DETERMINATION OF OPTICAL CONSTANTS AND
THICKNESS OF 3-Octylphenylthiophene/POPT
BY ROTATING ANGLE ELLIPSOMETRY

By
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TO WORDS OF GOD

For I know the plan that I have for you
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Abstract

When a plane polarized light is reflected from the surface of an organic sample or thin film mounted on a glass substrate called Poly 3-Octylphethylthiophene/POPT at some different angles of incidence around the brewster angle of the sample in between 57° up to 61°, then the reflected light which is elliptically polarized and its intensity is detected by photodetector. From its intensity curve the Fourier coefficients \( a \) and \( b \) are collected by fitting to Fast Non Linear Curve Fitting. Based on the Fourier Coefficients the ellipsometric angle \( \Psi \), \( \Delta \) and the amplitude coefficients (Fresnel Coefficients) for parallel and perpendicular light \( r_s \) and \( r_p \) from the upper and bottom surface of the film are computed. From the ellipsometric angles and Fresnel coefficients the optical parameters of the sample such as refractive index, extinction coefficients and the film thickness are determined[5].
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Ellipsometry theory is based on the Fresnel reflection or transmission theory for the polarized light that encounters boundaries in a multilayered materials. Evaluation of reflection or transmission coefficients is based on the Maxwell’s equations. The ellipsometric parameters are usually expressed in terms of $\Psi$ and $\Delta$. Where they are defined as amplitude and phase value of the ratio of the Fresnel reflection coefficients of $p$ and $s$ polarized light. These coefficients hold desirable information about the optical properties and the optical dimensions[1, 4].

The name ‘ellipsometry’ comes from the fact that polarized light often becomes ‘elliptical’ upon reflection. The change in light polarization upon reflection carries information of the optical properties of the medium under consideration. In general the spectroscopic ellipsometry measurement is carried out in the ultraviolet/visible region, but measurement in the infrared region has also been done[2].

Ellipsometry methods can be done with a single or multiple wavelengths for study the sample under experiment. The method used in this experiment is rotating angle ellipsometry RAE. Rotating Angle Ellipsometry is preferred for easy designed and data interpretation when accuracy and precisions are required[3, 10].

In this project, the RAEs are employed. This instrument based on the use of phase sensitive amplifiers for the processing of the optical signal to Fast Fourier Transform
analysis. This result is helpful to determine the ellipsometric parameter easily.

**Historical Development**

Ellipsometry was developed first by Drude in 1887. He also derived the equations of ellipsometry, which are used even today. Most ellipsometers were operated manually and the ellipsometry measurement was very time consuming. In 1975, however, Aspens realized the complete atomization of spectroscopic ellipsometry measurements. The development of this instrument improved not only the measurement time but also the measurement precision significantly.

Recently, optical anisotropic materials have been studied extensively by applying Mueller Matrix ellipsometry that allows the complete characterization of optical behavior in an anisotropic materials. For the characterization of the conventional isotropic sample, current spectroscopic ellipsometry instruments are highly satisfactory. Thus, most of recent ellipsometry studies have been made on a material characterization rather than the development of ellipsometry instruments.

**Objective of the Project**

The main objective of this project is:

- computing ellipsometric Angles $\Psi$ and $\Delta$ using the Fourier Coefficients $a$ and $b$

- computing the Fresnel’s Coefficients $r_p$ and $r_s$ which paving the way to calculate the complex quantity $\rho$

- calculating the real refractive index $n_R$ and the imaginary refractive index $n_I$ which is called the extinction coefficient

- calculating the thickness of the sample - POPT and the absorbance $A$

- determination of the Cauchy’s Coefficients
**Application**

Ellipsometry has been used for characterization of optical properties and physical dimensions. In addition, it has a wide applications in other fields of areas. Some of them are[9, 10]:

- Semiconductor -substrate, gate dielectrics
- Chemistry - polymer films, self assembled monolayers, proteins, DNA
- Optical Coating - high and low dielectric for anti reflection coating
- Data Storage - phase change media for CD, and DVD, magneto optic layers
Chapter 1

The Polarization of Light Wave

1.1 Polarization

An unpolarized beam of light, vibrating in all directions perpendicular to its path strikes a surface and is reflected. The reflected beam will be polarized with vibration directions parallel to the reflecting surface. If some of this light also enters the material and is refracted at an angle $90^\circ$ to the path of the reflected ray, it too will become partially polarized, with vibration directions again perpendicular to the path of the refracted ray, but in the plane perpendicular to the direction of vibration in the reflected ray. Polarization can also be achieved by passing the light through a substance that absorbs light vibrating in all directions except one. Anisotropic crystals have this property in certain directions, called privileged directions. The device used to make polarized light in modern microscopes is a Polaroid, a trade name for a plastic film made by the Polaroid Corporation. A Polaroid consists of long-chain organic molecules that are aligned in one direction and placed in a plastic sheet. They are placed close enough to form a closely spaced linear grid, that allows the passage of light vibrating only in the same direction as the grid. Light vibrating in all other
directions is absorbed. Such a device is also called a polarizer. If a beam on non-polarized light encounters a polarizer, only light vibrating parallel to the polarizing direction of the polarizer will be allowed to pass. The light coming out on the other side will then be plane polarized, and will be vibrating parallel to the polarizing direction of the polarizer. If another polarizer with its polarization direction oriented perpendicular to the first polarizer is placed in front of the beam of now polarized light, then no light will penetrate the second polarizer. In this case we say that the light has been extinguished. The direction of the displacement vector is called direction of polarization and the plane containing the direction of polarization and the propagation vector is called the plane polarization [11].

1.2 Representation of Elliptical Polarization Light

When a linearly plane polarized light strikes the surface of a thin film obliquely, the reflected light is elliptically polarized. The shape and the orientation of the ellipse depends on the angle of incidence, the direction of polarization of the incidence light wave, and the reflection properties of the surface.

Assuming a plane wave is propagating in $z$-direction and the electric field, determining the direction, is oriented in the $x$-$y$ plane. In complex notation, the plane wave is given by [12, 24]

$$\vec{E} = \vec{E}_0 e^{i(kz - wt + \Phi)}$$

(1.2.1)

This wave can be written in terms of the $x$ and $y$ components of $\vec{E}_0$

$$\vec{E} = A_x e^{i(kz - wt + \Phi_x)} \hat{i} + A_y e^{i(kz - wt + \Phi_y)} \hat{j}$$

(1.2.2)

The two harmonic oscillators in eq [1.2.2] have the same frequency $w$ and wave number
k, but differ in phase. The phase difference is given by:

\[ \delta = \Phi_y - \Phi_x \]  

(1.2.3)

We use the real part of the wave for manipulation to prevent errors. So Eq [1.2.2] can be written again as

\[ E_x = A_x \cos(kz - wt + \Phi_x) \]  

(1.2.4)

\[ E_y = A_y \cos(kz - wt + \Phi_y) \]  

(1.2.5)

First if the equations [1.2.4] and [1.2.2] are multiplied by \( \sin \Phi_y \) and \( \sin \Phi_x \), respectively and then carried out subtraction. Again, if the equations [1.2.4] and [1.2.2] are multiplied by \( \cos \Phi_y \) and \( \cos \Phi_x \), respectively, then subtraction is carried out. Performing some algebra, we obtain:

\[ \left[ \frac{E_x}{A_x} \right]^2 + \left[ \frac{E_y}{A_y} \right]^2 - \frac{2E_xE_y \cos \delta}{A_xA_y} = \sin^2 \delta \]  

(1.2.6)

Eq [2.2] has the same forms as equation of Conic section.

\[ Ax^2 + Bxy + cy^2 + Dx + Ey + F = 0 \]  

(1.2.7)

Geometry defines the conic as an ellipse when

\[ B^2 - 4AC = \frac{4(\cos^2 - 1)}{A_x^2A_y^2} < 0 \]  

(1.2.8)

The orientation of the ellipse with respect to x-axis is:

\[ \tan 2\Psi = \frac{B}{A - C} = \frac{2A_xA_y \cos \delta}{A_x^2 - A_y^2} \]  

(1.2.9)

The component of the electric field along the major axis of the ellipse is:

\[ E_M = E_x \cos \Psi + E_y \sin \Psi \]  

(1.2.10)
and along the minor axis of the ellipse is:

$$E_m = -E_x \sin \Psi + E_y \cos \Psi$$  \hspace{1cm} (1.2.11)

The ratio of the length of the minor to the major axis of the ellipse is equal to the ellipticity $\chi$, i.e indicating the amount of deviation of the ellipse from a circle. Mathematically:

$$\tan \chi = \pm \left( \frac{E_m}{E_M} \right)$$  \hspace{1cm} (1.2.12)

Another representation of the state of polarization and one in which leads quite naturally to the poincare sphere is by parameters which have the same physical dimensions, called stokes parameters.

