EFFECTIVE HAMILTONIAN APPROACH TO FERMION-ANTIFERMION BOUND STATES IN QUANTUM FIELD THEORY

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Abstract

We discuss the effective Hamiltonian approach to fermion-antifermion bound states, which imitates its description in terms of non-relativistic potential models. This approach is the relativistic generalization of the non-relativistic potential models. In this framework, observables such as, the ground state energy and the triplet-singlet mass differene of light mesons are calculated. We use both linear and funnel potential confinements comparatively. Though the constituents of mesons basically interact through the exchange of vector bosons, we also consider the constituents as if they interact through the exchange of pure scalar bosons.

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Chapter 1

Introduction

One of the most popular approaches to hadrons is to describe them by means of nonrelativistic potential models as a bound state of quarks which interact via some effective potential, since the confinement is not yet derivable from QCD. The overwhelming success of these nonrelativistic potential models not only for heavy quarkonia but also for light mesons remains up to now a miracle in hadron spectroscopy[1]. But in principle, at least bound states consisting of light constituents should be dealt with in a relativistic frame work. So we scrutinize relativistic wave equations for the description of quark-antiquark bound states. It does appear quite certain by now that most of the observed bosons are the bound states of fermions and antifermions which are presumably elementary[2]. It appears that some bosons such as π and ρ are so tightly bound that they have to be treated relativistically. As long as it makes sense to say that π, ρ etc are the bound states of a fermion and an antifermion, there must be a way to understand these bound states based up on some effective interaction which depends only on these fermions. In order to formulate this idea in a relativistic manner we begin with the conventional (and thus relativistic) field theory in which the fermions and antifermions interacts with the exchange of bosons.

We then simply replace the usual interaction Hamiltonian by an effective interaction Hamiltonian, which is invariant and depends only on the fermion and antifermions. Beyond doubt, the appropriate frame work for a relativistic description of quantum systems is quantum field theory. However, the treatment of bound states with in this frame work becomes a rather cumbersome and unrewarding task[3]. Therefore we would like to argue here in favor of an effective Hamiltonian method. This approach is reminiscent of the investigation of bound states in terms of nonrelativistic potential models. It allows one to remain from the very beginning on fully relativistic grounds,in contrast to, the nonrelativistic formalisms which incorporate only the static limit of the theory. The relativistic treatment of fermion-antifermion bound states by an effective Hamiltonian method, which we have used in this work, imitates their description in terms of non relativistic potential models where, the effective interaction potential to be used in schrodinger equation which incorporates relativistic kinematics, is derived from the underlying quantum field theory. Obviously this effective Hamiltonian method might be regarded as the relativistic generalization of the description of fermion-antifermion bound states in terms of non relativistic potential models. As far as the incorporation of relativistic kinematics is concerned, it provides a description of bound states which is as good as the treatment using the reduced Saltpeter equation[4]. The obvious advantage of the effective Hamiltonian approach is its physical transparency[4]. The basic idea of the proposed effective Hamiltonian method is to approximate by a potential (the perturbatively accessible part of the) interaction between particles which in fact are described by some quantum field theory (QFT). The main idea behind the effective Hamiltonian approach for treating

fermion-antifermion bound states is, the interaction between the bound state constituents is described by an effective potential. By considering the elastic scattering of the particles which build up the bound state, this potential is derived from the QFT which describes, in fact their basic interaction. With this effective potential at hand, the Hamiltonian controlling the bound state system is constructed. The strategy of this investigation is organized as follows: In chapter(2) we describe the gauge theories in QED and QCD for electromagnetic and strong interactions respectively, which will be used in the next chapters. In chapter(3) we present derivation of the effective potentials in electron-electron and nucleon-nucleon systems from QED and meson field theory. Towards this end we have checked the validity of this approach by deriving the static coulomb interaction and the spin-orbit interaction between two electrons. Then we derive the Yukawa and the one-pion exchange potentials between two nucleons, and we get the expected results. Chapter(4) is concerned about the investigation of fermion-antifermion bound states. We mainly focus on this chapter in deriving the transition amplitude, the ground state energy, and the squared mass difference between corresponding triplet and singlet of light mesons which serves as a tool to demonstrate the wide range of applicability of the effective Hamiltonian method as just relativistic kinematical effect. We use two types of confinements: (i) linear potential and (ii) funnel potential (Coulomb-plus-linear confinement) between quarks and antiquarks to derive the observables stated above. Next we propose an effective theory involving a scalar boson exchange between the constituents of a bound state and derive the ground state energy, and the squared mass difference between the spin-triplet and spin-singlet states, as an academic exercise in applications of QFT. Then finally chapter(5) gives us a conclusion about the study.

Chapter 2

Gauge Theory: QED and QCD

All the known interactions that occur in nature can be reduced to four fundamental interactions between material particles[5]. Listed in order of decreasing strength, these are: The strong interaction, electromagnetism, the weak interaction and gravitational interaction. This chapter is devoted to electromagnetic and strong interactions. The theory Corresponding to electromagnetic interactions is called Quantum Electrodynamics (QED). The mediators of this interaction are photons. And the theory corresponding to strong interactions is called Quantum Chromodynamics (QCD), and the corresponding mediators are called gluons.

Table(1) QED and QCD

Mediator	Charge	mass	Life time	Force	Theory
photon	0	0	∞	electromagnetic	QED
gluon	0	0	∞	strong	QCD

2.1 Quantum electrodynamics (QED)

According to quantum field theory (QFT) two charged particles interact through the exchange of photons, with each charged particle continually emitting them and the

other continually absorbing them. The Lagrangian for a free classical Dirac field is given by

$$\pounds_{Dirac} = -\bar{\psi}(x)(\gamma_\mu \partial_\mu + m)\psi(x). \tag{2.1.1}$$

Now lets see the gauge invariance of the Dirac free field. Lets take first global gauge transformation on $\psi(x)$. Such type of transformation does not depend on a specified space-time points. Let $\psi(x)$ transform to $\psi^{'}(x)$ as follows:

$$\psi(x) \rightarrow \psi^{'}(x) = U\psi(x); U = e^{i\alpha}, \tag{2.1.2}$$

where α is the same at all space-time points in the universe. This is called global gauge transformation on $\psi(x)$. And $\bar{\psi}(x)$ transforms to $\bar{\psi}^{'}(x)$ as

$$\bar{\psi}(x) \rightarrow \bar{\psi}^{'}(x) = e^{-i\alpha}\bar{\psi}(x). \tag{2.1.3}$$

Under this global gauge transformation the Dirac Lagrangian transforms as

$$\pounds_{Dirac} \rightarrow \pounds^{'}_{Dirac} = -\bar{\psi}^{'}(x)(\gamma_\mu \partial_\mu + m)\psi^{'}(x). \tag{2.1.4}$$

And can be easily checked that

$$\pounds^{'}_{Dirac} = \pounds_{Dirac}. \tag{2.1.5}$$

This implies that $\pounds_{Dirac}$ is invariant under global gauge transformation on $\psi(x)$. Now lets take local gauge transformation on $\psi(x)$. Such type of transformation depends on space time points and thus vary from point to point in space-time. Let $\psi(x)$ transforms to $\psi^{'}(x)$ as follows:

$$\psi(x) \rightarrow \psi^{'}(x) = U\psi(x); U = e^{i\alpha(x)}. \tag{2.1.6}$$

This is called local gauge transformation on $\psi(x)$. And $\bar{\psi}(x)$ transforms to $\bar{\psi}^{'}(x)$ as

$$\bar{\psi}(x) \rightarrow \bar{\psi}^{'}(x) = e^{-i\alpha(x)}\bar{\psi}(x). \tag{2.1.7}$$

The set of transformations $e^{i\alpha(x)}$ form a U(1) group and is an abelian group, since its elements commute with each other. Let under the above local gauge transformation, the Dirac lagrangian transforms as

$$\pounds_{Dirac} \rightarrow \pounds^{'}_{Dirac} = -\bar{\psi}^{'}(x)(\gamma_\mu \partial_\mu + m)\psi^{'}(x). \tag{2.1.8}$$

Then substituting $\psi^{'}(x)$ and $\bar{\psi}^{'}(x)$ in this equation, we get;

$$\pounds^{'}_{Dirac} = -\bar{\psi}(x)(\gamma_\mu \partial_\mu + m)\psi(x) - i\bar{\psi}(x)\gamma_\mu \partial_\mu \alpha(x)\psi(x). \tag{2.1.9}$$

$$\pounds^{'}_{Dirac} = \pounds_{Dirac} - i\bar{\psi}(x)\gamma_\mu \partial_\mu \alpha(x)\psi(x). \tag{2.1.10}$$

This tells us that $\pounds_{Dirac}$ is not invariant under local gauge transformation on $\psi(x)$, since there appears an additional term that spoils the gauge symmetry. Now lets modify $\pounds_{Dirac}$ to incorporate a term $ie\bar{\psi}(x)\gamma_\mu \psi(x)A_\mu(x)$ and also pull out a factor "e" from $\alpha(x)$ so that $\alpha(x) = e\Lambda(x)$. That means we introduce a term containing a gauge field $A_\mu(x)$ that transforms as $A_\mu(x) \rightarrow A^{'}_\mu(x) = A_\mu(x) + \partial_\mu \Lambda(x)$, so that we can write

$$\pounds = \pounds_{Dirac} + ie\bar{\psi}(x)\gamma_\mu \psi(x)A_\mu(x). \tag{2.1.11}$$

Now lets see the invariance of the above modified Lagrangian (2.1.11) under combined local gauge transformation on $\psi(x)$ and $A_\mu(x)$. Let under these transformations,

$$\pounds \rightarrow \pounds^{'} = \pounds_{Dirac} + ie\bar{\psi}^{'}(x)\gamma_\mu \psi^{'}(x)A_\mu(x). \tag{2.1.12}$$

Then substituting $\psi^{'}(x)$, $\bar{\psi}^{'}(x)$ and $A^{'}_\mu(x)$ in this Lagrangian we see that;

$$\pounds \rightarrow \pounds^{'} = -\bar{\psi}(x)(\gamma_\mu \partial_\mu + m)\psi(x) + ie\bar{\psi}(x)\gamma_\mu \psi(x)A_\mu(x). \tag{2.1.13}$$

This shows that the newly modified Lagrangian is invariant under local gauge transformation on both $\psi(x)$ and $A_\mu(x)$. Thus we can write the full QED Lagrangian by

incorporating a term $\pounds_{em} = -\frac{1}{4}(F_{\mu\nu})^2$, corresponding to free gauge field in the above Lagrangian as;

$$\pounds_{QED} = -\bar{\psi}(x)(\gamma_\mu \partial_\mu + m)\psi(x) - \frac{1}{4}(F_{\mu\nu})^2 + ie\bar{\psi}(x)\gamma_\mu \psi(x) A_\mu(x), \tag{2.1.14}$$

where $F_{\mu\nu}(x)$ is the electromagnetic field strength tensor written down in terms of electromagnetic four potential $A_\mu(x)$ as,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A\mu. \tag{2.1.15}$$

"e" which appears in the third term in (2.1.14) plays the role of a coupling constant between matter and gauge field (electromagnetic field). Now a simple prescription for making $\pounds_{Dirac}$ locally invariant is to replace; $\partial_\mu \rightarrow D_\mu = \partial_\mu - ieA_\mu(x)$. This is also called a minimal coupling prescription. All fundamental interactions in the physical world, originate by imposition of local gauge invariance on a free matter Lagrangian. This automatically gives rise to the terms in the Lagrangian describing the interaction between matter and gauge fields in a mathematically self consistent manner. Taking this in our mind the complete Lagrangian in quantum electrodynamics of an interacting field theory is given by: $\pounds_{QED} = \pounds_{Dirac} + \pounds_{em} + \pounds_{int}$, Where

$$\pounds_{int} = ie\bar{\psi}(x)\gamma_\mu \psi(x) A_\mu(x).$$

The QED Lagrangian can be written even more simply as

$$\pounds_{QED} = -\bar{\psi}(x)(\gamma_\mu D_\mu + m)\psi(x) - \frac{1}{4}(F_{\mu\nu})^2. \tag{2.1.16}$$

QED is a U(1) gauge theory, since the gauge group of this theory is U(1).

