THERMOELECTRIC OPTIMIZED EFFICIENCY
AT STRONG COUPLING IN NANO-SIZED
PHOTO-ELECTRIC DEVICE

By
SIMACHEW ENDALE ASHEBIR

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE IN PHYSICS
AT
ADDIS ABABA UNIVERSITY
ADDIS ABABA, ETHIOPIA
JUNE 2010

© Copyright by SIMACHEW ENDALE ASHEBIR, 2010
ADDIS ABABA UNIVERSITY
DEPARTMENT OF
PHYSICS

Supervisor: Dr. MULUGETA BEKELE

Examiners: Dr. MESFIN TSIGE

Dr. TATEK YERGOU
ADDIS ABABA UNIVERSITY

Date: JUNE 2010

Author: SIMACHEW ENDALE ASHEBIR

Title: THERMOELECTRIC OPTIMIZED EFFICIENCY AT STRONG COUPLING IN NANO-SIZED PHOTO-ELECTRIC DEVICE

Department: Physics

Degree: M.Sc. Convocation: JUNE Year: 2010

Permission is herewith granted to Addis Ababa University to circulate and to have copied for non-commercial purposes, at its discretion, the above title upon the request of individuals or institutions.

______________________________
Signature of Author

THE AUTHOR RESERVES OTHER PUBLICATION RIGHTS, AND NEITHER THE THESIS NOR EXTENSIVE EXTRACTS FROM IT MAY BE PRINTED OR OTHERWISE REPRODUCED WITHOUT THE AUTHOR’S WRITTEN PERMISSION.

THE AUTHOR ATTESTS THAT PERMISSION HAS BEEN OBTAINED FOR THE USE OF ANY COPYRIGHTED MATERIAL APPEARING IN THIS THESIS (OTHER THAN BRIEF EXCERPTS REQUIRING ONLY PROPER ACKNOWLEDGEMENT IN SCHOLARLY WRITING) AND THAT ALL SUCH USE IS CLEARLY ACKNOWLEDGED.
# Table of Contents

Table of Contents iv

List of Figures v

Acknowledgements vii

Abstract viii

1 Introduction 1

2 Efficiency at maximum power 7
   2.1 Introduction ................................................. 7
   2.2 Model of the system ........................................... 7
   2.3 Electron dynamics of the model ............................... 9
   2.4 Thermodynamics of the model ................................ 12

3 Optimization of solar energy converter 18
   3.1 Optimization Using objective function ....................... 18
   3.2 Optimized efficiency ......................................... 20

4 Summary and Conclusion 29

Appendix 31

Bibliography 36
List of Figures

1.1 The minimal model of solar energy converter. ........................................ 4

2.1 A nano structure photo-electric devices with 2 energy levels, contacts with
two electron reservoir leads at the same temperature but different chemical
potentials from ref.[5]. ................................................................. 8

2.2 Schematic view of the nano-sized photo-electric device. The grey arrows
show the different allowed electron transitions. Transitions between the
two energy levels are induced by solar photons (red curved arrows) and by
non-radiative processes (blue curved arrows) from ref.[5]. ........................ 9

2.3 The current of electrons through the device from ref.[5]. .......................... 11

2.4 The current of electrons through the device with the contribution to the
current due to the interaction with the sun and non-radiative processes
from ref.[5]. .................................................................................. 12

2.5 Plot of Efficiency at maximum power (green line) compared with Carnot
efficiency (red line)and Curzon Ahlborn efficiency (blue line) versus Carnot
efficiency ...................................................................................... 15

2.6 Plot of $\eta_C$ (green line ), $\eta_{CA}$ (blue line ) and $\eta_{MP}$ (red line) Versus $\Upsilon = \frac{T_c-T}{T}$ 15

2.7 Plot of time taken at maximum power for one complete cycle ($\tau_{mp}$) versus $\eta_c$ 17

3.1 Plot of objective function, $\hat{\Omega}$, versus the free parameter $f_N$ for $\eta_c=0.1$ .... 21

3.2 Plot of $\eta_c$ (red line), $\eta_{opt}$ (green line ), $\eta^{ca}$ (black line ), $\eta^{mp}$ (blue line),versus $\eta_c$ ................................................................. 23

3.3 Plot of $\eta_c$ (red line ) ,$\eta_{opt}$ (blue line )$\eta_{ca}$ (green line ) and $\eta_{mp}$ (black line)versus $\Upsilon = \frac{T_{ca}-T}{T}$ ......................................................... 24

3.4 The time taken to complete one full cycle efficiency at maximum power
(blue line) and for optimized efficiency (red line )versus Carnot efficiency . 26
3.5 plot of $\tau^{rel}$ versus $\eta_c$ ....................................................... 27
3.6 plot of $\eta^{rel}$ versus $\eta_c$ ....................................................... 27
Acknowledgements

First and foremost, I would like to thank the almighty, God. I would also like to express my heartfelt appreciation and gratitude to my advisor and instructor Dr. Mulugeta Bekele for his unreserved support, excellent and generous guidance throughout my work and making critical reading of my thesis. While working with him, I have got not only a chance to share his long research experience which benefited me a lot but also how to build up social life and friendly relationship with people.

I am grateful to all my colleagues of Statistical and Computational Physics group specially Fistum Borga and Solomon Worku who have delivered their unreserved help throughout my thesis work. I am also thanks to Yeneneh Yalew, Anteneh Getachew and my finest friend Mequannint Menuy.

Of course, I am very much pleased to foreword my unreserved affection and thank to my parents. Specially my brother Getasew Endale for his unlimited finical support to have my own laptop.