The stokes parameter of light wave are measurable quantities, defined as:

- $S_0$-Total flux density
- $S_1$-Difference between flux density transmitted by a linear polarizer oriented parallel to the x-axis and one oriented parallel to the y-axis.
• $S_2$-Difference between the flux density transmitted by a linear polarizer at $\frac{\pi}{4}$ to the x-axis and one oriented at $\frac{3\pi}{4}$

• $S_3$-Difference between flux density transmitted by a right circular/elliptical and a left circular/elliptical polarizer

Furthermore we can relate the stokes parameters with the value of the poynting vector of an electromagnetic field

$$ \overrightarrow{S} = \overrightarrow{E} \times \overrightarrow{H} = \epsilon_0 c^2 \overrightarrow{E} \times \overrightarrow{B} \quad (1.2.13) $$

The intensity of the propagated light within a time of $T$ is given by

$$ I = |\langle S \rangle| = \frac{1}{T} \int_{t_0}^{t_0+T} n\epsilon_0 c \cos^2(kz - wt + \delta) = \frac{n\epsilon_0 c E_0^2}{2} \quad (1.2.14) $$

The time average of Eq [1.2.6] is denoted by:

$$ \frac{\langle E_x^2 \rangle}{A_x^2} + \frac{\langle E_y^2 \rangle}{A_y^2} - \frac{2\langle E_x E_y \rangle}{A_x A_y} \cos \delta = \sin^2 \delta \quad (1.2.15) $$

In order to remove the terms in the denominator of eq [1.2.15], the equation should be multiplied by $(2A_xA_y)^2$, the result becomes:

$$ 4A_y^2 \langle E_x^2 \rangle + 4A_x^2 \langle E_y^2 \rangle - 8A_x A_y \langle E_x E_y \rangle \cos \delta = (2A_x A_y \sin \delta)^2 \quad (1.2.16) $$

According to Eq [1.2.14], the time averages for three terms in Eq [1.2.16] are

$$ \langle E_x^2 \rangle = \frac{A_x^2}{2}, \langle E_y^2 \rangle = \frac{A_y^2}{2}, \langle E_x E_y \rangle = \frac{A_x A_y \cos \delta}{2} \quad (1.2.17) $$

By inserting Eq [1.2.17] into Eq [1.2.16] yields:

$$ 4A_y^2 A_x^2 - (2A_x A_y \cos \delta)^2 = (2A_x A_y \sin \delta)^2 \quad (1.2.18) $$
If $A_x^2 + A_y^2$ is added to both sides of Eq [1.2.18], it can be written as:

$$[A_x^2 + A_y^2]^2 - [A_x^2 - A_y^2]^2 - [2A_xA_ycos\delta]^2 = [2A_xA_ysin\delta]^2$$

(1.2.19)

Each term in this equation can be identified with stokes parameters.

$$S_0 = A_x^2 + A_y^2, S_1 = A_x^2 - A_y^2, S_2 = 2A_xA_ycos\delta, S_3 = 2A_xA_y sin\delta$$

(1.2.20)

Eq [1.2.19] can be rewritten as:

$$S_0^2 - S_1^2 - S_2^2 = S_3^2$$

(1.2.21)

Using the parameters of elliptical polarized light $\Psi$ and $\chi$, the stokes parameters are further represented by:

$$S_1 = S_0cos2\chi cos2\Psi, S_2 = S_0cos2\chi sin2\Psi, S_0sin2\chi sin2\Psi$$

(1.2.22)

### 1.3 Elliptical polarized Light by Poincare’ Sphere

The state of polarization may be represented by a point on a sphere of radius $S_0$ by spherical coordinates $S_0, 2\chi, 2\Psi$ form the poincare’ sphere. Alternatively, the stokes parameters $S_1, S_2, S_3$ may be used as cartesian coordinate to describe the state of polarization. For example, for plane polarized light, the ellipticity $\chi$ and $S_3$ are zero. Therefore the plane polarized light is represented by points on the equator of the poincare’ sphere. Also, the ellipticity of circularly polarized light is $45^0$, where $S_1 = S_2 = 0, 2\chi = \pm \frac{\pi}{2}$. This circularly polarized light is represented by the poles of the sphere. The poincare’ sphere may be used to represent elliptically polarized light with respect to any physical axes[13]. The representation of polarizations are shown in two graphs of figs of 1.2 and 1.3.
1.4 The Jones-matrix Formulation

When a uniform monochromatic plane wave incident on optical system that consists of either a single optical system that consists of either a single optical device or successive series of optical devices, the emergent plane wave that comes out after propagating through optical system is modified as shown on figure 1.4 [14].

Let the incident and the out going plane waves be described by their appropriate the Jones vector $\vec{E}_i$ and $\vec{E}_0$ referenced to the input and out put coordinate systems respectively. The incident plane wave can be represented $2 \times 1$ column vector as

$$\vec{E}_i = \begin{pmatrix} E_{ix} \\ E_{iy} \end{pmatrix}$$ \hspace{1cm} (1.4.1)

The emergent plane wave can also be represented by a column vector as

$$\vec{E}_0 = \begin{pmatrix} E_{ox'} \\ E_{oy'} \end{pmatrix}$$ \hspace{1cm} (1.4.2)
Figure 1.3: the stokes parameters $S_1$, $S_2$ and $S_3$ used as cartesian coordinate to describe the polarization on poincare sphere.

The two planes wave can be related as

$$E_{0x'} = T_{11} E_{ix} + T_{12} E_{iy} \quad (1.4.3)$$

$$E_{0y'} = T_{21} E_{ix} + T_{22} E_{iy} \quad (1.4.4)$$

Equations [1.4.3] and [1.4.4] can be combined in matrix form as shown below.

$$\begin{pmatrix} E_{0x'} \\ E_{0y'} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} E_{ix} \\ E_{iy} \end{pmatrix} \quad (1.4.5)$$

more precisely

$$\vec{E}_0 = T \vec{E}_i \quad (1.4.6)$$

where

$$T = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \quad (1.4.7)$$
Figure 1.4: Incident and the emergent at and from the optical system $S$ are plane waves Jones vectors

The $2 \times 2$ transformation matrix $T$ is called the Jones matrix of the optical system.
Chapter 2

The propagation of Polarized Light Through Matter

2.1 General Theory

When light or electromagnetic wave propagates through matters, various optical phenomena or properties shown such as absorption, refraction, dispersion, polarization effects electro optical and magneto optical effects. These optical properties can be revealed through by utilizing Maxwell’s equations in media or matters. Because of these the electromagnetic state of matter at a point is described by four quantities[21, 22, 23, 26]. They are:

1. Electric charge density $\rho$

2. Electric dipoles density called the polarization $\vec{P}$

3. Magnetic dipoles density called the magnetization $\vec{M}$
4. The current density $\mathbf{J}$

These four quantities are macroscopically averaged in order to suppress the microscopic variations due to the atomic make up of all matter. The two perpendicular fields $\mathbf{E}$ and $\mathbf{H}$ are related by the following Maxwell’s Equations.

\begin{align}
\nabla \times \mathbf{E} &= -\mu_0 \frac{\partial \mathbf{H}}{\partial t} - \mu_0 \frac{\partial \mathbf{M}}{\partial t} \quad (2.1.1) \\
\nabla \times \mathbf{H} &= \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \quad (2.1.2) \\
\n\nabla \cdot \mathbf{E} &= -\epsilon_0 \nabla \cdot \mathbf{P} + \frac{\rho}{\epsilon_0} \quad (2.1.3) \\
\n\nabla \cdot \mathbf{H} &= -\nabla \cdot \mathbf{M} \quad (2.1.4)
\end{align}

Where $\mathbf{D}$ is the electric displacement stands for $\epsilon_0 \mathbf{E} + \mathbf{P}, \mathbf{B}$ is the magnetic induction stands for $\mu_0(\mathbf{H} + \mathbf{M})$. In this thesis we see the propagation of light in the thin film as non-magnetic non conducting medium, we can set $\mathbf{M}$ and $\rho$ are both zero. We can rewrite the Maxwell’s equation as follows:

\begin{align}
\nabla \times \mathbf{E} &= -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \quad (2.1.5) \\
\nabla \times \mathbf{H} &= \frac{\partial \mathbf{D}}{\partial t} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{P}}{\partial t} \quad (2.1.6) \\
\n\nabla \cdot \mathbf{E} &= -\frac{\nabla \cdot \mathbf{P}}{\epsilon_0} \quad (2.1.7) \\
\n\nabla \cdot \mathbf{H} &= 0 \quad (2.1.8)
\end{align}
Taking the curl of eq [2.1.5] and combining with eq [2.1.6], we obtain the following equation.

\[ \nabla \times (\nabla \times \vec{E}) + \frac{\partial^2 \vec{E}}{c^2 \partial t^2} = -\mu_0 \frac{\partial^2 \vec{P}}{\partial t^2} \]  

(2.1.9)

The terms on the right-hand side of the above equation called the source term. The way in which the propagation light affected by the source is revealed by the solution of the wave equation when the source term is included[21].

### 2.2 Propagation of Light in a Non-conducting Isotropic medium

Materials can be divided into 2 classes based on how the velocity of light of a particular wavelength varies in the material.

1. Materials whose refractive index not depend on the direction that the light travels are called isotropic materials. In these materials the velocity of light does not depend on the direction that the light travels. Isotropic materials have a single, constant refractive index for each wavelength. Minerals that crystallize in the isometric system, by virtue of their symmetry, are isotropic. Similarly, glass, gases, most liquids and amorphous solids are isotropic.

2. Materials whose refractive index does depend on the direction that the light travels are called anisotropic materials. These types of materials will have a range of refractive indices between two extreme values for each wavelength.

- Minerals that crystallize in the tetragonal and hexagonal crystal systems (as well as some plastics) are uniaxial and are characterized by 2 extreme
refractive indices for each wavelength.

- Minerals that crystallize in the triclinic, monoclinic, and orthorhombic crystal systems are biaxial and are characterized by 3 refractive indices, one of which is intermediate between the other two.

In an non-conducting, isotropic medium, the electrons are permanently bound to the atoms comprising the medium and there is no preferential direction. The interaction of the electromagnetic wave with the bounded electrons in the isotropic medium are well understood by a model called the Lorentz-model. This model helps to see the response of an atom to a perturbation. Under the influence of an electromagnetic field, the electron experiences the lorentz force and displaced from its equilibrium position. The lorentz force is given by:

\[ F = e(\overrightarrow{E} + \frac{d\overrightarrow{r}}{dt} \times \overrightarrow{B}) \]  \hspace{1cm} (2.2.1)

The second term which contributes the magnetic force in eq [2.2.1] can be dropped since optical phenomena do not actually involve relativistic particle velocities.

