A Numerical Model Simulation of Atmospheric Air Motion in Shallow Water Domain: performances with respect to global model

By

Shore Salle

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DEPARTMENT OF
PHYSICS

Supervisor: ______________________
Dr. Gizaw Mengistu

Examiners: ______________________
Dr. Mulugeta Bekele

____
Prof. A. V. Gholap

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Author: **Shore Salle**

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Signature of Author

Dedicated to

those who lose their life in natural disaster.

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Abstract

An atmosphere is a gaseous envelope around a planet where a wide variety of physical, chemical and dynamical processes takes place. As a result atmospheric researchers can not do controlled experiment in the large scale atmosphere rather they look at an alternative method so called atmospheric modelling to understand the atmosphere. Among different kinds of modelling, in this thesis we apply Numerical modelling, where the processes taking place in the earth's atmosphere are expressed in terms of mathematical equation (conservative principles), and experiments are done up on solving those mathematical equation. The validity of the model out put is checked by comparing it with the observed value or large global model which includes complete physics of the problem, by National Center for Environmental Prediction (NCEP) for this study. In this study, shallow water equation is used to simulate atmospheric air motion over mid latitude and Ethiopia on May 26, 2009. The result of the model out put for zonal velocity shows good agreement over mid latitude and Ethiopia. The discrepancy between our model and NCEP is ± 2.22273 m/s over mid latitude and ± 2.75427 m/s over Ethiopia on average. The result of model out put for absolute vorticity also agrees with NCEP data over mid latitude and Ethiopia. The discrepancy between our model and NCEP is ± 0.00001 1/s over mid latitude and ± 0.00004 1/s over Ethiopia. How ever, the discrepancy grows with prediction time. In general, this study shows that shallow water equation is good enough to capture many behaviors observed in earth's atmosphere, and hence it can be used over any other region of the earths atmosphere to describe the horizontal atmospheric structure.

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Introduction

The atmosphere is a fluid in which a wide variety of flows, physical and chemical processes take place. The gaseous envelope surrounding earth's atmosphere, is dominated by molecular nitrogen and molecular oxygen, which occupies approximately 99% of atmosphere (by mass as well as volume). Even though other gases have small amount in mass and volume, the minor constituents like carbon dioxide, ozone, water vapor play crucial roles in the atmosphere.

The atmosphere is continually bombarded by solar photons at infrared, visible and ultra-violet wavelengths. Some solar photons are scattered back to space by atmospheric gases or reflected back to space by clouds or earths surface. Some are absorbed by atmospheric molecules (especially water vapor and ozone) or clouds leading to heating of parts of the atmosphere; and some reaches earth's surface and heat it. Atmospheric gases (especially carbon dioxide, water vapor and ozone), clouds and earths surface also emit and absorb infrared photons, leading to further heat transfer between one region and another, or loss of heat to space. In general, the process taking place in the atmosphere are too difficult to understand it easily.

In general, each portion of the atmosphere is approximately in hydrostatic equilibrium, that is, its weight is supported by a difference in pressure between its lower and upper surfaces. An alternative statement of this physical fact is that there is a balance between vertical pressure gradients and gravitational force per unit volume acting on each portion of the atmosphere. Unlike the upper atmosphere, when considering dynamical processes in lower atmosphere that is, the response of atmospheric motion to applied forces we can average other physical quantities such as density and velocity over many molecules and regard the atmosphere as continuous fluid.

Among types of motions observed in the atmosphere, motion along straight line and oscillatory (wave) motion are common. Wave motions are oscillatory motion in field variables like pressure, velocity, and so on. These waves may propagate, allowing one part of the atmosphere to communicate over great distances with other parts, without corresponding transport of mass. This behavior of atmospheric air motion can be easily visualized through linearized atmospheric dynamic equation, by employing perturbation technique to the full dynamic equation [1].

Unlike laboratory physicist, atmospheric researchers can not perform controlled experiments on the large scale atmosphere. Therefore the standard 'scientific method' of observing phenomena formulating hypothesis, testing them by experiment, then formulating revised hypothesis, and so on cannot be applied directly in the atmosphere. Instead after an atmospheric phenomenon is discovered, perhaps by sifting through a great deal of data, we develop models, which incorporate a selection of processes that we hypothesize are significant for the phenomenon. Models act as surrogate atmospheres, on which 'thought experiment' can be performed. These models are usually formulated in terms of mathematical equations and the experiments are performed by solving the model equations (perhaps by computer, and the process of doing so is known as numerical modelling [2,3]) under various conditions and interpreting the solution in terms of physical behavior. The performance of the model is judged by comparing the model's behavior with that of the atmosphere.

Apart from complex chemical and physical processes taking place in the atmosphere, the general purpose of this study is to see how air motion behaves in the atmosphere using shallow water equation modelling, a simplified model which describes horizontal structure of the atmosphere in which air has almost uniform vertical motion and based on assumption of motions in shallow fluid layer, over mid latitude and Ethiopia. In particular, we want to study whether the simplified model can capture observation and the extent of deviation from observation.

Given initial and boundary condition of the model domain, specifically, this study aims in answering the questions:

'How much does our model result approximate the observed behavior of the atmospheric wind field?

'What is the difference between model out put over the region of mid latitude and over that of Ethiopia?'

'How reasonably vorticity from our model compares with that of NCEP?'

In the first chapter, the model equations that are responsible for describing the atmospheric motions are discussed. These equations are governed by three fundamental physical principles: conservation of mass, conservation of momentum, and conservation of energy, and are expressed in terms of partial differential equations involving the field variables as dependent variables and space and time as independent variables. Shallow water equation is then approximated from these principles at the end of the chapter.

The general set of partial differential equations governing the motions of the atmosphere is extremely complex, non linear; no general analytic solutions are known to exist for solving it till now. So the current method, that we have for solving such an equation is through numerical methods. Among numerical methods, finite difference technique is used to solve our model equation. The convergence criteria of the finite difference equation, after discretization of shallow water equation is then discussed in chapter two along with showing the region of model simulation.

In chapter three, the results and discussion of the model out put for zonal velociy, vorticity and stream functions for the two model regions are compared and discussed in terms of the current understanding of the atmosphere.

Finally, in chapter four, conclusions of the present study along with future direction for further study are given.

Chapter 1

DYNAMICAL EQUATIONS AND CONSERVATIONS LAWS

1.1 Introduction

Equations describing our atmospheric motion generally falls into three categories. These basic equations describe fundamental physical principle taking place in the atmosphere called conservation of mass, conservation of momentum and conservation of energy.

The forces that causes air motion are basically: pressure gradient force, the force of gravity and the force of friction(viscosity) [4]. With these forces applied to a parcel of moving air (some times called control volume) in rotating coordinate system (here earth), the basic dynamical equation that can describe conservation laws for the atmospheric air motions are derived. In this thesis we will use the final expression of these equation, the derivation can be found in any standard atmospheric books (e.g. Holton, 2004)

1.2 Momentum Equation

Applying Newton's second law of motion for a parcel of air in rotating coordinate system, an equation that describes conservation of momentum in the atmosphere, that is momentum equation can derived. In vector form, assuming the only real forces acting in the atmosphere are, the pressure gradient force, gravitation and friction, the equation takes the form

$$\frac{D\vec{U}}{Dt} = -2\Omega \times \vec{U} - \frac{1}{\rho}\nabla P + g + F_r \tag{1.2.1}$$

where

$$\frac{D}{Dt}(\ ) \equiv \frac{\partial}{\partial t}(\ ) + (\overrightarrow{U} \cdot \nabla)(\ ),$$

$\overrightarrow{U}$ is velocity, the first three right hand side terms are Coriolis force, pressure gradient force, effective gravity force (the sum of gravitational force and centrifugal force), and the last term is frictional force. Here we have two fictitious forces (coriolis and centrifugal forces) that arises due to expressing Newton's law in rotating coordinate system, in addition to the three basic forces.

In order to model an atmospheric motion, it is necessary if we could express the vector equations into their scalar equivalent. Neglecting the departure of the earth from the sphere, the best coordinate system, in describing the air motion in the earth's atmosphere in large scale is spherical coordinate system [5]. Using spherical coordinate system the above momentum vector equation can be written as

$$\frac{DU}{Dt} - \frac{UV\tan(\phi)}{R} + \frac{UW}{R} = \frac{-1}{\rho}\frac{\partial P}{\partial x} + 2\Omega V\sin(\phi) - 2\Omega W\cos(\phi) + F_{rx} \quad (1.2.2)$$

$$\frac{DV}{Dt} - \frac{U^2\tan(\phi)}{R} + \frac{VW}{R} = \frac{-1}{\rho}\frac{\partial P}{\partial \rho} - 2\Omega U\sin(\phi) + F_{ry} \quad (1.2.3)$$

$$\frac{DW}{Dt} - \frac{U^2 + V^2}{R} = \frac{-1}{\rho}\frac{\partial P}{\partial z} - g + 2\Omega U\cos(\phi) + F_{rz} \quad (1.2.4)$$

where

$$\vec{U} = (U, V, W)$$

is the wind velocity vector and ϕ is latitude.

1.3 Continuity Equation

The dynamical equation that describes the conservation of mass is the continuity equation. It is written as,

$$\frac{D\rho}{Dt} + \rho(\nabla \cdot \overrightarrow{U}) = 0 \quad (1.3.1)$$

in vector notation.

1.4 Thermodynamic Energy Equation

Thermodynamic energy equation describes energy conservation in the atmosphere. It states that in thermodynamic equilibrium, the change in internal energy of the system is equal to the difference between heat added to the system and the work done by the system. Considering once again a parcel of air (control volume in the fluid) of unit mass, under going reversible changes dS of entropy, dU of internal energy, and dV of volume in time dt, the thermodynamic equation can be written as

$$TdS = dU + PdV$$

For an ideal gas this takes the form

$$TdS = C_p dT - \frac{1}{\rho} dP$$

Dividing this equation by dt and letting $dt \rightarrow 0$ gives

$$\dot{Q} = T\frac{DS}{Dt} = C_p\frac{DT}{Dt} - \frac{1}{\rho}\frac{DP}{Dt} \tag{1.4.1}$$

where $\dot{Q}$ is the diabetic heating rate per unit mass. This is one of the form of first law of thermodynamics, as used in atmospheric physics.