Finally, a special thought also goes the department of physics and the school of graduate studies. I have derived materials from many research journals and books, and I am indebted to the authors of those publications and books.
Abstract

In this work we consider a model of solar energy converter which is composed of a nano-scale photo-electric device having two energy levels in contact with two electron reservoir leads at the same temperature but with different chemical potentials whose task is to convert radiation energy to electric energy. We analytically study the optimized thermodynamic efficiency, which lies between Carnot efficiency and efficiency at maximum power, of our model displaying strong coupling between the generated electron flux and the incoming photon flux from the sun. We also find the time taken to accomplish one complete cycle at the maximum power and at optimized efficiencies.
Chapter 1

Introduction

The area of physics concerned with the relationships between heat and work is thermodynamics. In its engineering applications thermodynamics has two major objectives. One of these is to describe the properties of matter when it exists in what is called an equilibrium state, a condition in which its properties show no tendency to change. The other objective is to describe processes in which the properties of matter undergo changes and to relate these changes to the energy transfers in the form of heat and work which accompany them. Thermodynamics is unique among scientific disciplines in that no other branch of science deals with subjects which are as commonplace or as familiar. Concepts such as "heat", "work", "energy", and "properties" are all terms in everyone's basic vocabulary. Thermodynamic laws which govern them originate from very ordinary experiences in our daily lives. In the development of the second law of thermodynamics, it is very convenient to have a hypothetical body with a relatively large thermal energy capacity that can supply or absorb finite amounts of heat without undergoing any change in temperature. We call such a body a reservoir. A reservoir that supplies energy in the form of heat is considered as a source, and one that absorbs energy in the form of heat is considered as a sink.

Basic thermodynamics tells us about heat engine. A heat engine is any device which, though a cyclic process, absorbs energy via heat and converts (some of) this energy to work. Many real world "engines" (e.g., automobile engine, steam engine) can be modeled
as heat engines. There are some fundamental thermodynamic features and limitation of heat engines which arises irrespective of the details of how the engines work. Even if heat engines differ considerably from one another, all can be characterized by the following common properties.

1. They receive heat from a high temperature source (e.g., solar energy, oil furnace, nuclear reactor, etc.).
2. They convert part of this heat to work.
3. They reject the remaining waste heat to a low temperature sink (e.g., the atmosphere, river, etc.).
4. They operate on a cycle.

A heat engine, while extracting some work, has to transform some amount of heat from a hot reservoir, at temperature $T_h$, to a cold reservoir, at a low temperature $T_c$. The ratio of the useful work, $W$, extracted to the heat taken from the hot reservoir, $Q_h$, is called the efficiency, $\eta$, of the heat engine; i.e.

$$\eta = \frac{W}{Q_h}$$  \hspace{1cm} (1.0.1)

It was Carnot who first showed that no heat engine could convert heat to work better than the Carnot engine whose efficiency, $\eta$, is given by

$$\eta_c = 1 - \frac{T_c}{T_h}.$$  \hspace{1cm} (1.0.2)

From thermodynamic point of view this efficiency increases as $T_c$ decreases. In other words, the lower the temperature of the cold system (to which heat is delivered), the higher the engine efficiency. The maximum possible efficiency, $\eta_c = 1$, occurs if the temperature of cold source is equal to zero and the temperature of the hot source is
nonzero. If the reservoir at zero temperature were available as a heat repository, heat could be freely and completely converted into work and the world "energy storage" would not exist [1].

Even though both macroscopic and nano-scale engines work at the same principle, extensive studies have been done in the performance of mainly macroscopic heat engines[2]. Nowadays, there is much interest in the study of nano-sized heat engines. Some of this interest lies on the need to have nano-sized engines in order to utilize energy resources available at these scales, and miniaturization of devices demanding tiny engines operating on the small scale. As such, modeling nano-sized engines and finding how well they work (efficiency) is the primary task that has to be considered at present.

To understand how nano-sized photo-electric devices operate as heat engine, we took the following model that has the minimum ingredients as solar energy converter. The main part of the solar energy converter is a light absorber (nano structure photo-electric device) with two energy levels $E_l$ and $E_r$ where the absorption of a photon of light promotes an electron from the ground state ($E_l$) to an excited state ($E_r$).

The solar energy conversion process is now modeled by adding two other states: an electron reservoir (which accepts an electron from the excited state) and a hole reservoir (which accepts a 'hole' from the ground state or, in other words, donates an electron to the ground state). The free energy per electron in the two reservoirs (in other words, the chemical potentials) will be denoted by $\mu_l$ and $\mu_r$ but the two lead reservoirs have the same temperature. In thermal equilibrium $\mu_l = \mu_r$ but in general, $\mu_l$ and $\mu_r$ will not be equal, and an amount of external work equal to $\Delta \mu = \mu_r - \mu_l = qV$ can be carried out by transferring an electron from hole reservoir (left lead) to the electron reservoir (right lead).
Figure 1.1: The minimal model of solar energy converter.

From the above diagram an electron enter from the left lead to the device since the left lead is an electron provider reservoir in addition to this the left lead is connected with the lower energy level and the absorption of a photon of light promotes an electron from the ground state to an excited state. Then an electron enter to the right lead since the right lead connected with the higher energy level as well as this lead act as an electron accepter reservoir and finally re-enter to the left lead by the external work equal to $\Delta \mu = \mu_r - \mu_l$ within the external circuit. This process helps the energy converter work as a heat engine.

To determine the efficiency of the nano-sized solar energy converter one can has to use sun as hot reservoir with temperature, $T_s$. The power $P$ generated by the solar cell to bring electrons from the left to the right lead with the net heat flux coming from the sun determines the efficiency of the engine.

The primary need for the efficient device arises either to decrease the energy consumption rate or to decrease the heat dissipation rate in the processes of operation. The latter concept is of more importance in a nonequilibrium state since there is always an unavoidable and irreversible transfer of heat via fluctuations thereby making the device less efficient as a device. Any irreversibility or finite entropy production will reduce the efficiency. Due to this the actual nano-sized heat engine that will be designed must consider all these. For practical application the Carnot efficiency has a limited significance, Indeed
a reversible power having no preferred direction in time has to be infinitely slow. Hence the corresponding power is zero (finite work divided by infinite time). Due to this Curzon and Ahlborn [3] investigated the problem of efficiency at nonzero power by constructing a Carnot cycle that operates in a finite time in which the only irreversible steps are assumed to be during the transfers of heat between reservoir and auxiliary work performing system (the so called endoreversible approximation). Also Van de Broeck [4] proposed a general derivation within the realm of linear irreversible thermodynamics. This is the most important feature because it paves the way for analyzing the nonisothermal heat engines using the linear irreversible thermodynamics frame work, a field upto now almost limited to isothermal energy converters. Curzon and Ahlborn [3] took an endoreversible engine that exchange heat linearly at finite rate with the two reservoirs and found its efficiency, $\eta_{CA}$, at maximum power to be

$$\eta_{CA} = 1 - \sqrt{\frac{T_c}{T_h}}.$$ (1.0.3)

On the other hand, a heat engine operating reversibly is quasistatic and takes infinite time eventhough its efficiency is maximum. Further more, a heat engine operating at maximum power takes shortest possible time while dissipating (wasting) high amount of the input energy and making the device less efficient. A compromise between these two extremes which leads to decreasing the wastage by relaxing the operating time enough to optimize energy utilization is of interest in studying such system.