The electron of charge \(-e\), in the medium is displaced a distance \(\overrightarrow{r}\) from its equilibrium position as shown in fig 2.1. The resulting polarization \(\overrightarrow{P}\) of the medium is given by

\[ \overrightarrow{P} = -Ne\overrightarrow{r} \]  \hspace{1cm} (2.2.2)

Where \(N\) is the number of electrons per unit volume.

To find the polarization, the bound electrons are considered as classical damped harmonic oscillator. The differential equation of motion of Newton’s second law is\([27]\):

\[ m \frac{d^2\overrightarrow{r}}{dt^2} + m\gamma \frac{d\overrightarrow{r}}{dt} + k\overrightarrow{r} = -e\overrightarrow{E} \]  \hspace{1cm} (2.2.3)
Figure 2.1: The electron oscillator/lorentz model of an atomic electron. An applied field displaces the electron from its equilibrium position. The electron were a charged mass on the spring.

Let the harmonic displacement of the electron in the medium is expressible the following equation and inserted in eq.[2.2.3]

$$\vec{r} = \hat{\epsilon} r_0 e^{i(kz - \omega t)}$$

The term $\gamma \frac{d\vec{r}}{dt}$ represents a frictional damping force. Since the applied electric field varies harmonically with time according to the usual factor $e^{-i\omega t}$. When eq [2.2.3] is simplified, it can be:

$$(mw^2 - iw\gamma + k) \vec{r} = -e \vec{E} \quad (2.2.4)$$

From eq [2.2.2], the polarization becomes:

$$\vec{P} = \frac{Ne^2}{-mw^2 - iw\gamma + k} \vec{E} \quad (2.2.5)$$

More precisely

$$\vec{P} = \frac{Ne^2}{w_0^2 - w^2 - iw\gamma} \quad (2.2.6)$$
where
\[ w_0 = \sqrt{\frac{k}{m}} \]  \hspace{1cm} (2.2.7)

Eq [2.2.7] tells the effective resonance frequency of the bound electrons. By inserting eq [2.2.6] into eq [2.1.5], it becomes:
\[
\hat{\nabla} \times (\hat{\nabla} \times \vec{E}) + \frac{\partial^2 \vec{E}}{c^2 \partial t^2} = -\frac{\mu_0 N e^2}{m(w_0^2 - w^2 - i\omega \gamma)} \frac{\partial^2 \vec{E}}{\partial t^2}
\]  \hspace{1cm} (2.2.8)

As the relation of $\vec{P}$ and $\vec{E}$ in isotropic media is linear, so $\hat{\nabla} \cdot \vec{E} = 0$. Consequently $\hat{\nabla} \times (\hat{\nabla} \times \vec{E}) = -\nabla^2 \vec{E}$. For a transverse plane wave propagating say along z-direction, $\nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial z^2}$ Eq [2.2.8] now becomes:
\[
-\frac{\partial^2 \vec{E}}{\partial z^2} + \frac{\partial^2 \vec{E}}{c^2 \partial t^2} = -\frac{\mu_0 N e^2}{m(w_0^2 - w^2 - i\omega \gamma)} \frac{\partial^2 \vec{E}}{\partial t^2}
\]  \hspace{1cm} (2.2.9)

Inserting $\vec{E} = \hat{\varepsilon} E_0 e^{i(kz - \omega t)}$ in eq [2.2.8], the result becomes:
\[
k^2 = \frac{w^2}{c^2} (1 + \frac{Ne^2}{m\varepsilon_0(w_0^2 - w^2 - i\omega \gamma)})
\]  \hspace{1cm} (2.2.10)

The presence of the imaginary term in the denominator implies that the wave number $k$ must be a complex number. On the other hand the complex refractive index is introduced as:
\[
\hat{\aleph}^2 = (n_R + i n_I)^2 = 1 + \frac{Ne^2}{m\varepsilon_0(w_0^2 - w^2 - i\omega \gamma)}
\]  \hspace{1cm} (2.2.11)
\[
\aleph = n_R + i n_I \simeq 1 + \frac{Ne^2}{2m\varepsilon_0(w_0^2 - w^2 - i\omega \gamma)}
\]  \hspace{1cm} (2.2.12)

### 2.3 Extinction and Absorption Coefficients, Dispersion

The fact that refractive indices differ for each wavelength of light produces an effect called dispersion. This can be seen by shining a beam of white light into a triangular
prism made of glass. White light entering such a prism will be refracted in the prism by different angles depending on the wavelength of the light. When light enters a transparent material some of its energy is dissipated as heat energy, and it thus looses some of its intensity. When this absorption of energy occurs selectively for different wavelengths of light, the light that gets transmitted through the material will show only those wavelengths of light that are not absorbed. The transmitted wavelengths will then be seen as color, called the absorption color of the material.

The rearrangement of the following equations are helpful in determining the the extinction and absorption coefficients indicating the decay down of the amplitudes of the field and the intensity when the light wave propagates in the media[28].

\[ \vec{E} = \hat{\varepsilon}E_0 e^{i(kz - \omega t)} \]  \hspace{1cm} (2.3.1)

\[ \vec{E} = \hat{\varepsilon}E_0 e^{i\omega \left(\frac{R}{c}\right) - t} \]  \hspace{1cm} (2.3.2)

\[ \vec{E} = \hat{\varepsilon}E_0 e^{i\omega \left(\frac{(n_R + in_I)z}{c}\right) - t} \]  \hspace{1cm} (2.3.3)

\[ \vec{E} = \hat{\varepsilon}E_0 e^{-\frac{2n_I w z}{c}} e^{i\omega \left(\frac{(n_R - z)}{c}\right) - t} \]  \hspace{1cm} (2.3.4)

In the propagating media, \( \vec{E} \) is no more purely oscillator due to the presence of the term \( e^{-n_I w z} \). This term makes the electric field decays with increasing distance of propagation. Since the intensity is proportional to the square of the electric field, the intensity also shows exponential decay with \( z \) as given below.

\[ I(z) = I_0 e^{-\frac{2n_I w z}{c}} = I_0 e^{-\alpha(w)z} \]  \hspace{1cm} (2.3.5)

Where \( \alpha(w) \) is the absorption coefficient, \( n_I \) is called extinction coefficient.

The extinction coefficient is directly related to the absorption coefficient by:

\[ \alpha(w) = \frac{2n_I w}{c} = 2n_I \kappa = \frac{4\pi n_I}{\lambda} \]  \hspace{1cm} (2.3.6)
Where \( \lambda \) is the wave length of the incident light.

From eq [2.2.12], the real part \( n_R \) helps to define the dispersion in the medium where as the imaginary or the extinction index \( n_I \) is to determine the absorption coefficient in the propagated media. When they are further expressed mathematically as follow.

\[
\begin{align*}
n_R &= 1 + \frac{Ne^2(w_0^2 - w^2)}{2m\epsilon_0((w_0^2 - w^2)^2 + (w\gamma)^2)} \\
n_I &= \frac{Ne^2\gamma w}{2m\epsilon_0((w_0^2 - w^2)^2 + (w\gamma)^2)} \\
\alpha &= \frac{2n_Iw}{c} = \frac{Ne^2\gamma w^2}{m\epsilon_0c((w_0^2 - w^2)^2 + (w\gamma)^2)}
\end{align*}
\] (2.3.7)

2.4 Reflection and Refraction of of a plane waves at Interface

When a plane waves are incident on a boundary between two media, some of its incident energy crosses the boundary and some is reflected. The reflection and refraction of light waves at the surface separating two media of different refractive incidences is a familiar phenomenon. Where as the geometry of reflected and transmitted waves has been obtained using only the wave character of light, but the amplitude of the waves has not been determined. We must use Maxwell’s equations and the boundary conditions associated with these to explain about the amplitudes and the of the reflected and transmitted waves. This will also show how optics is contained within the framework of Maxwell’s electrodynamic[22, 26].
2.5 Boundary Conditions

By taking a non conducting ($\sigma = 0$) dielectric media referred to as i the first medium and t the second media, characterized by permeability and permittivity of the first and the second medium are $\mu_i; \epsilon_i$ and $\mu_t \epsilon_t$ respectively. The two media are separated by a plane boundary $y=0$ shown the following figure.

Figure 2.2: The orientation of the electric field and wave vector in the coordinate system as they are selected.

Whose normal $\hat{n} = \hat{k}$is the unit vector along the z-direction. A plane wave is incident obliquely on the boundary.

The fields for the incident, reflected, and transmitted waves as:

$$\vec{E}_i = \vec{E}_0 e^{i(\vec{k}_i \cdot \vec{r} - w_i t)}, \quad \vec{H}_i = \frac{\vec{k}_i \times \vec{E}_i}{w_i \mu_i}$$  \hspace{1cm} (2.5.1)

$$\vec{E}_r = \vec{E}_0 e^{i(\vec{k}_r \cdot \vec{r} - w_r t)}, \quad \vec{H}_r = \frac{\vec{k}_r \times \vec{E}_r}{w_r \mu_r}$$  \hspace{1cm} (2.5.2)
and

\[ \vec{E}_t = \vec{E}_0 \exp(i(\vec{k}_t \cdot \vec{r} - w_t t)), \quad \vec{H}_t = \frac{\vec{k}_t \times \vec{E}_t}{w_t \mu_t} \] (2.5.3)

Where the subscripts i, r, and t represent incident reflected and transmitted waves respectively. The tangential components \( \vec{E} \) and \( \vec{H} \) can be continuous across the boundary at all points only if the exponents are the same at boundary for all fields.[29] This is possible if \( w_i = w_r = w_t \). That is the frequency is unchanged in the reflected and transmitted waves. It now follows

\[ \vec{k}_i \cdot \vec{r} = \vec{k}_r \cdot \vec{r} = \vec{k}_t \cdot \vec{r} \] (2.5.4)

If we choose \( \vec{r} \) lies on the boundary plane, \( \hat{n} \cdot \hat{k} = 0 \), The laws of reflection and refraction are easily extracted

\[ k_i \sin \Phi_i = k_r \sin \Phi_r = k_t \sin \Phi_t \] (2.5.5)

Therefore: Reflection Law

\[ \Phi_i = \Phi_r \] (2.5.6)

\[ \frac{\sin \Phi_i}{\sin \Phi_t} = \frac{k_i}{k_i} = \sqrt{\frac{\epsilon_t \mu_t}{\epsilon_i \mu_i}} \] (2.5.7)

For non-magnetic material, \( \mu_i = \mu_t \)

Snell’s Law

\[ \frac{\sin \Phi_i}{\sin \Phi_t} = \sqrt{\frac{\epsilon_t}{\epsilon_i}} = \frac{n_t}{n_i} \] (2.5.8)

The electric and propagating vectors are decomposed into their own respective components.

