ and $\lambda(r)$ are to be determined. Consistent with our assumption of spherical symmetry, let us assume that the material of the star has an equation of state which exhibits no transverse strains, so that all the off-diagonal elements of the stress energy tensor are zero and the first three spatial elements are equal to the matter equivalent of the energy density. The fourth diagonal component is just the matter density so the result will be:

$$T_{\mu\nu} = \begin{pmatrix} -e^{2\Phi}\rho & 0 & 0 & 0 \\ 0 & e^{2\lambda}p & 0 & 0 \\ 0 & 0 & r^2p & 0 \\ 0 & 0 & 0 & r^2\sin(\theta)p \end{pmatrix}, \tag{3.1.8}$$

This demonstrates that Einsteins field equations

$$G^{\mu}_{\nu} = \frac{8\pi G}{c^4} T^{\mu}_{\nu}, \tag{3.1.9}$$

This can be written as

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{3.1.10}$$

From the Einsteins field equations $G_{\mu\nu}$ is given by :

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{3.1.11}$$

where $R_{\mu\nu}$ is calculated from the cervature tensor

$$R_{\mu\nu} = \frac{\partial \Gamma^{\lambda}_{\mu\lambda}}{\partial x^{\kappa}} - \frac{\partial \Gamma^{\lambda}_{\mu\kappa}}{\partial x^{\lambda}} + \Gamma^{\eta}_{\mu\lambda}\Gamma^{\lambda}_{\kappa\eta} - \Gamma^{\eta}_{\mu\kappa}\Gamma^{\lambda}_{\lambda\eta}, \tag{3.1.12}$$

where the contracted recci tensor is:

$$R = g^{\mu\nu} R_{\mu\nu} \tag{3.1.13}$$

and the affine connection is given by:

$$\Gamma^{\delta}_{\lambda\mu} = \frac{1}{2} g^{\nu\delta} \left(\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\lambda}}{\partial x^{\nu}} \right) \tag{3.1.14}$$

The only non vanishing componensts are

$$\begin{aligned}
\Gamma^{r}_{rr} &= \frac{1}{2A}\frac{dA}{dr}, \\
\Gamma^{r}_{\theta\theta} &= \frac{-r}{A} \\
\Gamma^{r}_{\varphi\varphi} &= \frac{-r\sin^2\theta}{2A}, \\
\Gamma^{r}_{tt} &= \frac{1}{2A}\frac{dB}{dr} \\
\Gamma^{\theta}_{r\theta} &= \Gamma^{\theta}_{\theta r} = \frac{1}{r} \\
\Gamma^{\theta}_{\varphi\varphi} &= -\sin\theta\cos\theta \\
\Gamma^{\varphi}_{\varphi r} &= \Gamma^{\varphi}_{r\varphi} = \frac{1}{r} \\
\Gamma^{\varphi}_{\varphi\theta} &= \Gamma^{\varphi}_{\theta\varphi} = \cot\theta
\end{aligned}$$

$$\Gamma^{t}_{tr} = \Gamma^{t}_{rt} = \frac{1}{2B}\frac{dB}{dr} \tag{3.1.15}$$

Then the recci tensor will be:

$$R_{rr} = \frac{1}{2B}\frac{d^2B}{dr^2} - \frac{1}{4B}\frac{dB}{dr}\left(\frac{dA}{Adr} + \frac{dB}{Bdr}\right) - \frac{1}{rA}\frac{dA}{dr} \tag{3.1.16}$$

$$R_{\theta\theta} = -1 + \frac{r}{2A}\left(-\frac{dA}{Adr} + \frac{dB}{Bdr}\right) + \frac{1}{A} \tag{3.1.17}$$

$$R_{tt} = -\frac{1}{2A}\frac{d^2B}{dr^2} - \frac{1}{4A}\frac{dB}{dr}\left(\frac{dA}{Adr} + \frac{dB}{Bdr}\right) - \frac{1}{rA}\frac{dB}{dr}, \tag{3.1.18}$$

where $g_{rr} = A(r) = \exp 2\lambda$, $g_{\theta\theta} = r^2$, $g_{\varphi\varphi} = r^2 \sin^2\theta$ and $g_{tt} = -B(r) = -c^2 \exp 2\Phi$