The main purpose of this work is to find the optimized efficiency that lies between Carnot efficiency and efficiency at maximum power and also the corresponding optimized time taken to operate one complete cycle.
The rest of the thesis is organized as follows. In Chapter two we demonstrate a model which is considered by B. Cleuren, B. Rutten, M. Esposio of nano sized device having two single energy levels which is connected with two metal leads of the same temperature that acts as electrons reservoir where as the sun temperature as the other reservoir. But our consideration in the case of linear regime that means the sun temperature near to the leads temperature. We briefly summarize the derivation of efficiency at maximum power and finally we find the time taken to complete one cycle process. In Chapter three we develop an objective function based on the proposal of Hernandez et.al.[6] and using optimization principle we optimize the objective function with respect to the free parameter, $f_N$, after this we calculate the optimized efficiency of the model and the optimized time taken to perform one cycle. In the last Chapter we summarize and conclude the results of our work.
Chapter 2

Efficiency at maximum power

2.1 Introduction

In this chapter we are going to demonstrate the model and the derivation of maximum power as well as its corresponding efficiency which is presented by B. Cleuren, B. Rutten, M. Esposio. Finally we calculate the time taken to deliver one cycle process at maximum power.

2.2 Model of the system

We consider a nano structure photo electric device with two energy levels in contact with two electron reservoir leads at the same temperature but with different chemical potentials $\mu_r = \mu_l + qV$ whose task is conversion of radiation into electric energy where the efficiency at which the conversion takes place has universal upper bound given by the Carnot efficiency $\eta_c = 1 - \frac{T}{T_c}$ [1].

A nano structure photo-electric device with two energy levels in contact with two electron reservoir leads constructed as follows from B. Cleuren, B. Rutten, M. Esposio work.
Even if Carnot efficiency has fundamental theoretical implication, it is of poor practical use since it is only reached when the device is operated under reversible conditions. Hence the general power, as the output energy divided by the (infinite) operation time goes to zero. In realistic circumstances of finite power output, efficiency will necessarily be below the Carnot limit due to irreversible processes taking place in the device. Another source of possible efficiency decrease are energy losses within the device for example due to non-radiative recombination of charge carriers. Due to this Curzon and Ahlborn [3] investigated the problem of efficiency at nonzero power by constructing a Carnot cycle in finite time, in which the only irreversible steps are assumed to be during the transfer of heat between reservoir and auxiliary work performing system (the so called endoreversible approximation). Curzon and Ahlborn [3] took an endoreversible engine that exchanges heat linearly at finite rate with the two reservoirs and found its efficiency, $\eta_{CA}$, to be

$$\eta_{CA} = 1 - \sqrt{\frac{T_c}{T_h}}$$

(2.2.1)

The minimal model for solar energy converter of nano structure photo electric devices is shown below.
2.3 Electron dynamics of the model

As shown in the above fig 2.3 the nano device we consider is composed of two single particle levels of energy $E_l$ and $E_r$ (with $E_r > E_l$) which define the band gap energy $E_g = E_r - E_l$. Assume that coulomb interactions prevent two electrons to be present at the same time in the device. As a result the device is either empty or has an electron in level $E_l$ or $E_r$ with respective probabilities $p_i$ where $i \in \{0, l, r\}$.

Electron transition between $E_l$ and $E_r$ are induced by two possible mechanisms. The first is due to incoming sun (black body) radiation at the resonant energy $\lambda \nu = E_g$. The second is due to non-radiative processes at the same resonant transition. The dynamics of the cell is described using the master equation

$$\frac{\partial p_i(t)}{\partial t} = \sum_j [k_{ij}p_j(t) - k_{ji}p_i(t)], \quad (2.3.1)$$

which is the gain loss equation for the probability of each state $i$ where the first term of the above equation is the gain due to the transition of electrons from state $j$ to $i$ and the
second term is the loss due to transition of electron into j from i. Using the above relation we have the master equation to be

\[
\begin{pmatrix}
\dot{p}_0(t) \\
\dot{p}_l(t) \\
\dot{p}_r(t)
\end{pmatrix} =
\begin{pmatrix}
-k_{l0} - k_{r0} & k_{0l} & k_{0r} \\
k_{l0} & -k_{0l} - k_{rl} & k_{lr} \\
k_{r0} & k_{rl} & -k_{0r} - k_{lr}
\end{pmatrix}
\begin{pmatrix}
p_0(t) \\
p_l(t) \\
p_r(t)
\end{pmatrix}
\]  
(2.3.2)

where \(k_{ij}\) denotes the transition rate from state j to i, \(p_i\) and \(p_j\) are probability of state i and j. The rates describing the exchange of electrons with the leads are given by

\[k_{l0} = \Gamma_l f(x_l); k_{0l} = \Gamma_l [1 - f(x_l)]; k_{r0} = \Gamma_r f(x_r); k_{0r} = \Gamma_r [1 - f(x_r)],\]  
(2.3.3)

where \(f(x) = [exp(x) + 1]^{-1}\) is the Fermi distribution, \(\Gamma_l\) and \(\Gamma_r\) are inverse of spontaneous time taken. The arguments are the scaled energies \(x_l = \frac{(E_l - \mu_l)}{k_B T}\) and \(x_r = \frac{(E_r - \mu_r)}{k_B T}\) while \(k_B\) is the Boltzmann constant. The rates describing the transition between energy levels due to non-radiative (nr) effects and sun photons (s) are given by

\[k_{rl} = \Gamma_{nr} n(x_g) + \Gamma_s n(x_s); k_{lr} = \Gamma_{nr} [1 + n(x_g)] + \Gamma_s [1 + n(x_s)],\]  
(2.3.4)

where \(n(x) = [exp(x) - 1]^{-1}\) is the Bose-Einstein distribution, \(\Gamma_{nr}\) and \(\Gamma_s\) are inverse of spontaneous time taken with scaled energies \(x_g = \frac{E_g}{k_B T}\) and \(x_s = \frac{E_s}{k_B T_s}\). Notice that the ratio of the forward and backward transition rates associated to a given elementary process satisfies the detailed balance condition. This ensures that the equilibrium distribution (when \(\mu_l = \mu_r\) and \(T = T_s\)) has the corresponding grand-canonical form.

