DETERMINATION OF PARAMETERS FOR THE TANGENT MODULUS APPROACH

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NOTATIONS

\( a_v \)  Coefficient of Compressibility
\( C_c \)  Coefficient of Compression Index
\( C_v \)  Coefficient of Consolidation
\( e_o \)  Initial Void Ratio
\( e \)  Void Ratio
\( H \)  Soil Layer Thickness
\( k \)  Permeability
\( k' \)  Earth Pressure coefficient
\( m \)  Modulus number
\( a \)  Stress Exponent
\( m_v \)  Coefficient of Volume Compressibility
\( M \)  Tangent Modulus
\( OCR \)  Over Consolidation Range
\( NCR \)  Normally consolidated Range
\( q \)  Discharge
\( u \)  Pore Water Pressure
\( U \)  Degree of Consolidation
\( v_o \)  Nominal Velocity
\( V \)  Volume
\( \varepsilon \)  Strain
\( \gamma_w \)  Unit Weight of Water
\( \sigma \)  Total Stress
\( \sigma' \)  Effective Vertical Stress
\( \sigma_{pc} \)  Pre-consolidation Stress

\( \sigma_{ov} \)  over burden pressure

\( \sigma_{st} \)  Vertical pressure from foundation

\( p_a \)  atmospheric pressure
Abstract

The estimation of settlement often receive less attention than they deserve (or even not at all), with the result that excessive settlements cause far more problem than do bearing capacity failures. Settlement analysis is therefore one of the two most important types of analysis made by soil engineer for designing safe and economical structures.

This paper introduces different method for the determination of settlement, which is known as the tangent modulus approach, this approach deals with the use of a coherent definition of compression modulus leading to a unified procedure of practical settlement calculation for different types of soils ranging from rocks to very soft soil.

In this approach a compression modulus, the tangent to stress $V_s$ strain curves obtained from laboratory tests play a significant role. It is a measure of the resistance of soil against deformation due to application of loading. The compression modulus describes the resistance of the media or an isolated part of it against a forced change of equilibrium conditions. The resistance concept is very well developed in other field of engineering except its application for settlement analysis in soil mechanics is new. The procedure is therefore believed to give a much clearer mechanical understanding of what volumetric soil compression actually is. Moreover, the use of classical concepts have lead to a simpler, and more straight forward calculation procedure, where the practical advantage are most apparent when applied to layered soils.

Even though the classical approach for determining the consolidation settlement suggested by Karl Terzaghi has contributed more for the development of settlement computations, it has its limitations in that it assumes the value of the compression modulus to be constant, which is true only for elastic material. Further more, it was believed it is the initial pore water pressure that describes the consolidation characteristics of thick clay layer. But it is shown here in that a new theory for stress distribution, compatible with the actual soil properties is included; moreover, it is shown that the calculation of the time rate of consolidation of clay has to be based on strain distribution, instead of pore pressure distribution, to avoid fundamental misunderstandings.

It is therefore the application of this tangent modulus approach by introducing and fixing parameters for Addis Ababa clay soils that this paper introduces.
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1. Introduction

1.1 Background

High rise building, bridges towers, power plants and similar high cost structures are becoming more common. These heavily loaded structures generally induce high stresses on the soil underneath which undergoes compressive strain resulting in the settlement of structures.

Excessive or unequal settlement may prevent the structure from functioning properly. Which results in unnecessary cost of maintenance and even in extreme case become dangerously unstable and finally it may collapse.

Therefore, settlement determination has to be given due considerations by engineers as it affects engineered structures from functioning properly and has to be also managed before causing serious damage.

Different countries used different approaches to estimate settlement of structures. Here in our country the classical approach is used in order to estimate settlements, but this approach has many limitations, among others, the following are important

- The logarithmic scale in the eVs log $\sigma'$ hides away the details of the soil behaviors.

- The determination of the pre-consolidated pressure $\sigma'_c$, using the Casagrande approach is highly dependent on the human element and is empirical.

- The laboratory correction for field disturbance is not rational.

- The determination of the initial void ratio, $e_o$, is both tedious and subject to many variables. For instance, it requires knowledge of the specific gravity of the soil and
end of test water content. Hence, it is common knowledge that it is very sensitive to both water content and specific gravity

- The approach is limited to cohesive (clayey) soils only.

The other method, which is used in other countries, is the tangent modulus approach, which has many advantageous. These are:

- This method deals with the use of a coherent definition of compression modulus leading to a unified procedure of practical settlement calculations for different types of soils.

- The method also showed that the calculation of the time rate of consolidation of clays has to be based on strain distribution, instead of pore pressure distribution, to avoid fundamental misunderstanding.

- The concept simply and clearly relates the observed physical changes to the mathematical formulations; raw data is used to obtain desired answers with out recourse to assumptions.

The simplicity and rationality inherent in the resistance concept made it the preferred method of analysis of the compressibility soils.

Previous works show us that computation of soil compressibility using tangent modulus approach is applicable for our country. Hence, to further look at the applicability of these methods for different clay soils in Addis Ababa and to determine parameters for this method, and also to use the method for our country will simplify and rationalize the computation of settlement. This is the back ground of this thesis work.
1.2 Objective

The tangent modulus approach which gives a unified procedure of settlement calculations is based on elementary concepts and principle in classical mechanics. The procedure is therefore believed to give a much clearer mechanical understanding of what volumetric soil compression actually is. Moreover, the uses of classical concepts have lead to a simpler and more straightforward calculation procedure for the computation of settlements. Strict adherence to use the tangent modulus approach in the field of soil compressibility is being the primary advantage. The objective of this research work centers on developing and promoting the concept of tangent modulus in the field of soil compressibility. This helps specifically to study, in depth, the concept of tangent modulus approach to different clay soils found around Addis Ababa and determine parameters for the tangent modulus approach for the different types of soil.

1.3 Methodology

In order to meet the desire objective of the thesis a series of one dimensional consolidation tests were performed on undisturbed samples collected from different parts of Addis Ababa. Each test sample had 75mm diameter and 20mm thickness and was enclosed in a circular metal ring sandwiched between porous stones. Prior to applying normal loads each sample was allowed to saturate. After the saturation process was completed the samples were subjected to series of normal loads, while changes in thickness were recorded against time.

The vertical normal load increments were applied at regular time intervals for 24 hrs. The loads were doubled with each increment up to the required maximum loads (25, 50, 100, 200, 400, 800 kpa etc). From the change in thickness at the end of each normal load and from the change in thickness recorded against time during a load stage the compressibility and rate of consolidation of clay sample were calculated.

From the data obtained stress Vs strain graph were plotted. This plot was used to compute the resistance of the soil to deformation, and hence plots of tangent modulus Vs stress for all samples were made. It was this plots used for the analysis and computation of this thesis work.
2. Literature Review

2.1 General
The increase of stress in soil layers due to load imposed by various structures at the foundation level will always be accompanied by some strain. If the load is greater than the strength of the soil both elastic and residual deformation are absorbed (13). For soils the load deformation relationship is usually complex, varying widely with different soils, particularly in the plastic range of cohesive soils where time plays a major role. Yet it is such time-related soil deformation (i.e consolidation) that accounts for most of the settlement in cohesive soils and that the foundation engineer must cope with. (9)
Cohesionless soils (sand, gravel) will generally compress during a relatively short time. In fact, quite frequently most of anticipated settlement of a foundation resting on a cohesionless soil has taken place during the construction phase of the structure. Furthermore; the compression of such soil by induced vibration effect could be accomplished with much greater ease than of cohesive soils (9)
Unlike cohesionless soils, a saturated cohesive soil mass (e.g. fine silts, clays) with low permeability will compress quite slowly since the expulsion of the pore water from such soil occurs at a significantly slower rate than from cohesionless ones (4). Hence the deformation in such soil takes time after the construction of the structure.
Even though surface loads results in the vertical and horizontal deformations, one notes that it is the vertical compression or consolidation is the largest and it is frequently the most important component.
When cohesive soils are subjected to stress due to foundation loading the pore water pressure will immediately increase, however, because of the low permeability of the soil, there will be a time lag between the application of load and the extrusion of water and thus, settlement. This phenomenon, which is called consolidation is the subjects this chapter.

2.2 Theory of consolidation
As the purpose of this paper is to present a different approach for the determination of settlement for different types of soil it is worth to discuss first about consolidation of soil.
When a soil layer is subjected to a compressive stress, such as during the construction of a structure, it will exhibit a certain amount of compression. This compression is due to rearrangement of the soil solids or extrusion of the pore air and/or water. According to Terzaghi (1943), “a decrease of water content of a saturated soil with out replacement of the water by air is called processes of consolidation” (2).

The rate at which the air or water dissipated so that the soil particles closer towards each other due to the application of stress depends on the degree of saturation of the soil mass. If the soil is dry or partially saturated the air in the void gets easily compressed. That is the air is easily expelled out resulting in almost immediate settlement. In a saturated soil it is not easy for the soil particles to get closer, as the water in the voids is relatively incompressible. For reduction in voids to take place the water has to get dissipated. The rate of out flow of water depends on the permeability of the soil, and the length through which the pore water travels to reach when it drains quickly.

To understand the basic concept of consolidation, consider a clay layer of thickness H located below the ground water level in between two permeable sand layers as shown in Fig 1.1. If a surcharge of intensity \( \Delta \sigma \) applied over a large area the pore water pressure in the clay will increase. For a surcharge of infinite extent, the immediate increase of the pore water pressure, \( \Delta u \), at all depths of the clay layer will be equal to the increase of the total stress, \( \Delta \sigma \). Thus at \( t=0 \)

\[
\Delta \sigma = \Delta u \quad \text{.................................................................(2.1)}
\]

Since the total stress is the sum of the effective stress and the pore water pressure, at all depths of the clay layer the increase of the effective stress due to the surcharge at \( t=0 \) will be equal to zero (\( \Delta \sigma =0 \)). In other wards at \( t = 0 \) the entire stress is taken up by the pore water pressure and none by the soil skeleton. Here one has to note that for a load applied over a limited area, it may not be true that the increase of the pore water pressure is equal to the increase of a vertical stress at any depth at time \( t=0 \) (2).
After application of the surcharge (i.e. at time $t>0$) the water in the void space of the clay layer will be squeezed out and will flow towards both permeable sand layers, thereby reducing the excess pore water pressure. This will increase the effective stress, $\Delta \sigma'$ by an equal amount, since:

$$\Delta \sigma' + \Delta u = \Delta \sigma$$

Thus at time $t > 0$

$$\Delta \sigma' > 0$$  \hspace{1cm} \text{(2.2)}$$

And

$$\Delta u < \Delta \sigma$$  \hspace{1cm} \text{(2.3)}$$

Theoretically, at time $t = \infty$ the excess pore water pressure at all depths of the clay layer will be dissipated by gradual drainage. Thus at time $t = \infty$

$$\Delta \sigma' = \Delta \sigma$$  \hspace{1cm} \text{(2.4)}$$

$$\Delta u = 0$$  \hspace{1cm} \text{(2.5)}$$

It is this gradual increase in effective stress which results in settlement called process of consolidation.