The gravitational field equation's now reduce to three equations

$$\frac{1}{r^2} - \frac{e^{-2\lambda}}{r}(\frac{1}{r} - 2\frac{d\lambda}{dr}) = \frac{8\pi G\rho}{c^2} \tag{3.1.19}$$

$$-\frac{1}{r^2} + \frac{e^{-2\lambda}}{r}(\frac{1}{r} + 2\frac{d\Phi}{dr}) = \frac{8\pi GP}{c^4} \tag{3.1.20}$$

$$\frac{e^{-2\lambda}}{r}[-\frac{d\lambda}{dr}\frac{d\Phi}{dr} + (\frac{d\Phi}{dr})^2 + \frac{d^2\Phi}{dr^2} + \frac{1}{r}(\frac{d\Phi}{dr} - \frac{d\lambda}{dr})] = \frac{8\pi GP}{c^4} \tag{3.1.21}$$

The first two equations of the reduced gravitational field equation provides us two for the function $\lambda(r)$ and $\Phi(r)$ these are

$\frac{1}{r^2} - \frac{e^{-2\lambda}}{r}(\frac{1}{r} - 2\frac{d\lambda}{dr}) = \frac{8\pi G\rho}{c^2}$ and

$-\frac{1}{r^2} + \frac{e^{-2\lambda}}{r}(\frac{1}{r} + 2\frac{d\Phi}{dr}) = \frac{8\pi GP}{c^4}$ respectively.

The first reduced gravitational field equation is equivalent to

$$\frac{d(r\exp(-2\lambda))}{dr} = 1 - \frac{8\pi Gr^2}{c^2}\rho, \tag{3.1.22}$$

which can be integrated with the asymptotic flatness condition to yield the three space metric

$$\exp(-2\lambda) = 1 - \frac{2GM(r)}{rc^2}, \tag{3.1.23}$$

With the total mass inside radius r given as $M(r) \equiv 4\pi \int_0^r \rho(r) r^2 dr$.

adding $-\frac{1}{r^2} + \frac{e^{-2\lambda}}{r}(\frac{1}{r} + 2\frac{d\lambda}{dr}) = \frac{8\pi GP}{c^4}$ and $\frac{1}{r^2} - \frac{e^{-2\lambda}}{r}(\frac{1}{r} - 2\frac{d\lambda}{dr}) = \frac{8\pi G\rho}{c^2}$ we obtain

$$\exp(-2\lambda)(\frac{d\Phi}{dr} + \frac{d\lambda}{dr}) = \frac{4\pi G}{c^2}(\rho + P)r, \tag{3.1.24}$$

This gives

$$\frac{d\Phi(r)}{dr} = \frac{1}{(1 - \frac{2GM(r)}{rc^2})}(\frac{GM(r)}{r^2c^2} + \frac{4\pi GrP}{c^4}), \tag{3.1.25}$$

This demonstrates now, how the gravitational force is generalized in GR. In particular, pressure is a source of the gravitational field, and the Schwarzschild metric acts as a modification in the denominator of the force law.