The following figure shows the electron current entering the device from the left lead is given by

\[J = k_{l0}p_0 - k_{0l}p_l.\]  
(2.3.5)
Let us next find the current of the device at steady state defined by $\dot{p}_0(t) = \dot{p}_r(t) = \dot{p}(t) = 0$. $J$ becomes the current of electrons through the device (positive from the left to the right) with the corresponding electric current $qJ$. It can be decomposed as $J = J_s + J_{nr}$ with $J_s$ and $J_{nr}$ being the contribution to the current due to the interaction with the sun and the non-radiative processes, respectively, and are given by

$$J_s = \Gamma_s n(x)p_l - \Gamma_s [1 + n(x)]p_r; J_{nr} = \Gamma_{nr} n(x_y)p_l - \Gamma_{nr} [1 + n(x_y)]p_r,$$  \hspace{1cm} (2.3.6)
Figure 2.4: The current of electrons through the device with the contribution to the current due to the interaction with the sun and non-radiative processes from ref.[5].

### 2.4 Thermodynamics of the model

From a thermodynamic point of view, solar cells are heat engines converting part of the heat input from the hot reservoir (the sun) into work by moving electrons from lower to higher chemical potentials. The remaining heat gets transferred to the colder reservoir. Since all photons interacting with the solar cell have an energy $E_g$, the net heat flux coming from the sun (i.e. the net energy absorbed per unit time) is $\dot{Q}_s = E_g J_s$. The heat flux coming from the cold reservoir has three contributions: $\dot{Q}_l = (E_l - \mu_l)J$ and $\dot{Q}_r = -(E_r - \mu_r)J$ due to electron exchanges between the cell and the left and the right lead, respectively, while $\dot{Q}_{nr} = E_g J_{nr}$ is due to the non-radiative energy exchange.

The power $P$ generated by the solar cell to bring electrons from left to the right is given by

$$P = (\mu_r - \mu_l)J = T_s [x_s - (1 - \eta_c)(x_r - x_l)]J. \quad (2.4.1)$$

The efficiency at which this conversion takes place is given by
\[ \eta = \frac{P}{Q_s} = \frac{(\mu_r - \mu_l) J}{(E_r - E_l) J_s} = (1 - (1 - \eta_c)(\frac{x_r - x_l}{x_s}))(1 + \frac{J_{nr}}{J_s}) \] (2.4.2)

The entropy \( S(t) \) of the solar cell can be expressed in the usual form \( S(t) = -k_B \sum_i p_i(t) \ln p_i(t) \).

Its time evolution can be split into a reversible and an irreversible part, \( \dot{S} = \dot{S}_e + \dot{S}_i \), with \( \dot{S}_e = \frac{\dot{Q}_s}{T_s} + (\frac{\dot{Q}_l}{T} + \frac{\dot{Q}_r}{T} + \frac{\dot{Q}_{nr}}{T}) \) corresponding to the entropy change due to the heat exchange with the different reservoirs. \( \dot{S}_i (\geq 0) \) can be identified as the internal entropy production due to dynamical processes with the solar cell [5, 6, 7]. In the stationary regime \( \dot{S} = 0 \) so that \( \dot{S}_i = -\dot{S}_e \) and the entropy production takes on the familiar bilinear form (see more at the appendix)

\[ \dot{S}_i = Q_s f_U + J f_N = (x_r - x_l) - x_s J - x_r J_{nr} \] (2.4.3)

where \( f_U = \frac{1}{T} - \frac{1}{T_s} \) and \( f_N = \frac{(\mu_r - \mu_s)}{T} \) are the thermodynamic forces conjugate to the energy and matter fluxes, respectively.

From now on, we will focus on the linear regime close to thermal equilibrium. In this regime, characterized by small thermodynamics forces, the heat and the particle flows appearing in the entropy expression Eq. (2.4.3) can be expanded to first order:

\[ \dot{Q}_s \approx L_{UU} f_U + L_{UN} f_N; J \approx L_{NU} f_U + L_{NN} f_N, \] (2.4.4)

The coefficients \( L_{ij} \) appearing are the well known Onsager coefficients. The off-diagonal elements are responsible for energy conversion process and satisfy the Onsager symmetry relation \( L_{UN} = L_{NU} \) [5, see also the appendix]. For a given temperature difference (quantified by \( f_U \)), the power from Eq.(3.4.1) becomes \( P = -T f_N J \). So to find the efficiency at the maximum power, we first maximize the power with respect to the free parameter
$f_N$. The engine power takes its maximum value when

$$\frac{\partial P}{\partial f_N} \bigg|_{f_N^{mp}} = 0 \quad (2.4.5)$$

is satisfied. From Eqs.(2.4.1), (2.4.4) and (2.4.5) the power has maximum value at a finite value of $f_N^{mp}$ which is given by

$$f_N^{mp} = -\left(\frac{L_{NU}}{2L_{NN}}\right)f_U \quad (2.4.6)$$

Using Eqs. (2.4.2) and (2.4.6) then the corresponding efficiency, $\eta^{mp}$, at maximum power becomes

$$\eta^{mp} = \frac{\eta_c \kappa^2}{2 - \kappa^2} \quad (2.4.7)$$

This value of $\eta^{mp}$ is half the Carnot efficiency multiplied by a factor depending on the coupling parameter $\kappa = \frac{L_{NU}}{\sqrt{L_{NN}L_{NN}}} \quad [4]$ which has a numerical value between $-1$ and $+1$.

The coupling parameter $\kappa$ is given by [5]:

$$\kappa^2 = \frac{e^{x_1}(e^{x_2} - 1)\Gamma_l\Gamma_r\Gamma_s}{\Gamma_{nr}(\Gamma_l + \Gamma_r) + e^{x_1}(\Gamma_{nr} - \Gamma_l)\Gamma_r + e^{x_2}\Gamma_l(\Gamma_{nr} + \Gamma_r)(\Gamma_{nr} + \Gamma_s)} \quad (2.4.8)$$
Figure 2.5: Plot of Efficiency at maximum power (green line) compared with Carnot efficiency (red line) and Curzon Ahlborn efficiency (blue line) versus Carnot efficiency.