2.2 Quantum chromodynamics (QCD)

The nucleus of an atom is composed of protons and neutrons. A question naturally arises as to what holds the nucleus together. Positively charged protons should repel one another violently, but they are very strongly packed together as they are in such close proximity. Evidently there must be some other force, more powerful than the force of electrical repulsion, that binds the protons (and neutrons) together,and called strong force. The gauge theory corresponding to these strong interactions is called Quantum Chromodynamics. The mediators in such interactions are eight massless gluons. Initially, a theory on strong forces was proposed by Yukawa. He assumed that the protons and neutrons are attracted to one another by a strong force, just as the electron is attracted to the nucleus by an electric force. This field should properly, be quantized and he asked the question, what must be the properties of its "quantum", the particle (analogous to the photon) whose exchange should account for the known features of the strong force. For example, the short range of the force indicated that the mediator would be rather heavy. He calculated that its mass should be nearly 300 times that of the electron. Because it fell between the electron and the proton Yukawa's particle come to be known as the pi-meson (meaning "middle weight particle") where as the proton and neutron are baryons (heavy weight particles) and the electrons are called leptons (light weight particles). The leptons are characterized by the fact that they do not participate in strong interactions. The strongly interacting particles are, the mesons and baryons known collectively as hadrons. Protons, lambda and delta are some examples of baryons; pions and kaons are examples of mesons. In 1964 Gell-man and Zweig independently proposed that all hadrons are in fact composed of even more elementary constituents,

called quarks. The quarks come in various flavors(such as u, d, s etc). The "u" stands for up quark, "d" for down, and "s", for strange etc. To each quark (q) there corresponds an antiquark ($\bar{q}$),with additive quantum numbers such as charge, baryon number etc with reversed in sign. The quark model asserts that: (1) Every baryon is composed of three quarks (and every anti-baryon is composed of three anti quarks). (2) Every meson is composed of a quark and an antiquark. Since the three quarks in a given hadrons such as $\Delta^{++}(uuu)$ and $\Omega^{-}(sss)$ are in the same state, they seem to violate Pauli exclusion principle. Then O.W Greenberg, suggested that quarks not only come in three flavors (u, d, and s) but each of these flavors come in three different colors (Red, Green, and Blue say), due to which the problem of exclusion principle evaporates. So to study strong forces at the fundamental level, one should look, rather at the interaction between the individual quarks. The quarks and antiquarks interact through the exchange of gluons, which themselves carry color, and therefore (like the quarks) they should not exist as isolated particles, but they can form a bound state called glue balls. We are now in position to present the SU(3) gauge theory of color interactions.

2.3 The SU(3) gauge theory of color interactions

As stated in the previous section, each flavor of quark comes in three different colors R, B, and G. Then the Dirac Lagrangian for a given flavor of quark can be expressed as

$$\pounds_{Dirac} = -\bar{\psi}_R(\gamma_\mu\partial_\mu + m)\psi_R - \bar{\psi}_B(\gamma_\mu\partial_\mu + m)\psi_B - \bar{\psi}_G(\gamma_\mu\partial_\mu + m)\psi_G. \qquad (2.3.1)$$

Introducing a triplet of color fields as a 3×1 column matrix,

$$\psi = \begin{pmatrix} \psi_R \\ \psi_B \\ \psi_G \end{pmatrix}$$

and a 3×3 mass matrix,

$$m = \begin{pmatrix} m & o & o \\ o & m & o \\ o & o & m \end{pmatrix}$$

we can write the Lagrangian in (2.3.1) more compactly as,

$$\pounds_{Dirac} = -\bar{\psi}(\gamma_\mu \partial_\mu + m)\psi. \tag{2.3.2}$$

Lets consider global gauge transformation on $\psi(x)$,

$$\psi(x) \rightarrow \psi^{'}(x) = U\psi(x); U = e^{i\theta}. \tag{2.3.3}$$

It can be easily checked that under this transformation $\pounds^{'}_{Dirac} = \pounds_{Dirac}$. Now under local gauge transformation;

$$\psi(x) \rightarrow \psi^{'}(x) = U\psi(x); U = e^{i\theta(x)}, \tag{2.3.4}$$

where U should be a 3×3 transformation matrix, $\bar{\psi}^{'}(x)\psi^{'}(x) = \bar{\psi}(x)U^{+}U\psi(x) = \bar{\psi}\psi$; only if $U^{+}U = 1$. This implies that U has to be unitary, so that any unitary matrix can be written as; $U = e^{iH}$, where H is Hermitian, and is 3×3 matrix. So we can write H as: $H = \Theta(x)I + \lambda^a\theta^a(x)$, a= 1...8, where $\lambda^{a'}$s are Gell-man matrices which are eight generators of SU(3). So U can be written as: $U = e^{i\Theta(x)I}e^{i\lambda^a\theta^a(x)}$. Lets pull out a constant "g" from the arbitrary parameter $\theta(x)$. So that U is given by: $U = e^{i\Theta(x)I}e^{ig\lambda^a\theta^a(x)}$. Here I and λ^a are generators of U(3) group. Now lets consider a local SU(3) gauge transformation on $\psi(x)$:

$$\psi(x) \rightarrow \psi^{'}(x) = U\psi(x) = e^{ig\lambda^a\theta^a(x)}\psi(x). \tag{2.3.5}$$

Lets make use of minimal coupling prescription,

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ig\lambda^a A^a_\mu(x) \tag{2.3.6}$$

to make $\pounds_{Dirac}$ locally invariant. lets also impose prescription for the transformation of $D_\mu\psi(x)$ as,

$$D_\mu\psi \rightarrow (D_\mu\psi)' = U(D_\mu\psi). \tag{2.3.7}$$

$$(\partial_\mu - ig\lambda^a A'^a_\mu(x))U\psi(x) = U(\partial_\mu - ig\lambda^a A^a_\mu(x))\psi(x) \tag{2.3.8}$$

$$(\partial_\mu U)\psi(x) + U\partial_\mu\psi(x) - ig\lambda.A'_\mu(x)U\psi(x) = U\partial_\mu\psi(x) - igU(\lambda.A_\mu(x))\psi(x). \tag{2.3.9}$$

Dropping the common factor $\psi(x)$ on both sides; we get:

$$\partial_\mu U - ig\vec{\lambda}.\vec{A}'_\mu(x)U = -igU(\vec{\lambda}.\vec{A}_\mu(x). \tag{2.3.10}$$

Then multiplying both sides of this equation by U^{-1} from the right, we get:

$$\vec{\lambda}.\vec{A}'_\mu(x) = U\vec{\lambda}.\vec{A}_\mu(x)U^{-1} + \frac{1}{ig}(\partial_\mu U)U^{-1}. \tag{2.3.11}$$

Lets define $\vec{\lambda}.\vec{A}_\mu(x) \equiv A_\mu(x)$ where A_μ is now a 3×3 matrix. We see that to satisfy SU(3) local gauge invariance, $A_\mu(x)$ should transform as,

$$A_\mu(x) \rightarrow A'_\mu(x) = UA_\mu U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1}. \tag{2.3.12}$$

This is the transformation property of nonabelian gauge fields $A_\mu(x)$. Replacing $\partial_\mu \rightarrow D_\mu = \partial_\mu - ig\lambda^a A^a_\mu(x)$; we get the Lagrangian:

$$\pounds = -\bar{\psi}(x)(\gamma_\mu\partial_\mu + m)\psi(x) + ig\bar{\psi}(x)\gamma_\mu\lambda^a A^a_\mu\psi(x), \tag{2.3.13}$$

where the second term in this equation is simply the interaction Lagrangian ($\pounds_{int}$) which couples the matter field to the gluon fields through the coupling constant "g". The Lagrangian that governs the gluon fields is given by;

$$\pounds_{gluon} = -(\frac{1}{4})F^a_{\mu\nu}F^a_{\mu\nu}. \tag{2.3.14}$$

In general $F_{\mu\nu}$ can be written as:

$$F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]. \tag{2.3.15}$$

Substituting the values of D_μ and D_ν in this equation; we can write the gauge field tensor,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]. \tag{2.3.16}$$

Then the complete Lagrangian of QCD is given as

$$\pounds_{QCD} = \pounds_{free\ Dirac} + \pounds_{int} + \pounds_{gluon}$$

$$\pounds_{QCD} = -\bar{\psi}(\gamma_\mu \partial_\mu + m)\psi + ig\bar{\psi}\gamma_\mu \lambda^a A^a_\mu \psi - \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}, \tag{2.3.17}$$

which is invariant under local gauge transformation. Here, A^a_μ is the gluon field. Since $F_{\mu\nu}$ is given by;

$$F^a_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu) - ig[A_\mu, A_\nu]^a,$$

we can substitute this to $\pounds_{gluon}$ and we can write $\pounds_{QCD}$ as

$$\pounds_{QCD} = \pounds_{free\ Dirac} + \pounds_{int} + \pounds_{free\ gluon} + \pounds_{3-gluon} + \pounds_{4-gluon}, \tag{2.3.18}$$

where

$$\pounds_{free\ gluon} = -\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu),$$

$$\pounds_{3-gluon} = -\frac{1}{4}[g f_{ade}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)A^d_\mu A^e_\nu + g f_{abc} A^b_\mu A^c_\nu (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)],$$

$$\pounds_{4-gluon} = -\frac{1}{4}g^2 f_{abc} f_{ade} A^b_\mu A^c_\nu A^d_\mu A^e_\nu,$$

we have used $2if_{abc}\lambda^a = [\lambda^b, \lambda^c]$ and $2if_{ade}\lambda^a = [\lambda^d, \lambda^e]$. The f_{abc} and f_{ade} are structure constants of SU(3) group. The $\pounds_{3-gluon}$ and $\pounds_{4-gluon}$ shows the part of the Lagrangian showing the interaction between three gluon fields and four gluon fields

respectively. This suggests that gluons can form a bound state called glue balls. This self interaction between gluons also leads to the property of asymptotic freedom and confinement in QCD. We now go to derive the Feynman rules for QCD in the next section which will be used in our framework later.

2.4 Feynman rules for QCD

In this section we are going to derive free quark propagator, free gluon propagator, the quark-gluon vertex, three gluon vertex, and four gluon vertex from their corresponding Lagrangians. To find the free quark propagator lets start from $\pounds_{free\ Dirac}$,

$$\pounds_{free\ Dirac} = -\bar{\psi}(x)(\gamma_\mu \partial_\mu + m)\psi(x). \tag{2.4.1}$$

Substituting this to Euler Lagrange equation;

$$\frac{\partial \pounds}{\partial \bar{\psi}} - \partial_\nu(\frac{\partial \pounds}{\partial(\partial_\nu \bar{\psi})}) = 0 \tag{2.4.2}$$

we get, the free Dirac equation

$$(\gamma_\mu \partial_\mu + m)\psi(x) = 0. \tag{2.4.3}$$

Taking the fourier transformation; $\psi(x) = \frac{1}{(2\pi)^4}\int d^4p e^{ip.x}\psi(p)$ and substituting this in the free Dirac equation; we get the momentum representation of the free Dirac equation:

$$(i\gamma.p + m)\psi(p) = 0. \tag{2.4.4}$$

The free quark propagator from this equation is obtained as

$$S_F(p) = \frac{1}{i\gamma.p + m} = \frac{-i\gamma.p + m}{(\gamma.p)^2 + m^2}. \tag{2.4.5}$$

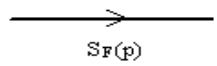

Figure 2.1: free quark propagation.