Incident waves:

\[ \vec{k}_i = k_i (\sin \Phi_i \hat{i} - \cos \Phi_i \hat{k}), \quad \vec{E}_i = E_{ip} (\cos \Phi_i \hat{i} + \sin \Phi_i \hat{k}) + E_{is} \hat{j} \] (2.5.9)
Reflected wave:
\[
\vec{k}_r = k_r (\sin \Phi \hat{i} - \cos \Phi \hat{k}), \quad \vec{E}_r = E_{rp} (-\cos \Phi \hat{i} + \sin \Phi \hat{k}) + E_{rs} \hat{j}
\] (2.5.10)

Transmitted wave:
\[
\vec{k}_t = k_t (\sin \Phi_t \hat{i} - \cos \Phi_t \hat{k}), \quad \vec{E}_t = E_{tp} (\cos \Phi_t \hat{i} + \sin \Phi_t \hat{k}) + E_{ts} \hat{j}
\] (2.5.11)

The subscripts p and s are stands for parallel and perpendicular or normal to the plane of incidence. By using the boundary conditions associated with Maxwell’s equations are the following.

1. From \(\vec{\nabla} \cdot \vec{D}\), if no surface current are present , by using \(\vec{D} = \varepsilon \vec{E}\)

\[
[\varepsilon_i (\vec{E}_i + \vec{E}_r) - \varepsilon_t \vec{E}_t] \cdot \hat{n} = 0
\] (2.5.12)

\[
\varepsilon_i \sin \Phi_i (E_{ip} + E_{rp}) = \varepsilon_t \sin \Phi_t (E_{tp})
\] (2.5.13)

2. From maxwell’s Equation containing \(\vec{\nabla} \times \vec{E}\), the tangential component of \(\vec{E}\) is continuous.

\[
[\vec{E}_i + \vec{E}_r - \vec{E}_t] \times \hat{n} = 0
\] (2.5.14)

By evaluating the cross product

\[
(E_{is} + E_{rs} - E_{ts}) \hat{i} - (E_{ip} \cos \Phi_i - E_{rp} \cos \Phi_i - E_{tp} \cos \Phi_t) \hat{j} = 0
\] (2.5.15)

By equating

\[
E_{is} + E_{rs} = E_{ts}
\] (2.5.16)

\[
(E_{ip} - E_{rp}) \cos \Phi_i = E_{tp} \cos \Phi_t
\] (2.5.17)
3. From $\nabla \cdot \vec{B} = 0$, the normal component of $\vec{B}$ must be continuous.

$$\vec{B} \cdot \hat{n} = \sqrt{\frac{\mu \epsilon}{k}} \vec{k} \times \vec{E} \cdot \hat{n}$$  \hspace{1cm} (2.5.18)$$

The boundary conditions using the above equation is

$$\left[ \sqrt{\frac{\mu_i \epsilon_i}{k_i}} (\vec{k}_i \times \vec{E}_i) + \vec{k}_r \times \vec{E}_r - \sqrt{\frac{\mu_t \epsilon_t}{k_t}} (\vec{k}_t \times \vec{E}_t) \right] \cdot \hat{n} = 0$$ \hspace{1cm} (2.5.19)$$

This equation yields

$$\sqrt{\mu_i \epsilon_i} [E_{is} + E_{rs} \sin \Phi_i] = \sqrt{\mu_t \epsilon_t} E_{ts} \sin \Phi_t$$ \hspace{1cm} (2.5.20)$$

By combining equations [2.5.16] and [2.5.20], snell’s law is obtained.

$$\frac{\sin \Phi_t}{\sin \Phi_i} = \sqrt{\frac{\mu_i \epsilon_i}{\mu_t \epsilon_t}}$$ \hspace{1cm} (2.5.21)$$

4. From Maxwell’s equation containing $\nabla \times \vec{H}$, the tangential component of $\vec{H}$ is continuous if there is no surface current. The tangential component of $\vec{H}$ can be written in terms of electric field.

$$\vec{H} \times \hat{n} = \frac{\vec{B} \times \hat{n}}{\mu} = \sqrt{\frac{\mu \epsilon}{\mu k}} (\vec{k} \times \vec{E}) \times \hat{n}$$ \hspace{1cm} (2.5.22)$$

By using the above equation, the boundary condition is then written as

$$\left[ \frac{1}{k_i} \sqrt{\frac{\epsilon_i}{\mu_i}} (\vec{k}_i \vec{E}_i + \vec{k}_r \times \vec{E}_r) - \frac{1}{k_t} \sqrt{\frac{\epsilon_t}{\mu_t}} (\vec{k}_t \times \vec{E}_t) \right] \times \hat{n} = 0$$ \hspace{1cm} (2.5.23)$$

This equation yields with its respective components

$$\sqrt{\frac{\epsilon_i}{\mu_i}} [E_{ip} + E_{rp}] = \sqrt{\frac{\epsilon_t}{\mu_t}} E_{tp}$$  \hspace{1cm} (2.5.24)$$

$$\sqrt{\frac{\epsilon_i}{\mu_i}} [E_{is} - E_{rs}] \cos \Phi = \sqrt{\frac{\epsilon_t}{\mu_t}} E_{ts} \cos \Phi_t$$ \hspace{1cm} (2.5.25)$$
Of the boundary conditions of the Maxwell’s equations only two are needed to obtain the relationships between the incident, reflected waves; the conditions utilized are the tangential components of $\vec{E}$ and $\vec{H}$ are continuous across the boundary. The boundary conditions place independent requirements on the polarizations parallel to and normal to the plane of incidence and generate two pairs of equations that are treated independently.

### 2.6 Fresnel’s equations

#### 2.6.1 perpendicular polarization

For the component of the polarization, $\vec{E}$ is perpendicular to the plane of incidence, this means that $\vec{E}$ is everywhere normal to $\hat{n}$ and parallel to the boundary surface between the two media. Using Snell’s law, equation [2.5.25] is written as

$$E_{is} - E_{rs} = \frac{\mu_i \sin \Phi_i \cos \Phi_t}{\mu_i \cos \Phi_i \sin \Phi_t} E_{rs} = \frac{\mu_i \tan \Phi_i}{\mu_i \tan \Phi_t} E_{ts}$$  \hspace{1cm} (2.6.1)

Adding equation [2.6.1] with [2.5.16] yields

$$2E_{is} = E_{is} = (1 + \frac{\mu_i \tan \Phi_i}{\mu_i \tan \Phi_t}) E_{ts}$$  \hspace{1cm} (2.6.2)

$$\frac{E_{ts}}{E_{is}} = \frac{2}{1 + \frac{\mu_i \tan \Phi_i}{\mu_i \tan \Phi_t}}$$  \hspace{1cm} (2.6.3)

For the majority of optical materials, $\mu_i \simeq \mu_t$ there for

$$t_s = \frac{E_{ts}}{E_{is}} = \frac{2 \sin \Phi_t \cos \Phi_i}{\sin (\Phi_t \Phi_i)}$$  \hspace{1cm} (2.6.4)

The amplitude ratio $t_s$ is called the amplitude transmission coefficient for perpendicular polarization.
With similar procedure, equations [2.6.2] and [2.5.16] are combined to produce the amplitude reflection coefficient for perpendicular polarization.

\[
E_{is} + E_{rs} = \frac{2E_{is}}{1 + \left(\frac{\mu_i \tan \Phi_i}{\mu_t \tan \Phi_t}\right)} \quad (2.6.5)
\]

\[
E_{rs} = \frac{1 - \left(\frac{\mu_i \tan \Phi_i}{\mu_t \tan \Phi_t}\right)}{1 + \left(\frac{\mu_i \tan \Phi_i}{\mu_t \tan \Phi_t}\right)} \quad (2.6.6)
\]

\[
r_s = \frac{E_{rs}}{E_{is}} = -\frac{\sin(\Phi_i - \Phi_t)}{\sin(\Phi_i + \Phi_t)} \quad (2.6.7)
\]

### 2.6.2 parallel polarization

For the component of polarization, \(\vec{E}\) os every where parallel of incident, however, \(\vec{B}\) and \(\vec{H}\) are every where normal to \(\hat{n}\) and parallel to the boundary between the media. From Eq [2.5.17], the following is obtained.

\[
E_{ip} - E_{rp} = \frac{\cos \Phi_t}{\cos \Phi_i} E_{tp} \quad (2.6.8)
\]

Applying Snell’s Law to eq [2.5.24] yields

\[
E_{ip} + E_{rp} = \sqrt{\frac{\mu_i \epsilon_i}{\mu_t \epsilon_t}} E_{tp} = \frac{\mu_i}{\mu_t} \frac{\sin \Phi_i}{\sin \Phi_t} E_{tp} \quad (2.6.9)
\]

Adding equation eq. [2.6.9] to eq [2.6.8] yields the desired ratio amplitude.

\[
\frac{E_{tp}}{E_{ip}} = \frac{2\cos \Phi_i}{\cos \Phi_i \sin \Phi_i + \frac{\mu_i}{\mu_t} \cos \Phi_i \sin \Phi_i} \quad (2.6.10)
\]

The amplitude transmission coefficient for parallel polarization is

\[
t_p = \frac{E_{tp}}{E_{ip}} = 2 \frac{\cos \Phi_i \sin \Phi_i}{\sin(\Phi_i + \Phi_t) \cos(\Phi_i + \Phi_t)} \quad (2.6.11)
\]


\[
r_p = \frac{E_{rp}}{E_{ip}} = \frac{\mu_t}{\mu_i} \left(\frac{\sin 2\Phi_i - \sin 2\Phi_t}{\sin 2\Phi_t + \frac{\mu_i}{\mu_t} \sin 2\Phi_i}\right) \quad (2.6.12)
\]
\[ r_p = \frac{E_{rp}}{E_{ip}} = \frac{\tan(\Phi_i - \Phi_t)}{\tan(\Phi_i + \Phi_t)} \]  

Equations [2.6.4][2.6.7] [2.6.11] [2.6.13] define the Fresnel equations or Formulae. They are commotional tools in computing ellipsometric parameters[17].
3.1 Introduction

Ellipsometry can be generally defined as the measurement of polarization of a polarized vector wave. The mathematical theory of ellipsometry is based on the Fresnel’s equations which are derived from Maxwell’s boundary equations discussed under chapter 2. Although measurement of the state of polarization of light wave is important in its own right, ellipsometry is generally conducted in order to obtain information about an optical system that modifies the state of polarization [15].