1.5 Shallow Water Equation

Motions in the atmosphere occur on different space and time scales. From molecular vibrations to very large planetary scale flow, there are a vast range of motions and each type of motion has its own characteristic scale of length, velocity and time. Although the equations of motion described above are quite general, it may be possible for one particular type of motion to simplify them by eliminating terms which may be irrelevant or vanishingly small, and retaining those which more closely represent the motion being considered. This process of simplification and approximation is called scale analysis.

An alternative way of looking for simplified equation of motion is obtained from the fully dynamical equations rather than determining the magnitude of terms in the equations of motions and ignoring smaller one as in scale analysis [6]. Shallow water equation are simplified and /or, a stripped down version of the full model equations that can describe the horizontal structure of an atmosphere on large horizontal scale and in the case where the air has almost uniform vertical motion. Even though it neglects some effect of the atmosphere, it gives a better understanding of atmosphere with out dealing with the whole dynamical equations (conservation laws) stated above.

1.5.1 Derivation of Shallow Water Equation from Basic Principles

Since frictional force does not play an important role unless we are dealing with narrow boundary layer close to the earths surface, we can neglect its effect. Up on neglecting it, the vector momentum equation reduces to

$$\rho\left[\frac{D\vec{U}}{Dt}+f(\hat{k}\times\vec{V})\right]=-\nabla P-\rho g\hat{k} \tag{1.5.1}$$

where $\vec{V}=(U,V,0)$, $\vec{U}=(U,V,W)$ in spherical coordinate.
The continuity equation is

$$\frac{D\rho}{Dt}+\rho(\nabla\cdot\overrightarrow{U}) \;=\; 0 \tag{1.5.2}$$

We need to show how this momentum and continuity equation reduces to shallow water equations.

It is convenient to use a fact that any field variables can be expressed as the sum of its basic part (a function of altitude only, independent of time and other space coordinates) and deviation part (a function of three space coordinates and time). That is

$$P = P_o(z) + P'(x, y, z, t) \tag{1.5.3}$$

$$\rho = \rho_o(z) + \rho'(x, y, z, t) \tag{1.5.4}$$

Using the fact that the real atmosphere is approximately in hydrostatic balance, the basic part satisfies the hydrostatic equation given by

$$\frac{dP_o}{dz} = -g\rho_o \tag{1.5.5}$$

substituting Eqn.(1.5.3) and Eqn.(1.5.5) in momentum Eqn.(1.5.1) above, we get

$$\left[\frac{D\vec{U}}{Dt} + f(\hat{k} \times \vec{V})\right] = -\nabla P' - \rho' g\hat{k}$$

The first assumption that shallow water equation uses is **incompressibility** of the fluid, that means the density of the fluid is assumed constant through out the fluid and hence the perturbation density is zero. Since water is nearly incompressible, the word 'water' in shallow water equation comes partly from this incompressibility assumption. Using this fact the momentum equation reduces to

$$\rho\left(\frac{D\vec{U}}{Dt} + 2\Omega \times \vec{V}\right) = -\nabla P'$$

Also continuity equation takes the form $\nabla \cdot \overrightarrow{U} = 0$.

The second assumption that shallow water equation uses is that, there is negligible effect of vertical shear of horizontal velocity in the fluid (**no vertical shear** of horizontal velocity). This assumption is in fact reasonable if we assume that thickness of the layer of the fluid is shallow, that is the vertical scale of the the fluid motion we are considering is too small as compared to that of its horizontal motion scale. Thus, the word 'shallow' in shallow water equation comes partly from this assumption.

To express the pressure gradient force that is exerted on the fluid assumed above (shallow water), consider that there exist a free surface of constant pressure or nearly constant pressure. And let the height of this free surface be h_f, that of surface topography be h_s and that of the depth be h^*. With free surface assumption(no fluid cross this surface), the height of the parcel embedded with this free surface satisfies

$$\frac{Dh_f}{Dt} = W(x, y, h_f, t) \tag{1.5.6}$$

and similarly, since no fluid crosses the lower boundary the fluid motion over there must follow the boundary i.e.

$$\frac{Dh_s}{Dt} = W_s \tag{1.5.7}$$

Using incompressibility assumption and integrating hydrostatic balance equation from some arbitrary depth with in the fluid up to free height, we obtain an expression of horizontal pressure gradient force as

$$\nabla P' = g\rho_o \nabla h_f$$

Hence the momentum equation for shallow water up on using this pressure gradient force reduces to

$$\frac{D\vec{V}}{Dt} + f(\hat{k} \times \vec{V}) = -g\nabla h_f \tag{1.5.8}$$

From no shear assumption one can see that the velocity is independent of height, so integrating the continuity equation from height h_s *to* h_f, we get

$$W(x, y, h_f, t) - W_s(x, y, h_s, t) = -(h_f - h_s)\nabla \cdot \vec{V} \tag{1.5.9}$$

substituting Eqn.(1.5.6) and Eqn.(1.5.7) into Eqn.(1.5.9), we obtain

$$\frac{\partial h*}{\partial t} + \nabla \cdot (\nabla h*) = 0 \qquad (1.5.10)$$

This is the continuity equation for shallow water equation

Therefore, in this thesis, momentum and continuity equations, Eqns.(1.5.9)and (1.5.10) respectively, will be used as our model equations so as to simulate atmospheric air motion and understand the behavior of the earth's atmosphere.

1.6 Vorticity and Stream function

Vorticity is a vector field that gives microscopic measure of rotation at any point in the fluid, and is defined as curl of a velocity. In large scale atmospheric motion, the most important component of vorticity in use is, the vertical one. The vertical component of relative vorticity is given, in vector form by

$$\zeta = \hat{k} \cdot (\nabla \times \vec{U})$$

While absolute vorticity is given by the sum of relative vorticity and coriolis parameter f, given by

$f = 2\Omega \sin \phi$ where ϕ is Latitude.

The quantity derived from wind, stream function, has introduced (especially in two dimensional flow) a great simplicity in expression of atmospheric governing equations and constitutes the basic predictor for many numerical weather prediction models [7,8]. Not only this but also this quantity has vivid physical meaning. Lines of constant stream function are stream lines of the flow, that is they are every where parallel to the local velocity vector (no flow can exist normal to stream line). In a steady flow, stream line represent the trajectories of particles. Mathematically stream function (ψ) can be found from an expression

$$\zeta = \nabla^2 \psi$$

We will use expressions of shallow water equations, absolute vorticity and stream function to study to what extent the simplified model mimics the real atmosphere.

Chapter 2

NUMERICAL SOLUTION OF SHALLOW WATER EQUATION AND THE THE STUDY AREA

2.1 Introduction

Many mathematical models of physical, chemical, biological, engineering phenomena, and many other process can be expressed in terms of partial differential equations, which involve rate of change of variables in terms of two or more independent variables (usually time and length or angle). The partial differential equation that result in describing such phenomena may be one or a number of such equations [9]. The most frequently encountered form of partial differential equation is second order partial differential equation in two dimension of the form

$$a\frac{\partial^2\Phi}{\partial x^2} + b\frac{\partial^2\Phi}{\partial y^2} + c\frac{\partial^2\Phi}{\partial z^2} + d\frac{\partial\Phi}{\partial x} + e\frac{\partial\Phi}{\partial y} + f\Phi + g = 0 \qquad (2.1.1)$$

where coefficients a, b, c, d, e, f and g can be functions of dependent variable Φ, and/or of independent variables x, y.

In general partial differential equations describing the above process and others fall into two from categories of problem, they are: Initial value problems and Boundary value problems [10]. Initial value problems are problems that involves time as one of their independent variables, where as Boundary value problems are problems that does not involves time as one of their independent variables. The partial differential equation that we encounter in our atmospheric phenomena are of Initial problem type.

As can be seen from the dynamical equations, and its simplified form shallow water equation in chapter one, all the partial differential equations describing the rate of change of atmospheric field variables are non linear. Such an equation has no easy analytic solution yet. In such cases approximation methods, whether analytic or numerical are needed to solve these partial differential equations. Analytical approximation methods often provides extremely useful information concerning the character of the solutions for critical values of the dependent variables but tend to be more difficult to apply than numerical methods [9]. Methods of solving partial differential equations by numerical method, that we have right now falls into three general categories. These are finite-difference, series expansion, and finite volume methods. With a finite difference method, each differential in a continuous partial differential equation is replaced with a difference analog written in terms of a finite number of values along a temporal or spatial direction. In a series expansion method, a dependent variable (e.g, u, v, w) in a partial differential equation is replaced with a finite series that approximates its value. Two series expansion methods are finite element and pseudo spectral methods. Finite volume methods are methods of solving partial differential equations that divide space into discrete volumes [11]. Series expansion methods and finite volume methods are generally used for the irregular grids. For the regular grid domain, finite difference method is the easiest and in fact the most widely used form of methods for solving the partial differential equation. In this thesis we are going to use this method for solving our model equation [8,12].

2.2 Finite Difference Method

Finite difference methods are an approximation methods in the sense that derivatives at a point are approximated by difference quotients over small interval. In this method the continuous area of integration of the partial differential equation is divided into a number of rectangular meshes (grids or nods or lattice points or pivotal points) formed by intersection of lines. An approximate solution to the differential equation is found at this points, using difference quotients approximating the derivatives, which leads to a number of algebraic equations (called difference equation). In general this difference quotients are obtained from Taylor series expansion, by truncating higher order terms from the expansion. The accuracy of the solutions are usually improved by increasing the number of mesh points and or by retaining more terms in Taylor series expansion [9,13]. Based on the distribution of field variables in the mesh points, the grids can be classified as staggered and non staggered. In the non staggered grids all the field variables are assumed to be found at the same grid points, where as in the staggered case the field variables are either partly or totally distributed among different mesh points rather than being all at the same mesh points. Arakawa categorized grids as Arakawan A, B, C, D and E where only grid A is non staggered [14]. Because of its relative simplicity, we employ this non staggered grid type to solve the shallow water equation in this study.

Finite differencing technique for time derivatives of field variables can be grouped into two, that is explicit technique and implicit technique. In an explicit time stepping scheme, final terms are calculated explicitly from known values. In an implicit time stepping scheme, final terms say at time t are evaluated from other terms at time t, which are initially unknown but which are solved simultaneously with the desired terms to get its value. Currently a new scheme called semi implicit scheme, which is in between these two schemes, are in use for global atmospheric circulation modelling [15]. In this scheme, final terms are evaluated from some terms that are known and other terms that are to be evaluated at unknown time. In this thesis we employ explicit time stepping scheme for shallow water equation, which is relatively simple and in fact applicable as long as short time integration is needed.