Fig 2.5 shows plot of efficiency versus $\eta_c$, that all the efficiencies increases as $\eta_c$ increase. Since as $\eta_c$ increase temperature difference between the two reservoirs increase that means heat dissipation decrease. Then one can extract more work at highest efficiency than at lower efficiency. To get finite work at high efficiency ($\eta_c$) one needs to wait for sufficiently long time but one can extract finite amount of work with in a finite time for efficiency at maximum power and Curzon and Ahlborn efficiency process.

Let us define dimensionless parameter that describe the model

$$\Upsilon = \frac{T_s}{T} - 1 \quad \text{(2.4.9)}$$

Figure 2.6: Plot of $\eta_C$ (green line), $\eta_{CA}$ (blue line) and $\eta_{MP}$ (red line) Versus $\Upsilon = \frac{T_s-T}{T}$.
Fig 2.6 shows plot of efficiency versus $\Upsilon = \frac{T_s - T}{T}$, that all the efficiencies increase as $\Upsilon = \frac{T_s - T}{T}$ increase. Since as $\Upsilon = \frac{T_s - T}{T}$ increase temperature difference between the two reservoirs increase that means heat dissipation decrease. Then one can extract more work at highest efficiency than at lower efficiency. To get finite work at high efficiency ($\eta_c$) one needs to wait for sufficiently long time but one can extract finite amount of work with in a finite time for efficiency at maximum power and Curzon and Ahlborn efficiency process.

Our next task is to calculate the time taken to complete one cycle when the device operates at maximum power. From Eqs.(2.4.4) and (2.4.6) with $f_U = \frac{1}{T} - \frac{1}{T_s} = \frac{\eta_c}{T}$ the corresponding flux at the maximum power becomes

$$J_{mp} = \frac{L_{NU} \eta_c}{2T}. \quad (2.4.10)$$

But it is known that the flux is

$$J = \frac{N}{\tau}. \quad (2.4.11)$$

where $N$ is number of electrons flowing in time $\tau$ across the device. So for one electron dynamics the flux becomes

$$J = \frac{1}{\tau}. \quad (2.4.12)$$

Using Eqs.(2.4.9) and (2.4.11) the period, $\tau^{mp}$, taken to accomplish one cycle at maximum power is

$$\tau^{mp} = \frac{2}{\eta_c L_{NU}}. \quad (2.4.13)$$
From Figure 2.7 we observe that the operation time goes to infinity as the Carnot’s efficiency goes to zero. This happen if the temperature of the reservoirs is the same. So we can not extract some amount of work with in a finite time for one complete cycle process to transfer heat from one system to the other. But as Carnot efficiency increase the time taken for one complete cycle becomes decrease.
Chapter 3

Optimization of solar energy converter

3.1 Optimization Using objective function

The subject of optimization of real devices has received continued attention in thermodynamics, engineering and recently, in biochemistry [6]. The main goal in optimization is to find the path way that yields optimum performance in processes operating at non zero rates. To achieve this goal, an objective function that depends on parameters of the problem must be optimized. In principle one has the freedom of choice of such function. It has been pointed out by Hernandez et.al.[6] that a thermodynamic criterion devoted to analyze the optimum regime of operation in a real process should meet the following requirements: (i) its dependance on the parameters of the process should be a guidance in order to improve the performance of that process; (ii) it should not depend on parameters of environment; and (iii) it should take into account the unavoidable dissipation of energy provoked by the process. The two most widely used ways in optimization of traditional thermodynamic heat devices are the entropy generation minimization and exergy analysis [6]. Exergetic methods additionally depend on the parameters of the environment which can be unknown or far from the average values [6]. In our work the optimization criteria as proposed by Hernandez et.al.[6] will be implemented.
Consider an energy converter that produces a useful energy $E_u(y; \{\alpha\})$, for a given input energy, $E_i(y, \{\alpha\})$ along a given real process, per cycle provided that it lies between the maximum, $E_{\text{max}}(y, \{\alpha\})$ and minimum, $E_{\text{min}}(y, \{\alpha\})$, amount of energy that can be extracted from the energy converter: $E_{\text{min}}(y, \{\alpha\}) \leq E_u(y; \{\alpha\}) \leq E_{\text{max}}(y, \{\alpha\})$. Here $y$ denotes an independent variable while $\{\alpha\}$ denotes a set of parameters which can be considered as controls. For an energy converter Hernandez et.al. defined two important quantities: effective useful energy $E_{u,\text{eff}}(y; \{\alpha\})$ as

$$ E_{u,\text{eff}}(y; \{\alpha\}) = E_u(y; \{\alpha\}) - E_{\text{min}}(y, \{\alpha\}), \quad (3.1.1) $$

and lost useful energy $E_{L,u}(y; \{\alpha\})$ as

$$ E_{L,u}(y; \{\alpha\}) = E_{\text{max}}(y; \{\alpha\}) - E_u(y; \{\alpha\}). \quad (3.1.2) $$

To evaluate the best compromise between useful energy and lost useful energy we introduce the objective function, defined by $\Omega$, function as the difference between these quantities

$$ \Omega(y, \{\alpha\}) = E_{u,\text{eff}}(y; \{\alpha\}) - E_{L,u}(y; \{\alpha\}). \quad (3.1.3) $$

Then the conventional efficiency of energy converter defined as the ratio between the useful energy and input energy:

$$ \eta(y; \{\alpha\}) = \frac{E_u(y; \{\alpha\})}{E_i(y, \{\alpha\})} \quad (3.1.4) $$
which satisfies the relation

$$\eta_{\min}(y; \{\alpha\}) \leq \eta(y; \{\alpha\}) \leq \eta_{\max}(y; \{\alpha\}) \quad (3.1.5)$$

where $\eta_{\min}(y; \{\alpha\})$ and $\eta_{\max}(y; \{\alpha\})$ are the minimum and maximum values of $\eta(y; \{\alpha\})$ respectively. Using Eqs.(3.1.1), (3.1.2), and (3.1.4) in Eq.(3.1.3), the expression for $\Omega$ is

$$\Omega(y, \{\alpha\}) = [2\eta(y; \{\alpha\}) - \eta_{\min}(y; \{\alpha\}) - \eta_{\max}(y; \{\alpha\})]E_i(y, \{\alpha\}). \quad (3.1.6)$$

This equation represents the proposal of Hernandez et.al.\cite{6} as the objective function to analyze the mode of operation of any energy converter giving the best compromise between energy benefits and losses. An important feature of the proposed criterion is that it gives an optimized efficiency that lies between Carnot efficiency and efficiency at maximum power condition.