The characterization of optical constants and thickness of organic thin films is major part of this project, and ellipsometry is the primary method of determining these quantities. The instatement used for this project is a rotating angle ellipsometry RAE because ellipsometry measures the ratio of its ellipsometric parameters that are identified during experiment. It is highly accurate and reproducible and no reference material is necessary. Because it is a measure a phase quantity $\Delta$ and amplitude ratio $\rho$, it is very sensitive to the presence of very thin film.
RAE provides new information because of different optical path length traversed, and it optimizes sensitivity to the unknown parameters.

3.2 The equations of Reflection Ellipsometry for Ambient film- Substrate system.

Basic mathematical tools of ellipsometry are built on the Maxwell’s wave equations at the boundaries of the interface[5, 6, 9, 14].

3.2.1 Basic mathematical Tools

\[
\begin{align*}
    r_p &= \frac{N_1 \cos \Phi_i - N_0 \cos \Phi_t}{N_1 \cos \Phi_i + N_0 \cos \Phi_t} \quad (3.2.1) \\
    r_s &= \frac{N_0 \cos \Phi_i - N_1 \cos \Phi_t}{N_0 \cos \Phi_i + N_1 \cos \Phi_t} \quad (3.2.2) \\
    t_p &= \frac{2N_0 \cos \Phi_i}{N_1 \cos \Phi_i + N_0 \cos \Phi_t} \quad (3.2.3) \\
    t_s &= \frac{2N_0 \cos \Phi_i}{N_0 \cos \Phi_i + N_1 \cos \Phi_t} \quad (3.2.4)
\end{align*}
\]

\( \Phi_i \) and \( \Phi_t \) are the incident and refracted (transmitted) angles respectively. They are related by Snell’s law.

\[
N_0 \sin \Phi_i = N_1 \sin \Phi_t \quad (3.2.5)
\]

Since, the reflection ellipsometry employing the Fresnel’s coefficients for the reflection ellipsometry is usually written as

\[
\begin{align*}
    r_p &= |r_p| e^{i \delta_{r_p}} \quad (3.2.6) \\
    r_s &= |r_s| e^{i \delta_{r_s}} \quad (3.2.7)
\end{align*}
\]
where $\delta_{rp}$ and $\delta_{rs}$ are the phase shift upon reflection for p and s polarized light respectively.

As it has been stated in the introduction part, reflection ellipsometry is a technique based on measurement of the states polarization of incident[16, 18] and reflected waves, leading to the determination of the ratio $\rho$ of the complex Fresnel reflection coefficients for p and s polarizations.

$$\rho = \frac{r_p}{r_s}$$  \hspace{1cm} (3.2.8)

It is often convenient to write $\rho$ in the form that

$$\rho = \tan \Psi e^{i\Delta}$$ \hspace{1cm} (3.2.9)

From eqs [3.2.6] and [3.2.8] it is readily seen that

$$\tan \Psi = \left| \frac{r_p}{r_s} \right|$$  \hspace{1cm} (3.2.10)

$$\Delta = \delta_{rp} - \delta_{rs}$$ \hspace{1cm} (3.2.11)

$\Psi$ and $\Delta$ are ellipsometric angle determining the differential changes in amplitude and phase, respectively, experienced upon reflection by component vibrations of the electric field vector parallel and perpendicular to the plane of incidence[16].

If we substitute for $r_p$ and $r_s$ of eqs [3.2.1] and [3.2.2] in eq [3.2.8], and using snell’s law [3.2.5], the complex refractive index of the second medium/film $N_1$ in terms of $\rho$ and $\Phi_i$ are solvable and given in the following equation.

$$N_1 = N_0 \sin \Phi_i \left[ 1 + \left( \frac{1 - \rho}{1 + \rho} \right)^2 \tan^2 \Phi_i \right]^{\frac{1}{2}}$$ \hspace{1cm} (3.2.12)
3.3 Reflection and Transmission by an ambient-substrate system/Multiple Reflections and Transmission from Thin Film

Let an electromagnetic wave of p and s polarized wave incident at the upper surface of thin film of thickness $d$ at angle $\Phi_i$ shown in figure 2. As the wave enters the film, the propagated wave reflected inside the film and transmitted to the surrounding media of the film. Inside the film the propagation wave makes a zigzag path. The phase angle $\beta$ or the film phase thickness is given by[17, 25]

$$\beta = 2\pi \frac{d}{\lambda} N_1 \cos \Phi_t$$

(3.3.1)

or

$$\beta = 2\pi \frac{d}{\lambda} (N_1^2 - N_0^2 \sin \Phi_i^2)^{\frac{1}{2}}$$

(3.3.2)
From figure 2: the total reflected amplitude $R$ are computed from an infinite geometric series for the waves comes from the upper face of the film.

$$R = r_{01} + t_{01}t_{10}r_{12}e^{-i2\beta} + t_{01}t_{10}r_{10}r_{12}^2e^{-i4\beta} + t_{01}t_{10}r_{10}^2r_{12}^3e^{-i6\beta} \quad (3.3.3)$$

This summation gives:

$$R = r_{01} + \frac{t_{01}t_{10}e^{-i2\beta}}{1-r_{10}r_{12}e^{i2\beta}} \quad (3.3.4)$$

From the principle of reversibility[25]: $r_{10} = -r_{01}$ and $t_{01}t_{10} = 1 - r_{01}^2$. Therefore,

$$R = \frac{r_{01} + r_{12}e^{-i2\beta}}{1 + r_{01}r_{12}e^{-i2\beta}} \quad (3.3.5)$$

Similarly, the total transmitted amplitude from the lower part of the film is computed in the following equation.

$$T = \frac{t_{01}t_{12}e^{-i\beta}}{1 + r_{01}r_{12}e^{-i2\beta}} \quad (3.3.6)$$

Reflection Ellipsometry utilizes eq [3.3.5] by introducing p and s subscripts as follow

$$R_p = \frac{r_{01p} + r_{12p}e^{-i2\beta}}{1 + r_{01p}r_{12p}e^{-i2\beta}} \quad (3.3.7)$$

$$R_s = \frac{r_{01s} + r_{12s}e^{-i2\beta}}{1 + r_{01s}r_{12s}e^{-i2\beta}} \quad (3.3.8)$$

The ellipsometric angles $\Psi$ and $\Delta$ are related to equations [3.3.7] and [3.3.8] by finding their ratio as follow

$$\rho = \tan\Psi e^{i\Delta} = \frac{R_p}{R_s} = \frac{r_{01p} + r_{12p}e^{-i2\beta}}{1 + r_{01p}e^{-i2\beta}} \times \frac{1 + r_{01s}r_{12s}e^{-i2\beta}}{r_{01s} + r_{12s}e^{-i2\beta}} \quad (3.3.9)$$

The corresponding amplitude reflection coefficients for p and s from the upper and
lower surface of the film is given by:

\[
\begin{align*}
    r_{01p} &= \frac{N_1 \cos \Phi_i - N_0 \cos \Phi_t}{N_1 \cos \Phi_i + N_0 \cos \Phi_t} \\
    r_{12p} &= \frac{N_2 \cos \Phi_t - N_1 \cos \Phi_{t1}}{N_2 \cos \Phi_t + N_1 \cos \Phi_{t1}} \\
    r_{01s} &= \frac{N_0 \cos \Phi_i - N_1 \cos \Phi_t}{N_0 \cos \Phi_i + N_1 \cos \Phi_t} \\
    r_{12s} &= \frac{N_1 \cos \Phi_t - N_2 \cos \Phi_{t1}}{N_1 \cos \Phi_t + N_2 \cos \Phi_{t1}}
\end{align*}
\]

By relating equations [3.3.1] and [3.3.9], one can obtain the film thickness $d$

### 3.4 Calculating The Fourier Coefficients $a$ and $b$, and $\tan \Psi$ and $\cos \Delta$

For non polarized source emits, an electromagnetic wave intensity[2]

\[
I(\lambda) = E.E^*
\]  \hspace{1cm} (3.4.1)

Let the first a polarizer selectees a beam with components

\[
E_p = E_0 \cos(\alpha), \quad E_s = E_0 \sin(\alpha)
\]  \hspace{1cm} (3.4.2)

Which after reflection becomes:

\[
E_{ref} = r_p E_p + r_s E_s = E_0 |r_p| \cos(\alpha) e^{i\delta_p} + E_0 |r_s| \sin(\alpha) e^{i\delta_s}
\]  \hspace{1cm} (3.4.3)