Numerical solution by finite difference method does not always gives good results, because of for instance due to truncating terms from Taylor series expansion (usually called 'truncation error'). In order to have a reasonable good result it should full fill the criteria called 'convergence'. The finite difference equation is said to be convergent if and only if the difference between the exact solution of partial differential equation and that of its difference equation at a fixed point or time (usually called descritization error) goes to zero as both the time step and grid step tends to zero. If this is satisfied then, the solution will be a good approximation of the solution of the given partial differential equation. But, directly checking this convergence is difficult because it requires another unknown value. An alternative way of doing so, for the linear Initial value problem is that, through checking its consistency and stability (given by Lax equivalence theorem which states that if a linear finite difference equation is consistent with a properly posed linear initial value problem, then stability is the necessary and sufficient condition for convergence) [9].

2.2.1 Stability and consistency of the finite difference method

If we have an information about the boundedness of the variable to be found from the partial differential equation, then stability of finite difference equations means that the corresponding exact solution of the finite difference equation should be also bounded as time progress. We have a number of methods for checking stability of finite difference equations. Among others, stability analysis by Fourier series method (Von Niemann's method) is the easiest one and widely used technique for analyzing stability. It uses the fact that the general independent solutions of the linear partial differential equation can be expressed as

$$U^q_{l,m,n} \quad = \quad \xi^q e^{i(K_x l\triangle x + K_y m\triangle y + K_z n\triangle z)} \tag{2.2.1}$$

where k is real spacial wave number (which can have any value) , and $\xi = \xi(k)$ is a complex number that depends on k and is called an amplification factor. The finite difference equation is said to be stable if the above value of U is substituted in the difference equation, and we get (in fact after a little algebra) the modulus of the amplification factor is less than or equal to one. Stability, in general further can be categorized as conditionally stable (for explicit and semi implicit finite difference scheme), in which the time step is restricted by Courant-Friedreich-Lewy(CFL) criteria, and unconditionally stable (for implicit finite difference scheme) which is always stable scheme, irrespective of time step.

If local truncation error at a mesh point tends to zero as the mesh grid lengths tends to zero, then the difference equation is said to be consistent.

2.3 Descretization of the Model Equation

In this section we need to descritisize the model equations: shallow water equations, vorticity, and stream function using finite difference technique.

2.3.1 Descritization of shallow water equation

The shallow water equations are given by in spherical coordinate system,

$$\frac{\partial U}{\partial t} = \frac{-U}{R\cos\phi}\frac{\partial U}{\partial \lambda} - \frac{V}{R}\frac{\partial U}{\partial \phi} + fV - \frac{g}{R\cos\phi}\frac{\partial h_f}{\partial \lambda} \tag{2.3.1}$$

$$\frac{\partial V}{\partial t} = \frac{-1}{R\cos\phi}[U\frac{\partial V}{\partial \lambda} + V\cos\phi\frac{\partial V}{\partial \phi}] - fU - \frac{g}{R}\frac{\partial h_f}{\partial \lambda} \tag{2.3.2}$$

$$\frac{\partial h^*}{\partial t} = \frac{-1}{R\cos\phi}\left[\frac{\partial(h^*U)}{\partial \lambda} + \frac{\partial(h^*V\cos\phi)}{\partial \phi}\right] \tag{2.3.3}$$

Descritisizing zonal component of wind speed (Eqn (2.3.1)) using leapfrog scheme in time and centered in space, that is using centered in time as well as centered in space (CTCS) method gives

$$\begin{aligned}\frac{U_{i,j}^{n+1} - U_{i,j}^{n-1}}{2\triangle t} &= -\frac{1}{(R\cos\Phi_j 2dl)}[U_{i,j}^n(U_{i+1,j}^n - U_{i-1,j}^n) + V_{i,j}^n\cos\Phi_j(U_{i,j+1}^n - U_{i,j-1}^n)] \\ &+ f_jV_{i,j}^n - \frac{g}{(R\cos\Phi_j 2dl)}(h_{i+1,j}^n - h_{i-1,j}^n)\end{aligned} \tag{2.3.4}$$

where dl is south-north Latitude increment (and also equals west-east Longitude increment, for this study). Up on rearranging terms, Eqn.(2.3.4) reduces to

$$\begin{aligned}U_{i,j}^{n+1} &= -2\triangle t\{\frac{1}{(R\cos\Phi_j 2dl)}[U_{i,j}^n(U_{i+1,j}^n - U_{i-1,j}^n) + V_{i,j}^n\cos\Phi_j(U_{i,j+1}^n - U_{i,j-1}^n)] \\ &- f_jV_{i,j}^n + \frac{g}{(R\cos\Phi_j 2dl)}(h_{i+1,j}^n - h_{i-1,j}^n)\} + U_{i,j}^{n-1}\end{aligned} \tag{2.3.5}$$

Descritisizing the meridional component of the wind speed, Eqn (2.3.2), as we did for zonal component by CTCS method, we obtain

$$\frac{V_{i,j}^{n+1} - V_{i,j}^{n-1}}{2\triangle t} = -\frac{1}{(R\cos\Phi_j 2dl)}[U_{i,j}^n(V_{i+1,j}^n - V_{i-1,j}^n) + V_{i,j}^n \cos\Phi_j(V_{i,j+1}^n - V_{i,j-1}^n)]$$
$$- \quad f_j U_{i,j}^n - \frac{g}{R}\frac{(h_{i,j+1}^n - h_{i,j-1}^n)}{2dl} \qquad (2.3.6)$$

which, up on rearranging terms, becomes

$$V_{i,j}^{n+1} = -2\triangle t\{\frac{1}{(R\cos\Phi_j 2dl)}[U_{i,j}^n(V_{i+1,j}^n - V_{i-1,j}^n) + V_{i,j}^n \cos\Phi_j(V_{i,j+1}^n - V_{i,j-1}^n)]$$
$$+ \quad f_j U_{i,j}^n + \frac{g}{R}\frac{(h_{i,j+1}^n - h_{i,j-1}^n)}{2dl}\} + V_{i,j}^{n-1} \qquad (2.3.7)$$

Similarly descritisizing the continuity equation (Eqn(2.3.3)) in similar manner using CTCS method, we get

$$\frac{h^{*n+1} - h^{*n-1}}{2\triangle t} = -\frac{1}{(R\cos\Phi_j 2dl)}[h_{i+1,j}^{*}U_{i+1,j}^n - h_{i-1,j}^{*n}U_{i-1,j}^n$$
$$+ \quad h_{i,j+1}^{*n}V_{i,j+1}^n \cos\Phi_{j+1} - h_{i,j-1}^{*n}V_{i,j-1}^n \cos\Phi_{i,j+1}] \qquad (2.3.8)$$

Up on setting $h^* = h - hs$ and using the fact that h_s is independent of time on the earths surface (because, in this thesis we are going to simulate atmospheric air motion of shallow layer in which the lower boundary is earth's surface and upper boundary of 500 mb level), we get

$$h_{i,j}^{n+1} = \frac{-2\triangle t}{R\cos\Phi_j 2dl}\{(h_{i+1,j}^n - hs_{i+1,j})(U_{i+1,j}^n) - (h_{i-1,j}^n - hs_{i-1,j})(U_{i-1,j}^n)$$
$$+ \quad (h_{i,j+1}^n - hs_{i,j+1})V_{i,j+1}^n \cos\Phi_{j+1} - V_{i,j-1}^n) - (h_{i,j-1}^n - hs_{i,j-1})(V_{i,j-1}^n \cos\Phi_{j-1}\} + h_{i,j}^{n-1}$$
$$(2.3.9)$$

The time step restriction for this finite differenced shallow water equation, up on applying Von Niemann method becomes [16,17]

$$\triangle t \leq \frac{1}{\sqrt{c^2(\frac{\sin^2 K_x\triangle x}{(\triangle x)^2} + \frac{\sin^2 K_y\triangle y}{(\triangle y)^2}) + f^2}} \qquad (2.3.10)$$

where c is the fastest phase speed in the domain and f is the coriolis parameter. This time step restriction (CFL condition) tells us that an information must not travel more than one grid length in a time step [18].

Because of using centered time step (leap frog scheme) we will have computational mode solution that oscillates in time during the integration. This computational mode should be suppressed during integration time if we need to have a reasonable solution. Among methods of suppressing this mode solution, in this thesis we started integrating with one forward time step and then compute 49 centered difference time steps. After this 50 time steps we break the process and repeat it again as many time steps that are needed [19,20].

This shallow water difference equation is found to be consistent with the corresponding continuous partial differential equation (shallow water equation).

An initial data from National center for Environmental prediction (NCEP) for field variables u, v, and h are used. Since our problem is sensitive to boundary condition, we use boundary value, at the time and place we need, from the global grid model, here from NCEP. This is a kind of boundary conditions used by a current nested limited area grid models [21,22].

Therefore by Lax-Equivalence theorem, our finite difference equation is convergent to the continuous shallow water equation and hence, our result is at a reasonable approximation to the solution of shallow water equation.

2.3.2 Descritisization of Relative Vorticity

The vertical component of relative vorticity expression is in spherical coordinate takes the form

$$\xi = \frac{1}{R\cos(\phi)}\frac{\partial V}{\partial \lambda} - \frac{1}{R\cos(\phi)}\frac{\partial (U\cos\phi)}{\partial \phi} \tag{2.3.11}$$

Descritisizing this by centered in space method, we obtain

(2.3.12)

$$\zeta_{i,j}^{n} = \frac{1}{(R\cos\Phi_j 2dl)}\{V_{i+1,j}^{n} - V_{i-1,j}^{n} - U_{i,j+1}^{n}\cos\Phi_{j+1} + U_{i,j-1}^{n}\cos\Phi_{j-1}\} \tag{2.3.13}$$

2.3.3 Descritisization of Stream Function

An expression for stream function is obtained from its relation ship with vorticity in spherical coordinate from

$$\zeta = \frac{1}{R^2\cos^2\phi}\left(\frac{\partial^2\psi}{\partial\lambda^2}\right) + \frac{1}{R^2\cos\phi}\frac{\partial(\cos\phi\frac{\partial\psi}{\partial\phi})}{\partial\phi} \tag{2.3.14}$$

Descritisizing this with centered in space method we get,

$$\begin{aligned}\zeta_{i,j}^{n} &= \frac{1}{R^2\cos\Phi_j}\{\frac{1}{\cos\Phi_j 2dl}(\Psi_{i+1,j}^{n} + \Psi_{i-1,j}^{n} - 2\Psi_{i,j}^{n}) + \frac{\cos\Phi_j}{dl^2}(\Psi_{i,j+1}^{n} + \Psi_{i,j-1}^{n} - 2\Psi_{i,j}^{n}) \\ &- \frac{\sin\Phi_j}{dl}(\Psi_{i,j+1}^{n} - \Psi_{i,j}^{n})\}\end{aligned} \tag{2.3.15}$$

Multiplying both sides of the equation by $R^2\cos\Phi_j$, we get

$$\zeta_{i,j}^{n}R^2\cos\Phi_j = \Psi_{i,j}^{n}\left[\frac{-2}{\cos\Phi_j dl^2} - \frac{2\cos\Phi_j}{dl^2} + \frac{\sin\Phi_j}{dl}\right] + \left(\frac{1}{\cos\Phi_j}\right) \tag{2.3.16}$$

$$\left(\frac{\Psi_{i+1,j}^{n} + \Psi_{i-1,j}^{n}}{dl^2}\right) + \cos\Phi_j\left(\frac{\Psi_{i,j+1}^{n} + \Psi_{i,j-1}^{n}}{dl^2}\right) - \frac{\sin\Phi_j\Psi_{i,j+1}^{n}}{dl}$$

(2.3.17)

Letting

$$x = \left(\frac{-2}{\cos\Phi_j dl^2} - \frac{2\cos\Phi_j}{dl^2} + \frac{\sin\Phi_j}{dl} \right) \qquad (2.3.18)$$

we get an expression for stream function up on rearranging terms

$$\Psi_{i,j}^n = \frac{\{\zeta_{i,j}^n R^2 \cos\Phi_j - \frac{1}{\cos\Phi_j}(\frac{\Psi_{i+1,j}^n + \Psi_{i-1,j}^n}{dl^2}) - (\cos\Phi_j/dl^2)(\Psi_{i,j+1}^n + \Psi_{i,j-1}^n) + \frac{\sin\Phi_j \Psi_{i,j+1}^n}{dl}\}}{x}$$

It has been shown in previous studies that boundary condition represent a major problem for the solution of vorticity and stream function [7]. To avoid this problem in this thesis, in finding stream function since there is no six hourly data for stream function in NCEP, we used the assumption that the out ward normal velocity at the boundary can be corrected to yield a net zero out ward mass flux. Letting V_{nc} be the out ward corrected normal velocity and V_n as out ward normal velocity, our assumption is expressed mathematically as

$$\oint (V_{nc}) ds = 0, \qquad (2.3.19)$$

where $V_{nc} = V_n + \varepsilon |V|_n$ and ε is the correction factor to be found.

2.4 Area of the Study

Even though shallow water equation is applicable any where in the globe on large horizontal scale, we focus in this study a region around mid latitude, and that of Ethiopia. For mid latitude we focus in a region bounded by latitude: 30^oN-60^oN, in steps of 2.5^o and longitude: 2.5^oE-30^oE, in steps of 2.5^o and for Ethiopia, the study area is bounded by, latitude: 2.5^oN to 20^oN in steps of 2.5^o and longitude: 30^oE to 50^oE in steps of 2.5^o.

The results obtained through these two regions are compared and discussed with NCEP reanalysis data in the next chapter.

Chapter 3

RESULTS AND DISCUSSION

3.1 Introduction

In this chapter the results obtained from our model equation as applied to two completely different regions of the world; mid latitude (a region where bounded by 2.5^oE-30^oE and 30^oN-60^oN) and Ethiopia (a region bounded by 30^oE-50^oE and 2.5^oN-20^oN) are discussed based on the current understanding of atmosphere. The results obtained through the model equation are also compared to that of the observed global data, by National Center for Environmental Prediction (NCEP), to validate how much the model result approaches to that of the globally observed value.

The initial data for integrating the model equation is taken from NCEP at 18 UTC on May 25, 2009. Even though our model equation is applicable over any height as far as hydrostatic approximation is valid, we prefer here to discuss at 500 mb level (a region around mid troposphere) for zonal velocity, absolute vorticity and stream function.

Among kinds of motions in atmosphere, wave motions are common. Waves like inertia gravity waves are among large scale waves observed in the atmosphere, the propagation of which is affected by not only gravity but also by the earth's rotation. In the case of such waves not all can propagate in vertical direction. The criterion for such vertical propagation is that the frequency of the waves must be greater than that of the coriolis frequency [4]. In mid latitude where the coriolis frequency is large, many long period waves (hence small frequency) fail to meet this criteria and so the waves are then said trapped by the atmosphere, showing approximate horizontal motion of the atmosphere over mid latitude. But in the equator the coriolis frequency is less and so that large horizontal scale waves are untrapped and moves vertically.

3.2 Atmospheric air motion over mid Latitude at 500 mb level on May 26, 2009

In this section the out put of the model result for zonal wind velocity over mid latitude is compared to that of the data from National Center for Environmental prediction. Not only this but also absolute vorticity and stream function are calculated and discussed.

Most of the general circulation of the atmospheric character, particularly the circulation around mid latitude are described by equations of dry air that we have described in chapter two in detail. By dry air it means that the air that contains no water vapor. Practically, except from the approximation involved in using shallow water equation, the dry air equations can explain the mid latitude circulation of atmosphere more approximately than that of its tropic counter part, where it is characterized by moist circulation [15].

Despite the fact that shallow water model is based on assumption of fluid incompressibility and negligible vertical shear of horizontal velocity (both meridional and zonal component), it can be seen from Fig. 3.1 (and from Fig. 3.2) that our model result are in almost good agreement with its corresponding result of NCEP zonal velocity.

Specifically one can see that from Fig. 3.1 of zonal velocity after 6hrs of simulation at 00 UTC of may 26, 2009, the region over which zonal velocity is reversed as compared to the mean flow in mid latitude is the same for both NCEP real analysis data and that of our model result, and is around a region centered on 50^{o}N and 30^{o}E. Similarly from Fig. 3.2 the zonal velocity after 12hrs of simulations at 6UTC of May 26, 2009, one can see that the region where the direction of zonal velocity is reversed as compared to the mean flow over mid latitude, which is in complete agreement with the NCEP reanalysis data.

The discrepancy that can be seen between our model results and that of the NCEP real analysis data can be possibly due to the fact that the assumption imposed on using shallow water equation. That is, the real atmosphere is no more incompressible and there is also vertical shear of horizontal velocity in real atmosphere. Not only this but also other factors like the effect of random perturbations on the field variables could not be ignored in the real atmosphere, as a recent study is showing [16]. In addition, the dynamics of the lower part of the troposphere (which is greatly influenced by friction) can be among the reasons why the model could not match fully to the observed structure of the wind field in the mid latitude from NCEP data. It can also be seen that from Fig. 3.2, the discrepancy between the model out put and that of NCEP data get increased as time increase, this is a fact that one can not go beyond

24 hour while simulating atmospheric air motion using shallow water equation [20].

It can be seen from Figs. 3.3 and 3.4 of absolute vorticity that the model out puts are in good agreement with that of the NCEP data. Specifically, the region of minimum value of absolute vorticity obtained through model is almost similar to that of the NCEP's data.

There is no NCEP reanalysis data six or twelve hourly for stream function so as to validate our model result.

Since a line of constant stream function is stream line of the wind flow (that is the line which is every where parallel to that of the wind flow), we can see from plot of stream function in Fig. 3.5 that how the wind blows in the mid latitude 500 mb level.

3.3 Atmospheric air motion over Ethiopia at 500 mb level on May 26, 2009

In general the atmospheric vertical structure is controlled by moist process, and hence it is necessary to use the equation of moist atmosphere to study atmospheric general circulation. In practice, however, the equation of dry atmosphere can be used if the moist effect is introduced only as a diabatic heating due to latent heat release. The large scale circulation in the tropical atmosphere (and hence Ethiopia) can be viewed as moist convective motions [15].

Although the vertical motion of the atmosphere is ignored in using shallow water equation modelling, which prone to large error in tropics as compared to that of mid latitude, the result obtained for Ethiopia through shallow water modelling is comparable to that of mid latitude.

As we can see from the contour plots in Fig. 3.6 of zonal velocity at 00 UTC after 6hrs of simulations from initial time prescribed above, the model captures many of properties of zonal velocity except some deviations which are observed in our model result as compared to that the reanalysis data. Specifically one can see that the region around 12^oN and 8^oN, where the direction of the zonal velocity is reversed is exactly the same as that of NCEP data.

As the time goes on, it can be seen from Fig. 3.7 contour plots, the discrepancy between the model results and that of the NCEP reanalysis data grows.

The magnified difference observed in tropics, as compared to mid latitude, can be, among others, due to the fact that shallow water equation ignores the effect of moist process, which is an important characteristic of atmospheric circulation on large scale motion in low latitude.

The common fact for both regions of our study (mid latitude and Ethiopia) is that we can not go beyond one day while using shallow water modelling to forecast field variables, because for time more than that of one day a lot of physical processes like convection process taking place in the atmosphere come to play an important role in the circulation of the atmosphere [15]. This is the fact that we have observed in the model out put as we approach towards 24 hr in the case study of time evolution of zonal velocity over Ethiopia and mid latitude.

The result obtained for vertical component of absolute vorticity after 6 and 12hrs of simulations from initial time are given in Fig. 3.8 and Fig. 3.9 respectively.

Since vorticity is the vector field that gives a microscopic measure of the turning of the air over a unit area of a surface about an axis normal to it, Fig. 3.8 and Fig.

3.9 show their contour plot over Ethiopia. Specifically, we can see that the region of maximum positive absolute vorticity for the model result are similar to the NCEP data.

Since we do not have NCEP reanalysis data six or twelve hourly for stream function for our study area, our model out put for this important parameter is not compared. However, we can infer from Fig. 3.10 that the wind blows from south west over southern and south western Ethiopia while it blows from north east over northern part of Ethiopia as expected during this time of the year.

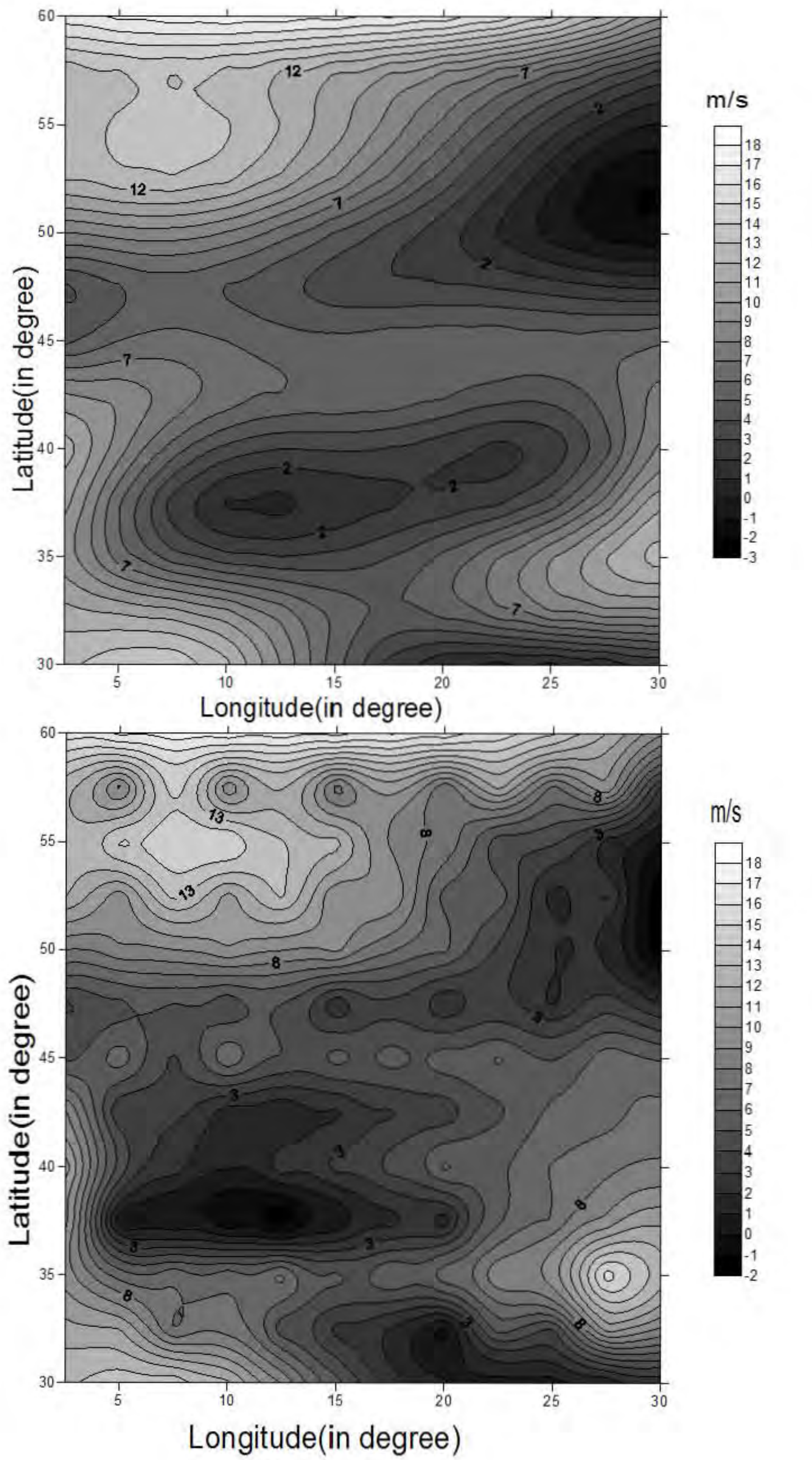

Figure 3.1: NCEP and our models out put after 6hrs of simulation for zonal velocity at 00 UTC over mid latitude. Top panel: NCEP data; Bottom panel: Model output data.

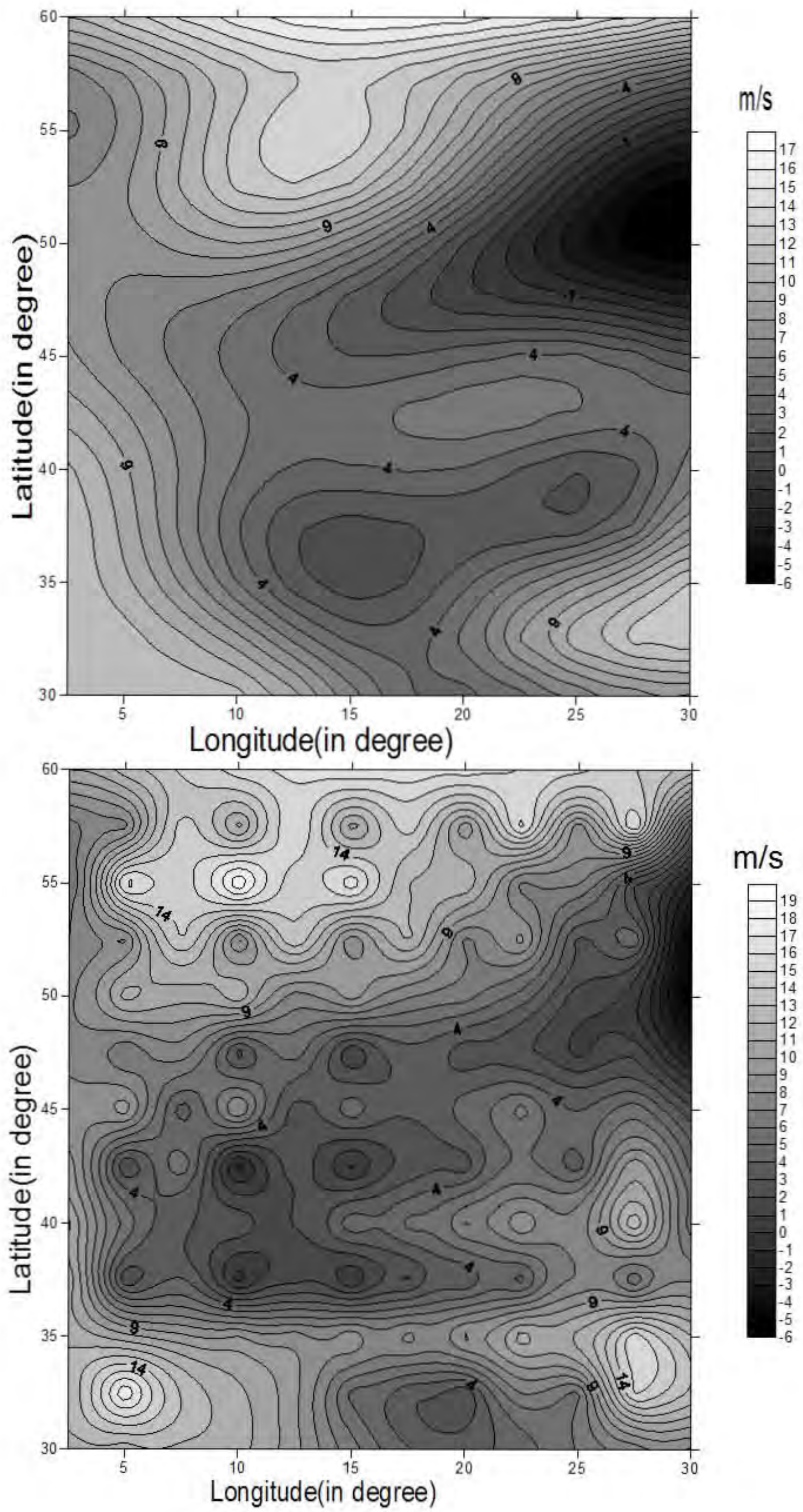

Figure 3.2: NCEP and our models out put after 12hrs of simulation for zonal velocity at 06 UTC over mid latitude. Top panel: NCEP data; Bottom panel: Model output data.

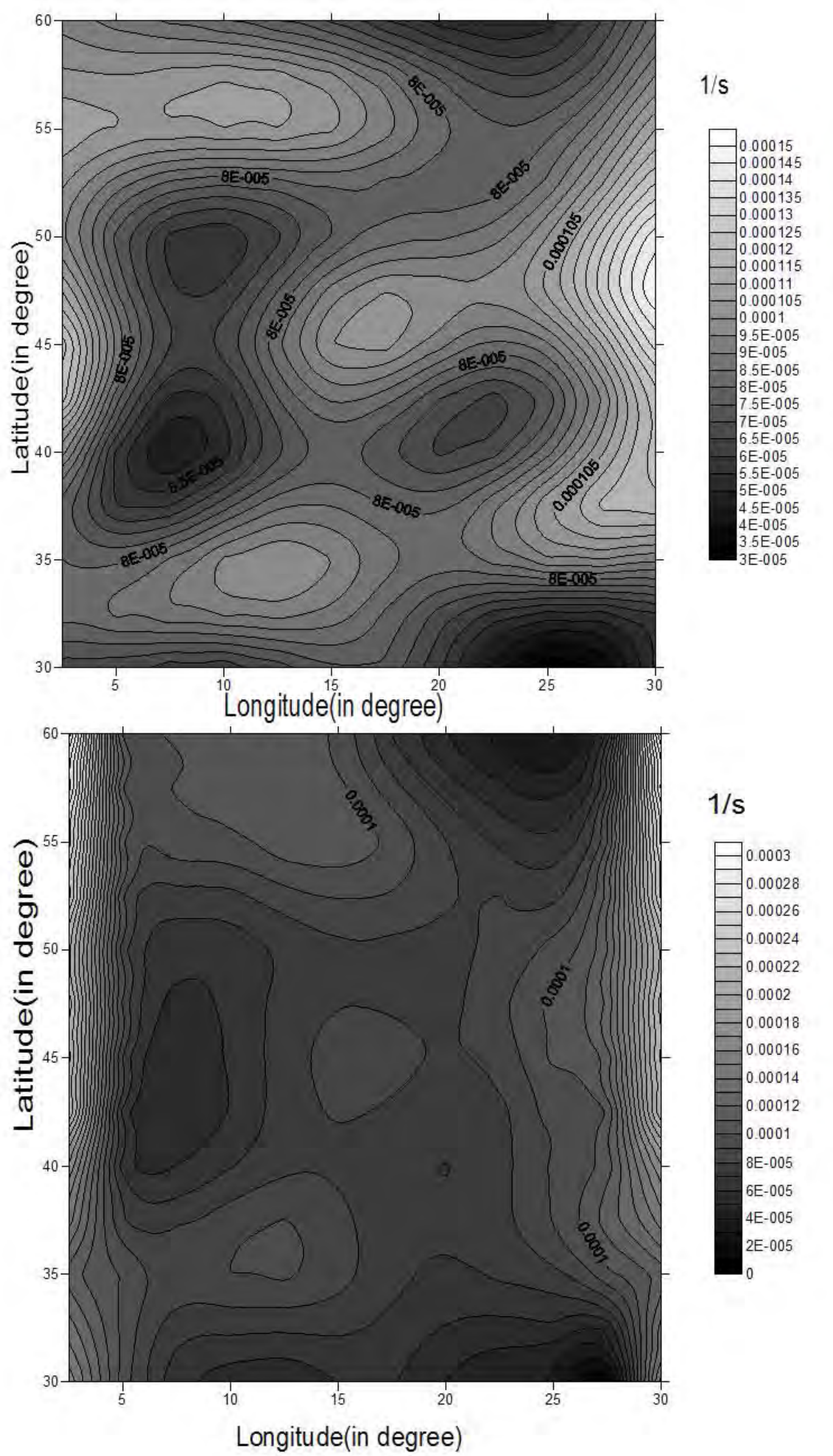

Figure 3.3: NCEP and our models out put after 6hrs of simulation for absolute vorticity at 00 UTC over midlatitude. Top panel: NCEP data; Bottom panel: Model output data.

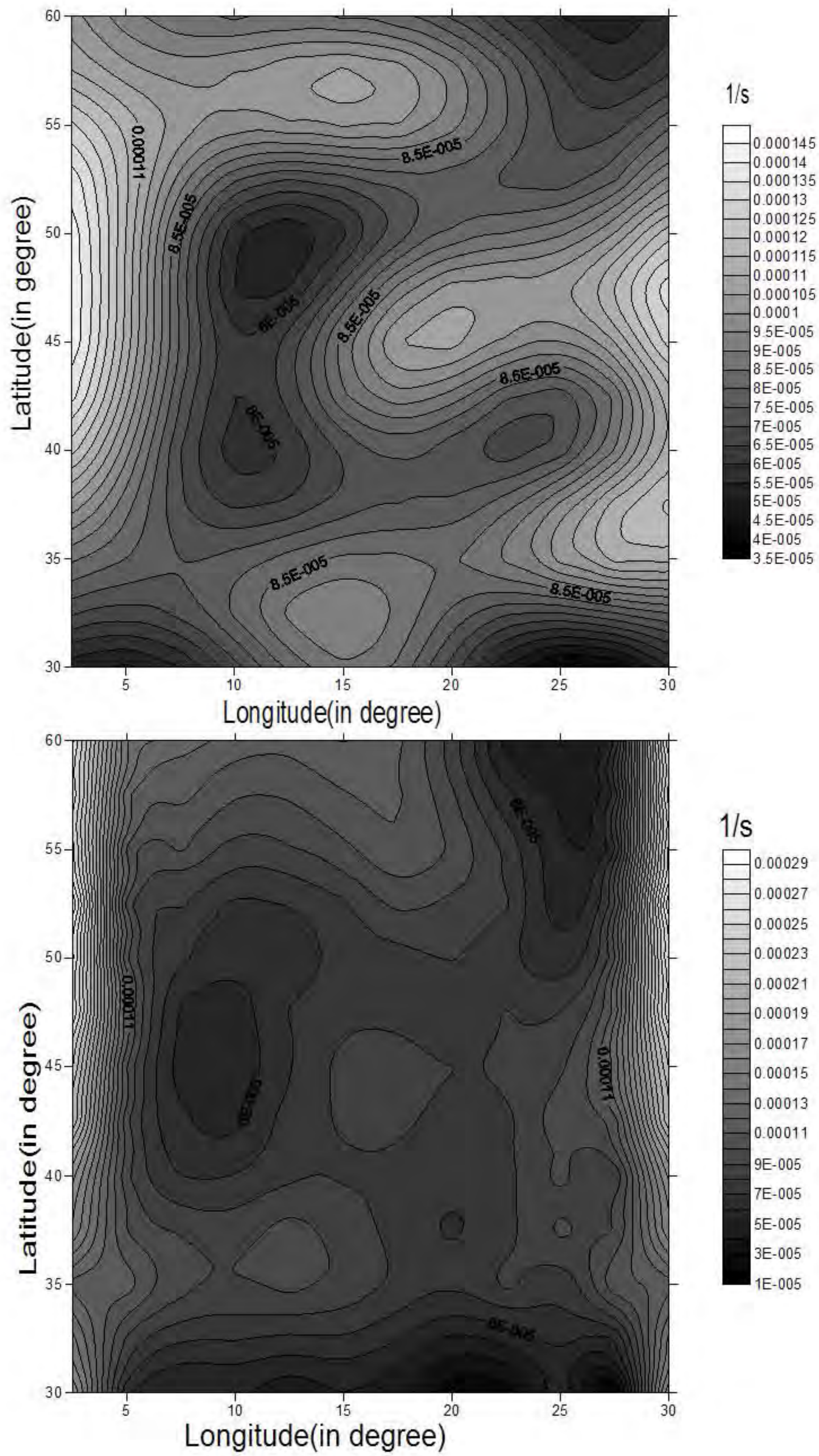

Figure 3.4: NCEP and our models out put after 12hrs of simulation for absolute vorticity at 06 UTC over Mid latitude. Top panel: NCEP data; Bottom panel: Model output data.

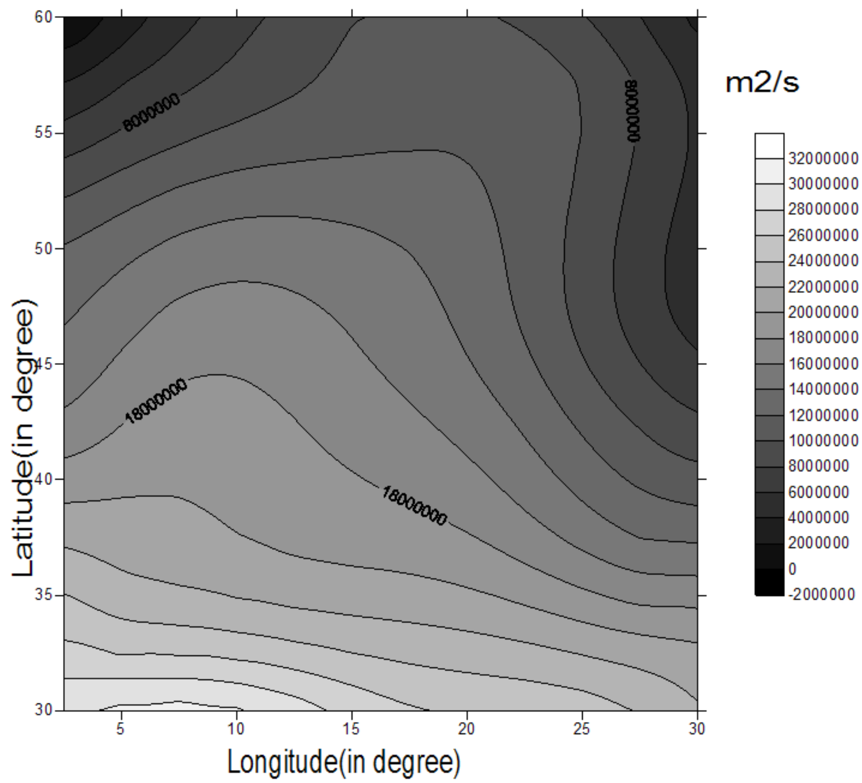

Figure 3.5: Model out of stream function after 6hrs of simulation at 00 UTC over mid latitude.

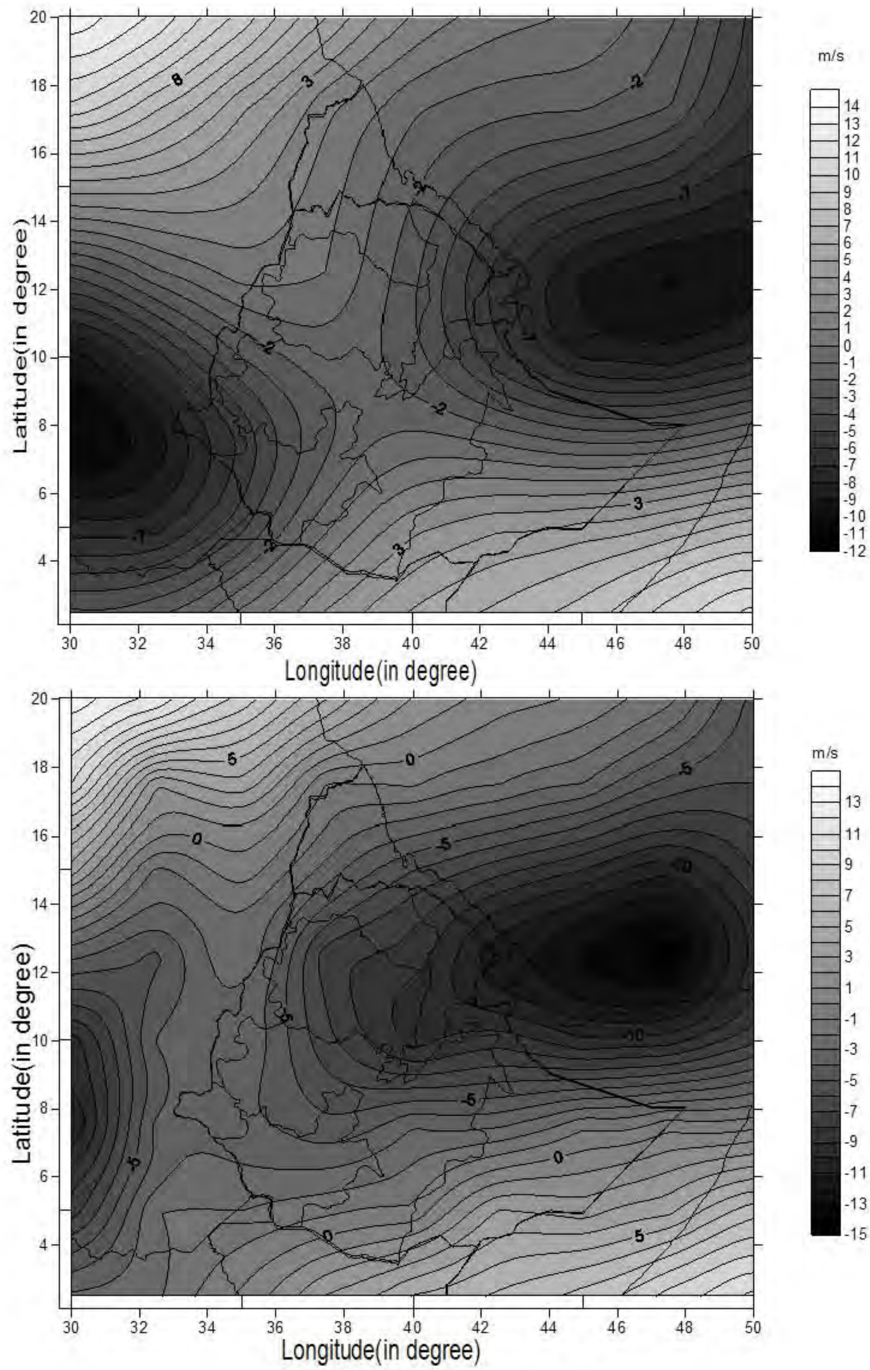

Figure 3.6: NCEP and our models out put after 6hrs of simulation for zonal velocity at 00 UTC over Ethiopia. Top panel: NCEP data; Bottom panel: Model output data.

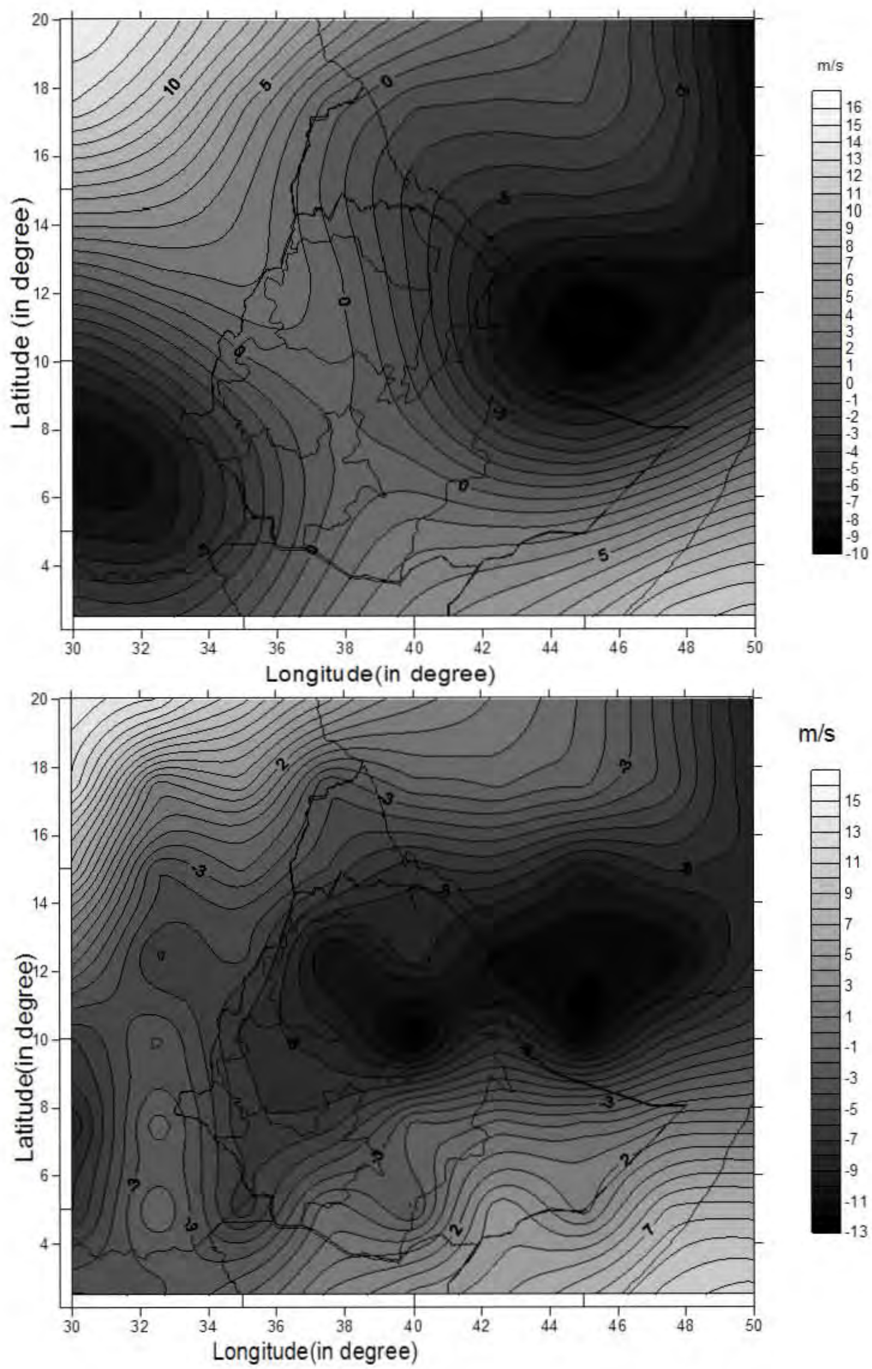

Figure 3.7: NCEP and our models out put after 12hrs of simulation for zonal velocity at 06 UTC over Ethiopia. Top panel: NCEP data; Bottom panel: Model output data.

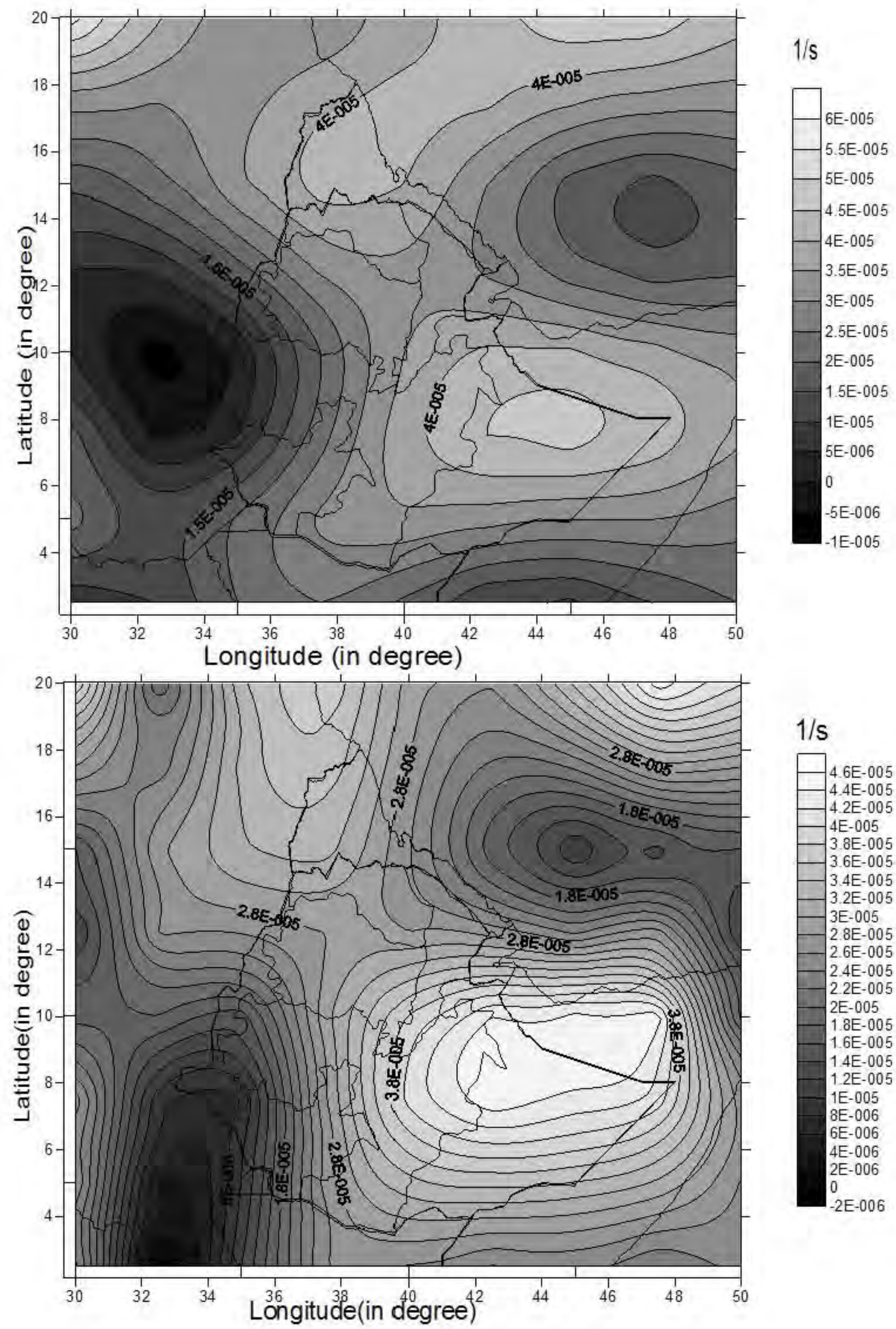

Figure 3.8: NCEP and our models data after 6hrs of simulation for absolute vorticity at 00 UTC over Ethiopia. Top panel: NCEP data; Bottom panel: Model output data.

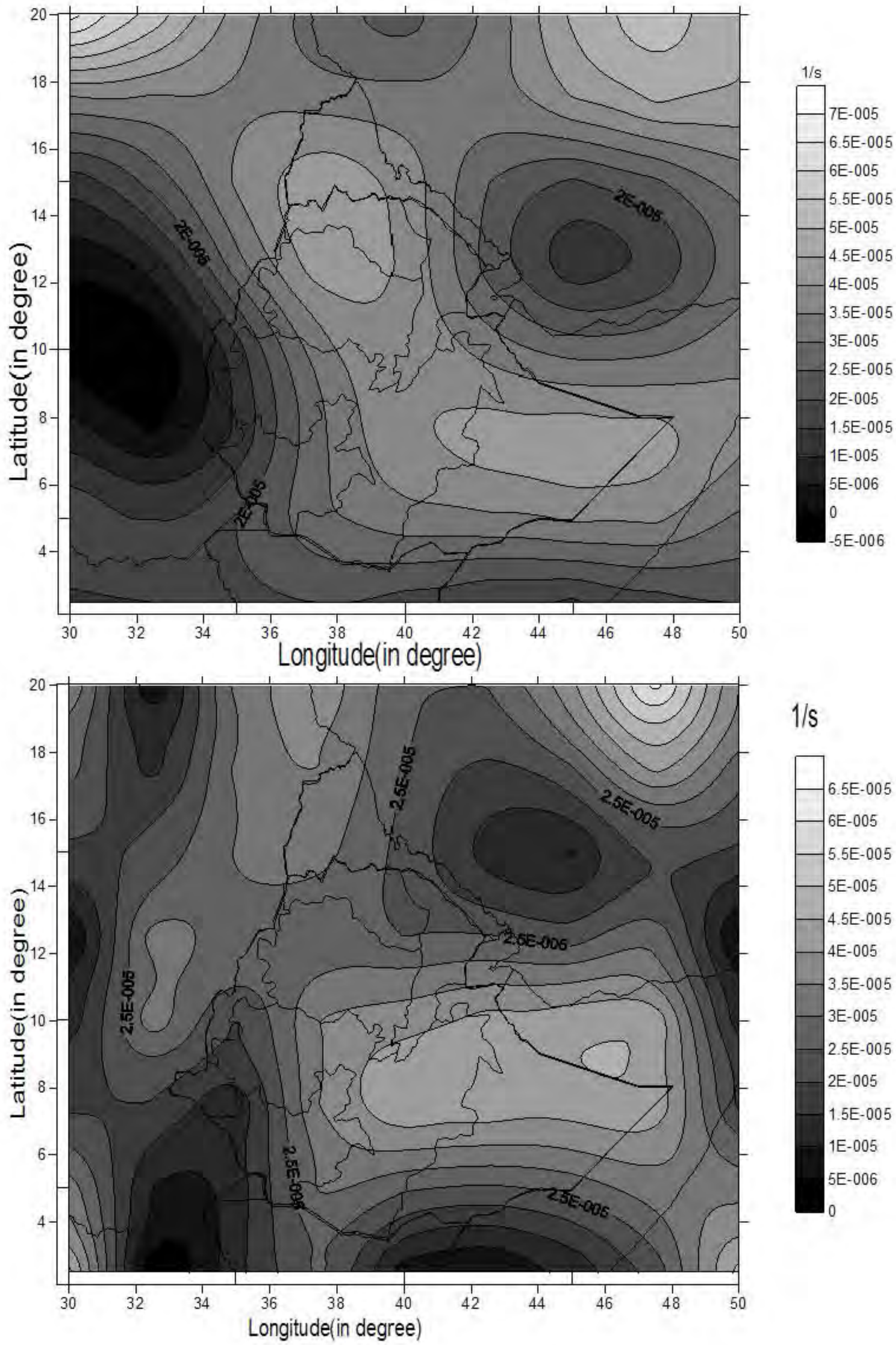

Figure 3.9: NCEP and our models out put after 12hrs of simulation for absolute vorticity at 06 UTC over Ethiopia. Top panel: NCEP data; Bottom panel: Model output data.

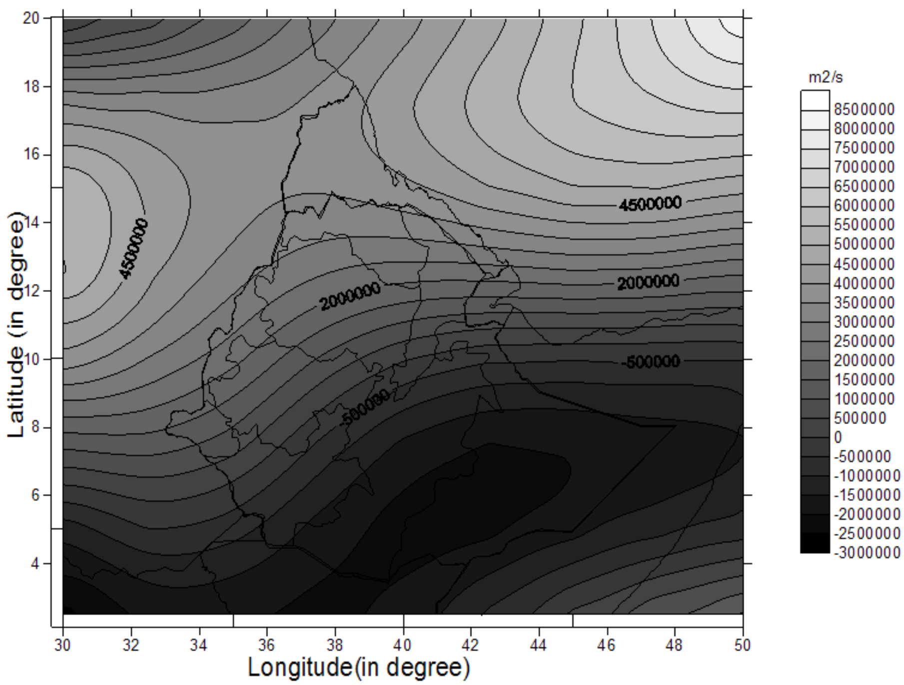

Figure 3.10: model out of stream function after 6hr of simulation at 00 UTC over Ethiopia.

Chapter 4

CONCLUSIONS AND FUTURE DIRECTION

4.1 Conclusions

An earth's atmosphere can be defined as a fluid, in natural laboratory, where wide variety of physical, chemical and dynamical processes are taking place. For instance transport by dynamical process carries chemicals from one part of the atmosphere to another, the heating due absorption of solar ultraviolet radiation by ozone may drive dynamical process. Solar radiation can also be energetic enough to disrupt chemical bonds, leading to photochemical reactions. In general these interactions taking place in the earths atmosphere are too difficult to be described easily.

In order to understand the earth's atmospheric behavior and to predict its future state, scientists are forced to model the behavior of the atmosphere because it is impossible to do controlled experiment in the process taking place in the earth's atmosphere on large scale till now. The best kind of modelling atmospheric researchers are using currently is, numerical modelling. Numerical modelling is a kind of modelling where the bulk properties of the atmosphere are expressed in the form of partial differential equation describing the behavior of the atmosphere (usually called conservative equations which shows conservation of mass, momentum and energy in the atmosphere) and the resulting nonlinear partial differential equation is solved approximately by numerical means (perhaps by computer).

In general the moist process, the radiation process, and turbulent process are the major physical process for atmospheric modelling while dynamical process is the second kind and is the one that we dealt with in this thesis. Atmospheric general circulation models (AGCMs) are numerical models that simulate three dimensional atmospheric flow. If these modelling of the atmosphere is coupled with that of oceanic general circulation model and a land surface model using various physical process, they form the coupled model that can be used for climate prediction of global warming.

Apart from the general circulation model, in this thesis the simplified form of atmospheric dynamic equation, shallow water equation, is used to simulate atmospheric air motion over mid latitude and Ethiopia. Shallow water equation can be viewed as a mathematical model of horizontal motions of the atmosphere in which the air has almost vertically uniform motions.

In this study the result of shallow water equation as applied to the region over mid latitude has shown good result in describing time evolution of zonal velocity, which is almost in complete agreement to the NCEP reanalysis data. This implies that shallow water equation can be used to simulate atmospheric air motion on large scale in mid latitude as long as the restriction imposed by shallow water equation is valid. Contrary to mid latitude the discrepancy in case study of the flow of zonal velocity over Ethiopia as compared to its NCEP reanalysis data counter part is large. And also it has been shown that as the time of forecasting the zonal velocity is increased, the difference between the model out put and that of NCEP reanalysis data gets larger and larger. This proves the observed fact that the circulation over Ethiopia (Tropics, in general) is not the same as that of Mid latitude, which is characterized by dry air motion. From this result we can conclude that as long as the time period for obtaining the zonal velocity is short, shallow water equation can be used to predict zonal velocity at 500 mb level in both regions as revealed from our study.

Our model simulation has also shown the result of an important parameters of the atmosphere, being used in lots of numerical weather prediction centers, like vertical component of absolute vorticity and stream function although we couldn't validate the goodness of stream function up on comparing with the data observed, because of lack of six or twelve hourly data of NCEP for stream function. Apart from this comparison, stream function has shown that the line parallel to the flow of an air in the two region of our study.

4.2 Future direction

While using shallow water modelling to simulate atmospheric air motion over mid latitude and Ethiopia in this study, there was a lots of restrictions used from the general atmospheric circulation. There is no doubt that inclusion of this effect will reduce the discrepancy between the model result and NCEP reanalysis data.

A kind of numerical approach in solving the non linear shallow water equation can be further improved to get best results that match with observed data. In this study finite difference technique is used for solving shallow water equation. If, for instance, spectral technique is used for solving shallow water equation we could get best result because this technique of solving the equation of atmospheric dynamics conserves global quantities like energy budget. But we should be much care full about in using finite difference technique for conserving this globally conserved quantities.

In the future we need to simulate the whole atmospheric behavior based on non hydrostatic primitive equation so as to understand a kind of circulation over the globe, rather than restricting our selves with shallow water equation (and also some times called one level primitive equation). The current primitive equation are equations based on hydrostatic assumption in the vertical directions (where the aspect ratio of typical resolution is about 1/100). Recently a very high resolution simulation with less than 10km in horizontal scale is becoming achievable for global models due to high computer performance development. In these models the hydrostatic assumptions is no longer acceptable and non hydrostatic equations will take the place of hydrostatic primitive equations for frame work of general circulation models. This model may well be called the next generation atmospheric general circulation model.

Explicit time stepping scheme used in this study can also be further improved, for instance, up on using semi implicit scheme which allows larger time step as compared to that of its explicit counter part (given by CFL condition) and also this scheme decreases the computational effort paid by implicit scheme.

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