### 3.2 Optimized efficiency

In this section we find that the optimized efficiency that lies between the maximum efficiency (Carnot efficiency) and the efficiency at maximum power condition by optimizing the objective function with respect free parameter. From Eq.(3.1.6) we have defined the objective function

$$\Omega(y, \{\alpha\}) = [2\eta(y; \{\alpha\}) - \eta_{\min}(y; \{\alpha\}) - \eta_{\max}(y; \{\alpha\})]E_i(y, \{\alpha\}) \quad (3.2.1)$$

In our case $\eta_{\min}$ is the efficiency at maximum power ($\eta_{mp}$) and $\eta_{\max}$ is the Carnot efficiency ($\eta_c$) then Eq. (3.2.1) becomes

$$\Omega(y, \{\alpha\}) = [2\eta - \eta_{mp} - \eta_c]E_i(y, \{\alpha\}) \quad (3.2.2)$$
where $E_i = \frac{W}{\eta}$. This objective function enable us to analyze the operational mode of our energy converter giving the best compromise between energy benefits and losses. After taking the time derivative of Eq.(3.2.2) becomes

$$\dot{\Omega} = \left[ \frac{2\eta - \eta_{mp} - \eta_c}{\eta} \right] \dot{W}$$  \hspace{1cm} (3.2.3)

after substituting Eqs. (2.4.2) and (2.4.4) in (3.2.3) our objective function becomes

$$\dot{\Omega} = -2T\left[ L_{NU} f_U f_N + L_{NN} f_N^2 \right] - (\eta_{mp} + \eta_c)\left[ L_{UU} f_U + L_{UN} f_N \right]$$  \hspace{1cm} (3.2.4)

The graph of the objective function is given below

Figure 3.1: Plot of objective function, $\dot{\Omega}$, versus the free parameter $f_N$ for $\eta_c=0.1$.

Figure 3.1 shows plot of $\dot{\Omega}$ versus $f_N$ in which the objective function has an optimum at a finite value of the free parameter, $f_N$. The efficiency of the device when it operates at this particular points in the parameter space gives the optimized efficiency, $\eta^{opt}$. 

After this we optimized the objective function with respect to the free parameter \( f_N \). The objective function takes its optimized value when

\[
\left. \frac{\partial \hat{\Omega}}{\partial f_N} \right|_{f_N^{\text{opt}}} = 0 \tag{3.2.5}
\]

is satisfied. Using Eqs.(2.4.2), (2.4.7), (3.2.4) and (3.2.5) we have

\[
f_N^{\text{opt}} = -\frac{(12 - 5\kappa^2)}{8(2 - \kappa^2)} \frac{L_{NU}}{L_{NN}} f_U \tag{3.2.6}
\]

Finally by using Eqs.(2.4.2), (2.4.4) and (2.2.6) the corresponding optimized efficiency becomes

\[
\eta^{\text{opt}} = \eta_c \left[ \frac{12 - 5\kappa^2}{8(2 - \kappa^2)} \right] \left[ \frac{\kappa^2 - \left( \frac{12 - 5\kappa^2}{8(2 - \kappa^2)} \right) \kappa^2}{1 - \left( \frac{12 - 5\kappa^2}{8(2 - \kappa^2)} \right) \kappa^2} \right] \tag{3.2.7}
\]

where \( \kappa \) is coupling strength. The optimized efficiency at strong coupling \( (\kappa = \pm 1) \) becomes

\[
\eta^{\text{opt}} = \frac{7}{8} \eta_c \tag{3.2.8}
\]
Fig. 3.2 shows that all the efficiencies increase as $\eta_c$ increase. This is because as we increase $\eta_c$ the temperature difference of the reservoirs increase. From this we can observe that the work extracted at Carnot cycle more than work extracted at optimized efficiency process which is greater than work extracted from Curzon and Ahlborn cycle process. Where as work extracted from the process of efficiency at maximum power is smaller than all of the others process. But the time taken to extract work at one complete cycle of Carnot process is much greater than all other process, and the time taken to extract work at maximum power the fastest of all the others for one complete cycle. This because at Carnot cycle process there is no dissipation of energy but there is an energy dissipation at the process of optimized efficiency and efficiency at maximum power process. We can therefore say that the operation of the device at optimized efficiency is indeed a compromise between energy benefits and losses. Since the graph of $\eta^{opt}$ lies between Carnot efficiency and Curzon and Ahlborn efficiency.
Figure 3.3: Plot of $\eta_c$ (red line), $\eta_{opt}$ (blue line), $\eta_{ca}$ (green line) and $\eta_{mp}$ (black line) versus $\Upsilon = \frac{T_s - T}{T}$.

From fig. 3.3 we see that all the efficiencies increase as $\Upsilon = \frac{T_s - T}{T}$ increase. This is because as we increase $\Upsilon = \frac{T_s - T}{T}$ the temperature difference of the reservoirs increase. From this we can observe that the work extracted at Carnot cycle more than work extracted at optimized efficiency process which is greater than work extracted from Curzon and Ahlborn cycle process. Where as work extracted from the process of efficiency at maximum power is smaller than all of the others process. But the time taken to extract work at one complete cycle of Carnot process is much greater than all other process, and the time taken to extract work at maximum power the fastest of all the others for one complete cycle. This because at Carnot cycle process there is no dissipation of energy but there is an energy dissipation at the process of optimized efficiency and efficiency at maximum power process. We can therefore say that the operation of the device at optimized efficiency is indeed a compromise between energy benefits and losses. Since the graph of $\eta^{opt}$ lies between Carnot efficiency and Curzon and Ahlborn efficiency. Finally we can observe that $\eta^{CA}$ and $\eta^{mp}$ are overlap for linear regime.

Where as the corresponding value of the flux at the optimized objective function of
our system becomes

\[ J_{\text{opt}} = \left[ \frac{4 - 3\kappa^2}{8(2 - \kappa^2)} \right] L_{NU} f_U \]  

(3.2.9)

From Eq.(2.4.11) of section (2.4) the period, \( \tau_{\text{opt}} \), taken to operate for one cycle at optimized efficiency becomes

\[ \tau_{\text{opt}} = \frac{8(2 - \kappa^2)}{4 - 3\kappa^2} \frac{1}{\eta c L_{NU}} T \]  

(3.2.10)

at strong coupling (\( \kappa = \pm 1 \)) the period is

\[ \tau_{\text{opt}} = \frac{8}{\eta c L_{NU}} T \]  

(3.2.11)

Relating Eqs.(2.4.12) and (3.2.11) we have \( \tau_{\text{opt}} = 4\tau_{mp} \) which means the time taken to complete a process at optimized efficiency four times the fastest process (the process at maximum power).