These equation representing the consolidation process shall be described for the concept of resistant / tangent modulus approach/ and for the classical consolidation theory.
2.3 Factors Affecting Consolidation

2.3.1 Compressibility of the soil
Granular materials exhibit compressibility behavior quite distinct from clay soils. The rate of compression decrease as the load increases. The major part of compression occurs almost instantaneously. As the permeability of granular soil is very high, the pore water dissipates with in a short duration. Most of the settlements have already taken place by the time the structure is complete.

For saturated fine - grained soils the major factor in the escape of pore water from the soil is the time required for dissipation. The extrusion of pore water takes time i.e. much time is required in fine grained soil for pore water to escape. Hence, in fine - grained soil the compression is larger than in granular soils. (10)

2.3.2 Permeability of the soil
Water from a soil is flowing through the voids between the soil particles if the at rest hydrostatic pore pressure changes in any way. This flow of water from the soil particles will produce a volume change and this is related directly to the permeability of the soil. The effect of consolidation settlement occurs in clays where the value of permeability is low so that it prevents the initial excess pore water pressure to dissipate immediately. The permeability of a soil, therefore, plays a significant role in the time taken for the process of consolidation (5).

The drainage of water in clay takes longer time when compared with that of free draining saturated sand. This is because that the permeability of clay is tens of thousands to million of times less than that of sands and the movement of water occurs very much slowly resulting in considerable time for water to squeeze out completely (10).

2.3.3 Role of stress history
Stresses are induced in the soil mass by past history such as surcharge load from soil later eroded away by natural causes, lowering of ground water table and desiccation by drying from the surface (16). A soil which is subjected to a certain effective stress for the first time in its history(normally consolidated) will obviously be more compressible than when it has been subjected to a large effective stress in it earlier history(over
consolidated). When a soil is stressed to a level greater than the maximum stress to which it was even subjected to in the past, some kind of break down in the soil structure occurs resulting in a much higher compressibility.

2.3.4 Effect of effective stress
Pore water starts to flow out from saturated soil due to the application of stress as long as there is a hydraulic gradient. When this flow continues, there is a reduction in pore water pressure and an increase in effective stress.

Free water and/ or gas bubbles in the voids of minerals soil can not transfer shear stress. Hence the soil skeleton alone transfers shear stress. Thus the effective normal stress governs the internal resistance of granular soils irrespective of the shear stress (4)

As a soil is subjected to a stress, the soil undergoes a decrease in void ratio. If the same stress is applied again the soil undergoes a decrease in void ratio which is less than the decrease in initial void ratio. This means compressibility of soil decrease as effective stress increase provided that there is no brakeage of the particles.
3. Evaluation of settlement

3.1 General
The increase of stress in soil layers due to the load imposed by various structures at the foundation level will always be accompanied by some strain, which causes deformation of the soil layer. If the load is greater than the structural strength of the soil, both elastic and residual deformations are observed. Though elasticity is a property inherent in all material bodies, soils can be regarded as elastic bodies only under definite conditions. With a repeated loading and unloading, however, the soil will generally acquire an elastically compacted state which is characterized by that its elastic property are constant for the given loading condition. If the load on the soil is increased above that causing elastically compacted state appreciable residual deformation will appear in it (13).

When thinking of a new construction, it is important to realize that due to change of soil condition settlement might be as equally critical as bearing capacity. It is the prediction of these changes that present the most difficult task (13).

The estimation of settlement often receives less attention than they deserve (or even non at all), with the result that excessive settlements cause far more problem than do bearing capacity failures (12)

The deformation of soils under loading is basically due to rolling, slipping and sliding and to some extent by crushing of particles at contact point and elastic deformation.

Some of the different causes of settlement are: compaction, consolidation, elastic distortions, moisture migration, lose of lateral support and ground water lowering. It is one or the combination of these causes which brings about settlement. (7)

3.2 Component of settlement
The response of stress applied to a soil brings about a total settlement, which is the sum of immediate or elastic settlement, primary consolidation settlement and secondary compression settlement.
Immediate settlement is the change in shape or distortion of the soil caused by the applied stress. This type of settlement is usually occurs in all coarse grained and dry or partially saturated fine grain soils. Immediate settlement is due to the elastic deformation of soil without any change in the soil water content.

Primary consolidation settlement occurs in cohesive or compressible soil during dissipation of excess pore fluid pressure, and it is controlled by the gradual expulsion of fluid from voids in soil leading to the associated compression of the soil skeleton. Excess pore pressure is pressure that exceeds the hydrostatic fluid pressure. The hydrostatic fluid pressure is the product of unit weight of water and the difference in elevation between the given point and elevation of free water (phreatic-surface).

Primary consolidation settlement is normally insignificant in cohesionless soil were settlement occurs rapidly because these soils have relatively large permeability. Hence this type of settlement takes substantial time in cohesive soils because they have relatively low permeability. Time of consolidation increase with thickness of the soil layer and inversely related to the coefficient of permeability of the soil (16).

Secondary compression settlement is a form of soil creep which is largely controlled by the rate at which the skeleton of compressible soils, particularly clays, silts, and peat, can yield and compress. Secondary compression is often conveniently identified to follow primary consolidation when excess pore fluid pressure can no longer be measured; however, both process may occurs simultaneously (16).

3.3 Consolidation settlement

3.3.1 Description
Vertical pressure $\sigma_{st}$ from foundation loads transmitted to a saturated compressible soil mass is initially carried by fluid or water in the pores because of water is relatively incompressible compared with that of the soil structure. The pore water pressure, $u$, induced in the soil by the foundation loads is initially equal to the vertical pressure $\sigma_{st}$ and it is defined as excess pore water pressure because this pressure exceeds that caused by the weight of water in the pores. Primary consolidation begins when water
starts to drain from the pores. The excess pressure and its gradient decrease with time as water drain from the soil causing the load to be gradually carried by the soil skeleton. This load transfer is accompanied by decrease in volume of the soil mass equal to the volume of water drained from the soil. Primary consolidation is complete when all excess pore water pressure has dissipated so that $u = 0$. Primary consolidation settlement is usually determined from the results of one-dimensional (1-D) consolidometer test. (13)

### 3.3.1.1 Normally consolidated soil

A normally consolidated soil is a soil which is subjected to an in situ effective vertical overburden stress $\sigma'_{ov}$ equal to or greater than the pre-consolidated stress $\sigma_{pc}$. Virgin consolidation settlement for applied stress exceeding $\sigma_{pc}$ can be significant in soft and compressible soil with a skeleton of low elastic modulus such as plastic CH and CL clays, silts, and organic MH and ML soils. (16)

### 3.3.1.2 Over consolidated soils

An over consolidated soil is a soil which is subjected to an in situ effective overburden stress $\sigma_{ov}$ less than the pre-consolidated stress $\sigma_{pc}$. Consolidation settlement will be limited to recompression from stress applied to the soil up to $\sigma_{pc}$ (16).

### 3.3.2 One dimensional consolidation

The theory of consolidation is developed based on one dimensional consolidation. One dimensional consolidation occurs under fills and embankment that are wide when compared with the thickness of the underlying compressible ground. When a large area is loaded uniformly, every element at every depth is confined by adjacent element that is subjected to the same state of stress. There is no horizontal deformation of the soil except near the boundaries of the loaded area. If the layer is overlain and confined by a desiccated crust or granular layer thick enough to minimized heave of the clay layer around the foundation, the condition also approximated as a state of one-dimensional consolidation. If the foundation is located deep enough, so that the surrounding overburden prevents lateral deformation of the clay located directly below the foundation, a similar boundary condition exists in one-dimensional condition is when the
compressible ground is overlain by a stiff layer such as dense granular soil or bed rock. The stretching in the horizontal direction is restrained by horizontal shearing resistance that develops at the top and bottom (15).

If the thickness of a compressible layer is large compared to the loaded area, the condition of one-dimensional consolidation does not exist. This is due to the fact that some settlement is caused by lateral displacement of the soil. But even if there is lateral displacement, it is very much smaller when compared with that of the vertical displacement.

Thus to determine settlement due to compression of clay stratum under confined conditions, that approximate one dimensional compression can be derived from compression of laterally confined specimen. This is carried out in consolidometer.

Hence, for the vast majority of practical settlement problems, it is sufficient to consider that both seepage and strain takes place in one direction only, the usually being vertical which is used throughout this thesis.

### 3.3.3 Mathematical model and Computation of 1-D consolidation settlement

#### 3.3.3.1 Time Rate of Settlement

The governing equation for one- dimensional consolidation theory and the distribution of excess pore water pressure, distributed within the soil when load is applied shall be presented here. In one- dimensional consolidation, where deformation takes place in the direction of loads, the theory for the time rate of consolidation was first proposed by Terzaghi (1925)

The mathematical derivations are based on the following assumptions.

- The clay layer is homogeneous
- The clay layer is saturated
- The compression of the soil layer is due to the change in volume of void only, which in turn, is due to the squeezing out of water from the void spaces
- Darcy's law is valid
- Deformation of soil occurs only in the direction of the load application
- The coefficient of consolidation \( C_v \) (in equation below) is constant during the consolidation.

Based on the above assumptions, let us consider a clay layer of thickness 2H as shown in the Fig 3.1. The layer is located between two highly permeable sand layers. When the clay is subjected to an increase of vertical pressure, \( \Delta \sigma \), the pore water pressure at any point A will increase by \( u \). For one dimensional consolidation, water will be squeezed out in the vertical direction towards the sand layer.