To find the free gluon propagator, lets start from its corresponding Lagrangian;

$$\pounds_{free\ gluon} = -\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu). \tag{2.4.6}$$

We can write this equation as;

$$-\frac{1}{2}(\partial_\lambda A^a_\sigma)[\partial_\lambda A^a_\sigma - \partial_\sigma A^a_\lambda]. \tag{2.4.7}$$

Then solving the Euler-Lagrange equation

$$\frac{\partial \pounds}{\partial A^a_\mu} - \partial_\nu(\frac{\partial \pounds}{\partial(\partial_\nu A^a_\mu)}) = 0, \tag{2.4.8}$$

we get

$$(\Box^2 \delta_{\mu\nu} - \partial_\mu \partial_\nu) A^a_\nu(x) = 0, \tag{2.4.9}$$

where $\Box^2 = \partial_\nu \partial_\nu$. Now substitute the fourier transformation of gluon field $A^a_\nu(x) = \frac{1}{(2\pi)^4}\int d^4k e^{ik.x} A_\nu(k)$ in this equation. Then finally we get;

$$(-k^2\delta_{\mu\nu} + k_\mu k_\nu) A^a_\nu(k) = 0. \tag{2.4.10}$$

Lets define $C_{\mu\nu} \equiv -k^2\delta_{\mu\nu} + k_\mu k_\nu$ and $C^{-1}_{\mu\nu} \equiv Ak^2\delta_{\mu\nu} + Bk_\mu k_\nu$, such that $C_{\mu\nu}C^{-1}_{\nu\lambda} = \delta_{\mu\lambda}$. Multiplying these we get,

$$C_{\mu\nu}C^{-1}_{\nu\lambda} = -Ak^4\delta_{\mu\lambda} + Ak^2 k_\mu k_\lambda. \tag{2.4.11}$$

From this equation we can not find value of A such that, $C_{\mu\nu}C^{-1}_{\nu\lambda} = \delta_{\mu\lambda}$ is satisfied. But under lorentz gauge condition, $\partial_\mu\partial_\nu A^a_\nu(x) = 0$. So that we remain with;

$$\Box^2\delta_{\mu\nu}A^a_\nu(x) = 0. \tag{2.4.12}$$

Substituting the fourier transformation of $A^a_\nu(x)$, we get

$$-k^2\delta_{\mu\nu}A^a_\nu(k) = 0. \tag{2.4.13}$$

Let $C_{\mu\nu} \equiv -k^2\delta_{\mu\nu}$, and $C^{-1}_{\mu\nu} \equiv Ak^2\delta_{\mu\nu}$. So $C_{\mu\nu}C^{-1}_{\nu\lambda} = -Ak^4\delta_{\mu\lambda}$. From this for $A = -k^{-4}$, we can get, $C_{\mu\nu}C^{-1}_{\nu\lambda} = \delta_{\mu\lambda}$. This implies that $C^{-1}_{\mu\nu} = -\frac{1}{k^2}$. Then the free gluon propagator is written down under lorentz gauge condition as;

$$D_{\mu\nu} = -\delta_{\mu\nu}/k^2. \tag{2.4.14}$$

Now lets find the quark-gluon vertex. To find this vertex, we drop all fields from;

Figure 2.2: free gluon propagation

$\pounds_{int} = ig\bar{\psi}\gamma_\mu\lambda^a A^a_\mu\psi$. So we get;

$$quark - gluon\ vertex = ig\gamma_\mu\lambda^a. \tag{2.4.15}$$

Now lets find the three gluon vertex. Lets recall first $\pounds_{3-gluon}$.

$$\pounds_{3-gluon} = -\frac{1}{4}[gf_{ade}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)A^d_\mu A^e_\nu + gf_{abc}A^b_\mu A^c_\nu(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)]. \tag{2.4.16}$$

This can be rewritten as;

$$\pounds_{3-gluon} = -\frac{1}{2}gf_{abc}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)A^b_\mu A^c_\nu. \tag{2.4.17}$$

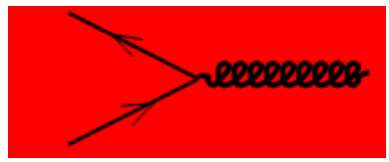

Figure 2.3: quark-gluon interaction

We can also write this more simply as;

$$\pounds_{3-gluon} = -gf_{abc}(\partial_\mu A^a_\nu)A^b_\mu A^c_\nu. \tag{2.4.18}$$

Lets change the indices; $a \to a^{'}$, $b \to b^{'}$, and $c \to c^{'}$ to write $\pounds_{3-gluon}$ as,

$$\pounds_{3-gluon} = -gf_{a^{'}b^{'}c^{'}}(\partial_\mu A^{a^{'}}_\nu)A^{b^{'}}_\mu A^{c^{'}}_\nu. \tag{2.4.19}$$

All the possible choices of $a^{'}$, $b^{'}$, and $c^{'}$ are $a^{'} = a, b^{'} = b, c^{'} = c$; $a^{'} = a, b^{'} = c, c^{'} = b$; $a^{'} = b, b^{'} = c, c^{'} = a$; $a^{'} = b, b^{'} = a, c^{'} = c$; $a^{'} = c, b^{'} = b, c^{'} = a$ and $a^{'} = c, b^{'} = a, c^{'} = b$. So, using these six possibilities, we have:

$$\pounds_{3-gluon} = -gf_{abc}(\partial_\mu A^a_\nu)A^b_\mu A^c_\nu - gf_{acb}(\partial_\mu A^a_\nu)A^c_\mu A^b_\nu - gf_{bca}(\partial_\mu A^b_\nu)A^c_\mu A^a_\nu - gf_{bac}(\partial_\mu A^b_\nu)A^a_\mu A^c_\nu$$

$$-gf_{cba}(\partial_\mu A^c_\nu)A^b_\mu A^a_\nu - gf_{cab}(\partial_\mu A^c_\nu)A^a_\mu A^b_\nu. \tag{2.4.20}$$

Now substituting $A^a_\nu(x) = \frac{1}{(2\pi)^4}\int d^4k_1 e^{-ik_1.x}A^a_\nu(k_1)$; $A^b_\nu(x) = \frac{1}{(2\pi)^4}\int d^4k_2 e^{-ik_2.x}A^b_\nu(k_2)$; $A^c_\nu(x) = \frac{1}{(2\pi)^4}\int d^4k_3 e^{-ik_3.x}A^c_\nu(k_3)$; in this equation we get

$$\pounds_{3-gluonfields} = -gf_{abc}(-ik_{1\mu})A^a_\nu(k_1)A^b_\mu(k_2)A^c_\nu(k_3) - gf_{acb}(-ik_{1\mu})A^a_\nu(k_1)A^c_\mu(k_3)A^b_\nu(k_2)$$

$$-gf_{bca}(-ik_{2\mu})A^b_\nu(k_2)A^c_\mu(k_3)A^a_\nu(k_1) - gf_{bac}(-ik_{2\mu})A^b_\nu(k_2)A^a_\mu(k_1)A^c_\nu(k_2)$$

$$-gf_{cba}(-ik_{3\mu})A^c_\nu(k_3)A^b_\mu(k_2)A^a_\nu(k_1) - gf_{cab}(-ik_{3\mu})A^c_\nu(k_3)A^a_\mu(k_1)A^b_\nu(k_2). \tag{2.4.21}$$

Using the symmetric property, $f_{abc} = f_{bca} = f_{cab}$ and anti-symmetric property, $f_{abc} = -f_{bac}$ etc., we can write this equation as;

$$\pounds_{3-gluon} = -gf_{abc}[-i(k_1.A^b)(A^a.A^c)+i(k_1.A^c)(A^b.A^c)-i(k_2.A^c)(A^a.A^b)+i(k_2.A^a)(A^b.A^c)$$

$$-i(k_3.A^b)(A^a.A^c) - i(k_3.A^a)(A^b.A^c)]. \tag{2.4.22}$$

Assigning the indices $a \to \alpha, b \to \beta, c \to \gamma$ and omitting the field variables; we get three gluon vertex:

$$3 - gluon\ vertex = -igf_{abc}[(k_3 - k_1)_\beta \delta_{\gamma\alpha} + (k_1 - k_2)_\gamma \delta_{\alpha\beta} + (k_2 - k_3)_\alpha \delta_{\beta\gamma}], \tag{2.4.23}$$

Figure 2.4: self interaction of three gluons

where k_i, is the propagation direction of the gluon field. Now lets obtain the four gluon vertex from its corresponding Lagrangian:

$$\pounds_{4-gluon} = -\frac{1}{4}g^2 f_{abc} f_{ade} A^b_\mu A^c_\nu A^d_\mu A^e_\nu. \tag{2.4.24}$$

Lets rewrite it as

$$\pounds_{4-gluon} = -\frac{1}{4}g^2 f_{na'b'} f_{nc'd'} A^{a'}_\mu A^{b'}_\nu A^{c'}_\mu A^{d'}_\nu. \tag{2.4.25}$$

Taking all possibilities as we did in the three gluon fields, we have;

$$\pounds_{4-gluon} = -\frac{1}{4}g^2[f_{nab}f_{ncd}A^a_\mu A^b_\nu A^c_\mu A^d_\nu + f_{nab}f_{ndc}A^a_\mu A^b_\nu A^d_\mu A^c_\nu$$

$$+f_{nac}f_{nbd}A^a_\mu A^c_\nu A^b_\mu A^d_\nu + f_{nac}f_{ndb}A^a_\mu A^c_\nu A^d_\mu A^b_\nu$$

$$+f_{nad}f_{nbc}A^a_\mu A^d_\nu A^b_\mu A^c_\nu + f_{nda}f_{nbc}A^d_\mu A^a_\nu A^b_\mu A^c_\nu], \tag{2.4.26}$$

which can be also written as

$$\pounds_{4-gluon} = -\frac{1}{4}g^2[f_{nab}f_{ncd}[(A^a.A^c)(A^b.A^d) - (A^a.A^d)(A^b.A^c)]+$$

$$f_{nac}f_{nbd}[(A^a.A^b)(A^c.A^d) - (A^a.A^d)(A^c.A^b)]+$$

$$f_{nad}f_{nbc}[(A^a.A^b)(A^d.A^c) - (A^d.A^b)(A^a.A^c)]]. \tag{2.4.27}$$

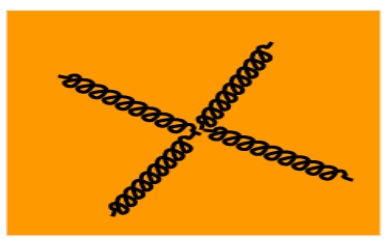

Figure 2.5: self interaction of four gluon fields

This implies;

$$4 - gluon\ vertex = -\frac{1}{4}g^2[f_{nab}f_{ncd}[\delta_{\mu\tau}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\tau}]+$$

$$f_{nac}f_{nbd}[\delta_{\mu\nu}\delta_{\tau\sigma} - \delta_{\mu\sigma}\delta_{\tau\nu}]+$$

$$f_{nad}f_{nbc}[\delta_{\mu\nu}\delta_{\tau\sigma} - \delta_{\nu\sigma}\delta_{\tau\mu}]] \tag{2.4.28}$$

Chapter 3

Effective potential from quantum field theory

In this chapter we are going to derive effective potential between two nonrelativistic interacting particles using quantum field theory. Specifically we will also derive effective interaction potential between two electrons (Moller scattering), and we will obtain both repulsive Coulomb potential as well as the spin-orbit interaction potential. We then derive one-pion exchange potentials in nucleon-nucleon interaction. First let us recall some general facts about the description of scattering processes in quantum field theory. We assume that the full-Hamiltonian H governing the dynamics of the quantum mechanical system under consideration can be split up into a free (or,more generally unperturbed) Hamiltonian H_0 and an interaction potential (or perturbation) V,

$$H(t) = H_0 + V(t). \tag{3.0.1}$$

In the absence of a scatterer, V would be zero, and an energy eigenstate would be just a free particle state. The presence of V causes the energy eigenstate to be different from a free-particle state. However, if the scattering process is to be elastic, that is, if there is no change in energy we can simply obtain a solution to the full-Hamiltonian with

the same energy eigenvalue. In scattering theory, the physical situation of interest is to specify the initial states $|i>$ at time $t = -\infty$, and final states $|f>$ at time $t = +\infty$ by some unperturbed state, i.e., eigenstates of the free Hamiltonian H_0 corresponding to some energy eigenvalue E. As a special case, we shall consider as an initial and final states two-particle states of definite momentum, i.e., the eigenstates: $|i>=|p_1, p_2>$, $|f>=|q_1, q_2>$ of the one particle momentum operators. The states may be built up from tensor products of corresponding single-particle momentum eigen states $|p>$ and $|q>$ with normalization:

$$< q|p >= \delta^3(q - p). \tag{3.0.2}$$

We now confine ourselves to the position basis wave function[6],

$$< x|p >= \frac{e^{ip.x}}{(2\pi)^{\frac{3}{2}}}. \tag{3.0.3}$$

$$< q|x >= \frac{e^{-iq.x}}{(2\pi)^{\frac{3}{2}}}. \tag{3.0.4}$$

The normalization (3.0.2) can also be written as;

$$\int d^3x < q|x >< x|p >= \delta^3(p - q). \tag{3.0.5}$$

Assume the potential $V(x)$ depends only on the relative distance of the two particles, i.e., only on the difference $x \equiv x_1 - x_2$ of their coordinate vectors x_1, x_2. Lets abbreviate the involved momentum transfer by $k \equiv q_1 - p_1$, (where q_1 and p_1 are final and initial momentum respectively). Then the Fourier transformation of the free two-particle matrix element of the potential operator $V(x)$ can be obtained with the use of equations (3.0.3) and (3.0.4) as

$$< q_1, q_2|V(x)|p_1, p_2 >= \frac{1}{(2\pi)^3}\delta^3(q_1 + q_2 - p_1 - p_2) \int d^3x e^{-ik.x} V(x), \tag{3.0.6}$$

where we have used $V(x)\delta^3(p-q) =< q|V(x)|p >$. Recalling the S-matrix element S_{fi} for the transition $i \to f$ as,

$$S_{fi} = \delta_{fi} - 2\pi i \delta(E_f - E_i) R_{fi}, \tag{3.0.7}$$

where R_{fi} is

$$R_{fi} =< f|V_s|i > + \lim_{\epsilon \to +0} \sum_n \frac{< f|V_s|n > R_{ni}}{E_i - E_n + i\hbar\epsilon}, \tag{3.0.8}$$

the Born approximation R^B_{fi} consists of retaining just the first term;

$$R^B_{fi} =< f|V_s|i > . \tag{3.0.9}$$

Factorizing of the overall momentum conservation from scattering amplitude R_{fi}, the reduced T-matrix element T_{fi} is defined by the decomposition

$$S_{fi} = \delta_{fi} + i(2\pi)^{(4)}\delta^{(4)}(p_f - p_i)T_{fi} \tag{3.0.10}$$

of the S-matrix, where p_i and p_f denote the total momentum of initial and final momentum states, respectively. Consequently, the reduced T-matrix element T_{fi} is related to the transition amplitude R_{fi} by;

$$R_{fi} = -(2\pi)^3\delta^{(3)}(p_f - p_i)T_{fi}. \tag{3.0.11}$$

The interaction of two particles which in fact are described by some QFT can be approximated by an effective potential at least as long as the Born approximation (3.0.9) makes sense. The simplest quantum field theoretical analog of the potential scattering considered previously is the elastic scattering;

$$P_1(p_1) + P_2(p_2) \to P_1(q_1) + P_2(q_2) \tag{3.0.12}$$

of the two particles P_1, P_2 with masses m_1, m_2. From equations (3.0.6), (3.0.9), and (3.0.11), the Born approximation T_{fi}^B to the T-matrix element for the above elastic two particle scattering is given by

$$T_{fi}^B = -\frac{1}{(2\pi)^6}\int d^3x e^{-ik.x}V(x). \tag{3.0.13}$$

In terms of the M_{fi}, which does not contain the kinematical factors, equation (3.0.13) can be easily written as

$$T_{fi}^B = -\frac{1}{(2\pi)^6}M_{fi}, \tag{3.0.14}$$

where

$$M_{fi} = \int Ve^{-ik.x}d^3x. \tag{3.0.15}$$

Fourier transformation of this one gives

$$V = \frac{1}{(2\pi)^3}\int M_{fi}e^{ik.x}d^3k. \tag{3.0.16}$$

This formula enables us to construct an effective three-dimensional potential (to be used in connection with the schrodinger equation) once we know the nonrelativistic limit of the covariant matrix element. **The essential point is that the effective potential is nothing more than the three-dimensional Fourier transform of the lowest order M-matrix element.** In the case of scattering of two identical fermions the M-matrix element that appears in the above equation is just that part of the covariant matrix element which corresponds to direct scattering. We now present the derivation of Coulomb and spin-orbit potentials from Moller scattering QED as well as nonrelativistic nucleon-nucleon potentials which are derived from meson field theory.

3.1 Derivation of Coulomb and spin-orbit potentials from QED

We are now prepared to make an attack on the scattering of two relativistic electrons, commonly known as Moller scattering and derive the Coulomb and spin orbit potentials which we present as a preliminary exercise before investigation of our main problem of quark-antiquark bound state systems. In quantum field theory we usually visualize the interactions between two charged particles, as arising from the exchange of quanta. The M_{fi} of this interaction contains two terms: that part of the covariant M-matrix which is due to the exchange of transverse photons ($M_{fi}^{(trans)}$) and M_{fi}^{Coul}, which represents the instantaneous Coulomb interaction (or for the Coulomb contribution). Looking at the Coulomb contribution, we can say that the so-called instantaneous Coulomb interaction is actually equivalent to an interaction due to the emission and subsequent absorption of a time-like and a longitudinal photon, considered together. Let us see how the rule expressed in (3.0.16) works in the case

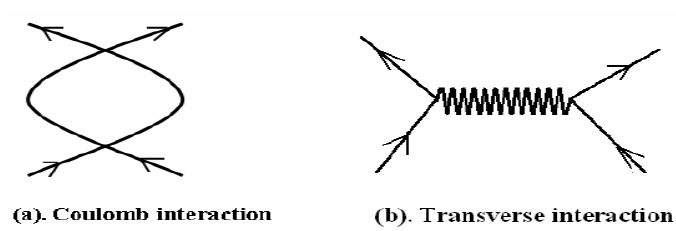

Figure 3.1: exchange of covariant photons

of Moller interaction. In the CM-system, the invariant amplitude corresponding to direct processes is given as

$$M_{fi}^{(direct)} = \frac{e^2(\bar{u_1'}\gamma_4 u_1)(\bar{u_2'}\gamma_4 u_2)}{|q|^2} + \frac{e^2(\bar{u_1'}\gamma_i u_1)(\bar{u_2'}\gamma_i u_2)}{|q|^2}, \tag{3.1.1}$$

where the first and second terms are for the Coulomb and transverse interaction respectively. At very low energies $\bar{u_1^{'}}\gamma_4 u_1$ can be replaced by $\delta_{s_1 s_2^{'}}$; $\bar{u_1^{'}}\gamma_i u_1$, by zero; similarly for $\bar{u_2^{'}}\gamma_\mu u_2$. Hence the insertion of (3.1.1) in (3.0.16) gives

$$V = \frac{e^2}{(2\pi)^3}\int \frac{e^{iq.r}}{q^2} d^3q. \tag{3.1.2}$$

To find the result of this integration lets put $d^3q = |q|^2 d|q| dcos\theta d\phi$, then we have

$$V = 2\pi \frac{e^2}{(2\pi)^3}\int_0^\infty d|q| \int_{-1}^{1} dcos\theta e^{i|q||r|cos\theta}, \tag{3.1.3}$$

$$= \frac{e^2}{2\pi^2 r}\int_0^\infty dq \frac{sin(qr)}{q}. \tag{3.1.4}$$

Now apply contour integration, by noting that it is an even function of q and hence the limits of integration can be extended to $+\infty$ and $-\infty$. we then write sin(qr) in terms of exponentials and then close the contour in the upper and lower half-plane appropriately (with poles at q=0). Remembering the sign arising from the direction of the contour, we have

$$V = \frac{-i}{8\pi^2 r} e^2 \int_{-\infty}^{+\infty} dq \frac{1}{q}[e^{iqr} - e^{-iqr}], \tag{3.1.5}$$

which finally implies the repulsive coulomb potential

$$V = \frac{e^2}{4\pi r}, \tag{3.1.6}$$

as expected[7]. To see the physical meaning of the second term of (3.1.1), we first recall that $\bar{u_1^{'}}\gamma_i u_1$ can be written as,

$$\bar{u_1^{'}}\gamma_i u_1 = \tilde{\chi}^{(s_1^{'})}[-\frac{i(p_1 + p_1^{'})}{2m} + \frac{\sigma^{(1)} \times (p_1^{'} - p_1)}{2m}]\chi^{(s_1)}. \tag{3.1.7}$$

Note that this vector is "transverse" in the sense that it has no component in the direction $p_1 - p_1^{'}$. Calling the first and second term of (3.1.7) respectively "current" and "dipole", we see that the second term of (3.1.1) gives rise to a current-current interaction, a dipole-dipole interaction, and a current-dipole interaction. As

an example, we shall consider the dipole-dipole interaction in detail. Remembering $q = p_2^{'} - p_2 = -p_1^{'} + p_1$, we can readily take the Fourier transform of the dipole-dipole matrix as follows:

$$-\frac{e^2}{(2\pi)^3}\int \frac{(\sigma^{(1)} \times q).(\sigma^{(2)} \times q)e^{iq.x}d^3q}{(2m)^2|q|^2}.$$
$$= (\frac{e}{2m})^2(\sigma^{(1)} \times \nabla).(\sigma^{(2)} \times \nabla)(\frac{1}{4\pi r}).$$
$$= -\frac{e\sigma^{(1)}}{2m}.\nabla \times [\frac{e\sigma^{(2)}}{2m} \times \nabla(\frac{1}{4\pi r})]. \tag{3.1.8}$$

We recognize that $[\frac{e\sigma^{(2)}}{2m} \times \nabla(\frac{1}{4\pi r})]$ is precisely the vector potential generated by the magnetic moment of electron 2; hence (3.1.8) is just the interaction energy between the magnetic moments of the two electrons expected from classical considerations. Using the identity

$$\int e^{-ip^{'}.x}\frac{1}{r}\frac{dV}{dr}\sigma.[x \times (-i\nabla)]e^{-ip.x}d^3x = -i\sigma.(p \times p^{'})\int Ve^{-iq.x}d^3x, \tag{3.1.9}$$

which valid for any spherically symmetric V, the spin-orbit interaction between two electrons is obtained as:

$$V^{L-S} = -\frac{3}{2m^2}\frac{1}{r}\frac{d}{dr}(\frac{e^2}{4\pi r})\vec{L}.(\frac{\vec{\sigma}^{(1)} + \vec{\sigma}^{(2)}}{2}), \tag{3.1.10}$$

which can be also rewritten[7] as

$$V^{L-S} = -\frac{3}{2m^2}\frac{1}{r}\frac{d}{dr}(\frac{e^2}{4\pi r})\vec{L}.\vec{S}, \tag{3.1.11}$$

with $\vec{S} = (\frac{\vec{\sigma}^{(1)}+\vec{\sigma}^{(2)}}{2})$, where L is orbital angular momentum. Prior to Moller's work, G.Breit worked out all these correction terms to the coulomb potential using classical arguments, hence they are collectively known as the Briet interaction between two electrons. We now present a derivation of one-pion exchange potential from meson field theory as a preliminary exercise.