The relative time taken under the process of optimized efficiency and efficiency at maximum power is

\[ \tau_{\text{rel}} = \frac{\tau_{\text{opt}} - \tau_{mp}}{\tau_{mp}} \]  

(3.2.12)

then using Eqs.(2.4.12) and (3.2.11) the relative time taken becomes

\[ \tau_{\text{rel}} = 3 \]  

(3.2.13)

The relative efficiency under the process of optimized efficiency and efficiency at maximum power is
Using Eqs.(2.4.7) and (3.2.8) the relative efficiency becomes

\[ \eta_{\text{rel}} = \frac{\eta^{\text{opt}} - \eta^{\text{mp}}}{\eta^{\text{mp}}} \]  

(3.2.14)

Using Eqs.(2.4.7) and (3.2.8) the relative efficiency becomes

\[ \eta_{\text{rel}} = \frac{3}{4} \]  

(3.2.15)

Figure 3.4: The time taken to complete one full cycle efficiency at maximum power (blue line) and for optimized efficiency (red line) versus Carnot efficiency

From fig. 3.4 we observe that as \( \eta_c \to 0 \) \( (T_s = T) \) both \( \tau^{\text{mp}} \) and \( \tau^{\text{opt}} \) goes to \( \infty \) which implies that there is no flow of heat from the hot reservoir to the the cold reservoir. Due to this one can not extract a certain amount of work with in finite time. When we go from zero to different finite value of \( \eta_c \), \( \tau^{\text{mp}} \) and \( \tau^{\text{opt}} \) become decrease with that of \( \tau^{\text{mp}} \) smaller than \( \tau^{\text{opt}} \). We can see that the time taken at maximum power process is faster than the time taken at optimized efficiency process for each corresponding cycle.
From fig. 3.5 we observe that whatever the value of $\eta_c$ changes, the relative time for each cycle process is constant that means in what way the temperature of the reservoirs changed the relative time taken at the process of optimized efficiency and efficiency at maximum power to extract work in a cycle is constant.
From fig. 3.6 plot of $\eta^{rel}$ versus $\eta_c$ tell us when we go from small value of $\eta_c$ ($T_s = T$) to very large value that is $T_s > T$ then $\eta^{rel}$ does not change everywhere.
In this work, we consider a nano structure photo-electric device with two energy levels in contact with two electron reservoir leads at the same temperature but with different chemical potential whose main task is to convert radiation energy to electric energy. We briefly re-derived the efficiency at maximum power using sun as hot reservoir with temperature $T_s$, power $P$ generated by the solar cell to bring electrons from the left lead to the right lead and the net heat flux coming from the sun. In the case of strong coupling ($\kappa = \pm 1$), where the heat and work producing fluxes are proportional, efficiency at maximum power becomes half of the Carnot efficiency. This result is the same as the work of Curzon and Ahlborn [3]. Furthermore, we calculate the time taken to extract work at maximum power for one complete cycle.

After we analyzed the efficiency at maximum power for the case of strong coupling and the corresponding time taken to complete one cycle, we introduce an objective function based on the proposal of Hernandez et.al.[6]. Using optimization principle we optimized the objective function with respect to the free parameter, $f_N$, and we found the point at which the objective function is maximum. We found the optimized efficiency to be exactly $\frac{7}{8} \eta_c$ which is true for strong coupling near the linear regime ($T_s \sim T$). We also calculated the corresponding optimized time taken to complete one cycle. After this we compared efficiencies: Carnot efficiency, Curzon and Ahlborn, efficiency at maximum power, and
optimized efficiency. Carnot efficiency which is the maximum efficiency but it takes infinite time to extract certain amount of work. Hence the corresponding power is zero (finite work divided by infinite time). So for practical application the Carnot efficiency has a limited significance. Curzon and Ahlborn [3] efficiency at nonzero power in a finite time but there is dissipation (wastage) of high amount of heat. Efficiency at maximum power which is the same as Curzon and Ahlborn efficiency at strong coupling as we observe from the graphs for linear regime. Finally, we have optimized efficiency which lies between Curzon and Ahlborn efficiency and Carnot efficiency which tells us it is the best compromise between useful energy and lost energy. But the time taken to complete one cycle at optimized efficiency four times the fastest time (the time taken to complete one cycle at maximum power).

In general, we believe that our work has, for the first time, found an efficiency which is between the efficiency at maximum power (and also Curzon and Ahlborn [3]) and maximum efficiency (Carnot efficiency) analytically, when the temperature of the sun is nearly the same as that of lead's temperature (in linear regime). We intend to extend our work to the non linear regime to find efficiency at maximum power, the corresponding time taken to complete one cycle and optimized efficiency as well as the time taken to accomplish on complete cycle.
Appendix

In this appendix we will derive the expression of entropy production in irreversible thermodynamics process and also the Onsager reciprocal relation.

Entropy production in irreversible thermodynamics process

Irreversible processes can be described in terms of thermodynamic forces and thermodynamic flows. The thermodynamic flows are a consequence of the thermodynamic forces. In general, the irreversible change $dS_i$ associated with a flow of $dX$ of a quantity such as heat or matter that has occurred in a time $dt$. For the flow of heat, we have $dX = dQ$, the amount of that followed in a time $dt$; for the case of matter, we have $dX = dN$, the number of moles of the substance that followed in a time $dt$. In each case the change in entropy can be written in the form of

$$dS_i = F dX \quad (4.0.1)$$

in which $F$ is the thermodynamic force. Where the force due to temperature gradient is given by

$$F = \frac{1}{T} - \frac{1}{T'} \quad (4.0.2)$$

For the flow of matter, the corresponding thermodynamic force is expressed in terms of
affinity,
\[ F = \frac{\mu}{T} - \frac{\mu'}{T'} \]  

(4.0.3)