Fig 3.1 shows the flow of water through prismatic element at A. For the soil element shown, (rate of out flow of water)-(rate of inflow of water) = (rate of change of volume). Thus,

\[
(v + (\partial v / \partial Z)dz)dx dy - v dx dy = \partial V / \partial t
\]

Where \( V \) = Volume of soil element and \( v \) = velocity of flow in the Z direction or

\[
\partial v / \partial Z(dx dy dz) = \partial V / \partial t
\]

Using Darcy’s law:

\[
v = Ki = -K(\partial h / \partial z) = -(K/\gamma_w)(\partial u / \partial z)
\]

Where \( u \) = excess pore water pressure due to the increase of stress.

![Fig. 3.1 Clay layer undergoing consolidation](image-url)
Substituting of Eq 2.1 in to Eq2.2 and rearranging gives:

\[-K/\gamma_w(\partial^2u/\partial^2Z)=1/(d_xd_yd_z)(\partial V/\partial t)\].....................................................(3.3)

During consolidation the rate of change of volume is equal to the rate of change of the volume of voids; therefore

\[\partial V/\partial t=\partial V_v/\partial t=\partial(V_s+eV_s)/\partial t=\partial V_v/\partial t+V_s(\partial e/\partial t)+e(\partial V_v/\partial t)\].............(3.4)

But \(\partial V_v/\partial t=0\)(assuming the soil solids are incompressible)

And:

\[V_s=V/(1+e_0)=(d_xd_yd_z)/ (1+e_0)\]

Substituting for \(\partial V_v/\partial t\) and \(V_s\) in Eq(2.4) yields

\[\partial V/\partial t= (d_xd_yd_z)/ (1+e_0)(\partial e/\partial t)\].....................................................(3.5)

Where \(e_0\) = initial void ratio . Combining Eqs (2.3)and(2.5)

\[(-K/\gamma_w)(\partial^2u/\partial^2Z)=1/(1+e_0)(\partial e/\partial t)\].....................................................(3.6)

The change in void ratio, \(\partial e\), is due to the increase of effective stress;

Assuming that these are linearly related; then

\[\partial e=a_v\partial(\Delta\sigma)\].....................................................(3.7)

Where \(\partial(\Delta\sigma)=\)change in effective stress and \(a_v=\)coefficient of compressibility.(It can be considered to be constant for narrow range of stress change).

Again, the increase of effective stress is due to the decrease of excess pore water pressure, \(\partial u\), hence:
\[ \partial e = -a_v \partial u \] \hspace{1cm} \text{(3.8)}

Combining Eq (2.5) and (2.8),

\[ (-K/\gamma_w)(\partial^2 u/\partial^2 Z) = a_v / (1+e_o) \left( \partial u / \partial t \right) = m_v \left( \partial u / \partial t \right) \] \hspace{1cm} \text{(3.9)}

Where \( m_v \) = coefficient of volume compressibility = \( a_v / (1+e_o) \)

Or:

\[ \partial u / \partial t = K / (\gamma_w m_v) \left( \partial^2 u / \partial^2 Z \right) = C_v \left( \partial^2 u / \partial^2 Z \right) \] \hspace{1cm} \text{(3.10)}

Where \( C_v \) = coefficient of consolidation = \( K / (\gamma_w m_v) \)

Equation (3.10) is the basic differential equation of Terzaghi’s consolidation theory and can be solved with proper boundary conditions.

### 3.3.4 Settlement Computation

#### 3.3.4.1 Evaluation of Void Ratio-Pressure Relationships

The standard one dimensional consolidation test is usually carried out to determine settlement. Settlement due to one dimensional compression results only from decrease in the volume of voids.

An estimate of the ultimate consolidation settlement following complete dissipation of hydrostatic excess pressure requires determination of the relationship between the in situ void ratio and effective vertical stress in the soil. The loading history of a test specimen taken from an undisturbed and saturated soil sample, for example, may be characterized by a void ratio versus logarithm pressure diagram (16).

This data is also used to plot the time-deformation curve, which helps for the determination of the end of primary consolidation. The graph of the time versus the effective stress is divided into three stapes as shown below in Fig (3.3) (2).

- Upper curved portion (Stage I). This is namely the result of pre-compression of the specimen.
- A straight-line portion (Stage II). This is referred to as a primary consolidation. At the end of the primary consolidation, the excess pore water pressure generated by the incremental loading is dissipated to a large extent.
- A lower straight-line portion (Stage III). This is called secondary consolidation. During this stage, the specimen undergoes small deformation with time. There is also
immeasurably small excess pore water pressure in the specimen during secondary consolidation (2)

Fig. 3.3 Typical specimen deformation verses log of time plot for a given load increment

According to the classical theory it is noted that at the end of the test for each incremental loading the stress on specimen is the effective stress $\sigma'$. Once the specific gravity of the soil solids, the initial specimen dimensions, and the specimen deformation at the end of each load has been determined, the corresponding void ratio can be calculated. A typical void ratio Vs effective pressure relation plotted on semi logarithmic graph paper is shown as in Figure (3.4) for both loading and unloading. The curve is some what curved with a flat shape, followed by a linear relationship with a steeper slope.
Removal of an impervious soil sample from its field location will reduce the confining pressure, but tendency of the sample to expand is restricted by the decrease in pore water pressure. The void ratio will remain constant at constant water content because the decrease in confinement is approximately balance by the decrease in water pressure, therefore, the effective stress remains constant by the above equation, and the void-ratio should not change. Classical consolidation assumes that elastic expansion is negligible and the effective stress is constant during release of the in situ confining pressure after the sample is taken from the field. Some sample disturbance, however, occurs hence the laboratory consolidation curve must be corrected as shown in Fig 3.5. Perfectly undisturbed soil should indicate a consolidation curve similar to line e₀ED as in Fig 3.5a or line e₀BFE as in Fig 3.5b. Soil disturbance increases the slope for stresses less than the...
pre consolidation stress illustrated by the laboratory consolidation curve of Fig 3.5. Pushing undisturbed sample into metal shell tubes and testing in the consolidometer without removing the horizontal restraint helps maintain the in situ horizontal confining pressure, and reduce any potential volume change following removal from the field, and helps the correction for sample disturbance (16).

A normally consolidated soil in situ will be at void ratio $e_o$ and effective over burden pressure $\sigma'_o$ equal to the pre consolidated stress $\sigma_p$. $e_o$ may be estimated as the initial void ratio prior to the test, if the water content of the sample had not changed during storage and soil expansion is negligible. In situ settlement from applied loads is determined from the field virgin consolidation curve. Reconstruction of the field virgin consolidation with slope $C_c$ shown on Fig 3.5a may be estimated by procedures involving the Casagrande construction procedure which requires care and judgment for determining the greatest curvature for evaluation of pre consolidated stress. This determination most of the time misleads which usually gives an exaggerated value than the actual pre consolidation stress. In this theory it is recommended to take high quality of undisturbed specimens for reducing the probable range of the pre consolidated $\sigma_p$.

Besides to this the scale of the plot may have also some influence on the evaluation of the parameters. Having these all parameters the ultimate consolidation settlement may be estimated by;

$$S_{cj}= \frac{\Delta e_j}{1 + e_{oj}} \cdot H_j ................................................................. (3-11)$$

Where:

- $S_{cj} = $ Consolidation settlement of stratum $j$
- $\Delta e_j = $ Change in void ratio of stratum $j = e_{fj} - e_{oj}$
- $e_{oj} = $ Initial void ratio of stratum $j$ at initial pressure $\sigma_{oj}$
- $e_{fj} = $ Final void ration of stratum $j$ at final pressure $\sigma_{fj}$
- $H_j = $ Height of Stratum $j$

The final void ratio may be found graphically using the final pressure $\sigma_{fj}$ illustrated on Fig 2-2a. The change in void ratio may be calculated by:
\[ \Delta e_j = C_c \cdot \log_{10} \frac{\sigma'_{\beta j}}{\sigma'_{oj}} \]  

Where, \( C_c \) is the slope of the field virgin consolidation curve or compression index.

One can put the following steps for calculating the ultimate primary consolidation settlement of a compressible stratum. (16)

**Procedure for Calculation of Ultimate Primary Consolidation Settlement of a Compressible Stratum using the Classical Method.**

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Evaluate the pre-consolidated stress ( \sigma'_{\rho o} ) from results of a one-dimensional (1-D) consolidation test on undisturbed soil specimens using the Casagrande Construction Procedure, by methods in the next paragraph.</td>
</tr>
<tr>
<td>2</td>
<td>Estimate the average initial effective overburden pressure ( \sigma'<em>{o\rho} ) in each compressible stratum using soil unit weights, depth of overburden on the compressible stratum, and the known groundwater level or given initial pore water pressure in the stratum. Refer to Equation, ( \sigma'</em>{o\rho} = \gamma z - u_w \cdot \sigma'<em>{o} = (\sigma'</em>{o\rho 1} + \sigma'<em>{o\rho 2})/2 ) where ( \sigma'</em>{o\rho 1} ) = effective pressure at top of compressible stratum and ( \sigma'_{o\rho 2} ) effective pressure at bottom of compressible stratum.</td>
</tr>
<tr>
<td>3</td>
<td>Determine the soil initial void ratio ( e_o ) as part of the 1-D consolidation test.</td>
</tr>
<tr>
<td>4</td>
<td>Evaluate the compression index ( c_c ) from results of a 1-D consolidation test using the slope of the field virgin consolidation line determined by the procedure of Casagrande as illustrated in Figures 3.5, or preliminary estimates may be made from some presumptive values. Determine the recompression index ( c_r ) for an over consolidated soil as illustrated in Figures 3.5; preliminary estimates may be made.</td>
</tr>
<tr>
<td>5</td>
<td>Estimate the final applied effective pressure ( \sigma'<em>{fj} ) where ( \sigma'</em>{fj} = \sigma'<em>{oV} + \sigma</em>{st} \cdot \sigma_{st}, \text{ soil} )</td>
</tr>
</tbody>
</table>
pressure caused by the structure, may be found from Bossiness solution.

Determine the change in void ratio $\Delta e_j$ of stratum $j$ for the pressure increment $\sigma'_f - \sigma'_o$ graphically from a data plot similar to Figure 3.5, from Equation 3-12 for a normally consolidated soil, or from Equation 3-14 for an over consolidated soil.