And finally, it is filtered by the second rotating analyzer rotating by $w$, and the emerging electric field becomes

\[
E_{out} = r_p E_p \cos(wt) + r_s E_s \sin(wt)
\]
\[ E_{\text{out}} = E_0|r_p|\cos(\omega t)\cos(\alpha)e^{i\delta_p} + E_0|r_s|\sin(\omega t)\sin(\alpha)e^{i\delta_s} \quad (3.4.4) \]

The intensity detected by the photo detector becomes now

\[ I_D = E_{\text{out}}^*E_{\text{out}} = |E_0|^2 [ |r_p|^2 \cos^2(\omega t)\cos^2(\alpha) + |r_s|^2 \sin^2(\omega t) \]
\[ + 2|r_p||r_s|\cos(\omega t)\cos(\alpha)\sin(\omega t)\sin(\alpha)\cos(\delta_p - \delta_s)] \quad (3.4.5) \]

\[ I_D = \frac{|E_0|^2}{2} \left\{ |r_p|^2 \cos^2(\alpha) + |r_s|^2 \sin^2(\alpha) \right. \]
\[ + \left. [ |r_p|^2 \cos^2(\alpha) - |r_s|^2 \sin^2(\alpha) ] \cos(2\omega t) + \frac{1}{2}|r_p||r_s| \sin(2\alpha)\cos(\Delta)\sin(2\omega) \right\} \quad (3.4.6) \]

By rearranging the terms in the equation and obtain it in the form

\[ I_D(\lambda) = g[1 + \frac{s_1}{s_0}\cos(2\omega t) + \frac{s_2}{s_0}\sin(2\omega t)] \quad (3.4.7) \]

Angle \( \alpha \) is a fixed angle for the polarizer. This angle is usually chosen as 45°

\[ s_2 = \frac{|r_p||r_s|\sin \alpha}{|r_p|^2 \cos^2 \alpha + |r_s|^2 \sin^2 \alpha} \]
\[ s_1 = \frac{|r_p|^2 \cos^2 \alpha - |r_s|^2 \sin^2 \alpha}{|r_p|^2 \cos^2 \alpha + |r_s|^2 \sin^2 \alpha} \]
\[ s_0 = |r_p|^2 \cos^2 \alpha + |r_s|^2 \sin^2 \alpha \]
\[ \tan \Psi = \frac{|r_p|}{|r_s|} \]
\[ a = \frac{s_1}{s_0} \]
\[ b = \frac{s_2}{s_0} \]

\( g \) is the the reflected intensity amplitude detected by the photodetector. Where as \( a \) and \( b \) are the Fourier coefficients are determined during the experiment by fitting to non linear curve. They depend on \( \tan \Psi \) and \( \cos \Delta \). Now:
\[ a = \frac{s_1}{s_0} = \frac{\tan^2 \Psi - \tan^2 \alpha}{\tan^2 \Psi + \tan^2 \alpha} \]  
\[ b = \frac{s_2}{s_0} = \frac{\tan \Psi \tan \alpha}{\tan^2 \Psi + \tan^2 \alpha \cos \Delta} \]  
\[ \tan \Psi = \sqrt{\frac{1 + a}{1 - a}}, \cos \Delta = \frac{b}{\sqrt{1 - a^2}} \]  
Therefore:
\[ I_D = g(1 + a \cos(2wt) + b \sin(2wt)) \]
Chapter 4

Instrument and Experiment

4.1 The sample

The sample on which the experiment was carried out is a poly (3-octalphenyl lithiumphene)POPT. This sample, a class of organic material was prepared in the Polymer Lab of Physics Department to investigate its electrical properties and optical absorption[7].

The sample has been formally used by coating on a glass substrate. The coating task has been done using a photoresist coater model 4000. This model spines the wafers on a DC motor driven chunk at speed that can be selected between 100 and 6200 rpm. This sample or film is spin coated on a glass substrate with speed of 1200 rpm. The thickness of the deposited polymer is controlled by the rate of the revolution and concentration. For a given concentration of polymer solution, the higher the speed (rpm) the thinner the deposited polymer film.

The optical absorption spectrum of spin coated thin film of POPT was measured using Perkin Elmer λ 19/UV/VIS/NIR spectrometer. This instrument is fast and a powerful machine for measuring the reflectance, transmittance, and absorbance of layers/films as a function of wavelength or energy. The PE λ 19 spectrometer allows
spectral measurement between 1700nm and 3200nm. It is a computer interfaced instrument with a software UV computerized spectroscopy software. The absorption spectrum of the sample was measured using this instrument[7]

4.2 Optical Materials

4.2.1 Optical Chopper

Optical chopper are mechanical device that physically block a light beam of some type. Rotating optical chopper are perhaps the most common form of metal disk in, it is which slots are etched in to its mounted on a DC motor. The disc is placed in the path of light beam which will then cause the beam to be periodically interrupted by the blocking part of the disc[31].

![Figure 4.1: Single beam Experiment](image)

Mechanical optical choppers are useful where it is not possible to control the light source directly or at the speeds required. For example a standard filament light bulb can be pulsed to a few 100Hz though depth of modulation is limited. If it is required
to switch the light on and off completely at 20 kHz, then the use of a mechanical optical chopper is required.

4.2.2 Polarizers

A polarizer is a device that converts unpolarized or mixed beam of electromagnetic wave (light) into a beam with a single polarization state usually a single linear polarization [32].

4.2.3 Lock-in Amplifier

A lock-in amplifier is able to measure a small signal even in the presence of a lot of noise. It does this with help of some signal processing [8, 30].

4.2.4 Stepper Motors

A stepper motor is an electromagnetic device which converts electric pulses

![Figure 4.2: Concept of stepper motor operation](image)

in to discrete mechanical movements [20, 33]. The shaft or spindle of the a stepper
motor rotates in discrete step increment when electrical command pulses are applied to it in the proper sequence. The motors rotation has several direct relationships to these applied input pulses. The sequence of the applied pulses is directly related to the direction of the motor shafts rotation. The speed of the motor shafts rotation is directly related to the frequency of the input pulses and the length of rotation is directly related to the number of input pulses applied. The simplified concept of an stepper motor operating using the principle of magnetic attraction and repulsion to convert digital pulses into mechanical shaft rotation is shown in the above figure.

4.3 Basic Instrument of rotating angle Ellipsometry RAE

At any particular angle of incidence and wavelength, two independent measurements can be made, the real and imaginary part of the complex reflectance ratio (or \(\tan \Psi\) and \(\cos \Delta\)). If more than two parameters are determined such as the thickness and the real part of the refractive index of the film then more than one set of observations will need to be made. This has lead to a number of forms of ellipsometry, of which two have assumed importance. Spectroscopic Ellipsometry SE and Multiple Angles of Incidence Ellipsometry MAIE [6, 7, 8]

- using a fixed angle of incidence the complex reflection ratio is measured over a range of wavelengths. This is generally termed as spectroscopic ellipsometry (SE) and is most widely used forms of ellipsometry.

- Using a fixed wavelength, the complex reflectance ratio is measured over a range of angle of incidence. This is generally known as multiple angles of incidence ellipsometry MAIE. MAIE systems generally employ lasers as the light source.
Lasers are highly monochromatic and have a much higher intensity at specific wavelength than a conventional source. Further more, they produce highly collimated light. Beam divergences is a major source of error in MAI systems. In this project four different wavelengths of laser sources were utilized. The 5mw He-Ne laser of wavelength 632.8nm, 1mw semiconductor Laser of wavelength 660nm, a temperature tunable semiconductor diode laser which has a maximum output power of 450mW at wavelength 808nm, Nd:YAG laser pumped by a semiconductor diode laser having a maximum power of 100mW at wavelength of 1064nm. But using a KTP crystal, the 1064 nm laser is converted to its second harmonic frequency, i.e 532nm.

Light from a source passes through a monochromator, and a narrow spectral band of collimated light passes through a polarizer, reflects at oblique angle off the sample under study, passes through a second, rotating polarizer (analyzer) and enters the detector. The angle of incidence in this ellipsometer is computer controlled, and generally is in the range of 50° and 80° depending on a sample.

For the set up of rotating angle ellipsometry, three stepper motors are needed at a time. The first stepper motor carries and rotates the sample so as to vary the angle of incidence at which the incoming rays striking the sample. The second stepper motor carries a cross bar holding both the detector and third stepper motor holding through which the reflected ray is passing and then being detected by photodetector via the analyzer. The task of this stepper motor is to rotate the photodetector and the third stepper motor upon which the analyzer is mounted rotates with twice a given rotational speed of the first stepper motor upon which the sample mounted. On the other hand the third stepper motor holding the analyzer rotates 360° for a
given angle of incidence shown in figure 4.3 or fig 4.4. The speeds and position of stepper motors are monitored by Advanced Positioning Technology ATP steppers motor controllers user interface software.

Figure 4.3: Systematic Set up of Ellipsometry

4.4 Procedure

This experiment has been conducted according to the following procedures.

1. The set up of the experiment should be carefully set according to figure 4.3 or 4.4.

2. The glass substrate is mounted on the first stepper motor. This stepper motor
Figure 4.4: The actual set up polarizer-sample-Analyzer ellipsometry makes the glass to be adjusted for different angles of incidence. For a given range of angle of incidence, the second stepper motor holding the detector should rotate at twice the speed of that of the first stepper motor. During this rotation, the detector picks the angle at which the minimum is intensity found. This angle is called Brewster angle. Determination of Brewster angle is made for four different laser sources. The same method is employed for Ambient Film Substrate system. Determination of the brewster angles for film substrate system is an initial guess for deciding the principal angles around the brewster angle. These principal angles are the angles where ellipsometer is sensitive.

3. The light source is a laser source which is plane polarized and the transmission axis of the polarizer should be set $\alpha = \frac{\pi}{4}$ to the plane of incidence.