3.2 One-pion exchange potential

The relation (3.0.16) can also be applied to meson theory to derive the one meson exchange potentials. The potential which we are going to derive is appropriate for describing the exchange of a neutral pseudoscalar meson, in p-p (or n-n) interaction due to π^0 exchange. The "rule" for writing $-iM_{fi}$ is remarkably simple. Just as we associate $\frac{(-i\gamma.q+m)}{[i(q^2+m^2-i\epsilon)]}$ with each virtual $spin-\frac{1}{2}$ fermions of four-momentum q, we associate $\frac{1}{[i(q^2+M_\pi^2-i\epsilon)]}$ with each virtual spinless meson of four-momentum q. In the case of scalar coupling the first term of the Fourier transform of the covariant matrix element for direct and exchange scattering for nucleon-nucleon interaction to order of interaction strength, G^2 is

$$-iM_{fi} = (-iG)^2[\frac{(\bar{u_1'}u_1)(\bar{u_2'}u_2)}{i[(p_1-p_1')^2+M_\pi^2-i\epsilon]} - \frac{(\bar{u_2'}u_1)(\bar{u_1'}u_2)}{i[(p_1-p_2')^2+M_\pi^2-i\epsilon]}. \tag{3.2.1}$$

At very low energies $\bar{u_1'}u_1$ can be replaced by $\delta_{s_1 s_2'}$, $\bar{u_1'}u_1$, by zero; similarly for $\bar{u_2'}u_2$. Substitution of (3.2.1) in (3.0.16) gives

$$V = \frac{(-iG)^2}{(2\pi)^3}\int \frac{e^{iq.r}}{q^2+M_\pi^2}. \tag{3.2.2}$$

To integrate this, lets put $d^3q = |q|^2 d|q| dcos\theta d\phi$. Then we have

$$V = \frac{(-iG)^2}{4\pi^2}\int_0^\infty d|q| \int_{-1}^{1} dcos\theta e^{i|q||r|cos\theta}\frac{|q|^2}{|q|^2+M_\pi^2} \tag{3.2.3}$$

$$= \frac{(-iG)2}{2\pi^2 r}\int_0^\infty dq \frac{q}{q^2+M_\pi^2} sin(qr). \tag{3.2.4}$$

It is now straight forward to do this integral by applying contour integration, by noting that it is an even function of q and hence the limits of integration can be extended to $+\infty$ and $-\infty$. we then write sin(qr) in terms of exponentials and then

close the contour in the upper and lower half-plane appropriately (with poles at q=0). Remembering the sign arising from the direction of the contour we have

$$V = \frac{-i}{8\pi r} e^2 (-iG)^2 \int_{-\infty}^{+\infty} dq \frac{q}{q^2 + M_\pi^2} [e^{iqr} - e^{-iqr}], \tag{3.2.5}$$

which immediately leads to the familiar Yukawa potential[7]

$$V^{(s)} = -\frac{G^2 e^{-M_\pi r}}{4\pi r}. \tag{3.2.6}$$

Note that the sign of the force between two nucleons due to the exchange of a scalar meson is attractive, in contrast to the coulomb case. It is known empirically that the lightest meson that can be exchanged between two nucleons is a pseudoscalar π meson (fig 3.2).

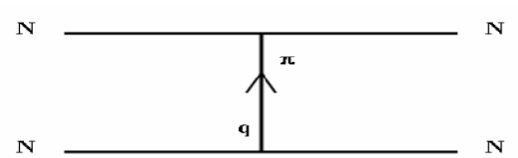

Figure 3.2: Feynman diagram that generates the one-pion exchange potential

If we use the pseudoscalar coupling

$$iG\bar{\psi}\gamma_5\psi\phi \tag{3.2.7}$$

in place of the scalar coupling, we obtain for direct scattering part of the covariant matrix element

$$-iM_{fi}^{(direct)} = \frac{(G)^2(\bar{u_1'}\gamma_5 u_1)(\bar{u_2'}\gamma_5 u_2)}{i[(p_1 - p_1')^2 + M_\pi^2]}. \tag{3.2.8}$$

Comparing this with the analogous expression in Moller case, we see that, apart from the finite mass of the exchanged boson, the only change necessary is $-e\gamma_\mu \rightarrow G\gamma_5$.

In the nonrelativistic limit we just have

$$\bar{u_1'}\gamma_5 u_1 = \begin{pmatrix} \chi^{(s_1')t} & -\chi^{(s_1')t}\frac{\sigma^{(1)}.p_1'}{2m} \end{pmatrix} \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \begin{pmatrix} \chi^{(s_1)} \\ \frac{\sigma^{(1)}.p_1}{2m}\chi^{(s_1)} \end{pmatrix}$$

$$= -\chi^{(s_1')t}\frac{\sigma^{(1)}.q}{2m}\chi^{(s_1)} \tag{3.2.9}$$

and

$$\bar{u_2'}\gamma_5 u_2 = \chi^{(s_2')t}\frac{\sigma^{(2)}.q}{2m}\chi^{(s_2)}. \tag{3.2.10}$$

So (3.2.8) leads to

$$M_{fi}^{(direct)} \rightarrow -\frac{G^2(\sigma^{(1)}.q)(\sigma^{(2)}.q)}{(2m)^2(|q|^2+M_\pi^2)}, \tag{3.2.11}$$

where m and M_π are respectively the masses of the nucleon and the meson. Inserting equation (3.2.11) in (3.0.16) gives rise to the potential

$$V_\pi(r) = \frac{G^2}{4\pi}\frac{1}{(2m)^2}(\sigma^{(1)}.\nabla)(\sigma^{(2)}.\nabla)\frac{e^{-M_\pi r}}{r}. \tag{3.2.12}$$

Now we have to evaluate the partial derivatives. Lets set up the partial derivatives so that it is clear where the various parts of the final expression come from, we see that

$$\nabla^i\nabla^j(\frac{e^{-M_\pi r}}{r}) = e^{-M_\pi r}\nabla^i\nabla^j(\frac{1}{r}) + [\nabla^i e^{-M_\pi r}\nabla^j(\frac{1}{r}) + \nabla^j e^{-M_\pi r}\nabla^i(\frac{1}{r})] + (\frac{1}{r})\nabla^i\nabla^j e^{-M_\pi r}. \tag{3.2.13}$$

From the first term, we get

$$e^{-M_\pi r}[\frac{3\hat{r}^i\hat{r}^j - \delta_{ij}}{r^3}], \tag{3.2.14}$$

and from the second term,

$$[\frac{2M_\pi\hat{r}^i\hat{r}^j}{r^2}]e^{-M_\pi r}, \tag{3.2.15}$$

finally from the third term we get,

$$[M_\pi(\frac{\hat{r}^i\hat{r}^j}{r^2} - \frac{\delta^{ij}}{r^2} + \frac{M_\pi\hat{r}^i\hat{r}^j}{r})]e^{-M_\pi r}. \tag{3.2.16}$$

Adding these three terms we get the expression

$$\nabla^i\nabla^j(\frac{e^{-M_\pi r}}{r}) = \frac{M_\pi^2}{3}\frac{e^{-M_\pi r}}{r}[(3\hat{r}^i\hat{r}^j - \delta^{ij})(1 + \frac{3}{M_\pi r} + \frac{3}{(M_\pi r)^2}) + \delta^{ij}], \tag{3.2.17}$$

where we have used $\partial^j r = \frac{x^j}{r} = \hat{r}$, $\partial^i x^j = \delta^{ij}$. The final result for the potential between two nucleons induced by the exchange of a virtual pion is given by

$$V_\pi(r) = \frac{G^2}{4\pi}\frac{1}{(2m)^2}\frac{M_\pi^2}{3}\frac{e^{-M_\pi r}}{r}[((\hat{r}.\sigma_1)(3\hat{r}.\sigma_2) - \sigma_1.\sigma_2)(1 + \frac{3}{M_\pi r} + \frac{3}{(M_\pi r)^2}) + \sigma_1.\sigma_2]. \tag{3.2.18}$$

Thus we get[7],

$$V_\pi(r) = \frac{G^2}{4\pi}(\frac{M_\pi}{2m})^2[(\sigma^{(1)}.\sigma^{(2)}) + (\frac{1}{(M_\pi r)^2} + \frac{1}{M_\pi r} + \frac{1}{3})S_{12}]\frac{e^{-M_\pi r}}{r} \tag{3.2.19}$$

where S_{12} is the tensor operator,

$$S_{12} = [\frac{3(\sigma^{(1)}.r)(\sigma^{(1)}.r)}{r^2}] - \sigma^{(1)}.\sigma^{(2)}. \tag{3.2.20}$$

The potential $V_\pi(r)$ (r is the separation between two nucleons) has the radial part $\frac{e^{-M_\pi r}}{r}$, which corresponds to Yukawa interaction potential between two nucleons and whose range varies as $\sim \frac{1}{M_\pi}$, leading to the short range of this potential. As far as the term in the square bracket is concerned, the first term corresponds to purely spin dependent interaction ($i.e., \vec{\sigma}^{(1)}.\vec{\sigma}^{(2)}$), where as the second part contains the non-central or tensor component S_{12} which is a characteristic feature of low energy nucleon-nucleon interactions.

Chapter 4

Investigation of mesons as fermion-antifermion systems

The primary aim of this chapter is to investigate: the transition amplitude, the bound state energy, and then the squared mass difference of the corresponding spin-singlet and spin-triplet systems of mesons as fermion-antifermion bound states. In the preceding section we find a prescription for the derivation of the (perturbatively accessible part of the) effective potential acting between two particles described by a field theory.

4.1 The transition amplitude

Let us now apply the above prescription to a fermion-antifermion system. Accordingly, we consider the elastic scattering;

$$f(p_1, \tau_1) + \bar{f}(p_2, \tau_2) \rightarrow f(q_1, \tau_3) + \bar{f}(q_2, \tau_4), \tag{4.1.1}$$

of the involved fermion f and antifermion $\bar{f}$ (with masses m_1 and m_2, respectively). expressing the four component object Dirac spinors (also known as bispinors) $u(p, \tau)$ and $\upsilon(p, \tau)$, the general form of the T-matrix element for a process of the type (4.1.1)

reads;

$$T = \frac{1}{(2\pi)^6}\frac{m_1 m_2}{(E_1E_2E_3E_4)^{\frac{1}{2}}}\bar{u}(q_1,\tau_3)\Gamma_1 u(p_1,\tau_1)\times\bar{\upsilon}(p_2,\tau_2)\Gamma_2\upsilon(q_2,\tau_4)K. \tag{4.1.2}$$

Where $\bar{u}(q_1,\tau_3)$ and $u(p_1,\tau_1)$ are spinors for the outgoing and incoming fermions respectively. Whereas $\bar{\upsilon}(p_2,\tau_2)$ and $\upsilon(q_2,\tau_4)$ are spinors for the out gong and incoming antifermions respectively. Γ_i, i=1,2 represent some product of gamma matrices (Dirac matrices), which are 4×4 matrices. K denotes an (unspecified) interaction kernel, which is usually assumed to depend only on the momentum transfer $k \equiv p_1 - q_1$. The fourier transform of just this kernel yields the static interaction potential. The product of gamma matrices, Γ, is either of the following[7];

$$\Gamma_i = 1, \gamma_\mu, \sigma_{\mu\nu}, i\gamma_5\gamma_\mu, \gamma_5, \tag{4.1.3}$$

corresponding to scalar, vector, tensor, axial vector and pseudoscalar interactions. So we can have five different choices of Γ_i which can insert into equation (4.1.2). Though we have five different choices of Γ_i as seen above, there are many indications that the dominant spin structure for quark-antiquark interaction, originating from quantum chromodynamics, is vector $\Gamma_1\bigotimes\Gamma_2 = \gamma_\mu\bigotimes\gamma_\mu$ plus scalar $\Gamma_1\bigotimes\Gamma_2 = 1\bigotimes 1$. In the Dirac representation the Dirac spinors are given by:

$$u(p_i,\tau_i) = (\frac{s_i}{2m_i})^{\frac{1}{2}}\begin{pmatrix}1\\ \frac{\sigma.p_i}{s_i}\end{pmatrix}\chi_{\tau_i} \tag{4.1.4}$$

$$\upsilon(p_i,\tau_i) = (\frac{s_i}{2m_i})^{\frac{1}{2}}\begin{pmatrix}\frac{\sigma.p_i}{s_i}\\ 1\end{pmatrix}\chi^c_{\tau_i}, \tag{4.1.5}$$

where χ_τ is the two component spinor corresponding to spin projection τ, and we define:

$$E_1 \equiv (p_1^2+m_1^2)^{\frac{1}{2}}, E_2 \equiv (p_2^2+m_2^2)^{\frac{1}{2}}, \chi^c_{\tau_i} = -i\sigma_2\chi_i^*$$

$$E_3 \equiv (q_1^2 + m_1^2)^{\frac{1}{2}}, E_4 \equiv (q_2^2 + m_2^2)^{\frac{1}{2}}$$

and $s_1 \equiv E_1 + m_1,\ s_2 \equiv E_2 + m_2\ s_3 \equiv E_3 + m_1,\ s_4 \equiv E_4 + m_2$. Lets insert these into equation (4.1.2), in the center-of mass system $p \equiv p_1 = -p_2$, $q \equiv q_1 = -q_2$. For $\Gamma_i = \gamma_\mu$,

$$T \to T_v = \frac{1}{(2\pi)^6} \frac{m_1 m_2}{(E_1 E_2 E_3 E_4)^{\frac{1}{2}}} \bar{u}(q_1, \tau_3)\gamma_\mu u(p_1, \tau_1) \times \bar{\upsilon}(p_2, \tau_2)\gamma_\mu \upsilon(q_2, \tau_4) K_v. \quad (4.1.6)$$

Where

$$\bar{u}(q_1, \tau_3) = (\frac{s_3}{2m_1})^{\frac{1}{2}} \begin{pmatrix} \chi_{\tau_3}^{+c} & \chi_{\tau_3}^{+c} \frac{\sigma.q_1}{s_3} \end{pmatrix} \quad (4.1.7)$$

$$u(p_1, \tau_1) = (\frac{s_1}{2m_1})^{\frac{1}{2}} \begin{pmatrix} 1 \\ \frac{\sigma.p_1}{s_1} \end{pmatrix} \chi_{\tau_1} \quad (4.1.8)$$

$$\bar{\upsilon}(p_2, \tau_2) = (\frac{s_2}{2m_2})^{\frac{1}{2}} \begin{pmatrix} \chi_{\tau_2}^{+c} \frac{\sigma.p_2}{s_2} & \chi_{\tau_2}^{+c} \end{pmatrix} \quad (4.1.9)$$

$$\upsilon(q_2, \tau_4) = (\frac{s_4}{2m_2})^{\frac{1}{2}} \begin{pmatrix} \frac{\sigma.q_2}{s_4} \\ 1 \end{pmatrix} \chi_{\tau_4}^{c}, \quad (4.1.10)$$

and $\gamma_\mu = (\gamma_k, \gamma_4)$, with

$$\gamma_k = \begin{pmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0 \end{pmatrix}$$

$$\gamma_4 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

where k=1,2,3, σ_k are the standard 2×2 Pauli matrices.

Substituting all these things in equation (4.1.6), we obtain the T-matrix element of vectorial spin structure.

$$T_v = N[\delta_{\tau_1\tau_3}\delta_{\tau_2\tau_4} + \frac{1}{s_1 s_3}\delta_{\tau_2\tau_4}(p.q\delta_{\tau_1\tau_3} - ip\times q.\sigma_1) + \frac{1}{s_2 s_4}\delta_{\tau_1\tau_3}(p.q\delta_{\tau_2\tau_4} - ip\times q.\sigma_2)$$
$$+\frac{1}{s_1 s_2}[p^2\delta_{\tau_1\tau_3}\delta_{\tau_2\tau_4} - p^2\sigma_1.\sigma_2 + (p.\sigma_1)(p.\sigma_2)] + \frac{1}{s_3 s_4}[q^2\delta_{\tau_1\tau_3}\delta_{\tau_2\tau_4} - q^2\sigma_1.\sigma_2$$
$$+(q.\sigma_1)(q.\sigma_2)] + \frac{1}{s_1 s_4}[p.q\delta_{\tau_1\tau_3}\delta_{\tau_2\tau_4} - ip\times q.\sigma_+ + (p.q)(\sigma_1.\sigma_2) - (q.\sigma_1)(p.\sigma_2)] + \frac{1}{s_2 s_3}$$
$$[p.q\delta_{\tau_1\tau_3}\delta_{\tau_2\tau_4} - ip\times q.\sigma_+ + (p.q)(\sigma_1.\sigma_2) - (p.\sigma_1)(q.\sigma_2)] + \frac{1}{s_1 s_2 s_3 s_4}$$
$$[(p.q)^2\delta_{\tau_1\tau_3}\delta_{\tau_2\tau_4} - i(p.q)(p\times q.\sigma_+) - (p\times q.\sigma_1)(p\times q.\sigma_2)]]K_v. \qquad (4.1.11)$$

where $N = N_1 N_2 N_3 N_4$. And for the T-matrix element of scalar spin structure,i.e., for $\Gamma_i = 1$ we obtain,

$$T \to T_s = \frac{1}{(2\pi)^6}\frac{m_1 m_2}{(E_1 E_2 E_3 E_4)^{\frac{1}{2}}}\bar{u}(q_1,\tau_3)u(p_1,\tau_1)\times\bar{\upsilon}(p_2,\tau_2)\upsilon(q_2,\tau_4)K_s. \qquad (4.1.12)$$

Substituting the spinor matrices, we have;

$$T_s = -N[\delta_{\tau_1\tau_3}\delta_{\tau_2\tau_4} - \frac{1}{s_1 s_3}\delta_{\tau_2\tau_4}(p.q\delta_{\tau_1\tau_3} - ip\times q.\sigma_1) - \frac{1}{s_2 s_4}\delta_{\tau_1\tau_3}(p.q\delta_{\tau_2\tau_4} - ip\times q.\sigma_2)$$
$$+\frac{1}{s_1 s_2 s_3 s_4}[(p.q)^2\delta_{\tau_1\tau_3}\delta_{\tau_2\tau_4} - i(p.q)(p\times q.\sigma_+) - (p\times q.\sigma_1)(p\times q.\sigma_2)]]K_s, \qquad (4.1.13)$$

where we have used; $(\sigma.p)(\sigma.q) = p.q + i\sigma.(p\times q)$, $\sigma_1 \equiv \chi^+_{\tau_3}\sigma\chi_{\tau_1}$, $\sigma_2 \equiv \chi^+_{\tau_4}\sigma\chi_{\tau_2}$, $\sigma_+ \equiv \sigma_1\delta_{\tau_2\tau_4} + \sigma_2\delta_{\tau_1\tau_3}$. The normalization factors are given by,

$$N_i \equiv \frac{1}{(2\pi)^{\frac{3}{2}}}(\frac{s_i}{2E_i})^{\frac{1}{2}}.$$

4.2 The relativistic limit

Let us now investigate in some more detail the scattering of massless fermions i.e., $m_i = 0$. In this case the factors s_i in the above reduces to; $s_1 = s_2 = E_1 = E_2 = \sqrt{p^2}$, $s_3 = s_4 = E_3 = E_4 = \sqrt{q^2}$; where as the normalization factors N_i are no longer p dependent;

$$N_i \equiv \frac{1}{(2\pi)^{\frac{3}{2}}} \frac{1}{\sqrt{2}}. \tag{4.2.1}$$

Defining the unit vectors; $\hat{p} = \frac{\vec{p}}{|p|} = \frac{\vec{p}}{\sqrt{p^2}}$, $\hat{q} = \frac{\vec{q}}{|q|} = \frac{\vec{q}}{\sqrt{q^2}}$. and abbreviating their difference by $\mathbf{k} \equiv \hat{p} - \hat{q} \Rightarrow \mathbf{k}^2 \equiv (\hat{p} - \hat{q})(\hat{p} - \hat{q}) \Rightarrow 2(1 - \hat{p}.\hat{q})$, this gives

$$\hat{p}.\hat{q} = 1 - \frac{\mathbf{k}^2}{2}. \tag{4.2.2}$$

Then substituting (4.2.1), (4.2.2), in equations (4.1.11) and (4.1.13) we get;

$$T_v = \frac{1}{(2\pi)^6} \frac{1}{4} [(3 + \hat{p}.\hat{q})(1 + \hat{p}.\hat{q}) \delta_{\tau_1 \tau_3} \delta_{\tau_2 \tau_4}$$

$$-i(3 + \hat{p}.\hat{q})(\hat{p} \times \hat{q}.\sigma_+) - \mathbf{k}^2 \sigma_1.\sigma_2 + (\mathbf{k}.\sigma_1)(\mathbf{k}.\sigma_2) - (\hat{p} \times \hat{q}.\sigma_1)(\hat{p} \times \hat{q}.\sigma_2)] K_v \tag{4.2.3}$$

and

$$T_s = -\frac{1}{(2\pi)^6} \frac{1}{4} [\frac{1}{4} (\mathbf{k}^2)^2 \delta_{\tau_1 \tau_3} \delta_{\tau_2 \tau_4} + \frac{i}{2} \mathbf{k}^2 (\hat{p} \times \hat{q}.\sigma_+) - (\hat{p} \times \hat{q}.\sigma_1)(\hat{p} \times \hat{q}.\sigma_2)] K_s \tag{4.2.4}$$

4.3 The bound state energy

Now lets obtain the energy eigenvalues E of the system, which give the mass spectrum of the composite particles. Lets Consider $|\psi(k) >$ as two-particle states which are simultaneous eigenstates of the two particle Hamiltonian with eigenvalues E_k and k,

$$H|\psi(k) >= E_k|\psi(k) >, \tag{4.3.1}$$

as well as of the total momentum $p_1 + p_2$,

$$(p_1 + p_2)|\psi(k) >= k|\psi(k) > . \tag{4.3.2}$$

E_k and k denote energy and momentum of the two-particle system, and the states are normalized as

$$< \psi(l)|\psi(k) >= \delta^3(l - k). \tag{4.3.3}$$

The energy eigenvalue E_k is thus obtained from

$$E_k\delta^3(l - k) =< \psi(l)|H|\psi(k) > . \tag{4.3.4}$$

We assume that the Hamiltonian H which governs the dynamics of the two-particle system under consideration is of the form (3.0.1), with H_0 describing the free motion of the particles and an interaction potential V which does not depend on the center-of-momentum coordinate of the two particles, that is, $V = V(x)$. This latter feature ensures that the Hamiltonian commutes with the total momentum, $[H, p_1 + p_2]$=0, which is unavoidable prerequisite in order to be able to define the simultaneous eigenstates considered above. In general the potential V will consist of V_p which can be grasped by perturbation theory, and a nonperturbative part V_{np} which is beyond reach of perturbation theory. The two-particle Hamiltonian operator we are dealing with thus reads

$$H = H_{0,1}(p_1) + H_{0,2}(p_2) + V_p(x) + V_{np}(x). \tag{4.3.5}$$

In the center-of-momentum system of the two particles, k=0, the expectation value of the Hamiltonian (4.3.5) is given by

$$E_0 = \int d^3p|\psi(p)|^2[H_{0,1}(p) + H_{0,2}(-p)] - (2\pi)^3 \int d^3pd^3q\psi^*(p)T^B_{fi}\psi(q)$$
$$+ \int d^3x|\psi(x)|^2V_{np}(x), \tag{4.3.6}$$

where we have used $V_p(x) = -(2\pi)^3 \int d^3k e^{ik.x} T^B_{fi}$, with $k = q - p$. As has been demonstrated in section (4.1) for the case of fermion-antifermion systems, T-matrix elements are more easily derived in momentum space, that is why we give the expectation value of V_p in momentum space representation in (4.3.6). The nonperturbative part V_{np} of the potential can not be derived from a scattering process but has to be obtained from somewhere else. The simplest possibility for this is to guess its form in configuration space, which is the reason why we give in (4.3.6) its expectation value in coordinate space representation. In the next section we are going to see the singlet-triplet mass differences of mesons, which basically interact through the exchange of vector bosons, for two interaction potential choices.