Determine the ultimate one-dimensional consolidation settlement of stratum $j$ with thickness $H_j$, from Equation (3-18)

$$S_{cj} = \frac{\Delta e_j}{1 + e_{oj}} H_j$$

Determine the total consolidation $S_c$ of the entire profile of compressible soil from the sum of the settlement of each stratum,

$$S_c = \sum_{j=1}^{n} S_{cj}$$
Fig 3.5 Construction of field virgin consolidation relationships
If the soil is in an over consolidated ones it will be at a void ratio \( e_o \) and effective vertical confining pressure \( \sigma_{ov}' \) represented by point B on Fig 3.5b. At some time in the past the soil was subjected to a pre consolidated pressure \( \sigma_{pc}' \), but the pressure was latter reduced, perhaps by soil erosion or removal of glacial ice to the existing over burden pressure \( \sigma_{ov}' \). The in situ settlement for an applied load will be the sum of recompression settlement between point B and F and any virgin consolidation from final effective vertical applied pressure \( \sigma_f' \) exceeding the pre consolidated stress \( \rho_{c}' \). Reloading a specimen in the consolidometer will give the laboratory curve shown in Fig 3.5b.

The virgin consolidation curve can be constructed by making use of Casagrande and some added procedures as it is discussed above.

The rebound loop in the laboratory curve is needed to develop the recompression line BF - from which one can evaluate the recompression index \( C_r \).

Hence, the settlement \( S_{cj} \) of stratum \( j \) may be estimated as the sum of recompression and virgin consolidation settlements. The final void ratio is found graphically from Fig 3.5b and the change in void ratio may be calculated by:

\[
\Delta e_j = C_r \cdot \log \frac{\sigma_{pj}'}{\sigma_{oj}'} + C_e \cdot \log_{10} \frac{\sigma_{fj}'}{\sigma_{pj}'} \quad \text{............... (3.14)}
\]

3.3.4.1 Pre consolidation Pressure

In the typical \( e \) Vs \( \log \sigma' \) plot it can be seen that the upper part is curved; however at higher pressure, \( e \) and \( \log \sigma' \) bear a linear relation. The interpretation of this can is: when the soil specimen was obtained from the field it was subjected to ascertain maximum effective pressure. During the process of soil exploration, the pressure is released. In the laboratory, when the soil specimen is loaded, it will show relatively
small decrease of void ratio is occur with load up to maximum effective stress to which the soil was subjected in the past. (16)
This is represented by the upper portion of the Fig 3.6. If the effective stress on the soil specimen is increased further, the decrease of void ratio with stress level will be larger. This is presented by the straight line portion in the \( e \text{vs.}\log\sigma' \) plot. This effect can also be demonstrated by unloading and reloading a soil specimen.

Fig. 3.6 Plot of void ratio vs. effective pressure showing unloading and reloading branches.

It is based on this stress history that most clay soils are characterized as normally consolidated or over consolidated ones. Therefore it is mandatory for geotechnical engineer to know the value of \( \sigma_{pc} \), (pre consolidated pressure), which represents the highest level of stress to which the soil has historically been subjected prior to the current application of load. It is determined from the same \( e \text{vs.}\log\sigma' \) graph using the most popular graphical procedure suggested by A Casagrande. The procedure is summarized as follows: the point of maximum curvature is determined from the consolidation curve by visual inspection. Then two lines are drawn passing through this point of maximum curvature. One is a tangent to the curve and the other is a line parallel to the stress axis and passing through the point of tangency. A line bisecting the angle obtained by the
intersection of the two lines is drawn. The straight-line part of the curve is extended back to meet the bisector line obtained. The projection of the point of intersection of these lines gives the approximate value for pre-consolidation stress $\sigma'_c$ as given on Fig 3-7 below.

Fig. 3.7 Typical $e$ vs. log $\sigma'$ plot showing procedures for determination of $\sigma'_c$ and $C_T$

As it can be seen above even though the determination of this $\sigma'_c$ (pre consolidated pressure) is very important for characterizing the soil, the determination procedure using the classical cassagrandi needs care and personal judgment. This will lead to end up with some what large errors as this curve normally hides the behavior of the soil.
4. The concept of resistance

4.1 General
The compressibility curve obtained from the consolidation may be expressed with sufficient accuracy by the equation of Ohde (14, 18) given by:

\[ E_s = \frac{d\sigma}{ds'} = \psi \left( \sigma' \right)^\omega \]  

(4.1)

In the above equation \( s' \) is the relative settlement, \( \psi \) and \( \omega \) are coefficients where \( \psi \) has a unit of \( \text{kpa} \). It depends on the void ratio, water (moisture) content and consistency of the sample. It could have values ranging from 50 to 30000 kpa. (18)

The coefficient \( \omega \) is a dimensionless quantity which depends on the soil type. It could have values (18) ranging from 0 to 1.

The tangent of the compressibility, which is a function of \( \sigma' \) gives the modulus of compressibility \( E_s \) (Fig 4.1)

Fig 4.1 Compressibility Curve (Linear Scale)
From Eq. (4.1)

\[ \frac{ds'}{d \sigma'} = \frac{1}{v (\sigma')^\omega} \]  .........................................................(4.2)

\[ s' = \frac{1}{v} \int (\sigma')^{-\omega} d \sigma' \]  .........................................................(4.3)

For the case \( \omega \neq 1 \)

\[ s' = \frac{1}{v (1-\omega)} \left( \sigma' \right)^{1-\omega} + C \]  .........................................................(4.4)

These equations given by Ohde later on developed and interpreted in the resistance concept /tangent modulus/ by Janbu as it is presented hereafter.

Any media possess a resistance against a forced change of existing equilibrium condition. This resistance of the media, or of an isolated part of it, can be determined by measuring the incremental effect of a given incremental cause, where after

\[ \text{Resistance} = \frac{\text{incremental cause (given)}}{\text{incremental effect (measured)}} \]  .........................................................(4.5)

Such resistance concept is well known from other fields in engineering. This is in the following definition equations in classical physics.

- Electric resistant (R) \[ \frac{\text{potential Change}}{\text{Current Change}} \] (ohm)
- Elastic resistant (E) \[ \frac{\text{Stress change}}{\text{Strain change}} \] (Hooke Youngd)
- Dynamic resist.mass (m) \[ \frac{\text{Force change}}{\text{Accelleration change}} \] (Newton)
- Hydraulic resistant (K^{-1}) \[ \frac{\text{Gradient change}}{\text{Velocity change}} \] (Darcy)
- Heat resistant (c) \[ \frac{\text{Heat change}}{\text{Temperature change}} \]
The well known technique of simulating one type of problem with another (say an hydraulic with an electrical) is based on the circumstance that over a given cause - effect ranges the corresponding resistances are either both constants or analogous in variation(11)

According to the outcome of a research work at the Technical University of Norway (11) the common term resistant can as well be applied to soil mechanics. The research revealed that the tangent to the stress - strain curve was found to be an appropriate and practical measure of deformation characteristics for all soils regardless of their type. The mechanical meaning of the Tangent Modulus, M, can therefore be defined as a resistance against deformation (Volumetric), expressed as

\[
\text{Deformation resistance, } M = \frac{\text{Stress Change(given)}}{\text{Strain Change(measured)}} \tag{4.6}
\]

This definition can be shown by the following Fig 4.2 showing stress - strain curve as obtained from an oedometer or triaxial test. The common case is represented by a non-linear stress - strain relationship.

The possibility of using a secant - or chord - value as an approximation was soon discarded, because both measures were unable to cover adequately the variable stress ranges encountered in practice. However, the tangent to the stress - strain curve was
found to be an appropriate and practical measure of the deformation characteristics of soil.

It is this value called the tangent Modulus, M, which is widely used in some parts of the world. The applicability of the tangent Modulus, M, is verified to our local red clay soil by one of the previous thesis work (Dessalegn Sisay, “Computation of soil compressibility using tangent modulus approach”) and it is also the basis of this research work. The tangent modulus, M, is given by

\[
M = \frac{d\sigma'}{d\varepsilon} \quad \text{................................................................. (4.7)}
\]

Where: 

\(M = \) Deformation modulus (Deformation Resistance)

\(\sigma' = \) Change in effective stress (in given direction)

\(\varepsilon = \) Change in strain (in that direction)

The tangent modulus, M, and the stress, \(\sigma'\) can be shown by a graph of M Vs \(\sigma'\) in the Fig 4-3 below

![Figure 4.3 tangent modulus Vs stress curve.](image)

Since deformation tests on soils are carried out as three-dimensional it is evident that any definition of a deformation modulus is meaningless unless the state of stress or strain is described more specifically (11)
Besides to this the branches of loading - unloading and reloading are entirely different. It is therefore necessary to be able to distinguish between the corresponding modules. It is therefore suggested that the following terms should be used:

\[ \begin{align*}
  M &= \text{Compression modulus} \\
  M_s &= \text{Swelling modulus} \\
  M_{rc} &= \text{Recompression modulus}.
\end{align*} \]

4.2 Application of the resistance concept in the field of soil compressibility.

The field condition for the deformation test in the laboratories usually carried out either by oedometer test or triaxial test. By using either of these tests the soil response is determined by subjecting representative undisturbed soil sample to an external action (e.g. changing external stress). The response of the sample to this external action is measured (either as strain and/or pore pressure) (6). The boundary conditions of the test can be explained by means of the following figure (4.4)

![Fig 4.4 test model](image)
The oedometer tests correspond to no lateral yield, or plain strain. Hence;

\[ K = K_0 \]

Where \( K_0 \) = effective earth pressure coefficient at rest such test corresponds to the field conditions.

From large number of oedometer and triaxial tests on different types of soils in Norway by N. Jonbu , A.C Stanmatopoulos and P.C Kotzius in Greece(11) they were able to deduce that the tangent modulus, derived from the vertical stress-strain curve, is suitable for direct measurement of soil compressibility. The plots of stress- strain and tangent modulus- stress on a linear scale have a different shape. This shape holds for soils of variable type and origin: clay, silt, sand, organic soils, sedimentary soils and residual soils etc. It appears, therefore, to constitute a general model of soil behavior to which reference can be made when evaluating the test result (1).

The plot of the tangent modulus on linear scale is very important for all types of soils as it has an embedded advantage by which it can characterize the type of soil we have.