Therefore, all the metric functions can be eliminated from the third of Einsteins equations by substitution of the above results. In this way, we obtained after some lengthy calculations the equation for the relativistic hydrostatic equilibrium, the entire structure of a compact star is then determined by the four equations called Tolman-Oppenheimer-Volkoff (TOV) equations [12].

$$\frac{dM(r)}{dr} = 4\pi\rho(r)r^2 \tag{3.1.26}$$

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}\left[1+\frac{P(r)}{\rho(r)c^2}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)c^2}\right]\left[1-\frac{2GM(r)}{rc^2}\right]^{-1} \tag{3.1.27}$$

This equation determines the structure of a relativistic star.

$$e^{-2\lambda(r)} = 1 - \frac{2GM(r)}{rc^2} \tag{3.1.28}$$

$$\frac{d\Phi(r)}{dr} = \frac{1}{1-\frac{2GM(r)}{rc^2}}\left[\frac{GM(r)}{r^2c^2} + \frac{4\pi GrP}{c^4}\right] \tag{3.1.29}$$

This equation could also be obtained directly from the equations of motion $T_{rr;r} = 0$. This equation is, however, a mere consequence of Einsteins equation and should not be considered as an independent equation. This demonstrates once again that $\Phi(r)$ is the analogue of the Newtonian potential $\Phi(r)$;

$$\frac{dP}{dr} = -(\rho c^2 + P)\frac{d\Phi}{dr} \tag{3.1.30}$$

A further consequence of this derivation is a relation for the gravitational force

$$\frac{d\Phi}{dr} = \left(\frac{GM(r)}{c^2r^2}\right)\left(\frac{1+\frac{4\pi r^3 P}{M(r)c^2}}{1-\frac{2GM(r)}{c^2 r}}\right) \tag{3.1.31}$$

For a given equation of state $P = P(\rho)$, the TOV equations can easily be integrated from the origin with initial conditions M(0) = 0 and an arbitrary value for the central density $\rho_c = \rho(0)$, until the pressure P(r) will vanish at some radius R. Notice that the equation which used to determine the the pressure has a singular at radius $r_s = \frac{2GM(r)}{c^2}$ called the schwarzschild radius and scales with mass. It is then obvious that these structure equations go over into the Newtonian analog for $P \ll \rho c^2$, i.e. roughly speaking for sound velocities much less than the velocity of light, for low compactness $\frac{2GM(r)}{c^2} \ll r$ and for low pressure-mass $4\pi r^3 P(r) \ll M(r)c^2$. The compactness parameter has a particular influence on the hydrostatic equilibrium (the last factor in the TOV equation). In this limit, three-space is flat,i.e. $\exp(\lambda) \equiv 1$ for all radii, and $\Phi(r) \equiv 1+\frac{\Phi(r)}{c^2}$ with the following structure equations, usually derived in the theory of stellar structure,

$$\frac{d\Phi(r)}{dr} = \frac{GM(r)}{r^2} \tag{3.1.32}$$

$$\frac{dM(r)}{dr} = 4\pi\rho(r)r^2 \tag{3.1.33}$$

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2} \tag{3.1.34}$$

the 'equation of hydrostatic equilibrium'. This equation implies that the pressure decreases outwards, since the right-hand side is always negative [15,11]. We can easily rewrite this Equation using m_r as an independent variable by noting that $dm_r = 4\pi r^2 \rho dr$, obtaining

$$\frac{dP}{dm_r} = -\frac{GM(r)}{4\pi r^4} \tag{3.1.35}$$

3.1.2 Exterior Solution

As a further consequence we see that for vanishing pressure, at r = R, P(R)= 0 i.e. in the exterior region of a star with $r > R$, the solution of the TOV equations is simply given by the Schwarzschild solution.

The mass distribution, $r \geqslant R$ is

$$M = M(R) = 4\pi \int_0^R \rho r^2 dr \tag{3.1.36}$$

$$\frac{d\Phi}{dr} = \frac{GM}{r^2(1-\frac{2GM}{rc^2})} \tag{3.1.37}$$

The second equation can be integrated with the boundary condition $\exp[\Phi(r)] \rightarrow 1$ for $r \rightarrow \infty$

$$e^{2\Phi(r)} = 1 - \frac{2GM}{rc^2} \tag{3.1.38}$$

$$e^{-2\lambda(r)} = 1 - \frac{2GM}{rc^2} \tag{3.1.39}$$

This is the famous Schwarzschild solution with its metric [2,33]:

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2dt^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 - r^2(d\theta + \sin^2\theta d\phi^2) \qquad (3.1.40)$$

uniquely determined by the mass M of the central object.