4. The reflected light passes through the analyzer rotated by the third stepper
motor controller. The angle of of incidence $\Phi_i$ is varied by computer control mechanism. The reflected light is detected and amplified by lock in amplifier at a frequency by which light is chopped. The intensity of the light received at the photo detector has the following form eq [3.4.11]

$$I = g(1 + a\cos(2\omega t) + b\sin(2\omega t))$$

(4.4.1)

Where:

$$\tan \Psi = \sqrt{\frac{1+a}{1-a}}, \cos \Delta = \frac{b}{\sqrt{1-a^2}}$$

(4.4.2)

Where $a$ and $b$, Fourier coefficients, are found from the data by fitting the plotted curve by the origin to the non linear curve fitting command from file menu of Analysis.

5. The ellipsometric angles $\Delta$ and $\Psi$ will be computed based on the value of Fourier coefficients $a$ and $b$. 
Figure 4.5: A figure showing the measurement of the Brewster Angles during Experiment for three Laser sources.
Chapter 5

Data Analysis And Discussion

5.1 Results for Optical characterization of POPT

In rotating analyzer ellipsometr a linear polarizer is used to bring the light to a linear polarization state just before reflection. A second rotating polarizer, the analyzer is to determine the polarization state after reflection. The change of polarization is characterized by $\Psi$ and $\Delta$, the relative amplitude change and the phase shift between two polarization directions respectively. Angle $\Psi$ and $\Delta$ are determined by measurement of the reflected intensity while the analyzer is rotated. The intensity $I$ then changes harmonically with analyzer $A$ and is written as

$$I = g(1 + a\cos(aA) + b\sin(2A))$$ (5.1.1)

The intensity wave forms obtained from ellipsometric measurement are sinusoidal in nature given by equation 5.1.1

where $A = \omega t$. Where $a$ and $b$, Fourier coefficients, are found from the data by fitting the plotted curve by the origin to the non linear curve fitting command from file menu of Analysis. Angle $\Psi$ and $\Delta$ are related to Fast Fitting Fourier Coefficients a
and b as follow.

\[ \tan \Psi = \sqrt{\frac{1 + a}{1 - a}} \]  
(5.1.2)

\[ \cos \Delta = \frac{b}{\sqrt{1 - a^2}} \]  
(5.1.3)

## 5.2 Calculating The refractive indices and Extinction coefficients

### 5.2.1 Relation between the refractive index and \( \Psi \) and \( \Delta \)

As it has been stated under chapter two, the Fresnel equations and refractive indices are related as follow on table 5.1:

\[
\begin{align*}
  r_p &= \frac{N_1 \cos \Phi_i - N_0 \cos \Phi_t}{N_1 \cos \Phi_i + N_0 \cos \Phi_t} \\
  r_s &= \frac{N_0 \cos \Phi_i - N_1 \cos \Phi_t}{N_0 \cos \Phi_i + N_1 \cos \Phi_t}
\end{align*}
\]  
(5.2.1, 5.2.2)

Where \( N_0 \) and \( N_1 \) are complex indices of the two media and \( \Phi_i \) and \( \Phi_t \) angle of incidence and refraction respectively. The ratio of these two coefficients defines a complex quantity \( \rho \) that is written as

\[ \rho = \tan \Psi \exp i \Delta = \tan \Psi \cos \Delta + itan \Psi \sin \Delta = \frac{r_p}{r_s} \]  
(5.2.3)

From chapter 2, Snell’ law is written as:

\[ N_0 \sin \Phi_i = N_2 \sin \Phi_t \]  
(5.2.4)

The refractive index of the the sample, the second medium is explicitly written as:

\[ N_1 = N_0 \sin \Phi_i \left[ 1 + \left( \frac{1 - \rho}{1 + \rho} \right)^2 \tan^2 \Phi_i \right]^{\frac{1}{2}} \]  
(5.2.5)
From equation 5.2.5, the real refractive index $n_R$ and the extinction coefficients $n_I$, which is the imaginary parts, are extracted.

Based on the above listed equations the calculated real refractive indices and the extinction coefficients are shown with their corresponding wavelengths.

### 5.2.2 Cauchy Formula or Dispersion Formula

Cauchy’s Formula is an empirical relationship between the refractive index $n$ and wavelength of light $\lambda$ for a particular transparent material. The Cauchy Formula are derived from an electron oscillator model discussed under chapter 2.

As it is shown on table 5.1 and fig 5.3 on page 53, the refractive index decreases with an increase of the wave length. The empirical Cauchy Formula is given in eq.[5.2.6] below

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} \tag{5.2.6}$$

A, B, and C are Cauchy coefficients which are unique values for different materials. Usually, it is sufficient to use a two term form of the equations to determine Cauchy’s coefficients by fitting to the second order of exponential decay[35]. The following results A and B are determined from by fitting the the graph of refractive index versus the wave length of fig 5.3 on page 54 to the second order of the exponential decay.
\[ A = 1.56034 \]
\[ B = 2.019 \times 10^{-7} m^2 \]

### 5.3 Absorbance

In spectroscopy, the absorbance \( A \) is defined as

\[ A_\lambda = \log \frac{I_0}{I} \]  \hfill (5.3.1)

Where \( I \) is the intensity of light at specified wavelength \( \lambda \) that passed through a sample and \( I_0 \) is the intensity of the light before it enters the sample or incident light intensity. The absorbance of the sample is proportional to the thickness of the sample and the concentration of the absorbing sample.

#### 5.3.1 Beer-Lambert Law

Beer-lambert law, also known as Beer’s law is an empirical relationships that relates the absorption of light to the properties of the material through which the light is traveling.

There are several ways in which the law can be expressed.

\[ A = adc \]  \hfill (5.3.2)

Where \( a \) is the absorption coefficient, \( d \) is the the distance that the light travelling , \( c \) is the concentration of the absorbing medium. From equation 5.3.1, the absorbance \( A \) is given by:

\[ A = \log \frac{I_0}{I} \]  \hfill (5.3.3)
From chapter one, the intensity I of the propagating wave in the medium moving a distance z within the sample is given by:

\[ I = I_0 e^{-a(w)z} \]  

(5.3.4)

The absorption coefficient is related to the extinction coefficient in the following equation.

\[ a(w) = 2n_I \frac{w}{c} \]  

(5.3.5)

where \( n_I \) is the extinction coefficient extracted from equation 5.2.5, \( w \) is the angular frequency at which the elections are oscillated.

Combing equations 5.3.3 and and 5.3.4, the absorbance A can be derived.

\[ A = \frac{4\pi d n_I}{\lambda \ln(10)} \]  

(5.3.6)

The computed absorbance and extinction coefficients are listed in the table 5.1. Furthermore the absorbance of the film measured using four laser sources of ellipsometry and UV/VIS/NIR is shown in fig 5.3 and fig 5.4.

### 5.4 Calculating the Thickness of the Film

To calculate the thickness of the sample the following equations may be helpful.

\[ \beta = 2\pi \frac{d}{\lambda} N_1 \cos \Phi_t \]  

(5.4.1)

\[ \rho = \frac{r_{01p} + r_{12p} e^{-i2\beta}}{1 + r_{01p} r_{12p} e^{-i2\beta}} \times \frac{1 + r_{01s} r_{12s} e^{-i2\beta}}{r_{01s} + r_{12s} e^{-i2\beta}} \]  

(5.4.2)

At this stage, \( \beta \), could not be extracted separately. To make \( \beta \) separate from equation [5.4.2], it is advisable to expand the right term using Taylor series to the first
order[36]. To facilitate the expansion let the Fresnel coefficients be represented as 
\( r_{01p} = r_1, r_{12p} = r_2, r_{01s} = r_3, r_{12s} = r_4 \). Then the right term can be expanded as:

\[
\rho = \frac{r_1 + r_2(1 - 2i\beta)}{1 + r_1r_2(1 - i2\beta)} \times \frac{1 + r_3r_4(1 - i2\beta)}{r_3 + r_4(1 - 2\beta)} \tag{5.4.3}
\]

Since the expansion is too long, the final result of the real \( R \) and the imaginary \( \text{Im} \) are presented here.

\[
p = (r_3 + r_4 + r_1r_2r_4 - 4r_2r_3r_4\beta^2)(r_3 + r_4 + r_1r_2r_3 - 4r_1r_2r_4\beta_2) \tag{5.4.4}
\]

\[
q = 4(r_1r_3r_4 + r_2r_3r_4 + r_2 + r_2r_3r_4)(r_4 + r_1r_2r_4 + r_1r_2r_3 + r_1r_2r_4)\beta^2 \tag{5.4.5}
\]

\[
v = (r_3 + r_4 + r_1r_2r_3 - 4r_1r_2r_4\beta^2)^2 + 4(r_4 + r_1r_2r_3 + r_1r_2r_4)\beta^2 \tag{5.4.6}
\]

Therefor the real part \( R \) can be technically written as:

\[
R = \frac{p + q}{v} \tag{5.4.7}
\]

For the imaginary Part we use technical representation of letters.

\[
s = 2(r_1 + r_2 + r_1r_3r_4 - 4r_1r_3r_4\beta^2)(r_4 + r_1r_2r_4 + r_1r_2r_3 + r_1r_2r_4) \tag{5.4.8}
\]

\[
t = (r_1r_2r_4 + r_2r_3r_4 + r_2 + r_2r_3r_4)\beta(r_3 + r_4 + r_1r_2r_4 - 4r_1r_2r_4\beta^2) \tag{5.4.9}
\]

\[
u = (r_3 + r_4 + r_1r_2r_3 - 4r_1r_2r_4\beta^2)^2 + 4(r_4 + r_1r_2r_4 + r_1r_2r_4)\beta^2 \tag{5.4.10}
\]

\[
\text{Im} = \frac{t - s}{u} \tag{5.4.11}
\]
Calculating the Fresnel coefficients is quite complicated. By preparing excel spreadsheet the numerical values of the Fresnel’s coefficients are handled and calculated. The full result is not presented here because of its lengthy. some of them are tabulated in the appendix.

The complex $\rho$ using equations [5.1.2],[5.1.3], and [5.2.3] is given by:

$$\rho = \tan \Psi \cos \Delta + i \tan \Psi \sin \Delta$$

(5.4.12)

Where

$$\tan \Psi = \sqrt{\frac{1 + a}{1 - a}}$$

$$\cos \Delta = \frac{b}{\sqrt{1 - a^2}}$$

a and b are the Fourier coefficients determined from the fitting of the data to the nonlinear curve.

By equating the real part of $\rho$, [5.4.12] and and equation [5.4.7]; $\beta$ is determined.

Using equation [5.4.1], the computed thickness of the film is tabulated in table 5.1.

5.5 Error Sources in The Ellipsometric Measurements

Some possible sources errors during experimental measurement in the ellipsometry are the following.

- Imperfections in polarizers in a residual ellipticity of the light.

- Errors in setting of the azimuth angle of the polarizer.

- Errors in the angle of incidence
• errors caused by beam divergence

Depending on the divergent beam of the light source used, the magnitude of the errors in $\Psi$ and $\Delta$ are big or small. The use of lasers with their highly collimated beam or less diverged reduces the error to the minimum level.

• Non uniformity on the surface of the sample

If the sample has non uniform surface properties, this would produce errors in the measurement value of $\Psi$ and $\Delta$.

• Improper set up of photo detector

Also, the reflected beam could miss walk off the surface of the detector where the light passing through the rotating analyzer is fully detected. This would contribute in accuracy measurements of the Fourier coefficients $a$ and $b$ light.

5.6 Discussion

From the work of my predecessors on different samples of polymer films using rotating angle ellipsometry, the optical constants dropped down dramatically with increasing wavelengths shown in figure 5.6. The same patterns were already obtained in this experiment. The physical thickness of their samples depend on speed coated and concentration. The computed thickness of the films from the experimental data analysis becomes less and less as the speed coated speed becoming larger and larger as shown in figure 5.7. The reader may expect the thickness of the films should be the same for each corresponding wavelengths. However there are some variations.
due to errors during computing the measured results .[36, 37] When the thickness of
the three thin polymer samples MDMO-PPV/PCBM[1:1] which is a mixture of
1-[3-Methoxycarbonyl] Proply-1 Phenyl-[6,6]-Metanofullerence (PCBM) and Poly[2-
Methoxy-5(3'7'-dimethyoctyloxy)-1-4-Phenylene-vineline](MDMO-PPV) with by equal
proportions;Poly[3-(4-octyphenyl)-2,2'-bytiioophene]/PTOPT; and poly [(3-octalpheny
lithiophene)]/POPT are compared, the formers two results are almost constants.
Where as the latter shows some disparity because of heavy approximation during
the expansion of eq.[5.4.2] to the first order of Taylor expansion.

Note: The computed refractive index, the extinction coefficients, the thickness ,
the absorbance, the ellipsometric angles (Ψ and Δ), the Fresnel’s coefficients for
p and s polarized light are displayed for each angle of incidence for the corresponding
wavelength on the appendix.
Figure 5.1: Intensity versus analyzer angle for five angles of incidence at one of the wavelengths-632.8nm. The Fourier coefficients $a$ and $b$ are determined from this intensity curve by fitting to Fast non linear for each angle of incidence. In turn, the ellipsometric angles $\Psi$ and $\Delta$ are determined using eq\[3.4.11\]
Figure 5.2: Intensity versus analyzer angle for five angles of incidence at one of the wavelengths - 532nm
Figure 5.3: wavelength versus refractive Index
Figure 5.4: A graph of Absorbance versus wavelength by ellipsometry using four different laser sources
Figure 5.5: The absorbance versus wavelength measured using UV/VIS/NIR Instrument
Figure 5.6: The refractive indices of the three polymer films versus the wavelength
Figure 5.7: The film thickness of the three polymer thin films versus the wavelengths.
Figure 5.8: The absorbance of the two polymer films versus the wavelengths
Chapter 6

Conclusion

The Rotating Angle Ellipsometry RAR is a useful instrument which can be used for determining the optical constants and the film thickness. In order to study a film by ellipsometry the complex reflection or Fresnel’s coefficients of the film substrate combination for p and s-polarized light should be determined. The optical constants and the thickness of the film are computed from these coefficients. The thickness of the film will decreases as spin coated speed increasing for a given amount of its concentrations. Furthermore, the refractive index of the film decreases as the wavelengths of the incident light becomes larger and larger. On the other hand full determination of the absorption spectra needs a number of sources with different wavelengths. However in this ellipsometry four different sources of different wavelengths are used for characterizing the sample. This perhaps limit the full absorption spectra of the sample.

If the Cauchy Coefficients A and B are determine for at least two wavelengths, the refractive index of the rest of the wavelengths within visible region can be determined with out using additional laser sources.
### Appendix

#### Table 5.1

For the wave length 532nm

<table>
<thead>
<tr>
<th>Angle</th>
<th>a</th>
<th>b</th>
<th>Tan((\Phi))=(\sqrt{1+a}/(1-a))</th>
<th>Cos((\delta))=b/(\sqrt{1-a^2})</th>
<th>(n_R)</th>
<th>(n_I)</th>
</tr>
</thead>
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<tr>
<td>58.5</td>
<td>0.97899</td>
<td>0.1733</td>
<td>0.10303633</td>
<td>0.848891991</td>
<td>1.633498057</td>
<td>-0.173499095</td>
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<tr>
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<td>0.1257</td>
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<td>0.849425047</td>
<td>1.767731057</td>
<td>-0.204754822</td>
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<tr>
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<td>0.20178</td>
<td>0.108486605</td>
<td>0.840819368</td>
<td>1.8095357</td>
<td>-0.139924617</td>
</tr>
</tbody>
</table>

For the wave length 632.8nm

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<th>Angle</th>
<th>a</th>
<th>b</th>
<th>Tan((\Phi))=(\sqrt{1+a}/(1-a))</th>
<th>Cos((\delta))=b/(\sqrt{1-a^2})</th>
<th>(n_R)</th>
<th>(n_I)</th>
</tr>
</thead>
<tbody>
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<td>58</td>
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For the wave length 660nm

<table>
<thead>
<tr>
<th>Angle</th>
<th>a</th>
<th>b</th>
<th>Tan((\Phi))=(\sqrt{1+a}/(1-a))</th>
<th>Cos((\delta))=b/(\sqrt{1-a^2})</th>
<th>(n_R)</th>
<th>(n_I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>59</td>
<td>0.98867</td>
<td>0.14346</td>
<td>0.075480296</td>
<td>0.955728442</td>
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<td>-0.060858569</td>
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<tr>
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<td>-0.062409912</td>
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<tr>
<td>59</td>
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<tr>
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For the wave length 808nm

<table>
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<th>Tan((\Phi))=(\sqrt{1+a}/(1-a))</th>
<th>Cos((\delta))=b/(\sqrt{1-a^2})</th>
<th>(n_R)</th>
<th>(n_I)</th>
</tr>
</thead>
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<tr>
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Average (\(n_R\), \(n_I\))

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<tr>
<th>Angle</th>
<th>(n_R), (n_I)</th>
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</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>60</td>
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</tr>
<tr>
<td>61</td>
<td>1.587604795, -0.07034357</td>
</tr>
<tr>
<td>61.5</td>
<td>1.571704292, -0.067036359</td>
</tr>
</tbody>
</table>

Table 5.1 This table shows the computed refractive indices and the extinction coefficients from the Fast Fourier Coefficients Fitting a and b.
<table>
<thead>
<tr>
<th>Angle</th>
<th>Tan(Phi)cos(del)</th>
<th>Tan(Phi)Sin(del)</th>
<th>Cos(phi)</th>
<th>r01p</th>
<th>r12p</th>
<th>r01s</th>
<th>r12s</th>
<th>Thickness d in nm</th>
<th>Absorbance</th>
</tr>
</thead>
<tbody>
<tr>
<td>58.5</td>
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<tr>
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<td>-0.97126702</td>
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Average, (d,A)

For the wavelength 532nm

<table>
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<tr>
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<th>Tan(Phi)Sin(del)</th>
<th>Cos(phi)</th>
<th>r01p</th>
<th>r12p</th>
<th>r01s</th>
<th>r12s</th>
<th>Thickness d in nm</th>
<th>Absorbance</th>
</tr>
</thead>
<tbody>
<tr>
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Average, (d,A)

26.45432441 -0.01957157

For the wavelength 632.8nm

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<tr>
<th>Angle</th>
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<th>Tan(Phi)Sin(del)</th>
<th>Cos(phi)</th>
<th>r01p</th>
<th>r12p</th>
<th>r01s</th>
<th>r12s</th>
<th>Thickness d in nm</th>
<th>Absorbance</th>
</tr>
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Average, (d,A)

26.04403224 -0.01516405

For the wavelength 660nm

<table>
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<tr>
<th>Angle</th>
<th>Tan(Phi)cos(del)</th>
<th>Tan(Phi)Sin(del)</th>
<th>Cos(phi)</th>
<th>r01p</th>
<th>r12p</th>
<th>r01s</th>
<th>r12s</th>
<th>Thickness d in nm</th>
<th>Absorbance</th>
</tr>
</thead>
<tbody>
<tr>
<td>56</td>
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<td>0.022000175</td>
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</tbody>
</table>

Average, (d,A)

33.84493223 -0.009612592

Table 5.1 This table shows the computed refractive indices and the extinction coefficients from the Fast Fourier Coefficients Fitting a and b
Bibliography


[27] Peter W. Milonni and Joseph H. Eberly: *Lasers*, Ch. 2, P21-39, John Wiley and Sons,

[28] Peter W. Milonni and Joseph H. Eberly: *Lasers*, Ch. 2, P65-75, John Wiley and Sons,


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