Fig 4.5 shows the different plots of tangent modulus- stress for different types of soils obtained form an oedameter test. The indicated variation of tangent modulus, M, with vertical effective stress \( \sigma' \) is typically for large number of different types of soil and sand of different porosity and grain size distribution characteristic. However the looser the soils the more will the M - \( \sigma' \) diagram approaches a straight line (6)
The above graphs show that the tangent modulus representing the behavior of soil is dependent on the nature of the soil. For low stress level on the loading branch the tangent modulus, $M$, against deformation is large while the stress increases this high resistance eventually decreases appreciably owing to the partial collapse of the grain skeleton. This break down of the resistance occurs around the pre-consolidation stress level $\sigma_{pc}$. This leads to a very practical and reasonable way of determining the pre-consolidated pressure from a linear scale plot (4).

The variation of tangent modulus, $M$, with effective stress, $\sigma'$ for a wide range of material type is plotted in fig.4.6. The general shapes of these plots are typical of the soil types.
As it can be seen from Fig 4.6 (a), for over consolidated clays (OC), the modulus \( M \) is almost independent of the stress change \( \Delta \sigma' \) between the present effective over burden \( \sigma'_{ov} \) and the pre-consolidated pressure \( \sigma_{pc} \), for stress above \( \sigma'_{ov} \) the modulus increases. But for normally consolidated Clays (NC), the modulus increases linearly with the effective stress change \( \Delta \sigma' \).

For sands and silts, the modulus varies with the effective stress raised to some power with an average of nearly 0.5 for practical purposes (11).

The above plots of the modulus generally show us a comprehensive behavior of soils ranging from clay to silt and sand. The general behavior and characteristics of soils which are hidden in the strain- stress curves are clearly shown.

From the study of a large number of stress - strain curve for different types of soils, ranging in porosity (zero) rock to about (90%), it is believed that, for engineering purposes, one can adequately cover the variation in compressibility by means of single definition, namely tangent modulus.
From the examples of compression module for individual soil types shown above, demonstrates that the modulus generally depend on (effective) stress, hence,

\[ M = f(\sigma'). \] \hspace{1cm} (4.8)

However, the function \( f(\sigma') \) is generally different for each new type of soil.

Janbu(1963) established that the variation of \( M \) with effective stress for the different soil types are adequately represented by a simple formula of the type(7):

\[ M = m p_a \left( \frac{\sigma'}{p_a} \right)^{1-a} \] \hspace{1cm} (4.9)

Where:
- \( M \) = Modulus
- \( m \) = modulus number (dimensionless)
- \( a \) = stress exponent (dimensionless)
- \( \sigma' \) = Vertical stress (effective)
- \( P_a \) = reference pressure = 1 atmosphere = 100kpa

The reference pressure \( p_a \) is introduced solely for the purpose of obtaining a dimensionally correct expression.

The governing parameters \( m \) and \( a \) have been the subject of experimental investigation over period of about 10 years and the general trend of variation is demonstrated in the Fig 4.7 and 4.8. The soil type considered ranges from sound rock with low porosity all the way up to the softest clays with porosity nearly 90% (11)

---

Fig 4.7 variation of \( a \) with porosity
Fig 4.7 shows that for rock the stress exponent $a=1$ corresponds to a modulus $M = \text{constant}$ i.e. independent of stress variation. For such soils the modulus number $m$ is generally high as we can see in the Fig 4.8 below.

For normally consolidated clays the stress exponent $a=0$, corresponding to a linear variation of modulus versus stress ($M = m \sigma'$). The modulus number is generally low.

For soils of intermediate porosity (sand) the stress exponent, $a$, varies around 0.5 as an average. This corresponds to a modulus which is about proportional to the square root of the stress. The corresponding modulus number $m$ is generally located between the low values assigned to clay on one hand and up to those rocks on the other hand.

The figures above (Fig 4.7 and Fig 4.8) are not sufficiently detailed for direct practical application. This is because that it is only meant as illustration of the general tendencies for variation of $a$ and $m$ with porosity. (11)

The main impression one gets from this figure is that it is the modulus number that actually determines the magnitude of $m$ while the value of $a$ is of less importunate as long as it is approximately known, say with in $\pm 25\%$. This fortunate circumstance make
it possible to assemble several soil types into a small number of soil groups, each of which can be studied in greater detail with respect to modulus number (11).

From the test results of Norwegian institute, soils are basically grouped in three principal groups according to the value of $a$ shown in the Fig 4.9 below.

![Fig 4.9principal types of soil](image)

For practical purposes the institute have found it adequate to assemble the various soil types in to the following three main categories with respect to modulus variation and specific value of $a$ as follows, (see Fig 4.9 above)

Type EL = "elastic" ($a=1$)
Type EP = elastic - plastic ($a = 0.5$)
Type PL = Plastic ($a=0$)

Each of these principal groups corresponds to a specific value of the exponent, $a$, and for each group the modulus formula is therefore a special case of equation. 4.9.

Rocky hard moraines, over consolidated clays, underained modulus of clay are typical examples of soil belonging to type EL defined by $a = 1$ corresponding to

$$M = m_p \quad \text{..................................................} \quad (4.10)$$
Type Ep are used for normally consolidated sand (NCS) where \( a = 0.5 \). In silty sandy soils (NC) the \( a \) - value may range typically between 0.4 to 0.65, so 0.5 should be considered as an average choice for practical purpose only. The formula for M is there for

\[
M = m \sqrt{\sigma'} \quad p_a 
\]  

(4.11)

Type PL where \( a = 0 \) corresponds to normally consolidated clays, NCC, for which

\[
M = m \sigma' 
\]  

(4.12)

Hence, it is only the modulus number that is required as a result of compression tests.

Classifying the vast character and soil type in to three main categories by a single non dimensional number will make the tangent modulus approach more preferable over the classical approaches.

### 4.3 Strain and settlement computation.

Independent of soil type, it is possible to establish a common definition for the compression modulus. Thus, the various steps of the settlement analysis can be derived from the same basic principle. This means that the entire calculation procedure will in principle becomes equal for all types of soils.

Besides, since the definition of the compression modulus utilizes familiar classical concepts, such as incremental stress and strain, the mechanical meaning of each step in the analysis will be familiar both to engineers in practice and to engineering students (11)

In utilizing the concept, resistance concept, for settlement analysis it is obvious that the vertical stress profile has to be determined. This profile consists of the effective overburden \( \sigma'_{ov} \) and net stress distribution with depth. An infinitesimal element having thickness \( dz \) at an arbitrary depth \( z \) below the foundation level undergoes deformation \( d\delta \) due to an additional stress \( d\sigma' \) and the effective overburden pressure prior to load application \( \sigma'_{ov} \), as strain is generally dependent on both these stresses.
The infinitesimal vertical compression \( d\delta \) of the layer \( dz \) by definition is the stain, \( \varepsilon \)

\[
d\delta = \varepsilon dz
\]

The total compression of the entire sediment of thickness \( H \) is then found by summing up the compression of each individual layer, hence

\[
\delta = \int_0^H \varepsilon dz \quad \text{................................. (4.13)}
\]

According to this equation the settlement is equal to the area of the \( \varepsilon - z \) diagram.

This elementary observation is of great practical importance for several reasons. Firstly, it may reduce to a minimum the number of \( \varepsilon \) - values that need to be calculated. Secondly, by plotting \( \varepsilon \) versus \( z \), the integration (4.13) is reduced to a simple area determination. Finally, the \( \varepsilon \) distribution with depth is of most importance in calculating the time rate of settlement of clays. It is therefore believed that \( \varepsilon - z \) plot should be a must in all settlement calculations.

The principle of calculating vertical strain (at an arbitrary depth in an arbitrary soil deposit) is based on the definition of the deformation modulus itself.

\[
d\varepsilon = \frac{d\sigma'}{M} \quad \text{.................................................. (4.14)}
\]

Therefore, as the effective stress increases from the overburden pressure \( \sigma'_{ov} \) until its final value \( \sigma'_{ov} + \Delta \sigma' \). The resulting strain is obtained by integrating equation (4.14) between these stress limits

\[
\varepsilon = \int_{\sigma'_{ov}}^{\sigma'_{ov} + \Delta \sigma'} \frac{d\sigma'}{M} \quad \text{.................................................. (4.15)}
\]

In order to obtain an explicit strain formula it is necessary to express \( M \) mathematically as a function of \( \sigma' \).

The generalized experimental value of the compression modulus \( M \) has been given as

\[
M = m p_a \left[ \frac{\sigma'}{P_a} \right]^{1-a} \quad \text{.................................................. (4.16)}
\]

Introducing (4.16) in to (4.15) one gets the following generalized strain formula
\[ \varepsilon = \frac{1}{ma} \left[ \left( \frac{\sigma'_{ov} + \Delta \sigma'}{P_a} \right)^a - \left( \frac{\sigma'_{ov}}{P_a} \right)^a \right] \] .................(4.17)

This is directly applicable for the entire range of \(a\) form 0 to 1, except \(a = 0\), which represents especial boundary case.

As suggested above this formula includes the majority of stress-strain relationships besides it satisfies the boundary condition \( \varepsilon = 0 \) for \( \Delta \sigma' = 0 \)

The formula (4.17) is also found advantageous to deal with the three principal categories of soils called EL (elastic analog), EP (elastic plastic) and PL (plastic). Each category corresponds to a given value of \(a\), equal to 1, 0.5 and 0, respectively. The strain formula corresponding to these types are as follows.