Supernova explosion is associated with neutron star, therefore in this section we deal mainly with the stellar structure of neutron stars. A typical $1.4M_{\odot}$ neutron star has a radius of 10km (or10-15 km), This means that a neutron star is small size and high density, central density of the order of 10^{14} - 10^{15}g cm^{-3}. The main source of the pressure needed to counterbalance the gravitational force is the degenerate neutrons, since electron degeneracy cannot support objects more massive than M_{ch} . Being degenerate objects, neutron stars follow a mass - radius relationship like WDs [15].

3.2 Neutron Stars

Neutron stars are one of the possible end states for a massive star. They result from massive stars which have mass greater than 8 $M_{\odot}$. After these stars have finished burning their nuclear fuel, they undergo a supernova explosion. This explosion blows off the outer layers of a star into a beautiful supernova remnant. The central region of the star collapses under gravity. It collapses so much that protons and electrons combine to form neutrons [2]. Neutron star is supported by the pressure of “cold“ degenerate neutrons, almost all electrons and protons having been converted in to neutrons through reaction. $p + e^- \rightarrow n + \nu$ the neutrinos escaping the star. A star whose mass is above the chandrasekhar limit reaches the end of its thermonuclear evolution and grows cold. Its internal pressure than fails to support it, and it collapses. In the process it either gravitionally collapses to a black hole or it will be come so heated during its collapse that it will explode becoming supernova. It might then blow off enough matter so that its mass drops below the chandrasekhar limit. It is believed that in this case the highly compressed core forms a superdense neutron star [12]. A dense ball of neutrons that remains after a supernova explosion has expelled the rest of the matterial in side the progenitor stars. Neutron stars may appear in supernova remnants, as isolated objects, or in binary systems. When a neutron star is in a binary system, astronomers are able to measure its mass. From a number of such binaries seen with radio or X-ray telescopes, the neutron star mass has been found to be close to have masses of about 1.4 times the mass of the Sun. For binary systems containing an unknown object, this information helps distinguish whether the object is a neutron star or a black hole, since black holes are more massive than neutron stars.

3.2.1 The Structure of a Neutron Star

The cross-section of a neutron star can roughly be divided into four distinct regions (see Fig. 3.1):

- The atmosphere which is only a few cm thick.

Neutron stars with masses below 1.2 $M_\odot$ have central densities not exceeding three times nuclear density. In these stars, the core merely consists of neutrons, protons, electrons and muons. In neutron stars with masses higher than this critical mass, the core has certainly a more complex structure and probably two cores. The structure of these stars can be summarized as follows:

- The outer crust is found underneath an atmosphere of a just a few meters of thickness. It consists of a lattice of ions (atomic nuclei) and degenerate, relativistic electrons. The electrons are strongly degenerate and forms an almost ideal Fermi gas, which is relativistic. The density in the crust increases from $10^6 gcm^{-3}$ to $10^{11} gcm^{-3}$. The ions form a strongly coupled Coulomb system, which is solid in most of the crust, but liquid at the lowest densities. The electron Fermi energy grows with increasing density and, as a consequence, nuclei tend to become richer in neutrons. The thickness of the outer crust is a few hundred meters [11].

• At the base of the outer crust neutrons begin to drip out of nuclei, thereby producing a neutron gas between the nuclei. The density increases from the neutron drip density of, $\sim 4 \times 10^{11}$ gcm^{-3} to a transition density, at which the density has reached $\rho_{tr} \backsimeq 2 \times 10^{14} gcm^{-3}$. In the inner crust, matter consists of electrons, free neutrons and neutron-rich atomic nuclei. The fraction of free neutrons increases with increasing density, and at the bottom of the crust, it also contains degenerate neutron gas (it may be super fluid), at a density of about half nuclear saturation density. Where nuclei occupy a significant fraction of space, nuclei are probably far from bieng spherical. At this density nuclei disappear, and matter then becomes a uniform fluid of neutrons, protons and electrons. The EoS throughout the crust is sufficiently well understood for the purpose of building neutron star models.

• Beyond the transition density one enters the core, where all atomic nuclei have been dissolved into their constituents, neutrons and protons. Due to the high Fermi pressure, the core might also contain hyperons, more massive baryon resonances, and possibly a gas of free up, down and strange quarks. At densities around nuclear density, matter consists of neutrons with a small admixture of protons, electrons and muons. All constituents are strongly degenerate. The neutrons and protons, which interact via nuclear forces, constitute a strongly non ideal liquid. The EoS is still not yet known with sufficient accuracy in the range above twice nuclear density.

• When the density reaches three times nuclear density, In the inner core, other particles probably appear (e. g., hyperons, pions, kaons and possibly quarks). The exact value of the transition density is still uncertain, but most probably it is in the range of two to four times nuclear density [2,11]. The region of the neutron star about which we know the least, the inner core, contains most of the stars mass [32].

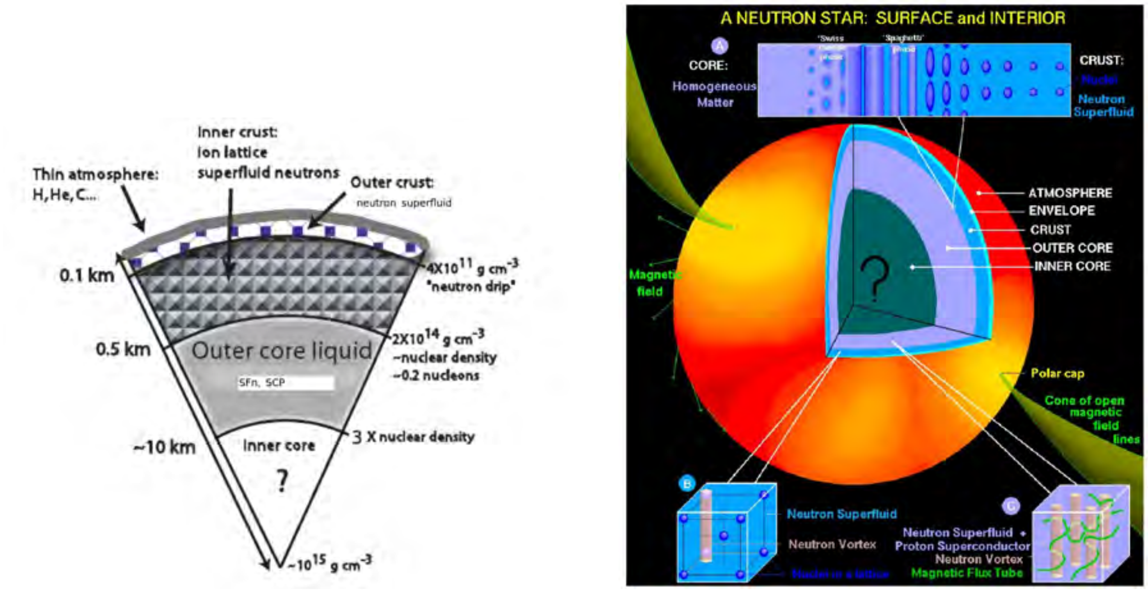

Figure 3.1: Cross-section through the interior of a neutron star. The neutron star is surrounded by a thin atmosphere and an outer crust consisting of heavy nuclei and electrons. The inner crust consists of nuclei, neutrons and electrons, which at nuclear density make a transition to a neutron fluid. The composition of the central core is still unclear, but certainly consists in the outer part only of neutrons, protons, electrons and muons [2].

3.2.2 Basic Equations of Stellar Structure of Neutron Stars

In the strongly degenerate case the ion pressure is much smaller than that of the degenerate electrons or neutros, and can usually be neglected. The study of pressure of a neutron stars begins with the successful description of their properties using Fermi-Dirac statistics. They are held up against gravitational collapse by degeneracy pressure of the neutrons [14]. Neutron stars of low mass are much like white dwarfs of the same mass, except that neutron degeneracy pressure replaces electron degeneracy pressure, and thus m_e should be replaced in all formulas with m_n (and μ should be set equal to unity). The central density at which the Fermi momentum k_F becomes equal to $m_n c$. where $2(4\pi k^2(2\pi\hbar)^{-3}dk)$ levels per unit volume (phase space factor) with momenta between k and dk and another factor of two comes from the spin of the neutrons. So the the number of neutron per unit volume (For a gas of fermions the number density of particles) will be related to the maximum momentum K_F by:

$$\frac{dn(k)}{dk} = \frac{8\pi f(k)k^2}{h^3}, \tag{3.2.1}$$

where f(k) is the Fermi-Dirac distribution. For a fully degenerate gas f(k) = 1 for $k < k_F$ and f(k) = 0 for $k > k_F$, allowing us to solve for k_F

$$n = \frac{8\pi}{(2\pi\hbar)^3}\int_0^{k_F} k^2 dk = \frac{k_F^3}{3\pi^2\hbar^3} \tag{3.2.2}$$

$$k_F = \hbar\left(\frac{3\pi^2\rho}{m_N}\right)^{\frac{1}{3}} \tag{3.2.3}$$

This can be written in terms of the Fermi energy using

$$E_F = \sqrt{k_F^2c^2 + m^2c^4} \tag{3.2.4}$$

$$k = \frac{m_n v}{\sqrt{1-\frac{v^2}{c^2}}} \tag{3.2.5}$$

For a non-relativistic gas $E_F = \frac{k_F^2}{2m_n}$, while for a relativistic gas $E_F = k_F c$. In order to formulate the quantitive theory of neutron stars we begin by writing down expresions for the total energy density and pressure of an ideal fermi gas of neutrons with maximam momentum,k_F.

$$\begin{aligned}\rho = \quad & \frac{8\pi}{(h)^3}\int_0^{k_F}(\frac{k^2}{c^2}+m_n^2)^{\frac{1}{2}}k^2 dk \\ & = 3\rho_c\int_0^{\frac{k_F}{m_n}}(u^2+1)^{\frac{1}{2}}u^2 du\end{aligned} \tag{3.2.6}$$

$$P = \frac{1}{3}\int_0^{\infty} v(k)k\frac{dn(k)}{dk}dk, \tag{3.2.7}$$

$$\begin{aligned}P = \quad & \frac{8\pi}{3(h)^3}\int_0^{k_F}\frac{k^2}{(\frac{k^2}{c^2}+m_n^2)^{\frac{1}{2}}}k^2 dk \\ & = \rho_c\int_0^{\frac{k_F}{m_n}}(u^2+1)^{\frac{-1}{2}}u^4 du\end{aligned} \tag{3.2.8}$$

where (in c.g.s units)

$$\rho_c \equiv \frac{8\pi m_n^4 c^3}{3(h)^3} = 6.1 \times 10^{15} gm(cm)^{-3} \tag{3.2.9}$$

By eliminating $\frac{k_F}{m_n}$ in the two above equation we obtain equation of state in the form.

$$\frac{P}{\rho_c} = F\frac{\rho}{\rho_c} \tag{3.2.10}$$

with F a transcendental function. From the fundamental differential equation we have

$$-r^2\frac{dP}{dr} = GM(r)\rho(r)\left[1 + \frac{P(r)}{\rho(r)}\right]\left[1 + \frac{4\pi r^3\rho(r)}{M(r)}\right]\left[1 - \frac{2GM(r)}{r}\right]^{-1} \tag{3.2.11}$$