4.4 Triplet-singlet mass differences of mesons

In the quark model, mesons are considered as being built up from quark-antiquark pair forming a "quarkonium" state. The quarks and antiquarks like any fundamental fermions and antifermions, carry spin $\frac{1}{2}$. When we compound the quark spin with the antiquark spin, the net result is $\vec{S} = \vec{1}$(spin triplet) or $\vec{S} = \vec{0}$ (spin singlet). And this coupling of quark spin with antiquark spin is known as **spin-spin coupling**. This strength is proportional to $(M_q M_{\bar{q}})^{-1}$. Each level of the fermion-antifermion bound state split further into two sublevels corresponding to these two possible values of $\vec{S}$, which leads to the hyperfine mass splitting. In addition to spin ($\vec{S}$) they can have orbital angular momentum 0,1,2... called S, P, D... states. Coupling the spin ($\vec{S}$) with the orbital angular momentum ($\vec{L}$) yields the total angular momentum of the system $\vec{J} = \vec{S} + \vec{L}$. The resulting energy levels in atomic physics are labeled using spectroscopic notation as ${}^{2S+1}L_J$, and so the lowest levels are ${}^1S_0$ and ${}^3S_1$ and so on. There are mesons observed corresponding to each of these levels, for example ${}^1S_0(\pi)$, ${}^3S_1(\rho)$. Just for the purpose of illustration we will apply here the ideas developed so far to the mass difference between mesons which differ only in the total spin of constituting quark-antiquark pair. Mesons with $\vec{S} = \vec{0}$, and $\vec{S} = \vec{1}$ are also called pseudoscalar and vector mesons, respectively. In the Breit-Fermi approximation, the mass difference between the spin-triplet and spin-singlet states of mesons comes from the hyperfine interaction which depends mainly on the short-range part of quark-antiquark potential. As already mentioned, we will treat the quarks as massless particles. Experimentally, the difference of the squared masses of corresponding spin-singlet and spin-triplet quarkonium states which contain **at least one light quark** which are not self-conjugate with zero isospin, have been found to be constant to a

surprisingly high degree of accuracy [8-10]:

$$[M_v(^3S_1)]^2 - [M_p(^1S_0)]^2 \simeq 0.56GeV^2. \tag{4.4.1}$$

Hence, for most mesons, it should be a good approximation to treat the hyperfine interaction as a small perturbation to the static effective potential which determines the spin-averaged energy levels of the meson system. The free relativistic Hamiltonian

$$\sum_i H_{0,i}(\pm p) = \sum_i (p^2 + m_i^2)^{\frac{1}{2}} \tag{4.4.2}$$

entering in equation (4.3.11) reduces in the massless case to $\sum_i H_{0,i}(\pm p) = \sqrt{p^2}$. We only consider ground states, with vanishing orbital angular momentum, which entails spherical symmetry, i.e.,

$$\psi(\mathbf{x}) = \psi(r), r \equiv \sqrt{\mathbf{x}^2}, \psi(\mathbf{p}) = \psi(p), p \equiv \sqrt{\mathbf{p}^2}.$$

The Fourier transform of the vector product $p \times q$ is the orbital angular momentum. Accordingly, all terms in the T-matrix element (4.1.11) and (4.1.13) involving this expression do not contribute to the energy expectation value (4.3.6) for ground states. In gauge theories the perturbative part of the interaction is very likely to originate from gauge-boson exchange. Consequently, the corresponding potential $V_p(x)$ is of vector type. For one vector-boson exchange the interaction kernel K_v[3] in (4.1.11) reads

$$K_v = \frac{k}{(p^2q^2)^{\frac{1}{2}}\mathbf{k}^2} \tag{4.4.3}$$

In quantum electrodynamics, for one-photon exchange, the parameter k equals $Q_1Q_2e^2$, where Q_i are the electric charges of the involved particles in units of the electron charge e. In quantum chromodynamics, for one-gluon exchange between quarks in a

color-singlet state, the parameter k equals $\frac{4}{3}\alpha^2$, where α is the strong coupling constant and the factor $\frac{4}{3}$ arises from the color. For an arbitrary scalar function f(p,q), due to spherical symmetry the relation

$$\int d^3p d^3q k_i k_j f(p,q) = \frac{1}{3}\delta_{ij} \int d^3p d^3q k^2 f(p,q)$$

holds. Decomposing the product $k_i k_j$ showing up in (4.2.3) like

$$k_i k_j = [k_i k_j - \frac{1}{3}\mathbf{k}^2\delta_{ij}] + \frac{1}{3}\mathbf{k}^2\delta_{ij} \tag{4.4.4}$$

into a traceless part (the term in parentheses) and a trace part, one finds that only the trace part contributes to the energy expectation value. Furthermore, we assume that the nonperturbative part $V_{np}(x)$ of the interaction is described by a central potential $V_{np}(\mathbf{x}) = V_{np}(r)$. We now discuss quark and antiquark bound states, where the quark and the antiquark are coupled by means of a linearly rising potential $V_{np}(r)$ in the next section.

4.4.1 Quark-antiquark bound states with linear potential confinement

It is reasonable to believe that for quark-antiquark bound states, the confining potential is a linear rise, $V_{np}(r)$=ar[3]. Using (4.4.4) and the assumption that the Fourier transformation of $p \times q$=0, T^B_{fi} in (4.2.3) is simplified as

$$T^B_{fi} = \frac{1}{(2\pi)^6}\frac{1}{4}[(3+\hat{p}.\hat{q})(1+\hat{p}.\hat{q})\delta_{\tau_1\tau_3}\delta_{\tau_2\tau_4} - \frac{2}{3}\mathbf{k}^2\sigma_1.\sigma_2]K_v. \tag{4.4.5}$$

Under the above assumptions and equation (4.4.5) the bound state energy (4.3.6) is given by

$$E_0 = 2\int d^3p|\psi(p)|^2 p - \frac{1}{(2\pi)^3}\frac{k}{4}|\int_0^\infty dpp\psi(p)|^2[\Omega^2 - \frac{2}{3}(4\pi)^2 < \sigma_1.\sigma_2 >] + a\int d^3x|\psi(r)|^2 r, \tag{4.4.6}$$

with $\Omega^2 \equiv \int d\Omega_p d\Omega_q \frac{1}{\mathbf{k}^2}(3+\hat{p}.\hat{q})(1+\hat{p}.\hat{q})$, where $d\Omega$ is solid angle and $\mathbf{k} = 2(1-\hat{p}.\hat{q})$. We adopt a standard variational technique, by evaluating the energy expectation value (4.4.6) with the help of some trial states $|\psi(\lambda)>$ characterized by a variational parameter λ,

$$E(\lambda) =< \psi(\lambda)|H|\psi(\lambda) > .$$

By minimizing the resulting expression with respect to λ, we get,

$$E \simeq E(\lambda_{min}), \frac{dE(\lambda)}{d\lambda}|_{\lambda_{min}} = 0. \tag{4.4.7}$$

Lets use the Gaussian trial wave functions

$$\psi(x) = \frac{\lambda^{\frac{3}{2}}}{\pi^{\frac{3}{4}}} exp[-\frac{\lambda^2 r^2}{2}], \psi(p) = \frac{1}{\lambda^{\frac{3}{2}}\pi^{\frac{3}{4}}} exp[-\frac{p^2}{2\lambda^2}], \tag{4.4.8}$$

with normalization

$$\int d^3x|\psi(x)|^2 = \int d^3p|\psi(p)|^2 = 1. \tag{4.4.9}$$

Already from dimensional considerations the general structure of $E(\lambda)$ as a function of λ is[2]

$$E(\lambda) = A\lambda + B\frac{a}{\lambda}. \tag{4.4.10}$$

Now substituting equation (4.4.8) in (4.4.6) and integrating we get

$$E_0 = (\frac{4}{\sqrt{\pi}} - \frac{k}{32\pi^4\sqrt{\pi}}[\Omega^2 - \frac{2}{3}(4\pi)^2 < \sigma_1.\sigma_2 >])\lambda + \frac{2}{\sqrt{\pi}}\frac{a}{\lambda}. \tag{4.4.11}$$

Comparing equation (4.4.11) with (4.4.10) we get the constants A and B as

$$A = \frac{4}{\sqrt{\pi}} - \frac{k}{32\pi^4\sqrt{\pi}}[\Omega^2 - \frac{2}{3}(4\pi)^2 < \sigma_1.\sigma_2 >]; B = \frac{2}{\sqrt{\pi}}. \tag{4.4.12}$$

Minimizing of $E(\lambda)$ in the form (4.4.10)

$$\frac{dE(\lambda)}{d\lambda}|_{\lambda_{min}} = 0 \Rightarrow \lambda_{min} = [\frac{Ba}{A}]^{\frac{1}{2}}, \tag{4.4.13}$$

then substituting of (4.4.13) to (4.4.10) we get the energy corresponding to λ_{min}

$$E(\lambda_{min}) = 2\sqrt{ABa}. \tag{4.4.14}$$

Squaring equation (4.4.14) and substituting of the values of A and B which are given in equation (4.4.12), we get squared energy

$$E(\lambda_{min})^2 = \frac{8}{\sqrt{\pi}}(\frac{4}{\sqrt{\pi}} - \frac{k}{32\pi^4\sqrt{\pi}}[\Omega^2 - \frac{2}{3}(4\pi)^2 < \sigma_1.\sigma_2 >]), \tag{4.4.15}$$

with the value $\Omega^2 = 195.7$.

The spin expectation value $< \sigma_1.\sigma_2 >$ depends on the total spin S of the two-fermion state $|\psi >$:

$$< \sigma_1.\sigma_2 >= \begin{cases} -3 \text{ for spin singlets}, S = 0, \\ +1 \text{ for spin triplets}, S = 1. \end{cases} \tag{4.4.16}$$

Now substituting (4.4.16) in (4.4.15) and taking the difference of both spin cases, we have

$$E^2_{S=1} - E^2_{S=0} \simeq \frac{32}{3\pi^3}ka. \tag{4.4.17}$$

The energy in the rest system of the bound state is, of course, nothing else but the mass of the composite system. So the squared mass difference we are looking for is obtained as

$$M^2_{s=1} - M^2_{s=0} \simeq \frac{32}{3\pi^3}ka. \tag{4.4.18}$$

By choosing the coupling constant $\alpha \sim 1$, and a is the usual string tension, whose value should be around $0.179GeV^2$, the ground sate energy can be easily obtained from equation (4.4.14) as (in units of GeV)

$$E_0 = \begin{cases} 1.344 \text{ for spin triplet}, S = 1, \\ 1.313 \text{ for spin singlet}, S = 0. \end{cases} \tag{4.4.19}$$

4.4.2 Quark-antiquark bound states with funnel potential confinement

In QCD the number that plays the role of coupling constant, α is in fact not constant at all, but depends on the separation distance between the interacting particles. Although at the relatively large distances $\sim 10^{-15}m$, $\alpha \simeq 1$, at very short distances (less than the size of the proton i.e.; $<< 10^{-15}$m), α becomes quite small. This phenomena is known as asymptotic freedom. It means that within a proton, the quarks move freely. Now taking this concept in our mind lets construct beautifully the confining potential. For small distances between the quarks, one expects from one-gluon exchange, by analogy to one-photon exchange in QED, a coulomb like contribution to the potential, that is V(r)$\approx \frac{1}{r}$. For large distances, in order to be able to describe confinement, the potential has to rise to infinity, and we can guess it simply as V(r) $\approx r$. The most reasonable possibility to construct an inter quark potential which satisfies both of the above constraints is simply add these two contributions. This leads to a funnel potential

$$V(r) = -\frac{k}{r} + ar, \tag{4.4.20}$$

which depends on just two parameters; on the coupling constant k and on the slope a of the linear term. Now lets find the ground state energy and the mass squared difference of singlet-triplet of the fermion-antifermion bound states using this potential.

Taking similar steps as in section (4.4.1) the bound state energy is given by

$$E_0 = 2\int d^3p|\psi(p)|^2 p - \frac{1}{(2\pi)^3}\frac{k}{4}|\int_0^\infty dpp\psi(p)|^2[\Omega^2 - \frac{2}{3}(4\pi)^2 < \sigma_1.\sigma_2 >]$$
$$+\int d^3x|\psi(r)|^2(-\frac{k}{r} + ar), \tag{4.4.21}$$

with $\Omega^2 \equiv \int d\Omega_p d\Omega_q \frac{1}{\mathbf{k}^2}(3+\hat{p}.\hat{q})(1+\hat{p}.\hat{q})$, with the same value as in the linear potential

case. By substituting the Gaussian wave functions (4.4.8) in (4.4.21) and integrating we get

$$E_0 = (\frac{4}{\sqrt{\pi}} - \frac{2k}{\sqrt{\pi}} - \frac{k}{32\pi^4\sqrt{\pi}}[\Omega^2 - \frac{2}{3}(4\pi)^2 < \sigma_1.\sigma_2 >])\lambda + \frac{2}{\sqrt{\pi}}\frac{a}{\lambda}. \tag{4.4.22}$$

Comparing equation (4.4.22) with (4.4.10) we get the constants A and B as

$$A = \frac{4}{\sqrt{\pi}} - \frac{2k}{\sqrt{\pi}} - \frac{k}{32\pi^4\sqrt{\pi}}[\Omega^2 - \frac{2}{3}(4\pi)^2 < \sigma_1.\sigma_2 >]; B = \frac{2}{\sqrt{\pi}}. \tag{4.4.23}$$

Now taking the use of (4.4.13) and substituting it to (4.4.10) the energy corresponding to λ_{min} is

$$E(\lambda_{min}) = 2\sqrt{ABa}. \tag{4.4.24}$$

Squaring equation (4.4.24) and substituting of the values of A and B which are given in equation (4.4.23), we get squared energy

$$E^2(\lambda_{min}) = \frac{8}{\sqrt{\pi}}(\frac{4}{\sqrt{\pi}} - \frac{2k}{\sqrt{\pi}} - \frac{k}{32\pi^4\sqrt{\pi}}[\Omega^2 - \frac{2}{3}(4\pi)^2 < \sigma_1.\sigma_2 >]). \tag{4.4.25}$$

By the use of (4.4.16) in (4.4.25) and taking the difference of both spin cases, we have

$$E^2_{S=1} - E^2_{S=0} \simeq \frac{32}{3\pi^3}ka. \tag{4.4.26}$$

The energy in the rest system of the bound state is, of course, nothing else but the mass of the composite system. So the squared mass difference we are looking for is obtained as

$$M^2_{s=1} - M^2_{s=0} \simeq \frac{32}{3\pi^3}ka. \tag{4.4.27}$$

By choosing the coupling constant $\alpha \sim 1$, and a is the usual string tension, whose value should be around $0.179GeV^2$, the ground sate energy can be easily obtained

from equation (4.4.24) as (in units of GeV)

$$E_0 = \begin{cases} 0.768 \text{ for spin triplet}, S = 1, \\ 0.713 \text{ for spin singlet}, S = 0. \end{cases} \tag{4.4.28}$$

Obviously, all predictions for the mass squared differences are independent of the mass of the particles which constitute the bound state in both potential cases. We see that the result (4.4.18) which is also obtained by Lucha[3], in the linear potential case is the same as our result (4.4.27), in the funnel potential case. So we can say that the squared mass difference between spin-triple and spin-singlet of mesons is independent of the choice of confinement. But the ground state energy for the light mesons obtained in our case, by taking the funnel potential, is different from the result when we take the linear potential, in which the squared ground state energy is lowered by an amount of $\frac{2k}{\sqrt{\pi}}$, see (4.4.15) and (4.4.25). The ground state energy which we obtain by taking the funnel potential interaction is significantly more closer to the experimental result[11], than when we take the linear potential (see table I).

TABLE I. The mass spectra (in units of GeV) of ρ and π predicted for the linear potential and the funnel potential, as well as the experimental values[11].

State	linear potential	funnel potential	experiment[11]
$\rho(^3S_1)$	1.344	0.768	0.768
$\pi(^1S_0)$	1.313	0.713	0.138

Now lets study the fermion-antifermion interaction which interact through the exchange of scalar bosons, hypothetically, for the two choices of interaction potentials.

4.5 Fermion-antifermion interaction through the exchange of scalar bosons

Lets now assume fermions and antifermions in mesons interact through the exchange of scalar bosons unlike vector bosons as we see above i.e.; we consider a hypothetical interaction which is governed by an effective theory. Since under Fourier transformation, the vector $p \times q$ is orbital angular momentum which do not contribute to the energy expectation value for ground states, the T_s we obtain in equation (4.2.4), can be written as

$$T_s = -\frac{1}{(2\pi)^6}\frac{1}{4}[\frac{1}{4}(\mathbf{k}^2)^2]K_s, \tag{4.5.1}$$

where

$$K_s = \frac{k}{\vec{p_1} - \vec{q_1}} = \frac{k}{(p^2q^2)^{\frac{1}{2}}\mathbf{k}^2}. \tag{4.5.2}$$

Taking the use of (4.5.1) the bound state energy for linear potential confinement can be obtained as

$$E_0 = 2\int d^3p|\psi(p)|^2p + \frac{1}{(2\pi)^3}\frac{k}{16}\int d^3p\mathbf{k}^2d^3q\psi^*(p)\frac{1}{(p^2q^2)^{\frac{1}{2}}}\psi(q)$$
$$+a\int d^3x|\psi(r)|^2r. \tag{4.5.3}$$

Now substituting the Gaussian wave functions (4.4.8) in (4.5.3) and integrating gives the result

$$E_0 = (\frac{4}{\sqrt{\pi}} + \frac{2k}{\pi^{\frac{5}{2}}})\lambda + (\frac{2a}{\sqrt{\pi}})\frac{1}{\lambda}. \tag{4.5.4}$$

Comparing equation (4.5.4) with (4.4.10) we get the constants A and B as

$$A = (\frac{4}{\sqrt{\pi}} + \frac{2k}{\pi^{\frac{5}{2}}}); B = \frac{2}{\sqrt{\pi}}. \tag{4.5.5}$$

Now squaring equation (4.4.14) and substituting of the values of A and B which are given in equation (4.5.5), we get

$$E(\lambda_{min})^2 = \frac{8}{\sqrt{\pi}}(\frac{4}{\sqrt{\pi}} + \frac{2k}{\pi^{\frac{5}{2}}}). \tag{4.5.6}$$

We see that equation (4.5.6) is independent of the total spin of the bound state, so the squared mass difference between spin triplet and spin singlet in the rest system of the bound system is simply given by

$$M^2_{s=1} - M^2_{s=0} = 0. \tag{4.5.7}$$

Now lets take the funnel potential instead of the linear potential. Taking the use of (4.5.1) the bound state energy for funnel confinement can be obtained as

$$E_0 = 2\int d^3p|\psi(p)|^2 p + \frac{1}{(2\pi)^3}\frac{k}{16}\int d^3p\mathbf{k}^2 d^3q\psi^*(p)\frac{1}{(p^2q^2)^{\frac{1}{2}}}\psi(q)$$
$$+\int d^3x|\psi(r)|^2(-\frac{k}{r} + ar). \tag{4.5.8}$$

Now substituting the Gaussian wave functions (4.4.8) in (4.5.8) and integrating gives the result

$$E_0 = (\frac{4}{\sqrt{\pi}} - \frac{2k}{\sqrt{\pi}} + \frac{2k}{\pi^{\frac{5}{2}}})\lambda + (\frac{2a}{\sqrt{\pi}})\frac{1}{\lambda}. \tag{4.5.9}$$

Comparing equation (4.5.9) with (4.4.10) we get the constants A and B as

$$A = (\frac{4}{\sqrt{\pi}} - \frac{2k}{\sqrt{\pi}} + \frac{2k}{\pi^{\frac{5}{2}}}); B = \frac{2}{\sqrt{\pi}}. \tag{4.5.10}$$

Now squaring equation (4.4.14) and substituting of the values of A and B which are given in equation (4.5.10), we get

$$E(\lambda_{min})^2 = \frac{8}{\sqrt{\pi}}(\frac{4}{\sqrt{\pi}} - \frac{2k}{\sqrt{\pi}} + \frac{2k}{\pi^{\frac{5}{2}}}). \tag{4.5.11}$$

We see that equation (4.5.11) is independent of the total spin of the bound state, so the squared mass difference between spin triplet and spin singlet in the rest system of the bound system is simply given by

$$M_{s=1}^2 - M_{s=0}^2 = 0. \tag{4.5.12}$$

This tells us that the squared mass difference between spin-triplet and spin-singlet mesons, in which the bound state constituents interact through the exchange of scalar bosons, vanish for both choices of potential confinements. So, in this hypothetical consideration we can say that, the scalar bosons that exchange between the two constituents of the system have no any contribution for the ground state squared mass difference between spin-triplet and spin-singlet of mesons. When we take the funnel potential, we understand that there is a decrease in the ground state energy of mesons, to that of taking the linear potential.

$$E_0 = \begin{cases} 1.395GeV \text{ for linear potential confinement} \\ 0.872GeV \text{ for funnel potential confinement.} \end{cases} \tag{4.5.13}$$

Chapter 5

Conclusion

In chapter (3) we presented derivation of the effective potentials in electron-electron and nucleon-nucleon systems from QED and meson field theory. Towards this end we have checked the validity of this approach by deriving the static coulomb interaction and the spin-orbit interaction between two electrons. Then we have derived the Yukawa and the one-pion exchange potentials between two nucleons, and we got the expected results. In chapter (4) we saw the investigation of fermion-antifermion bound states. We were mainly focusing on this chapter in deriving the transition amplitude, the ground state energy, and the squared mass difference between corresponding spin-triplet and spin-singlet of light mesons which serves as a tool to demonstrate the wide range of applicability of the effective Hamiltonian method, as just relativistic kinematical effect. We used two types of confinements:
(i) linear potential and (ii) funnel potential (incorporating one-gluon exchange and a linear confinement) between quarks and antiquarks to derive the observables stated above. We have also proposed an effective theory involving a scalar boson exchange between the constituents of a bound state. For this case we have derived the ground state energy, and the squared mass difference between the spin-triplet and spin-singlet

states of the bound states, as an academic exercise in applications of QFT. As far as the squared mass difference between the spin-triplet and spin-singlet states is concerned it comes out as

$$M_{S=1}^2 - M_{S=0}^2 \approx \frac{32}{3\pi^3} ka \tag{5.0.1}$$

for both linear as well as the funnel potentials needed as confinements. We see that the squared mass difference between the spin-triplet and spin-singlet of light mesons is independent of the mass of its constituents. The results of equations (4.4.18) and (4.4.27), tells us that the squared mass shift between mesons which differ only in their total spin is independent of the confining potential we choose. As far as the ground state energy is concerned we obtained that, the ground state energy of mesons with funnel potential confinement is smaller and much closer to the experiment[11] than the ground state energy with linear potential confinement. This is due to the fact that inclusion of one gluon exchange depresses the ground state energy of the mesons, as we demonstrated in our calculations. So we can arrive at a conclusion that, it is better to take the funnel interaction potential between quarks and antiquarks instead of linear confinements, see table(I). Looking at equations (4.5.7) and (4.5.12), we can say that the hypothetical scalar bosons which exchange between constituents of a bound state, does not cause ground state mass shift in bound states which differ only in their total spin. By comparing the ground state energy of light bound states obtained from a pure vector boson exchange and from a pure scalar boson exchange between the constituents, we can conclude that ground state energy of light bound states is higher when we consider pure scalar boson exchange than when we consider pure vector boson exchange. We calculated this as a purely academic exercise in quantum field theory.

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