For soil types corresponding to type EL
Where \(a = 1\), \(\varepsilon = \Delta \sigma / M\)
Where: \(M = mP_a = \text{constant.} \) -…………………………………………………..(4.18)

For soil type EP, when \(a = 0.5\), that is \(M = m\sqrt{aP_a}\) leads to:

\[ \varepsilon = \frac{2}{m} \left[ \sqrt{\frac{\sigma'_{ov} + \Delta \sigma'}{P_a}} - \sqrt{\frac{\sigma'_{ov}}{P_a}} \right] \] -…………………………………..(4.19)

Type PL, for which \(a = 0\) and \(M = m \sigma'\) is governed by

\[ \varepsilon = \frac{1}{m} \left( \frac{\sigma'_{ov} + \Delta \sigma'}{\sigma'_{ov}} \right) \] -…………………………………..(4.20)

Equation (4.20) is obtained by separate limit consideration.
The above equations can be employed for our local soils by determining the parameter \(m\) which is the basis of this paper.
In laboratories, all over the world, the stress-strain curve for normally consolidated clays becomes a straight line in a semi-logarithmic plot. These empirical findings appear now to have found clear mechanical explanation. According to Janbu the implication is that tangent modulus is a linear function of effective stress above the present effective overburden. This means there must be a close relationship between modulus number and compression index \( C_C \). It is moreover a simple matter to prove that

\[
m = \frac{1+e_0}{C_C} \log 10 \quad \text{.......................................................... (4.21)}
\]

4.4 Time rate of consolidation

In dealing with settlements not only is the prediction of the settlements that will occur when stress changes act over a given period of time is important but the question of how settlements develop with time have also great practical applications.

For course-grained soils such as sand and gravel the time-dependency is usually of little practical interest. This is because that the pores are sufficiently large to allow drainage almost simultaneously with stress change. Consequently, the practical interest is focused on the total settlement alone. Almost the same situation exists for layers of coarse silt (7)

For rock and heavily over consolidated clays the deformation behavior resembles some what elastic materials, and hence, in practice, the question of time rate is rarely considered necessary for these soils (7)

For non-saturated soils, and for organic soils, the time rate of settlement may very well be of considerable importance, but it is beyond the scope of this thesis.

For normally consolidated, fully saturated clays and very fine silts are the only soil types for which it is both practically necessary and theoretically possible to predict the time rate of settlement.
To define the degree of consolidation properly in the classical theory, which is discussed in the previous chapter, it was important to assume that the change of strain is proportional to the stress change, which is true only for the pre-consolidated soils. Hence, the degree of consolidation in the classical theory is only dependent on the shape of additional stress distribution diagram with no consideration given to stress existing prior to load application. Owing to this and practical experience the theory has frequently failed in predicting even roughly the settlement behavior of structures after completion (7).

Thus, the equation has to be modified so as to include the actual soil properties and at the same time it has to give values in good consent with already completed structures. However, decades of international experience have clearly shown that the compression of saturated, normally consolidated clays depends on stress history and that there is generally no proportionality between stress change and strain change. The immediate consequence is that the classical equation for degree of consolidation is in general inapplicable (5)

Thus the (U-T) solutions that are available in the literature for different stress distributions with depth can, therefore in general, not be applied to normally consolidated clays, but only to soils with constant modulus M (e.g. to over consolidated clays). The stress distribution diagram is needed only as an intermediate step in order to calculate the strain.

The distribution of strain, therefore, with depth bears no resemblance whatsoever with the stress distribution. Consequently, the shape of the stress distribution diagram is of no direct value for the time rate evaluation.

Since it is the area of the strain diagram that per definition gives the settlement, it is quite obvious that it is the $\varepsilon$ - distribution that determines the time - rate of consolidation.
Instead of defining and using the pore water pressure distribution as a function of time and depth it is rather realistic to define the variation of strain with respect to time and depth of the layer

\[ \varepsilon = f(\varepsilon, z) \]

The first step in obtaining a practical solution of (4.22) must be derived a differential equation on the basis of strain, instead of pore water pressure. Such equations have been obtained by Mikassa and Janbu (11) quite independent of one another, and probably at about the same time. Short outline of the aspect is presented below.

For completely saturated clays the continuity equation in terms of strain

\[ \frac{\partial \varepsilon}{\partial t} = \frac{\partial \nu}{\partial t} \]

In which \( \nu = \) vertical velocity of percolating water. Moreover, Darcy's law in terms of strain takes the form \( \gamma_w \)

\[ \nu = -\frac{K}{\gamma_w} \frac{\partial u}{\partial z} \]

\[ = -\frac{K}{\gamma_w} \left( \frac{\partial}{\partial z} (\sigma' - \sigma') \right) \]

Where: \( u = \)is the pore pressure

\( \sigma'_p = \)effective stress at the end of primary consolidation

\( \sigma' = \)effective stress at any time \( t \)

But: \( M = \frac{d\sigma}{d\varepsilon} \) or \( d\sigma' = Md\varepsilon \)

Then:

\[ \nu = -\frac{K}{\gamma_w} M \left( \frac{\partial}{\partial z} (\varepsilon_p - \varepsilon) \right) \]

But also: \( \varepsilon_p - \varepsilon = \bar{\varepsilon} \)

Then:

\[ \nu = -\frac{K}{\gamma_w} M \frac{\partial}{\partial z} \bar{\varepsilon} \]

Where:

\( \varepsilon_p = \)strain at the end of primary consolidation

\( \varepsilon = \)strain at any time \( t \)

\( \bar{\varepsilon} = \)Remaining strain
From: \( \varepsilon_p - \varepsilon = \varepsilon \)

\[
\frac{\partial \varepsilon}{\partial t} = - \frac{\partial \varepsilon}{\partial t} \quad \text{..............................................(4.28)}
\]

From continuity equation:

\[
\frac{\partial \nu}{\partial Z} \frac{dz}{dt} = \frac{\partial \varepsilon}{\partial t} \frac{dz}{dt}
\]

\[
\frac{\partial \nu}{\partial Z} = \frac{\partial \varepsilon}{\partial t} \quad \text{..............................................(4.29)}
\]

Substituting: \( \nu = -K \frac{M}{\gamma_w} \frac{\partial \varepsilon}{\partial z} \) and \( \frac{\partial \varepsilon}{\partial t} = - \frac{\partial \varepsilon}{\partial t} \)

The above equation yields:

\[
-\frac{K}{\gamma_w} \frac{M}{\gamma_w} \frac{\partial^2 \varepsilon}{\partial z^2} = \frac{\partial \varepsilon}{\partial t} \quad \text{..............................................(4.30)}
\]

\[
C_v \frac{\partial^2 \varepsilon}{\partial z^2} = \frac{\partial \varepsilon}{\partial t} \quad \text{..............................................(4.31)}
\]

Where: \( C_v = \frac{K}{\gamma_w} \frac{M}{\gamma_w} \)

This equation is called one-dimensional consolidation equation in terms of \( \varepsilon \)

In which: \( C_v \) = coefficient of consolidation

\( M \) = Modulus

\( K \) = permeability and

\( \gamma_w \) = unit weight of water

This is the one dimensional consolidation in terms of the remaining strain. But as it has been discussed in chapter 2 the one dimensional consolidation in terms of the excess pore water pressure is:

\[
C_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}
\]

The two are identical for constant modules and permeability. But in reality neither \( M \) nor \( K \) is constant. The difference of the results is due to the difference in the initial conditions.
4.5 Merits of tangent modulus to local soils

The advantage of the tangent modulus approach can be appreciated by first considering the limitations in the classical approach. The limitations of the classical approach can be summarized as follows.

♦ The logarithmic scale in the $e$-$\log \sigma$ hides away the details of the soil behavior
♦ The determination of the pre-consolidation pressure $\sigma^1_c$ using the Cassagrande approach is highly dependent on the Humana element and is empirical.
♦ The laboratory correction for field disturbance is not rational
♦ The determination of the initial void ratio, $e_0$ is both tedious and subject to many variables. For instance, it requires knowledge of the specific gravity of the soil and end of test water content. It also presumes that the water content at the end of the test is full saturation water content. It is common knowledge that it is very sensitive to both water content and specific gravity.
♦ The approach is limited to cohesive (clayey) soils only.

On the contrary, the tangent modulus concepts offer the following advantageous.

♦ The concept is fundamentally sound and is based on principles that have been successfully applied in other field on engineering and applied sciences.
♦ It is not sensitive to material type and hence presence in itself, a unifying framework for characterizing engineering material behavior. Janbu (1963,1998) demonstrated the versatility of the concept in handling the full range of materials from soft soils to hard rock including concrete.
♦ The concept simply and clearly relates the observed physical changes to the mathematical formulation raw data and is used to obtain the desired answers with out recourse to assumptions.

Thus, the simplicity and rationality inherent in the resistance concept made it the preferred method of analysis of the compressibility soil.
5. Application of the Tangent Modulus to Local Soils.

5.1 Material Tested

Even though the application of the tangent modulus is applicable to all types of soils ranging from rock to soft clays, in this thesis work a series of laboratory tests were carried out on clay soils in Addis Ababa for the verification of the method. Samples were collected from different sites that comprise the two common soil types in Addis Ababa. In order to meet the objective of the thesis, samples (disturbed and undisturbed) of different properties were taken. The trial test pits were usually dug to a depth of 2-3mts by giving enough space to digging and sampling. Two to three samples were taken from each test pit in order to represent the field condition. Red clay soil samples were obtained from the area where this soil has been prevalent namely, Addisu Gebeya, Asco, Paulos and from Bole area black cotton soils were obtained.

For the vast majority of practical settlement problems, both seepage and strain were considered so as to take place in one direction only, this is only being vertical. Hence these soils were tested in one dimensional consolidation apparatus called oedometer test where there was no lateral strains. Cylindrical specimen of clay soil, enclosed in metal ring was subjected to a series of increasing static loads, while changing in thickness were recorded against time. Through this test, the compression characteristics of these soils were measured.

From the change in thickness at the end of each load stage the compressibility of the clay soils, and from the changes in thickness recorded against time during a load stage the rate of consolidation, were calculated. It was these raw data that read from the oedometer which were used for the verification of the applicability and determination of constant for the tangent modulus approach to our local clay soils.

5.2 Laboratory procedures and test Result.

Most of the laboratory tests were carried out in accordance with the ASTM procedures for soil testing. Index properties such as Grain Size Analysis, Atterberg limit, and
Consistency Index for some of the samples were tested. The tested clay soils showed almost the same ranges of values to the previously studied clay soils of Addis Ababa.

By using ASTM designation D 422-63 the tested samples were fine grained soil in which the percentage of clay ranges from (51-62%), sand from 11-15% and silt from 26-36%.

The general index properties with respect to Atterberg limit was also tested. The red clay soils had Liquid Limit 47.3-82%, plastic limit 19.2-39.5% and plastic index 23.5-55.5%, but expansive clay soils have liquid limit 85-118%, plastic limit 24.5-41.9%, and plastic index 23.35-55.5%. From the above index property the consistency of the soil samples were computed as:

\[ I_c = \frac{w_l - w}{I_p} \times 100 \quad \text{(Eq. 4.1)} \]

Where:
- \( w \) = water contents of the soil in natural condition
- \( w_l \) = liquid limit
- \( I_p \) = plastic index

The consistency index indicates the consistency (firmness) of the soil. It shows the nearness of the water content of the soil to its plastic limit. A soil with a consistency index of zero is at a liquid limit. It is extremely soft and has negligible shear strength. On the other hand, a soil at a water content equal to the plastic limit has a consistency index of 100%, indicating that the soil is relatively firm. A consistency index of greater than 100% shows that the soil is relatively strong, as it is semi solid state. A negative value of consistency index is also possible, which indicates that the water content is greater than the liquid limit” (7).

Table 5.1 Consistency in terms of Consistency Index (7)

<table>
<thead>
<tr>
<th>Consistency</th>
<th>Consistency Index (%)</th>
<th>Characteristics of soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very soft</td>
<td>0 – 25</td>
<td>Fist can be pressed in to soil</td>
</tr>
<tr>
<td>Soft</td>
<td>25 – 50</td>
<td>Thumb can be pressed in to soil</td>
</tr>
<tr>
<td>Medium( Firm)</td>
<td>50 – 75</td>
<td>Thumb can be pressed with pressure</td>
</tr>
<tr>
<td>Stiff</td>
<td>75 – 100</td>
<td>Thumb can be pressed with great difficulty</td>
</tr>
<tr>
<td>Very Stiff( Hard)</td>
<td>&gt;100</td>
<td>The soil can be readily indented with thumb nail</td>
</tr>
<tr>
<td>Hard</td>
<td>&gt;100</td>
<td>The soil can be indented with difficulty by thumb nail</td>
</tr>
</tbody>
</table>
Hence, the tested samples had ranges of firmness from very stiff, stiff, to firm for red clay soil and very stiff to soft for expansive soil as shown in the table 5.2. The objective of this thesis as indicated earlier was determining the range of value for the tangent modulus number, $m$. This range of value was tabulated in accordance with the range of firmness.

Table 5.2 consistency index of samples

<table>
<thead>
<tr>
<th>Sample area</th>
<th>Consistency index (%)</th>
<th>firmness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addisu Gebeya(1)</td>
<td>59.89</td>
<td>firm</td>
</tr>
<tr>
<td>Asco 1</td>
<td>105.4</td>
<td>very stiff</td>
</tr>
<tr>
<td>Addisu Gebeya(2)</td>
<td>50.3</td>
<td>soft</td>
</tr>
<tr>
<td>Addisu Gebeya(3)</td>
<td>89</td>
<td>stiff</td>
</tr>
<tr>
<td>Gullale(1)</td>
<td>65.32</td>
<td>firm</td>
</tr>
<tr>
<td>Gullale(2)</td>
<td>68.7</td>
<td>firm</td>
</tr>
<tr>
<td>Gullale(3)</td>
<td>62.4</td>
<td>firm</td>
</tr>
<tr>
<td>Gullale(4)</td>
<td>89.65</td>
<td>stiff</td>
</tr>
<tr>
<td>Addisu Gebeya(4)</td>
<td>70.2</td>
<td>firm</td>
</tr>
<tr>
<td>Paulos (1)</td>
<td>87.3</td>
<td>stiff</td>
</tr>
<tr>
<td>Paulos (2)</td>
<td>50.35</td>
<td>soft</td>
</tr>
<tr>
<td>Bole (1)M.</td>
<td>101.2</td>
<td>very stiff</td>
</tr>
<tr>
<td>Bole (2)M.</td>
<td>111.27</td>
<td>very stiff</td>
</tr>
<tr>
<td>Bole (3)M.</td>
<td>111.3</td>
<td>very stiff</td>
</tr>
<tr>
<td>Bole (1)</td>
<td>97</td>
<td>stiff</td>
</tr>
<tr>
<td>Bole (2)</td>
<td>60.2</td>
<td>medium</td>
</tr>
<tr>
<td>Bole (3)</td>
<td>67.3</td>
<td>medium</td>
</tr>
<tr>
<td>Bole</td>
<td>63.54</td>
<td>medium</td>
</tr>
<tr>
<td>Bole</td>
<td>74.3</td>
<td>medium</td>
</tr>
</tbody>
</table>

In order to meet the desired objective of the thesis series of one dimensional consolidation tests were performed on undisturbed samples collected from different parts of Addis Ababa. Each test sample had 75mm diameter and 20mm thickness and was enclosed in a circular metal ring and sandwiched between porous stones as shown in Fig5.1.
Prior to applying normal loads each sample was allowed to saturation. After saturation process was completed the samples were subjected to series of normal loads, while changing in thickness were recorded against time.

The vertical normal load increments were applied at regular time intervals for 24 hrs. The loads were doubled with each increment up to the required maximum loads (25, 50, 100, 200, 400, 800 kpa etc). From the change in thickness at the end of each normal load and from the change in thickness recorded against time during a load stage the compressibility and rate of consolidation of clay sample were calculated.
Fig. 5.2. Typical (a) Effective Stress – Strain Curve
(b) Tangent Modulus Vs Stress for Red Clay Soil

\[ m_{\text{NCR}} = 10.724, \quad m_{\text{OCR}} = 20.17 \]
Fig. 5.3. Typical (a) Effective Stress – Strain Curve 

(b) Tangent Modulus Vs Stress for Red Clay Soil

\[ m_{\text{NCR}} = 8.375, m_{\text{OCR}} = 40 \]
Fig. 5.4. Typical (a) Effective Stress – Strain Curve
(b) Tangent Modulus Vs Stress for Expansive Soil

$m_{NCR} = 1.87$, $m_{ocR} = 4.939$
Fig. 5.5. Typical (a) Effective Stress – Strain Curve

(b) Tangent Modulus Vs Stress for Expansive Soil

\[ m_{NCR} = 3.25, m_{OCR} = 7.92 \]
5.3 Interpretation based on the tangent modulus approach

Stress - strain curves from oedometer tests are action reaction measurements. From the stress - strain curve the resistance of a medium was determined by measuring the incremental response to a given incremental action. As shown in the test result this resistance is compression modulus, M. The value of the tangent modulus M is plotted together with stress - strain curve from the oedometer tests, all in arithmetic scale. Even though for all three categories soils, it is possible to find several dozens, not to say hundreds of curves with similar shapes, here in this thesis, as the test result indicated, it was only one category of soil called the clay soils found around Addis Ababa was considered.

From the result shown above the tangent modulus M depends on effective stress, $\sigma'$. The over - consolidated (OC) and normal - consolidated (NC) range, and the pre consolidated pressure ($\sigma'_p$) are clearly discernible. We can see different behavior of OC and NC range. In particular for our soils, for stress less than the preconsolidated pressure the curve is steeper and in increasing order which is indifferent from the soil found in other part of the world. But for stress greater than the pre-consolidated the graph seems less steep and is also in increasing order having different slope than the former. Thus, one can say that two different ranges of slopes were obtained for both curves. Therefore in contrary to the classical approach one can determine settlement for both normally consolidated and over consolidated soils in one comprehensive formula.

Generally, even though both classical and tangent modulus shows the same characteristics, at a point of preconsolidated stress the same raw data are used for computation. It is the tangent modulus that clearly shows the point of demarcation for all soils and ease of settlement computation for all ranges of consolidation. To give an illustration to the procedure a simple example is shown below.
EXAMPLE:
Consider a layer of soil strata which comprises of three different layers of soil: dry crust, normally consolidated clay and sand as shown below. From the tangent modulus Vs stress plot the following modulus number were obtained.

\[ m_{\text{crust}} = 25, \; M = 2500 \text{t/m}^2 \]
\[ m_{\text{clay}} = 15, \; M = 15 \sigma' \]
\[ m_{\text{sand}} = 200, \; M = 200 (p_s \sigma')^{1/2} \]

The over burden pressure \( \sigma'_{ov} \) and the effective stress \( \sigma' \) through out the soil strata are shown below.
Using the modulus number and hence the tangent modulus one can determine the strain distribution easily using the tangent modulus approach as shown below.
SOLUTION:

\[ \varepsilon_1 = \frac{15}{2500} = 0.6\% \]
\[ \varepsilon_2 = \frac{1}{15} \ln\left(\frac{26.5}{11.5}\right) = 5.6\% \]
\[ \varepsilon_3 = \frac{2}{200} \left(\frac{32}{10}\right)^{1/2} - \left(\frac{17}{10}\right)^{1/2} = 0.5\% \]

Settlement of

Crust \( \delta = 0.006 \times 400 \text{cm} = 2.4 \text{cm} \)

Clay \( \delta = 0.056 \times 700 \text{cm} = 39.2 \text{cm} \)

Sand \( \delta = 0.005 \times 400 \text{cm} = 2.0 \text{ cm} \)

Total \( \delta = 43.6 \text{cm} \)

5.3.1 Determination of range of values for the test Result

According to the test results it is clearly seen that the characteristics of our local soil was consistent with the soil of the Scandinavian where the tangent modulus approach was developed, except in ranges of over consolidated. Hence, the only soil parameter required is the modulus number \( m \). But the value of the stress exponent \( a \) was equal to zero (0). The test results were consistent with the type PL, plastic. One can, therefore, use the same formula as developed by the Norwegian Institute but for ranges of values of \( m \) developed for our local soils. Table 5.1 below shows ranges which are typical for our local soils in both over consolidated and normally consolidated ranges. These ranges of values are obtained from the slope of tangent modulus Vs stress curve obtained from the test result.

**Table 5.1: Typical Ranges of Modulus Number and stress exponent**

<table>
<thead>
<tr>
<th>firmness</th>
<th>Modulus Number, m</th>
<th>Stress Exponent, a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red clay soil</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soft- firm</td>
<td>5-10</td>
<td>20-40</td>
</tr>
<tr>
<td>Stiff- Very Stiff</td>
<td>10-20</td>
<td>20-40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expansive soil</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>firmness</td>
<td>Modulus Number, m</td>
<td>Stress Exponent, a</td>
</tr>
<tr>
<td>Soft- firm</td>
<td>1-5</td>
<td>1-10</td>
</tr>
<tr>
<td>Stiff- Very Stiff</td>
<td>1-5</td>
<td>1-10</td>
</tr>
</tbody>
</table>
6. Discussion

As observed from the test results the tangent modulus approach clearly showed the over consolidated (OC) and normally consolidated (NC) ranges, and the pre-consolidated pressure, $\sigma_{pc}$ these are basic characteristic of the soil which required a lot of efforts to obtain them in the classical method. The pre-consolidated pressure, $\sigma_{pc}$ is specifically the most characteristics of clay soils by which their behavior is greatly dependent.

The ranges of over consolidated and normally consolidated are equally important to determine the soil behavior. In normally consolidated range the values almost correspond to the values found in the other part of the world. But in the over consolidated range the response of local clay soils is different from the soil found in the literatures. The steep slope in the over consolidated range gives a greater value of modulus number than in normally consolidated range. Hence, there were two straight lines having different slopes for both ranges. One can therefore use these values in one strain formula for computation of settlements in both over consolidated and normally consolidated ranges.

The simplicity of the graph gives an appreciated strain computation with out computing the void ratios as an intermediate step. The avoidance of computation of the void ratio gave a further step that it avoided the irrational concept of computing it with classical method.

The deformation time graph shows that the deformation after six hour reading is almost constant. Hence, primary consolidation for most of the loads took six hours. This is also compared with the $t_{90}$ point in the square root time fitting method. The reading of deformation after six (6) hours, therefore, means computing deformations in secondary consolidation range than in primary consolidation range.

The ease of finding the strain distribution through out the soil strata also gave us a more and precise way of solving problems of the time - rate of consolidation. This avoids the inconsistency of the argument made by the classical method that the stress is proportional
to strain and hence the time rate of settlement is dependent on the pore water pressure. In real sense this is not true as it can be understood from the test result. This means the stress and strain are not proportional and hence pore pressure distribution is different from strain distribution. Therefore, the U-T table that found in most of literatures, based on pore water pressure, gives us in general not be applied to normally consolidated clays, but to soils some part of the world with constant M, in the overconsolidated range. This is not applicable to our local soils as it shows a different graph in the range of over consolidation. Hence, a U-T table based on strain distribution for our local soils can be developed.

The range of data for the modulus number in normally consolidated red clay corresponds with soils in some part of the world for different consistency index, but for the expansive soils it doesn’t correspond with its firmness. All ranges of values of modulus number are between 1 to 5 irrespective of their consistency index. This implies one can not use the consistency index to determine the range of modulus number.

The ranges of data of modulus number for our local soils helps the geotechnical engineer in that if the settlement values used for assessment of the foundation design are not determined from a full analysis, the foundation should be evaluated to indicate what range of compressibility parameters (modulus numbers) the settlement value represent for the actual soil profile and condition of effective stress and load. For example, if the design of the superstructure indicates that a settlement of 35 mm is acceptable limit, the foundation design should calculate the modulus numbers that correspond to the limit under the given condition of soil profile and effective stress. This is a small effort that will provide a worth while check on the reasonableness of the result as well as assist in building up preference data base for future analysis.
7. Conclusion and Recommendation

7.1 Conclusion

In this thesis an investigation has been made to assess the applicability of the tangent modulus concept for our local soils and to establish the range of values of the soil parameters. In order to meet the objectives undisturbed and disturbed soil samples were collected from different parts of Addis Ababa and laboratory tests were conducted to determine the physical properties, and consolidation behavior of the soils.

Based on the test results the following conclusions were reached.

1. Grain size analysis tests revealed that the soils under investigation were fine grained soils in which the percentage of clay ranges from 51-62% silt from 26-36% and sand from 11-15%.

2. The general index properties with respect to Atterberg limit show the red clay soils had Liquid limit 47.3-82%, Plastic limit 19.2-39.5% and Plastic index 23.5-55.5%, but expansive clay soils had Liquid limit 85-118%, Plastic limit 24.5-41.9%, and Plastic index 23.35-55.5%.

3. It has been shown herein that the tangent modulus concept which has a good scientific explanation is applicable for both red and expansive clay soil to compute compressibility.

4. For the soil under investigation the following one simple formula, suggested by Janbu, can be used to calculate tangent modulus, $M$,

$$M = m_p a (\frac{\sigma^1}{P_a})^{1-a}$$
5. It has been found that, for clay soils, the value of the exponent, \( a \), is equal to zero and the modulus number, \( m \), have the following range of values.

Table 7.1 Typical and normally conservative modulus number and stress exponent

<table>
<thead>
<tr>
<th>Red clay soil</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>firmness</td>
<td>Modulus Number, ( m )</td>
<td>Stress Exponent, ( a )</td>
</tr>
<tr>
<td></td>
<td>NCR</td>
<td>OCR</td>
</tr>
<tr>
<td>Soft- firm(medium)</td>
<td>5-10</td>
<td>20-40</td>
</tr>
<tr>
<td>Stiff- Very Stiff</td>
<td>10-20</td>
<td>20-40</td>
</tr>
</tbody>
</table>

| Expansive soil                |                       |                      |
|                               | Modulus Number, \( m \) | Stress Exponent, \( a \) |
|                               | NCR                    | OCR                  |
| Soft- firm(medium)            | 1-5                    | 1-10                 | 0                    |
| Stiff- Very Stiff             | 1-5                    | 1-10                 | 0                    |

6. The value of the modulus number for expansive soil is not greatly dependent on the consistency index like the red clay soil. The resistance of such soil, therefore, does not greatly dependent on its index property just like other clay soils do. This leads us to conclude that in using this method for expansive soil, it requires different approach with respect to consistency index to analyze their responses.

7. In the tested local clay soils a linear relation (not constant) between stress and strain were found in the over consolidated range. The value of the modulus number, \( m \) is greater than in over consolidated range than in the normally consolidated range. The change in resistant between the two ranges leave the graph of tangent modulus versus stress to have a sharp and clear point of demarcation called the pre consolidated pressure, a pressure where major structural break down of the soil takes place. This indeed goes with the meaning of pre consolidated pressure.

8. The linear relationship between tangent modulus and stress in the over consolidated range made our local soil different from other clay soils found in other part of the world.
9. The tangent modulus approach can be used to compute the strain distribution throughout the soil strata for different modulus number in both ranges of consolidated in one compressive equation.

10. The deformation time graph shows that the deformation after six hour reading is almost constant. Hence, primary consolidation for most of the loads took six hours. This is also compared with the \( t_{90} \) point in the square root time fitting method. The reading of deformation after six (6) hours, therefore, means computing deformations in secondary consolidation range than in primary consolidation range.

11. The linear relationship between tangent modulus and stress in all ranges of consolidated enables us to conclude that the time rate of settlement is independent of pore water pressure distribution but it is on strain distribution.

### 7.2 Recommendation

The present work has attempted to assess the applicability of the tangent modulus concept. However, this investigation has not covered all areas of the concepts. In view of this, it would be desirable to extend the present work in the following areas.

1. One can develop reference data base for checking on the reasonableness of settlement results by calculating the one parameter, modulus number, for different acceptable value of settlement under the given condition of soil profile and effective stress.

2. A \( U - T \) reference depending on strain distribution for our local soil can be developed in order to determine the rate of settlements.

3. A detail analysis is recommended for refining the time of deformation by making especially across check with the \( t_{90} \) in the square root time fitting method for our local clay soils.
4. The study of this method can be applied for expansive soils by adding a third parameter called the swelling pressure.

5. A detail comparison of this method with the classical method, by using model soils for different soil strata, is left for future works.
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APPENDIX A: Test Results of stress Vs strain and tangent modulus Vs stress plot

Fig A-1 Typical (a) Effective Stress – Strain Curve
(b) Tangent Modulus Vs Stress for Red Clay Soil

\[ m = 15.348 \]
Fig A-2 Typical (a) Effective Stress – Strain Curve
(b) Tangent Modulus Vs Stress for Red Clay Soil

$m=16.423$
Fig A-3 Typical (a) Effective Stress – Strain Curve
(b) Tangent Modulus Vs Stress for Red Clay Soil

\[ m = 12.37 \]
Fig A-4 Typical (a) Effective Stress – Strain Curve
(b) Tangent Modulus Vs Stress for Red Clay Soil

\[ m = 11.098 \]
Fig A-5 Typical

(a) Effective Stress – Strain Curve

(b) Tangent Modulus Vs Stress for Red Clay Soil

\[ m = 6.5203 \]
Fig A-6 Typical (a) Effective Stress – Strain Curve
(b) Tangent Modulus Vs Stress for Red Clay Soil

\[ m = 15.817, \ \mu \sigma = 20.33 \]
Fig A-7 Typical (a) Effective Stress – Strain Curve
(b) Tangent Modulus Vs Stress for Red Clay Soil

$m=7.232$
Fig A-8 Typical (a) Effective Stress – Strain Curve
(b) Tangent Modulus Vs Stress for Red Clay Soil

\[ m = 13.792 \]
Fig A-9 Typical (a) Effective Stress – Strain Curve
(b) Tangent Modulus Vs Stress for Expansive soil

$m=6.31$
Fig A-10 Typical (a) Effective Stress – Strain Curve
(b) Tangent Modulus Vs Stress for Expansive soil

$m=1.44$
Fig A-11 Pressure Vs Void Ratio
Fig A-12 Typical

- (a) Effective Stress – Strain Curve
- (b) Tangent Modulus Vs Stress for Expansive soil

\[ m = 1.573 \]
Fig A-13 Pressure Vs Void Ratio
Fig A-14 Typical  
(a) Effective Stress – Strain Curve  
(b) Tangent Modulus Vs Stress for Expansive soil
Fig A-15 Pressure Vs Void Ratio
Fig A-16 Typical  (a) Effective Stress – Strain Curve
(b) Tangent Modulus Vs Stress for Expansive soil

\[ m = 1.78 \]
Fig A-17 Typical
(a) Effective Stress – Strain Curve
(b) Tangent Modulus Vs Stress for Expansive soil

\[ m = 1.57, \ m_{oe} = 2.16 \]
Fig A-18 Typical

(a) Effective Stress – Strain Curve

(b) Tangent Modulus Vs Stress for Expansive soil

$m=1.203$
Fig A-19 Typical (a) Effective Stress – Strain Curve
(b) Tangent Modulus Vs Stress for Expansive soil

\[ m = 1.21 \]