FURTHER DEVELOPMENT OF POLAR WIND MODEL

A thesis submitted to
Addis Ababa University
Faculty Of Science
Department Of Physics

by

Tadesse Terefe

In Partial Fulfillment
of the Requirements for the Degree of
MASTER OF SCIENCE IN PHYSICS

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FURTHER DEVELOPMENT OF POLAR WIND MODEL

Dr. Baylie Damtie, Advisor

Examiner:

Dr. A.K Chaubey, Committee Chair

Dr. Esayas Belay, Examiner

Dr. Gizaw Mengistu, Examiner

Date Approved __________________________
I dedicate this work to my parents.
This thesis is based on Akebono satellite data and different polar wind literatures. Basic familiarity with plasma physics would be useful to understand the thesis.

In the present thesis we have investigated different polar wind models including hydrodynamic and hybrid models. The pros and cons of different models are investigated numerically. We have also introduced a new model that can explain the observation made by Akebono Satellite.

Introduction, objective of the study and thesis organization is discussed in chapter one. In chapter two what makes physicists to study polar wind? is explained in detail by discussing sources and causes of ionospheric outflow followed by some ionospheric phenomenon.

The main work of the study are presented in chapter three, in this chapter magneto hydrodynamic MHD (fluid) approach is presented corresponding with graphical representation, advantage and limitation are also discussed. And other recent model (self consistent hybrid model) is reproduced in the Appendix part of the thesis.

I hope this thesis is important for any interested person to know the nature (characteristics) of polar wind. Moreover thoroughly, a relevant literature review has been included in two ways, where key ideas, are introduced in the text and they are associated with specific original publications, in addition, at the end I included a brief section entitled 'Bibliography'.

Finally I would like to thank Dr.Bayile Damtie for his suggestions and giving very important comment for the development of this thesis.

Tadesse Terefe
Addis Ababa University
ACKNOWLEDGEMENTS

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I would like to send my special thanks to my parents, brother and sister. They have known for a long time that I have been fascinated with the space and eager for studying space physics during undergraduate study. When I got a chance to study in Addis Ababa University, they just encouraged me to earn a degree instead of earning a lot of money. I would like to mention that I have spent a very fruitful time on my study, and with my colleague (partner in common hobbies, work and study). Finally I would like to thank among the many individuals who have generally offered suggestions for making this study more usable are Ato Luelseged Zekarias , (Director of st.Raguel school)is due to continuing appreciation and for his kind advice.

Tadesse Terefe
# TABLE OF CONTENTS

DEDICATION .......................................................... iii
PREFACE ............................................................. iv
ACKNOWLEDGEMENTS ............................................... v
LIST OF TABLES ..................................................... viii
LIST OF FIGURES ................................................... ix
ABSTRACT ............................................................. xi

1 INTRODUCTION .................................................. 1
   1.1 Objective of the study ...................................... 1
   1.2 Organization of the thesis ................................. 2

2 BACKGROUND INFORMATION ................................. 4
   2.1 Ionosphere .................................................. 4
   2.2 The solar wind and Earth’s magnetic field ............... 4
   2.3 Some observational evidence ............................. 5

3 EARLIER MODELS OF POLAR WIND ....................... 9
   3.1 Introduction ................................................ 9
       3.1.1 Transport equation .................................. 9
   3.2 Subsonic outflow ......................................... 12
       3.2.1 O\textsuperscript{+} outflow ............................. 14
       3.2.2 H\textsuperscript{+} outflow ............................. 16
   3.3 Supersonic outflow ....................................... 22
   3.4 Limitation of earlier model approach .................... 25

4 HYDRODYNAMIC DRIVEN POLAR WIND MODEL .......... 26
   4.1 Driven shock wave ....................................... 26
   4.2 Subsonic flow ............................................. 27
4.3 Supersonic flow ......................................................... 29

5 CONCLUSIONS .......................................................... 35

5.1 Future perspective ...................................................... 36

APPENDIX A — SOME BASIC CONCEPTS AND FORMULAE .... 37
APPENDIX B — IMPORTANT AREAS ................................. 41
APPENDIX C — HYDRODYNAMIC MODELLING .................... 44
APPENDIX D — HYBRID MODEL (REPRODUCE) ................. 48
DECLARATION ............................................................. 56
## LIST OF TABLES

1. A summary of the polar wind day-night asymmetries observed by the Akebono satellite. The average ion outflow velocities were measured between 5000 and 9000 km altitude, and the electron temperature ratios were obtained at about 1700 km altitude [Abe and et al., 1993].

2. Day-side polar wind observations by the Akebono satellite. Ion measurements were made at 5000-9000 km altitude [Abe and et al., 1993]. Electron observations were made at about 1700 km altitude [Yau and et al., 1997].

3. Boundary conditions for the four cases with different relative photoelectron density. Other parameters are common to these cases: $n_o(O^+ \text{ density})$, $n_h(H^+ \text{ density})$, $T_e \text{ (thermal electron temperature)}$.


5. The collision frequency coefficients $B_{ij}$ for ion-ion interaction [Schunk, 1978].

6. Momentum transfer collision frequencies for resonant ion-neutral interactions [Bank and et al., 1973].

7. The collision frequency coefficients $C_m \times 10^{10}$ for non resonant ion-neutral interaction (r means the collisional interaction is resonant) [Bank and et al., 1973].

8. Calculated result from numerical model.
## LIST OF FIGURES

1. Different mechanisms of polar wind (University of Alaska plot) (WWW.ann-geophys). .......................................................... 3
3. Ionospheric layers, the left panel: indicates the variation of electron density in night and day side (time), and the right panel: indicates the predominant ion populations with their respective heights above ground and variation of electron density with altitude (Fisica, 2000). ........................................... 6
4. Schematic diagram of the Sun, Earth’s magnetosphere and the Sun-Earth interaction (Original picture from NASA) (C. Lathuillere and et al, 2002) . 7
5. Ion outflow, obtained form satellite measurements ................................................. 8
6. $O^+$ density versus altitude, diffusive equilibrium distribution. .......................... 15
7. $O^+$ density versus altitude with relative drift velocity. ..................................... 16
8. $H^+$ density versus altitude. ............................................................................. 17
9. Neutral atoms density versus altitude. ................................................................. 19
10. Number density of $H^+$ versus altitude by consider momentum transfer collision frequency. ................................................................. 20
11. Number density of $H^+ and O^+$ by considering continuity equation with production and loss term. ......................................................... 21
12. Number density according to Akebono satellite data (data from [Abe and et al, 1993]). ......................................................................... 21
13. Mach number of $O^+$ versus altitude. ................................................................. 23
15. Mach number of $H^+ and O^+$ from Akebono satellite data (from [Abe and et al, 1993]). ................................................................. 24
16. Number density of $H^+$ by considering driven shock wave from conservation equations (momentum). ...................................................... 28
17. Number density of $H^+$ by considering driven shock wave from conservation equations (momentum). ...................................................... 29
<table>
<thead>
<tr>
<th>Page</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>Number density of $H^+$ and $O^+$ by considering driven shock wave from conservation equations (momentum).</td>
</tr>
<tr>
<td>19</td>
<td>Velocity according to Akebono satellite data (data from [Abe and et al, 1993]).</td>
</tr>
<tr>
<td>20</td>
<td>Mach number of $O^+$ by considering driven shock wave from conservation equations of ions.</td>
</tr>
<tr>
<td>21</td>
<td>Mach number of $H^+$ and $O^+$ by considering driven shock wave from conservation equations (momentum).</td>
</tr>
<tr>
<td>22</td>
<td>Statistical description.</td>
</tr>
<tr>
<td>23</td>
<td>Calculating distribution function.</td>
</tr>
<tr>
<td>24</td>
<td>Mach number of $O^+$ versus altitude by hybrid model.</td>
</tr>
</tbody>
</table>
This thesis deals with modeling of major ions outflow in the polar cap at altitudes higher than 400km. Different ion outflow models are numerical investigated and the pros and cons of these models are discussed. Observations are crucial for model validations. In this work we have used Akebono Satellite observations, which showed that between 5000 and 9000km altitudes $H^+$ and $O^+$ can have higher outflow velocities in the day-side region that on the night-side [Abe and et al,1993a, 1993b]. classical polar wind model cannot explain fully these observations [Schunk and etal,1981, 1988]. In this thesis, in order to address the observation we discuss hydrodynamic model by considering driven shock wave. Most of observed properties are well explained by using this model.
CHAPTER 1

INTRODUCTION

1.1 Objective of the study

The thesis contributes to the polar ion outflow research to calculate (numerically solved) the spatial (altitude) variation of number density and outflow velocity. Also we have to calculate the potential drop and heat flux along the magnetic field lines.

The fact that light ionospheric ions is able to escape from the Earth’s gravitational field in the polar region of the ionosphere was first propose by Axfored (1968). In the same year Banks and Holzer modelled that $H^+$ and $He^+$ are rapidly accelerated to supersonic speed and their model predicate that these ion species to be a dominant above 3000 km [Bank and Holzer, 1968, 1969a, 1969b]. Two years later, light ion outflows were confirmed by satellite observations [Hoffman, 1970, Brinton, 1971]. Since after wards for about 30 years researchers have been investigated different models to describe outflow characteristics observed by different satellites.

This thesis is concerned with the outflow of hydrogen and oxygen ions ($H^+$ and $O^+$) from the polar ionosphere region to the magnetosphere. The outflow of ions takes place along open magnetic field lines as shown in Figure 1.

Recently, Akebono satellite measurements between 5000 and 9000 km altitude have revealed unexpected ion transport properties in the polar region [Abe and et al, 1993a,1993b]. For example $O^+$ was most often found to be the major ion species, dominant over $H^+$, contrary to the classical polar wind model belief that due to their heavier mass very few $O^+$ ions are able to overcome the gravitational force and escape to such high altitudes. In fact, the measured $O^+$ outflow velocity (see Table 1 are much larger than the value expected from classical polar wind models [Schunk and Watkins, 1981]. All this ion outflow velocities, suggested a higher ambipolar electric field than that predicted by classical polar wind models [Ganguli, 1996], and are consistent with the values of field-aligned potential drop deduced by Winning and Gurgiolo (1982) based on the DE-2 satellite measurements.

Oxygen ion ($O^+$) gains a 1eV (equivalent to $11,000^0k$) from thermal ionospheric processes. This amount of energy, however, does not enable $O^+$ to escape the Earth’s gravitation field. Where does the extra energy for $O^+$ escape come from? This question has been a concern of many space scientists. So, our study on driven shock wave polar
Table 1: A summary of the polar wind day-night asymmetries observed by the Akebono satellite. The average ion outflow velocities were measured between 5000 and 9000km altitude, and the electron temperature ratios were obtained at about 1700km altitude [Abe and et al,1993].

<table>
<thead>
<tr>
<th></th>
<th>Day</th>
<th>Night</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_p$</td>
<td>12-13 km/s</td>
<td>4-5 km/s</td>
</tr>
<tr>
<td>$u_o$</td>
<td>6-7 km/s</td>
<td>2-3 km/s</td>
</tr>
<tr>
<td>$T_{e,up}/T_{e,down}$</td>
<td>1.5-2</td>
<td>$\approx 1$</td>
</tr>
</tbody>
</table>

wind is also address the observation or experimental indications by Akebono satellite.

1.2 Organization of the thesis

In the remainder of the previous section we discussed objective, development and problem of the thesis. Basic concept of ionosphere, magnetosphere, and open magnetic field lines in the polar region are discuss in chapter two.

Chapter three deals with earlier approach of polar wind models. And why they are inadequate to describe the Akebono satellite observations?

Chapter four deals with physics of driven shock wave, and we will discuss hydrodynamic driven polar wind model and we will discuss the result of the model, how to describe the flow as observed by Akebono satellite? Finally in Chapter five we will put the result of the study in short and we will indicate the future work.
Figure 1: Different mechanisms of polar wind (University of Alaska plot) (WWW.ann-geophys).
CHAPTER 2

BACKGROUND INFORMATION

2.1 Ionosphere

The ingredients for an ionosphere (Figure 2) are a neutral atmosphere, and source of ionization (photons and energetic particle precipitation). The process are called photoionization, and impact ionization consequently. Photons primarily originate from the Sun where as, energetic (ionizing) particles come from the galaxy (cosmic rays), the Sun, the magnetosphere. The requirement on the ionizing photons and particles is that their energies ($h\nu$ in the case of photons, and kinetic energy in the case of particles) exceed the ionization potential or binding energy of a neutral atmosphere gases. Impact ionization occurs typically at high altitude.

The ionosphere plasma density varies greatly with altitude and time as shown in left panel of Figure 3. Ionospheric layer are classified according to the taxonomy: D-layer (between about 60-90 km), E-layer (between about 90-150 km) and F-layer (about above 150 km).

The F-region is the thickest region of the ionosphere. It is divided in to $F_1$ and $F_2$ layers. During night time $F_1$ layer usually disappears due to recombination. As a result, of being closer to the sun, $F_2$ comes in to contact with more of the UV and X-ray radiations. As a result of ionization content of $F_2$ layer exceeds that of $F_1$ layer.

In addition to local ionization and recombination, large-scale transport process influence the chemistry and charge density. As a result, particle density reduced where as, electron densities are higher than the other ionospheric layers. The dominant particle species about 400 to 1000 km is ionized atomic oxygen $O^+$ and atomic hydrogen $H^+$.

2.2 The solar wind and Earth’s magnetic field

As we see from Figure 2, the supersonic solar wind (Appendix A) flow first encounters the Earth’s magnetic field which is a hard obstacle (bow shock) to the flow. This shock slows the solar wind and deflects it around the Earth in the magnetosheath. The subsequent interaction of the magnetosheath flow and the geomagnetic field results in the
formation of the magnetopause. However, some of the solar wind’s magnetic field penetrates the magnetopause and connects with the Earth’s magnetic field and extend deep into the tail of the magnetosphere to produce open magnetic field. Since the magnetic field lines that form the tail originate in the Earth’s polar regions and the pressure in the ionosphere is much greater than that in the distant tail, it was suggested that a continual escape of thermal plasma should occurs along these open field lines.

The flow of plasma \((H^+\) and \(O^+)\) from the ionosphere of the polar region along these open field lines to magnetosphere is called polar wind (see in Figure 4).

### 2.3 Some observational evidence

In addition to the information given in Table 1, we have showed velocity profiles of the various ion species observed by Akebono satellite in Figure 5. The figure shows the outflow the \(H^+\) ions starts at about 1500 km altitude where as, the flow of \(He^+\) ions at about 2400 km altitude. \(O^+\) ions do not indicate any upward motion before a height of about 5000 km. This high latitude vertical ion outflow has been observed up to 9 RE [Abe and et al, 1993].

A typical feature outflow velocity profiles is that the ions velocities increase with height. For all ion species, the bulk velocity at a given altitude is higher on the dayside.
Figure 3: Ionospheric layers, the left panel: indicates the variation of electron density in night and day side (time), and the right panel: indicates the predominant ion populations with their respective heights above ground and variation of electron density with altitude (Fisica, 2000).
than on the nightside [Tam and et al. 2006].

Table 2 shows general characteristics of different polar wind mechanisms, as measured by Akebono satellite.

Table 3 shows extracted average boundary conditions from Akebono satellite measurement by changing photoelectron density. This density tells us day-night transition.

Table 2: Day-side polar wind observations by the Akebono satellite. Ion measurements were made at 5000-9000km altitude [Abe and et al, 1993]. Electron observations were made at about 1700km altitude [Yau and et al, 1997].

<table>
<thead>
<tr>
<th>Qualitative features</th>
<th>Quantitative estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O^+$ dominance over $H^+$ ions</td>
<td>$O^+$ density: lower limit $\sim 10^8 m^{-3}$</td>
</tr>
<tr>
<td>Monotonically increasing ion outflow velocities</td>
<td>$H^+$ density: lower limit $\sim 10^7 m^{-3}$</td>
</tr>
<tr>
<td>Supersonic flows for both $H^+$ and $O^+$</td>
<td>Ion temperature: upper limit $\approx 10^4 k$</td>
</tr>
<tr>
<td>Anisotropy between upwardly and downwardly moving electron populations</td>
<td>$H^+$ outflow velocity about 12-13km/s</td>
</tr>
<tr>
<td>Upwardly directed total electron heat flux</td>
<td>$O^+$ outflow velocity about 6-7km/s</td>
</tr>
</tbody>
</table>
Figure 5: Ion outflow, obtained from satellite measurements.

Table 3: Boundary conditions for the four cases with different relative photoelectron density. Other parameters are common to these cases: $n_o(O^+\text{ density})$, $n_h(H^+\text{ density})$, $T_e$ (thermal electron temperature).

<table>
<thead>
<tr>
<th>Case, common parameters</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n_s/n_e$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>5 x 10^{-4}</td>
<td>1 x 10^{-3}</td>
<td>1.5 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>$n_o(m^{-3})$</td>
<td>4.0 x 10^{10}</td>
<td>5.0 x 10^{10}</td>
<td>6.5 x 10^{10}</td>
</tr>
<tr>
<td></td>
<td>$n_h(m^{-3})$</td>
<td>1.5 x 10^{9}</td>
<td>2 x 10^{9}</td>
<td>3 x 10^{9}</td>
</tr>
<tr>
<td></td>
<td>$T_e(k)$</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
</tr>
<tr>
<td>number flux at 10000km ($m^{-2}s^{-1}$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H^+$</td>
<td>9.49 x 10^{10}</td>
<td>1.24 x 10^{11}</td>
<td>2.35 x 10^{11}</td>
<td>2.61 x 10^{11}</td>
</tr>
<tr>
<td>$O^+$</td>
<td>3.25 x 10^{11}</td>
<td>1.82 x 10^{12}</td>
<td>3.20 x 10^{12}</td>
<td>3.19 x 10^{12}</td>
</tr>
</tbody>
</table>
CHAPTER 3

EARLIER MODELS OF POLAR WIND

3.1 Introduction

Plasma is used to describe a wide variety of macroscopically neutral substances containing many interacting free electrons and ionized atoms or molecules, which exhibit collective behavior due to long range coulomb force.

In order to describe the flow of plasma along open magnetic field lines numerous mathematical models have been constructed. There are hydrodynamic [Bank and Holzer, 1968, 1969, Schunk and et al, 1975, 1977, 1978], hydromagnetic [Holzer and et al, 1971], generalized transport [Schunk and et al, 1981, 1982, 1986, 1987, Ganguli, 1986, Ganguli and et al, 1987], Kinetic [Lemaire, 1972, 1973, Tam and et al, 2006], semi-kinetic [Barakat and Schunk, 1983,1984], and self consistent hybrid model [Tam and et al, 1995a, 1995b, 1998]. These models have been used primarily to study the study state characteristics of the flow, with the emphasis on elucidating physical processes. In this section, the common principles governing the models will be briefly discussed, and the simple hydrodynamic model (Appendix C) will be used to elucidate the basic flow characteristics.

3.1.1 Transport equation

The hydrodynamic, hydromagnetic, and generalized transport models are based on the use of conservation equations which describe the spatial and temporal evolution of physically relevant macroscopic parameters like concentration, drift velocity, and temperature of different species in the plasma. These conservation equations are obtained by taking velocity moments of Boltzmann’s equation (Appendix C), and the three lowest order equations are the continuity momentum and energy equations respectively,

\[
\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s u_s) = p'_s - L'_s n_s, \tag{1}
\]

\[
n_s m_s \left( \frac{D_s u_s}{Dt} \right) + \nabla p_s - n_s m_s G - n_s e_s [E + (1/c) u_s \times B] = \frac{\delta M_s}{\delta t}, \tag{2}
\]

\[
\left( \frac{D_s}{Dt} \right) \left( \frac{3p_s}{2} \right) + \left( \frac{5}{2} \right) p_s \nabla \cdot u_s + \nabla q_s = \frac{\delta E_s}{\delta t} + Q_s - L_s, \tag{3}
\]
where \( n_s \) and \( u_s \) are the number density and drift velocity of a specified species, \( m_s \) is the mass, \( p_s = n_s k T_s \) is the partial pressure, \( P'_s \) and \( L'_s \) are the production and loss rates, \( G \) is the gravitational acceleration, \( E \) is the electric field (zero for neutrals), \( k \) is the Boltzmann constant, \( e_s \) is the charge, \( T_s \) is the temperature, \( Q_s \) is the heating rate, \( D_s / Dt = d/dt + u_s \nabla \) is the convective of species \( s \), \( q_s \) is the heat flow vector and \( c \) the speed of light. The quantities \( \delta M_s / \delta t \) the rate of momentum and \( \delta E_s / \delta t \) the rate of energy exchange, in collisions between species \( s \) and the other species in the plasma. In order to calculate collision terms for the transport equations we use approximate expression for the species velocity distribution functions. For the simplest case of displaced maxwellians general collision terms have been evaluated that apply to arbitrary interparticle force laws (maxwell molecule interaction), large temperature difference, and large relative drift between the interacting species is [Schunk, 1977],

\[
\frac{\delta M_s}{\delta t} = \sum_t n_s m_s \nu_{sn} (u_n - u_s) \Phi_{sn},
\]

and,

\[
\frac{\delta E_s}{\delta t} = n_s m_s \sum_t \nu_{sn} \left[ 3 K_B (T_t - T_s) \Psi_{sn} + m_n (u_s - u_n)^2 \Phi_{st} \right] / (m_s + m_n).
\]

where \( \Psi_{sn} \) and \( \Phi_{sn} \) are velocity-dependent correction factors and \( \nu_{sn} \) is the momentum transfer collision frequency for gases \( s \) (ion or electron) and \( n \) (neutral). In the ionosphere, there are several ion (\( NO^+, O_2^+, N_2^+, O^+, N^+, He^+, H^+ \)) and neutral (\( N_2, O_2, O, N, He, H \)) species, and the relevant collision processes include coulomb interactions, nonresonant ion-neutral interactions, resonant charge exchange (resonant ion-neutral interaction), and electron-neutral interactions. The appropriate momentum transfer collision frequencies for two bodies elastic electron-neutral interactions are given in Table 4. With regard to coulomb interactions, one needs to consider electron-electron, electron-ion, and ion-ion collisions, appropriate expressions are given consequently [Schunk, 1983],

\[
\nu_{ee} = \frac{54.5 n_e}{[\sqrt{2 T_e^{3/2}}]},
\]

\[
\nu_{ei} = \frac{54.5 n_i}{[T_e^{3/2}]},
\]

and,

\[
\nu_{ij} = B_{ij} n_j /[T_i^{3/2}],
\]

where subscript \( e \) denotes electrons and subscripts \( i \) and \( j \) correspond to different ion species. The collision frequency coefficients \( B_{ij} \) are given in Table 5 for the ion species

10
Table 4: Momentum Transfer Collision Frequencies for Electron-Neutral interactions [Schunk, 1978].

<table>
<thead>
<tr>
<th>Species</th>
<th>$\nu_{en}s^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_2$</td>
<td>$2.33 \times 10^{-11}n(N_2)(1 - 1.21 \times 10^{-4}T_e/T_e)$</td>
</tr>
<tr>
<td>$O_2$</td>
<td>$1.82 \times 10^{-10}n(O_2)(1 + 3.6 \times 10^{-2}T_e^{1/2}/T_e^{1/2})$</td>
</tr>
<tr>
<td>$O$</td>
<td>$8.9 \times 10^{-11}n(O)(1 + 5.7 \times 10^{-4}T_e^{1/2}/T_e^{1/2})$</td>
</tr>
<tr>
<td>He</td>
<td>$4.6 \times 10^{-10}n(He)T_e^{1/2}$</td>
</tr>
<tr>
<td>H</td>
<td>$4.5 \times 10^{-9}n(H)(1 - 1.35 \times 10^{-4}T_e/T_e)$</td>
</tr>
</tbody>
</table>

Table 5: The collision frequency coefficients $B_{ij}$ for ion-ion interaction [Schunk, 1978].

<table>
<thead>
<tr>
<th>Species</th>
<th>$H^+$</th>
<th>$He^+$</th>
<th>$N^+$</th>
<th>$O^+$</th>
<th>$N_2^+$</th>
<th>$NO^+$</th>
<th>$O_2^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H^+$</td>
<td>0.90</td>
<td>1.14</td>
<td>1.23</td>
<td>1.23</td>
<td>1.25</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>$He^+$</td>
<td>0.28</td>
<td>0.45</td>
<td>0.56</td>
<td>0.57</td>
<td>0.59</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>$N^+$</td>
<td>0.088</td>
<td>0.16</td>
<td>0.24</td>
<td>0.25</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>$O^+$</td>
<td>0.077</td>
<td>0.14</td>
<td>0.22</td>
<td>0.22</td>
<td>0.25</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>$N_2^+$</td>
<td>0.045</td>
<td>0.085</td>
<td>0.14</td>
<td>0.15</td>
<td>0.17</td>
<td>0.17</td>
<td>0.18</td>
</tr>
<tr>
<td>$NO^+$</td>
<td>0.042</td>
<td>0.080</td>
<td>0.13</td>
<td>0.14</td>
<td>0.16</td>
<td>0.16</td>
<td>0.17</td>
</tr>
<tr>
<td>$O_2^+$</td>
<td>0.039</td>
<td>0.075</td>
<td>0.12</td>
<td>0.13</td>
<td>0.15</td>
<td>0.16</td>
<td>0.16</td>
</tr>
</tbody>
</table>

found in the ionosphere [Schunk, 1978].

The ion-neutral interactions in the ionosphere can be either resonant or non-resonant, depending on the species involved. Resonant charge exchange occurs when an ion collides with its parent neutral or it can occur accidentally as in the case of the reaction,

$$O^+ + H \leftrightarrow H^+ + O.$$  \hspace{1cm} (9)

The appropriate resonant ion-neutral momentum transfer collision frequencies are given Table 6 and $T_r$.

Table 6: Momentum transfer collision frequencies for resonant ion-neutral interactions [Bank and et al, 1973].

<table>
<thead>
<tr>
<th>Species</th>
<th>$T_rK$</th>
<th>$\nu_{in}s^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H^+, H(\nu_{hh})$</td>
<td>50</td>
<td>$2.65 \times 10^{-10}n(H)T_e^{1/2}(1 - 0.083 \log_{10} T_h)^2$</td>
</tr>
<tr>
<td>$He^+, He(\nu_{He2})$</td>
<td>50</td>
<td>$8.73 \times 10^{-11}n(He)T_e^{1/2}(1 - 0.093 \log_{10} T_h)^2$</td>
</tr>
<tr>
<td>$N^+, N(\nu_{nn})$</td>
<td>275</td>
<td>$3.83 \times 10^{-11}n(N)T_e^{1/2}(1 - 0.063 \log_{10} T_h)^2$</td>
</tr>
<tr>
<td>$O^+, O(\nu_{o+a})$</td>
<td>235</td>
<td>$3.67 \times 10^{-11}n(O)T_e^{1/2}(1 - 0.064 \log_{10} T_h)^2$</td>
</tr>
<tr>
<td>$N_2^+, N_2(\nu_{n2n})$</td>
<td>170</td>
<td>$5.14 \times 10^{-11}n(N_2)T_e^{1/2}(1 - 0.069 \log_{10} T_h)^2$</td>
</tr>
<tr>
<td>$O_2^+, O_2(\nu_{o2a})$</td>
<td>800</td>
<td>$2.59 \times 10^{-11}n(O_2)T_e^{1/2}(1 - 0.073 \log_{10} T_h)^2$</td>
</tr>
<tr>
<td>$H^+, O(\nu_{ho})$</td>
<td>300</td>
<td>$6.61 \times 10^{-11}n(O)T_h^{1/2}(1 - 0.047 \log_{10} T_h)^2$</td>
</tr>
</tbody>
</table>
Table 7: The collision frequency coefficients $C_{in} \times 10^{10}$ for non resonant ion-neutral interaction (r means the collisional interaction is resonant) [Bank and et al, 1973].

<table>
<thead>
<tr>
<th>$i^n$</th>
<th>H</th>
<th>He</th>
<th>N</th>
<th>O</th>
<th>$N_2$</th>
<th>$O_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H^+$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$He^+$</td>
<td>r</td>
<td>10.6</td>
<td>26.1</td>
<td></td>
<td>33.6</td>
<td>32.0</td>
</tr>
<tr>
<td>$N^+$</td>
<td>4.71</td>
<td>r</td>
<td>11.9</td>
<td>10.1</td>
<td>16.0</td>
<td>15.3</td>
</tr>
<tr>
<td>$O^+$</td>
<td>1.45</td>
<td>1.49</td>
<td>r</td>
<td></td>
<td>4.42</td>
<td>7.47</td>
</tr>
<tr>
<td>$N_2^+$</td>
<td>0.74</td>
<td>0.79</td>
<td>2.95</td>
<td></td>
<td>2.58</td>
<td>r</td>
</tr>
<tr>
<td>$NO^+$</td>
<td>0.69</td>
<td>0.74</td>
<td>2.79</td>
<td>2.44</td>
<td>4.34</td>
<td>4.27</td>
</tr>
<tr>
<td>$O_2^+$</td>
<td>0.65</td>
<td>0.70</td>
<td>2.64</td>
<td>2.31</td>
<td>4.13</td>
<td>r</td>
</tr>
</tbody>
</table>

$$T_r = (T_i + T_n)/2.$$ (10)

Non-resonant ion-neutral interactions occurs between unlike ion and neutrals, and they correspond to a long-range polarization attraction coupled with a short-range repulsion. In this case, the ion-neutral momentum transfer collision frequencies take a particularly simple form,

$$\nu_{in} = C_{in}n_n,$$ (11)

where subscript n corresponds to neutrals, $C_{in}$ is the numerical coefficients for different ion-neutral combinations found in the ionosphere are given in Table 7.

### 3.2 Subsonic outflow

The presence of density gradients in a plasma causes the particle to diffuse from dense regions to lower density. Diffusion of charged particles in weakly ionized (neutral-charge dominant) plasma contains one new aspect which is not encountered in neutral gas. Due to the different masses of ions and electrons, these two particle species tend to diffuse at different rates (electrons diffuse faster, this leads to charge separation). Because of the separation electric field will develop, (which will adjust itself to a value which forces ions and electrons to the same diffusion rate) once this electrostatic field has developed, the ions and electrons move together as a single gas under the influence of gravity and the density and temperature gradients. Such motion is called ambipolar outflow, and the polar wind corresponds to an ambipolar outflow of ionospheric ions.

Polar wind has been modeled by using the concept of ambipolar diffusion and it is more complicated in the presence of three ion species. The basic idea, can be understood when noticing that $O^+$ is much heavier than $H^+$, then the diffusion electric field is mainly
determined by $O^+$ ions and electrons. The direction of the electric field is upward and able to accelerating the light ion species $H^+$. Charge neutrality implies that a part of the electrons must follow this ion motion and the electron flow generate the diffusion electric field. In a simplified theory of polar wind, we consider two ion species and the geomagnetic field is taken to be vertical and only vertical motion. The ion continuity equation,

$$
\frac{D_i n_i}{Dt} + \nabla . (n_i u_i) = p_i' - L_i' n_i,
$$

where the subscripts $\parallel$ and $\perp$ refers to the directions parallel and perpendicular to B, $D_i/Dt = (d/dt + u_{i \perp} \cdot \nabla)$ is the convective derivative for horizontal motion (see Appendix A.1 and A.2 ($\omega << \nu$)), we shall consider steady state condition then in our case this term becomes zero, and with out loss and production, the equation becomes,

$$
\nabla . (n_i u_i) = 0.
$$

If the motion along B is subsonic, an important simplification is also possible. Throughout our model we use steady state ($\partial u_i/\partial t \approx 0$) flow. Other simplification can be seen by comparing the nonlinear inertial term with the pressure-gradient $\nabla || \sim 1/L$, where L is a characteristic length, the ratio of these two terms is,

$$
n_i m_i (u_{i ||} \cdot \nabla ||) (u_{i ||} / \nabla || p_i \sim (u_{i ||})^2 / (K_B T_i / m_i) = M_i^2,
$$

where, $M_i$ is the mach number of the ion flow. Therefore, if the ion flow is subsonic, the nonlinear inertial term can be neglected, and with the above simplifications, the ion momentum equation reduces to,

$$
n_i m_i \left( \frac{\partial}{\partial t} + u_i . \nabla \right) u_i = -n_i m_i g + e n_i (E + u_i \times B) - \nabla p_i - \sum n_i m_i \nu_{in} (u_i - u_n),
$$

$$
\nabla p_i + n_i m_i g - e n_i (E + u_i \times B) = - \sum n_i m_i \nu_{in} (u_i - u_n).
$$

The small electron mass also implies that, in the momentum equation, $n_e m_e \nu_{en} (u_e - u_n)$ as well as $m_e g$ can be put to zero. When the convective derivative is also put to zero (stationary state and weak spatial variation in electron velocity), the momentum equation for electrons is simply,

$$
n_e m_e \left( \frac{\partial}{\partial t} + u_e . \nabla \right) u_e = -n_e m_e g - e n_e (E + u_e \times B) - \nabla p_e - \sum n_e m_e \nu_{en} (u_e - u_n),
$$
\[ eE_\parallel = -\frac{1}{n_e} \nabla \parallel p_e, \quad (18) \]

where \( p_e = n_e kT_e \) is the electron pressure and \( n_e = \sum_i n_i \), with the sum being over the ion species, temperature has no spatial variation. The heavy ions stay stationary creating the diffusion electric field and the light ions are accelerated. Momentum equations are written for both ion species and electrons and the diffusion electric field is obtained from the electron momentum equation. It is then possible to derive a pair of coupled equations for the height derivatives of the velocity and density of the light ions. The equations can be solved numerically by making the following assumptions; (1) the neutral atmosphere is stationary \((u_n = 0)\), (2) steady state conditions prevail \((du_i/dt = 0)\), (3) the flow is subsonic \((u_i, \nabla \cdot u_i \to 0)\), (4) the ionosphere is isothermal.

### 3.2.1 \( O^+ \) outflow

We are considering \( O^+ \) outflow as one fluid motion, the momentum and continuity equation for \( O^+ \) becomes,

\[
n_o m_o \left( \frac{\partial}{\partial t} + u_o \nabla \right) u_o = -n_o m_o g + e n_o (E + u_o \times B) - \nabla p_o - \sum n_o m_o \nu_{on} (u_o - u_n), \quad (19)\]

and,

\[
\nabla \cdot (n_o u_o) = 0, \quad (20)\]

respectively.