All irreversible processes can be described in terms of thermodynamic forces, \( F_k \) and the thermodynamic flows, \( dX_k \). We then have the general expression
\[
 dS_i = \sum_k F_k dX_k \geq 0 \tag{4.0.4}
\]

or
\[
 \frac{dS_i}{dt} = \sum_k F_k \frac{dX_k}{dt} \geq 0 \tag{4.0.5}
\]

Eq.(4.0.5) embodies the second law of thermodynamics. The entropy production due to each irreversible process is a product of the corresponding thermodynamic force \( F_k \) and the flow \( J_k = \frac{dX_k}{dt} \). If we assume that the entire system is divided into two subsystems, we not only have
\[
 dS_i = dS^1_i + dS^2_i \geq 0 \tag{4.0.6}
\]

in which \( dS^1_i \) and \( dS^2_i \) are the entropy productions in each sub systems, but we have also
\[
 dS^1_i \geq 0, dS^2_i \geq 0 \tag{4.0.7}
\]

The total change in entropy, \( dS_i \) of the system is the sum of the changes of entropy in each part. The change in entropy due to the flow of heat:
\[
 dS_i = -\frac{dQ}{T_1} + \frac{dQ}{T_2} = (\frac{1}{T_1} - \frac{1}{T_2})dQ. \tag{4.0.8}
\]

Since the heat flows irreversibly from the hotter part to the colder part, \( dQ \) is positive if
$T_1 > T_2$. Hence $dS_i \geq 0$, in Eq.(4.0.8), $dQ$ and $(\frac{1}{T_2} - \frac{1}{T_1})$ respectively correspond to $dX$ and $F$ in Eq.(4.0.1). In terms of the rate flow of heat $\frac{dQ}{dt}$, the rate of entropy production can be written as

$$\frac{dS_i}{dt} = (\frac{1}{T_2} - \frac{1}{T_1}) \frac{dQ}{dt}. \quad (4.0.9)$$

Now the rate of heat flow or the heat current $J_Q \equiv \frac{dQ}{dt}$ is given by the laws of heat conduction. For example according to the Fourier law of heat conduction, $J_Q = \alpha(T_1 - T_2)$, in which $\alpha$ is the coefficient of heat conductivity. Note that the ”the thermodynamic flow” $J_Q$ is driven by the ”thermodynamic force” $F = (\frac{1}{T_1} - \frac{1}{T_2})$. For the rate of entropy production we have from Eq.(4.0.9) that

$$\frac{dS_i}{dt} = (\frac{1}{T_2} - \frac{1}{T_1})\alpha(T_1 - T_2) = \alpha\frac{(T_1 - T_2)^2}{T_1T_2} \geq 0 \quad (4.0.10)$$

Then the general form of entropy production due to irreversible processes take the quadratic form

$$\frac{dS_i}{dt} = \sum_k F_k \frac{dX_k}{dt} = \sum_k F_k J_k \quad (4.0.11)$$

in which $F_k$ are the ”thermodynamic forces” where we have represented $\frac{dX_k}{dt}$ as the ”flow” or ”current” $J_k$. The thermodynamic forces arise when there is non uniformity of temperature, pressure or chemical potential.
The Onsager Reciprocal relations

Onsager’s theory begins with the assumption that, where linear phenomenological laws valid, a deviation $\alpha_k$, decays according to the linear law

$$J_k = \frac{d\alpha_k}{dt} = \sum_j L_{kj} F_j. \quad (4.0.12)$$

From the thermodynamic theory of equilibrium fluctuation the entropy $\Delta S_i$ associated with fluctuations $\alpha_i$ can be written as

$$\Delta S_i = -\frac{1}{2} \sum_{i,j} g_{ij} \alpha_j \alpha_i = \frac{1}{2} \sum_i F_i \alpha_i \quad (4.0.13)$$

in which

$$F_k = \frac{\partial \Delta S_i}{\partial \alpha_k} = -\sum_j g_{kj} \alpha_j \quad (4.0.14)$$

is the conjugate thermodynamic force for the thermodynamic flow $\frac{d\alpha_k}{dt}$. Which, by the virtue of Eq.(4.0.14), can also be written as

$$J_k = \frac{d\alpha_k}{dt} = -\sum_{j,i} L_{kj} g_{ji} \alpha_i = \sum_i M_{ki} \alpha_i \quad (4.0.15)$$

in which the matrix $M_{ki}$ is the product of the matrices $L_{kj}$ and $g_{ij}$. According to the principle of detailed balance, the effect of $\alpha_i$ on the flow $(\frac{d\alpha_i}{dt})$ is the same as the effect of $\alpha_k$ on the flow $(\frac{d\alpha_k}{dt})$. This can be expressed in terms of the correlation function $\langle \alpha_i \frac{d\alpha_k}{dt} \rangle$ between $\alpha_i$ and $\frac{d\alpha_k}{dt}$ as

$$\langle \alpha_i \frac{d\alpha_k}{dt} \rangle = \langle \alpha_k \frac{d\alpha_i}{dt} \rangle \quad (4.0.16)$$

In a way, this correlation isolates that part of the flow $\frac{d\alpha_k}{dt}$ that depends on the variable $\alpha_i$. Using Eq.(4.0.15) and correlation property we have

$$\langle \alpha_i \frac{d\alpha_k}{dt} \rangle = \sum_j L_{kj} \langle \alpha_i F_j \rangle. \quad (4.0.17)$$
From Gaussian form of the probability distribution of thermodynamic theory of equilibrium fluctuation we have

\[ \langle \alpha_i F_j \rangle = -K_B \delta_{ij} \quad (4.0.18) \]

using Eqs.(4.0.17) and (4.0.18) we have

\[ \langle \alpha_i \frac{d\alpha_k}{dt} \rangle = -K_B \sum_j L_{kj} \delta_{ij} = -K_B L_{ki} \quad (4.0.19) \]

similarly

\[ \langle \alpha_k \frac{d\alpha_i}{dt} \rangle = \sum_j L_{ij} \langle \alpha_i F_j \rangle = -K_B L_{ik} \quad (4.0.20) \]

Finally using Eqs.(4.0.19) and (4.0.20)

\[ L_{ki} = L_{ik} \quad (4.0.21) \]

Eq.(4.0.21) Onsager reciprocal relation
Bibliography


Declaration
This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

Name: Simachew Endale

Signature:

Place and time of submission: Addis Ababa University, June 2010

This thesis has been submitted for examination with my approval as University advisor.
Name: Dr. Mulugeta Bekele

Signature: