PAIR PRODUCTION WITH NEUTRINOS AND HIGH-INTENSITY LASER FIELD

By

Eninges Asmare

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ADDIS ABABA UNIVERSITY
DEPARTMENT OF
PHYSICS

Advisor: ______________________
Dr. S. Bhatnagar

Examiners: ______________________
Dr. Fesseha

____
Dr. N. Behera

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Author: **Eninges Asmare**

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Abstract

Neutrinos participate in charged current reactions which are mediated by $W^{\pm}$ bosons and in neutral current reactions mediated by Z^0 bosons. The latter process allowed us to count the number of light neutrino species that have the usual electroweak interaction. In this work we examine electron-positron pair production with neutrinos in an intense laser field through the process $\nu \to \nu' e^- e^+$. We first review GWS model for electroweak interactions and the process $\nu \to \nu' e^- e^+$ which proceeds through neutral channel. We also treat this process through charged channel. Of course the process through the charged channel is relevant only for electron neutrino. A detailed calculation of these processes is carried out. For the charged current reaction the rate of electron-positron pair production is calculated, using the same procedure we do for the reaction through the neutral channel. Finally a tabulation of the rate of production for different neutrino energies is presented for the neutral current reaction. We also present graphs to show the variation of rate with the number of photons extracted from the laser field to run the reaction.

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I have no words to thank GOD for every thing he did for me, but I simply say thanks to Him.

Chapter 1

Introduction

One major success of the Standard Model is the accurate prediction of the masses of the intermediate vector bosons. This made a specific experimental search for these particles possible, to which end it is important to know the possible creation and decay mechanisms. We see immediately that in an electron-positron storage ring it is considerably easier to produce the neutral boson Z^0 than the charged $W^{\pm}$ bosons. In particular, decays involving neutrinos are experimentally characteristic, because a large fraction of the energy present in the scattering is transferred to the neutrino and therefore not seen in the detectors.

Laser peak power has climbed from gigawatts to petawatts in the previous 30 years and accessible focused intensity has increased by several orders of magnitude. Such a dramatic increase in intensity has accessed an entirely new phenomena. High repetition rate table top lasers can routinely produce intensity in excess of $10^{19} W/cm^2$, and intensities of up to $10^{20} W/cm^2$ are possible with latest petawatt class systems[1].

The problem of particle interaction with external electromagnetic fields is of considerable interest for modern physics. It is well known that processes forbidden in vacuum become possible in intense external fields; such as the photon decay into an electron-positron pair $\gamma \to e^- e^+$, the photon splitting into two photons $\gamma \to \gamma\gamma$, and the neutrino production of an electron-positron pair $\nu \to \nu' e^- e^+$ [2]. Apart from this, intense external electromagnetic fields catalyze some processes allowed in vaccum, for example, the radiative decay of a massive neutrino $\nu \to \nu' \gamma$. The method in which the external field effect is taken into account on the basis of exact solutions of the field theory equation for charged particles in an electromagnetic field rather than on the basis of perturbation theory, has become an important tool for studying some fundamental problems of particle interactions with an electromagnetic field.

Among many of the interactions of particles with intense external fields, in this thesis, we examine production of electron-positron pair from neutrinos in an intense laser field. The production of electron-positron pair with neutrinos in an intense laser field using the neutral channel had, of course, been presented in [3]. But here, in addition to the verification of the derivation of the field for that process, pair production through the charged channel is also treated. The thesis is organized as follows: in Chapter 2 we review the GWS model of leptons and the Lagrangian for interactions involving leptons. Chapter 3 deals with the physics of neutrinos. The effect an electromagnetic field has on particles is presented in Chapter 4. The process of production of electron-positron pair with neutrino in an intense laser field is treated in Chapter 5. Results of an experiment done by High Intensity Laser Science Group at the University of Austin based on Terawatt High-intensity Optical Research (THOR) laser is also presented in this chapter. Finally we summarize the work in Chapter 6.

Chapter 2

Introduction to Weak Interaction of Leptons

The Standard Model was born, in the end of sixties, in an attempt to build a renormalizable theory of weak interaction. This renormalizable theory of weak interaction was proposed by Glashow, Weinberg and Salam. It is built in the frame work of the unification of weak and electromagnetic interactions, which makes the theory a major breakthrough in the understanding of elementary particle physics[4]. The 'electroweak' theory of Glashow, Weinberg and Salam is based on a non-abelian $SU(2) \times U(1)$ gauge symmetry, which is broken down spontaneously to the $U(1)$ symmetry of electromagnetic interaction.

Studies of the helicity dependence of the weak interaction cross section and decay rate has shown that the weak interaction involves the current-current coupling between vector currents built of quark and lepton fields. It is thus natural to assume that the weak interaction is due to exchange of very heavy vector bosons.

2.1 GWS model of leptons

To formulate GWS model of weak and electromagnetic interactions among leptons and to study its properties, we have the following conditions[5].

1. There exist charged and neutral currents.

2. The charged currents contain only couplings between left-handed leptons.

3. The bosons ($W^+,\ W^-,\ Z^0$) mediating the weak interaction must be very massive.

To fulfill these conditions we introduce two vector fields, one isospin triplet $A^i_\mu (i = 1,2,3)$ and one singlet B_μ which should finally result as fields of the physical particles $W^+,\ W^-,\ Z^0$ and the photon through the symmetry breaking induced by the Higgs mechanism. The leptonic fields have to be distinguished according to their helicity.

Every fermion generation (e, μ, τ) contains two related left-handed leptons. These form an 'isospin' doublet of left-handed leptons,

$L_i (i = e, \mu, \tau)$

$$L_e = \frac{1-\gamma_5}{2}\begin{pmatrix}\Psi_{\nu e}\\ \Psi_e\end{pmatrix}, L_\mu = \frac{1-\gamma_5}{2}\begin{pmatrix}\Psi_{\nu\mu}\\ \Psi_\mu\end{pmatrix}, L_\tau = \frac{1-\gamma_5}{2}\begin{pmatrix}\Psi_{\nu\tau}\\ \Psi_\tau\end{pmatrix}$$

There are also right-handed components of the charged massive leptons. However, in the frame work of weak and electromagnetic interactions, a right-handed neutrino does not exist. Therefore the right-handed leptons can be represented by singlets:

$$R_e = \frac{1+\gamma_5}{2}\Psi_e, R_\mu = \frac{1+\gamma_5}{2}\Psi_\mu, R_\tau = \frac{1+\gamma_5}{2}\Psi_\tau$$

Now we consider the currents (charged, neutral and electromagnetic).
The charged weak currents have the following form:

$$\begin{aligned} J^{(l)\sigma}_{-} &= \bar{\Psi}_l \gamma^\sigma (1-\gamma_5)\Psi_{\nu l} \\ &= 2\bar{\Psi}_l \frac{1+\gamma_5}{2}\gamma^\sigma \frac{1-\gamma_5}{2}\Psi_{\nu l} \\ &= 2\bar{L}_l \gamma^\sigma \hat{T}_{-} L_l \end{aligned} \tag{2.1.1}$$

$$J^{(l)\sigma}_{+} = (J^{(l)\sigma}_{-})^\dagger = 2L_l \gamma^\sigma \hat{T}_{+} L_l \tag{2.1.2}$$

where $\hat{T}_\pm = \hat{T}_1 + iT_2$, $(\hat{T})^+ = \hat{T}_+$ and $l = (e, \mu, \tau)$

The electromagnetic current, since it is charged, exists only for e, μ and τ.

$$\begin{aligned} J^{(l)\sigma}_{em} &= \bar{\Psi}_l \gamma^\sigma \Psi_l \\ &= \frac{1}{2}\bar{\Psi}_l \gamma^\sigma (1-\gamma_5)\Psi_l + \frac{1}{2}\bar{\Psi}_l \gamma^\sigma (1+\gamma_5)\Psi_l \\ &= \bar{L}_l \gamma^\sigma (\frac{1}{2} - \hat{T}_3) L_l + \bar{R}_l \gamma^\sigma R_l \end{aligned} \tag{2.1.3}$$

This splits up into a part $-\bar{L}_l\gamma^\sigma T_3 L_l$ belonging to an isotriplet and $\frac{1}{2}\bar{L}_l\gamma^\sigma L_l + \bar{R}_l\gamma^\sigma R_l$ representing an isosinglet current. We rearrange the currents into an isotriplet with the isovector gauge field A_μ, $\bar{L}_l\gamma^\sigma T L_l$ and an isosinglet to be associated with the gauge field B_μ,

$$\frac{1}{2}\bar{L}_l\gamma^\sigma L_l + \bar{R}_l\gamma^\sigma R_l \tag{2.1.4}$$

These currents are minimally coupled to the corresponding gauge fields.

$$L^l_{int} = g(\bar{L}_l\gamma^\sigma T L_l)\cdot A_\mu - g'\left[\frac{1}{2}\bar{L}_l\gamma^\sigma L_l + (\bar{R}_l\gamma^\sigma R_l)\right] B_\mu \tag{2.1.5}$$

This equation characterizes the structure of interaction as demanded by the general-group theoretical considerations concerning the two gauge fields A_μ and B_μ. The real (physical) photon does not couple to the singlet current (2.1.4) but to J^σ_{em} (2.1.3). Therefore it has to be represented by a mixture of B_μ and A^3_μ fields. So the photon field is written as follows:

$$A_\mu = \cos\theta B_\mu + \sin\theta A^3_\mu \tag{2.1.6}$$

The physical neutral intermediate boson of weak interaction which is orthogonal to A_μ is expressed as

$$Z_\mu = -\sin\theta B_\mu + \cos\theta A^3_\mu \tag{2.1.7}$$

The neutral current is given by

$$J_0^{(l)\sigma} = 2\sqrt{2}\big[\cos\theta \bar{L}_l\gamma^\sigma \hat{T} L_l + \frac{g'}{g}\sin\theta(\frac{1}{2}\bar{L}_l\gamma^\sigma L_l + \bar{R}_l\gamma^\sigma R_l)\big]$$
$$= \frac{2\sqrt{2}}{\cos\theta}\left[\bar{L}_l\gamma^\sigma(\hat{T}_3\cos^2\theta + \frac{1}{2}\sin^2\theta)L_l + \sin^2\theta\bar{R}_l\gamma^\sigma R_l\right]$$
$$= \frac{\sqrt{2}}{\cos\theta}\left[\bar{L}_l\gamma^\sigma\begin{pmatrix}1 & 0\\ 0 & -\cos 2\theta\end{pmatrix}L_l + 2\sin^2\theta\bar{R}_l\gamma^\sigma R_l\right] \quad (2.1.8)$$

Written in terms of neutrino and the corresponding lepton fields the neutral weak current becomes

$$J_0^{(l)\sigma} = \frac{\sqrt{2}}{\cos\theta}\Big[\bar{\Psi}_{\nu l}\frac{1+\gamma_5}{2}\gamma^\sigma\frac{1-\gamma_5}{2}\Psi_{\nu l} - (1-\sin^2\theta)\bar{\Psi}_l\frac{1+\gamma_5}{2}\gamma^\sigma\frac{1-\gamma_5}{2}\Psi_l$$
$$+2\sin^2\theta\bar{\Psi}_l\frac{1-\gamma_5}{2}\gamma^\sigma\frac{1+\gamma_5}{2}\Psi_l\Big]$$
$$= \frac{1}{\sqrt{2}\cos\theta}\left[\bar{\Psi}_{\nu l}\gamma^\sigma(1-\gamma_5)\Psi_{\nu l} - \bar{\Psi}_l\gamma^\sigma(g'_V - g'_A\gamma_5)\Psi_l\right] \quad (2.1.9)$$

with $g'_A = 1$ and $g'_V = 1 - 4\sin^2\theta$

The neutrino part of the neutral current has pure $V - A$ coupling. It is of course what is expected, since the theory contains only left-handed components of the neutrino field.

2.2 Spontaneous symmetry breaking

In order to give $W_\mu^{(+)}$, $W_\mu^{(-)}$, and Z_μ^0 mass we apply Higgs mechanism. As the left-handed leptons form an isodoublet and the gauge fields A_μ an isovector, we need an isodoublet of Higgs fields.

$$\phi = \begin{pmatrix}\phi^{(+)}\\ \phi^{(0)}\end{pmatrix}, |\phi|^2 = |\phi^{(+)}|^2 + |\phi^{(0)}|^2 \quad (2.2.1)$$

The isospin T and hypercharge Y of the Higgs field must be given by $T = \frac{1}{2}$, $Y = 1$ [5]. To obtain a non-vanishing expectation value for the Higgs field we add a potential term;

$$U(\phi) = -\mu^2|\phi|^2 + h|\phi|^2$$

There is also a gradient term for the kinetic energy, where it is minimally coupled to the gauge fields A_μ and B_μ

$$|(i\partial_\mu + g\hat{T}\cdot A_\mu + \frac{g'}{2}B_\mu\hat{Y}\phi|^2 = |(\partial_\mu - ig\hat{T} - i\frac{g'}{2}B_\mu\hat{Y})\phi|^2 \tag{2.2.2}$$

The electromagnetic field A_μ does not couple to the lower component of the Higgs field.

The vacuum expectation value of the Higgs field is fixed by the condition that $U(\phi)$ attains a minimum. Demanding $\frac{dV}{d\lambda} = 0$, for the Higgs potential

$$V(\lambda) = U(\langle 0|\hat{\phi}|0\rangle) = -\frac{\mu^2}{2}\lambda^2 + \frac{h}{4}\lambda^4 \tag{2.2.3}$$

we get $\lambda^2 = \frac{\mu^2}{h}$, yielding the expectation value

$$\langle 0|\hat{\phi}|0\rangle = \frac{\mu}{\sqrt{2h}}\begin{pmatrix}0\\1\end{pmatrix}$$

The lower component of the Higgs field does not couple to the electromagnetic field. Therefore, the non-vanishing vacuum expectation value does not influence the photon field, that is in spite of the symmetry breaking the photon remains massless. Now we collect all parts of the Lagrangian density, i.e. the contribution of the free field, the interaction term, and contribution of Higgs field. Finally we add a term

$$-\sqrt{2}f_l(\bar{R}_l\phi^+L_l + \bar{L}_l\phi R_l) = -f_l(\lambda+\chi)\bar{\Psi}_l\Psi_l$$

This is to give the charged leptons the mass $m_l = f_l\lambda$.

The Lagrangian of the weak interaction is given by

$$
\begin{aligned}
L &= -\frac{1}{4}F_{\mu\nu}\cdot F^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - e\left(\sum_l \bar{\Psi}_l\gamma^\mu\Psi_l\right)A_\mu \\
&+\sum_l i\left(\bar{\Psi}_{\nu l}\gamma^\mu\frac{1}{2}(1-\gamma^5)\partial_\mu\Psi_{\nu l} + i\bar{\Psi}_l\gamma_\mu\partial_\mu\Psi_l - f_l\bar{\Psi}_l\Psi_l(\lambda+\chi)\right) \\
&+\frac{g}{2\sqrt{2}}\sum_l\left[\bar{\Psi}_l\gamma^\mu(1-\gamma^5)\Psi_{\nu l}W^{(-)}_\mu + \bar{\Psi}_{\nu l}\gamma^\mu(1-\gamma^5)\Psi_l W^{(+)}_\mu\right] \\
&+\frac{g}{4\cos\theta}\sum_l\left[\bar{\Psi}_{\nu l}\gamma^\mu(1-\gamma^5)\Psi_{\nu l} - \bar{\Psi}_l\gamma^\mu(g_V^{'}-\gamma_5)\Psi_l\right]Z_\mu \\
&+\frac{h\lambda^4}{4} + \frac{1}{2}(\partial_\mu\chi)^2 - h\lambda^2\chi^2 - h\chi^2(\lambda\chi + \frac{1}{4}\chi^2) \\
&+\frac{g^2}{8}\left(2W^{(+)}_\mu W^{(-)\mu} + \frac{Z_\mu Z^\mu}{\cos^2\theta}\right)(\lambda+\chi)^2
\end{aligned}
\tag{2.2.4}
$$

Rewriting the free field parts $F_{\mu\nu}\cdot F^{\mu\nu}$ and $B_{\mu\nu}B^{\mu\nu}$ in terms of the physical fields A_μ, Z_μ, and $W^{\pm}_\mu$, we find

1.

$$
\begin{aligned}
F_{\mu\nu}\cdot F^{\mu\nu} &= \sum_i\left(\partial_\mu A^i_\mu - \partial_\nu A^i_\mu + g\epsilon_{ikl}A^k_\mu A^l_\nu\right)\left(\partial^\mu A^{i\nu} - \partial^\nu A^{i\mu} + g\epsilon_{ikl}A^{k\mu}A^{l\nu}\right) \\
&= 2\Bigg[\partial_\mu W^{(-)}_\nu - \partial_\nu W^{(-)}_\mu - ig\cos\theta(W^{(-)}_\mu Z_\nu - W^{(-)}_\nu Z_\mu) \\
&-ie(W^{(-)}_\mu A_\nu - W^{(-)}_\nu A_\mu)\Bigg]\times\Bigg[\partial^\mu W^{(+)\nu} - \partial^\nu W^{(+)\mu} + ig\cos\theta(W^{(+)\mu}Z^\nu \\
&-W^{(+)\nu}Z^\mu) + ie(W^{(+)\mu}A^\nu - W^{(+)\nu}A^\mu)\Bigg] + \Bigg[\cos\theta(\partial_\mu Z_\nu - \partial_\nu Z_\mu) \\
&+\sin\theta(\partial_\mu A_\nu - \partial_\nu A_\mu) + ig(W^{(-)}_\mu W^{(+)}_\nu - W^{(-)}_\mu W^{(+)}_\nu)\Bigg] \\
&\times\Bigg[\cos\theta(\partial^\mu Z^\nu - \partial^\nu Z^\mu) + \sin\theta(\partial^\mu A^\nu - \partial^\nu A^\mu) \\
&+ig(W^{(-)\mu}W^{(+)}_\nu - W^{(-)\mu}W^{(+)\nu})\Bigg]
\end{aligned}
\tag{2.2.5}
$$

2.

$$\begin{aligned} B_{\mu\nu}B^{\mu\nu} &= (\partial_\mu B_\nu - \partial_\nu B_\mu)(\partial^\mu B^\nu - \partial^\nu B^\mu) \\ &= \cos^2\theta(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) + \sin^2\theta(\partial_\mu Z_\nu - \partial_\nu Z_\mu) \\ &(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - 2\sin\theta\cos\theta(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) \end{aligned} \tag{2.2.6}$$

Having determined all terms of the GWS Lagrangian describing the electromagnetic and weak interaction of leptons, we write the total expression, omitting the constant term in the Higgs sector $-\frac{1}{4}h\lambda^4$

$$L_{sw} = L_{sw}^{(2)} + \sum_l L_{sw,l}^{(3l)} + L_{sw}^{(3B)} + L_{sw}^{(4B)} + L_{sw}^{(H)} \tag{2.2.7}$$

1. Here $L_{sw}^{(2)}$ describes the part of the free boson and lepton fields.

$$\begin{aligned} L_{sw}^{(2)} &= -\frac{1}{2}\left(\partial_\mu W_\nu^{(+)} - \partial_\nu W_\mu^{(+)}\right)\left(\partial^\mu W^{(-)\nu} - \partial^\nu W^{(-)\mu}\right) + M_W^2 W_\mu^{(+)} W^{(-)\mu} \\ &-\frac{1}{4}\left(\partial_\mu Z_\nu - \partial_\nu Z_\mu\right)\left(\partial^\mu Z^\nu - \partial^\nu Z^\mu\right) + \frac{1}{2}M_Z^2 Z_\mu Z^\mu \\ &-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) \\ &+ \sum_{l=e,\mu,\tau}\left[i\bar{\Psi}_{\nu l}\gamma^\mu\partial_\mu\frac{1-\gamma_5}{2}\Psi_{\nu l} + \bar{\Psi}_l\left(i\gamma^\mu\partial_\mu - m_l\right)\Psi_l\right] \end{aligned} \tag{2.2.8}$$

2. $L_{sw,l}^{(3l)}$ represents the coupling between the leptons of the generation $l = (e, \mu, \tau)$ and the intermediate bosons.

$$\begin{aligned} L_{sw,l}^{(3l)} &= \frac{g}{2\sqrt{2}}\left[\bar{\Psi}_l\gamma^\mu(1-\gamma_5)\Psi_{\nu l}W_\mu^{(-)} + \bar{\Psi}_{\nu l}\gamma^\mu(1-\gamma_5)\Psi_l W_\mu^{(+)}\right] \\ &+\frac{g}{4\cos\theta}\left[\bar{\Psi}_{\nu l}\gamma^\mu(1-\gamma_5)\Psi_{\nu l} - \bar{\Psi}_l\gamma^\mu\left(1 - 4\sin^2\theta - \gamma_5\right)\Psi_l\right]Z_\mu \\ &-e\bar{\Psi}\gamma^\mu\Psi_l A_\mu \end{aligned} \tag{2.2.9}$$

3. $L_{sw}^{(3B)}$ and $L_{sw}^{(4B)}$ are third and fourth order terms of the bosonic fields describing their self coupling.

$$\begin{aligned} L_{sw}^{(3B)} &= ig\cos\theta\left[\left(\partial_\mu W_\nu^{(-)} - \partial_\nu W_\mu^{(-)}\right) W^{(+)\mu} Z^\nu - \left(\partial_\mu W_\nu^{(+)} - \partial_\nu W_\mu^{(+)}\right) W^{(-)\mu} Z^\nu\right] \\ &\quad -ie\left(\partial_\mu W_\nu^{(-)} - \partial_\nu W_\mu^{(-)}\right) W^{(+)\mu} A^\nu + ie\left(\partial_\mu W_\nu^{(+)} - \partial_\nu W_\mu^{(+)}\right) W^{(-)\nu} A^\nu \\ &\quad +ig\cos\theta\left(\partial_\mu Z_\nu - \partial_\nu Z_\mu\right) W^{(+)\mu} W^{(-)\nu} \\ &\quad -ie\left(\partial_\mu A_\nu - \partial_{nu} A_\mu\right) W^{(+)\mu} W^{(-)\nu} \end{aligned} \tag{2.2.10}$$

4.

$$\begin{aligned} L_{sw}^{(4B)} &= -g^2\cos^2\theta\left(W_\mu^{(+)} W^{(-)\mu} Z_\nu Z^\nu - W_\mu^{(+)} W_\nu^{(-)} Z^\mu Z^\nu\right) \\ &\quad -e^2\left(W_\mu^{(+)} W^{(-)\mu} A_\nu A^\nu - W_\mu^{(+)} W^{(-)\nu} A^\mu A^\nu\right) \\ &\quad +eg\cos\theta\left(2W_\mu^{(+)} W^{(-)\mu} Z_\nu A^\nu - W_\mu^{(+)} W_\nu^{(-)} Z^\mu A^\nu - W_\mu^{(+)} W_\nu^{(-)} Z^\nu A^\mu\right) \\ &\quad +g^2\left(W_\mu^{(+)} W^{(-)\mu} W_\nu^{(+)} W^{(-)\nu} - W_\mu^{(-)} W^{(-)\mu} W_\nu^{(+)} W^{(+)\nu}\right) \end{aligned} \tag{2.2.11}$$

5. $L_{sw}^{(H)}$ contains all terms of the Higgs field which are not contained in the mass term.

$$\begin{aligned} L_{sw}^{(h)} &= \frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi) - h\lambda^2\chi^2 \\ &\quad +\frac{1}{4}g^2\left[W_\mu^{(+)} W^{(-)\mu} + \frac{1}{2\cos\theta} Z_\mu Z^\mu\right](2\lambda\chi + \chi^2) - h\chi^2(\lambda\chi + \frac{1}{4}\chi^2) \\ &\quad -\sum_l f_l \bar{\Psi}_l \Psi_l \chi \end{aligned} \tag{2.2.12}$$

Chapter 3

Physics of Neutrinos

Historically, the neutrino was postulated to save the law of energy conservation[6]. It soon became the savior of the law of momentum conservation and angular momentum conservation. When a typical heavy radioactive nucleus emits an alpha particle the alpha particle shoots out with a well defined energy always the same for a particular type of nucleus. The difference in mass multiplied by the square of the speed of light is an energy which is exactly equal to the energy taken away by the alpha particle, that is energy is conserved. Similarly, in the phenomenon of gamma decay, a particular nucleus emits a characteristic-energy photon which carries away exactly the energy lost by the nucleus. In the case of beta decay, a collection of identical beta-active nuclei did not all emit beta particles(electrons) of the same energy, or even of a fixed set of energies; instead, the electrons come out in a 'continuous spectrum' with all possible energies from zero up to some maximum value. When an electron of the maximum value was ejected, it was just sufficient to account for the energy difference between parent and daughter nucleus. When a lower energy of electron was ejected the energy was not balanced-some energy remained unaccounted.

Pauli suggested that a neutral particle, which escaped detection, might be emitted from the nucleus along with the electron, with the available energy shared between this new particle (the neutrino) and the electron. This proposal could explain why the electrons observed in beta decay could fly off with any energy up to some maximum, for they could carry away any fraction of the total available energy, the neutrino taking the rest.

The neutrino must be assumed to be neutral for, if charged, its electric interaction would have caused it to be readily observable. More over, that would have led to non conservation of charge. In addition, the neutrino had to be assumed to have a very small mass. When the electron departed with its maximum energy, little or no energy would be left over for the neutrino.

3.1 Properties of neutrinos

Neutrinos are electrically neutral particles of spin $\frac{1}{2}$. There are at least 3 species of very light neutrinos, ν_e, ν_μ, and ν_τ, which are left-handed, and their antiparticles $\overline{\nu}_e$, $\overline{\nu}_\mu$, and $\overline{\nu}_\tau$, which are right-handed. Electron type neutrinos and anti neutrinos are produced in nuclear β decay[7],

$$A(Z,N) \to A(Z+1,N-1) + e^- + \overline{\nu}_e$$

$$A(Z,N) \to A(Z-1,N+1) + e^+ + \nu_e$$

and in particular in the neutron decay process $n \to p + e^- + \overline{\nu}_e$. They are also produced in muon and pion decays. Muon neutrino and anti neutrinos are produced in muon decays, pion decays and some other processes. The third type of neutrino, tau neutrino is expected to be produced in $\tau^\pm$ decays.

Neutrinos of each flavor participate in reactions in which the charged lepton of the corresponding type is involved; these reactions are mediated by $W^\pm$ bosons. Neutrinos can also participate in neutral current reactions mediated by Z^0 bosons. The latter process allowed us to count the number of light neutrino species that have the usual electroweak interactions.

It is known experimentally that neutrinos emitted in β^- decay cannot be captured in reactions which are caused by electron neutrinos; for example, the reaction

$$\overline{\nu}_e +^{37} Cl \to^{37} Ar + e^-$$

does not occur, whereas the reaction

$$\nu_e +^{37} Cl \rightarrow^{37} Ar + e^-$$

does. But, it does not necessarily mean that neutrinos are different from anti neutrinos, because the weak interactions that are responsible for neutrino interactions are chiral(V-A): in the $Cl - Ar$ reaction, only neutrinos of left-handed chirality can be detected. The particle $\overline{\nu}_e$ is right-handed, and so can not participate in the $Cl - Ar$ reaction.

3.2 Neutrino mass

For our discussion we need the particle-antiparticle conjugation operator $\hat{C}$. Its action on a fermion field is defined as

$$\hat{C} : \Psi \rightarrow \Psi^c = C\overline{\Psi}^T, C = i\gamma_2\gamma_0$$

The matrix C has the properties $C^\dagger = C^T = C^{-1} = -C$ and $C\gamma_\mu C^{-1} = -\gamma_\mu^T$

Based on these properties we have

$$(\Psi^c)^c = \Psi, \overline{\Psi}^c = \Psi^T C, \overline{\Psi}_1 \Psi_2^c = \overline{\Psi}_2^c \Psi_1,$$
$$\overline{\Psi}_1 A \Psi_2 = \overline{\Psi}_2^c (CA^T C^{-1}) \Psi_1^c$$

where Ψ, Ψ_i are 4-component fermion fields and A is an arbitrary 4×4 matrix. Using the commutation properties of the Dirac γ matrices it is easy to see that, acting on a chiral fermion, $\hat{C}$ flips its chirality.

$$\hat{C} : \Psi_L \rightarrow (\Psi_L)^c = (\Psi^c)_R, \Psi_R \rightarrow (\Psi_R)^c = (\Psi^c)_L$$

i.e. the antiparticle of a left-handed fermion is right-handed.

Now we discuss the Dirac and Majorana mass terms. For a massive fermion, the mass term in the Lagrangian has the form

$$-\mathcal{L} = m\overline{\Psi}\Psi = \overline{(\Psi_L + \Psi_R)}(\Psi_L + \Psi_R) = \overline{\Psi}_L \Psi_R + \overline{\Psi}_R \Psi_L \tag{3.2.1}$$

Thus, the mass terms couple the left-handed and right-handed components of the fermion field, and therefore a massive field must have both components: $\Psi = \Psi_L + \Psi_R$. Now, there are two possibilities. First, the right-handed component of a massive field can be completely independent of the left-handed one; in this case we have a Dirac field. Second, the right-handed field can be just a $\hat{C}$ conjugate of the left-handed one: $\Psi_R = (\Psi_L)^c$ or $\Psi = \Psi_L + \eta(\Psi^c)_R = \Psi_L + \eta(\Psi_L)^c$ with phase factor $\eta = e^{i\psi}$ for an arbitrary phase ψ which is the case of a Majorana field.

3.3 Neutrino oscillations

Neutrino oscillations are periodic transitions between different flavor neutrinos in neutrino beams. Discovery of neutrino oscillations signifies not only that neutrino mass-squared differences are different from zero but also that the states of flavor neutrino are superpositions ('mixtures') of states of neutrinos with definite masses[4]. Flavor neutrino states are connected with states of neutrinos with definite masses by the unitary mixing matrix which is characterized by three mixing angles and one phase.

If neutrinos possess mass then the phenomenon of neutrino mixing is possible. Neutrino mixing is a powerful method to probe for neutrino masses far below the kinematic limits. In general, the neutrino flavor eigenstates ν_e, ν_μ and ν_τ of the weak interaction may not be the same as the neutrino mass eigenstates that exist in the Standard Model Lagrangian. The weak eigenstates ν_α are therefore related to the mass eigenstates ν_i, by a unitary transformation matrix U such that $\nu_\alpha = U\nu_i$. In a neutrino oscillation experiment[8], a neutrino beam consisting of a particular flavor eigenstate ν_α is produced at $t = 0$ and sampled at a later time t, at a distance x from the source. The ν_α produced in the beam will be a linear combination of the mass eigenstates and if these possess finite and non-degenerate masses, they will propagate at different speeds in a vacuum. When the beam is sampled, there is a finite probability that a neutrino of flavor ν_β is detected, where $\beta \neq \alpha$. The transition probability can be written as:

$$P(\alpha \to \beta) = \left| \sum_i U_{\beta i} e^{-i(E_i t - p_i x)} U_{\alpha i}^* \right|^2$$

where the factor $U_{\alpha i}$ is the probability amplitude of mass eigenstate ν_i being produced at the source, the exponential factor describes the propagation of the mass eigenstate in space and time and the factor $U_{\beta i}$ is the probability amplitude of observing a ν_β interaction in the neutrino detector. This result is independent of the number of neutrino flavors.

The form of the oscillation probability is much simplified if only two neutrino generations are assumed to take part in the oscillation. In this case, the mixing matrix U takes the form:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{3.3.1}$$

The matrix U contains only one free parameter, the mixing angle θ. There is one Δm^2 parameter between the two neutrino mass eigenstates ν_1 and ν_2. Three-flavor neutrino oscillations are described by a 3×3 mixing matrix U and two independent Δm^2.

Chapter 4

Effect of Electromagnetic Field on Particles

4.1 Electron in an electromagnetic field

The wave equation of free particles express essentially only those properties which depend on the general requirements of space-time symmetry. However, physical processes involving the particles depend on their interaction properties. The wave equation of an electron in a given external field can be driven as follows: Let $A^\mu = (\phi, A)$ be the 4-potential of the external electromagnetic field. We obtain the desired equation on replacing the 4-momentum operator $\hat{p}$ in Dirac's equation by $\hat{p} - eA$

$$[\gamma (\hat{p} - eA) - m] \Psi = 0 \tag{4.1.1}$$

This equation can be solved for an electron moving in the field of an electromagnetic plane wave. The field plane wave with wave 4-vector $k(k^2 = 0)$ depends on the 4-coordinates only in the combination $\phi = kx$, so that the 4-potential is $A^\mu = A^\mu(\phi)$ and satisfies the Lorentz gauge condition

$$\partial_\mu A^\mu = k_\mu A'^\mu = 0$$

the prime denoting differentiation with respect to ϕ.

To solve this equation we start from the second order equation which we obtain by applying the oprator $\gamma(\hat{p} - eA) + m$ to the first order equation (4.1.1) to get

$$\left[\gamma^\mu \gamma^\nu (\hat{p}_\mu - eA)(\hat{p}_\nu - eA) - m^2\right] \Psi = 0 \tag{4.1.2}$$

Upon expansion we get

$$\left[-\partial^2 - 2ie(A\partial) + e^2 A^2 - m^2 - ie(\not{k})(\not{A}')\right] \Psi = 0 \tag{4.1.3}$$

We seek a solution of the form

$$\Psi = e^{-ipx} F(\phi) \tag{4.1.4}$$

where p is a constant 4-vector. The significance of the components of 4-vector p is more clearly seen in a particular frame of reference chosen so that $A_0 = 0$. Substituting equation (4.1.4) in (4.1.3) we obtain for $F(\phi)$ the equation

$$2i(kp)F' + \left[-2e(pA) + e^2 A^2 - ie(\not{k})(\not{A}')\right] F = 0$$

So Ψ becomes

$$\Psi_p = \left[1 + \frac{e}{2(kp)} \not{k}\not{A}\right] \frac{u}{\sqrt{2p_0}} e^{iS} \tag{4.1.5}$$

with

$$S = -px - \int_0^{kx} \left[\frac{e}{(kp)}(pA) - \frac{e^2}{2(kp)} A^2\right] d\phi$$

To determine the conditions to be imposed on the constant bispinor u, we must suppose that the wave is "switched on" with infinite slowness[9], starting from $t = -\infty$. Then $A \to 0$ when $kx \to -\infty$, and Ψ must become the solution of the free Dirac's equation. So that $u = u(p)$ must satisfy

$$(\gamma \cdot p - m)\, u = 0$$

Since u is independent of time, the condition remains valid for finite kx. Thus $u(p)$ is the same as the bispinor amplitude of a free plane wave[9].

In the next section we will see the propagator for massive bosons, $W^\pm$ and Z^0 and its relation with electron propagator.

4.2 Massive boson propagator

Let's now consider the problem of the propagator for particles with spin 1 and non-zero mass. There is then no arbitrariness of the gauge and the choice of the propagator is not ambiguous. Substituting the $\Psi-$operators given by

$$\begin{aligned} \Psi_\mu &= \sum_{p,\alpha} \frac{1}{\sqrt{2E}} \left(a_{p\alpha} u_\mu^{(\alpha)} e^{-ipx} + b_{p\alpha}^\dagger u_\mu^{(\alpha)*} e^{ipx} \right) \\ \Psi_\mu^\dagger &= \sum_{p,\alpha} \frac{1}{\sqrt{2E}} \left(a_{p\alpha}^\dagger u_\mu^{(\alpha)*} e^{ipx} + b_{p\alpha} u_\mu^{(\alpha)} e^{-ipx} \right) \end{aligned}$$

in the definition

$$G_{\mu\nu} = i\langle 0|T\Psi_\mu(x)\Psi_\nu^\dagger(x^{'})|0\rangle$$

we obtain the following expression

$$D_{ik}(\zeta) = \frac{1}{(2\pi)^3} \int \frac{2\pi i d^3k}{\omega} \left(\sum_\alpha u_i^{(\alpha)} u_k^{(\alpha)*} \right) e^{-i\omega|\tau| + ik\cdot\zeta} \tag{4.2.1}$$

Summation over polarizations is equivalent to averaging and multiplying by 3, the number of independent polarizations.

Thus we find for the propagator of vector particles

$$G_{\mu\nu}(p) = -\frac{1}{p^2 - m^2 + i\epsilon} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2} \right) \tag{4.2.2}$$

which has similar structure with electron propagator[9],

$$G(p) = \frac{\gamma p + m}{p^2 - m^2 + i\epsilon}$$

the denominators contain the difference $p^2 - m^2$, and the numerator is (apart from a factor) the density matrix of unpolarized particles with a given spin.

Chapter 5

Pair Production Using Neutrinos in an Intense Laser Field

The introduction of a background electromagnetic field can have very important consequences for neutrino physics. While the Standard Model neutrino does not itself couple to an electromagnetic field, the effects of such a field on neutrinos are made manifest through their interactions with charged particles. In this chapter we examine electron-positron pair production through the process $\nu \to \nu' e^- e^+$ in the presence of an intense laser field. Though normally forbidden, the electromagnetic field alters the final states of the electron-positron pair and frees the interaction to take place. Such a process could have very significant astrophysical and cosmological ramifications. We investigate the feasibility of detecting $\nu \to \nu' e^- e^+$ events using ultrashort pulsed lasers. Though not approaching the astronomical magnetic fields, todays femtosecond terawatt lasers can produce field strengths on the order of $E \sim 10^{11} V/cm (B \sim 10^9 G)$. We begin by presenting a review[3] of the derivation of the Volkov field operator solution for the electron(positron) in a circularly polarized electromagnetic field. We calculate the rate at which neutrinos in these fields produce pairs through the neutral channel. The process through the charged-current reaction is also considered independently. Next, we apply this Volkov solution to the calculation of the production rate for the process $\nu \to \nu' e^- e^+$. Lastly, we provide a tabulation of production rates for various incoming neutrino energies in the laser field and estimate the likelihood of detecting such a process.

5.1 Derivation of the field

The effect an electromagnetic field has on Standard Model neutrino physics is due to interaction with charged particles. For our process we incorporate the laser's intense field into the problem by using the field operator solution of the electron in the presence of an electromagnetic plane wave. We begin by solving the Dirac equation. The electromagnetic field is chosen to be circularly polarized and directed along the z-direction such that the 4-vector potential $A(x)$ is

$$A(x) = \begin{pmatrix} 0 \\ a\cos k\cdot x \\ a\sin k\cdot x \\ 0 \end{pmatrix} \tag{5.1.1}$$

where a is the magnitude of the vector potential and k is the 4-momentum directed along the z-direction. The choice of circularly polarized light makes the calculation much simpler than assuming linearly or elliptically polarized light.

The Dirac equation is given by

$$(i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu - m)\Psi(x) = 0 \tag{5.1.2}$$

To solve this equation we follow the derivation used in [10], employing light cone coordinates and light cone Dirac matrices. Equation (5.1.2) can be written as

$$\left[i\left(\gamma^0\partial_0 + \gamma^3\partial_3 + \gamma_\perp \cdot \partial_\perp\right) - e\left(\gamma^0 A_0 - \gamma^3 A_3 - \gamma_\perp \cdot A_\perp - m\right]\Psi(x) = 0 \tag{5.1.3}$$

with $\gamma_\perp = (\gamma_1, \gamma_2)$, $\partial_\perp = (\partial_1, \partial_2)$, $A_\perp = (A_1, A_2)$

We have

$$\gamma^0\partial_0 + \gamma^3\partial_3 = \frac{1}{2}(\gamma_+\partial_+ + \gamma_-\partial_-)$$

where

$$\gamma_+ = \gamma^0 + \gamma^3 \qquad \gamma_- = \gamma^0 + \gamma^3 \qquad \partial_+ = \partial_0 + \partial_3 \qquad \partial_- = \partial_0 - \partial_3$$

And, since $A_0 = A_3 = 0$, the term $\gamma^0 A_0 - \gamma^3 A_3$ vanishes.

Written in light-cone coordinate system the Dirac equation (5.1.2) now has the form

$$\big[\frac{i}{2}\big(\gamma_+\partial_+ + \gamma_-\partial_-\big) + i\gamma_\perp \cdot \partial_\perp + e\gamma_\perp \cdot A_\perp - m\big]\Psi(x) = 0 \tag{5.1.4}$$

The potential moves in the z-direction, hence it depends only on x_-

i.e.

$$A_\perp = A_\perp(x_-)$$

So we seek a solution of the form

$$\Psi \sim e^{-ip\cdot x}\phi(x_-) \tag{5.1.5}$$

Substituting to (5.1.4) we get

$$\begin{aligned}\Bigg[\frac{i}{2}\Bigg(\gamma_+(-i(p^0 - p^3)) + \gamma_-\big(-i(p^0 + p^3)\big)\Bigg) + (-\gamma_\perp \cdot p_\perp)\\ +\frac{i}{2}\gamma_-\partial_- + e\gamma_\perp \cdot A_\perp - m\Bigg]\phi(x_-) = 0\end{aligned} \tag{5.1.6}$$

which simplifies to

$$\big[\gamma_+p_- + \gamma_-p_+ + \frac{i}{2}\gamma_-\partial_- - \gamma_\perp \cdot p_\perp + e\gamma_\perp \cdot A_\perp - m\big]\phi(x_-) = 0 \tag{5.1.7}$$

Expressing the wave function in terms of its light-cone coordinate system $\phi = \frac{1}{2}(\phi_+ + \phi_-)$ with $\phi_\pm = \gamma_0\gamma_\pm\phi$ and $\gamma_\pm\phi_\mp = 0$, $\gamma_\pm\phi_\pm = 2\gamma_0\phi_\pm$ and plugging it to (5.1.7) gives

$$\big[\gamma_+p_- + \gamma_-p_+ + \frac{i}{2}\gamma_-\partial_- - \gamma_\perp \cdot p_\perp + e\gamma_\perp \cdot A_\perp - m\big]\frac{1}{2}(\phi_+ + \phi_-) = 0$$

$$\begin{aligned}\big[\gamma_+p_- - \gamma_\perp \cdot p_\perp + e\gamma_\perp \cdot A_\perp - m\big]\phi_+(x_-) + \big[\gamma_-p_+ + \frac{i}{2}\gamma_-\partial_- - \gamma_\perp \cdot p_\perp\\ +e\gamma_\perp \cdot A_\perp - m\big]\phi_-(x_-) = 0\end{aligned} \tag{5.1.8}$$

Multiplying by γ_- (from the left), and using the relations

$\gamma_-\phi_+ = 0$, and

$\gamma_-\gamma_+ = 2\gamma_0\gamma_+$ we obtain,

$$\big[2\gamma_o\gamma_+p_-\big]\phi_+(x_-) + \big[\big(\gamma_\perp\cdot p_\perp - e\gamma_\perp\cdot A_\perp - m\big)2\gamma_o\big]\phi_-(x_-) = 0 \qquad (5.1.9)$$

Thus,

$$\phi_+(x_-) = \frac{2\gamma_0}{4p_-}\big(\gamma_\perp\cdot p_\perp - e\gamma_\perp\cdot A_\perp + m\big)\phi_-(x_-) \qquad (5.1.10)$$

While multiplication with γ_+ gives

$$\big[\big(\gamma_\perp\cdot p_\perp - e\gamma_\perp\cdot A_\perp - m\big)2\gamma_0\big]\phi_+(x_-) + \big[4p_+ + 2i\partial_-\big]\phi_-(x_-) = 0 \qquad (5.1.11)$$

Substituting for ϕ_+ we get,

$$(\frac{e^2A_\perp^2 - 2ep_\perp\cdot A_\perp}{2p_-})\phi_-(x_-) = i\partial_-\phi_-(x_-) \qquad (5.1.12)$$

Here we used

$$4p_+p_- - p_\perp^2 = (p^0+p^3)(p^0-p^3) - p_\perp^2 = p^2 = m^2$$

This gives

$$\frac{\partial_-\phi_-}{\phi_-} = \frac{i}{2p_-}(2ep_\perp\cdot A_\perp - e^2A_\perp^2) \qquad (5.1.13)$$

Thus,

$$\phi_-(x_-) = exp\left(\int_0^{x_-} dx\frac{i}{2p_-}(2ep_\perp\cdot A_\perp - e^2A_\perp^2)\right)\phi_0 \qquad (5.1.14)$$

Substituting for $A(x)$ equation (5.1.1) we come up with

$$\phi_-(x_-) = exp\left(\frac{iea}{p\cdot k}(p_1\sin k\cdot x - p_2\cos k\cdot x) - \frac{e^2a^2k\cdot x}{2p\cdot k}\right)\phi_0 \qquad (5.1.15)$$

So we have

$$\begin{aligned}\phi_+(x_-) \;=\; & \frac{\gamma_0}{2p_-}\left(\gamma_\perp\cdot p_\perp - e\gamma_\perp\cdot A_\perp + m\right) exp\Bigg(\frac{iea}{p\cdot k}(p_1\sin k\cdot x - p_2\cos k\cdot x) \\ & -\frac{e^2a^2k\cdot x}{2p\cdot k}\Bigg)\phi_0 \qquad (5.1.16)\end{aligned}$$

where ϕ_0 is an integration constant satisfying $\gamma_+\phi_0 = 0$.

We choose it to be $\phi_0 = \gamma_0\gamma_- u^s(p)$

where $u^s(p)$ satisfies, as we saw in section (4.1)

$$(\not{p} - m)\, u^s(p) = 0$$

So we have

$$(\gamma_+ p_- + \gamma_- p_+)\, u^s(p) = (\gamma_\perp \cdot p_\perp)\, u^s(p)$$

This gives,

$$\phi(x_-) = \left(1 - \frac{e\gamma_-\gamma_\perp \cdot A_\perp}{4p_-}\right) exp\left(\frac{iea}{p\cdot k}(p_1 \sin k\cdot x - p_2 \cos k\cdot x) - \frac{e^2a^2k\cdot x}{2p\cdot k}\right) u^s(p) \tag{5.1.17}$$

but $\gamma_\perp \cdot A_\perp = -\not{A}$ and $\dfrac{\gamma_-}{4p_-} = \dfrac{\omega\gamma_-}{\omega 4p_-} = \dfrac{\not{k}}{2p\cdot k}$

Now (5.1.17) changes to

$$\phi(x_-) = \left(1 + \frac{e\not{k}\not{A}}{2p\cdot k}\right) exp\left(\frac{iea}{p\cdot k}(p_1 \sin k\cdot x - p_2 \cos k\cdot x) - \frac{e^2a^2k\cdot x}{2p\cdot k}\right) u^s(p) \tag{5.1.18}$$

Therefore the Dirac equation solution for electron in electromagnetic field in terms of Volkov solution is

$$\Psi_{e^-}(x) \sim \left(1 + \frac{e\not{k}\not{A}}{2p\cdot k}\right) exp\left(\frac{iea}{p\cdot k}(p_1 \sin k\cdot x - p_2 \cos k\cdot x) - \frac{e^2a^2k\cdot x}{2p\cdot k}\right) e^{-ip\cdot x} u^s(p) \tag{5.1.19}$$

Similarly, for the positron we have

$$\Psi_{e^+}(x) \sim \left(1 - \frac{e\not{k}\not{A}}{2p\cdot k}\right) exp\left(\frac{iea}{p\cdot k}(p_1 \sin k\cdot x - p_2 \cos k\cdot x) - \frac{e^2a^2k\cdot x}{2p\cdot k}\right) e^{-ip\cdot x} \nu^s(p) \tag{5.1.20}$$

In the relativistic treatment the usual normalization condition; $\int \rho(x)dV = 1$ is unsatisfactory[11]. For flying particles the volume element dV is Lorentz contracted; and the space-integral in the rest frame overestimates the volume by $\gamma = \dfrac{E}{m}$. It is better to adopt a normalization that is proportional to γ. We choose to set

$$\int \rho(x)dV = 2E$$

Writing plane wave solution obeying the Dirac equation as

$$\Psi = N u_s(p) e^{-p\cdot x}$$

the normalization condition is $\int \rho d^3x = \int d^3\Psi^*\Psi$ which implies $N = \frac{1}{\sqrt{V}}$. In the above treatment we integrated the density over a large but finite volume, $V = L^3$, which in turn means that the momentum is discrete. The number of wave states in the volume is given by $p_i L = 2\pi n_i$ i.e. $\Delta n_i = (\frac{L}{2\pi})^3 \Delta p$. The discrete sum becomes an integral in the large volume limit:

$$\sum_p = \sum_n \Delta n = (\frac{2}{2\pi})^3 \sum_p \Delta p \rightarrow V \int \frac{d^3p}{(2\pi)^3} \tag{5.1.21}$$

The integral of the plane wave in the discrete format is

$$\frac{1}{V}\int d^3x e^{i(p-p')\cdot x} = \delta_{p,p'} \xrightarrow{V\rightarrow\infty} \frac{(2\pi)^3}{V}\delta^3(p-p')$$

The one particle state orthogonality and completeness conditions are now

$$\langle p'|p\rangle = \frac{(2\pi)^3}{V} 2E_p \delta^3(p'-p)$$
$$1 = V\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E}|p'\rangle\langle p|$$

The commutator of the discrete creation/annihilation operators become in continuous limit

$$\left[a_{pr}, a^\dagger_{p's}\right] = \frac{(2\pi)^3}{V}\delta_{rs}\delta^3(p-p')$$

Then the 1-particle state is expressed in terms of the creation operator as

$$|p\rangle = \sqrt{2E}a^\dagger(p)|0\rangle$$

With the above definition, the expansion formula for the fields now become

$$\Psi(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E}}\sum_s \left[a_p u_s e^{-ip\cdot x} + b^\dagger \nu_s e^{ip\cdot x}\right] \tag{5.1.22}$$

So for our case the Volkov operator can be written as

$$\Psi(x) = \sum_{s,p}\left(a^s_p \Psi_{e^-}(x) + b^{\dagger s}_p \Psi_{e^+}(x)\right)$$

where a_p^s is annihilation operator for electron and $b_p^{\dagger s}$ is creation operator for positron. The normalized field operator, therefore, is

$$\begin{aligned}\Psi(x) &= \int \frac{d^3p}{(2\pi)^3} exp\left(\frac{iea}{p\cdot k}(p_1 \sin k\cdot x - p_2 \cos k\cdot x)\right) \\ &\times \sum_s \left[(1+\frac{e\not{k}\not{A}}{2p\cdot k}) exp\left(-i(p+\frac{e^2a^2}{2p\cdot k}k)\cdot x\right)\frac{u^s(p)}{\sqrt{2E}}a_p^s\right. \\ &\left.+(1-\frac{e\not{k}\not{A}}{2p\cdot k}) exp\left(i(p+\frac{e^2a^2}{2p\cdot k}k)\cdot x\right)\frac{\nu^s(p)}{\sqrt{2E}}b_p^{\dagger s}\right] \end{aligned} \tag{5.1.23}$$

The summation over s is a sum over possible spin states, p_1 and p_2 are the components of the momentum of the particle along x and y directions respectively. Here we see that

1. The momentum that is to be identified with the Volkov state is

$$q^\mu = p^\mu + \frac{e^2a^2}{2p\cdot k}k^\mu \tag{5.1.24}$$

2. Squaring this 4-momentum shows that the mass of the Volkov electron has a dependence on the strength of the electromagnetic field.

$$m_e^{*2} = m_e^2 + e^2a^2 \tag{5.1.25}$$

3. In the absence of electromagnetic field $a \to 0$, the Volkov field operator reduces to

$$\Psi(x) = \int \frac{d^3\overrightarrow{p}}{(2\pi)^3} \sum \left(e^{-ip\cdot x}\frac{u^s(p)}{\sqrt{2E}}a_p^s + e^{ip\cdot x}\frac{\nu^s(p)}{\sqrt{2E}}b_p^{\dagger s}\right) \tag{5.1.26}$$

and the Volkov mass m_e^* to the electron mass m_e.

5.2 Rate of production

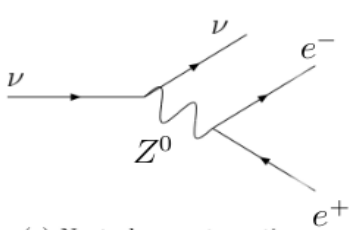
A quantity of interest to be calculated for this process is the rate of production Γ. Physically, the rate of production is the probability per unit time for the neutrino to emit an electron-positron pair in the presence of the laser field. We begin by finding the probability $\mathcal{P}$ defined by,

$$\mathcal{P} = \left(\prod_f \int \frac{d^3\overrightarrow{p}_f}{(2\pi)^3 2E_f}\right) \int \frac{d^3\overrightarrow{p}_{\nu'}}{(2\pi)^3 2E_{\nu'}} \frac{1}{2} \sum_{s_\nu} \sum_{s_{\nu'}, s_{e^-}, s_{e^+}} \left| \left\langle p_{\nu'}, s_{\nu'}; p_{e^-}, s_{e^-}; p_{e^+}, s_{e^+} \middle| \hat{S} \middle| p_\nu, s_\nu \right\rangle \right|^2 \tag{5.2.1}$$

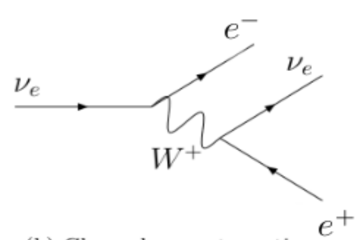
Here, we have summed over the spin state of the final neutrino ν', electron e^- and positron e^+, and averaged over the spin state of the incoming neutrino ν.

The phase-space integral over the final momentum $p_{e^-},\ p_{e^+},\ p_{\nu'}$ has been simplified into the form

$$\left(\prod_f \int \frac{d^3\overrightarrow{p}_f}{(2\pi)^3 2E_f}\right) = \int \frac{d^3\overrightarrow{p}_{\nu'}}{(2\pi)^3} \frac{1}{2E_{\nu'}} \int \frac{d^3\overrightarrow{p}_{e^-}}{(2\pi)^3} \frac{1}{2E_{e^-}} \int \frac{d^3\overrightarrow{p}_{e^+}}{(2\pi)^3} \frac{1}{2E_{e^+}}$$



(a) Neutral current reaction



(b) Charged current reaction

Figure 5.1: Possible diagrams considered for the process $\nu \to \nu' e^- e^+$

5.2.1 Rate of production through the neutral channel

According to equation (2.2.9) the scattering operator is given by

$$\hat{S} = \frac{-e^2}{4\cos^2\theta_w \sin^2\theta_w} \int d^4x \bar{\Psi}(x)\gamma^\mu\big(-\frac{1}{2} + 2\sin^2\theta_w + \frac{\gamma^5}{2}\big)\Psi(x)Z_\mu(x) \times \int d^4x \bar{\Psi}_{\nu'}(y)\gamma^\sigma\big(\frac{1}{2} - \frac{\gamma^5}{2}\big)\Psi_\nu(y)Z_\sigma(y) \quad (5.2.2)$$

where $\Psi(x)$ is the Volkov field operator given in equation (5.1.23), $\Psi_\nu(y)$ is the free-field operator for the neutrino and $Z_\mu(x)$ is the field operator of the Z boson, α is the fine-structure constant and θ_w is the weak-mixing angle.

The scattering matrix can be written as

$$S_{fi} = \langle p_{\nu'}, s_{\nu'}; p_{e^-}, s_{e^-}; p_{e^+}, s_{e^+}|\hat{S}|p_\nu, s_\nu\rangle \quad (5.2.3)$$

To evaluate this S-matrix with $\hat{S}$ given by (5.2.2), we apply the Wick's theorem[12]. In addition, we use the anti-commutation rules of creation and annihilation operators

$\{a_{rp}, a^\dagger_{sq}\} = \{b_{rp}, b^\dagger_{sq}\} = (2\pi)^3\delta^3(p-q)\delta^{rs}$

with all other anti-commutators equal to zero, and the vacuum $|0\rangle$ is defined to be the state such that

$a_{sp}|0\rangle = b_{sp}|0\rangle = 0$

With these requirements the only expressions that contribute to the S-matrix are :

$$\bar{\Psi}_{e^-}(x) = \bar{u}^s(p)exp\left[\frac{-iea}{p_{e^-}\cdot k}\left(p_{1e^-}\sin k\cdot x - p_{2e^-}\cos k\cdot x\right)\right]\left(1 + \frac{ekA}{2p_{e^-}\cdot k}\right)e^{iq_{e^-}\cdot x} \quad (5.2.4)$$

$$\Psi_{e^+} = exp\left[\frac{iea}{p_{e^+}\cdot k}\left(p_{1e^+}\sin k\cdot x - p_{2e^+}\cos k\cdot x\right)\right]\left(1 - \frac{ekA}{2p_{e^+}\cdot k}\right)e^{iq_{e^+}\cdot x}\nu^s(p) \quad (5.2.5)$$

$$\bar{\Psi}_{\nu'}(x) = \bar{u}^s_{\nu'}(p)e^{ip_{\nu'}\cdot x} \quad (5.2.6)$$

$$\Psi_\nu(x) = u^s_\nu(p)e^{-ip_\nu\cdot x} \quad (5.2.7)$$

While calculating the $S-$matrix, with these expressions substituting to equation (5.2.3), the integration in the free field case over the space-time variable y results in a 4-dimensional $\delta-$function that conserves energy and momentum. The boson propagator $\langle 0|TZ_\mu(x)Z_\sigma(y)|0\rangle$, as done in section (4.2), is given by

$$\langle 0|TZ_\mu(x)Z_\sigma(y)|0\rangle = \frac{g_{\mu\sigma} - \dfrac{(p_{\nu'}-p_\nu)_\mu(p_{\nu'}-p_\nu)_\sigma}{m_Z^2}}{(p_{\nu'}-p_\nu)^2 - m_Z^2 + i\Gamma_Z m_Z} \tag{5.2.8}$$

So the scattering matrix is

$$\begin{aligned} S_{fi} \quad = \quad & \frac{-4\pi\alpha}{2^4\cos^2\theta_w\sin^2\theta_w}\frac{g_{\mu\sigma} - \dfrac{(p_{\nu'}-p_\nu)_\mu(p_{\nu'}-p_\nu)_\sigma}{m_Z^2}}{(p_{\nu'}-p_\nu)^2 - m_Z^2 + i\Gamma_z m_Z}\bar{u}^s_{\nu'}(p)\gamma^\sigma(1-\gamma^5)u^s_\nu(p) \\ & \int d^4x\Bigg[\bar{u}^s_{e^-}(p)\left(1+\frac{e\not{k}\not{A}}{2p_{e^-}\cdot k}\right)\Gamma_0^\mu\left(1-\frac{e\not{k}\not{A}}{2p_{e^+}\cdot k}\right)\nu^s_{e^+}(p) \\ & exp\Big(-i(\frac{eap_{1e^-}}{p_{e^-}\cdot k}-\frac{eap_{1e^+}}{p_{e^+}\cdot k})\sin k\cdot x + i(\frac{eap_{2e^-}}{p_{e^-}\cdot k}-\frac{eap_{2e^+}}{p_{e^+}\cdot k})\cos k\cdot x\Big) \\ & exp\Big(i(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu)\Big)\cdot x\Bigg] \end{aligned} \tag{5.2.9}$$

$\Gamma_0^\mu = -1 + 4\sin^2\theta_w + \gamma^5$

But, the term

$$\bar{u}^s_{e^-}(p)\left(1+\frac{e\not{k}\not{A}}{2p_{e^-}\cdot k}\right)\Gamma_0^\mu\left(1-\frac{e\not{k}\not{A}}{2p_{e^+}\cdot k}\right)\nu^s_{e^+}(p) \tag{5.2.10}$$

simplifies to

$$\overline{u}^s_{e^-}(p)\left[\Gamma_0^\mu - \frac{e\Gamma_0^\mu\not{k}\not{A}}{2p_{e^+}\cdot k} + \frac{e\not{k}\not{A}}{2p_{e^-}\cdot k}\Gamma_0^\mu - \frac{e^2\not{k}\not{A}\Gamma_0^\mu\not{k}\not{A}}{4(p_{e^-}\cdot k)(p_{e^+}\cdot k)}\right]\nu^s_{e^+}(p) \tag{5.2.11}$$

Substituting the explicit values of A we come up with

$$\begin{aligned} \bar{u}^s_{e^-}(p)\Big[&\Gamma_0^\mu + \Gamma_x^\mu\cos k\cdot x + \Gamma_y^\mu\sin k\cdot x - \Gamma_{xx}^\mu\cos^2 k\cdot x - \Gamma_{yy}^\mu\sin^2 k\cdot x \\ & -\Gamma_{xy}^\mu\sin k\cdot x\cos k\cdot x\Big]\nu^s_{e^+} \end{aligned} \tag{5.2.12}$$

where Γ_i^μ are given by

$$\Gamma_0^\mu = \gamma^\sigma(-1 + 4\sin^2\theta_w + \gamma^5) \tag{5.2.13}$$

$$\Gamma_x^\mu = -\left(\frac{ea\not{k}\gamma^1}{2p_{e^-}\cdot k}\Gamma_0^\mu - \Gamma_0^\mu\frac{ea\not{k}\gamma^1}{2p_{e^+}\cdot k}\right) \tag{5.2.14}$$

$$\Gamma_y^\mu = -\left(\frac{ea\not{k}\gamma^2}{2p_{e^-}\cdot k}\Gamma_0^\mu - \Gamma_0^\mu\frac{ea\not{k}\gamma^2}{2p_{e^+}\cdot k}\right) \tag{5.2.15}$$

$$\Gamma_{xx}^\mu = \frac{e^2a^2\not{k}\gamma^1\Gamma_0^\mu\not{k}\gamma^1}{(2p_{e^-}\cdot k)(2p_{e^+}\cdot k)} \tag{5.2.16}$$

$$\Gamma_{xy}^\mu = \frac{e^2a^2\not{k}\gamma^1\Gamma_0^\mu\not{k}\gamma^2}{(2p_{e^-}\cdot k)(2p_{e^+}\cdot k)} + \frac{e^2a^2\not{k}\gamma^1\Gamma_0^\mu\not{k}\gamma^2}{(2p_{e^-}\cdot k)(2p_{e^+}\cdot k)} \tag{5.2.17}$$

$$\Gamma_{yy}^\mu = \frac{e^2a^2\not{k}\gamma^2\Gamma_0^\mu\not{k}\gamma^2}{(2p_{e^-}\cdot k)(2p_{e^+}\cdot k)} \tag{5.2.18}$$

γ^1 and γ^2 being Dirac matrices. Letting

$$\frac{eap_{1e^-}}{p_{e^-}\cdot k} - \frac{eap_{1e^+}}{p_{e^+}\cdot k} = \zeta\cos\sigma$$

and

$$\frac{eap_{2e^-}}{p_{e^-}\cdot k} - \frac{eap_{2e^+}}{p_{e^+}\cdot k} = \zeta\sin\sigma$$

we have

$$exp\Big(-i(\frac{eap_{1e^-}}{p_{e-}\cdot k} - \frac{eap_{1e^+}}{p_{e^+}\cdot k})\sin k\cdot x + i(\frac{eap_{2e^-}}{p_{e^-}\cdot k} - \frac{eap_{2e^+}}{p_{e^+}\cdot k})\cos k\cdot x\Big) = e^{-i\zeta\sin(k\cdot x-\sigma)}$$

Now equation (5.2.9) has the following form

$$\begin{aligned} S_{fi} \;=\; & \frac{-4\pi\alpha}{2^4\cos^2\theta_w\sin^2\theta_w}\frac{g_{\mu\sigma} - \dfrac{(p_{\nu'}-p_\nu)_\mu(p_{\nu'}-p_\nu)_\sigma}{m_Z^2}}{(p_{\nu'}-p_\nu)^2 - m_Z^2 + i\Gamma_Z m_Z}\bar{u}_{\nu'}\gamma^\sigma(1-\gamma^5)u_\nu\int d^4x\bar{u}^s_{e^-}(p) \\ & \Big[\Gamma_0^\mu + \Gamma_x^\mu\cos k\cdot x + \Gamma_y^\mu\sin k\cdot x - \Gamma_{xx}^\mu\cos^2 k\cdot x - \Gamma_{yy}^\mu\sin^2 k\cdot x \\ & -\Gamma_{xy}^\mu\sin k\cdot x\cos k\cdot x\Big]\nu^s_{e^+}(p)e^{-i\zeta\sin(k\cdot x-\sigma)}e^{i(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu)\cdot x} \end{aligned} \tag{5.2.19}$$

To make equation (5.2.19) analytically integrable, we apply Hansen's definition of the Bessel's function[13].

$$e^{\frac{1}{2}(z(t-\frac{1}{t}))} = \sum_{n=-\infty}^{\infty} J_n(z)t^n$$

Since equation (5.2.19) has terms containing the multiplication of $\sin(k \cdot x)$ and $\cos(k \cdot x)$ with $e^{-i\zeta \sin(k \cdot x - \delta)}$, we need to expand

$$\begin{pmatrix} \sin(k \cdot x) \\ \cos(k \cdot x) \\ \sin^2(k \cdot x) \\ \cos^2(k \cdot x) \\ \sin(k \cdot x)\cos(k \cdot x) \end{pmatrix} \times e^{-i\zeta \sin(k \cdot x - \delta)} \tag{5.2.20}$$

to Bessel function. So substituting $t = e^{-i(k \cdot x - \sigma)}$ terms containing products of trigonometric functions and exponentials will all be changed to Bessel functions. After these expansions are made, the space time integral in the S-matrix reduces to the normal 4-dimensional $\delta-$function[3].

$$S_{fi} \propto \int d^4x \; exp[i(q_{e^-} + q_{e^+} + p_{\nu'} - p_\nu - nk) \cdot x]$$

i.e.

$$S_{fi} \propto \delta^4 \left(q_{e^-} + q_{e^+} + p_{\nu'} - p_\nu - nk\right) \tag{5.2.21}$$

Physically, equation (5.2.21) suggests that it is the Volkov states that are to be considered when conserving momentum and energy. It also suggests that we interpret the nk term as the number of photons that are pulled from the laser field in order to drive this reaction. These photons are what allow this process to overcome the energy and momentum constraints that originally prohibited the reaction in the free field case.

In fact the $\delta-$function can be used to apply a constraint on the possible values for n

$$\begin{aligned} p_\nu^\mu + nk &= q_{e^-}^\mu + q_{e^+}^\mu + p_{\nu'}^\mu \\ (p_\nu^\mu + nk)^2 &= (q_{e^-}^\mu + q_{e^+}^\mu + p_{\nu'}^\mu)^2 \\ 2np_\nu \cdot k &\geq 4(m_e^*)^2 \end{aligned} \tag{5.2.22}$$

Taking the neutrino and laser fields to be oppositely directed, in the massless limit of the neutrino the constraint equation (5.2.22) becomes

$$\begin{aligned} 4nE_\nu E_\gamma &\geq 4(m_e^*)^2 \\ n &\geq \frac{(m_e^*)^2}{E_\nu E_\gamma} \end{aligned} \tag{5.2.23}$$

where E_γ is the energy carried per photon. It is easy to note that the reaction cannot emit photon ($n \not< 0$), nor can it proceed without absorbing some number of photons ($n \neq 0$) [3].

Now the scattering matrix is written as

$$S_{fi} = -\int d^4x \sum_{n=-\infty}^{\infty} e^{i(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu-nk)\cdot x}\mathcal{M}_n \tag{5.2.24}$$

This gives us that

$$S_{fi} = -i(2\pi)^4 \sum_{n=-\infty}^{\infty} \delta^4(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu-nk)\mathcal{M}_n \tag{5.2.25}$$

The couplings, spinors, Bessel functions, etc are absorbed into the scattering matrix $\mathcal{M}_n$ which is given by

$$\begin{aligned} \mathcal{M}_n \;=\; & \frac{4\pi\alpha}{2^4 sin_w^\theta \cos^2\theta_w}\bar{u}^s_{\nu'}(p_{\nu'})\gamma^\sigma(1-\gamma^5)u^s_\nu(p_\nu)\frac{g_{\mu\sigma}-(p_\nu-p_{\nu'})_\mu(p_\nu-p_{\nu'})_\sigma/m_z^2}{(p_\nu-p_{\nu'})^2-m_z^2+i\Gamma_z m_z} \\ & \times\bar{u}^s_{e^-}(p_{e^-})\Big(C_{0n}(\zeta,\delta)\Gamma_0^\mu + C_{xn}(\zeta,\delta)\Gamma_x^\mu + C_{yn}(\zeta,\delta)\Gamma_y^\mu + C_{xxn}(\zeta,\delta)\Gamma_{xx}^\mu \\ & + C_{xyn}(\zeta,\delta)\Gamma_{xy}^\mu + C_{yyn}(\zeta,\delta)\Gamma_{yy}^\mu\Big)\nu^s_{e^+}(p_{e^+}) \end{aligned} \tag{5.2.26}$$

where the $C_n(\zeta,\delta)'s$ are coefficients that depend on the Bessel functions

$$C_{0n}(\zeta,\delta) = J_n(\zeta)e^{in\delta} \tag{5.2.27}$$

$$C_{xn}(\zeta,\delta) = \frac{1}{2}\left(e^{i(n+1)\delta}J_{n+1}(\zeta) + e^{i(n-1)\delta}J_{n-1}(\zeta)\right) \tag{5.2.28}$$

$$C_{yn}(\zeta,\delta) = \frac{1}{2i}\left(e^{i(n+1)\delta}J_{n+1}(\zeta) - e^{i(n-1)\delta}J_{n-1}(\zeta)\right) \tag{5.2.29}$$

$$C_{xxn}(\zeta,\delta) = \frac{1}{2}(e^{in\delta}J_n(\zeta)) + \frac{1}{4}\left(e^{i(n+2)\delta}J_{n+2}(\zeta) + e^{i(n-2)\delta}J_{n-2}(\zeta)\right) \tag{5.2.30}$$

$$C_{xyn}(\zeta,\delta) = \frac{1}{4i}\left(e^{i(n+2)\delta}J_{n+2}(\zeta) - e^{i(n-2)\delta}J_{n-2}(\zeta)\right) \tag{5.2.31}$$

$$C_{yyn}(\zeta,\delta) = \frac{1}{2}(e^{in\delta}J_n(\zeta)) - \frac{1}{4}\left(e^{i(n+2)\delta}J_{n+2}(\zeta) + e^{i(n-2)\delta}J_{n-2}(\zeta)\right) \tag{5.2.32}$$

We substitute the square of the S-matrix (equation 5.2.25) into the probability equation (5.2.1).

Now the probability takes the form

$$\mathcal{P} = \left(\prod_f \int \frac{d^3\overrightarrow{p}_f}{(2\pi)^3}\frac{1}{2E_f}\right)\int \frac{d^3\overrightarrow{p}_\nu}{(2\pi)^3}\frac{1}{2E_\nu}\frac{1}{2}\sum_{s_\nu}\sum_{s_{\nu'},s_{e^-},s_{e^+}}\sum_{n,m=-\infty}^{\infty}$$
$$(2\pi)^8\delta^4(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu-nk)\delta^4(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu-mk)\mathcal{M}_n\mathcal{M}_m^* \tag{5.2.33}$$

By inspection, one can see that the two 4-dimensional $\delta-$functions imply that for there to be any contribution to the summation, either there is no incoming photon energy ($E_\gamma = 0$) or, more appropriately, that $m = n$. Therefore, we can replace $\mathcal{M}_m^*\mathcal{M}_n$ with the square of the norm of the scattering amplitude $|\mathcal{M}_n|^2$ and eliminate the sum over m.

Defining the square of the scattering matrix, after summing and averaging over spins to be[3]

$$\overline{|\mathcal{M}_n|^2} = \frac{1}{2}\sum_{s_\nu}\sum_{s_{e^-},s_{e^+},s_{\nu'}}|\mathcal{M}_n|^2$$

We have

$$\mathcal{P} = \left(\prod_f \int \frac{d^3\overrightarrow{p}_f}{(2\pi)^3}\frac{1}{2E_f}\right)\int \frac{d^3\overrightarrow{p}_\nu}{(2\pi)^3}\frac{1}{2E_\nu}\sum_{n=-\infty}^{\infty}(2\pi)^8$$
$$\left(\delta^4(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu-nk)\right)^2\overline{|\mathcal{M}_n|^2} \tag{5.2.34}$$

Thus,

$$\mathcal{P} = \left(\prod_f \int \frac{d^3\overrightarrow{p}_f}{(2\pi)^3}\frac{1}{2E_f}\right)\frac{1}{2E_\nu}\sum_{n=-\infty}^{\infty}(2\pi)^5\delta^4(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu-nk)$$
$$\delta(E_{e^-}+E_{e^+}+E_{\nu'}-E_\nu-nE_\gamma)\overline{|\mathcal{M}_n|^2} \tag{5.2.35}$$

Using

$$\int d^3\overrightarrow{p}_\nu\delta^4(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu-nk) = \int d^3\overrightarrow{p}_\nu\delta^3(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu-nk)$$
$$\times\delta(E_{e^-}+E_{e^+}+E_{\nu'}-E_\nu-nE_\gamma)$$
$$= \delta(E_{e^-}+E_{e^+}+E_{\nu'}-E_\nu-nE_\gamma)$$

the integral over the momentum of the incoming neutrino is eliminated. Then we have

$$\mathcal{P} = \left(\prod_f \int \frac{d^3\overrightarrow{p}_f}{(2\pi)^3}\frac{1}{2E_f}\right)\frac{1}{2E_\nu}\sum_{n=-\infty}^{\infty}(2\pi)^4\delta^4(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu-nk) \int dt\ e^{i(E_{e^-}+E_{e^+}+E_{\nu'}-E_\nu-nE_\gamma)t}\overline{|\mathcal{M}_n|^2} \tag{5.2.36}$$

Following the way outlined in[14]

i.e. $E_{e^-}+E_{e^+}+E_{\nu'}=E_\nu-nE_\gamma$ we get

$$\mathcal{P} = \left(\prod_f \int \frac{d^3\overrightarrow{p}_f}{(2\pi)^3}\frac{1}{2E_f}\right)\frac{1}{2E_\nu}\sum_{n=-\infty}^{\infty}(2\pi)^4\delta^4(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu-nk)\lim_{T\to\infty}\int_{-T/2}^{T/2}dt\overline{|\mathcal{M}_n|^2}$$

$$\mathcal{P} = \left(\prod_f \int \frac{d^3\overrightarrow{p}_f}{(2\pi)^3}\frac{1}{2E_f}\right)\frac{T}{2E_\nu}\sum_{n=-\infty}^{\infty}(2\pi)^4\delta^4(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu-nk)\overline{|\mathcal{M}_n|^2} \tag{5.2.37}$$

The rate of production is given by

$$\Gamma = \frac{\mathcal{P}}{T} = \left(\prod_f \int \frac{d^3\overrightarrow{p}_f}{(2\pi)^3}\frac{1}{2E_f}\right)\frac{1}{2E_\nu}\sum_{n=-\infty}^{\infty}(2\pi)^4\delta^4(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu-nk)\overline{|\mathcal{M}_n|^2} \tag{5.2.38}$$

Expressing the total production rate as the sum of the production rates for all of the individual processes involving n photons, the total rate is given by

$$\Gamma = \sum_n \Gamma_n \tag{5.2.39}$$

with

$$\Gamma_n = \left(\prod_f \int \frac{d^3\overrightarrow{p}_f}{(2\pi)^3}\frac{1}{2E_f}\right)\frac{1}{2E_\nu}(2\pi)^4\delta^4(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu-nk)\overline{|\mathcal{M}_n|^2} \tag{5.2.40}$$

We next present the rate of production for the charged current reaction.

5.2.2 Rate of production through the charged channel

The charged current reaction is relevant only for electron neutrino. The scattering matrix is expressed as

$$S_W = \langle p_{\nu_e'}, s_{\nu_e'}; p_e, s_e; p_{e^+}, s_{e^+}|\hat{S}_W|p_{\nu_e}, s_{\nu_e}\rangle \tag{5.2.41}$$

where the scattering operator, according to equation (2.2.9) is given by

$$\begin{aligned} \hat{S}_W &= -\frac{e^2}{2^3 \sin^2\theta_w}\int d^4x \overline{\Psi}(x)\gamma^\mu(1-\gamma_5)\Psi_{\nu_e}(x)W^-(x) \\ & \int d^4y \overline{\Psi}_{\nu_e}(y)\gamma^\sigma(1-\gamma_5)\Psi(y)W^+(y) \end{aligned} \tag{5.2.42}$$

To carry out the integration we use the same analysis as in section (5.2.1), which leads to the following expressions contributing to S_W.

$$\bar{\Psi}_e(x) = \bar{u}^s(p)exp\left[\frac{-iea}{p_e\cdot k}\left(p_{1e}\sin k\cdot x - p_{2e}\cos k\cdot x\right)\right]\left(1+\frac{ekA}{2p_e\cdot k}\right)e^{iq_e\cdot x} \tag{5.2.43}$$

$$\Psi_{e^+} = exp\left[\frac{iea}{p_{e^+}\cdot k}\left(p_{1e^+}\sin k\cdot x - p_{2e^+}\cos k\cdot x\right)\right]\left(1-\frac{ekA}{2p_{e^+}\cdot k}\right)e^{iq_{e^+}\cdot x}\nu^s(p) \tag{5.2.44}$$

$$\bar{\Psi}_{\nu_e'}(x) = \bar{u}^s_{\nu_e'}(p)e^{ip_{\nu_e'}\cdot x} \tag{5.2.45}$$

$$\Psi_{\nu_e}(x) = u^s_{\nu_e}(p)e^{-ip_{\nu_e}\cdot x} \tag{5.2.46}$$

Upon substitution to equation (5.2.41) we get

$$\begin{aligned} S_W &= -\frac{e^2}{2^3\sin^2\theta_w}\frac{g_{\mu\sigma}-(p_{\nu_e'}-p_{\nu_e})_\mu(p_{\nu_e'}-p_{\nu_e})_\sigma/m_W^2}{(p_{\nu_e'}-p_{\nu_e})^2-m_W^2+i\Gamma_W m_W}\int d^4x\Bigg[\bar{u}^s_e(p)\left(1+\frac{e\not{k}\not{A}}{2p_e\cdot k}\right) \\ & \gamma^\mu(1-\gamma^5)u^s_{\nu_e}(p)e^{(q_e-p_{\nu_e})\cdot y}exp\Big(-i(\frac{eap_{1e}}{p_e\cdot k})\sin k\cdot x + i(\frac{eap_{2e}}{p_e\cdot k})\cos k\cdot x\Big)\Bigg] \\ & \int d^4y\Bigg[\bar{u}^s_{\nu_e}(p)\gamma^\sigma(1-\gamma^5)\left(1-\frac{e\not{k}\not{A}}{2p_{e^+}\cdot k}\right)\nu^s_{e^+}(p)e^{i(p_{\nu_e'}+q_{e^+})\cdot y}exp\big(i\frac{eap_{1e^+}}{p_{e^+}\cdot k}\big) \\ & \sin k\cdot y - (\frac{ieap_{2e^+}}{p_{e^+}\cdot k})\cos k\cdot y\Bigg] \end{aligned} \tag{5.2.47}$$

To fashion this equation into integrable form, we need to simplify the expressions as follows:

$$\bar{u}^s_e(p)\left(1+\frac{e\not{k}\not{A}}{2p_e\cdot k}\right)\Gamma^\mu u^s_{\nu_e}(p) = \bar{u}^s_e(p)\Big(\Gamma^\mu + \Gamma^\mu_x\cos k\cdot x + \Gamma^\mu_y \sin k\cdot x\Big)u^s_{\nu_e}(p)$$

with $\Gamma^\mu = \gamma^\mu(1-\gamma^5)$,

$$\bar{u}^s_{\nu_e}(p)\Gamma^\sigma\left(1-\frac{e\not{k}\not{A}}{2p_{e^+}\cdot k}\right)\nu^s_{e^+}(p) = \bar{u}^s_{\nu_e}(p)\Big(\Gamma^\sigma - \Gamma^\sigma_x\cos k\cdot y - \Gamma^\sigma_y \sin k\cdot y\Big)\nu^s_{e^+}(p) \quad (5.2.48)$$

with $\Gamma^\sigma = \gamma^\sigma(1-\gamma^5)$,

$$exp\Big(\frac{-iea}{p_e\cdot k}p_{1e}\sin k\cdot x + \frac{iea}{p_e\cdot k}p_{2e}\cos k\cdot x\Big) = \sum_n e^{-ink\cdot x}e^{in\delta_1}J_n(\zeta) \quad (5.2.49)$$

and

$$exp\Big(\frac{iea}{p_{e^+}\cdot k}p_{1e^+}\sin k\cdot y + \frac{iea}{p_{e^+}\cdot k}p_{2e^+}\cos k\cdot y\Big) = \sum_m e^{imk\cdot y}e^{im\delta_2}J_m(\zeta) \quad (5.2.50)$$

Here we used

$$\frac{ea}{p_e\cdot k}p_{1e} = \zeta\cos\delta_1, \qquad \frac{ea}{p_e\cdot k}p_{2e} = \zeta\sin\delta_1$$
$$\frac{ea}{p_{e^+}\cdot k}p_{1e^+} = \zeta\cos\delta_2, \qquad \frac{ea}{p_{e^+}\cdot k}p_{2e^+} = \zeta\sin\delta_2$$

and $\Gamma^{\mu,\sigma}_i$ are given by

$$\Gamma^\mu = \gamma^\mu(1-\gamma^5) \quad (5.2.51)$$
$$\Gamma^\mu_x = \left(\frac{ea\not{k}\gamma^1}{2p_e\cdot k}\right)\Gamma^\mu \quad (5.2.52)$$
$$\Gamma^\mu_y = \left(\frac{ea\not{k}\gamma^2}{2p_e\cdot k}\right)\Gamma^\mu \quad (5.2.53)$$
$$\Gamma^\sigma = \gamma^\sigma(1-\gamma^5) \quad (5.2.54)$$
$$\Gamma^\sigma_x = \Gamma^\sigma\left(\frac{ea\not{k}\gamma^1}{2p_e\cdot k}\right) \quad (5.2.55)$$
$$\Gamma^\sigma_y = \Gamma^\sigma\left(\frac{ea\not{k}\gamma^2}{2p_e\cdot k}\right) \quad (5.2.56)$$

This simplifies S_W to

$$\begin{aligned} S_W \quad = \quad & -\frac{e^2}{2^3\sin^2\theta_w}\frac{g_{\mu\sigma}-(p_{\nu_e'}-p_{\nu_e})_\mu(p_{\nu_e'}-p_{\nu_e})_\sigma/m_W^2}{(p_{\nu_e'}-p_{\nu_e})^2-m_W^2+i\Gamma_W m_W}\sum_n \bar{u}^s_e(p) \\ & \Big(C^\mu_n\Gamma^\mu + C^\mu_{xn}\Gamma^\mu_x + C^\mu_{yn}\Gamma^\mu_y\Big)u^s_{\nu_e}(p)\int d^4x e^{i(q_e-p_{\nu_e}-nk)\cdot x} \\ & \sum_m \bar{u}^s_{\nu_e}(p)\Big(C^\sigma_m\Gamma^\sigma - C^\sigma_{xm}\Gamma^\sigma_x - C^\sigma_{ym}\Gamma^\sigma_y\Big)\nu^s_{e^+}(p)\int d^4y e^{i(q_{e^+}+p_{\nu_e'}+mk)\cdot y} \end{aligned} \quad (5.2.57)$$

Here we see that

$$S_W \propto \delta^4(q_e - p_{\nu_e} - nk)\delta^4(q_{e^+} + p_{\nu_e'} + mk)$$

As is done in section (5.2.1), nk is interpreted as the number of photons that are pulled from the laser field to drive the reaction. But here we have another term, mk which is opposite in sign to that of nk. Therefore we can say that in the case of the charged channel there is also emission of photons. The $\delta-$function can be used to apply a constraint on the possible values of r, for r being the net number of photons.
i.e. $r = n - m$

$$\begin{aligned} p_\nu^\mu + rk &= q_{e^-}^\mu + q_{e^+}^\mu + p_{\nu'}^\mu \\ (p_\nu^\mu + rk)^2 &= (q_{e^-}^\mu + q_{e^+}^\mu + p_{\nu'}^\mu)^2 \\ 2rp_\nu \cdot k &\geq 4(m_e^*)^2 \end{aligned} \tag{5.2.58}$$

Taking the neutrino and laser fields to be oppositely directed, in the massless limit of the neutrino this constraint equation becomes

$$\begin{aligned} 4rE_\nu E_\gamma &\geq 4(m_e^*)^2 \\ r &\geq \frac{(m_e^*)^2}{E_\nu E_\gamma} \end{aligned} \tag{5.2.59}$$

Now S_W takes the form

$$S_W = -\int d^4x \sum_{r=-\infty}^{\infty} e^{i(q_{e^-} + q_{e^+} + p_{\nu'} - p_\nu - rk)\cdot x} M_r \tag{5.2.60}$$

Where the scattering amplitude M_r is given by

$$\begin{aligned} M_r \quad = \quad & \frac{e^2}{2^3 \sin^2\theta_w} \frac{g_{\mu\sigma} - (p_{\nu_e'} - p_{\nu_e})_\mu (p_{\nu_e'} - p_{\nu_e})_\sigma / m_W^2}{(p_{\nu_e'} - p_{\nu_e})^2 - m_W^2 + i\Gamma_W m_W} \bar{u}_e^s(p) \Big(C_n^\mu \Gamma^\mu + C_{xn}^\mu \Gamma_x^\mu \\ & + C_{yn}^\mu \Gamma_y^\mu \Big) u_{\nu_e}^s(p) \bar{u}_{\nu_e}^s(p) \Big(C_m^\sigma \Gamma^\sigma - C_{xm}^\sigma \Gamma_x^\sigma - C_{ym}^\sigma \Gamma_y^\sigma \Big) \nu_{e^+}^s(p) \end{aligned} \tag{5.2.61}$$

with

$$C_n^\mu = e^{in\delta_1} J_n(\zeta) \tag{5.2.62}$$

$$C_{xn}^\mu = \frac{1}{2}\Big(e^{i(n+1)\delta_1} J_{n+1}(\zeta) + e^{i(n-1)\delta_1} J_{n-1}(\zeta)\Big) \tag{5.2.63}$$

$$C_{yn}^\mu = \frac{1}{2i}\Big(e^{i(n+1)\delta_1} J_{n+1}(\zeta) - e^{i(n-1)\delta_1} J_{n-1}(\zeta)\Big) \tag{5.2.64}$$

$$C_m^\sigma = e^{im\delta_2} J_m(\zeta) \tag{5.2.65}$$

$$C_{xm}^\sigma = \frac{1}{2}\Big(e^{i(m+1)\delta_2} J_{m+1}(\zeta) + e^{i(m-1)\delta_2} J_{m-1}(\zeta)\Big) \tag{5.2.66}$$

$$C_{yn}^\sigma = \frac{1}{2i}\Big(e^{i(m+1)\delta_2} J_{m+1}(\zeta) - e^{i(m-1)\delta_2} J_{m-1}(\zeta)\Big) \tag{5.2.67}$$

With the same procedure as section (5.2.1), the probability is given by

$$\mathcal{P} = \left(\prod_f \int \frac{d^3\overrightarrow{p}_f}{(2\pi)^3}\frac{1}{2E_f}\right)\frac{T}{2E_\nu}\sum_{n=-\infty}^{\infty}(2\pi)^4\delta^4(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu-rk)\overline{|M_r|^2} \tag{5.2.68}$$

The rate of production is then

$$\Gamma = \frac{\mathcal{P}}{T} = \left(\prod_f \int \frac{d^3\overrightarrow{p}_f}{(2\pi)^3}\frac{1}{2E_f}\right)\frac{1}{2E_\nu}\sum_{r=-\infty}^{\infty}(2\pi)^4\delta^4(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu-rk)\overline{|M_r|^2} \tag{5.2.69}$$

Expressing the total production rate as the sum of the production rates for all of the individual processes involving r photons, the total rate is given by

$$\Gamma = \sum_r \Gamma_r \tag{5.2.70}$$

with

$$\Gamma_r = \left(\prod_f \int \frac{d^3\overrightarrow{p}_f}{(2\pi)^3}\frac{1}{2E_f}\right)\frac{1}{2E_\nu}(2\pi)^4\delta^4(q_{e^-}+q_{e^+}+p_{\nu'}-p_\nu-rk)\overline{|M_r|^2} \tag{5.2.71}$$

5.3 Results

The calculations proceed by computing the individual production rates Γ_n and then summing to find the total rate Γ . Though the constraint equations (5.2.23) and (5.2.59) impose a lower bound on the summation over n for the total rate, there is no such upper bound. This means that the calculation should include computations of the individual production rates for n to infinity. Fortunately, the individual production rates Γ_n fall off exponentially for large n. The choice of incoming photon energy E_γ and the magnitude of the vector potential a, are deciding factors for how fast or slowly these rates fall off. For the case in which the individual rates fall off slowly, the rates are calculated[15] to sufficiently large n such that we can characterize the nature of the exponential decay. Use of this characterization to fit the rates at large n allows us to approximate the summation.

Based on the Terawatt High-intensity Optical Research (THOR) laser, which is a $20TW$ laser beam centered around a wave length of $800nm$ with an electric field strength of $|\overrightarrow{E}|$ on the order of $10^{10} \sim 10^{11} V/cm$, at the High Intensity Laser Science Group at the University of Austin, initial conditions that closely approximate the present high-intensity laser system are chosen[3]. For the calculation an electric field strength of $|\overrightarrow{E}| = 5 \times 10^{10} V/cm$ is used[3]. These choices of wavelength and electric field strength uniquely specify the electromagnetic field parameters in the problem.

The photon energy is given by the relation

$$E_\gamma = \frac{2\pi}{\lambda}$$

and the magnitude of the vector potential is

$$a = \frac{|\overrightarrow{E}|}{E_\gamma}$$

The profile of the individual production rates Γ_n as a function of the number of photons n is given in Figure 5.2 for incoming neutrino energies of $3GeV$, $30GeV$, and $1TeV$. We note that for a given choice of photon energy and vector potential magnitude, the profiles shown in this figure all follow the same exponential decay.

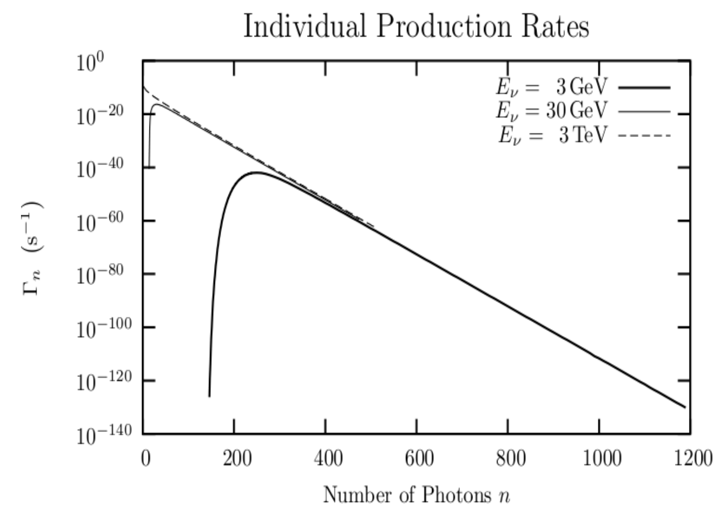


Figure 5.2: The profile of the individual production rates Γ_n as a function of n for incoming neutrino energies of $3GeV$, $30GeV$, and $3TeV$ [15].

The individual rates that make up these profiles are summed in order to find the total production rate. The results of this summation is presented in Table 5.1 for a range of neutrino energies from $10^9 eV$ to $10^{20} eV$. The total rate of production is also presented for a laser field that is able to generate the same field strength but with a much smaller wavelength of $\lambda = 100nm$. We note that the difference in wavelength has the most significant effect at low incoming neutrino energies.

Rather than considering the total rate of production for this process, it may have more physical significance if we consider production length Λ. The production length is the distance that a neutrino must travel in the laser field such that its likelihood of producing an electron-positron pair is $1-\frac{1}{e} \simeq 63\%$. That is, the probability for production is

$$p = 1 - e^{-l/n}$$

where l is the distance traveled in the field and the production length is simply $\Lambda = \frac{1}{\Gamma}$. The production length for the rates tabulated in Table 5.1 are shown in Figure 5.3.

From Figure 5.3 one can see how feeble this interaction is. For the production length to drop below the Hubble length, an estimate for the size of the universe, the incoming

$E_\nu(eV)$	$\Gamma(s^{-1})$	
	$\lambda = 800nm$	$\lambda = 100nm$
10^9	1.8×10^{-92}	2.6×10^{-57}
3×10^9	6.6×10^{-41}	1.6×10^{-28}
10^{10}	7.6×10^{-22}	2.2×10^{-17}
3×10^{10}	1.3×10^{-15}	1.6×10^{-13}
10^{11}	1.1×10^{-12}	1.6×10^{-11}
3×10^{11}	2.6×10^{-11}	1.1×10^{-10}
10^{12}	2.4×10^{-10}	6.0×10^{-10}
3×10^{12}	1.2×10^{-9}	2.4×10^{-9}
10^{13}	5.8×10^{-9}	1.0×10^{-8}
3×10^{13}	2.2×10^{-8}	3.6×10^{-8}
10^{14}	9.6×10^{-8}	1.4×10^{-7}
3×10^{14}	3.4×10^{-7}	4.8×10^{-7}
10^{15}	1.4×10^{-6}	1.8×10^{-6}
3×10^{15}	4.6×10^{-6}	6.2×10^{-6}
10^{16}	2.0×10^{-5}	2.2×10^{-5}
3×10^{16}	6.0×10^{-5}	7.4×10^{-5}
10^{17}	2.2×10^{-4}	2.6×10^{-4}
3×10^{17}	7.2×10^{-4}	8.4×10^{-4}
10^{18}	2.6×10^{-3}	3.0×10^{-3}
3×10^{18}	8.0×10^{-3}	9.0×10^{-3}
10^{19}	2.8×10^{-2}	3.0×10^{-2}
3×10^{19}	8.6×10^{-2}	8.2×10^{-2}
10^{20}	3.2×10^{-2}	2.0×10^{-1}

Table 5.1: The total production rate Γ for a given incoming neutrino energy E_ν in the presence of an intense laser field of electric field strength $|E| = 5 \times 10^{10}$ V/cm and wavelength λ [15]

neutrino energy must exceed $10GeV$. And at energies near $1PeV$ the production length is still on the order of light year.

To further stress the small likelihood of pair production under these conditions, this interaction was consider at a neutrino source such as the Neutrinos at the Main Injector (NuMI) facility at Fermi lab. Assuming the most ideal set of specifications for the neutrino beam and simplifying the beam to be uniform, it is estimated that the neutrino flux is on the order of $10^9/m^2/s$ per pulse with a pulse every $2s$ [3]. The energy profile of the beam is nontrivial. There is a low-energy component peaked at $3GeV$, a medium-energy

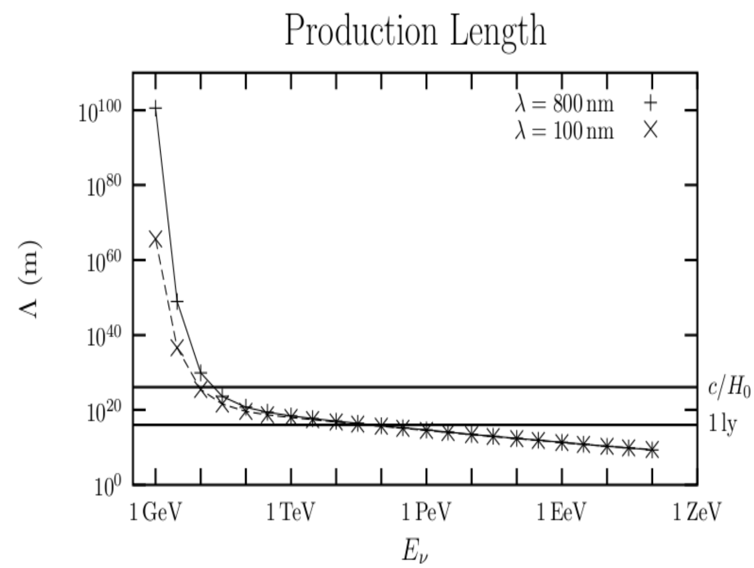


Figure 5.3: The production lengths Λ as a function of incoming neutrino energy E_ν for a laser field of wavelengths $100nm$ and $800nm$ at an electric field strength of $|E| = 5 \times 10^{10} V/cm$. The dark lines correspond to the Hubble length $c/H_0 \sim 1.2 \times 10^{26} m$ and one light year $1ly = 9.46 \times 10^{15} m$. [15]

component peaked at $7GeV$, and a high-energy component peaked at $15GeV$, but the beam will contain neutrinos with energy up to $120GeV$. However, for estimation purposes, one can simply assume that all of the neutrinos are at $15GeV$ and are uniformly distributed throughout the pulse. Using these conditions for the NuMI beam, realistic conditions for the laser beam, and assuming a detection region about $1m$ long, it is estimated that there would be an event once every 10^{22} years. In astronomical phenomena, where the magnetic field strengths can reach very high magnitudes, there exists the possibility of significant stimulation of this interaction.

As far as the reaction $\nu \to \nu' e^- e^+$ through the charged channel is concerned we see that the equation describing rate of production has similar structure to that of the reaction through the neutral channel. The scattering matrices $\mathcal{M}_n$ and $\mathcal{M}_r$ both contain Γ^μ matrices which have similar structures, combinations of γ^μ matrices. So it is reasonable to say that the rate of production in using the charged channel does not differ greatly from that of the neutral channel.

Chapter 6

Summary and Conclusion

In this thesis we first reviewed pair production with neutrinos in an intense laser field through the neutral channel. We made use of light-cone coordinate system to solve the Dirac equation for the process. A detailed calculation of all the steps was presented. We also studied the case of the reaction $\nu \to \nu' e^- e^+$ through the charged channel. We saw that the charged current reaction is relevant only for electron neutrino. We also noted that, following the same mathematical steps, one can see that the difference is that in the case of the charged current reaction, in contrast to the neutral current reaction, there is emission of photons.

In Chapter 2 we reviewed the weak interaction of leptons along with GWS model. The total Lagrangian describing the electromagnetic and weak interaction of leptons was also discussed. We had also discussed the properties of neutrinos such as neutrino mass and oscillation of neutrinos in Chapter 3.

We saw that, the process of pair production under normal conditions, is forbidden. However, the electromagnetic field alters the final states of the electron-positron pair and frees the interaction to take place. It is for this reason that we discussed the effect of electromagnetic field on particles, specifically on electrons. Using this analysis we solved the Dirac equation for our process.

Lastly, we compared the rates of pair production for the process $\nu \rightarrow \nu' e^- e^+$ for the two cases, neutral channel as well as charged channel. We noted that in the case of the reaction through the charged channel, in the presence of laser field, there is emission of photons. We also stated that, considering the net number of photons in the case of the charged-current reaction and looking at the general structures of equation (5.2.40) for neutral channel and equation (5.2.71) for the charged channel, it is possible to guess that the total rate of production for the two processes do not differ much. With this analysis, though we have no data for the charged-current reaction, we generalized the behavior of the rate of production from the results of the neutral-current reaction as is discussed in Section (5.3). We saw that for a detection region of $1m$ there would be an event once every 10^{22} years, which does not make sense. To over come this problem the process must be carried with astronomical and cosmological sources of neutrinos and fields. Because in astronomical phenomena, where the magnetic field strengths can reach very high magnitudes, there exists the possibility of significant stimulation of this interaction.

Appendix A

A.1 Bessel function expansion of trigonometric functions

Using the trigonometric relations:

$$\sin k \cdot x = \frac{e^{ik\cdot x} - e^{-ik\cdot x}}{2i}, \qquad \cos k \cdot x = \frac{e^{ik\cdot x} + e^{-ik\cdot x}}{2} \tag{A.1.1}$$

we get

$$\sin k \cdot x \times e^{-i\zeta \sin(k\cdot x - \sigma)} = \sum_{n=-\infty}^{\infty} \frac{e^{-ink\cdot x}}{2i} \left(e^{i(n+1)\delta} J_{n+1}(\zeta) - e^{i(n-1)\delta} J_{n-1}(\zeta)\right) \tag{A.1.2}$$

$$\cos k \cdot x \times e^{-i\zeta \sin(k\cdot x - \sigma)} = \sum_{n=-\infty}^{\infty} \frac{e^{-ink\cdot x}}{2} \left(e^{i(n+1)\delta} J_{n+1}(\zeta) + e^{i(n-1)\delta} J_{n-1}(\zeta)\right) \tag{A.1.3}$$

$$\begin{aligned}\sin^2 k \cdot x \times e^{-i\zeta \sin(k\cdot x - \sigma)} = \sum_{n=-\infty}^{\infty} e^{-ink\cdot x} \Bigg(\frac{1}{2} e^{in\delta} J_n(\zeta) - \frac{1}{4}\Big(e^{i(n+2)\delta} J_{n+2}(\zeta) \\ + e^{i(n-2)\delta} J_{n-2}(\zeta)\Big)\Bigg)\end{aligned} \tag{A.1.4}$$

$$\begin{aligned}\cos^2 k \cdot x \times e^{-i\zeta \sin(k\cdot x - \sigma)} = \sum_{n=-\infty}^{\infty} e^{-ink\cdot x} \Bigg(\frac{1}{2} e^{in\delta} J_n(\zeta) + \frac{1}{4}\Big(e^{i(n+2)\delta} J_{n+2}(\zeta) \\ + e^{i(n-2)\delta} J_{n-2}(\zeta)\Big)\Bigg)\end{aligned} \tag{A.1.5}$$

$$\sin k \cdot x \cos k \cdot x \times e^{-i\zeta \sin(k\cdot x - \sigma)} = \sum_{n=-\infty}^{\infty} \frac{e^{-ink\cdot x}}{4i} \left(e^{i(n+2)\delta} J_{n+2}(\zeta) - e^{i(n-2)\delta} J_{n-2}(\zeta)\right) \tag{A.1.6}$$

A.2 Notation

$$\partial^\mu = (\partial^0, -\bigtriangledown) \qquad \bigtriangledown = (-\frac{\partial}{\partial x_i})$$

$$x^\mu = (x^0, x) \qquad x_\mu = (x^0, -x)$$

$$x_\mu = g_{\mu\nu} x^\nu \qquad g_{00} = 1$$

$$g_{kk} = -1 \qquad g_{\mu\nu} = g^{\mu\nu}$$

$$p \cdot x = p^\mu x_\mu = g_{\mu\nu} p^\mu x^\nu = p^0 x^0 - p \cdot x \qquad p^2 = p^\mu p_\mu = E^2 - p^2 = m^2$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = (\frac{\partial}{\partial x^0}, \bigtriangledown) \qquad (\gamma^0)^2 = 1, (\gamma^i)^2 = -1$$

Light-cone coordinate system

$$x_+ = \frac{1}{2}(x^0 + x^3) \qquad x_- = \frac{1}{2}(x^0 - x^3)$$

$$\gamma_+ = \gamma^0 + \gamma^3 \qquad \gamma_- = \gamma^0 + \gamma^3$$

$$\partial_+ = \partial_0 + \partial_3 \qquad \partial_- = \partial_0 - \partial_3$$

$$x^0 = x_+ + x_- \qquad x^3 = x_+ - x_-$$

$$\gamma^0 = \frac{1}{2}(\gamma_+ + \gamma_-) \qquad \gamma^3 = \frac{1}{2}(\gamma_+ - \gamma_-)$$

$$\partial_0 = \frac{1}{2}(\partial_+ + \partial_-) \qquad \partial_3 = \frac{1}{2}(\partial_+ - \partial_-)$$

$$a \cdot b = a_\mu b^\mu = 2a_+ b_- + 2a_- b_+ - a_\perp \cdot b_\perp$$

$$\not{a} = \gamma_\mu a^\mu = \gamma_+ a_- + \gamma_- a_+ - \gamma_\perp \cdot a_\perp$$

$$\gamma_\pm \gamma_\pm = 0 \qquad \gamma_\pm \gamma_\mp = 2\gamma_0 \gamma_\mp$$

$$\gamma_0 \gamma_\pm = \gamma_\mp \gamma_0 \qquad \gamma_\pm \gamma_\perp = -\gamma_\perp \gamma_\pm$$

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Name: Dr. Bhatnagar

Signature:− − − − − − − − − − −