Under most circumstance, the relative drift velocity term on the right-hand-side of equation\( (19) \) can be neglected at high altitudes. \( P_o = n_o kT_o \), and use equation\( (18) \) for \( eE_\parallel \), let \( z \) be the coordinate along the vertical magnetic field, and \( g \) (gravity) is downward. The major ion executes ambipolar flow with \( n_o \simeq n_e \) and \( u_o \approx u_e \). Finally, it should be noted that in the altitude range 800-3000km, the minor ion effectively collides only with the major ion owing to the long-range nature of coulomb collisions. The simplified momentum equation becomes,

\[
\frac{k T_o}{m_o n_o} \frac{\partial n_o}{\partial z} + \frac{k}{m_o} \frac{\partial T_o}{\partial z} + g + \frac{k T_e}{m_o n_o} \frac{\partial n_o}{\partial z} + \frac{k}{m_o} \frac{\partial T_e}{\partial z} = \nu_{o+o} (u_o - u_o) + \nu_{o+h} (u_h - u_o), \quad (21)\]

from equation\( (21) \), when the relative drift velocity term neglected , and assuming \( O^+ \) is not greatly affected by the momentum transferred from the minor constituent \( (\nu_{o+h} \approx 0) \) the equation becomes,
Figure 6: $O^+$ density versus altitude, diffusive equilibrium distribution.

\[
\frac{1}{n_o} \frac{\partial n_o}{\partial z} = -\frac{1}{H_{oe}} \frac{\partial T_{oe}}{\partial z},
\]

(22)

where the plasma scale height $H_{oe}$ and temperature $T_{oe}$ are given by,

\[
H_{oe} = \frac{2kT_{oe}}{(m_o g)},
\]

(23)

and,

\[
T_{oe} = \frac{(T_e + T_o)}{2}.
\]

(24)

For an isothermal ionosphere, the numerical solution of equation (22) ($O^+$ density variation with altitude) can be obtain by using fourth order Runge-Kutta method as shown in Figure 6.

\[
\frac{\partial n_o}{\partial z} = -\left(\frac{1}{H_{oe}}\right)n_o = -\left(\frac{m_o g}{k(T_e + T_o)}\right)n_o,
\]

(25)

where, $g$,

\[
g = 9.8\left(\frac{R_E}{R_E + z}\right)^2,
\]

(26)

is gravitational acceleration, and $R_E$ is the Earth radius. When we consider the relative drift velocity term from equation (21), and assuming $O^+$ is not greatly affected by the momentum transferred from the minor constituent ($\nu_{o+h} \approx 0$), the velocity of $O^+$ becomes,
Figure 7: $O^+$ density versus altitude with relative drift velocity.

Let,

$$u_o = -DD_ob_o \frac{1}{n_o} \frac{\partial n_o}{\partial z} - b_og,$$

(27)

where,

$$DD_o = k(T_o + T_e)/m_o,$$

(28)

$$b_o = 1/\nu_o^{+o},$$

(29)

$\nu_o^{+o}$ and $\nu_o^{+h}$ are given in Table 6, and substituting equation (27) on equation (20), we get equation (30), and the numerical result is shown in Figure 7.

$$\frac{\partial^2 n_o}{\partial z^2} + \left( \frac{1}{b_o} \frac{\partial b_o}{\partial z} + \frac{g}{DD_o} \right) \frac{\partial n_o}{\partial z} + \left( \frac{1}{b_o} \frac{\partial b_o}{\partial z} \frac{g}{DD_o} + \frac{1}{DD_o} \frac{\partial g}{\partial z} \right) n_o = 0$$

(30)

3.2.2 $H^+$ outflow

The essential characteristics of an $H^+$ outflow can be derived using some important facts, first, at altitudes where the outflow is substantial, ion production and loss processes are not important. Second, when $H^+$ flows out of the topside ionosphere, it remains a minor ion to altitudes as high as 3000km, and we consider as other fluid,
Figure 8: $H^+$ density versus altitude.

$$n_h \ll n_o.$$ \hspace{1cm} (31)

In order to calculate $H^+$ number density in the case of diffusive equilibrium distribution, with the simplifying assumptions noted above, the $H^+$ continuity and momentum equation consequently becomes,

$$\frac{\partial}{\partial z} (n_h u_h) = 0,$$ \hspace{1cm} (32)

$$\frac{kT_h}{m_h n_h} \frac{\partial n_h}{\partial z} + g + \frac{kT_e}{m_h n_o} \frac{\partial n_o}{\partial z} = \nu_h (u_o - u_{h^+}) + \nu_{hh} (u_h - u_{h^+}).$$ \hspace{1cm} (33)

Where the polarization electrostatic field is set up by the major ions and electrons equation(18), neglect relative drift velocity term $P_h = n_h k T_h$, then, the momentum equation(33) can be written as,

$$\frac{\partial n_h}{\partial z} = - \frac{g m_h}{k T_h} n_h + \frac{T_e}{T_h k (T_e + T_o)} \frac{m_o g}{n_h},$$ \hspace{1cm} (34)

the numerical solution can be calculated by using fourth order Rung-Kutta method as shown in Figure 8.

And when we consider momentum transfer collision frequency and relative velocity term equation(33) becomes,
\begin{equation}
    u_h = u_o - DD_h b_h \frac{1}{n_h} \frac{\partial n_h}{\partial z} - gb_h - DD_h \frac{T_h b_h}{T_h n_o} \frac{\partial n_o}{\partial z},
\end{equation}

where, $D_h$ hydrogen diffusion coefficient, $\nu_{ho}$ momentum transfer collision frequency for hydrogen-oxygen neutral interaction, $\nu_{hh}$ momentum transfer collision frequency for hydrogen-hydrogen neutral interaction are given Table 6, and let,

\begin{equation}
    DD_h = \frac{kT_h}{m_h},
\end{equation}

let,

\begin{equation}
    b_h = \frac{1}{\nu_{ho} + \nu_{hh}} \approx \frac{1}{\nu_{ho}},
\end{equation}

\begin{equation}
    D_h = DD_h b_h.
\end{equation}

In order to calculate number density of $H^+$, first we should calculate $n(o)$ number of atomic oxygen (neutral), and $n(h)$ number density of atomic hydrogen (H) (neutral) analytically, from momentum conservation equation of ions, we have solved the equation and the solution is depicted in Figure 9,

\begin{equation}
    n(o) = 4 \times 10^{11} \exp\left(-\frac{z - 400}{H_o}\right),
\end{equation}

\begin{equation}
    n(h) = 6 \times 10^{10} \exp\left(-\frac{z - 400}{H_h}\right),
\end{equation}

where, $H_o$ scale height of oxygen neutral, $T_n$ temperature of atomic oxygen neutral, and $H_h$ scale height of atomic hydrogen,

\begin{equation}
    H_o = \frac{kT_n}{m_og},
\end{equation}

\begin{equation}
    H_h = \frac{kT_n}{m_hg}.
\end{equation}

From equation(35) the major ion affects the minor ion in three ways. First, if the major ion is flowing ($u_o \neq 0$), it tends to carry the minor ion along. However, the minor ion also tends to diffuse, but this diffusion is inhibited by the major ion through collisions. The polarization electrostatic field set up by the major ions and electrons exerts a force on the minor ions. But by the above argument the neutral is stationary above 400km ($u_n \approx 0$).

First, consider simplifying continuity equation (with out loss and production term) of equation (32), and by substituting equation(35) on equation(32) we get,
\[
\frac{\partial^2 n_h}{\partial z^2} + \left( \frac{1}{b_h} \frac{\partial b_h}{\partial z} + \frac{g}{DD_h} + \frac{T_e}{T_h n_o} \frac{\partial n_o}{\partial z} \right) \frac{\partial n_h}{\partial z} + \left( \frac{1}{DD_h} \frac{\partial g}{\partial z} + \frac{g}{DD_h b_h} \frac{\partial b_h}{\partial z} + \frac{T_e}{b_h T_h n_o} \frac{\partial n_o}{\partial z} \left( \frac{T_e}{T_h n_o} \frac{\partial n_o}{\partial z} \right) \right) = 0.
\]

The numerical solution of equation (43) (using fourth order Runge-Kutta method) is indicated in Figure 10.

When we consider simplifying continuity equation with production and loss term, for \(O^+\),

\[
\frac{\partial}{\partial z} (n_o u_o) = P_{r_o} - L_{o_o},
\]

and substitute equation (27) on equation (44) we get,

\[
\frac{\partial^2 n_o}{\partial z^2} + \left( \frac{1}{b_o} \frac{\partial b_o}{\partial z} + \frac{g}{DD_o} \right) \frac{\partial n_o}{\partial z} + \left( \frac{1}{b_o} \frac{\partial b_o}{\partial z} \frac{g}{DD_o} + \frac{1}{DD_o} \frac{\partial g}{\partial z} \right) n_o + \frac{P_o - L_o}{DD_o b_o} = 0.
\]

Similarly, for \(H^+\) outflow,

\[
\frac{\partial}{\partial z} (n_h u_h) = P_{r_h} - L_{o_h} n_h,
\]

where, production and loss term for hydrogen ion is given by [Schunk, 1981],

\[
P_{r_h} = 4.5 \times 10^{-11} T_n^{1/2} n(h)n_o \times 10^{12},
\]
Figure 10: Number density of $H^+$ versus altitude by consider momentum transfer collision frequency.

\[ L_{oh} = 4.2 \times 10^{-11} T_n^{1/2} n(o) \times 10^{12}, \]  

and substitute equation(35) on equation(46), we have,

\[
\begin{align*}
\frac{\partial^2 n_h}{\partial z^2} + \left( \frac{1}{b_h} \frac{\partial b_h}{\partial z} + \frac{g}{DD_h} + \frac{T_e}{T_h n_o} \frac{\partial n_o}{\partial z} \right) \frac{\partial n_h}{\partial z} + \left( \frac{1}{DD_h} \frac{\partial g}{\partial z} + \frac{g}{DD_h b_h} \frac{\partial b_h}{\partial z} \right)
\end{align*}
\]

\[
\begin{align*}
+ \frac{T_e}{b_h T_h n_o} \left( \frac{\partial b_h}{\partial z} \frac{\partial n_o}{\partial z} + T_e \frac{\partial^2 n_o}{\partial z^2} - \frac{T_e}{T_h n_o} \frac{(\partial n_o}{\partial z})^2 + \frac{L_{oh}}{DD_h b_h} \right) n_h - \frac{P_{rh}}{DD_h b_h} = 0.
\end{align*}
\]  

Equation(49) is second order ODE, then by using the same method we can calculate $n_h$ as shown Figure 11.

As we observe from Figure 6, Figure 7, Figure 8, Figure 10, and Figure 11 the minor ion ($H^+$) density decreases exponentially with altitude, and major ion ($O^+$) density decreases exponentially with altitude at a rate governed by plasma scale height, finally bounded about 3000km. However, the real observation by Akebono satellite indicates $O^+$ dominant over $H^+$ about up to 12000km as shown in Figure 12. From Figure 11 we observe production and loss term has no significant effect on the polar wind dynamics above 400 km, this means loss and production (out and in) are almost equal in high altitude. All solutions indicates both ion density decreases exponentially with altitude in different scale height. However, this solution corresponds to the maximum outflow that
Figure 11: Number density of $H^+$ and $O^+$ by considering continuity equation with production and loss term.

Figure 12: Number density according to Akebono satellite data (data from [Abe and et al., 1993]).
is possible. Since $n_iu_i=\text{constant}$, the flow velocity display the opposite behavior to the density, i.e., it increases exponentially with altitude. Based on this solution both ions flow supersonic at some altitude as shown Figure 19, hence, the assumption of subsonic flow becomes invalid.

### 3.3 Supersonic outflow

We assumed that both ions have supersonic flow at high-altitude, then, the nonlinear inertial term in the momentum equation of $O^+$ and $H^+$ cannot be neglected and the situation becomes more complex. To illustrate this case, it is convenient to make the following simplification, (1) the flow is ambipolar ($n_o = n_e$, and $u_o = u_e$), (2) the ionosphere is isothermal, (3) steady state conditions prevail, and (4) the neutrals are stationary.

The momentum equation of oxygen ion ($O^+$) (with out driven shock wave) becomes,

$$\frac{n_om_ou_o}{\partial z} + k(T_e + T_o)\frac{\partial n_o}{\partial z} + n_o m_o g = -n_o m_o \nu_{o+} u_o,$$

and, from the continuity equation,

$$\frac{\partial n_o}{\partial z} = -\frac{n_o}{u_o} \frac{\partial u_o}{\partial z},$$

then, substitute equation (51) on equation (50), we get,

$$u_o \frac{\partial u_o}{\partial z} - \frac{v_{to}^2}{u_o} \frac{\partial u_o}{\partial z} + g = -\nu_{o+} u_o,$$

where $v_{to}$ is the thermal speed of the ion-electron gas,

$$v_{to}^2 = \frac{k(T_e + T_o)}{m_o} = DD_o,$$

and, by introducing oxygen mach number, $M_o = u_o/v_{to}$, the equation becomes,

$$\frac{\partial M_o}{\partial z} = -\left[ \frac{g}{v_{to}^2} + \nu_{o+} M_o \frac{M_o}{v_{to}^2} \right](M_o - 1).$$

The numerical solution of equation (54) by using fourth order Runge-Kutta method is shown Figure 13,

Similarly the momentum equation of hydrogen ion becomes,

$$u_h \frac{\partial u_h}{\partial z} + \frac{v_{th}^2}{n_h} \frac{\partial n_h}{\partial z} + g + \frac{v_{th}^2 T_e}{T_h n_o} \frac{\partial n_o}{\partial z} = -\nu_{ho} u_h,$$

where $v_{th}$ is the thermal speed of hydrogen ion,
Figure 13: Mach number of $O^+$ versus altitude.

Figure 14: Mach number of $H^+$ and by using simple hydrodynamic view (from Schunk, 1988).
\[ \nu_{th}^2 = \frac{kT_h}{m_h} = D_h, \]  

(56)

and, introducing hydrogen mach number, \( M_h = u_h/v_{th} \), the equation becomes,

\[ \frac{\partial M_h}{\partial z} = -\left[ \frac{g}{v_{th}^2} + \frac{v_{ho}M_h}{v_{th}} - \frac{T_\infty n_o}{T_h M_o n_h} \frac{\partial M_o}{\partial z} \right] \left( \frac{M_h}{M_h^2 - 1} \right). \]

(57)

Equation (54) and equation (57) contains singularities at \( M_o = \pm 1 \), and \( M_h = \pm 1 \); at the point of transition from subsonic to supersonic flow as we observe from Figure 13 and Figure 14.

Figure 13 and Figure 14, shows schematically the different solutions that are possible for \( O^+ \) and \( H^+ \) outflow situations. The solutions are presented in an ion mach number (M) versus altitude format.

Where the velocity is measured in thermal Mach numbers (ratio of velocity and ion thermal velocity). It is found that the equations allow solutions of different type located at different parts of the height-velocity plane, and ion densities corresponding to each solution can also be calculated. Solutions giving two velocities for the same altitude are not physically meaningful. Only solutions starting from zero velocity at low altitudes can correspond to polar wind. Their main property is that ion velocity first increases with height, obtains a subsonic maximum at some altitude and then decreases towards zero.
This is not the case which is observed in polar wind. Solution increases with height, and the Mach number obtains unity at some altitude and increases more and more with heights, this are the case correspond to the situation in true polar wind where, in Akebono satellite observed Figure 15. Then, in order to solve the above problem, we consider driven shock wave from conservation (momentum) equations of ions.

3.4 Limitation of earlier model approach

There are two approach of polar wind modelling, collisionless kinetic calculations and moment equations each based on a different type of approximation. In collisionless kinetic calculations, one neglects the Coulomb collisional term. The collisionless kinetic approximation is valid at high altitudes (roughly $> 3000$ km). Near the polar wind source, the Coulomb collisional effect is definitely non-negligible. In order to take this effect into account, some investigators thus resort to the moment approach.

Moment-based models describe the particle transport using a few variables to characterize the species, such as density, velocity, temperature and heat fluxes. A variety (five, eight, ten, thirteen, sixteen, and even twenty) of moment-based models has been applied to the classical polar wind to take into account collisional effects. The intrinsically stiff nature of these systems of moment equations (due essentially to the high mass ratios), numerical solutions are not straightforward to obtain. In addition, these systems exhibit a number of singularities (can not provide transonic solutions easily). However, such transonic solutions are required for the appropriate description of the polar wind because observations indicate that both the $H^+$ and $O^+$ outflows are supersonic at high altitudes. The flow velocity for both these species are found out to be supersonic above 600 km for $O^+$ and 3000 km for $H^+$. In fact, the measured $O^+$ outflow velocities (by Akebono satellite) in Table 1 are much larger than the value expected from classical polar winds model. And it is not clear that the transition from collisional to collisionless for all species present will occur at the same altitudes. We can clearly see why the earlier models does not explain the Akebono observation.
CHAPTER 4

HYDRODYNAMIC DRIVEN POLAR WIND MODEL

4.1 Driven shock wave

Driven shock wave is a collective description of different polar wind mechanism or accelerating terms of polar wind. This shock wave can be produced when ever flow of plasma and field energy exists between any celestial bodies. Shocks are the most extensively studied nonlinear responses not proportional to input and dissipation (entropy increases) waves of plasmas. Field and plasma go through dramatic change in density temperature field strength, and flow speed, because of compression that is not reversible during shock wave produced in polar cusps, change of the state of the medium through which it travels, is called a shock wave.

The momentum transport by collisions plays a crucial role in the formation of gasdynamic shocks. In the rarefied plasmas of space, too few collisions occur between the constituents of the plasma to provide efficient momentum transport and allow for shock formation. Nonetheless, shocks do exist in space plasmas. These shocks are called collisionless shocks; here the collective behavior of the plasma is not guaranteed by collisions but by the collective effects of the electrical and magnetic properties of the plasma, which allow for frequent interactions between the particles and, consequently, also for the formation of a shock wave.

A shock wave is associated with a sudden change in properties of a continuous medium, in particular a sudden increase in gasdynamic pressure. A shock forms when a propagation speed exceeds the typical signal speed in the medium: a body might move through a medium faster than the signal speed (e.g., a supersonic jet, traveling shock) or a supersonic flow might be slowed down at an obstacle (e.g., the solar wind at a planetary magnetosphere, standing shock). Another shock wave is the blast wave following an explosion. General characteristics of shock waves are: (1) the disturbance propagates faster than the signal speed, (2) at the shock front, the properties of the medium change abruptly.

A driven shock occurs when there is always energy transformed into the gas flow,
or obstacle motions. Then in our model we use this force in order to explain the recent observation.

Above 3000km, gases are almost collisionless, and plasma consists fields and particles. The action of the magnetic field replaces that of collisions to "bind" the particles of the plasma together, and the resulting description is called magnetohydrodynamics (MHD).

MHD describes the plasma macroscopic field (electric, magnetic and other quantities density, bulk-velocity). MHD does not include effects due to individual particles (kinetic effect), it can not tell us any thing about how a shock provides, dissipation, structure of the shock, etc.

MHD relates the plasma states by conservation relation, in this description the shock is very much like a black box that changes the state of plasma (field, density, etc), although we can not tell any thing about what is actually happening at the shock.

4.2 Subsonic flow

In order to describe the ions outflow with driven shock wave first we can assume the flow is subsonic, then our mathematical description. The simplified momentum equation for oxygen ions (O\(^{+}\)) (by considering the above assumptions) becomes,

\[
n_i m_i \left( \frac{\partial}{\partial t} + u_i \cdot \nabla \right) u_i = -n_i m_i g + e n_i (E + u_i \times B) - \nabla p_i + \frac{B}{\mu_o} \frac{\partial B}{\partial z} - \sum n_i m_i \nu_{in} (u_i - u_n), \quad (58)
\]

\[
k (T_o + T_e) \frac{\partial n_o}{\partial z} + \frac{B}{n_o m_o \mu_o} \frac{\partial B}{\partial z} = -\nu_{o+o} u_o, \quad (59)
\]

where, \(B\) is the Earth’s magnetic field, \(\mu_o\) is permittivity of vaccum,

\[
B = B_o \left( \frac{R_E}{R_E + z} \right)^3, \quad (60)
\]

where, \(B_o = 3 \times 10^{-5} \text{T}\), and from equation(59), we can calculate \(u_o\),

\[
u_{o+o} u_o = -D_o \frac{b_o}{n_o} \frac{\partial n_o}{\partial z} - g b_o + \frac{B b_o}{n_o m_o \mu_o} \frac{\partial B}{\partial z}. \quad (61)
\]

Substituting equation(61) on equation(20) we get,

\[
\frac{\partial^2 n_o}{\partial z^2} + \left( \frac{1}{b_o} \frac{\partial b_o}{\partial z} + \frac{g}{D_o} \right) \frac{\partial n_o}{\partial z} + \left( \frac{1}{b_o} \frac{\partial b_o}{\partial z} \right) \frac{g}{D_o} + \frac{1}{D_o} \frac{\partial g}{\partial z} n_o - \frac{B}{b_o D_o m_o \mu_o} \frac{\partial B}{\partial z} - \frac{1}{D_o m_o \mu_o} \left( \frac{\partial B}{\partial z} \right)^2 - \frac{B}{D_o m_o \mu_o} \frac{\partial^2 B}{\partial z^2} = 0 \quad (62)
\]
then, the numerical solution of equation (62) by using fourth order Rung Kutta method is shown in Figure 16.

Similarity for $H^+$, the solution is as shown Figure 17 and for both ions Figure 18,

$$\frac{\partial^2 n_h}{\partial z^2} + \left( \frac{1}{b_h} \frac{\partial n_h}{\partial z} + \frac{g}{D_h} + \frac{T_e}{T_h n_o} \frac{\partial n_o}{\partial z} \right) \frac{\partial n_h}{\partial z} + \left( \frac{1}{D_h} \frac{\partial g}{\partial z} + \frac{g}{D_h b_h} \frac{\partial b_h}{\partial z} + \frac{T_e}{b_h T_h n_o} \frac{\partial b_h}{\partial z} \frac{\partial n_o}{\partial z} + \frac{T_e}{T_h n_o} \frac{\partial^2 n_o}{\partial z^2} - \frac{T_e}{T_h n_o^2} \left( \frac{\partial n_o}{\partial z} \right)^2 \right) n_h - \frac{B}{b_h D_h m_h \mu_o} \frac{\partial b_h}{\partial z} \frac{\partial B}{\partial z} - \frac{1}{D_h m_h \mu_o} \left( \frac{\partial B}{\partial z} \right)^2 - \frac{B}{D_h m_h \mu_o} \frac{\partial^2 B}{\partial z^2} = 0. \quad (63)$$

As we see from (Figure 16, Figure 17, Figure 18) both ions density decreases exponentially with altitude and $O^+$ is the major ion species, dominant over $H^+$. When we observe (Figure 16) the calculated number density initial value is in the order $5 \times 10^{10} m^{-3}$ at 400 km increases up to $7.8 \times 10^{10} m^{-3}$ at 1000 km, then decreases exponentially in to $2 \times 10^8 m^{-3}$ at 12000 km. And from Figure 17 we observe the calculated number density initial value is $2 \times 10^9 m^{-3}$ then decreases exponentially to $3 \times 10^7 m^{-3}$, both solutions indicated in Figure 18, and we can deduced that the problem that face in the previous section (i.e $O^+$ is bounded about 3000 km) is circumvented by including driven shock wave term from conservation of momentum equation. The solutions also tells us both
ions have supersonic speed at some altitude. Even though, our assumption for the present model is subsonic flow.

4.3 Supersonic flow

We argued that both ions have supersonic flow at high-altitude, in the previous section, and by considering similar simplifications of previous chapter, the momentum equation for \( O^+ \) ions by considering driven shock wave becomes,

\[
\frac{n_o m_o u_o}{\partial z} + k(T_e + T_o) \frac{\partial n_o}{\partial z} + n_o m_o g - \frac{B}{\mu_o} \frac{\partial B}{\partial z} = -n_o m_o \nu_{o^+o} u_o, \tag{64}
\]

and, by substituting equation(20) on equation(64), we have,

\[
\frac{\partial M_o}{\partial z} = -\left[ \frac{g}{v_{to}^2} + \frac{\nu_{o^+o} M_o}{v_{to}} - \frac{B}{n_o m_o \mu_o v_{to}^2} \frac{\partial B}{\partial z} \right] \left( \frac{M_o}{M_o^2 - 1} \right). \tag{65}
\]

The numerical solution of equation(65) by using fourth order Runge-Kutta method is shown in Figure 20.

Similarly for \( H^+ \),

\[
\frac{\partial u_h}{\partial z} + \frac{v_{th}^2}{n_h} \frac{\partial n_h}{\partial z} + \frac{v_{th}^2 T_e}{T_h n_o} \frac{\partial n_o}{\partial z} + g - \frac{B}{n_h m_h \mu_o} \frac{\partial B}{\partial z} = -\nu_{ho} u_o. \tag{66}
\]
Figure 18: Number density of $H^+$ and $O^+$ by considering driven shock wave from conservation equations (momentum).
Figure 19: Vellocity according to Akebono satellite data (data from [Abe and et al, 1993]).

Figure 20: Mach number of $O^+$ by considering driven shock wave from conservation equations of ions.
when, we substitute equation (32) and equation (20) on equation (66),

$$\frac{\partial M_h}{\partial z} = -\left[ \frac{g}{v_{th}^2} + \frac{\nu_{ho} M_h}{v_{th}} - \frac{T_e}{T_h M_o} \frac{\partial M_o}{\partial z} - \frac{B}{n_h m_h \mu_o v_{th}^2} \frac{\partial B}{\partial z} \right] \left( \frac{M_h}{M^2 - 1} \right).$$

(67)

In order to understand the impact of driven shock wave on the polar wind dynamics, we examine the contribution of each force as follows. Collision force from conservation of momentum,

$$n_i m_i \frac{d u_i}{d t} = - \sum n_i m_i \nu_{in} u_i,$$

(68)

where, $\frac{d u_i}{d t}$ is total time derivative.

$$u_i = u_{i_0} \exp(-\nu_i u_i),$$

(69)

the ion-neutral collision decreases the average ion velocity exponentially increases. Ambipolar electric field from momentum conservation equation,

$$n_i m_i \frac{d u_i}{d t} = e n_i E,$$

(68)
\[
\frac{du_i}{dt} = \frac{q}{m_i} E, \quad (70)
\]
\[
u_t = u_{t0} \exp\left(\frac{e}{m_i} E\right),
\]
if we integrate and solve by Simpson rule we will see the contribution of ambipolar electric field, i.e. when \(E\) decreases the average ion velocity also decreases. Or simply from the following relation we can observe the effect of ambipolar electric field.

\[
J = en_i u_i = \frac{n_i e^2}{m_i \nu_i} E. \quad (71)
\]

The contribution of gravitational force from conservation of momentum is so small when we compare with the other forces, and it decelerates the ion as follows,

\[
n_i m_i \frac{du_i}{dt} = -n_i m_i g,
\]
\[
\frac{du_i}{dt} = -g, \quad (72)
\]
\[
u_t = u_{t0} \exp(-g).
\]

Pressure gradient term from momentum conservation affects the outflow as follows,

\[
n_i m_i \frac{du_i}{dt} = -\nabla p_i,
\]
\[
\frac{du_i}{dt} = \frac{-kT_i}{m_i} \frac{1}{n_i} \frac{\partial n_i}{\partial z}, \quad (73)
\]
when we approximate both sides,

\[
u_i^2 = \frac{kT_i}{m_i}, \quad (74)
\]

from this equation when temperature increases average ion velocity also increases.

Similarly the impact of driven shock wave on the polar wind dynamics can be understood from the following simplified equations: from momentum conservation equation,

\[
m_s n_s u_s \frac{\partial u_i}{\partial z} - \frac{B}{\mu_o} \frac{\partial B}{\partial z} = 0, \quad (75)
\]

where, \(\mu_o\) is magnetic moment. And from Maxwell’s equation,

\[
\nabla \times E = -\frac{\partial B}{\partial t}, \quad (76)
\]
with the assumption that $\partial B/\partial t = 0$, the electric field is,

$$E = -u \times B,$$  \hspace{1cm} (77)

and from equation (77) we get,

$$u_s \frac{\partial B}{\partial z} - B \frac{\partial u}{\partial z} = 0, \hspace{1cm} (78)$$

from equation (75) and equation (78) we get,

$$\frac{\partial u_s}{\partial z} = \frac{u_s \partial B}{B \partial z}, \hspace{1cm} (79)$$

from this equation we can understand the effect of driven shock wave on the ions outflow velocity. When we calculate numerically by using fourth order rung kutta method the variation of speed with height is indicated in Table 8. And from conservation of energy,

$$\frac{\partial q_s}{\partial z} + p_s \frac{\partial u_s}{\partial z} + \frac{B^2 \partial u_s}{\mu_o \partial z} - \frac{u_s B}{\mu_o} \frac{B}{\partial z} = 0,$$

$$\frac{\partial q_s}{\partial z} = \left( -\frac{n_s k T_s u_s}{e} \right) \frac{\partial B}{\partial z},$$

where $q_s$ is heat flux ($Jm^{-2}s^{-1}$). And from conservation of momentum,

$$n_s m_s u_s \frac{\partial u_s}{\partial z} - \frac{B}{\mu_o} \frac{\partial B_s}{\partial z} + e n_s \frac{\partial \phi_s}{\partial z} = 0,$$

$$\frac{\partial \phi_s}{\partial z} = \left( \frac{B}{e \mu_o n_s} - \frac{m_s u_s^2}{e B} \right) \frac{\partial B}{\partial z},$$

and we can calculate potential drop due to ambipolar electric field along the magnetic field.

From our numerical results in Figure 21, our model is capable of explaining the major features and pattern of the polar wind observed by satellite. i.e number density decreases exponentially as we discuss in the previous section, and the out flow velocity of both ions becomes supersonic at some altitude. But the model does not show the exact transition for each ions, then we put this criterion as limitation of our model.
Our work on the hydrodynamic driven polar wind model is motivated by the Akebono satellite observations. The earlier models do not explain the observation, then in our model we added driven shock wave as accelerating mechanism on the earlier hydrodynamic model. Then by considering as a three fluid, a physical meaningful solutions is obtained.

From this model we have found evidences of additional polar wind mechanism (i.e driven shock wave) that has significant influence on the ion outflow processes in the dayside polar cusp region. The influence of driven shock wave are confirmed through a series of modeling by considering different assumptions like with and with out driven shock wave, and comparing the out put with the actual solutions.

In addition, our model gives a physical meaning for the discontinuity that happens during the transition from subsonic to supersonic flow, this criterion makes our model different from others. Because earlier models put the discontinuity as a limitation of their model, but in our model it is the description of driven shock wave.

Other success of our model is able to calculate the potential drop due to ambipolar electric field along the magnetic field line, and the upward heat flux as shown in Table 8.

As we see from Table 8 calculated potential drop indicates 9.8V at 400km decreases exponentially to 0.2ev around 12000km. From this ions gain energy, this energy enables ions (specially $O^+$) to over come gravitational barrier to escape in to magnetosphere. Not only shows the transformation of energy but also indicates how much ambipolar electric field is affected by this driven shock wave. Really we prove that driven shock wave has a significant influence on the polar wind dynamics.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>400</th>
<th>6000</th>
<th>12000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat flux ($Jm^{-2}s^{-1}$)</td>
<td>$\sim 10^{-6}$</td>
<td>$\sim 10^{-7}$</td>
<td>$\sim 10^{-7}$</td>
</tr>
<tr>
<td>Electric Potential energy(ev)</td>
<td>$\sim 9.8$</td>
<td>$\sim 4$</td>
<td>$\sim 2$</td>
</tr>
<tr>
<td>Velocity (km/s)</td>
<td>3</td>
<td>2</td>
<td>1.2</td>
</tr>
<tr>
<td>Electric field(V/m)</td>
<td>$\sim 10^{-6}$</td>
<td>$\sim 10^{-4}$</td>
<td>$\sim 10^{-4}$</td>
</tr>
<tr>
<td>Electric field with shock wave(V/m)</td>
<td>$\sim 10^{-3}$</td>
<td>$\sim 10^{-3}$</td>
<td>$\sim 10^{-5}$</td>
</tr>
</tbody>
</table>
5.1 Future perspective

Our work on the polar wind is motivated by the experimental evidence of Akebono satellite. According to measurement there are different polar wind mechanisms that may affect the dynamics of the polar wind. In order to study the impact of these mechanisms on the polar wind, we rely on a hydrodynamics (considering driven shock wave) and self-consistent hybrid model that provides a global kinetic collisional description for the polar wind photoelectrons. If we obtain more data from the satellite(s), the more we realize there are unanswered scientific questions on the space plasma (or polar cusp) physics. We have left many unanswered questions behind us: (1) We should identify the dayside low-latitude and high-latitude energization/acceleration process(es) acting on outflowing ionospheric ions: how large (how much) does process influence on the magnetospheric dynamics?, and atmospheric dynamics (like ozone layer)?, what is the upper energy which $O^+$ can obtain in the process?, (2) We have looked at maximum differential particle flux and its enhancement. Next we should examine the flow rate $[s^{-1}]$ on basis of the same observation region. Hence we can also estimate how much the dayside ions outflows contribute to the total outflows from the Earth, (3) In addition to point 2 above, we may be able to estimate the loss rate of outflowing ions in the magnetosheath. The list above is basically (and realistically) investigable using the Cluster data, or in conjugation with other satellite(s) (e.g. Double Star satellite, Akebono satellite).
APPENDIX A

SOME BASIC CONCEPTS AND FORMULAE

A.1 Collisions term and Effusion

The number of molecules scattered in to change of solid angle $d\Omega$ per second is equal to the number of molecules in the incident beam crossing an area $d\sigma / d\Omega$ per second. The total cross-section is the number of molecules scattered per second,

$$\sigma_{\text{tot}} = \int d\Omega \frac{d\sigma}{d\Omega}. \quad (82)$$

A.2 Effusion (Knudsen number ($K_n$))

An important quantity governing the behavior of a gas in the ratio of the mean free path (the distance travelled by a molecule between two successive collisions) to some other characteristic length such as, (1) the size of the box containing the gas, (2) the diameter of the hole through which gas molecules may pass, (3) the wavelength of density fluctuations.

When the mean free path is larger (the mean free path between collisions is greater) compared to any other length (the size of the system), the gas is said to be in the collisionless regime.

The opposite of the collisionless regime is one in which the mean free path is much smaller than the other characteristic length, (in this case the gas molecules will undergo many collisions as they pass through the hole), will "thermalize" locally hydrodynamic regime.

$$K_n = \frac{l}{L}, \quad (83)$$

when $K_n << 1$, we can neglect kinetic effect which occurs due to the thermal spread of the particles. Where $L$ is characteristic length, $l$ is mean free path, $T$ is characteristic period, $\omega$ is characteristic frequency, $u_t$ is thermal velocity, $\nu$ is collision frequency,

$$T \sim \frac{L}{u_t} \sim \frac{1}{\omega}, \quad (84)$$

$$\omega << \nu \Rightarrow l << L. \quad (85)$$
Mass density of specified species,

$$\rho_s = \sum_s n_s m_s,$$

(86)

electric charge density,

$$\rho_s = \sum_s n_s q_s,$$

(87)

total momentum density,

$$\rho_t u = \sum_s \rho_t u_s,$$

(88)

diffusion velocity,

$$W_s = u_s - u = u_s - \frac{1}{\rho_t} \sum_s \rho_t u_s,$$

(89)

mass current density or mass flux,

$$J_s = \sum_s n_s m_s u_s,$$

(90)

electric current density or charge flux,

$$J = \sum_s n_s q_s u_s,$$

(91)

kinetic pressure dyad for each particle is,

$$p_s = \rho_t u_s < c_s c_s >,$$

(92)

total scalar pressure p,

$$p = \sum_s p_s + \frac{1}{3} \sum_s \rho_t u_s^2,$$

(93)

total heat flux vector q,

$$q = \sum_s (q_s + \frac{5}{2} \rho_t W_s + \frac{1}{2} \rho_t u_s^2 W_s).$$

(94)

A.3 Maxwellian distributions

Many of the characteristic features of plasmas can be understood by knowing a specific property of the distribution function, i.e. a reduced six-dimensional phase space distribution function or simply velocity distribution function, assuming the plasma to be spatially homogeneous (or dependent on the velocity at a fixed position) and stationary (time-independent). The conditions in which the plasma does not change in time and
does not exhibit spatial variations can be realized when the plasma is in equilibrium. The
general equilibrium velocity distribution function of a collisionless plasma is called the
Maxwellian velocity distribution.

\[ f(x, u, t) \rightarrow f(u), \]  

(95)

\[ f(u) = n \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left( -\frac{mu^2}{2kT} \right), \]  

(96)

where \( n \) is the particle number density, \( m \) is the particle mass, \( k \) is the Boltzmann constant
\((1.38 \times 10^{-23} \text{J/K})\), and \( T \) is the particle temperature. Besides \( kT \) denotes the average
thermal energy and the velocity spread, \( u = (2kT/m)^{1/2} \), can be identified as the thermal
velocity. A Maxwellian plasma in thermal equilibrium implies that (a) it does not contain
any free energy, hence (b) there are no energy exchange processes between the particles in
the plasma, and (c) the velocities of particles are distributed randomly around the average
velocity which is zero for a rest frame of plasmas.

A.4 Adiabatic invariants guiding center motion of single particle

For particles in magnetic fields, an adiabatic invariant is associated with each of
three types of motions, (1) the gyration motion around \( B \), (2) the longitudinal motion
along \( B \), and (3) the drift motion perpendicular to \( B \). Regarding mirror folding effect by
diverging geomagnetic fields, the concept (1) is very important. First adiabatic invariant
This invariant is associated with the cyclotron motion (gyration) of the particle. Here we
derive the first invariant by a simple way instead of the approach for canonical momentum
of charged particles. In general, the force on a particle along \( B \) can be written as

\[ F_\parallel = \frac{mdu_\parallel}{dt} = -\mu \frac{\partial B}{\partial z} = -\mu \nabla_\parallel B, \]  

(97)

where \( z \) is along \( B \), and magnetic moment,

\[ \mu = \frac{mu_\perp^2}{2B}. \]  

(98)

If the angle between the particle velocity and the magnetic field is pitch angle \((\alpha)\),

\[ \mu = \frac{mu_\perp^2 \sin^2 \alpha}{2B}, \]  

(99)
where \( u = (u_\perp^2 + u_\parallel^2)^{\frac{1}{2}} \), because of that the energy of the particles is maintained due to the energy conservation, also the velocity vector, \( u \) stays constant. When a particle is coming from a weak-field region in to a strong field region, \( u_\perp \) must increases in order to keep \( \mu \) constant.

Multiply each side of the above equation by \( u_\parallel = \frac{dz}{dt} \), the above equation becomes,

\[
\frac{d}{dt} \left( \frac{1}{2} \mu u_\parallel^2 \right) = -\mu \frac{B}{\partial z} \frac{dz}{dt} = -\frac{\mu dB}{dt}.
\]  

The condition that \( B \) is not time-dependent is considered here, thus the convective term of \( dB/dt \) is remained, i.e. \( dB/dt = \partial B/\partial t + (u\Delta \nabla)B = u\partial B/\partial z \).

Conservation of the total energy of the particle requires,

\[
\frac{d}{dt} \left\{ \frac{1}{2} \mu u_\parallel^2 + \frac{1}{2} \mu u_\perp^2 \right\} = \frac{d}{dt} \left\{ \frac{1}{2} \mu u_\parallel^2 + \mu B \right\} = 0,
\]  

Combining equation(100) and equation(101) and obtain,

\[
-\mu \frac{dB}{dt} + \frac{d}{dt} (\mu B) = 0,
\]  

and, the differentiation of the second term results in,

\[
B \frac{d\mu}{dt} = 0.
\]  

Since \( B \neq 0 \), then equation(103) implies that the magnetic moment \( \mu \) is independent of time and is a constant in the guiding center motion, moreover that the total magnetic flux enclosed by the motion must also remain constant. The concept of first invariant leads to magnetic trapping of particles and the magnetic moment \( \mu \) is conserved as long as the perturbation time scale is much longer than the cyclotron period.

### A.5 Mirror force

The invariance of \( \mu \) is the basis for the magnetic mirror force. When a particle travels along the magnetic field lines from a weak field towards a strong field region it will meet a magnetic field gradient (the magnetic field lines are converging). The \( u_\perp \) velocity increases to conserve the magnetic moment, since the total energy must remain constant, \( u_\parallel \) must decrease when the magnetic field is increasing. If the magnetic field gradient is strong enough an incoming particle will stop and the particles will be reflected back to the weak-field region, this effect is often termed as **mirror force** (the geomagnetic field, which decreases with altitude, give rise to the mirror force that changes the particle pitch angle).
APPENDIX B

IMPORTANT AREAS

B.1 Magnetosphere

The magnetosphere refers to the region in which magnetic influence of a celestial body. When we move out from the Earth in the direction towards the Sun the magnetic field will decrease by distance of \( r^{-3} \) until about 5Re from the center of Earth. Between 5Re and 8Re the field is stronger than expected from the pure dipole field. At about 8Re a discontinuity is observed (MAGNETOPAUSE), and further out wards the field is irregular, turbulent and weak. On the Earth ward side is the magnetosphere and on the Sun ward side is the interplanetary space and solar wind as Shown in (Figure 2.3).

Out side the magnetopause there is a new discontinuity where the the turbulence in the field comes to a halt (SHOCK WAVE). Out side the shock wave the field is very weak but more regular. The volume between the shock wave and the magnetopause is often called the MAGNETOSHEATH. Total volume covered by the shock wave is called MAGNETOSPHERE.

The field just inside the magnetopause is nearly twice as large as the field expected from a dipole field, and immediately outside the discontinuity the field is close to zero (The same situation would arise if we had a magnetized sphere close to a conducting surface and magnetic field at a distance \( d \) would be,

\[
B = \frac{\mu_0 M}{4\pi d^3},
\]

where \( M \) is magnetic dipole moment, instead a conducting plate was placed the magnetic field becomes,

\[
B' = \frac{\mu_0 2M}{4\pi d^3}.
\]

The ionospheric plasma at the upper strata of the Earth’s atmosphere would then merge with the interplanetary plasma at some great distance. Let us assume that the Earth with it’s magnetic field is embedded in a stationary interplanetary plasma with the same density and temperature as the solar wind plasma. The border between magnetosphere and the outer region which would be controlled by the interplanetary plasma, would be
determined by the balance between the magnetic field pressure of the Earth’s magnetic field and the thermal pressure of the interplanetary plasma. When we neglect any magnetic field in the interplanetary plasma and the plasma density of the ionospheric plasma at this distance, then we have,

\[
\frac{B_m^2}{2\mu_0} = nk(T_e + T_i),
\]  

(106)

where \(B_m\) is the magnetic field strength at this border, and \(B_m\),

\[
B_m = B_o(R_e/r_m)^3.
\]

(107)

The extreme limit of the magnetosphere in static equilibrium with an interplanetary thermal plasma would be close to 50Re. But the situation is strongly altered because the solar wind is a highly conducting plasma that prevents the Earth’s magnetic field from penetrating the solar wind plasma, and the solar wind pushes the magnetic field back towards the Earth downstream the wind. Current are induced in the solar wind plasma as it move by the Earth, and these currents give rise to new magnetic field which add to the Earth’s dipole field. In the case of head on solar wind kinetic pressure must be balanced by the pressure (energy density) of the magnetic field,

\[
P_k = 2nmu^2 = \frac{B_{mp}^2}{2\mu_o},
\]

(108)

\[
B_{mp} = 2B_o(R_e/r_{mp}^o)^3;
\]

(109)

and, by using the above equations,

\[
2nmu^2 = \frac{2B_o^2}{\mu_o}(R_e/r_{mp}^o)^6,
\]

(110)

where \(u\) is the velocity of solar wind, then \(r_{mp}^o\) (distance from magneto pause to the center of the Earth in polar region (compressed)) is about 9Re, generally equation becomes,

\[
r_{mp}' = r_{mp}^o(c\cos\phi)^{-1/3}.
\]

(111)

**B.2 Polar cusps**

The solar wind particles are flowing from the Sun toward the Earth, when the solar wind particles meat the bow shock their directions change, the bow shock region reduces
the speed and change the motion of the particles. Most of particles go around the magnetosphere, the magnetosphere rejects most of particles because charged particles do not travel easy across a magnetic field, particles bends off and moving along the magnetic field. Some of the solar wind particles can travel along the magnetic field lines of the Earth. There are a funnel Shaped area between the sun ward magnetic field and the tail ward magnetic field. These funnel-shaped areas are called the polar cusp has a highly dynamic postion and geometry. There is one cusp in each hemisphere. The solar wind particles enter the exterior cusp and then follows the converging magnetic field down to to the ionosphere. The exterior cusp has a diameter is approximately 50000km and cusp size in the ionosphere is approximately 500km (ESA, 2003).

B.3 Polar cap

The polar cap is the area around the geomagnetic pole bounded by the auroral oval (Kivelson.et, 1995). Polar caps are the high latitude regions in both hemisphere with open magnetic field lines connected to the interplanetary magnetic field, IMF. At high altitudes the field lines from the polar caps form the tail lobes in the magnetotail. The auroral oval is eccentric to the magnetic pole. It is displaced towards the night side. The eccentric oval is fixed while the Earth rotates. The field lines that extend to the greatest distance are those who start or end near the magnetic poles. The most sensitive zone for magnetic effect on Earth is the polar caps.
The hydrodynamic model has been used extensively to study the polar wind characteristics. For this model first, we will consider the polar wind only at altitudes above 400km, at such altitudes, neutral densities are low enough to neglect the chemical reactions such as photoionization, recombination, etc. Second, the magnitude of the geomagnetic field is such that the gyration period and Larmor radius (see Appendix A), for all particle species, are much smaller than any relevant time or length scales, we can therefore use the guiding center approximation. Third, the gradients of the geomagnetic field are such that only transport along the geomagnetic field line is important. Fourth the ion and electron collision frequencies are much smaller than the corresponding cyclotron frequencies, and consequently, the plasma is constrained to move along geomagnetic field lines like beads on a string, except when electric fields cause the entire ionosphere to convect horizontally.

However, this latter motion is distinct from the field-aligned motion is influenced by gravity as well as vertical density and temperature gradients. In order to describe mathematically we use cartesian coordinate system (south to north (z-axis), west to east (x-axis), perpendicular to both (y-axis)).

### C.1 Kinetic approach

Plasma consists of a collection of particles, both particles with negative and positive charges. We have to deal with collection of particles, (large number of particles) with different velocities, then it is convenient to describe statistically. With Boltzmann’s approach, one is not interested the motion of individual particles but rather the distribution of particles. We describe the properties of a large number of particles by saying how many there are per unit volume of phase space. Assume $N_s(r, u, t)$ denotes the number of particles of type s inside the volume element drdu around the phase space coordinates (r,u) at the instant t as shown in Figure 6. The distribution function in phase space, $f_s(r, u, t)$ is defined,

$$f_s(r, u, t) = \frac{N_s(r, u, t)}{drdu},$$
where \( d^3r d^3u = drdu \).

The distribution function corresponds to the number of particles of species \( s \), that at a time \( t \) are located in a volume element \( dr \) and at the same time have the velocity in a velocity space element \( du \). The number density \( n_s(r, t) \) is a macroscopic variable defined in configuration space as the number of particles of type \( s \) per unit volume,

\[
n_s(r, t) = \frac{N_s(r, u, t)}{dr} = \int du f_s(r, u, t),
\]

and we can investigate over all possible velocities. Likewise, the average or drift velocity of a species, \( u_s(r, t) \), can be obtained by,

\[
u_s(r, t) = \frac{\int du u_s f_s(r, u, t)}{\int du f_s(r, u, t)}.
\]

If the bulk velocity is different for ions and electrons, then the electric current will flow in the system.

For collisionless plasma the total number density at \( t \) and at \( t+dt \) is equal as shown in Figure 7, the transformation from initial condition \((r,u)\) to final coordinate \((r',u')\) becomes,

\[
\text{dr} \text{du} = J \text{dr} \text{du},
\]

where \( J \) is jacobian of transformation from the initial coordinate \((r,u)\) to the final ones \((r',u')\), since the transformation of particles does not affect the volume element, because
we are assuming collisionless plasma, then the jacobian transformation \( (J) \) becomes one. The total number density at \( t \) and at \( t+\text{dt} \) is equal,

\[ N_s(r', u', t + \text{dt}) = N_s(r, u, t) \]

\[ [f_s(r', u', t + \text{dt}) - f_s(r, u, t)]\text{d}r\text{d}u = 0, \]

by Taylor series, the above equation,

\[ \frac{\partial f_s}{\partial t} + u_s \cdot \nabla f_s + a \cdot \nabla_{u_s} f_s = 0, \]

where acceleration produced by the net force, \( a = \frac{\mathbf{F}}{m}, \mathbf{F} \) force, \( m \) mass, and when we consider particle interaction during the time interval,

\[ \frac{\partial f_s}{\partial t} + u_s \cdot \nabla f_s + \frac{e_s}{m_s} (E + \frac{1}{c} u_s \times B) \cdot \nabla_{u_s} f_s = \left[ \frac{\delta f_s}{\delta \text{coll}} \right]. \]

The time evolution of the species distribution function is determined by the net effect of collisions and the flow in phase space of particles under the influence of external forces, the equation is called Boltzmann equation. Where \( e_s \) and \( m_s \) are the charge and mass of species \( s \), \( E \) is the electric field, \( B \) is the magnetic field, \( c \) is the speed of light, \( \partial/\partial t \) is the time derivative, \( \nabla \) is coordinate space gradient, and \( \nabla_u \) is the velocity space gradient.

Let \( \chi(u) \) represent some physical property of the particles in the plasma. The temporal and spatial variation of the average value of \( \chi(u) \) can be obtained by multiplying
the Boltzmann equation by the function \( \chi(u) \) and integrating the resulting equation over all of velocity space,

\[
\int_u \chi \frac{\partial f_s}{\partial t} du + \int_u \chi u_s \cdot \nabla f_s du + \int_u \chi a \cdot \nabla u f_s du = \int_u \chi \left( \frac{\partial f_s}{\partial t} \right)_{\text{coll}} du.
\]  
(114)

When we evaluate each term independently we reach the following equation,

\[
\frac{\partial}{\partial t} (n_s < \chi >) + \nabla \cdot (n_s < \chi u >) - n_s < a \cdot \nabla u \chi > = \left[ \frac{\delta}{\delta t} (n_s < \chi >) \right]_{\text{coll}},
\]  
(115)

The quantity \( \delta f_s/\delta t \) represents the rate of change of \( f_s \) in a given region of phase space as a result of collisions. For collisions governed by inverse power potentials and for resonant charge exchange collisions, the appropriate expression for \( \delta f_s/\delta t \) is the Boltzmann collision integral which is given [8],

\[
\frac{\delta f_s}{\delta t} = \sum_t \int dud\Omega g_{sn} \sigma_{sn}(g_{sn}, \theta) [f'_s f'_n - f_s f_n],
\]  
(116)

where \( du \) is the velocity-space volume element of species \( t \), \( g_{sn} \) is the relative velocity of the colliding particles \( s \) and \( n \), \( \sigma_{sn}(g_{sn}, \theta) \) is the differential scattering cross-section, \( d\Omega \) is the element of solid angle in the \( s \) particle reference frame, \( \theta \) is the scattering angle, and the primes denote quantities evaluated after a collision, the value \( \delta f_s/\delta t \) is depend up on the model approach (see Appendix A).
APPENDIX D

HYBRID MODEL (REPRODUCE)

The time evolution of the species distribution function is determined by the net effect of collisions and the flow in phase space of particles under the influence of external forces. The mathematical description of this evolution is given by the collisional gyrokinetic equation,

\[
\frac{\partial}{\partial t} + u_\parallel \frac{\partial}{\partial z} - (g - \frac{e}{m} E_\parallel) \frac{\partial}{\partial u_\parallel} - u_\perp^2 \frac{1}{2B} \frac{\partial B}{\partial z} (\frac{\partial}{\partial u_\parallel} - \frac{u_\parallel}{u_\perp} \frac{\partial}{\partial u_\perp}) f_s = \frac{\delta f_s}{\delta t},
\]

where, (||) parallel and (⊥) perpendicular component.

The model applies kinetic collisional description to the ions and the photoelectrons in a self-consistent model. The model was generate self-consistent global polar wind calculations whose solutions span continuously from a collisional subsonic regime at low altitudes to a collisionless supersonic regime at high altitudes.

The model is hybrid in that it consists of a kinetic and a fluid component. Photoelectrons treated as test particles because of their low relative density and both the $H^+$ and $O^+$ ions are described using a global kinetic collisional approach while thermal electron properties are determined from a simpler, massless neutralizing fluid approach that also calculates the self-consistent ambipolar electric field.

The ambipolar electric field, is obtained from the moment equations of the next two higher orders, i.e. the momentum equation(119) and energy equation(121) transfer equations for the whole electron population (S.W.Y. Tam, F. Yasseen, T. Chang, 1998).

The continuity equation of $O^+$,

\[
\frac{\partial}{\partial z} \left( \frac{n_0 u_o B}{B} \right) = \frac{\partial}{\partial z} \left( \frac{n_0 u_o}{B} \right) = 0,
\]

the momentum equation of the total electron population from equation(117), becomes,

\[
B \frac{\partial}{\partial z} \left( \frac{n_e T_e + n_e m_e u_e^2 + n_p e T_{pe||} + n_p e m_e u_{pe}^2}{B} \right) + (n_e + n_{pe})(m_e \frac{\partial \phi_G}{\partial z} - e \frac{\partial \phi_E}{\partial z}) +
\]

\[
\frac{1}{B} \frac{\partial B}{\partial z} (n_e T_e + n_{pe} T_{pe,\perp}) = \frac{\delta M_e}{\delta t} + \frac{\delta M_{pe}}{\delta t}.
\]

Energy conservation for every particle is,
\[ \frac{\partial}{\partial z} \left\{ \frac{1}{B}[Q_w + nu(m\phi_G + e\phi_E)] \right\} = 0, \quad (120) \]

from equation(120) or from equation(117),

\[ B \frac{\partial}{\partial z} \left[ \frac{n_e u_e}{B} \left( \frac{5}{2} T_e + \frac{m_e u_e^2}{2} \right) + \frac{Q_w}{B} + \frac{n_e u_e}{B} + \frac{n_p e u_p e}{B} \right] = \frac{\delta E_e}{\delta t} + \frac{\delta E_p e}{\delta t}. \quad (121) \]

Considering the assumptions of (section 3.2.1), we can calculate the electric field from momentum equation of total electron population,

\[ -T_e u_e \frac{\partial u_e}{\partial z} + m_e u_e \frac{\partial u_e}{\partial z} - m_e u_e^2 \frac{\dot{B}}{B} = eE. \quad (122) \]

The momentum equation of \( O^+ \),

\[ \frac{1}{n_o} \frac{\partial}{\partial z} (n_o T_{o||}) = -m_o u_o \frac{\partial u_o}{\partial z} + \frac{1}{B} \frac{\partial B}{\partial z} (T_{o||} - T_{o\perp}) + (eE - m_o g) - m_o \nu_o o u_o, \quad (123) \]

equation(123) includes the major forces a particle experiences as it travels along the geomagnetic field line: gravitational force, field-aligned electric force, mirror force, and forces that are due to coulomb collisions.

Then, by substituting equation(122) and equation(118) on equation(123),

\[ \frac{\partial M_o}{\partial z} = -\left[ \frac{g m_o}{v_{tep}^2 (m_o - m_e)} + \frac{\nu_{o} o m_o M_o}{v_{tep} (m_o - m_e)} + \left( \frac{m_o v_{tep}^2 m_o^2 + T_{o\perp} - T_{o||}}{B v_{tep}^2 (m_o - m_e)} \right) \frac{\partial B}{\partial z} \right] \left( \frac{M_o}{1 + M_o^2} \right), \quad (124) \]

where \( v_{tep} \) is thermal speed of the ion-total electron gas,

\[ v_{tep}^2 = \frac{(T_e - T_{o||})}{m_o - m_e}, \quad (125) \]

and mach number \( M_o = u_o/v_{tep} \), and the numerical solution by using fourth order Runge-Kutta method is given in Figure 31.

From equation(119) and equation(121) we can calculate the electric field,

\[ T_e \frac{1}{B} \frac{\partial B}{\partial z} - \frac{T_e}{u_e} u_e \frac{\partial u_e}{\partial z} + (T_s + m_e u_s^2) \frac{x}{B} \frac{\partial B}{\partial z} - \frac{x m_e u_e^2}{B} \frac{\partial B}{\partial z} = (1 + x) eE, \quad (126) \]

where, \( x = n_s / n_e \) and let \( y = 1/(1 + x) \), then mach number of \( O^+ \), becomes,

\[ \frac{\partial M_o}{\partial z} = -\left[ \frac{g m_o}{v_{teo} (m_o - y m_e)} + \frac{\nu_{o} o m_o M_o}{v_{teo} (m_o - y m_e)} + \left( m_e x y v_{teo}^2 M_o^2 + T_{o\perp} - y T_e \right) \frac{1}{B v_{teo}^2 (m_o - y m_e)} \frac{1}{\partial z} \right] \left( \frac{M_o}{1 + M_o^2} \right), \quad (127) \]
Figure 24: Mach number of O\(^+\) versus altitude by hybrid model.
where, $v_{teoh}^2 = (yT_e - T_o)/(m_o - ym_e)$.

Similarly continuity and momentum equation for $H^+$ consequently,

$$\frac{\partial}{\partial z} \left( \frac{n_h u_h B}{B} \right) = \frac{\partial}{\partial z} \left( \frac{n_h u_h}{B} \right) = 0$$

(128)

$$\frac{1}{n_h} \frac{\partial}{\partial z} (n_h T_{h//}) = -m_h u_h \frac{\partial u_h}{\partial z} + \frac{1}{B} \frac{\partial B}{\partial z} (T_{h//} - T_{h//}) + (eE - m_h g) - m_h v_{h/o} u_h,$$

(129)

from equation(126), equation(128) and equation(129) we have,

$$\frac{\partial M_h}{\partial z} = -\left[ \frac{g}{v_{teh}^2} + \frac{v_{h/o} M_h}{v_{teh}} + \frac{y}{M_o m_h v_{teh}^2} (m_e v_{teoh}^2 M_o^2 - T_e) \frac{\partial M_o}{\partial z} + \frac{y T_e}{m_h v_{teh}^2 B} \frac{\partial B}{\partial z} + (T_s + m_e u_s^2) \frac{y x}{m_h v_{teh}^2 B} \frac{\partial B}{\partial z} - \frac{T_{h//}}{m_h v_{teh}^2 B} \frac{\partial B}{\partial z} - \frac{y m_e v_{teoh}^2 M_o^2}{m_h v_{teh}^2 B} \frac{\partial B}{\partial z} \right] \left( \frac{M_h}{1 - M_h^2} \right),$$

(130)

where, $v_{teh}^2 = T_h/m_h$.

D.1 Summary

Even, from equation(130) we observe that the major problem for the previous section is solved, i.e. no singularity point (there is transonic solution).

The numerical solution of equation(130) can be obtain by using fourth order Runge Kutta method as shown Figure 31, and as we observe from figure we can conclude that $O^+$ (about 600km) and $H^+$ (about 3000km) have different transition height (from subsonic to supersonic). Then, solution indicates clearly the transonic region, and the polar wind characteristics observed by Akebono satellite.

D.2 Limitation of the model

Even though the model indicates $O^+$ is the dominant ion up to 12000km, and also indicate both ions have supersonic flow at high-altitude as observed by Akebono satellite, the assumption taken for the model, i.e considering similar behavioral plasma as a fluid and as a particle is not correct. Then in order to circumvented this problem we consider all species of plasma as a fluid, and considering driven shock wave as an accelerating polar wind mechanism. The new model i.e MHD with driven shock wave can explain the problem that face in the traditional MHD model, similar to hybrid model.
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DECLARATION

I declare that the thesis is my original work and has not been presented for a degree in any other university, and that all sources of materials have been duly acknowledged.

Name: Tadesse Terefe
Signature: 
Date: 

This thesis has been submitted for the examination with my approval as this thesis advisor.

Name: Dr. Baylie Damtie
Signature: 
Date: 

56