ADDIS ABABA UNIVERSITY
SCHOOL OF GRADUATE STUDIES

ANALYSIS OF SUSPENDED RECTANGULAR SLAB PANELS UNDER PARTIAL RECTANGULAR UNIFORM LOAD

By

Mikias Tesfaye

2006

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A Thesis Submitted to School of Graduate Studies, Addis Ababa University in Partial Fulfillment of the Requirements for the Degree of MASTER OF SCIENCE in CIVIL ENGINEERING

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Approved by Board of Examiners

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External Examiner	*Signature*	*Date*
Internal Examiner	*Signature*	*Date*
Chairman	*Signature*	*Date*

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Abstract

In structural engineering there are different types of duties; amongst them the analysis part is the most important and crucial part. Because of this it has been given a great focus. Analysis techniques for most structural elements with different arrangement of externally applied loads are well developed and sufficiently covered with modern analysis theories. Further more, the application of today's high-speed computers enable the analysis of complex structures with different load arrangements. In addition to these effective analysis tools, building design codes provide table of values and analysis charts for the analysis of different elements of structural systems. As one of the structural element in building structure, different analysis methods were proposed for the analysis of suspended slab panel subjected to uniform rectangular loads.

Among the possible arrangement of externally applied loads on suspended slab panel a uniform rectangular load is the one which can best represent the weight of heavy machineries, water tankers, etc. Different simplified analysis methods were proposed to consider the contribution of this load to the design action effects. The current practice of accounting these loads in the analysis of suspended slab panel is to change it to the 'equivalent' uniformly distributed load. The analysis of regular slab panel subjected to uniformly distributed load may be carried out using computer soft wares or using coefficients which are presented in many design codes, the Ethiopian Building Code of Standards for concrete structures, EBCS 2 – 1995 can be one example. These methods analysis need to be investigated whether they can represent the actual load or not.

Therefore, as it is important to address this problem, comparative analysis has been carried out. A new simplified method which considers the actual scenario has been developed. This newly proposed method makes use of coefficients derived from the basic principle of elastic analysis of plate. The results have been verified by comparing it with results of the finite element analysis. It enables us to make elastic analysis of suspended slab panel

subjected to the weight of a uniform rectangular load and avoids the uncertain use of approximate methods.

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Description of Symbols

Symbols	Descriptions
D	flexural rigidity of plate
E	young's modulus of plate material
ε	strain
γ	support restraint factor
I_a	second moment of inertia of the peripheral beam
L_x	the shorter dimension of the slab panel
L_y	the longer dimension of the slab panel
M_x	moment per meter along the longer side
M_y	moment per meter along the shorter side
M_{xs}	support moment per meter along the longer side
M_{xf}	field moment per meter in y-strip
M_{ys}	support moment per meter along the shorter side
M_{yf}	field moment per meter in x-strip
M_{xy}	twisting moment along the edge of the panel
q	load per sq. meter
σ_x	normal stress in the x-direction
σ_y	normal stress in the y-direction
σ_z	normal stress in the z-direction
t	thickness of the plate
θ_x	angle of rotation of normal line with respect to x-axis
θ_y	angle of rotation of normal line with respect to y-axis
τ_{xy}	shear stress along the face of the plate
υ	Poisson's ratio of the plate material
V_x	shear force per meter along the longer side
V_y	shear force per meter along the shorter side
u_0	displacement in the x-direction at $z = 0$
υ_0	displacement in the y-direction at $z = 0$
w_0	displacement in the z-direction at $z = 0$
u	displacement in the x-direction
u	displacement in the x-direction

v	displacement in the y-direction
w	displacement in the z-direction

Introduction

1.1 Problem Background

Most of the buildings in our country use two-way RC-panel slabs for their suspended floor structure. These two way slabs carry different type of loadings. For Instant wall load (line load), Column load (point load), container load (rectangular load), fortunately this paper is limited to the third type of non uniformity i.e. analysis of slab under equipment load (uniform rectangular load lied on part of the slab). Had these load been uniformly distributed all over the slab we would have used the coefficient method developed in our code (EBCS 2 -1995), but in a case where the uniform loading is on some part of the slab only, we need to solve a set of differential equations which satisfy equilibrium of the slab and boundary conditions, or we may use empirical approximate methods for analysis. We can clearly see that performing such analysis for each and every slab panel is too time taking and cumbersome. Therefore we can say that simplified Analysis methods on rectangular panels under such loads contribute a lot on saving designer's time.

When we are talking about the analysis of structural elements, we may refer to the analysis of beams, columns, slab panels, footings, truss elements and etc. Among this wide group of structural elements here it is intended to investigate the response of a slab panel under certain load arrangement. This flat structural element in the form of plate is an important component in the field of structural analysis. It is in most cases the primary structural element liable to externally applied loads in different arrangements. Those different arrangements may include uniformly distributed loads, point loads, line loads and triangularly distributed loads. The uniformly distributed load is due to dead weight of the slab and weight of finishing material imposed on it. That of a point load is may be as a result of leg-supported object on it. A rectangular uniform load may originate from the weight of water tankers, machinery weights, etc, and a triangularly distributed load may result from surface load. The arrangement of these externally applied loads leads the analysis of slab panel from relatively simple one to complex partial differential equations.

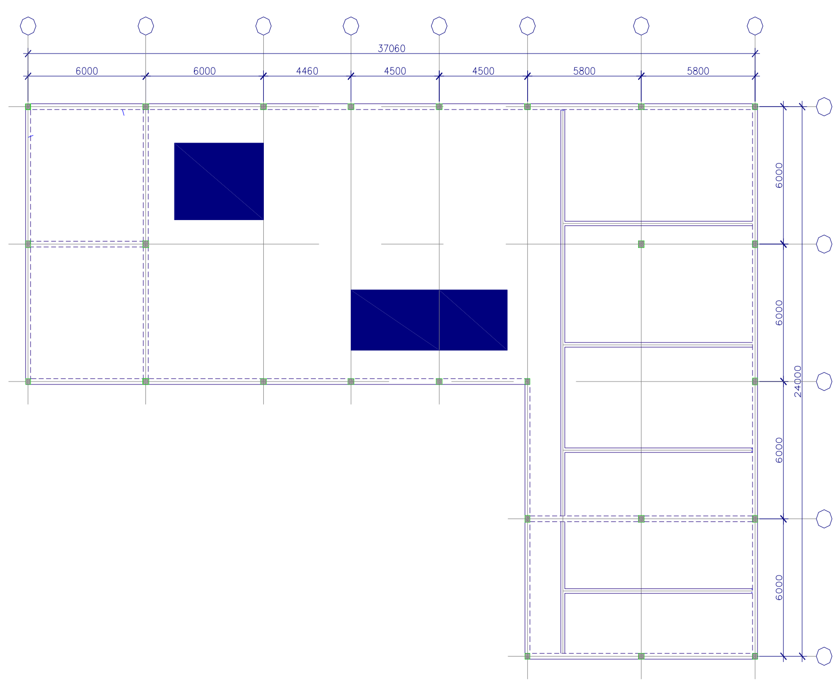


Figure 1-1: Typical application machinery loads on suspended slab of industrial building

From the above possible arrangement of externally applied loads on suspended slab panel a rectangular uniform load is the one, which can best represent the weight of water tankers, machinery weights on it. The water tankers, machinery weights are important element in roof slabs and factories respectively. Figure 1-1 shows a typical application of machinery loads on suspended slab.

Therefore to consider and to see the exact response scenario of the panel, we need to develop analysis methods, which consider this rectangular uniform load.

1.2 Objectives

The objective of this thesis is to develop a table of coefficients which is used for two way RC-Panel designers which are subjected to a uniform rectangular loading on some portion of the panel considering different boundary conditions. The paper also includes the following objectives.

1. To make comparative analysis of the existing methods
2. To propose alternative methods of accounting rectangular uniform load.
3. To provide tables of values or/and analysis chart for the analysis of slab panel under machinery loads.
4. To develop software that delivers coefficient of moments for rectangular suspended slab under rectangular uniform load.

1.3 Approaches

The following methods will be employed to achieve the objectives of this thesis.

- Literatures on the analysis techniques of suspended slab panel under different load arrangement will be reviewed. Different building design codes provisions for the problem specified will be studied thoroughly.
- Simplified alternative approach will be proposed.
- The result obtained using the existing methods, and the proposed method will be compared using statistical terms.

Chapter 2
Literature Review

2.1 Basic Theory of Plates

The classic theory of isotropic plates (some-times referred to as the Poisson-Kirchhoff theory of thin plates) is based on certain idealized assumptions and limiting conditions. These assumptions and limiting conditions relate to the behavior of the plate under the action of the loading as well as to the actual plate and the material of which it is made.

The assumptions concerning the material and the shape of the plate are the following:

1. The material of which the plate consists is completely elastic;
2. The material conforms to Hooke’s law and has the same elastic constants (modulus of elasticity, Poisson's ratio) for all kinds of loading;
3. The material of the plate is homogeneous and isotropic;
4. The thickness of the plate is constant;
5. The thickness is small in comparison with the other dimensions of the plate. No material is completely elastic, isotropic and homogeneous. However,

for most engineering materials the differences in relation to the "ideal" material are not too great to invalidate the assumptions.

The assumptions relating to the behavior of the plate under loading are;

6. Fibers which were perpendicular to the middle plane of the plate before bending occurs remain perpendicular to the (deformed) middle plane after the occurrence of bending;

7. The normal stress perpendicular to the plane of the plate is negligible;

8. The deflections of the plate are so small that the curvature in any particular direction is given by the second derivative of the deflection in that direction (thus the slopes are like-wise small);

9. No normal stresses act in the plane of the plate, i.e., there occurs no deformation in the middle plane (this assumption limits the deflection much more than does the preceding one);

10. The dead weight (self weight of the plate) is included in the plate loading;

11. The corners of the plate are secured against lifting, and in reinforced concrete slabs the corners are provided with torsion-resisting reinforcement.

2.2 Finite Element Analysis Method

For suspended Slab Panel under Rectangular uniform Load.

A. General

There are many structural design soft wares that use finite element analysis method for plane element structures under different load conditions. Some of them are ETABS, SAP, ANSYS, SAFE, etc. The finite element method is a widely accepted numerical procedure for solving the differential equations of engineering and other science fields. It is computational basis of many computer aided design programs. As its applications to solid mechanics problems are extensive, it is important tool in solving partial differential equations of plate bending theory. It has got a primary advantage of the ease with which it can be generalized to solve two-dimensional problem of plate bending theory with different irregular boundary conditions.

Here, in this study, Safe integrated slab analysis software is preferred and used because of its availability and sufficient capability of manipulating plate bending problems. It uses triangular and rectangular slab elements. Each of these slab elements can be isotropic or orthotropic, thin or thick plate bending element. The thin plate element is a three to four-node element and is based upon the classical linear thin plate bending theory, neglecting the effect of out-of-plane shear deformations. The thick plate is also a three to four-node element and accounts for the effect of out-of-plane shear deformations.

Safe Analysis Features

The Safe structural analysis program offers the following features:

- Static and dynamic analysis
- Linear and nonlinear analysis, including seismic analysis
- Vehicle live-load analysis for bridges
- P-Delta analysis
- Frame and shell structural elements, including beam-column, truss, membrane, and plate behavior
- Two- and three-dimensional and axisymmetric solid elements
- Nonlinear link and spring elements
- Multiple coordinate systems
- Many types of constraints
- A wide variety of loading options
- Alpha-numeric labels
- Large capacity
- Highly efficient and stable solution algorithms

These features and many more, make Safe the state-of-the-art in structural analysis programs.

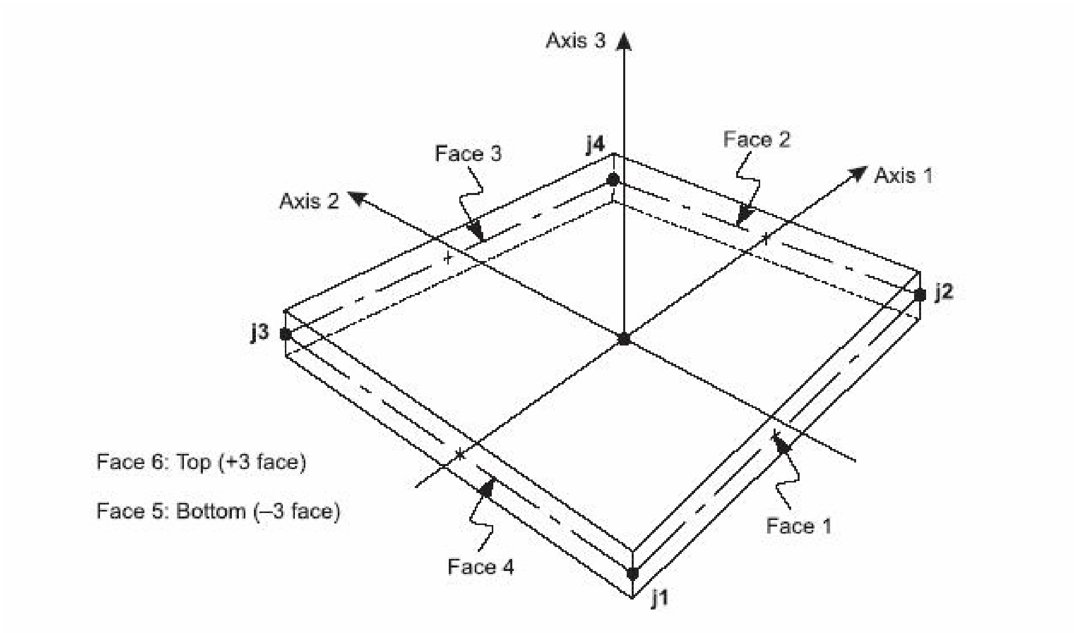


Figure 2-1: Four-node quadrilateral slab element

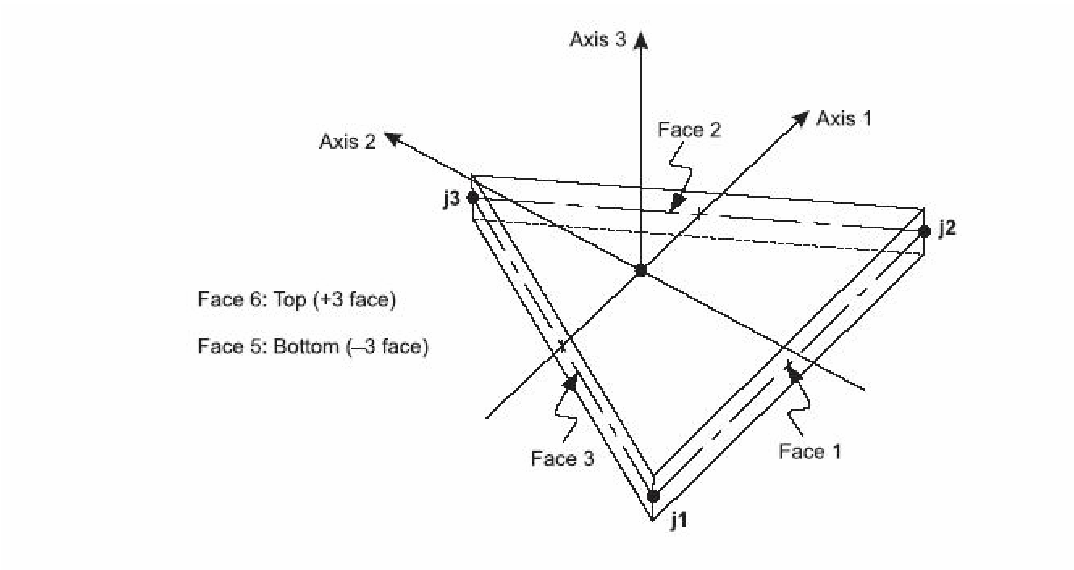


Figure 2-1: Three-node triangular slab element.

Some essential features of the plate elements that provided by Safe:

- Each of the element nodes has the three degree of freedom, w, θ_x, and θ_y.
- The material properties and thickness within each slab element are constant.

- Optionally, to model orthotropic effects, it is possible to specify three different effective thicknesses: x-direction bending, y-direction bending, and twist.
- The slab system must be planar and exist in xy plane. Changes in slab elevations the cause definite moment discontinuities may be reasonably captured using the release options.
- In-plane action is not allowed in the xy plane; therefore, membrane stresses in the plane of the slab system do not exist.
- The calculation of self-weight of the slab element is based upon the design thickness, the dimensions between mesh points in the x and y directions, and the unit weight of the material. The weight of the slab element is lumped (as concentrated loads) and distributed equally on to the mesh pints in to which the slab element frames.
- Slab element moments and shear are calculated at the mesh points of the element.

B. Analysis

Finite element method is so far the well-developed numerical method to deal with the partial differential equations of plate bending problems. Obviously, it is very tedious and difficult to think of the solutions of those partial differential equations using finite element method without applying any computer analysis software. For the purpose of this study, it has been investigated the internal response of a total number of 24 suspended slab panel models with different boundary conditions and span ratios.

To study the internal actions (M_x, M_y, V_x, and V_y) of a suspended slab panel loaded with a rectangular uniform load, the following model data are selected. Nine boundary conditions, which are shown in the figure 2-2 below, are selected since they simulate most of practically found slab boundary conditions.

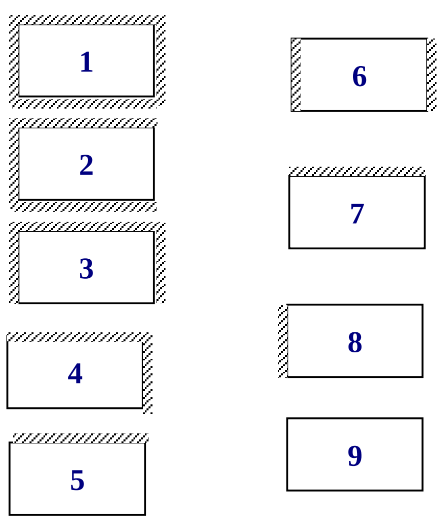


Figure 2-2: Boundary conditions.

To see the effect of span ratio to the internal action resulted from a rectangular uniform load eight span ratios are selected. Table 2-1 shows panel dimension and span ratio used for this study.

Table 2.1: Model panel dimensions and span ratio.

Panel dimension, L_x (m)	Panel dimension, L_y (m)	Span ratio, $\frac{L_y}{L_x}$
6	6	1.0
5	5.5	1.1
5	6	1.2
5	6.5	1.3
5	7	1.4
4	6	1.5
4	7	1.75
4	8	2.0

All the other remaining data are taken to be the same for all slab panel models including a rectangular uniform load of 10KN/m^2 at the center of the panel in the two orthogonal directions. Here it is shown only one case and the results of the other cases are attached under appendix A. For the purpose of demonstration a slab panel with all edges simply support and span ratio is equal to one is selected.

- **Suspended slab panel under rectangular uniform load**

Case 1: All sides simply supported with span ratio is equal to unity

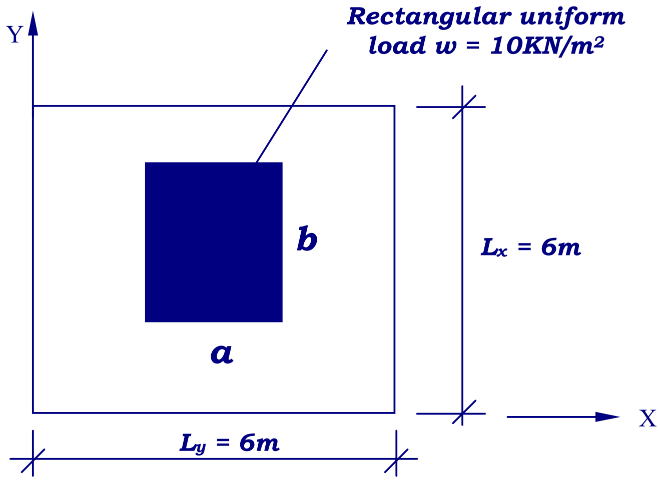


Figure 2-3: Simply supported slab Panel.

Data

- Panel size = L_xxL_y = 6mx6m
- Load width ratio = ***b/Lx*** x ***a/Ly*** = 0.5x0.5
- Edge beam on four side = *30*x*50cm*
- Corner support columns = *30*x*30cm*
- Slab panel thickness = *15cm*
- Modulus of elasticity = *29000000KN/*m^2
- Poisson's ratio = *0.2*

Load case:

- Rectangular Load (RL) = w = *10KN/*m^2

Modeling Procedure

To investigate the internal actions of a suspended slab panel under a rectangular uniform load, the problem is analyzed employing a maximum mesh size of 0.5m, as shown in the Figure 2-4. The slab is modeled using thin plate elements in Safe. The continuous edge is modeled by providing the same additional panel at that edge. Only one load case is considered. Self-weight is not included in these analyses.

To obtain internal actions, the plate is divided into three strips-two edge strips and one middle strip-each way, based on the definition of design strip widths for a two-way slab system as shown in the Figure 2-4.

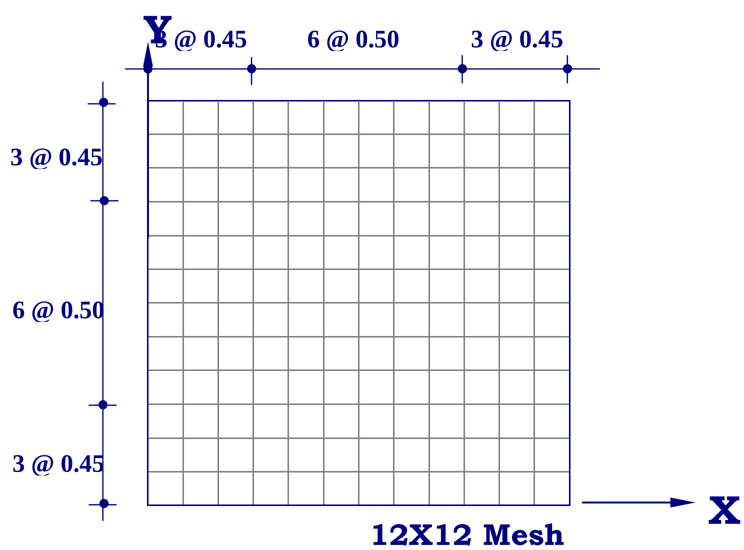


Figure 2-4: Safe Mesh.

Analysis Output

The analysis result of a slab panel under a rectangular uniform load is used as verification for this paper. Results are taken to plot a comparison graph on page 39.

Chapter 3

Analysis using coefficient method

3.1 Differential Equation of the Deflection Surface

We assume that the load acting on a plate is normal to its surface and that the deflections are small in comparison with the thickness of the plate. At the boundary we assume that the edges of the plate are free to move in the plane of the plate; thus the reactive forces at the edges are normal to the plate. With these assumptions we can neglect any strain in the middle plane of the plate during bending.

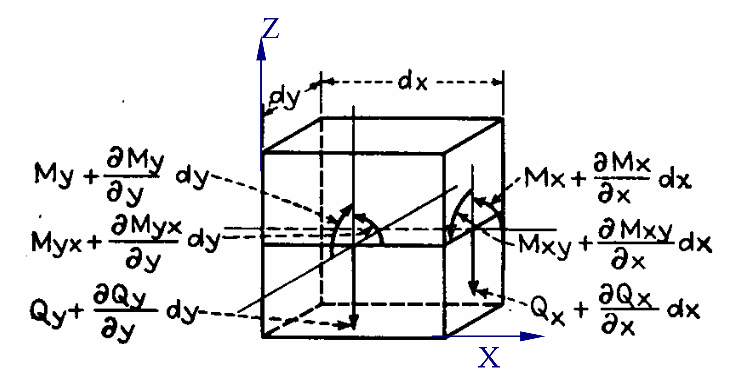


Figure 3-1: Infinitesimal element.

The coordinate axes x and y in the middle plane of the plate and the z axis perpendicular to that plane, let us consider an element cut out of the plate by two pairs of planes parallel to the xz and yz planes, as shown in Fig. 3.1. In addition to the bending moments Mx and My and the twisting moments Mxy which were considered in the pure bending of a plate there are vertical shearing forces1 acting on the sides of the element. The magnitudes of these shearing forces per unit length parallel to the y and x axes we denote by Qx and Qy, respectively, so that

$$Q_x = \int_{-h/2}^{h/2} \tau_{xz} d_z \qquad Q_y = \int_{-h/2}^{h/2} \tau_{xz} d_z \qquad \textbf{(a)}$$

Since the moments and the shearing forces are functions of the coordinates x and y, we must, in discussing the conditions of equilibrium of the element, take into consideration the small changes of these quantities when the coordinates x and y change by the small quantities dx and dy.

The middle plane of the element is represented in Fig. 3-2a and b, and the directions in which the moments and forces are taken as positive are indicated. We must also consider the load distributed over the upper surface of the plate. The intensity of this load we denote by q, so that the load acting on the element1 is q dx dy.

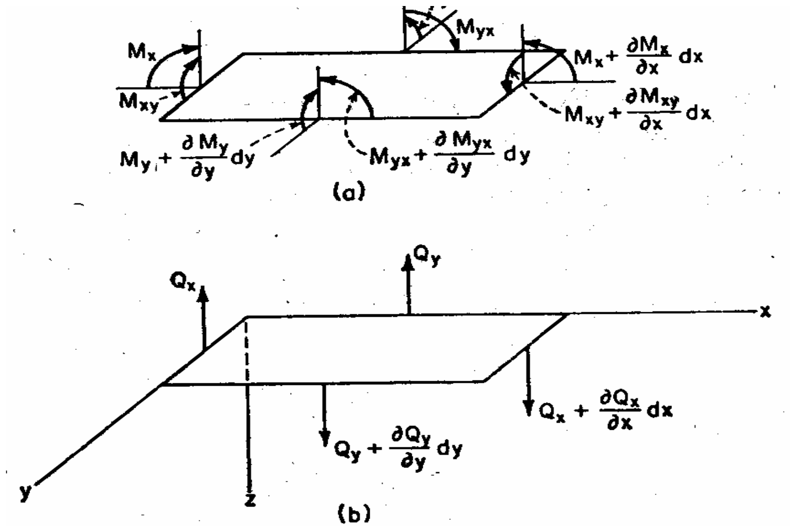


Figure 3-2: Rectangular Plate.

Projecting all the forces acting on the element onto the z axis we obtain the following equation of equilibrium:

$$\frac{\partial Q_x}{\partial x} d_x d_y + \frac{\partial Q_y}{\partial y} d_y d_x + q d_x d_y = 0 \qquad \textbf{(1)}$$

From which

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0$$

Taking moments of all the forces acting on the element with respect to the x axis, we obtain the equation of equilibrium

$$\frac{\partial M_{xy}}{\partial x} d_x d_y - \frac{\partial M_y}{\partial y} d_y d_x + Q_y d_x d_y = 0 \qquad \textbf{(b)}$$

The moment of the load q and the moment due to change in the force Qy are neglected in this equation, since they are small quantities of a higher order than those retained. After simplification, Eq. (b) becomes

$$\frac{\partial M_{xy}}{\partial x} - \frac{\partial M_y}{\partial y} + Q_y = 0 \qquad \textbf{(c)}$$

In the same manner, by taking moments with respect to the y axis, we obtain

$$\frac{\partial M_{xy}}{\partial x} - \frac{\partial M_y}{\partial y} + Q_y = 0 \qquad \textbf{(d)}$$

Since there are no forces in the x and y directions and no moments with respect to the z axis, the three equations (1), (c), and (d) completely define the equilibrium of the element. Let us eliminate the shearing forces Q_x and Q_y from these equations by determining them from Eqs. (c) And (d) and substituting into Eq. (1). In this manner we obtain

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_{yx}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \frac{\partial^2 M_{xy}}{\partial x \partial y} = -q \qquad \textbf{(e)}$$

Observing that M_{yx} = -M_{xy}, by virtue of τ_{xy} = τ_{yx}, we finally represent the equation of equilibrium (e) in the following form:

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -q \qquad \textbf{(2)}$$

To represent this equation in terms of the deflections w of the plate we use the expressions of pure bending. This assumption is equivalent to neglecting the effect on bending of the shearing forces Q_x and Q_y and the compressive stress σz produced by the load q. The errors in deflections obtained in this way are small provided the thickness of the plate is small in comparison with the dimensions of the plate in its plane.

Using x and y directions, we obtain

$$M_x = -D\left[\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right] \quad M_y = -D\left[\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right] \qquad \textbf{(3)}$$

$$M_{xy} = -M_{yx} = D(1-\nu)\frac{\partial^2 w}{\partial x \partial y} \qquad \textbf{(4)}$$

Substituting these expressions in Eq. (2), we obtain

$$\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} - \frac{q}{D} = 0 \qquad \textbf{(5)}$$

This latter equation can also be written in the symbolic form

$$\nabla\nabla w = q/D \qquad \textbf{(6)}$$

Where

$$\nabla w = \partial^2 w/\partial x^2 + \partial^2 w/\partial y^2 \qquad \textbf{(7)}$$

It is seen that the problem of bending of plates by a lateral load q reduces to the integration of Eq. (5). If, for a particular case, a solution of this equation is found that satisfies the conditions at the boundaries of the plate, the bending and twisting moments can be calculated from Eqs. (3) and (4). The corresponding normal and shearing stresses are found from the expression

$$(\tau_{xy})_{\max} = \frac{6M_{xy}}{h^2}$$

Equations (c) and (d) are used to determine the shearing forces Q_x and Q_y, from which

$$Q_x = \frac{\partial M_{yx}}{\partial y} + \frac{\partial M_x}{\partial x} = -D\frac{\partial}{\partial x}\left[\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right] \qquad \textbf{(8)}$$

$$Q_y = \frac{\partial M_y}{\partial y} - \frac{\partial M_{xy}}{\partial x} = -D\frac{\partial}{\partial y}\left[\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right] \qquad \textbf{(9)}$$

or, using the symbolic form,

$$Q_x = -D\frac{\partial}{\partial x}(\nabla w) \qquad Q_y = -D\frac{\partial}{\partial y}(\nabla w) \qquad \textbf{(10)}$$

The shearing stresses τ_{xz} and τ_{yz} can now be determined by assuming that they are distributed across the thickness of the plate according to the parabolic law.

Then $$(\tau_{xz})_{\max} = \frac{3Q_x}{2h} \qquad (\tau_{yz})_{\max} = \frac{3Q_y}{2h}$$

It is seen that the stresses in a plate can be calculated provided the deflection surface for a given load distribution and for given boundary conditions is determined by integration of Eq. (5).

3.2 Boundary Conditions

We begin the discussion of boundary conditions with the case of a rectangular plate and assume that the x and y axes are taken parallel to the sides of the plate.

Built-in Edge

If the edge of a plate is built in, the deflection along this edge is zero, and the tangent plane to the deflected middle surface along this edge coincides with the initial position of the middle plane of the plate. Assuming the built-in edge to be given by x = a, the boundary conditions are

$$(w)_{x=a} = 0 \qquad \left(\frac{\partial w}{\partial x}\right)_{x=a} = 0 \qquad \textbf{(11)}$$

Simply Supported Edge

If the edge x = a of the plate is simply supported, the deflection w along this edge must be zero. At the same time this edge can rotate freely with respect to the edge line; i.e., there are no bending moments Mx along this edge. The analytical expressions for the boundary conditions in this case are

$$(w)_{x=a} = 0 \qquad \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)_{x=a} = 0 \qquad \textbf{(12)}$$

Observing that $\partial^2 w/\partial y^2$ must vanish together with w along the rectilinear edge x = a, we find that the second of the conditions (12) can be rewritten as

$$\frac{\partial^2 w}{\partial x^2} = 0 \qquad \text{or also} \qquad \Delta w = 0 .$$

Equations (12) are there fore equivalent to the equations

$$(w)_{x=a} = 0 \qquad \Delta(w)_{x=a} = 0 \qquad \textbf{(13)}$$

Which do not involve Poisson's ratio ν

3.3 SIMPLY SUPPORTED RECTANGULAR PLATES

Taking the coordinate axes as shown in Fig. 3-5, we assume that the load distributed over the surface of the plate is given by the expression

$$q = q - \sin\left(x\Pi \big/ a\right)\sin\left(\Pi y \big/ b\right) \qquad \textbf{(a)}$$

in which q_0 represents the intensity of the load at the center of the plate. The differential equation (5) for the deflection surface in this case becomes

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q_0}{D}\sin\frac{\Pi x}{a}\sin\frac{\Pi y}{b} \qquad \textbf{(b)}$$

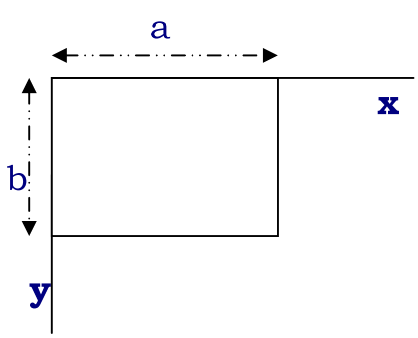


Figure 3-3: Simply supported.

The boundary conditions for simply supported edge are

$w = 0 \quad M_x = 0$ For $x = 0$ and $x = a$

$w = 0 \quad M_y = 0$ For $y = 0$ and $y = b$

Using expression (3) for bending moments and observing that, since w = 0 at the edges, $\partial^2 w/\partial_x^2$ = 0 and $\partial^2 w/\partial y^2$ for the edges parallel to the x and y axes, respectively, we can represent the boundary conditions in the following form:

$$(1)\quad w=0 \qquad (2)\quad \frac{\partial^2 w}{\partial x^2}=0 \qquad \text{For} \quad x=0 \quad \text{and} \quad x=a$$

(c)

$$(3)\quad w=0 \qquad (4)\quad \frac{\partial^2 w}{\partial x^2}=0 \qquad \text{For} \quad y=0 \quad \text{and} \quad y=b$$

It may be seen that all boundary conditions are satisfied if we take for deflections the expression

$$w = C\sin\frac{\Pi x}{a}\sin\frac{xy}{b} \qquad \textbf{(d)}$$

in which the constant C must be chosen so as to satisfy Eq. (b). Substituting expression (d) into Eq. (b), we find

$$\Pi^4\left[\frac{1}{a^2}+\frac{1}{b^2}\right]^2 C = \frac{q_0}{D}$$

and we conclude that the deflection surface satisfying Eq. (b) and boundary conditions (c) is

$$w = \frac{q_0}{\Pi^4 D\left(1/a^2+1/b^2\right)^2}\sin\frac{\Pi x}{a}\sin\frac{xy}{b} \qquad \textbf{(e)}$$

Having this expression and using Eqs. (3) and (4), we find

$$M_x = \frac{q_0}{\Pi^2 \left(1/a^2 + 1/b^2\right)^2} \frac{1}{a^2} + \nu \frac{\Pi x}{b^2} \sin\frac{\Pi x}{a} \sin\frac{\Pi y}{b}$$

$$M_x = \frac{q_0}{\Pi^2 \left(1/a^2 + 1/b^2\right)^2} \left[\frac{\nu}{a^2} + \frac{1}{b^2}\right] \sin\frac{\Pi x}{a} \sin\frac{\Pi y}{b} \qquad \textbf{(f)}$$

$$M_{xy} = \frac{q_0 (1-\nu)}{\Pi^2 \left(1/a^2 + 1/b^2\right)^2 ab} \cos\frac{\Pi x}{a} \cos\frac{\Pi y}{b}$$

It is seen that the maximum deflection and the maximum bending moments are at the center of the plate. Substituting x = a/2, y = b/2 in Eqs. (e) and (f), we obtain

$$w_{\max} = \frac{q_0}{\Pi^4 D \left(1/a^2 + 1/b^2\right)^2} \qquad \textbf{(14)}$$

$$\left(M_x\right)_{\max} = \frac{q_0}{\Pi^4 D \left(1/a^2 + 1/b^2\right)^2} \left[\frac{1}{a^2} + \frac{\nu}{b^2}\right] \qquad \textbf{(15)}$$

In the particular case of a square plate, a = b, and the foregoing formulas become

$$w_{\max} = \frac{q_0 a^4}{4\Pi^4 D}$$

$$\left(M_x\right)_{\max} = \left(M_y\right)_{\max} = \frac{(1+\nu) q_0 a^2}{4\Pi^2} \qquad \textbf{(16)}$$

We use Eqs. (8) and (9) to calculate the shearing forces and obtain

$$Q_x = \frac{q_0}{\Pi a\left(1/a^2 + 1/b^2\right)} \cos\frac{\Pi x}{a} \sin\frac{\Pi x}{b}$$

(g)

$$Q_y = \frac{q_0}{\Pi b\left(1/a^2 + 1/b^2\right)} \sin\frac{\Pi x}{a} \cos\frac{\Pi y}{b}$$

To find the reactive forces at the supported edges of the plate we proceed by taking the edge. For the edge x = a, we find

$$V_x = \left[Q_x - \frac{\partial M_{xy}}{\partial y}\right]_{x=a} = -\frac{q_0}{\Pi a\left(1/a^2 + 1/b^2\right)^2}\left[\frac{1}{a^2} + \frac{2-\nu}{b^2}\right]\sin\frac{\Pi y}{b} \qquad \textbf{(h)}$$

In the same manner, for the edge y = b,

$$V_y = \left[Q_y - \frac{\partial M_{xy}}{\partial x}\right]_{y=b} = -\frac{q_0}{\Pi b\left(1/a^2 + 1/b^2\right)^2}\left[\frac{1}{b^2} + \frac{2-\nu}{a^2}\right]\sin\frac{\Pi x}{a} \qquad \textbf{(i)}$$

Hence the pressure distribution follows a sinusoidal law. The minus sign indicates that the reactions on the plate act upward. From symmetry it may be concluded that formulas (h) and (i) also represent pressure distributions along the sides x = 0 and y = 0, respectively. The resultant of distributed pressures is

$$\frac{2q_0}{\Pi\left(1/a^2 + 1/b^2\right)^2}\left[\frac{1}{a}\left(\frac{1}{a^2} + \frac{2-\nu}{b^2}\right)\int_0^b \sin\frac{\Pi y}{b}\,dy + \frac{1}{b}\left(\frac{1}{b^2} + \frac{2-\nu}{a^2}\right)\int_0^a \sin\frac{\Pi x}{a}\,dx\right]$$

$$= \frac{4q_0 ab}{\Pi} + \frac{8q_0(1-\nu)}{\Pi^2 ab\left(a/a^2 + 1/b^2\right)^2} \qquad \textbf{(j)}$$

observing that

$$\frac{4q_0 ab}{\Pi^2} = \int_0^a \int_0^b q_0 \sin\frac{\Pi x}{a} \sin\frac{\Pi y}{b}\, dxdy$$

it can be concluded that the sum of the distributed reactions is larger than the total load on the plate given by expression (K). We obtain not only the distributed reaction but also reactions concentrated at the corners of the plate. These concentrated reactions are equal, from symmetry; and their magnitude is

$$R = 2\left(M_{xy}\right)_{x=a,y=b} = \frac{2q_0(1-\nu)}{\Pi^2 ab\left(1/a^2 + 1/b^2\right)^2} \qquad \textbf{(1)}$$

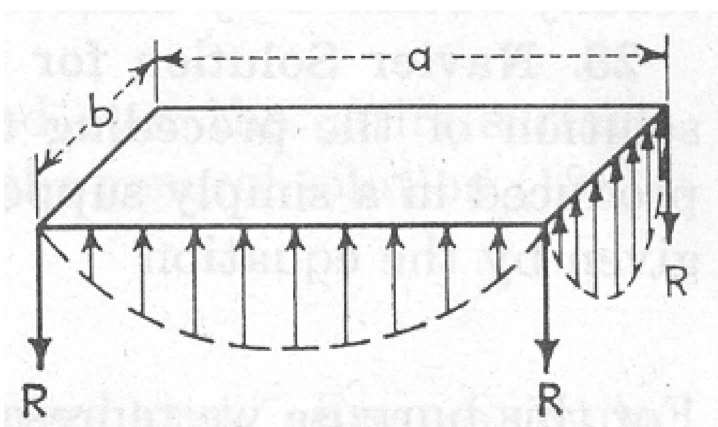


Figure 3-4: Reactions.

The positive sign indicates that the reactions act down ward. Their sum is exactly equal to the second term in expression (j). The distributed and the concentrated reaction which act on the plate and keep the load, defined by

Eq. (a), in equilibrium are shown graphically in Fig. 3-6. It may be seen that the corners of the plate have a tendency to rise up under the action of the applied load and that the concentrated forces R must be applied to prevent this.

The maximum bending stress is at the center of the plate. Assuming that a > b, we find that at the center My > Mx.

Hence the maximum bending stress is

$$(\sigma_v)_{\max} = \left[\frac{6(M_y)_{\max}}{h^2}\right] = \frac{6q_0(1-\nu)}{\Pi^2 h^2 \left(1/a^2 + 1/b^2\right)^2}\left[\frac{\nu}{a^2} + \frac{1}{b^2}\right]$$

The maximum shearing stress will be at the middle of the longer sides of the plate.

Observing that the total transverse force

$$V_y = Q_y - \frac{\partial M_{xy}}{\partial x}$$

is distributed along the thickness of the plate according to the parabolic law and using Eq. (i), we obtain

$$(\tau_{yz})_{\max} = \frac{3q_0}{2\Pi bh\left(1/a^2 + 1/b^2\right)^2}\left[\frac{1}{b^2} + \frac{2-\nu}{a^2}\right]$$

If the sinusoidal load distribution is given by the equation

$$q = q_0 \sin\frac{m\Pi x}{a}\sin\frac{n\Pi y}{b} \qquad \textbf{(m)}$$

Where m and n are integer numbers, we can proceed as before, and we shall obtain for the deflection surface the following expression:

$$w = \frac{q_0}{\Pi 4 D\left(m^2/a^2 + n^2/b^2\right)^2} \sin\frac{m\Pi x}{a} \sin\frac{n\Pi y}{b} \qquad \textbf{(17)}$$

from which the expressions for bending and twisting moments can be readily obtained by differentiation.

Now lets work general load case: The solution of this will be used in calculating the coefficients for the thesis work.

$$q = f(x, y) \qquad \textbf{(a)}$$

For this purpose we represent the function f(x,y) in the form of a double trigonometric series:

$$F(x, y) = \sum_0^\infty \sum_0^\infty a_{mn} \sin\frac{m\Pi x}{a} \sin\frac{n\Pi y}{b} \qquad \textbf{(17)}$$

To calcualate any particular coefficient am'n' of this series we multiply both sides of Eq. (18) by sin (n'πy/b) dy and integrate from 0 to b.

Observing that

$$\int_0^b \sin\frac{n\Pi y}{b} \sin\frac{n'\Pi y}{b} dy = 0 \qquad \text{where } n \neq n'$$

we find in this way

$$\int_0^b f(x, y) \sin\frac{n'\Pi y}{b} dy = \frac{b}{2} \sum_{m=1}^\infty a_{mn'} \sin\frac{m\Pi y}{a} \qquad \textbf{(b)}$$

Multiplying both sides of Eq. (b) by sin (m'πx/a) dx and integrating from 0 to a, we obtain

$$\int_0^a \int_0^b f(x, y) \sin \frac{m' \Pi x}{a} \sin \frac{n' \Pi y}{b} dxdy = \frac{ab}{4} a_{m'n'}$$

from which

$$a_{m'n'} = \frac{4}{ab} \int_0^a \int_0^b f(x, y) \sin \frac{m' \Pi x}{a} \sin \frac{n' \Pi y}{b} dxdy \qquad \textbf{(19)}$$

Performing the integration indicated in expression (19) for a given load distribution, i.e., for a given f(x,y), we find the coefficients of series (18) and represent in this way the given load as a sum of partial sinusoidal loadings. The deflection produced by each partial loading was discussed before, and the total deflection will be obtained by summation of such terms as are given by Eq. (17). Hence we find

$$w = \frac{1}{\Pi^4 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{a_{mn}}{\left(m^2/a^2 + n^2/b^2\right)^2} \sin \frac{m' \Pi x}{a} \sin \frac{n \Pi y}{b} \qquad \textbf{(20)}$$

Then this time we can consider a rectangular uniform load case

From the discussion in the preceding article it is seen that the deflection of a simply supported rectangular plate (Fig. 3-3) can always be represented in the form of a double trigonometric series (20), the coefficients a_{mn} being given by Eq. (19).

Let us apply this result in the case of a load uniformly distributed over the area of the rectangle shown in Fig. 3-5. By virtue of Eq. (19) we have

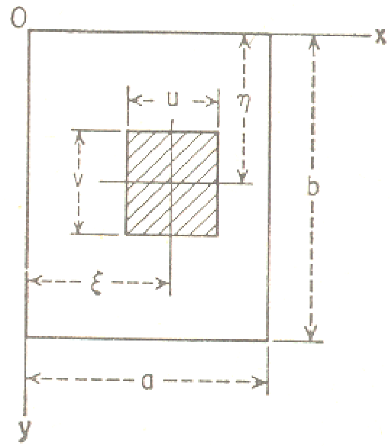


Figure 3-5: rectangular uniform load

$$a_{mn} = \frac{4p}{abuv} \int_{\varepsilon - u/2}^{\varepsilon + u/2} \int_{\eta - v/2}^{\eta + v/2} \sin \frac{m\Pi x}{a} \sin \frac{n\Pi y}{b} \, dxdy$$

or

$$a_{mn} = \frac{16p}{\Pi^2 mnuv} \sin \frac{m\Pi\zeta}{a} \sin \frac{n\Pi\eta}{b} \sin \frac{m\Pi u}{2a} \sin \frac{n\Pi v}{2b}$$

3.4 RECTANGULAR PLATES WITH VARIOUS EDGE CONDITIONS

Bending of Rectangular Plates by Moments Distributed along the Edges. Let us consider a rectangular plate supported along the edges and bent by moments distributed along the edges y = ± b/2 (Fig. 3-7) The deflections w must satisfy the homogeneous differential equation

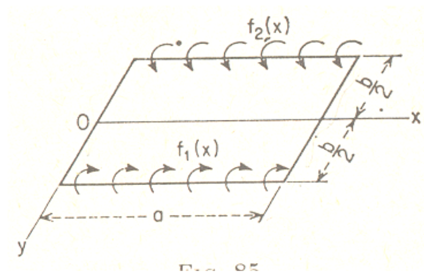


Figure 3-6: Applying resisting moment

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \qquad \textbf{(a)}$$

and the following boundary conditions:

$$w = 0 \qquad \frac{\partial^2 w}{\partial x^2} = 0 \qquad \text{For} \quad x = 0 \quad \text{and} \quad x = a$$

$$w = 0 \qquad \text{For} \quad y = \pm b/2$$

$$-D\left[\frac{\partial^2 w}{\partial y^2}\right]_{y=b/2} = f1(x) - D\left[\frac{\partial^2 w}{\partial y^2}\right]_{y=-b/2} = f2(x) \qquad \textbf{(d)}$$

in which f1 and f2 represent the bending moment distributions along the edges $y = \pm b/2$

We take the solution of Eq. (a) in the form of the series

$$w = \sum_{m=1}^{\infty} y_m \sin\frac{m\Pi x}{a} \qquad \textbf{(e)}$$

Each term of which satisfies the boundary conditions (b). The functions Y_m we take, as before, in the form

$$Y_m = A_m \sinh\frac{m\Pi y}{a} + B_m \cosh\frac{m\Pi y}{a} + C_m \frac{m\Pi y}{a}\sinh\frac{m\Pi y}{a} + D_m \frac{m\Pi y}{a}\cosh\frac{m\Pi y}{a}$$

(f)

which satisfies Eq. (a).

To simplify the discussion let us begin with the two particular cases:

1. The symmetrical case in which $(M_y)_{y=b/w} = (M_y)_{y=-b/2}$
2. The antisymmetrical case in which $(M_y)_{y=b/2} = -(M_y)_{y=-b/2}$

The general case can be obtained by combining these two particular cases. Necessary to put Am = Dm = 0 in expression (f). Then we obtain, from Eq. (e),

$$Y_m = \sum_{m=1}\left[B_m \cosh\frac{m\Pi y}{a} + C_m \frac{m\Pi y}{a}\right]\sinh\frac{m\Pi y}{a} \qquad \textbf{(g)}$$

To satisfy the boundary condition (c) we must put

$$B_m \cosh\alpha_m + C_m\alpha_m \sin\alpha_m = 0 \backslash$$

Where, as before,

$$\alpha_m = \frac{m\Pi y}{a}$$

Hence $B_m = -C_m\alpha_m \tanh\alpha_m$

And the deflection in the symmetrical case is

$$w = \sum_{m=1}^{\infty}\left[C_m \frac{m\Pi y}{a} + \sinh\frac{m\Pi y}{a} - \alpha_m \tanh\alpha_m \cosh\frac{m\Pi y}{a}\right]\sin\frac{m\Pi y}{a} \qquad \textbf{(h)}$$

We use the boundary conditions (d) to determine the constants Cm. Representing the distribution of bending moments along the edges y = ± b/2 by a trigonometric series, we have in the case of symmetry

$$f1(x) = f2(x) = \sum_{m=1}^{\infty} E_m \sin\frac{m\Pi x}{a} \qquad \textbf{(i)}$$

Where the coefficients E_m can be calculated in the usual way for each particular case, For instance, in the case of a uniform distribution of the bending moments we have,

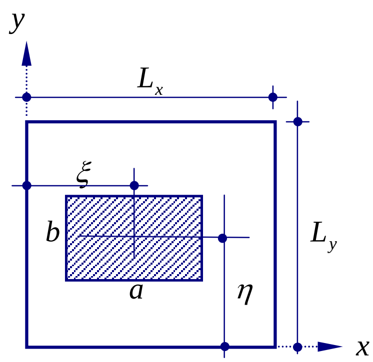
$$\left(M_y\right)_{y=b/2} = \frac{4M_0}{\Pi} \sum_{m=1,3,5,\ldots}^{\infty} \frac{1}{m} \sin\frac{m\Pi x}{a} \qquad \textbf{(j)}$$

Substituting expressions (h) and (i) into conditions (d), we obtain

$$-2D = \sum_{m=1}^{\infty} \frac{m^2\Pi^2}{a^2} C_m \cosh\alpha_m \sin\frac{m\Pi x}{a} = \sum_{m=1}^{\infty} E_m \sin\frac{m\Pi x}{a}$$

3.5 Tables for the analysis of suspended slab

Table 3.1 Bending Moment coefficient for a rectangular panel subjected to Arial load.



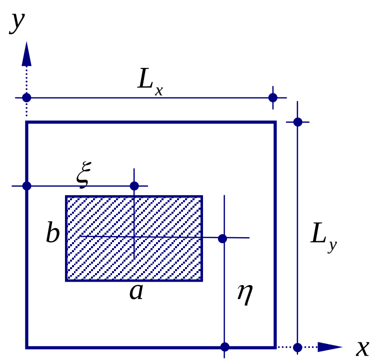
$$\psi = \frac{L_y}{L_x}$$

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

$$\psi_3 = \frac{a}{2L_x} = 0.25 \qquad \psi_4 = \frac{b}{2L_y} = 0.25$$

Boundary Conditions	ψ	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
1	α_{xs}	0.086	0.09	0.103	0.108	0.111	0.114	0.117	0.12
	α_{ys}	0.086	0.077	0.067	0.062	0.046	0.034	0.023	0.027
	α_{xf}	0.058	0.062	0.066	0.07	0.074	0.077	0.082	0.088
	α_{yf}	0.058	0.055	0.052	0.05	0.049	0.048	0.046	0.044
2	α_{xs}	0.125	0.119	0.110	0.096	0.079	0.055	0.044	0.058
	α_{ys}	0.135	0.094	0.080	0.062	0.046	0.035	0.023	0.027
	α_{xf}	0.086	0.081	0.077	0.067	0.054	0.043	0.035	0.044
	α_{yf}	0.096	0.077	0.058	0.048	0.037	0.028	0.021	0.025
3	α_{xs}	0.135	0.132	0.114	0.104	0.089	0.065	0.055	0.076
	α_{ys}	0.125	0.124	0.096	0.079	0.060	0.045	0.031	0.037
	α_{xf}	0.096	0.089	0.076	0.068	0.057	0.044	0.039	0.052
	α_{yf}	0.086	0.082	0.071	0.058	0.045	0.035	0.025	0.030
4	α_{xs}	0.149	0.133	0.127	0.113	0.095	0.068	0.057	0.076
	α_{ys}	0.149	0.134	0.102	0.081	0.062	0.045	0.031	0.037
	α_{xf}	0.098	0.091	0.083	0.073	0.060	0.048	0.039	0.053
	α_{yf}	0.098	0.091	0.071	0.058	0.045	0.035	0.025	0.030
5	α_{xs}	0.153	0.147	0.119	0.102	0.082	0.057	0.045	0.058
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.086	0.084	0.082	0.069	0.057	0.044	0.035	0.044
	α_{yf}	0.107	0.076	0.057	0.045	0.036	0.027	0.021	0.024
6	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	0.137	0.135	0.133	0.128	0.122	0.115	0.124	0.13
	α_{xf}	0.086	0.09	0.094	0.096	0.097	0.097	0.098	0.099
	α_{yf}	0.065	0.072	0.083	0.091	0.103	0.116	0.13	0.147
7	α_{xs}	0.178	0.176	0.144	0.124	0.103	0.073	0.060	0.077
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.104	0.098	0.092	0.080	0.066	0.051	0.040	0.053
	α_{yf}	0.114	0.092	0.069	0.057	0.044	0.032	0.024	0.029
8	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	0.178	0.156	0.133	0.110	0.087	0.066	0.048	0.058
	α_{xf}	0.114	0.108	0.092	0.083	0.072	0.058	0.051	0.071
	α_{yf}	0.104	0.095	0.087	0.073	0.058	0.044	0.032	0.042
9	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.107	0.115	0.124	0.128	0.132	0.136	0.141	0.146
	α_{yf}	0.107	0.104	0.102	0.099	0.097	0.095	0.093	0.091

Table 3.2 Shear force coefficient for a rectangular panel subjected to arial load.



$$\psi = \frac{L_y}{L_x}$$

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

$$\psi_3 = \frac{a}{2L_x} = 0.25 \qquad \psi_4 = \frac{b}{2L_y} = 0.25$$

Boundary Conditions	ψ	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
1	$\beta_{x,Con}$	0.842	0.973	1.209	1.365	1.514	1.649	1.352	1.425
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	0.842	0.839	0.913	0.916	0.915	1.175	0.780	0.706
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
2	$\beta_{x,Con}$	0.917	1.032	1.258	1.403	1.540	1.278	1.359	1.422
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	0.864	0.849	0.916	0.916	0.913	0.887	0.779	0.706
	$\beta_{y,Disc}$	0.325	0.318	0.342	0.343	0.354	0.391	0.392	0.389
3	$\beta_{x,Con}$	0.864	1.015	1.284	1.474	1.662	1.418	1.581	1.701
	$\beta_{x,Disc}$	0.325	0.387	0.498	0.581	0.666	0.630	0.711	0.771
	$\beta_{y,Con}$	0.917	0.939	1.049	1.077	1.095	1.102	0.987	0.899
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
4	$\beta_{x,Con}$	0.969	1.110	1.375	1.554	1.727	1.476	1.610	1.715
	$\beta_{x,Disc}$	0.380	0.443	0.555	0.634	0.713	0.663	0.730	0.782
	$\beta_{y,Con}$	0.969	0.976	1.077	1.095	1.107	1.107	0.987	0.897
	$\beta_{y,Disc}$	0.380	0.380	0.417	0.423	0.430	0.400	0.404	0.406
5	$\beta_{x,Con}$	0.999	1.097	1.310	1.441	1.567	1.299	1.363	1.420
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.320	0.311	0.336	0.339	0.356	0.388	0.387	0.384
6	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.320	0.384	0.501	0.595	0.693	0.683	0.813	0.922
	$\beta_{y,Con}$	0.999	1.056	1.218	1.291	1.353	1.412	1.336	1.255
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
7	$\beta_{x,Con}$	1.098	1.220	1.477	1.638	1.797	1.536	1.637	1.879
	$\beta_{x,Disc}$	0.444	0.500	0.611	0.686	0.758	0.699	0.750	0.791
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.386	0.380	0.415	0.421	0.427	0.408	0.406	0.403
8	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.386	0.454	0.579	0.675	0.773	0.751	0.863	0.959
	$\beta_{y,Con}$	1.098	1.143	1.300	1.361	1.412	1.458	1.359	1.265
	$\beta_{y,Disc}$	0.444	0.456	0.513	0.534	0.551	0.575	0.490	0.467
9	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.414	0.537	0.669	0.764	0.858	0.823	0.916	0.996
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.468	0.473	0.526	0.542	0.556	0.526	0.490	0.465

3.6 Verification of Analysis Results

As verification, the analysis method results are compared with the bares results. This is because bares develops the coefficients using similar methods and loading conditions. Bares results are shown on pages 116 to 191, from these only few data are taken for verification.

On this paper poisons ratio is taken to be **0.2**, but on bares I found a result for ψ?????????????On Bares Moments are the final result instead of coefficients, so we need to

The maximum difference is 3.4%, which verifies that the newly proposed method is acceptable and can be used for practical purposes

3.7 Comparison of Analysis Methods

In this section comparison analysis will be made for the newly developed coefficient method with a finite element software analysis and practical analysis.

1. **With safe out put**

   Hear the analysis is shown diagrammatically.

   Two panel types are taken, panel 1 and 8

   Load is located at the centre, $10Kn/m^2$ on 3mX3m

   Panel size 6mX6m

2. **With practical analysis**

   To compare with practical analysis we must apply the load on the entire panel, therefore lets take, $10Kn/m^2$ on 6mX6m

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

B/c the load is at the centre

$$\psi_3 = \frac{a}{2L_x} = 0.50 \qquad \psi_4 = \frac{b}{2L_y} = 0.50$$

Since, **$a=b=L_x=L_y=6m$**

Substituting these coefficients in the program results a coefficient table similar to that of concrete code. But for comparison purpose we pick up three panel types (1, 4 and 9). For Practical analysis EBCS-2, 1995 is used. Then a tabular and graphical comparison is carried out.

b) Comparison with incorrect practical analysis

Designers may use different analysis techniques to consider the previous loading types. Some may distribute the rectangular uniform load for the entire area. In this specific section I would like to see the comparative analysis with the coefficient method.

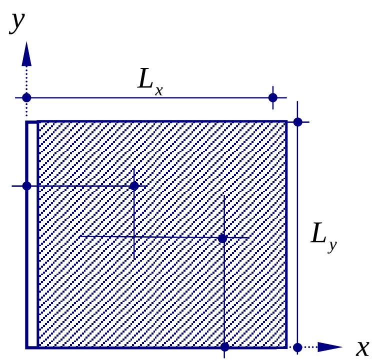
Example: a 3mX3m 10Kn/m^2 uniform rectangular load is placed on a 6mX6m panel. Let’s calculate the field and support moments for panel type 1 and 9

Panel Type	Using approximate practical analysis,	Using coefficient method	Difference in %
	M_{xs}=2.88	**M_{xs}=7.74**	**169 %**
1	**M_{ys}=2.88**	**M_{ys}=7.74**	**142 %**
	M_{xf}=2.16	**M_{xf}=5.22**	**169 %**
	M_{yf}=2.16	**M_{yf}=5.22**	**142 %**

Panel Type	Using approximate practical analysis,	Using coefficient method	Difference in %
9	**M_{xf}=5.04**	**M_{xf}=9.63**	**91 %**
	M_{yf}=5.04	**M_{yf}=9.63**	**91 %**

There fore it is generally **unsafe** to distribute the rectangular load for the entire area.

Table 3.3 Bending Moment coefficient for a rectangular panel subjected to **entire** aerial load.



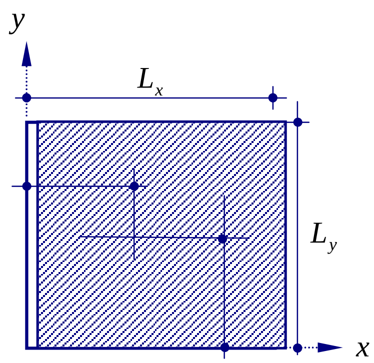
$$\psi = \frac{L_y}{L_x}$$

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

$$\psi_3 = \frac{a}{2L_x} = 0.50 \qquad \psi_4 = \frac{b}{2L_y} = 0.50$$

Boundary Conditions	ψ	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
1	α_{xs}	0.034	0.038	0.044	0.048	0.051	0.056	0.063	0.069
	α_{ys}	0.034	0.034	0.035	0.035	0.036	0.036	0.038	0.038
	α_{xf}	0.026	0.029	0.033	0.038	0.041	0.044	0.048	0.053
	α_{yf}	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029
2	α_{xs}	0.042	0.047	0.052	0.057	0.06	0.066	0.071	0.08
	α_{ys}	0.042	0.042	0.0429	0.043	0.044	0.045	0.047	0.47
	α_{xf}	0.032	0.036	0.0404	0.047	0.05	0.054	0.059	0.06
	α_{yf}	0.032	0.032	0.0328	0.033	0.034	0.035	0.036	0.036
3	α_{xs}	0.042	0.054	0.06	0.067	0.071	0.078	0.087	0.096
	α_{ys}	0.042	0.043	0.044	0.044	0.045	0.046	0.047	0.48
	α_{xf}	0.032	0.041	0.047	0.054	0.058	0.061	0.068	0.07
	α_{yf}	0.032	0.033	0.034	0.034	0.035	0.036	0.037	0.037
4	α_{xs}	0.05	0.063	0.071	0.074	0.078	0.086	0.092	0.1
	α_{ys}	0.05	0.051	0.052	0.053	0.053	0.054	0.055	0.056
	α_{xf}	0.037	0.041	0.046	0.052	0.058	0.063	0.071	0.076
	α_{yf}	0.037	0.038	0.039	0.04	0.042	0.043	0.044	0.044
5	α_{xs}	0.049	0.06	0.062	0.063	0.067	0.073	0.078	0.08
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.036	0.042	0.048	0.061	0.065	0.067	0.073	0.08
	α_{yf}	0.036	0.039	0.04	0.04	0.042	0.043	0.044	0.044
6	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	0.047	0.048	0.049	0.049	0.05	0.052	0.053	0.053
	α_{xf}	0.035	0.04	0.056	0.07	0.076	0.084	0.092	0.1
	α_{yf}	0.035	0.036	0.038	0.038	0.039	0.039	0.041	0.041
7	α_{xs}	0.059	0.075	0.084	0.087	0.09	0.099	0.104	0.11
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.059	0.059	0.061	0.061	0.063	0.064	0.066	0.07
	α_{yf}	0.044	0.048	0.05	0.053	0.056	0.058	0.061	0.061
8	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	0.06	0.061	0.063	0.063	0.065	0.066	0.068	0.07
	α_{xf}	0.047	0.055	0.063	0.071	0.08	0.089	0.101	0.112
	α_{yf}	0.047	0.048	0.048	0.048	0.049	0.051	0.052	0.052
9	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.058	0.064	0.071	0.078	0.087	0.095	0.104	0.116
	α_{yf}	0.058	0.058	0.059	0.059	0.06	0.06	0.061	0.061

Table 3.4 Shear force coefficient for a rectangular panel subjected to **entire** aerial load.



$$\psi = \frac{L_y}{L_x}$$

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

$$\psi_3 = \frac{a}{2L_x} = 0.50 \qquad \psi_4 = \frac{b}{2L_y} = 0.50$$

Boundary Conditions	ψ	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
1	$\beta_{x,Con}$	0.333	0.369	0.401	0.432	0.451	0.471	0.516	0.534
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	0.331	0.337	0.34	0.34	0.348	0.35	0.359	0.359
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
2	$\beta_{x,Con}$	0.38	0.44	0.546	0.567	0.573	0.592	0.633	0.654
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	0.372	0.378	0.381	0.381	0.391	0.4	0.403	0.4
	$\beta_{y,Disc}$	0.269	0.274	0.276	0.276	0.283	0.29	0.292	0.29
3	$\beta_{x,Con}$	0.378	0.407	0.443	0.474	0.522	0.544	0.604	0.643
	$\beta_{x,Disc}$	0.251	0.288	0.301	0.304	0.32	0.342	0.354	0.369
	$\beta_{y,Con}$	0.362	0.364	0.367	0.371	0.374	0.379	0.382	0.392
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
4	$\beta_{x,Con}$	0.417	0.452	0.489	0.502	0.531	0.571	0.614	0.657
	$\beta_{x,Disc}$	0.272	0.294	0.307	0.31	0.327	0.348	0.362	0.386
	$\beta_{y,Con}$	0.397	0.404	0.407	0.407	0.418	0.427	0.43	0.43
	$\beta_{y,Disc}$	0.274	0.279	0.281	0.282	0.284	0.286	0.29	0.293
5	$\beta_{x,Con}$	0.425	0.432	0.436	0.456	0.494	0.505	0.508	0.508
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.28	0.303	0.363	0.382	0.402	0.463	0.481	0.482
6	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.286	0.309	0.37	0.389	0.41	0.473	0.491	0.491
	$\beta_{y,Con}$	0.401	0.408	0.408	0.409	0.41	0.419	0.421	0.421
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
7	$\beta_{x,Con}$	0.49	0.529	0.552	0.596	0.627	0.67	0.695	0.696
	$\beta_{x,Disc}$	0.335	0.362	0.378	0.382	0.402	0.429	0.445	0.445
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.316	0.321	0.324	0.324	0.332	0.34	0.342	0.342
8	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.322	0.348	0.388	0.409	0.423	0.443	0.476	0.494
	$\beta_{y,Con}$	0.453	0.461	0.465	0.465	0.477	0.488	0.491	0.491
	$\beta_{y,Disc}$	0.306	0.314	0.316	0.316	0.325	0.332	0.334	0.334
9	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.332	0.361	0.407	0.428	0.445	0.461	0.502	0.521
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.33	0.345	0.348	0.348	0.357	0.365	0.367	0.367

Chapter 4

Conclusion and Recommendation

4.1 Conclusions

In this paper I have been developing a coefficient table for analyzing slab panel subjected to rectangular uniform load. The study of the paper is based on the basic concepts of plate bending theory. It has been also carried a comparison of this method with a safe analysis output and EBCS-2, 1995 moment coefficients. As it is shown on the comparison table, this coefficient method of analysis gives generally results with higher value. But it has a maximum of 3% deviation from safe analysis out put. This shows that the coefficient analysis method which is derived here can be used for analysis purpose. But to use for design purpose an extended study is required, because the comparison with practical conditions makes this result conservative. This may be

1. In practical conditions we don't take maximum values instead we use effective values which need some laboratory works.
2. Moment distribution is considered in practical conditions etc

In the newly proposed method coefficients are presented by taking the critical values without any modification. For example the bending moment coefficient for M_x is taken from the maximum ordinate at the longer edges. This bending moment has got its own distribution along the edges of the panel. It has to be taken some fraction of this moment to be distributed uniformly over the entire length of edge. Comparing the shear force with safe analysis out put and practical design methods, it has a wider variation than the bending moment. Since the coefficient method which is proposed in this thesis takes the peak value of the shear coefficient to be uniformly distributed over the entire length of the beam which results a higher value. But when we consider the distribution of the reaction shear force coming from orthogonal strip to the beams at the edges, the effective support length of the beam should be considered. By applying yield line theory this effective length is 75% of the total length, this is for panel fully loaded. But for other type of loading this percent decreases. So what we are looking for is that the effective length is less than the total length therefore the shear values are coming with greater values.

4.2 Recommendations

Generally the output of the newly developed method in this thesis is acceptable. Especially the coefficients of the moment are more realistic. But the shear force results on higher values, which leads to have a conservative outputs. But this analysis is for slaps we can neglect the effect of higher shear value. So regarding to the analysis of this thesis it is recommended to use for practical designs.

The thesis is assumed to be continued by some future studies considering the limitations made in the study. Therefore, one can extend this study to include the following subjects:

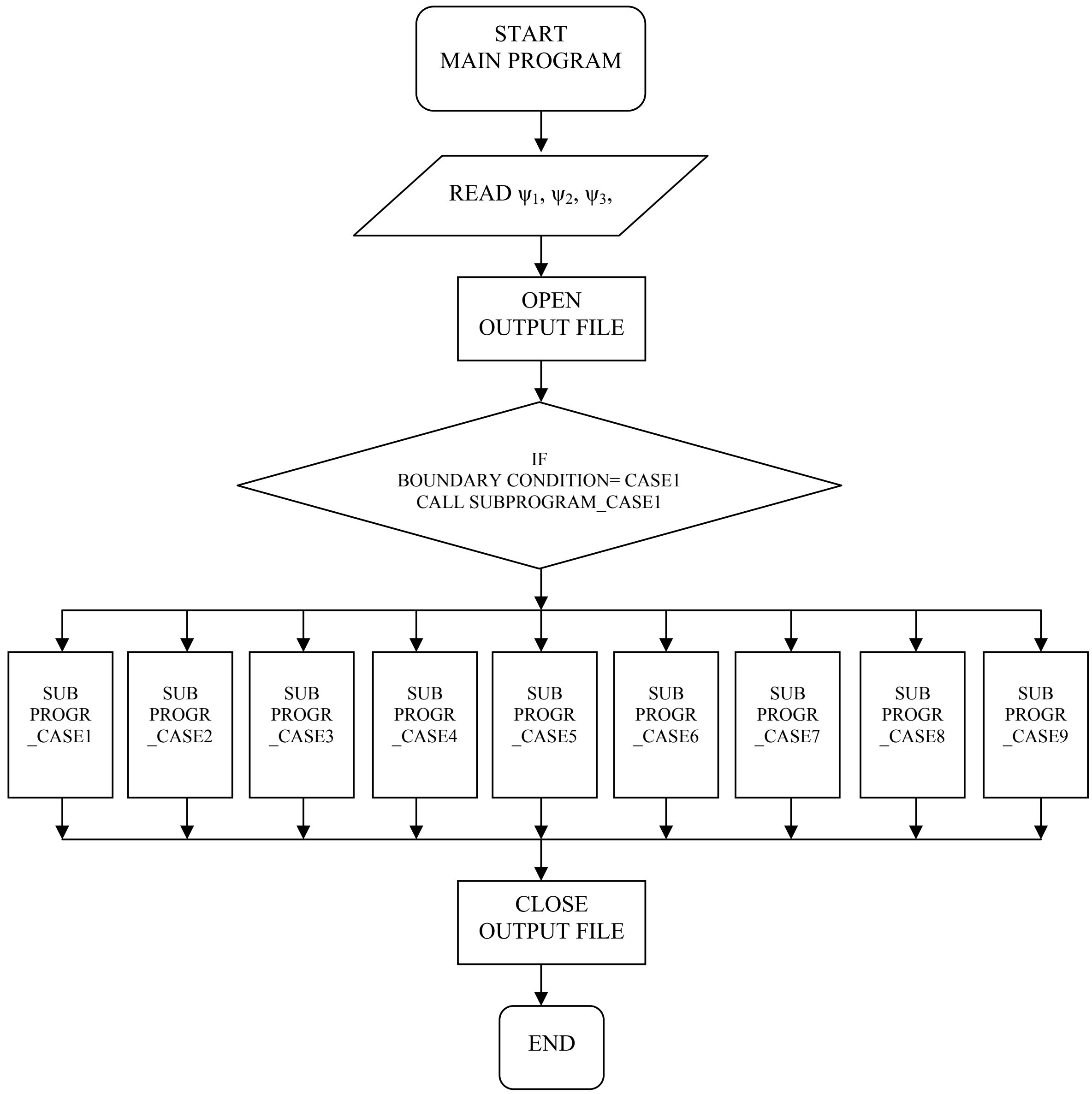
- This study dealt only for a rectangular load placed at the center, thus one can extend it to consider other positions.

- The coefficients produced in this study are derived from pure elastic analysis of plate, so one can make further study to include plastic moment redistribution.

- One can also make future studies to device some better means of taking the effective values of internal actions instead of taking peak values. This may require extensive study and laboratory experiment.

ANALYSIS PROGRAM

FLOW CHART


START
MAIN PROGRAM
READ ψ1, ψ2, ψ3,
OPEN
OUTPUT FILE
IF
BOUNDARY CONDITION= CASE1
CALL SUBPROGRAM_CASE1
SUB
PROGR
_CASE1
SUB
PROGR
_CASE2
SUB
PROGR
_CASE3
SUB
PROGR
_CASE4
SUB
PROGR
_CASE5
SUB
PROGR
_CASE6
SUB
PROGR
_CASE7
SUB
PROGR
_CASE8
SUB
PROGR
_CASE9
CLOSE
OUTPUT FILE
END

```
Private Sub Main_Click()

'---------------------------------------------------------
'Declaration

Dim Q As Integer
Dim eta As Double
Dim So As Double
Dim r As Double
Dim s As Double
Dim t As Double
Dim filenam As String
Dim E As Double
Dim D As Double
Dim nu As Double
Dim x As Double
Dim y As Double
Dim Lx As Double
Dim Ly As Double
Dim tex As Double
Dim c1 As Double
Dim c2 As Double
Dim c3 As Double
Dim c4 As Double
Dim c5 As Double
Dim c6 As Double
Dim c7 As Double
Dim c8 As Double
Dim Vx1 As Double
Dim Vx2 As Double
Dim Vx As Double
Dim Vy As Double
Dim Vy1 As Double
Dim Vy2 As Double
Dim cons1 As Double
Dim cons2 As Double
Dim cons3 As Double
Dim n As Integer
Dim m As Integer
Dim mx As Double
Dim mx1 As Double
Dim mx2 As Double
Dim my As Double
Dim my1 As Double
Dim my2 As Double
'------------------------------------------------
'Read Data

pi = 3.14159265358979
Q = Text5.Text
E = Text1.Text
```

```
nu = Text6.Text
s = Text2.Text
t = Text8.Text
Lx = Text4.Text
Ly = Text3.Text
filenam = Text7.Text
D = E * t ^ 3 / (12 * (1 - nu ^ 2))
'-----------------------------------------------
'Open Result File

Open filenam For Output As #1
'-----------------------------------------------
'Call Subprograms

If Combo1.Text = "Case1: All four edges built In." Then
Call Case1

ElseIf Combo1.Text = "Case2: One shorter edge simply supported and the
other three edges clamped." Then
Call Case2

ElseIf Combo1.Text = "Case3: One longer edge simply supported and the
other three edges clamped." Then
Call Case3

ElseIf Combo1.Text = "Case4: Two adjacent edges simply supported and the
other clamped." Then
Call Case4

ElseIf Combo1.Text = "Case5: Two opposite longer edges simply supported
and two edges clamped." Then
Call Case5

ElseIf Combo1.Text = "Case6: Two opposite shorter edges simply supported
and two edges clamped." Then
Call Case6

ElseIf Combo1.Text = "Case7: One longer edge clamped and the other three
edges simply supported." Then
Call Case7

ElseIf Combo1.Text = "Case8: One shorter edge clamped and the other three
edges simply supported." Then
Call Case8

ElseIf Combo1.Text = "Case9: Simply supported on four edges." Then
Call Case9
End If
Close 1
MsgBox " Analysis Completed"
End Sub
```

```
SUBPROGRAM_CASE1
pi = 3.14159265358979
Q = Text5.Text
E = Text1.Text
nu = Text6.Text
s = Text2.Text
t = Text8.Text
Lx = Text4.Text
Ly = Text3.Text
filenam = Text7.Text


For eta = 1 To Lx Step 1.5
For So = 1 To Ly Step 1.5

r = 1
Do While (r / 2 + eta <= Lx And r / 2 <= eta)

tex = "Eta = " & "," & FormatNumber(eta, 2) & "," & "Taw = " & "," &
FormatNumber(So, 2) & "," & "r = " & "," & FormatNumber(r, 2)
Print #1, tex

'******************************************************************
'MOMENT Mx

tex = "Moment per meter (Mx)"
Print #1, tex

cons1 = -16 * Q / ((pi ^ 4) * r * s)

tex = "y\x"
For x = 0 To Lx Step 1
tex = tex & "," & FormatNumber(x, 1)
Next
Print #1, tex

For y = 0 To Ly Step 1
 tex = FormatNumber(y, 1)
 For x = 0 To Lx Step 1
 mx1 = 0
 mx2 = 0
 '-----------------------------------------------
    n = 1
    Do While (n < 50)
     m = 1
      Do While (m < 50)
        c1 = (n * Lx ^ 2 * (((m ^ 2 / Lx ^ 2) + (n ^ 2 / Ly ^ 2)) ^ 2))
        mx1 = mx1 + ((m * Sin((m * pi * eta) / Lx) * Sin((n * pi * So) / Ly) * Sin((m
* pi * r * 0.5) / Lx) * Sin((n * pi * s * 0.5) / Ly) * Sin((m * pi * x) / Lx) * Sin((n *
pi * y) / Ly)) / c1)
        c2 = (m * Ly ^ 2 * (((m ^ 2 / Lx ^ 2) + (n ^ 2 / Ly ^ 2)) ^ 2))
```

```
        mx2 = mx2 + ((n * Sin((m * pi * eta) / Lx) * Sin((n * pi * So) / Ly) * Sin((m
* pi * r * 0.5) / Lx) * Sin((n * pi * s * 0.5) / Ly) * Sin((m * pi * x) / Lx) * Sin((n *
pi * y) / Ly)) / c2)
        m = m + 1
      DoEvents
      Loop
     n = n + 1
    Loop
 '-----------------------------------------------

  mx = cons1 * (mx1 + 0.2 * mx2)
 tex = tex & "," & FormatNumber(mx, 2)
 Next
 Print #1, tex
 Next

'*********************************************************************
'MOMENT My
tex = ""
Print #1, tex
tex = ""
Print #1, tex
tex = ""
Print #1, tex

cons2 = -16 * Q / ((pi ^ 4) * r * s)
tex = "Moment per meter (My)"
Print #1, tex

tex = "y\x"
For x = 0 To Lx Step 1
tex = tex & "," & FormatNumber(x, 1)
Next
Print #1, tex

For y = 0 To Ly Step 1
 tex = FormatNumber(y, 1)
 For x = 0 To Lx Step 1
 my1 = 0
 my2 = 0
 '-----------------------------------------------
    n = 1
    Do While (n < 50)
     m = 1
      Do While (m < 50)
        c3 = (n * Lx ^ 2 * (((m ^ 2 / Lx ^ 2) + (n ^ 2 / Ly ^ 2)) ^ 2))
        my1 = my1 + ((m * Sin((m * pi * eta) / Lx) * Sin((n * pi * So) / Ly) * Sin((m
* pi * r * 0.5) / Lx) * Sin((n * pi * s * 0.5) / Ly) * Sin((m * pi * x) / Lx) * Sin((n *
pi * y) / Ly)) / c3)
        c4 = (m * Ly ^ 2 * (((m ^ 2 / Lx ^ 2) + (n ^ 2 / Ly ^ 2)) ^ 2))
```

```
        my2 = my2 + ((n * Sin((m * pi * eta) / Lx) * Sin((n * pi * So) / Ly) * Sin((m
* pi * r * 0.5) / Lx) * Sin((n * pi * s * 0.5) / Ly) * Sin((m * pi * x) / Lx) * Sin((n *
pi * y) / Ly)) / c4)
        m = m + 1
      DoEvents
      Loop
     n = n + 1
    Loop
 '-----------------------------------------------


 my = cons2 * (0.2 * my1 + my2)
 tex = tex & "," & FormatNumber(my, 2)
 Next
 Print #1, tex
Next

'*********************************************************************
'SHEAR Vx

tex = ""
Print #1, tex
tex = ""
Print #1, tex
tex = ""
Print #1, tex

cons3 = -16 * Q / ((pi ^ 3) * r * s)
tex = "Shear Force per meter (Vx)"
Print #1, tex

tex = "y\x"
For x = 0 To Lx Step 1
tex = tex & "," & FormatNumber(x, 1)
Next
Print #1, tex

For y = 0 To Ly Step 1
 tex = FormatNumber(y, 1)
 For x = 0 To Lx Step 1
 vx1 = 0
 vx2 = 0
 '-----------------------------------------------
    n = 1
    Do While (n < 50)
     m = 1
      Do While (m < 50)
        c5 = (n * Lx ^ 3 * (((m ^ 2 / Lx ^ 2) + (n ^ 2 / Ly ^ 2)) ^ 2))
        vx1 = vx1 + ((m ^ 2 * Sin((m * pi * eta) / Lx) * Sin((n * pi * So) / Ly) *
Sin((m * pi * r * 0.5) / Lx) * Sin((n * pi * s * 0.5) / Ly) * Cos((m * pi * x) / Lx) *
Sin((n * pi * y) / Ly)) / c5)
```

```
        c6 = (Lx * Ly ^ 2 * (((m ^ 2 / Lx ^ 2) + (n ^ 2 / Ly ^ 2)) ^ 2))
        vx2 = vx2 + ((n * Sin((m * pi * eta) / Lx) * Sin((n * pi * So) / Ly) * Sin((m *
pi * r * 0.5) / Lx) * Sin((n * pi * s * 0.5) / Ly) * Cos((m * pi * x) / Lx) * Sin((n *
pi * y) / Ly)) / c6)
        m = m + 1
      DoEvents
      Loop
     n = n + 1
    Loop
 '-----------------------------------------------


 vx = cons3 * (vx1 + vx2)
 tex = tex & "," & FormatNumber(vx, 2)
 Next
 Print #1, tex
Next

'*********************************************************************
'SHEAR Vy


tex = ""
Print #1, tex
tex = ""
Print #1, tex
tex = ""
Print #1, tex

cons3 = -16 * Q / ((pi ^ 3) * r * s)
tex = "Shear Force per meter (Vy)"
Print #1, tex

tex = "y\x"
For x = 0 To Lx Step 1
tex = tex & "," & FormatNumber(x, 1)
Next
Print #1, tex

For y = 0 To Ly Step 1
 tex = FormatNumber(y, 1)
 For x = 0 To Lx Step 1
 vy1 = 0
 vy2 = 0
 '-----------------------------------------------
    n = 1
    Do While (n < 50)
     m = 1
      Do While (m < 50)
        c7 = (Ly * Lx ^ 2 * (((m ^ 2 / Lx ^ 2) + (n ^ 2 / Ly ^ 2)) ^ 2))
```

```
        vy1 = vy1 + ((m * Sin((m * pi * eta) / Lx) * Sin((n * pi * So) / Ly) * Sin((m *
pi * r * 0.5) / Lx) * Sin((n * pi * s * 0.5) / Ly) * Sin((m * pi * x) / Lx) * Cos((n *
pi * y) / Ly)) / c7)
        c8 = (m * Ly ^ 3 * (((m ^ 2 / Lx ^ 2) + (n ^ 2 / Ly ^ 2)) ^ 2))
        vy2 = vy2 + ((n ^ 2 * Sin((m * pi * eta) / Lx) * Sin((n * pi * So) / Ly) *
Sin((m * pi * r * 0.5) / Lx) * Sin((n * pi * s * 0.5) / Ly) * Sin((m * pi * x) / Lx) *
Cos((n * pi * y) / Ly)) / c8)
        m = m + 1
      DoEvents
      Loop
     n = n + 1
    Loop
 '-----------------------------------------------


 vy = cons3 * (vy1 + vy2)
 tex = tex & "," & FormatNumber(vy, 2)
 Next
 Print #1, tex
Next
tex = ""
Print #1, tex
tex = ""
Print #1, tex
tex = ""
Print #1, tex
tex = ""
Print #1, tex
tex = ""
Print #1, tex
tex = ""
Print #1, tex
r = r + 1
DoEvents
Loop
Next
Next


End Sub
```

References:

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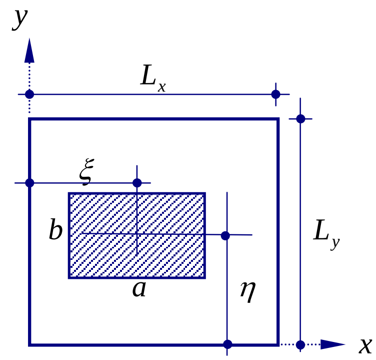
6. SAP2000, Integrated Finite Element Analysis and Design Structures Users Manual, Computers and Structures Inc., CSI, Version7.0, Berkeley, California, 1998.
7. SAFE, Integrated Analysis and Design of Slabs by Finite Element Method Users Manual, Computers and Structures Inc., CSI, Version6.0, Berkeley, California, 1998.

APPENDIX A

Internal Actions using Coefficient Method

Table A.1 Bending Moment coefficient for a rectangular panel subjected to arial load.



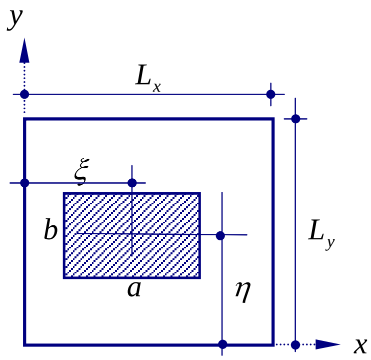
$$\psi = \frac{L_y}{L_x}$$

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

$$\psi_3 = \frac{a}{2L_x} = 0.30 \qquad \psi_4 = \frac{b}{2L_y} = 0.30$$

Boundary Conditions	ψ	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
1	α_{xs}	0.092	0.129	0.11	0.097	0.08	0.061	0.047	0.061
	α_{ys}	0.092	0.112	0.084	0.066	0.049	0.039	0.025	0.029
	α_{xf}	0.126	0.093	0.077	0.068	0.056	0.042	0.037	0.047
	α_{yf}	0.126	0.084	0.064	0.053	0.04	0.03	0.022	0.026
2	α_{xs}	0.133	0.143	0.117	0.102	0.083	0.058	0.047	0.061
	α_{ys}	0.144	0.1	0.085	0.066	0.049	0.037	0.025	0.029
	α_{xf}	0.092	0.116	0.082	0.071	0.057	0.045	0.037	0.047
	α_{yf}	0.102	0.082	0.062	0.051	0.039	0.03	0.022	0.026
3	α_{xs}	0.144	0.141	0.121	0.11	0.094	0.069	0.059	0.08
	α_{ys}	0.133	0.132	0.102	0.084	0.063	0.048	0.033	0.039
	α_{xf}	0.102	0.095	0.081	0.072	0.06	0.047	0.042	0.055
	α_{yf}	0.092	0.096	0.076	0.062	0.047	0.037	0.027	0.032
4	α_{xs}	0.159	0.161	0.135	0.12	0.1	0.072	0.061	0.08
	α_{ys}	0.159	0.143	0.109	0.086	0.065	0.048	0.033	0.039
	α_{xf}	0.104	0.106	0.088	0.078	0.063	0.051	0.042	0.056
	α_{yf}	0.104	0.097	0.076	0.062	0.047	0.037	0.027	0.032
5	α_{xs}	0.163	0.157	0.127	0.108	0.087	0.06	0.048	0.061
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.092	0.11	0.087	0.073	0.06	0.047	0.037	0.047
	α_{yf}	0.114	0.081	0.061	0.048	0.038	0.029	0.022	0.025
6	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	0.163	0.158	0.127	0.108	0.085	0.068	0.049	0.06
	α_{xf}	0.114	0.096	0.085	0.078	0.069	0.055	0.049	0.071
	α_{yf}	0.092	0.111	0.088	0.075	0.061	0.047	0.037	0.044
7	α_{xs}	0.19	0.188	0.153	0.132	0.109	0.077	0.064	0.082
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.111	0.12	0.098	0.085	0.07	0.054	0.043	0.056
	α_{yf}	0.122	0.098	0.073	0.061	0.046	0.034	0.026	0.031
8	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	0.19	0.113	0.142	0.117	0.092	0.07	0.051	0.061
	α_{xf}	0.122	0.179	0.098	0.088	0.076	0.061	0.054	0.075
	α_{yf}	0.111	0.116	0.093	0.078	0.061	0.047	0.034	0.044
9	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.154	0.148	0.147	0.143	0.14	0.14	0.136	0.131
	α_{yf}	0.104	0.113	0.119	0.125	0.128	0.132	0.14	0.141

Table A.2 Shear force coefficient for a rectangular panel subjected to arial load.



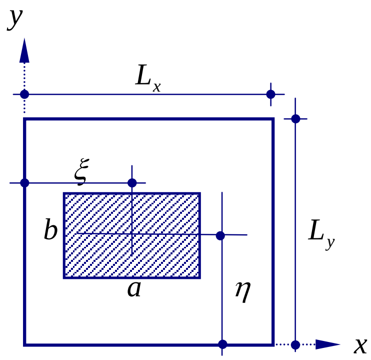
$$\psi = \frac{L_y}{L_x}$$

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

$$\psi_3 = \frac{a}{2L_x} = 0.30 \qquad \psi_4 = \frac{b}{2L_y} = 0.30$$

Boundary Conditions	ψ	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
1	$\beta_{x,Con}$	0.907	1.038	1.288	1.45	1.597	1.745	1.444	1.509
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	0.898	0.895	0.972	0.973	0.965	1.243	0.833	0.748
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
2	$\beta_{x,Con}$	0.978	1.101	1.34	1.49	1.625	1.352	1.451	1.506
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	0.921	0.906	0.976	0.973	0.963	0.938	0.832	0.748
	$\beta_{y,Disc}$	0.346	0.339	0.364	0.364	0.373	0.414	0.419	0.412
3	$\beta_{x,Con}$	0.921	1.083	1.367	1.565	1.753	1.5	1.689	1.801
	$\beta_{x,Disc}$	0.346	0.413	0.53	0.617	0.703	0.667	0.759	0.816
	$\beta_{y,Con}$	0.978	1.002	1.117	1.144	1.155	1.166	1.054	0.952
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
4	$\beta_{x,Con}$	1.033	1.184	1.464	1.65	1.822	1.562	1.719	1.816
	$\beta_{x,Disc}$	0.405	0.473	0.591	0.673	0.752	0.701	0.78	0.828
	$\beta_{y,Con}$	1.033	1.041	1.147	1.163	1.168	1.171	1.054	0.95
	$\beta_{y,Disc}$	0.405	0.405	0.444	0.449	0.454	0.423	0.431	0.43
5	$\beta_{x,Con}$	1.065	1.17	1.395	1.53	1.653	1.374	1.456	1.504
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.341	0.332	0.358	0.36	0.376	0.411	0.413	0.407
6	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.341	0.41	0.534	0.632	0.731	0.723	0.868	0.976
	$\beta_{y,Con}$	1.065	1.127	1.297	1.371	1.427	1.494	1.427	1.329
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
7	$\beta_{x,Con}$	1.17	1.302	1.573	1.74	1.896	1.625	1.748	1.99
	$\beta_{x,Disc}$	0.473	0.534	0.651	0.729	0.8	0.74	0.801	0.838
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.411	0.405	0.442	0.447	0.45	0.432	0.434	0.427
8	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.411	0.484	0.617	0.717	0.816	0.795	0.922	1.016
	$\beta_{y,Con}$	1.17	1.22	1.385	1.445	1.49	1.543	1.451	1.34
	$\beta_{y,Disc}$	0.473	0.487	0.546	0.567	0.581	0.608	0.523	0.495
9	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.441	0.573	0.712	0.811	0.905	0.871	0.978	1.055
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.499	0.505	0.56	0.576	0.587	0.557	0.523	0.492

Table A.3 Bending Moment coefficient for a rectangular panel subjected to arial load.



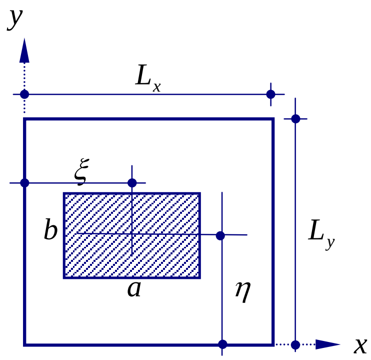
$$\psi = \frac{L_y}{L_x}$$

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

$$\psi_3 = \frac{a}{2L_x} = 0.20 \qquad \psi_4 = \frac{b}{2L_y} = 0.20$$

Boundary Conditions	ψ	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
1	α_{xs}	0.081	0.113	0.097	0.086	0.072	0.055	0.041	0.055
	α_{ys}	0.081	0.098	0.074	0.058	0.044	0.035	0.022	0.025
	α_{xf}	0.111	0.082	0.068	0.06	0.05	0.038	0.033	0.042
	α_{yf}	0.111	0.074	0.056	0.047	0.036	0.026	0.02	0.024
2	α_{xs}	0.117	0.126	0.103	0.09	0.075	0.052	0.041	0.055
	α_{ys}	0.127	0.088	0.075	0.058	0.044	0.033	0.022	0.025
	α_{xf}	0.081	0.102	0.072	0.063	0.051	0.041	0.033	0.042
	α_{yf}	0.09	0.072	0.054	0.045	0.035	0.026	0.02	0.024
3	α_{xs}	0.127	0.124	0.107	0.098	0.084	0.061	0.051	0.072
	α_{ys}	0.117	0.116	0.09	0.074	0.057	0.043	0.029	0.035
	α_{xf}	0.09	0.083	0.071	0.064	0.054	0.042	0.037	0.049
	α_{yf}	0.081	0.084	0.067	0.055	0.043	0.033	0.023	0.028
4	α_{xs}	0.14	0.142	0.119	0.106	0.09	0.064	0.053	0.072
	α_{ys}	0.14	0.126	0.096	0.076	0.059	0.043	0.029	0.035
	α_{xf}	0.092	0.093	0.078	0.069	0.057	0.045	0.037	0.05
	α_{yf}	0.092	0.085	0.067	0.055	0.043	0.033	0.023	0.028
5	α_{xs}	0.144	0.138	0.112	0.096	0.078	0.054	0.042	0.055
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.081	0.097	0.077	0.065	0.054	0.042	0.033	0.042
	α_{yf}	0.1	0.071	0.054	0.042	0.034	0.026	0.02	0.023
6	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	0.144	0.139	0.112	0.096	0.077	0.06	0.043	0.054
	α_{xf}	0.1	0.084	0.075	0.069	0.062	0.049	0.043	0.063
	α_{yf}	0.081	0.097	0.078	0.067	0.055	0.042	0.033	0.04
7	α_{xs}	0.167	0.165	0.135	0.117	0.098	0.069	0.056	0.073
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.98	0.105	0.086	0.075	0.063	0.048	0.037	0.05
	α_{yf}	0.107	0.086	0.065	0.054	0.042	0.03	0.022	0.027
8	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	0.167	0.099	0.125	0.104	0.082	0.062	0.045	0.055
	α_{xf}	0.107	0.157	0.086	0.078	0.068	0.055	0.048	0.067
	α_{yf}	0.098	0.102	0.082	0.069	0.055	0.042	0.03	0.04
9	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.135	0.13	0.13	0.127	0.126	0.125	0.119	0.117
	α_{yf}	0.092	0.099	0.105	0.111	0.115	0.118	0.123	0.126

Table A.4 Shear force coefficient for a rectangular panel subjected to arial load.



$$\psi = \frac{L_y}{L_x}$$

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

$$\psi_3 = \frac{a}{2L_x} = 0.20 \qquad \psi_4 = \frac{b}{2L_y} = 0.20$$

Boundary Conditions	ψ	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
1	$\beta_{x,Con}$	0.798	0.912	1.135	1.285	1.435	1.559	1.266	1.346
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	0.79	0.786	0.857	0.863	0.867	1.111	0.73	0.667
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
2	$\beta_{x,Con}$	0.86	0.967	1.181	1.321	1.46	1.208	1.272	1.343
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	0.811	0.796	0.86	0.863	0.865	0.838	0.729	0.667
	$\beta_{y,Disc}$	0.305	0.298	0.321	0.323	0.336	0.37	0.367	0.367
3	$\beta_{x,Con}$	0.811	0.951	1.206	1.388	1.575	1.34	1.48	1.606
	$\beta_{x,Disc}$	0.305	0.363	0.468	0.547	0.631	0.595	0.666	0.728
	$\beta_{y,Con}$	0.86	0.88	0.985	1.014	1.038	1.042	0.924	0.849
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
4	$\beta_{x,Con}$	0.909	1.04	1.291	1.463	1.637	1.395	1.507	1.619
	$\beta_{x,Disc}$	0.356	0.415	0.521	0.597	0.676	0.627	0.684	0.738
	$\beta_{y,Con}$	0.909	0.915	1.011	1.031	1.049	1.046	0.924	0.847
	$\beta_{y,Disc}$	0.356	0.356	0.392	0.398	0.408	0.378	0.378	0.383
5	$\beta_{x,Con}$	0.937	1.028	1.23	1.357	1.485	1.228	1.276	1.341
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.3	0.291	0.315	0.319	0.337	0.367	0.362	0.363
6	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.3	0.36	0.47	0.56	0.657	0.646	0.761	0.871
	$\beta_{y,Con}$	0.937	0.99	1.144	1.216	1.282	1.335	1.251	1.185
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
7	$\beta_{x,Con}$	1.03	1.143	1.387	1.542	1.703	1.452	1.533	1.774
	$\beta_{x,Disc}$	0.417	0.469	0.574	0.646	0.718	0.661	0.702	0.747
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.362	0.356	0.39	0.396	0.405	0.386	0.38	0.381
8	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.362	0.425	0.544	0.636	0.733	0.71	0.808	0.906
	$\beta_{y,Con}$	1.03	1.071	1.221	1.282	1.338	1.378	1.272	1.195
	$\beta_{y,Disc}$	0.417	0.427	0.482	0.503	0.522	0.543	0.459	0.441
9	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.388	0.503	0.628	0.719	0.813	0.778	0.858	0.941
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.439	0.443	0.494	0.51	0.527	0.497	0.459	0.439