Using $\rho_c = 6.1 \times 10^{15} gm(cm)^{-3}$ and for $\rho(o) \gg \rho_c$ the neutron near the center of the star have $k_F \gg m_n$, the equation for ρ and Pgives:

$$\rho = \frac{3\rho_c}{4}\left(\frac{k_F}{m_n}\right)^4 \tag{3.2.12}$$

$$P = \frac{\rho_c}{4}\left(\frac{k_F}{m_n}\right)^4 \tag{3.2.13}$$

and therefore we can get an equation of state

$$P = \frac{\rho}{3} \tag{3.2.14}$$

Using this equation of state in the fundamental differential equation we get:

$$-r^2\frac{d\rho}{dr} = GM(r)\rho(r)\left[1 + \frac{4\pi r^3\rho(r)}{3M(r)}\right]\left[1 - \frac{2GM(r)}{r}\right] \tag{3.2.15}$$

from which then follows

$$\rho(r) = \frac{3}{56\pi Gr^2} \tag{3.2.16}$$

This leads to the limit $\rho(0) \to \infty$. The equation of state, $P = \frac{\rho}{3}$ is not valid for the outer layers of any neutron star. The important point is that this equation leads to afinite radius R where ρ vanishes, and that the mass M with in this radius is finite as well.

Chapter 4

Discussion and Conclusion

A supernova explosion is the end of the life of most massive stars. Stellar evolution toward supernova can be divided into several phases the first being nuclear burning. This includes hydrogen burning, helium burning, carbon burning,neon burning, oxygen burning and Silicon burning. Because iron the ash form of Silcon burning is the most tightly bound nucleus. The star therefore is no longer able to produce energy in the core from the deposited iron. The Iron core begins to contract due to its own gravity. The contraction increases the density and temperature, and when the density reaches upto the onset of electron capture, the pressure decreases and the gravitational collapse set about. The collapse continues until the central density reaches nuclear density, in which the pressure at the center of the core becomes sufficiently large to prevail the local gravity with the help of the nuclear force. Under such extreme conditions electron degeneracy cannot support the stellar core, and the free electrons are forced to fuse with protons to form neutrons. Then bounce occurs and a shock wave is generated at the boundary of the inner and the outer core.

The idea of Maxwell stresses existing in an electromagnetic field has been reviewed, it has been used to analyses the magnetic pressure in connection with the magnetic force density, which was derived from the classical stand point that is the force of the Lorentz law. This magnetic pressure is used to support the star from gravitational collapse. We have found a magnetic stress tensor in magnetized matter that has the correct vacuum limit. The magnetic force density has two parts: first, the negative of the gradient of the magnetic pressure $\frac{B^2}{2\mu_o}$ orthogonal to $\mathbf{B}$, and second, an orthogonal curvature force $\frac{(\mathbf{B}\cdot\nabla)\mathbf{B}}{\mu_o}$, which has magnitude $\frac{\mathbf{B}^2}{\mu_o R}$, where R is the radius of curvature of a field line. This curvature force acts toward the field lines center of curvature and is the magnetic-field-line analog of the force that acts on a curved object under tension [10].

In a magnetized fluid, we show that the magnetic pressure is perpendicular to the magnetic field. The magnetic field which we found here is $\gtrsim 10^{18}$ G. Thus, a neutron star with an extremely strong magnetic field will definitely not collapse, as has been proposed. Neutron stars possess some of the strongest magnetic fields observed in nature. Observations of isolated radio pulsars and magnetars provide strong evidence that neutron-star magnetic fields range between $\sim 10^8$ G and $\sim 10^{15}$ G. However our calculation shows that magnetic fields. $\gtrsim 10^{18}$ G can be exist inside NSs.

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Declaration

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

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